WHAT FORM OF RELATIVE PERFORMANCE EVALUATION?*

Marco Celentani† and Rosa Loveira‡

April, 2003

Abstract: We study relative performance evaluation when managers have private information on their ability to make appropriate investment decisions. We assume that the joint distribution of an individual firm's profit and market movements depends on the ability of the manager that runs the firm. In the equilibrium of the executive labor market, compensation schemes exploit this fact to sort managers of different abilities. This implies that managerial compensation is increasing in own performance but may also be increasing in industry performance–a sharp departure from standard relative performance evaluation. This result provides an explanation for the scarcity of relative performance considerations in executive compensation documented by the empirical literature.

1 Introduction

Academics, practitioners and the business press frequently recommend the use of relative performance evaluation (RPE) in executive compensation packages. RPE is normally taken to mean that executive compensation is increasing in own performance and decreasing in performance of an appropriately defined peer group. RPE may be seen as a consequence of Holmström's (1979, 1982) informativeness principle that stipulates that in a principal-agent relationship all informative signals should be included in contracting: "one can improve risk sharing while at the same time retaining incentives."[1] When designing executive compensation, this principle may be argued to imply that "managers are not held responsible for events one can observe are outside of their control, and [...] their performance is always judged against information about what should be achievable given, say, the current economic situation."[2]

These principles are very popular in the business press. According to Rappaport (1999): "Shareholders expect boards to reward management for achieving superior returns–that is, for returns equal to or better than those earned by the company's peer group or by broader market indexes."[3] Dobbs and Koller (2000) warn that "compensations plans linking the pay of managers to the share values of their companies can reward or penalize them for events they don't control."[4] For *The Economist* (2002) "rewards linked to a company share price should probably be triggered only if the firm outperforms the market as a whole, or an industry peer group."[5]

Despite the general praise of RPE, the extensive empirical research dedicated to its study has produced surprisingly little evidence of a negative relationship between executive compensation and market movements and some studies have even found a positive relationship.[6] Summarizing the results of the empirical

*Marco Celentani gratefully acknowledges the financial support of DGES of MEC (Spain) under project PB98-00024. We thank Michele Boldrin, Wolfgang Pesendorfer, and Pablo Ruiz-Verdú for useful discussions and suggestions.

†Department of Economics, Universidad Carlos III de Madrid, Getafe (Madrid) 28903, Spain; fax +34-91-624 9875; celentan@eco.uc3m.es.

‡Department of Economics, Universidad Carlos III de Madrid, Getafe (Madrid) 28903, Spain and Department of Economics and Business, Universitat Pompeu Fabra, Carrer Ramón Trías Fargas, 23-27, 08005, Barcelona, Spain; fax: +34-93-542 1746; rosa.loveira@econ.upf.es.

[1] Holmström (1979), page 87.

[2] Holmström (1979), page 82.

[3] Rappaport (1999), page 92.

[4] Dobbs and Koller (2000), page 190.

[5] *The Economist* (2002), page 25.

[6] See, for instance Antle and Smith (1986), Gibbons and Murphy (1990), Barro and Barro (1990), Janakirman, Lambert

investigation on RPE, Murphy (1999) says that "the paucity of RPE considerations in [...] executive compensation remains a puzzle worth understanding." In a similar vein, Hall and Liebman (1998) say that "the near complete absence of relative pay seems to be a puzzle."[7]

The absence of RPE in executive compensation has also been commented by the business press which has frequently expressed dissatisfaction with current compensation practices. *The Economist* (2002) asks "If a firm's share price goes up for extraneous reasons-a fall in interest rates, say, or a rise in the stock market-why should the managers benefit?"[8] de Swaan and Harper (2003) note that "In case after case, investors have seen executives reap extraordinary rewards tied to share price increases that had little to do with management and everything to do with factors beyond its control, such as interest rate movements and changes in macroeconomic conditions."

The purpose of the present paper is to provide a theoretical model to study executive compensation and to show that RPE is an important compensation tool, but that it may take a form that differs from the one advocated by the existing literature. In particular, we show that there are cases in which executive compensation is increasing in market performance.

Contingent contracts serve two main purposes. On the one hand they may be used to provide incentives to act in an appropriate way (as in the principal-agent paradigm) and on the other, they may be employed to sort heterogeneous agents (as in the asymmetric information paradigm). Most of the literature on executive compensation has focused on performance-pay as an incentive instrument and has disregarded its sorting effects. This paper takes the complementary view and focuses on an environment in which firms use contingent compensation as a competitive tool in the market for executives with private information about their ability.

We study a situation in which each firm has two investment projects available. Project I is ex-ante superior to project II. Risk-averse managers have an innate ability to forecast the realization of projects. In most of the paper we assume that a manager is either good or bad. A good manager is able to forecast the realization of project I with certainty, but a bad manager has no such ability. If compensation is increasing in own performance, a good manager would invest in project I only if he foresees a success and would otherwise invest in project II. A bad manager would always invest in project I, which is ex-ante superior. We allow the probability of success for each of the two projects to depend on a common aggregate state. When the aggregate state is good, success is more likely. In such a situation compensation may be made dependent not only on own but also on aggregate performance.

We find that in contracts accepted in equilibrium executive compensation depends on both own and industry performance, but that it may be increasing in the latter. The intuition behind our result can be explained as follows. Managers of different abilities generate different probability distribution over states, i.e., over pairs of individual and aggregate performance. This implies that no equilibrium may exist in which both types of manager accept the same contract. If this were the case, a firm could profit from screening out a good manager by offering a contract that increases compensation in a state which is relatively more likely for the good manager and decreases it in a state which is relatively less likely for him.

If an equilibrium exists, then, it has to be separating in the sense that the contracts accepted by the two types are different. These results are reminiscent of the insurance market with adverse selection studied by Rothschild and Stiglitz (1976) and Wilson (1977). As in that case, risk aversion implies that in equilibrium the bad manager accepts a contract that fully insures him. The contract accepted by the good type is the contract that the good type most prefers among the ones that break even, and are such that the bad type prefers his own contract.

In Rothschild and Stiglitz (1976) the good type has incomplete insurance and therefore has higher consumption in the state that is relatively more likely for him, i.e., the no-loss state. In a similar way, we find that compensation to the good manager is higher in states that are relatively more likely for him than for the bad type. But we also find that, because of correlation between market and individual outcomes and because good managers have a higher ability to predict individual outcomes, the likelihood of individual success with a favorable aggregate state relative to success with an unfavorable aggregate state may be higher for a good manager than for a bad one. This implies that the contract accepted in equilibrium by a

and Larcker (1992), Joh (1999), Aggarwal and Samwick (1999a and b). For additional discussion of the empirical literature, refer to the surveys of Rosen (1992), Prendergast (1999) or Murphy (1999).

[7] Hall and Liebman (1999), page 683, footnote 34.

[8] *The Economist* (2002), page 25.

good manager may stipulate higher compensation with individual success and a favorable aggregate state than with individual success and an unfavorable aggregate state.

In other words, we find that executive compensation is increasing in the firm's absolute performance but is not necessarily decreasing with industry performance. It is decreasing in industry performance when individual performance is low, but–in a sharp departure from the prediction of classical RPE–it is increasing in industry performance when individual performance is high. We analyze the implications of our results by running a regression on the data generated by the model and we show that our results can generate a positive relationship between executive compensation and aggregate performance.

The apparent absence of RPE considerations in executive compensation contracts has received a great deal of attention as of late.

Garvey and Milbourn (2002) and Core and Guay (2002) have pointed out that the exposure to risk of executive compensation packages is only part of executives' total exposure to risk. When executive compensation does not filter out market risk, the executive can reduce his exposure to market risk by reducing the exposure to risk of the rest of his financial wealth. Garvey and Milbourn (2002) find some evidence of RPE for younger executives, who are likely to be less able to filter out market risk because they have less wealth.

Several authors have proposed the idea that, because compensation influences managers' decisions in product markets, optimal managerial compensation should take into account this effect when firms operate in imperfectly competitive markets. This idea, first proposed by Salas Fumás (1992), has been pursued by Aggarwal and Samwick (1999) and by Joh (1999) that have found empirical evidence of managerial compensation being increasing in industry performance rather than decreasing. They also find that the sensitivity of managerial compensation to industry performance is larger in more competitive environments. Aggarwal and Samwick (1999) reconcile this finding with price rather than quantity competition and Joh (1999) underlines that the usefulness of strategic group performance evaluation (in opposition to RPE) to sustain collusion in product markets is larger in a more competitive setting.

Bebchuk, Fried, and Walker (2002) argue that executive compensation is best explained as rent extraction subject to a constraint on the ""outrage" [executive] pay packages would create." Market or sector indexing are argued to be unattractive for several reasons, including that they are more likely to highlight inferior performance and that the value of indexed options has to be charged against earnings whereas standard option awards only appear in the footnotes of financial statements.

Himmelberg and Hubbard (2000) point out that aggregate shocks simultaneously raise firms' values and the marginal values of CEO services to firms. Given that the supply of talented CEO capable of running large and complex corporations is relatively inelastic, these shocks bid up the value of their compensation packages and make it appear as if RPE is violated. In a similar vein, Oyer (2002) points out that there may be positive correlation between workers' outside opportunities and firms' market values. In such an environment, when it is costly to revise labor contracts, a profit sharing scheme that does not filter out aggregate risk, is employed as a way to index compensation to market conditions and, therefore, automatically meet workers' participation constraints.

We see our research as most related to Himmelberg and Hubbard (2000) and Oyer (2002) because we also explain compensation scheme making reference to labor rather than product markets. But our work makes substantially different assumptions. According to Himmelberg and Hubbard (2000) and Oyer (2002) individual workers' outside opportunities are correlated with the market value of firms. This introduces a component of pay that has a positive rather than a negative correlation with the market. We instead assume that the profit of an individual firm has a correlation with the market that depends on the ability of the manager that runs it, and that in the equilibrium of the executive labor market compensation schemes exploits this fact to sort managers of different abilities.

The paper is organized as follows. Section 2 presents the model and introduces the equilibrium concept used in the paper. Section 3 characterizes the equilibrium. Section 4 studies the implications of the equilibrium analysis of section 3 on the link between managerial pay and firms' performance measures and discusses an extension of the model in which a manager may be one of a finite number of types. Section 5 concludes.

2 The Model

All firms are risk neutral and each has two investment projects available, I and II. Each project, if undertaken, may have two possible realizations, success, with revenue $s > 0$ or failure, with 0 revenue. The ex-ante probability of project I resulting in revenue equal to s is p when the firm is run by a manager and is $\widetilde{p}$ when the firm is not run by a manager. The ex-ante probability of project II resulting in revenue equal to s is q regardless of whether the firm is run by a manager or not. Both projects have a cost of 1. We assume that each firm can only invest in one investment project.

We assume that ex ante project I is efficient regardless of whether the firm is run by a manager or not, ($ps - 1 > qs - 1$ and $\widetilde{p}s - 1 > qs - 1$), that with project I the probability of success is higher when the firm is run by a manager, $p > \widetilde{p}$, and that the expected profit of any of the projects with or without a manager is strictly positive ($qs - 1 > 0$).[9] Summarizing we have $1 > p > \widetilde{p} > q > 1/s > 0$.

We introduce correlation among firms' performances by assuming that an aggregate state of nature $\Pi \in \left\{\underline{\Pi}, \overline{\Pi}\right\}$ determines the probability of success of individual firms' investment projects I. In the favorable aggregate state $\overline{\Pi}$ the probability of success is $\overline{p} > p$ and in in the unfavorable aggregate state $\underline{\Pi}$ it is $\underline{p} < p$. We denote the probability of state $\overline{\Pi}$ by $\overline{\beta}$ and the probability of state $\underline{\Pi}$ by $\underline{\beta} = 1 - \overline{\beta}$. Given that the unconditional probability of success is p,

$$\overline{p}\overline{\beta} + \underline{p}\left(1 - \overline{\beta}\right) = p.$$

In a similar way we assume that the aggregate state of nature $\Pi \in \left\{\underline{\Pi}, \overline{\Pi}\right\}$ determines the probability of success of individual firms' investment projects II. In state $\overline{\Pi}$ the probability of success is $\overline{q} > q$ and in state $\underline{\Pi}$ it is $\underline{q} < q$. Given that the unconditional probability of success is q,

$$\overline{q}\overline{\beta} + \underline{q}\left(1 - \overline{\beta}\right) = q.$$

Because the only purpose of assuming that a firm can function without a manager, although less effectively, is to establish reservation levels for firms, and given that we have no interest in the case of a firm that is not run by a (specialized) manager, we need not make any assumptions on whether and how the probabilities of success for a firm without a manager change as a function of the aggregate state.

Managers have innate abilities to forecast the realization of some projects. For simplicity we assume that only two types of managers exist, good and bad, $\tau \in \{G, B\}$. When a good manager is employed by a firm, he is able to forecast the realization of investment project I, whereas a bad manager is unable to improve his forecast beyond the prior probabilities of success. For the sake of simplicity we assume that neither good nor bad managers are able to make any forecast about the realization of project II. Given that project I is ex-ante superior to project II, this assumption could be justified if, for instance, the manager could devote a limited amount of time to the analysis of investment projects and it were optimal for him to dedicate it to project I.

