International Monetary Policy Coordination and Financial Market Integration*

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Abstract

The welfare gains from international coordination of monetary policy are analysed in a two-country model with sticky prices. The gains from coordination are compared under two alternative structures for financial markets: financial autarky and risk sharing. The welfare gains from coordination are found to be largest when there is risk sharing and the elasticity of substitution between home and foreign goods is greater than unity. When there is no risk sharing the gains to coordination are almost zero. It is also shown that the welfare gain from risk sharing can be negative when monetary policy is uncoordinated.
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1 Introduction

What are the gains from international coordination of monetary policy? This is a long-standing question in international macroeconomics which was the subject of an extensive literature in the 1980's (see for instance Canzoneri and Henderson (1991), Currie and Levine (1984), Miller and Salmon (1984), Oudiz and Sachs (1984) and Rogoff (1985)). More recently attention has returned to the topic following the development of new approaches to analysing the welfare effects of monetary policy in closed and open economies. The 'new open economy macroeconomics literature' emphasises the use of microfounded models and utility-based welfare measures.[1] Obstfeld and Rogoff (2002) analyse the welfare gains from monetary policy coordination in a model of this type. They show that welfare gains do exist but are likely to be very small, both in absolute terms and relative terms (when compared to the welfare costs of business cycle fluctuations).

But the model used by Obstfeld and Rogoff (2002) is special in two respects which are likely to have important implications for the welfare gains from policy coordination. Firstly, the elasticity of substitution between home and foreign goods is restricted to unity. This parameter determines the strength of the expenditure switching effect of exchange rate changes and is therefore an important determinant of the spillover effect of monetary policy. Secondly, Obstfeld and Rogoff assume that international financial markets do not exist. The trade balance is therefore forced into exact balance in all states of the world. Again this removes a potential source of international spillover effects of monetary policy.

The assumption of financial autarky is to some extent less extreme than it may seem at first. It is a well-known result that when the elasticity of substitution between home and foreign goods is unity and utility is logarithmic in consumption, the trade balance is always in balance in any case.[2] The structure of international financial markets is therefore irrelevant. It is only in the cases where Obstfeld and Rogoff consider non-logarithmic utility that the structure of financial markets becomes relevant.

The structure of financial markets does however become much more important when the elasticity of substitution between home and foreign goods differs from unity. In this case the trade balance does not automatically balance in all states of the world so the structure of financial markets will have an important influence on the behaviour of the exchange rate and the spillover effects of monetary policy. Benigno and Benigno (2001a) analyse a model similar to the Obstfeld and Rogoff (2002) model which allows for a non-unit elasticity of substitution between home and foreign goods and which assumes a financial structure which permits full international consumption risk sharing. They show that the gains from coordination depend on the degree of elasticity of substitution, but in general Benigno and Benigno are not able to solve explicitly for welfare or quantify the gains from coordination.

[1] See for instance Corsetti and Pesenti (2001a), Devereux and Engel (1998, 2000) and Obstfeld and Rogoff (1995, 1998, 2002). A recent survey of the literature is provided by Lane (2001).

[2] If all goods are traded then this result holds even when utility is not logarithmic in consumption.

A constraint that has hitherto hampered progress on this issue is the fact that it is not possible to obtain explicit exact solutions for welfare when the elasticity of substitution between home and foreign goods differs from unity. This paper adopts a second-order approximation technique to overcome this problem. Second-order accurate solutions for welfare are obtained for the general case where the elasticity of substitution differs from unity. This allows explicit solutions for the coordinated and non-coordinated policy rules to be obtained and explicit expressions for the welfare yielded by coordinated and non-coordinated policy to be derived. It is therefore possible to trace the spillover effects which give rise to gains from policy coordination and it is possible to quantify these gains.

The model is used to investigate the implications of the elasticity of substitution for the gains from policy coordination. The implications of financial market structure are also analysed. The gains from coordination that arise when there is no financial market are compared to the gains that arise when there is international risk sharing.[3]

In the financial autarky case it is found that a non-unit elasticity of substitution can indeed give rise to gains from coordination. But, as in the cases analysed by Obstfeld and Rogoff (2002), these gains are quantitatively very small. The spillover effects generated by the expenditure switching effect therefore seem to be unimportant when financial markets do not exist. But in the risk-sharing case it is found that the gains from coordination can be much higher. The existence of financial markets creates additional spillover effects which greatly increase the gains from policy coordination. Quantitatively these gains can be quite large in both absolute and relative terms.

Another way to look at the results presented in this paper is to consider the welfare gains from risk sharing. It is found that when monetary policy is coordinated the welfare level achieved in the risk-sharing case is unambiguously higher than the welfare level in the autarky case. But when monetary policy is not coordinated the answer is very different. In this case the gains from risk sharing are offset by the additional monetary policy spillover effects generated by the existence of financial markets. These spillover effects can be so strong that, for some parameter combinations, autarky yields higher welfare than risk sharing.

There have been a number of other contributions to the recent literature which are relevant to the subject of this paper. Corsetti and Pesenti (2001b) analyse the gains from monetary policy coordination when there is incomplete pass-through from exchange rate changes to local currency prices. They show that there are gains to coordination when there is incomplete pass-through even when the elasticity of substitution between home and foreign goods is unity. Clarida, Gali and Gertler (2001) analyse the welfare effects of monetary policy coordination in a model where there are non-optimal 'cost-push' shocks. Again they show that gains from coordination can arise even when the elasticity of substitution between home and foreign

[3] To be precise, the form of financial markets considered in this paper allow only full sharing of consumption risk. Agents in each country also face work-effort risk, but it is assumed that no financial instruments exist to allow this risk to be shared.

goods is unity. Benigno and Benigno (2001b) also consider cost-push shocks but do not consider non-coordinated policy. They show that the optimal coordinated policy can be sustained by each individual monetary authority pursuing a policy of flexible inflation targeting (when 'flexible inflation targeting' is of the form suggested by Svensson (1999)). Benigno (2001) analyses the implications of financial market structure for optimal coordinated policy. He compares an incomplete financial market (where trade is restricted to non-contingent bonds) with full risk sharing. Devereux (2001) also considers the implications of financial market structure. He compares the welfare implications of fixed and flexible exchange rates in the cases of financial autarky and full risk sharing. Tille (1999) analyses the role of the elasticity of substitution between home and foreign goods in the international transmission of shocks. He shows, using a deterministic model, that monetary policy can have a positive or a negative impact on foreign welfare depending on the degree of international substitutability.

This paper proceeds as follows. Section 2 presents the model. Section 3 briefly discusses the measurement of welfare. Section 4 analyses the gains from policy coordination in the special case where utility is logarithmic in consumption. Section 5 considers the more general case where the coefficient of relative risk aversion differs from unity. Section 6 analyses the welfare gains from risk sharing and Section 7 briefly considers the implications of the model for the optimality of price targeting. Section 8 concludes the paper.

2 The Model

2.1 Market Structure

The world exists for a single period[4] and consists of two countries, which will be referred to as the home country and the foreign country. Each country is populated by agents who consume a basket of goods containing all home and foreign produced goods.[5] Each agent is a monopoly producer of a single differentiated product. There is a continuum of agents of unit mass in each country. Home agents are indexed $h \in [0, 1]$ and foreign agents are indexed $f \in [0, 1]$. All agents set prices in advance of

[4] The model can easily be recast as a multi-period structure but this adds no significant insights. A true dynamic model, with multi-period nominal contracts and asset stock dynamics would be considerably more complex and would require much more extensive use of numerical methods. Newly developed numerical techniques are available to solve such models and this is likely to be an interesting line of future research (see Kim and Kim (2000), Sims (2000), Schmitt-Grohé and Uribe (2001) and Sutherland (2001)). However, the approach adopted in this paper yields useful insights which would not be available in a more complex model.

[5] In contrast to Obstfeld and Rogoff (2002) all goods in this model are traded goods. The presence of non-traded goods (or equivalently home bias in consumption preferences) is important in generating welfare gains from coordination in the Obstfeld and Rogoff model. The model presented in this paper generates gains to coordination when the elasticity of substitution between home and foreign goods differs from unity. These gains exist even when there are no non-traded goods.

the realisation of shocks and are contracted to meet demand at the pre-fixed prices.[6] Prices are set in the currency of the producer.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign real variables and foreign currency prices are indicated with an asterisk.

2.2 Preferences

All agents in the home economy have utility functions of the same form. The utility of agent h given by

$$U(h) = E\left[\frac{C(h)^{1-\rho}}{1-\rho} + \chi \log \frac{M(h)}{P} - K y_i(h)\right] \tag{1}$$

where $\rho > 0$, C is a consumption index defined across all home and foreign goods, M denotes end-of-period nominal money holdings, P is the consumer price index, $y(h)$ is the output of good h, E is the expectations operator, K is a log-normal stochastic labour-supply shock ($E[\log K] = 0$ and $Var[\log K] = \sigma_K^2$).

The consumption index C for home agents is defined as

$$C = \left[\left(\frac{1}{2}\right)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \left(\frac{1}{2}\right)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where $\theta \geq 1$. C_H and C_F are indices of home and foreign produced goods defined as follows

$$C_H = \left[\int_0^1 c_H(i)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}, \quad C_F = \left[\int_0^1 c_F(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}} \tag{3}$$

where $\phi > 1$, $c_H(i)$ is consumption of home good i and $c_F(j)$ is consumption of foreign good j. The parameter θ is the elasticity of substitution between home and foreign goods. This is the key parameter which will be the focus of the analysis in later sections. In Obstfeld and Rogoff (2002) this parameter is fixed at unity.

The budget constraint of agent h is given by

$$M(h) = M_0 + (1+\alpha)p_H(h)\,y(h) - PC(h) - T + PR(h) \tag{4}$$

[6] Obstfeld and Rogoff (2002) interpret their model as one where households supply labour to firms. They assume that each household is a monopoly supplier of a particular variety of labour and that wages are sticky (while goods prices are perfectly flexible). This is purely a matter of description. In terms of the analysis of this paper it makes no difference if households are described as supplying labour or supplying goods. In the first case it would be appropriate to regard wages as the sticky nominal variable, while in the second case it would be appropriate to regard prices as the sticky nominal variable.

where M_0 and $M(h)$ are initial and final money holdings, T is lump-sum government transfers, $p_H(h)$ is the price of home good h, P is the aggregate consumer price index and $R(h)$ is the income from a portfolio of state contingent assets (to be described in more detail below) and α is a production subsidy.[7]

The government's budget constraint is

$$M - M_0 - \alpha P_H Y + T = 0 \tag{5}$$

where P_H is the aggregate price of home produced goods and Y is the aggregate output of the home economy, defined as follows

$$Y = C_H + C_H^* \tag{6}$$

where C_H^* is aggregate foreign demand for home goods.

