Technology Shock and Employment: Do We Really Need DSGE Models with a Fall in Hours?

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Abstract

The recent empirical literature that uses Structural Vector Autoregressions (SVAR) has shown that productivity shocks identified using long–run restrictions lead to a persistent and significant decline in hours worked. This evidence calls into question standard RBC models in which a positive technology shock leads to a rise in hours. In this paper, we estimate and test a standard RBC model using Indirect Inference on impulse responses of hours worked after technology and non-technology shocks. We find that this model is not rejected by the data and is able to produce impulse responses in SVAR from simulated data similar to impulse responses in SVAR from actual data. Moreover, technology shocks represent the main contribution to the variance of the business cycle component of output. Our results suggest that we do not necessarily need DSGE models with a fall in hours to reproduce the results deriving from SVAR models.
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Introduction

Following the empirical strategy of Blanchard and Quah (1989), Galí (1999) shows that the response of hours to a technology shock is persistently and significantly negative in a Structural Vector Autoregression (SVAR) of labor productivity and hours with long–run restrictions. More precisely, resorting to a difference specification of hours (DSVAR) Galí (1999) shows that the level of hours significantly decreases in the short run in all G7 countries, with the exception of Japan. Galí (2004a) also finds similar qualitative results for the euro area as a whole. Conversely, with the level specification of hours[1] (LSVAR), Galí and Rabanal (2004) show that the point estimate of the impact response becomes positive, though very small and not significantly different from zero. Despite a hump shape, the response of hours is not significantly different from zero for each horizon.[2] In contrast, the negative response of hours in the DSVAR specification appears robust to various detrending methods of hours (see Galí and Rabanal, 2004) and to the inclusion of additional variables in the VAR model (see Galí 1999, Francis and Ramey 2004). If the measurement device, *i.e.* the DSVAR model, is taken seriously, this result is really challenging for a large part of the business cycle research program. Indeed, as pointed out by Galí and Rabanal (2004), the standard Real Business Cycle (RBC) model cannot reproduce this pattern, as hours worked increase after a positive technology shock. According to Francis and Ramey (2004), these empirical evidence reject unambiguously the RBC model and thus foretell the death of this paradigm. In light of their results, Galí (1999) and Galí and Rabanal (2004) suggest to abandon the frictionless approach in favor of models with nominal rigidities (sticky prices and/or sticky wages[3]). It is worth noting that flexible price models are able to reproduce a fall in hours following a technology shock, but they must include real frictions that deeply alter the structure of the original RBC model.[4]

However, recent contributions have questioned the ability of SVAR models to consistently measure the effect of a technology shock using long–run restrictions. Erceg, Guerrieri and Gust (2004),

[1] Christiano, Eichenbaum and Vigfusson (2004) argue convincingly that the DSVAR specification may induce distortions if hours worked are stationary in level.

[2] Chari, Kehoe and McGrattan (2004b) find similar results using various US datasets, with the exception of that of Francis and Ramey (2005).

[3] Galí and Rabanal (2004) propose a structural model with real frictions and nominal rigidities (the "triple" sticky model in the words of McGrattan, 2004) that is consistent with these evidence.

[4] Such frictions include, for example, habit persistence in consumption together with a high level of adjustment costs on physical capital (see Beaudry and Guay 1996, Boldrin, Christiano and Fisher 2001, Francis and Ramey, 2004).

using Dynamic Stochastic General Equilibrium (DSGE) models estimated on US data as their Data Generating Process (DGP), show that the effect of a technology shock on hours worked is not precisely estimated with SVAR. Moreover, when they adopt a DSVAR specification, the bias increases significantly.[5] Chari, Kehoe and McGrattan (2004b) provide similar results. They simulate an RBC model estimated by Maximum Likelihood on US data with two shocks (a permanent technology shock and a stationary labor tax shock), as well as measurement errors. They find that the DSVAR approach leads to a negative response of hours under an RBC model in which hours respond positively. These two papers caution the use of SVARs as a model–independent approach in order to identify the effect of technology shocks.[6] Moreover, these findings point out that it is not necessary to build macroeconomic models with a fall in hours after a technology shock so as to reproduce the predictions of a DSVAR model. These two papers highlight potential biases induced by long-run restrictions in VAR models (e.g. small sample biases, lag selection, SVAR specification). We build on these results to set up a complementary quantitative approach.

Our methodology departs from theirs in the following ways. First, the model's parameters are estimated so that impulse responses from the model are as close as possible to the impulse responses derived from the DSVAR estimated on actual data. More precisely, we use an original econometric methodology in order to estimate and test the model. Rather than using the popular Minimum Distance Estimation[7] (MDE), which consists in estimating directly the structural parameters from the impulse responses, we use an Indirect Inference approach.[8] The distinctive feature of Indirect Inference is to use an auxiliary model (or an auxiliary criterion) in order to indirectly estimate and test a DSGE model. The empirical strategy adopted in this paper uses the evidence from simulations experiments in Erceg, Guerrieri and Gust (2004) and Chari, Kehoe and McGrattan (2004b) as a starting point. Since the DSVAR specification used by Galí (1999) can deliver downward-biased responses of hours following a technology shock, we use precisely this DSVAR model as an auxiliary model. Second, we introduce a large number of over–identifying restrictions. This allows us to formally test the hypothesis: *Do We Really Need DSGE Models with a Fall in Hours?*, i.e. is a

[5]This is true in the case of an RBC model. However, a Sticky Price/Wage model delivers better results, as the LSVAR and DSVAR specifications provide a consistent estimate of the true (negative) response of hours.

[6]See also Cooley and Dwyer (1998) for an early criticism of SVARs.

[7]This limited information strategy is used by Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2004), and Altig, Christiano, Eichenbaum and Linde, (2005), among others.

[8]See Gouriéroux, Monfort and Renault (1993), Gouriéroux and Monfort (1996).

DSGE model in which hours worked increase after a technology shock necessarily incompatible with the predictions of a DSVAR specification? So as to test this hypothesis, our econometric approach is implemented as follows. The DSGE model is estimated indirectly so that the responses derived from an estimated DSVAR on simulated data from the DSGE model match as closely as possible their empirical counterpart from the DSVAR on actual data. In formal terms, the responses derived from an estimated DSVAR on simulated data from the DSGE model define a binding function from which the parameters of the DSGE can be consistently estimated. An attractive feature of this indirect approach is that it allows us to correct for biases and distortions, if any. If we keep in mind one of the main results of Chari, Kehoe and McGrattan (2004b), this means that a model in which hours worked increase after a technology shock is potentially able to match a DSVAR model in which hours decrease.

In a first step, so as to illustrate the importance of these points, we start by studying a streamlined model in which impulse responses can be directly computed. Using this simple model as the DGP, we show analytically that the DSVAR specification leads to biased estimated responses of hours worked. While hours do not respond to a technology shock in the theoretical model, they persistently decrease in the DSVAR. Moreover, the bias increases with the variance and the persistence of the non–technology shock. When this shock is highly persistent, adding more lags in the DSVAR does not allow us to correct for the bias in the estimated response. These results show that a direct quantitative evaluation of a DSGE model from a DSVAR specification, such as the MDE approach, can be highly misleading. However, the mere existence of this bias gives us the opportunity to estimate consistently the parameters of a DSGE model when resorting to an indirect approach such as Indirect Inference. In particular, this simple model allows us to characterize analytically the binding function from which the structural parameters of the underlying DSGE model can be identified and consistently estimated.

In a second step, we develop a more refined DSGE model which we formally take to the data. We consider a simplified version of the Kydland and Prescott (1982) model, where time non–separabilities include only one lag in leisure choices. We complement the Kydland and Prescott model by introducing an additional shock which shifts utility over periods. This preference shock accounts for persistent changes in the marginal rate of substitution between goods and work. Overall, our model can be viewed as an "old–fashioned" DSGE model and is thus representative of the first generation of frictionless RBC models. Our concern, implicitly, is to assess "how much" this type of model is dead. Not that much, according to our quantitative results.

In the empirical implementation of our methodology, we follow Christiano, Eichenbaum, and Vigfusson (2004), and first estimate impulse responses from DSVAR models using alternative measures (in logs) of productivity and hours worked with quarterly U.S. data for the period 1948:1-2002:4. We then estimate the structural parameters of our RBC model using Indirect Inference on the impulse responses of hours to technology and non–technology shocks.[9] For each dataset, our RBC model is able to produce impulse responses very close to those obtained from actual data. This means that our DSGE model in which hours worked persistently increase after a technology shock is consistent with Galí's findings, *i.e.* a technology shock in a DSVAR model leads to a fall in hours. Moreover, from the parameter estimates, we compute the contribution of the two shocks to aggregate fluctuations. Though under the DSVAR, identified technology shocks explain a tiny portion of the variance of output and hours (less than 10%), we find that these same shocks are the main source of output fluctuations at business cycle frequencies in the estimated RBC model, whereas the preference shock explains most of the fluctuations in hours.[10] Additionally, the correlation at business cycle frequencies between output and hours conditional on technology shocks only is very strong (more than 75%) in the estimated RBC model, instead of -0.08 with the DSVAR estimated on actual data. Since the RBC model matches very well the IRFs of hours in a DSVAR specification, our results cast serious doubts on variance decomposition and conditional correlation exercises conducted in SVARs. In particular, this contradicts the corollary of Galí and Rabanal (2004) that "technology shocks cannot be a quantitatively important (and, even less, a dominant) source of observed aggregate fluctuations". To the contrary, our findings are supportive of the claim that technology shocks matter, as argued by Prescott (1986).

We end our empirical exercise by inspecting the relative merits of LSVAR and DSVAR specifications in terms of empirical content. We show that the DSVAR specification, although biased, encompasses the LSVAR specification, while the converse is not true. This result suggests that the SVAR approach, if taken at face value, can lead to spurious stylized facts.

[9] We also complement these analyses by focussing on the IRFs of output and hours to both type of shocks.
[10] These results are consistent with findings reported by Chari, Kehoe, and McGrattan (2004a).

The paper is organized as follows. In a first section, we introduce a simple model that allows us to clearly show the main sources of distortions with the DSVAR approach. In section 2, we briefly present our simplified Kydland–Prescott model. The third section is devoted to the exposition of our econometric methodology. In section 4, we present the data and the results. The last section concludes.

1 Lessons from a Simple Model

In this introductory example, we consider a simple flexible prices equilibrium model without capital accumulation.[11] This model is deliberately stylized in order to deliver analytical results when a DSVAR model is estimated under this DGP. One can argue that the economy is highly stylized, so we cannot take its quantitative implications seriously. For example, the response of hours following a technology shock is zero. This is in contradiction with our results from SVARs in section 4 and previous quantitative findings.[12] This is not problematic for our purpose, as we simply try to evaluate the ability of SVARs (as a model–free statistical measurement method) to recover the effect of a technology shock. The zero response of hours to a technology shock should only be considered as a reference number for the analysis.

1.1 The Model

We consider a flex price version of the simple model analyzed in Galí (1999). The representative household seeks to maximize

$$\log(C_t) + \bar{\chi} \exp(\chi_t)(1 - N_t), \quad \bar{\chi} > 0, \tag{1}$$

subject to the per period budget constraint

$$C_t \leq W_t N_t + \Pi_t. \tag{2}$$

[11] A version of this model with capital accumulation is considered in the next section. Our main analytical findings are not qualitatively altered in this more general setup (see Erceg, Guerrieri and Gust, 2004, and Chari, Kehoe and McGrattan, 2004b)

[12] See Christiano, Eichenbaum and Vigfusson (2004)? Francis and Ramey (2004), Galí (1999), Galí and Rabanal (2004).

The quantity of good consumed in period t is C_t. The variable N_t denotes hours worked, W_t is the real wage, and Π_t represents the profit that the household receives from the firm. The utility function is separable, logarithmic in consumption, and following Hansen (1985), linear in leisure, implying an infinite labor supply elasticity. Without loss of generality, the time endowment is set to unity. Finally, χ_t is a random variable that shifts utility every periods. This variable is assumed to follow an AR(1) process

$$\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \varepsilon_{\chi,t},$$

where $\varepsilon_{\chi,t}$ is *iid* with zero mean and unit variance. As noticed by Galí (2004b), this shock can be an important source of fluctuations, as it accounts for persistent shifts in the marginal rate of substitution between goods and work (see Hall, 1997). Such shifts capture persistent fluctuations in labor supply following changes in labor market participation and/or changes in the demographic structure. Moreover, this preference shock allows us to generate persistence in hours.[13] It is worth noting that our assumption of linear labor supply has no consequence on our results. In what follows, the formula would be exactly the same, except for a scaling up of the variance of the preference shock by the square of one plus the inverse of the Frishian labor supply elasticity. The first order conditions of the household's problem (1)–(2) yield

$$\bar{\chi} \exp(\chi_t) C_t = W_t.$$

Consumption is an increasing function of the real wage, whereas it decreases after a positive preference shock on leisure.

The representative firm produces a homogenous good with a technology

$$Y_t = Z_t N_t^\alpha,$$

where $\alpha \in (0, 1]$. The variable Z_t is the aggregate technology, the growth rate of which is assumed to evolve according to

$$Z_t = Z_{t-1} \exp(\sigma_z \varepsilon_{z,t}),$$

where $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The first order condition of the firm is

$$W_t = \alpha \frac{Y_t}{N_t}.$$

[13]Note that this shock is observationally equivalent to a tax on labor income (see Erceg, Guerrieri and Gust, 2004, and Chari, Kehoe and McGrattan, 2004b). Additionally, it allows us to simply account for other distortions on the labor market, labelled *labor wedges* in Chari, Kehoe and McGrattan (2004a).

From the households and firms optimality conditions and market clearing $Y_t = C_t = Z_t N_t^{\alpha}$, the equilibrium employment is given by $N_t = \alpha \exp(-\chi_t)/\bar{\chi}$, and labor productivity is directly deduced from the production function. Taking logs and without loss of generality ignoring constant terms, we obtain the following log–linear representation of the economy

$$n_t = -\chi_t, \tag{3}$$

$$\Delta x_t = \sigma_z \varepsilon_{z,t} + (1-\alpha)\Delta\chi_t, \tag{4}$$

$$\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \varepsilon_{\chi,t}, \tag{5}$$

where Δ is the first difference operator and lower case letters represent the logarithms of the associated variables. In this economy, employment (3) does not react to a technological shock but decreases after a preference shock. Productivity (4) increases positively and permanently – one–for–one – after the technological shock. The stationary preference shock (5) has a positive impact effect on labor productivity, and no long–run effect.