Each manager knows his own type but it is common knowledge that firms believe that he is good with probability μ and bad with probability $1 - \mu$. When analyzing project I, a manager receives a signal $\rho \in \{V, L, H\}$. The bad manager receives signal V, the void signal, with probability 1 and the good manager receives signals L or H, the low and the high signal, with probabilities $1 - p$ and p, respectively. The probability of project I having the high return ($s > 0$) conditional on the received signal is

$$\Pr\left(r = s \mid \rho, I\right) = \begin{cases} p & \text{if } \rho = V \\ 0 & \text{if } \rho = L \\ 1 & \text{if } \rho = H \end{cases}.$$

In other words, while signals H and L ensure, respectively, the success or the failure of investment project I, the void signal, V, provides no additional information and the conditional probability of success is, therefore, equal to the prior, p.[10]

[9] Notice that $qs - 1 > 0$ implies $\widetilde{p}s - 1 > 0$ and $ps - 1 > 0$.

[10] Managers with high ability are often described in the literature as being able to generate high expected return investment projects, i.e., as being able to come up with good ideas. In contrast to this, we refer to managerial ability as the ability to forecast the realization of a given project. All investment projects are drawn from the same distribution, regardless of the manager's ability, but different managers may have different abilities to forecast their realizations.

Because our assumptions guarantee that it is always efficient to undertake one investment project, be it I or II, we can restrict the manager's action space to $\{I, II\}$. We assume that the manager's choice of the project is not observable but its ultimate realization is. We denote the observable final outcome of the investment project by $P \in \{F, S\}$, where F and S indicate a failure (revenue equal to 0) or a success (revenue equal to s). An investment profile for a manager is a vector $i = (i_V, i_L, i_H) \in \{I, II\}^3$ where i_V, i_L, and i_H denote the decision to invest in project I or II when the signal received by the manager is respectively V, L, or H. We denote by $i^E = (I, II, I)$ the efficient investment profile.

Managers of both types maximize expected utility of wage payments. We denote their Bernoulli utility function by $U(w)$. For the sake of simplicity we study a situation in which (at most) one equilibrium exists and is interior. For this purpose we assume that $U(.)$ is twice continuously differentiable, strictly increasing, $U'(.) > 0$, strictly concave, $U''(w) < 0$, and that the Inada conditions hold, $\lim_{w \to 0^+} U'(w) = +\infty$, $\lim_{w \to +\infty} U'(w) = 0$.

After managers learn their types, firms offer contracts to managers. A contract specifies that if the manager accepts it, he will receive a nonnegative payment from the firm for each subsequent public history of the game.

Once a manager accepts an offer, he privately observes signal ρ on project I and decides whether to invest in project I or II. A manager's choice of investment project is not publicly observed. After the manager has undertaken an investment project, its realization and the aggregate state are publicly observed.

Given that a contract offered by a firm to a manager conditions the manager's salary on the public history of the game following acceptance, a contract has to specify a payment for every $\sigma = (P, \Pi) \in \{F, S\} \times \{\underline{\Pi}, \overline{\Pi}\}$, where $P \in \{F, S\}$, denotes the realization of the investment project and $\Pi \in \{\underline{\Pi}, \overline{\Pi}\}$, denotes the realization of the aggregate state. To simplify notation we will denote a contract by

$$w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}_+^4$$

where $\underline{w}_P$ (respectively, $\overline{w}_P$) denotes the nonnegative payment to the manager when the outcome of the investment process is $P \in \{F, S\}$ and when $\Pi = \underline{\Pi}$ (respectively, when $\Pi = \overline{\Pi}$).

The informational structure described above, portrays a situation in which both individual and aggregate realizations convey information on the manager's likely behavior and in which shareholders may find it optimal to make managerial compensation dependent on both individual and aggregate results. To avoid unnecessary complications we assume that the aggregate state is directly observable. This means that we ignore the straightforward statistical problem of inferring the likely aggregate state from industry realizations and we concentrate on what firms may infer about a manager's likely behavior from the aggregate state and the implications that this may have on managerial compensation.

We finally make the assumption that a firm can function without a manager, but that in this case it will receive signal V with probability 1. Given that investment projects are assumed to be ex-ante profitable, investing in project I is the optimal decision for a firm with no manager. This implies that a firm with no manager gets an expected profit of $\widetilde{p}s - 1$.

We consider the case in which two firms compete for every single manager. In particular, we assume that every manager is offered a countable set of contracts $w \in \mathbb{R}_+^4$ by each of two firms and chooses one contract (if any) from them. All our results generalize to the case in which the measure of the set of managers is lower than the measure of the set of firms and firms are allowed to make offers to all managers.[11]

In the following we summarize the extensive form of the game.

N_1 *Nature* chooses:

- The aggregate state, $\underline{\Pi}$ with probability $\underline{\beta}$ and $\overline{\Pi}$ with probability $\overline{\beta}$.
- The type of each manager. Managers' types are i.i.d., and their realizations are G with probability μ and B with probability $1 - \mu$ (regardless of the aggregate state).

τ Each *manager* privately observes his type.

[11] In this case firms are allowed to offer different countable sets of contracts to managers with different prior probabilities of being good.

F Without observing nature's choices, each of the two *firms* competing for a given manager offers him a countable set of contracts, each of them of the form

$$w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}^4_+.$$

M The *manager* either accepts an offer or rejects them all.

R If the manager *rejects* all offers, he gets a 0 payoff and each of the firms chooses an investment project.

A If the manager *accepts* an offer, he is hired. The firm whose offers were not accepted chooses an investment project.

N_2 For each firm, Nature chooses

- The realization of investment project of type I. When the aggregate state is $\underline{\Pi}$, the realizations of different firms' investment projects of type I are i.i.d. and result in S or F with probabilities $\underline{p}$ and $1-\underline{p}$. When the aggregate state is $\overline{\Pi}$, the realizations of different firms' investment projects of type I are i.i.d. and result in S or F with probabilities $\overline{p}$ and $1-\overline{p}$.
- The realization of investment project of type II. Conditional on the aggregate state being $\underline{\Pi}$, the realizations of different firms' investment projects of type I are i.i.d. and result in S or F with probabilities $\underline{q}$ and $1-\underline{q}$. Conditional on the aggregate state being $\overline{\Pi}$, the realizations of different firms' investment projects of type I are i.i.d. and result in S or F with probabilities $\overline{q}$ and $1-\overline{q}$.

ρ The manager receives a private signal on investment project I, $\rho \in \{V, L, H\}$.

i The manager decides whether to invest in project I or II, $i \in \{I, II\}$.

P The realization of the investment project is publicly observed, $P \in \{F, S\}$.

Π The aggregate state $\Pi \in \left\{\underline{\Pi}, \overline{\Pi}\right\}$ is publicly observed.

w The firm pays the manager salary $\underline{w}_P$ or $\overline{w}_P$ when the public signal was $\underline{\Pi}$ or $\overline{\Pi}$, respectively.

Because the public signal Π is observed after the manager has decided which project to undertake, the manager's investment strategy can be denoted as

$$i(w) = (i_V(w), i_L(w), i_H(w)) \in \{I, II\}^3$$

Given that no additional use of notation will be made, we choose not to provide a full description of strategies and strategy spaces. Also, for notational convenience, we will occasionally omit arguments whenever this cannot cause any confusion.

The equilibrium concept we use is subgame perfect Nash equilibrium.

3 Equilibrium

Suppose that after a manager has accepted a contract $w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}^4_+$ he plays according to an investment profile $i \in \{I, II\}^3$. We denote by $\underline{\alpha}^\tau_P(i)$ the probability that the final public outcome is $(P, \underline{\Pi})$, $P \in \{F, S\}$ conditional on manager's type being τ. Similarly, we will denote by $\overline{\alpha}^\tau_P(i)$ the probability that the final public outcome is $\left(P, \overline{\Pi}\right)$, $P \in \{F, S\}$ conditional on manager's type being τ. The expected utility deriving from contract $w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}^4_+$ for a manager of type $\tau = G, B$ that plays according to $i \in \{I, II\}^3$ is therefore

$$V^\tau(w, i) = \underline{\alpha}^\tau_F(i)\, U(\underline{w}_F) + \underline{\alpha}^\tau_S(i)\, U(\underline{w}_S) + \overline{\alpha}^\tau_F(i)\, U(\overline{w}_F) + \overline{\alpha}^\tau_S(i)\, U(\overline{w}_S).$$

Marginal rates of substitution between salary payments in any two states of the world for a manager of type $\tau = G, B$ are defined in the obvious way. For instance the marginal rate of substitution of $\underline{w}_S$ for $\overline{w}_S$ is:

$$MRS^{\tau}_{\underline{w}_S,\overline{w}_S}(w,i) = -\frac{\frac{\partial V^{\tau}(w)}{\partial \underline{w}_S}}{\frac{\partial V^{\tau}(w)}{\partial \overline{w}_S}} = -\frac{\underline{\alpha}^{\tau}_S(i)}{\overline{\alpha}^{\tau}_S(i)}\frac{U'(\underline{w}_S)}{U'(\overline{w}_S)}$$

We now want to focus on the case in which the manager plays the efficient investment profile, $i^E = (I, II, I)$ and depict the indifference curves of the good and the bad manager in the planes $(\underline{w}_F, \overline{w}_F)$ and $(\underline{w}_S, \overline{w}_S)$. Simple calculations show that for all $w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}^4_+$ and for all $(\underline{p}, \overline{p}, \underline{q}, \overline{q})$ such that $\overline{p} > \underline{p}$, and that $\overline{q} > \underline{q}$

$$MRS^{G}_{\underline{w}_F,\overline{w}_F}(w, i^E) < MRS^{B}_{\underline{w}_F,\overline{w}_F}(w, i^E) \tag{1}$$

and that for all $w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S) \in \mathbb{R}^4_+$

$$\operatorname{sign}\left(MRS^{G}_{\underline{w}_S,\overline{w}_S}(w, i^E) - MRS^{B}_{\underline{w}_S,\overline{w}_S}(w, i^E)\right) = \operatorname{sign}\left(\frac{\underline{p}}{(1-\underline{p})\,\underline{q}} - \frac{\overline{p}}{(1-\overline{p})\,\overline{q}}\right). \tag{2}$$

Equation (2) lies at the heart of our results and therefore deserves careful discussion. Note first that

$$\frac{\underline{p}}{(1-\underline{p})\,\underline{q}} > \frac{\overline{p}}{(1-\overline{p})\,\overline{q}} \tag{3}$$

means that success is relatively less likely to derive from project I rather than II in state $\overline{\Pi}$ than in state $\underline{\Pi}$. Note also that (3) is equivalent to

$$\frac{\overline{p} + (1-\overline{p})\,\overline{q}}{\overline{p}} > \frac{\underline{p} + (1-\underline{p})\,\underline{q}}{\underline{p}} \tag{4}$$

or

$$\frac{\Pr\left(r = s \mid \tau = G, \Pi = \overline{\Pi}\right)}{\Pr\left(r = s \mid \tau = B, \Pi = \overline{\Pi}\right)} > \frac{\Pr\left(r = s \mid \tau = G, \Pi = \underline{\Pi}\right)}{\Pr\left(r = s \mid \tau = B, \Pi = \underline{\Pi}\right)}. \tag{5}$$

Conditions (4) or (5) offer an equivalent way of viewing condition (3). Condition (3) is satisfied if and only if, under individual success, the relative likelihood of the good type is higher in the favorable rather than the unfavorable aggregate state. Note that a bad manager has lower probability of success in either aggregate state. But conditions (4) or (5) only require that the probability of success deriving from the good rather than the bad manager is higher in the favorable rather than in the unfavorable aggregate state.

By (2), when condition (3) holds we have

$$MRS^{G}_{\underline{w}_S,\overline{w}_S}(w, i^E) > MRS^{B}_{\underline{w}_S,\overline{w}_S}(w, i^E). \tag{6}$$

Figure 1 depicts the indifference curves of the good and the bad manager in the planes $(\underline{w}_F, \overline{w}_F)$ and $(\underline{w}_S, \overline{w}_S)$ for a given contract $w = (\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S)$ in the case in which condition (3) holds. In such a case (1) and (6) imply that the (absolute) slope of the indifference curve of the good manager in the plane $(\underline{w}_F, \overline{w}_F)$ is higher than that of the bad manager (Figure 1(a)) and that the (absolute) slope of the indifference curve of the good manager in the plane $(\underline{w}_S, \overline{w}_S)$ is lower than that of the bad manager (Figure 1(b)). When the opposite of (3) holds, the indifference curves of types G and B in the Figure 1(b) would be inverted.

The indifference curves in Figure 1(a) clarify that, conditional on the investment project being a failure, the good type is more willing than the bad type to trade compensation when the aggregate state is unfavorable against compensation when the aggregate state is favorable. Similarly, the indifference curves in Figure 1(b) illustrate that when condition (3) holds, conditional on the investment project being successful, the good type is more willing than the bad type to trade compensation when the aggregate state is favorable against compensation when the aggregate state is unfavorable.

The goal of the rest of the paper is to show that, when condition (3) holds and, therefore, when the preferences of the two types of manager have the properties described above, in equilibrium the two types of manager accept different contracts, that the bad manager accepts a constant contract that insulates him from any kind of risk, and that the good manager accepts a contract that is decreasing in the aggregate state when the investment project is a failure but increasing in the aggregate state when the investment project is successful.

