Private Polling in Elections*

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Abstract

We consider a model of elections in which two office-motivated candidates receive private signals about the median voter's location prior to taking policy positions. When a pure strategy equilibrium exists, it is unique: after receiving a signal each candidate locates at the median of the distribution of the median voter's location, conditional on *both* candidates receiving that signal. That is, relative to their private information, candidates bias their positions towards the extremes of the policy space. A pure strategy equilibrium does not always exist. This leads us to develop a tractable environment in which we explicitly characterize the (possibly) mixed strategy equilibrium of the electoral game. Qualitatively, candidates who receive moderate signals locate more extremely than their information suggests, while candidates who receive more extreme signals temper their locations. From a welfare perspective, we show that although candidates extremize their locations, they do not do so by as much as voters would like. We further show that some noise in the polling technology raises voters' welfare. This suggests a novel justification for spending caps in election campaigns.

1 Introduction

"Extremism in the pursuit of victory is no vice." *Barely Goldwater*

The most familiar prediction in political science and political economy is that of platform convergence. The central result of the classical Downsian model (Hotelling (1929), Downs (1957), Black (1958)) is the median voter theorem: The unique Nash equilibrium in an election between two office-motivated candidates who are perfectly informed about the location of the median voter is that both candidates locate at the median voter's ideal point. This result extends to environments in which candidates share a common prior distribution about the location of the median voter. It is a 'folk theorem' that in probabilistic voting environments, the unique Nash equilibrium is for both candidates to locate at the *median of the prior distribution* of the median voter's ideal point.[1]

The intuition underlying this result is compelling: If candidate A does not locate at the median, then candidate B maximizes his or her probability of winning by locating near candidate A, but marginally closer to the median, thereby winning with a probability exceeding one-half.

This paper introduces private polling by candidates into an otherwise canonical probabilistic voting model. Private polling is empirically significant: 46 percent of all spending on U.S. Congressional campaigns in 1990 and 1992 was devoted to the hiring of political consultants, and the use of political pollsters was more common than any other type of consultant (Medvic (2001)).[2] In our model, before selecting a platform, each candidate receives a signal drawn from an arbitrary finite set of possible signals; each candidate updates about the location of the median voter and the platform of the opponent and then chooses a platform; the median voter's location is then realized, and the candidate closest to the median wins.

We then show that although giving candidates access to private polling seems innocuous, the nature of candidates' strategies is radically altered by polling, overturning the apparently robust result of platform convergence. Indeed, the characterization of the pure strategy equilibrium is simple: After receiving a signal, a candidate updates the prior distribution of the median voter, *conditioning on both candidates receiving that same signal,* and locates at the median of *that* posterior distribution.[3] This is starkly highlighted when candidates receive binary signals—left or

[1] Platform convergence extends qualitatively to models with additional candidate heterogeneity, so that candidates may have policy preferences (Wittman (1983), Calvert (1985), Duggan and Fey (2000), Duggan (2000), Banks and Duggan (2002), Bernhardt, Hughson, and Dubey (2003)). For example, in probabilistic voting environments if candidates care about the policy adopted by the winning candidate, candidates locate closer to the median than their most-preferred platforms, as they trade-off a less-preferred platform for an increased probability of winning.

[2] Medvic finds that of 805 U.S. House of Representatives races in 1990, 209 campaigns employed private pollsters; of 856 races in 1992, 396 campaigns did; and in races for open seats, about half of the candidates used pollsters. In addition, the major parties provide extensive polling services.

[3] This result follows from the common-value nature of this game and is reminiscent of the findings of Milgrom

right—about the likely median. Then, as long as signals are not negatively correlated, the pure strategy equilibrium exists and because the median given two "left" signals is more extreme than the median given one "left" signal, it follows that candidates bias their platforms away from the median voter in the direction of their private signals. Thus, private polling information necessarily leads to some variety in platforms because candidates receive different signals, and the equilibrium response by candidates to their information is to "overshoot" their private estimates of the median voter, leading to increased platform dispersion.

We then show that in a many-signal environment, if the correlation in candidates' signals is too low, the pure strategy equilibrium does not exist. This leads us to consider a tractable version of our model in which the median voter's ideal point is given by $\alpha+\beta$, candidates receive signals about the realization of β, and α is uniformly distributed. One possible interpretation of the model is that candidates learn about an initial location of the median voter, β, after which electoral preferences may shift by α. For example, after platforms have been selected, a weakening economy may change voters' views about increased fiscal spending; or terrorist attacks may alter voters' views about civil rights restrictions. Another interpretation of our model is that voters are unwilling or unable to provide pollsters accurate summaries about all of their views.

We solve for the mixed strategy equilibrium. Candidates with sufficiently moderate signals adopt their pure strategy equilibrium locations, locating more extremely than their information suggests. If signal correlation is low enough that the pure strategy equilibrium does not exist, then candidates who receive more and more extreme signals mix over their location, tempering their location by more and more toward the unconditional median. Thus, in a many signal environment, candidates who receive sufficiently moderate signals "extremize" their location, while candidates with sufficiently extreme signals moderate their location. We also derive comparative statics: We show, for example, that as signal quality improves, candidates locate more extremely.

Our welfare analysis allows us to assess a seemingly puzzling complaint in both the popular press and academic research. To whit, it is generally recognized that improved polling techniques lead to a convergence in candidate platforms, as candidates try to hone in on the median voter's preferred platform given their current information. Such convergence is often criticized as being bad for the median voter, and for voters in general—candidates are "too similar," there is "not enough choice" between candidates, and "they are all the same." Yet, on first pass, it is hard to understand why and how it can hurt the median voter for candidates to target his or her location

(1981), who shows that in a common-value second-price auction, the equilibrium bid of a type θ corresponds to the expected value of the good conditional on *both* types being equal to θ. Here, because candicates maximize the probability of winning, the relevant statistic is the median.

with increased precision; and it is hard to understand how this could reduce the welfare of risk averse voters with other ideologies, because such improved targeting is variance-reducing.

For simplicity, we take this issue up in the uniform model with two signals. We prove that although candidates extremize their location relative to their private information, they do not do so by as much as voters would like. Thus, the message of the Downsian model is fully reversed: There, platforms converge to the median voter's location, and this outcome is Pareto optimal when voters are risk averse. The contrast with the probabilistic voting model is equally sharp: There, platforms converge even though all voters would prefer variety. Here, private polling necessarily introduces some variety in equilibrium platforms, and the equilibrium response by candidates to their private signals is to further extremize their locations, but candidates still do not extremize their location by as much as voters would like.

The logic underlying why the electorate would prefer some dispersion in the candidates' platforms is that candidates cannot perfectly target the median voter's location. The platform ultimately implemented is the one closest to the median voter, however, and if voters' preferences are correlated with the median voter's, then some dispersion in candidate platforms is welfare enhancing because it gives voters greater choice. Because candidates care only about winning, they do not internalize the externality of providing voters greater choice. As a result, candidates do not collectively provide enough platform dispersion from the standpoint of the electorate.

A similar logic implies that greater signal correlation reduces voter welfare: Correlation reduces both the degree by which candidates extremize their platforms given their signals, as well as the probability that candidates receive different signals and, hence, choose distinct platforms. Finally, we show that the effect of signal precision on welfare is *not* monotonic: Increased polling accuracy raises the probability that candidates correctly identify the median voter's initial location, raising the welfare at any one candidate's platform, but it also raises the probability that the candidates adopt the same platform, reducing the probability that candidates give voters a choice.

This analysis provides a coherent explanation for why voters complain that candidates are too similar—while candidates respond strategically to private polling information by extremizing their location, they do not do so by as much as voters would like. This has an important policy implication: As long as polling precision rises with the resources expended, spending caps in elections may raise voter welfare.[4] In particular, even if campaign advertising were beneficial, the polling underlying the platforms advertised may be excessive.

Bernhardt, Duggan and Squintani (2003) provide a general analysis of the existence and conti-

[4] See Coate (2002) for a different argument for spending limits.

nuity properties of mixed strategy equilibria in the private polling model. Ledyard (1989) was the first to raise the issue of privately informed candidates. Recently, other papers have independently considered the issue. Chan (2001) develops a related three-signal model and then provides partial characterizations and welfare analyses under the assumption that a pure strategy equilibrium exists. Ottaviani and Sorensen (2002) numerically characterize a model of financial analysts who receive private signals of a firm's earnings and simultaneously announce forecasts, with rewards depending on the accuracy of their predictions. The case of two analysts can be interpreted as a model of electoral competition with privately informed candidates. Quite interestingly they show that increasing competition does not correct forecasters' strategic bias: as the number of forecasters increases, forecasts become more extreme. Other related papers include Razin (2003), Martinelli (2001, 2002) and Heidhues and Lagerlof (2001).

2 The Electoral Framework

2.1 The Model

Two political candidates, A and B, simultaneously choose policy platforms, x and y, on the real line. There is a unique median voter, whose preferred platform location is given by μ. Candidate A wins the election if his platform is closer to the median voter's preferred location than candidate B's, i.e., if $|x-\mu| < |y-\mu|$, and A loses if he locates further away. If $|x-\mu| = |y-\mu|$, then the election is decided by a fair coin toss, so that A wins with probability one half.

Candidates do not observe μ, but instead receive private, real-valued signals i and j from the same finite set I. Candidates share a common prior distribution on the joint distribution of μ and the signals. The distribution of μ conditional on candidate A receiving signal i and candidate B receiving signal j is $F_{i,j}$, and $f_{i,j}$ is the associated density, which we assume is strictly positive on its convex support. Given signals i and j, we let $m_{i,j}$ be the uniquely-defined median of $F_{i,j}$. Signals are ordered so that higher signals imply higher values of the conditional median. Specifically, letting $m_{i,K}$ denote the median given A receives signal i and B receives a signal from set $K \subseteq I$, signal $i < j$ implies that $m_{i,K} < m_{j,K}$ and $m_{K,i} < m_{K,j}$. In particular, signal $i < j$ implies that $m_{i,k} < m_{j,k}$ for all signals k.

The probability of signal pair (i, j) is $P(i, j)$, the marginal probability of signal i is $P(i)$, and the marginal probability of signal j is $P(j)$, where we assume that $P(i) > 0, P(j) > 0$ for all i and j. Conditional probabilities $P(\cdot|i)$ and $P(\cdot|j)$ are defined using Bayes rule. We stress that the model is completely general with respect to the correlation between candidates' signals, allowing for conditionally-independent signals and perfectly-correlated signals as special cases.

If $x < y$, then candidate A wins when $\mu < (x+y)/2$ and if $x > y$, A wins when $\mu > (x+y)/2$. The probability that B wins is, of course, just one minus the probability that A wins. Thus, the probability that candidate A wins when A adopts platform x and receives signal i, and B adopts platform y and receives signal j, is

$$\begin{aligned} \pi_A(x,y|i,j) &= F_{i,j}\Big(\frac{x+y}{2}\Big), && \text{if } x<y, \\ \pi_A(x,y|i,j) &= 1-F_{i,j}\Big(\frac{x+y}{2}\Big), && \text{if } y<x, \\ \pi_A(x,y|i,j) &= 1/2, && \text{if } x=y. \end{aligned} \tag{1}$$

We define a Bayesian game between the candidates in which pure strategies for the candidates are vectors $X = (x_i)$ and $Y = (y_j)$, and the solution concept is Bayesian equilibrium.

