Household Debt and Income Inequality, 1963-2003

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[PRELIMINARY. COMMENTS WELCOME]

Abstract

I construct a heterogeneous agents economy that mimics the time-series behavior of the US earnings distribution from 1963 to 2003. Agents face aggregate and idiosyncratic shocks and accumulate real and financial assets over time. I estimate the shocks driving the model using data on income inequality, on aggregate income and on measures of financial liberalization. I show how the model economy can replicate two empirical facts: the trend and cyclical behavior of household debt, and the diverging patterns in consumption and wealth inequality over time. In particular, I show that, while short-run changes in household debt can be explained by business fluctuations, the rise in household debt of the 1980s and the 1990s can be quantitatively explained only by the concurrent increase in income inequality.

KEYWORDS: Credit constraints, Incomplete Markets, Income Inequality, Household Debt

JEL : E31, E32, E44, E52, R21

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This paper uses a dynamic general equilibrium model with heterogenous agents to study the trend and the cyclical properties of household debt in a unified framework.[1] Having been relatively stable throughout the 1960s and the 1970s, household debt in the US has since the 1980s jumped out of proportion with real activity, rising between 1981 and 2003 from 67 percent to 113 percent of disposable personal income. This phenomenon has occurred alongside important changes in economic volatility. While aggregate volatility has fallen, microeconomic volatility and earnings inequality have strongly risen. For instance, the standard deviation of GDP growth has roughly halved between the period 1960-1983 and the period 1984-2002; instead, the cross-sectional standard deviation of log earnings, which had increased by only 4 basis points between 1963 and 1980, has risen by 15 basis points in the period between 1981 and 2003.[2]

Figure 1 illustrates the behavior of debt and the behavior of inequality in the period 1963-2003 and motivates this paper, asking the following question: can one construct a realistic model which explains over time the trend and the cycle in household debt? The answer of the paper is yes. Two key ingredients are needed: binding borrowing constraints for a fraction of the population, which explain the cyclicality of household debt; time-varying cross-sectional dispersion in earnings, which goes a long way in explaining, qualitatively and quantitatively, the trend.

The common explanations for the rise in household debt have referred to a combination of factors, including smaller business cycle fluctuations, the reduced costs of financial leveraging, changes in the regulatory environment for lenders, new technologies to control credit risk. Explanations for the decline in macroeconomic volatility have referred to good monetary policy, good practice (like better inventory management) and good luck (reduced volatility of the underlying economic shocks). Finally, explanations for the rise in volatility at the household level have included shifts in the relative supply of and demand for skilled workers, changes in

[1] Throughout the paper, I refer to household debt as the total outstanding debt of households and nonprofit organizations (which are grouped together in the Flows of Funds Accounts). Household debt is the broad category that includes all credit market instruments issued by households, mainly home mortgages (72% of the total as of year-end 2003) and consumer credit (21%). Residual categories include Municipal Securities, Bank Loans not elsewhere classified, other loans and advances and commercial mortgages.

The discussion here does not consider business and public debt nor does it take into account the net foreign asset position of the United States.

[2] The increase in earnings inequality has been apparent in any dimension of the data (pre-tax and post-tax, between and within groups, along the permanent and transitory components). The earnings inequality series I use is the one constructed by Eckstein and Nagypal (2004) using data drawn from the March Current Population Survey, and refers to the standard deviation of pre-tax log wages of full-time, full-year male workers. Measures of inequality constructed by other authors and based on different datasets or different samples show the same pattern.

economic institutions, and technological change.

To date, no study has tried to connect the various patterns in economic volatility with the behavior of household debt over time. There are several reasons, however, to believe that the forces driving aggregate and idiosyncratic developments in the economy play a major role in affecting the need of households to access the credit market. This is the perspective adopted here. At the aggregate level, macroeconomic developments should affect both the trend and the cyclical behavior of debt: over long horizons, as countries become richer, their financial systems better allocate the resources between those who have funds and those who need them; in addition, over the cycle, borrowers' balance sheets are strongly procyclical, thus generating credit to move together with economic activity.

At the cross-sectional level, the arguments are different: if permanent income does not change, but the income pattern becomes more erratic over time, agents will try and close the gap between actual income (which determines current period resources) and permanent income (which affects consumption) by accessing their financial assets more often. When one aggregates all financial assets across the population, market clearing implies that they sum to zero, but their cross-sectional dispersion increases. Aggregate debt - which is sum of all the negative financial positions - rises when income dispersion is greater.

The above stories are very stylized, but they serve to motivate the main question of the paper: how do the shocks hitting the economy's income and its distribution across agents affect the behavior of its credit flows? I address this question by constructing a dynamic general equilibrium model of the interaction between income volatility, household sector financial balances, and the distribution of expenditure and wealth. The model ingredients are extremely simple: heterogeneity in discount rates and market incompleteness (in the form of borrowing constraints for some of the agents). The economy is hit by idiosyncratic income shocks, financial shocks and aggregate shocks. I use these shocks because they can be somewhat easily backed out from the data and because they appear to be all plausible candidates to explain either the trend or the cycle in household debt over time.

Households are assumed to be representative of the US economy: they receive an exogenous income, consume durable and non-durable goods, and can buy and sell financial assets in order to smooth utility. An (exogenous) fraction of the households is assumed to have unrestricted access to the credit market, which they use in order to smooth expenditure in the face of a time-varying income profile. The remaining households are assumed to be impatient and credit constrained, in that they can only borrow up to a fraction of the collateral they own.

At each point in time, the economy features aggregates (like average income and average consumption) that move in line with macroeconomic aggregates; at the same time, one can see, given the time-varying behavior of income distribution, how the individual stories shape up the distribution of consumption, of wealth and financial assets. More in detail, and using annual observations on income inequality, I estimate the stochastic processes for the idiosyncratic income shocks which are capable of replicating the behavior of income inequality over time. Using data on loan-to-value ratios and productivity, I estimate processes for "financial" shocks and aggregate income shocks. I then consider the role of these shocks in explaining qualitatively and quantitavely the patterns in the data, in particular the trend and the cyclical behavior of household debt and the distribution of consumption and wealth across the population. The key finding of the paper lies in the ability of a heterogeneous agents model to explain two salient features of the data:

1. On the one hand, the model explains the timing and the magnitude of rise in household debt over income, and attributes its increase to a increase in income inequality.
2. On the other, the model can reconcile the sharp increase in income inequality over the period 1984-2003 with a smaller rise in consumption inequality, and a larger rise in wealth inequality.

The model is solved approximating the equations describing the economy (optimality conditions and market clearing conditions) around the deterministic steady state, and finding the decision rules for each agent by the method of undetermined coefficients. This approximate solution technique has the upshot that, even when the number of agents in the economy is very large (one hundred), the (linearized) decision rules can keep track of all the moments of the wealth distribution, and can then be aggregated to shed light on the distribution of gross debt. Yet the technique, albeit convenient, is subject to some caveats:

1. In the initial deterministic steady state of the model, the distribution of financial assets, and therefore the initial level of debt, is indeterminate. On the one hand, agents with high discount rates hit their borrowing limits, and this pins their total financial positions down. On the other, however, one needs to circumvent the problem that the distribution of financial assets among unconstrained agents is potentially indeterminate, unless one uses some trick. The one I pull out is to assume that agents who are not credit constrained face a very small quadratic cost of deviating from an exogenously given initial asset position. This asset position is carefully chosen in a way that aggregate net debt (the sum of all financial positions across all agents, constrained and unconstrained) is zero, while gross debt (the sum of the negative positions) is equal to what is observed in the data.
2. In the deterministic steady state, I rule out precautionary saving motives. Wealth accumulation is therefore lower than in the stochastic case for all agents. In addition, if

income uncertainty becomes very large, impatient agents might want to keep a buffer of resources in order not to hit the borrowing limit in all periods, despite the impatience motive.

3. In a neighborhood of the deterministic steady state, the assumption of certainty equivalence implies that patient agents behave like permanent income consumers. Impatient agents, instead, being borrowing constrained, behave in a "rule-of-thumb" fashion, consuming a constant fraction of their income and rolling their debt holdings over forever.

   How would the results change if one were to calculate the exact equilibrium of the model without resorting to linear approximations? Computational complexity is a major hurdle here. In a separate note (Iacoviello, 2005b), I provide evidence based on simulations of non-linear, recursive, equilibria for two-agent versions of the model presented here: an economy with two patient agents only, each bound by a natural debt limit; an economy with one patient and one impatient agent, bound by an ad-hoc collateral constraint. In the patient agents economy, the consumption differential between the two agents follows approximately a random walk, unless one of the two agents approaches (because of a series of bad income realizations) his natural debt limit.[3] When one of the agents starts with a debt to income ratio close to 1, for realistic income processes the natural debt limit is almost never approached.[4] In the patient-impatient economy, if the impatient agent starts at the borrowing constraint, he might escape from the constraint after a sufficiently long series of positive income shocks. How often this happens depends on impatience, income volatility, and risk aversion. However, if the agent is impatient enough, he hits the borrowing limit with probability one. These findings, of course, warrant further investigation. Taken together, however, they suggest that the certainty-equivalence solution of the 100 agents model can offer a good approximation of the full, non-linear model.

The structure of the paper is as follows. Section 1 briefly describes the patterns in the data. Section 2 presents the model. Section 3 describes the calibration and the simulation of the model. Section 4 presents the results. Section 5 discusses the results, and Section 6 concludes.

[3] Loosely speaking, the natural debt limit is the present discounted value of the worst income realization.

[4] In Zhang's (1997) two agents bond economy, agents rarely hit (the frequency is less than 1%) no default borrowing constraints which are much tighter than the constraints assumed here.

1. Patterns in household debt and earnings inequality

1.1. Household debt

Figure 1 illustrates the behavior of household debt to disposable personal income from 1963 to 2003. The ratio of debt to income was relatively stable throughout the 1960s and the 1970s, which led some economists to suggest that monetary policy should target broad credit aggregates in place of monetary aggregates. Debt to income expanded at a fast pace from the early 1980s on, fell slightly in the 1990-1992 recession, but began a gradual increase from 1994 on. At the end of 2003, the ratio of household debt to disposable personal income was 113 percent.