We now want to define and characterize a pair of contracts $\left(w^{RSW-B}, w^{RSW-G}\right)$ that are such that contract w^{RSW-B} maximizes the expected utility to the bad manager and contract w^{RSW-G} maximizes the expected utility to the good manager under the following constraints

1. Type B does not prefer w^{RSW-G} to w^{RSW-B};
2. Type G does not prefer w^{RSW-B} to w^{RSW-G};
3. A firm that offers w^{RSW-B} breaks even when w^{RSW-B} is accepted only by type B.
4. A firm that offers w^{RSW-G} breaks even when w^{RSW-G} is accepted only by type G.

We refer to $\left(w^{RSW-B}, w^{RSW-G}\right)$ as the *Rothschild-Stiglitz-Wilson (RSW) contracts* in reference to the work of Rothschild and Stiglitz (1976) and Wilson (1977) on insurance markets with adverse selection.[12]

Note that in the model we study and unlike in the literature on insurance with adverse selection, different contracts do not simply alter consumption in different states of nature, but also create different incentives for a manager when he has to make investment decisions and may therefore have an impact on the probabilities that different realizations of own performance occur. It is important to clarify that this is not the driving force behind our result, because the incentive compatibility constraints are not binding so long as contracts are weakly monotonic in own performance, a condition which is satisfied in equilibrium. The following definition serves to introduce notation that specifies what investment action profile should be expected once a manager of an unknown type accepts a given contract $w \in \mathbb{R}_+^4$.

Definition 1 *An investment profile is an induced investment profile given contract* $w \in \mathbb{R}_+^4$, $i^*(w) = \left(i_V^*(w), i_L^*(w), i_H^*(w)\right)$, *if*

$$\begin{aligned} i_V^*(w) &\in \arg \max_{i_V \in \{I, II\}} V^B\left(w, \left(i_V, i_L, i_H\right)\right) \\ \left(i_L^*(w), i_H^*(w)\right) &\in \arg \max_{(i_L, i_H) \in \{I, II\}^2} V^G\left(w, \left(i_V, i_L, i_H\right)\right) \end{aligned}$$

In words, $i^*(w)$ is the investment profile that would be played by the manager in the Nash equilibrium of the continuation game that starts after he accepts an offer $w \in \mathbb{R}_+^4$.

Once the dependence of investment profiles on accepted contracts is taken into account, we can define the following RSW-best responses:

Definition 2

$$\begin{aligned} w^{RSW-B}\left(w^G\right) &= \arg \max_{w \in \mathbb{R}_+^4} V^B\left(w, i^*(w)\right) \\ s.t.\ E\left[\pi - w \mid B, i^*(w)\right] &\geq \widetilde{p}s - 1 \\ V^G\left(w^G, i^*\left(w^G\right)\right) &\geq V^G\left(w, i^*(w)\right). \end{aligned}$$

$$\begin{aligned} w^{RSW-G}\left(w^B\right) &= \arg \max_{w \in \mathbb{R}_+^4} V^G\left(w, i^*(w)\right) \\ s.t.\ E\left[\pi - w \mid G, i^*(w)\right] &\geq \widetilde{p}s - 1 \\ V^B\left(w^B, i^*\left(w^B\right)\right) &\geq V^B\left(w, i^*(w)\right). \end{aligned}$$

[12] We follow standard terminology; See for instance Maskin and Tirole (1992).

In words, $w^{RSW-B}\left(w^{G}\right)$ is the contract that maximizes the expected payoff to a bad manager subject to the following three conditions (i) The bad manager selects the investment project that maximizes his expected utility; (ii) The expected payoff to a firm that offers this contract and whose offer is accepted only by the bad manager is no lower than the firm's reservation level, and (iii) The good manager does not prefer this contract to an arbitrary contract w^{G}. Similarly, $w^{RSW-G}\left(w^{B}\right)$ is the contract that maximizes the expected payoff to a good manager subject to the following three conditions (i) The good manager selects the investment project that maximizes his expected utility; (ii) The expected payoff to a firm that offers this contract and whose offer is accepted only by the good manager is no lower than the firm's reservation level, and (iii) The bad manager does not prefer this contract to an arbitrary contract w^{B}.

We can now define the Rothschild-Stiglitz-Wilson pair of contracts.

DEFINITION 3 *A pair of contracts is Rothschild-Stiglitz-Wilson,* $\left(w^{RSW-B}, w^{RSW-G}\right)$*, if and only if*

$$\begin{aligned} w^{RSW-B} &= w^{RSW-B}\left(w^{RSW-G}\right) \\ w^{RSW-G} &= w^{RSW-G}\left(w^{RSW-B}\right) \end{aligned}$$

In words, w^{RSW-G} and w^{RSW-B} are the contracts that maximize the payoff of the good and the bad manager respectively, under the constraints that neither type prefers to deviate and accept the offer for the other type and that firms offering each contract break even. Notice that the break-even constraints in Definition 2 imply that $w^{RSW-B} \neq w^{RSW-G}$.

The following Proposition provides a characterization of RSW pair of contracts.

PROPOSITION 1 *A unique RSW pair of contracts exists and is such that:*

1. $\left(i_V^*\left(w^{RSW-B}\right), i_L^*\left(w^{RSW-G}\right), i_H^*\left(w^{RSW-G}\right)\right) = i^E$.
2. *For all* $\left(\underline{p}, \underline{q}, \overline{p}, \overline{q}\right)$

$$\begin{aligned} \underline{w}_S^{RSW-B} &= \underline{w}_F^{RSW-B} = \overline{w}_S^{RSW-B} = \overline{w}_F^{RSW-B} = (p - \widetilde{p})\, s \\ \underline{w}_S^{RSW-G} &\geq \underline{w}_F^{RSW-G},\ \overline{w}_S^{RSW-G} \geq \overline{w}_F^{RSW-G},\ \textit{and}\ \underline{w}_F^{RSW-G} \geq \overline{w}_F^{RSW-G}; \end{aligned}$$

If $\frac{\overline{p}}{(1-\overline{p})\overline{q}} < \frac{\underline{p}}{(1-\underline{p})\underline{q}}$

$$\overline{w}_S^{RSW-G} \geq \underline{w}_S^{RSW-G};$$

If $\frac{\overline{p}}{(1-\overline{p})\overline{q}} = \frac{\underline{p}}{(1-\underline{p})\underline{q}}$

$$\overline{w}_S^{RSW-G} = \underline{w}_S^{RSW-G};$$

If $\frac{\overline{p}}{(1-\overline{p})\overline{q}} > \frac{\underline{p}}{(1-\underline{p})\underline{q}}$

$$\overline{w}_S^{RSW-G} \leq \underline{w}_S^{RSW-G}.$$

PROOF: Appendix.

Proposition 1 establishes that if the RSW pair of contracts $\left(w^{RSW-B}, w^{RSW-G}\right)$ is offered, the good manager accepts w^{RSW-G}, the bad manager accepts w^{RSW-B} and the investment profile that would follow would be the efficient one.

More importantly, part 2 of Proposition 1 characterizes the properties of the RSW contracts. The contract preferred by the bad manager, w^{RSW-B}, is a constant contract that insulates him from any source of risk. Notice, in particular, that because compensation is independent of performance, a bad manager that accepted w^{RSW-B} would have no reason to deviate from the efficient investment action (undertaking project I).

The contract preferred by the good manager, w^{RSW-G}, instead exposes him to risk in both own and aggregate performance. Two properties of w^{RSW-G} deserve careful discussion. First, w^{RSW-G} is such that compensation is (weakly) increasing in own performance. This implies that if a good manager accepted w^{RSW-G}, he would strictly prefer the efficient investment action profile, i.e., investing in project I when he anticipates that it will succeed and investing in project II when he anticipates that project I will fail.

Second, when condition (3) does not hold, compensation is (weakly) decreasing in aggregate performance, as is often postulated. When condition (3) holds, however, managerial compensation is not always decreasing in aggregate performance. When individual performance is low (i.e., with failure of the investment project) compensation is decreasing in aggregate performance. But when individual performance is high (i.e., with success of the investment project) compensation is increasing in aggregate performance, in contrast to the form of RPE that is normally taken for granted.

The next proposition characterizes the subgame perfect Nash equilibrium

PROPOSITION 2 *In the path of a subgame perfect Nash equilibrium*

1. *The bad manager accepts contract*
$$w^B = w^{RSW-B}.$$
2. *The good manager accepts contract*
$$w^G = w^{RSW-G}.$$
3. *The investment profile is efficient.*

PROOF: Appendix.

The proof of Proposition 2 follows standard arguments. It first establishes that in equilibrium firms earn zero profits on top of their reservation utility of $\widetilde{p}s - 1$. It then rules out the possibility of "pooling" equilibria in which both types of managers accept the same contract by showing that it is always possible to deviate and make an offer that would be accepted only by the good manager and which earns the firm positive profits. Finally, it shows that all "separating" offers different from the RSW contracts are also vulnerable to the same type of deviation. This shows (Parts 1 and 2 of Proposition 2) that if a subgame perfect Nash equilibrium exists, it has to be such that the two types of managers accept the RSW contracts. These results are reminiscent of the results on insurance with adverse selection of Rothschild and Stiglitz (1976) and Wilson (1977). Our result is complicated by the necessity of taking into account that managers' investment decisions respond to the contract they accept and part 3 establishes that in the path of the subgame perfect Nash equilibrium managers make efficient investment decisions.

Part 2 states that if a subgame perfect Nash equilibriums exists, the contract accepted by the good manager is such that his compensation is higher in states with a higher likelihood of the good manager relative to the bad. As in the principal-agent literature, this result evokes a statistical interpretation, but no actual statistical inference is drawn in equilibrium given that, once a manager has accepted contract w^{RSW-G}, he is known to be good with probability 1.

Proposition 2 in particular establishes that, if condition (3) holds, the contract accepted in equilibrium by the good manager is not monotonically decreasing in the aggregate state: When own performance is high ($r = s$), the manager receives a higher payment when the aggregate state is favorable rather than unfavorable. Note that this result does not depend on renegotiation and managers' outside opportunities being correlated with the market, because we assume that managers have commitment ability when they accept a contract. Instead, the result derives from the fact that, with asymmetric information, competing firms try to offer the best possible terms for the manager while trying to separate the two types. This implies that RSW contracts are among the contracts that are offered to managers and are accepted by the two types of manager.

We now want to focus on cases in which condition (3) holds. When this happens, the likelihood of individual success and a favorable aggregate state relative to individual success and an unfavorable aggregate state is higher for a good manager than a bad one. This implies that a good manager is more willing than a bad manager to trade compensation in the event of individual success and a favorable aggregate state against compensation in the event of individual success and an unfavorable aggregate state. This means that competition forces firms to offer a contract for the good manager with higher compensation in the first event than in the second.

The previous argument can be clarified by Figure 2. Suppose that, contrary to the claim of Proposition 2, $\overline{w}_S^{RSW-G} < \underline{w}_S^{RSW-G}$, as, for instance, in point $W2$ in Figure 2. Given that, when $\underline{w}_S = \overline{w}_S$,

$$MRS^G_{\underline{w}_S,\overline{w}_S}(w) = -\frac{\Pr\left(S, \underline{\Pi} \mid G, i^E\right)}{\Pr\left(S, \overline{\Pi} \mid G, i^E\right)},$$

$\overline{w}_S^{RSW-G} < \underline{w}_S^{RSW-G}$ implies that

$$MRS^G_{\underline{w}_S,\overline{w}_S}\left(w^{RSW-G}\right) > -\frac{\Pr\left(S,\underline{\Pi} \mid G, i^E\right)}{\Pr\left(S,\overline{\Pi} \mid G, i^E\right)},$$

i.e., GG, the indifference curve of the good manager, is less steep at w^{RSW-G} than gg, the set of contracts that have the same expected cost as w^{RSW-G} conditioned on only the good manager accepting it. Consider now a contract $\widehat{w}$ which differs from w^{RSW-G} in that $\left(\underline{w}_S^{RSW-G}, \overline{w}_S^{RSW-G}\right)$ is replaced by $(\underline{w}'_S, \overline{w}'_S)$ below gg, below BB, and above GG. This contract is strictly better for the good manager, strictly worse for the bad manager and is such that the expected compensation cost, conditional on only the good manager accepting it, is no higher than the expected compensation cost of w^{RSW-G}. This means that a profitable deviation exists and rules out that w^{RSW-G} can be such that $\overline{w}_S^{RSW-G} < \underline{w}_S^{RSW-G}$.

Notice that the same argument does not apply when $\overline{w}_S^{RSW-G} \geq \underline{w}_S^{RSW-G}$, as for instance in point $W1$ in Figure 2. In this case, gg, the set of contracts that have the same expected cost as w^{RSW-G} conditioned on only the good manager accepting it, is less steep than GG, the indifference curve of the good manager. Given that GG is less steep than BB, the set of contracts that are more profitable than $W1$ conditional on only the good manager accepting it, and above GG are also above BB. In other words, the contracts that lie above GG and below gg, also lie above BB.

