Strategic Liquidity Supply and Security Design*

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Abstract

We study how securities and trading mechanisms can be designed to mitigate the adverse impact of market imperfections on liquidity. In the line of DeMarzo and Duffie (1999), we consider asset owners who seek to obtain liquidity by selling their claims on future cash flows, on which they have private information. In contrast with them, we allow for strategic liquidity supply and take a mechanism design approach to characterize the optimal security *and* the optimal trading mechanism. For a given, arbitrary, security, issuers with cash-flow below a threshold entirely sell their securities, while issuers with greater cash-flows are excluded from trading. The optimal security design entirely avoids this partial market break-down phenomenon. Moreover, standard debt is optimal. Because of its low informational sensitivity, debt mitigates the adverse selection problem. Furthermore, by pooling all issuers with high cash-flows, it reduces the ability of strategic liquidity suppliers to exclude them from trade to better extract rents from agents with lower cash-flows. We also show that competition in non–exclusive schedules between finitely many oligopolistic liquidity suppliers implements the competitive trading mechanism.

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1. Introduction

This paper analyzes the link between the characteristics of securities and their liquidity. While corporate finance theory offers interesting insights in the design of optimal securities (see, e.g., Townsend (1979), Allen and Gale (1988), Harris and Raviv (1989)), market microstructure theory analyses how different trading mechanisms offer variable degrees of liquidity, emphasizing the consequences of adverse selection and strategic behaviour on the trading process. An important contribution of this literature has been to shown how such market imperfections can increase bid-ask spreads or the price impact of trades, and thus reduce the liquidity of the market. The present paper borrows from these two approaches to study the interaction between security design and market mechanisms. Our objective is to shed some light on how the design of securities can mitigate these market imperfections and thus enhance the liquidity and efficiency of the trading process.

Our analysis is directly in the line of the insightful recent paper by DeMarzo and Duffie (1999), with which it shares the following features. The potential security issuer owns a project yielding a random future cash-flow. Since her discount rate is lower than one, she could benefit from selling today claims on these future cash-flows and thereby obtaining liquidity. She faces financial intermediaries whose discount rate is greater than hers. This generates potential gains from trade between the issuer and these liquidity suppliers. The extent to which those gains can be reaped is limited, however, because, while the security is designed under homogeneous information, it is then traded after the issuer has observed a private signal on the future cash-flows (in the present paper, for simplicity, we assume the issuer's signal is perfectly informative about the future cash-flow). DeMarzo and Duffie (1999) study the optimal design of the security by an issuer facing Walrasian liquidity suppliers. They consider the situation where the issuer offers to sell a certain quantity of the security and the liquidity suppliers respond by quoting a price at which they earn zero expected profits. This is a signaling game similar to Kyle (1985). Because of adverse selection, the price is decreasing in the quantity. To mitigate this illiquidity, the issuer designs securities with limited sensitivity to her private information. This gives rise to the optimality of debt contracts. This is in the spirit of Myers and Majluf's (1984) pecking order theory, arguing that firms prefer to issue debt rather than equity because it is less sensitive to private information. The optimality of debt is also reminiscent of the results obtained by Innes (1990) in a moral hazard framework.

In contrast with DeMarzo and Duffie (1999), we analyze the consequences of market power on liquidity supply and security design. A growing body of empirical evidence documents strategic behaviour on the part of financial intermediaries and market makers (see, e.g., Christie and Schultz (1994), or Chen and Ritter (2000)). Furthermore, the banking literature has emphasized that financial intermediaries are likely to enjoy market power, for example because capital adequacy requirements limit entry, or because banks with preexisting relationships with firms are in a privileged position to lend to them (see, e.g., Freixas and Rochet (1997)). Against this backdrop, we endeavor to shed light on the following issues:

(i) Through what channels does market power affect liquidity ? How does it exacerbate the lemon's problem induced by adverse selection?

(ii) How do issuers react to the market power of financial intermediaries? How can they mitigate the illiquidity it induces? Does this alter qualitatively the type of security they issue?

By construction, the *signaling* trading game analyzed by DeMarzo and Duffie (1999) has a Walrasian flavor: liquidity suppliers quote a price equal to the expected value of the security, conditional on the fraction of the security sold by the issuer. As in Leland and Pyle (1977), albeit for different reasons, retention is a credible signal of the security's value because it is costly for the issuer. To analyze the consequences of market power, we consider instead a *screening* trading game, in the line of Glosten (1994), or Biais, Martimort and Rochet (2000). In this market structure, liquidity suppliers first place limit orders, specifying the prices at which they are willing to buy variable quantities, and then the issuer selects from this menu of offers the trade size maximizing her expected utility. (Because we want to disentangle the respective consequences of market power and the screening nature of our trading game, we analyze in turn competitive and strategic liquidity suppliers.) Limit order trading prevails in the German or French bourses, the internet based ECN Island, or the NYSE. This trading game also captures some of the features of the IPO market, where investors place offers to buy the shares at certain prices (either through indications of interest in the book building process (see e.g., Hanley and Wilhelm (1995) or Cornelli and Goldreich (1998)), or through bids in IPO auctions (see, e.g., Biais and Faugeron-Crouzet (2001)). Benveniste, Wilhelm and Yu (1999) present empirical evidence on the determination of quantities sold by issuing firms after these offers have been placed.

