Optimal Simple And Implementable Monetary and Fiscal Rules

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Abstract

The goal of this paper is to compute monetary and fiscal policy rules that are optimal within a family of implementable, simple rules in an empirically realistic model of the business cycle. The implementability condition requires policies to deliver uniqueness of the rational expectations equilibrium. Simplicity requires restricting attention to rules whereby policy variables are set as a function of a small number of easily observable macroeconomic indicators. Specifically, we study interest-rate feedback rules that respond to measures of inflation and output. We study six different specifications of those rules: backward-looking rules (where the interest rate responds to past inflation and output), contemporaneous rules (where the interest rate responds to current inflation and output), and forward-looking rules (where the interest rate responds to expected future inflation and output). For each of these three types of rule, we consider the cases of interest-rate smoothing (i.e., the past value of the interest rate enters as an additional argument into the rule) and no interest-rate smoothing. We analyze fiscal policy rules whereby the income tax rate is set as an increasing function of the level of public debt. Our baseline model of the business cycle features capital accumulation and sticky prices in product markets and is subject to technology and government purchases shocks. We then incorporate sequentially other features that have been identified as useful in improving the model's ability to explain actual business cycles, such as wage stickiness, habit formation, money, capital adjustment costs, and variable capital utilization. We depart from two unrealistic assumptions that are made in related studies. Namely, we do not assume the existence of production subsidies that offset the distortions introduced by the presence of imperfect competition in product and factor markets. Second, we do not restrict attention to zero-inflation steady-states. Two important consequences of doing away with these two assumptions are that one can no longer use linear approximations to the policy functions to evaluate a second-order expansion of the welfare function and, second, that a number of aggregation issues that can be ignored under the above assumptions must be tackled explicitly. Our findings indicate that in the class of models we consider the precise degree to which the central bank responds to inflation plays no role for welfare provided that the monetary policy stance is active (i.e., that the inflation coefficient in the interest rate rule is greater than one). Second, the optimal monetary policy features a muted response to output. More importantly, not responding to output is critical from a welfare point of view. In effect, our results show that interest rate rules that feature a positive response of the nominal interest rate to output can lead to significant welfare losses.

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1 Introduction

Recently, there has been an outburst of papers studying optimal monetary policy in economies with nominal rigidities.[1] Most of these studies are conducted in the context of highly stylized theoretical and policy environments. For instance, in most of this body of work it is assumed that the government has access to a subsidy to factor inputs financed with lump-sum taxes aimed at dismantling the inefficiency introduced by imperfect competition in product and factor markets. This assumption is clearly empirically unrealistic. But more importantly it undermines a potentially significant role for monetary policy, namely, stabilization of costly aggregate fluctuations around a distorted steady-state equilibrium.

A second notably simplification is the absence of capital accumulation. All the way from the work of Keynes (1936) and Hicks (1937) to that of Kydland and Prescott (1982) macroeconomic theories have emphasized investment dynamics as an important channel for the transmission of aggregate disturbances. It is therefore natural to expect that investment spending should play a role in shaping optimal monetary policy. Indeed it has been shown, that for a given monetary regime the determinacy properties of a standard Neo-Keynesian model can change dramatically when the assumption of capital accumulation is added to the model (Dupor, 200x)

A third important dimension along which the existing studies abstracts from reality is the assumed fiscal regime. It is standard practice in this literature to completely ignore fiscal policy. Implicitly, these models assume that the fiscal budget is balanced at all times by means of lump-sum taxation. In other words, the assumed fiscal policy is passive in the sense of Leeper (1991). Here again it is well known from the work of Leeper (1991), Sims (1994), Woodford (1994), and Schmitt-Grohe and Uribe (2000) among others, that given monetary policy the determinacy properties of the rational expectations equilibrium crucially depend on the nature of fiscal policy in place. It follows that the design of optimal monetary policy should depend upon the underlying fiscal regime as well.

Finally, analysis of optimal monetary policy is typically restricted to economies in which long-run inflation is nil or there is some form of wide-spread indexation. As a result, in the standard environments studied in the literature nominal rigidities have no real consequences for economic activity and thus welfare in the long-run. It follows that the assumptions of zero long-run inflation or indexation should not be expected to be inconsequential for the form that optimal monetary policy will take. Because from an empirical point of view, neither of these two assumptions is particularly compelling for economies like the United States, it is

[1]See Rotemberg and Woodford (1997, 1999), Clarida, Gali, and Gertler (1999), Gali and Monacelli (200x), Benigno and Benigno (200x), Woodford and Giannoni (200x), Svensson and Woodford (20xx), Schmitt-Grohe and Uribe (2001, 2003), and Erceg, Henderson, and Levin (200x) among many others.

of interest to investigate the characteristics of optimal policy in their absence.

Taken together the four assumptions listed above imply that business cycles are centered around an efficient non-distorted equilibrium. The main reason why these rather unrealistic features have been so widely adopted is not that they are the most empirically obvious ones to make nor that researchers believe that they are inconsequential for the nature of optimal monetary policy. Rather, the motivation is purely technical. Namely, the stylized models considered in the literature make it possible for a first-order approximation to the equilibrium conditions to be sufficient to accurately approximate welfare up to second order (Woodford 2003, chapter 6).[2] Any departure from the set of simplifying assumptions mentioned above, with the exception of the assumption of no investment dynamics, would require approximating the equilibrium conditions to second order. Moreover, to our knowledge, thus far it has not been shown that in models with capital accumulation evaluating a second-order approximation to the welfare function using a first-order approximation to the policy function yields an accurate second-order approximation to welfare.

Recent advancements in computational economics have delivered algorithms that make it feasible and simple to compute higher-order approximations to the equilibrium conditions of a general class of large stochastic dynamic general equilibrium models.[3] In this paper, we employ these new tools to analyze a model that relaxes all of the four questionable assumptions mentioned above. The central focus of this paper is to investigate whether the policy conclusions arrived at by the existing literature regarding the optimal conduct of monetary policy are robust with respect to more realistic specifications of the economic environment. That is, we study optimal policy in a world where there are no subsidies to undo the distortions created by imperfect competition, where there is capital accumulation, where the government may follow active fiscal policy and may not have access to lump-sum taxation, and where there are inefficiencies due to nominal rigidities even in the long-run.

Specifically, this paper characterizes monetary and fiscal policy rules that are optimal within a family of implementable, simple rules in an empirically realistic model of the business cycle. The implementability condition requires policies to deliver uniqueness of the rational expectations equilibrium. Simplicity requires restricting attention to rules whereby policy variables are set as a function of a small number of easily observable macroeconomic indicators. Specifically, we study interest-rate feedback rules that respond to measures of inflation and output. We study six different specifications of those rules: backward-looking

[2]We note that a first-order approximation to the utility function around the non-stochastic steady state is of little use. For the first-order approximation of the welfare function around the non-stochastic steady state equals the welfare function evaluated at the non-stochastic steady state. It follows that up to first-order all policies that preserve the non-stochastic steady state yield the same level of welfare.

[3]See, for instance, Kim et al. (2003) and Schmitt-Grohe and Uribe (2004).

rules (where the interest rate responds to past inflation and output), contemporaneous rules (where the interest rate responds to current inflation and output), and forward-looking rules (where the interest rate responds to expected future inflation and output). For each of these three types of rule, we consider the cases of interest-rate smoothing (i.e., the past value of the interest rate enters as an additional argument into the rule) and no interest-rate smoothing. We analyze fiscal policy rules whereby the income tax rate is set as an increasing function of the level of public liabilities.

Our baseline model of the business cycle features capital accumulation and sticky prices in product markets and is subject to technology and government purchases shocks. We then incorporate sequentially other features that have been identified as useful in improving the model's ability to explain actual business cycles, such as wage stickiness, habit formation, money, capital adjustment costs, and variable capital utilization.

Our preliminary findings indicate that in the class of models we consider the precise degree to which the central bank responds to inflation plays a minor role for welfare provided that the monetary policy stance is active in the sense of Leeper (1991) (i.e., that the inflation coefficient in the interest rate rule is greater than one). Second, the optimal monetary policy features a muted response to output. More importantly, not responding to output is found to be critical from a welfare point of view. In effect, our results show that interest rate rules that feature a positive response of the nominal interest rate to output can lead to significant welfare losses.

2 The Model

The starting point for our investigation into the welfare consequences of alternative policy rules is an economic environment featuring a blend of neoclassical and neo-Keynesian elements. Specifically, the skeleton of the economy is a standard real-business-cycle model with capital accumulation and endogenous labor supply driven by technology and government spending shocks. Four sources of inefficiency separate our model from the standard RBC framework: (a) nominal rigidities in the form of sluggish price adjustment. A later section incorporates sticky wages as a second source of nominal rigidity. (b) A demand for money motivated by a working-capital constraint on labor costs. (c) time-varying distortionary taxation. And (d) monopolistic competition in product markets. These four elements of the model provide a rationale for the conduct of monetary and fiscal stabilization policy.

2.1 Households

The economy is populated by a continuum of identical households. Each household has preferences defined over consumption, c_t, and labor effort, h_t. Preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave. The consumption good is assumed to be a composite good produced with a continuum of differentiated goods, c_{it}, $i \in [0,1]$, via the aggregator function

$$c_t = \left[\int_0^1 {c_{it}}^{1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{2}$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods. For any given level of consumption of the composite good, purchases of each variety i in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t. The optimal level of c_{it} is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t, \tag{3}$$

where P_t is a nominal price index given by

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}. \tag{4}$$

This price index has the property that the minimum cost of a bundle of intermediate goods yielding c_t units of the composite good is given by $P_t c_t$.

Households are assumed to have access to a complete set of nominal contingent claims. Their period-by-period budget constraint is given by

$$E_t r_{t,t+1} x_{t+1} + c_t + i_t + \tau_t^L = x_t + (1 - \tau_t^D)[w_t h_t + u_t k_t] + \tilde{\phi}_t, \tag{5}$$

where $r_{t,s}$ is a stochastic discount factor, defined so that $E_t r_{t,s} x_s$ is the nominal value in period t of a random nominal payment x_s in period $s \geq t$. The variable k_t denotes capital,

i_t denotes investment, $\tilde{\phi}_t$ denotes profits received from the ownership of firms net of income taxes, τ_t^D denotes the income tax rate, and τ_t^L denotes lump-sum taxes. The capital stock is assumed to depreciate at the constant rate δ, so the evolution of capital is given by

$$k_{t+1} = (1-\delta)k_t + i_t. \tag{6}$$

The investment good is assumed to be a composite good made with the aggregator function (2). Thus, the demand for each intermediate good $i \in [0,1]$ for investment purposes, denoted i_{it}, is given by $i_{it} = (P_{it}/P_t)^{-\eta}\, i_t$. Households are also assumed to be subject to a borrowing limit that prevents them from engaging in Ponzi schemes. The household's problem consists in maximizing the utility function (1) subject to (5), (6), and the no-Ponzi-game borrowing limit. The first-order conditions associated with the household's problem are

$$U_c(c_t, h_t) = \lambda_t, \tag{7}$$

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = w_t(1-\tau_t^D), \tag{8}$$

and

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[(1-\tau_{t+1}^D) u_{t+1} + 1 - \delta \right] \right\}. \tag{9}$$

It is apparent from these first-order conditions that the income tax distorts both the leisure-labor choice and the decision to accumulate capital over time.