Given pure strategies X and Y, candidate A's interim and ex-ante expected payoffs are:

$$\Pi_A(X,Y|i) = \sum_{j\in I} P(j|i)\,\pi_A(x_i,y_j|i,j) \quad \text{and} \quad \Pi_A(X,Y) = \sum_{i\in I} P(i)\Pi_A(X,Y|i).$$

The ex ante game that candidates play is a two-player, constant sum game: For all X and Y,

$$\Pi_A(X,Y) + \Pi_B(X,Y) = 1.$$

A consequence of the fact that the game has a constant sum is that equilibria are interchangeable in the sense that if (X,Y) and (X',Y') are equilibria, then so are (X,Y') and (X',Y).

While the general model allows for asymmetries between candidates because, for example, an incumbent has a more accurate polling technology, a natural assumption is to suppose that candidates have access to identical polling technologies.

(A1) Candidates have access to identical polling technologies if the conditional distributions are symmetric across players: for all signals $i,j \in I$, $P(i,j) = P(j,i)$ and $F_{i,j} = F_{j,i}$.

If, instead, candidates have different polling technologies, one way to conceptualize the informational advantage of one candidate over the other is to assume that first nature draws two signals with the same polling technology, but then scrambles the signal assigned to the least informed candidate.

2.2 Pure Strategy Equilibrium

We next characterize pure strategy equilibria. If a pure strategy Bayesian equilibrium exists, it is unique, and after receiving a signal, a candidate locates at the median of the distribution of

μ conditional on *both* candidates receiving that signal. A corollary is that candidates take policy positions that are extreme relative to their expectations of μ given their own information. That is, candidates who receive high signals overshoot μ, while those who receive low signals undershoot.

We use the following result, proved in the appendix, to characterize equilibrium outcomes.

Lemma 1 *Assume **(A1)** holds so that candidates have access to identical polling technologies. If (X, Y) is a symmetric pure-strategy Bayesian equilibrium, then $x_i = y_i = m_{i,i}$ for all $i \in I$.*

The intuition behind this lemma is simple. Suppose that candidates A and B locate at the same point following signal realizations, i and j, and suppose for simplicity that these are the only realizations for which they locate there. Then, if the candidates are not located at the median conditional on signals i and j, either candidate could raise their payoff by a small move toward that conditional median: If A does this, A's expected payoff given signal realization j for B jumps discontinuously, but for other signal realizations for B, A's payoff varies continuously with A's location. Hence, the slight deviation raises A's payoff, which is impossible in equilibrium.

Lemma 1 together with the properties of a symmetric, constant-sum game yield Theorem 1:

Theorem 1 (Necessity) *Assume **(A1)** holds so that candidates have access to identical polling technologies. If (X, Y) is a pure strategy Bayesian equilibrium, then $x_i = y_i = m_{i,i}$ for all $i \in I$.*

Proof: Suppose not. If there is an asymmetric equilibrium (X', Y') with $x'_i \neq m_{i,i}$ for some i, then because the game is symmetric, (Y', X') is an equilibrium. But then equilibrium interchangeability implies that (X', X') is an equilibrium. This contradicts Lemma 1, which requires that in a symmetric equilibrium, $x_i = m_{i,i}$ for all i. ■

We now provide conditions that ensure the pure strategy equilibrium described in Theorem 1 exists. Loosely speaking, the pure strategy equilibrium exists if conditional on receiving signal i, the probability the other candidate receives a signal at least as extreme as signal i is high enough. This limits the incentive to move away from $m_{i,i}$ after signal i to compete against candidates with more moderate signals. In a two-signal context, this is not a demanding criterion.

Theorem 2 (Sufficiency: Binary Signals) *Suppose that $S = \{-1, 1\}$. Then provided that*

$$f_{i,i}\left(\frac{z+m_{i,i}}{2}\right) \geq f_{i,-i}\left(\frac{z+m_{-i,-i}}{2}\right), \text{ for all } z \in [m_{-1,-1}, m_{1,1}], \tag{2}$$

a sufficient condition for the existence of the pure strategy equilibrium in which $x_i = y_i = m_{i,i}$ for all $i \in I$ is that candidate signals not be negatively correlated, so that for any i, $P(i|i) \geq P(-i|i)$. If these inequalities hold strictly, then this is the unique equilibrium.

Proof: We show that (X, Y) is an equilibrium, where $x_i = y_i = m_{i,i}$ for $i \in \{-1, 1\}$. Consider candidate A's best response problem given signal $i = 1$. If A deviates to $x \in [m_{-1,-1}, m_{1,1}]$, then the change in A's interim expected payoff is

$$\begin{aligned} & P(-1|1)\left[F_{1,-1}\left(\frac{m_{1,1}+m_{-1,-1}}{2}\right) - F_{1,-1}\left(\frac{x+m_{-1,-1}}{2}\right)\right] + P(1|1)\left[F_{1,1}\left(\frac{x+m_{1,1}}{2}\right) - \frac{1}{2}\right] \\ = & \int_x^{m_{1,1}} \left[P(-1|1) f_{1,-1}\left(\frac{z+m_{-1,-1}}{2}\right) - P(1|1) f_{1,1}\left(\frac{z+m_{1,1}}{2}\right)\right] dz, \end{aligned}$$

which is non-positive. Thus, the deviation does not increase A's expected payoff. It is easily verified that deviations $x < m_{-1,-1}$ and $x > m_{1,1}$ are also unprofitable. A similar argument holds for signal $i = -1$, and a symmetric argument for candidate B establishes that (X, Y) is an equilibrium. If $P(i|i) > \frac{1}{2}$, then X is the unique best response to $X = Y$, and hence (X, X) is the unique equilibrium: if (X', Y') is an equilibrium, then so is (X', Y) by interchangeability, and therefore $X' = X$. ■

Importantly, this result holds even if candidates do not have access to the same polling technology, say because the incumbent's polls are more accurate. Then, the median is more likely to be to the right when the incumbent receives a positive signal than when the challenger does, but each candidate locates at the same position when receiving a positive signal—the conditional median given that both candidates receive positive signals. Thus, asymmetry in polling technologies does not give rise to greater platform variety.

The regularity condition detailed in Theorem 2 is weak. Inequality (2) compares $f_{1,1}$, shifted to the left by $\frac{m_{1,1}}{2}$, with $f_{1,-1}$ shifted to the left by $\frac{m_{-1,-1}}{2}$. Hence, the condition requires that $f_{1,1}$ shifted to the left by $\frac{m_{1,1}-m_{-1,-1}}{2} > 0$ weakly exceed $f_{1,-1}$ on the relevant range. This holds, for example, if the signal merely shifts the median of the distribution, while preserving its shape. More generally, the condition holds if $f_{1,-1}$ has its median near $\frac{m_{1,1}+m_{-1,-1}}{2}$ and is somewhat more dispersed than $f_{1,1}$—as one would expect if identical signals decrease the variance of the posterior distribution of μ, and opposing signals offset each other, leading to a higher variance.

Our sufficiency conditions in the multi-signal case are more demanding. The following result is proved in the appendix.

Theorem 3 (Multiple Signals) *Assume **(A1)** holds so that candidates have access to identical polling technologies, and assume*

$$F_{i,j}\left(\frac{m_{i,i}+z}{2}\right) = F_{j,k}\left(\frac{m_{k,k}+z}{2}\right), \text{ for all signals } i \leq j \leq k \text{ and for all } z \in [m_{i,i}, m_{k,k}]. \quad (3)$$

Then a necessary and sufficient condition for the unique pure strategy equilibrium to exist is that for every signal i,

$$\sum_{j:j\leq i} P(j|i) \geq \sum_{j:j>i} P(j|i) \quad \text{and} \quad \sum_{j:j<i} P(j|i) \leq \sum_{j:j\geq i} P(j|i). \quad (4)$$

Condition (3) is a simple regularity condition on the distribution of μ conditional on signal realizations that eases characterizations. It holds when the median $m_{i,j}$ following signals i and j is the average of the two medians $m_{i,i}$ and $m_{j,j}$ following common signals and the signals simply shift the conditional median while preserving the shape of the distribution. Indeed, the shape of the conditional distributions only needs to be preserved between the conditional medians.

Condition (4) on the priors over signal pairs requires that conditional on a candidate's own signal i, the (net) probability the other candidate received the same signal i be sufficiently high. Condition (4) is most restrictive for the "extremal" signals, for which $P(i|i) \geq \frac{1}{2}$ is necessary, and its restrictiveness depends on the number of possible signals. With two signals, for example, it is satisfied when signals are not negatively correlated.

2.3 Mixed Strategy Equilibria

Condition (4) becomes relatively demanding when candidates receive many signals. This leads Bernhardt, Duggan, and Squintani (2003) to consider equilibria in mixed strategies, $G = \{G_i\}_{i\in I}$, where G_i is the cumulative distribution function identifying the mixed strategy of a player with signal i. They prove that mixed strategy equilibria always exist, and that the mixed strategy equilibrium payoffs are unique and vary continuously in the model's parameters. They use this last result to prove upper hemicontinuity of equilibrium mixed strategies, and they show that the support of mixed strategy equilibria is given by the interval defined by the smallest and largest conditional medians. It follows that in the unique equilibrium of the Downsian model, candidates locate at the median voter's ideal point—there are no mixed strategy equilibria. The upper hemicontinuity result implies that in models "close" to the Downsian model, mixed strategy equilibria must be "close" to the median voter's ideal point. However, in models where private information plays a more significant role, and which therefore do not approximate the Downsian model, the nature of mixed strategies used in equilibrium is difficult to characterize in general.

Bernhardt, Duggan, and Squintani (2003) prove that if the polling technology satisfies the monotone likelihood ratio property, then the only possible atoms of the equilibrium mixed strategy of a player with signal i is at the conditional median $m_{i,i}$. Because potential discontinuities of G_i are restricted to the conditional median $m_{i,i}$, it is natural to search for equilibria in which G_i is differentiable except at $m_{i,i}$. This leads us to the following concept.

Definition 1 *A mixed strategy G is regular if, for all signals $i \in I$ and all $z \in \Re$, either G_i is differentiable at z or it is discontinuous at z.*

In a symmetric regular equilibrium, for every signal i, the cumulative distribution function G_i must satisfy the following first-order condition for all $x_i \in Supp(G_i)$:

$$\begin{aligned} 0 \;=\; & \sum_{j: m_{j,j} < x_i} P(j|i) \left[-f_{i,j}\left(\frac{x_i + m_{j,j}}{2}\right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] \\ & + \sum_{j: x_i < m_{j,j}} P(j|i) \left[f_{i,j}\left(\frac{x_i + m_{j,j}}{2}\right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] \\ & + \sum_{j \in I} P(j|i) \left[\int_{-\infty}^{x_i} -f_{i,j}\left(\frac{x_i + z}{2}\right) \frac{g_j(z)}{2}\, dz + (1 - F_{i,j}(x_i)) g_j(x_i) \right. \\ & \left. + \int_{x_i}^{\infty} f_{i,j}\left(\frac{x_i + z}{2}\right) \frac{g_j(z)}{2}\, dz - F_{i,j}(x_i) g_j(x_i) \right], \end{aligned} \tag{5}$$

where here g_i is the density of G_i, wherever it is defined.

To learn how private information affects platform location when pure strategy equilibria do not exist, we would like to solve this system of ordinary differential equations for a mixed-strategy equilibrium, but more structure is needed. Accordingly, we consider the uniform model. In the uniform model we explicitly solve for the (essentially unique) mixed strategy, and we also characterize pure strategy equilibria of the multi-signal model when one candidate has access to better polling technology than the other candidate.

