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EXCHANGE RATES AND FUNDAMENTALS

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Abstract

Standard economic models hold that exchange rates are influenced by fundamental variables such as relative money supplies, outputs, inflation rates and interest rates. Nonetheless, it has been well documented that such variables little help predict changes in floating exchange rates — that is, exchange rates follow a random walk. We show that nonetheless the data do exhibit a related link suggested by standard models – that the exchange rate helps predict fundamentals. We also show analytically that in a rational expectations present value model, an asset price manifests near random walk behavior if fundamentals are I(1) and the factor for discounting future fundamentals is near one. We suggest that this may apply to exchange rates.

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A longstanding puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. Our theories state that the exchange rate is determined by such fundamental variables, but floating exchange rates between countries with roughly similar inflation rates are in fact well-approximated as random walks. Fundamental variables do not help predict future changes in exchange rates.

Meese and Rogoff (1983a, 1983b) first established this result. They evaluated the out-of-sample fit of several models of exchange rates, using data from the 1970s. They found that the mean-squared deviation between the exchange rate predicted by the model and the actual exchange rate was larger than the mean-squared error when the lagged exchange rate alone was used to predict the current exchange rate. In short, these models could not beat the random walk. While a large number of studies have subsequently claimed to find success for various versions of fundamentals-based models, sometimes at longer horizons, and over different time periods, the success of these models has not proven to be robust. A recent comprehensive study by Cheung, Chinn, and Pascual (2002) concludes, "the results do not point to any given model/specification combination as being very successful. On the other hand, it may be that one model will do well for one exchange rate, but not for another."

In this paper, we take a new line of attack on the question of the link between exchange rates and fundamentals. We work with a conventional class of exchange models, in which the exchange rate is the expected presented discounted value of a linear combination of observable fundamentals and unobservable shocks. Linear driving processes are posited for fundamentals and shocks.

We first put aside the question of why fundamentals seem not to help predict changes in exchange rates. We ask instead if these conventional models have implications for whether the exchange rate helps predict fundamentals. It is plausible to look in this direction. Surely much of the short-term fluctuations in exchange rates is driven by changes in expectations about the future. If the models are correct, and expectations reflect information about future fundamentals, the exchange rate changes will likely be useful in forecasting these fundamentals. So these models suggest that exchange rates Granger-cause the fundamentals. Using quarterly bilateral dollar exchange rates, 1974-2001, for the dollar versus the six other G7 countries, we find some evidence of such causality, especially for nominal variables.

The statistical significance of the predictability is not uniform, and suggests a link between exchange rates and fundamentals that perhaps is modest in comparison with the links between other sets of economic variables. But in our view, the statistical predictability is notable in light of the far weaker causality from fundamentals to exchange rates.

For countries and data series for which there is statistically significant evidence of Granger causality, we next gauge whether the Granger causality results are consistent with our models. We compare the correlation of exchange rate changes with two estimates of the change in the present discounted value of fundamentals. One estimate uses only the lagged value of fundamentals. The other uses both the exchange rate and own lags. We find that the correlation is substantially higher when the exchange rate is used in estimating the present discounted value.

We then ask how one can reconcile the ability of exchange rates to predict fundamentals with the failure of fundamentals to predict exchange rate changes. We show analytically that in the class of present value models that we consider, exchange rates will follow a process arbitrarily close to a random walk if (1) at least one forcing variable (observable fundamental or unobservable shock) has a unit autoregressive root, and (2) the discount factor is near unity. So, in the limit, as the discount factor approaches unity, the change in the time t exchange rate will be uncorrelated with information known at time t-1.

Intuitively, as the discount factor approaches unity, the model puts relatively more weight on fundamentals far into the future in explaining the exchange rate. Transitory movements in the fundamentals become relatively less important compared to the permanent components. Imagine performing a Beveridge-Nelson decomposition on fundamentals, expressing them as the sum of a random walk component and a transitory component. The class of theoretical models we are considering then express the exchange rate as the discounted sum of the current and expected future fundamentals. As the discount factor approaches one, the variance of the change of discounted sum of the random walk component approaches infinity, while the variance of the change of the stationary component approaches a constant. So the variance of the change of the exchange rate is dominated by the change of the random walk component as the discount factor approaches one.

We view as unexceptionable the assumption that a forcing variable has a unit root, at least as a working hypothesis for our study. The assumption about the discount factor is, however, open to debate. We note that in reasonable calibrations of some exchange rate models, this discount factor in fact is quite near unity.

Of course our analytical result is a limiting one. Whether a discount factor of .9 or .99 or .999 is required to deliver a process statistically indistinguishable a random walk depends on the sample size used to test for random walk behavior, and the entire set of parameters of the model.

Hence we present some correlations calculated analytically in a simple stylized model. We assume a simple univariate process for fundamentals, with parameters chosen to reflect quarterly data from the recent floating period. We find that discount factors above 0.9 suffice to yield near zero correlations between the period t exchange rate and period t-1 information. We do not attempt to verify our theoretical conclusion that large discount factors account for random walk behavior in exchange rates using any particular fundamentals model from the literature. That is, we do not pick specific models that we claim satisfy the conditions of our theorem, and then estimate them and verify that they produce random walks.

To prevent confusion, we note that our finding that exchange rates predict fundamentals is distinct from our finding that large discount factors rationalize a random walk in exchange rates. It may be reasonable to link the two findings. When expectations of future fundamentals are very important in determining the exchange rate, it seems natural to pursue the question of whether exchange rates can forecast those fundamentals. But one can be persuaded that exchange rates Granger cause fundamentals, and still argue that the approximate random walk in exchange rates is not substantially attributable to a large discount factor. In the class of models we consider, all our empirical results are consistent with at least one other explanation, namely, that exchange rate movements are dominated by unobserved shocks that follow a random walk. The plausibility of this explanation is underscored by the fact that we generally fail to find cointegration between the exchange rate and observable fundamentals, a failure that is rationalized in our class of models by the presence of an I(1) (though not necessarily random walk) shock. As well, the random walk also can arise in models that fall outside the class we consider. It does so in models that combine nonlinearities/threshold effects with small sample biases (see Taylor, Peel, and Sarno (2002), and Kilian and Taylor (2001).) Exchange rates will still predict fundamentals in such models, though a nonlinear forecasting process may be required.

Our suggestion that the exchange rate will nearly follow a random walk when the discount factor is close to unity does not mean that forecasting the exchange rate is hopeless. Some recent studies have found success at forecasting exchange rates at longer horizons, or using nonlinear methods, and further research along these lines may prove fruitful. Mark (1995), Chinn and Meese (1995), and MacDonald and Taylor (1994) have all found some success in forecasting exchange rates at longer horizons imposing long-run restrictions from monetary models. Groen (2000) and Mark and Sul (2001) find greater success using panel methods. Kilian and Taylor (2001) suggest that models that incorporate nonlinear mean-reversion can improve the forecasting accuracy of fundamentals models, though it will be difficult to detect the improvement in out-of-sample forecasting exercises.

The paper is organized as follow. Section 2 describes the class of linear present value models that we use to organize our thoughts. Section 3 presents evidence that changes in exchange rates help predict fundamentals. Section 4 discusses the possibility that the random walk in exchange rates results from a discount factor near one. Section 5 concludes. An Appendix has some algebraic details.

2. MODELS

Exchange rate models since the 1970s have emphasized that nominal exchange rates are asset prices, and are influenced by expectations about the future. A variety of models relate the exchange rate to economic fundamentals and to the expected rate of change of the exchange rate. We write this relationship as:

$$(2.1) \qquad s_t = f_{1t} + z_{1t} + \lambda(E_t s_{t+1} - s_t + f_{2t} + z_{2t})$$

Here, we define the exchange rate s_t as the home currency price of foreign currency (dollars per unit of foreign currency, if the U.S. is the home country.) The *f*'s and *z*'s are economic fundamentals that ultimately drive the exchange rate, such as money supplies, money demand shocks, productivity shocks, etc. We differentiate between fundamentals observable to the econometrician, f_{1t} and f_{2t}, and those that are not observable, z_{1t} and z_{2t}. One possibility is that the true fundamental is measured with error, so that the *f*'s are the measured fundamental and the *z*'s include the measurement error; another is the *z*'s are unobserved shocks.