In Section 3, we show that, for a given security design, the outcome of the trading interaction between the issuer and the liquidity suppliers has the following characteristics. The worse the private signal of the issuer, the more willing she is to sell the security, and the greater her trade. Reflecting this adverse selection problem, the (endogenous) cost function of the liquidity suppliers takes the form of lower-tail conditional expectations, as in Glosten (1994) and Biais, Martimort and Rochet (2000). This implies that the price at which the liquidity suppliers are willing to purchase any amount of the security is lower than the unconditional expectation of the value of the security. This is analogous to the small trade spread arising in screening models of market microstructure, in the line of Glosten (1994). As in Akerlof (1970), the "good types", i.e., the issuers with large future cash-flows, are those who suffer the most from the adverse selection problem. This can lead to a market break-down for these issuers, who do not trade at all. The spread, and correspondingly the fraction of issuer types excluded from trade, are greater in the monopolistic liquidity supply case than in its competitive counterpart. Very much in line with the classical IO paradigm, the monopolist, when setting his price schedule, trades off the increase in profit per unit (from widening the spread) with the decrease in volume traded (due to excluding potential issuers from trade).

The outcome of the trading process in our screening game contrasts with the market outcome in DeMarzo and Duffie (1999), where infinitesimal trade have an infinitesimal impact on prices, and where the good types are not entirely excluded from the market. Another difference between the market outcomes in our screening model and in the signaling game analysed by DeMarzo and Duffie (1999) is that while in the latter transaction size decreases smoothly with expected cash-flows, in the former there is a bang-bang solution: issuers with future cash-flows under a certain threshold sell 100% of the security, while issuers with cash-flows above the threshold are entirely excluded from trade. Thus, in contrast with DeMarzo and Duffie (1999) where the sale of the (a fraction) of the security is to be interpreted as collateralization or tranching, issuers who actually obtain liquidity in our model entirely sell the security they issue.

In Section 4, we turn to the analysis of the optimal security design. The key issue is how the security can be designed to mitigate the negative consequences of adverse selection and market power, to enhance liquidity, reduce the spread, improve market participation, and thus increase the gains from trade. We show that the optimal security is a debt contract. Debt mitigates the adverse selection problem, by making the payoff less sensitive to the high cash-flow realizations. This is in line with Myers and Majluf (1984), and DeMarzo and Duffie (1999). In addition, debt contracts mitigate the market power of the monopolistic liquidity supplier. This differs from DeMarzo and Duffie (1999) where liquidity suppliers are competitive. The intuition of our result is the following. The monopolistic liquidity suppliers extract rents from the agent by excluding good types from trade, which reduces the ability of the other issuers to earn rents by disguising their type. When the security's payoff increases smoothly with the cash-flow from the project (as with equity), the monopolistic liquidity supplier can fine-tune the determination of which issuers are excluded from trades. In particular, if he finds it advantageous, he can choose to exclude only a small fraction of issuers. In contrast, with debt contracts, the security's payoff are the same for all issuers with future cash-flows above the debt service. Hence the monopolistic liquidity supplier cannot discriminate across them. He must either include them all, or exclude them all from the market. Since the latter would be quite costly (as it would imply loosing a large fraction of the most profitable customers), the monopolistic liquidity supplier prefers to design his schedule so that all issuers participate to the market. Thus, the optimal design of the security enables to entirely avoid exclusion from the market. In this context, as all issuers types sell 100% of their securities, there is no informational content of trades, i.e., the expectation of the value of the security given a sale is equal to its unconditional value. This low information content of the sale of debt securities in our model is in line with the results of several empirical studies (see e.g., Dann and Mikkelson (1984), Eckbo (1986), and Mikkelson and Partch (1986)).

The methodological contribution of this stage of our analysis, is to study sequential mechanism design. Indeed, we analyze how the issuer optimally designs her mechanism (the security) to influence the mechanism subsequently designed by the monopolistic liquidity supplier (the trading game). In this context, the issuer first acts as a principal, anticipating that, at the next stage of the game, she will play the role of an agent. To analyze this situation we rely on variational techniques, both at the stage of the determination of the optimal schedule, and then for the determination of the optimal security, given the optimal schedule. This approach, where two mechanism design problems are nested, is somewhat in line with the recent literature on hierarchical mechanism design (see, e.g., McAfee and McMillan (1995), Melumad, Mookherjee and Reichelstein (1995), and Faure-Grimaud, Laffont and Martimort (2001)) .

Arguably, the framework considered here is quite general, since we make no parametric assumptions on the distribution of the random cash–flow, and since we consider a large class of trading mechanisms and securities. For example, while the class of securities we consider includes debt, equity, or convertible bonds, we do not restrict our analysis to these types of contract.

One could argue that there are three limitations to our analysis, however. First, we assume that the issuer initially designs one security, and is then restricted to that security.[1] Second, for technical reasons, we assume that both the security payoff and the residual claim of the agent must be increasing in the final cash–flow generated by the asset. This rules out mechanisms which have been shown to be optimal in other security design analyses, such as the "live–or–die" contract obtained by Innes (1990) for example. Third, one might wonder if the competitive case we analyze, which corresponds to an information–constrained Pareto optimum, could emerge from an actual trading game, where liquidity suppliers would post competing schedules.

