Competing for Ownership*

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Abstract

We study how the internal organization of firms – specifically, the allocation of ownership of assets and the distribution of shares among the firm's managers – is determined in a competitive market. We ask how scarcity of assets, skills or liquidity in the market translates into ownership and control allocations within organizations. Firms will be more integrated when the terms of trade are more favorable to the short side of the market, when liquidity is unequally distributed among existing firms and when there is a positive uniform shock to productivity. The model identifies a price-like mechanism whereby local liquidity or productivity shocks propagate and lead to widespread organizational restructuring.

1 Introduction

In the neoclassical theory of the firm, market signals affect choices of products, factor mixes, and production techniques: if the price of output rises, quantity produced increases; if wages rise, fewer workers will be hired and

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relatively greater use will be made of machines. Firms' decisions in turn feed back to the market: increases in the number of goods produced will lower the product price, reductions in the number of workers hired will induce wages to fall. Thus the market provides a mechanism whereby shocks to a few firms – say, an improvement in their technology – propagate to the rest of the economy, inducing other firms to readjust their production plans. Because these feedback effects are so well established, the neoclassical firm remains the backbone of much of applied economic analysis.

The modern theory of the firm emphasizes contractual incompleteness, agency problems, and the resulting importance of organizational design elements such as tasks allocation, asset ownership, and the assignment of authority and control. By introducing a rich set of new variables into economic analysis, it has made breakthroughs in our understanding of economic institutions as different as modern corporation and the sharecropped farm. Yet despite its original and still primary goal – clear since Coase (1937) asked his fundamental question on the boundaries of the firm – of understanding firms that operate in market economies, rather little has been done to investigate the influence of the price mechanism on internal organizational decisions, much less on how those decisions feed back to the market and to other firms.

The purpose of this paper is to provide a simple framework for the analysis of this kind of interaction. We focus on the allocation of ownership of assets, as in Grossman and Hart (1986), where owning an asset means having (residual) decision rights over it. Our main concern is with how scarcity in the market translates into ownership and control allocations within organizations and with how changes in the fundamentals of some firms – their technology and endowments – can spill over, via a pecuniary externality, to economy-wide reorganizations.

We consider an economy in which pairs of enterprises – each consisting of a managerial decision maker and a collection of assets – must produce together in order to generate marketable output. Managers care not only about income, but also have private "effort" costs that are affected by the way the firm's assets are tailored or fined-tuned to the joint production process. As contracts cannot specify what tailoring or effort decision will be made, they are limited to specifying *who* will make the tailoring decision for each asset (the ownership allocation), and to the package of cash transfers and revenue shares that will accrue to each manager.

Fine tuning assets enhances productivity, but is costly. In particular, the costs are incurred only by the manager who works with it, even if it is the other manager has decided how the asset will be tuned. If a manager retains control of an asset, he bears all the cost, but only shares in the benefit of the tailoring decision, and therefore underprovides tailoring. If he cedes control to his partner, she bears none of the cost but derives a positive benefit, and therefore overprovides. This tradeoff between retaining and ceding control of the asset will be one ingredient of the organizational design problem facing a firm.

Firms compete for partners in the supplier market; the contracts they sign upon matching specify the ownership allocation and sharing rule for their joint enterprise. We assume that the number of firms on each "side" of this market is unequal so that the terms of trade are determined by the willingness to pay of a marginal firm on the long side: that firm will be indifferent between matching and staying out of the market. The terms of trade determined in the equilibrium of this market govern the division of surplus between managers, and this in turn determines the way those managers will organize their firms.

In our model, "liquidity," for instance cash, can be transferred without any incentive distortions. In the special case in which everyone has enough liquidity, one need only focus on the contract structure that delivers the highest joint surplus to the managers: all firms will choose this and accomplish the surplus division with cash alone. This approach to predicting organizational structure has been popular in much of the recent literature on the firm.

However, in the general case in which liquidity is in short supply relative to the value of the transactions in question, firms will have to use the organizational variables – ownership allocations and sharing rules – to accomplish the surplus division commanded by the market. That output shares serve as a means of surplus division is obvious enough, but they cannot do so neutrally because arbitrary divisions will adversely affect incentives to provide high effort or tailor assets appropriately. The ownership allocation has a similar surplus-division role: awarding ownership to one manager gives him higher surplus, since it ensures that more of the ensuing production decisions will go in his preferred direction; the downside is that these decisions may pose significant costs on the other members of the firm.

Our model admits a continuum of ownership allocations as well as the usual continuum of sharing rules, and this feature facilitates studying how these two instruments co-vary in distributing surplus. Starting from the allocation in which managers get equal amounts of surplus, the first organizational variable to be distorted is the sharing rule; in this "Refinance zone," the firms remain non-integrated, and all that adjusts is the share or transfer price. Ownership allocations are distorted only at more uneven surplus shares; in this "Reorganization zone," the manager receiving the preponderance of the surplus also controls most of the assets; the share may not adjust at all in this zone in response to further surplus transfers. A manager for whom the market awards a share of the surplus from production, and whose partner has little cash, will receive a large output share and control most of the firm's assets.

The surplus that each partner obtains from a given contract is therefore a function of the characteristics of the relationship, in particular the production technology and the liquidity available. Higher productivity or more liquidity in the firm not only enlarges the feasible set but also "flattens" the frontier, that is it increases the transferability of surplus. If productivity is high, managers have a high opportunity cost of failing to maximize output; if firms have more liquidity, they can avoid using inefficient contractual instruments. Hence, a firm that receives a positive shock to its productivity or liquidity endowment will be able to accomplish surplus division more efficiently and reduce organizational distortions. We dub this the *internal effect.*[1]

But such a shock may have much wider effects than on the firm that experiences it. The internal effect implies indeed that a manager has effectively a higher "ability to pay" for a partner after a positive shock than before. He may therefore bid up the terms of trade in the supplier market: in order to meet the new price, firms which have not benefited from the shock will have to reorganize. Thus the shock may have an *external effect*: "local" shocks may propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium turns out to be amenable to a Marshallian supply-demand style of analysis, making the role of the external effect especially transparent. The internal effect of a positive shock to liquidity or technology is to decrease integration. The external effect is quite different, however. For instance, a uniform increase in the liquidity level of all agents lowers the degree of integration in all firms (the internal effect dominates the external effect). By contrast a uniform shock to productivity increases the degree of

[1]For a liquidity shock, this partial equilibrium result echoes the observations of Jensen-Meckling (1976) and Aghion-Bolton (1992) on the benefits of having greater wealth for the efficacy of financial contracting.

integration in all firms (the external effect dominates the internal effect). As we show in section 3, the model can capture quite simply the effects of more complex changes in the liquidity endowments or in productivity.

The external effect may also operate following a change in the relative scarcities of firms on the two sides. Suppose, for instance, that the short side of the market represents downstream producers and the long side their upstream suppliers. An increase in the supply of downstream firms will raise the share of surplus accruing to upstream firms:[2] downstream managers will find their shares of output lowered and upstream firms will acquire control over more assets.

Hence, the model offers a mechanism by which changes in traditional economic fundamentals – endowments, technology, or numbers participants – will manifest themselves as widespread organizational restructuring, sometimes in a direction opposite of what the internal effect would suggest. We view this as the main insight of the paper.

2 Model

In the economy we consider, output is generated through the cooperation of two production units, one of each type $\theta = 1, 2$. Each unit consists of a risk-neutral decision maker ("manager") and a collection of assets. Many interpretations are possible: one can think of the two types of units as a supplier and manufacturer, each with an associated manager and collection of physical assets, e.g., a coal mine and electric generating station; or as a chain restaurant manager and the franchising corporation, in which case some of the assets may be reputational; or as a corporation and its talented workforce (in which case the assets might also be thought of as duties or tasks).

Whatever the interpretation, we have in mind competitive outcomes, and so we suppose that there is a large number of both types of production unit: each side of the market is a continuum with Lebesgue measure, and we shall assume that the 2's are relatively scarce: the type 1's are represented by $i \in I = [0, 1]$ while the type 2's are represented by $j \in J = [0, n]$, where $n < 1$.

[2] This is because when the supply of downstream firms increases, the liquidity and ability to pay of the *marginal* upstream firm decreases; hence the equilibrium surpluses of downstream firms decrease.

The i-th type-1 manager will have at her disposal a quantity $l(i) \geq 0$ of cash (or "liquidity") which may be useful in contracting with production units of the opposite type; for the type 2's, the liquidity endowment is $\hat{l}(j)$. The indices i and j have been chosen in order of increasing liquidity; it is convenient to further assume that the corresponding liquidity distributions have strictly positive densities:

Assumption 1 The liquidity endowment functions $l(i)$ and $\hat{l}(j)$ are strictly increasing and continuous.

When discussing a generic plant or manager, we shall usually drop the indices.

2.1 The Basic Organizational Design Problem

Technology. The collection of assets in the type-θ production unit is represented by a continuum indexed by $k \in [\theta - 1, \theta)$. The expected output for the joint enterprise will be proportional to the integral of individual asset output over all assets in the two units. Each asset can be tailored to the specifics of the production relationship; the decision as to the value of $q(k) \in [0, 1]$ is taken by whichever manager owns the asset (i.e., whoever as been issued control by the contract).

At the same time assets are tailored, the managers apply effort that is complementary to the asset investment: the effort on asset k is a binary variable $e(k) \in \{0, 1\}$ and the productive contribution to total expected output of asset k is proportional to $e(k)q(k)$. The effort cost to the manager on asset k is $C\left(q(k), e(k)\right) = \frac{1}{2}q\left(k\right)^2 + ce(k)$,where $c \geq 0$.[3] This cost is always borne by the manager in whose unit the asset resides, even if the decision is taken by the other manager.

For instance, if e represents managerial attention devoted to overseeing assembly, supervising workers, and so on, q could index a choice of possible parts or input materials (the exact characteristics of which are difficult to specify, perhaps because they are unknown, at the time of contracting), ordered by the value they contribute to the final product. Each input choice requires solving a number of manufacturing process problems. We suppose that higher value inputs require greater learning and adaptation effort on the

[3] We could just as well suppose that $e(k)$ is continuous, with values taken in $[0, 1]$. With this specification; interior values of e will never be chosen in equilibrium: see Appendix.

part of the manufacturer's management, and the cost of adaptation is separable form the cost of supervision). If the input producer has control over the process, she is unlikely to fully internalize the adaptation cost, while if the manufacturer chooses the input, he may undervalue its contribution to the value of output. Thus, as in most of the literature on ownership since Grossman and Hart (1986), control allocations matter because of the payoff externalities imposed on other parties by the agent who has control.[4]

Since manager 1 bears cost on $k \in [0,1]$ and manager 2 bears cost on $k \in [1,2]$, we write

$$\begin{aligned} C_1(q,e) &= \int_0^1 C(q(k), e(k))\, dk \\ C_2(q,e) &= \int_1^2 C(q(k), e(k))\, dk. \end{aligned}$$

The managerial decisions contribute to the joint enterprise's performance as follows. The enterprise either succeeds, generating revenue $R > 1$, with probability $p(q,e)$; or it fails, generating 0, with probability $1 - p(q,e)$. Here $q : [0,2] \to [0,1]$ are the tailoring decisions and $e : [0,2] \to \{0,1\}$ are the effort decisions. The success probability functional is

$$p(q,e) = \frac{A}{R} \int_0^2 e(k) q(k) dk,$$

where A is a technological parameter. For the trade-off between monetary gains and private costs to be operative, that is to allow a variety of Pareto optimal contractual forms, A must not be too large (otherwise the revenue motive will make incentive provision trivial) nor too small with respect to c (otherwise it is impossible to provide effort incentives).

[4] In the 1960's, W. Corporation owned an electronic systems division that manufactured airplane cockpit voice recorders, and a composite materials division that made various compounds suitable for heat-resistant recording tape, a critical input for recorders. The electronic systems division had perfected a manufacturing process that used mylar tape, but W. ordered them to use a new metal-oxide tape developed by its materials division. The new tape was less flexible than mylar, and therefore subject to breakage, which raised serious manufacturing problems that required nearly a year of manufacturing redevelopment to resolve. A former manager of the systems division admits that had it been up to him, his division would have stuck with the mylar tape, simply because the experimentation and retooling costs were not (and because of verifiability and incentive problems could not be) reimbursed, even if the metal oxide tape arguably had slightly better heat-resistance and recording properties.

Assumption 2 $A \leq \frac{1}{2}$, $c \leq \frac{A^2}{8}$.

Contracts. We make the following contractibility assumptions:

Assumption 3 (1) The decisions (q, e) are not contractible.

(2) The ability to choose effort e is not alienable.

(3) The *right to decide* $q(k)$ on asset k is both alienable and contractible.[5]

The degree to which the q decisions have been allocated via contract to one or the other manager will be the distinguishing feature of organizational form. Represent the control allocation by a fraction ω of assets re-assigned to one of the managers. The type-1 manager controls all of the assets in $[0, 1-\omega)$, where $-1 < \omega \leq 1$ and the type-2 controls $[1-\omega, 2]$.

