Stock Prices and IPO Waves.

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Abstract

We develop a model of stock valuation and optimal IPO timing when investment opportunities are time-varying. IPO waves in our model are caused by declines in expected returns, increases in expected profitability, or increases in prior uncertainty about average profitability. The model predicts that IPO waves are preceded by high market returns, followed by low market returns, and accompanied by high stock prices. These as well as other predictions are supported empirically. Stock prices at the peak of the recent "bubble", which was associated with an IPO wave, are consistent with plausible parameter values in our rational valuation model.

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1. Introduction

The number of initial public offerings (IPOs) changes dramatically over time, as shown in Figure 1. For example, 845 firms went public in 1996, but there were only 83 IPOs in 2001 and 87 IPOs in 2002. What are the underlying causes of the wild swings in IPO volume? How is IPO volume related to market prices? Were the stock prices in March 2000 too high? These are the questions this paper aims to answer, theoretically as well as empirically.

We argue that IPO volume varies due to time variation in investment opportunities (or "market conditions"). We develop a model of stock valuation in which investment opportunities vary in three dimensions: expected market returns, expected aggregate profitability, and prior uncertainty about the post-IPO average profitability in excess of market profitability. We then set up a simple model of optimal IPO timing, and show that IPO volume responds to time variation in market conditions. IPO volume has important implications for stock prices – for the valuations of IPOs and of the market as a whole.

We consider an economy with a special class of agents, "inventors", who are uniquely equipped to invent new ideas that can lead to abnormal profits. New ideas are discovered at a constant frequency. Inventors patent each idea upon discovery and start a private firm that owns the patent. Inventors lack the capital necessary to begin production, so they must turn to capital markets. The decision faced by inventors is when to take their firms public and begin irreversible production to maximize the value of their patents. When investment opportunities are constant, it is optimal to go public as soon as the patent is secured. When investment opportunities vary over time, however, inventors may find it optimal to postpone their IPO in anticipation of more favorable economic conditions in the future.

We solve for the optimal time to go public, and show that new firms are attracted to capital markets especially when the cost of capital is low, when expected future cash flows are high, and when the uncertainty surrounding those flows is high. As a result, clusters of IPOs, or "IPO waves", occur after expected returns decline, after expected profitability increases, or after uncertainty about average profitability increases. This perspective seems novel in the finance literature. When investment opportunities improve, many inventors exercise their options to take their firms public at about the same time. The resulting IPO waves typically last several months, as all private firms rarely go public at exactly the same time because they differ in the time to expiration on their patents as well as in their firm-specific profitability. To demonstrate the properties of IPO waves implied by our model, we calibrate the model to the data and simulate it over long periods of time.

Our model is rich in empirical predictions. IPO waves caused by a decline in expected market return should be preceded by high market returns (because prices rise when expected return falls) and followed by low market returns (because expected return has declined). IPO waves caused by an increase in expected profitability should also be preceded by high market returns (because prices rise as cash flow expectations go up) and followed by high profitability (because expected profitability has risen). Finally, IPO waves caused by an increase in prior uncertainty about average profitability should be preceded by increased disparity between newly listed firms and seasoned firms in terms of their valuations and return volatilities (because prior uncertainty affects the valuations and volatilities of new firms only).

We test the model's implications using data between 1960 and 2002, and find considerable corroborating evidence. We find support for all three channels (discount rate, cash flow, and uncertainty) through which IPO waves are created in our model. IPO volume is positively related to recent market returns, consistent with a decline in expected returns as well as with an increase in expected profits before IPO waves. The discount rate channel is supported by two additional findings: IPO volume is negatively related to future market returns as well as to recent changes in market return volatility, which is itself positively related to expected market return in our model. The cash flow channel is also supported by two additional facts: IPO volume is positively related to changes in aggregate profitability as well as to revisions in analysts' forecasts of long-term earnings growth. Finally, IPO volume is also positively related to recent changes in the excess volatility and valuation of newly listed firms, consistent with an increase in prior uncertainty about average profitability.

In addition to optimal IPO timing, we also analyze stock prices, using a closed-form solution for a firm's ratio of market equity to book equity (M/B). We show that IPO waves in our model tend to be preceded by high valuations for the market as a whole. The reason is that IPO timing is endogenous – firms are induced to go public by improvements in investment opportunities (e.g. declines in expected returns or increases in expected profitability), and these improvements also have the effect of lifting the valuations of all firms. The fact that IPO waves follow high market valuations has been documented e.g. by Lerner (1994), Loughran, Ritter, and Rydqvist (1994), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000), and Lowry (2003).[1] Most of these papers provide behavioral explanations, but we show that this phenomenon is also consistent with a rational model.

[1] Schultz (2003) argues that equity issuers appear to time the market ex post but not ex ante, because market peaks are known only ex post. His crucial assumption is that firms issue equity at higher prices. (Note that issuance at higher prices arises endogenously in our model.) According to Schultz, IPO volume is correlated with future returns ex post, but not ex ante. In contrast, in our model firms go public after declines in expected returns, so that high IPO volume predicts low market returns also ex ante.

IPO valuations are especially high in our model, boosted not only by low discount rates and high expected cash flows, but also by prior uncertainty about average profitability. This uncertainty increases firm value, as shown by Pástor and Veronesi (2003).[2] M/B is predicted to decline after the IPO for two reasons. The first reason is learning. Upon observing realized profits, investors update their beliefs and the posterior uncertainty declines, resulting in a gradual decline in M/B over the lifetime of a typical firm. The second reason is mean reversion. Declines in expected returns induce private firms to go public and at the same time push M/B up, but sooner or later expected returns revert to their long-term averages, pulling M/B down. Expected profitability is also assumed to be mean-reverting. The same argument implies that the market's M/B should also decline after an IPO wave.

Did stock prices exhibit a "bubble" in the late 1990s? The late 1990s were a period of frenzied IPO activity, which in our model results from declines in expected returns or from increases in expected profitability or prior uncertainty. Fama and French (2002, 2003) and Lettau, Ludvigson, and Wachter (2003), among others, argue that expected returns declined recently. Expected profitability was high in the late 1990s, judging by the equity analysts' earnings forecasts. Low discount rates and high cash flow expectations clearly imply high valuation for the market as a whole. Moreover, prior uncertainty also appears to have been high. Technological revolutions, such as the one perceived in the late 1990s, are likely to be accompanied by high prior uncertainty, as the long-term prospects of new firms are particularly uncertain when new paradigms are entertained. Indeed, our empirical proxies reveal unusually high prior uncertainty in the late 1990s. This high uncertainty helped fuel the IPO wave observed at the time, and it also helps justify the astronomical valuations of many IPOs in the late 1990s.

We calibrate our valuation model to the circumstances of March 10, 2000, which is often referred to as the peak of the Nasdaq "bubble". We find that the observed valuations are consistent with levels of the equity premium, expected profitability, and prior uncertainty that are plausible given the high IPO volume in the 1990s. We do not attempt to rule out any behavioral explanations for the "bubble"; we only argue that they are not necessary, since the prices in March 2000 are consistent with our rational valuation model.

[2]The intuition relies on a simple convexity argument. As an example, consider the Gordon growth model, $P = D/(r-g)$, where P is stock price, D is tomorrow's dividend, r is the discount rate, and g is the growth rate of future dividends. Note that P is convex in g. If g is uncertain, P is equal to the expectation of the right hand side. This expectation increases with uncertainty about g, holding other things constant, due to the previously mentioned convexity. See Pástor and Veronesi (2003) for a more careful explanation. Interestingly, Bill Miller, portfolio manager of the Legg Mason Value Trust, used similar logic to justify the valuation of Amazon.com in 1999: "...being wrong isn't very costly, and being right has a high payoff... With Amazon, we believe the payoff for being right is high." Amazon's Allure..., Barron's, 15 Nov 1999.

Of course, this is not the first paper to analyze time variation in IPO volume. The clustering of IPOs was documented by Ibbotson and Jaffe (1975) and Ritter (1984), among others. One strand of the literature focuses on the adverse selection costs of issuing equity resulting from the differences between the managers' information and the market's information about firm value (e.g. Myers and Majluf, 1984). In this literature, IPO (and SEO) volume is driven by time variation in the amount of asymmetric information.[3] In contrast, IPO waves in our model are obtained under perfect information symmetry.

Another possibility is that many firms go public when they need capital for investment. Supporting empirical evidence is provided by Choe, Masulis, and Nanda (1993) and Lowry (2003), for example, although other studies, e.g. Helwege and Liang (2003), report evidence to the contrary. To the extent that firms invest more when they expect higher profits, this idea is captured through the cash flow channel in our model. Zingales (1995) focuses on the corporate control aspect of an IPO, which is absent from our model. Benninga, Helmantel, and Sarig (2003) model the tradeoff between private benefits of control and the diversification benefit of going public, and derive implications for optimal IPO timing that overlap with the implications of our cash flow channel. IPO waves can also obviously arise if technological innovations cluster in time. In our model, only one idea is born each period, and IPO waves instead arise due to clustering in the inventors' optimal IPO timing.

Some studies argue that IPO volume fluctuates in response to market mispricing.[4] The necessary assumption is that the periodic misvaluation can somehow be detected by the owners of the firms going public but not by the investors providing IPO funds. In this literature, market returns are predictable due to time variation in investor sentiment. We also assume that returns are predictable, but we differ on the source of this predictability. We model investor preferences using a habit formation model similar to Campbell and Cochrane (1999), so that time variation in expected market returns is driven by time-varying risk aversion of the representative investor. We do not need to take a firm stand on which of the two interpretations of predictability is closer to the truth, because whether expected returns vary for rational or behavioral reasons, their implications for IPO volume in our model are the same. Our modeling follows the rational avenue because it is tractable and because we feel no need to depart from the rational model when it is not necessary.

[3]See, for example, Lucas and McDonald (1990), Choe, Masulis, and Nanda (1993), Bayless and Chaplinsky (1996), Hoffmann-Burchardi (2001), and Lowry (2003). For other information-based models, see Persons and Warther (1997), Chemmanur and Fulghieri (1999), Subrahmanyam and Titman (1999), Stoughton, Wong, and Zechner (2001), Lowry and Schwert (2002), Benveniste, Busaba, and Wilhelm (2002), and Alti (2003).

[4]See, for example, Ritter (1991), Loughran and Ritter (1995), Rajan and Servaes (1997, 2003), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000), and Lowry (2003).

Jovanovich and Rousseau (2001) present a model in which delaying an IPO is valuable because it allows a private firm to learn about a parameter of its own production function. The idea that investment may be delayed due to learning is also discussed in the literature on irreversible investment under uncertainty (e.g. Cukierman, 1980, and Bernanke, 1983).[5] In our model, learning about the project does not begin until the IPO, and the option to delay an IPO is valuable due to time variation in aggregate market conditions.

The paper is organized as follows. Section 2 develops our model of stock valuation under time-varying investment opportunities. Section 3 discusses the decision to go public and analyzes some properties of optimal IPO timing together with IPO valuation. Section 4 uses a long simulated sample to investigate some properties of IPO waves in our model. Section 5 tests the main predictions of our model empirically. Section 6 examines the recent stock price "bubble" using our valuation model. Section 7 concludes.

2. Valuing Publicly Traded Firms

This section develops a stock valuation model that is used in Section 3 in the analysis of optimal IPO timing. We build on the model of Pástor and Veronesi (2003, henceforth PV), in which stock prices depend on expected profits, expected returns, and prior uncertainty about average profitability. We extend the PV model to allow for time variation in all three quantities at the aggregate level. This time variation generates IPO waves in Section 4.

2.1. Time-Varying Profitability

Consider a publicly traded firm i whose profits are protected by a patent until time T_i. Let $\rho_t^i = Y_t^i / B_t^i$ denote the firm's instantaneous profitability at time t, where Y_t^i denotes the earnings rate and B_t^i denotes the book value of equity. We take the firm's investment policy as given and assume that profitability follows a mean-reverting process until time T_i:

$$d\rho_t^i = \phi^i \left(\overline{\rho}_t^i - \rho_t^i \right) dt + \sigma_{i,0} dW_{0,t} + \sigma_{i,i} dW_{i,t}, \tag{1}$$

where $W_{0,t}$ and $W_{i,t}$ are uncorrelated Wiener processes capturing systematic ($W_{0,t}$) and firm-specific ($W_{i,t}$) components of the random shocks that drive the firm's profitability. Mean

[5] The option to delay investment is also studied by Brennan and Schwartz (1985), McDonald and Siegel (1986), Ingersoll and Ross (1987), Dixit (1992), Abel et al (1996), and Berk (1999). See also Shleifer (1986), Gale (1996), Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2001), and Novy-Marx (2003).

reversion in profitability is consistent with empirical evidence, as discussed in PV. We also assume that the firm's average profitability $\overline{\rho}_t^i$ can be decomposed as

$$\overline{\rho}_t^i = \overline{\psi}^i + \overline{\rho}_t.$$

The firm-specific component $\overline{\psi}^i$, which we refer to as the firm's "average excess profitability," reflects the firm's ability to capitalize on its patent and is assumed constant over time. The common component $\overline{\rho}_t$, referred to as "average aggregate profitability," is assumed to exhibit mean-reverting variation that reflects aggregate economic conditions:

$$d\overline{\rho}_t = k_L \left(\overline{\rho}_L - \overline{\rho}_t\right) dt + \sigma_{L,0} dW_{0,t} + \sigma_{L,L} dW_{L,t}, \tag{2}$$

where $W_{0,t}$ and $W_{L,t}$ are uncorrelated. Periods of high aggregate profitability (which often coincide with economic expansions) are characterized by $\overline{\rho}_t > \overline{\rho}_L$, and vice versa.

The firm is assumed to pay no dividends, to be financed only by equity, and to issue no new equity. These assumptions are made mostly for analytical convenience; relaxing them would add complexity with no obvious new insights.[6] The clean surplus relation then implies that book equity grows at the rate equal to the firm's profitability:

$$dB_t^i = Y_t^i dt = \rho_t^i B_t^i dt. \tag{3}$$

2.2. Time-Varying Equity Premium

We model agents' preferences using a habit formation model similar to Campbell and Cochrane (1999, henceforth CC), which implies that the equity premium varies over time due to time-varying risk aversion. Aggregate consumption C_t follows the process

$$dc_t = \left(b_0 + b_1 \overline{\rho}_t\right) dt + \sigma_c dW_{0,t}, \tag{4}$$

where $c_t = \log\left(C_t\right)$. Consumption growth is allowed to depend on average aggregate profitability because such a link might seem plausible ex ante, but none of our results rely on this link.[7] Consumption is not linked to production in any other way; as other recent studies, we

[6]Dividends can be easily introduced with no effect on our conclusions. In fact, the calibration of the "bubble" in Section 6 uses the version of the model that includes dividends. Debt financing can be allowed as long as its dynamics do not affect the firm's profitability process. New equity can also in principle be issued after the IPO. PV explain that if the firm raises more capital when expected profits are high and pays higher dividends when expected profits are low, then the firm's market value becomes even more convex in expected profitability, and prior uncertainty has an even bigger positive effect on firm value.

[7]As discussed later, our calibration uses the data-implied value of b_1. This value turns out to be small ($b_1 = 0.08$ in Table 1), and using $b_1 = 0$ leads to the same qualitative conclusions throughout.

assume that consumption is financed mostly by income (e.g. labor income) that is outside our model. Markets are complete, and all agents have identical information and preferences. All agents are endowed with the habit utility function

$$U\left(C_t, X_t, t\right) = e^{-\eta t} \frac{\left(C_t - X_t\right)^{1-\gamma}}{1-\gamma}, \tag{5}$$

where X_t is an external habit index, γ regulates the local curvature of the utility function, and η is a pure time discount parameter. Following CC, we work with the surplus consumption ratio $S_t = \left(C_t - X_t\right)/C_t$. The stochastic discount factor (SDF) π_t can then be written as

$$\pi_t = U_C\left(C_t, X_t, t\right) = e^{-\eta t}\left(C_t S_t\right)^{-\gamma} = e^{-\eta t - \gamma\left(c_t + s_t\right)} \tag{6}$$

where $s_t = \log\left(S_t\right)$. CC assume that s_t follows a mean-reverting process with time-varying volatility and perfect correlation with unexpected consumption growth. This specification allows CC to solve for market prices numerically. To obtain analytical solutions for prices, even in the presence of learning, we depart from CC and assume that

$$s_t \equiv s\left(y_t\right) = a_0 + a_1 y_t + a_2 y_t^2, \tag{7}$$

where y_t is a state variable driven by the following mean-reverting process:

$$dy_t = k_y\left(\overline{y} - y_t\right)dt + \sigma_y dW_{0,t}. \tag{8}$$

As shown in the appendix, high values of y_t imply a low volatility of the SDF and thus a low equity premium. The appendix also describes the parameter restrictions that we impose on $s\left(y\right)$ to ensure that the model is consistent with the habit formation setting, namely that $S_t \in (0,1)$ for all t, and that $s\left(y\right)$ in (7) is increasing in y for all plausible values of y.

2.3. Time-Varying Prior Uncertainty About Average Profitability

Average excess profitability $\overline{\psi}^i$ is unobservable, and all agents learn about $\overline{\psi}^i$ over time. For any firm i that goes public at time t, all agents have a common prior on $\overline{\psi}^i$, with prior uncertainty equal to $\widehat{\sigma}_t$. $\widehat{\sigma}_t$ is the same for all firms going public at time t, for simplicity. It seems plausible for $\widehat{\sigma}_t$ to vary over time. For example, uncertainty about the $\overline{\psi}^i$'s of new firms is large when these firms cluster in an industry that has experienced technological advances whose long-term impact is uncertain. We assume that average aggregate profitability $\overline{\rho}_t$ is observable, so it can also be referred to as "expected aggregate profitability."[8]

[8]Unobservable $\overline{\rho}_t$ can be incorporated at the cost of a significant increase in complexity but with little benefit for the purpose of this paper. It can be shown that higher uncertainty about $\overline{\rho}_t$ increases expected cash flow but also increases the discount rate, resulting in a relatively small net effect on prices. Veronesi (2000) discusses these effects in a different framework.

We assume that $\widehat{\sigma}_t$ takes values in the discrete set $\mathcal{V} = \{v^1, ..., v^n\}$, and that it switches from one value to another in each infinitesimal interval Δ according to the transition probabilities $\lambda_{hk}\Delta = \Pr\left(\widehat{\sigma}_{t+\Delta} = v^k | \widehat{\sigma}_t = v^h\right)$. A discrete state space model is used mainly to allow a reasonably convenient solution for optimal IPO timing in Section 3. In the calibration below, we assume that $\widehat{\sigma}_t$ moves relatively slowly over time between adjacent states.

All agents begin learning about $\overline{\psi}^i$ as soon as firm i begins producing (i.e. immediately after its IPO, as explained in Section 3). Agents learn by observing realized profitability ρ_t^i, as well as c_t, $\overline{\rho}_t$, and ρ_t^j for all firms j that belong to the set $\mathcal{I}_t$ of firms that are alive at time t. The learning process is described by the following lemma, proved in the appendix.

Lemma 1. Suppose the prior of $\overline{\psi}^i$ at time t_0 is normal, $\overline{\psi}^i \sim \mathcal{N}\left(\widehat{\psi}^i_{t_0}, \widehat{\sigma}^2_{t_0}\right)$, and the priors are uncorrelated across firms. Then the posterior of $\overline{\psi}^i$ at any time $t > t_0$ conditional on $\mathcal{F}_t = \{(\rho^j_s, c_s, \overline{\rho}_s) : t_0 \leq s \leq t, j \in \mathcal{I}_t\}$ is also normal, $\overline{\psi}^i|_{\mathcal{F}_t} \sim \mathcal{N}\left(\widehat{\psi}^i_t, \widehat{\sigma}^2_{i,t}\right)$, where

(a) The conditional mean $\widehat{\psi}^i_t = E[\overline{\psi}^i_t|\mathcal{F}_t]$ evolves according to the process

$$d\widehat{\psi}^i_t = \widehat{\sigma}^2_{i,t}\frac{\phi^i}{\sigma_{i,i}}d\widetilde{W}_{i,t}, \tag{9}$$

where $\widetilde{W}_{i,t}$ is the idiosyncratic component of the Wiener process capturing the agents' perceived expectation errors (see equation (31) in the appendix).

(b) The mean squared error $\widehat{\sigma}^2_{i,t} = E\left[\left(\overline{\psi}^i - \widehat{\psi}^i_t\right)^2 |\mathcal{F}_t\right]$ is non-stochastic and given by

$$\widehat{\sigma}^2_{i,t} = \frac{1}{\frac{1}{\widehat{\sigma}^2_{t_0}} + \frac{(\phi^i)^2}{\sigma^2_{i,i}}(t - t_0)} \tag{10}$$

The uncertainty about $\overline{\psi}^i$ declines deterministically over time due to learning. Note that the assumption of uncorrelated priors on $\overline{\psi}^i$ can be relaxed. For example, good news about a firm's $\overline{\psi}^i$ might be bad news for the $\overline{\psi}^i$'s of the firm's competitors, in which case negative prior correlation would seem appropriate. Correlated priors on $\overline{\psi}^i$ would also lead to tractable solutions for prices, but they would have no obvious effect on any of our conclusions.

2.4. Stock Prices and Returns

After its IPO, firm i earns abnormal profits $(\overline{\psi}^i)$ until its patent expires at time T_i. Any abnormal earnings after T_i are assumed to be eliminated by competitive market forces, so that the firm's market value at T_i equals its book value, $M^i_{T_i} = B^i_{T_i}$. (See PV, pp.5-6, for additional discussion.) The firm's market value at any $t \leq T_i$ is $M^i_t = E_t\left[(\pi_{T_i}/\pi_t)B^i_{T_i}\right]$, where the SDF π_t is given in (6). The following proposition is proved in the appendix.

Proposition 1. Let $h_i = T_i - t$ be the time to expiration of the patent of firm i. Then

(a) The firm's ratio of market value of equity to book value of equity is given by

$$\frac{M^i_t}{B^i_t} = G^i\left(h_i, \rho^i_t, \overline{\rho}_t, \widehat{\psi}^i_t, y_t, \widehat{\sigma}_{i,t}\right), \tag{11}$$

where

$$G^i\left(h_i, \rho^i_t, \overline{\rho}_t, \widehat{\psi}^i_t, y_t, \widehat{\sigma}_{i,t}\right) = Z^i\left(h_i, \rho^i_t, \overline{\rho}_t, \widehat{\psi}_t, y_t\right) \times e^{\frac{1}{2}K_\phi(h_i)^2\widehat{\sigma}^2_{i,t}} \tag{12}$$

and $Z^i\left(h_i, \rho^i_t, \overline{\rho}_t, \widehat{\psi}_t, y_t\right)$ is defined in equation (32) in the appendix.

(b) The firm's excess stock returns follow the process

$$dR^i_t = \mu_R\left(y_t, h_i\right)dt + \sigma_{R,0}\left(y_t, h_i\right)d\widetilde{W}_{0,t} + \sigma_{R,L}\left(h_i\right)d\widetilde{W}_{L,t} + \sigma_{R,i}\left(\widehat{\sigma}_{i,t}, h_i\right)d\widetilde{W}_{i,t}, \tag{13}$$

where $d\widetilde{W}_{j,t}$'s are Wiener processes defined by the agents' expectation errors (see equation (31) in the appendix), and the explicit formulas for expected excess return μ_R, systematic volatility $\sigma_{R,0}$, and the components of idiosyncratic volatility $\sigma_{R,L}$ and $\sigma_{R,i}$ are given in equations (33) through (36) in the appendix.