In Section 5, we show that our analysis is robust to these three limitations:

We analyze, in the competitive case, the situation where, instead of one security, the issuer initially designs at time a menu of securities, among which she will be able to choose at the trading stage. Furthermore, in that analysis, we relax the monotonicity assumptions. Quite strikingly we find that the equilibrium allocations arising in this more general setting are exactly the same as those arising in our basic model.

We also analyze the case where instead of a single liquidity supplier, there are several liquidity suppliers posting non–exclusive competing transfer schedules. We show that the trades arising in the competitive case also are a Nash equilibrium of the oligopolistic liquidity supply game.

2. The basic model

2.1. The extensive form of the game

Assumptions There are two agents: a financial intermediary, and an agent who owns assets generating random cash-flows in the future. Both agents are risk-neutral and the market interest rate is normalized to zero.

At time $t+1$, the random cash flow, denoted X, is obtained. X is drawn according to an absolutely continuous c.d.f. G with positive density g over a compact interval $\mathcal{X} = [\underline{x}, \overline{x}] \subset \mathbb{R}_{++}$.

[1]This differs from Nachman and Noe (1994), where the security is designed by the agent after she has observed her private information, and thus conveys a a signal of the profitability of her assets.

At time t, trading can take place between the two agents. There are potential gains from trade, as the agent who owns X is more impatient than the financial intermediary. The discount factor of the former is $\delta < 1$, while that of the latter is normalized to 1. The financial intermediary is referred to as the liquidity supplier, while the agent owning the asset is referred to as the issuer. Because of the linearity of the preferences and the difference in discount factors, social optimality requires full transfer of the asset from the issuer to the liquidity supplier at time t. Market imperfections can prevent these gains from trade from being fully exploited, however, as discussed below.

At time $t-1$, before trading can take place, the issuer observes a private signal on the future realization of the cash flows. This creates an adverse selection problem, limiting the extent to which gains from trade can be reaped. For simplicity, we consider the case where the private signal observed by the issuer is perfectly informative.[2]

At time $t-2$, the trading mechanism is designed. It is a mapping: $T : [0,1] \to \mathbb{R}$, from the fraction q of the security sold at time t by the issuer into a transfer T.

At time $t-3$, rationally anticipating the following stages of the game, the issuer designs the security that will be exchanged at time t. The payoff F of that security is contingent on the realized cash–flow. We assume the issuer has limited liability, i.e., $0 \leq F \leq X$. The security is a measurable mapping $\varphi : \mathcal{X} \to \mathbb{R}_+$ such that $F = \varphi(X) \in [0, X]$. In the first part of our analysis, as in Harris and Raviv (1989) or Nachman and Noe (1994), we shall restrict the set of admissible securities to the set of functions satisfying the following limited liability and monotonicity conditions:

(LL) $\varphi(x) \in [0, x]$ for all $x \in \mathcal{X}$;

(M) φ is non-decreasing on $\mathcal{X}$;

(MR) $\mathrm{Id}_{\mathcal{X}} - \varphi$ is non-decreasing on $\mathcal{X}$,

where $\mathrm{Id}_{\mathcal{X}}$ is the identity function on $\mathcal{X}$. Conditions (M)–(MR) require that both the payment to the liquidity supplier and to the issuer be non-decreasing in the realized cash-flow. We impose (MR) mainly for technical convenience. Together with (LL)–(M), it implies that the set Φ of admissible securities payments is a subset of Lipschitz, hence absolutely continuous functions on $\mathcal{X}$. Denote by $\mathcal{F}$ the set of possible values for f. Our assumptions on φ imply that this set is an interval $[\underline{f}, \overline{f}]$.

[2]This differs from DeMarzo and Duffie (1999) who allow for noisy private signals.

Our assumption that, at time $t-3$, the issuer designs a single security (φ), rather than a menu of securities will be relaxed in Section 5. We believe that, in certain contexts, it is quite reasonable, however. One example is offered by the following setting: A manufacturing firm will generate cash flows X in the future (at time $t+1$), and needs cash now (at time $t-3$). It initially sells to a financier a security φ with payoff contingent on these cash flows. The financier can be a bank providing a loan, a supplier providing trade credit, or a venture capitalist purchasing convertible bonds. After this initial exchange, the financier naturally receives information about the project. Correspondingly, it observes a private signal on the cash flows. Then it is subject to a liquidity shock, and demands liquidity from the market.[3] To obtain this liquidity, it sells the security it holds in its portfolio: φ. The price at which it initially purchases the security from the manufacturing firm reflects its rational expectation about future market liquidity. The security is initially designed to maximize market liquidity, and correspondingly the initial sale price.

Comparison with DeMarzo and Duffie (1999) The two major differences between our analysis and theirs are the following: First, we take a different approach to modeling the trading game. DeMarzo and Duffie (1999) consider a signaling game, whereby the issuer, after observing her signal, chooses the size (q) of the trade, and the liquidity suppliers react to this quantity by quoting prices. In contrast, we take a mechanism design approach. The trading mechanism is a menu of pairs $\{q, T(q)\}$, from which the informed agent selects the optimal trade. This menu of trades, designed (at time $t-2$) before the private signal is observed, can be interpreted as a screening mechanism. If the transfer schedule ($T(q)$) is convex, it amounts to a sequence of limit orders, as in Biais, Martimort and Rochet (2000). As will be shown below, the allocation arising in the signalling equilibrium is implementable in the trading mechanism. It is not the optimal allocation, however, as established in the next section. This is because the screening mechanism yields more commitment power, as the liquidity supplier can commit to a menu of trades before the quantity q is observed.[4]

[3] In the case of a bank, the need for liquidity can be due to prudential rules (see Dewatripont and Tirole, 1994). In the case of a trade creditor it can stem from a transient cash flow gap, or an investment oppportunity, combined with credit rationing constraints. In the case of a venture capitalist it can reflect an opportunity to invest in new projects combined with constraints on raising new funds.