When $\omega = 0$, each manager retains control of his original assets, and, following the literature, we refer to this situation as non-integration. As ω increases beyond 0, we have increasing degree of integration with control by 2, until with $\omega = 1$ we have *full 2-control.* (The symmetric cases with $\omega < 0$ correspond to 1-control, but will not occur in any competitive equilibrium of our model, given the greater scarcity of 2's.)[6] Note that when 2 has control on task $k \in [0, 1]$, 2 chooses $q(k)$ while 1 chooses $e(k)$; hence the control allocation modifies the strategy sets of both managers.

The managers sign a contract specifying the allocation of control and a sharing rule. Contracting is subject to the following two basic constraints.

Assumption 4 (Limited Liability) Incomes in all states must weakly exceed zero.

(Budget balance) Liquidity and output must be shared between the two managers.

[5] See Aghion et al. (2004) for similar assumptions. Given the timing of decisions in our model, we could just as well assume $e(k)$ is private information and that $q(k)$ is not observable to third parties. Even if it is reasonable to assme that $q(k)$ is revealed to the other manager after it is chosen, it is then too late to renegotiate or use a message game to enforce a decision: at that point, the two parties' preferences are independent of what has gone on before.

[6] This leaves out another logical possibility, namely that the managers "swap" assets; in additon to ω, which indicates how many of 1's assets are shifted to 2, the contract would have an additional variable ψ indicating how many of 2's assets are shifted to 1. However, as we show in the appendix, under Assumption 2, asset swapping will never be Pareto efficient.

Assumption 4 is made on plausibility as well as tractability grounds. Limited liability is a fundamental assumption common to many models of financial contracting. The budget balance assumption rules out contracts in which liquidity is pledged to a third party, to be forfeited in case of failure. Though such contracts would in principle strengthen the managers' incentives, they are somewhat implausible (because of third party incentives to sabotage the operation either on its own or via collusion with one of the managers). Moreover, ruling them out considerably simplifies the analysis of competitive equilibrium: the single-market supply-demand analysis we develop here would instead be rendered into a full-blown assortative matching problem with nontransferabilities (e.g., Legros and Newman, 2003), which nonetheless would yield similar conclusions.

These assumptions imply a simple characterization of the set of contracts. First, there is no use of outside finance. In principle, managers might decide to borrow from the financial market in order to increase their liquidity at the time of contracting and therefore increase the lump sum transfer to the other manager. However, Assumption 4 implies that such outside financing is strictly Pareto dominated by contracts that make no use of outside financing: if the upstream manager borrows, his incentives are weakened since he owes money to the lender, while for the downstream manager, having received the borrowed sum up front, there is no effect on his incentives; it would be better for the upstream manager simply to reduce his share of the output, which has the same impact in the first instance on his incentives, but now strengthens those of his partner, thereby making both of them better off. (All proofs not appearing in the text are in the Appendix.)

Proposition 1 *Payoffs on the Pareto frontier are attained without the use of outside finance.*

Second, there is a *separation property*: in order to describe the set of utility levels that two managers can attain, it is enough to first calculate the set payoffs that they could achieve if they had no liquidity and then to add to this a lump-sum division of their total liquidity. In other words, after signing, there will be a transfer of cash from one manager to the other; then decisions are taken, output is realized, and shares distributed. The separation property says that the liquidity transfer has no effect on what is feasible with respect to the other aspects of the contract. Since there are only two output levels, 0 and R, the sharing rule can be fully described by a single parameter s, $0 \leq s \leq 1$.

Proposition 2 *(Separation) Consider two managers 1 and 2 with total liquidity L. There is no loss of generality in restricting attention to contracts in which L is distributed in a lump sum fashion and in which manager 1 receives a fraction* $(1-s)$ *of the realized output and manager 2 receives* s *of the realized output.*

Solution of the game induced by (s, ω)**.** From Proposition 2, it is enough to consider contracts (s, ω) where s is the share of R accruing to manager 2. Such a contract induces a game between the managers in tailoring and effort decisions, and to characterize the payoff possibilities, we seek the most efficient equilibrium for each choice of (s, ω).[7] For $\omega \geq 0$, the type-1 chooses $e_1(\cdot)$ and $q_1(\cdot)$ to maximize

$$(1-s)A[\int_0^{1-\omega} e_1(k)q_1(k)dk + \int_{1-\omega}^1 e_1(k)q_2(k)dk + \int_1^2 e_2(k)q_2(k)dk] \\ -[\frac{1}{2}\int_0^{1-\omega} q_1(k)^2 dk + \frac{1}{2}\int_{1-\omega}^1 q_2(k)^2 dk + c\int_0^1 e_1(k)dk],$$

taking $e_2(\cdot)$, $q_2(\cdot)$ as given. Manager 2 takes $e_1(\cdot), q_1(\cdot)$ as given and maximizes

$$sA[\int_0^{1-\omega} e_1(k)q_1(k)dk + \int_{1-\omega}^1 e_1(k)q_2(k)dk + \int_1^2 e_2(k)q_2(k)dk] \\ -[\frac{1}{2}\int_1^2 q_2(k)^2 dk + c\int_1^2 e_2(k)dk],$$

Consider first situations where the manager both controls the asset and chooses the effort level to expend with it. For $k \geq 1$, manager 2 chooses $e_2(k) = 1$ only if

$$sAq_2(k) - \frac{1}{2}q_2(k)^2 - c \geq 0$$

[7]We are making the assumption that all assets in the joint enterprise are "up and running," i.e., $e(k) = 1$ for all k. Although conceivably there are circumstances in which it would be desirable to shut down some of the assets in order to avoid imposing excessive costs on one of the managers, mild parameter restrictions will ensure such an arrangement is not Pareto optimal; details are in the appendix.

and will choose $q_2(k) = sA$ in this case. A necessary and sufficient condition for $e_2(k) = 1$ is then that $\frac{1}{2}s^2A^2 - c \geq 0$, or $s \geq \sqrt{2c}/A$. Similarly, for $k < 1 - \omega$, manager 1 chooses $e_1(k) = 1$ and $q_1(k) = (1-s)A$ if and only if $1 - s \geq \sqrt{2c}/A$. The effect of these constraints is to bound feasible values of s away from 0 and 1. Define

$$\bar{s} \equiv 1 - \sqrt{2c}/A, \tag{1}$$

and note that by Assumption 2 $\bar{s} \geq \frac{1}{2}$ and therefore that the set $[1 - \bar{s}, \bar{s}]$ is not empty.

Next, for the assets $k \in [1 - \omega, 1]$ that 2 controls but 1 works with, manager 1 exerts effort $e_1(k) = 1$ only if $(1-s)Aq_2(k) \geq c$. Since 2 gets the benefit from a higher level of q on such assets, it is optimal for 2 to set $q_2(k) = 1$, its highest possible value, for all $k \in [1 - \omega, 1]$., The incentive compatibility condition for 1 effort to be $e_1(k)$ reduces to $(1-s)A \geq c$, which is satisfied when the previous incentive compatibility conditions hold.[8]

Proposition 3 *Pareto efficient contracts satisfy* $s \in [1 - \bar{s}, \bar{s}]$.

As we said earlier, part of the role of an organizational variable is to transfer surplus. We have already discussed the limits of this with respect to s. We now show that in the present model, ω broadly plays the surplus-transfer role as well. Indeed, from the individual point of view, more control is better:

Proposition 4 *Given a sharing rule s, a decision maker's payoff is nondecreasing in the degree of control.*

The argument is by revealed preference. The person with control over an asset can always replicate the decisions that would be made on that asset if he didn't control it; (unlike in hold-up models, there is no strategic decision by his partner *before* the asset decision is made that might make having control worse for him). The result then follows from the additivity of the payoffs over the assets.

Given a feasible contract (s, ω) 1's payoff taking into account the play of the induced game can be written

[8] By Assumption 2, $c < 1$; hence $\sqrt{2c} > c$ and the incentive compatibility condition $(1-s)A \geq \sqrt{2c}$ implies indeed $(1-s)A > c$ for any $s \in [1 - \bar{s}, \bar{s}]$.

$$u_1(s,\omega) = \frac{1-s^2}{2}A^2 - c - \omega\frac{((1-s)A-1)^2}{2} \tag{2}$$

while 2 gets

$$u_2(s,\omega) = \frac{s(2-s)}{2}A^2 - c + \omega s A(1-(1-s)A), \tag{3}$$

The next step is to build up the payoff possibility frontier for any pair of type-1 and type-2 managers. Each point on the frontier will be generated by a different organizational arrangement, i.e., choice of sharing rule s and control structure ω. The separation result of Proposition 2 facilitates the derivation, since the complete frontier can be constructed by first considering the payoffs from all feasible organizations and then adding the liquidity in at the end.

2.1.1 Surplus maximizing contracts

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which $q(k) = A$ and $e(k) = 1$ for all k) cannot be attained. However, we can still ask what are the second-best contracts, given the constraints in contractibility. The total surplus generated by a contract (s,ω) is $W(s,\omega) = u_1(s,\omega) + u_2(s,\omega)$.

The optimal ownership structure trades off underprovision against overprovision of asset tuning. As we have said, having the decision right over an asset one works with entails underprovision of tailoring, since $q(k) < A$, while ceding control on that asset entails overprovision by the new owner since $q(k) = 1 > A$. When the productivity parameter A is large enough, output matters more than costs to the managers, and total surplus is maximized by giving (nearly) full control to one manager.[9] This is simply a variant of the oft-noted point that ceding control may be a useful commitment device.

However, this case is of less interest here, both because it overstates the benefits of integration (taken to its logical extreme, there ought to be only one giant firm in the economy), and because it implies there is no tradeoff between surplus generation and surplus division. As the type 2's are

[9] We say "nearly" full control, because putting $\omega = 1$, which in turn implies $s = \bar{s}$, typically violates manager 1's individual rationality.

relatively scarce, the market will tend to assign the preponderance of surplus to them, and this would be accomplished by giving them control without much loss of efficiency. Moderate changes in market conditions would have no effect on the internal organization of the firm, and large changes, say by reversing the relative scarcity of 1's and 2's, would change the identity of those in control but not the form of organization.

Things are quite different, however, under Assumption 2. In this case, the costs of asset adjustment figure prominently enough in the managers' calculations to render the surplus production/surplus division trade-off nontrivial. The surplus-maximizing contract is $(1/2, 0)$, i.e., *non-integration with equal shares*. From the symmetry of the problem, this contract allocates equal surplus to the two parties, and we denote it $u_=$. Deviating from this contract, i.e., allocating control, or giving a share greater than $1/2$, to one or the other party will entail a loss of total surplus, although it will of course tend to make one of the parties better off at the expense of the other. Allocating control can be interpreted as (partial) *integration* or *centralization,* increasing the share beyond $1/2$ can be interpreted as *refinancing*.

Absent sufficient liquidity, organizational choices will be made to accomplish the surplus division called for by the market. In order to see precisely how this occurs, we shall need to derive the "pre-liquidity" Pareto frontier for the pair of managers, each point of which will correspond to a different organizational arrangement.

2.1.2 The Pre-liquidity Frontier U

We call $u_\theta(s,\omega)$ the type θ's surplus generated by the organization, or *generated surplus* for short, since it represents the surplus the type θ reaps from the organizational variables s and ω net of any ex-ante liquidity transfers.

The pre-liquidity frontier $\phi(u_1)$ is constructed by maximizing 2's surplus over s and ω subject to the guarantee of a surplus of u_1 to 1:

$$\begin{gathered}\phi(u_1) \equiv \max_{s\in[1-\bar{s},\bar{s}],\omega} \frac{s(2-s)}{2}A^2 - c + \omega s A(1-(1-s)A) \qquad (4) \\ \text{s.t. } \frac{1-s^2}{2}A^2 - c - \omega\frac{(1-(1-s)A)^2}{2} \geq u_1\end{gathered}$$

We denote the set of payoffs $(u_1, \phi(u_1))$ by U. We shall describe the solution to this problem here; details are in the Appendix. Starting at the

$45°$-line, where $\phi(u_1) = u_1 = u_=$ with $s = 1/2$ and $\omega = 0$, decreasing u_1 and therefore increasing 2's surplus can be accomplished through two instruments, namely *refinancing* (increasing s) and *reorganization* (increasing ω). For small deviations from equal payoffs, the best way to transfer surplus is via refinance alone: though this will distort incentives on the q's, this is preferable to shifts of control, which result in large changes in the q's on the assets that change ownership; eventually, though, one begins to use reorganization: there is a share level $s^* > 1/2$ above which it is optimal to raise ω as well as s when u_1 falls, *provided s doesn't already equal $\bar{s}$.*[10] Above $\bar{s}$, only reorganization is available as an instrument, so that if $\bar{s} < s^*$, one uses either refinancing or reorganization, never both together. We will focus on this parametric case .

Assumption 5 $s^* > \bar{s}$.[11]

Proposition 5 *Under Assumption 5, the solution to problem (4) is characterized by two intervals $[0, \underline{u}]$, $(\underline{u}, u_=]$, such that*

(i) $s = \bar{s}$ and ω is linear and strictly decreasing in u_1 on $[0, \underline{u}]$;

(ii) $\omega = 0$ and s is strictly decreasing and strictly concave in u_1 on $[\underline{u}, u_=]$;

(iii) ϕ is linear on $[0, \underline{u}]$, strictly concave on $[0, u_=]$;

(iv) The total surplus W is increasing concave in u_1 on $[0, u_=]$.