In equation (2.1), λ is a strictly positive parameter. The equation relates the current level of the exchange rate to fundamentals. But since $\lambda > 0$, it also says that the value of the currency is lower (the exchange rate is higher) when the currency is expected to depreciate. In other words, the value of the dollar is less when the dollar is expected to lose value.

Let $b = \lambda/(1+\lambda)$. Since $\lambda > 0$, we have $0 < b < 1$. We can rewrite equation (2.1) as:

$$(2.2) \qquad s_t = (1-b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + bE_t s_{t+1}$$

Upon imposing the Ano bubbles@ condition that $b^j E_t s_{t+j}$ goes to zero as $j \to \infty$, we have the present value relationship

$$(2.3) \qquad s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(f_{1t+j} + z_{1t+j}) + b\sum_{j=0}^{\infty} b^j E_t(f_{2t+j} + z_{2t+j})$$

We have distinguished between f_{1t} and z_{1t} on the one hand and f_{2t} and z_{2t} on the other in anticipation of our discussion of the behavior of the exchange rate as the discount factor *b* approaches 1. We see in (2.2) that as *b* goes to one, the coefficient on $f_{1t} + z_{1t}$ goes to zero and the coefficient on $f_{2t} + z_{2t}$ goes to one. This has implications for the technical conditions required to obtain a random walk in s_t in the limit, as will be discussed in section 4 below.

At this stage, it is convenient to fix *b*, and rewrite equation (2.1) as

(2.4) $$s_t = f_t + z_t + bE_t s_{t+1},$$

where $f_t = (1-b)f_{1t} + bf_{2t}$ and similarly for z_t.

We now outline three models that fit into the framework of equations (2.1) - (2.4). We will not attempt to estimate directly the models that we are about to outline. Rather, we use these to motivate alternative measures of observable fundamentals, f_t.

A. Money-Income Model

Consider first the familiar monetarist model of Frenkel (1976), Mussa (1976), and Bilson (1978). Assume in the home country there is a money market relationship given by:

(2.5) $$m_t = p_t + \gamma y_t - \alpha i_t + v_{mt}.$$

Here, m_t is the log of the home money supply, p_t is the log of the home price level, i_t is the level of the home interest rate, and v_{mt} is a shock to money demand. Here and throughout we use the term Ashock@ in a somewhat unusual sense. Our Ashocks@ potentially include constant and trend terms, may be serially correlated, and may include omitted variables that in principle could be measured. The log of output, y_t, is assumed in the monetarist model to be exogenous.

Assume a similar equation holds in the foreign country:

$$(2.6) \qquad m_t^* = p_t^* + \gamma y_t^* - \alpha i_t^* + v_{mt}^*.$$

The nominal exchange rate equals its purchasing power parity value plus an exogenous shock to the real exchange rate:

$$(2.7) \qquad s_t = p_t - p_t^* + q_t.$$

In financial markets, the interest parity relationship is

$$(2.8) \qquad E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t$$

Here ρ_t can be interpreted as an exogenous risk premium. Alternatively, uncovered interest parity might hold exactly with no risk premium, but ρ_t represents an expectational error. That is, the market=s expectation of the future exchange rate is $E_t^m s_{t+1}$, and uncovered interest parity holds:

$$(2.9) \qquad E_t^m s_{t+1} - s_t = i_t - i_t^*,$$

but the market=s expectations are not rational:

$$E_t^m s_{t+1} = E_t s_{t+1} - \rho_t.$$

Putting these equations together and rearranging, (2.10) $$s_t = \frac{1}{1+\alpha}\left[m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_{mt} - v_{mt}^*) + \alpha\rho_t\right] + \frac{\alpha}{1+\alpha}E_t s_{t+1}$$

This equation takes the form of equation (2.4) when the discount factor is given by $b = \frac{\alpha}{1+\alpha}$, the observable fundamentals are given by $f_t = \frac{m_t - m_t^* - \gamma(y_t - y_t^*)}{1+\alpha}$, and the unobservables – the shocks to the real exchange rate, to money demand, and to the risk premium – are: $z_t = \frac{1}{1+\alpha}\left[q_t - (v_{mt} - v_{mt}^*) + \alpha\rho_t\right]$.

Following Mark (1995), we set $\gamma = 1$. Under some conditions, the model implies that the exchange rate should Granger cause $m_t - m_t^* - (y_t - y_t^*)$ in a bivariate Granger causality test—namely, if the optimal forecast of $m_t - m_t^* - (y_t - y_t^*)$ does not depend only on own lags. Failure to find such a relationship is not, however, inconsistent with equation (2.10), because the presence of the shocks q_t and ρ_t breaks what would otherwise be a singular relationship. (It may help readers familiar with Campbell and Shiller's (1987) work on equity and bond markets to stress that the presence of the unobservable shocks relaxes many restrictions of a present value model, including the one just noted relating to Granger causality.)

In addition to considering the bivariate relationship between s_t and $m_t - m_t^* - (y_t - y_t^*)$, we will also investigate the relationship between s_t and $m_t - m_t^*$. That is, we also use (2.10) to motivate setting $f_t \propto m_t - m_t^*$ and, moving the other variable to z_t. We do so largely because we wish to conduct a relatively unstructured investigation into the link between exchange rates and various measures of fundamentals. But we could argue that we focus on $m_t - m_t^*$ because financial innovation has made standard income measures poor proxies for the level of transactions. Similarly, we investigate the relationship between s_t and $y_t - y_t^*$.

We also test for whether s_t Granger causes $p_t - p_t^*$. It is probably more natural to motivate the Granger causality tests between the exchange rate and prices in a sticky-price model. The sticky-price model has the feature that exchange rates are forward looking asset prices that immediately reflect changes in expectations about future variables. Nominal prices adjust only slowly. So, the exchange rate may help forecast future prices.

B. Sticky-Price Model

The sticky-price models of exchange rates, such as Dornbusch (1976) and Frankel (1979) begin, as do the monetarist models, with money market relations like equations (2.5) and (2.6). However, in those models, goods prices do not respond immediately to shocks to fundamentals, but instead are largely predetermined. That is, p_t is a function of past prices, exchange rates and fundamentals (and perhaps a small contemporaneous shock), but not affected by current fundamentals. Purchasing power parity does not hold in the short run. Output is explicitly modeled, with an aggregate demand curve that depends on the real exchange rate:

(2.11) $$y_t = \frac{\delta}{2}(s_t + p_t^* - p_t) + v_{yt},$$

where v_{yt} captures omitted terms such as trend output. An analogous equation holds for the foreign country:

(2.12) $$y_t^* = \frac{-\delta}{2}(s_t + p_t^* - p_t) + v_{yt}^* .$$

Equations (2.5), (2.6), (2.8), (2.11), and (2.12) now give us:

(2.13) $$s_t = \frac{1}{1+\alpha}[m_t - m_t^* + (1-\delta\gamma)q_t - (v_{mt} - v_{mt}^*) - \gamma(v_{yt} - v_{yt}^*) + \alpha\rho_t] + \frac{\alpha}{1+\alpha}E_t s_{t+1} .$$

We now add a price-adjustment equation as in Obstfeld and Rogoff (1996):

(2.14) $$p_{t+1} - p_t - (p_{t+1}^* - p_t^*) = \theta q_t + E_t s_{t+1} - s_t + v_{pt+1}, \quad 0 < \theta < 1 .$$

Home and foreign prices for time *t*+1 are set in advance at time *t*, except for a small error term v_{pt+1}. Home relative to foreign inflation is set equal to a trend – the expected depreciation of the home currency – plus an error-correction term. The error correction term implies that when home prices are low relative to foreign prices (expressed in a common currency), home inflation will be higher. Note that this equation implies $E_t q_{t+k} = (1-\theta)^k q_t$.

Using (2.13) and (2.14), we can write

(2.15) $$s_t = \tilde{s}_t + \frac{(1-\delta\gamma)}{1+\theta\alpha} q_t ,$$

where

$$\tilde{s}_t \equiv \frac{1}{1+\alpha}\sum_{s=t}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^{s-t} E_t[m_s - m_s^* - (v_{ms} - v_{ms}^*) - \gamma(v_{ys} - v_{ys}^*) + \alpha\rho_s] .$$

Note that $\tilde{s}_t$ is very similar to the solution for the nominal exchange rate under flexible prices (see equation (2.10)), and would be exactly the same if we had set $q_t = -(v_{yt} - v_{yt}^*)$ in the money-income model, so that the real exchange rate would be determined by relative demand shocks.