2.3 Price Indices

The aggregate consumer price index for home agents is

$$P = \left[\frac{1}{2}P_H^{1-\theta} + \frac{1}{2}P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7}$$

where P_H and P_F are the price indices for home and foreign goods respectively defined as

$$P_H = \left[\int_0^1 p_H(i)^{1-\phi}\, di\right]^{\frac{1}{1-\phi}}, \quad P_F = \left[\int_0^1 p_F(j)^{1-\phi}\, dj\right]^{\frac{1}{1-\phi}} \tag{8}$$

The law of one price is assumed to hold. This implies $p_H(i) = p_H^*(i)\,S$ and $p_F(j) = p_F^*(j)\,S$ for all i and j where an asterisk indicates a price measured in foreign currency and S is the exchange rate (defined as the domestic price of foreign currency). Purchasing power parity holds in terms of aggregate consumer price indices, $P = P^*S$.

2.4 Consumption Choices

Individual home demand for representative home good, h, and foreign good, f, are given by

$$c_H(h) = C_H\left(\frac{p_H(h)}{P_H}\right)^{-\phi}, \quad c_F(f) = C_F\left(\frac{p_F(f)}{P_F}\right)^{-\phi} \tag{9}$$

[7] The production subsidy is introduced as a modelling device which makes it possible to set the 'baseline' or average level of output of the two economies. In most cases it proves convenient to set the subsidy so that the distortions created by monopoly are completely offset and average output is at its first-best level. But in one case it proves convenient to assume that the production subsidy in each country is chosen as part of a Nash game between fiscal authorities which are attempting to maximise national welfare.

where

$$C_H = \frac{1}{2} C \left(\frac{P_H}{P} \right)^{-\theta}, \quad C_F = \frac{1}{2} C \left(\frac{P_F}{P} \right)^{-\theta} \tag{10}$$

Foreign demands for home and foreign goods have an identical structure to the home demands. Individual foreign demand for representative home good, h, and foreign good, f, are given by

$$c_H^* (h) = C_H^* \left(\frac{p_H^* (h)}{P_H^*} \right)^{-\phi}, \quad c_F^* (f) = C_F^* \left(\frac{p_F^* (f)}{P_F^*} \right)^{-\phi} \tag{11}$$

where

$$C_H^* = \frac{1}{2} C^* \left(\frac{P_H^*}{P^*} \right)^{-\theta}, \quad C_F^* = \frac{1}{2} C^* \left(\frac{P_F^*}{P^*} \right)^{-\theta} \tag{12}$$

Each country has a population of unit mass so the total demands for goods are equivalent to individual demands.

2.5 Optimal Price Setting

Individual agents are each monopoly producers of a single differentiated good. They therefore set prices as a mark-up over marginal costs. The mark-up (net of the production subsidy α) is given by $\Phi = \phi / [(\phi - 1)(1 + \alpha)]$. The first-order condition for price setting is derived in Appendix A and implies the following

$$P_H = \Phi \frac{E[KY]}{E[Y/(PC^{\rho})]} \tag{13}$$

where Y is the total output of the home economy.[8]

Notice that prices will contain a form of risk premium which will depend on the variances and covariances of the variables on the right hand side of (13). The risk premium reflects the fact that prices are set before shocks are realised. This risk premium plays a role in the link between shocks, monetary policy and welfare. An increase in the variance of KY for instance will (other things being equal) increase the risk premium and therefore increase the price of home produced goods. This lowers the expected level of output of home goods and therefore reduces the expected level of consumption for both home and foreign consumers. Home and foreign welfare is therefore reduced. Monetary policy can be used to affect the variances and covariances which determine the risk premium and can therefore also affect welfare.[9]

[8] For some of the cases analysed below the production subsidy is assumed to be chosen so that the net markup is unity (i.e. $\Phi = 1$). But in one case it proves convenient to assume that the production subsidy in each country is chosen as part of a Nash game between fiscal authorities which are attempting to maximise national welfare. The derivation of the Nash equilibrium subsidy is discussed in Appendix D.

[9] Note however that the risk premium is not the only link between monetary policy and welfare.

2.6 Home and Foreign Shocks

The foreign economy has a structure identical to the home economy. The foreign economy is subject to labour-supply shocks of the same form as the home economy. For simplicity it is assumed that the variances of the shocks are identical across the two countries, i.e.

$$\sigma_K^2 = \sigma_{K^*}^2 \tag{14}$$

The cross-country coefficient of correlation of shocks is given by υ where $-1 \leq \upsilon \leq 1$.

2.7 Money Demand and Supply

The first order condition for the choice of money holdings is

$$\frac{M}{P} = \chi C^{\rho} \tag{15}$$

It is assumed that the monetary authority in each country chooses a rule for the setting of the money supply. These rules may depend on the realisations of the supply shocks in each country and will take the form

$$M = M_0 K^{\delta_K} K^{*\delta_{K^*}} \text{ and } M^* = M_0^* K^{\delta_K^*} K^{*\delta_{K^*}^*} \tag{16}$$

The feedback parameters δ_K, δ_{K^*}, δ_K^* and $\delta_{K^*}^*$ are chosen by policymakers before prices are set and shocks are realised. It is assumed that policymakers are able to commit to their choice of rule.[10]

2.8 Financial Markets and Risk Sharing

When there are no financial markets portfolio payoffs are zero by assumption, i.e. $R(h) = R^*(f) = 0$ for all h and f. Thus the current account must balance in all states of the world, i.e.

$$P_H C_H^* = P_F C_F \tag{17}$$

where $P_H C_H^*$ is the value of home sales to the foreign country valued in home currency and $P_F C_F$ is the value of foreign sales to the home country valued in home currency.

When there is risk sharing it is assumed that sufficient contingent financial instruments exist to allow efficient sharing of consumption risks. All consumption is financed out of real income so the only source of consumption risk is variability in real income. Efficient sharing of consumption risk can therefore be achieved

[10] As discussed in Corsetti and Pesenti (2001b) and Benigno and Benigno (2001a), policymakers face an *ex post* temptation to deviate from any pre-announced policy rule. This can generate either an inflationary or a deflationary bias depending on the balance between the monopoly distortion, the production subsidy and other factors affecting the expected level of output and the terms of trade. In this paper the complications arising from these issues are avoided by assuming that policymakers can commit to the *ex ante* choice of policy rules.

by allowing trade in two state-contingent assets, one which has a payoff correlated with home aggregate real income and one with a payoff correlated with foreign real income.[11]

For simplicity it is assumed that each asset pays a return equal to the relevant country's real income, i.e. a unit of the home asset pays $y = YP_H/P$ and a unit of the foreign asset pays $y^* = Y^*P_F/P$.[12] The portfolio payoffs for home and foreign agents are given by the following

$$R(h) = \zeta_H(h)(y - q_H) + \zeta_F(h)(y^* - q_F) \tag{18}$$

$$R^*(f) = \zeta_H^*(f)(y - q_H) + \zeta_H^*(f)(y^* - q_F) \tag{19}$$

where $\zeta_H(h)$ and $\zeta_F(h)$ are holdings of home agent h of the home and foreign assets, $\zeta_H^*(f)$ and $\zeta_H^*(f)$ are the holdings of foreign agent f of home and foreign assets and q_H and q_F are the unit prices of the home and foreign assets. The derivation of solutions for portfolio shares and asset prices is explained in Appendix B.

It is important to specify the point in time at which asset trade takes place. There are two possible structures. In the first structure asset markets open *after* policymakers have made their choice of monetary policy rules. In the second structure asset markets open *before* policy rules have been chosen. The first structure implies a more limited form of insurance because agents can not insure against the choice of policy rules - they can only insure against the risk implied by a particular pair of rules.

The distinction between the two risk-sharing structures is important from the point of view of policymakers. In the first structure policymakers are aware that agents are not fully insured against the potential negative impact of the choice of policy rule. Policymakers therefore internalise these costs. In the second case policymakers do not fully internalise the costs of policy rule choice. Not surprisingly this can greatly increase the cross-country spillover effects of monetary policymaking and can generate very large welfare gains from monetary policy coordination. The two alternative risk-sharing cases are analysed separately.[13]

[11] It is important to emphasise that trade in income contingent assets is only sufficient to allow full sharing of consumption risk. It does not allow agents to share work-effort risk. And it does not allow agents to undo the constraints of nominal price contracts. The situation modelled here is therefore *not* one of 'complete markets'.

[12] Note that asset payoffs are correlated with aggregate income. Individual agents therefore treat payoffs as exogenous. This implies that the existence of contingent assets has no direct impact on optimal price setting.

[13] An earlier version of this paper focused only on the case where asset trade takes place after policy rules are chosen. In that version of the paper the contingent assets necessary to support risk sharing were not modelled explicitly. It was argued (incorrectly) that the contingent assets necessary to allow insurance against the choice of policy rule would be implausibly complicated. The structure adopted here makes it clear that a simple asset structure does exist which allows agents to insure against the choice of policy rules.

3 Welfare

One of the main advantages of the model just described is that it provides a very natural and tractable measure of welfare which can be derived from the aggregate utility of agents. Following Obstfeld and Rogoff (1998, 2002) it is assumed that the utility of real balances is small enough to be neglected. It is therefore possible to measure aggregate welfare of home agents using the following

$$\Omega = E\left[\frac{C^{1-\rho}}{1-\rho} - KY\right] \tag{20}$$

It is not possible to derive an exact expression for welfare (except in special cases). The complication arising in this model (which does not arise in other models used in recent literature) is contained in equations (6) and (7). When θ is greater than unity neither of these equations is linear in logs. The model is therefore solved as a second-order approximation around a non-stochastic equilibrium. This allows a second-order accurate solution for welfare to be derived.

Define the non-stochastic equilibrium of the model to be the solution which results when $K = K^* = 1$ with $\sigma_K^2 = \sigma_{K^*}^2 = 0$ and for any variable X define $\hat{X} = \log\left(X/\bar{X}\right)$ where $\bar{X}$ is the value of variable X in the non-stochastic equilibrium. A second-order approximation of the welfare measure is given by

$$\tilde{\Omega} = E\left\{\bar{C}^{1-\rho}\left[\hat{C} + \frac{1}{2}(1-\rho)\hat{C}^2\right] - \bar{Y}\left[\hat{Y} + \frac{1}{2}\left(\hat{Y} + \hat{K}\right)^2\right]\right\} + O\left(\|\xi\|^3\right) \tag{21}$$

where $\tilde{\Omega}$ is the deviation in the level of welfare from the non-stochastic equilibrium and the term $O\left(\|\xi\|^3\right)$ contains all terms of third order and higher in deviations from the non-stochastic equilibrium.[14] Notice that, to evaluate welfare, it is necessary to solve for both the first and second moments of output and consumption. Appendix C describes some of the details of the solution process and shows how the first moments of output and consumption can be written in terms of the second moments of the model.