3 Uniform Model

Let the median voter's location be given by $\mu = \alpha + \beta$, where α is uniformly distributed on $[-a, a]$ and β is an independently-distributed discrete random variable with support on $b_1 < b_2 < \cdots < b_N$.[5] Let $Q(b_k)$ denote the probability of b_k. Candidates share the same set of signals, although the signal quality can differ across candidates, so that signal i may be more informative to candidate A than candidate B. Signals depend stochastically on the realization of β:

$$P(i,j) \quad = \quad \sum_{k=1}^{N} Q(i,j|b_k)Q(b_k),$$

where $Q(i,j|b_k)$ is the probability conditional on b_k that candidate A receives signal i and candidate B receives signal j. To simplify the computation of the equilibrium, we assume that $a > b_N - b_1$, so that the conditional distribution of the median given signals i and j is *uniformly* distributed on $[b_N - a, b_1 + a]$:

$$F_{i,j}(z) \quad = \quad \frac{a - m_{i,j} + z}{2a}, \text{ for all } z \in [b_N - a, b_1 + a].$$

We first show that the sufficient conditions for the existence of an equilibrium in symmetric environments detailed in Theorem 3 are also sufficient in the uniform environment even when one candidate has access to a more accurate polling technology than the other. We prove the next result in the appendix.

Theorem 4 (Pure-Strategy Equilibrium: Asymmetric Case) *In the uniform model, a necessary and sufficient condition for the pure strategy equilibrium in which $x_i = y_i = m_{i,i}$ for all $i \in I$ to exist is that for all signals i of candidate A,*

$$\sum_{j:j\leq i} P(j|i) \geq \sum_{j:j>i} P(j|i) \quad \text{and} \quad \sum_{j:j<i} P(j|i) \leq \sum_{j:j\geq i} P(j|i),$$

and similarly for all signals j of candidate B,

$$\sum_{i:i\leq j} P(i|j) \geq \sum_{i:i>j} P(i|j) \quad \text{and} \quad \sum_{i:i<j} P(i|j) \leq \sum_{i:i\geq j} P(i|j).$$

If these inequalities are strict, then this is the unique equilibrium.

Even though signal qualities may differ across candidates, candidates order the signals in a common way: For signals $i > j$, both candidates believe that the median is more likely to be

[5] We assume finiteness only for simplicity. Our analysis extends if β were drawn from an arbitrary probability measure space.

further to the right following signal i than following signal j, even though the conditional median given that candidate A receives signal i may differ from the conditional median when candidate B receives signal i. Theorem 4 says that in a pure strategy equilibrium, following a signal i, candidates locate at the conditional median given that both receive the same ordered signal, $m_{i,i}$.

If the correlation in signals is not high enough and there are too many possible signals, then the pure strategy equilibrium does not exist. To solve for the resulting mixed strategy equilibrium, we need to impose more structure on the signals that candidates receive.

First, for simplicity, we now assume that candidates have access to the same polling technology, i.e., assumption **(A1)** holds. We also impose a natural ordering on the implications of receiving a higher signal. Assumption **(A2)** below implies that the higher is a candidate's signal (i) the more likely his signal is to exceed his opponent's, but (ii) the higher is the likely signal of his opponent. So, too, if a candidate draws a lower signal, then the probability his signal is less than his opponent's rises, but his opponent is also more likely to receive a lower signal.

(A2) For all signals i, k,

$$\begin{aligned} (a) \sum_{j:j<i} P(j|i) \;\geq\; \sum_{j:j<k} P(j|k) \;&\geq\; \sum_{j:j<k} P(j|i) \\ (b) \sum_{j:j>k} P(j|k) \;\geq\; \sum_{j:j>i} P(j|i) \;&\geq\; \sum_{j:j>i} P(j|k). \end{aligned}$$

Loosely speaking, receiving a higher signal raises the probability that the opponent's signal is higher, but by less than one for one.

We let C be the set of signals i that satisfy Condition (4) in Theorem 3:

$$\sum_{j:j\leq i} P(j|i) \;\geq\; \sum_{j:j>i} P(j|i) \quad \text{and} \quad \sum_{j:j<i} P(j|i) \;\leq\; \sum_{j:j\geq i} P(j|i).$$

Looking at the sufficient conditions (Theorem 3) for pure strategy equilibrium existence, we see that if all signals are in C, then the pure strategy equilibrium exists. An implication of **(A2)** is that the set C is an interval. Let $\bar{c} = \max C$, and $\underline{c} = \min C$.

Our third assumption implies that the set C is non-empty. The implicit normalization is made only to ease presentation.

(A3) $0 \in C$.

Our fourth assumption imposes the natural restriction that the probability a candidate draws signal i is greatest when his opponent draws that same signal.

(A4) For all signals $i, j \in I$, $P(i|i) \geq P(i|j)$.

Our last assumption is a technical assumption. Its role in our proof is to ensure that the supports of the mixed strategies that candidates choose in equilibrium following sufficiently extreme signals i and j are non-overlapping.

(A5) For all pairs of signals $i, j \notin C$ such that either $\overline{c} < j < i$, or $i < j < \underline{c}$, the conditional probability weighted conditional medians decline for more extreme signals, $P(j|i)m_{i,j} \leq P(j|j)m_{j,j}$.

Clearly, **(A5)** only matters if there are more than three signals, so that there are multiple extreme signals on the same side of C.

We search for symmetric equilibria satisfying a natural monotonicity condition. Specifically, we assume that the supports of the distributions G_i are non-overlapping and maintain the same order as the signals i; and further that the consecutive supports of non-degenerate distributions G_i are adjacent. Also, in line with our findings on pure-strategy existence, candidates play a pure strategy if and only if they receive a signal $i \in C$:

Definition 2 *A mixed strategy G is ordered if (a) a candidate who receives signal i plays a pure strategy if and only if $i \in C$, and (b) for all $i > \bar{c}$, $Supp(G_i) = [\underline{x}_i, \overline{x}_i]$ and $\underline{x}_i = \overline{x}_{i-1}$; and for all $i < \underline{c}$, $Supp(G_i) = [\underline{x}_i, \overline{x}_i]$ and $\underline{x}_{i+1} = \overline{x}_i$.*

Theorem 5 below explicitly solves for the closed-form solution of the unique ordered equilibrium. Remarkably enough, the closed-form solution obtains with minimal assumptions about the precision and correlation of the signals. The theorem details that candidates who receive moderate signals in the central set C extremize their location, playing the pure strategy $m_{i,i}$. However, candidates with more extreme signals likely face an opponent who draws a more moderate signal than theirs, and this leads them to moderate their location. Because locating to compete against opponents with more moderate signals is inconsistent with pure strategy equilibria, it follows that candidates with more extreme signals must mix. Figure 1 illustrates this equilibrium. This figure highlights that, as we will later show, candidates who receive extreme signals outside of C adopt convex mixed strategy densities that are steeper for more extreme signals. Thus, the degree of moderation is increasing in the candidates' signals.

Theorem 5 *In the uniform model, if conditions **(A1)-(A5)** hold, there is a unique symmetric regular ordered equilibrium (G, G). In that equilibrium, for all $i \in C$, candidates locate at $m_{i,i}$, and*

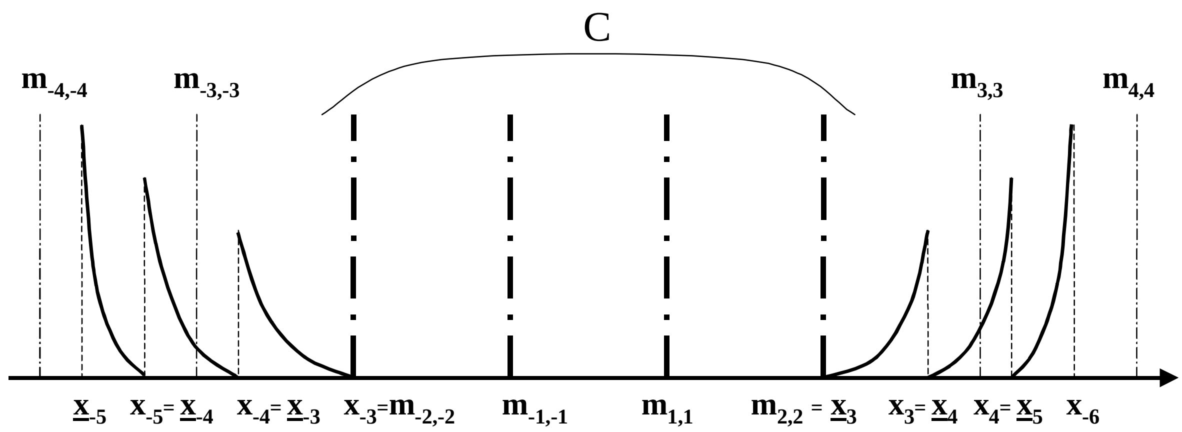


Figure 1: Equilibrium Diagram

for all $i > \bar{c}$, candidates mix according to an increasing, convex density,

$$g_i(x) \quad = \quad \frac{\Phi_i}{2}\left[\frac{(m_{i,i} - \underline{x}_i)^{\frac{1}{2}}}{(m_{i,i} - x)^{\frac{3}{2}}}\right] \tag{6}$$

on an interval $[\underline{x}_i, \overline{x}_i]$ with $\overline{x}_i < m_{i,i}$. Here,

$$\Phi_i \;=\; \frac{\sum_{j:j\leq i} P(j,i) - 1/2}{P(i,i)} > 0 \quad \text{and} \quad \underline{x}_i \;=\; m_{i,i}\left[1 - \left(\frac{\Phi_i}{\Phi_i + 1}\right)^2\right] + \underline{x}_{i-1}\left(\frac{\Phi_i}{\Phi_i + 1}\right)^2, \tag{7}$$

with $\underline{x}_{\bar{c}} = m_{\bar{c},\bar{c}}$. The associated cumulative distribution function is

$$G_i(x) \quad = \quad \Phi_i\left[\sqrt{\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x}} - 1\right], \tag{8}$$

and the expected location of a candidate with signal $i > \bar{c}$ is given by a weighted average of $\underline{x}_i$ and $m_{i,i}$,

$$E[x_i] \quad = \quad \frac{\Phi_i}{\Phi_i + 1}\underline{x}_i + \frac{1}{\Phi_i + 1}m_{i,i}. \tag{9}$$

An analogous characterization controls G_i for $i < \underline{c}$.

The proof, in the appendix, is tedious but straightforward. If there is a regular ordered mixed strategy equilibrium, then the equilibrium mixed strategy solves the first order condition, equation (5). The conjectured strategies are simply the solution to that first order condition given that a candidate's posterior distribution following a signal is uniformly distributed on the relevant support. The guts of the proof is tediously verifying that a candidate does not want to deviate to any possible alternative platform. Finally, integration reveals the properties of the equilibrium mixed strategy. We conjecture that the restriction to regular ordered strategies is unnecessary, and that the equilibrium we uncover is, in fact, unique.

Inspection of the closed-form solution for the equilibrium mixed strategy following signal $i > \bar{c}$, G_i, in equation (8) reveals several comparative statics results that are easy to establish.

- Ceteris paribus, increasing the conditional median, $m_{i,i}$, leads to a first order stochastic increase in the equilibrium distribution, i.e., candidates tend to locate further away from the unconditional median.
- So, too, reducing Φ_i—for example, by increasing the probability that the candidates draw the same signal, $P(i,i)$, and reducing the probability of competing against a candidate with a more moderate signal, $\sum_{j:j<i} P(j,i)$—leads to a first order stochastic increase in $G_i(\cdot)$.
- In particular, because $\Phi_i \to 0$ as $\sum_{j:j\leq i} P(j,i) \to 1/2$, it follows that as the condition for candidates to adopt their pure strategy location of $m_{i,i}$ becomes close to being satisfied, candidates place almost all probability close to their pure strategy location of $m_{i,i}$.
- Under reasonable structural assumptions, increasing signal precision implies that $P(i,i)$ grows and that $\sum_{j:j<i} P(j,i)$ decreases for all i, which implies a declining Φ_i. Thus, increasing signal precision leads to candidates locating more extremely.