[4] This commitment power makes it possible to engineer cross-subsidization between the issuer's types. Indeed, in the equilibrium of our screening model, the liquidity supplier will earn profits when trading with issuers whose cash-flows are high, while he will make losses when trading with issuers

The second difference is that, while in the context of the signaling model, competitive liquidity supply is warranted, we allow for strategic liquidity supply. We consider two polar cases. In the monopoly case, the trading mechanism is designed, at time $t-2$, by the liquidity supplier, to maximize his expected profit, under the incentive and participation constraints of the informed agent. The latter constraint requires that the informed agent accepts to participate in the trading mechanism, at time t. For the sake of realism, we impose that, if the informed agent chooses not to trade (or equivalently not to participate in the trading mechanism), the liquidity supplier cannot demand any payment from this agent. In the alternative case, referred to as the competitive case, the trading mechanism is designed by the issuer to maximize her expected utility, subject to the participation constraint of the liquidity suppliers. By comparing the allocations arising in the monopoly case and in the competitive case, we shed some light on the consequences of market power for market liquidity.

2.2. Incentive Compatibility and Individual Rationality Conditions

Consider a given a security design F and transfer schedule T. Conditional on her private information about future cash flows, the issuer selects what fraction q of the security to sell to the liquidity supplier. At this interim stage, since the issuer has perfect advance knowledge of the cash-flows, and since the security's payoff is only contingent on these, she also perfectly knows the realization $f = \varphi(x)$ of F. Her utility is: $T(q) + \delta(x - fq)$, while the profit of the liquidity supplier is$qf - T(q)$. Thus, for a given security design, i.e., a given mapping φ, the type of the informed agent is entirely summarized by f, and the set of possible types is $\mathcal{F}$.

The issuer finds it attractive to trade q rather than not to trade at all and consume δx if and only if:

$$f \leq \frac{T(q)}{\delta q}. \qquad (1)$$

This condition holds if the unit price of the security $\frac{T(q)}{q}$ is high enough, δ is low enough (i.e., the issuer is sufficiently impatient), and the security payoff f is low enough. Thus, the willingness to trade signals a relatively low type. This underscores the adverse selection problem arising in our model, very much in line with Akerlof's (1970) lemons problem.

whose cash-flows are low. This cross-subsidization of the bad types by the good types also takes place in the screening games analyzed by Glosten (1994) and Biais, Martimort and Rochet (2000).

The trading mechanism must be incentive compatible and satisfy the issuer's individual rationality constraint for all realizations of f. There is no loss of generality in applying the revelation principle (Myerson (1979)): any implementable allocation achieved via a transfer schedule $T(q)$ can also be achieved via a truthful direct mechanism $(\tau, q) : \mathcal{F} \to \mathbb{R} \times [0,1]$ that stipulates a transfer and a trading volume as a function of the issuer's report of her type $f \in \mathcal{F}$. Incentive compatibility requires that:

$$f \in \arg\max_{\hat{f} \in \mathcal{F}} \tau(\hat{f}) - \delta f q(\hat{f}); \quad f \in \mathcal{F}. \tag{2}$$

We denote by U_F the corresponding informational rent:

$$U_F(f) = \sup_{\hat{f} \in \mathcal{F}} \tau(\hat{f}) - \delta f q(\hat{f}). \tag{3}$$

U_F is analogous to the informational rent of a regulated firm with privately observed marginal cost δf, as in Baron and Myerson (1982). We take the dual approach and characterize the set of pairs (U_F, q) that correspond to an incentive compatible mechanism. This set is characterized in the following lemma.

LEMMA 1 *A pair (U_F, q) is implementable if and only if:*

(i) U_F is convex on $\mathcal{F}$;

(ii) For almost every $f \in \mathcal{F}$, $\dot{U}_F(f) = -\delta q(f)$.

Lemma 1 simply reflects the fact that U_F is the upper enveloppe of a family of affine and decreasing functions of f. This characterization of the incentive compatibility condition via the dual variable U_F is in the line of Mirrlees (1971), and has been used in financial contracting settings by Rochet and Vila (1994), Biais, Martimort and Rochet (2000) and Biais, Bossaerts and Rochet (2001). Convexity of U_F together with $\dot{U}_F = -\delta q$ implies the following important property.

LEMMA 2 *In any implementable allocation, the volume of trade q is non-increasing in the security payoff f, and consequently in the cash-flow x.*

The intuition is in the line of Akerlof (1970). As discussed above, issuers with relatively large future cash-flows are relatively less eager to trade at a given price than issuers with lower future cash-flows. That issuers with low cash-flows are always ready to trade depresses the price, which makes issuers with high cash-flows even less eager to trade. In the limit this can lead to a market breakdown, where the issuers with the largest cash-flows obtain zero gains from trade. Lemmas 1 and 2, and their intuition are similar to Proposition 1 in DeMarzo and Duffie (1999). The similarity of incentive compatibility conditions in screening trading games and signaling trading games also arises in microstructure models à la Kyle (1985) and Glosten (1989). See Biais and Rochet (1997) for a discussion of this point.