It is now possible analyse the welfare gains from policy coordination.

4 The Welfare Gains from Policy Coordination: The Logarithmic Utility Case

It is useful first to consider the case where utility is logarithmic in consumption. In this case the coefficient of relative risk aversion, ρ, is set equal to unity.

[14] In most cases considered below the production subsidy is chosen such that the net markup, Φ, is unity so the non-stochastic equilibrium implies $\bar{Y} = \bar{C} = 1$. But in the case where risk sharing takes place before the setting of policy it is necessary to consider a non-stochastic equilibrium where the production subsidy is chosen as part of a Nash game between the two countries. The derivation of $\bar{Y}$ and $\bar{C}$ in this case is explained in Appendix D.

4.1 Monetary Policy, the Exchange Rate and Output

Many of the implications of this model can be understood by examining the links between monetary policy in the two countries, the exchange rate and output. It is sufficient for this purpose to consider a log-linearised version of the model.[15] First note that

$$\hat{P}_H = \hat{P}_F^* = 0 + O\left(\|\xi\|^2\right) \tag{22}$$

where $O\left(\|\xi\|^2\right)$ is a residual which contains all terms of order two and above. Equation (22) implies that the deviation of goods prices from their non-stochastic equilibrium values is zero (to a first-order approximation) so

$$\hat{P} = \frac{1}{2}\hat{S} + O\left(\|\xi\|^2\right), \quad \hat{P}^* = -\frac{1}{2}\hat{S} + O\left(\|\xi\|^2\right) \tag{23}$$

When these expressions are combined with the demands for home and foreign goods it is simple to show that home and foreign aggregate outputs are given by

$$\hat{Y} = \frac{1}{2}\left(\hat{C} + \hat{C}^*\right) + \frac{\theta}{2}\hat{S} + O\left(\|\xi\|^2\right) \tag{24}$$

and

$$\hat{Y}^* = \frac{1}{2}\left(\hat{C} + \hat{C}^*\right) - \frac{\theta}{2}\hat{S} + O\left(\|\xi\|^2\right) \tag{25}$$

Thus aggregate output is determined by aggregate world consumption and the exchange rate. The exchange rate term is the "expenditure switching effect". A depreciation of the exchange rate increases demand for home goods and reduces demand for foreign goods. Notice that the strength of the expenditure switching effect is determined by θ (which is the elasticity of substitution between home and foreign goods). These expressions hold regardless of the structure of financial markets.

Now consider the money market equations. When combined with the expressions for aggregate prices the money market equations imply

$$\hat{C} = \hat{M} + \frac{1}{2}\hat{S} + O\left(\|\xi\|^2\right), \quad \hat{C}^* = \hat{M}^* - \frac{1}{2}\hat{S} + O\left(\|\xi\|^2\right) \tag{26}$$

so

$$\hat{C} + \hat{C}^* = \hat{M} + \hat{M}^* + O\left(\|\xi\|^2\right) \tag{27}$$

Thus aggregate world consumption is determined by the sum of home and foreign monetary policy. Again this expression holds regardless of the structure of financial markets.

The structure of financial markets comes into play in the determination of the exchange rate. When there is no financial market the current account has to balance

[15] As already noted, Appendix C shows how a second-order approximation of welfare can be written in terms of the second moments of the model. Second-order accurate solutions to second moments can be obtained from first-order accurate solutions to the variables of the model. It is therefore sufficient to consider a log-linearised version of the model when considering the links between monetary policy and welfare.

in all states of the world. Using the expressions for aggregate prices and the demands for home and foreign goods, current account balance implies

$$\hat{S} = \frac{1}{\theta - 1}\left(\hat{C} - \hat{C}^*\right) + O\left(\|\xi\|^2\right) \tag{28}$$

Thus, when home consumption exceeds foreign consumption the exchange rate must depreciate in order to maintain current account balance (and vice versa when foreign consumption exceeds home consumption). When this expression is combined with the expressions for aggregate consumption it is found that

$$\hat{S} = \frac{1}{\theta}\left(\hat{M} - \hat{M}^*\right) + O\left(\|\xi\|^2\right) \tag{29}$$

Thus the exchange rate depends on relative money supplies.

When there is risk sharing (of either form) Appendix B shows that realised consumption levels are related as follows

$$\hat{C} = \hat{C}^* + O\left(\|\xi\|^2\right) \tag{30}$$

When combined with the money market relationships this implies

$$\hat{S} = \hat{M} - \hat{M}^* + O\left(\|\xi\|^2\right) \tag{31}$$

Thus again the exchange rate depends on relative monetary supplies. But notice that the exchange rate is more sensitive to monetary policy when there is risk sharing (provided $\theta > 1$).

When the exchange rate expressions are combined with the expressions for aggregate consumption and outputs it is found that in the case of financial autarky

$$\hat{Y} = \hat{M} + O\left(\|\xi\|^2\right), \quad \hat{Y}^* = \hat{M}^* + O\left(\|\xi\|^2\right) \tag{32}$$

while in the case of risk sharing (of either form)

$$\hat{Y} = \frac{1+\theta}{2}\hat{M} + \frac{1-\theta}{2}\hat{M}^* + O\left(\|\xi\|^2\right), \quad \hat{Y}^* = \frac{1+\theta}{2}\hat{M}^* + \frac{1-\theta}{2}\hat{M} + O\left(\|\xi\|^2\right) \tag{33}$$

The important point to note from these expressions is that in the financial autarky case monetary policy has no international spillover effects. A change in home monetary policy only affects home output and a change in foreign monetary policy only affects foreign output. This is because the effects of monetary policy on aggregate world demand are just enough to offset the expenditure switching effect. But in the risk-sharing case monetary policy does have international spillover effects. In this case monetary policy has a larger effect on the exchange rate so the expenditure switching effect outweighs the effect of monetary policy on aggregate world consumption. Thus an increase in the home money supply causes an expansion of home output and a contraction of foreign output (and vice versa for an expansion of the foreign money supply).

The expressions for output and the exchange rate just derived will prove useful for understanding the source of the gains from coordination. The welfare gains to monetary coordination are now analysed in the financial autarky and the two risk-sharing cases.

4.2 Financial Autarky

Appendix C shows that in this case home and foreign welfare can be written as follows

$$\tilde{\Omega} = -\frac{1}{4}E\left[\frac{1}{\theta}(2\theta-1)\left(\hat{Y}+\hat{K}\right)^2 + \frac{1}{\theta}\left(\hat{Y}^*+\hat{K}^*\right)^2 + \frac{1}{2}(1-\theta)\hat{S}^2\right] \tag{34}$$

and

$$\tilde{\Omega}^* = -\frac{1}{4}E\left[\frac{1}{\theta}(2\theta-1)\left(\hat{Y}^*+\hat{K}^*\right)^2 + \frac{1}{\theta}\left(\hat{Y}+\hat{K}\right)^2 + \frac{1}{2}(1-\theta)\hat{S}^2\right] \tag{35}$$

while the previous section showed that output levels and the exchange rate are linked to monetary policy by the following simple relationships

$$\hat{Y} = \hat{M}, \quad \hat{Y}^* = \hat{M}^* \tag{36}$$

$$\hat{S} = \frac{1}{\theta}\left(\hat{M} - \hat{M}^*\right) \tag{37}$$

To simplify notation the residual terms $O\left(\|\xi\|^3\right)$ and $O\left(\|\xi\|^2\right)$ have been omitted from these and all subsequent expressions. It should be understood, however, that the welfare expressions are second-order approximations and the output and exchange rate expressions are first-order approximations.

The structure of the welfare functions can easily be understood. Notice that welfare depends negatively on the variances of $\hat{Y}+\hat{K}$ and $\hat{Y}^*+\hat{K}^*$. These terms are effectively the (log deviations of the) disutility of work effort for home and foreign producers. A higher variance of the disutility of work effort tends to raise the risk premium in goods prices. This reduces the expected level of output and consumption. Agents consume both home and foreign goods so welfare in both countries depends on the variance of the disutility of work effort in both countries. But notice that when $\theta > 1$ the variance of home disutility matters more for home welfare than does the variance of foreign disutility (and vice versa for foreign welfare). This is because a rise in the variance of home disutility not only raises the price of home goods for home agents it also results in a switch in world expenditure towards foreign goods and this reduces the income of home agents. The same mechanism means that the variance of foreign disutility has a greater impact on foreign welfare than the variance of home disutility.

Welfare depends positively on the variance of the exchange rate (when $\theta > 1$). This can be understood by considering the definition of the consumer price index. The consumer price index is concave in the price of home and foreign goods. Any volatility in the relative price of home and foreign goods (which would result from exchange rate volatility) will reduce the expected level of aggregate consumer prices. This has a positive effect on utility and welfare. (Another way to understand this effect is to note that, when home and foreign goods are substitutable, agents can reduce the average cost of their consumption basket by switching expenditure towards whichever set of goods are cheapest *ex post*. Relative price volatility is therefore a utility benefit.)

It is assumed that monetary authorities choose money supply rules of the following form

$$\hat{M} = \delta_K \hat{K} + \delta_{K^*} \hat{K}^* \tag{38}$$

and

$$\hat{M}^* = \delta^*_K \hat{K} + \delta^*_{K^*} \hat{K}^* \tag{39}$$

In the case of coordinated policymaking it is assumed that a single world monetary authority chooses the feedback parameters of both rules to maximise world welfare, where world welfare is given by the average of national welfare levels, i.e.

$$\tilde{\Omega}^W = \frac{1}{2}\left(\tilde{\Omega} + \tilde{\Omega}^*\right) \tag{40}$$

In the case of non-coordinated policymaking it is assumed that the feedback parameters of the home monetary rule are chosen by the home monetary authority in an attempt to maximise home welfare and the parameters of the foreign monetary rule are chosen by the foreign monetary authority in an attempt to maximise foreign welfare. Each monetary authority acts as a Nash player and takes as given the parameters of the other country's rule when choosing their own feedback parameters.