Let R_t denote the gross one-period, risk-free, nominal interest in period t. Then by a no-arbitrage condition, R_t must equal the inverse of the period-t price of a portfolio that pays one dollar in every state of period $t+1$. That is,

$$R_t = \frac{1}{E_t r_{t,t+1}}.$$

Combining this expression with the optimality conditions associated with the household's problem yields

$$\lambda_t = \beta R_t E_t \lambda_{t+1} \frac{P_t}{P_{t+1}}. \tag{10}$$

2.2 The Government

The consolidated government prints money, M_t, issues one-period nominally risk-free bonds, B_t, collects taxes in the amount of $P_t \tau_t$, and faces an exogenous expenditure stream, g_t. Its period-by-period budget constraint is given by

$$M_t + B_t = R_{t-1}B_{t-1} + M_{t-1} + P_t g_t - P_t \tau_t.w$$

The variable g_t denotes per capita government spending on a composite good produced via the aggregator (2). We assume, maybe unrealistically, that the government minimizes the cost of producing g_t. Thus, we have that the public demand for each type i of intermediate goods, g_{it}, is given by $g_{it} = (P_{it}/P_t)^{-\eta} g_t$. Let $\ell_{t-1} \equiv (M_{t-1} + R_{t-1}B_{t-1})/P_{t-1}$ denote total real government liabilities outstanding at the beginning of period t in units of period $t-1$ goods. Also, let $m_t \equiv M_t/P_t$ denote real money balances in circulation and $\pi_t \equiv P_t/P_{t-1}$ denote the gross consumer price inflation. Then the government budget constraint can be written as

$$\ell_t = (R_t/\pi_t)\ell_{t-1} + R_t(g_t - \tau_t) - m_t(R_t - 1) \tag{11}$$

We wish to consider various alternative fiscal policy specifications that involve possibly both lump sum and distortionary income taxation. Total tax revenue, τ_t, consist of revenue from lump-sum taxation, τ_t^L, and revenue from income taxation, $\tau_t^D y_t$. That is,

$$\tau_t = \tau_t^L + \tau_t^D y_t. \tag{12}$$

The fiscal regime is defined by the following rule

$$\tau_t = \gamma_0 + \gamma_1(\ell_{t-1} - \ell) + \gamma_2 \left[g_t + \left(\frac{R_{t-1} - 1}{R_{t-1}} \right) \left(\frac{\ell_{t-1} - m_{t-1}}{\pi_t} \right) \right], \tag{13}$$

where γ_0, γ_1, γ_2, and ℓ are parameters. According to this rule, the fiscal authority sets total tax receipts as a function of two variables, the deviation of total government liabilities ℓ_{t-1} from a target level ℓ and the level of the real secondary deficit, $g_t + \left(\frac{R_{t-1}-1}{R_{t-1}} \right) \left(\frac{\ell_{t-1}-m_{t-1}}{\pi_t} \right)$. We consider four different fiscal policy regimes. In the first two regimes all taxes are lump sum at all times, and in the latter two all taxes are distortionary at all times. For each case, lump-sum or distortionary taxation, we consider two different feedback rules. One feedback rule postulates that each period tax receipts are adjusted in response to variations in the secondary fiscal deficit in such a way that the secondary deficit is zero. We refer to this rule as a balanced-budget rule. Under the second policy total tax collection is set as a linear function of the deviation of the stock of government liabilities from their target value. We refer to this policy as liability targeting. The parameterizations associated with the four cases then are: (i) lump-sum taxes and balanced-budget rule: $\tau_t^D = \gamma_0 = \gamma_1 = 0$ and $\gamma_2 = 1$; (ii) lump-sum taxes and liability targeting: $\tau_t^D = 0$, $\gamma_2 = 0$; (iii) income taxation and balanced-budget rule:

$\tau_t^L = \gamma_0 = \gamma_1 = 0$ and $\gamma_2 = 1$; and (iv) income taxation and liability targeting: $\tau_t^L = 0$ and $\gamma_2 = 0$. The a fiscal policy consisting of lump-sum taxation and a balanced-budget rule a is Ricardian policy in the sense that fiscal variables play no role for price level determination.[4] The fiscal policy featuring lump-sum taxes and liability targeting is motivated by the one considered in Leeper (1991). As Leeper shows depending on the size of the coefficient γ_1, this fiscal policy regime is active or passive. In particular for γ_1 greater than but close to the real rate of interest, fiscal policy will be passive, or Ricardian. We consider liability targeting because it allows for the possibility that fiscal policy is non-Ricardian, or in the terminology of Leeper (1991) active. In that case fiscal considerations will play an important role for price level determination. This feature distinguishes our analysis from most of the existing related literature where it is assumed from the outset (either explicitly or implicitly) that fiscal policy is passive. It then follows that optimal monetary policy must be active by construction because otherwise a determinate equilibrium usually does not exist. Our analysis is thus broader because it allows for the possibility that a combination of active fiscal and passive monetary policy is optimal.

We assume that the monetary authority sets the short-term nominal interest rate according to a simple feedback rule belonging to the following class of Taylor (1993)-type rules

$$\ln(R_t/R^*) = \alpha_R \ln(R_{t-1}/R^*) + \alpha_\pi E_t \ln(\pi_{t-i}/\pi^*) + \alpha_y E_t \ln(y_{t-i}/y); \quad i = -1, 0, 1 \tag{14}$$

where R_t denotes the gross one-period nominal interest rate, y_t denotes output in period t, y denotes the non-stochastic steady-state level of output, and R^*, π^*, α_R, α_π, α_y are parameters. The index i can take three values 1, 0 -1. In the case that $i = -1$, we refer to the interest rate rule as backward looking, when $i = 0$ we call the rule contemporaneous, and when $i = 1$ the rule is said to be forward looking. The reason why we focus on interest rate feedback rules belonging to this class is that they are easily implemented. In fact, there may be an issue of whether the policy maker knows the current level of output and inflation at the time he fixes the short-term interest rate or is sophisticated enough to determine what the equilibrium level of inflation and output would be for any particular setting of the nominal interest rate. Similarly, one may wonder how the policy maker is able to form the correct expectations about future values of aggregate activity and inflation since they depend on the current value of the central bank's policy instrument. Clearly, in the case that $i = -1$ all arguments of the feedback rule are in the information set of the policy maker at time t and more importantly are independent of current and future settings of the policy instrument.

[4] As shown in Schmitt-Grohé and Uribe (2000), this claim is correct only if the nominal interest rate is expected to be strictly positive in the long-run, which is an assumption we will maintain throughout the paper.

Implementation of this type of rule thus require the least amount of sophistication on the side of the policy maker.

Because the variables included in the interest-rate feedback rule are deviations from the non-stochastic steady state, implementation of the rule requires knowledge of the non-stochastic steady state by the central bank. The non-stochastic steady state is, however, non-observable. Thus, the assumed rule presumes a degree of sophistication that the central may not posses. A way to avoid this problem would be to postulate a rule that includes only observable, such as one of the form $\ln(R_t/R_{t-1}) = \alpha_\pi E_t \ln(\pi_{t-i}/\pi^*) + \alpha_y E_t \ln(y_{t-i}/y_{t-1-i})$. We study this family of rules in a later section.

We note that the type of monetary policy rules that are typically analyzed in the related literature require no less information on the part of the policymaker than the feedback rule given in equation (14). This is because the rules most commonly studied feature an output gap measure defined as deviations of output from the level that would obtain in the absence of nominal rigidities. Computing the flexible-price level of aggregate activity requires the policymaker to know not just the deterministic steady state of the economy, but also the joint distribution of all the shocks driving the economy and the current realizations of such shocks.

2.3 Firms

Each good's variety $i \in [0, 1]$ is produced by a single firm in a monopolistically competitive environment. Each firm i produces output using as factor inputs capital services, k_{it}, and labor services, h_{it}. The production technology is given by

$$z_t F(k_{it}, h_{it}),$$

where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments. The variable z_t denotes an exogenous and stochastic productivity shock.

It follows from our analysis of private and public absorption behavior that the aggregate demand for good i, $a_{it} \equiv c_{it} + i_{it} + g_{it}$ is given by

$$a_{it} = (P_{it}/P_t)^{-\eta} a_t,$$

where $a_t \equiv c_t + i_t + g_t$ denotes aggregate absorption.

We introduce money in the model by assuming that wage payments are subject to a cash-in-advance constraint of the form

$$m_{it} \geq \nu w_t h_{it}, \tag{15}$$

where m_{it} denotes the demand for real money balances by firm i in period t and $\nu \geq 0$ is a parameter denoting the fraction of the wage bill that must be backed with monetary assets. Real profits of firm i at date t expressed in terms of the composite good are given by[5]

$$\phi_{it} \equiv \frac{P_{it}}{P_t} a_{it} - u_t k_{it} - w_t h_{it} - (1 - R_t^{-1}) m_{it}. \tag{16}$$

We assume that the firm must satisfy demand at the posted price. Formally, we impose

$$z_t F(k_{it}, h_{it}) \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} a_t. \tag{17}$$

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, k_{it} and m_{it} so as to maximize the present discounted value of profits, given by

$$E_t \sum_{s=t}^{\infty} r_{t,s} P_s \phi_{is}.$$

Throughout our analysis, we will focus on equilibria featuring a strictly positive nominal interest rate. This implies that the cash-in-advance constraint (15) will always be binding. Then, letting mc_{it} be the Lagrange multiplier associated with constraint (17), the first-order conditions of the firm's maximization problem with respect to capital and labor services are, respectively,

$$\mathrm{mc}_{it} z_t F_h(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]$$

and

$$\mathrm{mc}_{it} z_t F_k(k_{it}, h_{it}) = u_t.$$

Notice that because all firms face the same factor prices and because they all have access to the same homogenous-of-degree-one production technology, the capital-labor ratio, k_{it}/h_{it} and marginal cost, mc_{it}, are identical across firms.

Prices are assumed to be sticky à la Calvo (1983) and Yun (1996). Specifically, each period a fraction $\alpha \in [0, 1)$ of randomly picked firms is not allowed to change the nominal price of the good it produces. The remaining $(1 - \alpha)$ firms choose prices optimally. Suppose firm i gets to choose the price in period t, and let $\tilde{P}_{it}$ denote the chosen price. This is set so

[5]Appendix A derives this expression.

as to maximize the expected present discounted value of profits. That is, $\tilde{P}_{it}$ maximizes

$$E_t \sum_{s=t}^{\infty} r_{t,s} P_s \alpha^{s-t} \left\{ \left[\left(\frac{\tilde{P}_{it}}{P_s} \right)^{1-\eta} a_s - u_s k_{is} - w_s h_{is} [1 + \nu(1 - R_s^{-1})] \right] + \text{mc}_{is} \left[z_s F(k_{is}, h_{is}) - \left(\frac{\tilde{P}_{it}}{P_s} \right)^{-\eta} a_s \right] \right\}.$$

The associated first-order condition with respect to $\tilde{P}_{it}$ is

$$E_t \sum_{s=t}^{\infty} r_{t,s} \alpha^{s-t} \left(\frac{\tilde{P}_{it}}{P_s} \right)^{-1-\eta} a_s \left[\text{mc}_{is} - \frac{\eta - 1}{\eta} \frac{\tilde{P}_{it}}{P_s} \right] = 0. \tag{18}$$

According to this expression, firms whose price is free to adjust in the current period, pick a price level such that some weighted average of current and future expected differences between marginal costs and marginal revenue equals zero.

2.4 Equilibrium and Aggregation

We limit attention to a symmetric equilibrium in which all firms that get to change their price in each period indeed choose the same price. We thus drop the subscript i. So the firm's demands for capital and labor aggregate to

$$\text{mc}_t z_t F_h(k_t, h_t) = w_t \left[1 + \nu \frac{R_t - 1}{R_t} \right] \tag{19}$$

and

$$\text{mc}_t z_t F_k(k_t, h_t) = u_t. \tag{20}$$

Similarly, the sum of all firm-level cash-in-advance constraints holding with equality yields the following aggregate relationship between real balances and the wage bill:

$$m_t = \nu w_t h_t. \tag{21}$$

From (4), it follows that the aggregate price index can be written as

$$P_t^{1-\eta} = \alpha P_{t-1}^{1-\eta} + (1 - \alpha) \tilde{P}_t^{1-\eta}$$

Dividing this expression through by $P_t^{1-\eta}$, one obtains

$$1 = \alpha \pi_t^{-1+\eta} + (1-\alpha)\tilde{p}_t^{1-\eta}, \tag{22}$$

where $\tilde{p}_t$ denotes the relative price of any good whose price was adjusted in period t in terms of the composite good.

At this point, most of the related literature using the Calvo-Yun apparatus, proceeds to linearize equations (18) and (22) around a deterministic steady state featuring zero inflation. This strategy yields the famous simple (linear) neo-Keynesian Phillips curve involving inflation and marginal costs (or the output gap). In the present study one cannot follow this strategy for two reasons. First, we do not wish to restrict attention to the case of zero long-run inflation. For we believe it is unrealistic, as it is contradicted by the post-war economic history of most industrialized countries. Second, we refrain from making the set of highly special assumptions that allow welfare to be approximated accurately from a first-order approximation to the equilibrium conditions. One of these assumptions is the existence of factor-input subsidies financed by lump-sum taxes aimed at ensuring the perfectly competitive level of long-run employment. Another assumption that makes it appropriate to use first-order approximations to the equilibrium conditions for welfare evaluation is that of a cashless economy. In the model under study we introduce a demand for money and calibrate its size to US postwar experience.