It is straightforward to check that $\underline{u} = A\sqrt{2c} - 2c$, $u_= = \frac{3}{8}A^2 - c$.

Thus in the upper half quadrant, the frontier can be divided into two "zones": the "refinance" zone, in which movements along the frontier are accomplished via changes in s alone, and the "reorganization" zone where it is ω that varies. Notice that as 1's payoff decreases, 2's degree of control (weakly) increases. At the same time total surplus is decreasing; thus it is fair to say that here *reallocations of control are used to transfer surplus, not to generate it.*

[10] The cutoff value s^* is the unique level of s for which both first order conditions of a relaxed version of (4) – in which the constraint $s \in [1 - \bar{s}, \bar{s}]$ is ignored – are satisfied as equalities at $\omega = 0$.

[11] The restriction is that c is not too small in terms of A, specifically that $c > \frac{1}{2}\left[\frac{2}{3}A + \frac{1}{6} - \frac{1}{6}\sqrt{(1 + 8A - 8A^2)}\right]^2$.

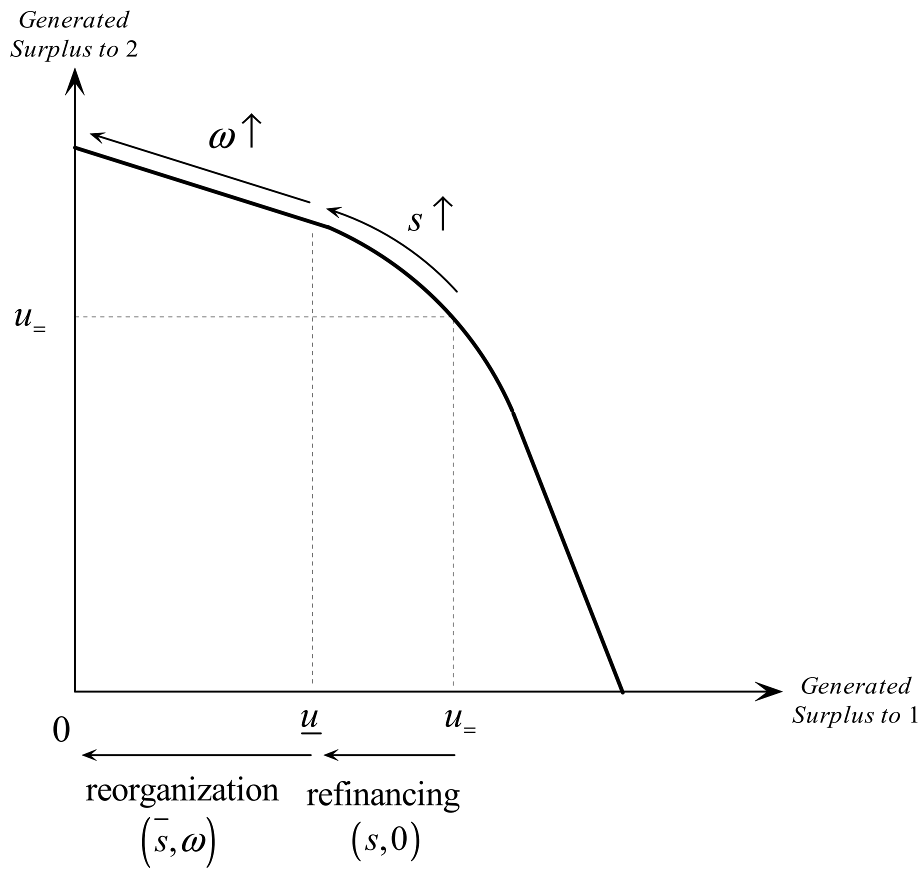

Figure 1: Two regimes of reorganizations

Assumption 5 has the analytic advantage that it keeps the frontier function ϕ concave and the degree of integration ω convex.[12] While we shall focus on this case, most of our results do not depend on this simplification, and we discuss the alternate case in the Appendix. A further strengthening of Assumption 5 is to set parameters so that $\bar{s} = 1/2$ (this is equivalent to $c = A^2/8$); then $\underline{u} = u_=$, and the frontier in each half quadrant is linear.

2.2 Completing the Picture: Adding Liquidity

The endowments of a matched pair of managers with liquidities $(l, \hat{l})$ expands their joint surplus possibilities relative to those generated by the set of contracts (s, ω) by allowing for one-for-one utility transfers between these managers (as we have observed, organizational and financial changes do not allow for such efficient transfers except perhaps locally). But by Proposition 2 the liquidity levels do *not* affect the surpluses generated from the contracts (s, ω) themselves. Thus, modifying the frontier $\phi(\cdot)$ constructed so far to take account of the liquidity endowments of the managers is quite simple: one need only add it to a line segment whose endpoints are $(-l, l)$ and $(\hat{l}, -\hat{l})$, since the line segment describes all possible liquidity transfers between the managers.

Notice that in the special case that agents with sufficient levels of liquidity (in particular, $l \geq u_=$ for the type-1) achieve full transferability: the Pareto frontier is linear with unit slope magnitude. In this case, no matter what the division of surplus might be, the plants always remain separate firms with equal output shares accruing to each manager; the desired surplus division is accomplished ex-ante with a liquidity transfer. The study of organizational arrangements in the special case of full transferability reduces to the calculation of what maximizes total surplus.

But in the general case, where liquidity is scarce, partnerships with different levels of liquidity will choose different organizational forms and achieve different levels of total surplus, given a fixed level going to the type 2's. This is the "internal" liquidity effect: in general, more is better in the sense that the firm can generate higher surplus. With liquidity in short supply, there is

[12] When $s^* < \bar{s}$, we show in the Appendix that the solution to (4) is characterized by three intervals $[0, \underline{u}]$, $(\underline{u}, u^*]$, $(u^*, u_=]$ such that (1) $s = \bar{s}$ and ω is strictly decreasing in u_1 in $[0, \underline{u}]$; (2) s and ω are strictly decreasing in u_1 in $(\underline{u}, u^*]$; and (3) $\omega = 0$ and s is strictly decreasing on $(u^*, u_=]$. While ϕ is still concave locally in the two extreme intervals, it is convex when $u_1 \in (\underline{u}, u^*]$.

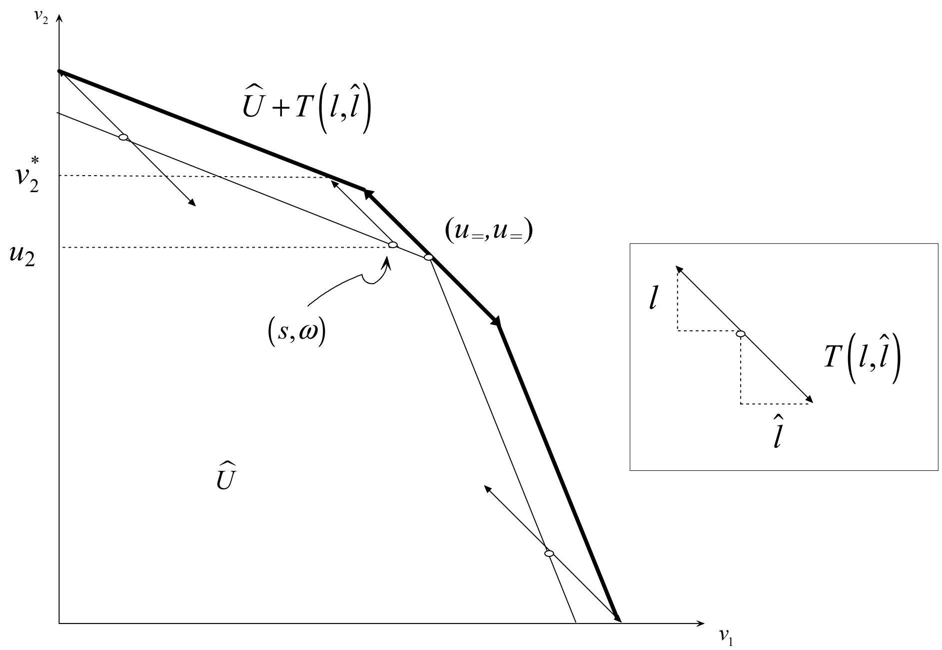

Figure 2: Feasible Set

still a trade-off between surplus division and surplus production.

Let U be the set of payoffs (u_1, u_2) on the pre-liquidity frontier and $\hat{U} = U - \Re_+$ its comprehensive extension. For liquidity levels $\left(l, \hat{l}\right)$, the set of lump-sum transfers is

$$T\left(l, \hat{l}\right) = \left\{(t_1, t_2) : t_i \geq 0,\ t_1 + t_2 = l + \hat{l}\right\}$$

For a partnership $(i, j) \in I \times J$; feasible payoffs are in the set $\hat{U} + T\left(l\left(i\right), \hat{l}\left(j\right)\right)$; we denote by v the surpluses in this set. Figure 1 is an illustration of this construction for the special case in which $\bar{s} = \frac{1}{2}$, so that U is piecewise linear. Note that in the upper half quadrant, the set of feasible surpluses is equivalently described by $\hat{U} + T\left(l\left(i\right), 0\right)$, that is the liquidity of type 2 does not matter.

3 Market Equilibrium

Since the problem of market equilibrium involves "matching" the type 1's and type 2's into partnerships of two (with some of the 1's necessarily left unmatched), we have an assignment game with nontransferable utility. In thinking about equilibrium in this matching market it is convenient to use the core as a solution concept. This concept states that firms form, share surplus on the Pareto frontier, and are stable in the sense that no new firm could form and strictly improve the payoffs to its members.

A firm consists of one type 1 plant $i \in I$ and one type 2 plant $j \in J$. Since there is excess supply of type 1 plants, there is at least a measure $1-n$ of type 1 managers who do not find a match and who therefore obtain a surplus of zero. Stability imposes that these unmatched type 1 managers cannot bid up the surplus of type 2 managers while getting for themselves a positive surplus. Necessary conditions for this are that all type 2 managers are matched and that they have a generated surplus not smaller than $u_{=}$. As we have shown, for such generated surpluses, the 2's liquidity does not matter. Thus all 2's are equally good as far as a 1 is concerned and they must therefore receive the same surplus.[13]

This "equal treatment" property for the 2's is an important simplification that does not normally appear in assignment models of this sort.[14] It enables us to identify the set of firms F with the index of the type 1 manager in the firm, and we refer to "firm i" to indicate that the firm consists of the i-th type 1 plant and a type 2 manager.[15]

Definition 6 *An equilibrium consists of a set $F \subset I$ of firms where $\lambda(F) = n$, a surplus v_2^* to the type 2 managers and a surplus function $v_1^*(i)$ for type 1 managers such that:*

(ii) (feasibility) For all $i \in F$, $(v_1^*(i), v_2^*) \in \hat{U} + T(l(i), 0)$. For all $i \notin F$, $v_1^*(i) = 0$.

[13] If in firm (i,j) type 2 j has a strictly larger surplus than type 2 j' in the firm (i',j'), the firm (i,j') could form and both i and j' could be better off since the Pareto frontier is strictly decreasing.

[14] As we pointed out above, this is where the budget balance assumption comes in. Without it, the 2's liquidity *would* figure in generating the utility possibilities, and it would not generally be possible to treat all 2's the same.

[15] For each firm $i \in F$ corresponds a type 2 manager $j(i)$; this matching function must be measure consistent; see Legros and Newman (2003).

(ii) (stability) For all $i \in I$, for all $j \in J$, for all $(v_1, v_2) \in \hat{U} + T(l(i), 0)$, either $v_1 \leq v_1^*(i)$ or $v_2 \leq v_2^*$.

The common equilibrium surplus for the type-2 managers, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2's. We construct the demand as follows. The amount of surplus a 1 is willing *and able* to transfer to a 2 depends on how much liquidity he has. The most he would offer of course is the entire maximum surplus $2u_=$, which he could do provided his liquidity exceeds $u_=$. A 1 with zero liquidity can offer $\phi(0)$. In general, agent i with $l(i) < u_=$ can offer $\phi(l(i)) + l(i)$, since this gives her zero surplus.[16] Since the frontier has slope magnitude less than unity above the 45^o-line, this effective *willingness to pay* is nondecreasing in i; since l is increasing in i, we have a (weakly) downward sloping "demand" schedule given by

$$\text{willingness to pay: } \min\{2u_=, \phi(l(1-x)) + l(1-x)\},$$

where x is the quantity of 2's. The supply is vertical at n, the measure of 2's. Equilibrium is at the intersection of the two curves: this indicates that n of the 1's are matched, as claimed above, and that the marginal 1 is receiving zero surplus.

Proposition 7 *The equilibrium set of firms is $F = [1-n, 1]$ and the equilibrium surplus of type 2 managers is*

$$v_2^* = \min\{2u_=, \phi(l(\bar{\imath})) + l(\bar{\imath})\},$$

where $\bar{\imath} = 1 - n$ is the marginal type 1 manager .