The coordinated equilibrium results in the following choices of feedback parameters

$$\delta^C_K = \delta^{*C}_{K^*} = \frac{-1+\theta-2\theta^2}{2\left(1-\theta+\theta^2\right)} \tag{41}$$

$$\delta^C_{K^*} = \delta^{*C}_K = \frac{-1+\theta}{2\left(1-\theta+\theta^2\right)} \tag{42}$$

where the superscript 'C' indicates the coordinated equilibrium. The non-coordinated equilibrium results in

$$\delta^N_K = \delta^{*N}_{K^*} = \frac{1-3\theta+4\theta^2}{-2\left(1-2\theta+2\theta^2\right)} \tag{43}$$

$$\delta^N_{K^*} = \delta^{*N}_K = \frac{-1+\theta}{-2\left(1-2\theta+2\theta^2\right)} \tag{44}$$

where the superscript 'N' indicates the non-coordinated equilibrium. The world welfare level yielded by coordinated policy is

$$\tilde{\Omega}^C_A = \frac{(\theta-1)}{4\left(1-\theta+\theta^2\right)}(1-\upsilon)\sigma^2_K \tag{45}$$

where again the superscript 'C' indicates the coordinated equilibrium and the subscript 'A' indicates the financial autarky case. The welfare yielded by non-coordinated policy is

$$\tilde{\Omega}^N_A = \frac{\left(-2+7\theta-9\theta^2+4\theta^3\right)}{4\left(1-2\theta+2\theta^2\right)^2}(1-\upsilon)\sigma^2_K \tag{46}$$

As a point of reference it is useful to consider an inactive policy regime, where feedback parameters are all set to zero. (This is equivalent to a money targeting regime.) The welfare level yielded by this regime is

$$\tilde{\Omega}_A^M = -\frac{1}{2}\sigma_K^2 \tag{47}$$

where the superscript 'M' indicates the case of non-active policy (or money targeting).

Two propositions can now be stated. (Proofs follow from a simple comparison of the above expressions and are omitted.)

Proposition 1 *If $\upsilon < 1$ and $\theta > 1$ then $\tilde{\Omega}_A^C > \tilde{\Omega}_A^N$, i.e. there are gains from coordination.*

It is clear from expressions (34) to (37) that there will be gains to coordination provided $\theta > 1$. When $\theta > 1$ each monetary authority cares about the variance of the exchange rate, and monetary policy in each country affects the exchange rate. In addition, when $\theta > 1$, each monetary authority cares more about the volatility of the disutility of work effort in its own country than it does about the volatility of the disutility of work effort in the other country. There is therefore a policy spillover (operating through the exchange rate) and an incentive to bias policy to the benefit of domestic welfare.

The gains from coordination disappear in two circumstances. The first case is when $\theta = 1$. In this case exchange rate volatility does not affect welfare so there is no policy spillover. Each monetary authority therefore maximises the welfare of its population by minimising the variance of the disutility of work effort in its own country. This also maximises world welfare. The second case where there are no gains from coordination is when the shocks in the two countries are perfectly correlated, i.e. when $\upsilon = 1$. This corresponds to a result noted and emphasised by Obstfeld and Rogoff (2002). When shocks are perfectly correlated the use of monetary policy to stabilise the disutility of work effort in one country will automatically also stabilise disutility of work effort in the other country. There is therefore no difference between coordinated and non-coordinated policymaking.

Proposition 2 *If $\theta > 1$ then: (a) $\left|\delta_K^C\right| = \left|\delta_{K*}^{*C}\right| > \left|\delta_K^N\right| = \left|\delta_{K*}^{*N}\right|$ and $\left|\delta_{K*}^C\right| = \left|\delta_K^{*C}\right| > \left|\delta_{K*}^N\right| = \left|\delta_K^{*N}\right|$ and (b) $Var\left(\hat{S}^N\right) < Var\left(\hat{S}^C\right)$, $Var\left(\hat{Y}^N\right) < Var\left(\hat{Y}^C\right)$ and $Var\left(\hat{Y}^{*N}\right) < Var\left(\hat{Y}^{*C}\right)$ (where the superscripts 'C' and 'N' indicate values in coordinated and non-coordinated equilibria respectively).*

This proposition shows that non-coordinated policymaking is less active than coordinated policymaking. It also shows that the exchange rate and output levels are less volatile with non-coordinated policymaking. In other words non-coordinated policymaking has a bias towards over-stabilisation.

θ	1	2	3	4	6
$\tilde{\Omega}_A^C - \tilde{\Omega}_A^M$	0.500	0.583	0.571	0.558	0.540
$\tilde{\Omega}_A^N - \tilde{\Omega}_A^M$	0.500	0.580	0.568	0.555	0.539
$\tilde{\Omega}_A^C - \tilde{\Omega}_A^N$	0.000	0.003	0.003	0.002	0.001
$\frac{100\times\left(\tilde{\Omega}_A^C - \tilde{\Omega}_A^N\right)}{\left(\tilde{\Omega}_A^C - \tilde{\Omega}_A^M\right)}$	0.0	0.6	0.6	0.4	0.2

Table 1: The welfare effects of coordination: Financial autarky

Proposition 1 establishes that there are gains to coordination when $\theta > 1$. But in order to determine the size of these gains it is necessary to perform some numerical exercises with different values of θ. Table 1 reports some values for welfare with $\sigma_K^2 = \sigma_{K^*}^2 = 0.01$ and $\upsilon = 0$. A range of values of θ has been suggested in previous literature, for instance Benigno and Benigno (2001a) suggest $\theta = 6$. Table 1 shows welfare calculations for $\theta = 1$ to $\theta = 6$. The first row shows the welfare gain from coordinated policy relative to an inactive policy (i.e. $\tilde{\Omega}_A^C - \tilde{\Omega}_A^M$). The figures in the first row therefore represent the maximum possible gain from following an active policy. The second row shows the welfare gain from non-coordinated policy relative to an inactive policy (i.e. $\tilde{\Omega}_A^N - \tilde{\Omega}_A^M$). The third row shows the absolute gains from coordination (i.e. $\tilde{\Omega}_A^C - \tilde{\Omega}_A^N$). In each case these figures are measured as a percentage of (first-best) non-stochastic equilibrium consumption. The fourth row shows the gains from coordination as a percentage of the maximum possible gain from an active policy (i.e. row 3 as a percentage of row 1). It is apparent from Table 1 that the welfare gain from coordination is positive when θ is greater than unity. But the gain is never large, either in absolute or relative terms. This is very similar to the result emphasised by Obstfeld and Rogoff (2002).

4.3 Risk sharing - asset trade after monetary policy

This section analyses the case where risk sharing takes place after monetary policy is set. The procedure described in Appendix C can be used to show that home and foreign welfare in this case can be written as follows

$$\tilde{\Omega} = -\frac{1}{4}E\left[\frac{1}{\theta}(2\theta-1)\left(\hat{Y}+\hat{K}\right)^2 + \frac{1}{\theta}\left(\hat{Y}^*+\hat{K}^*\right)^2 + \frac{1}{2}\theta(1-\theta)\hat{S}^2\right] \tag{48}$$

and

$$\tilde{\Omega}^* = -\frac{1}{4}E\left[\frac{1}{\theta}(2\theta-1)\left(\hat{Y}^*+\hat{K}^*\right)^2 + \frac{1}{\theta}\left(\hat{Y}+\hat{K}\right)^2 + \frac{1}{2}\theta(1-\theta)\hat{S}^2\right] \tag{49}$$

while it was shown above that output levels and the exchange rate are linked to monetary policy by the following simple relationships

$$\hat{Y} = \frac{1+\theta}{2}\hat{M} + \frac{1-\theta}{2}\hat{M}^*, \quad \hat{Y}^* = \frac{1+\theta}{2}\hat{M}^* + \frac{1-\theta}{2}\hat{M} \tag{50}$$

$$\hat{S} = \hat{M} - \hat{M}^* \tag{51}$$

The form of the welfare function for each country is almost identical to the autarky case. The only difference is a small change to the coefficient on the variance of the exchange rate. The main difference between the this case and the previous case is contained in the determination of output. There is now a spillover effect from monetary policy in one country to the level of output in the other country. It is clear that this creates more scope for gains from coordinated policy. The quantitative implications of this spillover effect are considered below. First consider the expressions for the coefficients of the policy rules and welfare levels.

The coordinated equilibrium results in the following choices of feedback parameters

$$\delta_K^C = \delta_{K^*}^{*C} = -1 \tag{52}$$

$$\delta_{K^*}^C = \delta_K^{*C} = 0 \tag{53}$$

while the non-coordinated equilibrium results in

$$\delta_K^N = \delta_{K^*}^{*N} = \frac{1 - 3\theta^2}{-2\theta(1 - 2\theta)} \tag{54}$$

$$\delta_{K^*}^N = \delta_K^{*N} = \frac{1 - 2\theta + \theta^2}{2\theta(1 - 2\theta)} \tag{55}$$

The world welfare level yielded by coordinated policy is

$$\tilde{\Omega}_{RAP}^C = \frac{(\theta - 1)}{4}(1 - \upsilon)\sigma_K^2 \tag{56}$$

where the subscript 'RAP' indicates "risk sharing after policy". The welfare yielded by non-coordinated policy is

$$\tilde{\Omega}_{RAP}^N = \frac{\left(-1 + 3\theta - \theta^2 - 4\theta^3 + 3\theta^4\right)}{4\theta(1 - 2\theta)^2}(1 - \upsilon)\sigma_K^2 \tag{57}$$

Again, as a point of reference it is useful to consider an inactive policy regime. The welfare level yielded by this regime is

$$\tilde{\Omega}_{RAP}^M = -\frac{1}{2}\sigma_K^2 \tag{58}$$

Two propositions can now be stated (and again the proofs are omitted).

Proposition 3 *If $\upsilon < 1$ and $\theta > 1$ then $\tilde{\Omega}_{RAP}^C > \tilde{\Omega}_{RAP}^N$, i.e. there are gains from coordination.*

It is clear from the expressions (48) to (51) that gains from coordination will arise. All the factors that were present in the autarky case are also present in this case. When $\theta > 1$ both monetary authorities care about the volatility of the exchange rate and both monetary authorities can affect the exchange rate using monetary policy. Also welfare in each country is affected more by the volatility of the disutility of work effort within the country than the volatility in the other country. But now there is an extra spillover effect of monetary policy. When $\theta > 1$ a monetary expansion in the home country reduces output in the foreign country because of the expenditure switching effect. Likewise a monetary expansion in the foreign country reduces output in the home country.

Again notice that there are two cases where the gains from coordination disappear. The first is where $\theta = 1$. In this case the spillover effect from monetary policy to foreign output disappears. The second case is where $\upsilon = 1$. Correlated shocks do not create any conflicts between optimal policy in each country.

Proposition 4 *If* $\theta > 1$ *then: (a)* $\left|\delta_K^C\right| = \left|\delta_{K^*}^{*C}\right| > \left|\delta_K^N\right| = \left|\delta_{K^*}^{*N}\right|$ *and* $\left|\delta_{K^*}^N\right| = \left|\delta_K^{*N}\right| > \left|\delta_{K^*}^C\right| = \left|\delta_K^{*C}\right|$ *and (b)* $Var\left(\hat{S}^N\right) < Var\left(\hat{S}^C\right)$, $Var\left(\hat{Y}^N\right) < Var\left(\hat{Y}^C\right)$ *and* $Var\left(\hat{Y}^{*N}\right) < Var\left(\hat{Y}^{*C}\right)$.