Our approach makes it necessary to retain the non-linear nature of the equilibrium conditions and in particular of equation (18). It is convenient to rewrite this expression in a recursive fashion that does away with the use of infinite sums. To this end, we define two new variables, x_t^1 and x_t^2. Let

$$\begin{aligned}
x_t^1 &\equiv E_t \sum_{s=t}^{\infty} r_{t,s}\alpha^{s-t}\left(\frac{\tilde{P}_t}{P_s}\right)^{-1-\eta} a_s \mathrm{mc}_s \\
&= \left(\frac{\tilde{P}_t}{P_t}\right)^{-1-\eta} a_t \mathrm{mc}_t + E_t \sum_{s=t+1}^{\infty} r_{t,s}\alpha^{s-t}\left(\frac{\tilde{P}_t}{P_s}\right)^{-1-\eta} a_s \mathrm{mc}_s \\
&= \left(\frac{\tilde{P}_t}{P_t}\right)^{-1-\eta} a_t \mathrm{mc}_t + \alpha E_t r_{t,t+1}\frac{\tilde{P}_t}{\tilde{P}_{t+1}} E_{t+1} \sum_{s=t+1}^{\infty} r_{t+1,s}\alpha^{s-t-1}\left(\frac{\tilde{P}_{t+1}}{P_s}\right)^{-1-\eta} a_s \mathrm{mc}_s \\
&= \left(\frac{\tilde{P}_t}{P_t}\right)^{-1-\eta} a_t \mathrm{mc}_t + \alpha E_t r_{t,t+1}\left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}}\right)^{-1-\eta} x_{t+1}^1 \\
&= \tilde{p}_t^{-1-\eta} a_t \mathrm{mc}_t + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}^{\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-1-\eta} x_{t+1}^1.
\end{aligned} \tag{23}$$

Similarly, let

$$
\begin{aligned}
x_t^2 &\equiv E_t \sum_{s=t}^{\infty} r_{t,s}\alpha^{s-t}\left(\frac{\tilde{P}_t}{P_s}\right)^{-1-\eta} a_s \frac{\tilde{P}_t}{P_s} \\
&= \tilde{p}_t^{-\eta} a_t + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}^{\eta-1}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2, \qquad (24)
\end{aligned}
$$

Using the two auxiliary variables x_t^1 and x_t^2, the equilibrium condition (18) can be written as:

$$
\frac{\eta}{\eta-1} x_t^1 = x_t^2. \qquad (25)
$$

Naturally, the set of equilibrium conditions includes a resource constraint. Such a restriction is typically of the type $z_t F(k_t, h_t) = c_t + i_t + g_t$. In the present model, however, this restriction is not valid. This is because the model implies relative price dispersion across varieties. This price dispersion, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. To see this, start with equilibrium condition (17) stating that supply must equal demand at the firm level:

$$
z_t F(k_{it}, h_{it}) = (c_t + i_t + g_t)\left(\frac{P_{it}}{P_t}\right)^{-\eta}.
$$

Integrating over all firms and taking into account that the capital-labor ratio is common across firms, we obtain

$$
h_t z_t F\left(\frac{k_t}{h_t}, 1\right) = (c_t + i_t + g_t)\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di,
$$

where $h_t \equiv \int_0^1 h_{it} di$ and $k_t \equiv \int_0^1 k_{it} di$ denote the aggregate per capita levels of labor and capital services in period t. Let $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$. Then we have

$$
\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \\
&= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} + (1-\alpha)\alpha\left(\frac{\tilde{P}_{t-1}}{P_t}\right)^{-\eta} + (1-\alpha)\alpha^2\left(\frac{\tilde{P}_{t-2}}{P_t}\right)^{-\eta} + \dots \\
&= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\tilde{P}_{t-j}}{P_t}\right)^{-\eta} \\
&= (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\pi_t^{\eta} s_{t-1}
\end{aligned}
$$

Summarizing, the resource constraint in the present model is given by the following two expressions

$$y_t = \frac{z_t}{s_t} F(k_t, h_t) \tag{26}$$

$$y_t = c_t + i_t + g_t \tag{27}$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \pi_t^{\eta} s_{t-1}, \tag{28}$$

with s_{-1} given. The state variable s_t summarizes the resource costs induced by the inefficient price dispersion present in the Calvo-Yun model in equilibrium.

Three observations are in order about the dispersion measure s_t. First, s_t is bounded below by 1. That is, price dispersion is always a costly distortion in this model. Second, in an economy where the non-stochastic level of inflation is nil, i.e., when $\pi = 1$, up to first order the variable s_t is deterministic and follows a univariate autoregressive process of the form $\hat{s}_t = \alpha \hat{s}_{t-1}$. Thus, the underlying price dispersion, summarized by the variable s_t, has no real consequences up to first order in the stationary distribution of endogenous variables. This means that studies that restrict attention to linear approximations to the equilibrium conditions around a noninflationary steady-state are justified to ignore the variable s_t. But this variable must be taken into account if one is interested in higher-order approximations to the equilibrium conditions or if one focuses on economies without long-run price stability ($\pi^* \neq 1$). Omitting s_t in higher-order expansions would amount to leaving out certain higher-order terms while including others. Finally, when prices are fully flexible, $\alpha = 0$, we have that $\tilde{p}_t = 1$ and thus $s_t = 1$. (Obviously, in a flexible-price equilibrium there is no price dispersion across varieties.).[6]

A stationary competitive equilibrium is a set of processes c_t, h_t, λ_t, w_t, τ_t^D, u_t, mc_t, k_{t+1}, R_t, i_t, y_t, s_t, $\tilde{p}_t$, π_t, τ_t, τ_t^L, ℓ_t, m_t, x_t^1, and x_t^2 for $t = 0, 1, \ldots$ that remain bounded in some neighborhood around the deterministic steady-state and satisfy equations (6)-(14), (19)-(28) and either $\tau_t^L = 0$ (in the absence of lump-sum taxation) or $\tau_t^D = 0$ (in the absence of distortionary taxation), given initial values for k_0, s_{-1}, and ℓ_{-1}, and exogenous stochastic processes g_t and z_t.

[6] Here we add a further note on aggregation. The variable $\tilde{\phi}_t$ introduced in the household's budget constraint (5) is related to aggregate profits, $\phi_t \equiv \int_0^1 \phi_{it} di$ by the relation $\tilde{\phi}_t = (1-\tau_t^D)\phi_t - \tau_t^D(1-R_t^{-1})m_t$. This relationship states that working-capital expenditures are not tax deductible. We introduce this twist in the tax code so that the base for distortionary taxation is simply value added, or aggregate demand, y_t.

3 Computation, Calibration, and Welfare Measure

We wish to find the monetary and fiscal policy rule combination that is optimal and implementable within the simple family defined by equations (13) and (14). For a policy to be implementable, we require that it ensure local uniqueness of the rational expectations equilibrium. In turn, for an implementable policy to be optimal, the contingent plans for consumption and hours of work associated with that policy must yield the highest level of lifetime utility, within the particular class of policy rules considered, given the current state of the economy. Formally, we look for implementable policies that maximize

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j U(c_{t+j}, h_{t+j}),$$

given that at time t all state variables take their steady-state values. That is to say, these policies are optimal conditional on the current state being the steady state.

3.1 Computation

Given the complexity of the economic environment we study in this paper, we are forced to characterize an approximation to lifetime utility. Up to first-order accuracy, V_t is equal to its non-stochastic steady-state value. Because all the monetary and fiscal policy regimes we consider imply identical non-stochastic steady states, to a first-order approximation all of those policies yield the same level of welfare. To determine the higher-order welfare effects of alternative policies one must therefore approximate V_t to a higher order than one. For an expansion of V_t to be accurate up to second order, it is in general required that the solution to the equilibrium conditions—the policy functions—also be accurate up to second order. In particular, approximations to the policy functions based on a first-order expansion of the equilibrium conditions would result in general in an incorrect second-order approximation of the welfare criterion V_t. In this paper, we compute second-order accurate solutions to policy functions using the methodology and computer code of Schmitt-Grohé and Uribe (2004).

In characterizing optimal policy we search over the coefficients α_R, α_π, and α_y of the monetary policy rule (14) and, when we consider fiscal policies other than balanced-budget rules, over the coefficient γ_1 of the fiscal policy rule (13).

3.2 Calibration

We compute a second-order approximation to the policy functions around the non-stochastic steady state of the model. The coefficients of the approximated policy functions are themselves functions of the deep structural parameters of the model. Therefore, one must assign numerical values to these structural parameters.

The time unit is meant to be a quarter. We calibrate the model to the U.S. economy. We assume that the period utility function is given by

$$U(c,h) = \frac{[c(1-h)^{\gamma}]^{1-\sigma} - 1}{1-\sigma}. \tag{29}$$

We set $\sigma = 2$, so that the intertemporal elasticity of consumption, holding constant hours worked, is 0.5. In the business-cycle literature, authors have used values of $1/\sigma$ as low as 1/3 (e.g., Rotemberg and Woodford, 1992) and as high as 1 (e.g., King, Plosser, and Rebelo, 1988). Our choice of σ falls in the middle of this range.

The production function is assumed to be of the Cobb-Douglas type

$$F(k,h) = k^{\theta}h^{1-\theta},$$

where θ describes the cost share of capital. We set θ equal to 0.3, which is consistent with the empirical regularity that in the U.S. economy wages represent about 70 percent of total cost.

We assign a value of 0.9902 to the subjective discount factor β, which is consistent with an annual real rate of interest of 4 percent (Prescott, 1986). We set η, the price elasticity of demand, so that in steady state the value added markup of prices over marginal cost is 28 percent (see Basu and Fernald, 1997). We require the share of government purchases in value added to be 17 percent in steady state, which is in line with the observed U.S. postwar average. The steady-state inflation rate is assumed to be 4.2 percent per year. This value is consistent with the average U.S. GDP deflator growth rate over the period 1960-1998. The annual depreciation rate is taken to be 10 percent, a value typically used in business-cycle studies.

Based on the observations that two thirds of M1 are held by firms (Mulligan, 1997) and that annual GDP velocity is 0.17 in U.S. data (for a 1960 to 1999 sample), we calibrate the ratio of working capital to quarterly GDP to $0.45 (= 0.17 \times 2/3 \times 4)$. This parameterization implies that $\nu = 0.82$, which means that firm's must pay 82 percent of their wage bill with cash.

We set the ratio of tax revenues to GDP to 0.2, which is consistent with the 1997-2001 average of the US federal budget receipts to GDP ratio.[7]

[7]Together with the assumed value for the share of government purchases in value added, the value assigned to the tax-to-GDP ratio implies a long-run debt-to-GDP ratio of about 90 percent. This value is high relative to the US out-of-war experience, but closer to what is observed in other G7 countries. A lower steady-state

Following Sbordone (2002) and Galí and Gertler (1999), we assign a value to α, the fraction of firms that cannot change their price in any given quarter, that implies that on average firms change prices every 3 quarters. We set the preference parameter γ so that in the simple economy without money and lump-sum taxes, agents allocate on average 20 percent of their time to work, as is the case in the U.S. economy according to Prescott (1986). Given the other calibrated parameters and the steady-state conditions, the implied value of γ is 3.4080. The associated Frisch elasticity of labor supply then is about 1.5, which lies well within the range of values typically used in the real business cycle literature.

We equate the parameters R^*, π^*, and y appearing in the monetary policy rule (14) to the steady-state values of R, π, and y, respectively.

Government purchases are assumed to follow a univariate autoregressive process of the form

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_t^g,$$

where $\hat{g}_t \equiv [\ln g_t - \ln G]$ denotes the percentage deviation of government purchases from steady state and G denotes the steady-state level of government purchases. The first-order autocorrelation, ρ_g, is set to 0.9 and the standard deviation of ϵ_t^g to 0.0074. The second source of uncertainty in the model are productivity shocks. They are also assumed to follow a univariate autoregressive process

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z,$$

where $\rho_z = 0.82$ and the standard deviation of ϵ_t^z is 0.0056. Table 1 summarizes the calibration of the model.

3.3 The Welfare Measure

We measure the level of utility associated with a particular monetary and fiscal policy specification as follows. Let the contingent plans for consumption and hours associated with a particular monetary and fiscal regime be denoted by c_t^r and h_t^r. Then we measure welfare as the conditional expectation of lifetime utility as of time zero, that is,

$$\text{welfare } = V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^r, h_t^r).$$

In addition, we assume that at time zero all state variables of the economy equal their respective steady-state values. Note that we are departing from the usual practice of iden-

debt-to-GDP ratio could be accommodated by allowing for government transfers.

