

Endogenous Lobbying*

LEONARDO FELLI
*London School of Economics
and CEPR*

ANTONIO MERLO
*University of Pennsylvania
and CEPR*

Revised December 2001

ABSTRACT. In this paper, we present a citizen-candidate model of representative democracy with endogenous lobbying. We find that lobbying induces policy compromise and always affects equilibrium policy outcomes. In particular, even though the policy preferences of lobbies are relatively extreme, lobbying biases the outcome of the political process toward the center of the policy space, and extreme policies cannot emerge in equilibrium. Moreover, in equilibrium, not all lobbies participate in the policy-making process.

ADDRESS FOR CORRESPONDENCE: Leonardo Felli, London School of Economics, Houghton Street, London WC2A 2AE, England. E-mail: lfelli@econ.lse.ac.uk.

*The authors are grateful to two anonymous referees, Tim Besley, Steve Coate, James Dow, Elhanan Helpman, Steve Matthews, George Mailath, Michele Piccione and Andy Postlewaite for their very helpful comments and suggestions. Seminar and conference participants at several institutions provided useful comments. Felli acknowledges the financial support of the E.S.R.C. while Merlo acknowledges the financial support of the National Science Foundation. The first draft of the paper was completed while the first author was visiting the Department of Economics at the University of Pennsylvania. This revision was completed while both authors were visiting the Institute for International Economic Studies at Stockholm University. Their generous hospitality is gratefully acknowledged.

1. Introduction

A long tradition in political economy builds on the assumption that the main objective of politicians is to win an election (Downs 1957). Within this framework, known as the “downsian” paradigm, political candidates shape their policy platforms to please the (policy-concerned) electorate so as to maximize their probability of winning. In other words, a building block of the downsian paradigm is that the preferences of political candidates differ from the preferences of the citizens, or equivalently, the (pre-specified) set of political candidates is not a subset of the citizenry.

Several authors have challenged this view by proposing alternative models of electoral competition where politicians are assumed to be not only office-motivated, but also policy-motivated (Alesina 1988, Hibbs 1977, Wittman 1977). Within this framework, known as the “partisan” paradigm, political candidates choose their policy platforms by trading-off their policy concerns with their desire to win the election. As in the downsian framework, however, the set of political candidates is exogenous.

Recently, Besley and Coate (1997) and Osborne and Slivinski (1996) have proposed an alternative approach to the study of political competition known as the “citizen-candidate” paradigm. This framework removes the artificial distinction between citizens and candidates prevalent in the other approaches. This is accomplished by assuming that politicians are selected by the people from those citizens who choose to become candidates in an election. Once in office, elected candidates implement their most preferred policies.

While ultimately implemented by elected representatives, policies are typically the outcome of a political process that also involves non elected political actors. In particular, lobbying is an important part of the policy-making process in representative democracies. This raises the question: To what extent does lobbying affect policy?

Several authors have addressed this issue in the context of models of electoral competition where lobbies (or interest groups) compete to influence policy-makers.¹

¹This literature originates from the work by Tullock (1967) on rent-seeking. For a partial account of the large literature on lobbying see, for example, Grossman and Helpman (2001) or Chapter 7 in Persson and Tabellini (2000) and the references therein. A substantial part of the literature has focused on the incentives for lobbies to gather information and provide it to the policy-makers (Austen-Smith and Wright 1992, Grossman and Helpman 2001, e.g.). Like Besley and Coate (2001), Grossman and Helpman (1996), Persson and Helpman (1998) and many others, we abstract from the informational role of lobbies and focus instead on their influence-seeking activities.

In most of the recent literature, lobbying is modelled as a “menu-auction,” where exogenously given lobby groups offer policy-makers contribution schedules, representing binding promises of payment, conditional on the chosen policy (Bernheim and Whinston 1986, Besley and Coate 2001, Dixit, Grossman, and Helpman 1997, Grossman and Helpman 1994, Grossman and Helpman 1996, Persson and Helpman 1998).²

An implicit assumption of the menu-auction model of lobbying is that all lobbies participate in the policy-making process. We find this assumption problematic for at least two reasons. First, casual observations suggest that while a number of lobby groups may be willing to offer favors to elected politicians in exchange for policy compromise, policy-makers have a choice as to whom to accept favors from. Second, empirical evidence suggests that many existing lobby groups are often dormant and make no contributions (Wright 1996).

In this paper, we propose an alternative model of lobbying where the elected policy-maker chooses the lobbies that participate in the policy-making process. In our framework, policy is the outcome of efficient bargaining between the elected policy-maker and a coalition of lobbies selected by the policy-maker.³ This is the sense in which lobbying is endogenous in our model.

We consider a citizen-candidate model of electoral competition that builds on the work by Besley and Coate (1997) and Besley and Coate (2001). As in Besley and Coate (1997), we model the political process as a multi-stage game that begins with the citizens’ decisions to participate in the political process as candidates for public office. Given the set of candidates, citizens vote in an election that selects the plurality winner to choose policy for one period. When casting their ballot, citizens are assumed to be strategic.⁴

As in Besley and Coate (2001), we assume that after the election lobbies try to influence the policy choice of the elected candidate by offering him transfers in exchange for policy compromise. Contrary to Besley and Coate (2001), however, we do not model lobbying as a menu-auction, where all lobbies are (exogenously)

²In same models, payments take the form of campaign contributions (Grossman and Helpman 1996, e.g.). In other models, they take the form of lobbying expenditures that provide post-election support to officeholders (Besley and Coate 2001, Persson and Helpman 1998, e.g.).

³Diermeier and Merlo (2000) use a similar framework to analyze the process of government formation in parliamentary democracies.

⁴This assumption differentiates the citizen-candidate model of Besley and Coate (1997) from the one of Osborne and Slivinski (1996) where citizens are assumed to vote sincerely.

assumed to participate in the policy-making process.⁵ Rather, we assume that given the set of existing lobbies, the elected candidate (endogenously) chooses the coalition of lobbies he will bargain with over policy in exchange for transfers.

Although the citizen-candidate framework generates equilibria where any number of candidates participate in the electoral competition, in our analysis we restrict attention to the characterization of the set of two-candidate equilibria. Since we consider an electoral system where candidates are elected using plurality rule (like, for example, in the United States), we justify this choice on grounds of realism.⁶

Our main results can be summarized as follows. First, lobbying induces policy compromise. The equilibrium policy outcome is always a compromise between the policy preferences of the elected candidate and the policy preferences of the lobbies that participate in the policy-making process. The extent of the compromise depends on the relative intensity of the policy motivation of the elected candidate *vis-a-vis* the lobbies. We believe that compromise is a natural consequence of lobbying and is also an implication of the menu-auction model of lobbying (Besley and Coate 2001, Grossman and Helpman 1996, e.g.).⁷

Second, not all lobbies participate in the policy-making process. In equilibrium, no elected candidate ever includes all lobbies in the bargaining process that determines the policy outcome. This result is consistent with the empirical evidence cited above. Moreover, it highlights the fact that assuming that all lobbies participate in the decision-making process is not without consequences, and sets our framework apart from the menu-auction approach.

Third, lobbying matters. In our model, even though the policy preferences of all potential candidates span the entire policy space, the lobbying process reduces the set of policies that can be implemented in equilibrium. This result is in contrast with the findings of Besley and Coate (2001). In their model of exogenous lobbying, the presence of lobbies in the political process need have little or no effect on equilibrium

⁵In the remainder of the paper, we refer to this approach as exogenous lobbying.

⁶As discussed in Section 4 below, even though our analysis abstracts from the role of political parties, some of our results can be usefully interpreted in the context of a two-party system. We thank a referee for suggesting this interpretation.

⁷Notice, however, that Grossman and Helpman (1996) consider a Downsian model of electoral competition where candidates choose policies to maximize their probability of winning. In their model, lobbying induces candidates to adopt policies that represent a compromise between the policy preferences of lobbies and those of voters.

policy outcomes. In particular, they show that it is possible to construct examples where the equilibrium sets of policy outcomes of the games with and without lobbying coincide with the set of feasible policies. The reason for the result is that voters can restrict the influence of lobbies via strategic delegation by supporting candidates with offsetting policy preferences. In other words, in the game where lobbies are allowed to influence policy, voters can strategically elect a candidate who (after lobbying takes place) implements exactly the same policy that a different candidate would implement in the game where lobbying is ruled out. The feature of their model that is critical to obtain this result is the freedom to choose the characteristics of the lobbies that participate in the policy-making process (i.e., the menu-auction game). In our model, lobbying takes the form of bargaining between the elected candidate and a coalition of lobbies of his choice. In equilibrium, not all lobbies are selected to participate in the policy-making process for any elected candidate, and not all feasible policies can be implemented.