In the autarky case it was clear that non-coordinated policy was less active than coordinated policymaking. In this case coordinated policymaking implies a stronger monetary policy reaction to shocks occurring within a country but a smaller reaction to shocks occurring in the other country. In other words non-coordinated policy involves a shifting of the burden of policy adjustment onto the other country. It remains true however that non-coordinated policy implies less volatility in the exchange rate and output levels.

The quantitative implications of risk sharing for the gains from coordination are illustrated in Table 2. The parameter values are the same as those used to construct Table 1 and the structure of the table is identical. It is apparent that the gains from coordination are much larger than in the autarky case in both absolute and relative terms. For instance when $\theta = 6$ the gains from coordination are worth 0.2 percent of equilibrium consumption which represents 12.3 percent of the gains from optimal stabilisation. These figures obviously can not be described as large, but they are also not trivial.[16]

4.4 Risk Sharing - asset trade before monetary policy

In this section risk sharing is assumed to take place before monetary policy rules are determined. This greatly increases the potential sources of international spillover effects of policy because now policymakers believe that their populations are more fully insured against the negative effects of monetary policy.

[16] Notice from (56), (57) and (58) that the size of the welfare effects is proportional to the aggregate variance of the shocks. In a more general model, with more sources of shocks and some persistence in the shock processes, the size of the welfare effects will depend on some aggregate of all shock variances and the degree of persistence of the shocks. This may generate larger welfare effects than reported here.

θ	1	2	3	4	6
$\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^M_{RAP}$	0.500	0.750	1.000	1.250	1.750
$\tilde{\Omega}^N_{RAP} - \tilde{\Omega}^M_{RAP}$	0.500	0.736	0.947	1.147	1.535
$\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^N_{RAP}$	0.000	0.014	0.053	0.103	0.215
$\frac{100\times\left(\tilde{\Omega}^C_{RAP}-\tilde{\Omega}^N_{RAP}\right)}{\left(\tilde{\Omega}^C_{RAP}-\tilde{\Omega}^M_{RAP}\right)}$	0.0	1.8	5.3	8.3	12.3

Table 2: The welfare effects of coordination: Risk sharing - asset trade after monetary policy

The new mechanisms at work in this case can be more easily understood by considering the factors determining the expected level of consumption. Appendix B shows that, when asset trade takes place *after* policy rules are chosen (i.e. the case considered in section 4.3), the expected level of home consumption is given by

$$E\left[\hat{C}\right] = E\left[\hat{y} + \frac{3-2\rho}{8}\hat{y}^2 - \frac{1-2\rho}{8}\hat{y}^{*2} - \frac{1}{4}\hat{y}\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{59}$$

But when asset trade takes place *before* policy rules are chosen (i.e. the case to be considered in this section) the expected level of home consumption is given by

$$E\left[\hat{C}\right] = \frac{1}{2}E\left[\hat{y} + \hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{60}$$

Equation (59) shows that, when asset trade takes place after policy rules are chosen, any change in the expected level of home real income must produce a one-for-one change in the expected level of home consumption. The home policymaker therefore fully internalises the welfare cost of changes in the expected level of home income. Equation (60), on the other hand, shows that, when asset trade takes place before policy rules are chosen, the expected level of home consumption is tied to the expected average level of world income. Thus, for instance, the home policymaker can choose a policy rule which depresses the *expected* level of home output (or increases the *expected* level of foreign output) while knowing that the expected level of home consumption is tied to the average level of world income. Home agents therefore benefit from an increase in the expected level of leisure time while receiving the world average level of consumption. In other words the home policymaker believes that it is possible to shift the (expected) burden of production onto the foreign population. This mechanism clearly creates a additional spillover effect of monetary policy which potentially increases the gains from monetary policy coordination.[17]

[17] In addition, the fact that each policymaker perceives a potential net benefit from manipulating expected output levels introduces a further mechanism which is not present in the previous cases analysed. This is a link between the non-stochastic equilibrium level of output and the equilibrium of the monetary policy game. If, for instance, the non-stochastic equilibrium output level is set at the world first-best level each policymaker has a strong incentive to attempt to use monetary

The solution procedure described in Appendix C can be used to show that home and foreign welfare can be written as follows

$$\tilde{\Omega} = -\frac{1}{2(1+\theta)}E\left[\left(\hat{Y}+\hat{K}\right)^2+\theta\left(\hat{Y}^*+\hat{K}^*\right)^2 -\theta\left(\hat{Y}^*-\hat{M}^*\right)^2+\frac{1}{2}\theta(1-\theta)\hat{S}^2\right] \quad (61)$$

and

$$\tilde{\Omega}^* = -\frac{1}{2(1+\theta)}E\left[\left(\hat{Y}^*+\hat{K}^*\right)^2+\theta\left(\hat{Y}+\hat{K}\right)^2 -\theta\left(\hat{Y}-\hat{M}\right)^2+\frac{1}{2}\theta(1-\theta)\hat{S}^2\right] \quad (62)$$

while output levels and the exchange rate are given by (50) and (51). Equations (61) and (62) when compared to (48) and (49) show that the change in the timing of asset trading has a significant effect on the structure of the welfare function.

The coordinated equilibrium results in the following choices of feedback parameters

$$\delta_K^C = \delta_{K^*}^{*C} = -1 \quad (63)$$

$$\delta_{K^*}^C = \delta_K^{*C} = 0 \quad (64)$$

while the non-coordinated equilibrium results in

$$\delta_K^N = \delta_{K^*}^{*N} = -\frac{(1+\theta)^2}{4\theta} \quad (65)$$

$$\delta_{K^*}^N = \delta_K^{*N} = \frac{(\theta-1)^2}{4\theta} \quad (66)$$

The world welfare level yielded by coordinated policy is

$$\tilde{\Omega}_{RBP}^C = \frac{(\theta-1)}{4}(1-\upsilon)\sigma_K^2 \quad (67)$$

policy to depress output (and thereby shift the burden of production onto the other country). But if the non-stochastic equilibrium output is set at a lower level the contractionary incentive is reduced. The equilibrium of the monetary policy game therefore has a less contractionary bias. Thus, in general, the equilibrium of the monetary policy game depends on the assumed level of output in the non-stochastic equilibrium. And it follows that the welfare gains from monetary policy coordination also depend on the non-stochastic equilibrium.

Output in the non-stochastic equilibrium is determined by the choice of the production subsidies (α and α^*). In previous sections it was assumed that the production subsidies where chosen to yield the first-best in the non-stochastic equilibrium. In this section it is assumed that the production subsidies are chosen as a Nash game between the policymakers. (See Appendix D for details.) This implies that non-stochastic output levels are already at a Nash equilibrium. Each policymaker therefore has no incentive to use monetary policy to further depress output. Thus the contractionary bias of the monetary policy game is reduced to a minimum. Nevertheless, the results reported in this section show that the spillover effects arising from risk sharing can create quite substantial welfare gains from monetary policy coordination.

where the subscript 'RBP' indicates "risk sharing before policy". The welfare yielded by non-coordinated policy is

$$\tilde{\Omega}^{N}_{RBP} = -\frac{\left(1 + 2\theta^{2} - 4\theta^{3} + \theta^{4}\right)}{16\theta}(1 - \upsilon)\sigma^{2}_{K} \tag{68}$$

Again, as a point of reference it is useful to consider an inactive policy regime. The welfare level yielded by this regime is

$$\tilde{\Omega}^{M}_{RBP} = -\frac{1}{2}\sigma^{2}_{K} \tag{69}$$

As in the pervious cases two propositions can now be stated (and again the proofs are omitted).

Proposition 5 *If $\upsilon < 1$ and $\theta > 1$ then $\tilde{\Omega}^{C}_{RBP} > \tilde{\Omega}^{N}_{RBP}$, i.e. there are gains from coordination.*

It is clear from the expressions (61) and (62) that gains from coordination will arise. All the factors that were present in the previous cases are also present in this case.

Again notice that there are two cases where the gains from coordination disappear. The first is where $\theta = 1$. In this case the spillover effect from monetary policy to foreign output disappears. The second case is where $\upsilon = 1$. Correlated shocks do not create any conflicts between optimal policy in each country.

Proposition 6 *If $\theta > 1$ then: (a) $\left|\delta^{N}_{K}\right| = \left|\delta^{*N}_{K^{*}}\right| > \left|\delta^{C}_{K}\right| = \left|\delta^{*C}_{K^{*}}\right|$ and $\left|\delta^{N}_{K^{*}}\right| = \left|\delta^{*N}_{K}\right| > \left|\delta^{C}_{K^{*}}\right| = \left|\delta^{*C}_{K}\right|$ and (b) $Var\left(\hat{S}^{N}\right) > Var\left(\hat{S}^{C}\right)$, $Var\left(\hat{Y}^{N}\right) > Var\left(\hat{Y}^{C}\right)$ and $Var\left(\hat{Y}^{*N}\right) > Var\left(\hat{Y}^{*C}\right)$.*

In this case, in contrast to the previous cases, Nash policymaking implies a stronger monetary policy reaction to shocks than coordinated policymaking. And now non-coordinated policy implies *more* volatility in the exchange rate and output levels.

The quantitative implications of the additional spillover effects arising in this case are illustrated in Table 3.[18] The parameter values and construction of the table are identical to the previous cases. It is clear that the gains from coordination can now be very large, both in relative and absolute terms. When compared to the case where risk trading takes place after policy rules are chosen non-coordinated policymaking now yields much lower levels of welfare.

[18] No values are shown for non-coordinated policy for $\theta = 6$. This is because the second-order conditions for a maximum of the individual countries' policy problems are not satisfied for values of θ larger than approximately 4.5.