We can rewrite (2.15) as:

$$s_t = \frac{1+\theta\alpha}{\gamma\delta+\theta\alpha}\tilde{s}_t - \frac{(1-\delta\gamma)}{\gamma\delta+\theta\alpha}(p_t - p_t^*)\,. \tag{2.16}$$

The nominal exchange rate is composed of a weighted sum of two parts: a part that is the discounted sum of expected future fundamentals, $\tilde{s}_t$, and the slow-adjusting price levels, $p_t - p_t^*$. Then using the price adjustment equation, (2.14), we get

$$p_t - p_t^* = \frac{(\gamma\delta+\theta\alpha)(1-\theta)}{1+\theta\alpha}(p_{t-1} - p_{t-1}^*) - \frac{(\gamma\delta+\theta\alpha)(1-\theta)}{1+\theta\alpha}s_{t-1} + E_{t-1}\tilde{s}_t + \frac{(\gamma\delta+\theta\alpha)}{1+\theta\alpha}v_{pt}\,. \tag{2.17}$$

From equation (2.17), we see how s_t might be useful in forecasting $p_t - p_t^*$. New information about future fundamentals, acquired at time *t*-1, is immediately incorporated into $\tilde{s}_{t-1}$ and, therefore, s_{t-1}. But that information is only incorporated with a lag into prices, which adjust slowly to their long-run level. So s_{t-1} may help predict $p_t - p_t^*$.

Finally note that the real exchange rate in this model is stationary, and can be written, using equations (2.16) and (2.17) as:

$$q_t = (1-\theta)q_{t-1} + \frac{1+\theta\alpha}{\gamma\delta+\theta\alpha}(\tilde{s}_t - E_{t-1}\tilde{s}_t) - v_{pt}\,. \tag{2.18}$$

We will refer to some of these features of the sticky-price model later, when discussing whether the main theorem of section 4 applies to this model. For now, the main point we wish to emphasize is that these models – monetarist and sticky price – have the feature that exchange rates might incorporate useful information for predicting future variables such as $m_t - m_t^* - (y_t - y_t^*)$, $m_t - m_t^*$, $y_t - y_t^*$, $p_t - p_t^*$, and $i_t - i_t^*$. We include the interest differential because if the exchange rate can be helpful in forecasting prices, it might also forecast the difference in the inflation premium that is incorporated in nominal interest rates. The next subsection develops alternative motives for including prices and the interest differential.

C. Taylor-Rule Model

Here we draw on the burgeoning literature on Taylor rules. Let $\pi_t = p_t - p_{t-1}$ denote the inflation rate, and y_t^g be the "output gap". We assume that the home country (the U.S. in our empirical work) follows a Taylor rule of the form:

(2.19) $$i_t = \beta_1 y_t^g + \beta_2 \pi_t + v_t .$$

In (2.19), $\beta_1 > 0$, $\beta_2 > 1$, and the shock v_t contains omitted terms.

The foreign country follows a Taylor rule that explicitly includes exchange rates:

(2.20) $$i_t^* = -\beta_0 (s_t - \bar{s}_t^*) + \beta_1 y_t^{*g} + \beta_2 \pi_t^* + v_t^* .$$

In (2.20), $0 < \beta_0 < 1$, and $\bar{s}_t^*$ is a target for the exchange rate. We will assume that monetary authorities target the PPP level of the exchange rate:

(2.21) $$\bar{s}_t^* = p_t - p_t^* .$$

Since s_t is measured in dollars per unit of foreign currency, the rule indicates that *ceteris paribus* the foreign country raises interest rates when its currency depreciates relative to the target. Clarida, Gali and Gertler (1998) estimate monetary policy reaction functions for Germany and Japan (using data from 1979-1994) of a form similar to equation (2.20). They find that a one percent real depreciation of the mark relative to the dollar led the Bundesbank to increase interest rates (expressed in annualized terms) by five basis points, while the Bank of

Japan increased rates by 9 basis points in response to a real yen depreciation relative to the dollar.

As the next equation makes clear, our argument still follows if the U.S. were also to target exchange rates. We omit the exchange rate target in (2.19) on the interpretation that U.S. monetary policy has virtually ignored exchange rates except, perhaps, as an indicator.

Subtracting the foreign from the home money rule, we obtain

(2.22) $$i_t - i_t^* = \beta_0(s_t - \bar{s}_t^*) + \beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^*$$

Use interest parity (2.8) to substitute out for $i_t - i_t^*$, and (2.21) to substitute out for the exchange rate target:

(2.23) $$s_t = \frac{\beta_0}{1+\beta_0}(p_t - p_t^*) - \frac{1}{1+\beta_0}(\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t) + \frac{1}{1+\beta_0}E_t s_{t+1}.$$

This equation is of the general form (2.4) of the expected discounted present value models. The model provides another motivation (along with the sticky-price model) for why the exchange rate might Granger cause $p_t - p_t^*$ (treating $\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t$ as unobserved forcing variables.)

Equation (2.22) can be expressed another way, again using interest parity (2.8), and the equation for the target exchange rate, (2.21):

(2.24)

$$s_t = \beta_0(i_t - i_t^*) + \beta_0(p_t - p_t^*) - \beta_1(y_t^g - y_t^{*g}) - \beta_2(\pi_t - \pi_t^*) - v_t + v_t^* - (1-\beta_0)\rho_t + (1-\beta_0)E_t s_{t+1}$$

This equation is very much like (2.23), except that it shows that the exchange rate may be useful in forecasting future $i_t - i_t^*$. The intuition is that when the exchange rate is above its target, for example, the gap between the exchange rate and target will be eliminated only gradually. As long as the gap persists, *ceteris paribus* $i_t - i_t^*$ will be above average. So, high s_t may predict high future values of $i_t - i_t^*$.

As with the money-income model and the sticky-price model, we will not estimate explicitly the Taylor-rule model. We do not take a stand on the particular form of the Taylor rule. We use equations (2.23) and (2.24) merely to motivate our unstructured empirical work in the next section.

3. EMPIRICAL FINDINGS

A. Data and Basic Statistics

We use quarterly data, usually 1974:1-2001:3 (with exceptions noted below). With one observation lost to differencing, the sample size is $T = 110$.

We study bilateral US exchange rates versus the other six members of the G7: Canada, France, Germany, Italy, Japan and the United Kingdom. The *International Financial Statistics* (IFS) CD-ROM is the source for the end of quarter exchange rate s_t and consumer prices p_t. The OECD's *Main Economic Indicators* CD-ROM is the source for our data on the seasonally adjusted money supply, m_t (M4 in the U.K., M1 in all other countries; 1978:1-1998:4 for France, 1974:1-1998:4 for Germany, 1975:1-1998:4 for Italy). The OECD is also the source for real, seasonally adjusted GDP, y_t, for all countries but Germany, which we obtain by combining IFS (1974:1-2001:1) and OECD (2001:2-2001:3) data, and Japan, which combines data from the OECD (1974:1-2002) with 2002:3 data from the web site of the Japanese Government's

Economic and Social Research Institute. Datastream is the source for the interest rates, i_t, which are 3 month Euro rates (1975:1-2001:3 for Canada, 1978:3-2001:3 for Italy and Japan). We convert all data but interest rates by taking logs and multiplying by 100. Through the rest of the paper, the symbols defined in this paragraph (s_t, m_t, y_t, p_t) refer to the transformed data.

Let f_t denote a measure of "fundamentals" in the U.S. relative to abroad (for example, $f_t = m_t - m_t^*$.) Using Dickey-Fuller tests with a time trend included, we were generally unable to reject the null of a unit root in f_t with the following measures of f_t: m_t, p_t, i_t, y_t, $m_t - y_t$. Hence our analysis presents statistics on Δf_t for all measures of fundamentals. Even though we fail to reject unit roots for interest differentials, we are uneasy using interest differentials only in differenced form. So we present statistics for both levels and differences of interest rates.