We now exploit the closed-form solution for G_i to derive a key characterization result: Candidates with more moderate signals expect to locate more extremely relative to their information than do candidates with more extreme signals. Specifically, we provide simple conditions under which candidates with extreme signals $i > \bar{c}$ expect to locate further away from $m_{i,i}$ as i increases.

First note that under reasonable conditions, the Φ_i coefficients in our equilibrium characterization of $G_i(\cdot)$ are increasing in i, for $i > \bar{c}$. To see this, note that we can write Φ_i as

$$\Phi_i = \frac{\sum_{j:j\leq i} P(j|i) - \sum_{j:i<j} P(j|i)}{P(i|i)}.$$

Φ_i is the net probability that when candidates A's signal is i, B's signal is not larger than i, all normalized by the probability that B also receives exactly signal i. Under **(A2)**, the numerator of Φ_i rises with i. Hence as a candidate's signal becomes more extreme, Φ_i rises as long as the conditional probability that the other candidate receives the same signal, $P(i|i)$, does not rise too sharply with i (and one might expect that $P(i|i)$ should fall with i). We assume precisely this regularity condition.

(A6) For all $i > \bar{c}$, $\Phi_{i+1} \geq \Phi_i$ and for all $i \leq \underline{c}$, $\Phi_{i-1} \leq \Phi_i$.

Because Φ_i is increasing in i, for $i > \bar{c}$, it follows immediately that $G_i(m_{i,i} - x)$ rises faster with i. Hence, to prove that candidates with more extreme signals expect to locate more moderately relative to their information, we just need to show that $m_{i,i} - \underline{x}_i$ is strictly increasing in i. To do this, use the difference equation (7) describing the relationship between $\underline{x}_{i+1}$ and $\underline{x}_i$ to solve for

$$m_{i+1,i+1} - \underline{x}_{i+1} = m_{i+1,i+1} - m_{i,i} + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 (m_{i,i} - \underline{x}_i), \qquad \text{for } i \geq \bar{c}.$$

At $i = \bar{c}$, $\underline{x}_{\bar{c}} = m_{\bar{c},\bar{c}}$, so that $\left(m_{\bar{c}+1,\bar{c}+1} - \underline{x}_{\bar{c}+1}\right) - (m_{\bar{c},\bar{c}} - \underline{x}_{\bar{c}}) = m_{\bar{c}+1,\bar{c}+1} - m_{\bar{c},\bar{c}} > 0$. Continuing, inductively

$$\begin{aligned}
&(m_{i+1,i+1} - \underline{x}_{i+1}) - (m_{i,i} - \underline{x}_i) \\
&= [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}] + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 (m_{i,i} - \underline{x}_i) - \left(\frac{\Phi_i}{\Phi_i+1}\right)^2 (m_{i-1,i-1} - \underline{x}_{i-1}) \\
&\geq [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}] + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 [(m_{i,i} - \underline{x}_i) - (m_{i-1,i-1} - \underline{x}_{i-1})] \\
&> [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}],
\end{aligned}$$

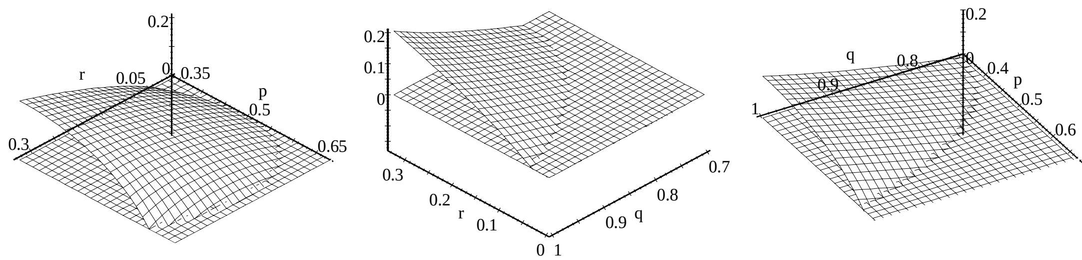
where the first inequality follows because $\Phi_{i+1} \geq \Phi_i$, and the second inequality follows from the induction hypothesis. We have shown that when $[m_{i+1,i+1} - m_{i,i}]$ is constant, $m_{i,i} - \underline{x}_i$ is strictly increasing with significant slack. Thus, a gross sufficient condition for candidates with increasingly extreme signals to locate increasingly moderately relative to their information is that the distance between successive conditional medians, $[m_{i+1,i+1} - m_{i,i}]$, not fall too quickly with i for $i \geq \bar{c}$:

Theorem 6 *If conditions **(A1)-(A6)** hold in the uniform model, then there exists a $\delta > 0$ such that $[m_{j+1,j+1} - m_{j,j}] - [m_{j,j} - m_{j-1,j-1}] > -\delta$ for all $j \geq \bar{c}$, implies that $m_{i,i} - E[x_i]$ is increasing in i for $i \geq \bar{c}$.*

We end this section by exploring how the probability of extremal electoral platforms varies with model parameters. Example 1 highlights both how likely extreme location is and how increased signal precision or increased signal correlation makes extreme location by candidates more likely.

Example 1 *Three Spatial Signals with Conditional Correlation.* Consider a uniform model with three equally-likely values of $\beta \in \{-1, 0, 1\}$, and three possible signals, $i \in \{-1, 0, 1\}$. With probability q, candidates receive conditionally-independent signals where with probability p the signal is correct, and with probability r the signal is "off by one":

$$\begin{aligned}
p &= Pr(i = 0|\beta = 0) = Pr(i = 1|\beta = 1) = Pr(i = -1|\beta = -1) \\
r &= Pr(i = 1|\beta = 0) = Pr(i = -1|\beta = 0) = Pr(i = 0|\beta = 1) = Pr(i = 0|\beta = -1).
\end{aligned}$$



(a) q = 1 (b) p = 0.35 (c) r = 0.3

Figure 2: Probability that candidates locate moderately in Example 1.

We assume that $p > r > 1 - p - r$, which implies that $r < 1/3$. With probability $1 - q$ candidates receive identical signals drawn from the same distribution.

We index the medians by the parameterization, (p, q, r) so that the median given only signal $i = 1$ is $m_1^{p,q,r}$, and the median given $i = j = 1$ is $m_{1,1}^{p,q,r}$. Following a positive signal of 1, a candidate locates more moderately than his private information if he locates at $x \in (0, m_1^{p,q,r}]$, and extremizes his location relative to his private information if he biases his location further from the center of the policy space in the direction of his private information so that $x > m_1^{p,q,r}$.

We use Maple to calculate the probability $\mathcal{P}(p, q, r) = G_1^{p,q,r}(m_1^{p,q,r})$ that a candidate chooses a moderate platform. We find that $\mathcal{P}(p, q, r)$ is maximal for $q = 1$, p approaching 1/3 from above, and r approaching 1/3 from below, with

$$\lim_{p \downarrow \frac{1}{3}, r \uparrow \frac{1}{3}, q \to 1} \mathcal{P}(p, q, r) = \frac{1}{2}(\sqrt{2} - 1) \approx 0.207.$$

Thus, the probability that candidates adopt extreme platforms is quite high for all values of p, q, r. Figure 2 illustrates how the probability candidates moderate their location relative to their private information varies with model parameters.

Figure 2a graphs outcomes when signals are conditionally-independent, presenting $\mathcal{P}(p, q = 1, r)$. Figure 2b fixes the signal's precision at $p = .35$ and presents $\mathcal{P}(p = 0.35, q, r)$; and Figure 2c fixes $r = .3$ and presents $\mathcal{P}(p, q, r = .3)$. Negative values correspond to parameter combinations for which G_1 is degenerate on $m_{1,1}$. These figures highlight the key point that as signals become more accurate, i.e., p rises, extreme platforms become more likely.

Example 2 highlights that the probability candidates locate extremely rises with the informational content of their signals.

Example 2 : *Conditional Independence with an Uninformative Signal.* Suppose that realiza-

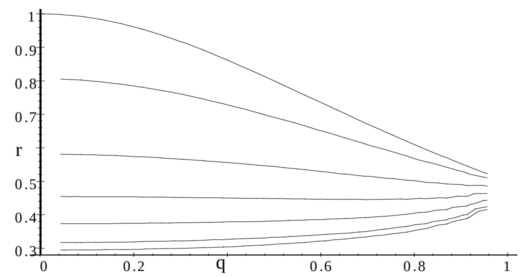


Figure 3: Level Curves of $G(m_1)$ in Example 2.

tions of $\beta \in \{-1, 1\}$ are equally likely and there are three possible signals, $i \in \{-1, 0, 1\}$ that are conditionally-independently distributed given β. With probability r, a candidate receives an informative signal $i \in \{-1, 1\}$ of quality q, where given β, the probability $i = \beta$ is $\frac{1+q}{2}$. If $q = 1$, then a signal of $i \in \{-1, 1\}$ is completely informative, and if $q = 0$, the signal is uninformative. With probability $1 - r$, a candidate receives signal $i = 0$, independently of β, so that this signal is uninformative. A pure strategy equilibrium exists if and only if $r \geq 1/(1 + q^2)$; i.e., fixing the informativeness of signals, a pure strategy equilibrium exists if and only if the probability of an informative signal is high enough. If, instead, $r < 1/(1 + q^2)$, then in the unique regular mixed strategy equilibrium, candidates locate at $m_{0,0} = 0$ upon receiving the uninformative signal 0, and mix according to Theorem 5 upon receiving signals 1 or -1. Figure 3 plots level sets of $\mathcal{P}(q, r)$, the probability of locating at least as moderately as $m_1 = r$ following signal 1.

Here, the higher is a level curve, the lower is $\mathcal{P}(q, r)$, i.e., the more likely is the candidate to choose a location that extremizes the informational content of his signal. The top level set represents $\mathcal{P}(q, r) = 0$, and the bottom level set represents $\mathcal{P}(q, r) = 1$. Note that increasing q (the informativeness of signals 1 and -1) has an ambiguous effect depending on r (the probability of an informative signal). Increasing r, however, reduces the probability of a moderate platform.

4 Welfare

We now explore the determinants of voter welfare. We separate the strategic effect of candidate competition from the statistical effects of polling technology, including the precision and correlation of the candidates' signals. We find that strategic interaction between the candidates leads them to take more extreme platforms than they would if they simply targeted the median voter using their private information, though voter welfare would be increased if the candidates took even more extreme positions. Not surprisingly, voter welfare is decreasing in the correlation of the candidates' signals, but the effect of precision is non-monotonic: welfare at first rises with precision but then falls after a point, so that the optimal amount of precision for the voters is strictly positive.

The median voter's preferred platform is $\mu = \alpha + \beta$, where α is uniformly distributed on $[-a, a]$, and for simplicity there are two equally likely values of $\beta \in \{-1, 1\}$. Candidates receive signals $i \in \{-1, 1\}$. We allow for asymmetric polling technologies. Given β, candidates receive the same signal with probability q, and with probability $1-q$ they receive conditionally-independent signals. The common signal is accurate with probability p, i.e., $\Pr(i|\beta = i) = p$, while candidate j's individual signal is accurate with probability p_j, where $1/2 < p_B \leq p \leq p_A < 1$. Thus, p_A, p_B and p capture the precision of the candidates' signals, and q is a measure of signal correlation. Because $P(i|i) \geq 1/2$ for each i, by Theorem 2, the unique pure strategy equilibrium obtains.

Because we want to characterize how the polling technology affects *each* voter's welfare, we assume that shifts in preferences in the electorate affect all voters in a common way and that voters have quadratic utilities. Voter v's preferred platform, θ_v, is defined relative to the median voter's preferred platform μ: $\theta_v = \mu + \delta_v$. That is, a voter's "relative ideal point", δ_v, represents the position of v's ideal point relative to μ. A change in μ simply shifts the distribution of voter ideal points. A voter with ideal point θ receives utility $u(\theta, z) = -(\theta - z)^2$ from policy outcome z, and we assume each voter votes for the candidate whose platform is closest.