If $l(\bar{\imath}) \geq u_=$, efficiency is obtained since each matched type 1 is able to pay $u_=$ to the type 2 manager; note that in this case the equilibrium surplus of all type 1 managers is zero. We will consider below situations in which $l(\bar{\imath}) < u_=$.

The type 1 manager with liquidity $l(\bar{\imath})$ has a surplus of 0; his *generated surplus* however is $u_1(l(\bar{\imath}), v_2^*) = l(\bar{\imath})$. An inframarginal type 1 manager

[16] The pair choose an organizational form that generates $(l, \phi(l))$, and since type-1 transfers l to the 2 we obtain a surplus of 0 for 1 and $\phi(l) + l$ for 2.

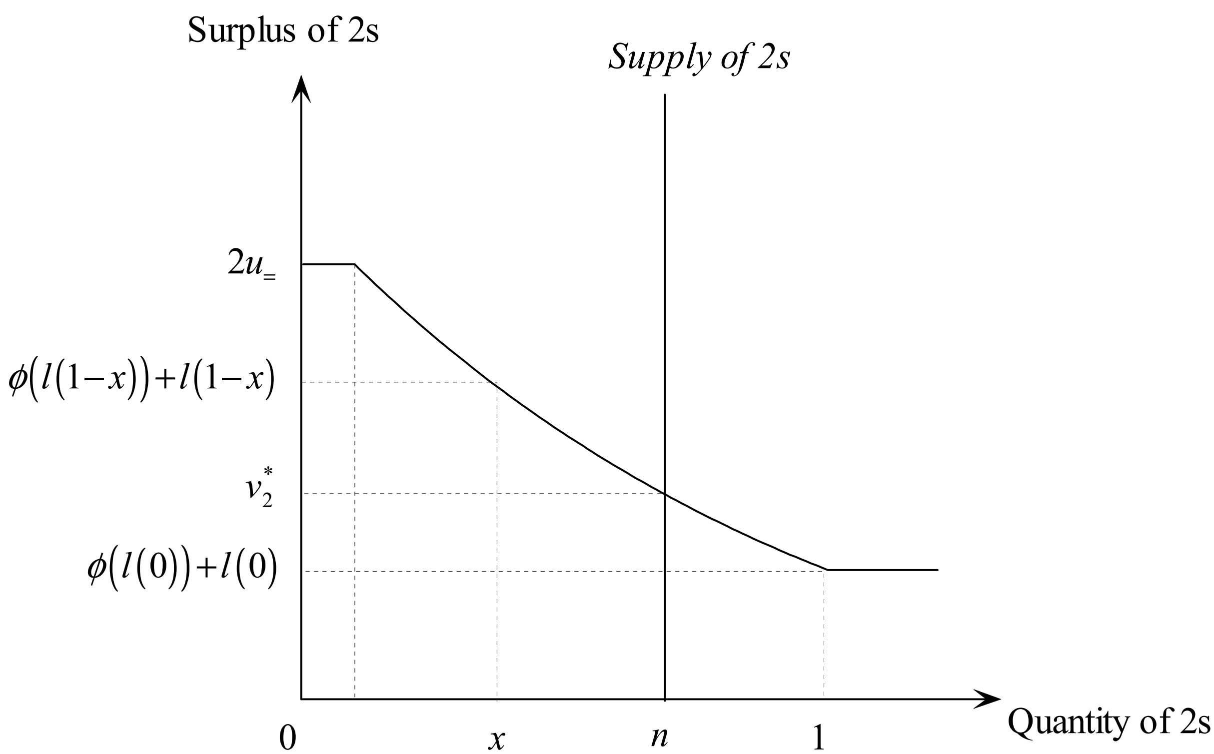
Surplus of 2s
Supply of 2s
$2u_=$
$\phi(l(1-x))+l(1-x)$
v_2^*
$\phi(l(0))+l(0)$
0
x
n
1
Quantity of 2s

Figure 3:

with liquidity $l > l(\bar{\imath})$ will be able to generate a higher surplus for himself since he can transfer more liquidity than the marginal type 1. If $l \geq v_2^* - u_=$, the inframarginal type 1 has a generated surplus of $2u_= - v_2^*$ and the contract is the efficient contract $s = \frac{1}{2}, \omega = 0$. If $l < v_2^* - u_=$, type 1 has a *generated surplus* of $u_1(l, v_2^*)$ such that

$$\text{Generated Surplus of Type 1}: \ \phi(u_1(l, v_2^*)) + l = v_2^*. \tag{5}$$

Properties of the generated surplus are easily derived from Proposition 5(iv).

Lemma 8 *The generated surplus $u_1(l, v_2^*)$ of an inframarginal type 1 is decreasing concave in v_2^* and increasing concave in l.*

Proposition 5(i)-(ii) and Lemma 8 show that there is a simple relationship between the generated surplus and the contractual terms, in particular the degree of control. Indeed, from small changes in the generated surplus accruing to the 1 will *either* result in changes in s or in ω, but not both simultaneously. Since the generated surplus $u_1(l, v_2^*)$ is increasing in l, there will be a critical liquidity level $L(v_2^*)$, increasing in v_2^*, that separates firms in the "refinance zone" from those in the "reorganization zone": above $L(v_2^*)$, firms are nonintegrated and only output shares vary with l, while below $L(v_2^*)$, output shares are fixed at $\bar{s}$ and variation in l leads to variation in ownership structure. Since $L(v_2^*)$ is increasing in v_2^*, the higher is the surplus accruing to the 2's, the fewer firms will be nonintegrated.

Lemma 9 *(i) If $l(\bar{\imath}) \geq \underline{u}$ all firms choose nonintegration.*

(ii)If $l(\bar{\imath}) < \underline{u}$, define $L(v_2^*)$ by $\phi(\underline{u}) + L(v_2^*) = v_2^*$. Firms with type 1 liquidity of $l \in [l(\bar{\imath}), L(v_2^*)]$ choose integration contracts $(\bar{s}, \omega)$, ω decreasing in l. Firms with type 1 liquidity of $l > L(v_2^*)$ choose nonintegration contracts $(s, 0)$, s decreasing in l.

Thus the model captures two aspects of the influence of market conditions on internal organization: not only do firms respond to v_2^* in making organizational decisions, but the way they respond to internal shocks (e.g., small liquidity windfalls) will also depend on v_2^*. When 2's command high payoffs, small increases in a 1's liquidity will be likely to result in a reorganization, specifically a (partial) reacquisition of control by the 1, whereas when the 2's are less well compensated the same shock will more likely simply lead to a greater output share for the 1. Of course, to study these effects systematically, we must take account of the fact that v_2^* itself is endogenous, which we do in the next section.

4 Comparative Statics of Market Equilibrium

In equilibrium, there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, "richer" firms are more decentralized, accrue smaller shares of output to the type-2, and generate greater surplus for the managers. Similar types of results have been found in the literature on financial contracting (Jensen and Meckling 1976, Aghion and Bolton 1992): more liquidity inside the firm improves the efficiency of contracting in the sense that all agents are better off. We refer to this as an *internal* effect. However it is also possible that more liquidity leads to *less* decentralization: if the liquidity of the marginal firm also increases, the equilibrium surplus to type 2 increases and even if the liquidity of the inframarginal firm increases, it may be insufficient to compensate for the increase in v_2^*, leading to less decentralization as claimed. This effect is new and we refer to it as an *external* effect.

We consider below three types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity and changes in the technological parameter A.

4.1 Relative Scarcity

In order to isolate the "external effect" our first comparative statics exercise involves changes in the tightness of the supplier market, i.e., in the relative scarcities of 1's and 2's.

Suppose that the measure of 2's increases, for instance from entry of downstream firms into the domestic market from overseas. Then just as in the standard textbook analysis, we represent this by a rightward shift of the "supply" schedule: the "price" of 2's decreases. Indeed, as n increases the marginal liquidity of type 1 decreases since $l(\bar{\imath})$ is decreasing with n. What of course is different from the standard textbook analysis is that this change in price entails (widespread) refinancing or corporate re-organization.

Let $F(n)$ be the set of firms when there is a measure n of type 2 firms. As n increases to $\hat{n}$, there is an equilibrium set $F(\hat{n})$ where $F(n) \subset F(\hat{n})$, that is after the increase in supply, new firms are created but we can consider that previously matched managers stay together. Since $l(\bar{\imath}) > l(\bar{\imath})$, from Lemma 8, the generated surplus of *all* type 1 managers in firms in $F(n)$ increases. Managers in a firm in $F(n)$ with unequal output shares or partial integration will either refinance (decrease s) or reorganize (decrease ω) in response to the reduction in the equilibrium value of v_2^*. The analysis is similar in the opposite direction: a decrease in the measure of 2's leads to an increase in v_2^*. Thus, we have

Proposition 10 *In response to an increase in the measure of 2's, the firms remaining in the market become (weakly) less integrated and output shares accruing to the 2's weakly decrease. Total surplus generated by each firm of these firms does not decrease.*

It is worth remarking that if the relative scarcity changes so drastically that the 2's become more numerous, then 1's get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have seen, with the role of 1's and 2's reversed. The point is that the owners of the integrated enterprise gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

As an application, if we interpret the assets as tasks or duties, the model can suggest a simple explanation for the "empowerment of talent" that has been noted by several authors (see Marin and Verdier, 2003; and references therein). Empowerment then means giving the highly skilled and professional workers decision rights over more of these tasks, i.e., more discretion. A large literature in labor economics has shown that in the last thirty years the demand for skilled workers in North America and Western Europe has outstripped the (nonetheless growing) supply. Interpreting the 1's in our model as the corporate demanders of talent and the 2's as the talented workforce, relative rightward shifts in demand mean more surplus to the type 2's, which will manifest itself variously as bigger liquidity payments, greater shares of output (use of bonus schemes or possibly stock options), and greater "empowerment," often in combination. As long as firms' liquidity is restricted (relative to the scale of operations), tighter labor markets mean more control by these workers, not merely higher wages. However, this story is so far heuristic: increases in demand for the talented workforce most likely emanate from entry of new firms (which in turn implies a change in the liquidity distribution among the active firms) and from increases in productivity (e.g., "skill-biased technical change"). We address each of these effects next.

4.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the presence of two countervailing effects. First, as we have noted, there is an *internal effect:* if the liquidity of an individual type 1 increases, he can "afford" a more efficient organization, which typically entails an increase in his share of the output and control of assets. But increasing liquidity also increases the 1's effective willingness to pay, so if a distributional change increases the value of the *marginal* liquidity, it creates an *external effect* via an increase in v_2^* that results in efficiency-reducing shifts in control and shares in potentially *all* the relationships.

In light of Proposition 7, we ignore the distribution of type 2 agents. The dependence of the organizational variables on the type1 liquidity l and the equilibrium surplus v_2^* is summarized in the following simple corollary of Proposition 5 and Lemma 8.

Lemma 11 *Under Assumption 5:*
(i) The share s is nondecreasing in v_2^* and nonincreasing in l
(ii) The degree of integration ω is nondecreasing convex in v_2^*, nonincreasing convex in l
(iii) The total surplus W is decreasing concave in v_2^* and increasing concave in l.

Equipped with this result, we can derive simple comparative statics. We focus on the aggregate degree of integration in the market, but will also summarize the effects on the average sharing rule and the aggregate surplus generated by the economy.

Suppose the initial liquidity endowment is $l(i)$ and that the economy receives a "shock" that transforms $l(i)$ into $\psi(l(i))$; the shock function $\psi(\cdot)$ is assumed continuous and increasing. We wish to compare the degree of integration before and after the shock. Let $\omega(l, v_2^*)$ be the degree of integration in a firm with a type 1 manager having liquidity l when the equilibrium surplus to 2 is v_2^*.

The change in average degree of integration is

$$\int_{\bar{\imath}}^{1} \omega\left(\psi(l(i)), v_2^*(\psi(l\left(\bar{\imath}\right)))\right) di - \int_{\bar{\imath}}^{1} \omega\left(l(i), v_2^*\left(l\left(\bar{\imath}\right)\right)\right) di, \tag{6}$$

where $\psi(l\left(\bar{\imath}\right))$ and $l\left(\bar{\imath}\right)$ are the respective marginal liquidity levels and the notation $v_2^*(\cdot)$ reflects the dependence of the 2's equilibrium surplus on the marginal liquidity.

We now derive simple comparative-static results for some special cases that place more structure on the problem.

4.2.1 Positive Shocks to Liquidity

Suppose that the shocks $\psi(l) - l$ are both positive and nondecreasing in l. Note that a *uniform* shock in which every type 1 receives the same increase to his endowment is a special case. So is a multiplicative shock in which the percentage increase to the endowment is the same for all 1's. The impact of this shock is to increase both the "purchasing power" of the type 1's, which, via the internal effect, reduces the degree of integration, but also to increase the equilibrium surplus to 2, which, via the external effect, has the opposite effect. However, it is a simple matter to demonstrate that in this case, the internal effect dominates: more liquidity implies less integration. Heuristically, the change in v_2^* is $\phi'(l(\bar{\imath})) + 1$ times the change in $\bar{\imath}$'s liquidity; since $-1 < \phi' < 0$, this is smaller than the liquidity increase and thus $\bar{\imath}$ can cover the new price and still buy back some control; all $i > \bar{\imath}$ have at least as large an increase in their endowments and can therefore do the same. Of course, negative, nonincreasing shocks yield the opposite changes in surplus and organization.