Fourth, lobbying biases the outcome of the policy-making process toward the center of the policy space. In our model, even though the policy preferences of lobbies are relatively extreme, lobbying has a moderating effect on policy, and extreme policies never emerge as an equilibrium outcome of the political process. This result is at odd with the findings of other existing models where lobbying tends to induce policy outcomes that are relatively extreme (Austen-Smith 1987, Baron 1994, Groseclose and Snyder 1996, Grossman and Helpman 1996, e.g.).⁸ The intuition for this result is as follows. In equilibrium, the candidates who run for office are citizens with relatively extreme policy preferences. If elected, they include in their bargaining coalition lobbies whose policy preferences are on the opposite end of the policy spectrum than their own preferences. This maximizes the transfers they receive for compromising on their policy choices. The outcomes of the compromise are policies that are relatively moderate (that is, policies that are near the center of the policy space). This implication of our model is consistent with the empirical evidence presented by Austen-Smith and Wright (1994), which shows that special interest groups often lobby legislators whose policy positions, prior to any lobbying, are diametrically opposed to theirs.

⁸Notice that this literature treats candidate entry as exogenous and hence ignores the effects of lobbying on the type of citizens who run for public office.

2. The Model

Each citizen $i \in \{1, \dots, N\}$ has quasi-linear preferences over a one-dimensional policy outcome $x \in X = [-1, 1]$ that has a public good nature and distributive benefits $y_i \in \mathbb{R}$ that have a private good nature. Citizens differ with respect to their policy preferences. We assume there exists a continuum of types of citizens indexed by $j \in X$, where j denotes the most preferred policy outcome of all citizens of that type.⁹ Let f denote the density of citizens' types over the support X . We take f to be continuous and symmetric around 0. We assume that the number of citizens N is large. Moreover, to guarantee that every $j \in X$ is represented in the citizenry, we abuse notation and refer to the population of citizens as a unit mass with density f on X .

The utility function of citizen i of type j (henceforth, citizen i^j) is given by

$$U(x, y_i, j) = u(x, j) + \lambda y_i \quad (1)$$

where $u(x, j)$ is strictly concave in x , single-peaked and symmetric around j , and $\lambda > 0$ measures the intensity of each citizen's preferences over money with respect to policy.¹⁰

As discussed in the Introduction, we model policy-making as the outcome of a political process that involves not only the citizen who is elected by the citizenry to represent them, but also non elected political agents known as lobbies. We assume there is a finite number of lobbies H that differ with respect to their policy preferences. Each lobby $h \in \{1, \dots, H\}$ has a most preferred policy outcome $l_h \in X$ and preferences represented by

$$V(x, y_h, l_h) = v(x, l_h) + \mu y_h \quad (2)$$

where $v(x, l_h)$ is strictly concave in x , single-peaked and symmetric around l_h , and $\mu > 0$ measures the intensity of each lobby's preferences over money with respect to policy. To capture the idea that lobbies care relatively more about money than citizens we assume that $\mu \geq \lambda$.¹¹

⁹As in Besley and Coate (1997) there is no incomplete information in our model. In particular, the type of each citizen is publicly observable.

¹⁰Notice that if $\lambda = 0$ citizens are purely policy-motivated and lobbying is irrelevant. This is the case studied in Besley and Coate (1997). We therefore restrict attention to the case where λ is strictly positive.

For ease of exposition — in order to obtain closed-form solutions to the model — in what follows we take:¹²

$$u(x, j) = -(x - j)^2 \quad (3)$$

and

$$v(x, l_h) = -(x - l_h)^2. \quad (4)$$

We normalize aggregate transfers to be zero (i.e., $\sum_{i \leq N} y_i + \sum_{h \leq H} y_h = 0$). Also, we assume that any policy $x \in X$ is costless to implement. Furthermore, we restrict attention to the case where there are three lobbies labelled L , C , and R , with most preferred policy outcomes $l_L = -1$, $l_C = 0$, and $l_R = 1$, respectively. Notice that l_L , l_C , and l_R denote, respectively, the left, center, and right of the policy space X , and l_C is the median of the distribution of the citizens' types. In the remainder of the paper, we denote $\mathcal{L} = \{L, C, R\}$ the set of lobbies.

We typically think of lobbies as representing a wide range of policy preferences. In particular, while some lobbies may hold rather extreme views on either side of the political spectrum, other lobbies may hold more moderate views. Our specification of the set of lobbies $\mathcal{L} = \{L, C, R\}$ is the simplest one that captures this insight. However, our setup can be extended to include any finite number of lobbies.

We assume that the political process has three stages. In the first stage, all citizens choose whether to run for office. Given the set of candidates that have entered the electoral competition an election follows in the second stage. The election selects one candidate that is delegated the policy decision for one period. In the third and final stage, lobbying takes place and policy is chosen. We describe below the structure of each stage of the political process.

2.1. Entry of Candidates

Each citizen must decide simultaneously and independently whether or not to run for office. If a citizen enters the electoral competition as a candidate he has to pay a (small) monetary cost $\delta > 0$. The decision to run for office yields benefits to

¹¹This assumption seems natural in light of the fact that lobbies are typically corporations. However, our analysis can be easily extended to the case where $\mu < \lambda$ without changing the main thrust of our results.

¹²While the details of the derivation presented in the paper clearly depend on the quadratic form of the functions $u(\cdot, \cdot)$ and $v(\cdot, \cdot)$, all of our results hold true for all strictly concave, single-peaked and symmetric functions.

the citizen either directly from winning or indirectly by affecting the identity of the winner.

Let $\sigma(i^j) \in \{0, 1\}$ denote the decision by citizen i^j whether to become a candidate: $\sigma(i^j) = 1$ indicates citizen i^j 's decision to enter the electoral competition.¹³ Let $\sigma = (\sigma(1), \dots, \sigma(N))$ denote the vector of all citizens' entry decisions. For any given σ let $\mathcal{C}(\sigma) = \{i^j \mid \sigma(i^j) = 1\}$ denote the set of candidates with typical element e . This set is the outcome of the entry-of-candidates subgame.

In the event that no citizen runs for office, we assume that a default policy $x_0 \in X$ is implemented.

2.2. Voting

Elections are structured so that all citizens have one vote that, if used, must be cast for one of the candidates.

In particular, given a set of candidates $\mathcal{C}(\sigma)$, each citizen simultaneously and independently decides to vote for any candidate in $\mathcal{C}(\sigma)$ or abstains. Let $\gamma(i^j)$ denote citizen i^j 's choice: if $\gamma(i^j) = e$ then citizen i^j casts a vote for candidate $e \in \mathcal{C}(\sigma)$; while if $\gamma(i^j) = 0$ he abstains. The vector of all citizens' voting decisions is denoted by $\gamma = (\gamma(1), \dots, \gamma(N))$.

The candidate who receives the most votes is elected, and in the event of ties, the winning candidate is chosen with equal probability from among the tying candidates. We denote $P^E \in \mathcal{C}(\sigma)$ the elected candidate, where $E \in X$ denotes the elected candidate's most preferred policy outcome.

We assume that citizens correctly anticipate the outcome of the lobbying stage that follows an election and vote strategically: each citizen i^j makes his voting decision $\gamma(i^j)$ so as to maximize his expected utility given the decisions of all other citizens.

2.3. Lobbying

Each lobby $h \in \mathcal{L}$ is assumed to be able to sign binding contracts on policy choices with the elected candidate P^E in exchange for transfers. Notice that the elected

¹³In principle, we could allow candidates to randomize on their entry decision. However, as in Besley and Coate (1997), we restrict attention to equilibria in pure strategies.

candidate P^E has the option of not signing any contract and implement his most preferred policy E .¹⁴

Let

$$\Delta = \{\{\emptyset\}, \{L\}, \{C\}, \{R\}, \{L, C\}, \{C, R\}, \{L, R\}, \{L, C, R\}\}$$

denote the power set of \mathcal{L} with typical element ℓ . The set Δ is the collection of all possible coalitions of lobbies with whom the elected candidate P^E may choose to bargain over policy and transfers.