θ	1	2	3	4	6
$\tilde{\Omega}^{C}_{RBP} - \tilde{\Omega}^{M}_{RBP}$	0.500	0.750	1.000	1.250	1.750
$\tilde{\Omega}^{N}_{RBP} - \tilde{\Omega}^{M}_{RBP}$	0.500	0.719	0.667	-0.016	-
$\tilde{\Omega}^{C}_{RBP} - \tilde{\Omega}^{N}_{RBP}$	0.000	0.031	0.333	1.266	-
$\frac{100 \times \left(\tilde{\Omega}^{C}_{RBP} - \tilde{\Omega}^{N}_{RBP}\right)}{\left(\tilde{\Omega}^{C}_{RBP} - \tilde{\Omega}^{M}_{RBP}\right)}$	0.0	4.2	33.3	101.2	-

Table 3: The welfare effects of coordination: Risk-sharing - asset trade after monetary policy

ρ	1/4	1/2	1	2	4	6	8
$\tilde{\Omega}^{C}_{A} - \tilde{\Omega}^{M}_{A}$	1.444	0.900	0.583	0.375	0.229	0.167	0.131
$\tilde{\Omega}^{N}_{A} - \tilde{\Omega}^{M}_{A}$	1.421	0.898	0.580	0.359	0.208	0.147	0.113
$\tilde{\Omega}^{C}_{A} - \tilde{\Omega}^{N}_{A}$	0.023	0.002	0.003	0.016	0.021	0.020	0.018
$\frac{100 \times \left(\tilde{\Omega}^{C}_{A} - \tilde{\Omega}^{N}_{A}\right)}{\left(\tilde{\Omega}^{C}_{A} - \tilde{\Omega}^{M}_{A}\right)}$	1.589	0.227	0.571	4.167	9.276	12.00	13.64

Table 4: Risk aversion and the welfare effects of coordination: Financial autarky

5 Risk Aversion and the Welfare Gains from Policy Coordination

The analysis so far has focused on the case where utility is logarithmic in consumption. The coefficient of relative risk aversion, ρ, is therefore unity. This section briefly considers the implications of varying the degree of risk aversion. In what follows the explicit derivation is omitted and the discussion focuses on numerical examples.

Table 4 illustrates the quantitative implications of varying the degree of risk aversion in the autarky case. In this table $\theta = 2$ and the value of ρ is varied between 1/4 and 8. The baseline parameter values are the same as in previous examples. It is apparent that the size of the welfare gain is increasing as the degree of risk aversion deviates from unity. The size of the welfare gain is now rather larger in relative terms but it remains small in absolute terms.

Tables 5 and 6 conduct the same exercise in the two risk-sharing cases (again with $\theta = 2$). Again the welfare gains from coordination are increasing in the degree of risk aversion.[19]

[19] The results reported in Table 5 differ from the results reported in the equivalent table in a previous version of this paper. As already noted, the previous version of the paper did not explicitly model the asset structure necessary to support risk sharing. The explicit asset structure adopted in this version of the paper allows a more rigorous derivation of the equilibrium conditions implied by risk sharing. This reveals an additional term (which was omitted in the calculations reported in the previous version of the paper) which comes into play when ρ differs from unity. See Appendix B for more details.

ρ	1/4	1/2	1	2	4	6	8
$\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^M_{RAP}$	1.500	1.000	0.750	0.625	0.563	0.542	0.531
$\tilde{\Omega}^N_{RAP} - \tilde{\Omega}^M_{RAP}$	1.486	1.000	0.736	0.580	0.487	0.452	0.434
$\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^N_{RAP}$	0.014	0.000	0.014	0.045	0.076	0.089	0.097
$\frac{100\times\left(\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^N_{RAP}\right)}{\left(\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^M_{RAP}\right)}$	0.9	0.0	1.8	7.2	13.4	16.5	18.3

Table 5: Risk aversion and the welfare effects of coordination: Risk sharing - asset trade after monetary policy

ρ	1/4	1/2	1	2	4	6	8
$\tilde{\Omega}^C_{RBP} - \tilde{\Omega}^M_{RBP}$	1.500	1.000	0.750	0.625	0.563	0.542	0.531
$\tilde{\Omega}^N_{RBP} - \tilde{\Omega}^M_{RBP}$	1.375	1.000	0.719	0.555	0.467	0.437	0.421
$\tilde{\Omega}^C_{RBP} - \tilde{\Omega}^N_{RBP}$	0.125	0.000	0.031	0.070	0.096	0.105	0.110
$\frac{100\times\left(\tilde{\Omega}^C_{RBP} - \tilde{\Omega}^N_{RBP}\right)}{\left(\tilde{\Omega}^C_{RBP} - \tilde{\Omega}^M_{RBP}\right)}$	8.3	0.0	4.2	11.2	17.0	19.4	20.7

Table 6: Risk aversion and the welfare effects of coordination: Risk sharing - asset trade before monetary policy

6 The Welfare Gains from Risk Sharing

The main focus of this paper is on the welfare gains from policy coordination. But the model also yields estimates of the welfare gains from risk sharing. Table 7 repeats some of numerical welfare results from the previous sections in a way which allows a comparison across financial market structures. Table 7 focuses on the effects of varying θ when $\rho = 1$ (i.e. the case of logarithmic utility). The first row shows the welfare gains from risk sharing when monetary policy is coordinated. The second row shows the same results for the non-coordinated policy regime where risk sharing takes place after monetary policy rules are chosen. And the third row shows the same results for the non-coordinated policy regime where risk sharing takes place before monetary policy rules are chosen.

It is clear from expressions (45) and (56) that there is an unambiguous welfare gain to risk sharing when policy is coordinated (provided $\theta > 1$ and $\upsilon < 1$). That there should be such a welfare gain is not *a priori* obvious in a model where there are

θ	1	2	3	4	6
$\tilde{\Omega}^C_{RAP} - \tilde{\Omega}^C_{A}$	0.000	0.167	0.492	0.692	1.210
$\tilde{\Omega}^N_{RAP} - \tilde{\Omega}^N_{A}$	0.000	0.156	0.379	0.592	0.996
$\tilde{\Omega}^N_{RBP} - \tilde{\Omega}^N_{A}$	0.000	0.139	0.099	-.571	-

Table 7: The welfare effects of risk sharing: Logarithmic utility

several market distortions (such as monopoly power and sticky nominal prices). The figures in the first row in Table 6 provide a quantitative measure of the potential welfare gain from risk sharing. These figures are within the range of estimates suggested by previous literature.[20]

The welfare effects of risk sharing are somewhat smaller when monetary policy is not coordinated and when risk sharing takes place after monetary policy rules are chosen. A comparison of (46) and (57) shows that risk sharing again provides an unambiguous welfare gain when $\rho = 1$. But figures in the second row of Table 6 show that the welfare gain is smaller than when monetary policy is coordinated. The monetary policy spillover effects created by risk sharing (and the consequent welfare losses generated by uncoordinated monetary policy) partly offset the welfare benefits of risk sharing.

The welfare gains from risk sharing are, however, very sensitive to the timing of risk trading. This is illustrated in the third row of Table 6. Here it becomes apparent that risk sharing can have a negative effect on welfare when θ is large. In this case the policy spillover effects created by risk sharing are so strong that they completely offset the welfare benefits of risk sharing.

7 The Optimality of Price Targeting

One theme in the recent literature on monetary policy has been the welfare implications of price (or inflation) targeting. A number of authors have argued that price or inflation targeting is desirable from a welfare point of view (see for instance, King and Wolman (1999), Goodfriend and King (2001), Woodford (2001)). The final section of this paper briefly discusses the implications of the model for the optimality of price targeting.

The model assumes that all prices are fixed in advance so it is not possible directly to analyse a price targeting policy. But it is possible to gain some indirect insight into the implications for prices by considering the first-order condition for price setting that would be relevant if agents were able to set prices after shocks are realised. The first-order condition for the choice of prices in a flexible-price equilibrium is derived in Appendix A and implies the following

$$\hat{P}_H = \hat{K} + \hat{P} + \rho\hat{C}, \quad \hat{P}^*_F = \hat{K}^* + \hat{P}^* + \rho\hat{C}^* \tag{70}$$

A price targeting policy implies $\hat{P}_H = \hat{P}^*_F = 0$ so, when expressions (70) are combined with the money demand equations the following monetary rules are obtained

$$\hat{M} = -\hat{K}, \quad \hat{M}^* = -\hat{K}^* \tag{71}$$

[20] For instance Cole and Obstfeld (1991) suggest a welfare gain from risk sharing of the order of 0.2 percent of steady state consumption while van Wincoop (1994) suggests a gain closer to 5 percent of steady state consumption.

These rules are relevant for all values of θ and ρ and for all financial market structures. So any equilibrium which implies policy rules of the above form is consistent with price targeting.

It is immediately clear that neither coordinated nor non-coordinated policymaking is consistent with price targeting in the case of financial autarky (as is argued in Benigno (2001) and Obstfeld and Rogoff (2002)). It is however clear that coordinated policy is consistent with price targeting when there is risk sharing (of either form). Uncoordinated policymaking is only consistent with price targeting in the risk-sharing case for particular parameter combinations (as shown in Benigno and Benigno (2001a).

8 Conclusion

This paper has analysed the welfare effects of monetary policy coordination in a model where the elasticity of substitution between home and foreign goods can differ from unity. It is shown that welfare gains to policy coordination can arise when the elasticity is greater than unity, but these gains are quantitatively small when there is no international financial market. When, however, there is a sufficiently sophisticated financial market to allow full consumption risk sharing the gains from policy coordination are found to be much larger. This is particularly true when asset trade takes place before monetary policy rules are chosen.

The model also yields results concerning the welfare impact of financial market integration (i.e. a move from financial autarky to risk sharing). It is found that the additional monetary policy spillover effects created by financial markets can be so strong that financial market integration can have a negative impact on welfare if monetary policy is not coordinated.

This paper has considered two extreme forms of financial market structure. The gains from coordination are found to differ significantly between the two extremes. But neither extreme is entirely satisfactory as a representation of reality. An obvious next step in this line of research is to investigate the welfare gains to coordination in some intermediate financial market structure. A possible example of an intermediate structure is one where financial trade only takes place in the form of non-contingent bonds. This type of model will inevitably involve asset stock dynamics and will therefore require more extensive use of numerical simulation techniques.[21] An alternative way to model an intermediate degree of risk sharing has recently been proposed by Ligon, Thomas and Worrall (1997, 2000) and Kehoe and Perri (2000). It may also be interesting to consider the gains from monetary policy coordination in this alternative 'endogenous incomplete market' framework.

[21] Techniques which make this form of analysis possible have recently been developed by Kim and Kim (2000), Sims (2000), Schmitt-Grohé and Uribe (2001) and Sutherland (2001).

Appendix

A. Optimal Price Setting

The price-setting problem facing a fixed-price producer is the following:

$$MaxU(h) = E\left\{\frac{C^{1-\rho}(h)}{1-\rho} + \chi \log \frac{M(h)}{P} - Ky(h)\right\} \tag{72}$$

subject to

$$PC(h) = (1+\alpha)\, p_H\,(h)\, y(h) + M_0 - M(h) - T + R(h) \tag{73}$$

$$y(h) = c_H(h) + c_H^*(h) = (C_H + C_H^*)\left(\frac{p_H\,(h)}{P_H}\right)^{-\phi} \tag{74}$$

The first order condition with respect to $p_H\,(h)$ is[22]

$$E\left\{(1+\alpha)\,\frac{y(h)}{PC^{\rho}(h)} - \phi\left[(1+\alpha)\,\frac{p_H\,(h)}{PC^{\rho}(h)} - K\right]\frac{y(h)}{p_H\,(h)} = 0\right\} = 0 \tag{75}$$

In equilibrium all agents choose the same price and consumption level so

$$E\left\{(1+\alpha)\,\frac{Y}{PC^{\rho}} - \phi\left[(1+\alpha)\,\frac{P_H}{PC^{\rho}} - K\right]\frac{Y}{P_H} = 0\right\} = 0 \tag{76}$$

where

$$Y = C_H + C_H^* \tag{77}$$

Rearranging yields the expression in the main text.