Some basic statistics are presented in Table 3.1. Row 1 is consistent with much evidence that changes in exchange rates are serially uncorrelated, and quite volatile. The standard deviation is 5 to 10 times the size of the mean. First order autocorrelations are small, under 0.15 in absolute value. Under the null of no serial correlation, the standard error on the estimator of the autocorrelation is approximately $1/\sqrt{T} \approx 0.1$, so none of the estimates are significant at even the 10 percent level.

Rows 2 through 7 present statistics on our measures of fundamentals. A positive value for the mean indicates that the variable has been growing faster in the U.S. than abroad. For example, the figure of -0.92 for the mean value of the U.S.- Italy inflation differential means that quarterly inflation was, on average, 0.92 percentage points lower in the U.S. than in Italy during the 1974-2001 period. Of particular note is that the vast majority of estimates of first order autocorrelation coefficients suggest a rejection of the null of no serial correlation at the 10% level, and most do at the 5% level as well (again using an approximate standard error of 0.1). An exception to this pattern is in output differentials in row (7). None of the autocorrelations are significant at the 5% level, and only one (France, for which the estimate is 0.19) at the 10% level.

For each country we conducted four cointegration tests, between s_t and each of our measures of fundamentals, $m_t - m_t^*, p_t - p_t^*, i_t - i_t^*$, $y_t - y_t^*$ and $m_t - y_t - (m_t^* - y_t^*)$. We used Johansen's (1991) trace and maximum eigenvalue statistics, with critical values from Osterwald-Lenum (1992). Each bivariate VAR contained four lags. Of the 30 tests (6 countries, 5 fundamentals), we rejected the null of no cointegration at the 5 percent level in 5 instances using the trace statistic. These were for $m_t - m_t^*, p_t - p_t^*$, and $i_t - i_t^*$ for Italy, and, $p_t - p_t^*$, and $i_t - i_t^*$ for the U.K. Of the 30 tests using the maximum eigenvalue statistic, the null was rejected only once, for the U.K. for $p_t - p_t^*$. We conclude that it will probably not do great violence to assume lack of cointegration, recognizing that a complementary analysis using cointegration would be useful.

We take the lack of cointegration to be evidence that unobserved variables such as real demand shocks, real money demand shocks, or possibly even interest parity deviations have a permanent component, or at least are very persistent. Alternatively, it may be that the data we use to measure the economic fundamentals of our model have some errors with permanent or very persistent components. For example, it may be that the appropriate measure of the money supply has permanently changed because of numerous financial innovations over our sample, so that the M1 money supply series vary from the "true" money supply by some I(1) errors.

B. Granger-Causality Tests

Table 3.2 summarizes the results of our Granger causality tests on the full sample. We see in panel A that at the five percent level of significance, the null that that Δs_t fails to Granger cause $\Delta(m_t - m_t^*)$, $\Delta(p_t - p_t^*)$, $i_t - i_t^*$, $\Delta(i_t - i_t^*)$, $\Delta(y_t - y_t^*)$, and $\Delta[m_t - y_t - (m_t^* - y_t^*)]$, can be rejected in 9 cases at the 5 percent level, and 3 more cases at the 10 percent level. There are no rejections for Canada and the U.K., but rejections in 12 of the 24 tests for the other four countries. The strongest rejections are for prices, where the null is rejected in three cases at the one percent level.

In a sense, this is not particularly strong evidence that exchange rates predict fundamentals. After all, even if there were zero predictability, one would expect a handful of significant statistics just by chance. We accordingly are cautions in asserting that the posited link is well established. But one statistical (as opposed to economic) indication that the results are noteworthy comes from contrasting these results with ones for Granger causality tests running in the opposite direction. We see in panel B of Table 3.2 that the null that the fundamentals fail to Granger cause Δs_t can be rejected at the 5 percent level in only one test, and at the 10 percent level in only one more test. So, however modest is the evidence that exchange rates help to predict fundamentals, the evidence is distinctly stronger than that on the ability of fundamentals to predict exchange rates.

There were some major economic and non-economic developments during our sample that warrant investigation of sub-samples. Several of the European countries' exchange rates and monetary policies became more tightly linked in the 1990s because of the evolution of the European Monetary Union. Germany's economy was transformed dramatically in 1990 because of reunification. We therefore look at causality results for two subsamples. Table 3.3 presents results for 1974:1-1990:2, and Table 3.4 for the remaining part of the sample (1990:3-2001:2).

The results generally go the same direction as for the whole sample. In Table 3.3, we see that for the first part of the sample, we reject the null of no Granger causality from exchange rates to fundamentals at the five percent level in 10 cases, and at the ten percent level in 2 more cases. There are no cases in which we can reject the null of no Granger causality from fundamentals to exchange rates at the five percent level, and only 2 cases at the ten percent level.

Table 3.4 reports results for the second part of the sample. Now we reject the null of no Granger causality from exchange rates to fundamentals in 9 cases at the five percent level, and five more cases at the 10 percent level. But for the test of no causality from fundamentals to exchange rates, we reject nine times at the five percent level, once at the 10 percent level. In the 1990s, then, there appears to be more evidence of exchange-rate predictability. This perhaps is not entirely surprising given the effort by the European countries to stabilize exchange rates. We note, however, that several of the rejections of the null are for the yen/dollar rate.

To summarize, while the evidence is far from overwhelming, there does appear to be a link from exchange rates to fundamentals, going in the direction that exchange rates help forecast fundamentals.

C. Correlation between Δs and the Present Value of Fundamentals

Here we propose a statistic similar to one developed in Campbell and Shiller (1987). The modification of the Campbell-Shiller statistic is necessary for two reasons. First is that, unlike Campbell and Shiller, our variables are not cointegrated. Second is that we allow for unobservable forcing variables, again in contrast to Campbell and Shiller.

Write the present value relationship (2.4) as

$$s_t = \sum_{j=0}^{\infty} b^j E_t f_{t+j} + \sum_{j=0}^{\infty} b^j E_t z_{t+j} \equiv F_t + U_t \ . \tag{3.1}$$

Now $\sum_{j=0}^{\infty} b^j E_t f_{t+j} = \frac{1}{1-b}(f_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j})$. Thus

$$s_t - \frac{1}{1-b} f_{t-1} = \frac{1}{1-b} \sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j} + U_t \ . \tag{3.2}$$

Our unit root tests indicate that Δf_t, and hence $\sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j}$ are $I(0)$, and that s_t and f_t are not cointegrated. For (3.2) to be consistent with lack of cointegration between s_t and f_t, we must have $U_t \sim I(1)$. A stationary version of (3.1) is then

$$\Delta s_t = \Delta F_t + \Delta U_t \ . \tag{3.3}$$

Let F_{it} be the present value of future Δf's computed relative to an information set indexed by the i subscript. The two information sets we use are univariate and bivariate:

$$F_{1t} \equiv E(\sum_{j=0}^{\infty} b^j f_{t+j} \mid f_t, f_{t-1}, \ldots), \tag{3.4}$$

$$F_{2t} \equiv E(\sum_{j=0}^{\infty} b^j f_{t+j} \mid s_t, f_t, s_{t-1}, f_{t-1}, \ldots). \tag{3.5}$$

We hope to get a feel for whether either of these information sets yield economically meaningful present values by estimating $corr(\Delta F_{it}, \Delta s_t)$, the correlation between ΔF_{it} and Δs_t.

We estimate $corr(\Delta F_{it}, \Delta s_t)$ using estimates of ΔF_{it} constructed from univariate autoregressions (F_{1t}) or bivariate vector autoregressions (F_{2t}). If the estimated correlation is substantially stronger using the bivariate estimate, we take that as evidence that the coefficients of Δs_t in the VAR equation for Δf_t are economically reasonable and important. We limit our analysis to the variables in which there is a statistically significant relationship between Δf_t and Δs_t, as indicated by the Granger causality tests in Table 3.2.

Note that a low value of the correlation is not necessarily an indication that s_t is little affected by the present value of f_t. A low correlation will result from a small covariance between ΔF_{it} and Δs_t. But since $\text{cov}(\Delta F_{it}, \Delta s_t) = \text{cov}(\Delta F_{it}, \Delta F_t) + \text{cov}(\Delta F_{it}, \Delta U_t)$, this covariance might be small because a sharply negative covariance between ΔF_{it} and ΔU_t offsets a positive covariance between ΔF_{it} and ΔF_t. Conversely, of course, a high correlation might reflect a tight relationship between ΔF_{it} and ΔU_t with little connection between ΔF_{it} and ΔF_t.[1]

We do, however, take as reasonable the notion that if the correlation is higher for the bivariate than for the univariate information set, the coefficients on lags of Δs_t in the Δf_t equation are economically meaningful.