What determines market prices? Proposition 1 implies that M/B is high if cash flows are high, if the discount rate is low, and if uncertainty is high. As for cash flows, M/B increases with expected aggregate profitability, $\overline{\rho}_t$, with expected excess profitability, $\widehat{\psi}^i_t$, as well as with current profitability, ρ^i_t. As for the discount rate, M/B depends on the state variable y, which captures risk tolerance. This dependence is nontrivial, but we find numerically that M/B increases with y in the calibrated model, as expected. High y implies low risk aversion, and thus a low equity premium and high prices. The fact that prices are high as a result of high expected cash flow or low discount rate is not surprising, of course. What seems more interesting is that M/B also increases with $\widehat{\sigma}_{i,t}$, as clearly seen from equation (12). The intuition behind this relation, first documented in PV, is provided in footnote 2.

Return volatility in equation (13) has three components. Systematic volatility $\sigma_{R,0}\left(y_t, h_i\right)$, given in equation (34) in the appendix, declines with y in our calibrated model. Firm-specific volatility $\sigma_{R,i}$ depends crucially on uncertainty $\widehat{\sigma}_{i,t}$, as shown in equation (36). Higher $\widehat{\sigma}_{i,t}$ makes the perceived $\overline{\psi}^i$ more volatile, which increases return volatility. This additional volatility is idiosyncratic because $d\widehat{\psi}^i_t$ in equation (9) is uncorrelated with consumption. The third component of volatility, $\sigma^i_{R,L}\left(h_i\right)$, is small and essentially constant over time.

2.5. A Long-Lived Firm

The analysis so far pertains to patent-owning firms that recently went public. Newly listed firms constitute only a small fraction of the market (e.g. Lamont, 2002). To be able to relate IPO waves to the market as a whole, we assume the existence of a "long-lived" firm, which represents the rest of the market. Let B_t^m denote this firm's book value, and D_t^m its dividends at time t. The firm's dividend yield, $c^m = D_t^m / B_t^m$, is constant, and its instantaneous profitability is $\overline{\rho}_t$. Clean surplus implies that $dB_t^m = (Y_t^m - D_t^m)\, dt = (\overline{\rho}_t - c^m)\, B_t^m dt$, and the pricing formula is $M_t^m = E_t \left[\int_t^\infty \pi_s / \pi_t D_s^m ds\right]$. It is shown in the appendix that

$$\frac{M_t^m}{B_t^m} \equiv c^m \int_0^\infty Z^m (s, \overline{\rho}_t, y_t)\, ds, \tag{14}$$

where $Z^m (s, \overline{\rho}_t, y_t)$ is given in equation (37) in the appendix. Excess returns follow

$$dR_t^m = \mu_R^m (y_t, \overline{\rho}_t)\, dt + \sigma_{R,0}^m (y_t, \overline{\rho}_t)\, d\widetilde{W}_{0,t} + \sigma_{R,L}^m (y_t, \overline{\rho}_t)\, d\widetilde{W}_{L,t}, \tag{15}$$

where μ_R^m, $\sigma_{R,0}^m$, and $\sigma_{R,L}^m$ are given in equations (38) and (39) in the appendix.

2.6. Some Proxies

Our explicit pricing formulas help us construct useful empirical proxies. Two key variables of interest that are unobservable in the data are the equity premium and prior uncertainty $\widehat{\sigma}_t$. Note from equations (14) and (15) that neither the market value nor the volatility of the long-lived firm depends on $\widehat{\sigma}_t$. In contrast, both M/B and the volatility of IPOs are strongly positively related to $\widehat{\sigma}_t$. This distinction suggests two proxies for $\widehat{\sigma}_t$. One proxy, $\text{NEWVOL}_t = \sigma_{R,t}^{ipo} - \sigma_{R,t}^m$, compares the return volatilities of IPOs and the long-lived firm, and the other proxy compares their M/B ratios: $\text{NEWMB}_t = \log\left(M_t^{ipo}/B_t^{ipo}\right) - \log\left(M_t^m/B_t^m\right)$.[9] The intuition that both NEWVOL and NEWMB should increase with $\widehat{\sigma}_t$ is confirmed numerically in our calibrated model. In the long simulated sample discussed in Section 4., NEWVOL and NEWMB exhibit high positive correlations (0.84 and 0.75) with $\widehat{\sigma}_t$, while their correlations with the other two state variables are much lower: 0.04 with expected market return and zero with $\overline{\rho}_t$ for NEWVOL, -0.28 with expected market return and 0.12 with $\overline{\rho}_t$ for NEWMB. (All correlations are computed for first differences because those are used in the empirical work.) Our proxies for prior uncertainty thus have solid theoretical motivation.

[9] Jovanovich and Rousseau (2002, 2003) focus on a related quantity, the dispersion in Q (which is closely related to M/B) across firms. They find that most merger waves in the 20th century were preceded by a rise in this dispersion. They also argue that this dispersion increases due to arrival of new technologies.

One of our proxies for the equity premium is market return volatility (MVOL). The equity premium and MVOL are highly correlated in our model because both variables decrease with y_t for all plausible values of y_t. Both variables also increase with $\overline{\rho}_t$, but the effect is weak. In our simulation, MVOL is highly positively correlated (0.91) with the equity premium, but not with the other two state variables (the correlation is 0.06 with $\overline{\rho}_t$, and zero with $\widehat{\sigma}_t$). The usefulness of MVOL as a proxy is underlined by the fact that its empirical estimates have reasonably high precision. Another proxy for changes in the equity premium is realized market returns, motivated by the fact (e.g. Campbell and Ammer, 1993) that market returns seem to respond more to news about discount rates than to news about cash flows.

2.7. Calibration

This subsection describes the parameters chosen to calibrate the model. All parameters are summarized in Table 1, together with some implied aggregate quantities. We use data on quarterly real aggregate consumption and aggregate profitability between 1966Q1 and 2002Q1 to estimate the parameters for c_t in equation (4) and for $\overline{\rho}_t$ in equation (2). Both series are described in the appendix. The Kalman filter is applied to the discretized versions of the respective processes. The estimated parameters imply expected consumption growth of 2.37% and volatility of 0.94% per year. For profitability, we obtain $\overline{\rho}_L = 12.16\%$ per year, $k_L = 0.1412$, and $\sigma_{LL} = 0.64\%$ per year.[10] We impose $\sigma_{L,0} = 0$, which implies zero correlation between $\overline{\rho}_t$ and y_t. This innocuous restriction (unconstrained estimation produces $\sigma_{L,0}$ very close to zero) makes it easier to interpret our subsequent results, as all three state variables that drive IPO volume ($\overline{\rho}_t$, y_t, and $\widehat{\sigma}_t$) are independent of each other.

The agents' preferences are characterized by the processes for s_t in equation (7), y_t in equation (8), and by the utility parameters η and γ. The parameters are chosen to calibrate the expected return, volatility, and M/B of the long-lived firm to their respective empirical values for the market, while producing reasonable properties for the real risk-free rate. Our values $\overline{y} = -.0017$ and $\sigma_y = .5156$ imply the average equity premium of 6.8% and market volatility of 15% per year, and the speed of mean reversion $k_y = .073$ implies a half-life of 9.5 years for y_t. We use a 10% dividend yield c^m for the long-lived firm. The resulting average aggregate M/B is 1.7, equal to the time-series average in the data. The average risk-free rate is 3.3% per year. The volatility of the risk-free rate is 3.9%, which is slightly higher than in the data (as is common in models with habit utility) but still reasonable.

[10]The speed of mean reversion k_L implies a half-life of about 4.9 years. That is, given any starting value $\overline{\rho}_0$, it takes on average 4.9 years for $\overline{\rho}_t$ to cover half the distance between $\overline{\rho}_0$ and its central tendency $\overline{\rho}_L$.

The parameters for individual firm profitability ρ_t^i in equation (1) are chosen to match the median firm in the data. We use $\phi^i = 0.3968$, estimated by PV. PV also report an 8.34% per year median volatility of the AR(1) residuals for individual firm profitability. We decompose this volatility into $\sigma_{i,0} = 4.79\%$ and $\sigma_{i,i} = 6.82\%$ per year, which implies a M/B of 1.7 for a firm with 15 years to patent expiration and $\widehat{\psi}_t^i = 0$ when $\widehat{\sigma}_t = 0$, $y_t = \bar{y}$, and $\rho_t^i = \overline{\rho}_t = \overline{\rho}_L$. Finally, prior uncertainty $\widehat{\sigma}_t$ moves along the grid $\mathcal{V} = \{0, 1, ..., 12\}$ % per year. The transition probabilities are such that there is 10% probability in any given month of $\widehat{\sigma}_t$ moving up or down to an adjacent value in the grid. If $\widehat{\sigma}_t$ hits the boundary of the grid, there is a 20% probability of moving away from the boundary. This specification is adopted for simplicity. One alternative approach would link $\widehat{\sigma}_t$ to the number of recent IPOs, as those might in principle help agents learn about the prospects of new firms. Such an approach would complicate the optimal IPO timing problem, but it would lead to the same pricing formula, and we believe that none of the model's implications would change.

3. Optimal IPO Timing

There are two classes of agents, "inventors" and "investors". Investors are endowed with the stream of consumption good given in equation (4). Inventors are endowed with the ability to invent ideas that can deliver abnormal profits. Inventors compete so it is always optimal to patent a new idea as soon as it is discovered. Upon patenting his idea, an inventor starts a private firm that owns the patent. The private firm produces no revenue because the inventor lacks the capital necessary to begin production. This capital is raised in an IPO, in which the private firm is sold to investors. Production begins immediately after the IPO, generating profits described in (1). The inventor times the IPO to maximize the value of his patent. The basic tradeoff is that delaying the IPO forfeits abnormal profits, but it may be optimal if investment opportunities are expected to improve sufficiently in the future.

Maximizing the value of the patent is optimal for the inventor as it increases the value of his endowment, and thus maximizes his life-time utility of consumption, given in equation (5). Complete markets allow investors to insure the pre-IPO consumption of inventors using contingent claims. Assuming that their endowments are equally valuable, investors and inventors consume equally, justifying the representative agent framework in Section 2.2.

The capital necessary for production is raised by issuing equity because the inventor has a strong incentive to diversify. If he instead borrowed and began producing, his entire wealth would become driven by idiosyncratic shocks ($d\widetilde{W}_{i,t}$) that cannot be hedged unless equity is issued in an IPO. Standard risk-sharing arguments imply that the inventor wants to issue some equity, so the IPO takes place. To simplify the exposition, we assume that the inventor sells all of his ownership in the IPO, but it is easy to show that all implications of our model are identical if the inventor retains any fraction of ownership after the IPO.

Figure 2 summarizes the sequence of events. At time t_i, the idea is invented and patented. The patent enables the owner to earn average excess profitability $\overline{\psi}^i$ until time T_i, but production requires capital B^{t_i} that must be raised in an IPO.[11] At time τ_i, $t_i \leq \tau_i \leq T_i$, the inventor decides to go public and files the IPO. The IPO itself takes place at time $\tau_i + \ell$. The lag ℓ reflects mostly the time required by the underwriter to conduct the "road show". We choose $\ell = 3$ months.[12] Only the inventor knows how to invest B^{t_i} (e.g. what machine to buy or construct), and this knowledge is too complex to be sold, so B^{t_i} must be invested by the inventor at the IPO. This assumption rules out pre-IPO patent sales. Once the investment B^{t_i} is made, it is irreversible in that the project cannot be abandoned.[13]

In the IPO, the inventor sells the firm to investors for its fair market value $M^i_{\tau_i+\ell}$, given in Proposition 1, and pays a proportional underwriting fee $f = 0.07$.[14] The inventor's payoff is thus $M^i_{\tau_i+\ell}(1-f) - B^{t_i}$, the value of the patent net of fees. The inventor chooses the time to go public to maximize his expected payoff, properly discounted. The value of the patent net of fees at any time t, $t_i \leq t \leq T_i$, is given by

$$V\left(\overline{\rho}_t, y_t, \widehat{\sigma}_t, T_i - t\right) = \max_{\tau_i} E_t \left(\frac{\pi_{\tau_i+\ell}}{\pi_t} \left(M^i_{\tau_i+\ell}(1-f) - B^{t_i} \right) \right) \tag{16}$$

The optimal IPO timing problem bears resemblance to the pricing of American options, as the inventor solves for the optimal stopping time, or the best time to exercise his option/patent. Due to the time-inhomogeneity of the problem and the complexity of the pricing function, we solve for V and the optimal stopping time numerically. The value of the patent must satisfy the standard Euler equation $E_t\left[d\left(\pi_t V_t\right)\right] = 0$. The appendix shows that this condition

[11]For simplicity, we assume that B^{t_i} is proportional to the book value of the long-lived firm, $B^{t_i} = qB^m_{t_i}$, with $q = 0.0235\%$. Every month between January 1960 and December 2002, the book value of new lists (ordinary common shares that first appear on CRSP in that month) is divided by the total book value. The time-series average of the monthly ratios is 0.0235%, excluding the spikes in July 1962 and December 1972 when AMEX and Nasdaq were added to CRSP. The exact value of q is not important for any of our conclusions. As long as q is reasonably small, the long-lived firm accounts for the bulk of the market portfolio.

[12]Lowry and Schwert (2002) report that the average time between the IPO filing and offer dates between 1985 and 1997 is 72 days. The median is 63 days, the minimum 11 days, and the maximum 624 days.

[13]Schwartz (2001) finds that the option to abandon a patent-protected project often represents a substantial part of the project's value. This option is not considered here, for tractability (options on options are not easy to value), but if it were, it would further increase IPO valuations, supporting one of our main conclusions.

[14]Chen and Ritter (2000) find that in 91% of the U.S. IPOs raising between \$20 and \$80 million (and in 77% of all IPOs) between 1995 and 1998, the gross spreads received by underwriters were exactly 7%. IPO underpricing can also be incorporated by using a bigger f without affecting our qualitative results.

translates into a system of partial differential equations, one for each possible uncertainty state $\widehat{\sigma}_t \in \mathcal{V} = \{v^1, ..., v^n\}$. Using the final condition that the patent is worthless at T_i if not exercised before, we work backwards to compute V_t for every combination of the state variables on a fine grid. See the appendix for details.

3.1. When Do Firms Go Public?

Figure 3 plots the combinations of the equity premium and $\overline{\rho}_t$ for which the inventor optimally decides to go public. Each line denotes the locus of points that trigger the IPO decision, or the "entry boundary." Firms go public for all pairs of the equity premium and $\overline{\rho}_t$ that lie inside the "entry region" north-west of the entry boundary. If the idea is born when market conditions are inside the entry region, an IPO is filed immediately. Otherwise, the inventor waits until market conditions improve, and an IPO is filed as soon as the entry boundary is reached. If this never happens before the patent expires, the firm never goes public.

The top left panel considers a firm with $\widehat{\psi}_t^i = 0$ and a patent with $T = 15$ years to expiration. The entry boundary is upward sloping, so if the equity premium increases, $\overline{\rho}_t$ must also increase to trigger entry. The entry boundary moves south-east as prior uncertainty $\widehat{\sigma}_t$ increases. Both effects are intuitive. At any point in time, the inventor compares the option value of delaying the IPO with the value of the abnormal profits given up by waiting. The option to wait is valuable when investment opportunities are bad and the patent's market value is low. IPOs take place when investment opportunities improve so that the option to wait is not valuable enough. Indeed, Figure 3 shows that IPOs take place when the cost of capital is low, when expected profitability is high, or when prior uncertainty is high.

The other three panels in Figure 3 tell the same story, with some additional insights. The top right panel plots the entry boundaries for three different values of expected excess profitability $\widehat{\psi}_t^i$, with $\widehat{\sigma}_t = 0$. Higher values of $\widehat{\psi}_t^i$ expand the entry region by shifting the entry boundary south-east, which is intuitive because a more profitable patent has a higher opportunity cost of waiting for an improvement in the investment environment. The bottom panels focus on the effects of time to the patent's expiration, T. As time passes and T declines, the entry boundary moves south-east, lowering the hurdle for entry. This is also intuitive. As the patent ages, expected improvements in investment opportunities become smaller and the need to begin capitalizing on the patent becomes more pressing.

3.2. Valuing IPOs

Figure 4 plots a firm's M/B at the time of optimal entry. For each value of the equity premium on the x axis, $\bar{\rho}_t$ is chosen from the optimal entry boundary (Figure 3). Note that M/B depends nontrivially on the underlying state variables. If the entry decision were made based on a simple cutoff rule, such as 'go public as soon as M/B exceeds 3', the plots of M/B would be overlapping flat lines. Instead, M/B increases with the equity premium along the entry boundary. As the premium increases, $\overline{\rho}_t$ also increases (see Figure 3), and the resulting gain in market value more than offsets the loss due to the larger premium. In sum, inventors pay attention to the overall market conditions, not just to current market prices.

The main message from Figure 4 is that IPOs command high market prices in our model. Recall from the calibration that when y_t, ρ_t^i, and $\overline{\rho}_t$ are at their unconditional values and $\widehat{\sigma}_t = 0$, a firm with $\widehat{\psi}_t^i = 0$ and $T = 15$ has a M/B of 1.7. Most M/B values in Figure 4 are higher, with M/B of 5 in some cases (for higher M/Bs, see Section 6). The reason is the endogeneity of IPO timing, as IPOs take place when market conditions are good. For example, when $\widehat{\sigma}_t = \widehat{\psi}_t^i = 0$, $T = 15$, and $\overline{\rho}_t$ is at its unconditional value of $\overline{\rho}_L = 12.16\%$, optimal entry in Figure 3 occurs at the equity premium of about 3%, substantially below its unconditional value of 6.8%. An immediate implication is that M/B should decline after the IPO, on average, as the underlying state variables revert to their central tendencies.

IPO market values are high not only due to low discount rates and high expected profits, but also thanks to prior uncertainty about $\overline{\psi}^i$. As shown in the top left panel of Figure 4, M/B at the IPO increases with $\widehat{\sigma}_t$, as an outcome of two countervailing effects. Prior uncertainty raises M/B, holding other things equal, but other things are not equal because the entry boundary shifts south-east as $\widehat{\sigma}_t$ increases (Figure 3). In this shift, $\overline{\rho}_t$ goes down for any given equity premium, reducing the market value at entry. The reduced $\overline{\rho}_t$ apparently matters less due to mean reversion in $\overline{\rho}_t$. Also note that prior uncertainty gives a second reason why M/B should decline after the IPO, on average. As soon as the firm begins generating observable profits, the market begins learning about $\overline{\psi}^i$, which reduces uncertainty (see equation 10) and thus also M/B (see Proposition 1).[15] Despite their projected decline, the high IPO valuations are perfectly rational, of course. Investors buy expensive IPO shares because the fair value of expected future cash flows is high when $\widehat{\sigma}_t$ is high (Proposition 1).

More profitable patents clearly result in higher IPO valuations, as shown in the top right

[15] This channel was first discussed in PV, who also find empirically that younger firms have higher M/B than older firms, ceteris paribus, and attribute this finding to learning about average profitability.

panel of Figure 4. For example, with a 4% equity premium, a firm with $\widehat{\psi}^i = 0$ commands M/B of about 3, while a firm with $\widehat{\psi}^i = 4\%$ goes public at M/B of 4.5. Finally, the bottom panels of Figure 4 confirm the intuition that patents with less time to expiration (lower T) are less valuable, leading to lower M/B at the IPO.

4. IPO Waves

IPO waves arise naturally in our model as a result of optimal IPO timing. This section analyzes the properties of IPO waves in a simulated environment. To make IPO waves endogenous, we assume that the pace of innovation is constant, so that exactly one idea is invented and patented each month. New patents do not make existing patents obsolete, but ideas become obsolete (i.e. unable to generate abnormal profits) immediately after their patent expires. To introduce heterogeneity across ideas in their excess profitability, we draw $\widehat{\psi}_t^i$ randomly from the set $\{-6, -4, \ldots, 4, 6\}$ % per year with equal probabilities. Ideas are born into different economic conditions, as investment opportunities change month to month. To decide when to go public, patent owners solve the optimal timing problem described in the previous section. To obtain population values for the variables of interest, we simulate 10,000 years (120,000 months) of data.

Figure 5 illustrates the dependence of IPO volume on the three state variables: the equity premium (left panels), expected aggregate profitability $\overline{\rho}_t$ (center panels), and prior uncertainty $\widehat{\sigma}_t$ (right panels), all in percent per year. To focus on the individual effects, only one state variable is allowed to vary over time in each panel; the other two variables are fixed to their long-term averages (for equity premium and $\overline{\rho}_t$) or to zero (for $\widehat{\sigma}_t$). For illustration purposes, we plot typical 100-year segments of simulated data.

The panels on the left show that IPO volume responds strongly to time variation in the equity premium. Few if any firms go public after increases in the premium, but many firms go public when the cost of capital declines. A decline in the premium represents an improvement in investment opportunities, which may draw some patents into their entry region (Figure 3). Most of the time, there is a 'backlog' of private firms waiting for economic conditions to improve. A drop in expected return that is either dramatic or persistent induces many of these firms to go public at about the same time, creating an IPO wave. The figure shows periods as long as five years in which no IPOs take place (as the equity premium rises), but also months of feverish IPO activity, with over 20 IPOs per month (as the premium drops).[16]

[16]Since new ideas arrive at the rate of one per month, the average number of IPOs in our simulations

The center panels show that increases in expected aggregate profitability $\overline{\rho}_t$ tend to be followed by more IPOs, and vice versa. However, this cash flow effect is substantially weaker than the effect of changing expected returns, for two reasons. First, $\overline{\rho}_t$ exhibits less variation than the equity premium. The statistical process for $\overline{\rho}_t$ is calibrated using parameters obtained from aggregate profitability data, as explained earlier, and aggregate profitability in the data is relatively stable (see Figure 6, to be discussed). Second, $\overline{\rho}_t$ reverts to its mean faster than the variable y_t that drives expected returns (see Table 1). Since changes in $\overline{\rho}_t$ are perceived as shorter-lived, their impact on prices is weaker. The inventor's option to wait for an increase in $\overline{\rho}_t$ is thus less valuable, and he often finds it optimal to file an IPO as soon as the idea is patented. Hence, IPOs in the top center panel of Figure 5 are distributed much more evenly across time than IPOs in the other panels.

The panels on the right show that IPO waves are also caused by increases in prior uncertainty $\widehat{\sigma}_t$. Increases in $\widehat{\sigma}_t$ shift the entry boundary to the right (see Figure 3), inducing some inventors sitting on a fence to enter capital markets. Given the dynamics of $\widehat{\sigma}_t$ specified in Section 2.7., $\widehat{\sigma}_t$ has a powerful effect on IPO volume.