The price-setting problem facing a flexible-price producer is the following:

$$MaxU(h) = \frac{C^{1-\rho}(h)}{1-\rho} + \chi \log \frac{M(h)}{P} - Ky(h) \tag{78}$$

subject to

$$PC(h) = (1+\alpha)\, p_H\,(h)\, y(h) + M_0 - M(h) - T + R(h) \tag{79}$$

$$y(h) = c_H(h) + c_H^*(h) = (C_H + C_H^*)\left(\frac{p_H\,(h)}{P_H}\right)^{-\phi} \tag{80}$$

The first order condition with respect to $p_H\,(h)$ is

$$(1+\alpha)\,\frac{y(h)}{PC^{\rho}(h)} - \phi\left[(1+\alpha)\,\frac{p_H\,(h)}{PC^{\rho}(h)} - K\right]\frac{y(h)}{p_H\,(h)} = 0 \tag{81}$$

[22] Notice that this first-order condition is the unaffected by the existence of income contingent assets because the asset returns are assumed to be correlated with aggregate real income. Asset returns are therefore treated as exogenous from the point of view of individual agents.

In equilibrium all agents choose the same price and consumption level so

$$(1+\alpha)\frac{Y}{PC^{\rho}}-\phi\left[(1+\alpha)\frac{P_H}{PC^{\rho}}-K\right]\frac{Y}{P_H}=0 \tag{82}$$

where

$$Y=C_H+C_H^* \tag{83}$$

Rearranging yields the expression in the main text.

B. Portfolio Allocation, Asset Prices and Risk Sharing

It will prove convenient to denote the expectations of private agents with the operator $\tilde{E}$ where these expectations are conditional on information at the time at which asset trade takes place. The first-order conditions (two for each country) for the choice of asset holdings imply the following four equations

$$\tilde{E}\left[C^{-\rho}y\right]=\tilde{E}\left[C^{-\rho}\right]q_H \tag{84}$$

$$\tilde{E}\left[C^{-\rho}y^*\right]=\tilde{E}\left[C^{-\rho}\right]q_F \tag{85}$$

$$\tilde{E}\left[C^{*-\rho}y\right]=\tilde{E}\left[C^{*-\rho}\right]q_H \tag{86}$$

$$\tilde{E}\left[C^{*-\rho}y^*\right]=\tilde{E}\left[C^{*-\rho}\right]q_F \tag{87}$$

The combination of the private and government budget constraints and the portfolio payoff functions for each country imply that aggregate home and foreign consumption levels are given by

$$C=y+\zeta_H\left(y-q_H\right)+\zeta_F\left(y^*-q_F\right) \tag{88}$$

$$C^*=y^*+\zeta_H^*\left(y-q_H\right)+\zeta_F^*\left(y^*-q_F\right) \tag{89}$$

where in a symmetric equilibrium $\zeta_H(h)=\zeta_H$ and $\zeta_F(h)=\zeta_F$ for all h and $\zeta_H^*(f)=\zeta_H^*$ and $\zeta_F^*(f)=\zeta_F^*$ for all f. Equilibrium in asset markets implies $\zeta_H+\zeta_H^*=0$ and $\zeta_F+\zeta_F^*=0$. Using these eight equations it is possible to solve for eight unknowns (q_H, q_F, ζ_H, ζ_F, ζ_H^*, ζ_F^*, C and C^*) in terms of y and y^*.

After using the asset market equilibrium conditions to eliminate ζ_H^* and ζ_F^* the remaining six equations can be replaced by second-order approximations in terms of log-deviations from a non-stochastic equilibrium as follows

$$\hat{q}_H=\tilde{E}\left[\hat{y}+\frac{1}{2}\hat{y}^2-\rho\hat{C}\hat{y}\right]+O\left(\|\xi\|^3\right) \tag{90}$$

$$\hat{q}_F=\tilde{E}\left[\hat{y}^*+\frac{1}{2}\hat{y}^{*2}-\rho\hat{C}\hat{y}^*\right]+O\left(\|\xi\|^3\right) \tag{91}$$

$$\hat{q}_H=\tilde{E}\left[\hat{y}+\frac{1}{2}\hat{y}^2-\rho\hat{C}^*\hat{y}\right]+O\left(\|\xi\|^3\right) \tag{92}$$

$$\hat{q}_F = \tilde{E}\left[\hat{y}^* + \frac{1}{2}\hat{y}^{*2} - \rho\hat{C}^*\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{93}$$

$$\tilde{E}\left[\hat{C}\right] = \tilde{E}\left[\hat{y} + \zeta_H\left(\hat{y} - \hat{q}_H\right) + \zeta_F\left(\hat{y} - \hat{q}_F\right) + \lambda_C\right] + O\left(\|\xi\|^3\right) \tag{94}$$

$$\tilde{E}\left[\hat{C}^*\right] = \tilde{E}\left[\hat{y}^* - \zeta_H\left(\hat{y} - \hat{q}_H\right) - \zeta_F\left(\hat{y} - \hat{q}_F\right) + \lambda_{C^*}\right] + O\left(\|\xi\|^3\right) \tag{95}$$

where

$$\lambda_C = \frac{1}{2}\left[-\zeta_H\left(1+\zeta_H\right)\hat{y}^2 + \zeta_F\left(1-\zeta_F\right)\hat{y}^{*2} - 2\zeta_F\left(1+\zeta_H\right)\hat{y}\hat{y}^*\right] \tag{96}$$

$$\lambda_{C^*} = \frac{1}{2}\left[-\zeta_H\left(1+\zeta_H\right)\hat{y}^2 + \zeta_F\left(1-\zeta_F\right)\hat{y}^{*2} - 2\zeta_H\left(1-\zeta_F\right)\hat{y}\hat{y}^*\right] \tag{97}$$

The solution of these equations yields the following expressions for the portfolio shares and asset prices

$$-\zeta_H = \zeta_F = \frac{1}{2} + O\left(\|\xi\|^3\right) \tag{98}$$

$$q_H = \tilde{E}\left[\hat{y} + \frac{1}{2}\left(1-\rho\right)\hat{y}^2 - \rho\hat{y}\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{99}$$

$$q_F = \tilde{E}\left[\hat{y}^* + \frac{1}{2}\left(1-\rho\right)\hat{y}^{*2} - \rho\hat{y}\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{100}$$

It is now necessary to distinguish between the two risk-sharing cases.

When asset trade takes place after policy rules are chosen then the $\tilde{E}$ operator obviously contains information on monetary policy. Thus monetary policy can affect asset prices. If the E operator is used to denote expectations at the time that policy rules are chosen then policymakers' expectations of asset prices can be formed simply by replacing the $\tilde{E}$ operators with E operators. It is then possible to show that the expected level of home and foreign consumption are

$$E\left[\hat{C}\right] = E\left[\hat{y} + \frac{3-2\rho}{8}\hat{y}^2 - \frac{1-2\rho}{8}\hat{y}^{*2} - \frac{1}{4}\hat{y}\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{101}$$

$$E\left[\hat{C}^*\right] = E\left[\hat{y}^* + \frac{3-2\rho}{8}\hat{y}^{*2} - \frac{1-2\rho}{8}\hat{y}^2 - \frac{1}{4}\hat{y}\hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{102}$$

and that

$$E\left[\hat{C} - \hat{C}^*\right] = E\left[\hat{y} - \hat{y}^* + \frac{1}{2}\left(1-\rho\right)\left(\hat{y}^2 - \hat{y}^{*2}\right)\right] + O\left(\|\xi\|^3\right) \tag{103}$$

After some further manipulation using the expressions for the demand for home and foreign goods it is possible to show that

$$E\left[\hat{P}_H + \hat{C}_H^* - \hat{P}_F - \hat{C}_F\right] = E\left[\frac{1}{2}\left(\rho-1\right)\left(\hat{y}^2 - \hat{y}^{*2}\right)\right] + O\left(\|\xi\|^3\right) \tag{104}$$

Notice that the expression on the left hand side of (104) is a measure of the expected balance of trade. Furthermore, notice that the expression on the right hand side of this equation is zero when $\rho = 1$. Thus, when asset trade takes place after policy rules are chosen and $\rho = 1$, the expected trade balance is zero.[23]

When asset trade takes place before policy rules are chosen the $\tilde{E}$ operator obviously does not contain information on monetary policy. Thus monetary policy can not affect asset prices. In this case private agents must form expectations of the equilibrium in the monetary policy game. Notice, however, that in this case it is not necessary to obtain explicit expressions for asset prices. It is sufficient to note that, regardless of the equilibrium in the monetary policy game, the symmetry of the model implies that $q_H = q_F$ and $-\zeta_H = \zeta_F = 1/2$. Thus, using (88) and (89) it follows that, at the time at which policy is formed,

$$E\left[\hat{C}\right] = E\left[\hat{C}^*\right] = \frac{1}{2}E\left[\hat{y} + \hat{y}^*\right] + O\left(\|\xi\|^3\right) \tag{105}$$

and

$$E\left[\hat{C} - \hat{C}^*\right] = 0 + O\left(\|\xi\|^3\right) \tag{106}$$

The important differences between the two forms of risk sharing are revealed in the contrast between equations (103) and (106). Equation (106) shows that, when asset trade takes place before policy rules are chosen, expected consumption levels are equal regardless of the choice of policy rules . But when asset trade takes place after policy rules are chosen equation (103) shows that the choice of policy rules can affect the cross-country distribution of expected consumption levels by affecting the first and second moments of real incomes.

Finally notice that regardless of when asset trade takes place a first-order approximation of realised consumption yields the following

$$\hat{C} = \hat{C}^* = \frac{1}{2}\left(\hat{y} + \hat{y}^*\right) + O\left(\|\xi\|^2\right) \tag{107}$$

and thus

$$\hat{C} - \hat{C}^* = 0 + O\left(\|\xi\|^2\right) \tag{108}$$

C. Model Solution

The solution procedure is described using the autarky case as an illustration. The amendments necessary to derive the risk sharing solution are then described.

[23] In a previous version of this paper this term was incorrectly set to zero for all values of ρ. Notice that when $\rho > 1$ equation (104) implies that the home country must run an expected balance of trade surplus if home real income is more volatile than foreign real income and an expected deficit if home real income is less volatile than foreign real income. In the former case the surplus represents compensation to foreign agents for taking on the extra risk implied by volatile home income, and in the latter case the deficit represents compensation to home agents for taking on the extra risk implied by volatile foreign real income.