The unique equilibrium is symmetric: Candidates locate at the conditional medians given two like signals,[6]

$$\begin{aligned} m_{1,1} &= -m_{-1,-1} = Q(1|1,1) - Q(-1|1,1) \\ &= \frac{qp + (1-q)p_A p_B - q(1-p) + (1-q)(1-p_A)(1-p_B)}{q(1-p) + (1-q)(1-p_A)(1-p_B) + qp + (1-q)p_A p_B}. \end{aligned}$$

We calculate voter welfare not only in equilibrium, but also for any symmetric strategy profile, $(m, -m)$, where $m > 0$ is the location that candidates adopt following signal $i = 1$, and $-m$ is their location following $i = -1$. Lemma 2, proved in the appendix, shows how the expected utility of voter δ_v compares with that of the median voter's.

Lemma 2 *Voter δ_v's expected utility is just δ_v^2 less than the expected utility of the median voter:*

$$W(\delta_v; m, p_A, p_B, p, q, a) = -\delta_v^2 + W(0; m, p_A, p_B, p, q, a).$$

Lemma 2 implies that all voters agree on the welfare ranking of different polling technologies (p_A, p, p_B, q) and the degree of dispersion when the candidates choose distinct platforms. As a

[6] For comparison, $m_1 = -m_{-1} = P(1|1)m_{1,1}$. If the candidate have access to the same polling technology, so that Assumption **(A1)** holds and $p = p_A = p_B$, this simplifies to $m_{1,1} = \frac{2p-1}{2p^2+2q(p-p^2)+1-2p}$ and $m_1 = 2p - 1$.

result, we need only consider the median voter's welfare:

$$\begin{aligned}
&W(m, p_A, p, p_B, q, a) \\
&= -\frac{1}{2}(qp + (1-q)p_A p_B)\left(\int_{-a}^{a} \frac{(1+\alpha-m)^2}{2a} d\alpha + \int_{-a}^{a} \frac{(-1+\alpha+m)^2}{2a} d\alpha\right) \\
&\quad -\frac{1}{2}(q(1-p) + (1-q)(1-p_A)(1-p_B))\left(\int_{-a}^{a} \frac{(1+\alpha+m)^2}{2a} d\alpha + \int_{-a}^{a} \frac{(-1+\alpha-m)^2}{2a} d\alpha\right) \\
&\quad -\frac{1}{2}(1-q)\left[p_A(1-p_B) + p_B(1-p_A)\right]\left(\int_{-a}^{-1} \frac{(1+\alpha+m)^2}{2a} d\alpha + \int_{-1}^{a} \frac{(1+\alpha-m)^2}{2a} d\alpha\right) \\
&\quad -\frac{1}{2}(1-q)\left[p_A(1-p_B) + p_B(1-p_A)\right]\left(\int_{-a}^{1} \frac{(-1+\alpha+m)^2}{2a} d\alpha + \int_{1}^{a} \frac{(-1+\alpha-m)^2}{2a} d\alpha\right).
\end{aligned}$$

The first line captures the median voter's expected payoff when both candidates receive the correct signal $i = \beta$; the second line is the median voter's expected payoff when both candidates receive the wrong signal, $i = -\beta$; and the last two lines are the median voter's expected payoff when the candidates receive opposing signals. For example, in the first line, $[qp + (1-q)p_A p_B]$ is the probability that both candidates receive the correct signal β; with probability $\frac{1}{2}$, $\beta = 1$ in which case both candidates locate at m yielding the payoff in the first integral; and with probability $\frac{1}{2}$, $\beta = -1$ in which case both candidates locate at $-m$ yielding the payoff in the second integral.

Three major forces determine voter welfare. Two of these forces, signal precision and correlation, are statistical properties of polling technology. The third is the equilibrium effect due to the common-value nature of the game, which causes candidates to extremize their location past the median given only their private information.

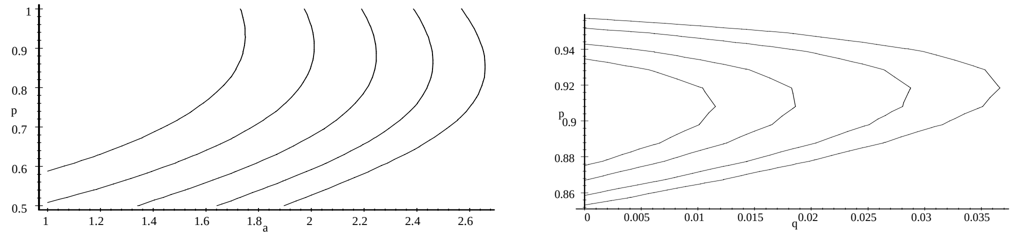
We first analyze the equilibrium effect. We prove that if the voters could choose the amount by which candidates "biased" their location following a signal realization, then voters would choose to *increase* the bias. Specifically, we compare the equilibrium candidate choices with (i) the medians conditional only on their private signals, and (ii) the level of dispersion m^* that maximizes $W(m, p_A, p, p_B, q, a)$. Lemma 2 implies that all voters agree on the optimal level of dispersion m^*.

Theorem 7 *For any (p_A, p, p_B, q, a), the voters' unique optimal level of dispersion m^* satisfies:*

$$m^* > m_{1,1} > m_1.$$

Proof: Let $R = (1-q)\left[p_A(1-p_B) + p_B(1-p_A)\right]$, $S = (qp + (1-q)p_A p_B)$, and $T = (q(1-p) + (1-q)(1-p_A)(1-p_B))$. Differentiate $W(m, p_A, p, p_B, q, a)$ with respect to m to obtain:

$$\frac{\partial W}{\partial m} = -2S(m-1) - 2T(m+1) + R\frac{1+a^2-2ma}{a}$$



(a) q = 0 (b) a = 2

Figure 4: Welfare Level Curves

and

$$\frac{\partial^2 W}{\partial m^2} = -2(R+S+T) < 0,$$

which shows that there is a unique optimal location $m_1^* > 0$. Furthermore, because $m_{1,1} = \frac{S-T}{S+T}$, it follows that

$$\begin{aligned}\left.\frac{\partial W}{\partial m}\right|_{m=m_{1,1}} &= -2\left((S+T)\frac{S-T}{S+T} + T - S\right) + R\frac{1+a^2-2m_{1,1}a}{a} \\ &= R\frac{1+a^2-2m_{1,1}a}{a} > \frac{1+a^2-2a}{a} > 0.\end{aligned}$$

Because $\frac{\partial^2 W}{\partial m^2} < 0$, it follows that $m_{1,1} < m_1^*$. ■

Turning our consideration to the statistical properties of the polling technology, it is easy to show that increased signal correlation, in the sense of higher q, is always bad in terms of welfare, as it reduces the likelihood that candidates take different locations. The next result is proved in the appendix.

Theorem 8 *For any* (p_A, p, p_B, a), *voter welfare* $W(m_{1,1}, p_A, p, p_B, q, a)$ *is decreasing in* q.

The welfare effect of increasing signal precision on welfare is subtle: welfare first rises with precision over a certain range and then falls beyond that. Increasing p raises the probability that both candidates receive the "correct" signal, $i = \beta$, but it also increases the probability that candidates receive the same signal, and hence do not offer voters variety. As a result, increasing signal precision eventually reduces voter welfare. This is illustrated in Figure 4, where we set $p_A = p_B = p$ (i.e., i.i.d. signals), and portray level sets of W as functions of (p, a) when $q = 0$, as well as level sets of W in (p, q) when $a = 2$.

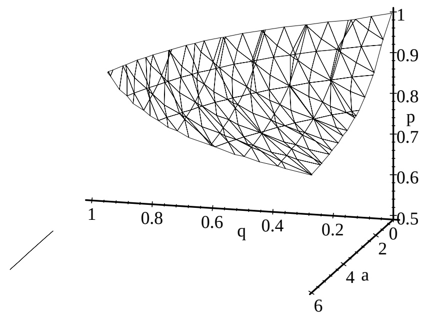


Figure 5: Welfare-Maximing p Plot

Figure 5 plots the optimal precision, $p^*(q, a)$, as a function of q and a. As long as signals are not perfectly correlated, voters prefer that there be some noise in the polling technology. The optimal precision $p^*(q, a)$ rises with signal correlation q, and $p^*(q, a) = 1$ when $q = 1$. Regardless of the noise level, when signals are perfectly correlated, there cannot be any platform variety, and hence voters are better off if candidates target μ as accurately as possible. Because a measures the degree to which the median voter's preferred platform may shift, the value of precisely targeting β decreases in a, and hence the optimal precision $p^*(q, a)$ decreases in a. Voters want less accurate polls, so that candidates are less likely to receive the same signal, and providing greater choice.

This discussion underscores a rationale for campaign spending limits—such limits reduce expenditures on polling, thereby reducing the precision of candidate's signals, and possibly raising voter welfare. So, too, this discussion suggests that if voters were able to coordinate, they may want to give dishonest answers to political pollsters in order to add noise to their polling technology.

5 Conclusion

This paper shows how private polling radically alters the nature of the strategies of office-motivated candidates, overturning the apparently robust result of platform convergence. Specifically, in any pure strategy equilibrium, candidates' platforms over-emphasize their private information: candidates locate at the median given that *both* receive the same signal. When candidates are not sufficiently likely to receive the same signal, equilibrium is characterized by mixed strategies. In the mixed strategy equilibrium, candidates who receive sufficiently moderate signals locate more extremely than their information suggests, while candidates who receive more extreme signals temper their locations. Finally, we characterize voter welfare. We show that voters would prefer that candidates extremize their location by even more. We also show that, from the perspective of voters, there is an optimal amount of noise in the polling technology. That is, the social value of better information for candidates about voters may be negative. Therefore, to the extent that polling accuracy depends on funding, it may be optimal to impose electoral spending caps.

Our analysis suggests fruitful directions for future research. First, because the strategic value of better information is always positive for candidates, it is straightforward to endogenize the choice of costly polling technologies by candidates. Second, it would be worthwhile to determine how outcomes are affected when candidates have ideological preferences, and to endogenize contributions by ideologically-motivated lobbies to fund polling by candidates. Finally, as Ledyard (1989) observes, it would be useful to uncover how equilibrium outcomes are affected when candidates sequentially choose platforms, so the second candidate can see where the first locates, and hence can unravel the latter's signal, before locating.