We model lobbying as a two stage bargaining game. In the first stage, each possible coalition $\ell \in \Delta$ is associated with a willingness to pay, $W_\ell(x, E)$, for any policy $x \in X$ the elected candidate P^E may choose to implement instead of his most preferred policy E :

$$W_\ell(x, E) = \sum_{h \in \ell} w_h(x, E), \quad (5)$$

where $w_h(x, E)$ is the willingness to pay of lobby h measured in units of the private good and $W_\emptyset(x, E) \equiv 0$.

Given the preferences of a lobby specified in equation (2) above, the willingness to pay of lobby $h \in \mathcal{L}$ for any policy $x \in X$ implemented by the elected candidate P^E is:

$$w_h(x, E) = \frac{v(x, l_h) - v(E, l_h)}{\mu} \quad (6)$$

This is the monetary value of the utility gain (or loss) with respect to the *status quo* that lobby h obtains if the elected candidate P^E 's policy choice is x . The *status quo* is here defined to be P^E 's policy choice in the absence of any lobbying, E .¹⁵

From (5) and (6) we obtain the total willingness to pay of coalition $\ell \in \Delta$ for a given policy choice $x \in X$ by the elected candidate P^E :

$$W_\ell(x, E) = \sum_{h \in \ell} \frac{v(x, l_h) - v(E, l_h)}{\mu}. \quad (7)$$

In the second stage of the bargaining game, the elected candidate P^E first chooses

¹⁴If the elected candidate chooses this option, then the model coincides with the original model of Besley and Coate (1997) where lobbying is not allowed.

¹⁵A direct implication of (6) is that for any policy $x \in X$, the willingness to pay of a lobby with the same most preferred policy as the elected candidate is non positive.

an optimal policy $x_{PE}(\ell)$ for any potential coalition $\ell \in \Delta$:

$$x_{PE}(\ell) \in \operatorname{argmax}_{x \in X} u(x, E) + \lambda W_\ell(x, E) \quad (8)$$

and then chooses a bargaining coalition ℓ_{PE} :

$$\ell_{PE} \in \operatorname{argmax}_{\ell \in \Delta} u(x_{PE}(\ell), E) + \lambda W_\ell(x_{PE}(\ell), E) \quad (9)$$

Hence, an outcome of the bargaining game between the elected candidate P^E and a selected coalition ℓ_{PE} is a policy choice $x_{PE}(\ell_{PE})$ and transfers $W_{\ell_{PE}}(x_{PE}(\ell_{PE}), E)$.

Implicit in the statement of problems (8) and (9) is the assumption that the elected candidate appropriates the entire willingness to pay of the selected bargaining coalition. This is equivalent to assuming that at the lobbying stage the elected candidate has all the bargaining power.¹⁶

3. Results

We proceed backward to solve for the *subgame perfect equilibria* of the three-stage political game described in Section 2 above. We start from the last stage of the game: lobbying.

3.1. Equilibria of the Lobbying Subgame

Let P^E be the candidate elected in the voting subgame. We begin our analysis by characterizing the elected candidate P^E 's optimal coalition choice $\ell_{PE} \in \Delta$.

We first show that for any coalition $\ell \in \Delta$ the equilibrium policy choice that the lobbying process generates is uniquely determined.¹⁷

Lemma 1. *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ and any coalition $\ell \in \Delta$, there exists a unique optimal policy choice $x_{PE}(\ell)$ that solves problem (8):*

$$x_{PE}(\ell) = \frac{1}{1 + \rho |\ell|} \left(E + \rho \sum_{h \in \ell} l_h \right), \quad (10)$$

¹⁶This assumption is not critical for our results. The equilibrium characterization of the lobbying subgame remains the same (up to the size of the transfers) if the gains from trade are shared between the elected candidate and the members of the coalition in any fixed proportion.

¹⁷This result is similar to the one obtained by Diermeier and Merlo (2000) in the context of government coalition bargaining.

where

$$\rho \equiv \frac{\lambda}{\mu}. \quad (11)$$

The proof of Lemma 1 is presented in the Appendix.

The outcome of the bargaining is a compromise between the policy most preferred by the elected candidate and the policy preferences of the lobbies included in the bargaining coalition. Given the quadratic specification of preferences we adopt, this policy compromise takes the form of a weighted average of the most preferred policies of the parties involved in the negotiation. Since, by assumption, $\rho \leq 1$, the elected candidate's policy preferences are weighted more favorably than the policy preferences of the lobbies included in the bargaining coalition. This implies that the stronger the policy motivation of the elected candidate relative to that of the lobbies, the closer the equilibrium policy is to the one most preferred by the elected candidate.

We can now complete our characterization of the lobbying stage of the model by analyzing the elected candidate P^E 's choice of the optimal lobbying coalition $\ell_{PE} \in \Delta$.

Lemma 2. *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ the optimal coalition choice $\ell_{PE} \in \Delta$ that solves problem (9) is:*

If $-1 \leq E \leq -\tau(\rho)$, then $\ell_{PE} = \{C, R\}$;

If $-\tau(\rho) \leq E \leq 0$, then $\ell_{PE} = \{R\}$;

If $0 \leq E \leq \tau(\rho)$, then $\ell_{PE} = \{L\}$;

If $\tau(\rho) \leq E \leq 1$, then $\ell_{PE} = \{L, C\}$;

where

$$\tau(\rho) \equiv \frac{\sqrt{2\rho^2 + 3\rho + 1} - 1}{2\rho + 3} \quad (12)$$

The proof of Lemma 2 is presented in the Appendix.

An immediate consequence of Lemma 2 is that no elected candidate ever chooses to implement his most preferred policy. Thus, lobbying always occurs in equilibrium and influences the policy choice of any elected candidate.

Another consequence of Lemma 2 above is that in all equilibria no candidate ever includes all lobbies in his bargaining coalition. In equilibrium, there always exists at least one lobby that is excluded from the policy-making process and does not make any transfer to the elected candidate. Which lobbies are excluded depends on the policy preferences of the elected candidate.

We have now all the elements to present our first result. This result summarizes the outcome of the lobbying subgame for any possible elected candidate P^E .

Proposition 1. *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ the optimal coalition choice $\ell_{P^E} \in \Delta$, policy choice $x_{P^E} \in X$ and transfers $W_{\ell_{P^E}}$ are:*

If $-1 \leq E \leq -\tau(\rho)$, then:

$$\ell_{P^E} = \{C, R\}, \quad x_{P^E} = \frac{E + \rho}{1 + 2\rho}, \quad W_{\ell_{P^E}} = \frac{(2E - 1)^2 (1 + \rho) 2\rho}{\mu (1 + 2\rho)^2};$$

If $-\tau(\rho) \leq E \leq 0$, then:

$$\ell_{P^E} = \{R\}, \quad x_{P^E} = \frac{E + \rho}{1 + \rho}, \quad W_{\ell_{P^E}} = \frac{(E - 1)^2 (2 + \rho) \rho}{\mu (1 + \rho)^2};$$

If $0 \leq E \leq \tau(\rho)$, then:

$$\ell_{P^E} = \{L\}, \quad x_{P^E} = \frac{E - \rho}{1 + \rho}, \quad W_{\ell_{P^E}} = \frac{(E + 1)^2 (2 + \rho) \rho}{\mu (1 + \rho)^2};$$

If $\tau(\rho) \leq E \leq 1$, then:

$$\ell_{P^E} = \{L, C\}, \quad x_{P^E} = \frac{E - \rho}{1 + 2\rho}, \quad W_{\ell_{P^E}} = \frac{(2E + 1)^2 (1 + \rho) 2\rho}{\mu (1 + 2\rho)^2}.$$

Proof: The proof follows directly from Lemma 1, Lemma 2, and equation (7). ■

As shown in Proposition 1, the equilibrium of the lobbying subgame is such that the elected candidate P^E receives strictly positive transfers $W_{\ell_{P^E}}$ from coalition ℓ_{P^E} for implementing policy x_{P^E} . It is easy to show that the more lobbies value money over policy (that is, the higher is μ) and the less the elected candidate values money over policy (that is, the lower is λ), the smaller are these transfers.