In a similar way, but independently of whether condition (3) holds or not, the likelihood of individual failure and a favorable aggregate state relative to individual failure and an unfavorable aggregate state is lower for a good manager than a bad one. This implies that a good manager is more willing than a bad manager to trade compensation in the event of individual failure and an unfavorable aggregate state against compensation in the event of individual failure and a favorable aggregate state. This means that competition forces firms to offer a contract for the good manager with higher compensation in the first event than in the second.

It is important to clarify that Proposition 2 does not establish existence of a subgame perfect Nash equilibrium, because it shows that profitable deviations that attract only the good manager exist for all contracts different from the RSW contracts. When the RSW contracts are offered, such a deviation is not profitable, but other deviations may be, in which case no subgame perfect Nash equilibrium exists. To see this, notice first that no profitable deviation may exist that is such that only the bad manager accepts a different contract, because w^{RSW-B} is the contract that the bad manager prefers among all contracts that break even. This means that if a contract is offered that the bad manager strictly prefers to w^{RSW-B}, it has to give negative expected profit. But two other types of deviations may be profitable:

1. A deviation that is such that both types accept the same contract. In this case the firm makes a positive profit if the manager is good and a negative profit if the manager is bad.

2. A deviation that is such that each of the two types accepts a different contract. In this case the deviation has to be such that the firm makes a positive profit if the manager is good and a negative profit if the manager is bad.[13]

While it is impossible to guarantee that such deviations are in general unprofitable, a sufficient condition for that to happen is that the probability of the manager being good is not too high. Note that the possibility that an equilibrium does not exist for certain parameter constellations is pervasive in models of contracting with adverse selection and has been the object of careful analysis. We do not want to review the literature on the issue, but we want to mention that under appropriate modifications of the extensive form of the game, equilibria exist in which the main results of this paper hold.

Maskin and Tirole (1992) have analyzed a situation in which the informed party (the manager in our case) offers a set of contracts to the uninformed party (the firm); the latter may accept the set or reject it; if the uninformed party accepts the set of contracts, the informed party then chooses one contract from this set. With this extensive form, the RSW allocation described in Proposition 2 would be the unique equilibrium outcome.

[13] A deviation that is such that each of the two types accepts a different contract and such that the firm makes a negative profit if the manager is good and a positive profit if the manager is bad is dominated by a deviation in which only the bad manager accepts a different contract and this type of deviation has already been argued to be unprofitable.

Hellwig (1987) modifies the extensive form by introducing an extra stage in which after the informed party has accepted a contract, the uninformed party may withdraw the contract (if it believes that it will lead to a loss). If we also introduced an additional stage in which firms can withdraw offers that have been accepted, an equilibrium would always exist. In particular, if an equilibrium does not exist in which the managers accept the RSW contracts, an equilibrium exists in which both types of managers accept the same contract. The contract accepted in this equilibrium would be the contract that the good manager most prefers among all contract that break even when accepted by both types. But as for the case of the RSW contract for the good type, if condition (3) is satisfied, the contract accepted by both types would be decreasing in the aggregate state when own performance is low and increasing in the aggregate state when own performance is high.

4 Discussion of the results

4.1 An example

We now consider the case in which the manager has constant relative risk aversion, $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma = 0.5$. We assume that probabilities of success are $p = 0.6$, $\overline{p} = 0.7$, $\underline{p} = 0.5$, $\overline{q} = 0.35$, $\underline{q} = 0.01$, $\tilde{p} = 0.4$, that the favorable and the unfavorable aggregate states are equally likely $\overline{\beta} = \underline{\beta} = 0.5$, and that revenue under success is $s = 30$. Notice that these parameter values satisfy the conditions of the model and in particular are such that (3) holds so that $\overline{w}_S^{RSW-G} > \underline{w}_S^{RSW-G}$.

The purpose of this example is to show that when (3) holds, and, therefore, when $\overline{w}_S^{RSW-G} > \underline{w}_S^{RSW-G}$, the aggregate relationship between executive compensation and market movements may be positive. This may help explain the conflicting empirical evidence on RPE that has been discussed in the introduction.

We find that the RSW contracts are

$$\begin{aligned} w^{RSW-B} &= \left(\underline{w}_F^{RSW-B}, \underline{w}_S^{RSW-B}, \overline{w}_F^{RSW-B}, \overline{w}_S^{RSW-B}\right) = (6, 6, 6, 6) \\ w^{RSW-G} &= \left(\underline{w}_F^{RSW-G}, \underline{w}_S^{RSW-G}, \overline{w}_F^{RSW-G}, \overline{w}_S^{RSW-G}\right) = (6.6000, 7.0993, 0.0024, 10.4936). \end{aligned}$$

The bad manager accepts a constant contract that fully insures him and the good manager accepts a contract which gives him compensation which is increasing in own performance, decreasing in industry performance when individual performance is low, but increasing in industry performance when individual performance is high. Notice that in this example the good manager receives higher compensation when firm performance is in line with industry performance. This might suggest that the result arises because this contract is more appealing to the good manager rather than to the bad because the good manager has higher correlation with the industry than the bad. This is incorrect, because the good manager may have higher correlation with the industry also when condition (3) is not satisfied, and in this case his compensation package would not give him higher compensation when own performance is in line with the industry but rather when industry performance is low. The driving force behind our results is therefore not managers' different correlations with the market but their different relative likelihoods of success under different aggregate states.

We now analyze whether the RSW contracts derived above may lead to a positive relationship between executive compensation and aggregate performance. To do this we compute the joint probability distributions over executives' salaries, firms' own profits, and industry profits generated by equilibrium play for different values of μ. We then turn to the link between pay, individual performance and industry performance by performing regression analyses on the equilibrium distribution over salaries, profits and industry profits. In particular we compute OLS coefficients for the following linear specification

$$w_i = \beta_0 + \beta_1 \cdot FP_i + \beta_2 \cdot IP + \xi \tag{7}$$

where i denotes a manager, w_i his salary realization, FP_i the profit realization of the firm he works for and IP denotes the realization of industry profit. We approximate the joint distribution with 10,000 data points.

We have repeated the exercise for different values of the prior probability of the manager being good and we have always found the same qualitative results. In Table 1 we report the estimation results for five different values of the probability of being good. In columns 2 to 4 we report the regression coefficients. Column 5 reports the 95% confidence interval for the coefficient of industry profit, $\widehat{\beta}_2$. The results summarized in Table 1, suggest that wages are increasing in own performance (column 3) but they are also increasing in industry performance (columns 4 and 5). This result appears as a violation of the shape that is normally taken for granted for RPE but is consistent with the results documented by the empirical literature.

4.2 Extension: n types of managers

The model we have used so far has made the assumption that only two types of managers exist, good and bad. The purpose of this subsection is to clarify that this assumption is made only for the sake of simplicity and that the results of section 3 can be generalized to a case in which an arbitrary but finite number of types exist.

Assume that a manager can be of type $\tau = \{1, ..., T\}$ and that each type τ gets a signal $\rho \in \{L, H\}$ about the realization of project I. Types differ in the informativeness of the signal they receive. In particular, assume that type τ has probability $\lambda(\tau)$ of receiving a high signal when project I is doomed to success and to receive a low signal when project I is doomed to failure and that

$$\begin{aligned}\lambda(\tau) &= \Pr(\rho = H \mid r = s, I, \tau) = \Pr(\rho = L \mid r = 0, I, \tau) = \\ &= \frac{T-\tau}{T-1}\frac{p(1-q)}{p(1-q)+(1-p)q} + \frac{\tau-1}{T-1}\end{aligned}$$

Notice that $\lambda(\tau)$ is increasing in τ, that $\lambda(T) = 1$ and that

$$\lambda(1) = \frac{p(1-q)}{p(1-q)+(1-p)q} > \frac{1}{2}.$$

This implies that higher types receive more informative signals, that the best type, T, is able to perfectly forecast the realization of project I and that the worst possible type, 1, receives a signal which has positive although limited informativeness about the realization of project I.

The probability of project I having the high return ($s > 0$) conditional on the high signal is

$$\Pr(r = s \mid \rho = H, I, \tau) = \frac{\lambda(\tau)p}{\lambda(\tau)p + (1-\lambda(\tau))(1-p)}$$

and the probability of project I having the high return ($s > 0$) conditional on the low signal is

$$\Pr(r = s \mid \rho = L, I, \tau) = \frac{(1-\lambda(\tau))p}{(1-\lambda(\tau))p + \lambda(\tau)(1-p)}$$

Given that $\lambda(\tau)$ is increasing in τ, $\Pr(r = s \mid \rho = L, I, \tau)$ is decreasing in τ. Notice also that for the best type $\tau = T$

$$\Pr(r = s \mid \rho, I, T) = \begin{cases} 1 & \text{if } \rho = H \\ 0 & \text{if } \rho = L \end{cases}$$

and for the worst type $\tau = 1$,

$$\Pr(r = s \mid \rho, I, 1) = \begin{cases} \frac{p^2(1-q)}{p^2(1-q)+(1-p)^2 q} & \text{if } \rho = H \\ q & \text{if } \rho = L \end{cases}$$

Because $\Pr(r = s \mid \rho = L, I, \tau = 1) = q$, we have that for all $\tau > 1$

$$\Pr(r = s \mid \rho = L, I, \tau) < q.$$

From the above it follows that, if executive compensation is increasing in absolute performance (which implies that the manager wants to maximize expected absolute performance) all types of manager prefer to invest in project I if they receive the high signal, $\rho = H$, that all $\tau > 1$ prefer to invest in II if they receive the low signal and that only the worst type $\tau = 1$ is indifferent between investing in project I or II if he receives the low signal. Assume, that when indifferent between project I and II, type $\tau = 1$ chooses project I, and let i^E denote the efficient investment profile that specifies that type $\tau = 1$ always invests in project I, and that all types $\tau > 1$ invest in project I when they receive the high signal, and in project II when they receive the low signal.

Under the assumptions on the joint distribution of individual and aggregate states made in 3, straightforward calculations show that for all types $\tau = \{1, ...T\}$

$$\begin{aligned}
\underline{\alpha}_S^\tau\left(i^E\right) &= \underline{\beta}\left\{\underline{\lambda}(\tau)\underline{p} + \underline{q}\left[(1-\underline{\lambda}(\tau))\underline{p} + \underline{\lambda}(\tau)(1-\underline{p})\right]\right\} \\
\overline{\alpha}_S^\tau\left(i^E\right) &= \overline{\beta}\left\{\overline{\lambda}(\tau)\overline{p} + \overline{q}\left[(1-\overline{\lambda}(\tau))\overline{p} + \overline{\lambda}(\tau)(1-\overline{p})\right]\right\} \\
\underline{\alpha}_F^\tau\left(i^E\right) &= \underline{\beta}\left\{(1-\underline{\lambda}(\tau))\left(1-\underline{p}\right) + \left(1-\underline{q}\right)\left[(1-\underline{\lambda}(\tau))\underline{p} + \underline{\lambda}(\tau)(1-\underline{p})\right]\right\} \\
\overline{\alpha}_F^\tau\left(i^E\right) &= \overline{\beta}\left\{\left(1-\overline{\lambda}(\tau)\right)(1-\overline{p}) + (1-\overline{q})\left[(1-\overline{\lambda}(\tau))\overline{p} + \overline{\lambda}(\tau)(1-\overline{p})\right]\right\}.
\end{aligned}$$

Tedious calculations show that for all $\tau' > \tau$, for all $w = \left(\underline{w}_F, \underline{w}_S, \overline{w}^F, \overline{w}^S\right) \in R_+^4$ and for all $\left(\underline{p}, \overline{p}, \underline{q}, \overline{q}\right)$ such that $\overline{p} > \underline{p}$, $\overline{q} > \underline{q}$, $\overline{p} > \overline{q}$ and $\underline{p} > \underline{q}$

$$MRS^{\tau'}_{\underline{w}_F, \overline{w}_F}\left(w, i^E\right) < MRS^{\tau}_{\underline{w}_F, \overline{w}_F}\left(w, i^E\right) \tag{8}$$

and that for all $w = \left(\underline{w}_F, \underline{w}_S, \overline{w}_F, \overline{w}_S\right) \in \mathbb{R}_+^4$

$$\operatorname{sign}\left(MRS^{\tau'}_{\underline{w}_S, \overline{w}_S}(w) - MRS^{\tau}_{\underline{w}_S, \overline{w}_S}(w)\right) = \operatorname{sign}\left(\frac{\underline{p}}{\left(1-\underline{p}\right)\underline{q}} - \frac{\overline{p}}{(1-\overline{p})\overline{q}}\right). \tag{9}$$

Given that conditions (8) and (9) generalize conditions (1) and (2) for the two-type model of section 3, it is easy to see that the same qualitative results also hold in the case of an arbitrary but finite number of types.

5 Conclusion

The principle that an optimal incentive scheme should neither reward nor punish managers for things outside their control flies in the face of the lack of support for a negative relationship between executive compensation and market performance documented by empirical research. We present a simple theoretical model that attempts to reconcile this apparent discrepancy.

Most of the literature on the topic has concentrated on contingent contracts as tools to endow managers with incentives to make appropriate decisions. We focus instead on a situation in which contingent contracts may be used to sort heterogeneous managers in an asymmetric information setting. We find that in this environment equilibrium contracts include RPE considerations, but their form may differ from that which is obtained by the existing literature.