6 Appendix

Proof of Lemma 1: Say that (X, Y) with $x_i = y_i$ for all $i \in I$ is an equilibrium. For (X, Y) to be an equilibrium, it is necessary that:

$$\lim_{z \uparrow x_i} \Pi_A(z, Y|i) \le \Pi_A(x_i, Y|i) \quad \text{and} \quad \lim_{z \downarrow x_i} \Pi_A(z, Y|i) \le \Pi_A(x_i, Y|i). \tag{10}$$

Suppose that upon receiving signal i, A locates at $z > x_i$. Then his payoff is

$$\Pi_A(z, Y|i) = P(i|i)\left[1 - F_{i,i}\left(\frac{z + x_i}{2}\right)\right] + \sum_{j \ne i} P(j|i)\pi_A(z, y_j|i, j).$$

If, instead, candidate A locates at $z < x_i$, then

$$\Pi_A(z, Y|i) = P(i|i)F_{i,i}\left(\frac{z + x_i}{2}\right) + \sum_{j \ne i} P(j|i)\pi_A(z, y_j|i, j),$$

Finally if A locates exactly at x_i, his payoff is

$$\Pi_A(x_i, Y|i) = \sum_{k \in K_i} P(k|i)\frac{1}{2} + \sum_{j \notin K_i} P(j|i)\pi_A(z, y_j|i, j),$$

with $K_i = \{k \in I : y_k = y_i\}$. Note that for every $j \notin K_i$, continuity of the conditional distributions implies

$$\lim_{z \uparrow x_i} \Pi_A(z, y_k|i, j) = \Pi_A(x_i, y_k|i, j) = \lim_{z \downarrow x_i^-} \Pi_A(z, y_k|i, j),$$

conditions (10) simplify as follows:

$$\lim_{z \downarrow x_i} \sum_{k \in K_i} P(k|i)\left[1 - F_{i,k}\left(\frac{z + x_i}{2}\right)\right] \le \sum_{k \in K_i} P(k|i)\frac{1}{2}$$
$$\lim_{z \uparrow x_i} \sum_{k \in K_i} P(k|i)F_{i,k}\left(\frac{z + x_i}{2}\right) \le \sum_{k \in K_i} P(k|i)\frac{1}{2}.$$

After some algebra, we obtain

$$\frac{\sum_{k \in K_i} P(k|i)\left[1 - F_{i,k}(x_i)\right]}{\sum_{k \in K_i} P(k|i)} \le \frac{1}{2} \quad \text{and} \quad \frac{\sum_{k \in K_i} P(k|i)F_{i,k}(x_i)}{\sum_{k \in K_i} P(k|i)} \le \frac{1}{2}.$$

Because $F_{i,k}$ is continuous with connected support, these conditions imply that $x_i = m_{i,K_i}$ for all i. We conclude by showing that $K_i = \{i\}$ for any i. If not, because $y_i = x_i$ for all i, and $m_{i,K} \ne m_{k,K}$ for any $k \ne i$, a contradiction obtains: $y_i = x_i = m_{i,K_i} \ne m_{k,K_i} = x_k = y_k$ for all $k \in K_i$. ■

Proof of Theorem 3: The profile (X, Y) is an equilibrium if and only if for all $j \in I$, candidate A is not willing to deviate and play $z \ne m_{j,j}$. Since deviations outside $[\min m_{i,i}, \max m_{i,i}]$, this amounts to the requirement that for all $k \in \{j, \ldots, n-1\}$, and for all $z \in (m_{k,k}, y_{k+1}]$,

$$\begin{aligned}
0 \quad \leq \quad & \sum_{l=1}^{j-1} P\left(l|j\right)\left[F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{m_{j,j}+m_{l,l}}{2}\right)\right] \qquad (11)\\
& +P\left(j|j\right)\left[F_{j,j}\left(\frac{z+m_{j,j}}{2}\right)-F_{j,j}\left(\frac{m_{jj}+m_{j,j}}{2}\right)\right]\\
& +\sum_{l=j+1}^{k} P\left(l|j\right)\left[F_{j,l}\left(\frac{m_{j,j}+m_{l,l}}{2}\right)-\left[1-F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)\right]\right]\\
& +\sum_{l=k+1}^{n} P\left(l|j\right)\left[F_{j,l}\left(\frac{m_{j,j}+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)\right];
\end{aligned}$$

and that for all $k \in \{1, \ldots, j-1\}$, and for all $z \in [m_{k,k}, y_{k+1})$,

$$\begin{aligned}
0 \quad \leq \quad & \sum_{l=1}^{k} P\left(l|j\right)\left[F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{m_{jj}+m_{l,l}}{2}\right)\right] \qquad (12)\\
& +\sum_{l=k+1}^{j-1} P\left(l|j\right)\left[1-F_{j,l}\left(\frac{m_{jj}+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)\right]\\
& +\Pr(j|j)\left[F_{j,j}\left(\frac{m_{jj}+m_{j,j}}{2}\right)-F_{j,j}\left(\frac{z+m_{j,j}}{2}\right)\right]\\
& +\sum_{l=j+1}^{n} P\left(l|j\right)\left[F_{j,l}\left(\frac{m_{jj}+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)\right].
\end{aligned}$$

By (3), it follows that $F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)=F_{j,j}\left(\frac{z+m_{j,j}}{2}\right)$ and $F_{j,l}\left(\frac{m_{j,j}+m_{l,l}}{2}\right)=\frac{1}{2}$, and inequality (11) is equivalent to:

$$0 \ \leq \ \left[F_{j,j}\left(\frac{z+m_{j,j}}{2}\right)-1/2\right]\left[\sum_{l=1}^{j} P\left(l|j\right)-\sum_{l=j+1}^{n} P\left(l|j\right)\right].$$

Because $z > m_{k,k} \geq m_{j,j}$, it follows that $F_{j,j}\left(\frac{z+m_{j,j}}{2}\right) > 1/2$, so this holds if and only if:

$$0 \ \leq \ \left[\sum_{l=1}^{j} P\left(l|j\right)-\sum_{l=j+1}^{n} P\left(l|j\right)\right],$$

which corresponds to equation (4). An analogous argument for inequality (12) establishes the result. ■

Proof of Theorem 4: The profile (X, X) is an equilibrium if and only if for each candidate A, B, for all $j \in I$, for all $k \in \{j, ..., n-1\}$, and for all $z \in (m_{k,k}, y_{k+1}]$, Condition (11) is satisfied; and for all $k \in \{1, ..., j-1\}$, and for all $z \in [m_{k,k}, y_{k+1})$, condition (12) is satisfied. In the uniform model, for all signals j, l,

$$F_{j,l}\left(\frac{z+m_{l,l}}{2}\right)-F_{j,l}\left(\frac{m_{j,j}+m_{l,l}}{2}\right) \ = \ \frac{z-m_{jj}}{2},$$

and therefore condition (11) reduces to

$$0 \;\leq\; \left[\frac{z - m_{jj}}{2}\right]\left[\sum_{l=1}^{j} P\left(l|j\right) - \sum_{l=k+1}^{n} P\left(l|j\right)\right],$$

regardless of the identity of the player who receives signal j. An analogous simplification applies to condition (12). If the simplified versions of (11) and (12) hold with strict inequalities, then X is the unique best response to X, and hence (X, X) is the unique equilibrium: if (X', Y') is an equilibrium, then so is (X', X) by interchangeability, and therefore $X' = X$. ■

Proof of Theorem 5: We begin by deriving necessary conditions for a symmetric equilibrium in regular ordered strategies, and we will see that these pin down the solution exactly. Because a regular ordered mixed strategy G has non-overlapping supports, it must be that for every $x \in (\underline{x}_i, \overline{x}_i)$, $g_j(x) > 0$ implies $j = i$. As a consequence, for $i > \overline{c}$, the first order condition (5) simplifies to

$$P(i|i)g_i(x)(2F_{i,i}(x) - 1) \;=\; -\sum_{j\in C}\frac{P(j|i)}{2} f_{i,j}\left(\frac{x + m_{j,j}}{2}\right)$$
$$+\sum_{j\in I\setminus C}\frac{P(j|i)}{2}\left[\int_{-\infty}^{x} -f_{i,j}\left(\frac{x+z}{2}\right)g_j(z)\,dz + \int_{x}^{\infty} f_{i,j}\left(\frac{x+z}{2}\right)g_j(z)\,dz\right]$$

for all $x \in [\underline{x}_i, \overline{x}_i]$, unless $i = \overline{c} + 1$, in which case the condition holds on the half-open interval $(m_{\overline{c},\overline{c}}, \overline{x}_{\overline{c}+1}]$. Since the candidate's expected payoff is constant over the relevant interval, it must in particular be linear over this interval, so the second order condition must be satisfied with equality. Because α is uniformly distributed, the second order condition simplifies to

$$3g_i(x)f_{i,i}(x) + g_i'(x)(2F_{i,i}(x) - 1) \;=\; 0$$

for all $x \in [\underline{x}_i, \overline{x}_i]$, with the same qualification as before if $i = \overline{c} + 1$. Because a candidate chooses the platform $m_{\overline{c},\overline{c}}$ with probability zero when the candidate receives signal $\overline{c} + 1$, we include it in the interval as well, yielding a differential equation in g_i that is easily solved. We find that

$$g_i(x) \;=\; g_i(\underline{x}_i)\left(\frac{1 - 2F_{i,i}(\underline{x}_i)}{1 - 2F_{i,i}(x)}\right)^{\frac{3}{2}} \tag{13}$$

for all $x \in [\underline{x}_i, \overline{x}_i]$, with associated distribution

$$G_i(x) \;=\; g_i(\underline{x}_i)((1/2) - F_{i,i}(\underline{x}_i))^{3/2}\int_{\underline{x}_i}^{x}\frac{1}{((1/2) - F_{i,i}(z))^{3/2}}\,dz. \tag{14}$$

Substituting for $F_{i,j}(z) = (a - m_{i,j} + z)/2a$, equations (13) and (14) become

$$g_i(x) \;=\; g_i(\underline{x}_i)\left(\frac{\underline{x}_i - m_{i,i}}{x - m_{i,i}}\right)^{\frac{3}{2}} \tag{15}$$

and

$$G_i(x) \;=\; g_i(\underline{x}_i)(\underline{x}_i - m_{i,i})^{3/2}\left(\frac{2}{\sqrt{m_{i,i} - x}} - \frac{2}{\sqrt{m_{i,i} - \underline{x}_i}}\right). \tag{16}$$

Thus, the second order condition pins down the density g_i and distribution G_i up to the location of $\underline{x}_i$ and the initial condition $g_i(\underline{x}_i)$.

These parameters are determined by the first order condition for each signal realization and by our assumption of non-overlapping adjacent supports. In the uniform model, the first order condition can be written as

$$g_i(x) \;=\; \frac{\sum_{j:j<i} P(j|i) + P(i|i)(1-2G_i(x)) - \sum_{j:i<j} P(j|i)}{4P(i|i)(m_{i,i}-x)}.$$

Evaluated at $\underline{x}_i$, the first order condition yields

$$g_i(\underline{x}_i) \;=\; \frac{\sum_{j:j\leq i} P(j|i) - \sum_{j:i<j} P(j|i)}{4P(i|i)(m_{i,i}-\underline{x}_i)} \;=\; \frac{\Phi_i}{2}\frac{1}{(m_{i,i}-\underline{x}_i)},$$

which is well-defined for $\underline{x}_i < m_{i,i}$ and positive for $i > \overline{c}$, as required. Substituting for $g_i(\underline{x}_i)$ in (15) and (16) yields the following expressions for the density g_i and distribution G_i on the support $[\underline{x}_i, \overline{x}_i]$,

$$g_i(x) \;=\; \frac{\Phi_i}{2}\left[\frac{(m_{i,i}-\underline{x}_i)^{\frac{1}{2}}}{(m_{i,i}-x)^{\frac{3}{2}}}\right] \quad \text{and} \quad G_i(x) \;=\; \Phi_i\left[\sqrt{\frac{m_{i,i}-\underline{x}_i}{m_{i,i}-x}}-1\right].$$

That g_i is increasing and convex is apparent from the functional form of the density. The condition $G_i(x) = 1$ determines the upper bound of the support $\overline{x}^i$, which coincides with the lower bound of the support of G_{i+1} and is denoted $\underline{x}_{i+1}$. Note that for $\underline{x}_i < m_{i,i}$, a solution to $G_i(x) = 1$ does indeed exist for all $i > \overline{c}$, since $\frac{2}{\sqrt{m_{i,i}-x}}$ goes to infinity as x increases to $m_{i,i}$. For signal $\overline{c}+1$, by construction, $\underline{x}_{\overline{c}+1} = m_{\overline{c},\overline{c}}$. Therefore, the lower bounds are pinned down recursively by the difference equation:

$$\underline{x}_{i+1} \;=\; m_{i,i}\left[1-\left(\frac{\sum_{j\leq i} P(j,i) - 1/2}{\sum_{j\leq i} P(j,i) + P\,(i,i) - 1/2}\right)^2\right] + \underline{x}_i\left(\frac{\sum_{j\leq i} P(j,i) - 1/2}{\sum_{j\leq i} P(j,i) + P\,(i,i) - 1/2}\right)^2,$$

with the initial condition $\underline{x}_{\overline{c}+1} = m_{\overline{c},\overline{c}}$, as in (7). These observations, with an induction argument starting with $\underline{x}_{\overline{c}+1} = m_{\overline{c},\overline{c}}$, yield $\overline{x}_i < m_{i,i}$ for all $i \in I$. The expectation (9) is derived simply by integrating.