In summary, IPO volume in our model is driven by time variation in the equity premium, prior uncertainty, and expected profits. The first two channels have been largely neglected in the extant literature. The literature has examined the third channel, but this cash flow channel seems to be the weakest of the three in our calibration. Now that we know how IPO waves are generated in our model, we turn to the various properties of IPO waves.

4.1. Simulation Evidence Around IPO Waves

IPO waves are periods in which the number of IPOs is consistently high. Following Helwege and Liang (2003), we calculate three-month centered moving averages in which the number of IPOs in each month is averaged with the numbers of IPOs in the months immediately preceding and following that month. "Hot markets" are defined as months in which the moving average falls into the top quartile across the whole simulated sample. IPO waves are then defined as all sequences of consecutive hot-market months.[17] In our simulated 10,000-year-long sample, there are 4,790 IPO waves whose length ranges from 1 to 23 months, with

is just under one IPO per month (less than one because some patents never go public). In the data, the number of IPOs between January 1960 and December 2002 averages 28.78 per month. Thus, to convert IPO volume in our simulation into a comparable number in the data, multiply it roughly by a factor of 30.

[17]Rarely, a month with zero IPOs can be designated as the first or last month of a wave if the large IPO volume in the neighboring month inflates the moving average. Such months are excluded from the wave.

the median of 2 months and the average of 2.8 months. The maximum number of IPOs in any given month is 65, while the median across months is zero and the average is 0.9.

Since we assume a three-month lag between an IPO filing and the IPO itself, it is also useful to define an IPO "pre-wave" as an IPO wave shifted back in time by three months. Each IPO wave in our model is driven by state variable changes that occur in the respective pre-wave. Let "b" denote the beginning of a wave, or more precisely the last month before the wave begins, and let "e" denote the end of the wave's last month. An IPO wave then begins at the end of month b and ends at the end of month e, whereas an IPO pre-wave begins at the end of month b-3 and ends at the end of month e-3.

Table 2 reports the averages of selected variables around IPO waves. Due to the enormous size of the simulated sample, all averages can be treated as population values, so no p-values are shown. Column 1 of Panel A reports the average change in the given variable during a pre-wave. First, IPO waves are generated by pre-wave changes in the discount rate, as expected total market return declines during a pre-wave by 0.92% per year on average. This decline is due to declines in expected excess return (0.51%) as well as the risk-free rate (0.41%). Expected return declines consistently before the wave and begins increasing shortly before the end of a wave. Waves tend to occur when the cost of capital is low – expected market return during a wave averages 7.00% per year, while the average outside a wave is 10.61%. Second, IPO waves are triggered by pre-wave changes in expected cash flows. Expected profitability rises by 0.06% per year during a pre-wave, and waves tend to occur in periods of above-average expected profitability. The cash flow channel is substantially weaker than the discount rate channel, as observed earlier. Third, IPO waves are initiated by pre-wave changes in prior uncertainty. This uncertainty increases by 0.35% per year during a pre-wave, and waves tend to occur in periods of above-average prior uncertainty. Table 2 thus illustrates the importance of all three channels in generating IPO waves.

Table 2 also examines how our proxies for changes in investment opportunities vary around simulated IPO waves. Market volatility, MVOL, declines during pre-waves by 0.62% per year on average, and waves tend to occur in periods of low MVOL (14.13% during a wave versus 15.38% outside a wave). This result follows from the strong positive relation between expected market return and MVOL in our model, and it helps justify MVOL as a proxy for expected market return in the empirical analysis. The excess volatility and M/B of new firms (NEWVOL and NEWMB) both increase during pre-waves. Both variables are closely related to prior uncertainty in our model, and they will proxy for prior uncertainty in the empirical analysis.[18] The table also shows that the market's aggregate M/B (defined as the sum of earnings divided by the sum of book values across all firms) increases during pre-waves, by 0.13 on average. IPO waves take place when the market is valued highly – the aggregate M/B equals 2.02 during a wave, on average, versus 1.69 outside a wave.

Realized market returns should be unusually high before IPO waves due to declines in expected returns and increases in expected profitability. Indeed, Panel B of Table 2 shows that average returns are significantly higher during pre-waves than outside pre-waves: 38.76% versus 6.79% per year. Market returns are low during and after IPO waves – total returns average 7.69% per year during a wave and 8.60% over the first three post-wave months, substantially less than the 10.72% average outside a wave. There are two reasons behind the low market returns after pre-waves. First, these returns are expected to be low if the wave is caused by a pre-wave decline in expected return. Second, investment opportunities typically begin deteriorating during the wave in light of the endogeneity of IPO timing – if investment opportunities didn't get worse, the wave would have continued.

4.2. Regression Analysis

Table 3 further analyzes the determinants of IPO volume. Each column reports the coefficients from a regression of the number of IPOs on the variables listed in the first column, all simulated from our calibrated model. Although the model is simulated at a monthly frequency, all variables are cumulated to the quarterly frequency so that Table 3 matches its empirical counterpart, Table 5. No p-values are shown because all coefficients are highly statistically significant due to the size of the simulated sample (40,000 quarters).

Let us first examine the discount rate channel for generating IPOs. As shown in column 1, IPO volume increases after declines in expected market return over the previous two quarters. Column 5 shows that IPO volume also increases after declines in the risk-free rate. As seen from column 6, declines in market return volatility (which proxy for declines in expected market return) also tend to be followed by more IPOs. The results in column 9 also support the discount rate channel: IPO volume is positively related to past market returns, but negatively related to future and current returns. Realized returns are high while

[18] Computing NEWVOL and NEWMB requires at least one IPO in the given month. Since only one idea is born each month, our simulated sample includes many months with zero IPOs, especially before IPO waves. To avoid missing observations in the months with the biggest improvements in investment opportunities, we assume that only one firm with $T = 15$ and $\widehat{\psi}_t^i = 0$ is born in any month t into the current market conditions summarized by y_t, $\overline{\rho}_t$, and $\widehat{\sigma}_t$. This assumption is made for the purpose of constructing NEWVOL and NEWMB only, and it provides a cleaner assessment of these proxies for $\widehat{\sigma}_t$ than any obvious alternatives.

expected return drops, but they are low after the drop stops.

The cash flow and uncertainty channels are also clearly demonstrated in Table 3. As shown in column 2, IPO volume is high after increases in expected profitability $\bar{\rho}$. The results in column 9, discussed in the previous paragraph, are also consistent with the cash flow channel. IPO volume is also high after increases in prior uncertainty $\widehat{\sigma}$, as shown in column 3, as well as after increases in the excess volatility and M/B of new firms (columns 7 and 8), both of which proxy for $\widehat{\sigma}$ in our empirical work.

All regressions in Table 3 also include a lag of IPO volume on the right-hand side, to be consistent with the subsequent empirical regressions. This lag is always highly significant, but its removal does not alter any of the above relations. Also note that the R^2's are relatively low, between 0.06 and 0.19, because the true relations between IPO volume and the given variables are complex and nonlinear in our model. We run linear regressions for two reasons – to be consistent with the subsequent empirical regressions, and because they suffice to clearly demonstrate the presence of all three channels that produce IPOs in our model.

5. Empirical Analysis

This section empirically investigates some implications of the model presented earlier. The model suggests three determinants of IPO volume: time-varying expected returns (the discount rate channel), time-varying expected profitability (the cash flow channel), and time-varying uncertainty about average profitability (the uncertainty channel). The empirical evidence is consistent with the existence of all three channels, as shown below.

5.1. Data

The data on the number of IPOs, obtained from Jay Ritter's website, cover the period January 1960 through December 2002. To avoid potential concerns about nonstationarity (see Lowry, 2003), we deflate the number of IPOs by the number of public firms at the end of the previous month.[19] In the rest of the paper, "the number of IPOs" and "IPO volume"

[19] All individual stock price data is obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. We define public firms as ordinary common shares (CRSP sharecodes 10 or 11) with positive market values. The number of CRSP-listed firms jumps in July 1962 and December 1972 due to the addition of Amex and Nasdaq firms. Following Lowry (2003), we use the actual number of public firms after December 1972, but estimate the number of public firms prior to that by assuming that this number grew at the compounded growth rate of 0.45% per year before December 1972.

both refer to the deflated series, whose values range from zero to 2.1% per month, with an average of 0.5%. The pattern of time variation in the deflated series looks so similar to the pattern in the raw series plotted in Figure 1 that it is not worth plotting separately.

The data on our proxies for changes in investment opportunities are also constructed monthly for January 1960 through December 2002, unless specified otherwise. We use all available data at the time of this writing. Market returns (MKT) are total returns on the value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks, extracted from CRSP. Market volatility (MVOL) is computed each month after July 1962 as standard deviation of daily market returns within the month. The aggregate M/B ratio (M/B) is the sum of market values of equity across all ordinary common shares divided by the sum of the most recent book values of equity. The risk-free rate (RF) is the nominal yield on a one-month T-bill, obtained from Ken French's website. Aggregate profitability (return on equity, ROE) is computed quarterly for 1966Q1 through 2002Q1 using data from Compustat, as described in the appendix. As another measure of cash flow expectations, we use the I/B/E/S summary data on equity analysts' forecasts of long-term earnings growth. These forecasts have horizons of five years or more, which makes them suitable given the relatively long-term nature of $\bar{\rho}$. For each firm and each month, the average forecast of long-term earnings growth is computed across all analysts covering the firm. The forecast of average earnings growth (IBES) is then computed by averaging the average forecasts across all ordinary common shares. The resulting series is available for November 1981 through March 2002. The monthly time series of M/B, MVOL, ROE, and IBES are plotted in Figure 6.

We construct both proxies for prior uncertainty discussed in Section 2.6. New firm excess volatility (NEWVOL) in a given month is computed by subtracting market return volatility from the median return volatility across all new firms, defined as firms whose first appearance in the CRSP daily file occured over the previous month. A given firm's return volatility in each month is the standard deviation of daily stock returns within the month. NEWVOL has 464 valid monthly observations in the 486-month period between July 1962 and December 2002. New firm excess M/B ratio (NEWMB) is computed for each month between January 1950 and March 2002 as follows. First, we compute the median M/B across all new firms, defined as those that appeared in the CRSP monthly file over the previous year.[20] NEWMB is computed as the natural logarithm of that median minus the log of the median M/B across all firms. The construction of M/B for individual firms is described in the appendix.

[20]This definition of new firms ensures availability of their valid M/B ratios. Few firms have valid M/B ratios in the first few months after listing because M/B is computed using lagged book equity, which is often available only on an annual basis and generally available only after market equity becomes available. For both NEWMB and NEWVOL, we require at least three new firms to compute a valid median.

NEWMB has eight missing values between January 1960 and March 2002. The monthly time series of NEWVOL and NEWMB are plotted in Figure 7.

5.2. Empirical Evidence Around IPO Waves

Table 4 reports the averages of selected variables around IPO waves, defined in the same way as in Table 2. Between 1960 and 2002, there are 16 IPO waves whose length ranges from one to 21 months, with a median of 5 months and an average of 8.1 months. The t-statistics, reported in parentheses, capture the significance of the difference between the variable's average in the given period and outside that period. For example, the t-statistic for average MVOL during a wave (-3.18) is computed from a monthly regression of market volatility on a dummy variable equal to one if the month is part of an IPO wave and zero otherwise. A positive (negative) t-statistic reveals that the average in the given period is bigger (smaller) than the variable's average in the full sample. All variables except for the unitless M/B-related variables are expressed in percent per year.

The average pre-wave change in MVOL is significantly negative at -2.81% ($t = -2.27$), consistent with our hypothesis that IPO waves are often caused by declines in expected returns. Moreover, the average MVOL during a wave (12.67%) is significantly lower than outside a wave (15.24%). Aggregate M/B ratio increases during pre-waves by 0.08 ($t = 1.79$), as predicted by the model. M/B is above its full-sample average of 1.7 in and around IPO waves, suggesting that the waves tend to occur when prices are high. Aggregate profitability and IBES both increase before IPO waves, as predicted by the cash flow channel, but neither increase is statistically significant. NEWVOL increases significantly during pre-waves, consistent with the uncertainty channel, but NEWMB does not.

Panel B of Table 4 shows that average market returns are high before IPO waves and low thereafter, as predicted by the model. Market excess returns are significantly higher two quarters before a wave (25.83% annualized with $t = 2.81$), while they are below average during and especially after IPO waves (-5.35% with $t = -1.36$ in the first quarter). This pattern is quite similar to the pattern observed in Table 2, supporting the model.

Since the averages in Table 4 are computed across only 16 IPO waves, only a few relations are statistically significant.[21] More detailed empirical analysis is therefore performed in the

[21]Note that the t-statistics are generally higher in absolute value during IPO waves than in a given month simply as a result of a larger number of observations. For example, there are 129 months that belong to an IPO wave, but only 16 months that correspond to the beginning of a wave.

following section, which focuses on IPO volume rather than on IPO waves alone.

5.3. Regression Analysis

Each column of Table 5 corresponds to a separate regression, in which the number of IPOs in the current quarter is regressed on proxies for changes in investment opportunities. Lagged IPO volume is included on the right-hand side to capture persistence in IPO volume that is unexplained due to any potential misspecification in the regressions. Lowry (2003) also includes lagged IPO volume on the right-hand side of her regressions. She also always includes a first-quarter dummy that captures an apparent seasonality in IPO volume, and we follow her treatment. Both variables are significant in each regression. The t-statistics are in parentheses. Note that Table 5 is an empirical counterpart of Table 3.

First, we test the discount rate channel, in which new IPOs are triggered by declines in expected returns. Column 1 shows that IPO volume is significantly positively related to total market returns over the previous two quarters ($t = 3.34$ and 3.25), consistent with both the discount rate and cash flow channels. Moreover, IPO volume is significantly negatively related to market returns in the subsequent quarter ($t = -2.23$), consistent with the discount rate channel.[22] The relation with current returns is positive, not negative as in Table 3, but this difference does not contradict the model. IPO waves in the data tend to last longer than our simulated IPO waves, perhaps due to clustering in the production of new ideas, which is outside the model. As a result, actual IPO waves have more overlap than simulated waves with the declines in expected returns that caused the waves, and therefore also with high realized returns. Column 2 shows that IPO volume is significantly negatively related to current ($t = -4.41$) as well as past ($t = -3.59$) changes in market volatility, again consistent with the discount rate channel. Since changes in the risk-free rate in column 3 seem unrelated, IPO volume appears to be driven by time variation in the risk premia.

Second, the cash flow channel is also supported by the data. Column 5 shows that IPO volume is positively related to current ($t = 2.50$) as well as future changes in aggregate profitability, suggesting that firms tend to go public when cash flow expectations improve.[23] The same conclusion is reached in column 6: IPO volume is significantly higher ($t = 5.07$) when equity analysts on average upgrade their forecasts of long-term earnings growth.

[22] This negative relation is also reported by Lamont (2002), Schultz (2003), and Lowry (2003).

[23] We include leads rather than lags of ROE in the regression because changes in expectations of future cash flows should be detectable in future cash flows.

Third, prior uncertainty also seems to go up before firms go public. In columns 8 and 9, IPO volume is positively related to recent changes in the excess M/B ratio of new firms ($t = 3.18$ and 2.35) as well as to recent changes in the excess volatility of new firms ($t = 2.23$), both of which are strongly associated with prior uncertainty in our model.

Some of the relations described above lose their statistical significance when realized market returns are included in the regression. The reason goes beyond the simple lost-degrees-of-freedom effect. The right-hand side variables are merely proxies for unobservable changes in expectations and uncertainty. In reasonably efficient markets, where prices reflect much of the available information, realized returns are the best proxy for changes in investment opportunities – when investment opportunities improve, prices go up, and vice versa. It is thus not surprising that including market returns drives some of the weaker proxies below the threshold of significance. The role of these other proxies is simply to provide additional evidence on the likely causes of the observed price changes. To summarize, we confirm empirically that firms go public especially after capital becomes cheaper, after cash flow expectations improve, and after prior uncertainty regarding future growth increases.

6. The Recent Stock Price "Bubble"

On March 10, 2000, the Nasdaq Composite Index closed at its all-time high of 5,048.62. For comparison, the same index reached 1,114 in August 1996 as well as in October 2002. Many practitioners and some academics refer to the time period culminating in March 2000 as the "Nasdaq bubble" because they find it difficult to rationally explain the high valuations. In this section, we calibrate our rational valuation model to the circumstances of March 10, 2000, and show that the observed valuations can be explained using plausible values of prior uncertainty, expected returns, and expected profitability.

To judge the plausibility of our choices, it is important to realize that the peak of the "bubble" was preceded by a decade of frenetic IPO activity. More than 500 firms per year went public in every year between 1992 and 1999 (except for 1998 with 344 IPOs), and 123 firms went public in the first quarter of 2000. There was at least one IPO wave in each year between 1991 and 1997, according to the strict definition presented earlier, and the last IPO wave in our sample occurred in the summer of 1999. The 1990s can thus be loosely characterized as one big IPO wave. According to our model, the unusually high IPO volume in the 1990s must have been caused either by a prolonged decline in expected market returns or by prolonged increases in expected profitability or prior uncertainty. If all three channels were at work, then expected market return in March 2000 was quite low, and both expected profitability and prior uncertainty were quite high. Low discount rates and high cash flow expectations imply high valuations for the market as a whole, and high prior uncertainty implies especially high valuations for firms that recently went public. The high market prices in March 2000 should therefore not be surprising, at least qualitatively.

Several recent studies also argue that expected market return in March 2000 was low. Fama and French (2002) report recent estimates of the equity premium of 2.6% and 4.3% per year. Fama and French (2003) argue that a declining cost of equity capital attracted weaker firms to capital markets recently. At the extreme, Glassman and Hassett (1999) argue that the equity premium declined to zero. Pástor and Stambaugh (2001) estimate the premium of 4.8% at the end of 1999. Welch (2001) surveys 510 academics in 2001 and reports a median equity premium forecast of 3%. Based on this evidence, the values of the equity premium between 2% and 5% per year receive the most attention in our calibration.

At the peak of the "bubble", many Nasdaq firms were expected to deliver high future profitability. Firm profits were strong, as shown in Figure 6, and equity analysts expected unusually high long-term earnings growth. The average forecast of long-term earnings growth (IBES) in March 2000 was 22.77%, higher than ever before. This average forecast is even higher, 28.82%, when computed across Nasdaq firms only. There is also abundant anecdotal evidence that cash flow expectations at the time were very optimistic.[24]

Prior uncertainty was also high, according to its proxies, plotted in Figure 7. NEWMB rises substantially in the late 1990s and declines after 2000. NEWVOL exhibits an even more remarkable pattern: In 1998, it triples from about 2% per day to about 6%, it remains around 6% through the end of 2000, and then it drops back to about 2% after 2000. These extraordinary humps indicate abnormally high prior uncertainty in 1998 through 2000. This is not surprising. The long-term prospects of new firms are uncertain when old paradigms are fading away and a "new era" is being embraced. During the "Internet revolution", investors appeared to be unusually uncertain about future productivity and growth.[25] [26] The high

[24]E.g. "Applegate: This business cycle is extraordinary... Today tech earnings are growing 24%, which is significantly better than the rest of the market. Their prices are accordingly richer... I'm comfortable right here, sticking with the Ciscos, Microsofts and Intels." David Henry, USA Today, 16 December 1999.

[25]E.g. "...the projections of revenue growth were, by and large, wild guesses." New Economy, Bad Math, ... Avital Louria Hahn, Investment Dealers Digest, 23 October 2000. E.g. "The problem is that since we know so little about where the Net is headed, predicting cash flow so far into the future is largely meaningless... investing in this new technology was a bet..." You Believe? ... Fortune Magazine, June 7, 1999.

[26]In 1999, 57.4% of IPOs were carried out by Internet firms, according to Ljungqvist and Wilhelm (2003). Ofek and Richardson (2002, 2003) analyze the rise and fall of Internet stock prices in 1998 through 2000. They also discuss the high return volatility of Internet firms, which is consistent with high prior uncertainty.

prior uncertainty in the late 1990s may have attracted many firms to go public, and it might also have contributed to the exorbitant valuations of many IPOs at that time.

6.1. Matching Nasdaq's Valuation

For each Nasdaq-traded firm, the market value of equity on March 10, 2000 is computed by multiplying the share price by the number of shares outstanding, using the CRSP daily file. The book value of equity at the end of 1999 is computed as described in the appendix. Nasdaq's M/B, computed as the sum of market values of all Nasdaq firms divided by the sum of book values, is equal to 6.85. Matching this large number is the task of this section. Nasdaq's profitability, $\rho_t = 12.79\%$, is computed as the sum of earnings of all Nasdaq firms in 1999 divided by the sum of book values of equity at the end of the previous year. Similarly, Nasdaq's dividend yield, $c = 2.06\%$, is computed as the sum of dividends available to common stockholders of all Nasdaq firms in 1999 divided by the sum of book values at the end of the previous year. Expected aggregate profitability is set equal to the 1999Q4 value of aggregate market profitability (the sum of earnings of all NYSE/Nasdaq/Amex firms divided by the sum of most recent book values of equity), so that $\bar{\rho} = 15.66\%$ per year.

In our model, M/B is assumed to converge to one after T years. As discussed in PV, this theoretical treatment ignores the practical issues of conservative accounting and of intangible assets missing from the books, which imply that M/B might exceed one even after profits are competed away. From the practical standpoint, therefore, it seems reasonable to assume that M/B converges to the average M/B of mature firms whose patents have expired. We assume that Nasdaq's M/B after $T = 15$ years converges to 1.29, which is the average across years 1962 through 1999 of the median M/B across "old" firms, defined as those that first appeared in the CRSP monthly file no later than December 1947 (i.e. 15 years before 1962). All other parameters needed to calibrate the model are taken from Table 1.

Panel A of Table 6 reports the model-implied M/B for Nasdaq on March 10, 2000 under zero prior uncertainty ($\widehat{\sigma} = 0$) for different values of the equity premium and expected excess profitability ($\widehat{\psi}^i$). The model-implied M/B increases with $\widehat{\psi}^i$ and decreases with the equity premium, as expected. When Nasdaq is expected to deliver the same average profitability as the market ($\widehat{\psi}^i = 0$), not even the equity premium of 1% per year is able to match Nasdaq's M/B of 6.85.[27] With $\widehat{\psi}^i > 0$, though, the model often produces M/Bs that match or exceed

[27] Using an inflation-adjusted residual income model, Ritter and Warr (2002) argue that stocks in the Dow Jones 30 index were overvalued in the late 1990s even if the equity premium was zero.

6.85. For example, with $\widehat{\psi}^i = 4\%$ per year, the equity premium needed to match Nasdaq's M/B is about 2.5% per year, and with $\widehat{\psi}^i = 6\%$, the required premium is about 4.5%. A high enough $\widehat{\psi}^i$ and a low enough equity premium will do the trick, of course.