1.2 Identification from DSVAR(1)

We use the system (3)–(5) as the DGP. Given the realization of the equilibrium, we seek to evaluate the quantitative implications of DSVAR specifications when the econometrician uses long–run restrictions on labor productivity in order to recover the effect of a technology shock on employment. Notice that the hours process can be highly persistent and indistinguishable from a unit root in small sample when ρ_χ is close to one. Indeed, many studies suggest that hours can display non–stationarity.[14]

We consider the identification and estimation of technology shocks using long–run restrictions in a DSVAR model. The VAR(1) model to be estimated has the following form

$$\mathbf{z}_t = \mathbf{A}_1 \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{z}_t$ includes the following variables $\mathbf{z}_t = (\Delta x_t, \Delta n_t)'$. In order to get analytical results, we only consider a VAR(1) model. Despite its simplicity, this assumption allows us to shed light on the main mechanisms at work during the course of identification. Increasing the number of lags does

[14]See Francis and Ramey (2004), Galí and Rabanal (2004) and Galí (2004b) among others.

not modify the main results, especially when the preference shock is highly serially correlated, i.e. $\rho_\chi \approx 1$. The estimated VARs' parameters and associated IRF allow us to determine the mapping between the structural parameters and those of the DSVAR, the so-called binding function. We impose the long–run restriction that only the productivity shocks have a permanent effect on labor productivity (see subsection 3.1 for more details about the identification using long–run restrictions). The following proposition characterizes impulse responses of hours to a technology shock.

Proposition 1 *Let $\boldsymbol{\eta}_{1,t}$ denote the technology shock identified by the DSVAR. When $\alpha \in (0,1)$ and $\sigma_z, \sigma_\chi > 0$, the impulse responses of hours worked to a technology shock $\boldsymbol{\eta}_{1,t}$ in a DSVAR(1) model under the DSGE model (3)–(5) is given by*

$$\frac{\partial n_{t+k}}{\partial \boldsymbol{\eta}_{1,t}} = -\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}} \frac{1-\left(\frac{\rho_\chi - 1}{2}\right)^{k+1}}{1+\frac{1-\rho_\chi}{2}}.$$

Proposition 1 shows that the estimated responses of hours in the DSVAR(1) are always negative, at all horizons. Notice that, when $k \to \infty$, the IRF of hours is

$$-\left(\frac{2}{3-\rho_\chi}\right)\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}.$$

In other words, these responses are downward-biased for any horizon $k \geq 0$. Notice that these results are asymptotic and do not hinge on small sample biases. When the variance of the non–technology shock increases relative to that of the technology shock and/or when the persistence of the non technology shock increases, the negative response is more pronounced. Moreover, when the labor share α decreases, the negative response is amplified. Conversely, when this share tends to one, the response is zero. In this latter case, productivity growth depends only on the technology shock and the impulse responses of hours are uniformly zero. This simply means that when productivity growth is an appropriate measure of total factor productivity growth, the DSVAR specification permits us to recover the true IRF of hours after a technology shock. Another interesting result concerns the estimated response to a technology shock when hours display persistence. Proposition 1 states that the persistence of the preference shock χ_t does not qualitatively affect the results in the DSVAR specification. The responses of hours is always negative in the DSVAR(1) for any value $\rho_\chi \in [0,1]$.

Note that when hours are non–stationary, the difference specification of hours does not allow us to recover the true response of hours. In the limit case $\rho_\chi \to 1$, the impulse responses at any horizon $k \geq 0$ are given by

$$\lim_{\rho_\chi \to 1} \frac{\partial n_{t+k}}{\partial \boldsymbol{\eta}_{1,t}} = -(1-\alpha)\sigma_\chi^2 \left(\sigma_z^2 + (1-\alpha)^2 \sigma_\chi^2\right)^{-1/2}.$$

The response of hours is asymptotically biased in the DSVAR model even when the preference shock –thus hours too– follows a random walk. This result can be easily explained, since the preference shock has a permanent effect on labor productivity when $\rho_\chi = 1$. In this case, the long run restriction in the DSVAR model (only the technology shock has a permanent effect) is not correct.

Finally, we have considered for simplicity only a VAR(1) model. The inclusion of further lags will affect our quantitative results in the DSVAR specification, because the first difference of hours introduces a unit root in the moving average of the preference shock, especially when ρ_χ is close to zero. This point is illustrated in figure 1–(a), which reports the responses of hours to a technology shock for various lags ($p = 1, ..., 12$) in the DSVAR(p) model when the preference shock is *iid*. In this case, increasing the number of lags allows us to weaken the negative response of hours. Except on impact where the response is always negative, the responses of hours are almost zero. This result does not hold when the preference shock (and thus hours) is persistent. Figure 1–(b) reports the impulse responses of hours for various lags when $\rho_\chi = 0.98$. As this figure makes clear, increasing the number of lags has a very negligible effect. To get a feel for this result, consider the DSGE model when $\rho_\chi \approx 1$. In this case, employment and productivity in first difference are *iid* and estimated coefficients from any VAR(p) model would be zero. Thus, the number of lags does not affect the response of hours.

Finally, it is worth noting that despite its simplicity, the identification of technology shock from the model possesses some empirical contents. Indeed, the IRFs from the DSVAR model under the DSGE model roughly match the IRFs from the data, since the response of hours is persistently negative, as predicted by Proposition 1.

This latter remark suggests another way to evaluate quantitatively business cycle models from a DSVAR model. Rather than a direct evaluation from impulse responses of hours in DSVAR, as in the MDE approach, Proposition 1 suggests an indirect approach. To illustrate this point, let us consider the following simple exercise. Let $\widehat{\psi}_T$ denote the estimated impact response of hours to a technology shock using a DSVAR(1) specification with actual data. Following Proposition 1, the impact response of a DSVAR(1) model estimated under the DSGE model is

$$\psi(\alpha, \sigma_z, \rho_\chi, \sigma_\chi) = -\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}.$$

Assume for simplicity that $\rho_\chi = 0$ and α and σ_z are set prior to estimation. One can thus determine a value of σ_χ such that the following equality holds

$$\psi(\sigma_\chi) = \widehat{\psi}_T.$$

The binding function $\psi(\sigma_\chi)$ offers the opportunity to estimate a value of σ_χ such that the impact response in a DSVAR(1) model under the DSGE model matches exactly the impact response in a DSVAR(1) model estimated on actual data. To illustrate this property, figure 2 reports the binding function in the $(\psi(\sigma_\chi), \sigma_\chi)$ plane.[15] This figure illustrates the results of Proposition 1. When the standard–error of the non–technology shock increases, the negative response of hours in the DSVAR(1) model is more pronounced. Using the point estimate[16] $\widehat{\psi}_T \simeq -0.27$ of Galí and Rabanal (2004), we can directly deduce the value of σ_χ such that $\psi(\sigma_\chi) = \widehat{\psi}_T$ using the binding function. In this simple setting, a value $\sigma_\chi \simeq 0.014$ allows us to match the IRFs of a DSVAR model, while hours in the DSGE model never respond to a technology shock.

The previous example simply shows how to quantitatively investigate the ability of business cycle models to match impulse responses of hours in DSVAR models. We now introduce an RBC model in the line of Kydland and Prescott (1982) in order to conduct a quantitative analysis in a canonical model.

2 A Kydland–Prescott Type Model

We consider a simplified and modified version of the Kydland and Prescott (1982) model. The model includes two shocks: a random walk productivity shock (Z_t) and a stationary preference shock (χ_t). We consider that intertemporal leisure choices are not time separable – as in Kydland and Prescott –,

[15]For illustrative purpose, this figure is drawn with $\alpha = 0.6$ and $\sigma_z = 0.025$.

[16]See also the subsection 4.1 and figure 4.

and assume that the service flows from leisure are a linear function of current and once-lagged leisure choices. More precisely, the intertemporal expected utility function of the representative household is given by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \log(C_{t+i}) + \bar{\chi} \exp\left(\chi_{t+i}\right) \log(\mathcal{L}^{\star}_{t+i}) \right\},$$

where $\bar{\chi} > 0$, $\beta \in (0,1)$ denotes the discount factor and E_t is the expectation operator conditional on the information set available as of time t. C_t is the consumption at t and $\mathcal{L}^{\star}_t$ represents the service flows from leisure $\mathcal{L}_t$. As in the simple model, the labor supply $N_t \equiv 1 - \mathcal{L}_t$ is subjected to a stochastic shock χ_t, that follows a stationary stochastic process

$$\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \varepsilon_{\chi,t},$$

where $|\rho_\chi| < 1$, $\sigma_\chi > 0$, and $\varepsilon_{\chi,t}$ is *iid* with zero mean and unit variance. The service flow of leisure is assumed to evolve according to the law of motion

$$\mathcal{L}^{\star}_t = \mathcal{L}_t - b\mathcal{L}_{t-1}.$$

This form of the utility function – although simpler – is very similar to that considered by Kydland and Precott (1982)[17]. The main difference with Kydland and Precott is the sign of b. Kydland and Prescott require that b be strictly negative, implying that current and future leisure choices are intertemporally substitutable. We do not a priori impose this restriction and let the data select b.

The representative firm uses capital K_t and labor N_t to produce the homogeneous final good Y_t. The technology is represented by the following constant returns–to–scale Cobb–Douglas production function

$$Y_t = K_t^{1-\alpha} \left(Z_t N_t\right)^{\alpha},$$

where $\alpha \in (0,1)$. Z_t is assumed to follow an exogenous process of the form

$$\log(Z_t) = \gamma_z + \log(Z_{t-1}) + \sigma_z \varepsilon_{z,t},$$

where $\sigma_z > 0$ and $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The constant γ_z is a drift term in the random walk process of Z_t. The capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta) K_t + I_t,$$

[17]We consider only one lag whereas Kydland and Prescott assume that habits in leisure gradually react to past leisure choices.

where $\delta \in (0, 1)$ is the constant depreciation rate. Finally, the final good can be either consumed or invested

$$Y_t = C_t + I_t.$$

We first apply a stationary–inducing transformation for variables that follow a stochastic trend. Output, consumption and investment are divided by Z_t, and the capital stock is divided by Z_{t-1}. The approximate solution of the model is computed from a log–linearization of the stationary equilibrium conditions around the deterministic steady state using the numerical algorithm of Anderson and Moore (1985).

3 Econometric Methodology

To estimate and evaluate the RBC model, we resort to Indirect Inference. In doing so, we depart from a large strand of the literature that employs a limited information MDE strategy. The idea of the MDE strategy is to estimate the structural parameters so that the impulse responses of a DSGE model directly match as closely as possible the impulse responses from a SVAR model estimated on actual data. We do not employ this empirical strategy as we have shown in the introductory example of subsection 1.2 that the responses estimated from the DSVAR specification can be severely downward-biased. Additionally, Chari, Kehoe, and McGrattan (2004b) show that the DSVAR specification has trouble recovering the true responses of hours to a technology shock when a DSGE model with capital accumulation is used as the DGP. In contrast, the principle of Indirect Inference is to use an auxiliary criterion (or an auxiliary model) in order to estimate the DSGE model's parameters. Rather than directly estimating the model using the theoretical impulse responses, we estimate a DSVAR model on simulated data from the DSGE model and compute the responses of hours using long–run restrictions. The responses, averaged over simulations, are compared to those obtained from actual data. The structural parameters are then estimated in order to make the discrepancy between the two responses as small as possible.

In order to present our econometric approach, we first consider the following VAR(p) model

$$\mathbf{z}_t = \mathbf{A}_1 \mathbf{z}_{t-1} + \cdots + \mathbf{A}_p \mathbf{z}_{t-p} + \boldsymbol{\varepsilon}_t, \qquad \mathrm{E}\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' = \boldsymbol{\Sigma}, \tag{6}$$

with $\mathbf{z}_t = (\Delta x_t, \Delta n_t)'$, where n_t is logged total hours worked per capita and x_t is logged labor productivity. For the sake of simplifying the exposition, we abstract from constant terms in the above VAR model. This specification with hours worked in first difference is used by Galí (1999, 2004a, 2004b), Galí and Rabanal (2004), and Francis and Ramey (2004, 2005). We follow Galí and Rabanal (2004) and assume that $p = 4$.

3.1 Identification of Impulse Responses

Let us define $\mathbf{B}(L) = (\mathbf{I}_2 - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p)^{-1}$, so that

$$\mathbf{z}_t = \mathbf{B}(L)\,\boldsymbol{\varepsilon}_t,$$

where $\mathbf{I}_2$ is the identity matrix. Now, we assume that the canonical innovations are linear combinations of the structural shocks $\boldsymbol{\eta}_t$, i.e. $\boldsymbol{\varepsilon}_t = \mathbf{S}\boldsymbol{\eta}_t$, for some non singular matrix $\mathbf{S}$. As usual, we impose an orthogonality assumption on the structural shocks which, together with a scale normalization, implies $\mathrm{E}\boldsymbol{\eta}_t\boldsymbol{\eta}_t' = \mathbf{I}_2$. This gives us three constraints out of the four needed to completely identify $\mathbf{S}$.

To setup the last identifying constraint, let us define $\mathbf{C}(L) = \mathbf{B}(L)\,\mathbf{S}$. Given the ordering of $\mathbf{z}_t$, we simply require that $\mathbf{C}(1)$ be lower triangular, so that only technology shocks can affect the long-run level of labor productivity. This amounts to imposing that $\mathbf{C}(1)$ is the Cholesky factor of $\mathbf{B}(1)\,\boldsymbol{\Sigma}\mathbf{B}(1)'$. Given consistent estimates of $\mathbf{B}(1)$ and $\boldsymbol{\Sigma}$, we easily obtain an estimate for $\mathbf{C}(1)$. Retrieving $\mathbf{S}$ is then a simple task using the formula $\mathbf{S} = \mathbf{B}(1)^{-1}\,\mathbf{C}(1)$. The impulse responses are then deduced from the VMA(∞) representation

$$\mathbf{z}_t = \mathbf{B}(L)\,\mathbf{B}(1)^{-1}\,\mathbf{C}(1)\,\boldsymbol{\eta}_t \tag{7}$$

with $\boldsymbol{\eta}_t = (\boldsymbol{\eta}_{1,t}, \boldsymbol{\eta}_{2,t})'$, where $\boldsymbol{\eta}_{1,t}$ is the identified technology shock, whereas $\boldsymbol{\eta}_{2,t}$ is the non–technology one. The standard–errors of the IRFs are computed numerically using the δ-function method.[18]

3.2 Estimation Method

This section presents the econometric methodology. We partition the model parameters $\boldsymbol{\theta}$ into two groups $\boldsymbol{\theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2\}$.

[18] See appendix B for further details relating to the computation of the impulse response functions as well as their standard errors.