The increase in household debt has been common to both home mortgage debt and consumer debt, although it has been more pronounced for the former. Mortgage debt (which includes home equity lines of credit and home equity loans) to personal disposable income averaged around 40% in the 1960-1980 period and rose to about 75% in the late 1990s. Consumer debt averaged around 20% in the early period and rose to about 25% in later periods.

1.2. Volatility and Inequality

Several papers have documented upward trends in income and earnings inequality in the US (see for instance Katz and Autor, 1999, Moffitt and Gottschalk, 2002, and Piketty and Saez, 2003). Increased earnings dispersion has been apparent in every dimension of the data. Inequality was little changed in the 1960s, increased slowly in the 1970s and sharply in the early 1980s, and continued to rise, although at a modest pace, since the 1990s.

The next section sketches a general equilibrium model of debt, inequality and business cycles that can help making sense of these results as well as being consistent with the observations of the introduction.

2. The model

2.1. The environment and the nature of income inequality

Time is discrete. The economy consists of a large class of infinitely-lived agents (for computational purposes, they will be $N = 100$) who are distinguished by the scale of their income, by their discount rates and by the access to the credit market. Agents are indexed by i. Each agent receives an income endowment and accumulates financial assets and real assets (a house) over time. Each agent is subject to idiosyncratic and aggregate income risk.

The credit market works as follows. A fraction of the agents (unconstrained, patient agents) can freely trade one-period consumption loans. The remaining agents (constrained, impatient agents) cannot commit to repay their loans, and need to post collateral to secure access to the credit market. Instead, unbacked claims are enforceable within patient agents, whose credit limits are so large that they never bind. For all agents, the amounts that they are allowed to borrow can be repaid with probability one, and there is no default.

On the income side, agents differ in the scale of their total endowment which, absent shocks, can be thought as the source of permanent inequality in the economy. Income differentials across agents are completely exogenous.

For each agent, the log income process is the sum of three (normally distributed) components: (1) an individual-specific fixed effect, which is constant over time; (2) a time-varying aggregate component, common to every agent, which follows an autoregressive component; (3) a time-varying, individual component, which follows an autoregressive component.

2.2. Patient Agents

A fraction $\frac{n}{N}$ of the agents have a low discount rate (patients) and do not face borrowing constraints (unconstrained): alternatively, one can think that their borrowing limits are so large that they do not bind. Each of the patient agents maximize a lifetime utility function given by

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_{it} + j \log h_{it}\right)$$

where $i = \{1, 2, 3, ..., n\}$, where c_t is consumption and h_t denotes holdings of housing. The flow wealth constraint and the income process are respectively:

$$c_{it} + \left(h_{it} - (1-\delta)\, h_{it-1}\right) + R_{t-1} b_{it-1} = y_{it} + b_{it} + \psi_{it} \tag{1}$$

$$y_{it} = f_i a_t z_{it} \tag{2}$$

where b_{it} denotes borrowing of agent i at the market interest rate R_t.

In the income process above, f_i is an individual specific fixed effect; a_t denotes a macroeconomic component, which is assumed to follow an $AR(1)$ process in logs; and z_{it} denotes an individual-specific, idiosyncratic component that follows an $AR(1)$ process in logs and generates a time-varying, cross-sectional dispersion of earnings. Shocks to the macro component, instead, are perfectly correlated across all agents.

The term

$$\psi_{it} = \phi \left(b_{it} - b_i\right)^2$$

represents a very small quadratic cost of holding a quantity of debt different from b_i (that will be the steady state debt): this cost is needed in order to pin down steady state asset holdings of each patient agent, but has no effect on the dynamics of the model.[5]

For each agent, the first order conditions for this problem involve standard Euler equations for consumption and durables as follows:

$$\frac{1}{c_{it}} = E_t\left(\frac{\beta}{c_{it+1}}R_t\right) \quad (3)$$

$$\frac{1}{c_{it}} = \frac{j}{h_{it}} + \beta E_t\left(\frac{1-\delta}{c_{it+1}}\right). \quad (4)$$

In the solution procedure, I obtain the decision rules by assuming that these agents' asset position is such that they are never close to their maximum borrowing limits. This procedure is safe if their natural borrowing limit (the one which is consistent with positive steady state consumption) is large enough relative to their wealth, a condition which is assumed to hold throughout the paper.

2.3. Impatient agents

A fraction $\frac{N-n}{N}$ of the agents is assumed to discount the future more heavily than the patient agents and to face a liquidity constraint that limits the amount of borrowing to a time-varying fraction of their durables assets. With this simple assumption, I want to capture the idea that for some agents enforcement problems are such that only real assets can be used as a form of collateral. The problem the impatient agents solve is:

$$\max E_0 \sum_{t=0}^{\infty} \gamma^t \left(\log c_{it} + j \log h_{it}\right)$$

where $i = \{n+1, n+2, ..., N\}$, where $\gamma < \beta$, subject to the following budget constraint:

$$c_{it} + \left(h_{it} - (1-\delta)\, h_{it-1}\right) + R_{t-1} b_{it-1} = y_{it} + b_{it} \quad (5)$$

where again

$$y_{it} = f_i a_t z_{it}$$

and the borrowing constraint is:

$$b_{it} \leq m_t h_{it}. \quad (6)$$

For each unit of h they own, impatient agents can borrow at most m_t: exogenous time variation in m proxies for any shock to the economy-wide supply of credit which is independent of income,

[5] In the data, there is secular growth in incomes. I detrend log real income in the data using a bandpass filter procedure that isolates the frequencies between 1 and 8 years. The same trend in income is used to detrend real debt, so that the ratio detrended real debt over detrended real GDP is identical to the ratio of the nonfiltered series.

as in Ludvigson (1999). The first order conditions can be written as:

$$\frac{1}{c_{it}} = E_t\left(\frac{\gamma}{c_{it+1}}R_t\right) + \lambda_{it} \tag{7}$$

$$\frac{1}{c_{it}} = \frac{j}{h_{it}} + \gamma E_t\left(\frac{1-\delta}{c_{it+1}}\right) + m_t\lambda_{it}. \tag{8}$$

The first-order conditions for the impatient agents are thus isomorphic to those of patient ones, with the crucial addition of λ_{it}, the Lagrange multipliers on the borrowing constraint. It is straightforward to show that, in a neighborhood of the non-stochastic steady state, these agents will be borrowing constrained: so long as $\gamma < \beta$, the multiplier λ on the borrowing constraint will be strictly positive.[6]

2.4. Equilibrium

In equilibrium, all the markets clear and the interest rate works to equate demand and supply in the goods market and in the market for bonds. That is

$$\sum_{i=1}^{N}\left(c_{it} + \left(h_{it} - (1-\delta)\, h_{it-1}\right)\right) + \sum_{i=1}^{n}\psi_{it} = \sum_{i=1}^{N} y_{it}.$$

Notice that only patient agents pay the bondholding adjustment cost. When the goods market clears, Walras' law also implies that the bond market clears, that is $\sum_{i=1}^{N} b_{it} = 0$. Operationally, I solve for the recursive equilibrium in a neighborhood of its deterministic steady state when the variances of all shocks are zero. In addition, I rule out Ponzi schemes by imposing the appropriate transversality conditions.

At the aggregate level, the economy is hit by two aggregate shocks: an aggregate income shock and a financial shock. They are assumed to vary according to an AR(1) processes, that is:

$$\log a_t = \rho_a \log a_{t-1} + e_{at}$$

$$m_t = \rho_m m_{t-1} + e_{mt}$$

where the e's are normally distributed with zero mean and constant variance.

At the individual level, agents are also hit by idiosyncratic income shocks that follow:

$$\log z_{it} = \rho_z \log z_{it-1} + e_{it}$$

where $e_{it} \sim N\left(-x_t, v_t^2\right)$. The variable e_{it} is iid across agents but not over time: in practice, the cross-sectional variance of the individual income shocks is allowed to be time-varying. By

[6] See Iacoviello (2005) for a related application and for a discussion in the context of a monetary business cycle model with heterogeneous agents.

virtue of the law of large numbers, these shocks only affect the distribution of income, but not its mean level (see the appendix B for more on this: because the cross-sectional variance of the shocks is time varying, one needs to correct the cross-sectional mean of e_{it} so that the mean level of income remains constant over time; otherwise aggregate income would be high in periods of high idiosyncratic variance).

3. Calibration and simulation

3.1. Overview

The model is solved using a certainty-equivalence approximation and standard linearization procedures. At best, this is an approximation: in particular, the non-stochastic state is such that the distribution of financial assets across unconstrained agents would be indeterminate in absence of the ad-hoc bond quadratic cost. While unappealing for some purposes, this property of the model gives me one degree of freedom, since it allows choosing the steady state initial asset position in a way to match the gross household debt to disposable personal income ratio which is observed in the data.

To check whether the model can account for the main stylized facts in the data, I then use the following procedure:

1. First, I set the fixed effects in the income process in a way to match the year-1963 standard deviation of log incomes.

2. Next, I calibrate the structural parameters of the model, so that the initial steady state matches key observations of the US economy in the year 1963. In detail, I set the parameters describing preferences and technology $(\beta, \gamma, \delta, j)$ so that in the initial steady state the ratio of durable wealth to disposable income roughly matches the data.