Matching Nasdaq's M/B is easier when we recognize that $\bar{\psi}^i$ is unknown. Panel B reports "implied prior uncertainty", or the prior uncertainty $\widehat{\sigma}$ that equates the model-implied M/B to the observed M/B.[28] Implied uncertainty is listed as zero for all pairs of $\widehat{\psi}^i$ and the equity premium that deliver M/B > 6.85 in Panel A. When $\widehat{\psi}^i = 0$ and the equity premium is 4%, matching M/B of 6.85 requires prior uncertainty of 9.42% per year. It seems more reasonable to use $\widehat{\psi}^i > 0$, though, as investors apparently expected excess profits on Nasdaq stocks (e.g. footnote 24). Raising $\widehat{\psi}^i$ to 2% per year, implied uncertainty drops to 7.50%. When $\widehat{\psi}^i$ and the equity premium are both at the plausible value of 4%, the required uncertainty is 4.86%, which seems plausible as well. To summarize, Nasdaq's M/B on March 10, 2000 appears consistent with reasonable parameter values in our valuation model.

In Table 6, the Nasdaq firms are expected to earn abnormal profits over $T = 15$ years after March 2000. Table 7 is an equivalent of Panel B of Table 6 with $T = 10$ and 20 years. This table shows that implied prior uncertainty is highly sensitive to T. With $T = 10$, matching Nasdaq's M/B requires a fair amount of optimism. With $\widehat{\psi}^i = 8\%$ (i.e. expected total profitability of $\bar{\rho} + \widehat{\psi}^i = 23.66\%$) and a 2% equity premium, implied prior uncertainty is 7.15%. Raising the equity premium to 4% raises implied prior uncertainty to almost 10%. In contrast, with $T = 20$, matching Nasdaq's M/B is easy. Even with $\widehat{\psi}^i = 2\%$ and a 4% equity premium, implied uncertainty is only 3.83%.

What choice of T is appropriate here? Taking our model literally, T should reflect patent duration. According to the U.S. law, patents issued before June 8, 1995 typically last for 17 years from the date of issuance, while patents granted after June 8, 1995 last for 20 years from the date of filing. The effective life of a patent is often shorter than 20 years, because some products such as drugs require various regulatory approvals before coming to the market, but patent extensions can frequently be obtained to compensate for the time lost in regulatory review (see Schwartz, 2001). Values of T between 10 and 20 years thus seem reasonable. We report results for $T = 10$, 15, and 20 years, with the most emphasis on $T = 15$. Of course, many patents of the Nasdaq firms in 2000 had less than 15 years to expiration. On the other hand, many firms had R&D in progress that almost guaranteed new patents in the near future. PV also choose $T = 15$ in their analysis.

[28] The idea of backing out the prior uncertainty needed to match the observed evidence is not new. In a mean-variance framework where investors can invest in U.S. as well as non-U.S. stocks, Pástor (2000) computes the prior uncertainty about mispricing necessary to explain the observed degree of home bias.

6.2. Matching the Valuations of Individual Firms

Table 8 reports the implied prior uncertainty for selected high-profile technology firms. Each firm's M/B is computed on March 10, 2000, profitability and dividend yield are computed over 1999, and $T = 15$ throughout. The valuations of Compaq ($M/B = 3.20$), Dell (55.96), and Lucent (16.07) are easy to match using our valuation model. For example, while Dell's and Lucent's M/Bs appear huge, their most recent annual ROEs are over 110% and 85%, respectively. As a result, the implied prior uncertainty for all three firms is under 6% even with $\widehat{\psi}^i = 0$ and the equity premium as high as 5%. The valuations of HP (8.31), IBM (8.97), Intel (11.09), Microsoft (18.79), and Motorola (5.81) can also be reconciled with our model for plausible parameter values. With $\widehat{\psi}^i = 5\%$ and the equity premium of 4%, HP's implied prior uncertainty is 6%, Intel's and Motorola's is 3%, and IBM's and Microsoft's valuations are matched even with zero prior uncertainty.

The firms whose valuations are more difficult to match are Cisco ($M/B = 39.02$), Oracle (62.23), and Yahoo (78.41). Now we need to resort to the equity premium of 3% (or less). With $\widehat{\psi}^i = 10\%$, which implies expected total profitability of $\bar{\rho} + \widehat{\psi}^i = 25.66\%$, the implied prior uncertainty is 7.48% for Cisco, 8.41% for Oracle, and 14.37% for Yahoo. These parameter combinations do not seem implausible to us, but Yahoo's valuation is clearly pushing the limits. With the above parameter values, the market's 95% confidence interval for Yahoo's average profitability in excess of aggregate profitability over the following 15 years is $(-18.74\%, 38.74\%)$ per year. Fixing $\bar{\rho}$ at 15.66%, this translates into a 95% confidence interval for Yahoo's average total profitability of $(-3.08\%, 54.50\%)$ per year.

Is such uncertainty implausibly large? Not necessarily. Oracle's and Microsoft's average annual ROEs over the previous 14 years (over which their data is available on Compustat, 1987–2000) were 52.35% and 45.45%, respectively. Two other firms from Table 8, Cisco and Dell, also delivered average annual ROE in excess of 45% over the previous ten years or more. If investors in March 2000 believed that Yahoo might possibly turn into the next Oracle but that it might also fail, then the prior uncertainty must have been huge, and even values as large as 14.37% should be considered seriously.[29] In summary, even Yahoo's astronomical valuation can be justified by sufficiently high prior uncertainty.

After March 2000, the valuations dropped significantly and the IPO activity cooled down. In our model, such events are caused by positive shocks to expected returns or by negative

[29] Some related evidence is reported by PV, who extract prior uncertainty from the estimated cross-sectional dispersion of average annual ROE. They report the dispersion of 13.68% (7.28%) per year when at least 10 (20) valid ROEs are required to compute the average. They choose $\widehat{\sigma}_t = 10\%$ as a round-number compromise.

shocks to expected cash flows or prior uncertainty. Both realized and expected profits came down after 2000 (Figure 6), as did both proxies for prior uncertainty (Figure 7), but it seems difficult to establish causality. We prefer to avoid speculation as to why the "bubble" burst, and simply pin it on unexpected worsening of investment opportunities.

To summarize, the high market valuations observed in March 2000 can be rationalized in our model using plausible values for expected returns, profitability, and prior uncertainty. March 2000 was preceded by a decade of extensive IPO activity, and the economic conditions that entice firms to go public also imply high market valuations.

7. Conclusions

In their recent survey of the IPO literature, Ritter and Welch (2002) conclude that "market conditions are the most important factor in the decision to go public." We agree, and we point out which dimensions of market conditions appear the most relevant: the equity premium, expected aggregate profitability, and prior uncertainty. Ritter and Welch (2002) also argue that "perhaps the most important unanswered question is why issuing volume drops so precipitously following stock market drops." Our model provides a simple answer. Whether prices drop due to increases in expected returns or decreases in expected cash flows, IPO volume declines as private firms wait for more favorable market conditions.

Our model has numerous asset pricing implications for IPO volume. IPO waves should be preceded by high market returns, followed by low market returns, and accompanied by high stock prices and by increases in aggregate profitability. IPO waves should also be preceded by increased disparity between new firms and old firms in terms of their valuations and return volatilities. All of these implications are confirmed in the data.

A broad objective of this paper is to establish IPO volume as a useful tool in asset pricing. IPO volume helps us better understand prices because it proxies for unobservable changes in investment opportunities. For example, observing high IPO volume should lead us to infer that investment opportunities have improved, justifying higher prices. We point to the 1990s IPO wave and argue that stock prices in March 2000 were high thanks to some combination of high prior uncertainty, low expected returns, and high expected profitability.

This paper highlights the importance of prior uncertainty about average profitability. We show that this uncertainty helps explain high IPO valuations, and that its increases can lead to IPO waves. We argue that prior uncertainty is high during technological revolutions.

According to its proxies, prior uncertainty was unusually high in the late 1990s.

We cannot rule out behavioral explanations for time-varying IPO volume, but two of our empirical findings seem more in line with our rational model than with stories based on mispricing. High IPO volume tends to be accompanied by increased aggregate profitability as well as increased difference between the volatilities of new firms and old firms. Both facts are consistent with our model, but neither appears to be predicted by the mispricing story in which firms go public in response to market overvaluation. Differentiating mispricing from rational variation in expected returns clearly merits more work.

This paper links IPO volume to market returns, but not to the returns on individual IPOs; thus we make no contribution to the literature on the long-run IPO performance.[30] The arrival of new ideas is exogenous in our model, but endogenizing it could be interesting. If capital must be raised to produce an idea, then low cost of capital might accelerate the pace of technological innovation, leading to IPO clustering. We show that IPOs cluster in time as a result of optimal IPO timing even if the rate of technological change is constant. Our model can also explain why some IPO waves exhibit industry concentration: increases in industry-specific prior uncertainty or excess profitability can lead to IPO waves concentrated in the given industry, without triggering IPOs in other industries.

Our focus is on IPO waves, but our model can be modified in principle to address a broader issue of cyclicality of investment. Suppose public firms can invent ideas or purchase them from inventors. A public firm solving for the optimal time to make an irreversible investment is considering tradeoffs similar to those of our inventor, and "investment waves" could obtain after investment opportunities improve. If the capital for investment is raised in a seasoned equity offering (SEO), we might also see SEO waves. We do not focus on SEOs because public firms are more likely than the inventor to seek and obtain debt financing, and capital structure issues are beyond the scope of this paper. More generally, we do not focus on investment by public firms because such firms often invest simply to maintain competitive stock of physical capital rather than to embark on new projects with uncertain and perishable abnormal profits, making some of the key features of our framework less relevant. Prior uncertainty, for example, is clearly higher for IPOs than for the investment projects of existing public firms. Nonetheless, properties of aggregate investment in time-varying market conditions certainly deserve to be investigated further.

[30]For important contributions to that literature, see Ritter (1991), Loughran and Ritter (1995), Brav and Gompers (1997), Brav, Geczy, and Gompers (2000), and Schultz (2003), among others.

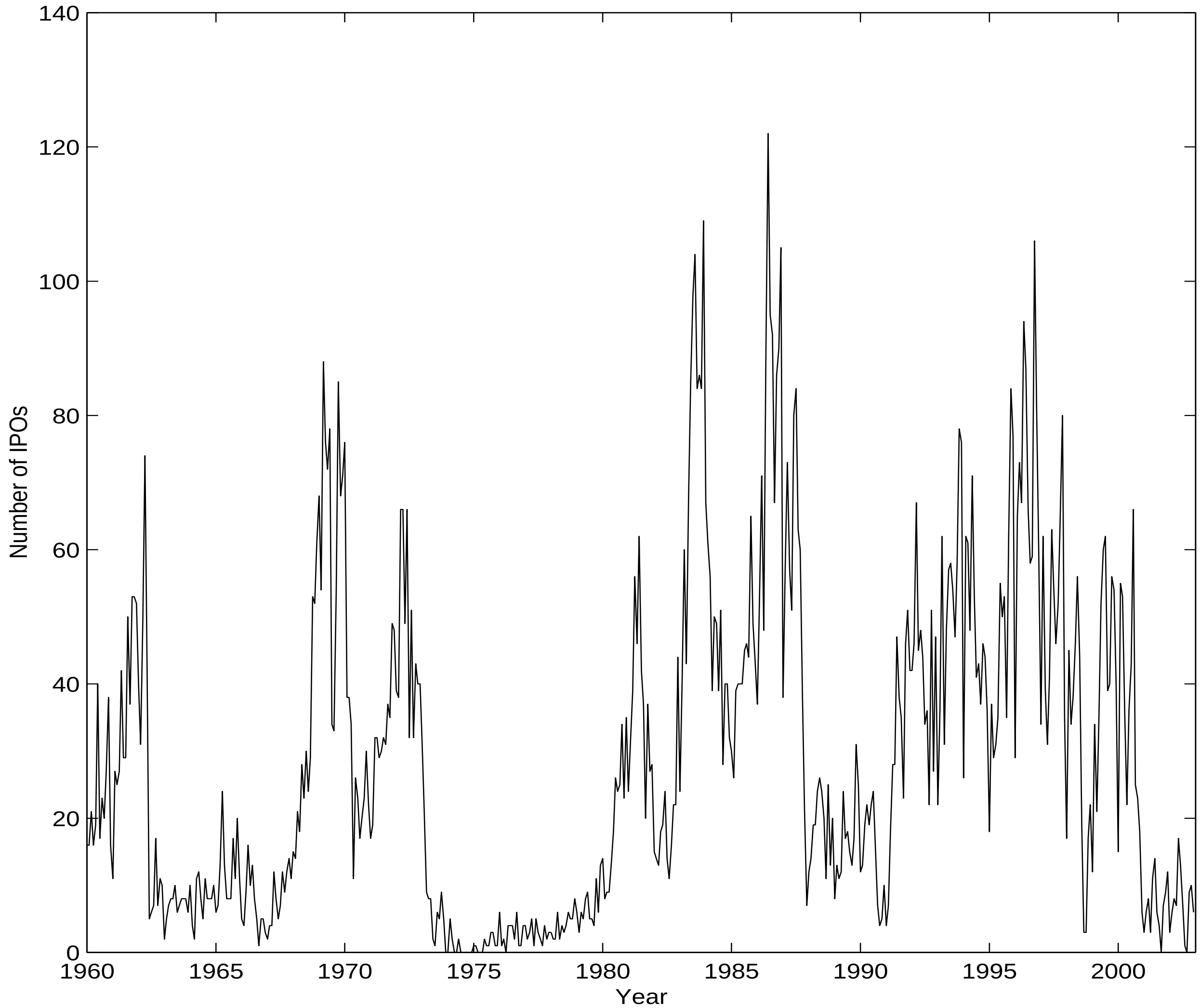

Figure 1. IPO volume. The figure plots the number of IPOs in each month between January 1960 and December 2002. The data is obtained from Jay Ritter's website.

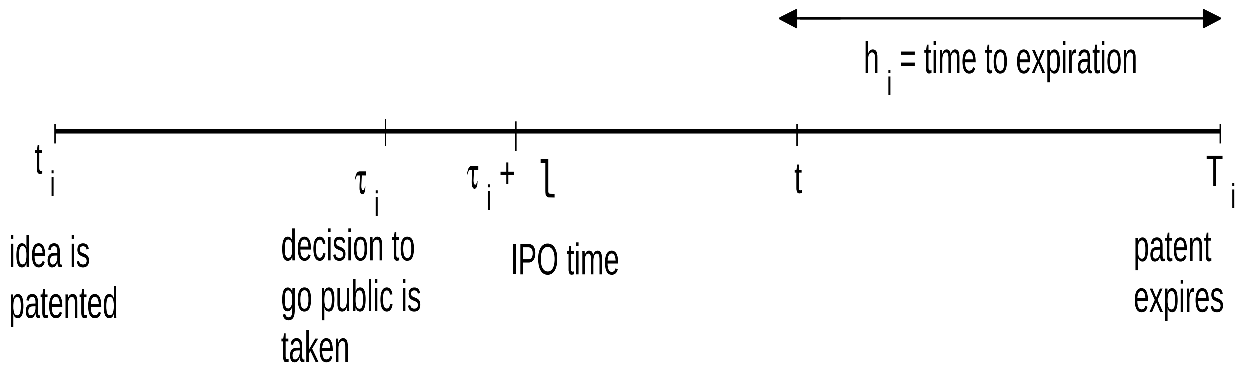

Figure 2. The timing of the events in our model.

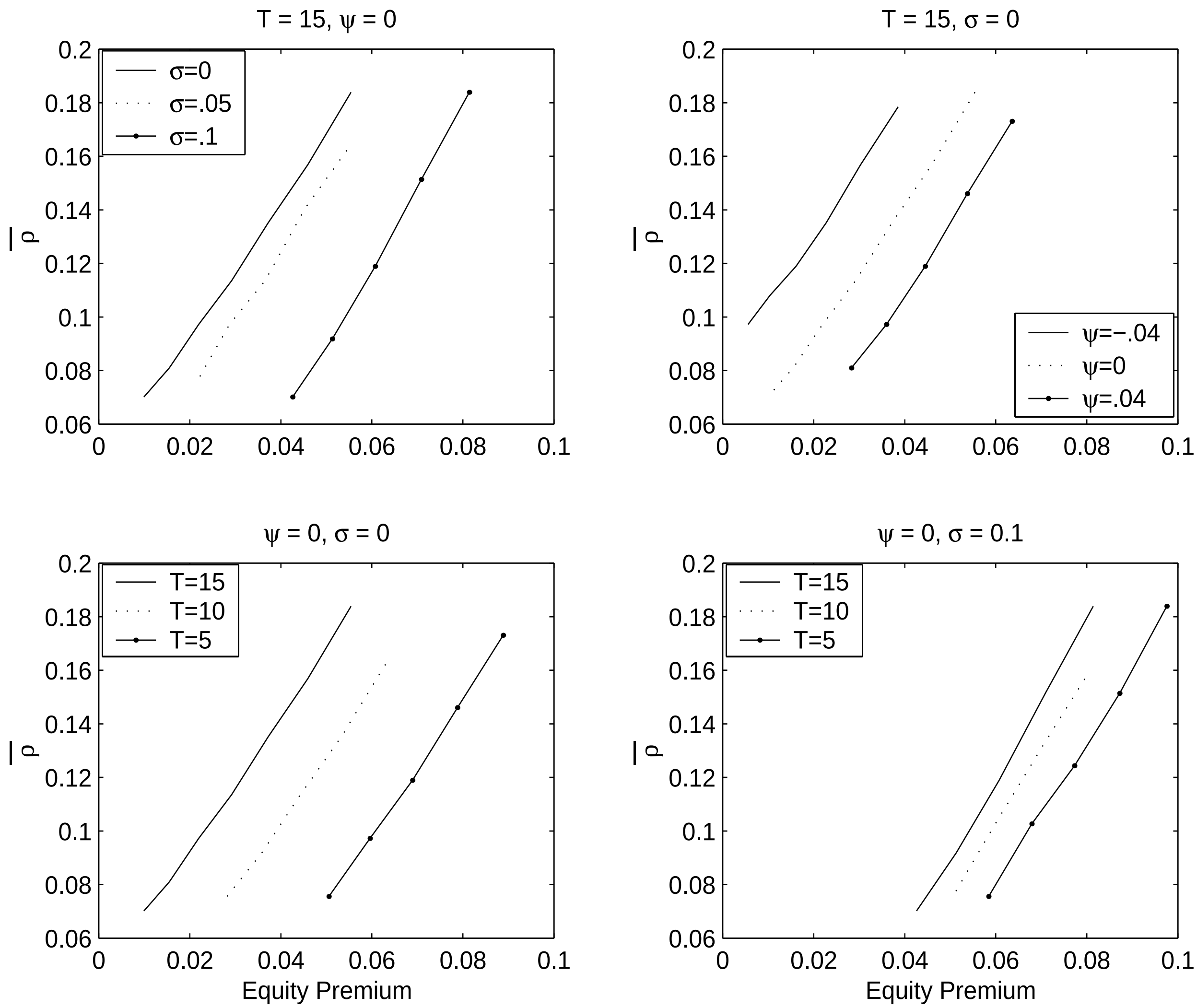

Figure 3. Optimal IPO Timing. Each panel plots the entry boundary, the set of pairs of the equity premium (horizontal axis) and expected aggregate profitability $\overline{\rho}$ (vertical axis) for which the inventor optimally decides to go public. The entry boundaries are reported for three levels of prior uncertainty $\widehat{\sigma}_t = 0$, 5%, and 10% per year (top left panel), firm-specific excess profitability $\hat{\psi} = 0$ and $\pm 4\%$ per year (top right panel), and time $T = 5$, 10, and 15 years to the patent's expiration (bottom panels). An IPO takes place for values of the equity premium and $\overline{\rho}$ north-west of each boundary. Unless noted otherwise, the parameter values used to compute the optimal IPO timing decision are given in Table 1.

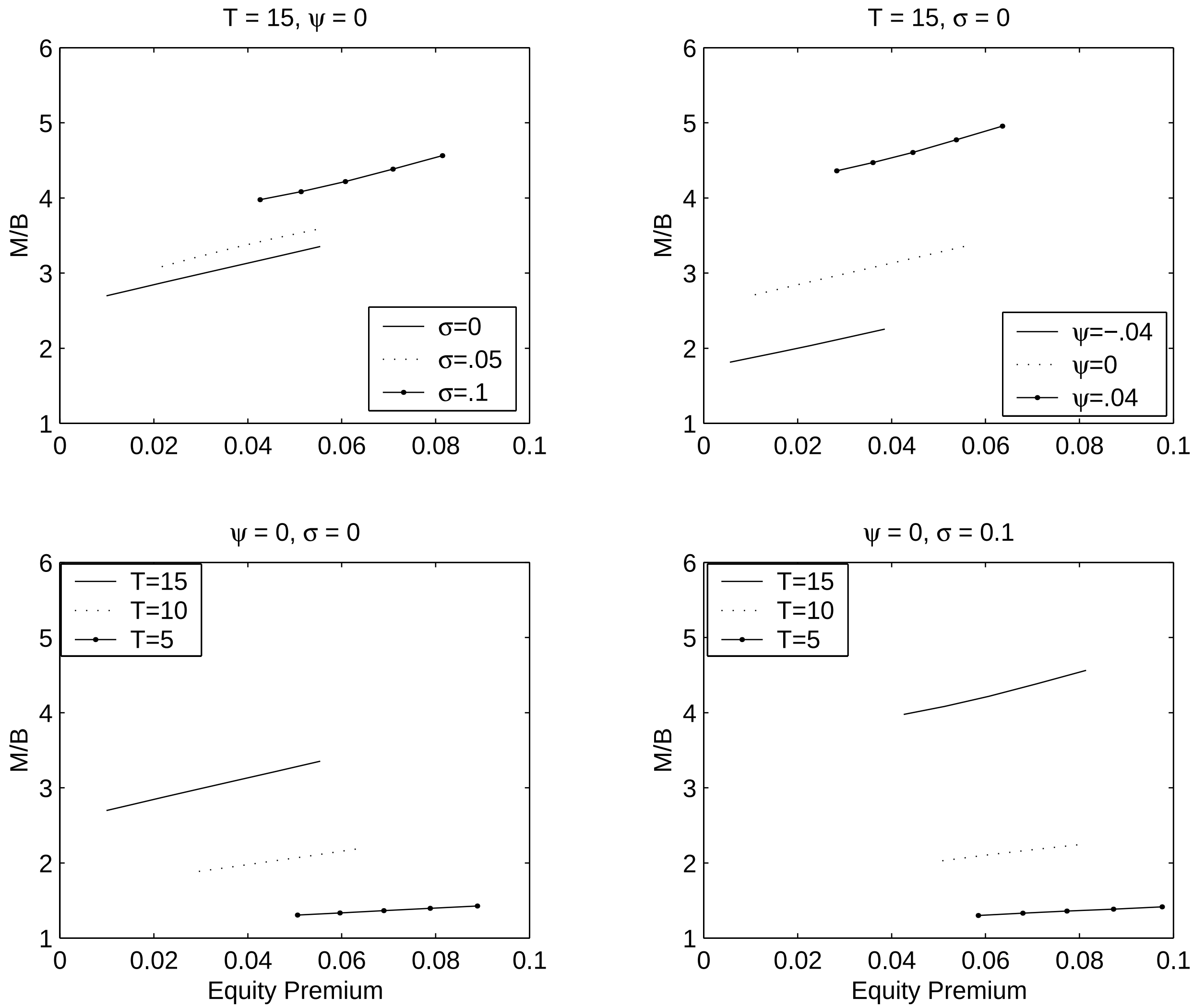

Figure 4. IPO valuations. Each panel plots the firm's market-to-book ratio (M/B) at the time of the IPO. For each value of the equity premium, the value of expected aggregate profitability $\overline{\rho}$ is chosen from the optimal entry boundary shown in Figure 3. The M/Bs are reported for three levels of prior uncertainty $\widehat{\sigma}_t = 0$, 5%, and 10% per year (top left panel), firm-specific excess profitability $\hat{\psi} = 0$ and ±4% per year (top right panel), and time $T = 5$, 10, and 15 years to the patent's expiration (bottom panels). Unless noted otherwise, the parameter values used to compute the optimal IPO timing decision are given in Table 1.