The first group, denoted $\boldsymbol{\theta}_1$, is composed of $\bar{\chi}$, γ_z, β, α, and δ, which are calibrated prior to estimation. The parameter $\bar{\chi}$ is pinned down so that the steady state labor supply amounts to one third of the time endowment. The growth rate of Z_t, γ_z, is equal to 0.0036. We set $\beta = 1.03^{-0.25}$, which implies a steady state annualized real interest rate of 3 percent. We set $\alpha = 0.60$, that implies a steady state capital's share in output equal to 40%, as in Cooley and Prescott (1995). Finally, we set $\delta = 0.025$, which implies an annual rate of depreciation of capital equal to 10 percent.

Given these a priori values, we first check that the impulse responses of hours to a technology shock are positive. We conduct this exercise for several alternative values of b, since we have no prior on this parameter. The results of this sensitivity analysis are reported on figure 3, where we focus on normalized technology shocks (one percent shocks). This graphic clearly shows that hours never respond negatively to such a shock. When $b < 0$, as in Kydland and Prescott (1982), agents are willing to substitute intertemporally leisure after a technology shock, so as to exploit the increase in productivity. This implies a substantial response of hours on impact, though a smaller persistence. To the contrary, when $b > 0$, the labor supply is complementary in adjacent periods, due to habit formation. In this case, agents are less willing to increase their labor supply, conducing to a smaller response of hours on impact and a gradual increase of hours over time.

The second group of model parameters is $\boldsymbol{\theta}_2 = \{b, \sigma_z, \rho_\chi, \sigma_\chi\}$. These four parameters are estimated using Indirect Inference. The empirical IRFs of hours computed from eq. (7), are used as an auxiliary criterion to estimate these structural parameters. The DSVAR model is thus considered as an auxiliary model, that allows us to identify and estimate $\boldsymbol{\theta}_2$ through simulations. More precisely, we consider in a first step the impulse responses of hours to a technology shock $\partial n_{t+k}/\partial \boldsymbol{\eta}_{1,t}$ and a non–technology shock $\partial n_{t+k}/\partial \boldsymbol{\eta}_{2,t}$, deduced from (7) for $k = 1, ..., h$ where h is the selected horizon.[19] There are two reasons to justify this choice. First, in doing so, we make sure that the model is estimated in order to match hours fluctuations generated by these two shocks. Second, this allows us to overcome possible identification failures. To see this, assume that we only focus on the responses of hours to a technology shock. In this case, if the DSVAR model does a good job of identifying technology shocks, the parameters ρ_χ and σ_χ cannot be estimated on this basis.

[19] In a complementary exercise, we also estimate and test the model using as an indirect criterion the responses of output and hours to both types of shocks estimated from this DSVAR.

The estimation method is implemented as follows.

Step 1: We estimate a q-dimensional vector of IRFs, denoted $\widehat{\boldsymbol{\psi}}_T$, from actual data, where q denotes the number of selected impulse responses (horizon $\times$ number of selected IRFs).

Step 2: From the model' solution, and given the vector of structural parameters, $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$, and initial conditions on capital, labor and the shocks, S simulated paths for productivity and employment, denoted $\widetilde{x}^i_T(\boldsymbol{\theta})$, $\widetilde{n}^i_T(\boldsymbol{\theta})$, $i = 1, \cdots, S$, are recursively computed.

Step 3: From these simulations, we estimate a VAR model from the simulated data $\widetilde{\mathbf{z}}^i_t = (\Delta\widetilde{x}^i_t, \Delta\widetilde{n}^i_t)'$,

$$\widetilde{\mathbf{z}}^i_t = \widetilde{\mathbf{A}}^i_1 \widetilde{\mathbf{z}}^i_{t-1} + \cdots + \widetilde{\mathbf{A}}^i_p \widetilde{\mathbf{z}}^i_{t-p} + \widetilde{\boldsymbol{\varepsilon}}^i_t, \qquad \mathrm{E}\tilde{\boldsymbol{\varepsilon}}^i_t \tilde{\boldsymbol{\varepsilon}}^{i\prime}_t = \widetilde{\boldsymbol{\Sigma}}^i, \qquad i = 1, \cdots, S,$$

with the same number of lags ($p = 4$) as in the DSVAR on actual data from step 1. We then compute the associated vector of IRFs, denoted $\tilde{\boldsymbol{\psi}}^i_T(\boldsymbol{\theta})$ ($i = 1, \cdots, S$) using the exact same long–run restrictions as in step 1

$$\widetilde{\mathbf{z}}^i_t = \widetilde{\mathbf{B}}^i(L) \, \widetilde{\mathbf{B}}^i(1)^{-1} \, \widetilde{\mathbf{C}}^i(1) \, \widetilde{\boldsymbol{\eta}}^i_t,$$

and we construct their average over simulations

$$\tilde{\boldsymbol{\psi}}_{T,S}(\boldsymbol{\theta}) = \frac{1}{S} \sum_{i=1}^{S} \tilde{\boldsymbol{\psi}}^i_T(\boldsymbol{\theta}).$$

Step 4: An Indirect Inference estimate $\hat{\boldsymbol{\theta}}_{2,T,S}$ for $\boldsymbol{\theta}_2$ minimizes the quadratic form

$$J(\boldsymbol{\theta}_2) = \mathbf{g}'_{T,S} \mathbf{W}_T \, \mathbf{g}_{T,S},$$

where $\mathbf{g}_{T,S} = (\widehat{\boldsymbol{\psi}}_T - \widetilde{\boldsymbol{\psi}}_{T,S}(\boldsymbol{\theta}_2))$ and $\mathbf{W}_T$ is a symmetric nonnegative matrix defining the metric.

Steps 2 to 4 are conducted repeatedly until convergence — *i.e.* until a value of $\boldsymbol{\theta}_2$ that minimizes the objective function is obtained. Let $\boldsymbol{\psi}_0$ denote the pseudo–true value of $\boldsymbol{\psi}$ and $\boldsymbol{\theta}_{2,0}$ the pseudo–true value of $\boldsymbol{\theta}_2$. Under standard regularity conditions,[20] for S held fixed and as T goes to infinity, $\sqrt{T}(\hat{\boldsymbol{\theta}}_{2,T,S} - \boldsymbol{\theta}_{2,0})$ is asymptotically normally distributed, with a covariance matrix equal to $(1 + S^{-1}) \, (\mathbf{D}'_\theta \mathbf{W}_T \mathbf{D}_\theta)^{-1}$ where $\mathbf{D}_\theta = \partial \mathbf{g}_{T,S} / \partial \boldsymbol{\theta}_2$.

[20]See Gouriéroux, Monfort, and Renault (1993).

A preliminary consistent estimates of the weighting matrix $\mathbf{W}_T$ is required for the computation of $\hat{\boldsymbol{\theta}}_{2,T,S}$. It may be directly based on actual data, and corresponds to the inverse of the covariance matrix of $\sqrt{T}(\widehat{\boldsymbol{\psi}}_T - \boldsymbol{\psi}_0)$, which is obtained from step 1. Here, $\mathbf{W}_T^{-1}$ is a diagonal matrix with the sample variances of $\sqrt{T}(\widehat{\boldsymbol{\psi}}_T - \boldsymbol{\psi}_0)$ along the diagonal. This choice of weighting matrix ensures that $\boldsymbol{\theta}_2$ is effectively chosen so that $\widetilde{\boldsymbol{\psi}}_{T,S}(\boldsymbol{\theta}_2)$ lies as much as possible inside the confidence intervals of $\widehat{\boldsymbol{\psi}}_T$.

For the sake of identification, we impose that the number of IRFs exceeds the number of structural parameters. This enables us to conduct a global specification test in the lines of Hansen (1982), denoted $J - stat = TSJ(\boldsymbol{\theta}_2)/(1 + S)$, which is asymptotically distributed as a chi–square, with degrees of freedom equal to the number of over–identifying restrictions ($q - \dim \boldsymbol{\theta}_2$).

4 Empirical Results

In this section, we present the empirical results obtained with US data. We first document the data and discuss the impulse responses of hours to technology and non-technology shocks. Second, we present the estimation results of the structural parameters using Indirect Inference on the DSVAR specification. Finally, we investigate the ability of the RBC model to encompass LSVARs and DSVARs.

4.1 Data and the Responses of Hours

We first present results based on a simple bivariate DSVAR $(\Delta\hat{x}_t, \Delta\hat{n}_t)'$. As in Christiano, Eichenbaum and Vigfusson (2004), we use alternative measures (in logs) of productivity and hours worked: *(i)* non–farm business output divided by non–farm business hours worked, non–farm business hours worked divided by civilian population over the age of 16 (NFB sector hereafter); *(ii)* business output divided by business hours worked, business hours worked divided by civilian population over the age of 16 (B sector); *(iii)* real GDP divided by non-farm business hours worked, non-farm business hours worked divided by civilian population over the age of 16 (mixed NFB sector), and *(iv)* real GDP divided by total business hours, business hours worked divided by civilian population over the age of

16 (mixed B sector). The empirical analysis uses quarterly U.S. data for the period 1948:1-2002:4. The dataset *(i)* is that used by Galí and Rabanal (2004) while the dataset *(ii)* corresponds to the benchmark dataset of Christiano, Eichenbaum and Vigfusson (2004).

The response of hours worked to a technology shock for each measure of productivity and hours are qualitatively similar (see figure 4). For instance, as in Galí (1999), hours worked decrease significantly on impact. Moreover, the negative effect appears rather persistent. In the case of NFB sector data, as noticed by Galí and Rabanal (2004), hours do eventually return to their original level. Conversely, the response of hours is persistently below zero for the considered horizon for datasets *(ii)*, *(iii)* and *(iv)*. Another noticeable difference concerns the confidence intervals. For measures *(i)* and *(ii)* (NFB sector and B sector), the negative response is not significantly different from zero after two periods, whereas the negative response remains significant at any horizon for measures *(iii)* and *(iv)*. The impulse responses of hours to the non–technology shock[21] is persistent and hump–shaped. Moreover, the response of hours is precisely estimated for each horizon.

4.2 Estimation Results from DSVARs

Apparently, the previous evidence does not support the empirical relevance of standard RBC models, as they cannot reproduce a persistent and negative response of hours after a transitory technology shock. We now investigate this issue using the econometric methodology discussed previously. Table 1 reports the estimation results in four cases. Each case is associated to a particular measure of productivity and hours worked. In each situation, we use a bivariate VAR with a first difference specification and four lags. $S = 30$ simulations were used for a sample size equal to 219 quarters. Simulated values are redrawn from the same seed values for each function evaluation. In order to reduce the effect of initial conditions, the simulated samples include 200 initial points which are subsequently discarded in the estimation. The minimization of the simulated criterion function is carried out using the sequential dynamic programming algorithm provided by the MATLAB Optimization Toolboox. We first estimate the four structural parameters $\theta_2 = (b, \sigma_z, \rho_\chi, \sigma_\chi)$ using the responses of hours to technology and non–technology shock for a horizon equal to 21. We hereby introduce

[21]More precisely, figure 4 reports the responses of hours to a shock without long run effect on labor productivity. Having in mind our DSGE model, this shock can be viewed as a preference shock that reduces output and hours and increases labor productivity. In the SVAR literature, this shock is usually interpreted as a negative demand shock.

$2 \times 21 - 4 \equiv 38$ over–identifying restrictions. Our estimation results are reported on table 1.

We first discuss the parameters estimates. The labor supply parameter b is significantly positive, indicating that labor supply is subject to intertemporal complementarities. This result is in accordance with previous results of Eichenbaum, Hansen and Singleton (1988), Bover (1991), and Wen (1998). The estimated value is greater than 0.8 for each dataset and very close to the estimated values in Eichenbaum, Hansen and Singleton (see Table II, p. 65). Our results clearly suggest that today's leisure significantly reduces leisure services in the subsequent time period. Notice that the estimated value is very similar in each case, *i.e.* the same degree of habit persistence in leisure habit allows us to match different datasets. The estimated value of the standard–error of the technology shock σ_z is rather large (0.0256 and 0.0228), when Indirect Inference with DSVARs as auxiliary models is applied on NFB or B sector data. This partly results from the high volatility of productivity and hours in these two sectors. However the point estimates are not significantly different from what would obtain in simple growth accounting exercises.[22] When the data are mixed, the estimated value is similar to those obtained in previous studies (0.0126 and 0.0137).[23] Relatively high values of σ_z can be explained by the estimated value of b. When b is positive and large, the responses of hours to any shock are small on impact and increase gradually with the horizon. This is a direct consequence of habit in leisure choices that tends to smooth labor supply. The estimated value of the autoregressive parameter ρ_χ is not large, especially if we compare it to previous estimates. Our estimations suggest values between 0.65 and 0.70, whereas Chari, Kehoe and McGrattan report estimated values between 0.94 and 0.97.[24] These differing figures result partly from the specification of the utility function. In Chari, Kehoe and McGrattan, the utility function is time separable, so most of the persistence in the fluctuations of hours worked is the result of the forcing variable, which consequently requires a high degree of serial correlation. Conversely, when $b > 0$ in our Kydland-Prescott type model, hours can respond more persistently to any transitory shock. This explains that a large value of ρ_χ is no longer necessary in order to match the persistence of hours. Finally, the estimated value of σ_χ (between 0.025 and 0.034) are similar to what Erceg, Guerrieri and Gust (2004) obtain for their composite

[22] In this type of calculation, we first use the calibrated values of δ and a series of aggregate investment (private investment plus durable goods expenditures) to construct a capital stock series assuming an initial capital-output ratio of 6. In a second step, using the calibrated value of α, we determine the Solow residual from observed output and hours and the calculated capital stock. For NFB and B sectors, we obtain, respectively, $\sigma_z = 0.0169$ and $\sigma_z = 0.0162$.

[23] See, for example, Hansen (1997).

[24] Erceg, Guerrieri and Gust (2004) calibrate ρ_χ to 0.95.

fiscal and preference shock.

Table 1 reports the global specification test statistic $(J - stat)$. For each dataset, the model is not globally rejected by the data, since the p-values associated to the J-stat are large. One may argue that the standard–errors of the responses of hours to a technology shock are so large that the RBC model can easily match the data. For example, in the case of NFB and B sectors datasets, the response of hours is not significantly different from zero after 2 periods. But in the case of mixed datasets, the response of hours is significantly different from zero for each horizon and the RBC model still matches the data very well. Moreover, the four structural parameters are estimated in order to match simultaneously the responses of hours to technology and non–technology shocks. Since the latter are very precisely estimated, any departure from the empirical response is highly penalized in the objective function.

Figures 5–8 report the IRFs of hours to technology and non–technology shocks under actual data and the model. These figures also include the true response of hours in the RBC model. As these figures show, the response of hours to a technology shock is always positive. Note that the RBC model is able to reproduce a hump–shaped positive response of hours, as the maximal response is obtained after ten periods. The implied response of hours from the DSVAR estimated on simulated data is negative and does not display a hump–shaped profile. These figures also illustrate the downward bias implied by the DSVAR specification. Another interesting quantitative feature of the RBC model is its ability to display a persistent response of hours to a non–technology shock.