We construct $\hat{F}_{1t}$ from estimates of univariate autoregressions, and $\hat{F}_{2t}$ from bivariate VARs, imposing a value of the discount factor *b*. The lag length is four in both the univariate

[1] Since s_t is an element of the bivariate information set, projection of both sides of (3.1) onto this information set yields $s_t = F_{2t} + E(U_t \mid s_t, f_t, s_{t-1}, f_{t-1}, \ldots)$. It may help readers familiar with Campbell and Shiller (1987) to note that because our models include unobserved forcing variables (i.e., because U_t is present), we may not have $s_t = F_{2t} = F_t$. These equalities hold only if $E(U_t \mid s_t, f_t, s_{t-1}, f_{t-1}, \ldots) = 0$.

and bivariate estimates. We then estimate the correlations $corr(\Delta F_{it}, \Delta s_t)$ using these estimated $\hat{F}_{it}$. We report results only for data that show Granger causality from Δs_t to Δf_t at the 10 percent level or higher in the whole sample (Table 3.2, panel A). When f_t is measured by the interest rate differential, we construct F_{1t} and F_{2t} with a VAR in the level but not difference of $i_t - i_t^*$ and thus we do not report separate results for $i_t - i_t^*$ and $\Delta(i_t - i_t^*)$.

We tried three values of the discount factor, $b = 0.5$, $b = 0.9$, and $b = 0.98$, and report results for these values of the discount factor in Panels A, B, and C, respectively, of Table 3.5. For the univariate information set (F_{1t}), the three discount factors give very similar results. Of the 10 estimated correlations, only two are positive for each value of *b*. (All of the relations should be positive for the four variables reported in Table 3.5 -- $\Delta(m_t - m_t^*)$, $\Delta(p_t - p_t^*)$, $\Delta(i_t - i_t^*)$, and $\Delta[m_t - y_t - (m_t^* - y_t^*)]$ -- according to the models of section 2, if the contribution of ΔU_t is sufficiently small.) So if one relies on univariate estimates of the present value, one would find little support for the notion that changes in exchange rates reflect changes in the present value of fundamentals.

The bivariate estimates lend rather more support for this notion, especially for $b = 0.9$ and $b = 0.98$. The estimated correlation between ΔF_{2t} and Δs_t is positive in 6 of the 10 cases for $b = 0.5$; 7 of the 10 cases for $b = 0.9$ and $b = 0.98$. The median correlation is 0.10 for $b = 0.5$; 0.24 for $b = 0.9$; and 0.30 for $b = 0.98$. These compare to median correlations of -0.04 for $b = 0.5$; -0.05 for $b = 0.9$; and -0.05 for $b = 0.98$ when the univariate information set is used.

It is clear that using lags of Δs_t to estimate the present value of fundamentals results in an estimate that is more closely tied to Δs_t itself than when the present value of fundamentals is based on univariate estimates. But even for $b = 0.98$, and even limiting ourselves to data in which there is Granger causality from Δs_t to Δf_t, the largest single correlation in the full sample is 0.59 (Germany, for $\Delta(p_t - p_t^*)$, when $b = 0.98$.) A correlation less than one may be due to omitted forcing variables, U_t. In addition, we base our present values on the expected present discounted value of fundamental variables one at a time, instead of trying to find the appropriate linear combination (except when we use $m - y$ as a fundamental.) So we should not be surprised that the correlations are still substantially below one.

The long literature on random walks in exchange rates causes us to interpret the correlations in Table 3.5 as new evidence that exchange rates are tied to fundamentals. We recognize, however, that these estimates leave a vast part of the movements in exchange rates not tied to fundamentals. The results may suggest a direction for future research into the link between exchange rates and fundamentals – looking for improvements in the definition of fundamentals used to construct F_{2t}. But why is it so difficult to find a link going the other direction – using the fundamentals to forecast exchange rates? We turn to that question in the next section.

4. RANDOM WALK IN s_t AS $b \to 1$

In the class of models that we consider, one simple and direct explanation for s_t following a random walk is that the observable fundamentals variables, f_t, and the unobservable forcing variables, z_t, each follow random walks. We saw in Table 3.1 that this is not an appealing argument for our candidates for f_t, since Table 3.1's estimates indicate that most of our measures of Δf_t have significant autocorrelation. Nonetheless, it is possible the exchange rate is dominated by unobservable shocks that are well-approximated by random walks – that is, that z_t is well-approximated by a random walk, and the variance of Δs_t is dominated by the changes in z_t rather than by changes in f_t. In such a case it may be difficult to reject the null of a random walk in small samples. We put this possibility aside to consider a more appealing (to us) explanation – an explanation that is less reliant on assumptions about unobservable shocks.

A. Theoretical Statement

We begin by spelling out the sense in which the exchange rate should be expected to follow a random walk for a discount factor *b* that is near 1. We assume that f_t and z_t are forecast using current and lagged values of an $(n\times 1)$ I(1) vector x_t whose Wold innovation is the $(n\times 1)$ vector ε_t. In the money-income in section 2.A, for example, x_t would include m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, ρ_t, and any other variables used by private agents to forecast f_t and z_t. Our result requires that either (1) $f_{1t}+z_{1t}\sim$ I(1), $f_{2t}+z_{2t}\equiv 0$, or (2) $f_{2t}+z_{2t}\sim$ I(1), with the order of integration of $f_{1t}+z_{1t}$ essentially unrestricted (I(0), I(1) or identically 0). In either case, for *b* near 1, Δs_t will be well approximated by a linear combination of the elements of the unpredictable innovation ε_t. In a sense made precise in the Appendix, this approximation is arbitrarily good for b arbitrarily near 1. This means, for example, that any and all autocorrelations of Δs_t will be very near zero for b very near 1.

Of course, there is continuity in the autocorrelations in the following sense: for b near 1, the autocorrelations of Δs_t will be near zero if the previous paragraph's condition that certain variables are I(1) is replaced with the condition that those variables are I(0) but with an autoregressive root very near one. For a given autoregressive root less than one, the autocorrelations will not converge to zero as b approaches 1. But they will be very small for b very near 1.

Table 4.1 gives an indication of just how small "small" is. The table gives correlations of Δs_t with time t-1 information when x_t follows a scalar univariate AR(2). (One can think of $x_t = f_{1t} + z_{1t}$, or $x_t = f_{2t} + z_{2t}$. One can think of these two possibilities interchangeably since for given $b < 1$, the autocorrelations of Δs_t are not affected by whether or not a factor of 1-b multiplies the present value of fundamentals.) Lines (1)-(9) assume that $x_t \sim$ I(1) – specifically, $\Delta x_t \sim$ AR(1) with parameter φ. We see that for $b = 0.5$ the autocorrelations in columns (4)-(6) and the cross-correlations in columns (7)-(9) are appreciable. Specifically, suppose that one uses the conventional standard error of $1/\sqrt{T}$. Then when $\varphi = 0.5$, a sample size larger than 55 will likely suffice to reject the null that the first autocorrelation of Δs_t is zero (since row (2), column (5) gives $corr(\Delta s_t, \Delta s_{t-1}) = 0.269$, and $0.269/[1/\sqrt{55}] \approx 2.0$). (In this argument, we abstract from sampling error in estimation of the autocorrelation.) But for $b = 0.9$, the autocorrelations are dramatically smaller. For $b = 0.9$, $\varphi = 0.5$, a sample size larger than 1600 will be required, since $0.051/[1/\sqrt{1600}] \approx 2.0$. We see in lines (10)-(13) in the table that if the unit root in x_t is replaced by an autoregressive root of 0.9 or higher, the auto- and cross-correlations of Δs_t are not much changed.