Proposition 12 *Under positive, nondecreasing, shocks to the liquidity distribution of type 1:*
(i) the aggregate degree of integration decreases;
(ii) the output shares between 1 and 2 become more equal;
(iii) total welfare rises.

To maintain this conclusion, the proviso that the shocks are monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is higher overall degree of integration. Intuitively, if the positive shock hits only a small neighborhood of the marginal type 1, the price v_2^* will increase and the inframarginal unshocked firms will choose to integrate more in response to the increase in v_2^*. In the Appendix, we provide an example to demonstrate

Proposition 13 *There exist first order stochastic dominant shifts in the distribution of type-1 liquidity that lead to more integration and lower surplus in the aggregate.*

We turn now to consider other types of distributional changes.

4.2.2 Inequality and the Integration-Minimizing Distribution

It is helpful to compare distributions with a common marginal liquidity level, as this restricts attention to the internal effect. Thus in this subsection we compare two endowment functions $l(i)$ and $\psi(l(i))$ that are equal for the marginal type-1, i.e. $l(\bar{\imath}) = \psi\left(l\left(\bar{\imath}\right)\right)$.

Suppose first that l and $\psi \circ l$ have a single crossing property at $l\left(\bar{\imath}\right)$: $\psi(l(i)) < l(i)$ for $i < \bar{\imath}$ and $\psi(l(i)) > l(i)$ for $i > \bar{\imath}$. Since all *matched* 1's have greater liquidity and the equilibrium surplus v_2^* is by construction fixed, the generated surplus to 2 falls in every firm and the economy becomes less integrated. If one supposes further that $\int_0^1 l(i)di = \int_0^1 \psi(l(i))di$, then in fact the new liquidity distribution (which is essentially the inverse of the liquidity endowment function) is riskier then the old one in the sense of second order stochastic dominance (equivalently, it is more unequal in the sense of Lorenz dominance). This is an instance in which *increasing inequality may lower integration and raise efficiency.*

Now maintain the common marginal liquidity assumption, and denote the inverses of the restrictions of $l\left(\cdot\right)$ and $\psi\left(l\left(\cdot\right)\right)$ to $[\bar{\imath}, 1]$ as $\bar{l}^{-1}$ and $\overline{(l \circ \psi)}^{-1}$: these are just the conditional distributions of liquidity above $l\left(\bar{\imath}\right)$. Suppose that $\bar{l}^{-1}$is more unequal than $\overline{(l \circ \psi)}^{-1}$. Then because ω is convex in l (Lemma 11) and v_2^* is the same for both distributions, there is less integration under the new distribution.

This suggests the opposite of the previous conclusion: *increasing inequality may raise integration and lower efficiency.* These two results may appear to contradict each other, but they are easily reconciled: while the single-crossing result refers to the distribution for the economy as a whole, the second result refers to the distribution *only among the existing firms.*

If one is interested in the optimal distribution of liquidity for the economy as a whole, it is clear that one wants the marginal liquidity as low as possible, so as to minimize the equilibrium price. But from the previous result, the distribution among the firms must be as egalitarian as possible. And finally, one wants to maximize the liquidity of the inframarginal firms. Taking these three factors into account, along the with the fact that the liquidity of the 2's has no effect on organization or efficiency, one arrives at the following

Proposition 14 *Under Assumption 5, a distribution of liquidity that minimizes the aggregate level of integration and maximizes the aggregate surplus consists of two atoms:* $1 - n$ *of the type 1's and all of the 2's get zero; the remaining* n *type 1's each get* $1/n$ *times the mean liquidity.*

This likely is a very unequal distribution indeed.[17] From the empirical point of view the important distinction is between overall inequality and inequality among the selected sample of matched firms, which in this model at least can work in opposite directions.

4.2.3 A Global Condition when the Frontier is Linear

When the frontier ϕ is linear in the upper half quadrant, as it will be in the parametric case mentioned above in which $\bar{s} = 1/2$, we are able to obtain global necessary and sufficient conditions for aggregate organizational and surplus comparisons. This case is useful to consider because it separates clearly the internal and external effects of changes in liquidity distributions.

Proposition 15 *Assume that* ϕ *is linear above the* 45^o*-line with slope* $-\alpha$ *(*$\alpha < 1$*). Consider two continuous endowments* l *and* $\psi(l)$*, with marginal liquidity levels* $l(\bar{\imath})$ *and* $\psi(l(\bar{\imath}))$ *and conditional mean liquidity levels* μ *and* $\hat{\mu}$ *on* $[l(\bar{\imath}), l(1)]$ *and* $[\psi(l(\bar{\imath})), \psi(l(1))]$ *respectively. Total welfare improves (and average integration decreases) when the distribution changes from* l *to* $\psi(l)$ *if and only if*

$$\hat{\mu} - \mu \geq (1 - \alpha)\left(\psi(l(\bar{\imath})) - l(\bar{\imath})\right).$$

[17] This distribution doesn't satisfy Assumption 1, of course, but equilibrium is perfectly well defined nonetheless. It is true that there is an indeterminacy in the value of v^* with this distribution; the optimum is achieved at the lowest value, which is $\phi(0)$.

If the marginal agent's liquidity increases ($\psi(l(\bar{\imath})) > l(\bar{\imath})$), for instance, the average level of integration falls only if the mean liquidity increases enough. Otherwise, the other agents' increased ability to transfer surplus, and thereby reduce distortionary reassignments of control, will be offset by the increase in the required level of surplus transfer. In addition to isolating the role of the internal and external effects of the change in distribution, the condition in Proposition 15 emphasizes the role of the degree of inefficiency in transferring surplus via control structure rather than via monetary transfers. Indeed, as α increases (for instance, with increases in A), the inefficiency (as measured by $(1-\alpha)$) decreases, and for a given change in marginal agent's liquidity, the condition on the change in mean liquidity becomes less stringent.

4.3 Productivity Shocks

The "external effect" outlined in the previous section offers a propagation mechanism whereby local shocks that affect only a few firms initially may nevertheless entail widespread re-organizations. This is important empirically because it implies that in order to explain why a particular reorganization happens, there is no need to find a smoking gun in the form of a change *within* that organization: instead it may happen elsewhere in the economy. The same logic applies to other types of shocks and most prominently to innovating productivity shock. These are often thought to be the basis of large scale restructuring such as merger waves (Jovanovic and Rousseau, 2002).

We model a (positive) productivity shock or technological innovation as an increase in A, which is consistent either with increased reliability (probability of success) or of higher output or revenue (increases in R). We suppose the shock inheres in the type 1's and hits only a small fraction of firms.

In the initial economy, all firms have technology A; after a shock, a small subset of them, an interval $[i_0, i_1]$, have access to a better technology $\hat{A} > A$. Raising A modifies the game that managers play given a contract (s, ω) : it is clear from (2) and (3) that both managers obtain a larger surplus from a given contract. Hence the feasible set expands and the willingness to pay of type 1 agents who receive the positive shock also increases. What is less immediate however is that there is also more transferability in the firm. In what follows, we restrict ourselves to considering "small" shocks in the sense that Assumptions 2 and 5 continue to hold for $\hat{A}$, so that we can still use Proposition 5.

Lemma 16 *(i) A positive productivity shock increases* $\underline{u}$*, increases* $u_=$ *on the pre-liquidity frontier, and raises* $\bar{\omega}$*, the maximum individually rational level of* ω*.*

(ii) There is more transferability in the sense that for a contract (s, ω) *the slope of the frontier is steeper in the region* $u_2 \geq u_1$ *when* A *increases.*

Let $\phi(\cdot; A)$ be the function defining the Pareto frontier when the technology is A.The maximum willingness to pay depends now on the technology available inside the firm,

$$\text{willingness to pay: } w(i) = \begin{cases} \min\{2u_=(A), \phi(l(i); A) + l(i)\} & i \notin [i_0, i_1] \\ \min\left\{2u_=\left(\hat{A}\right), \phi\left(l(i); \hat{A}\right) + l(i)\right\} & i \in [i_0, i_1] \end{cases}.$$

Let

$$\begin{aligned} \pi &: [0,1] \to [0,1] \\ \pi(i) &\geq \pi(\hat{\imath}) \Leftrightarrow w(i) \geq w(\hat{\imath}). \end{aligned}$$

be a reordering of the indexes of type 1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type 1 agent is i_π such that (λ is the Lebesgue measure)

$$\text{marginal} : \lambda(\{i : w(i) \geq w(i_\pi)\}) = n,$$

and the set of equilibrium firms is $F = \{i : \pi(i) \geq \pi(i_\pi)\}$.

Lemma 16(ii) implies that – for a *fixed* equilibrium surplus to the 2 – a shocked firm integrates less since it is able to transfer surplus in a more efficient way. Hence when the equilibrium surplus to the 2 is the same, technological shocks lead to less integration in the economy. When the equilibrium surplus of the 2 increases there is a force towards more integration. Unshocked firms certainly integrate more; for shocked firms, we show below that while they benefit internally from the technological shock, the negative effect of an increase in equilibrium to the 2 dominates. The net effect is towards more integration *for all firms in the economy* if the marginal firm is a shocked firm.

Proposition 17 *(i) (Inframarginal shocks) If* $i_\pi = 1 - n$ *and* $w(i_\pi) = v_2^*(A)$, *then* $F = [1-n, 1]$, *the shocked firms become less integrated and the 2's shares fall, while the unshocked firms remain unaffected* *(ii) (Marginal shocks) If* $1-n \in [i_0, i_1]$, *and* $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1+\varepsilon)$, *then* $i_\pi = 1-n$, $F = [1-n, 1]$, *the equilibrium price* $v_2^*\left(\hat{A}\right)$ *increases and all firms, shocked and unshocked, integrate more.*

(iii) If there is a uniform shock to the technology ($i_0 = 0, i_1 = 1$), *then* $i_\pi = 1-n$, $F = [1-n, 1]$ *and each firm integrates more.*

Thus the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in "rich" firms (case (i)), only the innovating firms are affected, and they decentralize. But innovations that occur in "poor" firms (case (ii)) may affect the whole economy: even firms that don't possess the new technology become more centralized. Note that new technologies are often introduced by new, small firms – the very ones that are likely to be liquidity poor. This result suggests that widespread reorganizations (such as a merger wave) are more likely to be set off by the entry of new firms embodying new technologies, while "local" reorganizations involving established firms originate with those firms themselves.

Proposition 17 (iii) emphasizes that in contrast to reduced integration, more equal shares and greater efficiency after a positive uniform liquidity shock, a uniform positive productivity shock will have the *opposite* effects. In this sense the external effect of productivity shocks is more powerful than that for liquidity shocks.

5 Discussion

If one asks the question "who gets organizational power in a market economy?," one is tempted to answer "to the scarce goes the power." There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) which ascribes power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore, 1990; Stole and Zweibel, 1996). Our results suggest that organizational power may emanate from scarcity *outside* the organization, i.e., from market power. But this result has to be qualified somewhat: Proposition 13 suggests that having more liquidity may actually cause one to *lose* power, via what we have called the external effect of shocks to fundamentals. Similarly, the possessors of a new technology, if they are inframarginal, will gain control over assets (Proposition 17 (i)), but if they are marginal may lose it. This is evidence of the importance of market effects for the allocation of power inside firms and more generally of their importance for the study of organizations.

We now discuss some other implications of the model.

5.1 Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or the technology available to firms. One can easily extend the model to allow for liquidity that yields a positive return though the period of production. Because liquidity in this model is used only as a means of surplus transfer, and not as a means to purchase new assets, the effects of this can be somewhat surprising. Raising this interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before: formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and by Proposition 12, firms will integrate less if the interest rate increases, and will integrate more if interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity, by lowering interest rates, may be consitute a force for integration above and beyond that suggested by the example in Proposition 13. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward; whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

5.2 Product Market

If we imagine all the firms sell to a competitive product market, then the selling price inheres in A, which we have thus far viewed as exogenous (for instance the supplier market is contained in a small open economy, with prices determined in the world market). But if instead price is determined endogenously in the product market, then shocks to product demand will change the price, which has the effect of changing A for all firms. Suppose the price increases. Then from the analysis of productivity shocks, all firms become more integrated (on the one hand raising A decreases centralization, but since everyone's willingness to pay increases, the surplus to the 2's increases, and as we saw the second effect dominates).

Next, notice that expected output is $A(sA + (1 - \omega)(1 - s)A + \omega)$; this is constant and independent of s for all nonintegrated firms ($\omega = 0$), and is increasing in ω for all integrated firms. Thus integration raises expected output and the product supply curve is upward sloping. Thus an increase in consumer demand raises equilibrium price, and we conclude that increasing demand results in greater integration.