Table 1: Calibrated Parameters

Parameter	Value	Description
$1/\sigma$	$\frac{1}{2}$	Intertemporal elasticity of consumption, $U(c,h) = \frac{[c(1-h)^{\gamma}]^{1-\sigma}-1}{1-\sigma}$
θ	0.3	Cost Share of capital, $F(k,h) = k^{\theta}h^{1-\theta}$
β	$1.04^{-1/4}$	Quarterly subjective discount rate
η	5	Price elasticity of demand
s_g	0.17	Steady-state share of government purchases, $\frac{g}{y}$
π^*	$1.042^{(1/4)}$	Gross quarterly inflation rate
δ	$1.1^{(1/4)} - 1$	Quarterly depreciation rate
s_m	$0.17 \times \frac{2}{3} \times 4$	Ratio of M1 held by firms to quarterly GDP
α	$\frac{2}{3}$	Share of firms that can change their price each period
γ	3.4080	Preference Parameter
s_τ	0.2	Steady-state tax revenue to GDP ratio
ρ_g	0.9	first-order serial correlation of g_t
σ^{ϵ^g}	0.0074	Standard Deviation of government purchases shock
ρ_z	0.82	first-order serial correlation of z_t
σ^{ϵ^z}	0.0056	Standard Deviation of technology shock

tifying the welfare measure with the unconditional expectation of lifetime utility. Because different policy regimes will in general be associated with a different stochastic steady state, using unconditional expectations of welfare amounts to not taking into account the transitional dynamics leading to the stochastic steady state. Because the non-stochastic steady state is the same across all policy regimes we consider, our choice of computing expected welfare conditional on the initial state being the nonstochastic steady state ensures that the economy begins from the same initial point under all possible polices. Therefore, our strategy will deliver the constrained optimal monetary/fiscal rule associated with a particular initial state of the economy. It is of interest to investigate the robustness of our results with respect to alternative initial conditions. For, in principle, the welfare ranking of the alternative polices will depend upon the assumed value for (or distribution of) the initial state vector.[8]

We compute the welfare cost of a particular monetary and fiscal regime relative to the optimized rule as follows. Consider two policy regimes, a reference policy regime denoted by r and an alternative policy regime denoted by a. Then we define the welfare associated with policy regime r as

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^r, h_t^r),$$

where c_t^r and h_t^r denote the contingent plans for consumption and hours under policy regime

[8]For further discussion of this issue, see Kim et al., 2003.

r. Similarly, define the welfare associated with policy regime a as

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a, h_t^a).$$

Let λ denote the welfare cost of adopting policy regime a instead of the reference policy regime r. We measure λ as the fraction of regime r's consumption process that a household would be willing to give up to be as well off under regime a as under regime r. Formally, λ is implicitly defined by

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda)c_t^r, h_t^r).$$

For the particular functional form for the period utility function given in equation (29), the above expression can be written as

$$\begin{aligned} V_0^a &= E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda)c_t^r, h_t^r) \\ &= (1-\lambda)^{1-\sigma} V_0^r + \frac{(1-\lambda)^{1-\sigma} - 1}{(1-\sigma)(1-\beta)}. \end{aligned}$$

Solving for λ we obtain the following expression for the welfare cost associated with policy regime a vis-á-vis the reference policy regime r in percentage terms

$$\text{welfare cost} = \lambda \times 100 = \left[1 - \left(\frac{(1-\sigma)V_0^a + (1-\beta)^{-1}}{(1-\sigma)V_0^r + (1-\beta)^{-1}}\right)^{1/(1-\sigma)}\right] \times 100. \tag{30}$$

4 A Cashless Economy

We first consider a non-monetary economy by setting

$$\nu = 0$$

in equation (15). The fiscal authority is assumed to have access to lump-sum taxes and to follow a balanced-budget rule. That is, the fiscal policy rule is given by equations (12) and (13) with

$$\gamma_0 = \gamma_1 = \tau_t^D = 0,$$

and

$$\gamma_2 = 1.$$

This case is of interest for it most resembles the case studied in the related literature on optimal policy (see Clarida, Galí, and Gertler, 1999, Woodford, 2003, chapter 4, and the references cited therein). This body of work studies optimal monetary policy in the context of a cashless economy with nominal rigidities and no fiscal authority. For analytical purposes, the absence of a fiscal authority is equivalent to modeling a government that operates under a perpetual balanced-budget rule and collects all of its revenue via lump-sum taxation. We wish to highlight, however, two important differences between the economy studied here and the one typically considered in the related literature. Namely, in our economy there is capital accumulation and there do not exist subsidies to factor inputs that undo the distortions arising from monopolistic competition. The latter difference is of consequence for the solution method that can be applied to the optimal policy problem. As shown by Woodford (2003, chapter 6), one can use a first-order approximation to the policy function to obtain an accurate second-order approximation to the utility function under certain assumptions. One of the necessary assumptions is that the government has access to factor input subsidies to undo the monopolistic distortion. Without this ad-hoc subsidy scheme, first-order approximations to the policy functions no longer deliver a second-order accurate approximation to the utility function. Thus, in this case one must approximate the policy functions up to second order to obtain a second-order accurate approximation to the level of welfare, which is what we do in this paper.

The top panel of table 2 presents the coefficients of some optimized policy rules and of some other monetary policy specifications. For this economy, we consider five different monetary policies. Two of those are constrained optimal rules. In one case, we search over the monetary feedback rule coefficients α_π and α_y while restricting α_R to be zero. This case is labeled no smoothing in the table. For each parameter we search over a grid from -3 to 3 with a step of 0.1, that is, we consider 61 values for each parameter. We find that the best no-smoothing rule requires that the monetary authority not respond to output and choose an inflation coefficient of 3. Note that this is the largest value of α_π that we allow in our search. Our conjecture is that if we left this parameter unconstrained, then optimal policy would call for an arbitrarily large inflation coefficient.[9] The reason is that in that case under the optimal policy inflation would in effect be forever constant so that the economy would be characterized by zero inflation volatility.

One might wonder why the representative household prefers to live in a world with constant positive inflation rather than in one with varying inflation. This question is motivated by the fact that the non-stochastic steady-state level of inflation in our model is positive, which means that the distortions introduced by price stickiness are present even in the

[9]We experimented enlarging the α_π range up to $[-7, 7]$. We found that the optimal rule always picks the highest value allowed for the inflation coefficient.

Table 2: Optimal Interest-Rate Rules in the Sticky-Price Model

Interest-Rate Rule	$\hat{R}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \alpha_R \hat{R}_{t-1}$					
	α_π	α_y	α_R	γ_1	Welfare	Welfare Cost
No Money, Lump-Sum Taxes, Balanced Budget ($\nu = \tau_t^D = \gamma_0 = \gamma_1 = 0$; $\gamma_2 = 1$)						
No smoothing	3	0	–	–	-628.2193	0.0002
Smoothing	3	0	0.9	–	-628.2180	0
Inflation Targeting ($\hat{\pi}_t = 0$)	–	–	–	–	-628.2175	-0.00007
Taylor Rule	1.5	0.5	–	–	-634.1565	0.8061
Simple Taylor Rule	1.5	0	–	–	-628.2383	0.0028
Money, Lump-Sum Taxes, Balanced Budget ($\nu = 0.82$, $\tau_t^D = \gamma_0 = \gamma_1 = 0$, $\gamma_2 = 1$)						
No smoothing	3	0	–	–	-629.6905	0.0002
Smoothing	3	0	0.9	–	-629.6892	0
Inflation Targeting ($\hat{\pi}_t = 0$)	–	–	–	–	-629.6889	-0.00005
Taylor Rule	1.5	0.5	–	–	Too close to bifurcation	
Simple Taylor Rule	1.5	0	–	–	-629.7077	0.0025
Fiscal Feedback Rule: $\tau_t^L = 0.2 + \gamma_1(\ell_{t-1} - \ell)$; ($\nu = 0.82$, $\tau_t^D = \gamma_2 = 0$)						
Optimized Rule	3	0	–	1.9*	-629.6905	0
Inflation Targeting ($\hat{\pi}_t = 0$)	–	–	–	1.9*	-629.6889	-0.0002
Money Growth Rate Peg ($M_{t+1} = \mu M_t$)	–	–	–	1.9*	-629.7319	0.0057
Simple Taylor Rule	1.5	0	–	1.9*	-629.7077	0.0023
Distorting Taxes: $\tau_t^D y_t = \gamma_0 + \gamma_1(\ell_{t-1} - \ell)$. ($\nu = 0.82$, $\tau_t^L = 0$, $\gamma_2 = 0$)						
Optimized Rule	-3	0.1	–	-3	-710.7907	0
Taylor Rule	1.5	0.5	–		Too close to bifurcation	
Simple Taylor Rule	1.5	0	–	0.1	-710.7978	.0009
Inflation Targeting ($\hat{\pi}_t = 0$)	–	–	–	0.1	-710.7558	-0.0043

Notes: (1) R_t denotes the gross nominal interest rate, π_t denotes the gross inflation rate, and y_t denotes output. (2) For any variable x_t, its non-stochastic steady-state value is denoted by x, and its log-deviation from steady state by $\hat{x}_t \equiv \ln(x_t/x)$. (3) In all cases, the parameters α_π, α_y, and α_R are restricted to lie in the interval $[-3, 3]$. (4) Welfare is defined as follows: Let $V(g_t, z_t, R_{t-1}, \ell_{t-1}, s_{t-1}, k_t)$ denote the equilibrium level of lifetime utility of the representative household in period t given that period's state $(g_t, z_t, R_{t-1}, \ell_{t-1}, s_{t-1}, k_t)$. Then welfare is defined as $V(g, z, R, \ell, s, k)$. (5) The welfare cost is measured relative to optimized rule and is defined as the percentage decrease in the consumption process associated with the optimal rule necessary to make the level of welfare under the optimized rule identical to that under the considered policy. Thus, a positive figure indicates that welfare is higher under the optimized rule than under the alternative policy.

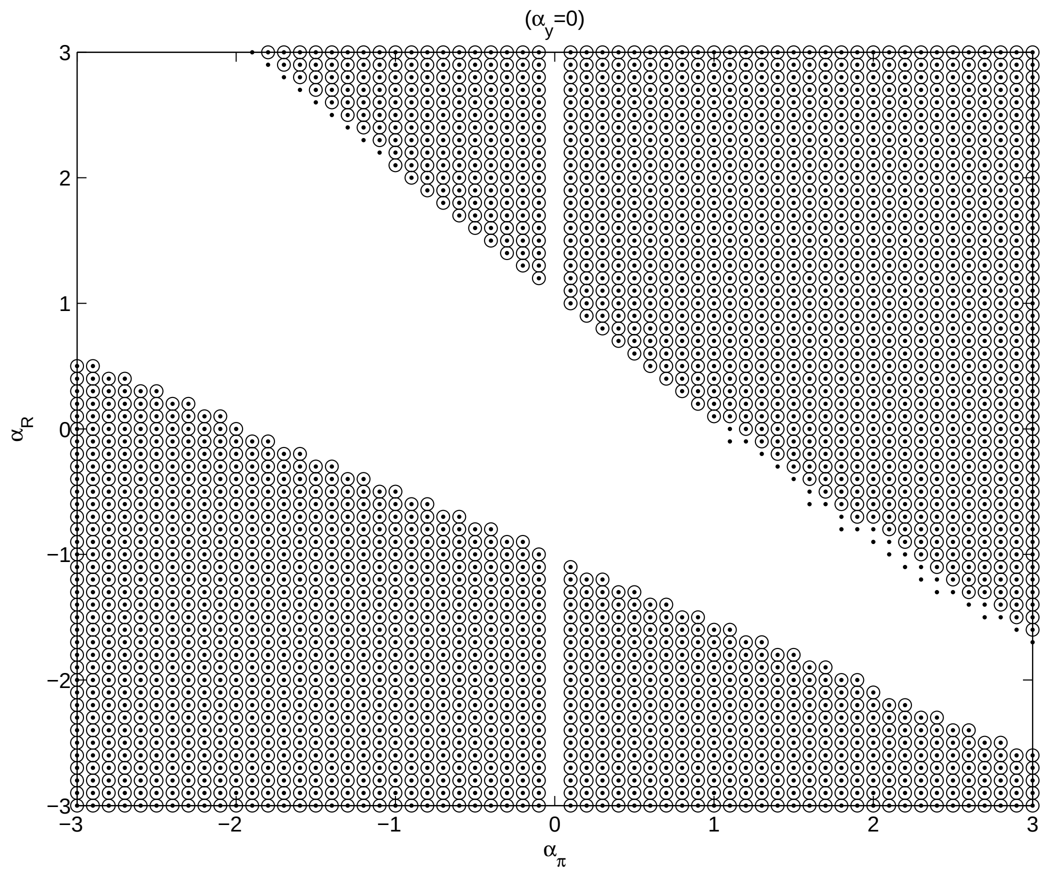
* In the economy with a fiscal feedback rule for lump-sum taxes, any passive fiscal policy yields the identical level of welfare, that is, any $\gamma_1 \in [0.1, 1.9]$ is optimal.

steady state. Some intuition for why constant inflation is optimal when the long-run level is constrained exogenously to be positive can be gained from the fact that in our model the non-stochastic steady-state level of welfare is globally concave in the steady-state inflation rate with a maximum at zero inflation. Thus, loosely speaking households dislike to randomize around the constant level of long-run inflation.

We next study a case in which the central bank can smooth interest rates over time, formally, we allow the coefficient α_R on the lagged interest rate to take any value between -3 and 3. Our grid search yields that the optimal policy coefficients are $\alpha_\pi = 3$, $\alpha_y = 0$, and $\alpha_R = 0.9$. These coefficients imply that the long-run coefficient on inflation is 30, the largest value it can take given our grid size. So, again, as in the case without smoothing optimal policy calls for a large response to inflation deviations in order to stabilize the inflation rate and for no response to deviation of output from the steady state. The level of welfare associated with this policy is -628.2180. This is slightly higher than -628.2193, the level of welfare associated with the optimal policy without smoothing. But the difference is not very large. As shown in column 7 of table 2, agents would be willing to give up just 0.0002, that is, 2 one-thousands, of one percent of their consumption stream under the optimized rule with smoothing to be as well off as under the optimized policy without smoothing. For all practical purposes we regard this difference in the level of welfare as negligible.