3. Once I choose m, the model endogenously generate a steady-state ratio of aggregate debt holdings for the constrained agents. Next, I choose the b_i's in the bond holding cost function so that the aggregate bond market clears

$$\sum_{i=1}^{N} b_i = 0,$$

and that the gross household debt to total income matches the data in the initial steady state, where gross debt is defined as:

$$D_t = \sum_{i=1}^{N} (b_{it} | b_{it} > 0)$$

. In 1963, household debt to personal disposable income was 0.64. Hence, I choose a sequence of b_i such that:

$$\underbrace{\sum_{i=1}^{n}(b_i|b_i>0)}_{\text{debt held by unconstrained agents}} + \underbrace{\sum_{i=n+1}^{N}(b_i)}_{\text{debt held by constrained agents}} = 0.64 \underbrace{\sum_{i=1}^{N} y_i}_{\text{total income}}$$

4. I estimate from the data sequences of income shocks (time variation in aggregate income), financial shocks (time variation in the loan-to-value ratio m_t) and idiosyncratic income shocks (time variation in the cross-sectional earnings dispersion) for the period in exam.[7]

5. I feed the estimated shocks into the model decision rules (calculated under the assumption that the shocks that hit the economy are drawn by the same distribution from which the decision rules are calculated) starting from the year 1963, and check whether the time series generated from the model can replicate the cyclical and trend behavior of debt, consumption inequality and wealth inequality which are observed in the data.

3.2. Calibration

The time period is set equal to one year. This reflects the lack of higher frequency measures of income inequality over time, which are needed to recover the processes for the idiosyncratic shocks.

Table 3.1 summarizes the calibrated parameters. As explained above, these parameters are meant to capture the initial steady state distribution of income and financial assets, as well as the ratio of durable wealth to output. Given that patient agents are unconstrained in the nonstochastic steady state, I set their discount factor to 0.965: this pins down the steady state real interest rate at 3.5% per year.

The durable/housing preferences weight j is chosen to match the steady state stock of structures over output that is found in the data. A choice of $j = 0.1$ implies that housing stock is worth 1.4 times annual income in the initial steady state. Together with the housing depreciation rate of $\delta = 0.03$, this also ensures that steady state residential investment is about 5% of annual income.

The discount factor for impatient agents is set at 0.9 (see Iacoviello, 2005 for a discussion).

The fixed effects in the earnings process are chosen so that the standard deviation of log earnings is 0.5173 in the initial steady state.

The share of unconstrained agents is set to 65%: this is a value in between the range of estimates in the literature: using aggregate data, Campbell and Mankiw (1989) estimate a

[7] In this, I depart from the traditional RBC approach that normally treats exogenous shocks as unobservable.

$$
\begin{aligned}
\gamma &= 0.9 \\
\beta &= 0.965 \\
j &= 0.1 \\
\delta &= 0.03 \\
m &= 0.729 \\
\sigma_{\log y} &= 0.5173 \\
\frac{n}{N} &= \frac{65}{100}
\end{aligned}
$$

Table 3.1: Calibrated Parameter Values

fraction of rule-of-thumb/constrained consumers around 40 percent. Using the 1983 Survey of Consumer Finances, Jappelli estimates a 20 percent of the population to be liquidity constrained. Iacoviello (2005) finds that a share of constrained consumers of 34% is necessary to account for the positive response of spending to an aggregate housing price shock.

I then calibrate the loan-to-value ratios. In 1963, the ratio of household debt over disposable personal income was 0.64. Letting the beginning of period loan-to-value ratios to be equal to 0.73 (a number which is taken from the 1963 loan-to-value ratios for new homes), this generates a ratio of "constrained" debt to total output of 31%.

As outlined in the previous sections, the distribution of financial assets among unconstrained agents is chosen so that net household debt is zero, whereas gross total household debt matches its 1963 value of 0.64. To obtain such distribution, I proceed as follows. Unconstrained agents are 65% of the population. I split them in unconstrained creditors and unconstrained debtors, and assume that creditors are 35%, debtors 30% of the total. This is roughly in line with data from the Survey of Consumer Finances (SCF) that indicate that only a small fraction of the population has positive net financial assets.[8] Next, I assume that financial assets and liabilities are lognormally distributed with same standard deviation as that of log incomes: this way, the overall wealth distribution is more skewed that the income distribution, as in the data. Once the distributions are created, I have to decide the joint probability distribution of income and net financial assets for the unconstrained agents. Data from the 1998 SCF show a strong positive correlation between incomes and net financial as-

[8] I construct net financial assets from the SCF data as the difference between positive financial assets (like stocks. bonds and checking accounts) and financial debts (like mortgages, car loans and credit card debt). Because my model does not differentiate among financial assets, it is plausible to look at this variable in the data as the counterpart to net financial assets (that is, minus b) in my model.

	Total y	c/y	h/y	gross b/y	net b/y
Impatient agents	0.35	0.932	1.20	0.31	0.31
Patient agents	0.65	0.977	1.51	0.35	−0.31
Aggregate economy	1	0.96	1.4	0.66	0

Table 3.2: Initial Income, Wealth and Financial Positions of the Agents.

Note: Total income is normalized to 1.

sets, mainly driven by the large positive correlation between income and net financial assets at the top end of the income distribution. However, the same data from the 1983 SCF show an opposite pattern. The 1962 survey (the only survey conducted before 1983) is less detailed and harder to interpret, because the data classifications exclude mortgage debt from the financial liabilities. Because of this conflicting evidence, I assume that the net financial position of unconstrained agents is uncorrelated with their initial income, but I report the results using alternative assumptions in Section 6. The left panel of Figure 7 is a scatter plot of the income - debt combinations for all the agents in the initial steady state.

Table 3.2 illustrates some issues related to the distribution of income, financial assets and real assets at the beginning of the sample period.

3.3. Recovering the stochastic processes for the shocks

3.3.1. The income shock

I extract the income shock from the log real personal disposable income series. First, I use a bandpass filter which isolates frequencies between 1 and 8 years to remove the trend component. The resulting series is then assumed to follow an $AR(1)$ process and used to construct the $\log(a_t)$ process, shown in the top panel of Figure 2. The resulting series has the following properties:

$$\log a_t = 0.54 \log a_{t-1} + e_{at},\ \sigma_{ea} = 0.024$$

and is positively correlated with the usual business cycle indicators: in particular, it shows declines in the periods associated with NBER-dated recessions.

3.3.2. The financial shock

It is hard to construct a single indicator of the degree of financial intermediation and of the ability of impatient agents to access the credit market. In practice, financial liberalization in the United States has been a combination of a variety of forces which no single indicator can easily capture. Therefore, any indicator is likely to proxy only imperfectly for the time-variation in the degree of tightness of the borrowing constraint. Because it comes closest to proxying for the model counterpart, I take the loan-to-price ratio for newly-built homes as a measure of financial shocks and assume that it varies stochastically over time according to an $AR(1)$ process. This way, I can construct a measure of time-varying liquidity constraints, which gives me the process for m_t. As shown in the bottom panel of Figure 2, loan-to-value ratios have increased only by a small amount relative to the 1963 baseline (which was 0.729) over the sample period, rising by a roughly 5%. A sharp increase occurred in the early 1980s, when the Monetary Control Act of 1980 and the Garn-St. Germain Act of 1982 expanded households' options in mortgage markets, thus relaxing collateral constraints. The resulting series for the financial shock has the following properties:

$$m_t = 0.84 m_{t-1} + e_{mt},\ \sigma_{em} = 0.011.$$

3.3.3. The idiosyncratic shocks

Appendix B describes in detail how I use the observed measures of time-varying income inequality (when measured by the cross-sectional variance of log incomes) in order to recover the primitive idiosyncratic shocks that are consistent with given variations in income variance, once assumptions are made about the persistence of the individual income process. In the initial steady state, income dispersion is given by the variance of the log-fixed effects, $var(\log y_{ss})$. Over time, the cross-sectional log-income dispersion evolves according to:

$$var(\log y_t) = \rho_z^2 var(\log y_{t-1}) + \left(1 - \rho_z^2\right) var(\log y_{ss}) + v_t^2 \quad (1)$$

that is, income dispersion comes partly from the past, partly by new innovations. A crucial parameter determining the behavior of the model is therefore ρ_z, the autocorrelation in the individual income process. Various authors have estimated this parameter using data from the PSID. Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects, and find a value of $\rho_z = 0.53$. Recent studies, like Storesletten, Telmer and Yaron (2004), estimate a much higher value of $\rho_z = 0.95$. These studies also estimate v_t, the variance of the innovation process, which here is time-varying in order to match observed cross-sectional variation in incomes. I take a value in between these number and choose $\rho_z = 0.75$. In Section 5, I document the robustness of my results to various alternatives values of ρ_z.

Given the data of Figure 1 on $var(\log y_t)$ and given the assumption on $\rho_z = 0.75$, I can construct the individual income processes that exactly replicate the time series behavior of income inequality.

An important caveat is in order here: an assumption that I am making is that the steady state in 1963 features no cross-sectional dispersion in earnings due to temporary factors. As inequality grows over time and shows lots of persistence, I can almost in every period back out the sequences of iid shocks $\{e_{it}\}$ with variance v_t^2 that solve equation 1 given the observed behavior of $var(\log y_t)$. This is doable so long as inequality is not falling over time; suppose, in fact, that inequality were below its baseline in $t+1 = 1964$: to match the model with the data, I should assume negative correlation between the $\{e_{it}\}$ shocks and the fixed effects.

Figure 2 plots the implied time series for the shock processes normalized to zero in the year 1963, which is taken to be the base year.

4. The behavior of the model and its (successful) empirical properties

4.1. Model behavior

The model behavior is remarkably simple. At the individual level, a large chunk of the forecast error variance of incomes is driven by the idiosyncratic component. Unconstrained agents behave like permanent income consumers, and respond to positive income shocks by increasing consumption only a bit and by reducing their debt holdings (increasing their financial assets). Instead, constrained/impatient agents behave like hand-to-mouth consumers, reacting to positive income shocks by borrowing more, and being forced to cut back on borrowing in the face of negative income shocks. Figure 3 plots two typical income-debt profiles over the simulation periods for one randomly chosen constrained and one randomly chosen unconstrained agent.