What is more, the product market price effect now means that more local shocks will result in widespread restructuring: more than just the very poorest firms in the economy may be "marginal." To see this, suppose a number of perfectly nonintegrated firms innovate. With fixed prices, nothing happens, except that these firms produce more output. With endogenous prices, the increased output in the first instance lowers product price; all other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of "amplification" effect on organizational restructuring.[18]

Previous work has analyzed how the intensity of product market competition may act as an incentive tool for managers.[19] In this literature the set of firms and their internal organization are exogenous. Here we wish to emphasize a causal relation in the opposite direction that becomes apparent once organization is allowed to be endogenous: organizations may affect product market prices, even when there is perfect competition. As discussed in Legros and Newman (2004), the fact that product market is affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

6 Appendix I: Proofs

6.1 Proof of Proposition 1

Suppose that manager 1 borrows B from a third party who transfers it to 2 and that the contract specifies an additional lump sum transfer $t_1 \geq 0$ to 2 and uses share s and control structure ω, with resulting equilibrium (q, e).

[18] Of course the effect is self-limiting because as they become less integrated, they lower their output, causing the price to go up again. As shown in Legros-Newman (2004), these product market effects can be more pronounced in models that rely on somewhat different trade-offs in their basic organizational model than the one considered here.

[19] The first models are Hart (1983) and Scharfstein (1988); more recent papers are Schmidt (1997), Aghion et al. (1999).

Since the creditor must make nonnegative profits, he must get a payoff of D when output is R (and 0 when output is 0), $p(q,e)\,D \geq B$. Resulting payoffs to 1 and 2 are

$$\begin{aligned} u_1 &= -t_1 + p(q,e)\left[(1-s)\,R - D\right] - C_1(q,e) \\ u_2 &= B + t_1 + p(q,e)\,sR - C_2(q,e) \end{aligned}$$

Consider the contract without borrowing consisting of the same transfer of t_1 and a share to 2 of $sR + D$ and to 1 of $(1-s)\,R - D$ where D assumes the value it did in the first contract. If $p(e,q)$ were still the equilibrium probability of success, this contract pays both partners exactly the same as the first contract. But since 2's ex-post share is now larger, he raises $q_2(k)$ for $k \in [1,2]$; by revealed preference he is better off, and 1, by virtue of the increase in the success probability is also better off. Any borrowing is Pareto dominated by a contract that involves no borrowing.

6.2 Proof of Proposition 2

Consider a contract (s,ω) where the sharing rule gives contingent shares to manager 2 of $s(R)$ and $s(0)$; by budget balance, manager 1 gets contingent shares of $R+L-s(R)$ and $L-s(0)$. Let $p(q,e)$ be the resulting probability of success in the equilibrium of the game induced by the contract (s,ω). In choosing decisions and effort, each manager considers his marginal share, that is $s(R)-s(0)$ for manager 2 and $R-(s(R)-s(0))$ for manager 1. Utility payoffs are then

$$\begin{aligned} u_1 &= p(q,e)\left[R-(s(R)-s(0))\right] + L - s(0) - C_1(q) - ce_1 \\ u_2 &= p(q,e)\left[s(R)-s(0)\right] + s(0) - C_2(q) - ce_2. \end{aligned}$$

Consider now the contract in which the two managers share first the total liquidity L in such a way that manager 2 gets $s(0)$ and manager 1 gets $L-s(0)$ and then agree to a contract $(\hat{s},\omega)$ such that manager 2 gets $\hat{s}(R) = \hat{s}R$, $\hat{s}(0)=0$ and manager 1 gets $R-\hat{s}(R) = (1-\hat{s})\,R$ and $-\hat{s}(0)=0$. It is immediate that the marginal shares are the same with $\hat{s}$ and with s and moreover, since the control structure has not changed, that the equilibrium of the induced game is the same.

6.3 Derivation of Pre-Liquidity Pareto Frontier and Proof of Proposition 5

Recall that payoffs are (2) and (3) or

$$
\begin{aligned}
u_1(s,\omega) &= \frac{1-s^2}{2}A^2 - c - \omega\frac{((1-s)A-1)^2}{2} \\
u_2(s,\omega) &= \frac{s(2-s)}{2}A^2 - c + \omega s A(1-(1-s)A),
\end{aligned}
$$

It is immediate that u_1 is a linear decreasing function of ω and u_2 is a linear increasing function of ω.

Let λ be the Lagrange multiplier on the constraint involving 1's payoff in problem (4) of $\max\{u_2(s,\omega) : u_1(s,\omega) = u\}$. Note that in the problem we have ignored the incentive constraint that $s \in [1-\bar{s},\bar{s}]$. The first-order conditions are

$$
s\left\{\begin{array}{c} = 1-\bar{s} \\ \in (1-\bar{s},\bar{s}) \\ = \bar{s} \end{array}\right\}
$$

$$
\Rightarrow (1-\omega)(1-2s)A^2 + \omega A + sA^2 - \lambda((1-\omega)(1-s)A^2 + \omega A - (1-2s)A^2)\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}0 \tag{7}
$$

$$
\omega\left\{\begin{array}{c} = 0 \\ \in (0,1) \\ = 1 \end{array}\right\}
$$

$$
\Rightarrow sA - s(1-s)A^2 - \lambda(\frac{(1-s)^2A^2}{2} - (1-s)A + \frac{1}{2})\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}0 \tag{8}
$$

Let $\Lambda(s,\omega)$ be the value of λ for which (7) holds with an equality and let $K(s)$ be the value of λ for which (8) holds with an equality. After some simple algebra,

$$
\begin{aligned}
\Lambda(s,\omega) &= \frac{(1-\omega-s+2\omega s)A+\omega}{(s-\omega+\omega s)A+\omega} \qquad (9) \\
K(s) &= \frac{2sA}{1-(1-s)A}.
\end{aligned}
$$

We note that $K(s)$ is increasing in s and that $\Lambda(s,0) = \frac{1-s}{s}$ is decreasing in s. Therefore, there exists a unique value of s, that we denote by s^* for which $K(s) = \Lambda(s,0)$.[20] When $s < s^*$, $\Lambda(s,0) > K(s)$ and therefore when $\lambda = \Lambda(s,0)$, we have indeed $\lambda > K(s)$ and therefore $\omega = 0$ by (8). The frontier has slope $-\Lambda(s,0)$; since $\Lambda(s,0)$ is decreasing with s, the frontier is concave. In this regime, $\omega = 0$ and $s = \sqrt{1 - \frac{2(u_1+c)}{A^2}}$ and s is decreasing and concave in u_1.

Two cases are of interest.

Case $s^* < \bar{s}$. Noting that $\Lambda(s,\omega)$ is increasing in ω, it is necessary that $s > s^*$ and $\omega > 0$ in order to have $K(s) = \Lambda(s,\omega)$; keeping $s = s^*$ while increasing ω would lead to a contradiction since $\Lambda(s^*,\omega) > \Lambda(s^*,0) = K(s^*)$ and therefore if $\lambda = \Lambda(s^*,\omega)$, $\lambda > K(s^*)$ and we should have $\omega = 0$. Replicating this argument, s and ω must jointly increase in order to satisfy the equality $K(s) = \Lambda(s,\omega)$. This continues until $s = \bar{s}$, after which the frontier becomes linear. Note that when $s \in (s^*, \bar{s})$ the slope of the frontier is given by $\frac{du_2}{du_1} = -K(s)$ and since $K(s)$ is increasing in s, $-K(s)$ is decreasing in s, which shows that the frontier is convex.

Case $\mathbf{s^* \geq \bar{s}}$. choosing $\omega = 0$ and $s = \bar{s}$ implies that $\Lambda(\bar{s},0) > K(\bar{s})$. If 1 must get a lower surplus, ω must increase and the Pareto frontier is linear with slope equal to $\frac{\partial u_2/\partial\omega}{\partial u_1/\partial\omega} = 2\frac{\bar{s}A(1-(1-\bar{s})A)}{((1-\bar{s})A-1)^2} = \frac{2(A-\sqrt{2c})\left(1-\sqrt{2c}\right)}{A\left(1-\sqrt{2c}\right)^2}$. When $\omega = 0$, $\underline{u} = u_1(\bar{s},0) = \frac{1-\bar{s}^2}{2}A^2 - c,$. or using (1), $\underline{u} = \frac{2\bar{s}}{1-\bar{s}}c = A\sqrt{2c} - 2c$.

Hence, when $u_1 \in [0,\underline{u}]$, $s = \bar{s}$ and $u_1(\bar{s},\omega)$ is linear and decreasing in ω, which proves that ω is linear and decreasing in u_1. When $u_1 \in [\underline{u}, u_=]$, the frontier has slope $\Lambda(s,0)$, decreasing in s, and the frontier is concave. However we cannot conclude immediately that the frontier is globally concave because there is a kink at $(\underline{u}, \phi(\underline{u}))$. Indeed, the absolute value of the right derivative is $\Lambda(\bar{s},0)$ while the absolute value of the left derivative is $K(\bar{s})$. The frontier is globally concave if $K(\bar{s}) < \Lambda(\bar{s},0)$. Since K is increasing in s and since $s^* < \bar{s}$, $K(\bar{s}) < K(s^*)$. Since $\Lambda(s,0)$ is decreasing in s, $\Lambda(s^*,0) < \Lambda(\bar{s},0)$. Now, by definition of s^*, $K(s^*) = \Lambda(s^*,0)$; hence $K(\bar{s}) < \Lambda(\bar{s},0)$ and the frontier is concave as claimed.

[20] One can check that $s^* = \frac{1}{3A}\left(A + \frac{1}{2}\sqrt{8A - 8A^2 + 1} - \frac{1}{2}\right)$.

6.4 Implications of Relaxing Assumption 5

The only results this change affects are those that relied on concavity of the frontier and convexity of ω, namely the discussion in section **??** leading up to Proposition 14. Relaxing Assumption 5 doesn't change the fact that giving 2 more surplus entails giving him a greater degree of control, so that changes in market conditions that give 2's more surplus will continue to increase the degree of integration. Also the slope remains less than unity in magnitude, increasing 2's generated surplus still lowers total surplus.

6.5 Proof of Lemma 9

(i) The marginal firm chooses non integration since the generated surplus of the type 1 manager is $\min\{l(\bar{\imath}), u_=\} > \underline{u}$; from Proposition 5, all inframarginal firms choose also nonintegration.

(ii) The marginal firm chooses integration since the generated surplus of the marginal type 1 is less than $l(\bar{\imath}) < \underline{u}$. The cutoff value $L(v_2^*)$ is well defined and corresponds to an inframarginal agent: by Proposition **??** $v_2^* = \phi(l(\bar{\imath})) + l(\bar{\imath})$ and $v_2^* - \phi(\underline{u}) = \phi(l(\bar{\imath})) - \phi(\underline{u}) + l(\bar{\imath})$; since ϕ is decreasing $\phi(\underline{u}) < \phi(l(\bar{\imath}))$ and therefore $v_2^* - \phi(\underline{u}) > l(\bar{\imath})$.

6.6 Proof of Propositions 12

To see this, it is enough to show that $\omega(\psi(l(i)), v_2^*(\psi(l(\bar{\imath})))) \leq \omega(l(i), v_2^*(l(\bar{\imath})))$ for all i. Now ω depends on v_2^* and l only via its dependence on the generated surplus u_1. In firm i the generated surplus is $\hat{u}_1$ after the shock and solves

$$\begin{aligned}\phi(\hat{u}_1) &= \min\{u_=, v_2^*(\psi(l(\bar{\imath}))) - \psi(l(i))\} \\ &= \min\{u_=, \phi(\psi(l(\bar{\imath}))) + \psi(l(\bar{\imath})) - \psi(l(i))\}.\end{aligned}$$

Before the shock it was u_1 solving

$$\begin{aligned}\phi(u_1) &= \min\{u_=, v_2^*(l(\bar{\imath})) - l(i)\} \\ &= \min\{u_=, \phi(l(\bar{\imath})) + l(\bar{\imath}) - l(i)\}.\end{aligned}$$

We note that $\phi(\psi(l(\bar{\imath}))) - \phi(l(\bar{\imath})) \leq 0 \leq [\psi(l(i)) - l(i)] - [\psi(l(\bar{\imath})) - l(\bar{\imath})]$: the left hand side is nonpositive since ϕ is decreasing and $\psi(l(\bar{\imath})) \geq l(\bar{\imath})$, while the right hand side is nonnegative because the shocks are nondecreasing. Rewriting this expression, as $\phi(\psi(l(\bar{\imath}))+\psi(l(\bar{\imath}))-\psi(l(i)) \leq \phi(l(\bar{\imath}))+l(\bar{\imath})-l(i)$ then implies $\min\{u_=, v_2^*(\psi(l(\bar{\imath}))) - \psi(l(i))\} \leq \min\{u_=, v_2^*(l(\bar{\imath})) - l(i)\}$. Therefore, $\phi(\hat{u}_1) \leq \phi(u_1)$ and $\hat{u}_1 \geq u_1$. It follows from Proposition 5 that the firm will reorganize and choose a lower value of ω. This proves (i); (ii) and (iii) are direct consequences of (i) and Proposition 5.

6.7 Proof of Proposition 13

It is enough to provide an example. Consider the liquidity distribution $l(i) = i$ where $i \in [0,1]$; and suppose that $n = 1$, that is that the marginal liquidity is 0. The contract for the marginal firm is therefore an integration contract with $s = \bar{s}$ and $\omega < 0$; from Proposition 5 the frontier is linear and can be written $\phi(u_1) = -\alpha u_1 + \phi(0)$, where $\alpha \in (0,1)$.[21] To simplify assume that $1 < \underline{u}$; this will insure that when the equilibrium surplus is $v_2^* = \phi(0)$ the firm with $i = 1$ chooses an integration contract.