Additionally, we report in Table 1 the variance decomposition for the business cycle components of output and hours after HP filtering the series simulated from the estimated RBC model. In each case, the fraction of the variance of output explained by the technology shock is always close to or higher than 70%. For instance, it exceeds 85% with NFB sector data. These results are in sharp contrast with the predictions drawn from the DSVAR estimated on actual data. In this specification, the variance of output explained by technology shock is 7%, and it is 5% for hours. The same result qualitatively applies to B sector data. In this case, the DSVAR attributes a mere 10% of output variance to technology shocks while the RBC model estimated on this same DSVAR model attributes 84% of output variance to these shocks. These very contrasted results suggest that care should be taken when interpreting results from DSVAR models. It is worth noting that the preference shock explains most of hours fluctuations (between 75% and 90% of the variance). This is in accordance with the business cycle accounting exercise of Chari, Kehoe and McGrattan (2004a). Galí and Rabanal (2004) also compute the correlation between the business cycle components of output and hours conditional on technology shocks only, as implied by their DSVAR model. This correlation is very small (-0.08), whereas it is much larger when all the shocks are taken into account (0.88). They conclude from this exercise that technology shocks cannot be the main source of business cycle fluctuations. Once again, our results contradict their findings, since the estimated RBC model, consistent with their DSVAR model, implies a correlation between output and hours at business cycle frequencies of 83% conditional on technology shocks only.

Finally, one may argue that our results are derived so as to reproduce the dynamic behavior of hours only, and thus compel us to remain silent on the dynamic behavior of output. To answer this legitimate concern, we also estimate the RBC model using IRFs of output and hours to both types of shocks, as implied by the DSVAR of productivity growth and hours growth. As in the previous estimation, we select an IRF horizon equal to 21, hereby introducing $4 \times 21 - 4 \equiv 80$ over-identifying restrictions. The impulse response functions are reported on figure 9. On the the right column of this figure, the empirical responses of hours (plain lines) are identical to those previously reported. In contrast, the simulated and theoretical IRFs differ, since the model has been reestimated. The left column contains the responses of output to technology shocks (top panel) and non technology shocks (bottom panel). As expected, output increases permanently in response to technology shocks. The non technology shock, interpreted as a negative demand shock, induces a persistent decline in output. Since the long-run restrictions in the DSVAR specification only apply to labor productivity (as opposed to output) and since hours decline permanently in response to a non technology shock, the response of output to this same shock is also characterized by a negative long-run effect. Notice that the response of output to this shock is very precisely estimated.

The estimation results are reported in the last column of table 1. All the estimated parameters are found significant. Though still implying a strong intertemporal complementarity of the labor supply, the habit parameter b is somewhat lower than previously estimated. Conversely, the persistence of the preference shock increases significantly. Our results suggest lower values for the standard errors of the structural shocks. Notice also that in this case, the value of σ_z is consistent with that implied by a simple growth accounting exercise.[25] The over-identification test statistic does not reject the model, with a p-value of rougly 88%. Notice that this result was not a priori warranted since the objects to be matched (IRFs to non technology shocks) are very precisely estimated, with narrow confidence bands. The DSVAR estimated on simulated data from the RBC model closely matches the DSVAR estimated from actual data. In contrast, the true RBC IRFs differ significantly from those implied by the DSVAR model. In particular, the response of hours, while positive in the RBC model, is found persistently negative in the DSVAR model. Using this estimation, we compute once again the variance of the business cycle components of output and hours explained by the technology shock. We find that the latter accounts for roughly 82.4% of output and 25.7% of hours, in sharp contrast with the DSVAR results. Finally, we obtain that the conditional correlation between output and hours is very large.

4.3 Estimation Results from SVARs

The estimation of impulse responses of hours critically depends on the SVAR specification. Christiano, Eichenbaum and Vigfusson (2004) argue that a difference specification of hours may create severe distortions in the DSVAR specification if hours are truly stationary. Using an LSVAR specification, they obtain a positive and hump–shaped response of hours following a technology shock, though not precisely estimated (see Chari, Kehoe and Mac Grattan, 2004b). We report in Figure 10, the responses of hours to technology and non–technology shocks in the LSVAR specification with NFB sector data. The response to technology shocks is always positive, hump-shaped, but not significantly different from zero at each horizon. In contrast, the response of hours to a non–technology shock is persistent and significant. Notice that in the short-run, there is little difference between IRFs of hours to a non-technology shock from DSVAR (see figure 4) or LSVAR specifications (see figure 10).

We now investigate the ability of the structural model to replicate such patterns with NFB sector data. To do this, we conduct three experiments. In the first one, we compute the IRFs of hours in an LSVAR specification using the estimations of Table 1 (column NFB sector). This counterfactual experiment allows us to quantify the ability of the RBC model estimated from a DSVAR to replicate

[25] See footnote 22, as well as Erceg, Guerrieri, and Gust (2004).

the impulse responses obtained from the LSVAR specification. This exercise can be viewed as an indirect assessment of the ability of a DSVAR specification to encompass an LSVAR specification. In the second experiment, we estimate the RBC model using the responses of hours to technology and non–technology shocks obtained in an LSVAR specification. We also compute the IRF of hours in a DSVAR specification under the estimated RBC model. Finally, in a third experiment, we estimate the four structural parameters in order to match simultaneously the impulse responses of hours in DSVAR and LSVAR specifications.

Table 2 reports the empirical results. The first column is identical to that of table 1. Given these parameters estimates, we simulate the RBC model and estimate an LSVAR specification on these simulated data so as to identify the associated IRFs. Figure 11 reports the IRFs from the actual data, from simulated data as well as the true IRF from the RBC model. The left column of figure 11 is the same as that of figure 5. The right column of figure 11 allows us to assess the ability of the RBC model estimated from the DSVAR auxiliary criterion to reproduce IRFs estimated in the LSVAR specification on actual data. As this figure makes clear, the RBC model matches well the responses of hours. More precisely, the responses of hours from simulations display a similar hump–shaped pattern to those obtained from the actual data. To test the match between the two IRFs, we compute the following statistic Q_i for various horizons

$$Q_i = \left(\widehat{\psi}_{[1:i],T} - \widetilde{\psi}_{[1:i],T,S}(\widetilde{\theta}_{2,T,S})\right)' \Omega_{[1:i],T} \left(\widehat{\psi}_{[1:i],T} - \widetilde{\psi}_{[1:i],T,S}(\widetilde{\theta}_{2,T,S})\right),$$

where $\widehat{\psi}_{[1:i],T}$ are the IRFs of hours deduced from an LSVAR specification on actual data, $\widetilde{\psi}_{[1:i],T,S}(\widetilde{\theta}_{2,T,S})$ is $\widehat{\psi}_{[1:i],T}$'s simulated counterpart, and $\Omega_{[1:i],T}$ is the inverse of the covariance matrix of $\widehat{\psi}_{[1:i],T}$. This simple test (see the Q_i, $i = 6, 11, 21$, statistic in Table 2) shows that the RBC model, estimated using the DSVAR model as an auxiliary criterion, generates responses of hours that are not significantly different from those obtained under the empirical LSVAR model at horizons 6 and 11 (see the P-values of Q_6 and Q_{11}). However, for a longer horizon, the RBC model has trouble reproducing the response of hours to a non–technology shock (see Q_{21} in the table).

We now investigate whether the model is able to match the responses of hours in an LSVAR specification using the Indirect Inference approach. The second column of Table 2 reports the parameter estimates. Note that the standard errors of the two shocks are larger than those of the first column.

The global specification tests indicates that the RBC model easily matches the responses of hours. Given the large confidence interval of the response to a technology shock (see the left column of figure 12), this is not surprising. Moreover, the LSVAR specification is a priori in accordance with the RBC model, since hours increase after a technology shock both in the LSVAR model and in the RBC model. However, the response to a non–technology shock is very precisely estimated, making any departure from it very penalizing in the objective function. Using these estimated values, we now compute the responses of hours using a DSVAR specification applied on simulated paths from the RBC model. The right column of figure 12 reports the impulse responses from the DSVAR model. The responses from simulated data depart significantly from those of actual data. Indeed, the response of hours to a technology shock is zero on impact and becomes persistently positive. Moreover, the response to a non–technology shock does not display the hump–shaped profile seen in the empirical responses. The Q_i statistic (see the second column of table 2) indicates that the model estimated from an LSVAR specification fails to reproduce a DSVAR specification. This result, together with the previous ones, show that the DSVAR specification, although providing biased responses of hours, indirectly encompasses (i.e. through the RBC model) the LSVAR specification, while the converse is not true. This result suggests that results obtained from the SVAR approach should be taken cautiously.

Finally, we estimate the structural model using the two SVAR specifications as auxiliary models for indirect estimation, *i.e.* the responses of hours to technology and non–technology shocks with hours in difference and in level. The J-statistic in the third column of Table 2 shows that the RBC model is able to match very well the responses of hours. In the DSVAR specification, the response under the model is negative, whereas it is persistently positive in the LSVAR specification. These results show that a simple RBC model in which hours increase after a technology shock easily encompasses SVAR models with contradictory results.

5 Concluding Remarks

The identification of the response of hours worked after a technology shock using SVAR has renewed the debate on the relative contributions of various shocks to the business cycle. More precisely, the

DSVAR approach documents a striking evidence against the standard RBC model: after a positive technology shock, hours worked decrease. For researchers that use the SVAR approach, this evidence suggests to abandon the RBC model in favor of models with important (real) frictions and (nominal) rigidities.

This paper shows that DSVAR specifications poorly identify the impulse responses of hours and suggests another way to evaluate DSGE model. Using an indirect approach (Indirect Inference), we show that a Kydland–Prescott type model matches indirectly very well impulse responses of DSVAR. Moreover, the estimated technology shock accounts for a large part of output fluctuations at business cycle frequencies. Finally, the proposed RBC model encompasses both the LSVAR and DSVAR models.

References

Altig, D., Christiano, L., Eichenbaum, M. and J. Lindé (2005) “Firm-Specific Capital, Nominal Rigidities and the Business Cycle”, *mimeo* Northwestern University.

Anderson, G. and G. Moore (1985) “A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models”, *Economic Letters*, 17, pp. 247–252.

Beaudry, P. and A. Guay (1996) “What Do Interest Rates Reveal of the Function of Real Business Cycle Model?” , *Journal of Economic Dynamics and Control*, 20, pp. 1661–1682.

Blanchard, O.J. and D. Quah (1989) “The Dynamic Effects of Aggregate Demand and Supply Disturbances” , *American Economic Review*, 79(4), pp. 655-673.

Boldrin, M., Christiano, L. and J. Fisher (2001) “Habit Persistence, Asset Returns and the Business Cycle” , *American Economic Review*, 91, pp. 149–166.

Bover, O. (1991) “Relaxing intertemporal separability: a Rational Habit Model of Labor Supply Estimated from Panel Data”, *Journal of Labor Economics*, 9, pp. 85–100.

Chari, V., Kehoe, P. and E. Mc Grattan (2004a) “Business Cycle Accounting”, Federal Reserve Bank of Minneapolis, Research Department, Staff Report 328.

Chari, V., Kehoe, P. and E. Mc Grattan (2004b) “A Critique of Structural VARs Using Real Business Cycle Theory”, Federal Reserve Bank of Minneapolis, Research Department, Working Paper 631.

Christiano, L., Eichenbaum, M. and R. Vigfusson (2004) “What Happens after a Technology Shock”, NBER Working Paper Number 9819, *revised version 2004*.

Christiano, L., Eichenbaum, M. and C. Evans (2004) “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *forthcoming Journal of Political Economy*.

Cooley, T. and M. Dwyer (1998) "Business Cycle Analysis without Much Theory: A Look at Structural VARs", *Journal of Econometrics*, 83(1–2), pp. 57–88.

Cooley, T., and Prescott, E., (1995). "Economic Growth and Business Cycles", in Cooley T. (Ed.) Frontiers of Business Cycle Research, Princeton University Press, Princeton.

Erceg, C., Guerrieri, L. and C. Gust (2004) "Can Long–Run Restrictions Identify Technology Shocks", Board of Governors of the Federal Reserve System, International Finance Discussion paper, Number 792.

Eichenbaum M., Hansen L. and K. Singleton (1988) "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty", *Quarterly Journal of Economics*, 103(1), pp. 51–78.

Francis, N. and V. Ramey (2004) "Is the Technology–Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited", University of California, San Diego, *mimeo*, *forthcoming Journal of Monetary Economics.*

Francis, N. and V. Ramey (2005) "The source of Historical Economic Fluctuations: an Analysis Using Long–Run Restrictions", *forthcoming Journal of the European Economic Association.*

Galí, J. (1999) "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?", *American Economic Review*, 89(1), pp. 249–271.

Galí, J. (2004a) "On the Role of Technology Shocks as a Source of Business Cycles: Some New Evidence", *Journal of European Economic Association*, 2(2–3), pp. 372–380.

Galí, J. (2004b) "Trends in Hours Worked and the Role of Technology in the Business Cycle Theory: Theory and International Evidence", prepared for the 29th Economic Policy Conference on *Productivity, Labor and the Business Cycle*, Federal Reserve Bank of St. Louis, October 20–21, 2004.

Galí, J., and P. Rabanal (2004) "Technology Shocks and Aggregate Fluctuations; How Well does the

RBC Model Fit Postwar U.S. Data?", *forthcoming NBER Macroeconomics Annual.*

Gourieroux, C., Monfort, A. and E. Renault (1993) "Indirect Inference", *Journal of Applied Econometrics*, Supplement.

Gourieroux, C. and A. Monfort (1996) "Simulation Based Inference Methods", *Cambridge University Press.*

Hall, R. (1997) "Macroeconomic Fluctuations and the Allocation of Time", *Journal of Labor Economics*, 15(1), pp. 223–250.

Hamilton, J. (1994) *Time Series Analysis*, Princeton University Press.

Hansen, L. (1982) "Large Sample Properties of Generalized Method of Moments", *Econometrica*, 50, pp. 1029–1054.

Hansen, G. (1985) "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics*, 16, pp. 309–327.

Hansen, G. (1997). "Technical Progress and Aggregate Fluctuations", *Journal of Economic Dynamics and Control*, 21, pp.1005-1023.

McGrattan (2004) "Comment on Galí and Rabanal' "Technology Shocks and Aggregate Fluctuations; How Well does the RBC Model Fit Postwar U.S. Data?", *forthcoming NBER Macroeconomics Annual.*

Prescott, E., (1986). "Theory Ahead of Business Cycle Measurement", *Quarterly Review*, 10, pp. 9-22, Ferderal Reserve Bank of Minneapolis.