Aside from these individual stories, there is one important consequence of the aggregate implication of varying cross-sectional income dispersion. In the plots of Figure 3, one can notice how income become more erratic from the 1980's on. Some agents will borrow more, some will borrow less, but the increased dispersion of earnings will imply over time an increased dispersion of financial assets. Below, I summarize the main findings:

1. *The model successfully captures the trend behavior of debt over disposable personal income.*

   The data show a rise of the debt-income ratio from 66 percent in 1963 to 113 percent in 2003. In the model simulations, the increase over the same time-period is of the same magnitude, and its timing is very simuilar. As shown by the Figure 4, the model predicts a debt to income ratio of 111 percent in 2003! In particular, just like in the data, the model predicts, starting from the 1963 steady state, flat household debt income ratios up to 1985, and a sharp increase from 1985 on. Below, I decompose the total variation in debt in its candidate causes.

2. *The model roughly captures the cyclical behavior of debt.*

Figure 5 compares year on year debt growth in the model and in the data. In the benchmark calibration, the correlation coefficient between the two series is positive (0.37) and different from zero at conventional significance levels.[9]

3. *The model predicts a modest rise of consumption inequality and a large rise in wealth inequality, as in the data.*

   Figure 6 plots simulated time profiles for income, consumption and wealth inequality.

   While the standard deviation of log incomes rises by 0.2 units over the sample period, the increase in the standard deviation of log non-durable consumption is only half as much, about 0.1.[10] Finally, the huge rise in wealth inequality is also in line with the data: this is explained by the fact that rich people in the model become on average even richer and accumulate this way positive financial assets over time.

4. *The model attributes the trend increase in debt to a rise in inequality.*

   A closer look at the sources of shocks in the model highlights the role of income inequality as the leading cause of the increase in debt over income from 1984 on.

   To understand why, consider the time pattern in income inequality. In particular, the strong acceleration in inequality of the early 1980s is what drives up, according to the model, the pattern of debt growth in the 1980s.

   To disentangle the relative contribution of each of the shocks in explaining the cyclical behavior of debt, Figure 9 and 10 show the historical decomposition of debt over personal disposable income and of real debt growth in the model in terms of the three model shocks: as shown in the figures, the trend variation in debt is driven by the change income inequality. In particular, while aggregate productivity shocks and financial shocks generate positive correlation between model debt growth and the data counterpart and can explain its cyclical movements, the observed behavior of income inequality goes a long way in making sense of the trend.

   Figure 9 confirms that, had income inequality not changed from its baseline value, the ratio of debt to disposable personal income would not have increased to its current levels. Financial shocks - as measured by the model - can explain about 5 percent of the debt/DPI increase. And cyclical variations in productivity, by their own nature, should not have affected long-run trends in debt over DPI.

[9] Clearly, it would be possible to match the data even better by carefully choosing the model parameters. For instance, one can pick the structural parameters of the model in a way to maximize the correlation between actual debt growth and simulated debt growth: by doing so, the correlation between simulated and actual debt growth can rise even futher, without significantly altering the end-of-period value of debt to income.

[10] Krueger and Perri (2003b) obtain a similar result in a model of endogenous credit markets developments.

Corr b_{63}, y_{63} creditors	Corr b_{63}, y_{63} all	d_{63}/y_{63}	d_{83}/y_{83}	d_{03}/y_{03}	Corr $\frac{\Delta d}{d}$ model,data
-0.30	-0.05	0.66	0.71	1.08	0.41
0	0.12	0.66	0.70	1.11	0.37
0.30	0.33	0.66	0.69	1.15	0.41
DATA		0.66	0.66	1.13	

Table 5.1: Sensitivity of trend and cycle in debt to initial correlation between income and net financial assets

To conclude: given the calibrated income processes, the model successfully captures the cyclical and trend dynamics over time of debt on the one hand, and consumption and wealth inequality on the other: this is especially remarkable, since I have not used these data as an input of my calibration.

5. Robustness analysis

5.1. Varying the correlation between income and net financial assets

Because the model does not endogenously generate an initial distribution of financial assets, it is natural to ask how its results depend on the initial steady state, namely on who is a borrower and who is a lender in the initial steady state of the model. Table 5.1 illustrates how the results are largely independent of the initial distribution of financial assets. In other words, they show that the large increase in debt is largely a simple by-product of the larger dispersion of earnings.

5.2. Varying the persistence of the income process

The model results depend on the persistence of the individual income processes. Were income shocks very persistent, the need for consumption smoothing would be absent, and there would be less need to access the credit market. Table 5.2 shows that one can get an increase in debt over income similar to what observed in the data for persistence of income shocks ranging in between 0.65 and 0.95. Very transitory income shocks fail to generate enough rise in debt because reshuffle continuously the income and wealth distribution. Very persistent income shocks also fail to explain the increase in debt, because agents would not need to access the credit market to smooth shocks which are thought to be very persistent.

An important conclusion that one can draw is that the increase in debt of the period 1984$-$

Persistence of shock ρ_Z	d_{63}/y_{63}	d_{83}/y_{83}	d_{03}/y_{03}	$\text{Corr}\frac{\Delta d}{d}_{\text{model,data}}$
0.5	0.66	0.68	0.94	0.24
0.65	0.66	0.69	1.03	0.32
0.75	0.66	0.70	1.11	0.37
0.85	0.66	0.72	1.17	0.36
0.95	0.66	0.68	1.08	0.44
0.99	0.66	0.62	0.67	0.28
DATA	0.66	0.66	1.13	

Table 5.2: Sensitivity to the persistence of the income process.

2003 can only be explained if the increased dispersion in earnings is perceived to be a persistent but not permanent phenomenon. But, is it? Long-term data offer conflicting hypotheses: using computations based on income tax returns, Piketty and Saez (2003) construct data for the top decile wage income share in the US going from 1919 until 1998. Figure 11 plots their inequality measure against the inequality measure used in this paper. When seen from a long-term perspective, it is hard to tell whether the recent rise in inequality is a permanent phenomenon (which could be accounted for by long-term shifts in the demand for skilled workers) or a more temporary one (which could be explained by fiscal policies, changes in social norms, or other factors). Indeed, seen from a long-term perspective and as argued for instance by Piketty and Saez (2003), one could claim that the rise in inequality of the 1980s and 1990s is only part of long-term cycle, just like the rise in inequality between the two World Wars.[11]

6. Conclusions

Credit aggregates and their evolution have always occupied an important role in macroeconomic thinking. This paper has investigated to which extent a heterogeneous agents model that mimics the distribution of income over time can explain the dynamics of household debt which have characterized the US economy in the period 1963-2003. The main finding of the paper

[11] Gottschalk and Moffitt (1995) distinguish between temporary and permanent changes in the variance of earnings, and attribute between 1/3 and 1/2 of the increased cross-sectional earnings dispersion of the 1980s to temporary phenomena. The statistical model for incomes proposed in my specification does not distinguish between temporary and permanent changes in inequality (with a value of $\rho_z = 0.75$, everything is very persistent, and nothing is very temporary, nothing is forever): it would be nice to extend the model to account for permanent and transitory components in the income process.

is that the rise in income inequality of the 1980s and the 1990s can explain at the same time all the increase in household debt, the large widening of wealth inequality and the relative stability of consumption inequality.

On the consumption inequality result, one paper which is related to mine is Krueger and Perri (2003a).[12] They argue that, in the data, consumption inequality has risen much less than income inequality. They present a model of endogenous market incompleteness in which the incentive to trade assets is directly related to the uncertainty faced at the individual level. They show that only such a model is able to predict a modest *decrease* in within-group consumption inequality alongside an increase in between-group consumption inequality.

I regard this model as a first step into making sense of the links between income inequality and the behavior of credit flows. Future extensions will consider the role of monetary policy, asset price fluctuations and debt indexation in better explaining the patterns highlighted here: in particular, relative prices are constant here; in reality, collateral prices (like housing prices) are much more volatile and procyclical than the numeraire; it would be interesting to see whether variable collateral prices can better explain the cyclical properties of debt.

[12] See also the work by Campbell and Hercowitz (2005), although their main focus is on collateralized debt.

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Appendix A: Data description and treatment

Description

- The disposable personal income series are produced by the BEA.
  The nominal and real series are respectively at http://research.stlouisfed.org/fred2/series/dpi and http://.../series/dpic96
  The ratio between the two series is used to construct the deflator of nominal debt.
- Data on household (end of period, outstanding) debt are from the Flow of Funds Z1 tables (release: December 9, 2004). An updated version of the series is available through FRED2 database at
  http://research.stlouisfed.org/fred2/series/CMDEBT
- Data on loan-to-value ratios are taken from the Federal Housing Finance Board. The loan-to-price ratio measure refers to newly built homes (see
  http://www.fhfb.gov/MIRS/mirs_t2.xls)
- Data on inequality are from Eckstein and Nagypal (2004), using data drawn from the March Current Population Survey, and refers to the standard deviation of pre-tax log wages of full-time, full-year male workers. Measures of inequality constructed by other authors and based on different datasets or different samples show the same pattern. The Eckstein-Nagypal series ends in 2002. I construct data for 2003 using data drawn from the U.S. Census Bureau website.

Treatment

In the data, there is trend growth in disposable personal income (DPI), which I account for by detrending real DPI using a band-pass filter. I then construct a deflated, detrended household debt series subtracting the trend in DPI from the original household debt series. In other words

$$\begin{aligned}
\frac{B}{Y} &= \frac{\text{nominal debt}}{\text{nominal PDI}} \\
P &= \text{deflator } = \frac{\text{nominal PDI}}{\text{real PDI}} = \frac{Y}{y} \\
\overline{y} &= \text{detrended real PDI} = \frac{\text{real PDI}}{\text{trend real DPI}} = \frac{y}{\widetilde{y}} \\
\overline{b} &= \text{detrended real debt} = \frac{\text{nominal debt/deflator}}{\text{detrended real PDI}} = \frac{B/P}{\overline{y}}
\end{aligned}$$

the advantage of this procedure is that detrended real debt shows the same trend over time as the original B/Y series plotted in Figure 1. The first difference of log detrended real debt $\Delta\left(\log \overline{b}\right)$ can then be used to compare debt growth in the data with debt growth in the model.