In order to derive a solution for the welfare measure it is necessary to derive solutions for both the first and second moments of the model. The first step in the solution process is to replace each equation of the model with a second-order approximation in terms of log-deviations from the non-stochastic steady state. Most of the equations of the model are linear in logs so this process does not involve any approximation for those equations. There are just three pairs of equations where approximations are necessary.

The log-deviation form of the money market equations implies

$$\hat{M} = \hat{P} + \rho\hat{C}, \quad \hat{M}^* = \hat{P}^* + \rho\hat{C}^* \tag{109}$$

For home and foreign demand equations the log-deviation forms are

$$\hat{C}_H = \hat{C} - \theta\left(\hat{P}_H - \hat{P}\right), \quad \hat{C}_F = \hat{C} - \theta\left(\hat{P}_F - \hat{P}\right) \tag{110}$$

and

$$\hat{C}_H^* = \hat{C}^* - \theta\left(\hat{P}_H^* - \hat{P}^*\right), \quad \hat{C}_F^* = \hat{C}^* - \theta\left(\hat{P}_F^* - \hat{P}^*\right) \tag{111}$$

The log-deviation form of current account balance implies

$$\hat{P}_H + \hat{C}_H^* = \hat{P}_F + \hat{C}_F \tag{112}$$

And the log-deviation form of purchasing power parity implies

$$\hat{P} = \hat{P}^* + \hat{S} \tag{113}$$

None of the above equations require any approximation when converting to log-deviation form.

The expressions for total outputs, aggregate prices and price setting do require approximation. The second-order approximation for the total output equations are

$$\hat{Y} = \frac{1}{2}\hat{C}_H + \frac{1}{2}\hat{C}_H^* + \lambda_Y + O\left(\|\xi\|^3\right), \quad \hat{Y}^* = \frac{1}{2}\hat{C}_F + \frac{1}{2}\hat{C}_F^* + \lambda_{Y^*} + O\left(\|\xi\|^3\right) \tag{114}$$

where

$$\lambda_Y = \frac{1}{8}\left(\hat{C}_H - \hat{C}_H^*\right)^2, \quad \lambda_{Y^*} = \frac{1}{8}\left(\hat{C}_F - \hat{C}_F^*\right)^2$$

The second-order approximations for the aggregate price indices are

$$\hat{P} = \frac{1}{2}\hat{P}_H + \frac{1}{2}\hat{P}_F + \lambda_P + O\left(\|\xi\|^3\right), \quad \hat{P}^* = \frac{1}{2}\hat{P}_H^* + \frac{1}{2}\hat{P}_F^* + \lambda_{P^*} + O\left(\|\xi\|^3\right) \tag{115}$$

where

$$\lambda_P = \frac{1}{8}\left(\hat{P}_H - \hat{P}_F\right)^2, \quad \lambda_{P^*} = \frac{1}{8}\left(\hat{P}_H^* - \hat{P}_F^*\right)^2$$

And the second-order approximations for the price setting conditions are

$$\hat{P}_H = E\left[\hat{K} + \hat{P} + \rho\hat{C}\right] + \lambda_{P_H} + O\left(\|\xi\|^3\right) \tag{116}$$

$$\hat{P}_F^* = E\left[\hat{K}^* + \hat{P}^* + \rho\hat{C}^*\right] + \lambda_{P_F^*} + O\left(\|\xi\|^3\right) \tag{117}$$

where

$$\lambda_{P_H} = \frac{1}{2}E\left[\hat{K}^2 + 2\hat{K}\hat{Y} - \hat{P}^2 - \rho^2\hat{C}^2 + \hat{Y}\hat{P} + \rho\hat{Y}\hat{C} - \rho\hat{P}\hat{C}\right]$$

$$\lambda_{P_F^*} = \frac{1}{2}E\left[\hat{K}^{*2} + 2\hat{K}^*\hat{Y}^* - \hat{P}^{*2} - \rho^2\hat{C}^{*2} + \hat{Y}^*\hat{P}^* + \rho\hat{Y}^*\hat{C}^* - \rho\hat{P}^*\hat{C}^*\right]$$

Notice that second-order terms are collected in the six terms λ_Y, λ_{Y^*}, λ_P, λ_{P^*}, λ_{P_H} and $\lambda_{P_F^*}$. Using the above equations it is possible to solve for the first moments of all the variables of the model in terms of these second-order terms. In this way the following expression is obtained for the first-order terms in the home welfare function

$$\begin{aligned} E\left[\hat{C} - \hat{Y}\right] &= \frac{1}{2\left[1 + \rho\left(\theta - 1\right)\right]}E\left\{\lambda_{P_H} - \lambda_{P_F^*} - 2\left[1 + \rho\left(\theta - 1\right)\right]\lambda_Y \right. \\ &\quad \left. + \left(1 - 2\theta\right)\left(1 + \rho\theta\right)\lambda_P - \left(1 - \rho\theta\right)\lambda_{P^*}\right\} + O\left(\|\xi\|^3\right) \end{aligned} \tag{118}$$

Notice now that welfare can be written entirely in terms of second moments. The remaining task is therefore to derive expressions for the second moments of the variables of the model. This task is made easier by noting that second-order accurate solutions for second moments can be derived from first-order accurate solutions for the realisations of variables. First-order accurate solutions for *ex post* realisations can be obtained from equations (109) to (116) by ignoring second-order terms. In the case where $\rho = 1$ the resulting set of equations can be used to derive the following expressions for the λs

$$E\left[\lambda_{P_H}\right] = \frac{1}{2}E\left[\left(\hat{Y} + \hat{K}\right)^2\right] + O\left(\|\xi\|^3\right) \tag{119}$$

$$E\left[\lambda_{P_F^*}\right] = \frac{1}{2}E\left[\left(\hat{Y}^* + \hat{K}^*\right)^2\right] + O\left(\|\xi\|^3\right) \tag{120}$$

$$E\left[\lambda_Y\right] = E\left[\lambda_{Y^*}\right] = \frac{1}{8}\left(1 - \theta\right)^2 E\left[\hat{S}^2\right] + O\left(\|\xi\|^3\right) \tag{121}$$

$$E\left[\lambda_P\right] = E\left[\lambda_{P^*}\right] = \frac{1}{8}\left(1 - \theta\right)E\left[\hat{S}^2\right] + O\left(\|\xi\|^3\right) \tag{122}$$

Home welfare can therefore be written as follows

$$\tilde{\Omega} = -\frac{1}{4}E\left[\frac{1}{\theta}\left(2\theta - 1\right)\left(\hat{Y} + \hat{K}\right)^2 + \frac{1}{\theta}\left(\hat{Y}^* + \hat{K}^*\right)^2 + \frac{1}{2}\left(1 - \theta\right)\hat{S}^2\right] + O\left(\|\xi\|^3\right) \tag{123}$$

which is the expression used in the main text. The expression for foreign welfare follows immediately by symmetry.

The procedure for deriving welfare expressions for the two risk-sharing cases is identical. The only amendment required is to replace the current account equation with the relevant risk-sharing conditions. In the case where risk sharing takes place after monetary rules are chosen *ex ante* expected consumption levels are related by condition (103) while in the case where risk sharing takes place before monetary rules are chosen *ex ante* expected consumption levels are related by condition (106). In both risk sharing cases *ex post* consumption levels are related by (107).

D. Nash Equilibrium Production Subsidies

A simple way to derive the non-stochastic equilibrium output level implied by the Nash equilibrium in subsidies is to consider a non-stochastic flexible-price version of the model. A first-order approximation of the model around an arbitrary non-stochastic equilibrium shows that consumption and output are related to the production subsidies as follows

$$\hat{C} = \hat{C}^* = -\frac{\hat{\alpha} + \hat{\alpha}^*}{2\rho} + O\left(\|\xi\|^2\right) \tag{124}$$

$$\hat{Y} = -\frac{(1+\rho\theta)\,\hat{\alpha} + (1-\rho\theta)\,\hat{\alpha}^*}{2\rho} + O\left(\|\xi\|^2\right) \tag{125}$$

$$\hat{Y}^* = -\frac{(1-\rho\theta)\,\hat{\alpha} + (1+\rho\theta)\,\hat{\alpha}^*}{2\rho} + O\left(\|\xi\|^2\right) \tag{126}$$

where $\hat{\alpha} = \log(\alpha/\bar{\alpha})$ and $\hat{\alpha}^* = \log(\alpha^*/\bar{\alpha}^*)$. And where $\bar{\alpha}$ and $\bar{\alpha}^*$ are the production subsidies in the arbitrary non-stochastic equilibrium around which the approximation is taken.

A second-order approximation of the home and foreign welfare functions yields

$$\tilde{\Omega} = \hat{C} + \frac{1}{2}(1-\rho)\,\hat{C}^2 - \bar{Y}^\rho\left(\hat{Y} + \frac{1}{2}\hat{Y}^2\right) + O\left(\|\xi\|^3\right) \tag{127}$$

$$\tilde{\Omega}^* = \hat{C}^* + \frac{1}{2}(1-\rho)\,\hat{C}^{*2} - \bar{Y}^\rho\left(\hat{Y}^* + \frac{1}{2}\hat{Y}^{*2}\right) + O\left(\|\xi\|^3\right) \tag{128}$$

where $\bar{Y}$ is the level of output in the arbitrary non-stochastic equilibrium around which the approximation is taken.[24]

Using these expressions it is simple to show that the Nash equilibrium in the choices of $\hat{\alpha}$ and $\hat{\alpha}^*$ implies

$$\hat{\alpha} = \hat{\alpha}^* = \frac{\rho\left[1 - (1+\rho\theta)\,\bar{Y}^\rho\right]}{1 - (1+\rho\theta)\,\bar{Y}^\rho - \rho} \tag{129}$$

This expression shows how the Nash equilibrium values of α and α^* will deviate from some arbitrary values of $\bar{\alpha}$ and $\bar{\alpha}^*$. But now suppose that $\bar{\alpha}$ and $\bar{\alpha}^*$ are themselves the result of a Nash equilibrium. Then by definition $\hat{\alpha} = \hat{\alpha}^* = 0$. Thus, by setting

[24] Note that $\bar{Y} = \bar{Y}^* = \bar{C} = \bar{C}^*$ in a symmetric equilibrium.

the expression on the right hand side of (129) equal to zero, it is possible to show that

$$\bar{Y} = \left(\frac{1}{1 + \rho\theta} \right)^{\frac{1}{\rho}} \tag{130}$$

This is the level of output yielded by a Nash equilibrium in the choice of $\bar{\alpha}$ and $\bar{\alpha}^*$. This is the non-stochastic equilibrium used in the calculations reported in Section 4.4.

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