¿From Proposition 9, the equilibrium surplus is $v_2^*(0) = \phi(0)$. Let $\varepsilon < 2\underline{u}$ and define $\psi(l)$ by

$$\psi(l) = \begin{cases} \delta, \text{if } l \leq \delta \\ l \text{ if } l \geq \delta. \end{cases}$$

$\psi(l)$ is increasing and continuous. The marginal liquidity is now $\psi(0) = \delta$ and the new equilibrium surplus is $v_2^*(\delta) = \phi(\delta)+\delta$. The generated surpluses in a firm with type 1 of index i is before the shock (distribution l)

$$u_1(i) : \phi(u_1)(i) = \phi(0) - i$$

and after the shock (distribution ψ)

$$\hat{u}_1(i) : \phi(\hat{u}_1(i)) = \phi(\delta) + \delta - \psi(i).$$

Firms with $i \geq \delta$ have the same liquidity but a higher equilibrium surplus accrues to type 2, and by Lemma 8 $\hat{u}_1 < u_1$. Precisely,

$$\begin{aligned} \phi(\hat{u}_1(i)) - \phi(u_1(i)) &= \phi(\delta) - \phi(0) + \delta \\ &\Leftrightarrow \\ u_1(i) - \hat{u}_1(i) &= \frac{1-\alpha}{\alpha}\delta \end{aligned}$$

[21] $\alpha < 1$ follows concavity of ϕ and the fact that total surplus is $\phi(u)+u$ and is maximum at $u = u_=$.

For firms with $i < \delta$, $\psi(l(i)) = \delta$, and

$$\begin{aligned}
\phi(\hat{u}_1(i)) - \phi(u_1(i)) &= \phi(\delta) - \phi(0) + i \\
&\Leftrightarrow \\
u_1(i) - \hat{u}_1(i) &= -\delta + \frac{i}{\alpha}.
\end{aligned}$$

Therefore, for all firms $i \geq \alpha\delta$ the generate surplus decreases and these firms are more integrated. For firms with $i < \alpha\delta$, the generated surplus increases and these firms are less integrated.

For the linear part of the frontier, the generated surplus is also a linear function of ω (see 2) and we can write $\omega(i) = -\beta u_1(i) + \omega(0)$, where $\beta > 0$. Hence, the change in the degree of control is $\hat{\omega}(i) - \omega(i) = \beta(u_1(i) - \hat{u}_1(i))$, and in the aggregate,

$$\begin{aligned}
\int_0^1 (\hat{\omega}(i) - \omega(i))\, di &= \beta\left[\int_0^\delta \left(-\delta + \frac{i}{\alpha}\right) di + \int_\delta^1 \frac{1-\alpha}{\alpha}\delta di\right] \\
&= \beta\frac{\delta}{\alpha}\left(1 - \alpha - \frac{\delta}{2}\right).
\end{aligned}$$

Hence as long as $\delta < 2(1-\alpha)$, average integration increases in the economy.

Note that $\psi(l)$ does not satisfy Assumption 1 since it is constant on $l \leq \delta$. However, by continuity, there exists $\tilde{\psi}(l)$ satisfying Assumption 1 for which average integration increases.

6.8 Proof of Proposition 15

The pre-liquidity frontier is $\phi(u_1) = -\alpha u_1 + \phi(0)$, where $\alpha = \frac{2A}{2-A}$ (substitute $\bar{s} = \frac{1}{2}$ in (2)-(3)). The equilibrium surplus is $v_2^* = (1-\alpha) l(\bar{\imath}) + \phi(0)$ and welfare is $W = \phi(u_1) + u_1 = (1-\alpha) u_1 + \phi(0)$. The generated surplus is $u_1(l, v_2^*)$ solving $\phi(u_1) = v_2^* - l$; hence, $u_1 = -\frac{1-\alpha}{\alpha} l(\bar{\imath}) + \frac{l}{\alpha}$, and therefore welfare in a firm with type 1 manager i is

$$W_i(l) = \phi(0) + \frac{1-\alpha}{\alpha} l(i) - \frac{(1-\alpha)^2}{\alpha} l(\bar{\imath})$$

and total surplus is $W(l) = \int_{\bar{\imath}}^{1} W_i(l)\,di$. Hence, change of welfare when going from l to $\psi \circ l$ is

$$W(\psi \circ l) - W(l) = \int_{\bar{\imath}}^{1} \frac{1-\alpha}{\alpha}\left(\psi(l(i)) - l(i)\right) - n\frac{(1-\alpha)^2}{\alpha}\left(\psi(l(\bar{\imath})) - l(\bar{\imath})\right)$$

Noting that the conditional means are $\mu = \int_{\bar{\imath}}^{1} l(i)\frac{di}{n}$ and $\hat{\mu} = \int_{\bar{\imath}}^{1} \psi(l(i))\frac{di}{n}$ we have

$$W(\psi \circ l) - W(l) \geq 0 \Longleftrightarrow \hat{\mu} - \mu \geq (1-\alpha)\left(\psi(l(\bar{\imath}) - l(\bar{\imath})\right)$$

as claimed. Since the degree of integration is linear in the generated surplus, integration decreases when the condition holds.

6.9 Proof of Lemma 16

Going back to Proposition 5 the Pareto frontier of the feasible set is characterized by levels of generated surpluses $\underline{u}(A) = A\sqrt{2c} - 2c$ and $u_{=}(A) = \frac{3}{8}A^2 - c$ such that :

For $u_1 \in [0, \underline{u}(A)]$, the contract is an integration contract $(\bar{s}, \omega)$. Hence from (2) and $\bar{s} = 1 - \frac{\sqrt{2c}}{A}$, we obtain after some computations,

$$\begin{aligned} u_1(\bar{s}, \omega; A) &= A\sqrt{2c} - 2c - \omega\frac{\left(1-\sqrt{2c}\right)^2}{2} \\ u_2(\bar{s}, \omega; A) &= \frac{A^2}{2} - 2c + \omega(A - \sqrt{2c})\left(1 - \sqrt{2c}\right) \end{aligned} \tag{10}$$

Note that the maximum degree of control consistent with individual rationality of 1 is

$$\bar{\omega}(A) = 2\frac{A\sqrt{2c} - 2c}{\left(1-\sqrt{2c}\right)^2}$$

which is increasing in A. From Proposition 5 $\underline{u}(A), u_{=}(A)$ are both increasing in A. Note that for a given value of ω, the absolute value of the slope of the frontier is $\left|\frac{du_2/d\omega}{du_1/d\omega}\right| = \frac{2(A-\sqrt{2c})}{\left(1-\sqrt{2c}\right)}$ which is increasing in A.

For $u_1 \in [\underline{u}(A), u_=(A)]$, the contract is a non-integration contract $(s,0)$ and

$$\begin{aligned} u_1(s,0;A) &= \frac{1-s^2}{2}A^2 - c \\ u_2(s,0;A) &= \frac{s(2-s)}{2}A^2 - c. \end{aligned} \tag{11}$$

Here the slope of the frontier is $\frac{du_2/ds}{du_1/ds} = -\frac{1-s}{s}$ and independent of A.

6.10 Proof of Proposition 17

¿From Proposition 5, a pair of payoffs (u_1, u_2) on the pre-liquidity frontier is generated by a unique contract (s, ω). Let $(s(l, v_2^*; A), \omega(l, v_2^*; A))$ be the contract chosen in a firm with technology A when 1 has liquidity l and 2 must obtain v_2^*; with this contract the generated payoffs solves $\phi(u_1; A) = v_2^* - l$.

Remark 18 *Note that the other possibilities than the two considered in the Proposition..*

Case 1: A first possibility is $i_1 < 1-n$, that is shocked firms were not matched in the initial economy but because $w(i_1) > v_2^*(A)$, some of these firms will be matched. In this case, the set of "new entrants" are firms with $i \in [i_\pi, i_1]$ while the set of "old firms" are those with index $i \geq k$, where $k \geq 1-n$ satisfies $w(k) = w(i_\pi)$ and $i_1 - i_\pi = k - (1-n)$ (hence firms $i \in [i_\pi, i_1]$ "replace" firms $i \in [1-n, k]$).[22] Since $w(i_\pi) > v_2^*(A)$,the degree of control to 2 increases in old firms. For new firms, the question is whether the increase in price $w(i_\pi) - w(1-n)$ is large enough to overcome the internal effect of technology shock pushing towards less integration.

Case 2: Another possibility is $1-n \in (i_0, i_1)$ and $w(1-n) > \lim_{\varepsilon \downarrow 0} w(i_1+\varepsilon)$. Then there exists $k > i_1$ such that $w(k) = w(1-n)$, and either $i_\pi \in (i_1, k]$ or $i_\pi \in [i_0, 1-n)$. In either case, if $l(i_\pi)$ is low enough, the increase in equilibrium surplus to the 2 may be small enough that the internal effect dominates and shocked firms integrate less.

[22] The existence of such values of $\bar{\imath}$ and k is insured if indeed $\bar{\imath} \in (i_0, i_1)$. By assumption $w(i_1) > w(1-n.)$ If $w(i_0) \geq w(1-n)$, but $w(1-n+\lambda) < w(i_0)$, where $\lambda = i_1 - i_0$ is the measure of shocked type 1 firms; the marginal type is then $\bar{\imath} = 1-n+\lambda < i_0$ which contradicts our assumption. If $w(i_0) \geq w(1-n)$ we need therefore that $w(1-n+\lambda) \geq w(i_0)$, in which case there exists k such that $k = 1-n+i_1-\bar{\imath}$ and $w(k) = w(\bar{\imath})$. If $w(i_0) < w(1-n)$, there exists $i \in (i_0, i_1)$ such that $w(i) = w(1-n)$ and we can replicate the previous argument with $\lambda = i_1 - i$.

We continue with the proof of the Proposition.

(i) (Inframarginal shocks) If $i_\pi = 1 - n$ *and* $w(i_\pi) = v_2^*(A)$, *then the shocked firms become less centralized and the 2's shares fall, while the unshocked firms remain unaffected*

This is a direct consequence of Lemma 16(ii) and Proposition 5.

(ii) (Marginal shocks) If $1-n \in [i_0, i_1]$, *and* $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1 + \varepsilon)$, $i_\pi = 1 - n$, $F = [1-n, 1]$, *the equilibrium price* $v_2^*\left(\hat{A}\right)$ *increases and all firms, shocked and unshocked, integrate more.*

If $1 - n \in (i_0, i_1)$ and $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1 + \varepsilon)$, $1 - n$ minimizes $w(i)$ over $i \geq 1 - n$; therefore $i_\pi = 1 - n$ and $F = [1-n, 1]$.

¿From Lemma 16, $\phi(u; A)$ is increasing in A for any value of u. Since $v_2^*\left(\hat{A}\right) - v_2^*(A) = \phi\left(l(1-n); \hat{A}\right) - \phi(l(1-n); A)$, $v_2^*(A)$ is increasing in A; it follows from Proposition 5 that all unshocked firms $[i_1, 1]$ integrate more.

If the firm $1 - n$ did not integrate before the shock (that is chose $\omega = 0$ and $s < \bar{s}$), all $i > 1 - n$ firms also chose not to integrate since the degree of control is decreasing in the liquidity of type 1. Hence, it is immediate that an increase in A can only lead to more integration.

Consider now the case where firm $1 - n$ integrated before, that is chose a contract $(\bar{s}, \omega)$. If i_1 chose initially a contract $(s, 0)$, there exists $k \in (1 - n, i_1)$ choosing the contract $(\bar{s}, 0)$ and all firms with $i < k$ integrate $(\omega > 0)$ and all firms with $i > k$ do not integrate; firms with $i > k$ will necessarily integrate more after the shock. Hence without loss of generality assume that all firms $i \in [1-n, i_1]$ integrated before the shock, i.e., that the equilibrium contracts lead to surpluses on the linear part of the frontier. By Proposition 5, the pre-liquidity Pareto frontier is given by the map

$$\phi(u_1) = \phi(0; A) - \alpha(A) u_1, \tag{12}$$

where $\alpha(A) = 2\frac{A - \sqrt{2c}}{1 - \sqrt{2c}}$ is increasing in A. Let $u_1(i; A)$ be the equilibrium generated surplus of type 1 when the technology is A and let $\omega(i; A)$ be the degree of control chosen by firm i in equilibrium. Recall that, $\phi(u_1(i; A)) = v_2^*(A) - l(i)$. Since $1 - n$ is the marginal type 1,

$$u_1(1-n; A) = l(1-n), \tag{13}$$

$$\begin{aligned} v_2^*(A) &= \phi(l(1-n); A) + l(1-n) \\ &= \phi(0; A) - \alpha(A) l(1-n; A) - l(1-n). \end{aligned} \tag{14}$$

and therefore using (12) and (14) we have

$$\alpha(A)\left[u_1(i)-u_1(1-n)\right]=l(i)-l(1-n)\,. \tag{15}$$

Since $\alpha(A)$ is increasing in A, it follows from (15) that $u_1(i)-u_1(1-n)$ must decrease after the shock.