Appendix B: Recovering the idiosyncratic shocks

Notation and assumptions

This section describes how one can back out the idiosyncratic income shocks that are able to replicate the observed behavior of income dispersion over time. There are N individuals, for T periods.

Starting at time $t = 1$, I specify the following law of motion for individual incomes:

$$\log y_{it} = \log a_t + \log f_i + \log z_{it}$$

where f_i is an individual specific fixed-effect, a_t is a log-normally distributed aggregate disturbance, and the time-varying effect follows an AR(1) process of the form:

$$\log z_{it} = \rho \log z_{it-1} + e_{it}$$

At $t = 1$, I normalize $\log a_1 = 1$, so the level of aggregate productivity is 1. The other two terms have the following representation:

$$\begin{aligned} e_{it} &\sim N\left(-x_t, v_t^2\right) \\ \log f_i &\sim N\left(-\frac{s^2}{2}, s^2\right). \end{aligned}$$

The variance of the time-varying shocks v_t is allowed to change over time, thus affecting the cross-sectional dispersion of earnings over time. The term x_t is a time-varying factor that ensures that the mean level of z is unity for all t.[13]

At time 1 I let the economy to be in steady state, that is, I assume that $\log a_t = 0$ and $e_{i1} = 0$ for all i's. This implies that:

$$\begin{aligned} \log y_{i1} &= \log f_i \\ E\left(y_1\right) &\equiv \frac{1}{N}\sum_{i=1}^{N} y_{i1} = 1 \end{aligned}$$

Backing out the x's and the v's

Absent aggregate shocks (which, by construction, do not affect the dispersion of log earnings), it is straightforward to calculate the conditions under which mean level income will be unity for all t. At time $t = 2$:

$$\begin{aligned} E\left(\log z_2\right) &= E\left(e_2\right) \\ \log z_{i2} &\sim N\left(-x_2, v_2^2\right) \\ E\left(z_2\right) &= \exp\left(-x_2 + \frac{v_2^2}{2}\right) = 1 \text{ if } x_2 = \frac{v_2^2}{2} \\ &\implies e_2 = \log z_2 \sim N\left(-\frac{v_2^2}{2}, v_2^2\right). \end{aligned}$$

Next period, when $t = 3$, we have:

$$\begin{aligned} E\left(\log z_3\right) &= \rho E\left(\log z_2\right) + E\left(e_3\right) \\ \log z_2 &\sim N\left(-\frac{v_2^2}{2}, v_2^2\right) \\ e_3 &\sim N\left(-x_3, v_3^2\right) \\ \log z_3 &\sim N\left(-\rho\frac{v_2^2}{2} - x_3, \rho^2 v_2^2 + v_3^2\right) \\ E\left(z_3\right) &= \exp\left(-\rho\frac{v_2^2}{2} - x_3 + \frac{\rho^2 v_2^2 + v_3^2}{2}\right) = 1 \text{ if } x_3 = \frac{1}{2}\left(v_3^2 - \rho\left(1-\rho\right)v_2^2\right) \\ &\implies e_3 \sim N\left(-\frac{1}{2}\left(v_3^2 - \rho\left(1-\rho\right)v_2^2\right), v_3^2\right) \\ &\implies \log z_3 \sim N\left(-\frac{1}{2}\left(\rho^2 v_2^2 + v_3^2\right), \rho^2 v_2^2 + v_3^2\right). \end{aligned}$$

[13] Were x_t equal to zero in all periods, the properties of the lognormal distribution would imply that a higher dispersion of log-incomes would increase the mean of income.

By the same reasoning, at time $t = 4$, one can calculate

$$
\begin{aligned}
E(\log z_4) &= \rho E(\log z_3) + E(e_4) \\
\log z_3 &\sim N\left(-\frac{1}{2}\left(\rho^2 v_2^2 + v_3^2\right), \rho^2 v_2^2 + v_3^2\right) \\
e_4 &\sim N\left(-x_4, v_4^2\right) \\
\log z_4 &\sim N\left(-\frac{\rho}{2}\left(\rho^2 v_2^2 + v_3^2\right) - x_4, \rho^2\left(\rho^2 v_2^2 + v_3^2\right) + v_4^2\right) \\
E(z_4) &= 1 \text{ if } x_4 = \frac{1}{2}\left(v_4^2 - \rho(1-\rho) v_3^2 - \rho^3(1-\rho) v_2^2\right) \\
E(\log z_4) &= -\frac{1}{2}\left(v_4^2 + \rho^2 v_3^2 + \rho^4 v_2^2\right).
\end{aligned}
$$

Hence the pattern of the x's over time obeys the following formulas:

$$
\begin{aligned}
x_1 &= 0 \\
x_2 &= \frac{1}{2} v_2^2 \\
x_3 &= \frac{1}{2}\left(v_3^2 - \rho(1-\rho) v_2^2\right) \\
x_4 &= \frac{1}{2}\left(v_4^2 - \rho(1-\rho) v_3^2 - \rho^3(1-\rho) v_2^2\right) \\
&\ldots \\
x_t &= \frac{1}{2}\left(v_t^2 - \rho(1-\rho)\sum_{i=0}^{t-1}\left(\rho^{2(t-1-i)} v_i^2\right)\right)
\end{aligned}
$$

The implied volatility of earnings

In each period t, assuming that the v_{it} shocks are *iid* and uncorrelated with the fixed effect, the cross-sectional variance of log earnings will be given by:

$$var(\log y_t) = var(\log f) + var(\log z_t)$$

where

$$var(\log z_t) = \rho^2 var(\log z_{t-1}) + v_t^2$$

and for each variable x_{it} the variance is taken with respect to the i, that is:

$$var(x_t) \equiv \frac{1}{N}\left(\sum_{i=1}^{N} x_{it} - \frac{1}{N}\sum_{i=1}^{N} x_{it}\right).$$

Let the economy be in the non-stochastic steady state at time $t = 1$. At time $t = 1$, if $e_{i1} = 1$ for all i, we have that $v_1 = 0$ and

$$var(\log y_1) = s^2$$

At time $t = 2$, instead, let $v_2 > 0$ (notice how this procedure works easily *only* when income inequality rises from the initial steady state), so that

$$var(\log y_2) = s^2 + v_2^2$$

at time $t = 3$, $var(\log z_3) = \rho^2 v_2^2 + v_3^2$, so that

$$var(\log y_3) = s^2 + \rho^2 v_2^2 + v_3^2$$

Given observations over time on $var(\log y_{it})$, the last three equations and so on for each period t can be used to construct in each period the vector of individual income shocks $\mathbf{v}$ which generates the desired pattern of log-income variances. That is

$$\begin{aligned}
v_1 &= 0 \\
v_2^2 &= var\left(\log y_2\right) - s^2 \\
v_3^2 &= var\left(\log y_3\right) - \rho^2 var\left(\log y_2\right) - \left(1-\rho^2\right) var\left(\log f\right) \\
&\ldots \\
v_t^2 &= var\left(\log y_t\right) - \rho^2 var\left(\log y_{t-1}\right) - \left(1-\rho^2\right) s^2
\end{aligned}$$

Practical implementation of the algorithm used to calculate the vector of shocks

The practical implementation of the algorithm used to back out the individual income shocks goes through the steps outlined below. Some precautions need to be followed to take care of sampling error.

1. Given a $T \times 1$ time-series vector of data on income dispersion $var\left(\log y_t\right)$, set the variance of the initial fixed effects so that

$$var\left(\log f\right) = var\left(\log y_1\right).$$

   This is done by using a random number generator that creates a $N \times 1$ vector of observations on $\log f$. Sampling uncertainty is taken care using the zscore Matlab function, which standardizes a given vector so that it has sample mean and variance exactly equal to 0 and 1, respectively.

2. Assume a value for ρ, the autocorrelation of the income shocks. Construct the $T \times 1$ vector of cross-sectional variances v_t^2 using data on $var\left(\log y_t\right)$ using the formulas of the previous subsection.

3. Using the time-series vector v_t^2, construct recursively the series x_t.

4. Construct the $T \times N$ matrix (e) of iid shocks over time having, for each period t, variance equal to v_t^2 and mean equal to x_t. To correct for sampling error, go as follows:

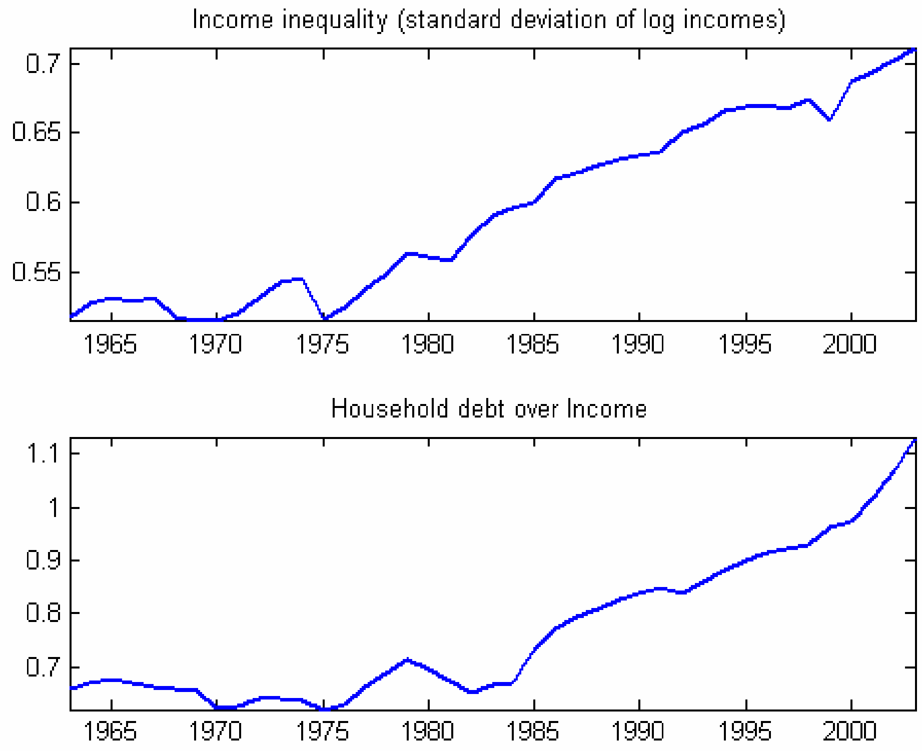
    1. at time 1, all the e's are equal to zero.
    2. construct the second row (e_2) of iid shocks corresponding to $t = 2$, by generating a random vector of length N.
    3. Ensure that e_2 is orthogonal to $\log f$ by constructing $\widehat{e_2}$, the residuals of a regression of e_2 on a constant term and $\log f$. Normalize $\widehat{e_2}$ so that it has mean equal to x_2 and variance equal to v_2^2. Let the resulting vector be $\widetilde{e_2}$
    4. For each successive period, construct e_t in a way that is orthogonal to $\log f$ and $\widetilde{e_{t-1}}$, $\widetilde{e_{t-2}}$ and so on.