In addition to the above incentive compatibility constraint, a feasible trade mechanism must also satisfy the issuer's ex-post participation constraint. Specifically, since the issuer has always the option not to trade, and since in this case she cannot be compelled to pay anything to the liquidity supplier, the issuer's informational rent U_F must always be non-negative:

$$U_F(f) \geq 0; \quad f \in \mathcal{F}. \tag{4}$$

Since U_F is non-increasing by Lemma 1, this simplifies to:

$$U_F(\overline{f}) \geq 0. \tag{5}$$

2.3. *The Expected Utilities of the Agents*

Given a security F and a schedule T, the expected profit of the liquidity supplier is:

$$\int_{\mathcal{F}} (fq(f) - T(q(f))\, dG^{\varphi}(f), \tag{6}$$

where G^{φ} is the c.d.f. of the random variable $F = \varphi(X)$. Similarly, the ex-ante expected informational rent of the issuer is :

$$\int_{\mathcal{F}} (T(q(f)) - \delta f q(f))\, dG^{\varphi}(f). \tag{7}$$

Adding the expected profits of the liquidity supplier and the expected rent of the issuer, we obtain the total gains from trade:

$$(1-\delta) \int_{\mathcal{F}} f q(f)\, dG^{\varphi}(f). \tag{8}$$

The greater the difference between the discount rate of the liquidity suppliers and that of the issuer, the greater the potential gains from trade. The gains from trade are also increasing in the cash-flows transferred from the second period to the first.

2.4. *Ex-Ante Efficiency*

As a benchmark, we first consider the case where a benevolent social planner chooses a trading mechanism so as to maximize social welfare. Following Holmström and Myerson (1983), efficiency is defined at an ex-ante stage, i.e., before the issuer learns the value of the future cash-flows. Thus an ex-ante optimal mechanism maximizes the expected rent of the issuer:

$$\sup_{(T,q)} \int_{\mathcal{F}} (T(q(f)) - \delta f q(f))\, dG^{\varphi}(f)$$

subject to the liquidity supplier's participation constraint:

$$\int_{\mathcal{F}} (f q(f) - T(q(f))\, dG^{\varphi}(f) \geq \underline{\pi}$$

for some $\underline{\pi} \geq 0$. The only difference between this program, and the problem of the issuer analyzed in the following sections is that, the incentive compatibility and ex–post individual rationality conditions of the issuer are not imposed.

Solving this program is immediate. The participation constraint of the liquidity supplier is binding, and the optimal trading volume is constant: $q = 1$. It follows then that, with an equity contract, the issuer fully reaps all the gains from trade.

3. Liquidity Supply

In this section, we analyze the optimal price-quantity schedule for a given security design F. We consider the two polar cases of competitive and monopolistic liquidity supply. In the competitive case, the schedule T is designed by the issuer, who extends a take-it-or-leave it offer to the liquidity supplier; the situation is reversed in the monopolistic case.

The Competitive Case. We first consider the case where the issuer has all the bargaining power. Given a security F, the issuer's problem is to design the transfer schedule T to maximize her expected rent (7), subject to her incentive compatibility condition (2), her ex-post individual rationality condition (4), and the participation constraint of the liquidity supplier, that his expected profit be non-negative. Recall that the expected rent of the issuer is equal to the expected total gains from trade minus the expected profit of the liquidity supplier:

$$\int_{\mathcal{F}} (1-\delta) f q(f)\, dG^{\varphi}(f) - \int_{\mathcal{F}} (f q(f) - T(q(f)))\, dG^{\varphi}(f).$$

To maximize her rent, the issuer designs the schedule so as to saturate the participation constraint of the liquidity supplier and set his expected profit to zero. The liquidity supplier's zero-profit condition simplifies the program of the issuer to the choice of a trading volume q which maximizes the overall expected gains from trade (8) under her incentive compatibility condition, characterized in Lemma 1, and her ex-post participation constraint (5). The only difference between this problem and the design of the ex-ante efficient allocation is the ex-post participation constraint, since the ex-ante efficient trading profile is incentive compatible.

The Monopolistic Case. Now turn to the case where the liquidity supplier has all the bargaining power. The liquidity supplier's task is to choose a transfer schedule T in order to maximize his expected profit (6), subject to the incentive compatibility condition (2) and the ex-post individual rationality condition (4). Recall that the expected profit of the liquidity supplier is equal to the expected total gains from trade minus the expected informational rent of the issuer:

$$\int_{\mathcal{F}} (1-\delta) f q(f)\, dG^{\varphi}(f) - \int_{\mathcal{F}} (T(q(f)) - \delta f q(f))\, dG^{\varphi}(f).$$

The relevant constraints are again the incentive compatibility conditions, characterized in Lemma 1, and the ex-post participation contraint (5). Since the informational rent is non-increasing, the participation constraint of the issuer must be binding at the upper end of the support $\mathcal{F}$.

The optimal trading mechanism. The menus (τ^c, q^c) and (τ^m, q^m) offered respectively by the issuer and by the liquidity supplier are characterized in the following proposition.