B. Discussion

Two conditions are required for s_t to follow an approximate random walk. The first is that fundamentals variables be very persistent – I(1) or nearly so. This is arguably the case with our data. We saw in section 3 that we cannot reject the null of a unit root in any of our data. Further, there is evidence in other research that the unobservable variable z_t is very persistent. For the money-income model (equation (2.10)), this is suggested for v_{mt}, q_t, and ρ_t by the literature on money demand, e.g., Sriram (2000); purchasing power parity, e.g., Rogoff (1996); and, interest parity, e.g., Engel, (1996). (We recognize that theory suggests that a risk premium like ρ_t is I(0); our interpretation is that if ρ_t is I(0), it has a very large autoregressive root.)

A second condition for s_t to follow an approximate random walk is that b is sufficiently close to 1. We take Table 3.1's estimates of first order autocorrelations as suggesting that the lines in Table 4.1 most relevant to our data are those with $\varphi = 0.3$ or $\varphi = 0.5$. If so, Table 4.1 suggests that the second condition holds if b is around 0.9 or above. This condition seems plausible in the models sketched in section 2.

In the first two models presented in section 2 (the money-income model and the sticky-price model), b is related to the interest semi-elasticity of money demand: $b = \frac{\alpha}{1+\alpha}$. Bilson (1978) estimates $\alpha \approx 60$ in the monetary model, while Frankel (1979) finds $\alpha \approx 29$. The estimates from Stock and Watson (1993, Table 2, panel I, page 802) give us $\alpha \approx 45$.[2] They imply a range for b of 0.97 to 0.98 for quarterly data.

To get a sense of the plausibility of this discount factor, compare it to the discount factor implied in a theoretical model in which optimal real balance holdings are derived from a money-in-the-utility-function framework. Obstfeld and Rogoff (1998) derive a money demand function that is very similar to equation (2.1), when utility is separable over consumption and real balances, and money enters the utility function as a power function: $\frac{1}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon}$. They show that $\alpha \approx 1/\varepsilon\bar{i}$, where $\bar{i}$ is the steady-state nominal interest rate in their model. They state (p. 27), "Assuming time is measured in years, then a value between 0.04 and 0.08 seems reasonable for $\bar{i}$. It is usually thought that ε is higher than one, though not necessarily by a large margin. Thus, based on a priori reasoning, it is not implausible to assume $1/\varepsilon\bar{i} = 15$." For our quarterly data, the value of α would be 60, which is right in line with the estimate from Bilson cited above.

We note that while the money-income model (equation (2.10)) neatly fits the present-value framework of (2.1)-(2.3), the fit is less precise for the sticky-price model, since (from equation (2.15)), $s_t = \tilde{s}_t + \frac{(1-\delta\gamma)}{1+\theta\alpha} q_t$. While $\tilde{s}_t$ is explicitly a sum of discounted current and expected future fundamentals, the second term does not take such a simple form. However, we are interested in the properties of this model as α gets large. We note from equation (2.18) that

[2] Bilson uses quarterly interest rates that are annualized and multiplied by 100 in his empirical study. So his actual estimate of $\alpha = 0.15$ should be multiplied by 400 to construct a quarterly discount rate. MacDonald and Taylor (1993) estimate a discounted sum of fundamentals and test for equality with the actual exchange rate – following the methods of Campbell and Shiller (1987) for equity prices. MacDonald and Taylor rely on the estimates of Bilson to calibrate their discount factor, but mistakenly use 0.15 instead of 60 as the estimate of α. Stock and Watson's data estimates also use annualized interest rates multiplied by 100, so we have multiplied their estimate by 400.

q_t is a stationary random variable whose variance is bounded as $\alpha \to \infty$. Hence, the variance of $\frac{(1-\delta\gamma)}{1+\theta\alpha} q_t$ goes to zero as $\alpha \to \infty$, so that the real exchange rate behaves as a discounted sum of current and expected future fundamentals as α gets large. So our theorem applies also if the sticky-price model determines exchange rates.

In the Taylor-rule model of section 2, the discount factor is large when the degree of intervention by the monetary authorities to target the exchange rate is small. The strength of intervention is given by the parameter β_0 from (2.20), and the discount factor is either $\frac{1}{1+\beta_0}$ in the formulation of (2.23), or $1-\beta_0$ in the representation in (2.24). In practice, it seems as though foreign exchange intervention within the G7 has not been very active. For example, if the exchange rate were 10 percent above its PPP value, it is probably an upper bound to guess that a central bank would increase the short-term interest rate by one percentage point (expressed on an annualized basis.) With quarterly data, this would imply a value of *b* of about 0.975, which is consistent with the discount factors we imputed in the monetary models. Clarida, Gali and Gertler's (1998) estimates of the monetary policy reaction functions for Germany and Japan over the 1979-1994 period find that a 10 percent real depreciation of the currency led the central banks to increase annualized interest rates by 50 and 90 basis points, respectively. This translates to quarterly discount factors of 0.988 and 0.978.

5. CONCLUSIONS

We view the results of this paper as providing some counterbalance to the bleak view of the usefulness (especially in the short run) of rational expectations present value models of exchange rates that has become predominant since Meese and Rogoff (1983a, 1983b). On the other hand, our findings certainly do not provide strong direct support for these models, and indeed there are several caveats that deserve mention.

First, while our Granger causality results are consistent with the implications of the present value models – that exchange rates should be useful in forecasting future economic variables such as money, income, prices and interest rates – there are other possible explanations for these findings. It may be, for example, that exchange rates Granger cause the domestic consumer price level simply because exchange rates are passed on to prices of imported consumer goods with a lag. Exchange rates might Granger cause money supplies because monetary policy-makers react to the exchange rate in setting the money supply. In other words, the present value models are not the only models that imply Granger causality from exchange rates to other economic variables. The findings of Table 3.5, concerning the correlation of exchange rate changes with the change in the expected discounted fundamentals, provide some evidence that the Granger causality results are generated by the present value models, but it is far from conclusive.

Second, the empirical results are not uniformly strong. Moreover, we have produced no evidence of out-of-sample forecasting power for the exchange rate.

Third, we acknowledge a role for "unobserved" fundamentals – money demand shocks, real exchange rate shocks, risk premiums – that others might label as failures of the model. We do not find much evidence that the exchange rate is explained only by the "observable" fundamentals. Our bivariate cointegration tests generally fail to find cointegration between exchange rates and fundamentals. Moreover, we know from Mark (2001) that actual exchange rates are likely to have a much lower variance than a discounted sum of observable fundamentals. Our view is that it is perhaps unrealistic to believe that only fundamentals that are observable by the econometrician should affect exchange rates, but it is nonetheless important to note that observables are not explaining most of exchange rate changes.

Finally, we emphasize that our discussion linking the near random walk behavior of exchange rates to large discount factors is not meant to preempt other possible explanations. As we have noted, it is certainly possible that a major role is played by unobservable determinants of the exchange rate that themselves nearly follow random walks.

But perhaps our findings shift the terms of the debate. If discount factors are large (and fundamentals are I(1)), then it may not be surprising that present value models cannot outforecast the random walk model of exchange rates. If that is the case, then the more promising location for a link between fundamentals and the exchange rate is in the other direction – that exchange rates can help forecast the fundamentals. There we have found that the evidence is somewhat supportive of the link.