Let $\bar{\rho} \in (0, 1)$ be implicitly defined by the following equation

$$(1 + 2\bar{\rho})\tau(\bar{\rho}) = 3\bar{\rho}^2 + \bar{\rho} - 1. \quad (13)$$

where $\tau(\cdot)$ is defined in (12) above.

Corollary 1. *The lobbying process implies that for every $\rho > 0$ not all policy choices $x \in X$ can be implemented in equilibrium. In particular, if $\rho \leq \bar{\rho}$ the set of policy choices $X_1 \subset X$, $X_1 \neq \emptyset$:*

$$X_1 = \left[-1, \min \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho}{1+\rho} \right\} \right] \cup \left[\max \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho}{1+\rho} \right\}, 1 \right] \quad (14)$$

cannot be implemented in equilibrium.

If instead $\rho \geq \bar{\rho}$ the set of policy choices $X_2 \subset X$, $X_2 \neq \emptyset$:

$$\begin{aligned} X_2 = & \left[-1, -\frac{\rho}{1+\rho} \right] \cup \left[-\frac{\rho - \tau(\rho)}{1+\rho}, \min \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho - \tau(\rho)}{1+2\rho} \right\} \right] \\ & \cup \left[\max \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho - \tau(\rho)}{1+2\rho} \right\}, \frac{\rho - \tau(\rho)}{1+\rho} \right] \cup \left[\frac{\rho}{1+\rho}, 1 \right] \end{aligned} \quad (15)$$

cannot be implemented in equilibrium.

Proof: The result follows from Proposition 1. ■

Corollary 1 shows that even though the set of potential candidates spans the entire policy space, the lobbying process reduces the set of policies that are implementable. Hence, we conclude that lobbying matters: lobbying changes the set of implementable policy outcomes. In particular, lobbying prevents the political process from implementing policies that are relatively extreme.

We can now turn our attention to the analysis of the voting stage of the model.

3.2. Equilibria of the Voting Subgame

As discussed in the Introduction we restrict our analysis to the set of two-candidate equilibria of our electoral model. This implies that when analysing the voting subgame we focus exclusively on voting when two or at most three candidates enter the electoral

competition.¹⁸ Our analysis of the voting subgame parallels the analysis of Besley and Coate (1997). In particular, we rule out weakly dominated voting strategies.

A voting strategy $\gamma(i^j)$ is weakly dominated for citizen i^j if there exists an alternative voting strategy $\hat{\gamma}(i^j)$ for i^j such that for every configuration of the voting profile of the other citizens, citizen i^j 's payoff associated with $\gamma(i^j)$ is less than or equal to the payoff associated with $\hat{\gamma}(i^j)$.

Restricting attention to equilibria that survive one round of elimination of weakly dominated voting strategies greatly simplifies the analysis of the voting subgame when there is more than one candidate. In particular, we can prove the following proposition.

Proposition 2. *Assume that $\mathcal{C}(\sigma)$ contains at least two candidates. All equilibria of the voting subgame that survive one round of elimination of weakly dominated strategies are such that no citizen i^j ever votes for any candidate $\underline{e} \in \mathcal{C}(\sigma)$ that, if elected, implements i^j 's least preferred policy within the set of equilibrium policy choices of the candidates in $\mathcal{C}(\sigma)$:*

$$\underline{e} \in \operatorname{argmax}_{e \in \mathcal{C}(\sigma)} |x_e(\ell_e) - j| \quad (16)$$

The proof of Proposition 2 is presented in the Appendix. This proposition states that when there are at least two candidates running for office no citizen i^j ever votes for his least preferred candidate \underline{e} . Given our assumptions about each citizen's preferences, \underline{e} is any candidate whose equilibrium policy choice is the one farthest away (among the equilibrium policies implemented by the candidates in $\mathcal{C}(\sigma)$) from citizen i^j 's most preferred policy.

Proposition 2 implies that in a two-candidate voting subgame where the two candidates implement different policies each citizen votes for his most preferred candidate. In other words, strategic voting coincides with sincere voting in this instance. This is not necessarily the case in a three-candidate voting subgame.

¹⁸The analysis of the case where three candidates compete for election is needed in order to consider the consequences of a deviation in the entry-of-candidates subgame.

3.3. Equilibria of the Entry-of-Candidates Subgame

We focus on the characterization of the set of two-candidate equilibria. Let $C(\sigma) = \{e_1, e_2\}$ be the equilibrium set of candidates where $j_1, j_2 \in X$ denote the type of candidate e_1 and e_2 , respectively.

We first show that in all two-candidate equilibria the candidates' policy choices are symmetric around the median policy 0.

Lemma 3. *All two-candidate equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, are such that*

$$x_{e_1}(\ell_{e_1}) = -x_{e_2}(\ell_{e_2}). \quad (17)$$

The proof of Lemma 3 is presented in the Appendix. The intuition behind this result is that to enter the electoral competition and pay the entry cost δ each candidate must have a strictly positive probability of winning. In the contest of our model, this implies that neither candidate can win with probability one and both candidates have to win with equal probability. Since, in two-candidate electoral competitions, citizens vote sincerely, then necessarily the population of voters has to split equally between the two candidates. This cannot occur if the distance of each candidate from the median policy differs.

There are two types of two-candidate equilibria. There are equilibria in which the two candidates are of the same type (that is, they have identical policy preferences), and equilibria in which the two candidates are of different types. We start from the latter, clearly more interesting, case.

We distinguish among three classes of two-candidate equilibria depending on whether the equilibrium policy choices exhibit *reversal*. An equilibrium policy choice exhibits reversal if it is on the opposite side of the median than the candidate's type. On the basis of this criterion we identify: *no-reversal equilibria*, where the policy choices of both candidates exhibit no reversal; *reversal equilibria*, where the policy choices of both candidates exhibit reversal; and *hybrid equilibria*, where the policy choice of one of the candidates exhibits reversal while the policy choice of the other candidate does not.

We start from the characterization of the set of two-candidates no-reversal equilibria.

Proposition 3. *All two-candidate no-reversal equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, are such that:*

The candidates' types are:

$$j_1 \in [-1, -\rho] \quad j_2 \in [\rho, 1] \quad \text{and} \quad j_1 = -j_2, \quad (18)$$

The equilibrium coalition choices are:

$$\ell_{e_1} = \{C, R\} \quad \ell_{e_2} = \{L, C\}, \quad (19)$$

The equilibrium policy choices are:

$$x_{e_1} \in \left[-\frac{1-\rho}{1+2\rho}, 0 \right] \quad x_{e_2} \in \left[0, \frac{1-\rho}{1+2\rho} \right], \quad (20)$$

and

$$x_{e_1} = -x_{e_2}. \quad (21)$$

The proof of Proposition 3 is presented in the Appendix. Intuitively, the two-candidate equilibria characterized in Proposition 3 are such that both candidates are elected with equal (and strictly positive) probability. This is enough to guarantee that each candidate wants to run for office since in our framework, as in Besley and Coate (2001), there are rents from being the elected candidate that are generated by the lobbying process. Of course this relies on our assumption, mentioned in Subsection 2.1 above, that the cost δ of running for office is small relative to the rents that an elected candidate can capture through the lobbying process. Moreover, no other candidate is willing to enter the electoral competition provided that, in the event of a new entry, the population of voters splits so that the new entrant has zero probability of winning. Notice that in our framework it is possible to construct an off-the-equilibrium path behaviour for voters that has this feature. Indeed, following a new entry we are in a three-candidate voting subgame and according to Proposition 2 above it is enough that voters, when they are non-pivotal, do not vote for the candidate that implements their least preferred policy choice.

A key feature of the characterization of the two-candidate no-reversal equilibria presented in Proposition 3 is that the two candidates that run for office are citizens with rather extreme policy preferences, as shown in equation (18). However, as a result of the lobbying process, they implement policies that are biased toward the center, as shown in equation (20).

Next we characterize the set of two-candidates reversal equilibria.