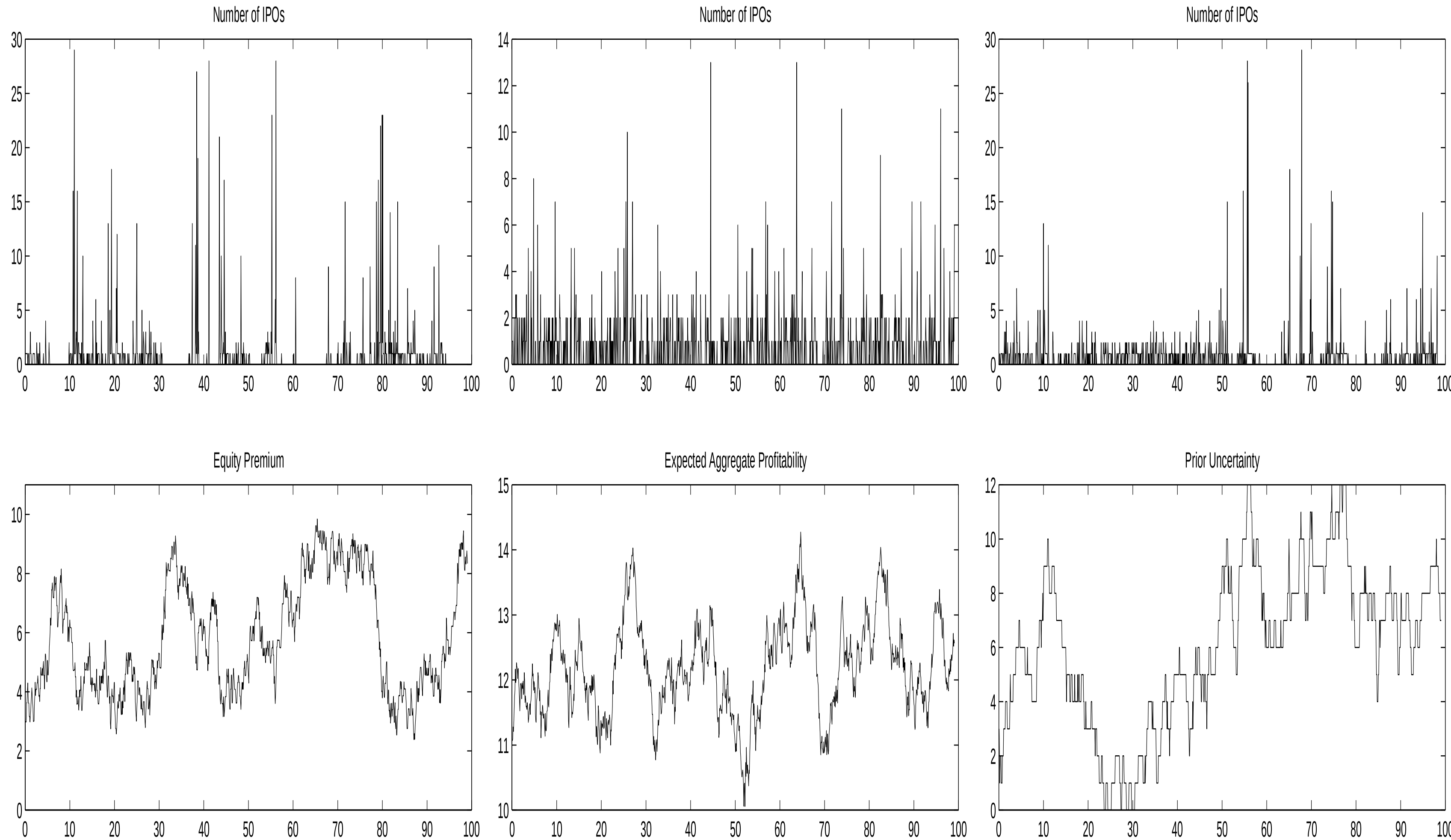

Figure 5. Simulated IPO Waves. This figure illustrates the dependence of IPO volume on the time variation in equity premium (left panels), expected aggregate profitability $\bar{\rho}_t$ (center panels), and prior uncertainty (right panels). The figure plots a typical 100-year segment of simulated data. In each panel, the indicated state variable is allowed to vary over time, but the other two variables are fixed to their long-term averages (for expected returns and profitability) or zero (for uncertainty). The parameter values used in the simulation are given in Table 1.

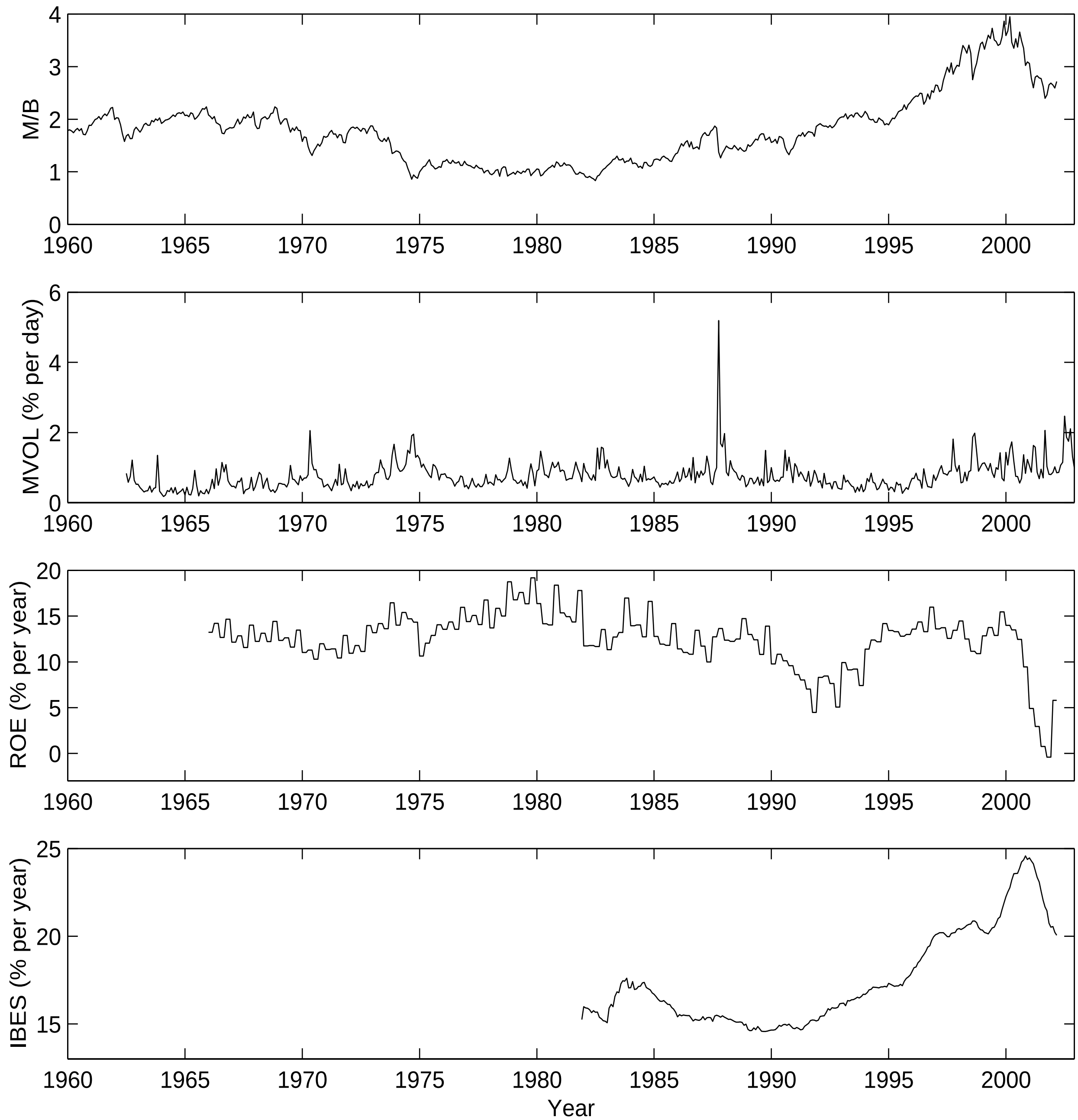

Figure 6. Monthly time series of selected aggregate variables. The top panel plots aggregate M/B (M/B), the sum of market values of equity across all firms divided by the sum of the most recent book values of equity. The second panel plots market return volatility (MVOL), the standard deviation of daily market returns within the month, which is available since July 1962. The third panel plots aggregate profitability (ROE), the sum across stocks of earnings in the current quarter divided by the sum of book values of equity at the end of the previous quarter. The monthly ROE series is created from the quarterly series for 1966Q1 through 2002Q1 by intrapolation. The bottom panel plots the average analyst long-term earnings growth forecast (IBES). For each firm, forecasts of long-term earnings growth are averaged across all analysts covering the firm, and IBES is computed as the average of such averages across firms. The IBES series is available for November 1981 through March 2002.

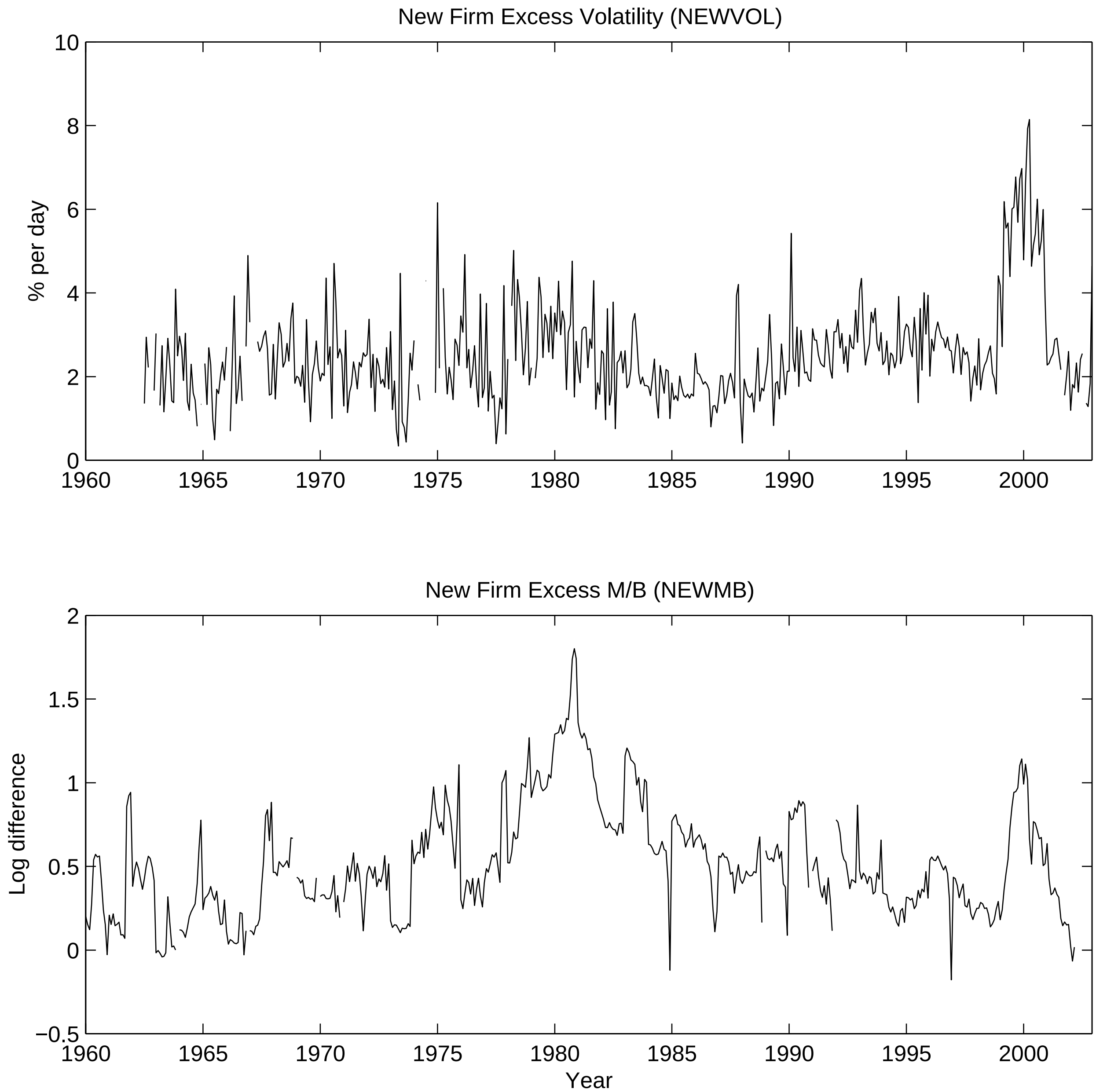

Figure 7. Monthly time series of proxies for prior uncertainty. The top panel plots NEWVOL, the median return volatility (standard deviation of daily returns) across all newly listed firms in excess of market return volatility. NEWVOL is available between July 1962 and December 2002. The bottom panel plots NEWMB, the log median M/B across all newly listed firms in excess of the log median M/B across all firms. NEWMB is available between January 1960 and March 2002.

Table 1
Parameter Values in the Calibrated Model.

The table reports the parameter values used to calibrate our model. The parameters of the processes for expected aggregate profitability and consumption growth are estimated from the consumption and aggregate profitability data using the Kalman filter. $\sigma_{L,0}$ is restricted to zero to eliminate correlation across the three state variables ($\overline{\rho}_t$, y_t and $\widehat{\sigma}_t$). The parameters of the individual profitability process are calibrated to the median firm in our sample. The utility parameters (η and γ), the parameters defining the log surplus consumption ratio $s(y) = a_0 + a_1 y_t + a_2 y_t^2$, and those characterizing the state variable y_t are calibrated to match the observed levels of the equity premium, market volatility, aggregate M/B, and the interest rate. The transition probabilities $\lambda_{i,i\pm1}$ that characterize the uncertainty process $\widehat{\sigma}_t$ on the grid $\mathcal{V} = \{0, .1, ..., .12\}$ are chosen to obtain plausible dynamics for $\widehat{\sigma}_t$. λ_b denotes the transition probability at the boundaries of the grid. All entries are annualized.

Aggregate Profitability				Consumption Growth			Individual Profitability		
k_L	$\overline{\rho}_L$	σ_{LL}	$\sigma_{L,0}$	b_0	b_1	σ_c	ϕ^i	$\sigma_{i,0}$	$\sigma_{i,i}$
0.1412	12.16 %	0.64 %	0	1.40 %	0.0812	0.94 %	0.3968	4.79 %	6.82 %
Utility		**Surplus Consumption Ratio**						**Uncertainty**	
η	γ	k_y	$\overline{y}$	σ_y	a_0	a_1	a_2	$\lambda_{i,i\pm1}$	λ_b
0.0475	3.70	0.073	-0.0017	0.5156	-2.8779	0.2132	-0.0198	10%	20%
Unconditional Moments from Calibration									
$E\left[R_t^{mkt}\right]$	$\sigma\left(R_t^{mkt}\right)$	$E\left[r_{f,t}\right]$	$\sigma\left(r_{f,t}\right)$	$E\left[M/B\right]$	$\sigma\left(M/B\right)$	$E\left[\widehat{\sigma}_t\right]$	$\sigma\left(\widehat{\sigma}_t\right)$	$E\left[\overline{\rho}_t\right]$	$\sigma\left(\overline{\rho}_t\right)$
6.8%	15%	3.3%	3.9%	1.7	.614	6.11%	3.5%	12.1%	1.2%

Table 2
Simulation Evidence Around IPO Waves.

The table reports averages of selected variables and market returns around simulated IPO waves, which are defined in the text. "b" stands for the beginning of an IPO wave, more precisely the end of the last month before the wave begins. "e" stands for the end of the wave's last month. "b(e)$\pm n$" denotes n months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b–3 and ends at the end of month e–3. Expected excess and total returns are computed for the market portfolio, the value-weighted portfolio of all existing simulated firms. Expected profitability stands for $\overline{\rho}$, and prior uncertainty stands for $\widehat{\sigma}$. MVOL denotes market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, NEWVOL is the difference between the return volatility of a new firm and market volatility, and NEWMB is the log difference between the M/B of a new firm and the M/B of the market. All variables except for M/B and NEWMB are expressed in percent per year.

Panel A. Averages of first-column variables.

	Avg change	Before wave			Wave		After wave		Outside
	in pre-wave	b-6	b-3	b	b+1:e	e	e+3	e+6	wave
Expected total return	-0.92	8.48	8.12	7.48	7.00	7.56	7.58	7.65	10.61
Expected profitability	0.06	12.23	12.25	12.28	12.29	12.28	12.28	12.27	12.20
Prior uncertainty	0.35	6.26	6.26	6.56	6.68	6.54	6.53	6.52	6.27
Expected excess return	-0.51	6.40	6.25	5.89	5.63	5.93	5.93	5.95	7.05
RF	-0.41	2.08	1.88	1.59	1.37	1.63	1.65	1.69	3.56
M/B	0.13	1.83	1.87	1.95	2.02	1.95	1.95	1.95	1.69
MVOL	-0.62	15.00	14.85	14.42	14.13	14.49	14.48	14.50	15.38
NEWVOL	3.54	51.58	51.20	54.34	55.03	54.08	53.84	53.76	53.15
NEWMB	0.08	0.41	0.42	0.48	0.51	0.48	0.47	0.47	0.32

Panel B. Average realized market returns.

	Pre-wave	Outside	Before wave		Wave	After wave		Outside
	b–2:e–3	pre-wave	b–5:b–3	b–2:b	b+1:e	e+1:e+3	e+4:e+6	wave
Total return	38.76	6.79	18.61	28.29	7.69	8.60	7.46	10.72
Excess return	37.23	3.25	16.58	26.58	6.34	6.96	5.79	7.16

Table 3
Simulation Evidence: Regressions of IPO Volume on Selected Variables.

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. All variables are taken from a 10,000-year-long sample simulated from our calibrated model. No t-statistics are given because all reported numbers are highly significant. "Δ" denotes changes (first differences), and "$-n$" ("$+n$") denotes quarterly lags (leads). ER denotes expected total market return, and all other variables are defined in Table 2. The units were chosen to ensure some significant digits for all coefficients in the table: IPO volume is measured as the number of firms that went public this quarter, MKT is measured in decimals per month, and all other variables in percent per year.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept	1.98	1.94	1.95	2.02	1.96	1.94	1.94	1.95	1.49	1.94
ΔER-2	-0.40			-0.57						
ΔER-1	-1.09			-1.12						
$\Delta\bar{\rho}$-2		0.08		0.44						
$\Delta\bar{\rho}$-1		0.97		1.34						
$\Delta\hat{\sigma}$-2			0.13	0.20						
$\Delta\hat{\sigma}$-1			0.81	0.81						
ΔRF-2					-0.38					
ΔRF-1					-1.22					
ΔMVOL-2						-0.38				-0.41
ΔMVOL-1						-2.49				-1.92
ΔNEWMB-2							0.13			0.44
ΔNEWMB-1							12.06			6.68
ΔNEWVOL-2								0.02		0.02
ΔNEWVOL-1								0.09		0.02
MKT-2									8.11	5.81
MKT-1									18.63	3.62
MKT									-2.03	-1.89
MKT+1									-2.36	-2.15
MKT+2									-2.57	-2.23
IPO(t-1)	0.23	0.24	0.24	0.21	0.24	0.25	0.25	0.24	0.22	0.21
T	40000	40000	40000	40000	40000	40000	40000	40000	40000	40000
R^2	0.13	0.06	0.07	0.16	0.10	0.15	0.11	0.07	0.15	0.19

Table 4
Empirical Evidence Around IPO Waves

The table reports averages of selected variables and market returns around IPO waves, which are defined in the text. "b" stands for the beginning of an IPO wave, more precisely the end of the last month before the wave begins. "e" stands for the end of the wave's last month. "b(e)$\pm n$" denotes n months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b–3 and ends at the end of month e–3. MVOL denotes market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, ROE is aggregate profitability (return on equity), IBES is the average analyst forecast of long-term earnings growth, NEWVOL is the difference between the median return volatility of new firms and market volatility, and NEWMB is the log difference between the median M/B of new firms and the median M/B across all firms. All variables except for M/B and NEWMB are expressed in percent per year. The t-statistics, reported in parentheses, assess the significance of the difference between the variable's averages in the given period and outside that period.

Panel A. Averages of selected variables.

	Avg change in pre-wave	Before wave b-6	b-3	b	Wave b+1:e	e	After wave e+3	e+6	Outside wave
MVOL	-2.81	12.36	13.66	12.70	12.67	12.73	15.63	15.00	15.24
	(-2.27)	(-1.14)	(-0.48)	(-0.97)	(-3.18)	(-0.96)	(0.54)	(0.21)	(3.18)
M/B	0.08	1.78	1.87	1.94	1.81	1.93	1.90	1.97	1.78
	(1.79)	(-0.06)	(0.50)	(0.96)	(0.50)	(0.89)	(0.72)	(1.15)	(-0.50)
RF	-0.01	5.41	5.39	5.49	5.68	5.74	5.79	5.34	5.68
	(-0.03)	(-0.42)	(-0.44)	(-0.29)	(0.02)	(0.09)	(0.17)	(-0.52)	(-0.02)
ROE	0.32	11.69	11.59	11.71	12.24	12.11	13.10	12.02	12.57
	(0.54)	(-1.02)	(-1.15)	(-1.00)	(-1.01)	(-0.48)	(0.81)	(-0.60)	(1.01)
IBES	0.39	16.93	16.88	16.86	16.81	17.45	17.58	17.57	17.83
	(1.32)	(-0.66)	(-0.72)	(-0.74)	(-2.90)	(-0.03)	(0.12)	(0.12)	(2.90)
NEWVOL	9.87	46.92	42.26	47.65	44.82	47.19	50.12	49.87	49.46
	(2.27)	(-0.24)	(-1.06)	(-0.11)	(-1.97)	(-0.19)	(0.34)	(0.28)	(1.97)
NEWMB	-0.05	0.44	0.56	0.55	0.55	0.46	0.49	0.46	0.51
	(-0.69)	(-0.91)	(0.50)	(0.41)	(1.18)	(-0.67)	(-0.31)	(-0.74)	(-1.18)

Panel B. Average realized market returns.