The generated surplus is linear in the degree of control (see (10)) and we have:

$$u_1(i;A)-u_1(1-n;A)=\frac{\left(1-\sqrt{2c}\right)^2}{2}\left(\omega(1-n;A)-\omega(i;A)\right) \tag{16}$$

and (15)-(16) imply

$$\omega(1-n;A)-\omega(i;A)\ \text{ is decreasing in } A. \tag{17}$$

By (13) and (10),

$$\omega(1-n;A)=2\frac{A\sqrt{2c}-2c-l(1-n)}{\left(1-\sqrt{2c}\right)^2}$$

is clearly increasing in A; it follows from the previous observation that that $\omega(1-n;A)-\omega(i;A)\,\omega(i;A)$ is also increasing in A, therefore by (17), $\omega(i;A)$ is increasing in A for all $i>1-n$.

(iii) If $i_0=0$ and $i_1=1$, the arguments for (ii) apply since $1-n$ is still the marginal type 1 manager.

7 Appendix II: Parameter Restrictions

Here we provide sufficient conditions on the parameters that yield the concave frontiers – along with the simple description of organizational forms – described in the text. We consider them roughly in order of increasing strength.

In order for there to be a trade-off between surplus division and surplus production (and thus a role for the market to determine internal organization), we need

- nonintegration with $s = 1/2$ produces more surplus than giving full control to one party along with a full share of output (this being the most efficient given full control). Simple calculations reveal that the necessary and sufficient condition for this is that $A \leq 2 - \sqrt{2}$; however this is stronger than necessary, since if $c > 0$, this isn't even feasible, and in any case isn't individually rational for the partner ceding control. It's easier elsewhere just to take $A \leq 1/2$.

- A feasibility requirement that all assets can be operated is that the maximum interim rational share is at least 1/2 (otherwise both partners cannot be incentive compatible); this is essentially a requirement that c not be "too big": $\bar{s} \geq 1/2 : \sqrt{2c} \leq A/2$ or $c \leq A^2/8$; this is not quite sufficient to guarantee that all assets *will* be operated; see below.

- No shut down (see below): it is an interesting logical possibility, which may have empirical counterpart; that some of the firms' assets will be shut down in order to compensate a manager who is ceding control; though worthy of further research, we choose here to rule it out in order to focus on other issues. What is required is a simple condition on the costs of operating assets that is approximately the same as the previous one, but neither implies nor is implied by it

- no swapping (see below): another interesting logical possibility: managers "swap" assets as a commitment device. This makes sense if productivity is far more important to payoffs than costs, and turns out to be ruled out by the other assumptions (in particular the first and third, which together guarantee that the slope of the frontier above the 45°-line is less than unity in magnitude).

- concavity of the frontier: this facilitates some of the aggregate computations in Section **??**, but does not otherwise affect most of the conclusions. What is required here is that ω and s do not truly covary, for which the necessary and sufficient condition is that $s^* \geq \bar{s}$: a simple computation shows that this is equivalent to $\sqrt{2c} \geq \frac{2}{3}A + \frac{1}{6} - \frac{1}{6}\sqrt{(1 + 8A - 8A^2)}$, in other words, c is "not too small."

It is this last condition that is really most stringent, but it should be clear that its role is more of expositional rather than conceptual importance.

7.1 No Shutting Down of Assets

As in the text, we continue to maintain the assumption $A \leq 1/2$. If $\hat{s} \geq s > \bar{s}$, any asset under 1's control will be shut down; if $s > \hat{s}$, then all of the type-1 assets are shut down. Thus for a contract (s, ω) the payoffs will be

$$u_1 = \begin{cases} (1-s)A[sA+\omega] - \frac{\omega}{2} - \omega c, \ \hat{s} \geq s > \bar{s} \\ (1-s)sA^2 - \frac{\omega}{2}, \ s > \hat{s} \end{cases}$$
$$u_2 = \begin{cases} sA[\frac{sA}{2} + \omega] - c, \ \hat{s} \geq s > \bar{s} \\ \frac{s^2A^2}{2} - c, \ s > \hat{s} \end{cases}$$

If $\bar{s} < s < \hat{s}$, **then** $\omega = 0$. Solve the Pareto problem $\max\limits_{s,\omega} u_2$ s.t. $u_1 \geq v$ with multiplier λ and obtain from the first-order conditions

$$\omega \left\{ \begin{array}{c} = 0 \\ \in (0,1) \\ = 1 \end{array} \right\} \text{as } \lambda \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{sA}{c + \frac{1}{2} - (1-s)A}$$
$$s \in (1/2, \hat{s}) \Longrightarrow \lambda = \frac{\omega + sA}{\omega - (1-2s)A}$$

Since $A \leq c + \frac{1}{2}$, $\frac{\omega+sA}{\omega-(1-2s)A} > \frac{sA}{c+\frac{1}{2}-(1-s)A}$ for any $\omega \in [0,1]$, so we must have $\omega = 0$.

If $s = \hat{s}$ then $\omega = 0$. Varying ω above 0 here generates a frontier of slope magnitude less than 1 ($= 2\hat{s}A = 2(A-c) < 1$, since $A \leq c + \frac{1}{2}$), while varying s above $\hat{s}$ with $\omega = 0$ generates a steeper frontier (slope $= \frac{s}{2s-1} > 1$), so Pareto dominates $\omega > 0$ with $s = \hat{s}$

If $s > \hat{s}$, **then** $\omega = 0$. This is immediate by inspection: 2's payoff is independent of ω in this case, and he therefore does not benefit from $\omega > 0$, while 1 is hurt.

We conclude that if s is large enough that any 1-assets are shut down, they will all be kept under 1's control ($\omega = 0$), and the ($s > \bar{s}$)-frontier is continuous.

To ensure that no assets are shut down, we simply impose that the best payoff to 2 on the ($s > \bar{s}$)-frontier is smaller than his best payoff on the ($s \leq \bar{s}$)-frontier. That this suffices depends on noting that the ($s > \bar{s}$)-frontier begins below the ($s \leq \bar{s}$)-frontier and is smooth, with negative slope greater than unity in magnitude, and yielding its maximum payoff to 2 at $s = 1$, while the $(s \leq \bar{s})$-frontier has slope less than unity. The maximal payoff to 2 on the $(s > \bar{s})$-frontier is $\frac{A^2}{2} - c$, where 1 gets 0. On the $(s \leq \bar{s})$-frontier, the maximal payoff to 2 obtains when 1 gets zero, which entails that $(1-\bar{s})A[\bar{s}A + \omega] - \frac{\omega}{2} - \omega c = 0$, or $\omega = \frac{\bar{s}(1-\bar{s})A^2}{\frac{1}{2}+c-(1-\bar{s})A}$. In this case, 2 obtains $\frac{\bar{s}^2A^2}{2} + \frac{\bar{s}(1-\bar{s})A^2(\frac{1}{2}+c-(1-\bar{s})A-\bar{s}(1-\bar{s})A^2)+\bar{s}^2(1-\bar{s})A^3}{\frac{1}{2}+c-(1-\bar{s})A} - c$, which exceeds $\frac{A^2}{2} - c$ if and only if $\bar{s}^2A\frac{1-(1-\bar{s})A}{\frac{1}{2}+c-(1-\bar{s})A} \geq \frac{1-\bar{s}}{2}$, or, using the definition of $\bar{s}$, $4\bar{s}^2A \geq (1-\bar{s})(1-(1-\bar{s})A)$. In terms of c and A, this reduces to $4A^2 - (1+8A)\sqrt{2c} + 10c \geq 0$.

Thus shutting down assets is Pareto dominated when $4A^2-(1+8A)\sqrt{2c}+10c \geq 0$ and $A \leq \frac{1}{2}+c$, the Pareto frontier is therefore as described in the text.

There is a positive measure of the parameter space satisfying all of the above conditions. In particular, the case $c = A^2/8$ with A large enough is included, and thus admits the expositionally useful case in which the Pareto frontier is piecewise linear with the constant share $\bar{s} = 1/2$.

7.2 No Swapping of Assets

Asset swapping is a means of effectively committing the managers to high levels of the decisions q. This commitment is only worthwhile if productivity is sufficiently high relative to costs; our parametric case of interest rules this out.

To see this, note that if assets are to be swapped, we can characterize the situation via two control parameters ψ and ω : manager 1 controls $k \in [0, 1-\omega)$ and $k \in [1, 2-\psi)$, and 2 controls $k \in [1-\omega, 1)$ and $[2-\psi, 2]$, where $\psi \in [0,1]$ and now $\omega \in [0,1]$ instead of $[-1,1]$. Thus full control by 2 (1) involves $\psi = \omega = 1$ ($\psi = \omega = 0$), nonintegration is represented by $\psi = 1$, $\omega = 0$, and full swapping by $\psi = 0$, $\omega = 1$.

Given a contract (s, ψ, ω) with utility allocation above the 45°-line (we restrict attention to this case; the other one is similar), it is straightforward to check that the payoffs are now (assuming $s \leq \bar{s}$; if $\bar{s} < s \leq \hat{s}$, the $(1-\omega)$ terms vanish, and if $s > \hat{s}$, so do the ω terms)

$$u_1 = (1-s)A[\frac{(1-\omega)(1-s)A}{2} + \psi sA + (1-\psi) + \omega] - \frac{\omega}{2} - c$$
$$u_2 = sA[\frac{\psi sA}{2} + (1-\psi) + \omega + (1-\omega)(1-s)A] - \frac{1-\psi}{2} - c$$

Total surplus is

$$W = \psi s^2 A^2 (\frac{1+\psi}{2}) + (1-\psi)A + \omega A + (1-\omega)(\frac{1-s^2}{2})A^2 - \frac{1-\psi}{2} - \frac{\omega}{2} - 2c$$

Asset swapping then entails $\psi < 1$ (with $\omega \geq 0$), and we need to rule it out. Observe that both u_2 and W are increasing in ψ ($\frac{\partial u_2}{\partial \psi} = \frac{1}{2} + \frac{s^2 A^2}{2} - sA = \frac{1}{2}(1-sA)^2 > 0$; $\frac{\partial W}{\partial \psi} = s(1-\frac{s}{2})A^2 - A + \frac{1}{2} > 0$ for $s \geq 1/2$ and $A < 2/3$), while u_1 is decreasing ($\frac{\partial u_1}{\partial \psi} = (1-s)A(sA-1) < 0$); this is true even if $s > \bar{s}$. From the previous derivations, we know that the Pareto frontier above the 45°-line for the set of contracts restricted by $\psi = 1$ has slope magnitude less than one.

Take an arbitrary contract (s, ψ, ω) with $u_1(s, \psi, \omega) \leq u_2(s, \psi, \omega)$. Then $u_1(s, 1, \omega) \leq u_2(s, 1, \omega)$ as well. Let U be the restricted utility possibility set above the 45°-line, that is points generated by the set of restricted contracts (or Pareto inferior points generated from a restricted contract plus free disposal).

For any contract $(s, 1, \omega)$ in the restricted set of contracts with utilities above the 45°-line, the set

$$P(s,\omega) = \{(u_1, u_2) | u_1 \leq u_2, u_1 \geq u_1(s, 1, \omega), u_2 \leq u_2(s, 1, \omega), u_1 + u_2 \leq u_1(s, 1, \omega) + u_2(s, 1, \omega)\}$$

lies in U, since U's frontier has slope less than 1. Moreover, $(u_1(s, \psi, \omega), u_2(s, \psi, \omega)) \in P(s, \omega)$ by construction. Thus, (s, ψ, ω) is in U, and is therefore generated by or is Pareto inferior to some contract $(s', 1, \omega')$.

Implicit in this are parametric restrictions, of course, the same ones used to generate a frontier slope less than one, i.e., Assumption 2.

7.3 Continuous effort

We show here that allowing for continuous effort with linear cost doesn't change anything. Suppose $e \in [0, 1]$ instead of $\{0, 1\}$, and keep the effort cost equal to ec. First consider assets that 1 owns. In order to implement an interior e, we must have at the ex-post stage $eq_1 A(1-s) - ec = 0$ or $q_1 = c/(1-s)A$. But at the interim stage, 1's payoff is $e(1-s)Aq_1 - \frac{1}{2}q_1^2 - ec$; if he somehow chose q_1 to satisfy the ex-post constraint, he would obtain a negative payoff of $-\frac{1}{2}q_1^2 = -\frac{c^2}{(1-s)^2A^2}$. He would do better to pick $e = q_1 = 0$, and better still to pick $e = 1$, $q_1 = (1-s)A$ (provided $s < \bar{s}$).

Thus interior e's are not implementable because we can only satisfy the interim rationality constraint if we satisfy the ex-post constraint strictly, which necessitates $e = 1$ (or $e = 0$ if $s \geq \bar{s}$).

For assets owned by 2, the only way to sustain an interior e is to have $(1-s)A = c$, i.e. $s = \hat{s}$. If $\omega > 0$, then payoff to 2 is strictly increasing in e, while 1 is indifferent, so $e = 1$ is Pareto optimal.

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