Proposition 4. *All two-candidate reversal equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, are such that:*

The candidates' types are:

$$j_1 \in [-\rho, 0] \quad j_2 \in [0, \rho] \quad \text{and} \quad j_1 = -j_2, \quad (22)$$

If $j_1 \in [-\rho, -\tau(\rho)]$ — equivalently $j_2 \in [\tau(\rho), \rho]$ — the equilibrium coalition choices are:

$$\ell_{e_1} = \{C, R\} \quad \ell_{e_2} = \{L, C\} \quad (23)$$

and the equilibrium policy choices are:

$$x_{e_1} \in \left[0, \frac{\rho - \tau(\rho)}{1 + 2\rho}\right] \quad x_{e_2} \in \left[-\frac{\rho - \tau(\rho)}{1 + 2\rho}, 0\right]. \quad (24)$$

If instead $j_1 \in [-\tau(\rho), 0]$ — equivalently $j_2 \in [0, \tau(\rho)]$ — the equilibrium coalition choices are:

$$\ell_{e_1} = \{R\} \quad \ell_{e_2} = \{L\} \quad (25)$$

and the equilibrium policy choices are:

$$x_{e_1} \in \left[\frac{\rho - \tau(\rho)}{1 + \rho}, \frac{\rho}{1 + \rho}\right] \quad x_{e_2} \in \left[-\frac{\rho}{1 + \rho}, -\frac{\rho - \tau(\rho)}{1 + \rho}\right]. \quad (26)$$

In both cases

$$x_{e_1} = -x_{e_2}. \quad (27)$$

The proof of Proposition 4 is presented in the Appendix. As in the case of the no-reversal equilibria (Proposition 3), the lobbying process biases the candidates' policy choices in all reversal equilibria. However, in contrast to the no-reversal equilibria, it is possible to have reversal equilibria where both candidates have rather moderate policy preferences (close to the median policy 0) and the lobbying process leads them to implement less moderate policies that exhibit reversal, as shown in equation (26). Notice that in the case of two-candidate reversal equilibria the set of policies that may be implemented in equilibrium is not connected. In other words, the intervals in (24) and (26) are disjoint.

Finally, we characterize the set of two-candidate hybrid equilibria.

Proposition 5. *There exist two-candidate hybrid equilibria of the electoral competition model if and only if $\rho \geq \bar{\rho}$ where $\bar{\rho}$ is defined in (13) above. All the two-candidate hybrid equilibria, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, are such that:*

The candidates' types either satisfy

$$j_2 = -j_1 \left(\frac{1 + \rho}{1 + 2\rho} \right) - \rho \left(\frac{2 + 3\rho}{1 + 2\rho} \right) \quad (28)$$

and

$$j_1 \in [-1, -\rho] \quad j_2 \in [-\tau(\rho), 0], \quad (29)$$

or

$$j_1 = -j_2 \left(\frac{1 + \rho}{1 + 2\rho} \right) + \rho \left(\frac{2 + 3\rho}{1 + 2\rho} \right) \quad (30)$$

and

$$j_1 \in [0, \tau(\rho)] \quad j_2 \in [\rho, 1]. \quad (31)$$

If $j_1 \in [-1, -\rho]$ and $j_2 \in [-\tau(\rho), 0]$, the equilibrium coalition choices are:

$$\ell_{e_1} = \{C, R\} \quad \ell_{e_2} = \{R\}. \quad (32)$$

and the equilibrium policy choices are:

$$\begin{aligned} x_{e_1} &\in \left[\max \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho}{1+\rho} \right\}, -\frac{\rho-\tau(\rho)}{1+\rho} \right] \\ x_{e_2} &\in \left[\frac{\rho-\tau(\rho)}{1+\rho}, \min \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho}{1+\rho} \right\} \right]. \end{aligned} \quad (33)$$

If instead $j_1 \in [0, \tau(\rho)]$ and $j_2 \in [\rho, 1]$ the equilibrium coalition choices are:

$$\ell_{e_1} = \{L\} \quad \ell_{e_2} = \{C, L\}. \quad (34)$$

and the equilibrium policy choices are:

$$\begin{aligned} x_{e_1} &\in \left[\max \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho}{1+\rho} \right\}, -\frac{\rho-\tau(\rho)}{1+\rho} \right] \\ x_{e_2} &\in \left[\frac{\rho-\tau(\rho)}{1+\rho}, \min \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho}{1+\rho} \right\} \right]. \end{aligned} \quad (35)$$

In both cases

$$x_{e_1} = -x_{e_2}. \quad (36)$$

The proof of Proposition 5 is presented in the Appendix. Notice that, in contrast to the no-reversal and reversal equilibria, hybrid equilibria do not exist for every value of $\rho > 0$. The distinctive feature of these equilibria is that while the types of both candidates are on the same side of the median policy, their equilibrium policy choices are symmetrically located around the median. Hence, unlike in the two classes of equilibria characterized in Propositions 3 and 4 above, the lobbying process biases the policy choice of both candidates in the same direction.

We conclude our characterization of the set of two-candidate equilibria of the electoral competition model by presenting the equilibria where the two candidates are of the same type.¹⁹

Proposition 6. *All two-candidate equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, where $j_1 = j_2$, are such that:*

¹⁹The policy choices in this type of two-candidate equilibria coincide with the policy choices that would arise in all one-candidate equilibria of the model.

The candidates' type is either $j_1 = j_2 = -\rho$ or $j_1 = j_2 = \rho$.

If $j_1 = j_2 = -\rho$ the equilibrium coalition choices are: $\ell_{e_1} = \ell_{e_2} = \{C, R\}$, and the equilibrium policy choices are: $x_{e_1} = x_{e_2} = 0$.

If instead $j_1 = j_2 = \rho$ the equilibrium coalition choices are: $\ell_{e_1} = \ell_{e_2} = \{L, C\}$ and the equilibrium policy choices are: $x_{e_1} = x_{e_2} = 0$.

The proof of Proposition 6 is presented in the Appendix. Intuitively, the reason why there does not exist an equilibrium with two identical candidates that through the lobbying process implement a policy that differs from the median policy is that in this case a candidate with policy preferences equal to ρ or $-\rho$ can enter the electoral competition and win with probability one. The reason why such a candidate would win is that although following the deviation we are considering a three-candidate equilibrium, two candidates are identical and hence only two policies may be implemented. Since from Proposition 2 above a citizen never votes for the candidate that implements his least preferred policy we conclude that in this situation citizens vote sincerely. Hence, the candidate that implements the median policy receives the majority of the votes.

4. Discussion

To analyze the full set of implications of our model we begin by characterizing the set of equilibria of the benchmark model where lobbying is not allowed. This analysis is based on Besley and Coate (1997). Consistently with the focus of our analysis above we restrict attention to the set of two-candidate equilibria.

When lobbying is not allowed (or equivalently when $\lambda = 0$), there exists a continuum of two-candidate equilibria. These equilibria are such that the two candidates who run for election have equal probabilities of winning and, if elected, implement policies that are symmetric around the median policy 0 (see Proposition 7 in Besley and Coate (1997) or Lemma 3 above for the case when $\lambda = 0$). The set of policies that can be implemented in equilibrium is the entire policy space X .²⁰

²⁰This characterization differs slightly from the one of Besley and Coate (1997). We assume that the cost of running for office δ is a monetary cost. Therefore, when $\lambda = 0$ this cost does not enter the payoff of a potential candidate. Besley and Coate (1997), instead, assume that δ is a utility cost. The set of policies that can be implemented in equilibrium is then the entire policy space X with the exception of a symmetric interval around the median policy 0, whose size depends on δ . When this cost is arbitrarily small (that is $\delta \rightarrow 0$) every policy $x \in X$ can be implemented in equilibrium.

This characterization of the set of two-candidate equilibria survives in the citizen-candidate model with exogenous lobbying of Besley and Coate (2001). In particular, they show that it is possible to construct equilibria of such a model where the policy choices coincide with the ones that would emerge in the equilibria of the citizen-candidate model without lobbying. The equilibria of the two models are, however, different with respect to the identity of the elected candidate who implements such policies. In particular, in the model with exogenous lobbying citizens neutralize the influence of lobbies over policy by strategically electing a candidate with offsetting preferences.

We can now discuss the main implications of our analysis.

Remark 1. Lobbying induces policy compromise.

In all the equilibria of our model the policy outcome is a compromise between the policy most preferred by the elected candidate and the policy preferences of the lobbies that participate in the policy-making process. This is a natural consequence of lobbying. A similar result is derived by Grossman and Helpman (1996) and Besley and Coate (2001).

Remark 2. Not all lobbies participate in the policy-making process.