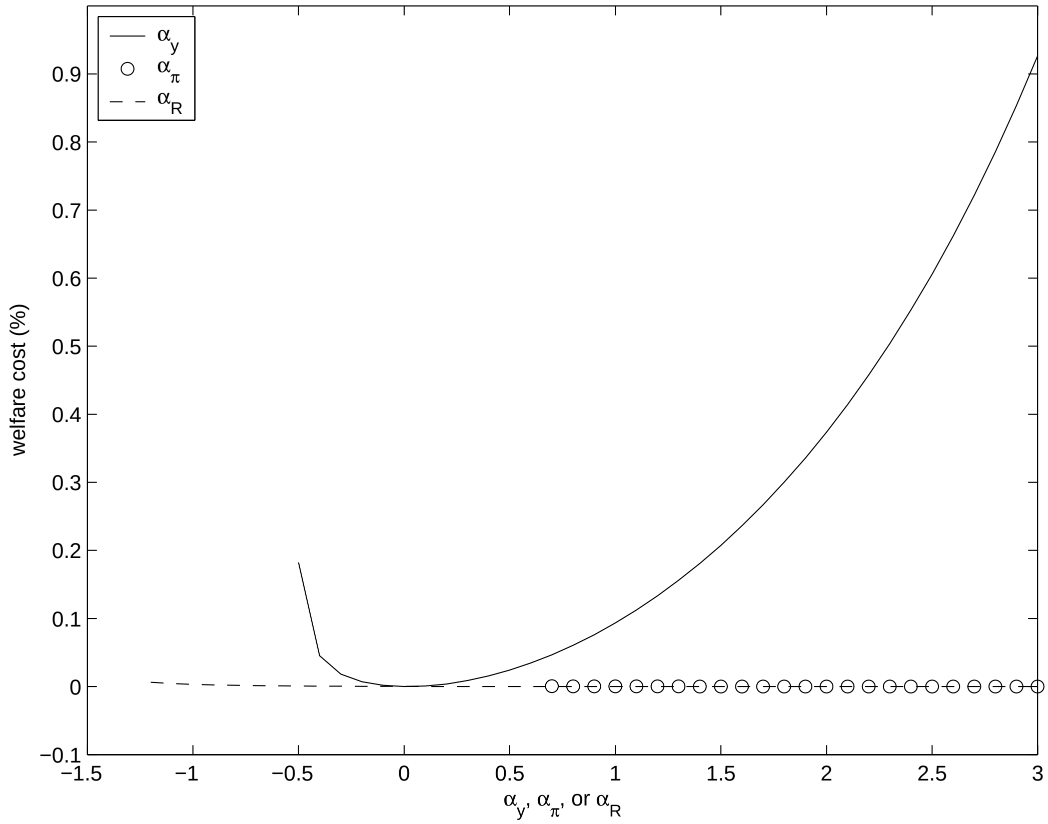
This finding let us to investigate by how much welfare indeed changes as we vary the coefficients of the policy rule. Figure 1 shows that given that the central bank does not respond to output, $\alpha_y = 0$, varying α_π and α_R between the -3 and 3 typically leads to welfare losses of less than five one-hundredth of one percent. The graph shows with a dot the combinations of α_π and α_R that render the rational expectations equilibrium determinate and with a circle the combinations for which the welfare costs are less than 0.05 percent. The figure makes two important points. First, it shows that there are quite a large number of α_π and α_R combinations for which the equilibrium fails to be locally unique (the blank area in the figure). This is for example the case for positive values of α_π and α_R such that the policy stance is passive in the long run, that is, for α_π and α_R combinations such that $0 < \alpha_\pi/(1-\alpha_R) < 1$. This finding is consistent with those obtained in economic environments that abstract from capital accumulation. It is thus reassuring that this particular abstraction appears to be of no consequence for the finding that long-run passive policy is inconsistent with local uniqueness of the rational expectations equilibrium. Similarly, with rules in which the response to inflation and past interest rates is positive we find that determinacy obtains for policies that are active in the long run ($\alpha_\pi/(1-\alpha_R) > 1$). Second, and more importantly, the graph shows that basically all parameterization of the monetary feedback rule that deliver determinacy yield welfare differences in the order of at most five one-hundredth of one percent

Figure 1: Determinacy Regions and Welfare in the Cashless Economy



Note: A dot represents a parameter combination for which the equilibrium is determinate. A circle denotes that the welfare cost of the policy relative to the optimal policy (i.e. $\alpha_\pi = 3$, $\alpha_y = 0$, and $\alpha_R = 0.9$) is less than 0.05 percent.

Figure 2: The Importance of Not Responding to Output: The Cashless Economy



Note: The welfare cost is relative to the optimized policy rule, i.e., $\alpha_\pi = 3$, $\alpha_y = 0$, and $\alpha_R = 0.9$. See equation (30).

of the consumption stream associated with the optimized rule. This implies a simple policy prescription, namely, that any parameter combination that implies that the policy stance is active in the long run is equally desirable from a welfare point of view.

One possible reaction to the finding that determinacy preserving variations in α_π and α_R have little welfare consequences may be that in the class of model we consider welfare is always very flat in a rather large neighborhood around optimum, so that it does not really matter what the government does. However, this is not the case in our economy. Recall that in the welfare calculations underlying figure 1 the response coefficient on output, α_y, was kept constant at zero.

Figure 2 studies the consequences of varying α_y. It demonstrates that the welfare costs of varying α_y can be large, thus it underlines the importance of not responding to output. The solid line shows the welfare cost of deviating from the optimal output coefficient, which is

$\alpha_y = 0$. For positive α_y, the welfare cost of the suboptimal rule is monotonically increasing in α_y. For values of $\alpha_y = 1$, the welfare cost is one tenth of one percent of the consumption stream associated with the optimized rule. For negative output response coefficients, the welfare cost also rapidly rises. For an α_y of -0.5 the welfare cost is already two tenth of one percent. For values below -0.5, the equilibrium ceases to be locally unique and thus the solid line ends.

To make the importance of not responding to output more transparent, the graph also shows the welfare consequences of varying either α_π, shown with the circled line, or α_R, shown with the dashed line. For both parameters the welfare costs are negligible so that in the scale of the graph they appear to be indistinguishable from zero. Thus these findings suggest that bad policy can have huge welfare costs in our model and that big policy mistakes are committed when policy makers are unable to resist the temptation to respond to output fluctuations. It follows that good policy calls for sticking to the basics of responding to inflation alone.

Our arguments presented above suggest that a policy of complete inflation stabilization may be the optimal policy prescription in our economy. Thus, we were led to compute the level of welfare associated with inflation targeting. Under inflation targeting the central bank is assumed to do something that results in a constant inflation rate over the business cycle. We do not discuss how such a policy may actually be implemented. The level of welfare for this regime is -628.2175, which is higher than the level of welfare associated with the optimized rule with smoothing. But the welfare benefit is only 0.00007, which means that one would have to raise the consumption stream under the optimized rule by 0.00007 percent to make agents as happy as they are under an inflation targeting regime.

Finally, we show the welfare costs associated with a Taylor rule featuring an inflation coefficient of 1.5 and an output coefficient of either 0.5 or of 0. In the former case, the welfare costs are large (0.8 percent) as expected from the analysis presented in figure 2 whereas in the latter case the welfare costs are negligible as was already implicit in figure 1.

5 A Monetary Economy

We next introduce money into the model by assuming that the parameter ν denoting the fraction of the wage bill that must be cash financed takes the value shown on table 1. All other aspects of the model, including the fiscal policy specification, are as in the cashless economy analyzed in the previous section. Unlike in the cashless economy, in this model complete inflation stabilization may not continue to be optimal because it is associated with fluctuations in the nominal interest rate, which in turn now distort the effective wage rate via the working-capital constraint. So, there will be a trade off between inflation stabilization to neutralize the distortions stemming from sluggish price adjustment and nominal interest rate stabilization to dampen the distortions introduced by the working capital constraint.

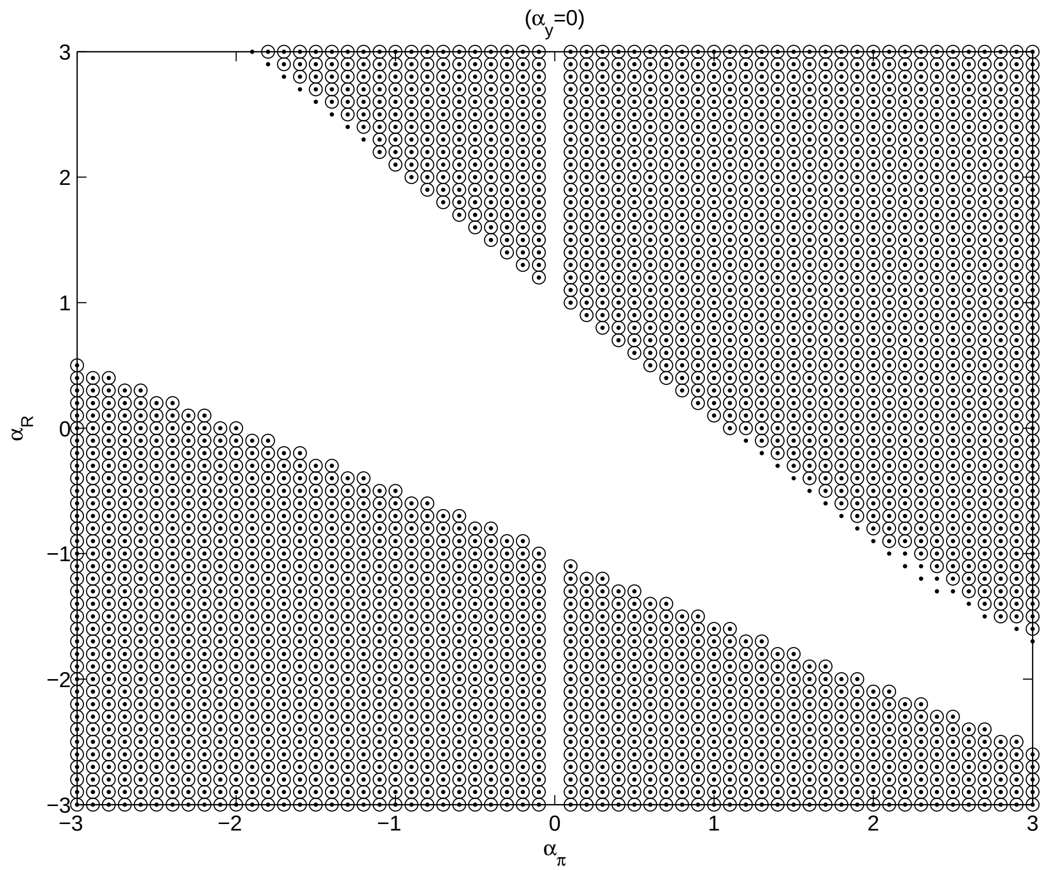
This tradeoff, however, does not seem to be quantitatively important. In effect, when we search over the coefficients of the interest rate feedback rule, α_π, α_y, and α_R, we recover the same optimal coefficient values as in the economy without money, that is, α_π takes the largest value included in our grid, 3, the output coefficient is zero, $\alpha_y = 0$, and the central bank makes intensive use of interest rate smoothing, $\alpha_R = 0.9$. The level of welfare under the optimal rule is -629.6892.[10] If we do not allow for interest rate smoothing, that is, if we constrain α_R to be zero, it is still optimal not to respond to output, $\alpha_y = 0$ and to make the inflation response of the interest rate as large as possible ($\alpha_\pi = 3$). Utility falls slightly to -629.6905. The welfare cost of eliminating smoothing is just 0.0002 percent of consumption, which is again economically negligible.

As in the cashless case, we find that the precise magnitude of the inflation coefficient and the smoothing coefficient play no rule, provided that they imply a locally unique rational expectations equilibrium and α_y is held at zero. This point is clearly communicated by figure 3. As before, a dot in the figure indicates that this particular (suboptimal) combination of α_π and α_R results in a determinate equilibrium and a circle indicates that the welfare cost associated with it is less than 0.05 percent of the optimal consumption stream. Variations in the output response coefficient of the interest rate feedback rule, α_y continue to be associated with large welfare losses particularly if α_y is large. Figure 4 plots with a solid line the welfare losses as a function of α_y. Equilibrium is locally unique only for values of α_y between -0.3 and 2.4, given $\alpha_\pi = 3$ and $\alpha_R = 0.9$. The welfare costs exceed 0.05 percent for α_y greater than 0.6. Consider $\alpha_y = 0.6$. Then, given $\alpha_R = 0.9$ the long-run coefficient on output is 6 and the welfare loss is only 0.0424 percent. On the other hand, for $\alpha_y = 2$, for example, the welfare cost is 1.15 percent of consumption, which is a relatively large number. By contrast, variations in α_R and α_y over the range $[-3, 3]$ lead to welfare costs of at most 0.0013 and 0.0004 percent, respectively.