Rotemberg, J. and M. Woodford (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", *National Bureau of Economic Research Macroeconomics Annual 1997*, S. Bernanke and J. Rotemberg eds, Cambridge, MA, *MIT Press.*

Wen, Y. (1998) "Can a Real Business Cycle Model Pass the Watson Test?", *Journal of Monetary Economics*, 42, pp. 185–203.

Appendix

A Proof of Proposition 1

We consider the estimation of a VAR(1) model with data generated by the structural model of section 1 (see equations (3)–(5)). The VAR(1) model has the form:

$$\begin{pmatrix} \Delta x_t \\ \Delta n_t \end{pmatrix} = \mathbf{A}_1 \begin{pmatrix} \Delta x_{t-1} \\ \Delta n_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \quad \mathbf{A}_1 = \begin{pmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{pmatrix}.$$

The OLS regression from the first equation yields:

$$\begin{pmatrix} \widehat{\boldsymbol{a}}_{11} \\ \widehat{\boldsymbol{a}}_{12} \end{pmatrix} = \begin{pmatrix} V(\Delta x_t) & Cov(\Delta x_t, \Delta n_t) \\ Cov(\Delta x_t, \Delta n_t) & V(\Delta n_t) \end{pmatrix}^{-1} \begin{pmatrix} Cov(\Delta x_t, \Delta x_{t-1}) \\ Cov(\Delta x_t, \Delta n_{t-1}) \end{pmatrix}$$

whereas the OLS regression from the second equation yields

$$\begin{pmatrix} \widehat{\boldsymbol{a}}_{21} \\ \widehat{\boldsymbol{a}}_{22} \end{pmatrix} = \begin{pmatrix} V(\Delta x_t) & Cov(\Delta x_t, \Delta n_t) \\ Cov(\Delta x_t, \Delta n_t) & V(\Delta n_t) \end{pmatrix}^{-1} \begin{pmatrix} Cov(\Delta n_t, \Delta x_{t-1}) \\ Cov(\Delta n_t, \Delta n_{t-1}) \end{pmatrix}$$

The variances and covariances that enter in the $\mathbf{A}_1$ matrix are given by $V(\Delta x_t) = \sigma_z^2 + 2\sigma_\chi^2(1-\alpha)^2/(1+\rho_\chi)$, $V(\Delta n_t) = 2\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta n_t) = -2(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta x_{t-1}) = -(1-\rho_\chi)(1-\alpha)^2\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta n_{t-1}) = (1-\rho_\chi)(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta n_t, \Delta x_{t-1}) = (1-\rho_\chi)(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$ and $Cov(\Delta n_t, \Delta n_{t-1}) = -(1-\rho_\chi)\sigma_\chi^2/(1+\rho_\chi)$. The OLS estimator $\widehat{\mathbf{A}}_1$ of $\mathbf{A}_1$ is then deduced

$$\widehat{\mathbf{A}}_1 = \begin{pmatrix} 0 & \frac{(1-\rho_\chi)(1-\alpha)}{2} \\ 0 & -\frac{1-\rho_\chi}{2} \end{pmatrix}$$

The residuals of each equation are given by

$$\begin{aligned} \varepsilon_{1,t} &= \Delta x_t - \frac{(1-\rho_\chi)(1-\alpha)}{2}\Delta n_{t-1} \\ \varepsilon_{2,t} &= \Delta n_t + \frac{1-\rho_\chi}{2}\Delta n_{t-1} \end{aligned}$$

and the associated covariance matrix is

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_z^2 + \frac{(3-\rho_\chi)(1-\alpha^2)}{2}\sigma_\chi^2 & -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 \\ -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 & \frac{3-\rho_\chi}{2}\sigma_\chi^2 \end{pmatrix}$$

We thus compute the long–run covariance matrix

$$\begin{aligned} \left(\left(\mathbf{I}_2 - \widehat{\mathbf{A}}_1\right)^{-1}\right)\boldsymbol{\Sigma}\left(\left(\mathbf{I}_2 - \widehat{\mathbf{A}}_1\right)^{-1}\right)' &= \begin{pmatrix} 1 & \frac{(1-\rho_\chi)(1-\alpha)}{3-\rho_\chi} \\ 0 & \frac{2}{3-\rho_\chi} \end{pmatrix} \begin{pmatrix} \sigma_z^2 + \frac{(3-\rho_\chi)(1-\alpha^2)}{2}\sigma_\chi^2 & -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 \\ -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 & \frac{3-\rho_\chi}{2}\sigma_\chi^2 \end{pmatrix} \\ &\quad \begin{pmatrix} 1 & 0 \\ \frac{(1-\rho_\chi)(1-\alpha)}{3-\rho_\chi} & \frac{2}{3-\rho_\chi} \end{pmatrix} \\ &= \begin{pmatrix} \sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2 & -\frac{2(1-\alpha)}{3-\rho_\chi}\sigma_\chi^2 \\ -\frac{2(1-\alpha)}{3-\rho_\chi}\sigma_\chi^2 & \frac{2}{3-\rho_\chi}\sigma_\chi^2 \end{pmatrix} \end{aligned}$$

The matrix $\boldsymbol{C}(1)$ is the Choleski decomposition of the long–run covariance matrix

$$\mathbf{C}(1)=\begin{pmatrix} \left(\sigma_z^2+\frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2} & 0 \\ \\ -\frac{2(1-\alpha)\sigma_\chi^2}{(3-\rho_\chi)\left(\sigma_z^2+\frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}} & \left(\frac{2\sigma_z^2\sigma_\chi^2}{(3-\rho_\chi)\left(\sigma_z^2+\frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)}\right)^{1/2} \end{pmatrix}$$

The IRF for labor productivity and hours are then deduced from $\mathbf{C}(L)$

$$\mathbf{C}(L)=(\mathbf{I}_2-\widehat{\mathbf{A}}_1L)^{-1}(\mathbf{I}_2-\widehat{\mathbf{A}}_1)\mathbf{C}(1)$$

The response on impact of hours to a technology shock is negative

$$-\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2+\frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}$$

and the response at horizon k of the level of hours is

$$-\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2+\frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}\sum_{j=0}^{k}\left(-\left(\frac{1-\rho_\chi}{2}\right)\right)^j.$$

Since

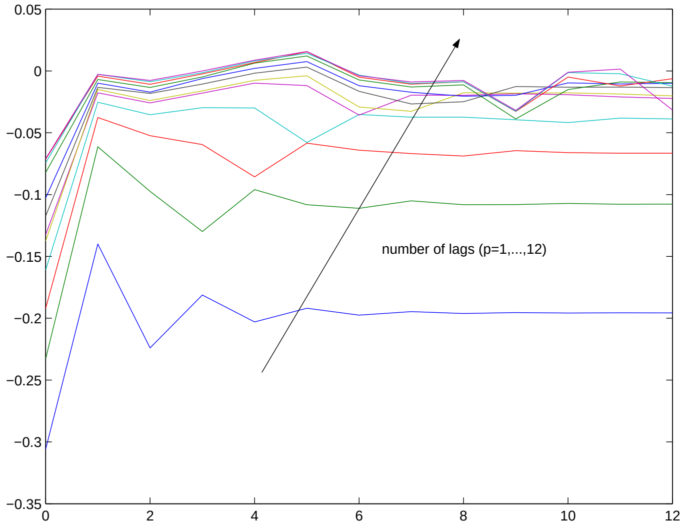
$$\left|\frac{1-\rho_\chi}{2}\right|<1,$$

we obtain

$$\sum_{j=0}^{k}\left(-\left(\frac{1-\rho_\chi}{2}\right)\right)^j=\frac{1-\left(\frac{\rho_\chi-1}{2}\right)^{k+1}}{1+\frac{1-\rho_\chi}{2}}>0$$

Figure 1: IRF of hours

(a) *iid* preference shock ($\rho_\chi = 0$)



(b) Persistent preference shock ($\rho_\chi = 0.98$)

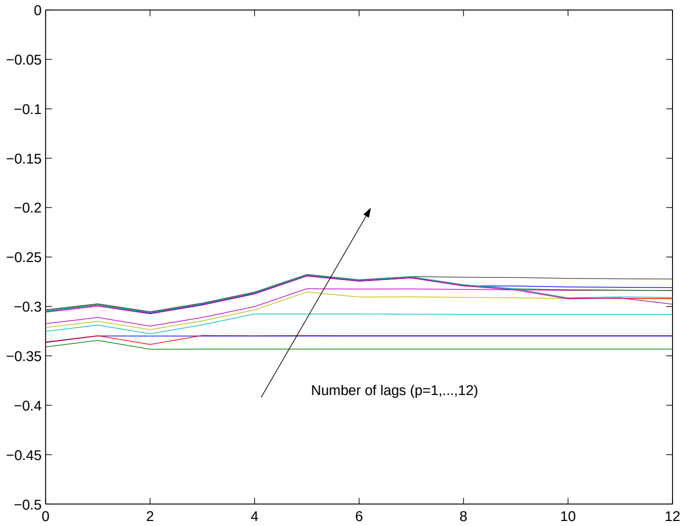
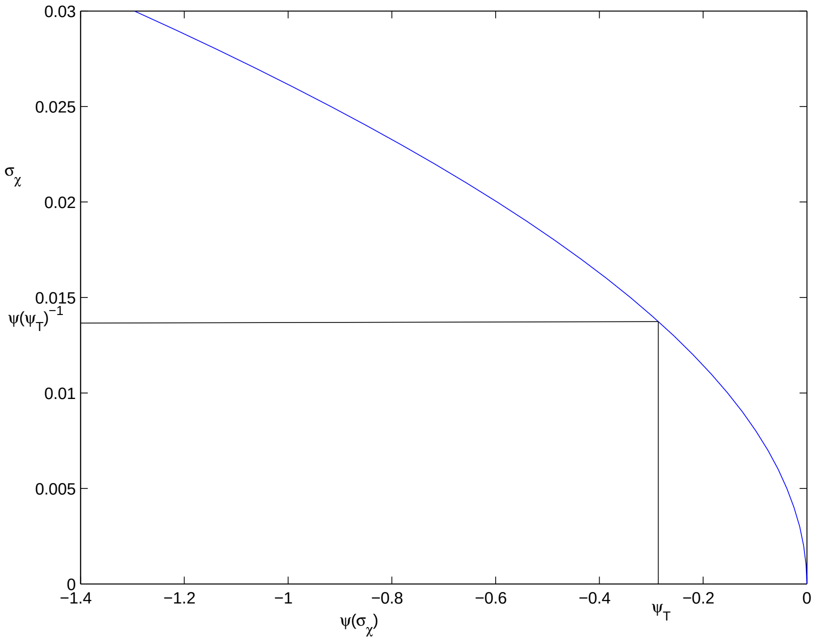


Figure 2: The Binding Function and Indirect Estimation



B Indirect Inference Weighting Matrix

This appendix describes how were computed the impulse response functions and their asymptotic confidence intervals. It is convenient to define

$$\begin{aligned}\mathbf{\Pi}' &= \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_\ell \end{pmatrix} \\ \mathbf{Q} &= \mathrm{E}\left(\begin{pmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \\ \vdots \\ \mathbf{z}_{t-p} \end{pmatrix}\begin{pmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \\ \vdots \\ \mathbf{z}_{t-p} \end{pmatrix}'\right)\end{aligned}$$

Now let $\hat{\mathbf{\Pi}}$ and $\hat{\mathbf{\Sigma}}$ denote the empirical estimates of $\mathbf{\Pi}$ and $\mathbf{\Sigma}$, respectively. We regroup the VAR parameters in the vector $\boldsymbol{\beta}$:

$$\boldsymbol{\beta} = (\mathrm{vec}(\mathbf{\Pi})', \mathrm{vech}(\mathbf{\Sigma})')', \quad \hat{\boldsymbol{\beta}} = (\mathrm{vec}(\hat{\mathbf{\Pi}})', \mathrm{vech}(\hat{\mathbf{\Sigma}})')',$$

where $\mathrm{vec}(\cdot)$ is the operator transforming an $(n \times n)$ matrix into an $(n^2 \times 1)$ vector by stacking the columns, $\mathrm{vech}(\cdot)$ is the operator transforming an $(n \times n)$ matrix into an $(n(n+1)/2 \times 1)$ vector by vertically stacking those elements on or below the principal diagonal. For later purpose, define $m = n(n\ell + (n+1)/2)$, so that $\boldsymbol{\beta}$ is an $(m \times 1)$ vector. Following Hamilton (1994) (proposition 11.2, page 301), it can be shown that

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \underset{a}{\sim} \mathrm{N}\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \mathbf{\Sigma}_\beta\right),$$

where T is the sample size and

$$\mathbf{\Sigma}_\beta = \begin{pmatrix} \mathbf{\Sigma} \otimes \mathbf{Q}^{-1} & 0 \\ 0 & \mathbf{\Sigma}_{22} \end{pmatrix},$$

with $\mathbf{\Sigma}_{22}$ defined as

$$\mathbf{\Sigma}_{22} = 2\left(\mathbf{D}_n^+\right)(\mathbf{\Sigma} \otimes \mathbf{\Sigma})\left(\mathbf{D}_n^+\right)'.$$

Here $\mathbf{D}_n^+$ is the unique matrix such that $\mathrm{vech}(\mathbf{\Sigma}) = \mathbf{D}_n^+ \mathrm{vec}(\mathbf{\Sigma})$. In practice, we replace $\mathbf{\Sigma}$ and $\mathbf{Q}$ in the above formula with their empirical counterparts

$$\begin{aligned}\hat{\mathbf{\Sigma}} &= \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_t \hat{\boldsymbol{\epsilon}}_t' \\ \hat{\mathbf{Q}} &= \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'\end{aligned}$$

We assume that the canonical innovations are linear combinations of the structural shocks $\boldsymbol{\eta}_t$, *i.e.*

$$\boldsymbol{\varepsilon}_t = \mathbf{S}\boldsymbol{\eta}_t,$$

for some non singular matrix $\mathbf{S}$. We impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies $\mathrm{E}\boldsymbol{\eta}_t\boldsymbol{\eta}_t' = \mathbf{I}_n$. Now, let us define

$$\begin{aligned}\mathbf{B}(L) &= (\mathbf{I}_n - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p)^{-1} \\ \mathbf{C}(L) &= \mathbf{B}(L)\mathbf{S}\end{aligned}$$

Now, let us define the vector collecting the dynamic response of the components of $\mathbf{z}_t$ to a technology/supply shock $\boldsymbol{\eta}_{1,t}$

$$\boldsymbol{\psi}_k = \frac{\partial \mathbf{z}_{t+k}}{\partial \boldsymbol{\eta}_{1,t}}.$$

Formally, $\boldsymbol{\psi}_k$ is the first column of $\mathbf{C}_k$, where $\mathbf{C}_k$ is the k-coefficient of $\mathbf{C}(L)$. In the sequel, we define $\boldsymbol{\psi}$ as

$$\boldsymbol{\psi} = \mathrm{vec}([\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_k]').$$

Recall that $\mathbf{C}_k = \mathbf{B}_k\mathbf{S}$, where $\mathbf{B}_k$ is the upper leftmost $(n \times n)$ block of $\mathbf{F}^k$ (Hamilton, 1994, p. 260), where

$$\underset{(np\times np)}{\mathbf{F}} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \cdots & \mathbf{A}_{p-1} & \mathbf{A}_p \\ \mathbf{I}_n & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{I}_n & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \mathbf{I}_n & \mathbf{0}_{n\times n} \end{pmatrix}$$

In practice, we use this formula with $\hat{\mathbf{\Sigma}}$, $\hat{\mathbf{A}}_1$,..., and $\hat{\mathbf{A}}_p$ substituted for $\mathbf{\Sigma}$, $\mathbf{A}_1$,..., and $\mathbf{A}_p$ to estimate the $\boldsymbol{\psi}_k$. In the sequel, we let $\hat{\boldsymbol{\psi}}_k$ denote the empirical estimates of $\boldsymbol{\psi}_k$ and $\hat{\boldsymbol{\psi}}$ denote the empirical estimate of $\boldsymbol{\psi}$. To compute the confidence intervals of $\hat{\boldsymbol{\theta}}$, we resort to the δ-function method. It can be shown that $\boldsymbol{\theta}$ is an implicit function of $\boldsymbol{\beta}$. Then, we obtain the formula

$$\sqrt{T}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}) \underset{a}{\sim} \mathrm{N}\left(0, \frac{\partial\boldsymbol{\psi}(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}'}\mathbf{\Sigma}_\beta\frac{\partial\boldsymbol{\psi}(\boldsymbol{\beta})'}{\partial\boldsymbol{\beta}}\right).$$

In practice, the derivatives $\partial\boldsymbol{\psi}(\boldsymbol{\beta})/\partial\boldsymbol{\beta}'$ are computed numerically at the point estimate $\hat{\boldsymbol{\beta}}$.