In all the equilibria of our model no candidate ever includes all lobbies in the policy-making process. In equilibrium, there is always at least one lobby that is excluded from the bargaining process that determines the policy outcome. This is the sense in which lobbying is endogenous in our model. This implication of our analysis is consistent with the evidence presented by Wright (1996). According to Wright, many of the existing lobbies in the United States are often dormant, raising and contributing no money at all. For example, between 1991 and 1992, 35% of all registered lobbies in the United States spent zero dollars (Wright 1996, p. 125). The fact that not all lobbies participate in the policy-making process is a key feature of our approach that distinguishes it from the menu-auction approach to lobbying (Grossman and Helpman 1996, Besley and Coate 2001, e.g.), where by assumption all lobbies participate in the policy-making process.²¹

²¹While in this paper we restrict attention to the case where there are only three potential lobbies we believe that increasing the number of lobbies would only complicate the analysis without affecting our main results.

Remark 3. Lobbying matters.

In our model, even though the policy preferences of all potential candidates span the entire policy space, the lobbying process reduces the set of policies that can be implemented in equilibrium. This is the sense in which lobbying matters in our model. For example, as discussed at the beginning of this section, there exists a two-candidate equilibrium in the model without lobbying where, if elected, the two candidates implement policies $x = -1$ and $x = 1$, respectively. As Corollary 1 above shows, this is not an equilibrium in the model with endogenous lobbying. As discussed above this distinguishes our framework from the one of Besley and Coate (2001), where (exogenous) lobbying can have no effect on equilibrium policy outcomes. In their model, the lobbies that participate in the policy-making process can be arbitrarily chosen to guarantee that any feasible policy is implementable in equilibrium. In our model, lobbies are endogenously selected to participate in the policy-making process by the elected candidate, and not all feasible policies can be implemented in equilibrium.

Remark 4. Lobbying biases the outcome of the policy-making process toward the center of the policy space.

As illustrated in our analysis above, lobbying may induce elected candidates to implement policies that are on the opposite side of the median than their most preferred policy. This phenomenon, that we label reversal, is more severe the more candidates care about money over policy. We argue that equilibria that display any form of reversal are pathological. In reality, the political process has means to discipline candidates so as to prevent reversal. For example, we may think of political parties as playing such a role (Levy 2001).²² Even though our model abstracts from the role of parties, these considerations lead us to focus on equilibria without reversal.

A key feature of all two-candidate equilibria without reversal is that although the two candidates who run for office are citizens with relatively extreme policy preferences the lobbying process induces them to implement policies that are biased toward

²²In the citizen-candidate model of Levy (2001) political parties act as commitment devices for candidates running for office. For another citizen-candidate model with political parties see Riviere (1999).

the center of the policy space. The reason for this result is that in equilibrium, elected candidates always include in their bargaining coalition lobbies whose policy preferences are on the opposite end of the policy spectrum than their own preferences. This implication of our model is consistent with the evidence presented by Austen-Smith and Wright (1994). The empirical findings of Austen-Smith and Wright are that, in the United States, special interest groups often lobby legislators who are predisposed to vote against their favored positions.

In our model, even though the policy preferences of lobbies are relatively extreme, lobbying has a moderating effect on policy, and extreme policies never emerge as an equilibrium outcome of the political process. This result distinguishes our model from other existing models where lobbying has a tendency to induces policy outcomes that are relatively extreme (Austen-Smith 1987, Baron 1994, Groseclose and Snyder 1996, Grossman and Helpman 1996, e.g.). The key difference with this literature is that in our model candidate entry is endogenous. Thus, lobbying affects the type of citizens who choose to run for elections as well as the policy choices of the elected policy-makers.

Appendix

Proof of Lemma 1: From equation (3) the objective function in (8) is strictly concave. The first order conditions of problem (8) are:

$$(x - E) + \rho \sum_{h \in \ell} (x - l_h) = 0. \quad (\text{A.1})$$

Then the unique solution of equation (A.1) is (10). ■

Proof of Lemma 2: Given that our model is completely symmetric around the median policy 0 we prove the result for the case in which the most preferred policy choice z^E by the elected candidate P^E is such that $z^E \geq 0$. The case $z^E \leq 0$ is completely symmetric and therefore the proof is omitted.

Notice first that P^E 's optimal policy choices for every $\ell \in \Delta$ — the solution to problem (8) above — are:

$$x_{PE}(\emptyset) = E, \quad x_{PE}(\{L\}) = \frac{E - \rho}{1 + \rho}, \quad x_{PE}(\{C\}) = \frac{E}{1 + \rho}, \quad x_{PE}(\{R\}) = \frac{E + \rho}{1 + \rho} \quad (\text{A.2})$$

together with

$$x_{PE}(\{L, C\}) = \frac{E - \rho}{1 + 2\rho}, \quad x_{PE}(\{L, R\}) = \frac{E}{1 + 2\rho}, \quad x_{PE}(\{C, R\}) = \frac{E + \rho}{1 + 2\rho} \quad (\text{A.3})$$

and

$$x_{PE}(\{L, C, R\}) = \frac{E}{1 + 3\rho}. \quad (\text{A.4})$$

It is now possible to evaluate the elected candidate's payoff for every coalition choice $\ell \in \Delta$ using (A.2), (A.3) and (A.4). These payoffs are:

$$u(x_{PE}(\emptyset), E) = 0 \quad (\text{A.5})$$

if $\ell = \{\emptyset\}$,

$$u(x_{PE}(\{L\}), E) + \lambda W_{\{L\}}(x_{PE}(\{L\}), E) = \left(\frac{\rho}{1 + \rho}\right) (1 + E)^2 \quad (\text{A.6})$$

if $\ell = \{L\}$,

$$u(x_{PE}(\{C\}), E) + \lambda W_{\{C\}}(x_{PE}(\{C\}), E) = \left(\frac{\rho}{1 + \rho}\right) (E)^2 \quad (\text{A.7})$$

if $\ell = \{C\}$ and

$$u(x_{PE}(\{R\}), E) + \lambda W_{\{R\}}(x_{PE}(\{R\}), E) = \left(\frac{\rho}{1 + \rho}\right) (E - 1)^2 \quad (\text{A.8})$$

if $\ell = \{R\}$. The payoff in (A.6) weakly dominates all the payoffs in (A.5), (A.7) and (A.8) for every $E \in [0, 1]$ and is therefore the only relevant payoff among the one computed above. The elected candidate's payoffs for the remaining coalitions $\ell \in \Delta$ are:

$$u(x_{PE}(\{L, C\}), E) + \lambda W_{\{L, C\}}(x_{PE}(\{L, C\}), E) = \left(\frac{\rho}{1 + 2\rho}\right) [1 + 4E + 4(E)^2] \quad (\text{A.9})$$

if $\ell = \{L, C\}$,

$$u(x_{PE}(\{L, R\}), E) + \lambda W_{\{L, R\}}(x_{PE}(\{L, R\}), E) = \left(\frac{\rho}{1 + 2\rho}\right) 4(E)^2 \quad (\text{A.10})$$

if $\ell = \{L, R\}$,

$$u(x_{PE}(\{C, R\}), E) + \lambda W_{\{C, R\}}(x_{PE}(\{C, R\}), E) = \left(\frac{\rho}{1 + 2\rho}\right) [1 - 4E + 4(E)^2] \quad (\text{A.11})$$

if $\ell = \{C, R\}$ and finally

$$u(x_{PE}(\{L, C, R\}), E) + \lambda W_{\{L, C, R\}}(x_{PE}(\{L, C, R\}), E) = \left(\frac{\rho}{1 + 3\rho}\right) 9(E)^2 \quad (\text{A.12})$$

if $\ell = \{L, C, R\}$. The payoff in (A.9) weakly dominates all the payoffs in (A.10), (A.11) and (A.12) for every $E \in [0, 1]$. Therefore the only relevant comparison is the one between the payoffs in (A.6) and in (A.9) above.

The payoff in (A.9) is greater or equal than the payoff in (A.6) for every $E \in [0, \tau(\rho)]$ where $\tau(\rho)$ is defined in (12) above. In other words for every $E \in [0, \tau(\rho)]$ the coalition choice that, in this case, solves problem (9) is $\ell_{PE} = \{L, C\}$. Conversely, the payoff in (A.6) is greater or equal than

the payoff in (A.9) for every $E \in [\tau(\rho), 1]$. In other words, the coalition choice that, in this case, solves problem (9) is $\ell_{PE} = \{L\}$. ■

Proof of Proposition 2: Assume by way of contradiction that there exists an equilibrium of the voting subgame that survives one round of elimination of weakly dominated strategies and is such that citizen i^j votes for candidate \underline{e} . The voting profiles of all the citizens but i^j can be partitioned into two sets. The set of profiles such that citizen i^j is *not* pivotal and the set of profiles such that citizen i^j is pivotal. Citizen i^j is pivotal if, when $\sigma(i^j) = \underline{e}$, candidate \underline{e} is elected, while when $\gamma(i^j) \neq \underline{e}$ the elected candidate is $e \in \mathcal{C}(\sigma)$ with $e \neq \underline{e}$.