A further similarity between the cashless and the cash-in-advance economies is that inflation targeting dominates all other policies considered. In sum, in this economy, the tradeoff between inflation stabilization and interest rate stabilization introduced by nominal rigidities

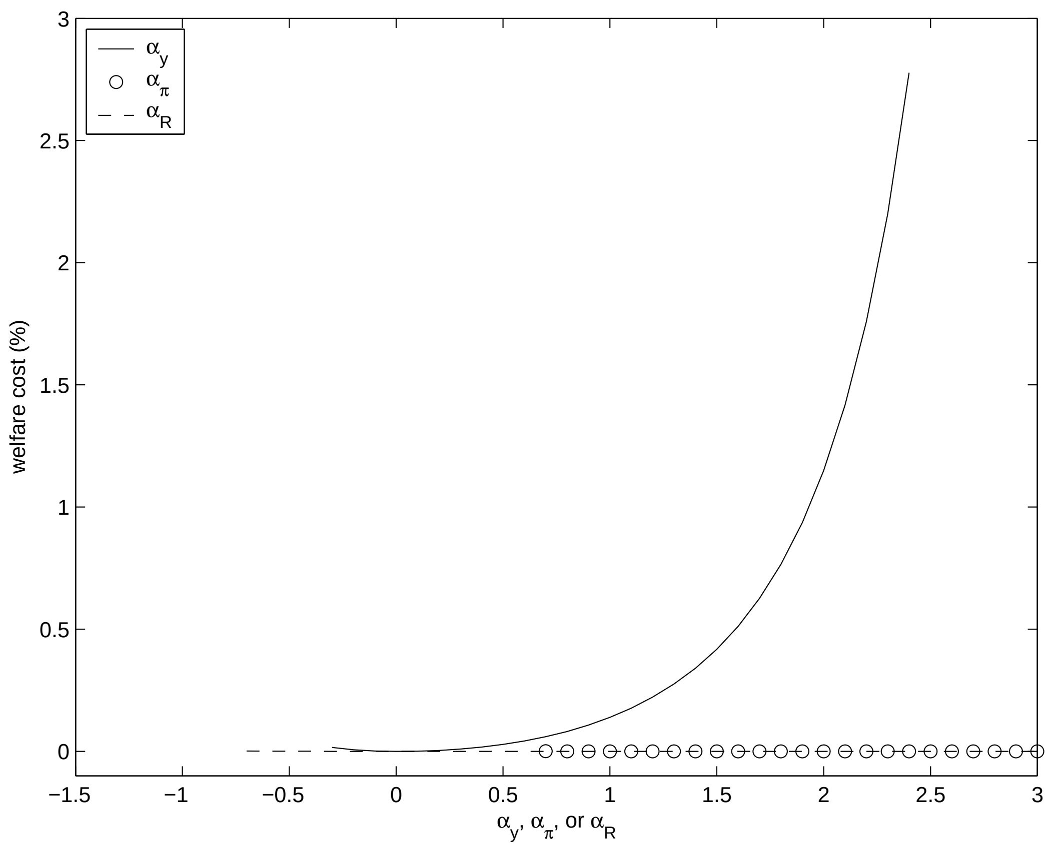
[10] In this economy the deterministic steady-state level of welfare is -629.7040 compared to -628.2323 for the economy without money. Given our assumption that the nominal interest rate is positive in the non-stochastic steady state welfare must be lower in the economy with money than in the one without money. Both in the cashless economy and in the model with money, welfare under the optimized rule is higher than in the non-stochastic steady state. The reason must be that the presence of monopolistic competition induces higher output and consumption on average in a stochastic economy than in a non-stochastic one.

Figure 3: Determinacy Regions and Welfare in the Monetary Economy



Note: See note to figure 1.

Figure 4: The Importance of Not Responding to Output: The Monetary Economy



Note: See note to figure 2.

on the one hand and the monetary exchange friction on the other hand, is overwhelmingly resolved in favor of inflation stabilization.

6 An Economy With A Fiscal Feedback Rule (To be Completed)

Thus far, we have restricted attention to the case of a fiscal authority that takes a passive stance in the sense that fiscal policy has no effect on the price level and inflation. The motivation for this treatment of fiscal policy is in part that this is what is typically assumed in the related literature. But it is worthwhile to ask whether from a welfare point of view a passive fiscal policy stance is desirable and moreover even if it turns out that optimal policy calls for a passive fiscal stance, it is of interest to know how close one can get to the level of welfare associated with the optimized monetary and fiscal rule in a world where fiscal policy is active. For this reason, in this section, we study a simple fiscal policy rule that allows for the possibility that fiscal policy is either active or passive. The rule is similar to the one studied in Leeper's (1991) seminal paper. Initially, we assume, as we did in previous sections, that the fiscal authority can levy lump-sum taxes. However, the level of lump-sum taxes is no longer assumed to be set so as to balance the budget but rather according to the rule

$$\tau_t^L = \gamma_0 + \gamma_1(\ell_{t-1} - \ell^*).$$

That is, fiscal policy is defined by equations (12) and (13) with $\tau_t^D = 0$ for all t and $\gamma_2 = 0$. Combining the above fiscal policy with the government sequential budget constraint, equation (11), one obtains $\ell_t = R_t/\pi_t(1 - \pi_t\gamma_1)\ell_{t-1} + \text{rest}$. Loosely speaking, this expression states that the feedback parameter γ_1 controls the rate of growth of total real government liabilities. If $1 - \gamma_1\pi^*$ is less than one in absolute value, then real government liabilities grow at a rate less than the real rate of interest. In this case, fiscal solvency is guaranteed regardless of the stance of monetary policy and fiscal concerns play no role for the determination of the price level, that is, fiscal policy is passive. On the other hand, if $1 - \gamma_1\pi^*$ is greater than unity in absolute value, then the size of government liabilities grows without bounds in absolute value. In this case, existence of a stationary equilibrium requires that the initial price level adjusts to a value that is consistent with a bounded path for government liabilities. This would be an example of an active, or Non-Ricardian fiscal policy.

To save on computing time, in this section, we only consider interest rate feedback rules that depend on the current value of inflation and output. That is, we restrict α_R to be equal to zero in equation (14).[11]

The third panel of table 2 presents the numerical results. We find that the optimal monetary/fiscal rule combination features an active monetary policy and a passive fiscal policy. The optimal coefficients are $\alpha_\pi = 3$, $\alpha_y = 0$, and any $\gamma_1 \in [0.1, 1.9]$. Under the optimal policy, utility is equal to -629.6905, slightly above the steady state level of -629.7040. Note that the level of utility under the optimized rule is the same as in the monetary economy discussed in the previous section. This is because if fiscal policy is passive and taxation is lump-sum—which is the case in the economies analyzed in this and the previous sections—then the real allocation is the same regardless of the precise nature of the passive fiscal policy. It follows that any feedback rule coefficient γ_1 such that fiscal policy is passive (i.e., values of γ_1 satisfying $|1 - \gamma_1 \pi^*| < 1$, or, under our calibration (and grid size), $0 < \gamma_1 < 1.9$) implement, ceteris paribus, the same real allocation as the balanced-budget rule analyzed in the previous section.

The intuition for why the optimal monetary and fiscal rule combination features passive fiscal and active monetary policy as opposed to active fiscal and passive monetary policy is the following. Recall that this is an economy in which the government has access to lump-sum taxation. Thus, strategies to ensure fiscal solvency that involve the use of lump-sum taxes should be non-distorting. Under passive fiscal policy this is exactly what happens. If government liabilities are, say, above their target level, then lump-sum taxes are increased and with time government liabilities return to their long-run level. A rather different strategy for bringing about fiscal solvency is to use unexpected variations in the price level as a lump-sum tax/subsidy on nominal asset holdings of private households. This is what happens under active fiscal policy. For example, consider the simple case in which $\gamma_1 = 0$, so that primary fiscal deficits are exogenous, and monetary policy is passive pegging the nominal interest rate. The only way in which fiscal solvency of the government can be brought about in this case is through variations in real government liabilities, which in turn require and adjustment in the price level. However, in the economy under study unexpected movements in the price level increase the distortions stemming from the presence of nominal rigidities. This is why this strategy of reigning in government finances is distorting. For these reasons, from a qualitative point of view, optimal policy is one in which the non-distorting rather than the distorting fiscal instrument is chosen.

Furthermore, we find that under inflation targeting for an equilibrium to exist and be locally unique fiscal policy must be passive, that is, $\gamma_1 \in [.1, 1.9]$. This is because under

[11] To determine the optimal monetary and fiscal policy stance we search over a grid of values of α_π, α_y and γ_1 in the interval $[-3, 3]$ with a step size of 0.1. This involves computing the conditional expectation of welfare, V_t, 226,981 times. Were we to consider in addition, the possibility of interest-rate smoothing, we would have to compute welfare for $61^4 = 13,845,841$ possible parameter combinations.

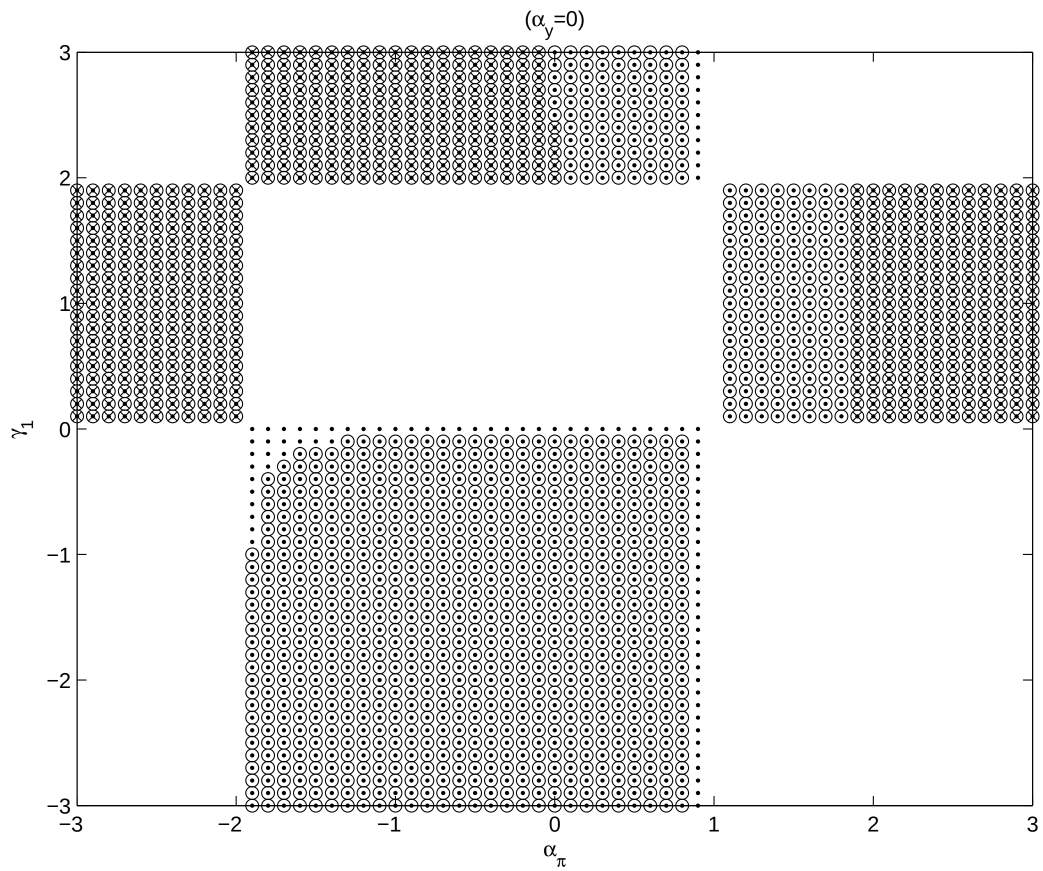
inflation targeting variations in the price level are unavailable as fiscal instruments. Again, for any passive fiscal policy the real allocation is the same, as it should be. As in the previous sections, under inflation targeting the level of welfare is marginally higher than under the optimized rule. Under a simple Taylor rule that responds only to inflation (with a coefficient of 1.5) optimal fiscal policy is passive with $\gamma_1 \in [.1, 1.9]$. Welfare under the simple Taylor rule is slightly below the level of welfare associated with the constrained optimal rule. But the difference is small. A decrease of a mere 0.0023 percent in the optimal consumption stream leaves agents with the same utility than under the Taylor rule.

We now turn our attention to the question how costly from a welfare point of view it is to follow a rule within the general class we consider that is not the optimal rule. In general, figure 5 shows that variations in α_π and γ_1 have little effect of the level of welfare, provided α_y is held constant at 0. The figure shows with dots the fiscal/monetary rule parameter combinations that result in a locally unique equilibrium. In the positive orthant, we see that equilibrium is determinate only for combinations of active fiscal policy and passive monetary policy or a combination of passive fiscal policy and active monetary policy. Clearly, one requirement for sound policymaking is that the decision makers agree on a joint monetary-fiscal policy that renders the equilibrium unique. In the absence of any such coordination, the policies fail to have their intended effects because equilibrium may either not exist or if it exists, it may not be unique. And as the graph shows there are many parameterizations of policy for which this undesirable outcome holds.