Figure 3: Model Impulse Responses of Hours to Technology Shocks

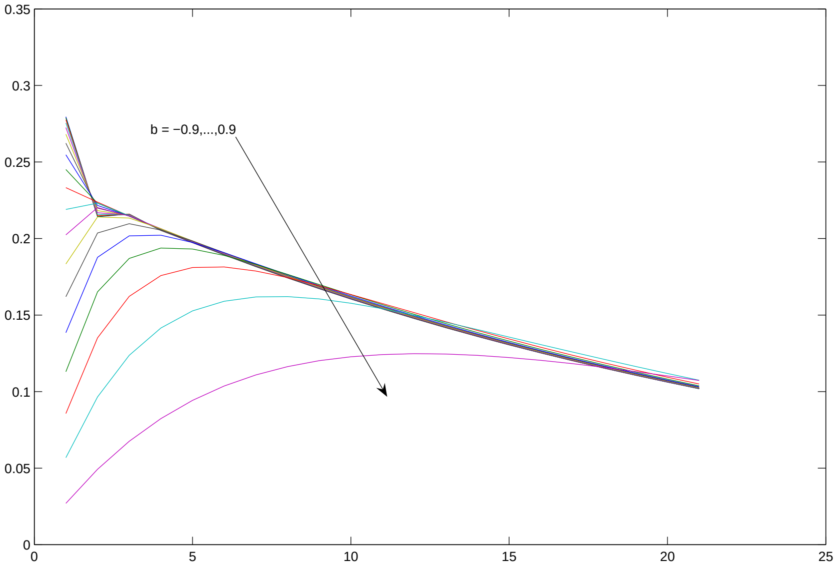
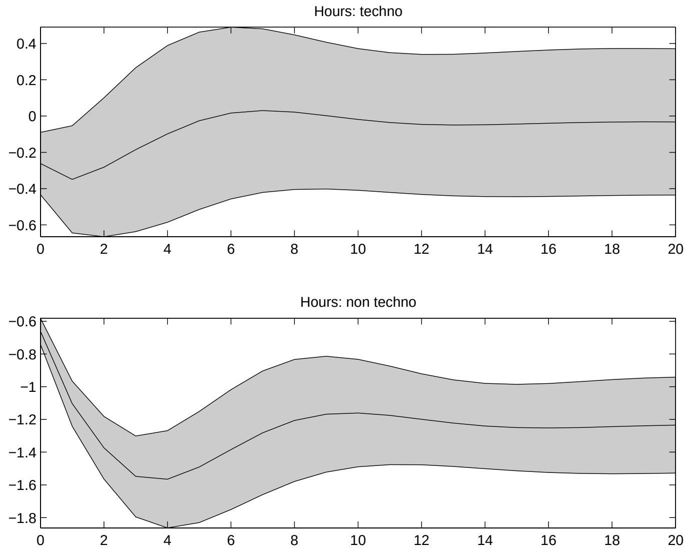
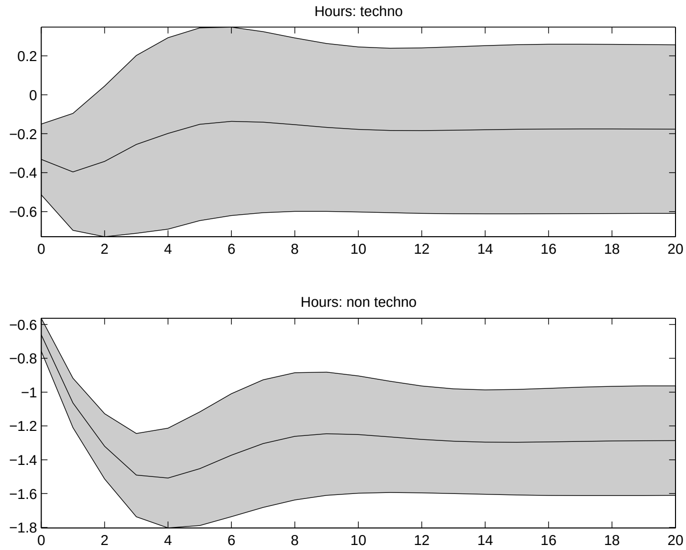


Figure 4: IRF of hours

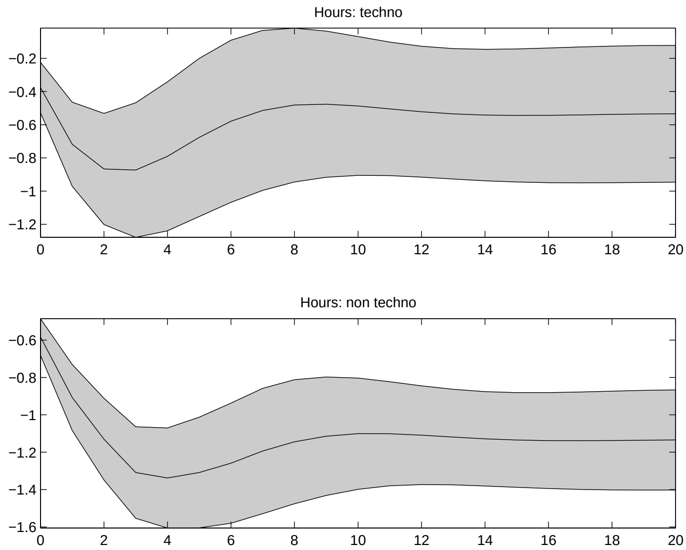
(a) NFB sector data



(b) B sector data



(c) Mixed NFB sector data



(d) Mixed B sector data

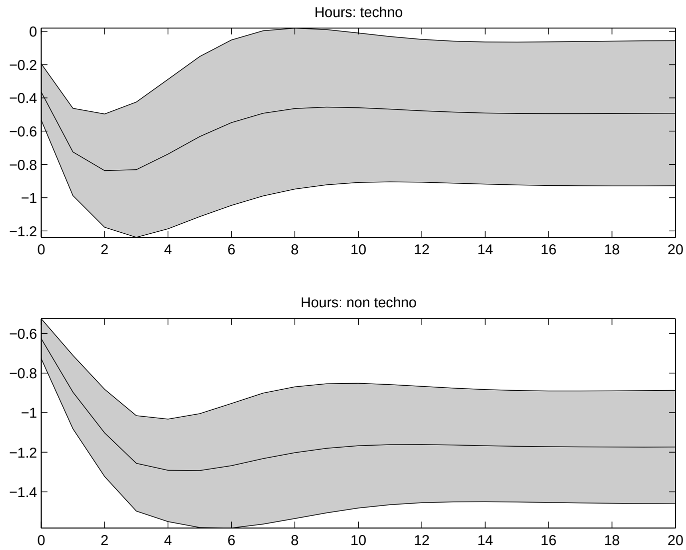
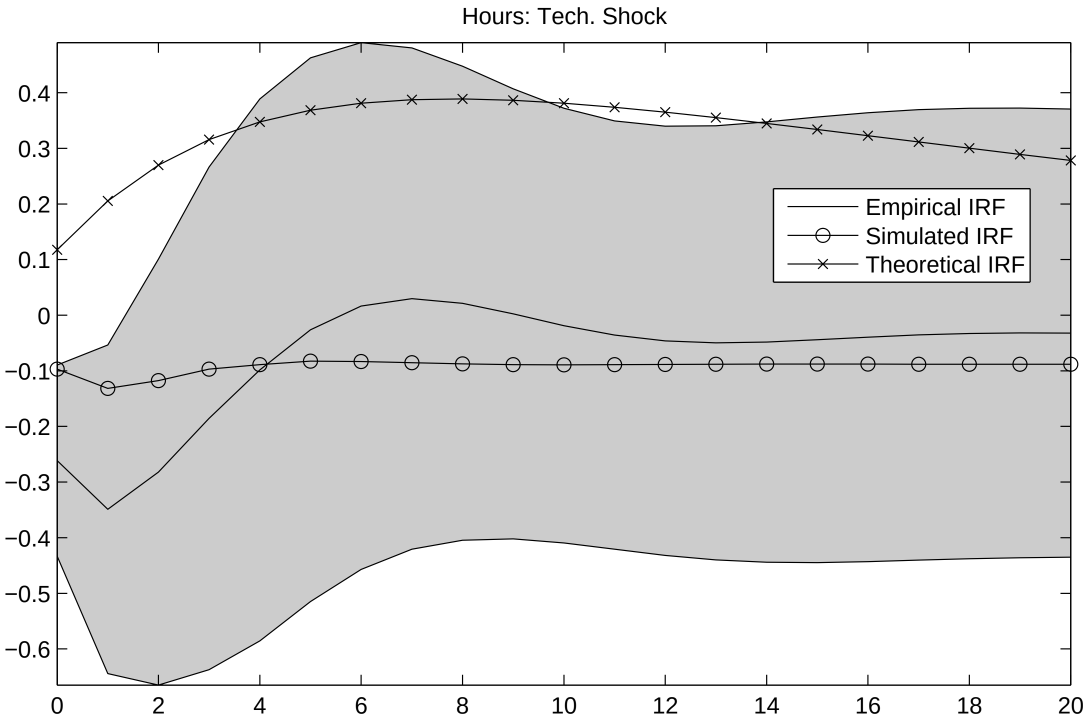


Table 1: Results from DSVARs

Data	NFB sector	B sector	Mixed NFB sector	Mixed B sector	NFB sector
Variable	Hours	Hours	Hours	Hours	Output-Hours
b	0.8367 (0.2106)	0.8563 (0.2100)	0.8482 (0.2806)	0.8816 (0.1678)	0.6361 (0.0772)
σ_z	0.0256 (0.0079)	0.0228 (0.0065)	0.0126 (0.0020)	0.0137 (0.0017)	0.0168 (0.0006)
ρ_χ	0.6855 (0.2981)	0.6837 (0.3073)	0.7004 (0.4016)	0.6507 (0.2636)	0.9680 (0.0642)
σ_χ	0.0255 (0.0281)	0.0287 (0.0367)	0.0270 (0.0422)	0.03381 (0.0433)	0.0147 (0.0006)
$J - stat$	11.82 [100]	6.23 [100]	9.88 [100]	9.13 [100]	65.36 [88.2]
$V(y/\varepsilon_z)$ (in %)	87.2	84.0	68.5	70.5	82.4
$V(n/\varepsilon_z)$ (in %)	23.1	18.4	10.9	9.9	25.7
$Corr(y, n/\varepsilon_z)$	0.83	0.81	0.82	0.76	0.95

Note: standard–errors in parentheses; P–values in brackets

Figure 5: IRF of hours (NFB sector data)