PROPOSITION 1 *There exist $f_F^c \geq f_F^m$ and $\tau_0^c \geq \tau_0^m = 0$ such that, for all $f \in \mathcal{F}$, and for $i \in \{c, m\}$,*

(i) $\tau^i(f) = \tau_0^i + \delta f_F^i$ whenever $f \leq f_F^i$ and $\tau^i(f) = \tau_0^i$ otherwise;

(ii) $q^i(f) = 1$ whenever $f \leq f_F^i$ and $q^i(f) = 0$ otherwise.

Moreover, $\tau_0^c = 0$ whenever $f_F^c < \overline{f}$.

In both the competitive and the monopolistic cases, issuers with cash-flows below a the threshold f_F^i sell 100% of the security, while issuers above this threshold do not trade at all. Correspondingly, issuers with small future cash-flows obtain large gains from trade, while issuers with large future cash-flows can face a market break-down, and obtain no gains from trade. This "bang–bang" solution differs markedly from the signalling equilibrium analyzed by DeMarzo and Duffie (1999), where the trade smoothly decreases with the issuer's type.[5] As in the monopoly pricing model of Riley and Zeckhauser (1983), it arises because of the combined effect of the linearity of the preferences and the screening nature of the trading game.

In order to saturate the liquidity supplier's break-even constraint in the competitive case, it can be necessary to allow for a lump-sum transfer τ_0^c given to the issuer independently of her trade. This can however only arise when no type of the issuer is excluded from trade. Indeed, when some types are excluded from trade, it is preferable to increase the price of the security, in order to make trading more attractive for the good types and thus minimize the extent of the market break-down, rather than giving a lump-sum transfer.

The optimal transfer schedule can be implemented with a limit order to buy, or bid price, posted by the liquidity supplier, at which he stands ready to buy up to one unit of the security.[6] Saturating the participation constraint of the liquidity supplier (and, for smplicity, neglecting the lump-sum tax τ_0^i), we obtain the price at which the competitive liquidity supplier purchases the security:

$$\frac{\int_{\underline{f}}^{f_F^c} f \, dG^{\varphi}(f)}{G^{\varphi}(f_F^c)} = E(F|F \leq f_F^c).$$

This is reminiscent of the result obtained by Glosten (1994) in a screening model with competitive market makers, where the bid is equal to the lower tail expectation of the final value of the security.

In the competitive case, the threshold f_F^c above which issuers opt out from trading, and the bid price are pinned down by combining this lower tail expectation and the ex-post rationality condition of the issuer:

$$\delta f_F^c = E(F|F \leq f_F^c).$$

[5]Note that, by Lemma 2, such a smoothly decreasing trade would be implementable in the screening mechanism we consider.

[6]In line with the analogy drawn in the market microstructure literature between limit orders and options (Copeland and Galai (1983)), we can also interpret this arrangement as the option, for the issuer, to sell her securities at a predetermined price.

In line with basic price theory, the left–hand–side this equates the valuation of the marginal issuer for the security with the security's price.

The difference between the bid price and the unconditional expectation of the value of the security is similar to the bid-ask spread. The greater the probability mass corresponding to low cash-flow realizations, the lower the bid price, the wider the spread, and, consequently, the greater the mass of high–cash flows issuers who are deterred from trading. This is similar to the result obtained in screening models of market microstructure (Glosten (1989, 1994), Biais, Martimort and Rochet (2000)) that the small trade spread maps into the set of investors' types who are excluded from trade.

As the threshold value of the cash-flow above which the issuer exits the market is greater in the competitive than in the monopoly case, more gains from trade are achieved in the former than in the latter. This bears some analogy with credit rationing models such as Bolton and Scharfstein (1990), or market microstructure models such as Biais, Martimort and Rochet (2000). The intuition is that the monopolistic liquidity supplier trades off the benefits of a high volume of trade against the incentive costs of inducing the issuer to reveal truthfully low realizations of the cash-flows. This rent-efficiency trade-off is less acute when the issuer designs the trading mechanism, since the rent extraction motive is not present.

4. Security Design

In both the competitive and the monopolistic environments, the issuer's problem is to choose a security F, or equivalently a function $\varphi \in \Phi$, in order to maximize her expected rent, anticipating the equilibrium price at which she will be able to sell the securities. For simplicity we assume hereafter that:

$$\frac{d}{dx}\left(\frac{G(x)}{g(x)}\right) \geq \frac{1-\delta}{\delta}; \quad x \in \mathcal{X}. \tag{9}$$

This condition is slightly stronger than the standard assumption of log-concavity of the density g. It ensures that one may neglect the constraint that U_F be convex when solving for the optimal transfer schedule. In other terms, it enables us to focus on the first-order conditions of the mechanism design problem, while warranting that the second-order conditions hold.

4.1. Debt and equity

Equity. If the issuer designs a pure equity contract, i.e., $\varphi = \mathrm{Id}_{\mathcal{X}}$, the optimal schedules (τ^c, q^c) and (τ^m, q^m) are as stated in the next proposition.