The figure also conveys the idea that if a particular policy combination ensures determinacy, it is likely that it yields almost the same level of welfare as that associated with the optimized policy rules. Specifically, figure 5 shows with a circle values for the feedback parameter α_π and the fiscal rule parameter γ_1 such that the welfare cost of that policy is at most 0.05 percent. Even if we require a policy configuration to be associated with at most a 0.001 percent welfare cost (shown with a crossed circle), the figure shows that there are many such combinations. In particular as long as α_{pi} is greater than 1.8 and fiscal policy is passive, welfare is within one thousands of percent of the optimal rule. That is, from a welfare point of view it does not matter whether, given a passive fiscal policy stance, whether $\alpha_\pi = -3$ or $+3$.

Also, note that there exist some parameter constellations that imply welfare costs of below one one-thousands of one percent of the optimal consumption stream and feature an active fiscal policy. In particular, for a pure interest rate peg, $\alpha_\pi = 0$, and γ_1 values between 2 and 2.4 this is the case. Given our previous discussion of the intuition for why passive fiscal policy is optimal, this result is somewhat surprising. But here is exactly where the contribution of our paper lies. We ask quantitatively how harmful are policies other

Figure 5: Determinacy Regions and Welfare in the Model with a Fiscal Feedback Rule for Lump-Sum Taxes ($\tau_t^L = \gamma_0 + \gamma_1(\ell_{t-1} - \ell^*)$)



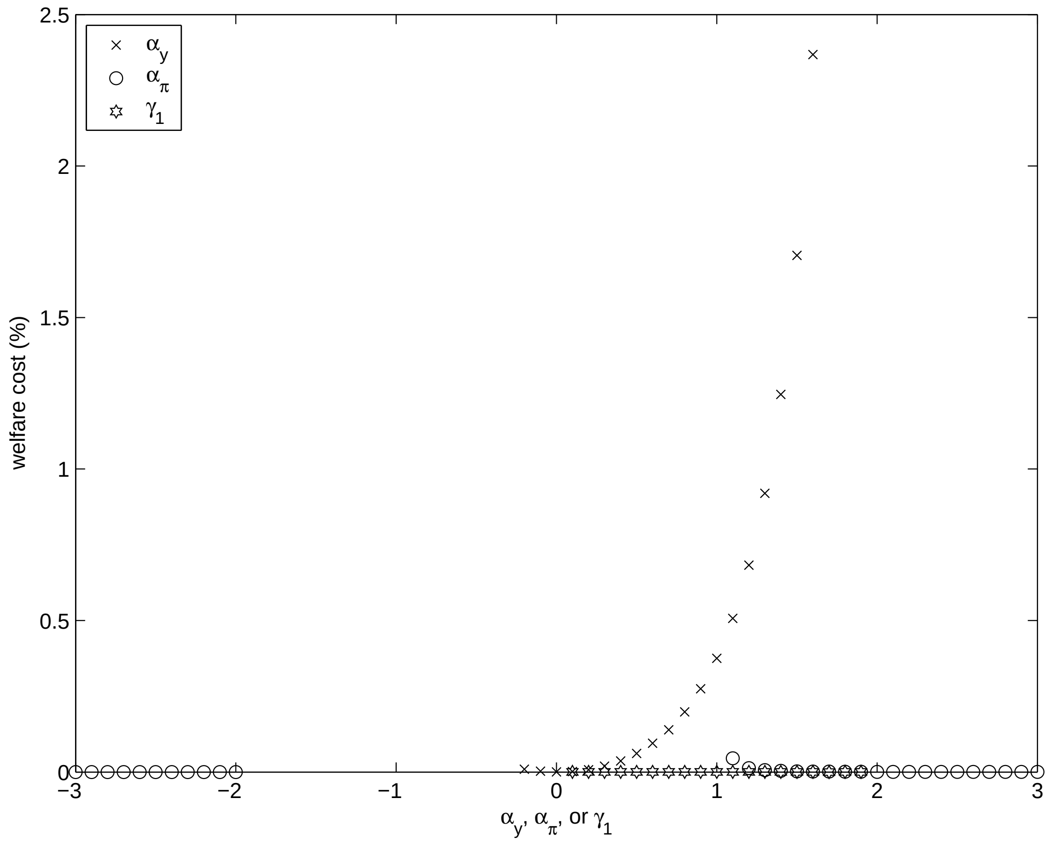
Note: A dot represents a policy parameterization for which the equilibrium is locally unique. A circle indicates that the welfare cost relative to the optimized policy is less than 5 one-hundredth of one percent. A cross with a circle indicates that the welfare cost relative to the optimized policy is less than 1 one-thousands of one percent.

than the optimal one. And our quantitative results show that even if in equilibrium fiscal policy is active and hence price level variations are used to some extend to bring about fiscal solvency, despite the fact that this could be done less costly with lump-sum taxes, we find the welfare differences are small as long as there is some response in lump-sum taxes to deviations of government liabilities from target, that is, as long as $\gamma_1 \neq 0$. We conclude from this analysis that the exact setting of policy parameters, other than α_y, matters only insofar as it guarantees determinacy of equilibrium provided that there is some response of taxes to the level of government liabilities, that is, provided, $\gamma_1 \neq 0$. About the same level of welfare can be achieved with a combination of active fiscal policy and passive monetary policy as with passive fiscal and active monetary policy.

The previous analysis was conducted under the assumption that $\alpha_y = 0$, as prescribed by the optimized policy rule. In figure 6 we consider the consequences of varying α_y between -3 and +3 holding α_π and γ_1 at their optimized values of 3 and 1.9 (or any other value implying a passive fiscal policy), respectively. The figure also considers variations in α_π and γ_1 for comparison. Variations in α_y are shown with the symbol x. Clearly, for $\alpha_y = 0$ the welfare cost is zero, since this corresponds to the optimized rule. For values of $\alpha_y < -0.2$, we find that no locally unique equilibrium exists. The graph indicates that the welfare cost have a minimum at $\alpha_y = 0$ as it should be. For positive values of α_y we found equilibrium to exist. Welfare is highly sensitive to the value of α_y. These findings are consistent with those obtained for the previous models and reinforce the conclusion that conditioning monetary policy on the level of economic active can potentially lead to significant welfare losses.

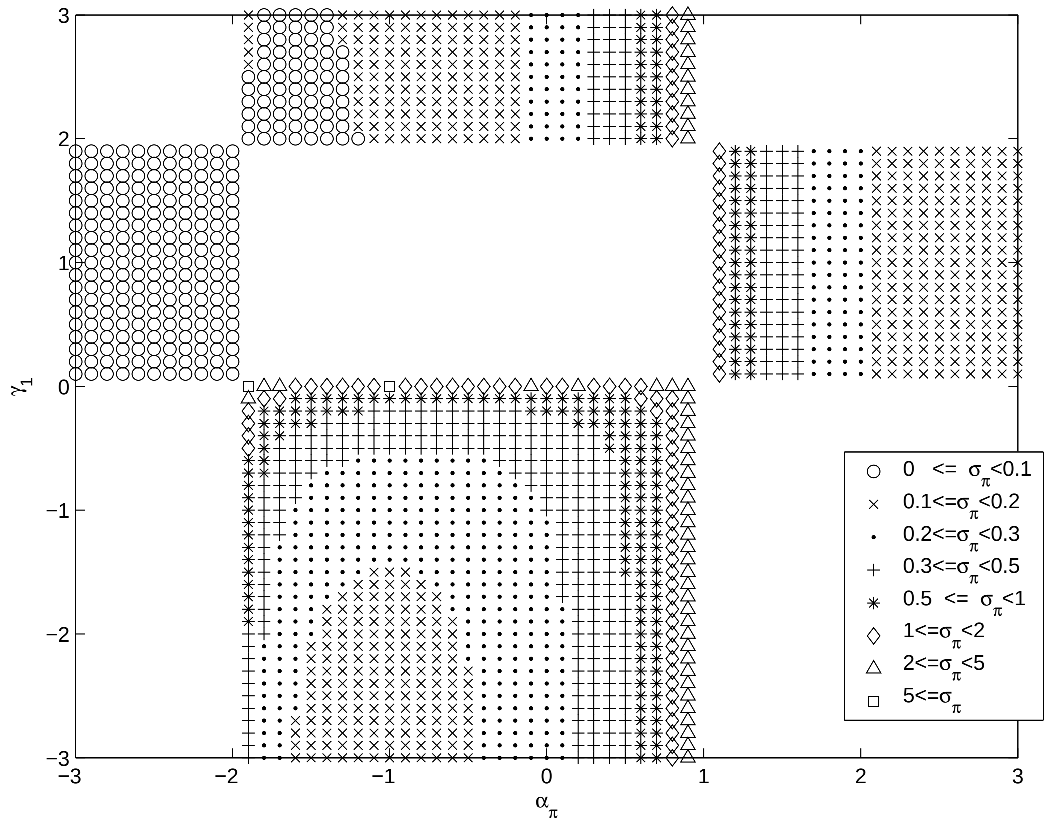
The obvious question is why is responding to output so costly in terms of welfare, in particular, in light of the fact, that deviating from the optimal rule by making fiscal policy Non-Ricardian had turned out to be of limited welfare consequences. While at this point, we do not understand this point as fully as we would like to the following observations may be somewhat clarifying. Under Non-Ricardian fiscal policy, there are potentially large surprises in the price level in response to innovations in the government's budget constraint. However, and this point is, we believe, important, the path of expected inflation should not be much affected by the fact that fiscal policy is Non-Ricardian as opposed to Ricardian. As a result, there should be high inflation volatility at very high frequencies but not much difference in the inflation volatility at lower frequencies. This could in principle translate into the unconditional variance of inflation being not much higher under Non-Ricardian fiscal policy than under Ricardian. Figure 7 shows the standard deviation of inflation (expressed in percent per year) for all 61 values of γ_1 and α_π considered in our analysis, holding α_y constant at zero. At the constrained optimal rule, we have that the standard deviation of inflation is between one and two tenth of one percent. Under a Non-Ricardian fiscal policy,

Figure 6: The Importance of Not Responding to Output in the Model with the Lump-sum Tax Feedback Rule: $\tau_t^L = \gamma_0 + \gamma_1(\ell_{t-1} - \ell^*)$



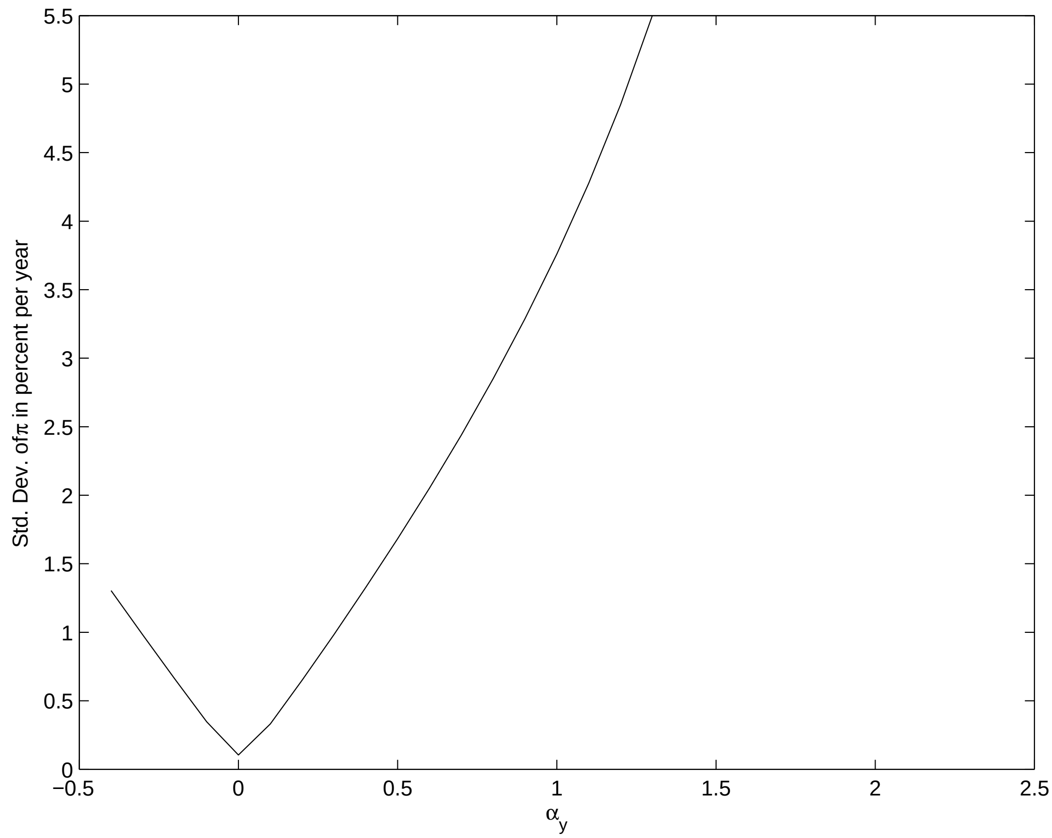
Note: See note to figure 2.