	Pre-wave b–2:e–3	Outside pre-wave	Before wave b–5:b–3	b–2:b	Wave b+1:e	After wave e+1:e+3	e+4:e+6	Outside wave
Total return	15.20	9.13	31.17	21.45	9.51	0.49	18.86	11.03
	(1.11)	(-1.11)	(2.77)	(1.45)	(-0.28)	(-1.35)	(1.09)	(0.28)
Excess return	9.54	3.44	25.83	15.74	3.83	-5.35	13.42	5.35
	(1.12)	(-1.12)	(2.81)	(1.44)	(-0.28)	(-1.36)	(1.12)	(0.28)

Table 5
Empirical Evidence: Regressions of IPO Volume on Selected Variables

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. "Δ" denotes changes (first differences), and "$-n$" ("$+n$") denotes quarterly lags (leads). All variables are defined in Table 4. Their units were chosen to ensure some significant digits for all coefficients in the table: Scaled IPO volume is measured in percent per month, MKT in decimals per month, ROE and RF in percent per month, MVOL and NEWVOL in percent per day, and IBES in percent per year. The t-statistics, given in parentheses, are computed using standard errors that are robust to heteroskedasticity and serial correlation of residuals (Newey-West with five lags).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Intercept	0.23	0.31	0.33	0.20	0.34	0.57	0.47	0.31	0.37	0.21	0.35
	(3.03)	(4.48)	(4.71)	(2.50)	(3.92)	(5.04)	(3.09)	(4.33)	(4.67)	(2.79)	(3.76)
MKT-2	1.67			1.70			2.33			2.09	
	(3.25)			(2.81)			(3.06)			(3.40)	
MKT-1	2.09			2.01			2.67			2.00	
	(3.34)			(2.60)			(2.71)			(2.68)	
MKT	2.06			1.88			2.31			3.32	
	(4.51)			(3.50)			(3.08)			(4.67)	
MKT+1	-0.95			-0.69			-1.56			-0.97	
	(-2.23)			(-1.68)			(-2.38)			(-1.52)	
MKT+2	-0.49			-0.16			-0.81			-0.04	
	(-0.74)			(-0.21)			(-1.26)			(-0.09)	
ΔMVOL-2		-0.31		-0.00							-0.18
		(-1.91)		(-0.02)							(-1.06)
ΔMVOL-1		-0.63		-0.11							-0.58
		(-3.59)		(-0.48)							(-2.64)
ΔMVOL		-0.60		-0.20							-0.78
		(-4.41)		(-1.14)							(-4.92)
ΔRF-2			0.50	0.80							0.16
			(0.89)	(1.69)							(0.26)
ΔRF-1			-0.38	0.05							-0.44
			(-1.03)	(0.15)							(-1.17)
ΔRF			0.44	0.55							0.61
			(1.02)	(1.76)							(1.50)
ΔROE					0.90		0.78				0.01
					(2.50)		(2.83)				(0.04)
ΔROE+1					0.55		0.68				0.07
					(1.43)		(2.06)				(0.19)
ΔROE+2					0.64		0.30				0.96
					(1.90)		(0.71)				(3.30)
ΔIBES-2						-0.16	0.22				
						(-0.87)	(0.99)				
ΔIBES-1						-0.43	-0.46				
						(-1.37)	(-1.81)				
ΔIBES						0.78	0.26				
						(5.07)	(1.41)				
ΔNEWMB-2								0.46		0.53	0.54
								(2.35)		(3.31)	(2.45)
ΔNEWMB-1								0.52		0.30	0.66
								(3.18)		(1.70)	(3.36)
ΔNEWMB								0.11		-0.23	-0.15
								(0.59)		(-1.22)	(-0.62)
ΔNEWVOL-2									0.12	0.05	0.12
									(2.23)	(1.10)	(2.61)
ΔNEWVOL-1									0.03	-0.08	-0.01
									(0.57)	(-1.75)	(-0.41)
ΔNEWVOL									0.01	0.01	-0.03
									(0.19)	(0.18)	(-0.67)
IPO(t-1)	0.84	0.87	0.85	0.85	0.84	0.79	0.75	0.87	0.84	0.82	0.88
	(23.09)	(21.47)	(20.20)	(23.66)	(18.30)	(11.16)	(11.04)	(19.79)	(18.75)	(19.19)	(17.90)
Q1 Dummy	-0.48	-0.42	-0.42	-0.45	-0.26	-0.67	-0.60	-0.51	-0.43	-0.52	-0.44
	(-4.92)	(-4.52)	(-5.00)	(-4.73)	(-2.22)	(-4.59)	(-4.45)	(-5.40)	(-4.42)	(-4.95)	(-3.41)
T	169	159	171	157	142	79	77	136	144	113	105
R^2	0.78	0.75	0.72	0.79	0.72	0.70	0.80	0.76	0.71	0.83	0.81

Table 6
Nasdaq Valuation on March 10, 2000

Panel A reports the model-implied M/B for the Nasdaq Composite Index on March 10, 2000 under zero prior uncertainty for different values of the equity premium and expected profitability in excess of aggregate profitability ($\widehat{\psi}^i$). Panel B reports the prior uncertainty $\hat{\sigma}$ that equates the model-implied M/B to the observed M/B. All variables (equity premium, $\widehat{\psi}^i$, and $\hat{\sigma}_t$) are expressed in percent per year. The observed M/B for Nasdaq on March 10, 2000 is 6.85, its observed profitability (ROE) in 1999 is $\rho_t = 12.79\%$ per year, and its dividend yield (dividends over book equity) in 1999 is $c = 2.06\%$ per year. We set $\bar{\rho} = 15.66\%$ per year, equal to the aggregate market profitability in 1999Q4. M/B for Nasdaq is assumed to converge after $T = 15$ years to the long-run value of 1.29, equal to the average M/B of old firms, as defined in the text. The remaining parameters needed to calibrate the model are given in Table 1.

Excess ROE	Equity premium (% per year)									
$\widehat{\psi}^i$ (% per year)	1	2	3	4	5	6	7	8	9	10
	Panel A. Model-implied M/B with zero prior uncertainty.									
-5	2.70	2.51	2.30	2.05	1.81	1.57	1.33	1.11	0.91	0.72
0	4.82	4.46	4.07	3.62	3.18	2.74	2.31	1.91	1.55	1.20
2	6.10	5.64	5.14	4.56	4.01	3.45	2.90	2.39	1.93	1.49
4	7.73	7.14	6.51	5.77	5.06	4.35	3.65	3.01	2.42	1.86
6	9.82	9.07	8.26	7.31	6.41	5.50	4.61	3.79	3.05	2.33
8	12.49	11.52	10.49	9.28	8.13	6.96	5.83	4.78	3.84	2.93
10	15.91	14.67	13.35	11.80	10.33	8.84	7.40	6.06	4.86	3.69
	Panel B. Prior uncertainty needed to match the observed M/B.									
-5	11.40	11.87	12.39	13.04	13.72	14.47	15.28	16.14	17.05	18.12
0	6.97	7.71	8.49	9.42	10.34	11.31	12.34	13.40	14.48	15.73
2	3.99	5.18	6.29	7.50	8.62	9.77	10.94	12.12	13.31	14.66
4	0.00	0.00	2.63	4.86	6.46	7.93	9.34	10.70	12.03	13.51
6	0.00	0.00	0.00	0.00	3.01	5.50	7.39	9.05	10.60	12.25
8	0.00	0.00	0.00	0.00	0.00	0.00	4.69	7.03	8.93	10.85
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.10	6.87	9.23

Table 7
Implied Prior Uncertainty for Nasdaq With Different Horizons

The table is an equivalent of Panel B of Table 6 with $T = 10$ and $T = 20$. It reports the prior uncertainty $\hat{\sigma}$ that equates the model-implied M/B to the observed M/B. All variables (equity premium, $\widehat{\psi}^i$, and $\hat{\sigma}_t$) are expressed in percent per year. The observed M/B for Nasdaq on March 10, 2000 is 6.85, its observed profitability (ROE) in 1999 is $\rho_t = 12.79\%$ per year, and its dividend yield (dividends over book equity) in 1999 is $c = 2.06\%$ per year. We set $\bar{\rho} = 15.66\%$ per year, equal to the aggregate market profitability in 1999Q4. M/B for Nasdaq is assumed to converge after T years to the long-run value of 1.29, equal to the average M/B of old firms, as defined in the text. The remaining parameters needed to calibrate the model are given in Table 1.

Excess ROE	Equity premium (% per year)									
$\widehat{\psi}^i$ (% per year)	1	2	3	4	5	6	7	8	9	10
					T=10					
-5	19.70	19.93	20.25	20.75	21.38	22.22	23.33	24.81	25.70	26.88
0	15.93	16.31	16.89	17.70	18.60	19.62	20.46	21.47	22.81	24.87
2	14.15	14.57	15.22	16.12	17.09	18.21	19.44	20.45	21.61	23.37
4	12.10	12.59	13.34	14.36	15.44	16.67	18.00	19.43	20.57	22.09
6	9.63	10.24	11.14	12.34	13.59	14.97	16.45	17.99	19.59	20.98
8	6.25	7.15	8.40	9.93	11.45	13.06	14.73	16.43	18.17	20.03
10	0.00	0.00	4.09	6.70	8.80	10.81	12.77	14.71	16.63	18.78
					T=20					
-5	8.23	8.74	9.24	9.83	10.41	11.04	11.71	12.41	13.14	13.98
0	3.04	4.22	5.18	6.17	7.07	7.97	8.88	9.79	10.70	11.73
2	0.00	0.00	1.84	3.83	5.15	6.34	7.45	8.51	9.55	10.69
4	0.00	0.00	0.00	0.00	1.74	4.08	5.66	7.01	8.24	9.54
6	0.00	0.00	0.00	0.00	0.00	0.00	2.93	5.07	6.67	8.22
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.51	4.60	6.66
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.58

Table 8
Implied Prior Uncertainty on March 10, 2000 for Selected Technology Firms

The table is an equivalent of Panel B of Table 6 for selected technology firms. It reports the prior uncertainty about average profitability ($\hat{\sigma}$) that equates the model-implied M/B to the observed M/B. All variables (equity premium, $\widehat{\psi}^i$, and $\hat{\sigma}_t$) are expressed in percent per year. Each firm's name is accompanied by the firm's observed M/B on March 10, 2000 as well as the firm's realized profitability ρ_t and dividend yield c in 1999. We set $\bar{\rho} = 15.66\%$ per year, equal to the aggregate market profitability in 1999Q4. M/B for each firm is assumed to converge after $T = 15$ years to the long-run value of 1.29, equal to the average M/B of old firms, as defined in the text. The remaining parameters needed to calibrate the model are from Table 1.

Excess ROE	Equity premium (% per year)									
$\widehat{\psi}^i$ (% per year)	1	2	3	4	5	6	7	8	9	10
	Cisco, $M/B = 39.02$, $\rho_t = 29.49\%$, $c = 0$									
0	13.88	14.26	14.69	15.24	15.81	16.46	17.15	17.92	18.72	19.68
5	10.61	11.11	11.66	12.34	13.04	13.81	14.64	15.53	16.45	17.54
10	5.70	6.59	7.48	8.51	9.49	10.52	11.59	12.68	13.80	15.08
	Compaq, $M/B = 3.20$, $\rho_t = 5.01\%$, $c = 1.27\%$									
0	0.00	0.00	0.00	1.53	4.51	6.44	8.10	9.63	11.09	12.67
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.42	6.48	8.93
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Dell, $M/B = 55.96$, $\rho_t = 110.67\%$, $c = 0$									
0	0.00	0.00	0.77	4.12	5.89	7.44	8.89	10.27	11.63	13.12
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.06	7.43	9.60
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.48
	Hewlett-Packard, $M/B = 8.31$, $\rho_t = 18.35\%$, $c = 3.84\%$									
0	8.93	9.52	10.17	10.97	11.77	12.65	13.58	14.57	15.59	16.76
5	0.00	2.82	4.57	6.14	7.50	8.81	10.12	11.41	12.69	14.12
10	0.00	0.00	0.00	0.00	0.00	0.00	4.46	6.92	8.88	10.83
	IBM, $M/B = 8.97$, $\rho_t = 37.16\%$, $c = 4.15\%$									
0	5.22	6.17	7.14	8.25	9.30	10.41	11.55	12.70	13.87	15.20
5	0.00	0.00	0.00	0.00	1.83	5.03	7.09	8.86	10.48	12.19
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.18	8.13
	Intel, $M/B = 11.09$, $\rho_t = 29.53\%$, $c = 1.48\%$									
0	7.06	7.78	8.56	9.48	10.38	11.35	12.36	13.41	14.49	15.72
5	0.00	0.00	0.00	2.97	5.17	6.92	8.48	9.95	11.36	12.91
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.24	6.95	9.26
	Lucent, $M/B = 16.07$, $\rho_t = 85.04\%$, $c = 4.73\%$									
0	0.00	0.00	0.00	0.00	1.85	5.09	7.18	8.97	10.60	12.33
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.32	8.27
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Microsoft, $M/B = 18.79$, $\rho_t = 49.57\%$, $c = 0$									
0	5.85	6.71	7.59	8.60	9.57	10.60	11.66	12.75	13.86	15.14
5	0.00	0.00	0.00	0.00	3.41	5.68	7.48	9.09	10.59	12.21
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.60	5.67	8.31
	Motorola, $M/B = 5.81$, $\rho_t = 6.09\%$, $c = 2.17\%$									
0	7.09	7.82	8.59	9.52	10.42	11.39	12.41	13.47	14.55	15.78
5	0.00	0.00	0.00	3.00	5.20	6.95	8.52	10.00	11.42	12.97
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.30	7.00	9.32
	Oracle, $M/B = 62.23$, $\rho_t = 43.45\%$, $c = 0$									
0	14.40	14.77	15.19	15.72	16.27	16.90	17.59	18.32	19.12	20.02
5	11.28	11.76	12.28	12.93	13.60	14.33	15.14	16.00	16.90	17.96
10	6.88	7.63	8.41	9.33	10.23	11.20	12.21	13.26	14.33	15.57
	Yahoo, $M/B = 78.41$, $\rho_t = 10.52\%$, $c = 0$									
0	18.53	18.81	19.14	19.57	20.01	20.19	20.44	20.79	21.26	22.03
5	16.22	16.56	16.93	17.40	17.91	18.48	19.10	19.78	20.19	20.60
10	13.53	13.93	14.37	14.93	15.51	16.16	16.88	17.65	18.48	19.45

8. Appendix

(A) Data

Aggregate consumption data is obtained quarterly from NIPA. Consumption is defined as real per capita consumption expenditures on non-durables plus services, seasonally adjusted. The series is deflated by the personal consumption expenditure deflator (PCE), also taken from NIPA.

The following data is obtained from CRSP and Compustat. Quarterly aggregate profitability (ROE) is computed as the sum across stocks of earnings in the current quarter divided by the sum of book values of equity at the end of the previous quarter. Quarterly earnings, which are generally available from 1966Q1, denote income before extraordinary items available for common (Compustat item 25) plus deferred taxes from the income account (item 35, if available). If either value is indicated as .A (annual) or .S (semi-annual) in the quarterly file, these values are divided by four (if .A) or two (if .S). When quarterly book equity is missing, it is replaced by the most recent annual book equity. Following Fama and French (1993), annual book equity is constructed as stockholders' equity plus balance sheet deferred taxes and investment tax credit (item 35) minus the book value of preferred stock. Depending on availability, stockholder's equity is computed as Compustat item 216, or 60+130, or 6-181, in that order, and preferred stock is computed as item 56, or 10, or 130, in that order. Quarterly book equity, which is generally available from 1972Q1, is constructed analogously. Stockholders' equity is item 60, or 59+55, or 44-54, preferred stock is item 55, and deferred taxes and tax credit is item 52. If the quarterly values are indicated as .A (annual) or .S (semi-annual) in the SAS datafile, the respective annual or semiannual values are used. Monthly ROE values are intrapolated from quarterly values. Market equity is computed monthly by multiplying the common stock price by common shares outstanding, both obtained from CRSP. M/B ratio is computed as market equity divided by book equity from the most recent quarter. We eliminate the values of market equity and book equity smaller than \$1 million, as well as M/B ratios smaller than 0.01 and larger than 100. All variables that require Compustat data (e.g. ROE, M/B) are constructed through the end of 2002Q1.

(B) Preferences and the Stochastic Discount Factor

This appendix describes in detail the properties of the process of log surplus consumption

$$\log(S_t) \equiv s_t \equiv s(y_t) = a_0 + a_1 y_t + a_2 y_t^2 \tag{17}$$

The process for y_t implies a normal unconditional distribution for y_t with mean $\overline{y}$ and variance $\sigma_{y,0}^2/2k_y$. Let $y_D = \overline{y} - 4\sigma_y/\sqrt{2k_y}$ and $y_U = \overline{y} + 4\sigma_y/\sqrt{2k_y}$ be the boundaries between which y_t lies 99.9% of the time. To ensure that log surplus s_t conforms to the economic intuition of a habit formation model, we impose the following parametric restrictions: $a_2 < 0$, $a_1 > -2a_2 y_U$ and $a_0 < 1/4\left(a_1^2/a_2\right)$. These restrictions ensure that for all t, $s_t < 0$, and thus $S_t \in (0,1)$, and that $s(y)$ is increasing in y for all $y \in [y_D, y_U]$. The process for log surplus is given by

$$ds_t = \mu_s(y)\,dt + \sigma_s(y)\,dW_{0,t} \tag{18}$$

with

$$\begin{aligned}\mu_s(y) &= k_y(\bar{y} - y_t)(a_1 + 2a_2 y) + a_2\sigma_y^2 \\ \sigma_s(y) &= (a_1 + 2a_2 y)\,\sigma_y\end{aligned}$$

The restrictions above imply that $\sigma_s(y)$ is positive and decreasing in y, for all $y \in [y_D, y_U]$. Since s increases with y in the relevant range, surplus is perfectly correlated with innovations to aggregate consumption, and its volatility is higher for low surplus levels.

Given the dynamics of consumption (4) and log surplus (18), the process for the stochastic discount factor $\pi_t = U_C(C_t, X_t, t) = e^{-\eta t}(C_t S_t)^{-\gamma} = e^{-\eta t - \gamma(c_t + s_t)}$ is given by

$$d\pi_t = -r_t \pi_t dt - \pi_t \sigma_{\pi,t} dW_{0,t}$$

where

$$r_t = R_0 + R_1 \overline{\rho}_t + R_2 y_t + R_3 y_t^2$$

with

$$\begin{aligned} R_0 &= \eta + \gamma b_0 + \gamma a_1 k_y \overline{y} - \frac{1}{2}\gamma^2 \sigma_c^2 + (\gamma a_2 - \frac{1}{2}\gamma^2 a_1^2)\sigma_y \sigma_y' - \gamma^2 a_1 \sigma_c \sigma_y' \\ R_1 &= \gamma b_1 \\ R_2 &= \gamma\left(2a_2 k_y \overline{y} - a_1 k_y - \gamma a_2\left(2\sigma_c \sigma_y' + a_1 2\sigma_y \sigma_y'\right)\right) \\ R_3 &= 2a_2\gamma\left(-k_y - \gamma a_2 \sigma_y \sigma_y'\right) \end{aligned}$$

and

$$\sigma_{\pi,t} = \gamma\left(\sigma_c + (a_1 + 2a_2 y_t)\sigma_y\right)$$

The parameter restrictions imposed earlier imply that $\sigma_{\pi,t}$ decreases as y_t (and hence also the surplus S_t) increases. As a result, expected returns and return volatility are low when y_t is high.

(C) Proofs:

The following Lemma is instrumental to most of the results:

Lemma A1: Let $\mathbf{Z} \sim N(\mu, \mathbf{\Sigma})$. Let $\mathbf{a}_0$ denote an $(n \times 1)$ vector and $\mathbf{a}_1$ denote an $(n \times n)$ matrix such that $\left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)^{-1}$ exists and is a valid covariance matrix. Then we have

$$E\left[e^{\mathbf{a}_0'\mathbf{Z} + \mathbf{Z}'\mathbf{a}_1\mathbf{Z}}\right] = c_2 e^{\mathbf{a}_0'\mu + \mu'\mathbf{a}_1\mu + \frac{1}{2}\left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)\left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)^{-1}\left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)'} \tag{19}$$

where

$$c_2 = |\mathbf{\Sigma}|^{-\frac{1}{2}} \; |-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}|^{-\frac{1}{2}}$$

Proof: By definition

$$E_t\left[e^{\mathbf{a}_0'\mathbf{Z} + \mathbf{Z}'\mathbf{a}_1\mathbf{Z}}\right] = c_1 \int_{\mathcal{R}^n} e^{\mathbf{a}_0'\mathbf{Z} + \mathbf{Z}'\mathbf{a}_1\mathbf{Z}} e^{-\frac{1}{2}(\mathbf{Z}-\mu)'\mathbf{\Sigma}^{-1}(\mathbf{Z}-\mu)} d\mathbf{Z} \tag{20}$$

where $c_1 = (2\pi)^{-\frac{n}{2}} |\mathbf{\Sigma}|^{-\frac{1}{2}}$. Since

$$-(\mathbf{Z}-\mu)'\mathbf{\Sigma}^{-1}(\mathbf{Z}-\mu) = -\mathbf{Z}'\mathbf{\Sigma}^{-1}\mathbf{Z} + 2\mu\mathbf{\Sigma}^{-1}\mathbf{Z} - \mu'\mathbf{\Sigma}^{-1}\mu$$

the whole exponent in the integrand of (20) is given by

$$\text{Exponent} = \mathbf{a}_0'\mathbf{Z} + \mathbf{Z}'\mathbf{a}_1\mathbf{Z} - \frac{1}{2}\mathbf{Z}'\mathbf{\Sigma}^{-1}\mathbf{Z} + \mu'\mathbf{\Sigma}^{-1}\mathbf{Z} - \frac{1}{2}\mu'\mathbf{\Sigma}^{-1}\mu$$

By developing the term $-\frac{1}{2}(\mathbf{Z}-\mu)'\left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)(\mathbf{Z}-\mu)$ the following identity obtains

$$\mathbf{Z}'\mathbf{a}_1\mathbf{Z} - \frac{1}{2}\mathbf{Z}'\mathbf{\Sigma}^{-1}\mathbf{Z} + \mu'\mathbf{\Sigma}^{-1}\mathbf{Z} - \frac{1}{2}\mu'\mathbf{\Sigma}^{-1}\mu = -\frac{1}{2}(\mathbf{Z}-\mu)'\left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)(\mathbf{Z}-\mu) + 2\mu'\mathbf{a}_1\mathbf{Z} - \mu'\mathbf{a}_1\mu$$

Thus, the exponent in (20) can be rewritten as

$$\text{Exponent} = \left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)\mathbf{Z} - \mu'\mathbf{a_1}\mu - \frac{1}{2}\left(\mathbf{Z}-\mu\right)'\left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)\left(\mathbf{Z}-\mu\right)$$

implying

$$\begin{aligned} E\left[e^{\mathbf{a}_0'\mathbf{Z}+\mathbf{Z}'\mathbf{a}_1\mathbf{Z}}\right] &= c_1\int_{\mathcal{R}} e^{\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mathbf{Z}-\mu'\mathbf{a_1}\mu}e^{-\frac{1}{2}\left(\mathbf{Z}-\mu\right)'\left(-2\mathbf{a}_1+\mathbf{\Sigma}^{-1}\right)\left(\mathbf{Z}-\mu\right)}d\mathbf{Z} \\ &= c_2 e^{-\mu'\mathbf{a_1}\mu}E^*\left[e^{\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mathbf{Z}}\right] \end{aligned}$$

where $c_2 = c_1\left(2\pi^{\frac{n}{2}}|\mathbf{G}|^{\frac{1}{2}}\right) = |\mathbf{\Sigma}|^{-\frac{1}{2}}|\mathbf{G}|^{\frac{1}{2}}$ and $\mathbf{G} = \left(-2\mathbf{a}_1 + \mathbf{\Sigma}^{-1}\right)^{-1}$, and $E^*(.)$ denotes the expectation computed assuming that $\mathbf{Z} \sim \mathcal{N}\left(\mu, \mathbf{G}\right)$. Since we then have

$$\left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)\mathbf{Z} \sim \mathcal{N}\left(\left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)\mu, \left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)\mathbf{G}\left(\mathbf{a}_0' + 2\mu'\mathbf{a}_1\right)'\right)$$

we finally obtain that the expectation is

$$E^*\left[e^{\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mathbf{Z}}\right] = e^{\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mu+\frac{1}{2}\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mathbf{G}\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)'}$$

thereby yielding

$$E\left[e^{\mathbf{a}_0'\mathbf{Z}+\mathbf{Z}'\mathbf{a}_1\mathbf{Z}}\right] = c_2 e^{\mathbf{a}_0'\mu+\mu'\mathbf{a}_1\mu+\frac{1}{2}\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)\mathbf{G}\left(\mathbf{a}_0'+2\mu'\mathbf{a}_1\right)'}$$

■

The following Lemma is applied repeatedly to price stocks and bonds.