5. Consistently with the value of ρ, for each i, the time series of length T of income sequences $\log z_{it}$ will be formed using:

$$\log z_{it} = \rho \log z_{it-1} + e_{it}.$$

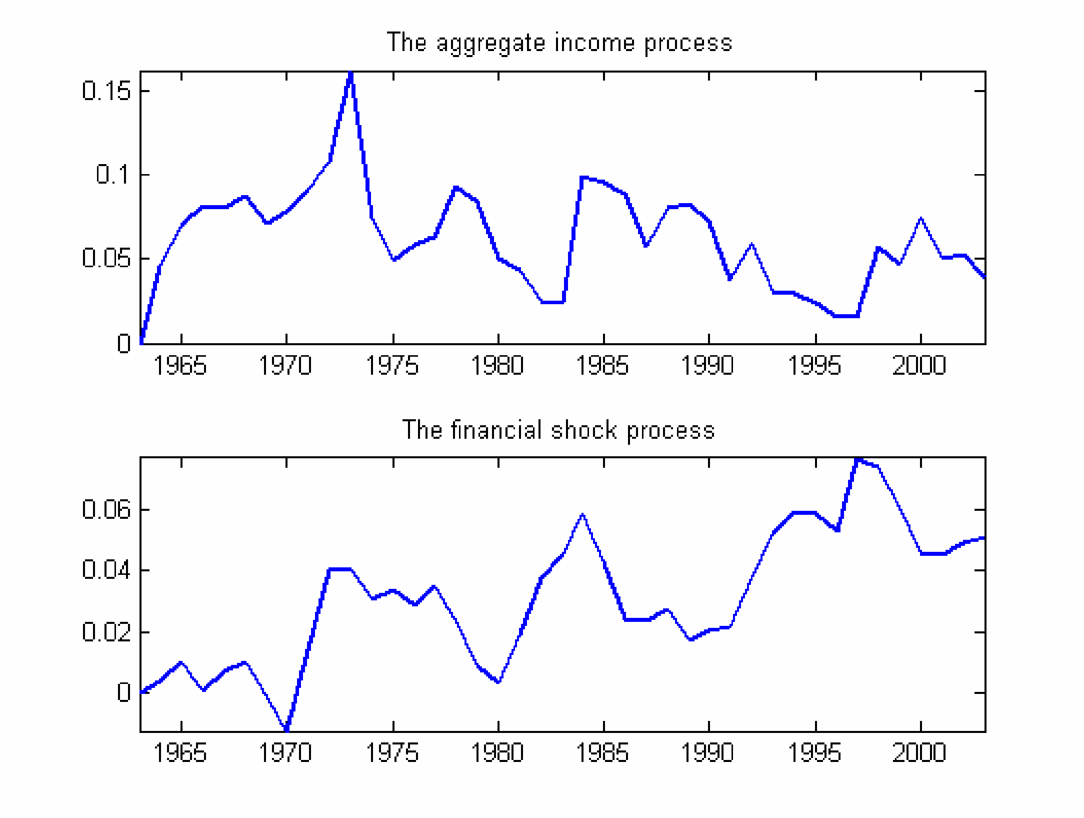
Figures

FIGURE 1
Top panel: Income Inequality
Bottom panel: Household Debt divided by Personal Disposable Income.



Note: See Appendix A for data definitions and sources.

FIGURE 2: The stochastic processes for aggregate income and the loan-to-value ratio



Notes: The variables are expressed in deviations from the steady state.