PROPOSITION 2 *If the issuer designs an equity contract, then, in the competitive case, the threshold level of cash–flows above which issuers exit the market is:* $f_E^c = \min\{\overline{x}, f^c\}$ *where* f^c *is the highest value of* f *such that:*

$$\delta f = \frac{\int_{\underline{x}}^{f} \phi\, g(\phi)\, d\phi}{G(f)}, \tag{10}$$

while in the monopoly case it is $f_E^m = \min\{\overline{x}, f^m\}$*, where* f^m *is the highest value of* f *such that:*

$$\frac{1-\delta}{\delta} f - \frac{G(f)}{g(f)} \geq 0.$$

When $f_E = \overline{x}$, all issuer types achieve gains from trade. Otherwise, issuers with high cash-flows are excluded from the market. To determine if issuers with type f should be excluded from the market, the monopolistic liquidity supplier compares the gains from trade $(1-\delta) f g(f)$ that can be achieved with these agents, with the rent $\delta G(f)$ they must be left. This rent increases with the cumulative distribution of types up to f, since, as incentive compatible rents are decreasing with types, rents left to type f must be left to all types below f.

Debt. Now consider the case where the security traded is debt with face value d, i.e., $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d\}$. First consider the competitive case. As above, to determine which issuers trade, and at what price, we need to study the consistency between the liquidity supplier's participation constraint, and the issuer ex-post rationality condition. The highest face value such that these two conditions can both hold is d^c, the maximum value of $d \in \mathcal{F}$ such that:

$$\int_{\underline{x}}^{d} f g(f)\, df + (1 - G(d))d = \delta d.$$

The following proposition characterizes the set of issuers who participate to the market for a given face value d:

PROPOSITION 3 *If the issuer designs a debt contract with face value* d*, then* $f_D^c = d$ *if* $d \leq d^c$ *and* $f_D^c = f_E^c$ *otherwise.*

The intuition is the following. If the face value of the debt is too high, $d > d^c$, then the participation constraint of the liquidity supplier cannot hold jointly with the ex-post rationality constraint of all issuers. Market equilibrium then requires that the highest types, with cash-flow between f_D^c and d, be excluded from the market. This effectively converts the debt contract into an equity contract, since for all the issuers who participate to the market, $\varphi(x) = x$. Therefore the threshold above which issuers exit the market is the same as under the equity contract, f_E^c. On the other hand, if $d \leq d^c$ then the participation constraint of the liquidity supplier is consistent with the ex-post rationality condition of all issuer types. In that case, all issuers sell their security, and thus reap gains from trade.

In analogy with d^c, let d^m be the highest possible value of $d \in \mathcal{F}$ such that:

$$\int_{\underline{x}}^{d} f g(f)\, df + (1 - G(d))d - \delta d = \int_{\underline{x}}^{f_E^m} (f - \delta f_E^m)\, g(f)\, df.$$

d^m is the highest face value of debt such that the liquidity supplier obtains the same expected profit than under an equity contract. Then we have the following result.

Proposition 4 *If the issuer designs a debt contract with face value d, then $f_D^m = d$ if $d \leq d^m$ and $f_D^m = f_E^m$ otherwise.*

The intuition is the following. When the issuer designs a debt contract with face value d, the liquidity supplier has the option to shut-down the upper tail of the payoff distribution by setting the price at which the issuer can sell his security to $f_D^m < \delta d$. If d is large enough, it may be optimal for the monopolistic liquidity supplier to do so. The shut-down threshold is then optimally set at f_E^m. Just as the issuer in the competitive case, the liquidity supplier is effectively converting a debt contract into an equity one. On the other hand, if $d \leq d^m$, then all issuer types find it preferable to sell their security.

Remark. In DeMarzo and Duffie (1999), the interpretation of $\min\{\mathrm{Id}_{\mathcal{X}}, d\}$ as a standard debt contract requires the assumption that the unsold fraction of the security is not held on the balance sheet of the issuer at the time of default. This does not arise in our model, since, when the issuer trades, the security is entirely transferred to the liquidity supplier.

4.2. The optimal security

First, note that risk-free cash-flows are not subject to adverse selection problems. Hence, in the line of Myers and Majluf (1984), it is always optimal to sell these, in order to maximize trade and thus the gains from trade. Consequently, it is optimal to design the security to yield at least the worst possible realization of the cash-flow. This yields the following lemma.

LEMMA 3 *There exists an optimal security F such that $\varphi(\underline{x}) = \underline{x}$.*

Our next proposition is key to our results. Let $i \in \{c, m\}$, and consider a security F with payoff φ such that issuers above a certain threshold f_F^i do not trade. What this proposition asserts is that the issuer could strictly gain by offering instead an alternative security, with payoff capped at a level slightly above the shut-down level f_F^i. That alternative security F_ε would have payoff $\varphi_\varepsilon = \min\{\varphi, f_F^i + \varepsilon\}$, and would be such that all issuers would trade. Rationally anticipating the participation of all issuer types, including the better ones, the liquidity supplier would be ready to pay a slightly better price, $\delta(f_F^i + \varepsilon)$, than for F, as long as ε is not high enough to make the issuer's incentive prohibitively expensive. At that price, issuers with high cash-flows would be willing to sell the security, given that its payoff is capped just above f_F^i. The increase in price implies that the security F_ε strictly dominates the original security F from the issuer's point of view. Given our assumptions on F, this implies that it is not optimal for the issuer to design a security involving shut-down for good types. Thus, we can state the following proposition.