If citizen i^j is *not* pivotal then citizen i^j 's payoff is the same whatever his vote. If instead citizen i^j is pivotal then by definition (16) of \underline{e} citizen i^j 's payoff is weakly lower if his vote is $\gamma(i^j) = \underline{e}$ than if it is $\gamma(i^j) \neq \underline{e}$. This implies that $\gamma(i^j) = \underline{e}$ is a weakly dominated strategy. This is clearly a contradiction to the hypothesis that the equilibrium of the voting subgames survives one round of elimination of weakly dominated strategy. ■

Proof of Lemma 3: Assume by way of contradiction that the two-candidate equilibria of the electoral competition model, $\mathcal{C}(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, are such that

$$x_{e_1}(\ell_{e_1}) \neq -x_{e_2}(\ell_{e_2}). \quad (\text{A.13})$$

In particular, without any loss of generality we assume that

$$|x_{e_1}(\ell_{e_1})| < |x_{e_2}(\ell_{e_2})|. \quad (\text{A.14})$$

By Proposition 2 above all citizens will vote sincerely. In other words, the citizens of type j^* , where

$$j^* = \frac{x_{e_1}(\ell_{e_1}) + x_{e_2}(\ell_{e_2})}{2}$$

are indifferent between voting for one candidate or the other. All the citizens of type $j > j^*$ in equilibrium will vote for the candidate whose policy $x_e(\ell_e) > j^*$ and all the citizens of type $j < j^*$ in equilibrium will vote for the other candidate. This implication together with assumption (A.14) imply that more than half of the population will vote for candidate e_1 . Therefore, candidate e_1 wins the vote with probability one. This implies that candidate e_2 has a profitable deviation. By not running for office he does not change the policy choice but increases his payoff of the cost of running $\delta > 0$. This contradicts the hypothesis that there exists a two candidate equilibrium where (A.14) is satisfied. ■

Proof of Proposition 3: We start from (18). Proposition 1 implies that the policy selected by the elected candidate P^E is equal to the median policy choice 0 if and only if $E = -\rho$ and $E = \rho$. Since from Proposition 1 the optimal policy choice is monotonic increasing in E , if $E \leq 0$ and monotonic decreasing in E if $E \geq 0$, we do not observe any policy reversal for $j_1 \in [-1, -\rho]$ and $j_2 \in [\rho, 1]$. From the definition (12) of $\tau(\rho)$ for every $\rho \in (0, 1]$ we have $0 < \tau(\rho) < \rho$ and $\lim_{\rho \rightarrow 0} \tau(\rho) = 0$. This

implies that, from Proposition 1, for $j_1 \in [-1, -\rho]$ and $j_2 \in [\rho, 1]$ the equilibrium policy choices are such that:

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} < 0, \quad x_{e_2} = \frac{j_2 - \rho}{1 + 2\rho} > 0. \quad (\text{A.15})$$

Lemma 3 and the symmetry of the policy choices (A.15) around the median policy 0 yield $j_1 = -j_2$, which completes the proof of (18).

Condition (19) follows directly from Proposition 1 and the observation that for every $\rho \in (0, 1]$ $\tau(\rho) < \rho$. While Lemma 3 and the policy choices in (A.15) imply (20) and (21).

If δ is small enough the two candidates of types j_1 and j_2 that satisfy (18) run for office and are elected with probability 1/2. This implies that neither candidate is willing to withdraw from the electoral race since both

$$u(x_{e_1}, j_1) + \lambda [W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1)$$

and

$$u(x_{e_1}, j_1) + \lambda [W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1)$$

are strictly positive if δ is small enough.

To conclude the proof we still need to specify the out-of-equilibrium path behaviour of non-pivotal voters such that no other candidate is willing to enter the electoral competition. We assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy choice that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. In this case citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e' is strictly better off by not running and saving the cost δ . No citizen is pivotal in determining whether candidate e' wins the election. Therefore the specified voting behaviour is compatible with strategic voting. Finally, no citizen votes for the one, among the three candidates e' , e_1 and e_2 , that implements the least preferred policy choice. Therefore, the specified voting behaviour is compatible with Proposition 2 above. ■

Proof of Proposition 4: We start from condition (22). As argued in the proof of Proposition 3 above, Proposition 1 implies that the policy selected by the elected candidate P^E is equal to the median policy choice 0 if and only if $E = -\rho$ and $E = \rho$. Proposition 1 also implies that the optimal policy choice is monotonic increasing in E , if $E \leq 0$ and monotonic decreasing in E if $E \geq 0$. Therefore we observe reversal of policy choices for $j_1 \in [-\rho, 0]$ and $j_2 \in [0, \rho]$. Since $\tau(\rho) < \rho$ from Proposition 1 we need to distinguish between the case $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$ and the case $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$. In the first case, $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$ the equilibrium policy choices are such that:

$$x_{e_2} = \frac{j_2 - \rho}{1 + 2\rho} < 0, \quad x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} > 0. \quad (\text{A.16})$$

while in the second case $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$ the equilibrium policy choices are such that:

$$x_{e_2} = \frac{j_2 - \rho}{1 + \rho} < 0, \quad x_{e_1} = \frac{j_1 + \rho}{1 + \rho} > 0. \quad (\text{A.17})$$

Lemma 3 and the symmetry of the policy choices in (A.16) and (A.17) imply that $j_1 = -j_2$, which completes the proof of (18).

Conditions (23) and (25) follow directly from Proposition 1 and $\tau(\rho) < \rho$. While Lemma 3 and the policy choices in (A.16) and (A.17) imply (24), (26) and (27).

If δ is small enough the two candidates of types j_1 and j_2 that satisfy (18) run for office and are elected with probability 1/2. This implies that neither candidate is willing to withdraw from the electoral race since for δ small enough when $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$ we have

$$\begin{aligned} u(x_{e_1}, j_1) + \lambda [W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\ u(x_{e_2}, j_2) + \lambda [W_{\{L,C\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0 \end{aligned} \quad (\text{A.18})$$

while when $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$ we have

$$\begin{aligned} u(x_{e_1}, j_1) + \lambda [W_{\{R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\ u(x_{e_2}, j_2) + \lambda [W_{\{L\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0 \end{aligned} \quad (\text{A.19})$$

To conclude the proof we still need to specify the out-of-equilibrium path behaviour of non-pivotal voters such that no other candidate is willing to enter the electoral competition. As in the proof of Proposition 3 we assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy choice that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. Then citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e' is strictly better off by not running and saving the cost δ . No citizen is pivotal in determining whether candidate e' wins the election. Therefore the specified voting behaviour is compatible with strategic voting. Finally, no citizen votes for the one, among the three candidates e' , e_1 and e_2 , that implements the least preferred policy choice. Therefore, the specified voting behaviour is compatible with Proposition 2 above. ■

Proof of Proposition 5: Given that our model is completely symmetric around the median policy 0 we prove the result in the case $j_1 \leq j_2 \leq 0$. The case $j_2 \geq j_1 \geq 0$ is completely symmetric and therefore the proof is omitted.

Recall that by definition hybrid equilibria are such that candidate e_1 chooses a policy that does not exhibit policy reversal, it is to the left of the median policy 0, while candidate e_2 chooses a policy that does exhibit policy reversal, it is to the right of the median policy 0. Proposition 1 implies that for this to be the case we need $j_1 \in [-1, -\rho]$ and $j_2 \in [-\rho, 0]$. Notice first that if $j_2 \in [-\rho, -\tau(\rho)]$

Proposition 1 implies that Lemma 3 cannot hold since

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} > -x_{e_2} = -\frac{j_2 + \rho}{1 + \rho}$$

for $j_1 \in [-1, -\rho]$ and $j_2 \in [-\rho, -\tau(\rho)]$. Therefore a necessary condition for a hybrid equilibrium to exist is $j_2 \in [-\tau(\rho), 0]$.