FIGURE 3: Income and Asset Choices for an Unconstrained and a Constrained Agent

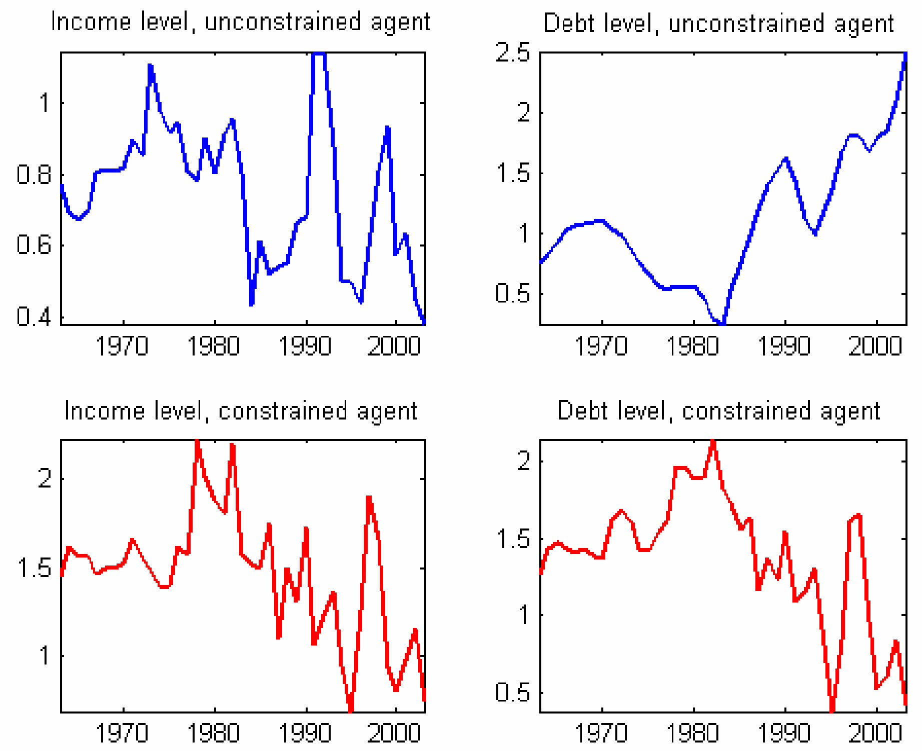


FIGURE 4: Comparison between model and data: Household Debt over Income

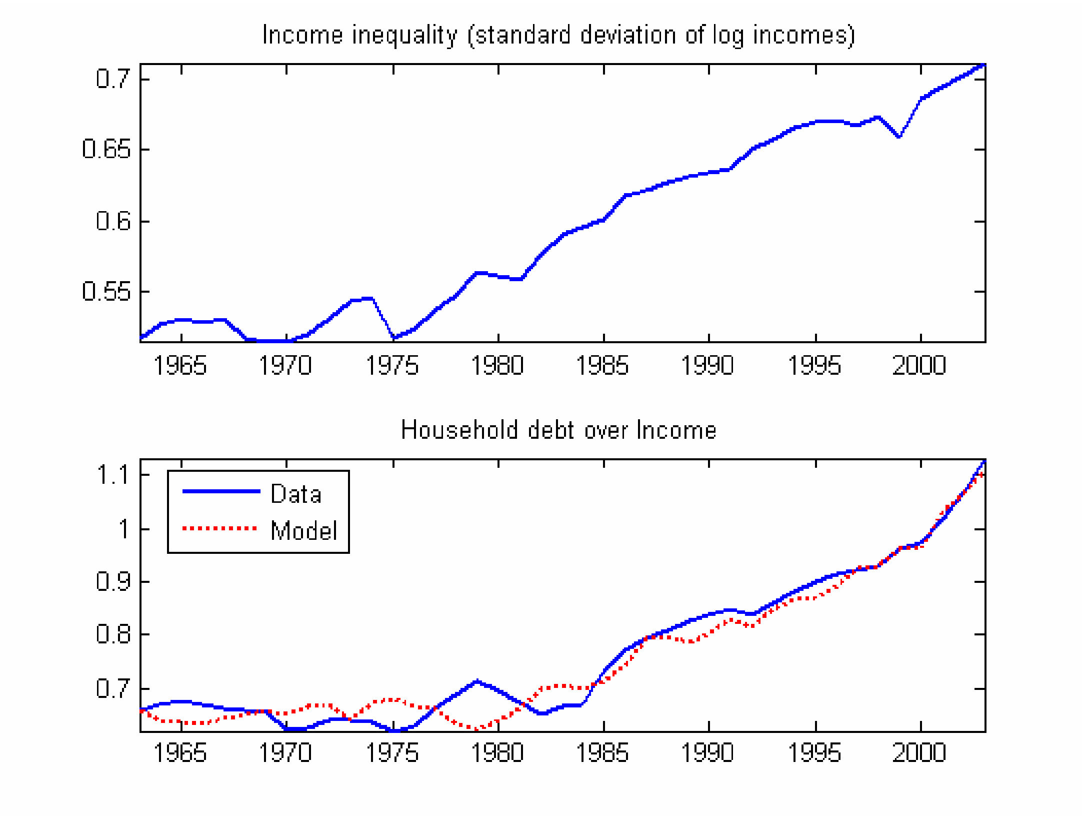


FIGURE 5: Comparison between model and data: Household Debt Growth

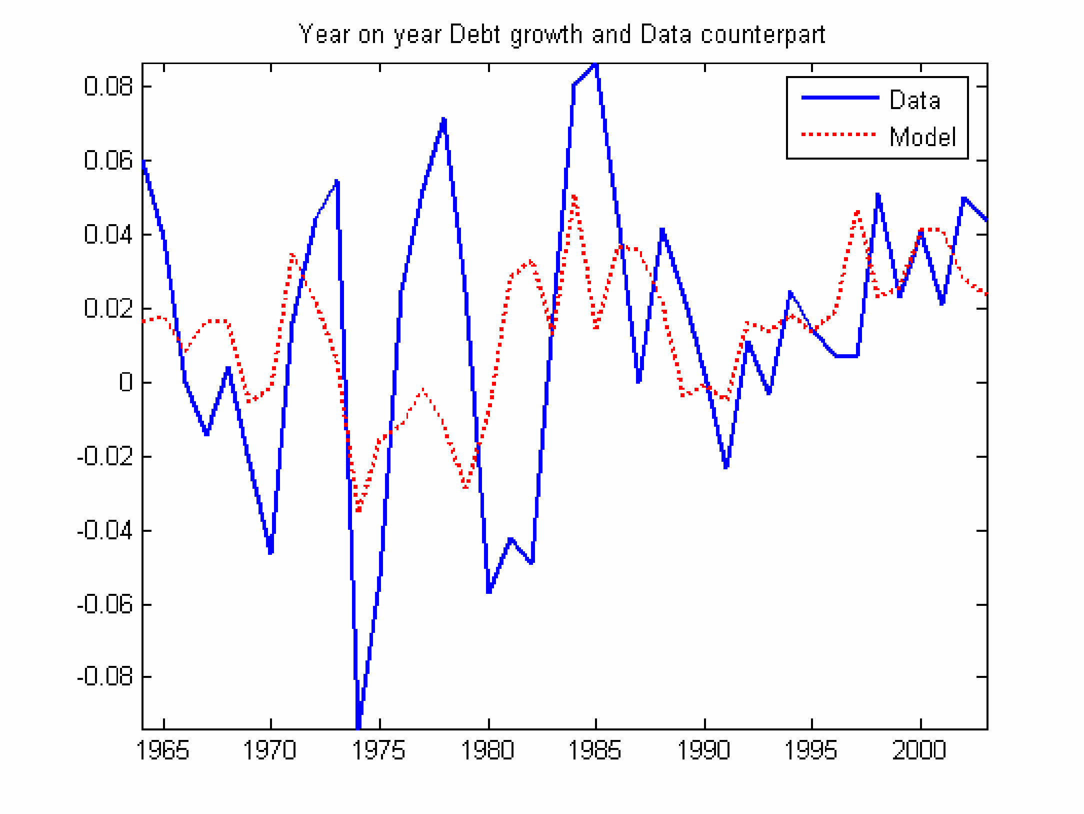


FIGURE 6: Simulated time series for Income, Consumption and Wealth Inequality

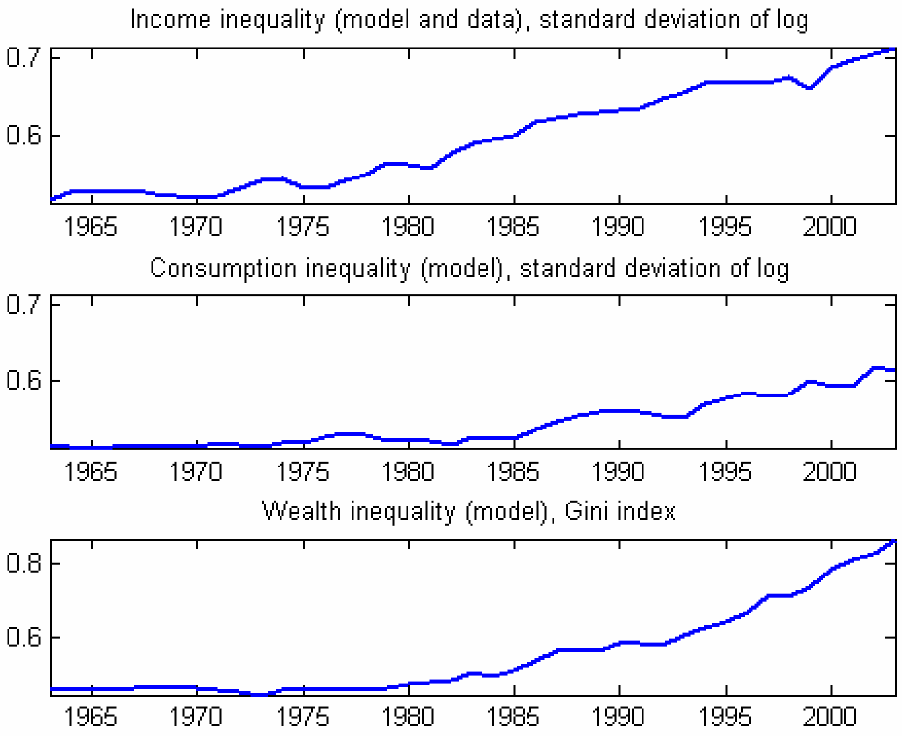


FIGURE 7: Initial and Final Income and Debt Positions

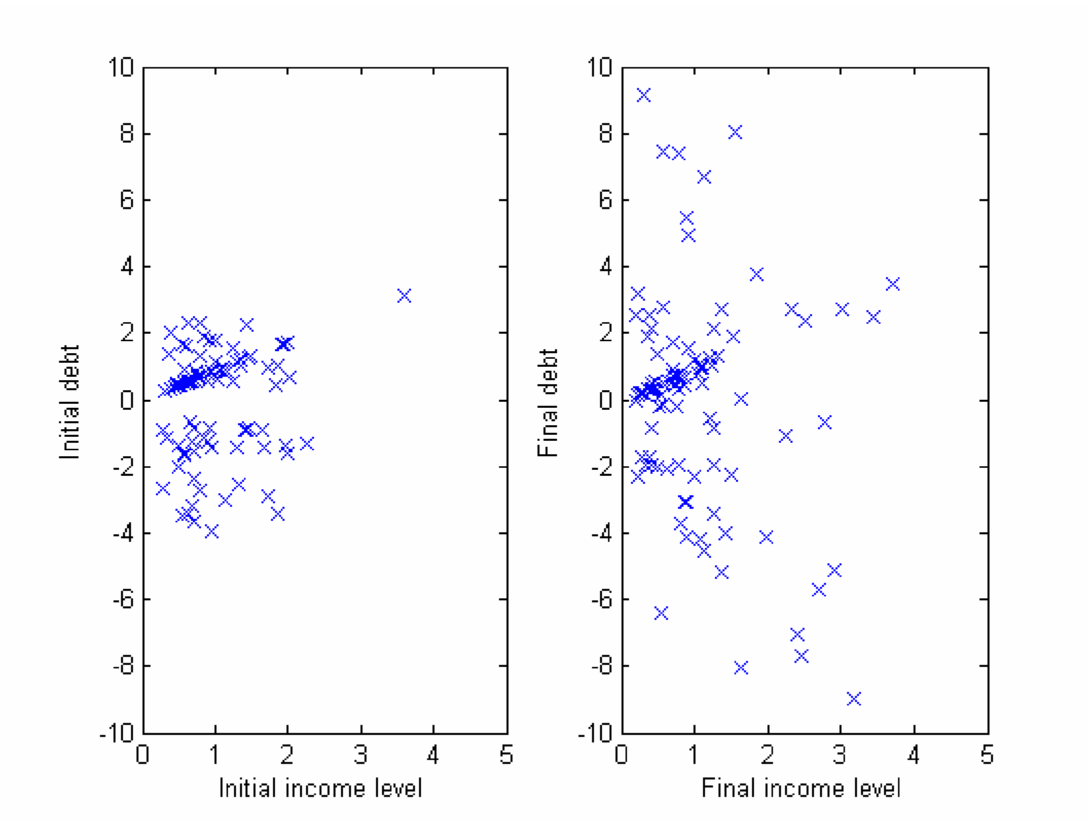


FIGURE 8: Simulated Time Series for the Macroeconomic Aggregates

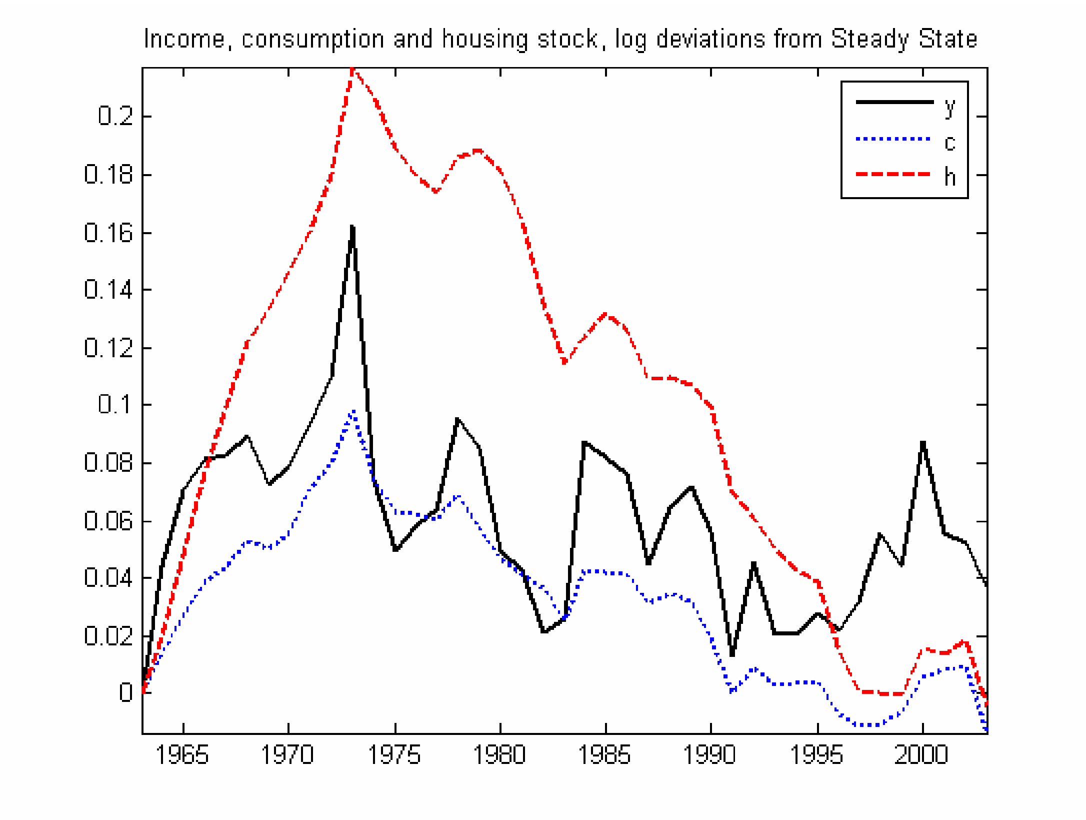


FIGURE 9: Counterfactual Experiment: Simulated Time Series for Household Debt over Income