Lemma A2: Consider an asset with payoff $e^{\nu\widetilde{b}_T}$ where ν is a constant,

$$d\widetilde{b}_t = \left(\zeta_0\overline{\rho}_t + \zeta_1\rho_t^i - \zeta_2\right)dt,$$

and $\overline{\rho}_t$ and ρ_t^i follow the processes (2) and (1). For every $T > t$, with $h = T - t$, we obtain

$$E_t\left[\frac{\pi_T}{\pi_t}e^{\nu\left(\widetilde{b}_T-\widetilde{b}_t\right)}\right] = Z^g\left(h, \overline{\rho}_t, \rho_t^i, \overline{\psi}^i, y_t\right) \equiv c_2(h)\, e^{q_0(h)+\Psi(h)\overline{\rho}_t+\nu\zeta_1 Q_{\phi^i}(h)\rho_t^i+K_\phi(h)\nu\zeta_1\overline{\psi}^i+q_1(h)y_t+q_2(h)y_t^2} \tag{21}$$

where

$$Q_{\phi^i}(h) = \left(1-e^{-\phi^i h}\right)/\phi^i, \qquad K_{\phi^i}(h) = \left(h - Q_{\phi^i}(h)\right), \tag{22}$$

$$\Psi(h) = \frac{\nu\zeta_1+\nu\zeta_0-\gamma b_1}{k_L} + \frac{\nu\zeta_1 e^{-\phi^i h}}{\phi^i - k_L} - \frac{\phi^i\nu\zeta_1 + \left(\nu\zeta_0-\gamma b_1\right)\left(\phi^i-k_L\right)}{k_L\left(\phi^i-k_L\right)}e^{-k_L h}, \tag{23}$$

and $c_2(h)$, $q_i(h)$, $i = 0, 1, 2$, are provided in equations (26) and (27)-(29), respectively.

Proof: Consider $\mathbf{N}_t = \left(\nu\widetilde{b}_t - \gamma c_t, y_t, \overline{\rho}_t, \rho_t^i\right)$; this follows the Gaussian process

$$d\mathbf{N}_t = \left(\mathbf{A}_N + \mathbf{B}_N\mathbf{N}_t\right)dt + \mathbf{\Sigma}_N d\mathbf{W}_t$$

with

$$\mathbf{A}_N = \begin{pmatrix} -\gamma b_0 - \nu\zeta_2 \\ k_y\overline{y} \\ k_L\overline{\rho}_L \\ \phi^i\overline{\psi} \end{pmatrix}; \mathbf{B}_N = \begin{pmatrix} 0 & 0 & \nu\zeta_0-\gamma b_1 & \nu\zeta_1 \\ 0 & -k_y & 0 & 0 \\ 0 & 0 & -k_L & 0 \\ 0 & 0 & \phi^i & -\phi^i \end{pmatrix}; \mathbf{\Sigma}_N = \begin{pmatrix} -\gamma\sigma_c & 0 & 0 \\ \sigma_y & 0 & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 \\ \sigma_{i,0} & 0 & \sigma_{i,i} \end{pmatrix};$$

Standard results on Gaussian processes imply that $\mathbf{N}_T|\mathbf{N}_t \sim \mathcal{N}\left(\mu_N\left(\mathbf{N}_t, h\right), \mathbf{S}_N\left(h\right)\right)$, where

$$\begin{aligned} \mu_N\left(\mathbf{N}_t, h\right) &= \mathbf{\Psi}\left(h\right)\mathbf{N}_t + \int_0^h \mathbf{\Psi}\left(h-s\right)\mathbf{A}_N ds & (24) \\ \mathbf{S}_N\left(h\right) &= \int_0^h \mathbf{\Psi}\left(h-s\right)\mathbf{\Sigma}_N\mathbf{\Sigma}_N'\mathbf{\Psi}\left(h-s\right)' ds & (25) \end{aligned}$$

and

$$\mathbf{\Psi}\left(h\right) = \begin{pmatrix} 1 & 0 & \Psi\left(h\right) & \nu\zeta_1 Q_{\phi^i}\left(h\right) \\ 0 & e^{-k_y h} & 0 & 0 \\ 0 & 0 & e^{-k_L h} & 0 \\ 0 & 0 & \frac{\phi^i}{\phi^i - k_L}\left(e^{-k_L h} - e^{-\phi^i h}\right) & e^{-\phi^i h} \end{pmatrix}$$

with $\Psi\left(h\right)$ defined in (23). For any x, denote $Q_x\left(h\right) = x^{-1}\left(1-e^{-xh}\right)$ and $K_x\left(h\right) = \left(h - Q_x\left(h\right)\right)$.

We can now apply the result of Lemma A1 to obtain an analytical expression for the expectation (21). Define $\mathbf{n}_t = \left(\nu\widetilde{b}_t - \gamma c_t, y_t\right)' = [1,1,0,0] \times \mathbf{N}_t$. We have $\mathbf{n}_T|\mathbf{N}_t \sim \mathcal{N}\left(\mu_n\left(\mathbf{N}_t, h\right), \mathbf{S}_n\left(h\right)\right)$ with $\mathbf{S}_n\left(h\right) = [1,1,0,0] \times \mathbf{S}_N\left(h\right) \times [1,1,0,0]'$,and

$$\mu_n\left(\mathbf{N}_t, h\right) = [1,1,0,0] \times \mu_N\left(\mathbf{N}_t, h\right) = \begin{pmatrix} \nu\widetilde{b}_t - \gamma c_t + \Psi\left(h\right)\overline{\rho}_t + \nu\zeta_1 Q_{\phi^i}\left(h\right)\rho_t^i + \xi\left(h\right) \\ e^{-k_y h} y_t + k_y \overline{y} Q_{k_y}\left(h\right) \end{pmatrix}$$

where, computing the integral in (24) explicitly, we find

$$\begin{aligned} \xi\left(h\right) &= \left(-\gamma b_0 - \nu\zeta_2 + \left(\nu\zeta_1 + \nu\zeta_0 - \gamma b_1\right)\right)\overline{\rho}_L h + \overline{\rho}_L \frac{\nu\zeta_1}{\phi^i - k_L}\left(k_L Q_\phi\left(h\right) - \phi^i Q_{k_L}\left(h\right)\right) \\ &\quad -\overline{\rho}_L\left(\nu\zeta_0 - \gamma b_1\right) Q_{k_L}\left(h\right) + \overline{\psi}\nu\zeta_1 K_\phi\left(h\right). \end{aligned}$$

Defining $\mathbf{a}_0 = \left(1, -\gamma a_1\right)'$ and $\mathbf{a}_1 = -\gamma a_2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, we can write

$$E_t\left[\pi_T e^{\nu\widetilde{b}_T}\right] = E_t\left[e^{-\eta T} e^{-\gamma\left(c_T + a_0 + a_1 y_T + a_2 y_T^2\right) + \nu\widetilde{b}_T}\right] = e^{-\eta T - \gamma a_0} E_t\left[e^{\mathbf{a}_0\mathbf{n}_T + \mathbf{n}_T'\mathbf{a}_1\mathbf{n}_T}\right]$$

Using Lemma A1, we have

$$E_t\left[e^{\mathbf{a}_0'\mathbf{n}_T + \mathbf{n}_T'\mathbf{a}_1\mathbf{n}_T}\right] = c_2\left(h\right) e^{\mathbf{a}_0'\mu_n\left(\mathbf{N}_t,h\right) + \mu_n\left(\mathbf{N}_t,h\right)'\mathbf{a}_1\mu_n\left(\mathbf{N}_t,h\right) + \frac{1}{2}\left(\mathbf{a}_0' + 2\mu_n\left(\mathbf{N}_t,h\right)'\mathbf{a}_1\right)\mathbf{G}\left(h\right)\left(\mathbf{a}_0' + 2\mu_n\left(\mathbf{N}_t,h\right)'\mathbf{a}_1\right)'}$$

with

$$c_2\left(h\right) = |\mathbf{G}\left(h\right)|^{\frac{1}{2}} / |\mathbf{S}\left(h\right)|^{\frac{1}{2}}, \tag{26}$$

and $\mathbf{G}\left(h\right) = \left(-2\mathbf{a}_1 + \mathbf{S}_n\left(h\right)^{-1}\right)^{-1}$. Tedious but straightforward algebra shows

$$\begin{aligned} \text{Exponent} &= \nu\widetilde{b}_t - \gamma c_t + \Psi\left(h\right)\overline{\rho}_t + \nu\zeta_1 Q_{\phi^i}\left(h\right)\rho_t^i + \zeta_1\left(h\right) + \frac{1}{2}G_{11}\left(h\right) - G_{12}\left(h\right)\gamma a_1 \\ &\quad - G_{12}\left(h\right) 2\gamma a_2 e^{-k_y h} y_t - G_{12}\left(h\right) 2\gamma a_2 k_y \overline{y} Q_{k_y}\left(h\right) + \frac{1}{2}G_{22}\left(h\right)\gamma^2 a_1^2 \\ &\quad + \left(2G_{22}\left(h\right)\left(\gamma a_2\right)^2 - \gamma a_2\right)\left(e^{-2k_y h} y_t^2 + k_y^2 \overline{y}^2 Q_{k_y}^2\left(h\right) + 2e^{-k_y h} k_y \overline{y} Q_{k_y}\left(h\right) y_t\right) \\ &\quad + \left(2G_{22}\left(h\right)\gamma^2 a_1 a_2 - \gamma a_1\right)\left(e^{-k_y h} y_t + k_y \overline{y} Q_{k_y}\left(h\right)\right) \end{aligned}$$

Substituting into $E_t\left[\pi_T/\pi_t e^{\nu\widetilde{b}_T}\right] = \pi_t^{-1}e^{-\eta T-\gamma a_0}E_t\left[e^{\mathbf{a}_0\mathbf{n}_T+\mathbf{n}_T'\mathbf{a}_1\mathbf{n}_T}\right]$ yields (21) with

$$\begin{aligned}
q_0(h) &= (-\gamma b_0-\nu\zeta_2-\eta+(\nu\zeta_1+\nu\zeta_0-\gamma b_1))\,\overline{\rho}_L h+\overline{\rho}_L\frac{\nu\zeta_1}{\phi^i-k_L}\left(k_L Q_\phi(h)-\phi^i Q_{k_L}(h)\right) && (27)\\
&\quad -\overline{\rho}_L(\nu\zeta_0-\gamma b_1)\,Q_{k_L}(h)+\frac{1}{2}G_{11}(h)-G_{12}(h)\gamma a_1-G_{12}(h)\,2\gamma a_2 k_y\overline{y}Q_{k_y}(h)\\
&\quad +\frac{1}{2}G_{22}(h)\gamma^2a_1^2+\left(2G_{22}(h)(\gamma a_2)^2-\gamma a_2\right)k_y^2\overline{y}^2Q_{k_y}^2(h)\\
&\quad +\left(2G_{22}(h)\gamma^2a_1a_2-\gamma a_1\right)k_y\overline{y}Q_{k_y}(h),\\
q_1(h) &= -2G_{12}(h)\gamma a_2e^{-k_yh}+\left(2G_{22}(h)(\gamma a_2)^2-\gamma a_2\right)2e^{-k_yh}k_y\overline{y}Q_{k_y}(h) && (28)\\
&\quad +\left(2G_{22}(h)\gamma^2a_1a_2-\gamma a_1\right)e^{-k_yh}+\gamma a_1,\\
q_2(h) &= \left(2G_{22}(h)(\gamma a_2)^2-\gamma a_2\right)e^{-2k_yh}+\gamma a_2. && (29)
\end{aligned}$$

■

Proof of Lemma 1: Consider the vector $\mathbf{z}_t=\left(c_t,\overline{\rho}_t,\rho_t^1,...,\rho_t^n\right)$ of signals to identify the unobservable variables, stacked in the vector $\overline{\psi}=(\overline{\psi}^1,...,\overline{\psi}^n)'$. The assumptions in the text imply

$$d\mathbf{z}_t=\left(\mathbf{A}+\mathbf{B}\mathbf{z}_t+\mathbf{C}\overline{\mathbf{\Psi}}\right)dt+\mathbf{b}d\mathbf{W}_t$$

where $\mathbf{W}_t=(W_{0,t},W_{L,t},W_{1,t},...,W_{n,t})$ and

$$\mathbf{A}=\begin{pmatrix} b_0\\ k_L\overline{\rho}_L\\ \vdots\\ 0\end{pmatrix},\ \mathbf{B}=\begin{pmatrix} 0 & b_1 & 0 & 0 & 0\\ 0 & -k_L & 0 & 0 & 0\\ 0 & \phi^1 & -\phi^1 & 0 & 0\\ 0 & \vdots & & \ddots & 0\\ 0 & \phi^n & 0 & 0 & -\phi^n\end{pmatrix}$$

$$\mathbf{C}=\begin{pmatrix} 0 & 0 & \cdots & \\ 0 & 0 & \cdots & \\ \phi^1 & & & \\ & \ddots & & \\ & & \phi^n & \end{pmatrix},\ \mathbf{b}=\begin{pmatrix} \sigma_{c,1} & 0 & \cdots & \cdots & \cdots & 0\\ \sigma_{L,0} & \sigma_{L,L} & 0 & \cdots & \cdots & 0\\ \sigma_{0,1} & 0 & \sigma_{1,1} & & & \\ \sigma_{0,2} & \vdots & & \sigma_{2,2} & & \\ \vdots & \vdots & & & \ddots & \\ \sigma_{0,n} & 0 & & & & \sigma_{n,n}\end{pmatrix}$$

The Kalman-Bucy filter implies the following result:

Lemma A3: Let there be n firms in the time interval $[t_0,t_1]$, and assume that investors have a prior distribution at time t_0 on the $n+1$ dimensional vector $\overline{\psi}$ given by $\overline{\psi}\sim\mathcal{N}\left(\widehat{\psi}_{t_0},\widehat{\mathbf{\Sigma}}_{t_0}\right)$, with $\widehat{\mathbf{\Sigma}}_{t_0}$ diagonal. Then, for any $t\in[t_0,t_1]$ the posterior distribution on $\overline{\psi}$ is also normally distributed, and given by $\overline{\psi}\sim\mathcal{N}\left(\widehat{\psi}_t,\widehat{\mathbf{\Sigma}}_t\right)$ where $d\widehat{\psi}_t=\widetilde{\mathbf{\Sigma}}_t d\widetilde{\mathbf{W}}_t$ with $\widetilde{\mathbf{\Sigma}}_t=\widehat{\mathbf{\Sigma}}_t\mathbf{C}'(\mathbf{b}')^{-1}$ and

$$\frac{d\widehat{\mathbf{\Sigma}}_t}{dt}=-\left[\widehat{\mathbf{\Sigma}}_t\mathbf{C}'\right](\mathbf{b}\mathbf{b}')^{-1}\left[\mathbf{C}\widehat{\mathbf{\Sigma}}_t'\right] \tag{30}$$

Finally,

$$d\widetilde{\mathbf{W}}_t=\mathbf{b}^{-1}\left\{d\mathbf{z}_t-E\left[d\mathbf{z}_t|\mathcal{F}_t\right]\right\}=\mathbf{b}^{-1}\left\{d\mathbf{z}_t-\left[\mathbf{A}+\mathbf{B}\mathbf{z}_t+\mathbf{C}\widehat{\mathbf{s}}_t\right]dt\right\} \tag{31}$$

is a Brownian motion in the filtered space.

Let $b_{11} = \sigma_{c,1}$, $b_{12} = (\sigma_{L,0}, \sigma_{0,1}, ..., \sigma_{0,n})$, $b_{22} = diag\,(\sigma_{LL}, \sigma_{1,1}, ..., \sigma_{n,n})$, $b_{21} = (0, .., 0)'$, and

$$\mathbf{b}' = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

Using known results about the inversion of partitioned matrices (see e.g. Magnus and Neudacker (1991, page 11)), we obtain

$$\left(\mathbf{b}'\right)^{-1} = \begin{pmatrix} b_{11}^{-1} & -b_{11}^{-1} b_{12} b_{22}^{-1} \\ 0 & b_{22}^{-1} \end{pmatrix}$$

which leads to

$$\mathbf{C}'\left(\mathbf{b}'\right)^{-1} = \begin{pmatrix} 0 & 0 & \phi^1 \sigma_{1,1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \phi^n \sigma_{n,n}^{-1} \end{pmatrix}$$

Thus, since $\widehat{\mathbf{\Sigma}}_{t_0}$ is a diagonal matrix, we find

$$\widetilde{\mathbf{\Sigma}}_t = \widehat{\mathbf{\Sigma}}_t \mathbf{C}'\left(\mathbf{b}'\right)^{-1} = \begin{pmatrix} 0 & 0 & \widehat{\sigma}_{1,t}^2 \phi^1 \sigma_{1,1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \widehat{\sigma}_{n,t}^2 \phi^n \sigma_{n,n}^{-1} \end{pmatrix}$$

This implies

$$\frac{d\widehat{\mathbf{\Sigma}}_t}{dt} = -\left[\widehat{\mathbf{\Sigma}}_t \mathbf{C}'\right]\left(\mathbf{b}\mathbf{b}'\right)^{-1}\left[\mathbf{C}\widehat{\mathbf{\Sigma}}_t'\right] = -\widetilde{\mathbf{\Sigma}}_t \widetilde{\mathbf{\Sigma}}_t' = -diag\left(\left(\widehat{\sigma}_{i,t}^2\right)^2 \left(\phi^i \left(\sigma_{i,i}\right)^{-1}\right)^2\right)$$

Thus $\widehat{\mathbf{\Sigma}}_t$ is diagonal for every t, and in addition, each diagonal element i satisfies the differential equation $d\widehat{\sigma}_{i,t}^2/dt = -\left(\widehat{\sigma}_{i,t}^2\right)^2 \left(\phi^i \sigma_{i,i}^{-1}\right)^2$, whose solution is $\widehat{\sigma}_{i,t}^2 = \left(\frac{1}{\widehat{\sigma}_{t_0}^2} + \frac{\left(\phi^i\right)^2}{\sigma_{i,i}^2}\left(t - t_0\right)\right)^{-1}$. ■

Proof of Proposition 1: Part (a) stems from Lemmas A1 and A2, and the law of iterated expectations. We must compute

$$E_t\left[\frac{\pi_T}{\pi_t} B_T^i\right] = E_t\left[\frac{\pi_T}{\pi_t} e^{b_T^i}\right] = E_t\left[E_t\left[\frac{\pi_T}{\pi_t} e^{b_T^i} | \overline{\psi}^i\right]\right]$$

The inner expectation is immediate from Lemma A2 for the special case where $\nu = 1$, $\zeta_0 = 0$, $\zeta_1 = 1$ and $\zeta_2 = 0$. That is, $E_t\left[\frac{\pi_T}{\pi_t} e^{b_T^i} | \overline{\psi}^i\right] = B_t^i Z^i\left(h, \overline{\rho}_t, \rho_t^i, \overline{\psi}^i, y_t\right)$, where

$$Z^i\left(h, \overline{\rho}_t, \rho_t^i, \overline{\psi}^i, y_t\right) = c_2\left(h\right) e^{q_0(h) + \Psi(h)\overline{\rho}_t + Q_{\phi^i}(h)\rho_t^i + K_{\phi^i}(h)\overline{\psi}^i + q_1(h) y_t + q_2(h) y_t^2}, \tag{32}$$

and where all functions of h, namely $Q_{\phi^i}\left(h\right)$, $K_{\phi^i}\left(h\right)$, $c_2\left(h\right)$, $q_i\left(h\right)$ for $i = 0, 1, 2$, $\Psi\left(h\right)$ are as in Lemma A2 with $\nu = 1$, $\zeta_0 = 0$, $\zeta_1 = 1$ and $\zeta_2 = 0$. The last step is to realize that from Lemma 1, $\overline{\psi}^i \sim N\left(\widehat{\psi}_t^i, \widehat{\sigma}_{i,t}^2\right)$, which implies $K_{\phi^i}\left(h\right)\overline{\psi}^i \sim \mathcal{N}\left(K_{\phi^i}\left(h\right)\widehat{\psi}^i, K_{\phi^i}\left(h\right)^2 \widehat{\sigma}_{i,t}^2\right)$. This in turn yields

$$\begin{aligned} E_t\left[E_t\left[\frac{\pi_T}{\pi_t} e^{b_T^i} | \overline{\psi}^i\right]\right] &= B_t^i c_2\left(h\right) e^{q_0(h) + \Psi(h)\overline{\rho}_t + Q_{\phi^i}(h)\rho_t^i + K_{\phi^i}(h)\widehat{\psi}^i + q_1(h) y_t + q_2(h) y_t^2 + \frac{1}{2} K_{\phi^i}(h)^2 \widehat{\sigma}_{i,t}^2} \\ &= B_t^i Z^i\left(h, \overline{\rho}_t, \rho_t^i, \widehat{\psi}^i, y_t\right) e^{\frac{1}{2} K_{\phi^i}(h)^2 \widehat{\sigma}_{i,t}^2} \end{aligned}$$

Part (b): From the pricing function $M_t^i = B_t^i G^i \left(\tau, \rho_t^i, \overline{\rho}_t, \widehat{\psi}_t^i, y_t, \widehat{\sigma}_{i,t}\right)$, using Ito's Lemma yields

$$dM_t^i = M_t^i \left(\frac{\partial G/\partial \rho_t^i}{G} d\rho_t^i + \frac{\partial G/\partial \overline{\rho}_t}{G} d\overline{\rho}_t + \frac{\partial G/\partial \widehat{\psi}_t^i}{G} d\widehat{\psi}_t^i + \frac{\partial G/\partial y_t}{G} dy_t \right) + o(dt)$$

where $o(dt)$ collects all the "dt" terms. From the definition of $G(.)$ in (11) we find

$$\begin{aligned} \partial G/\partial \rho_t^i &= Q_{\phi^i}(h) G; \ \partial G/\partial \overline{\rho}_t = \Psi(h) G \\ \partial G/\partial \widehat{\psi}_t^i &= K_{\phi^i}(h) G; \ \partial G/\partial y_t = (q_1(h) + 2q_2(h) y_t) G \end{aligned}$$