Again for Lemma 3 to hold we need that the policy choices x_{e_1} and x_{e_2} satisfy condition (17) or equivalently (36). For this to be the case we need that the distance from the median policy 0 of the smallest policy choice that candidate e_1 can implement is greater or equal than the distance from the median policy 0 of the smallest policy choice that candidate e_2 can implement. From Proposition 1 this condition implies that a necessary condition for an equilibrium to exist is that ρ is such that

$$-\frac{1 - \rho}{1 + 2\rho} \leq -\frac{\rho - \tau(\rho)}{1 + \rho}. \quad (\text{A.20})$$

In other words, from the definition (12) of $\bar{\rho}$, a necessary conditions for a hybrid two-candidate equilibrium to exist is $\rho \leq \bar{\rho}$. To show that this is also a sufficient condition it is enough to observe that if $\rho = \bar{\rho}$ then $j_1 = -1$ and $j_2 = -\tau(\rho)$ is an hybrid two-candidate equilibrium of the electoral competition model that satisfies conditions (28), (29), (32), (33) and (36).

Lemma 3 also implies that the two policy choices x_{e_1} and x_{e_2} must satisfy:

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} = -x_{e_2} = -\frac{j_2 + \rho}{1 + \rho} \quad (\text{A.21})$$

Solving (A.21) for j_1 we obtain (28).

Conditions (32) follows directly from Proposition 1 while Lemma 3 and the necessary and sufficient condition for the existence of a hybrid equilibrium $\rho \leq \bar{\rho}$ imply (33) and (36). Notice that the sets in (33) are non-empty if and only if $\rho \leq \bar{\rho}$.

If δ is small enough the two candidates of types j_1 and j_2 that satisfy (28) and (29) run for office and are elected with probability 1/2. Neither candidate is willing to withdraw from the electoral race since for δ small enough when $j_1 \in [-1, -\rho]$ and $j_2 \in [-\tau(\rho), 0]$ we have

$$\begin{aligned} u(x_{e_1}, j_1) + \lambda [W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\ u(x_{e_2}, j_2) + \lambda [W_{\{R\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0 \end{aligned} \quad (\text{A.22})$$

To conclude the proof we still need to specify the out-of-equilibrium path behaviour of non-pivotal voters such that no other candidate is willing to enter the electoral competition. As in the proof of Propositions 3 and 4 we assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. Then citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e' is strictly better off by not running and saving the cost δ . No citizens is pivotal in

determining whether candidate e' wins the election. Therefore the specified voting behaviour is compatible with strategic voting. Finally, no citizen votes for the one, among the three candidates e' , e_1 and e_2 , that implements the least preferred policy choice. Therefore, the specified voting behaviour is compatible with Proposition 2 above. ■

Proof of Proposition 6: Given that our model is completely symmetric around the median policy 0 we prove the result in the case $j_1 = j_2 = -\rho$. The case $j_2 = j_1 = \rho$ is completely symmetric and therefore the proof is omitted.

Proposition 1 implies that the policy selected by the elected candidates e_1 and e_2 is equal to the median policy choice if $j_1 = j_2 = -\rho$, the coalition selected by both candidates is $\ell_{e_1} = \ell_{e_2} = \{C, R\}$ and the policy choice is, of course, $x_{e_1} = x_{e_2} = 0$.

If δ is small enough the two candidates of type $j_1 = j_2 = -\rho$ run for office and are elected with probability 1/2. Neither candidate is willing to withdraw from the electoral race since for δ small enough when $j_1 = j_2 = -\rho$ we have

$$u(0, -\rho) + \lambda [W_{\{C,R\}}(0, -\rho) - \delta] - u(0, -\rho) = \lambda [W_{\{C,R\}}(0, -\rho) - \delta] > 0 \quad (\text{A.23})$$

To conclude the proof we need to specify the out-of-equilibrium path behaviour of non-pivotal voters such that no other candidate is willing to enter the electoral competition. If a candidate e' of type $j' \neq -\rho$ enters the electoral competition Proposition 2 implies that citizens will vote for the one among the three candidates e' , e_1 and e_2 , that will choose his most preferred policy choice. This is because the two candidates e_1 and e_2 choose the same policy choice and hence only two policy outcomes will be observed following the entry of e' . However, we assume that among the candidates e_1 and e_2 all the citizens that prefer the median policy 0 to policy j' will concentrate their vote on just one candidate, for example e_1 . Then citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e' is strictly better off by not running and saving the cost δ . No citizen is pivotal in determining whether candidate e_1 or e_2 wins the election. Therefore the specified voting behaviour is compatible with strategic voting.

If instead a candidate e'' of type $j'' = -\rho$ enters the electoral competition we assume that no citizen will vote for e'' and all the citizens will vote for either e_1 or e_2 . Then citizen e'' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e'' is strictly better off by not running and saving the cost δ . No citizen is pivotal in determining whether candidate e'' wins the election. Therefore the specified voting behaviour is compatible with strategic voting. Finally, since all three candidates e'' , e_1 and e_2 , implement the same policy choice the specified voting behaviour is obviously compatible with Proposition 2 above. ■

References

- ALESINA, A. (1988): "Credibility and Policy Convergence in a Two-Party System with Rational Voters," *American Economic Review*, 78, 796–806.
- AUSTEN-SMITH, D. (1987): "Interest Groups, Campaign Contributions, and Probabilistic Voting," *Public Choice*, 54, 123–39.
- AUSTEN-SMITH, D., AND J. R. WRIGHT (1992): "Competitive Lobbying for a Legislator's Vote," *Social Choice and Welfare*, 9, 229–57.
- (1994): "Counteractive Lobbying," *American Journal of Political Science*, 38, 25–44.
- BARON, D. P. (1994): "Electoral Competition with Informed and Uninformed Voters," *American Political Science Review*, 88, 33–47.
- BERNHEIM, B., AND M. WHINSTON (1986): "Menu Auctions, Resource Allocation, and Economic Influence," *Quarterly Journal of Economics*, 101, 1–31.
- BESLEY, T., AND S. COATE (1997): "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 108, 85–114.
- (2001): "Lobbying and Welfare in a Representative Democracy," *Review of Economic Studies*, 68, 67–82.
- DIERMEIER, D., AND A. MERLO (2000): "Government Turnover in Parliamentary Democracies," *Journal of Economic Theory*, 94, 46–79.
- DIXIT, A., G. GROSSMAN, AND E. HELPMAN (1997): "Common Agency and Coordination: General Theory and Application to Government Policy Making," *Review of Economic Studies*, 105, 752–69.
- DOWNS, A. (1957): *An Economic Theory of Democracy*. New York: Harper Collins.
- GROSECLOSE, T., AND J. M. SNYDER (1996): "Buying Supermajorities," *American Political Science Review*, 90, 303–15.
- GROSSMAN, G., AND E. HELPMAN (1994): "Protection for Sale," *American Economic Review*, 84, 833–50.

- (1996): “Electoral Competition and Special Interest Politics,” *Review of Economic Studies*, 63, 265–86.
- (2001): *Special Interest Politics*. Cambridge: M.I.T. Press.
- HIBBS, D. (1977): “Political Parties and Macroeconomic Policy,” *American Political Science Review*, 71, 1467–87.
- LEVY, G. (2001): “The Role of Parties in Multi-dimensional Policy Space,” mimeo.
- OSBORNE, M., AND A. SLIVINSKI (1996): “A Model of Political Competition with Citizen Candidates,” *Quarterly Journal of Economics*, 111, 65–96.
- PERSSON, T., AND E. HELPMAN (1998): “Lobbying and Legislative Bargaining,” NBER Working paper no. 6589.
- PERSSON, T., AND G. TABELLINI (2000): *Political Economics: Explaining Economic Policy*. Cambridge: M.I.T. Press.
- RIVIERE, A. (1999): “Citizen Candidacy, Party Formation and Duverger’s Law,” mimeo.
- TULLOCK, G. (1967): “The Welfare Costs of Tariffs, Monopolies and Theft,” *Western Economic Journal*, 5, 224–32.
- WITTMAN, D. (1977): “Candidates with Policy Preferences: A Dynamic Model,” *Journal of Economic Theory*, 14, 180–89.
- WRIGHT, J. R. (1996): *Interest Groups and Congress*. Boston: Allyn & Bacon.