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Table 3.1

Basic Statistics

(1)	(2) Canada mean	ρ_1	(3) France mean	ρ_1	(4) Germany mean	ρ_1	(5) Italy mean	ρ_1	(6) Japan mean	ρ_1	(7) U.K. mean	ρ_1
(1) Δs	-0.44	-0.03	-0.35	0.10	0.15	0.07	-1.11	0.14	0.76	0.13	-0.44	0.15
	(2.20)		(5.83)		(6.06)		(5.79)		(6.22)		(5.26)	
(2) $\Delta(m-m^*)$	-0.56	0.19	0.03	0.25	-0.55	0.28	-1.19	0.28	-0.39	0.46	-1.34	0.54
	(2.59)		(2.41)		(2.38)		(2.24)		(2.18)		(1.94)	
(3) $\Delta(p-p^*)$	-0.04	0.47	-0.13	0.62	0.49	0.42	-0.92	0.62	0.50	0.16	-0.54	0.27
	(0.58)		(0.68)		(0.77)		(1.17)		(0.86)		(1.29)	
(4) $i-i^*$	-0.92	0.75	-1.89	0.62	2.02	0.84	-4.33	0.66	3.64	0.78	-2.40	0.76
	(1.72)		(3.70)		(3.01)		(4.25)		(2.78)		(2.88)	
(5) $\Delta(i-i^*)$	-0.01	-0.39	0.06	-0.37	-0.01	-0.34	0.06	-0.35	-0.04	-0.15	0.06	-0.13
	(1.21)		(3.23)		(1.70)		(3.51)		(1.83)		(2.00)	
(6) $\Delta(m-m^*)$	-0.60	0.17	-0.24	0.17	-0.72	0.13	-1.42	0.24	-0.43	0.35	-1.53	0.41
$-\Delta(y-y^*)$	(2.65)		(2.59)		(2.92)		(2.35)		(2.54)		(2.19)	
(7) $\Delta(y-y^*)$	0.04	-0.08	0.21	0.19	0.17	0.08	0.20	0.14	0.04	0.06	0.19	-0.04
	(0.79)		(0.88)		(1.47)		(1.01)		(1.21)		(1.06)	

Notes:

1. Variable definitions: Δs = percentage change in dollar exchange rate (higher value indicates depreciation). In other variables a "*" indicates a non-U.S. value, absence of "*" a U.S. value: Δm = percentage change in M1 (M2 for U.K.); Δy = percentage change in real GDP; Δp = percentage change in consumer prices; i = short term rate on government debt. Money and output are seasonally adjusted.

2. Data are quarterly, generally 1974:2-2001:3. Exceptions include an end date of 1998:4 for $m-m^*$ for France, Germany and Italy, start dates for $m-m^*$ of 1978:1 for France, 1974:1 for Germany and 1975:1 for Italy, and start dates for $i-i^*$ of 1975:1 for Canada and 1978:3 for Italy and Japan. See the text.

3. ρ_1 is the first-order autocorrelation coefficient of the indicated variable.

Table 3.2

Bivariate Granger Causality Tests, Different Measures of Δf_t
Full Sample: 1974:1-2001:3

A. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δs_t fails to cause Δf_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$		*		**	**	
(2) $\Delta(p-p^*)$			***	***	***	
(3) $i-i^*$		**			**	
(4) $\Delta(i-i^*)$		**			***	
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$		*		*		
(6) $\Delta(y-y^*)$						

B. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δf_t fails to cause Δs_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$						
(2) $\Delta(p-p^*)$	*					
(3) $i-i^*$					**	
(4) $\Delta(i-i^*)$						
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$						
(6) $\Delta(y-y^*)$						

Notes:

1. See notes to earlier tables for variable definitions.

2. Statistics are computed from fourth order bivariate vector autoregressions in $(\Delta s_t, \Delta f_t)'$. Because four observations were lost to initial conditions, the sample generally is 1975:2-2001:3, with exceptions as indicated in the notes to Table 3.1.

Table 3.3

Bivariate Granger Causality Tests, Different Measures of Δf_t
Early Part of Sample: 1974:1-1990:2

A. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δs_t fails to cause Δf_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$		**		*		
(2) $\Delta(p-p^*)$			**	***	**	
(3) $i-i^*$		***				*
(4) $\Delta(i-i^*)$		***			**	**
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$		**		**		
(6) $\Delta(y-y^*)$					**	

B. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δf_t fails to cause Δs_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$						
(2) $\Delta(p-p^*)$	*		*			
(3) $i-i^*$						
(4) $\Delta(i-i^*)$						
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$						
(6) $\Delta(y-y^*)$						

Notes:

1. See notes to earlier tables for variable definitions.

Table 3.4

Bivariate Granger Causality Tests, Different Measures of Δf_t
Later Part of Sample: 1990:3-2001:3

A. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δs_t fails to cause Δf_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$		**			***	
(2) $\Delta(p-p^*)$	*	***	*			
(3) $i-i^*$			*		**	**
(4) $\Delta(i-i^*)$			**		**	**
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$		*				**
(6) $\Delta(y-y^*)$					*	

B. Rejections at 1%(***), 5% (**), and 10% (*) level of H_0: Δf_t fails to cause Δs_t

(1)	(2) Canada	(3) France	(4) Germany	(5) Italy	(6) Japan	(7) U.K.
(1) $\Delta(m-m^*)$			**		**	
(2) $\Delta(p-p^*)$		***		**		
(3) $i-i^*$					***	
(4) $\Delta(i-i^*)$		**			***	
(5) $\Delta(m-m^*)$ $-\Delta(y-y^*)$			**		**	
(6) $\Delta(y-y^*)$			*			

Notes:

1. See notes to earlier tables for variable definitions.

Table 3.5

Correlation between Δs_t and ΔF_t

A. Discount factor $b = 0.5$

(1)	(2) Information set	(3) France	(4) Germany	(5) Italy	(6) Japan
(1) $\Delta(m-m^*)$	(a) F_{1t}	-0.02		-0.13	0.24
	(b) F_{2t}	0.10		-0.05	0.23
(2) $\Delta(p-p^*)$	(a) F_{1t}		-0.03	0.19	-0.20
	(b) F_{2t}		0.10	0.27	-0.12
(3) $\Delta(i-i^*)$	(a) F_{1t}	-0.19			-0.06
	(b) F_{2t}	-0.09			0.10
(4) $\Delta(m-m^*)$	(a) F_{1t}	-0.01		-0.10	
$-\Delta(y-y^*)$	(b) F_{2t}	0.10		-0.05	

B. Discount factor $b = 0.9$

(1)	(2) Information set	(3) France	(4) Germany	(5) Italy	(6) Japan
(1) $\Delta(m-m^*)$	(a) F_{1t}	-0.05		-0.13	0.19
	(b) F_{2t}	0.25		-0.03	-0.05
(2) $\Delta(p-p^*)$	(a) F_{1t}		-0.01	0.17	-0.16
	(b) F_{2t}		0.48	0.51	0.31
(3) $\Delta(i-i^*)$	(a) F_{1t}	-0.20			-0.06
	(b) F_{2t}	0.12			0.39
(4) $\Delta(m-m^*)$	(a) F_{1t}	-0.03		-0.11	
$-\Delta(y-y^*)$	(b) F_{2t}	0.23		-0.04	

Table 3.5 (continued)

C. Discount factor $b = 0.98$

(1)	(2) Information set	(3) France	(4) Germany	(5) Italy	(6) Japan
(1) $\Delta(m-m^*)$	(a) F_{1t}	-0.06		-0.13	0.17
	(b) F_{2t}	0.30		-0.04	-0.16
(2) $\Delta(p-p^*)$	(a) F_{1t}		0.00	0.16	-0.14
	(b) F_{2t}		0.59	0.57	0.45
(3) $\Delta(i-i^*)$	(a) F_{1t}	-0.20			-0.05
	(b) F_{2t}	0.19			0.48
(4) $\Delta(m-m^*)$	(a) F_{1t}	-0.05		-0.11	
$-\Delta(y-y^*)$	(b) F_{2t}	0.29		-0.04	

Notes:

1. F_{1t} and F_{2t} are the expected discounted values of the fundamental listed in column (1), computed from a fourth-order univariate autoregression in Δf_t (F_{1t}) or a fourth order bivariate autoregression in $(\Delta s_t, \Delta f_t)'$ (F_{2t}). See equations (3.4) and (3.5) of the text.

2. Results are presented only when Δs_t Granger causes Δf_t according to Table 3.2. For this reason, no results are presented for Canada or the U.K., or for $\Delta f_t = \Delta(y_t - y_t^*)$.