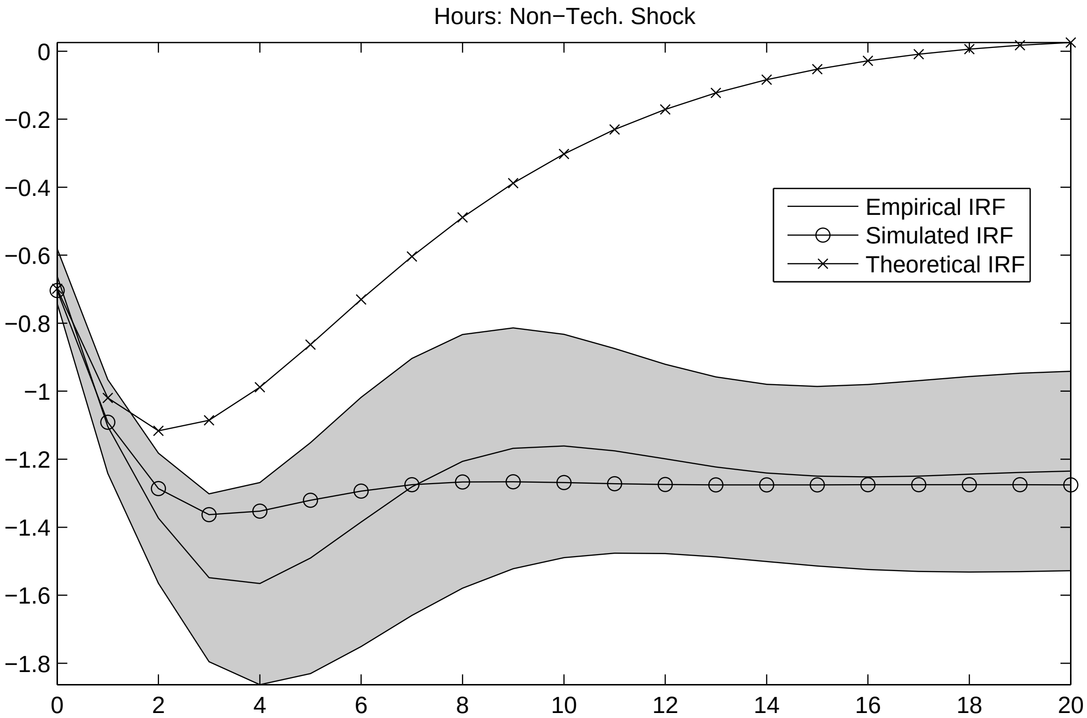
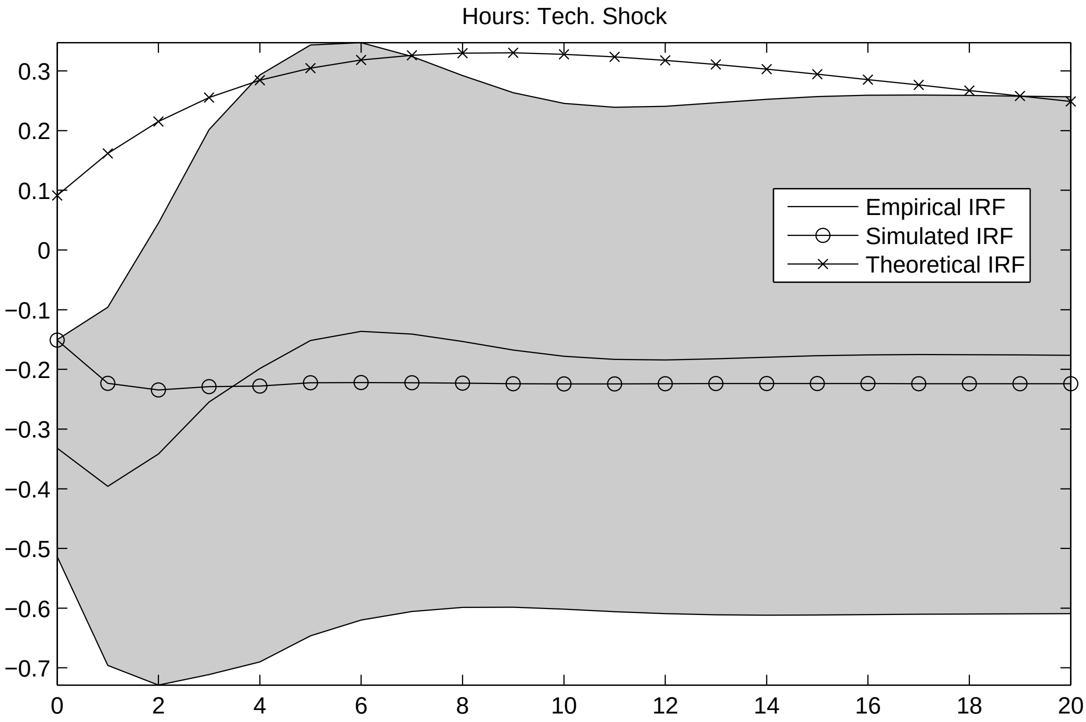


Figure 6: IRF of hours (B sector data)



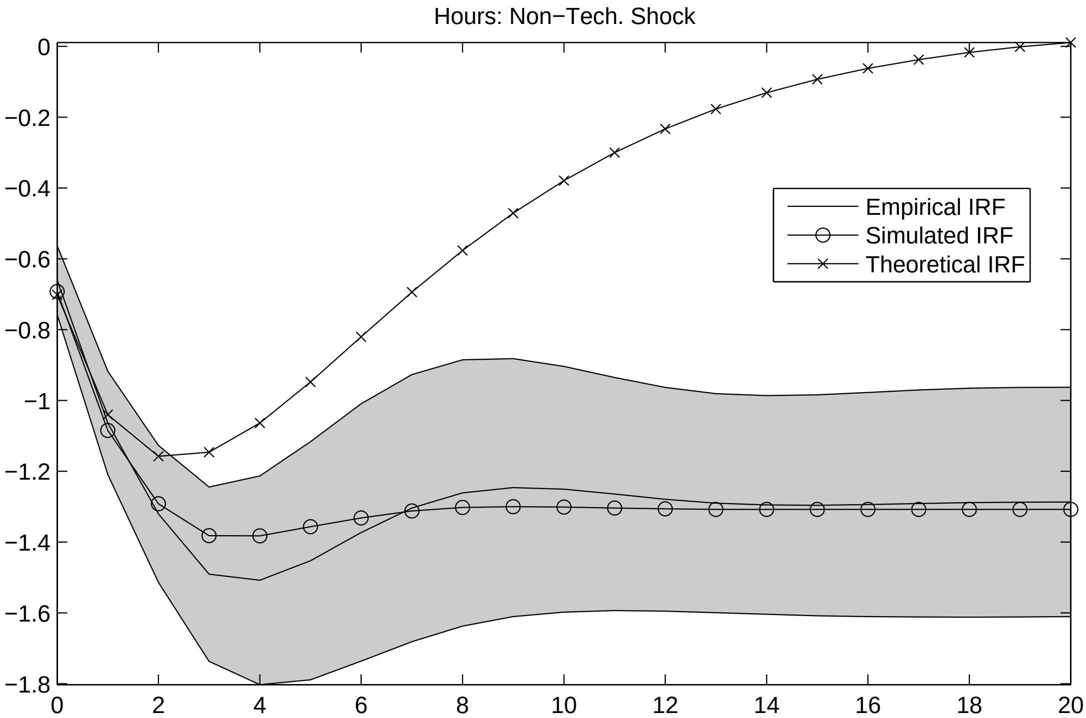
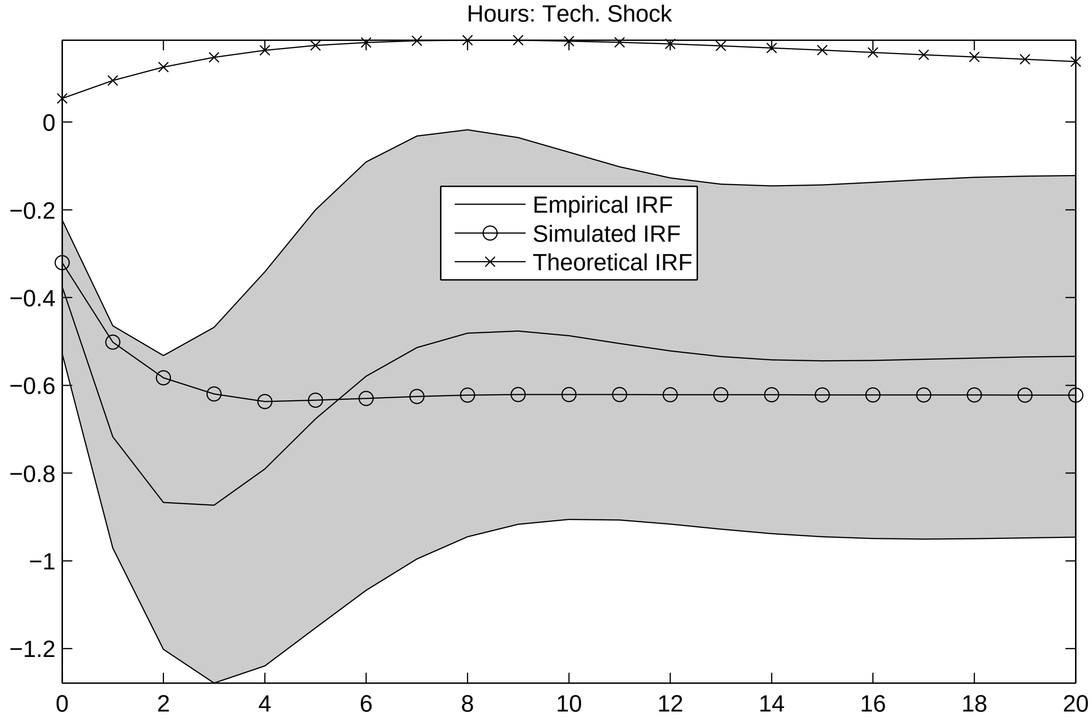


Figure 7: IRF of hours (mixed NFB sector data)



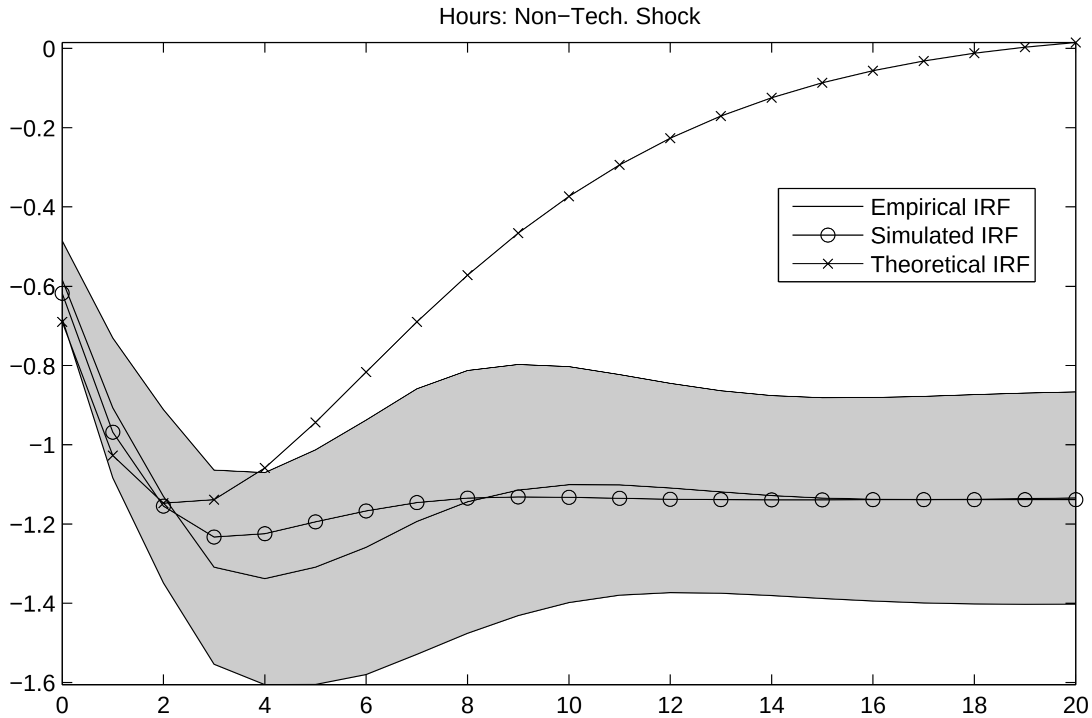


Figure 8: IRF of hours (mixed B sector data)

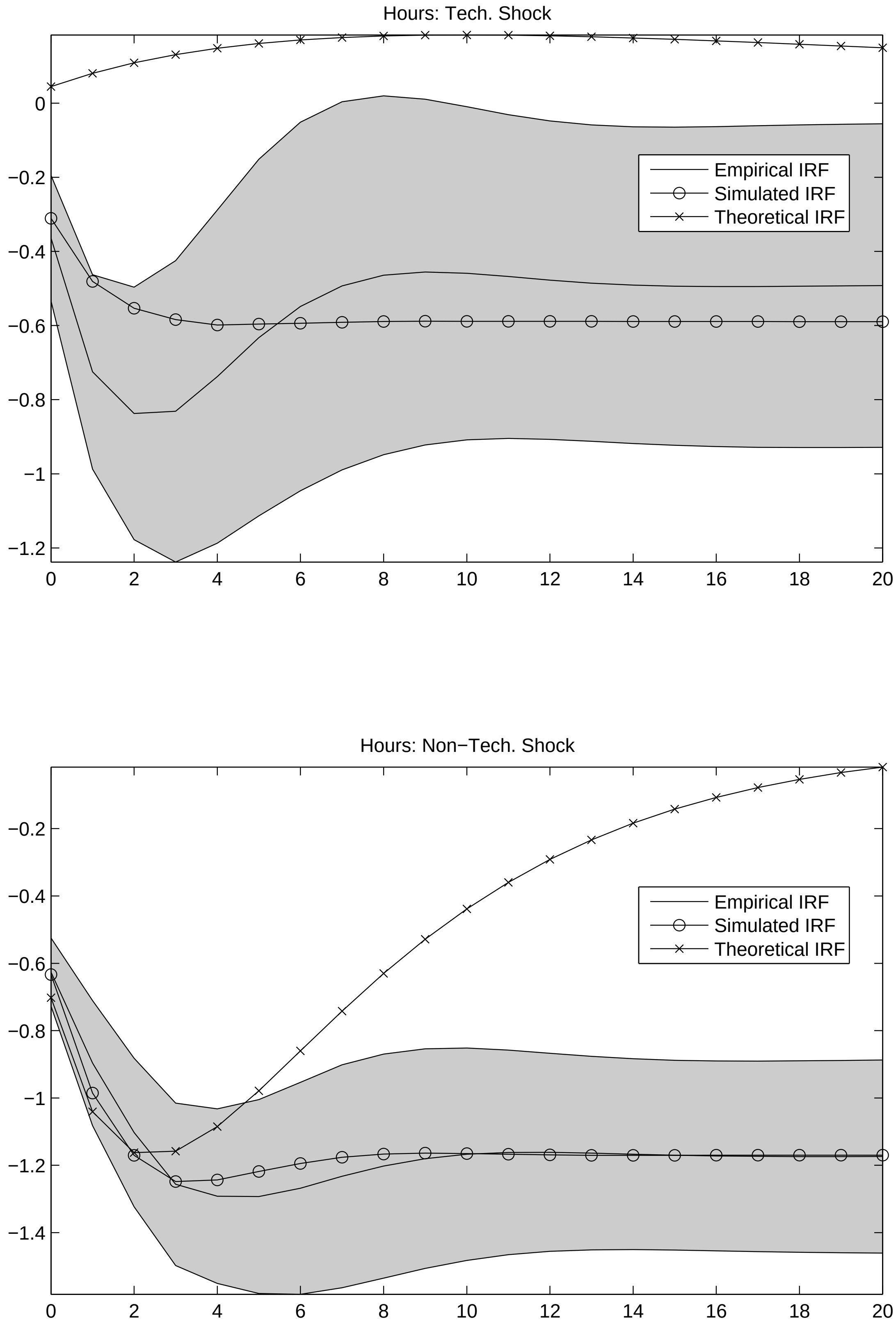


Figure 9: IRF of output and hours (NFB sector data)