Figure 7: The Standard Deviation of Inflation (in percent per year) in the Model with the Lump-sum Tax Feedback Rule: $\tau_t^L = \gamma_0 + \gamma_1(\ell_{t-1} - \ell^*)$



Note: The computation of the variance of inflation assumes that $\alpha_y = 0$. Numbers shown are based on a first-order approximation to the policy function, which results in a second-order accurate approximation of the variance of inflation.

Figure 8: The Relation between the Standard Deviation of Inflation (in percent per year) and α_y in the Model with the Lump-sum Tax Feedback Rule



Note: Graph is truncated at a 5.5 percent standard deviation.

for example, one consisting of a pure interest rate peg ($\alpha_\pi = 0$) and an active fiscal feedback rule ($\gamma_1 = 2.1$), the standard deviation of inflation lies between 0.2 and 0.3 percentage points.[12] We view these differences in standard deviation as economically small. Figure 8 shows the standard deviation of inflation for various values of α_y holding α_π and γ_1 constant at their optimal values. The figure is truncated at a 5.5 percent standard deviation to keep the scale comparable to the numbers shown in figure 7.[13] At $\alpha_y = 0$, the standard deviation reaches the minimum standard deviation of 0.1042 and then rises steeply. For example, at $\alpha_y = 0.5$ the standard deviation of inflation is already 1.7 percent. One reason why an interest rate feedback rule with a non-zero coefficient on output leads to such a rapid rise

[12]The exact difference is 0.2217-0.1042=0.1176.

[13]The standard deviation keeps rising at an accelerating speed until it reaches about 25 percent at $\alpha_y = 2$, the higher value of α_y for which a unique equilibrium exists.

in inflation volatility is that for such a policy inflation and the nominal interest rate remain for a long period away from their target value. The idea of an active monetary policy rule roughly speaking is to set the coefficient on inflation so high that any inflation value other than its long-run level would give rise to an explosive path for inflation. In this way an active policy forces inflation to return to its target fast and results in low inflation volatility, so that there is an inverse relation between α_{pi} and inflation volatility. However, the same type of relationship does not exist between α_y and inflation volatility. On the contrary, a high output feedback coefficient in our model is associated with a large inflation volatility. This is because a large value of α_y does not necessarily force inflation to explode if it is above target and thus does not force the equilibrium to be such that inflation is back at target almost immediately. In fact, large values of α_y lead to highly persistent (and non-explosive) deviations of inflation from target. Those persistent deviation then show up in high inflation volatility.

Incomplete

6.1 Distortionary Taxation

In this economy, we searched over the three policy parameters α_π, $\alpha_y 0$ and τ_1 in the following monetary and fiscal feedback rules, respectively,

$$\ln(R_t/R^*) = \alpha_\pi \ln(\pi_t/\pi^*) + \alpha_y \ln(y_t/y)$$

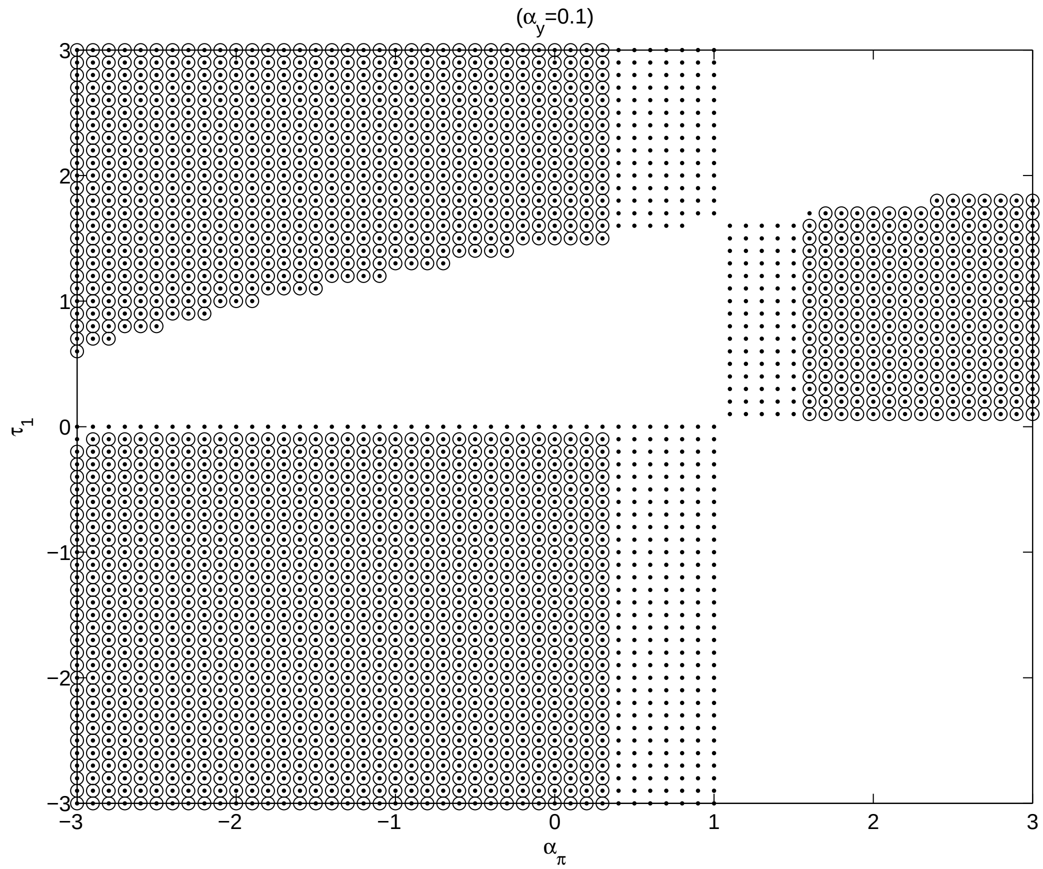
and

$$\tau_t^D y_t = \gamma_0 + \gamma_1(\ell_{t-1} - \ell).$$

Following the procedure described above to exclude parameter configuration that are near a bifurcation point, we find that the conditional expectation of welfare is largest when $\alpha_\pi = -3$, $\alpha_y = 0.1$, and $\gamma_1 = -3$. The conditional expectation of welfare under the optimized rules is -710.7907. Although, this is not a paper about the detriments of distortionary taxation, we would nevertheless like to point out that this level of welfare is significantly below that associated with economies in which the fiscal authority has access to lump-sum taxation. The steady-state level of welfare is -710.7351 whereas in the economy with lump-sum taxes it is -629.7040. For an agent to be indifferent between living in the steady state of the economy with distorting taxes and the one with lump-sum taxes, he must be forced to give up 10 percent of the steady-state consumption that he enjoys in the lump-sum tax world.

The optimal monetary policy rule coefficients are in line with the previous economies studied in that they are characterized by inflation coefficients that are large in absolute

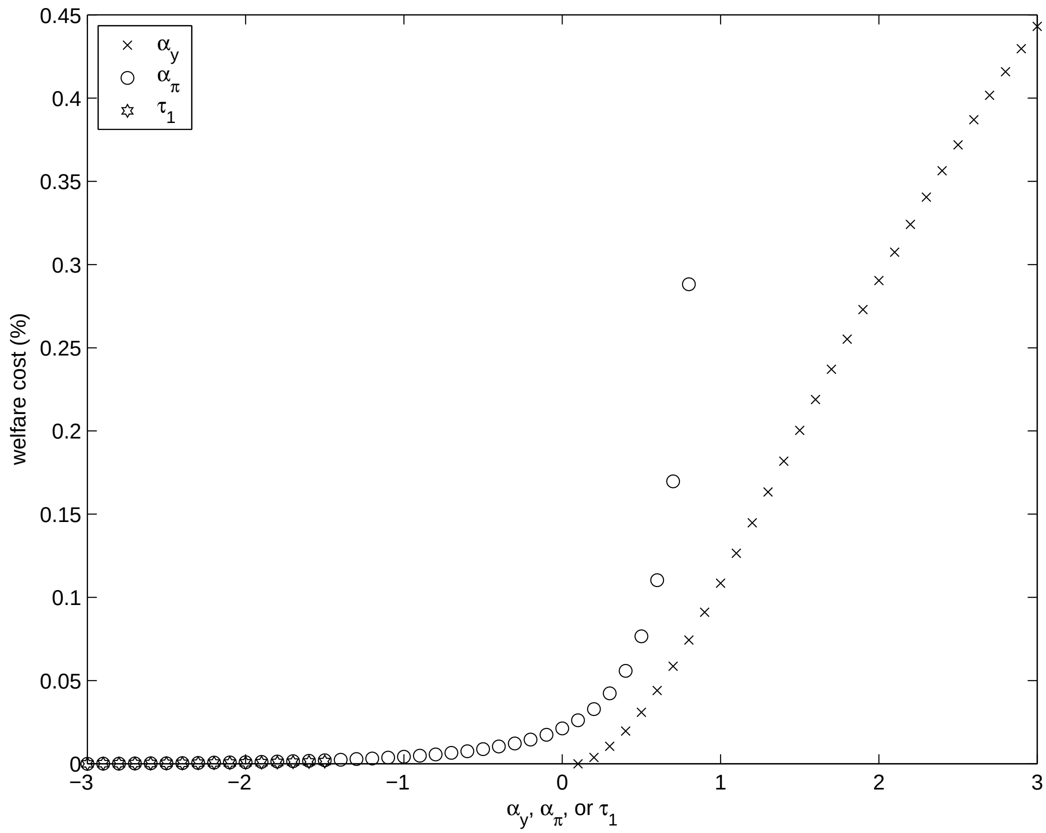
Figure 9: Determinacy Regions and Welfare in the Model with Distorting Taxes ($\tau_t^D y_t = \gamma_0 + \tau_1(\ell_{t-1} - \ell)$)



Note: See note to figure 1.

value and output coefficients that are close to zero in absolute value. The fiscal policy rule coefficient at -3 indicates that in response to positive deviations of total government liabilities from their long run level total tax revenues would fall significantly, suggesting that fiscal policy is active. However, figure 9 shows that as long as the output coefficient, α_y, is held constant at 0.1 many other combinations of fiscal and monetary policy result only in welfare differences of at most 0.05 percent of consumption. In particular, as the figure shows there exist many combinations of active monetary policy with $\alpha_\pi > 1$ and fiscal policy with small and positive values for γ_1, which one may regard as passive fiscal rules, that provide about the same level of welfare as the optimized rule. Our computations furthermore show that a pure interest rate peg, that is, $R_t = R^*$ at all times, if accompanied by sufficiently active fiscal policy can yield a real allocation that is associated with a welfare cost of at most

Figure 10: The Importance of Not Responding to Output in the Model with Distortionary Taxation ($\tau_t^D y_t = \gamma_0 + \tau_1(\ell_{t-1} - \ell)$)



Note: See note to figure 2.

0.05 percent of consumption.

As with the other models, we could not compute the level of welfare associated with a standard Taylor rule. The reason is that for this parameter configurations the equilibrium is too close to a bifurcation point for our numerical approximation technique to produce a reliable answer. We were able, though, to approximate the level of welfare associated with a simple Taylor rule ($\alpha_{pi} = 1.5$ and $\alpha_y = 0$. In this case it is optimal to set $\gamma_1 = 0.1$. The resulting level of welfare is marginally below the optimum at -710.7978 implying a welfare cost of 0.0009 percent of consumption. A magnitude that we regard as negligible. Inflation targeting continues to be a good policy, it slightly dominates the optimized rule yielding welfare gains of 0.0043 percent of consumption.

Figure 10 demonstrates that variations in the optimal policy coefficients may involve welfare losses in excess of 0.1 percent if either policymakers react strongly to output, $\alpha_y > .8$

or for α_π approaching unity from below. For α_y of 3 the welfare losses amount to 0.45 percent of consumption, which is a sizeable number.

Incomplete

7 An Economy with Sticky Wages

To be added.

8 Conclusion

To be added

Appendix A: Derivation of ϕ_{it}, equation (16)

Firms in our model can hold money, M_t, and bonds B_t^f. Total wealth of the firm, W_t evolves over time according to the following law of motion

$$W_{t+1} = R_t \left[P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} \right] + M_t + R_t B_t$$

Wealth will then be used to buy bonds and money, that is,

$$W_{t+1} = M_{t+1} + B_{t+1}$$

Rewriting the evolution of firm wealth we then have:

$$\begin{aligned} W_{t+1} &= R_t \left[P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} \right] + R_t W_t + M_t(1 - R_t) \\ &= R_t \left[P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} + W_t + M_t(R_t^{-1} - 1) \right] \\ &= R_t \left[W_t + P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} - M_t(1 - R_t^{-1}) \right] \end{aligned}$$

So the change in the present value of wealth of the firm from one period to the next is:

$$\frac{W_{t+1}}{R_t} - W_t = P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} - M_t(1 - R_t^{-1})$$

Thus we define profits as:

$$\phi_{it} = \frac{W_{t+1}}{R_t} - W_t = P_{it}a_{it} - P_t u_t k_{it} - P_t w_t h_{it} - M_t(1 - R_t^{-1}),$$

which is equation (16).

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