Thus, the total return on the asset is $\left(dM_t^i + D^i dt\right)/M_t^i = \mu_M^i(t,h) dt + \sigma_M^i(t,h) d\widetilde{\mathbf{W}}_t$, with

$$\begin{aligned} \mu_M^i(t,h) &= r_t + \left(Q_{\phi^i}(h) \sigma^i \sigma_{\pi,t}' + \Psi_{13}(h) \sigma_L \sigma_{\pi,t}' + (q_1(h) + 2q_2(h) y_t) \sigma_y \sigma_{\pi,t}'\right) \\ \sigma_M^i(t,h) &= \left(Q_{\phi^i}(h) \sigma^i + \Psi_{13}(h) \sigma_{L,t} + K_{\phi^i}(h) \widetilde{\sigma}_t^i + (q_1(h) + 2q_2(h) y_t) \sigma_y\right) \end{aligned}$$

The process for excess return (13) therefore has

$$\begin{aligned} \mu_R^i(t,h) &= \gamma \left(Q_{\phi^i}(h) \sigma_0^i + \Psi_{13}^i(h) \sigma_{L,0}\right) (\sigma_c + (a_1 + 2a_2 y_t) \sigma_y) \\ &\quad + \gamma \left(q_1^i(h) + 2q_2^i(h) y_t\right) \left(\sigma_y \sigma_c' + (a_1 + 2a_2 y_t) \sigma_y^2\right) \end{aligned} \tag{33}$$

$$\sigma_{R,0}^i(t,h) = Q_{\phi^i}(h) \sigma_0^i + \Psi_{13}^i(h) \sigma_{L,0} + \left(q_1^i(h) + 2q_2^i(h) y_t\right) \sigma_y \tag{34}$$

$$\sigma_{R,L}^i(t,h) = \Psi_{13}^i(h) \sigma_{L,L} \tag{35}$$

$$\sigma_{R,i}^i(t,h) = Q_{\phi^i}(h) \sigma_i^i + K_{\phi^i}(h) \frac{\phi}{\sigma_c} \widehat{\sigma}_{i,t}^2 \tag{36}$$

■

The Long Lived Firm: The market price of the long lived firm is $M_t^m = E\left[\int_t^\infty \pi_s/\pi_t D_s^m ds\right] = c^m E\left[\int_t^\infty \pi_s/\pi_t B_s^m ds\right]$. Assuming integrability, and applying Fubini theorem to invert the order of integration, we can solve for the expectation by computing $E_t\left[\pi_T B_T^m\right]$ when $b_t^m = \log(B_t^m)$ follows the process $db_t^m = (\overline{\rho}_t - c^m) dt$. This is a special case of Lemma A2, and thus equation (14) is immediately verified, with

$$Z^m(s, \overline{\rho}_t, y_t) = Z^g(s, 0, \overline{\rho}_t, 0, y_t)\,|_{\nu=1,\zeta_0=1,\zeta_1=0,\zeta_2=c^m} \tag{37}$$

Expected returns and volatility are then computed analogously as in the proof of Proposition 1. In particular, from the pricing function $M_t^i = B_t^i G^m(\overline{\rho}_t, y_t)$ with $G^m(\overline{\rho}_t, y_t) = c^m \int_0^\infty Z^m(s, \overline{\rho}_t, y_t) ds$, and Ito's Lemma, we find

$$dM_t^i = M_t^i \left(\frac{\partial G^m/\partial \overline{\rho}_t}{G^m} d\overline{\rho}_t + \frac{\partial G^m/\partial y_t}{G^m} dy_t \right) + o(dt)$$

where $o(dt)$ collects all the "dt" terms. From the definition of $G^m(.)$ and $Z^m(.)$, we find

$$\begin{aligned} \partial G^m/\partial \overline{\rho}_t &= c^m \int_0^\infty \Psi^m(s) Z^m(s, \overline{\rho}_t, y_t) ds \\ \partial G^m/\partial y_t &= c^m \int_0^\infty (q_1^m(s) + 2q_2^m(s) y_t) Z^m(s, \overline{\rho}_t, y_t) ds \end{aligned}$$

where $\Psi^m(s)$, and $q_i^m(s)$ are equal to $\Psi(s)$, and $q_i(s)$ in Lemma A2 for the parametrization $\nu = 1, \zeta_0 = 1, \zeta_1 = 0, \zeta_2 = c^m$. Defining the quantities

$$\begin{aligned}
F_{\overline{\rho}}^m(t) &= \frac{\int_0^\infty \Psi^m(s) Z^m(s, \overline{\rho}_t, y_t) \, ds}{\int_0^\infty Z^m(s, \overline{\rho}_t, y_t) \, ds} \\
F_{y,1}^m(t) &= \frac{\int_0^\infty q_1^m(s) Z^m(s, \overline{\rho}_t, y_t) \, ds}{\int_0^\infty Z^m(s, \overline{\rho}_t, y_t) \, ds} \\
F_{y,2}^m(t) &= 2\frac{\int_0^\infty q_2^m(s) y_t Z^m(s, \overline{\rho}_t, y_t) \, ds}{\int_0^\infty Z^m(h, \overline{\rho}_t, y_t) \, ds}
\end{aligned}$$

we find that excess returns and diffusion are given by

$$\mu_R^m(t) = \gamma \left\{ F_{\overline{\rho}}^m(t) \left(\sigma_L \sigma_c' + (a_1 + 2a_2 y_t) \sigma_L \sigma_y \right) + \left(F_{y,1}(t) + F_{y,2}(t) y_t \right) \left(\sigma_y \sigma_c' + (a_1 + 2a_2 y_t) \sigma_y \sigma_y' \right) \right\} \tag{38}$$

$$\sigma_R^m(t) = F_{\overline{\rho}}^m(t) \sigma_{L,t} + \left(F_{y,1}(t) + F_{y,2}(t) y_t \right) \sigma_y, \tag{39}$$

■

Payoff computation: Exercising at time τ, the expected payoff at time $\tau + \ell$ is

$$\begin{aligned}
EPay_{\tau,\tau+\ell}^i &= E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \left(M_{\tau+\ell}^i (1 - f) - B^{t_i} \right) \right) = B^{t_i} E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \left(\frac{M_{\tau+\ell}^i}{B^{t_i}} (1 - f) - 1 \right) \right) \qquad (40) \\
&= B^{t_i} (1 - f) E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \frac{M_{\tau+\ell}^i}{B^{t_i}} \right) - B^{t_i} E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \right)
\end{aligned}$$

Using (11) for $\widetilde{h} = T - (\tau + \ell)$, we have

$$M_{\tau+\ell}^i = B^{t_i} c_2 \left(\widetilde{h} \right) e^{q_0^i(\widetilde{h}) + \Psi^i(\widetilde{h}) \overline{\rho}_{\tau+\ell} + Q_{\phi^i}(\widetilde{h}) \rho_{\tau+\ell}^i + K_\phi(\widetilde{h}) \widehat{\psi}_{\tau+\ell}^i + q_1^i(\widetilde{h}) y_{\tau+\ell} + q_2^i(\widetilde{h}) y_{\tau+\ell}^2} e^{\frac{1}{2} K_\phi(\widetilde{h})^2 \widehat{\sigma}_{i,t+\ell}^2}$$

The initial profitability at the time of the IPO is unknown when the IPO decision is made, so it is set equal to its unconditional expectation $\rho_{\tau+\ell}^i = \overline{\rho}_{t+\ell} + \widehat{\psi}_{\tau+\ell}^i$. Then

$$\begin{aligned}
E_\tau \left(\pi_{\tau+\ell} \frac{M_{\tau+\ell}^i}{B^{t_i}} \right) &= c_2^i \left(\widetilde{h} \right) e^{q_0^i(\widetilde{h}) + \left(K_\phi(\widetilde{h}) + Q_{\phi^i}(\widetilde{h}) \right) \widehat{\psi}_{\tau+\ell}^i} e^{-\eta(\tau+\ell) - \gamma a_0} \times E_\tau \left[e^{\frac{1}{2} K_\phi(\widetilde{h})^2 \widehat{\sigma}_{i,t+\ell}^2} \right] \qquad (41) \\
&\quad \times E_\tau \left(e^{-\gamma \left(c_{t+\ell} + a_1 y_{t+\ell} + a_2 y_{t+\ell}^2 \right)} e^{\left(\Psi^i(\widetilde{h}) + Q_{\phi^i}(\widetilde{h}) \right) \overline{\rho}_{\tau+\ell} + q_1^i(\widetilde{h}) y_{\tau+\ell} + q_2^i(\widetilde{h}) y_{\tau+\ell}^2} \right)
\end{aligned}$$

where the term $e^{\left(K_\phi(\widetilde{h}) + Q_{\phi^i}(\widetilde{h}) \right) \widehat{\psi}_{\tau+\ell}^i}$ can be taken out of the expectation, as agents do not change their prior mean between τ and $\tau + \ell$ (there is no new information, and thus $\widehat{\psi}_{\tau+\ell}$ is known to all agents, by definition). Instead, $\widehat{\sigma}_{i,t+\ell}^2$ could still move stochastically according to transition matrix, but since its movement is independent of everything else, we can pull it out the expectation of $e^{\frac{1}{2} K_\phi(\widetilde{h})^2 \widehat{\sigma}_{i,t+\ell}^2}$ and compute it separately. Since $\widehat{\sigma}_{i,\tau}^2$ follows a continuous time Markov Chain process, we have $E_\tau \left[e^{\frac{1}{2} K_\phi(\widetilde{h})^2 \widehat{\sigma}_{i,t+\ell}^2} | \widehat{\sigma}_{i,t}^2 = v_j \right] = [\mathbf{\Lambda}(\ell)]_j \mathbf{E}(v)$ where $\mathbf{\Lambda}(\ell) = \mathbf{W}^{-1} diag\left(e^{\omega_j \ell} \right) \mathbf{W}$ and $\mathbf{E}_i(v) = e^{\frac{1}{2} K_\phi(\widetilde{h})^2 v_i}$, and ω_j are the eigenvalues of the infinitesimal transition matrix $\mathbf{\Lambda}$, and $\mathbf{W}$ is the matrix of corresponding eigenvectors.

Finally, the last term in (41) can be written as $E_\tau\left(e^{-\gamma c_{\tau+\ell}+\xi_{\overline{\rho}}\overline{\rho}_{\tau+\ell}+\xi_{y,1}y_{\tau+\ell}+\xi_{y,2}y^2_{\tau+\ell}}\right)$, where $\xi_{\overline{\rho}}=\left(\Psi^i\left(\widetilde{h}\right)+Q_{\phi^i}\left(\widetilde{h}\right)\right)$, $\xi_{y,1}=\left(q^i_1\left(\widetilde{h}\right)-\gamma a_1\right)$, and $\xi_{y,2}=\left(q^i_2\left(\widetilde{h}\right)-\gamma a_2\right)$. Consider the vector $\mathbf{N}_t=\left(-\gamma c_t,y_t,\overline{\rho}_t\right)$, similar to the one used in the proof of Lemma A2, with $\nu=0$. As in that proof, $\mathbf{N}_{\tau+\ell}|\mathbf{N}_\tau\sim\mathcal{N}\left(\mu_N\left(\mathbf{N}_\tau,\ell\right),\mathbf{S}_N\left(\ell\right)\right)$, where $\mu_N\left(\mathbf{N}_\tau,\ell\right)$ and $\mathbf{S}_N\left(\ell\right)$ are defined as in (24) and (25) for the appropriate $\mathbf{\Psi}\left(\ell\right)$. By following exactly the same steps as in the proof of Lemma A2, we find

$$E_\tau\left(e^{-\gamma c_{\tau+\ell}+\xi_{\overline{\rho}}\overline{\rho}_{\tau+\ell}+\xi_{y,1}y_{\tau+\ell}+\xi_{y,2}y^2_{\tau+\ell}}\right)=c^\ell_2\left(\widetilde{h}\right)e^{-\gamma c_\tau+E^\ell_1(y_\tau,\overline{\rho}_\tau)+E^\ell_2(y_\tau)+\frac{1}{2}E^\ell_3(y_\tau)}$$

where $c^\ell_2\left(\widetilde{h}\right)=|\mathbf{G}\left(\ell,\widetilde{h}\right)|^{\frac{1}{2}}/|\mathbf{S}_N\left(\ell\right)|^{\frac{1}{2}}$, $\mathbf{G}\left(\ell,\widetilde{h}\right)=\left(-2\mathbf{a}_{\ell,1}+\mathbf{S}_N\left(\ell\right)^{-1}\right)^{-1}$, $\mathbf{a}_{\ell,0}=[1,\xi_{1,y},\xi_{\overline{\rho}}]$, $\mathbf{a}_{\ell,1}=\xi_{y,2}\left[0,1,0\right]'\left[0,1,0\right]$, and

$$\begin{aligned}
E^\ell_1\left(y_\tau,\overline{\rho}_\tau\right) &= -\gamma b_1 Q_{k_L}\left(\ell\right)\overline{\rho}_t-\gamma b_0\ell-\overline{\rho}_L\gamma b_1 K_{k_L}\left(\ell\right)+\xi_{1,y}\left(e^{-k_y\ell}y_t+k_y\overline{y}Q_{k_y}\left(\ell\right)\right)\\
&\quad+\xi_{\overline{\rho}}\left(e^{-k_L\ell}\overline{\rho}_t+k_L\overline{\rho}_L Q_{k_L}\left(\ell\right)\right)\\
E^\ell_2\left(y_\tau\right) &= \xi_{y,2}\left(e^{-k_y\ell}y_\tau+k_y\overline{y}Q_{k_y}\left(\ell\right)\right)^2\\
E^\ell_3\left(y_t\right) &= \left[1,\xi_{1,y}+2\xi_{y,2}\left(e^{-k_y\ell}y_t+k_y\overline{y}Q_{k_y}\left(\ell\right)\right),\xi_{\overline{\rho}}\right]\mathbf{G}\left(\ell,\widetilde{h}\right)\\
&\quad\left[1,\xi_{1,y}+2\xi_{y,2}\left(e^{-k_y\ell}y_t+k_y\overline{y}Q_{k_y}\left(\ell\right)\right),\xi_{\overline{\rho}}\right]'
\end{aligned}$$

This finally leads to

$$\begin{aligned}
E_\tau\left(\frac{\pi_{\tau+\ell}}{\pi_\tau}\frac{M^i_{\tau+\ell}}{B^t}|\widehat{\sigma}_{i,t}=v_j\right) &= e^{-\eta\ell+\gamma\left(a_1y_\tau+a_2y^2_\tau\right)}c^i_2\left(\widetilde{h}\right)e^{q^i_0\left(\widetilde{h}\right)+\left(K_\phi\left(\widetilde{h}\right)+Q_{\phi^i}\left(\widetilde{h}\right)\right)\widehat{\psi}^i_{\tau+\ell}}\times\\
&\quad\times c^\ell_2 e^{E^\ell_1(y_\tau,\overline{\rho}_\tau)+E^\ell_2(y_\tau)+\frac{1}{2}E^\ell_3(y_\tau)}\times\left[\mathbf{\Lambda}\left(\ell\right)\right]_j\mathbf{E}\left(v\right)
\end{aligned}$$

Similarly, we can compute $E_\tau\left(\frac{\pi_{\tau+\ell}}{\pi_\tau}\right)$ immediately from Lemma A2, under the assumption $\nu=0$.■

(D) The optimal IPO time and the numerical solution of the system of PDEs

Given the stochastic discount factor π_t, the value of the patent must satisfy the standard Euler equation $E_t\left[d\left(\pi_tV_t\right)\right]=0$. Recalling that $\widehat{\sigma}_t$ before exercise moves on the grid $\mathcal{V}=\left\{v^1,...,v^n\right\}$, define $V^k_t=V\left(\overline{\rho}_t,y_t,v^k,T-s\right)$. For each k, we then obtain the PDE

$$\begin{aligned}
r_tV^k_t &= \frac{\partial V^k_t}{\partial t}+\frac{\partial V^k_t}{\partial y}k\left(\overline{y}-y_t\right)+\frac{\partial V^k_t}{\partial\overline{\rho}}k_L\left(\overline{\rho}_L-\overline{\rho}_t\right)+\frac{1}{2}\frac{\partial^2V^k_t}{\partial y^2}\sigma_y\sigma'_y+\frac{1}{2}\frac{\partial^2V^k_t}{\partial\overline{\rho}^2}\sigma_L\sigma'_L\\
&\quad+\frac{\partial^2V^k_t}{\partial\overline{\rho}\partial y}\sigma'_L\sigma'_y+\frac{\partial V^k_t}{\partial y}\sigma_y\sigma'_\pi+\frac{\partial V^k_t}{\partial\overline{\rho}}\sigma_L\sigma'_\pi+\sum_{h\neq k}\lambda_{kh}\left(V^h_t-V^k_t\right)
\end{aligned}$$

Recall that $\sigma_\pi=\gamma\left(\sigma_c+\left(a_1+2a_2y_t\right)\sigma_y\right)$, which yields

$$\begin{aligned}
r\left(\overline{\rho}_t,y_t\right)V^k_t &= \frac{\partial V^k_t}{\partial t}+\frac{\partial V^k}{\partial y}\left(A_y+B_yy_t\right)+\frac{\partial V^k_t}{\partial\overline{\rho}}\left(A_{\overline{\rho}}+B_{\overline{\rho}}y_t+C_{\overline{\rho}}\overline{\rho}_t\right)\\
&\quad+\frac{1}{2}\frac{\partial^2V^k_t}{\partial y^2}\sigma_y\sigma'_y+\frac{1}{2}\frac{\partial^2V^k_t}{\partial\overline{\rho}^2}\sigma_L\sigma'_L+\frac{\partial^2V^k_t}{\partial\overline{\rho}\partial y}\sigma'_L\sigma'_y+\sum_{h\neq k}\lambda_{kh}\left(V^h_t-V^k_t\right)
\end{aligned}$$

where $r(\overline{\rho}, y_t) = R_0 + R_1\overline{\rho}_t + R_2 y_t + R_3 y_t^2$ and

$$\begin{aligned} A_y &= k\overline{y} + \gamma(\sigma_y\sigma_c + a_1\sigma_y\sigma_y);\ B_y = 2a_2\gamma\sigma_y\sigma_y - k \\ A_{\overline{\rho}} &= k_L\overline{\rho}_L + \gamma\left(\sigma_L\sigma_c' + a_1\sigma_L\sigma_y'\right);\ B_{\overline{\rho}} = \gamma\sigma_L\sigma_y 2a_2;\ C_{\overline{\rho}} = -k_L \end{aligned}$$

Thus, we finally obtain

$$\begin{aligned} \left(r(\overline{\rho}_t, y_t) + \sum_{h\neq k}\lambda_{kh}\right)V_t^k &= \frac{\partial V_t^k}{\partial t} + \frac{\partial V^k}{\partial y}(A_y + B_y y_t) + \frac{\partial V_t^k}{\partial \overline{\rho}}(A_{\overline{\rho}} + B_{\overline{\rho}} y_t + C_{\overline{\rho}}\overline{\rho}_t) \\ &+ \frac{1}{2}\frac{\partial^2 V_t^k}{\partial y^2}\sigma_y^2 + \frac{1}{2}\frac{\partial^2 V_t^k}{\partial \overline{\rho}^2}\left(\sigma_{L,0}^2 + \sigma_{LL}^2\right) + \frac{\partial^2 V_t^k}{\partial\overline{\rho}\partial y}\sigma_{L,0}\sigma_y + \sum_{h\neq k}\lambda_{kh}V^h \end{aligned}$$

From equation (16), the following must also hold at the time of exercise τ

$$V(\overline{\rho}_\tau, y_\tau, \upsilon_\tau, T-\tau) = B^{t_i}E_\tau\left(\frac{\pi_{\tau+\ell}}{\pi_\tau}\left(G^i(\tau+\ell, T)(1-f) - 1\right)\right)$$

Since the problem is then homogeneous, we can renormalize it by setting $B^{t_i} = 1$. We use an explicit finite difference method to solve this system of PDE numerically backward. Explicitly, let $V_t^{ijk} = V^k(T-t, y(i), \overline{\rho}(j))$, so that we can write

$$\begin{aligned} V_{t+1}^{ijk} &= V_t^{ijk}A_0^{ijk} + V_t^{i+1,j,k}A_1^{ijk} + V_t^{i-1,j,k}A_2^{ijk} + V_t^{i,j+1,k}A_3^{ijk} + V_t^{i,j-1,k}A_4^{ijk} \\ &+ \left(V_t^{i+1,j+1,k} + V_t^{i-1,j-1,k} - V_t^{i+1,j-1,k} - V_t^{i-1,j+1,k}\right)\left(\frac{\sigma_L\sigma_y'\Delta t}{4\Delta y\Delta\overline{\rho}}\right) + \sum_{h\neq k}\lambda_{kh}V_t^{i,j,h} \end{aligned}$$

where

$$\begin{aligned} A_0^{ijk} &= 1 - \left(r(\widehat{\rho}(j), y(i)) + \sum_{h\neq k}\lambda_{kh}\right)\Delta t - \frac{\Delta t}{(\Delta y)^2}\sigma_y\sigma_y' - \frac{\Delta t}{(\Delta\widehat{\rho}^2)}\sigma_L\sigma_L' \\ A_1^{ijk} &= \frac{\Delta t}{2\Delta y}(A_y + B_y y(i)) + \frac{1}{2}\frac{\Delta t}{(\Delta y)^2}\sigma_y\sigma_y' \\ A_2^{ijk} &= \frac{1}{2}\frac{\Delta t}{(\Delta y)^2}\sigma_y\sigma_y' - \frac{\Delta t}{2\Delta y}(A_y + B_y y(i)) \\ A_3^{ijk} &= \frac{\Delta t}{2\Delta\widehat{\rho}}(A_{\overline{\rho}} + B_{\overline{\rho}}y(i) + C_{\overline{\rho}}\overline{\rho}(j)) + \frac{1}{2}\frac{\Delta t}{(\Delta\widehat{\rho}^2)}\sigma_L\sigma_L' \\ A_4^{ijk} &= \frac{1}{2}\frac{\Delta t}{(\Delta\widehat{\rho}^2)}\sigma_L\sigma_L' - \frac{\Delta t}{2\Delta\widehat{\rho}}(A_{\overline{\rho}} + B_{\overline{\rho}}y(i) + C_{\overline{\rho}}\overline{\rho}(j)) \end{aligned}$$

Let $EPay_{t+1}^{ijk}$ denote the expected payoff computed in (40). The American option component is taken into account by requiring that $V_{t+1}^{ijk} = \max\left(V_{t+1}^{ijk}, EPay_{t+1}^{ijk}\right)$ at every $t+1$. The boundary conditions around the grid are computed by extrapolating the values from the interior of the grid.

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