3. See notes to Table 3.1.

Table 4.1

Population Auto- and Cross-correlations of Δs_t

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				---------	Correlation of Δs_t with:				----------
	b	φ_1	φ	Δs_{t-1}	Δs_{t-2}	Δs_{t-3}	Δx_{t-1}	Δx_{t-2}	Δx_{t-3}
(1)	0.50	1.0	0.3	0.15	0.05	0.01	0.16	0.05	0.01
(2)			0.5	0.27	0.14	0.07	0.28	0.14	0.07
(3)			0.8	0.52	0.42	0.34	0.56	0.44	0.36
(4)	0.90	1.0	0.3	0.03	0.01	0.00	0.03	0.01	0.00
(5)			0.5	0.05	0.03	0.01	0.06	0.03	0.01
(6)			0.8	0.09	0.07	0.06	0.13	0.11	0.09
(7)	0.95	1.0	0.3	0.02	0.01	0.00	0.02	0.01	0.00
(8)			0.5	0.03	0.01	0.01	0.03	0.01	0.01
(9)			0.8	0.04	0.04	0.03	0.07	0.05	0.04
(10)	0.90	0.90	0.5	0.04	-0.01	-0.03	0.02	-0.03	-0.05
(11)	0.90	0.95	0.5	0.05	0.01	-0.01	0.04	-0.00	-0.02
(12)	0.95	0.95	0.5	0.02	-0.00	-0.01	0.01	-0.02	-0.03
(13)	0.95	0.99	0.5	0.02	0.01	0.00	0.03	0.01	-0.00

Notes:

1. The model is $s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t x_{t+j}$ or $s_t = b\sum_{j=0}^{\infty} b^j E_t x_{t+j}$. The scalar variable x_t follows an AR(2) process with autoregressive roots φ_1 and φ. When $\varphi_1 = 1.0$, $\Delta x_t \sim$ AR(1) with parameter φ.

2. The correlations in columns (4)-(9) were computed analytically. If $\varphi_1 = 1.0$, as in rows (1) to (9), then in the limit, as $b \to 1$, each of these correlations approaches zero.

APPENDIX

In this Appendix, we prove the statement in the text concerning random walk behavior in s_t as the discount factor $b \to 1$.

We suppose there is an $(n \times 1)$ vector of fundamentals x_t. This vector includes all variables, observable as well as unobservable (to the economist), that private agents use to forecast f_{1t}, f_{2t}, z_{1t}, and z_{2t}. For example, we may have $n = 9$, $x_t = (m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, \rho_t, u_t)'$, $f_t = m_t - m_t^* - (y_t - y_t^*)$, with u_t a variable that helps predict one or more of m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, and ρ_t. We assume that u_t is a scalar only as an example; there may be a set of variables like u_t. We assume that Δx_t follows a stationary finite order ARMA process (possibly with one or more unit moving average roots – we allow x_t to include stationary variables, as well as cointegrated I(1) variables.) Let ε_t denote the $(n \times 1)$ innovation in Δx_t, and L the lag operator, $Lx_t = x_{t-1}$. For notational simplicity we assume tentatively that Δx_t has zero mean. Write the Wold representation of Δx_t as

(A.1) $\Delta x_t = \theta(L)\varepsilon_t = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j}$, $\theta_0 \equiv I$.

We define $E_t \Delta x_{t+j}$ as $E(\Delta x_{t+j} \mid \varepsilon_t, \varepsilon_{t-1}, \ldots)$, and assume that mathematical expectations and linear projections coincide.

Define the $(n \times 1)$ vectors w_{1t} and w_{2t} as

(A.2) $w_{1t} = (1-b)\sum_{j=0}^{\infty} b^j E_t x_{t+j}$, $w_{2t} = b\sum_{j=0}^{\infty} b^j E_t x_{t+j}$, $w_t = (w_{1t}', w_{2t}')'$.

Then s_t is a linear combination of the elements of the elements of w_{1t} and w_{2t}, say

(A.3) $$s_t = a_1' w_{1t} + a_2' w_{2t}.$$

for suitable $(n \times 1)$ a_1 and a_2. We assume that either (a) $a_1' w_{1t} \sim$ I(1) and $a_2 \equiv 0$, or (b) if $a_2 \neq 0$, $a_2' w_{2t} \sim$ I(1) with $a_1' w_{1t}$ essentially unrestricted (stationary, I(1) or identically zero).

We show the following below.

1. Suppose that $a_2 \equiv 0$ (that is, $\rho_t = 0$ in the monetary model). Then

(A.4) $\underset{b \to 1}{\text{plim}}[\Delta s_t - a_1' \theta(1) \varepsilon_t] = 0$.

Here, $\theta(1)$ is an ($n \times n$) matrix of constants, $\theta(1) = \sum_{j=0}^{\infty} \theta_j$, for θ_j defined in (A.1). We note that if $a_1' x_t$ were stationary (contrary to what we assume when $a_2 = 0$), then $a_1' \theta(1) = 0$, and (A.3) states that as b approaches1, s_t approaches a constant. But if $a_1' x_t$ is I(1), as is arguably the case in our data, we have the claimed result: for b very near 1, Δs_t will behave very much like the unpredictable sequence $a_1' \theta(1) \varepsilon_t$.

2. Suppose that $a_1 \equiv 0$, $a_2 \neq 0$. Then

(A.5) $\underset{b \to 1}{\text{plim}}\{[(1-b)\Delta s_t] - b a_2' \theta(1) \varepsilon_t\} = 0$.

By assumption, $a_2' x_t \sim$ I(1), so $a_2'\theta(1) \neq 0$. Then for *b* near one, $(1-b)\Delta s_t$ will behave very much like the unpredictable sequence $a_2'\theta(1)\varepsilon_t$. This means in particular that the correlation of $(1-b)\Delta s_t$ with any information known at time *t*-1 will be very near zero. Since the correlation of Δs_t with such information is identical to that of $(1-b)\Delta s_t$, Δs_t will also be almost uncorrelated with such information.

Let us combine (A.4) and (A.5). Then for *b* near 1, Δs_t will be approximately uncorrelated with information known at *t*-1, since for *b* near 1

(A.6) $\Delta s_t \approx \{a_1 + [ba_2 /(1-b)]\}'\theta(1)\varepsilon_t$.

Two comments. First, for any given $b < 1$, the correlation of Δs_t with period *t*-1 information will be very similar for (1) x_t processes that are stationary, but barely so, in the sense of having autoregressive unit roots near 1, and (2) x_t processes that are I(1). This is illustrated in the calculations in Table 4.1.

Second, suppose that Δx_t has non-zero mean μ ($n \times 1$). Then (A.6) becomes

(A.7) $\Delta s_t \approx \{a_1 + [ba_2 /(1-b)]\}'[\mu + \theta(1)]\varepsilon_t$.

Thus the exchange rate approximately follows a random walk with drift $\{a_1 + [ba_2 /(1-b)]\}'\mu$, if $\{a_1 + [ba_2 /(1-b)]\}'\mu \neq 0$.

<u>Proof of A.4</u>:

With elementary rearrangement, we have

(A.8) $w_{1t} = x_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta x_{t+j}$.

Project (A.8) on period *t*-1 information and subtract from (A.8). Since $w_{1t} - E_{t-1}w_{1t} = \Delta w_{1t} - E_{t-1}\Delta w_{1t}$ and $x_{t-1} - E_{t-1}x_{t-1} = 0$, we get

(A.9) $\Delta w_{1t} - E_{t-1}\Delta w_{1t} = \sum_{j=0}^{\infty} b^j (E_t \Delta x_{t+j} - E_{t-1}\Delta x_{t+j}) = \theta(b)\varepsilon_t$,

the last equality following from Hansen and Sargent (1981). Next, difference (A.8). Upon rearranging the right hand side, we get $\Delta w_{1t} = \sum_{j=0}^{\infty} b^j (E_t \Delta x_{t+j} - bE_{t-1}\Delta x_{t+j})$. Project upon period *t*-1 information and rearrange to get

(A.10) $E_{t-1}\Delta w_{1t} = (1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$.

From (A.3) (with $a_2 = 0$, by assumption), (A.8) and (A.9),

(A.11) $\Delta s_t = a_1'\theta(b)\varepsilon_t + a_1'(1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$.

Since $a_1'\Delta x_t$ is stationary, $a_1'\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$ converges in probability to a stationary variable as $b \to 1$. Since $\lim_{b\to 1}(b-1) = 0$, $(1-b)a_1'\sum_{j=0}^{\infty} b^j E_{t-1}\Delta x_{t+j}$ converges in probability to zero as $b \to 1$. Hence $[\Delta s_t - a_1'\theta(b)\varepsilon_t]$ converges in probability to zero, from which (A.2) follows.

Result (A.5) results simply by noting that when $a_1 \equiv 0$, $(1-b)s_t = a_2'(1-b)b\sum_{j=0}^{\infty} b^j E_t x_{t+j}$, and the argument for (A.2) may be applied to $(1-b)s_t$.