In particular we show that in equilibrium, when the likelihood of individual success with a favorable aggregate state relative to success with an unfavorable aggregate state is higher for a good manager than for a bad one, better managers accept contracts that may make larger payments when the aggregate state is favorable rather than unfavorable. In other words, we show that executive compensation is increasing in the firm's absolute performance but is not necessarily decreasing with industry performance.

An example shows that this effect may lead to a positive relationship between executive compensation and aggregate performance and that our results may resolve the apparent discrepancy between the empirical and the theoretical literature.

Summarizing, we believe that the form that is normally defended for RPE is a principle that implicitly relies on the following assumptions:

- Firms know managers' abilities;

- Firms only need to make sure that the manager's expected utility is at least as high as the one he would receive in an alternative employment, but they need not worry about the form of the compensation scheme.

Our paper shows that with asymmetric information on managers' talent, competition implies restrictions on compensation schemes that may turn contracts with the classical form of RPE into inferior choices. In other words, our paper argues that the classical form of RPE is probably appropriate as a way to retain a manager of known ability, but that it may have adverse implications when used to induce a manager of unknown ability to accept an employment contract.

References

[1] Aggarwal, R.K. and A.A. Samwick (1999) "Executive compensation, strategic competition, and relative performance evaluation: Theory and evidence", *Journal of Finance*, LIV, 1999-2043.

[2] Bebchuk, L.A., J.M. Fried and D.I. Walker (2002) "Managerial power and rent extraction in the design of executive compensation", *The University of Chicago Law Review*, 69, 751-846.

[3] Core and Guay (2002) "When Contracts Require Risk-Averse Executives to Hold Equity", mimeo.

[4] de Swaan, J.C. and N.W.C. Harper (2003) "Getting what you pay for with stock options", *The McKinsey Quarterly*, 2003-1, Closing Views, 2-5.

[5] Dobbs and Koller (2000) "Stock options aren't enough" *The McKinsey Quarterly*, 2000-3, 190-192.

[6] Garvey G.T. and T.T. Milbourn (2002) "Incentive Compensation When Executives Can Hedge the Market: Evidence of Relative Performance Evaluation in the Cross-Section" *Journal of Finance*, forthcoming.

[7] Hellwig, M. (1987) "Some recent developments in the theory of competition in markets with adverse selection" *European Economic Review*, 31, 319-325.

[8] Himmelberg, C.P. and R.G. Hubbard (2000) "Incentive Pay and the Market for CEO's: An Analysis of Pay-for-Performance Sensitivity", mimeo., Columbia University.

[9] Holmström, B. (1979) "Moral hazard and observability" *Bell Journal of Economics*, 10, 231-259.

[10] Holmström, B. (1979) "Moral hazard in teams" *Bell Journal of Economics*, 13, 324-340.

[11] Holmström, B. and J. Ricart i Costa (1986) "Managerial incentives and capital management" *Quarterly Journal of Economics*, 101, 835-860.

[12] Jensen, M. and K. Murphy (1990) "Performance pay and top-management incentives" *Journal of Political Economy*, 98, 225-264.

[13] Kanodia, C., R. Bushman, and J. Dickhaut (1989) "Escalation errors and the sunk cost effect: An explanation based on reputation and information asymmetries", *Journal of Accounting Research*, 27, 59-77.

[14] Murphy (1999) "Executive compensation" in O. Ashenfelter and D. Card (1999) (eds.) *Handbook of Labor Economics*, Vol. 3, North Holland, Amsterdam.

[15] Rappaport (1999) "New thinking on how to link executive pay with performance", *Harvard Business Review*, March-April, 91-101.

[16] *The Economist* (2002) "Is Greed Good?" in "Capitalism and Its Troubles. A Survey of International Finance", May 18, 2002, 23-26.

[17] Wilson, C. (1977) "A model of insurance markets with incomplete information" *Journal of Economic Theory*, 16, 167-207.

A Appendix

A.1 Proof of Proposition 1

We prove Proposition 1 through a sequence of Lemmas. We first define a pair of RSW contracts with the additional constraint that the investment profile played by managers after they accept their contracts is a prespecified profile i' that may differ from the one that maximizes managers' expected utilities, $\left(i_V^*\left(w^B\right), i_L^*\left(w^G\right), i_H^*\left(w^G\right)\right)$.

Definition 4

$$\begin{aligned} w^{RSW-B}\left(w^G, i'\right) &= \arg\max_{w \in \mathbb{R}_+^4} V^B\left(w, i'\right) && (10) \\ s.t.\ E\left[\pi - w \mid B, i'\right] &\geq \widetilde{p}s - 1 && (11) \\ V^G\left(w^G, i'\right) &\geq V^G\left(w, i^*\left(w\right)\right). && (12) \end{aligned}$$

$$\begin{aligned} w^{RSW-G}\left(w^B, i'\right) &= \arg\max_{w \in \mathbb{R}_+^4} V^G\left(w, i'\right) && (13) \\ s.t.\ E\left[\pi - w \mid G, i'\right] &\geq \widetilde{p}s - 1 && (14) \\ V^B\left(w^B, i'\right) &\geq V^B\left(w, i^*\left(w\right)\right). && (15) \end{aligned}$$

Notice that the incentive compatibility conditions (12) and (15) constrain each type of manager to play according to i' if he accepts the contract that is meant for his type but allow each type of manager to choose the investment profile that maximizes his expected utility if he deviates and chooses the contract meant for the other type.

Definition 5 *A pair of contracts is i-Rothschild-Stiglitz-Wilson,* $\left(w^{RSW-B}\left(i\right), w^{RSW-G}\left(i\right)\right)$*, if and only if*

$$\begin{aligned} w^{RSW-B}\left(i\right) &= w^{RSW-B}\left(w^{RSW-G}\left(i\right), i\right) \\ w^{RSW-G}\left(i\right) &= w^{RSW-G}\left(w^{RSW-B}\left(i\right), i\right) \end{aligned}$$

Lemma 1 *A* i^E*-RSW pair of contracts exists and is such that:*

1. $w^{RSW-B} = \left(\left(p - \widetilde{p}\right)s, \left(p - \widetilde{p}\right)s, \left(p - \widetilde{p}\right)s, \left(p - \widetilde{p}\right)s\right)$
2. $w^{RSW-G} \neq w^{RSW-B}$
3. *For all* $\left(\underline{p}, \underline{q}, \overline{p}, \overline{q}\right)$

$$\begin{aligned} \underline{w}_S^{RSW-G} &\geq \underline{w}_F^{RSW-G} && (16) \\ \overline{w}_S^{RSW-G} &\geq \overline{w}_F^{RSW-G} && (17) \\ \underline{w}_F^{RSW-G} &\geq \overline{w}_F^{RSW-G}; && (18) \end{aligned}$$

If $\frac{\overline{p}}{(1-\overline{p})\overline{q}} < \frac{\underline{p}}{(1-\underline{p})\underline{q}}$

$$\overline{w}_S^{RSW-G} \geq \underline{w}_S^{RSW-G}; \qquad (19)$$

If $\frac{\overline{p}}{(1-\overline{p})\overline{q}} > \frac{\underline{p}}{(1-\underline{p})\underline{q}}$

$$\overline{w}_S^{RSW-G} \leq \underline{w}_S^{RSW-G}. \qquad (20)$$

Proof: Under the assumption that the manager invests according to i^E the expected profits gross of compensation generated by the two types of manager are

$$\begin{aligned} E\left[\pi \mid G, i^E\right] &= p\left(s-1\right) + \left(1-p\right)\left(qs-1\right) > \widetilde{p}s - 1 \\ E\left[\pi \mid B, i^E\right] &= ps - 1 > \widetilde{p}s - 1 \end{aligned}$$

with

$$E\left[\pi \mid G, i^E\right] > E\left[\pi \mid B, i^E\right]. \tag{21}$$

Given that $U'(.) > 0$, constraints (11) and (14) have to be satisfied with equality. This implies that w^{RSW-G} cannot be a constant contract, because if it were, type B would prefer it to any contract satisfying (11).

We now want to show that (12) cannot be satisfied with equality. To see this, notice that an upper bound on the utility that type G could get from accepting w^{RSW-B} is the utility he would get by accepting a constant contract satisfying (11) with equality, i.e.,

$$w = ((p-\widetilde{p})\, s, (p-\widetilde{p})\, s, (p-\widetilde{p})\, s, (p-\widetilde{p})\, s).$$

Suppose that the good manager accepts this contract and consider the indifference curves of the two types of manager passing through w on the $(\underline{w}_S \overline{w}_S)$ plane as depicted in Figure 3. Notice that given that the good manager generates a higher expected profit, his break-even line, gg, lies strictly above w. Figure 3 shows that the contracts that differ from w in that $(\underline{w}_S \overline{w}_S) = ((p-\widetilde{p})\, s, (p-\widetilde{p})\, s)$ is replaced by a pair strictly above GG , strictly below BB and below gg are strictly better for type G and strictly worse for type B. This implies that (12) cannot be satisfied with equality.

Given that (12) is slack type B's risk aversion implies that w^{RSW-B} is constant and satisfies (11) with equality so that

$$w^{RSW-B} = ((p-\widetilde{p})\, s, (p-\widetilde{p})\, s, (p-\widetilde{p})\, s, (p-\widetilde{p})\, s)$$

as in Part 1. Given that w^{RSW-G} cannot be constant, Part 2 follows.

To prove part 3 notice first that simple calculations show that for all $\left(\underline{p}, \underline{q}, \overline{p}, \overline{q}\right)$

$$\begin{aligned}
MRS^G_{\underline{w}_F,\underline{w}_S}\left(w, i^E\right) &> MRS^B_{\underline{w}_F,\underline{w}_S}\left(w, i^E\right)\\
MRS^G_{\overline{w}_F,\overline{w}_S}\left(w, i^E\right) &> MRS^B_{\overline{w}_F,\overline{w}_S}\left(w, i^E\right)\\
MRS^G_{\underline{w}_F,\overline{w}_F}\left(w, i^E\right) &< MRS^B_{\underline{w}_F,\overline{w}_F}\left(w, i^E\right)
\end{aligned}$$

and that

$$MRS^G_{\underline{w}_S,\overline{w}_S}\left(w, i^E\right) > MRS^B_{\underline{w}_S,\overline{w}_S}\left(w, i^E\right)$$

if and only if $\frac{\overline{p}}{(1-\overline{p})\overline{q}} < \frac{\underline{p}}{(1-\underline{p})\underline{q}}$.

Given this, to prove part 3, it suffices to show that for any two states σ' and σ'':

$$MRS^G_{w(\sigma'),w(\sigma'')}\left(w, i^E\right) > MRS^B_{w(\sigma'),w(\sigma'')}\left(w, i^E\right) \Rightarrow w^{RSW-G}(\sigma'') \geq w^{RSW-G}(\sigma') \tag{22}$$

$$MRS^G_{w(\sigma'),w(\sigma'')}\left(w, i^E\right) < MRS^B_{w(\sigma'),w(\sigma'')}\left(w, i^E\right) \Rightarrow w^{RSW-G}(\sigma'') \leq w^{RSW-G}(\sigma') \tag{23}$$

Contrary to 22 suppose that in equilibrium $w^{RSW-G}(\sigma'') < w^{RSW-G}(\sigma')$, such as for instance $W2$ in Figure 4. Given that

$$MRS^G_{w(\sigma'),w(\sigma'')}(w) = -\frac{\Pr(\sigma' \mid G)}{\Pr(\sigma'' \mid G)},$$

when $w(\sigma'') = w(\sigma')$, $w^{RSW-G}(\sigma'') < w^{RSW-G}(\sigma')$ implies that

$$MRS^G_{w(\sigma'),w(\sigma'')}(w^*) > -\frac{\Pr(\sigma' \mid G)}{\Pr(\sigma'' \mid G)},$$

i.e., GG, the indifference curve of the good manager, is less steep at w^{RSW-G} than gg, the set of contracts that have the same expected cost as w^{RSW-G} conditioned on only the good manager accepting it. Consider now a contract $\widehat{w}$ which differs from w^{RSW-G} in that $\left(w^{RSW-G}(\sigma'), w^{RSW-G}(\sigma'')\right)$ is replaced by $(\widehat{w}(\sigma'), \widehat{w}(\sigma''))$ below gg, below BB and above GG. This contract is strictly better for the good manager and strictly worse for the bad manager and is such that the expected compensation cost, conditional on only the good manager accepting it, is no higher than the expected compensation cost of w^*. This implies that contract $\widehat{w}$ gives a higher expected utility to the good manager while satisfying (14) and (15) a contradiction to the hypothesis that w^{RSW-G} could be such that $w^{RSW-G}(\sigma'') < w^{RSW-G}(\sigma')$.

Notice that the same argument does not apply when $w^{RSW-G}(\sigma'') \geq w^{RSW-G}(\sigma')$, such as for instance point $W1$ in Figure 4. In this case, gg, the set of contracts that have the same expected cost as w^G conditioned on only the good manager accepting it, is less steep than GG, the indifference curve of the good manager. Given that GG is less steep than BB, the set of contracts that are more profitable than $W1$ conditional on only the good manager accepting it, and above GG are also above BB. In other words, the contracts that lie above GG and below gg, such as contract $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$, also lie above BB. This implies that this contract would violate (15).