There remains only to establish that this mixed strategy does determine an equilibrium. Take any $i, k \in I$ with $\overline{c} < k \leq i$. Let X be a pure strategy such that $x_i = \overline{x}_k$, and let X' be a pure strategy such that $x'_i = x' \in (\underline{x}_k, \overline{x}_k]$. Define

$$\psi_k(x) \;\equiv\; F_{i,j}\left(\frac{\overline{x}_k + z}{2}\right) - F_{i,j}\left(\frac{x' + z}{2}\right) = \frac{\overline{x}_k - x}{4a},$$

which is independent of j and z, and note that this quantity is positive. The change in candidate

A's expected payoff, conditional on signal i, upon moving from $\overline{x}_k$ to x' is

$$\begin{aligned}\Pi_A(X', G|i) - \Pi_A(X, G|i) \;&=\; \sum_{j\in C: j<k} P(j|i)\psi_k(x') - \sum_{j\in C: j>k} P(j|i)\psi_k(x') \\ + \sum_{j\notin C: j<k} P(j|i) \int_{-\infty}^{\underline{x}_k} \psi_k(x') g_j(z)\, dz &- \sum_{j\notin C: k<j} P(j|i) \int_{\overline{x}_k}^{\infty} \psi_k(x') g_k(z)\, dz \\ +P(k|i)\left[\int_{\underline{x}_k}^{x'} \psi_k(x') g_k(z)\, dz + \int_{x'}^{\overline{x}_k} \left[F_{i,k}\left(\frac{x'+z}{2}\right) + F_{i,k}\left(\frac{\overline{x}_k+z}{2}\right) - 1\right] g_k(z)\, dz\right].\end{aligned}$$

Substituting for $F_{i,k}(z) = \frac{1}{2} + \frac{z-m_{i,k}}{2a}$, this simplifies to

$$\begin{aligned}\Pi_A(X', G|i) - \Pi_A(X, G|i) \;&=\; \psi_k(x') \left[\sum_{j:j<k} P(j|i) - \sum_{j:k<j} P(j|i)\right] \\ +P(k|i)&\left[\psi_k(x')G_k(x') + \int_{x'}^{\overline{x}_k} \frac{\overline{x}_k + x' + 2z}{4a} g_k(z)\, dz - \left(\frac{1-G_k(x')}{a}\right) m_{i,k}\right] \\ &=\; \psi_k(x') \left[\sum_{j:j<k} P(j|i) - \sum_{j:k<j} P(j|i)\right] + P(k|i)[D - Em_{i,k}],\end{aligned}$$

where

$$\begin{aligned} D &= \psi_k(x')G_k(x') + \int_{x'}^{\overline{x}_k} \frac{\overline{x}_k + x' + 2z}{4a} g_k(z)\, dz \\ E &= \frac{1-G_k(x')}{a}.\end{aligned}$$

Note that by construction, $\Pi_A(X', G|i) - \Pi_A(X, G|i)$ is equal to zero if $i = k$. The first term on the right-hand side, namely

$$\psi_k(x') \left[\sum_{j:j<k} P(j|k) - \sum_{j:k<j} P(j|k)\right],$$

is positive because $\overline{c} < k$, implying that $P(k|k)[D - Em_{k,k}] < 0$. If $k < i$, then

$$\sum_{j:j<k} P(j|i) - \sum_{j:j>k} P(j|i) \;\leq\; \sum_{j:j<k} P(j|k) - \sum_{j:j>k} P(j|k)$$

by **(A2)**, so the first term decreases relative to $i = k$. The change in the second term,

$$P(k|i)[D - Em_{i,k}] - P(k|k)[D - Em_{k,k}],$$

is generally indeterminate, because $P(k|k) > P(k|i)$ and $m_{i,k} > m_{k,k}$. But we now claim that **(A4)** and **(A5)** imply that it is non-positive, or equivalently,

$$E[P(k|k)m_{k,k} - P(k|i)m_{i,k}] \;\leq\; D[P(k|k) - P(k|i)].$$

To see this, note that D is positive since $0 \in [m_{\underline{c}}, m_{\overline{c}}]$ by **(A3)**, so that $x' > 0$. Clearly, E is also non-negative. But **(A5)** implies that the term multiplying E is non-positive, and **(A4)** implies that the term multiplying D is non-negative, as claimed. Therefore,

$$\Pi_A(X', G|i) - \Pi_A(X, G|i) \;\leq\; \Pi_A(X', G|k) - \Pi_A(X, G|k) \;=\; 0,$$

and we conclude that

$$\Pi_A(X', G|i) \quad \leq \quad \Pi_A(X, G|i) \tag{17}$$

whenever $x' \in (\underline{x}_k, \overline{x}_k]$. Indeed, if $\overline{c} + 1 < k$, then $\underline{x}_k$ is a continuity point of A's expected payoff function, so the inequality also holds at $\underline{x}_k$ as well.

If $k = \overline{c} + 1$, then $\underline{x}_k = m_{\overline{c},\overline{c}}$ is not a continuity point, and the last remark no longer holds. In this case, let X be a pure strategy such that $x_i = \overline{x}_{\overline{c}+1}$, and let X' be a pure strategy such that $x_i' = x' = m_{\overline{c},\overline{c}}$. Let $\{x^n\}$ be a sequence decreasing to $m_{\overline{c},\overline{c}}$, and let X^n be such that $x_i^n = x^n$. Then

$$\begin{aligned}
&[\Pi_A(X', G|i) - \Pi_A(X, G|i)] - \lim_{n\to\infty}[\Pi_A(X^n, G|i) - \Pi_A(X, G|i)] \\
&= \; P(\overline{c}|i)\left[\frac{1}{2} - \left[1 - F_{i,\overline{c}}\left(\frac{\overline{x}_{\overline{c}+1} + m_{\overline{c},\overline{c}}}{2}\right)\right]\right] \\
&\quad + P(\overline{c}+1|i)\left[\int_{m_{\overline{c},\overline{c}}}^{\overline{x}_{\overline{c}+1}} \left[2F_{i,\overline{c}+1}\left(\frac{z + m_{\overline{c},\overline{c}}}{2}\right) - 1\right] g_{\overline{c}+1}(z)\, dz\right] \\
&\quad - P(\overline{c}|i)\left[1 - F_{i,\overline{c}+1}(m_{\overline{c},\overline{c}}) - \left[1 - F_{i,\overline{c}+1}\left(\frac{\overline{x}_{\overline{c}+1} + \overline{m}_{\overline{c},\overline{c}}}{2}\right)\right]\right] \\
&\quad - P(\overline{c}+1|i)\left[\lim_{n\to\infty}\int_{x^n}^{\overline{x}_{\overline{c}+1}} \left[2F_{i,\overline{c}+1}\left(\frac{z + x^n}{2}\right) - 1\right] g_{\overline{c}+1}(z)\, dz\right] \\
&= \; P(\overline{c}|i)\left[F_{i,\overline{c}}\left(\frac{\overline{x}_{\overline{c}+1} + m_{\overline{c},\overline{c}}}{2}\right) - \frac{1}{2} + F_{i,\overline{c}+1}(m_{\overline{c},\overline{c}}) - F_{i,\overline{c}+1}\left(\frac{\overline{x}_{\overline{c}+1} + m_{\overline{c},\overline{c}}}{2}\right)\right] \\
&= \; P(\overline{c}|i)\left[\left(\frac{\overline{x}_{\overline{c}+1} - m_{i,\overline{c}}}{4a}\right) + \left(\frac{m_{\overline{c},\overline{c}} - \overline{x}_{\overline{c}+1}}{4a}\right)\right] \\
&= \; P(\overline{c}|i)\left(\frac{m_{\overline{c},\overline{c}} - m_{i,\overline{c}}}{4a}\right),
\end{aligned}$$

which is non-positive since $\overline{c} < i$. Therefore,

$$\Pi_A(X', G|i) - \Pi_A(X, G|i) \;\leq\; \lim_{n\to\infty}[\Pi_A(X^n, G|i) - \Pi_A(X, G|i)] \;\leq\; 0,$$

where the second inequality follows from the arguments of the previous paragraph. We conclude that (17) holds here too.

Now take any $i, k \in I$ with $k \leq i$ and $\underline{c} < k \leq \overline{c}$. Let X be a pure strategy such that $x_i = m_{k,k}$, and let X' be a pure strategy such that $x_i' = x' \in (m_{k-1,k-1}, m_{k,k}]$. Then, defining

$$\psi_k(x') \;=\; F_{i,k}(m_{k,k}) - F_{i,k}((x' + m_{k,k})/2) \;=\; \frac{m_{k,k} - x'}{4a},$$

we have

$$\begin{aligned}
&\Pi_A(X', G|i) - \Pi_A(X, G|i) \\
&= \psi_k(x') \left[\sum_{j:j<k} P(j|i) - \sum_{j:k<j} P(j|k) \right] + P(k|i) \left(F_{i,k}\left(\frac{x' + m_{k,k}}{2} \right) - \frac{1}{2} \right).
\end{aligned}$$

Note that since $m_{k,k} \leq m_{i,k}$, we have

$$F_{i,k}\left(\frac{x' + m_{k,k}}{2} \right) - \frac{1}{2} \quad \leq \quad -\psi_k(x').$$

Therefore,

$$\Pi_A(X', G|i) - \Pi_A(X, G|i) \quad \leq \quad \psi_k(x') \left[\sum_{j:j<k} P(j|i) - \sum_{j:k\leq j} P(j|k) \right],$$

which is non-positive if $i = k$, by definition of $k \in C$. If $i > k$, then the inequality follows from **(A2)**. We again conclude that (17) holds whenever $x' \in (m_{k-1,k-1}, m_{k,k}]$. That the inequality actually holds for $x' = m_{k-1,k-1}$ as well follows from an argument similar to that used in the previous paragraph.