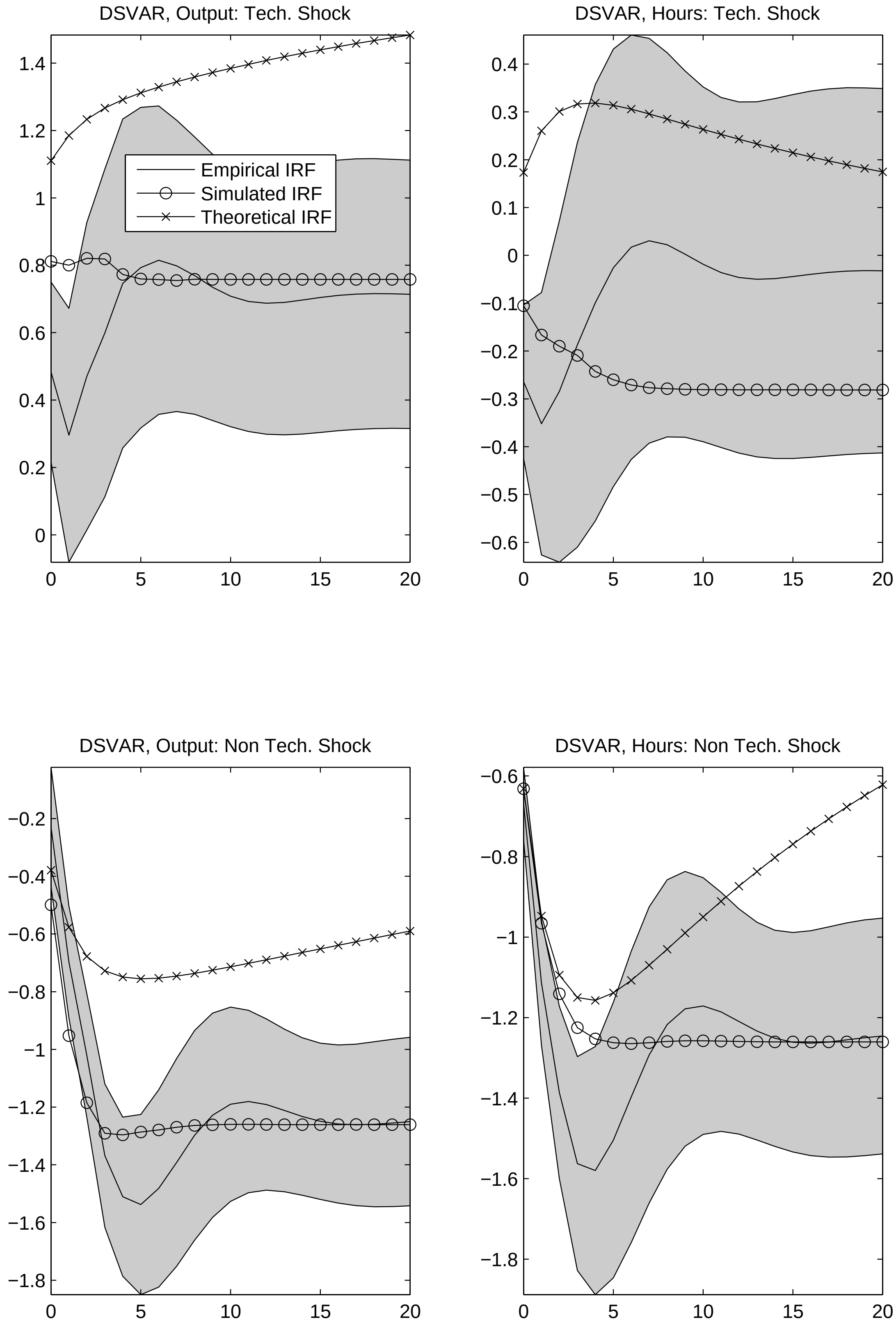


Figure 10: IRF of hours in LSVAR (NFB sector data)

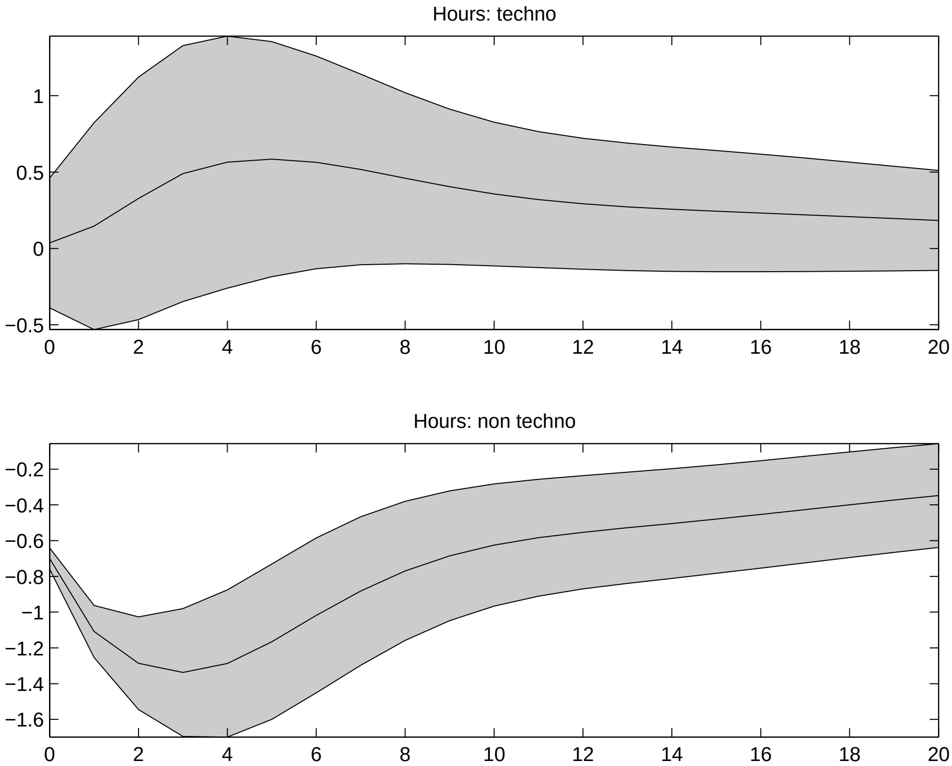


Table 2: Results from SVARs

Model	DSVAR	LSVAR	DSVAR–LSVAR
Data	NFB	NFB	NFB
Variable	Hours	Hours	Hours
b	0.8367	0.9323	0.9469
	(0.2106)	(0.0154)	(0.0125)
σ_z	0.0256	0.0600	0.0376
	(0.0079)	(0.0107)	(0.0043)
ρ_χ	0.6855	0.5597	0.5153
	(0.2981)	(0.0669)	(0.0399)
σ_χ	0.0255	0.0633	0.0760
	(0.0281)	(0.0143)	(0.0184)
$J - stat$	11.82	4.25	32.87
	[100]	[100]	[100]
Q_6	16.26	26.67	–
	[17.9]	[0.9]	–
Q_{11}	23.28	39.10	–
	[38.6]	[1.37]	–
Q_{21}	61.38	80.27	–
	[2.70]	[0.03]	–
$V(\Delta y/\varepsilon_z)$ (in %)	87.2	94.8	91.3
$V(n/\varepsilon_z)$ (in %)	23.1	23.6	13.0
$Corr(y, n/\varepsilon_z)$	0.83	0.64	0.58

Note: standard–errors in parentheses; P–values in brackets

Figure 11: IRF of hours in DSVAR and LSVAR (estimations from DSVAR)

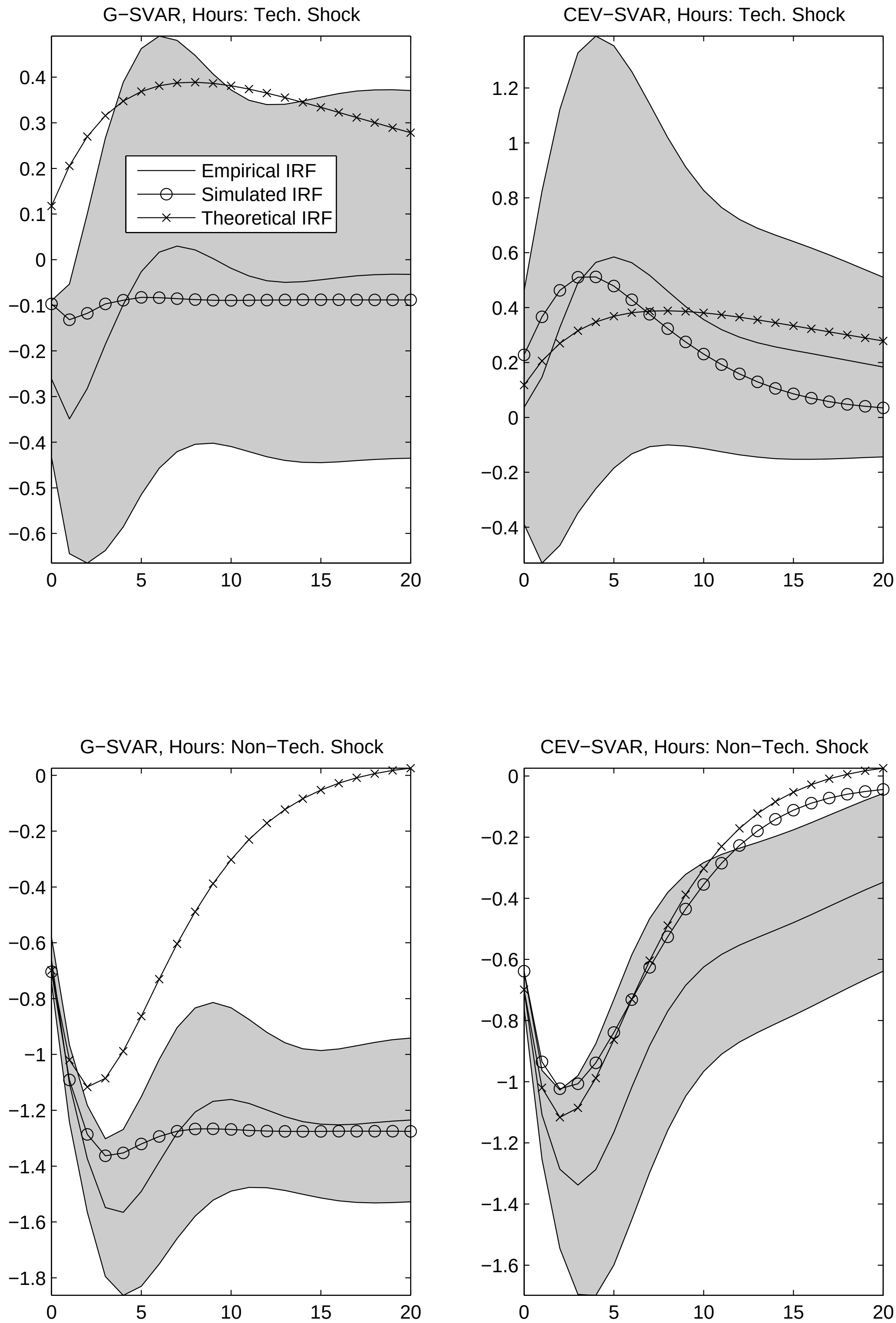


Figure 12: IRF of hours in DSVAR and LSVAR (estimations from LSVAR)

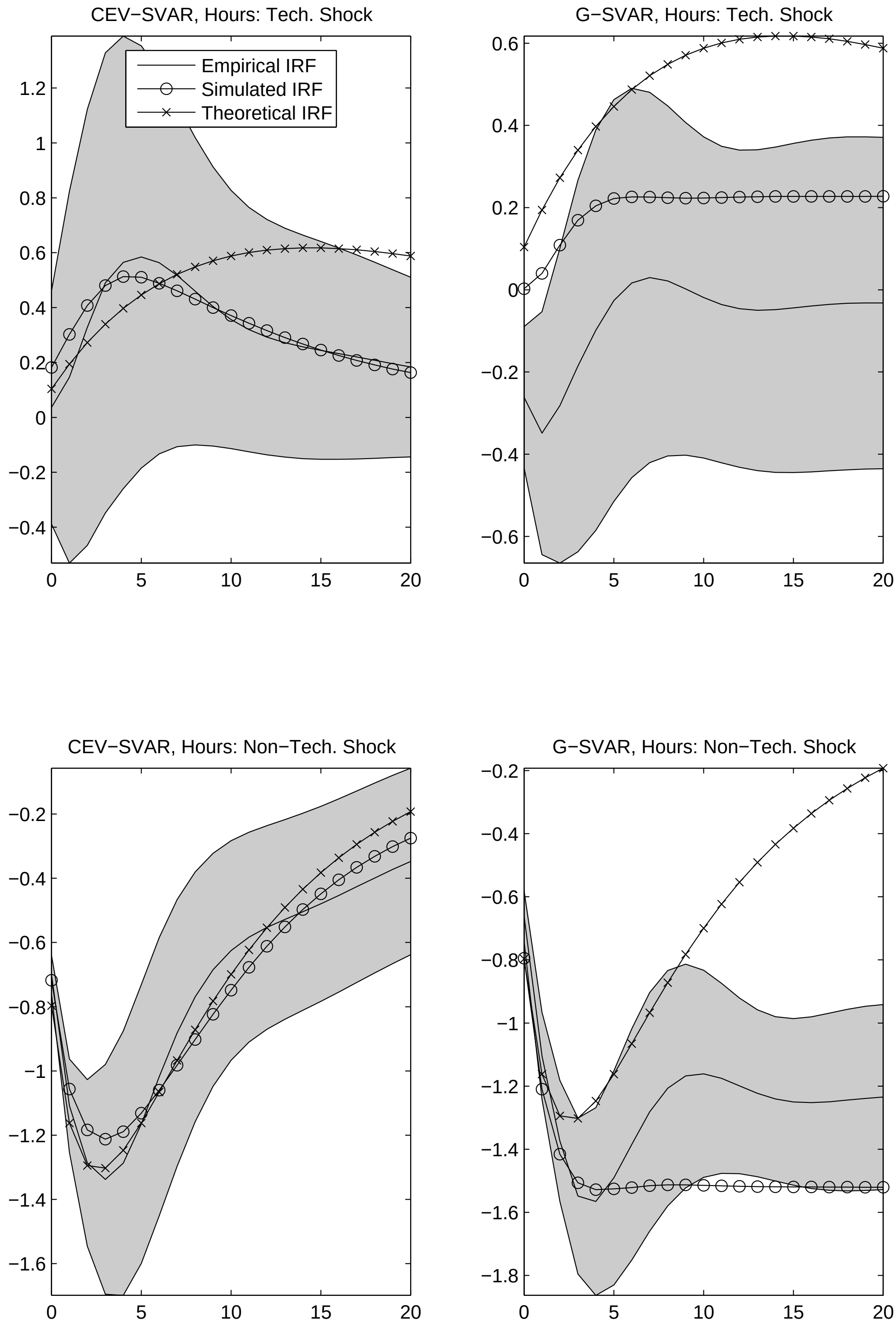


Figure 13: IRF of hours in DSVAR and LSVAR (estimations from DSVAR and LSVAR)