Part 23: Similar arguments apply to this case. We will not repeat the full argument, but inspection of Figure 5 should clarify that when $MRS^G_{w(\sigma'),w(\sigma'')}(w) < MRS^B_{w(\sigma'),w(\sigma'')}(w)$, w^{RSW-G} cannot be like $W3$ and can be like $W4$. Suppose first that w^{RSW-G} is such $w^{RSW-G}(\sigma') < w^{RSW-G}(\sigma'')$, as in point $W3$ in Figure 5. In this case a contract $\widehat{w}$ that differs from w^{RSW-G} in that $\left(w^{RSW-G}(\sigma'), w^{RSW-G}(\sigma'')\right)$ is replaced by $(\widehat{w}(\sigma'), \widehat{w}(\sigma''))$, that lies below gg, below BB and above GG would improve upon w^{RSW-G} for type G while satisfying (14) and (15).

Suppose instead that w^{RSW-G} is such $w^{RSW-G}(\sigma') \geq w^{RSW-G}(\sigma'')$, as in point $W4$ in Figure 5. Consider a contract $\widetilde{w}$ that differs from w^{RSW-G} in that $\left(w^{RSW-G}(\sigma'), w^{RSW-G}(\sigma'')\right)$ is replaced by $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$, that lies below gg and above GG. Given that GG is steeper than BB, however, all contracts that lie on or below gg and above GG, such as $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$, also lie above BB and therefore violate (15). ■

LEMMA 2 *A i^E-RSW pair of contracts is an RSW pair of contracts.*

PROOF: We want to show that

$$\left(i^*_V\left(w^{RSW-B}\right), i^*_L\left(w^{RSW-G}\right), i^*_L\left(w^{RSW-G}\right)\right) = (I, II, I) \tag{24}$$

and that all i-RSW pairs of contracts $\left(w^B, w^G\right)$ with $i \neq i^E$ are dominated by the i^E-RSW pair of contracts.

To prove (24) it is sufficient to notice that Lemma 1 establishes the weak monotonicity in firm absolute performance of w^{RSW-B} and w^{RSW-G} and this together with the assumption that a manager who is indifferent between two investment decision chooses the one which has a higher expected profit is sufficient to guarantee that both types of managers always make the efficient investment decision.

Suppose now that i-RSW pairs of contracts exist for all $i \neq i^E$. We will show that, even if they exist, they are dominated by (I, II, I)-RSW pair of contracts. To do that we will make use of the following observation: An i-RSW pair of contracts gives each type an expected wage equal to his value to the firm (when he invests according to i), and an expected utility at least as large as the expected utility from accepting the other type's contract; given that the expected wage is no larger than the expected value of a manager, each type gets an expected utility which is no larger than the expected utility of a constant wage equal to his own value.

1. (I, I, I): In this case the expected utility to types B and G would be the same that they would get with type B's (I, II, I)-RSW contract. Given that the incentive compatibility condition for that case, (12), has been shown to be slack, the expected utility to type G would be strictly lower and the expected utility to type B would be the same, so that (I, II, I)-RSW pair of contracts dominates any (I, I, I)-RSW pair of contracts, if the latter exists.

2. (I, II, II): The expected utility to type B would be no higher than in the previous case but the expected utility to type G would be lower than in the previous case. This implies that the (I, II, I)-RSW pair of contracts dominates any (I, I, I)-RSW pair of contracts, if the latter exists.

3. (I, I, II): The expected utility to type G would be even lower than in the previous case. This implies that the (I, II, I)-RSW pair of contracts dominates any (I, I, II)-RSW pair of contracts, if the latter exists.

4. (II, I, II): The expected utility to type B would be strictly lower than with the (I, II, I)-RSW pair of contracts. This implies that the (I, II, I)-RSW pair of contracts dominates any (II, I, II)-RSW pair of contracts, if the latter exists.

5. (II, II, II): The expected utility to both types would be strictly lower than with the (I, II, I)-RSW pair of contracts. This implies that (I, II, I)-RSW pair of contracts dominates any (II, II, II)-RSW pair of contracts, if the latter exists.

6. (II, II, I): The expected utility to type B would be lower than with his (I, II, I)-RSW contract and the expected utility to type G would be no higher than with his (I, II, I)-RSW contract. This implies that (I, II, I)-RSW pair of contracts dominates any (II, II, I)-RSW pair of contracts, if the latter exists.

7. (II, I, I): The expected utility to both types would be strictly lower than with the (I, II, I)-RSW pair of contracts. This implies that (I, II, I)-RSW pair of contracts dominates any (II, I, I)-RSW pair of contracts, if the latter exists.

■

A.2 Proof of Proposition 2

We prove Proposition 2 through a sequence of Lemmas.

Lemma 3 *In a SPNE firms' expected profits are* $\widetilde{p}s - 1$.

Proof: Immediate from a standard Bertrand pricing argument. ■

Lemma 4 *No SPNE exists in which both following conditions hold:*

1. *Both types accept the same contract;*
2. *The equilibrium path investment profile is different from* (I, I, I) *or* (II, II, II).

Proof: Contrary to the claim suppose that there is a SPNE in which both types of managers accept a given contract w^* and in which the equilibrium path investment profile is different from (I, I, I) or (II, II, II). By Lemma 3, if this contract is accepted in equilibrium by both types of managers, the payoff to a firm offering it is $\widetilde{p}s - 1$. Without loss of generality assume that the manager accepts such a contract offered by firm 1.

Notice that given that we are assuming that the equilibrium path investment profile is different from (I, I, I) or (II, II, II), the probability distributions over final outcomes depend on the manager's type. This means that, for all $(i_V, i_L, i_H) \notin \{(I, I, I), (II, II, II)\}$ there has to exist a pair of states (σ', σ'') such that $MRS^{\tau}_{w(\sigma'),w(\sigma'')}(w^*, i(w^*)) \neq MRS^{\tau'}_{w(\sigma'),w(\sigma'')}(w^*, i(w^*))$. In the following we will focus without loss of generality on two states, σ' and σ'' such that $MRS^{\tau}_{w(\sigma'),w(\sigma'')}(w^*, i(w^*)) < MRS^{\tau'}_{w(\sigma'),w(\sigma'')}(w^*, i(w^*))$.

Consider the following 3 cases:

1. The expected profit to a firm whose offer w^* is accepted by type τ is $\pi_\tau > \widetilde{p}s - 1$.

   Consider Figure 6, depicting TT and $T'T'$, the indifference curves for types τ and τ' passing through $(w^*(\sigma'), w^*(\sigma''))$ and $\tau\tau$, the set of contracts differing from w^* only in that $(w^*(\sigma'), w^*(\sigma''))$ is replaced by $(w(\sigma'), w(\sigma''))$ and that conditional on being accepted only by type τ have the same expected profit as w^* when this is accepted by both types, i.e., $\widetilde{p}s - 1$. Notice that given that $\pi_\tau > \widetilde{p}s - 1$, line $\tau\tau$ lies strictly above $(w^*(\sigma'), w^*(\sigma''))$. This implies, that there are pairs that lie below $\tau\tau$ above TT and below $T'T'$. This means that if firm 2 offers a contract differing from w^* only in that $(w^*(\sigma'), w^*(\sigma''))$ is replaced by a pair such as $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$ in Figure 6, it would get a profit strictly larger than $\widetilde{p}s - 1$ because this contract would be accepted by type τ only. Given that this offer would be a profitable deviation for firm 2, a contradiction arises. Notice that while Figure 6 depicts a case in which $w^*(\sigma'') > w^*(\sigma')$, this inequality is inessential for the result, because the result depends on the difference between the marginal rates of substitution of the two type of manager and the fact that $\tau\tau$, the break-even line for type τ, lies strictly above point $(w^*(\sigma'), w^*(\sigma''))$ but it does not depend on the relative magnitudes of the slopes of TT and $\tau\tau$.

2. The expected profit to a firm whose offer w^* is accepted by type τ is $\pi_\tau < \widetilde{ps} - 1$.

   By Lemma 3 the expected profit to a firm whose offer w^* is accepted by type τ' is

$$\pi_{\tau'} = \frac{\widetilde{ps} - 1 - \Pr(\tau)\pi_\tau}{1 - \Pr(\tau)} > \widetilde{ps} - 1.$$

   Consider Figure 7, depicting TT and $T'T'$, the indifference curves for types τ and τ' passing through $(w^*(\sigma'), w^*(\sigma''))$ and $\tau'\tau'$, the set of contracts differing from w^* only in that $(w^*(\sigma'), w^*(\sigma''))$ is replaced by $(w(\sigma'), w(\sigma''))$ and that conditional on being accepted only by type τ' have the same expected profit as w^* when this is accepted by both types, i.e., $\widetilde{ps} - 1$. Notice that given that $\pi_{\tau'} > \widetilde{ps} - 1$, $\tau'\tau'$ lies strictly above $(w^*(\sigma'), w^*(\sigma''))$. This implies, that there are pairs that lie below $\tau'\tau'$ above $T'T'$ and below TT. This means that if firm 2 offers a contract differing from w^* only in that $(w^*(\sigma'), w^*(\sigma''))$ is replaced by a pair such as $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$ in Figure 7, it would get a profit strictly larger than $\widetilde{ps} - 1$ because this contract would be accepted by type τ' only. Given that this offer would be a profitable deviation for firm 2, a contradiction arises. As before, notice that Figure 7 depicts a case in which $w^*(\sigma'') > w^*(\sigma')$, but this inequality is inessential for the result. In fact the result holds despite the fact that $w^*(\sigma'') > w^*(\sigma')$ implies that $T'T'$ is steeper at w^* than $\tau'\tau'$, because $\tau'\tau'$ lies strictly above point $(w^*(\sigma'), w^*(\sigma''))$.

3. The expected profit to a firm whose offer w^* is accepted by type τ is $\pi_\tau = \widetilde{ps} - 1$.

   By Lemma 3 the expected profit to a firm whose offer w^* is accepted by type τ' is

$$\pi_{\tau'} = \frac{\widetilde{ps} - 1 - \Pr(\tau)\pi_\tau}{1 - \Pr(\tau)} = \widetilde{ps} - 1.$$

   Notice that w^* cannot be a constant contract, because otherwise $\pi_\tau \neq \pi_{\tau'}$. This implies that there have to be two states of nature, σ' and σ'' such that the salaries are different from each other $w^*(\sigma') \neq w^*(\sigma'')$. Without loss of generality suppose that $w^*(\sigma'') > w^*(\sigma')$ and recall that we are assuming that $MRS^{\tau}_{w(\sigma'),w(\sigma'')}(w^*) < MRS^{\tau'}_{w(\sigma'),w(\sigma'')}(w^*)$. Consider Figure 8. Given that

$$MRS^{\tau}_{w(\sigma'),w(\sigma'')}(w) = -\frac{\Pr(\sigma' \mid \tau)}{\Pr(\sigma'' \mid \tau)},$$

   when $w(\sigma'') = w(\sigma')$, $w^*(\sigma'') > w^*(\sigma')$ implies that

$$MRS^{\tau}_{w(\sigma'),w(\sigma'')}(w^*) < -\frac{\Pr(\sigma' \mid \tau)}{\Pr(\sigma'' \mid \tau)},$$

   i.e., TT, the indifference curve of type τ, is steeper at w^* than $\tau\tau$, the set of contracts that have the same expected cost as w^* conditioned on only type τ accepting it. Consider now a contract w' which differs from w^* in that $(w^*(\sigma'), w^*(\sigma''))$ is replaced by $(\widetilde{w}(\sigma'), \widetilde{w}(\sigma''))$ below $\tau\tau$, below $T'T'$ and above TT, as shown in Figure 8. This contract is strictly better for type τ and strictly worse for τ' and is such that the expected profit, conditional on only type τ accepting it, is higher than $\widetilde{ps} - 1$, the expected profit from offering w^* and having it accepted by both types. This implies that offering contract w' would be a profitable deviation for firm 2, and a contradiction arises.

■

We now turn to the proof of Proposition 2.

Suppose first that in the path of a SPNE, the investment profile is either (I, I, I) or (II, II, II) and recall that firms' equilibrium payoffs are $\widetilde{ps} - 1$. Given under (I, I, I) or (II, II, II) the two types generate the same probability distributions over outcomes, and because of risk-aversion, both types would accept the same constant contract. Denote the contract accepted in equilibrium by $\widehat{w}$. Notice that $\widehat{w} = w^{RSW-B}$ when the investment profile is (I, I, I) and $\widehat{w} < w^{RSW-B}$ when the investment profile is (II, II, II). Suppose without loss of generality that firm 1 offers $\widehat{w}$ and consider the following possible deviation for firm 2: offering only contracts

$$\left(w^B, w^G\right) = \left(w^{RSW-B}, w^{RSW-G} - \varepsilon\right) \in \mathbb{R}^4_+$$

Notice that there exists an $\varepsilon > 0$ sufficiently small that:

1. The profit of w^{RSW-B} conditional on only type B accepting it is $\widetilde{p}s-1$;
2. The profit of $w^{RSW-G}-\varepsilon$ conditional on only type G accepting it is $\widetilde{p}s-1+\varepsilon$;
3. Type G strictly prefers $w^{RSW-G}-\varepsilon$ to $\widehat{w}$ or w^{RSW-B}.
4. Type B strictly prefers w^{RSW-B} to $w^{RSW-G}-\varepsilon$.