Finally, take any $i, k \in I$ with $k \leq i$ and $k < \underline{c}$. Let X be a pure strategy such that $x_i = x \in [\underline{x}_k, \overline{x}_k)$, and let X' be a pure strategy such that $x_i' = \underline{x}_k$. Then, defining

$$\psi_k(x) \;=\; F_{i,j}\left(\frac{z + x}{2} \right) - F_{i,j}\left(\frac{z + \underline{x}_k}{2} \right) \;=\; \frac{x - \underline{x}_k}{4a},$$

which is independent of j and z, we have

$$\begin{aligned}
\Pi_A(X', G|i) - \Pi_A(X, G|i) \;&=\; \psi_k(x) \left[\sum_{j:j<k} P(j|i) - \sum_{j:k<j} P(j|i) \right] \\
&+ P(k|i) \left[-\psi_k(x)(1 - G_k(x)) + \int_{\underline{x}_k}^{x} \left[F_{i,k}\left(\frac{\underline{x}_k + z}{2} \right) + F_{i,k}\left(\frac{x + z}{2} \right) - 1 \right] g_k(z)\, dz \right].
\end{aligned}$$

If $i = k$, then, by $k < \underline{c}$, we know that the first term on the right-hand side is negative. Since $\Pi_A(X', G|k) - \Pi_A(X, G|k) = 0$, by construction, the second term must then be positive. If $k < i$, then, by **(A2)**, the first term decreases relative to $i = k$. Furthermore,

$$\begin{aligned}
&P(k|i) \left[-\psi_k(x)(1 - G_k(x)) + \int_{\underline{x}_k}^{x} \left[F_{i,k}\left(\frac{\underline{x}_k + z}{2} \right) + F_{i,k}\left(\frac{x + z}{2} \right) - 1 \right] g_k(z) dz \right] \\
&\leq \; P(k|k) \left[-\psi_k(x)(1 - G_k(x)) + \int_{\underline{x}_k}^{x} \left[F_{k,k}\left(\frac{\underline{x}_k + z}{2} \right) + F_{k,k}\left(\frac{x + z}{2} \right) - 1 \right] g_k(z)\, dz \right]
\end{aligned}$$

follows from **(A4)**, from the fact that the right-hand side is positive, and from the fact that $F_{i,k}$ stochastically dominates $F_{k,k}$. Thus, the second term decreases relative to $i = k$ as well. Once again, we conclude that (17) holds whenever $x \in [\underline{x}_k, \overline{x}_k)$. Indeed, if $k < \underline{c} - 1$, then $\overline{x}_k$ is a continuity point, so the inequality also holds for $x = \overline{x}_k$ as well. If $k = \underline{c} - 1$, then $\overline{x}_k = m_{\underline{c},\underline{c}}$, which is not a continuity point of A's expected payoffs. In this case, we may argue as above that the inequality remains true for $x = m_{\underline{c},\underline{c}}$.

We now argue that the candidate has no profitable deviation, conditional on signal i, from the mixed strategy G. First, consider the case $i > \overline{c}$. Let X satisfy $x_i = \overline{x}_i$, and let X' satisfy $x_i' = x' < \overline{x}_i$. Suppose that x' does not lie below the supports of all distributions G_j. (That case is easily checked and is omitted.) Then either $x' \in [\underline{x}_k, \overline{x}_k]$ for some $k \notin C$, or $x' \in [m_{k-1}, m_{k-1}]$ for some $k \in C$. Suppose the former. Indeed, suppose $k < \underline{c}$, and let the pure strategies X^j satisfy: For $j \notin C$, we have $x_i^j = \underline{x}_j$, and for $j \in C$, we have $x_i^j = m_{j,j}$. Our above arguments show that

$$\begin{aligned}
&\Pi_A(X', G|i) - \Pi_A(X, G|i) \\
&\quad = \; [\Pi_A(X', G|i) - \Pi_A(X^{k+1}, G|i)] + \sum_{j:k<j<\underline{c}} [\Pi_A(X^j, G|i) - \Pi_A(X^{j+1}, G|i)] \\
&\qquad + \sum_{\underline{c} \leq j < \overline{c}} [\Pi_A(X^j, G|i) - \Pi_A(X^{j+1}, G|i)] + \sum_{j:\overline{c}<j<i} [\Pi_A(X^j, G|i) - \Pi_A(X^{j+1}, G|i)]
\end{aligned}$$

is non-positive. Because $\overline{x}_i$ is a continuity point in the support of G_i, we have $\Pi_A(X, G|i) = \Pi_A(G, G|i)$, so that X' is not profitable. Other cases are proved the same way, by decomposing a deviation to the left after signal i into a finite number of moves across the supports for other signal realizations. This proves that candidate A has no profitable deviation to the left after signal i, and a symmetric argument shows that there are no profitable deviations to the right. Finally, the same argument for candidate B establishes that the strategy pair (G, G) is a mixed strategy equilibrium.

■

Proof of Lemma 2: To calculate the welfare of a voter with relative ideal point δ_v, suppose first that $\beta = 1$. If both candidates receive signal $i = 1$, then both locate at m, yielding voter δ_v expected utility of $(1/2a) \int u(\alpha + 1 + \delta_v, m) d\alpha$. If both receive signal $i = -1$, then δ_v's expected utility is $(1/2a) \int u(\alpha + 1 + \delta_v, -m) d\alpha$. Finally if candidates receive different signals, one locates at m, while the other locates at $-m$, and the candidate closest to the median voter wins the election. That is, the candidate at $-m$ wins if $\mu < 0$, or equivalently if $\alpha < -1$; and the candidate at m wins if $\mu > 0$, or equivalently if $\alpha > -1$. Thus, voter δ_v's expected utility is

$$\frac{1}{2a} \int_{-a}^{-1} u(\alpha + 1 + \delta_v, m)\, d\alpha + \frac{1}{2a} \int_{-1}^{a} u(\alpha + 1 + \delta_v, m)\, d\alpha.$$

A similar analysis holds for $\beta = -1$. With probability $qp + (1-q)p_A p_B$, both candidates receive the signal corresponding to β; with probability $(1-q)\,[p_A(1-p_B) + p_B\,(1-p_A)]$, candidates receive distinct signals; and with probability $q(1-p)+(1-q)(1-p_A)(1-p_B)$. Therefore,

$$\begin{aligned} W(\delta_v;p,q,a) &= \frac{1}{2}\Bigg[(qp+(1-q)p_Ap_B)\frac{1}{2a}\int_{-a}^{a}u(\alpha+1+\delta_v,m)\,d\alpha \\ &\quad +(q(1-p)+(1-q)(1-p_A)(1-p_B))\frac{1}{2a}\int_{-a}^{a}u(\alpha+1+\delta_v,-m)\,d\alpha \\ &\quad +(1-q)\left[p_A(1-p_B)+p_B(1-p_A)\right]\Bigg(\frac{1}{2a}\int_{-a}^{-1}u(\alpha+1+\delta_v,-m)\,d\alpha \\ &\quad +\frac{1}{2a}\int_{-1}^{a}u(\alpha+1+\delta_v,m)\,d\alpha\Bigg)\Bigg] \\ &\quad +\frac{1}{2}\Bigg[(qp+(1-q)p_Ap_B)\frac{1}{2a}\int_{-a}^{a}u(\alpha-1+\delta_v,-m)\,d\alpha \\ &\quad +(q(1-p)+(1-q)(1-p_A)(1-p_B))\frac{1}{2a}\int_{-a}^{a}u(\alpha-1+\delta_v,m)\,d\alpha \\ &\quad +(1-q)\left[p_A(1-p_B)+p_B(1-p_A)\right]\Bigg(\frac{1}{2a}\int_{-a}^{1}u(\alpha-1+\delta_v,-m)\,d\alpha \\ &\quad +\frac{1}{2a}\int_{1}^{a}u(\alpha-1+\delta_v,m)\,d\alpha\Bigg)\Bigg]. \end{aligned}$$

Gathering the two terms multiplied by $(qp+(1-q)p_Ap_B)$, corresponding to the candidates both receiving the correct signal, yields

$$\begin{aligned} &\frac{1}{4a}\int_{-a}^{a}u(\alpha+1+\delta_v,m)\,d\alpha+\frac{1}{4a}\int_{-a}^{a}u(\alpha-1+\delta_v,-m) \\ &= -\frac{1}{4a}\int_{-a}^{a}(\delta_v-(m-\alpha-1))^2\,d\alpha-\frac{1}{4a}\int_{-a}^{a}(\delta_v-(-m-\alpha+1))^2\,d\alpha \\ &= -\frac{\delta_v^2}{2}+\delta_v(m-1)-\frac{1}{4a}\int_{-a}^{a}(m-\alpha-1)^2d\alpha-\frac{\delta_v^2}{2}+\delta_v(-m+1)-\frac{1}{4a}\int_{-a}^{a}(-m-\alpha+1)^2d\alpha \\ &= -\delta_v^2+\frac{1}{4a}\int_{-a}^{a}u(\alpha+1,m)\,d\alpha+\frac{1}{4a}\int_{-a}^{a}u(\alpha-1,-m)\,d\alpha, \end{aligned} \tag{18}$$

where we use $E[\alpha]=0$. Similar manipulations for the remaining terms establish that the voter's welfare is just δ_v^2 less than the welfare of the median voter ($\delta_v=0$). ■

Proof of Theorem 8: Differentiating, we have

$$\frac{dW}{dq} = \frac{\partial W}{\partial q}+\frac{\partial W}{\partial m_{1,1}}\frac{\partial m_{1,1}}{\partial q}.$$

Note that because $\frac{\partial W(p,q,a)}{\partial m_{1,1}}>0$ as $m_{1,1}<m^*$, it is enough to show that $\frac{\partial m_{1,1}}{\partial q}<0$ and $\frac{\partial W}{\partial q}<0$.

$$\frac{\partial m_{1,1}}{\partial q} = 2\frac{p(1-p_A-p_B)+p_Ap_B(2p-1)}{(q(1-p)+(1-q)(1-p_A)(1-p_B)+qp+(1-q)p_Ap_B)^2},$$

because

$$\frac{\partial}{\partial p}\left[p\left(1-p_A-p_B\right)+p_Ap_B\left(2p-1\right)\right] \quad > \quad 0,$$

we can set $p=p_A$, its upper bound, and find

$$p\left(1-p_A-p_B\right)+p_Ap_B\left(2p-1\right)\big|_{p=p_A} \quad = \quad \left(1-p_A\right)p_A\left(1-2p_B\right),$$

which is negative for $p_B>1/2$. Because

$$\begin{aligned}\frac{\partial}{\partial p}\frac{\partial W(p,q,a)}{\partial q} \quad = \quad & -\int_{-a+1}^{a+1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha-\int_{-a-1}^{a-1}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha \\ & +\int_{-a+1}^{a+1}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha+\int_{-a-1}^{a-1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha\end{aligned}$$

is positive and $m_{1,1}>0$, it is enough to set $p=p_A$, its upper bound, and obtain:

$$\begin{aligned}\frac{\partial W(p,q,a)}{\partial q}\bigg|_{p=p_A} \quad = \quad & -p_A(1-p_B)\left(\int_{-a+1}^{a+1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha+\int_{-a-1}^{a-1}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha\right) \\ & -p_B(1-p_A)\left(\int_{-a+1}^{a+1}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha+\int_{-a-1}^{a-1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha\right) \\ & +\left[p_A(1-p_B)+p_B\left(1-p_A\right)\right]\left(\int_{-a+1}^{0}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha+\int_{0}^{a+1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha\right) \\ & +\left[p_A(1-p_B)+p_B\left(1-p_A\right)\right]\left(\int_{-a-1}^{0}\frac{(\alpha+m_{1,1})^2}{2a}d\alpha+\int_{0}^{a-1}\frac{(\alpha-m_{1,1})^2}{2a}d\alpha\right) \\ = \quad & p_A(1-p_B)\left[\int_{-a+1}^{0}\left(\frac{(\alpha+m_{1,1})^2}{2a}-\frac{(\alpha-m_{1,1})^2}{2a}\right)d\alpha\right. \\ & \left.+\int_{0}^{a-1}\left(\frac{(\alpha-m_{1,1})^2}{2a}d\alpha-\frac{(\alpha+m_{1,1})^2}{2a}\right)d\alpha\right] \\ & +p_B(1-p_A)\left[\int_{0}^{a+1}\left(\frac{(\alpha-m_{1,1})^2}{2a}-\frac{(\alpha+m_{1,1})^2}{2a}\right)d\alpha\right. \\ & \left.+\int_{-a-1}^{0}\left(\frac{(\alpha+m_{1,1})^2}{2a}-\frac{(\alpha-m_{1,1})^2}{2a}\right)d\alpha\right],\end{aligned}$$

which is positive for $m_{1,1}>0$. ■

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