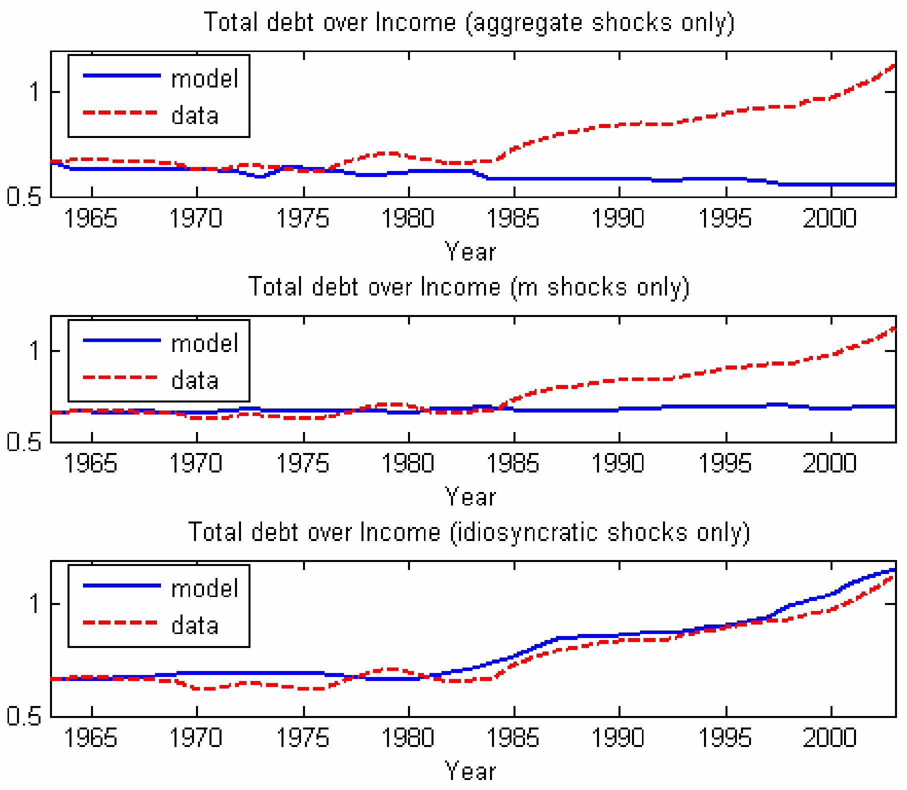


FIGURE 10: Counterfactual experiment: Simulated time series for household debt growth over income

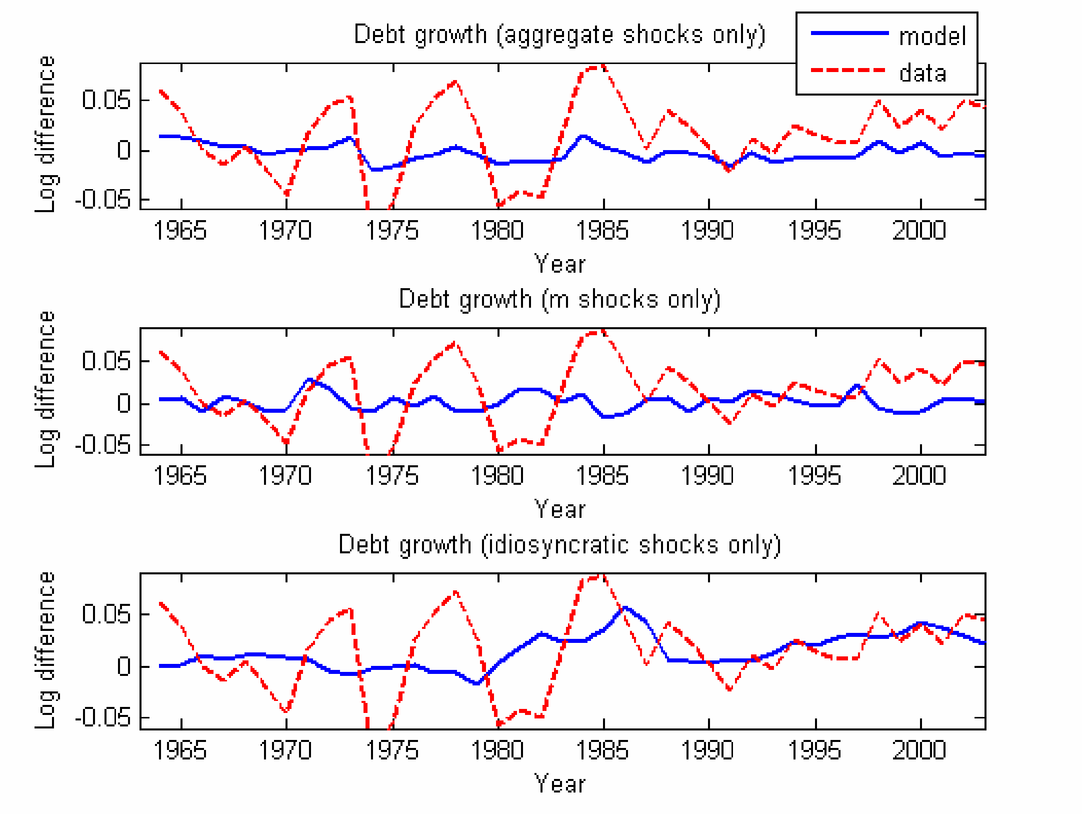
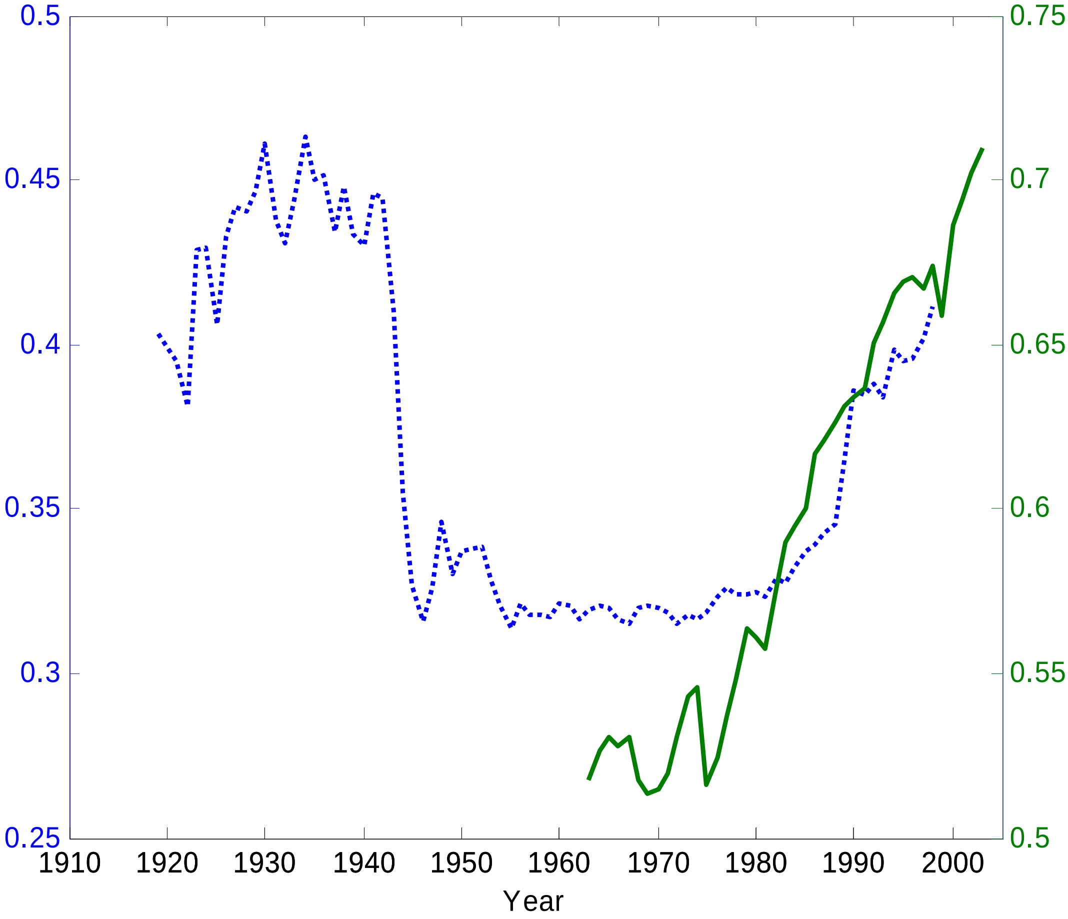


FIGURE 11: Income Inequality over Long Periods of Time



Notes: Dashed line, left scale: Top Decile Income Share in the United States, from Piketty (2003). Solid line, right scale: Standard deviation of log-earnings (used in this paper), from Eckstein and Nagypal (2004).