This implies that if firm 2 offers these contracts, type G would accept $w^{RSW-G}-\varepsilon$ and type B would accept either w^{RSW-B} or $\widehat{w}$. Firm 2 would therefore get an expected payoff strictly above $\widetilde{p}s-1$ if the manager is good and exactly equal to $\widetilde{p}s-1$ if the manager is bad. This implies that unconditional expected payoff from such a deviation is strictly above the equilibrium payoff and a contradiction is obtained.

Consider now the case in which the investment profile is neither (I,I,I) nor (II,II,II).

From Lemma 4 we know that in the path of a subgame perfect Nash equilibrium types B and G accept different contracts, w_B and w_G. This requires that

$$\begin{aligned} V^G\left(w^G,i^*\left(w^G\right)\right) &\geq V^G\left(w,i^*\left(w\right)\right) && (25)\\ V^B\left(w^B,i^*\left(w^B\right)\right) &\geq V^B\left(w,i^*\left(w\right)\right). && (26)\end{aligned}$$

From Lemma 3 we know that firms' equilibrium profits are $\widetilde{p}s-1$ and this implies that

$$\begin{aligned} E\left[\pi-w\mid G,i^*\left(w\right)\right] &= \widetilde{p}s-1 && (27)\\ E\left[\pi-w\mid B,i^*\left(w\right)\right] &= \widetilde{p}s-1 && (28)\end{aligned}$$

Notice first that $\left(w^{RSW-B},w^{RSW-G}\right)$ satisfy conditions (25)-(28). Suppose now that

$$\left(w^B,w^G\right)\neq\left(w^{RSW-B},w^{RSW-G}\right). \quad (29)$$

Notice that given that (12) is slack, w^{RSW-B} maximizes type B's utility subject to (11) and this implies that the utility to B with w^{RSW-B} can be no lower than the utility from w^B. This in turn implies that the utility to G with w^{RSW-B} can be no lower than the utility from w^G. Given (29), moreover the utility to at least one of the two types has to be strictly lower than with $\left(w^{RSW-B},w^{RSW-G}\right)$. This implies that there exists a contract that is no worse for any of the two types, is strictly better for at least one of the two and is such that the unconditional expected profit from offering it is strictly larger than $\widetilde{p}s-1$, a contradiction.
■

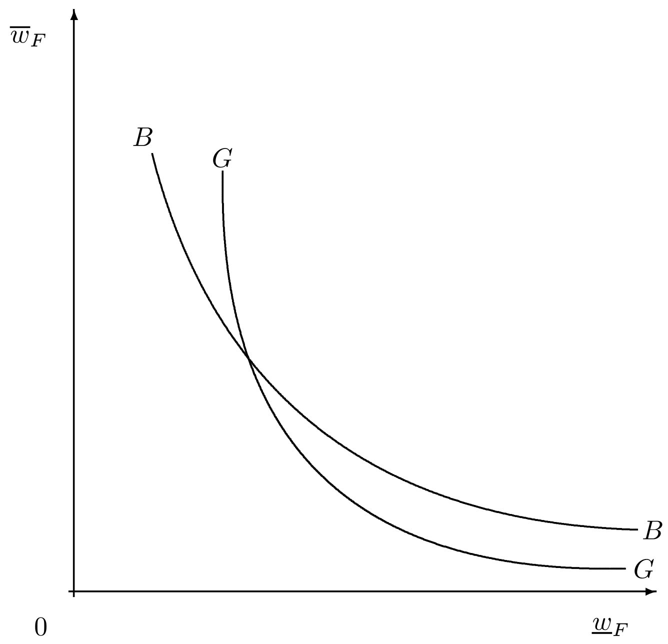

Figure 1(a): Indifference curves of the good and the bad manager with failure ($P = F$)

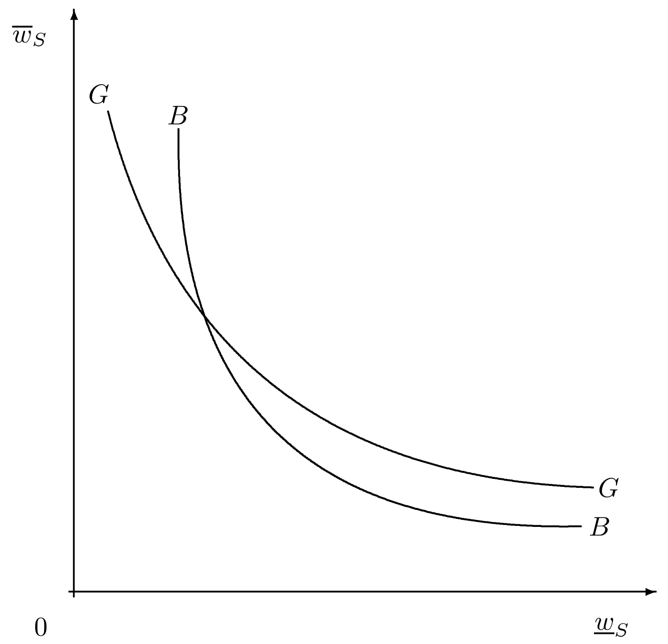

Figure 1(b): Indifference curves of the good and the bad manager with success ($P = S$)

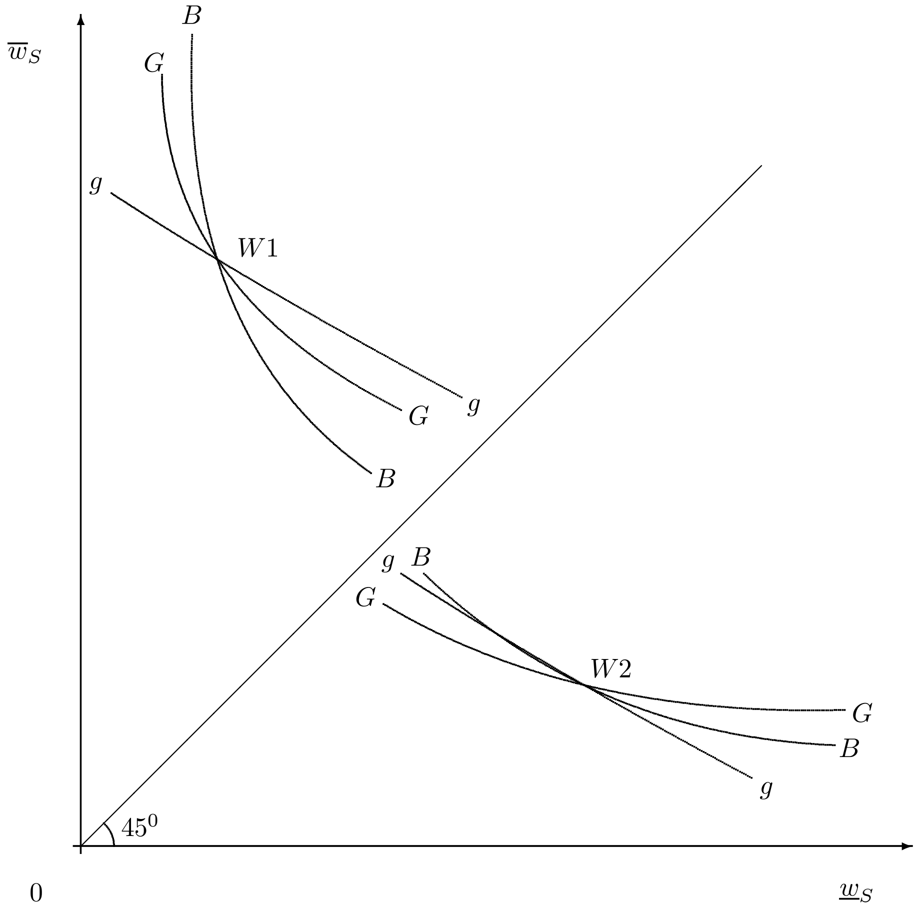

Figure 2

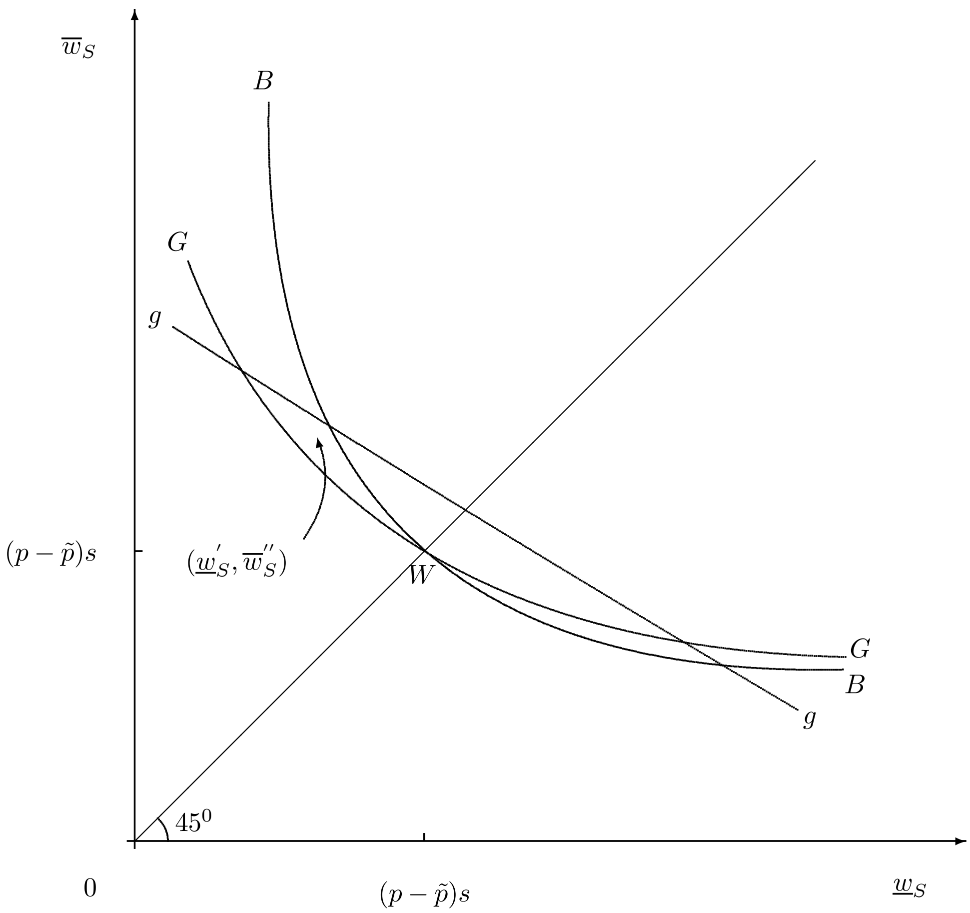
$\overline{w}_S$
B
G
g
$(p-\tilde{p})s$
$(\underline{w}_S', \overline{w}_S'')$
W
G
B
g
45^0
0
$(p-\tilde{p})s$
$\underline{w}_S$

Figure 3

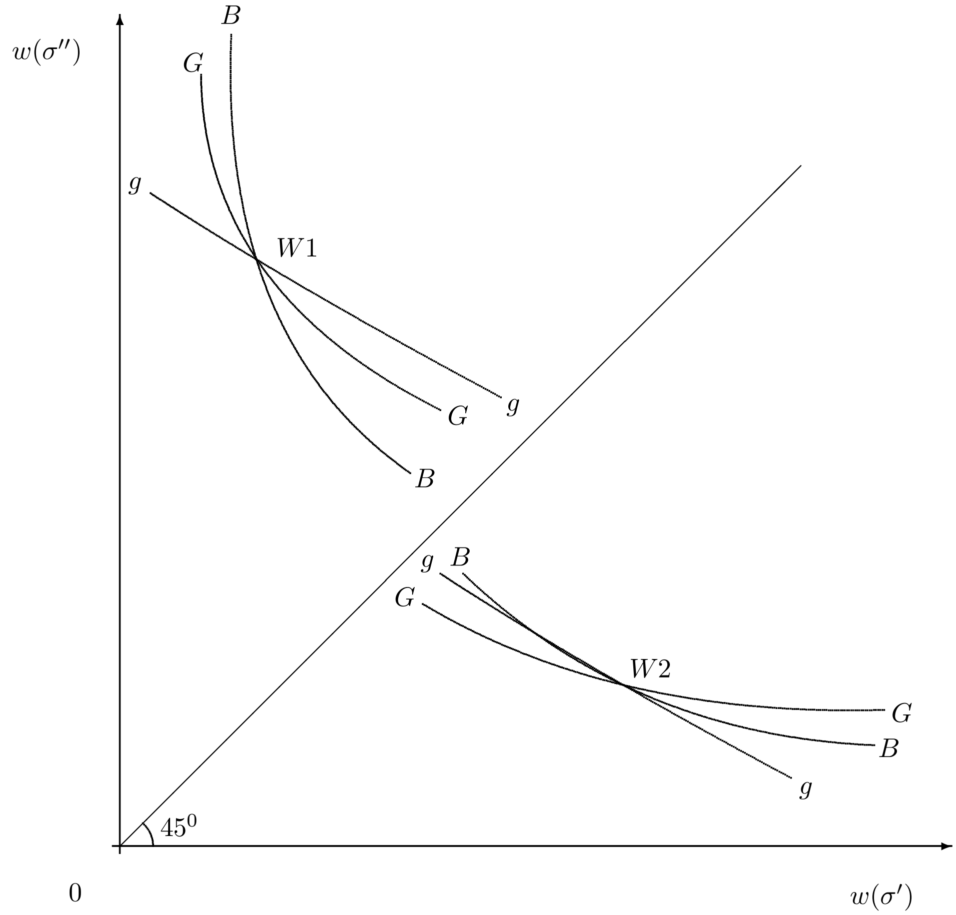

Figure 4

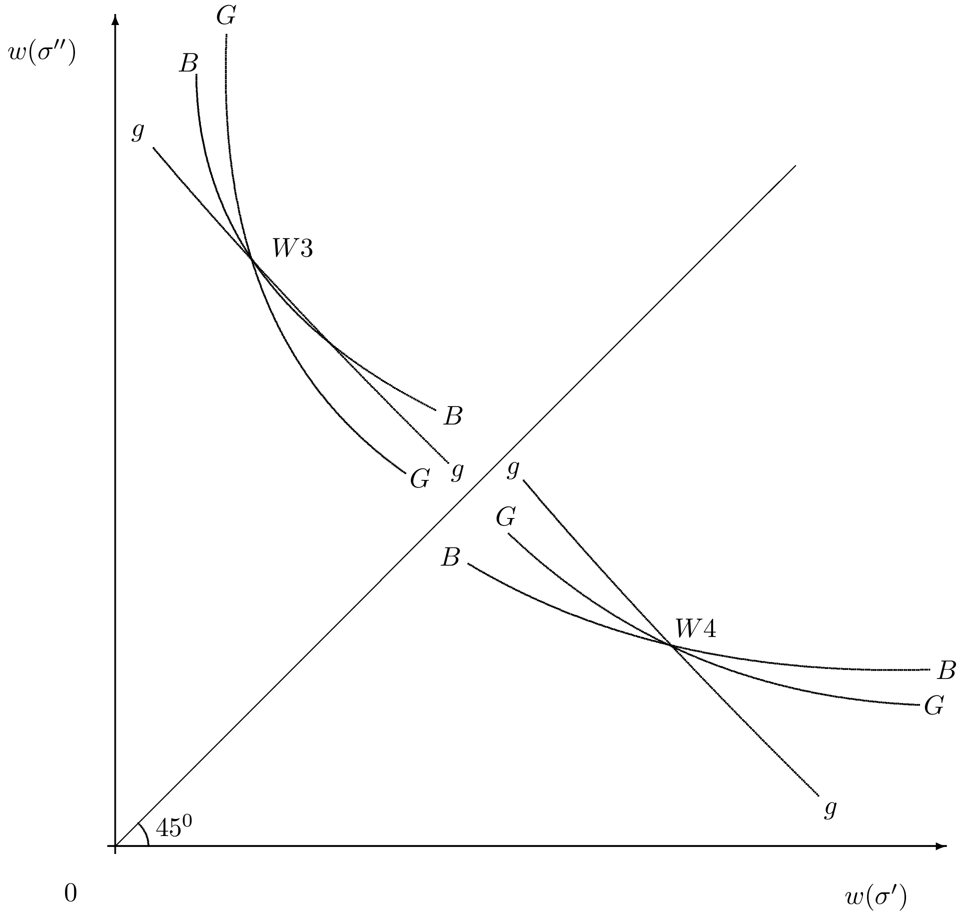

Figure 5

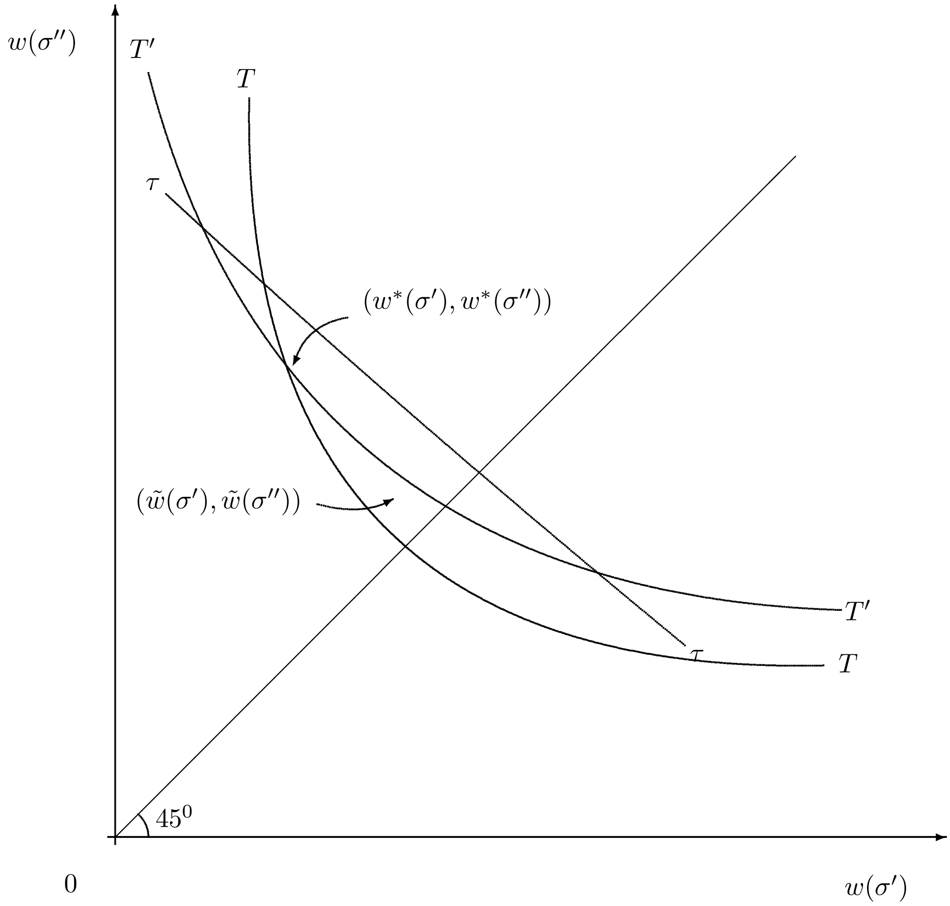
$w(\sigma'')$
T'
T
τ
$(w^*(\sigma'), w^*(\sigma''))$
$(\tilde{w}(\sigma'), \tilde{w}(\sigma''))$
T'
τ
T
45^0
0
$w(\sigma')$

Figure 6

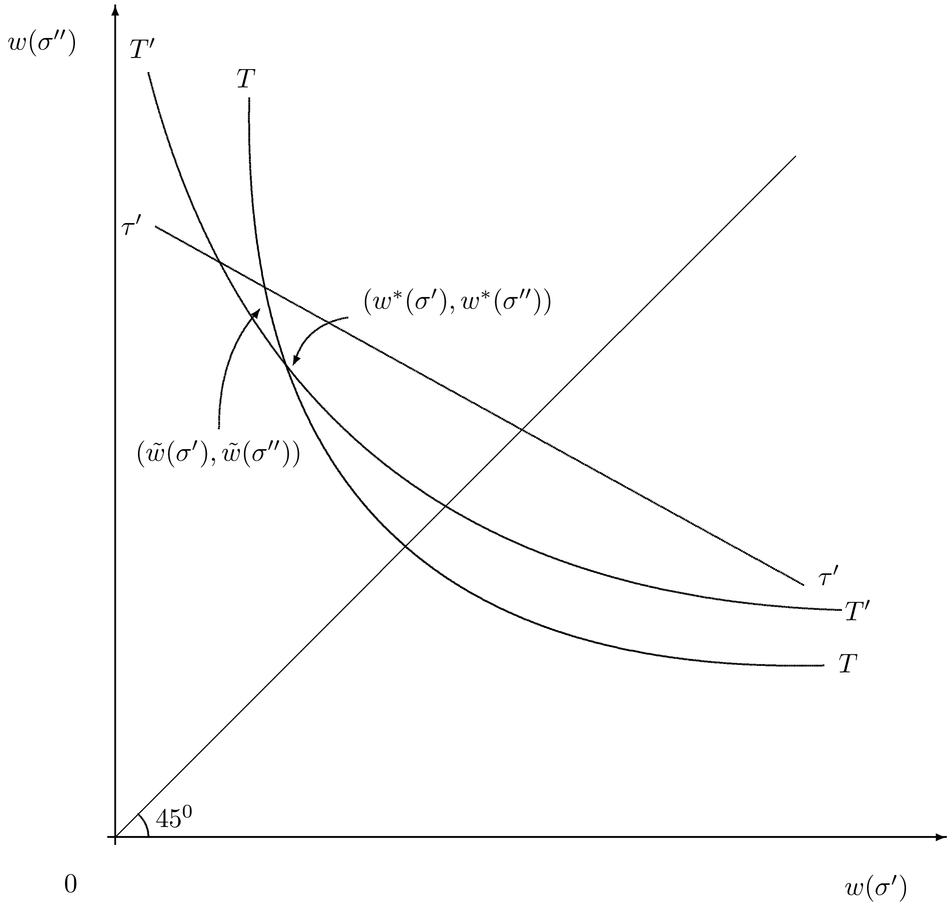
$w(\sigma'')$
T'
T
τ'
$(w^*(\sigma'), w^*(\sigma''))$
$(\tilde{w}(\sigma'), \tilde{w}(\sigma''))$
τ'
T'
T
45^0
0
$w(\sigma')$

Figure 7

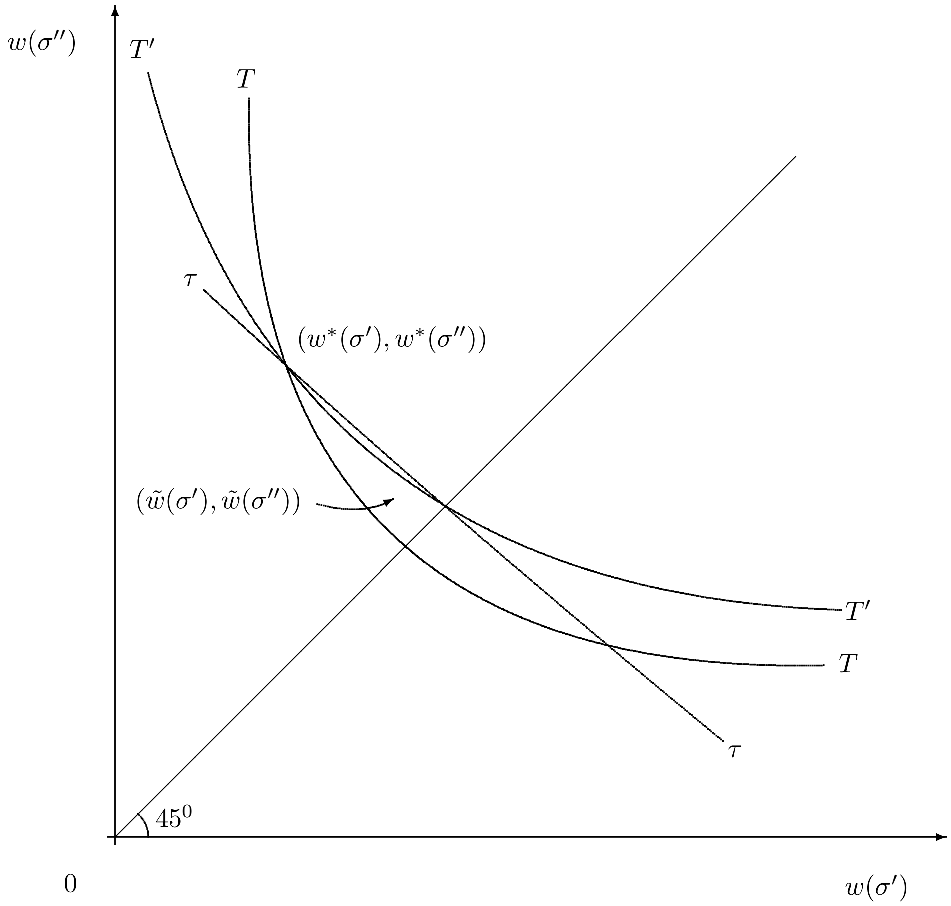
$w(\sigma'')$
T'
T
τ
$(w^*(\sigma'), w^*(\sigma''))$
$(\tilde{w}(\sigma'), \tilde{w}(\sigma''))$
T'
T
τ
45^0
0
$w(\sigma')$

Figure 8

Table 1

OLS coefficients for specification (7)

μ	$\widehat{\beta}_0$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$CI\left(\widehat{\beta}_2\right)$
0.2500	5.3851	0.0365	0.0225	$(0.0130, 0.0320)$
0.3660	5.1568	0.0533	0.0289	$(0.0184, 0.0394)$
0.5000	4.9306	0.0727	0.0336	$(0.0225, 0.0446)$
0.6340	4.7360	0.0920	0.0360	$(0.0249, 0.0472)$
0.7500	4.5919	0.1087	0.0364	$(0.0254, 0.0473)$