PROPOSITION 5 *The optimality of security F requires that all issuer types entirely sell their holdings to the liquidity supplier.*

This result underscores the difference between our screening model and the signaling models of Leland and Pyle (1977) and DeMarzo and Duffie (1999). In these models, an informed agent can credibly signal the quality of a project only by retaining part of the cash-flows generated by this project. For an arbitrarily chosen security, the analogue of this phenomenon in our screening model is the possibility of market break-down. From the issuer's point of view, this way of signaling the quality of her assets is however very costly. Hence she is better off designing her security to avoid market break-down altogether. As a consequence, the market for an optimal security will be very liquid. For instance, in the competitive case, the price at which such a security will be traded will just be equal to the unconditional expectation $\int_{\mathcal{F}} f\,dG^{\varphi}(f)$ of the value of this security, thereby eliminating the bid-ask spread.

The Competitive Case. The program of the issuer is to maximize the total gains from trade, subject to her own incentive compatibility and ex-post participation constraints at the trading stage, and to the zero-profit constraint of the liquidity suppliers. The analysis of Section 3 implies that these constraints simplify to a bang-bang trading structure, whereby issuers with types above a certain threshold do not trade, while those below entirely sell their security at a price equal to a lower tail expectation. Proposition 7 simplifies the situation further by mandating to concentrate only on securities such that there is no shut-down. Thus we can restate the issuer's problem of choosing an optimal security as an infinite-dimensional linear programming problem:

$$\sup_{\varphi\in\Phi}\,(1-\delta)\int_{\mathcal{X}}\varphi(x)\,g(x)\,dx$$

subject to the no shut-down condition:

$$\int_{\mathcal{X}}\varphi(x)\,g(x)\,dx\geq\delta\varphi(\overline{x}).$$

This inequality can alternatively be seen as an ex-post participation constraint for the issuer, requiring that the price $\int_{\mathcal{X}}\varphi(x)\,g(x)\,dx$ of the security be greater than its present value for all issuer types, even for the issuer with the greatest possible cash-flow, i.e., $\delta\varphi(\overline{x})$. Note that in formulating the issuer's problem, we have already taken into account the liquidity supplier's break-even constraint, which must be saturated at the optimum. The issuer's security design problem can then be analyzed as follows. Let us form the Lagrangian:

$$L(\varphi,\lambda)=(1-\delta)\int_{\mathcal{X}}\varphi(x)\,g(x)\,dx+\lambda\left(\int_{\mathcal{X}}\varphi(x)\,g(x)\,dx-\delta\varphi(\overline{x})\right),$$

where λ is the Lagrange multiplier of the issuer's ex-post participation constraint. By (LL)–(M)–(MR), any $\varphi\in\Phi$ is absolutely continuous, the derivative $\dot{\varphi}$ is a.e. well-defined with $0\leq\dot{\varphi}\leq 1$, and $\varphi(x)=\int_{\underline{x}}^{x}\dot{\varphi}(\xi)\,d\xi$ for all $x\in\mathcal{X}$. Hence, integrating by parts, we get:

$$L(\varphi,\lambda)=-\,(1+\lambda-\delta)\int_{\mathcal{X}}\dot{\varphi}(x)\,G(x)\,dx+(1-\delta)(1+\lambda)\,\varphi(\overline{x}).$$

The maximization of $L(\varphi,\lambda)$ with respect to φ can thus be treated as a standard optimal control problem. We have then the following result.

Proposition 6 *Suppose that (9) holds. Then the debt contract with face value d^c is an optimal security from the issuer's point of view.*

The intuition is that a debt contract trades-off in an optimal way two conflicting objectives. On the one hand, it is efficient to transfer as much cash-flows from the second period to the first. On the other hand, the lemons problem limits the extent to which this can be done. By imposing a cap on the security payoff, a debt contract minimizes this adverse selection cost, in support of Myers and Majluf's (1984) pecking-order hypothesis.

The Monopolistic Case. In the monopoly case, the issuer designs the security to maximize her expected gain from trade, anticipating the optimal response of the monopolistic liquidity supplier, and her own reaction, reflected in her incentive compatibility and ex-post participation constraints. The issuer anticipates that the liquidity supplier will design his schedule to maximize his expected profit. She designs the optimal security to mitigate the adverse consequences of this rent extraction strategy on the gains from trade. From the previous section, we know that the transfer schedule optimally designed by the monopolistic liquidity supplier is a simple take-it-or-leave-it offer to buy all the security at a given price. In designing this offer the liquidity supplier trades-off the benefit from a large market, from which no issuer would be excluded, with the benefits of a smaller market, excluding issuers with high cash-flows, but extracting more rents from the others. Proposition 7 implies that with the optimal security there is no shut-down. Thus we can re-state the issuer's problem of choosing an optimal security as an infinite-dimensional linear programming problem:

$$\sup_{\varphi \in \Phi} \delta \int_{\mathcal{X}} (\varphi(\overline{x}) - \varphi(x))\, g(x)\, dx,$$

subject to the no shut-down condition:

$$\int_{\mathcal{X}} (\varphi(x) - \delta\varphi(\overline{x}))\, g(x)\, dx \geq \int_{\underline{x}}^{\tilde{x}} (\varphi(x) - \delta\varphi(\tilde{x}))\, g(x)\, dx; \qquad \tilde{x} \in \mathcal{X}.$$

This can be interpreted by comparing the security design problem to a principal-agent problem with moral hazard. The principal is the issuer, who designs the security, while the agent is the liquidity supplier, and the moral-hazard variable is the decision by the agent to shut-down the market or not. The issuer's security design problem can then be analyzed as follows. Let us form the Lagrangian:

$$L(\varphi,\Lambda) \;=\; \delta\int_{\mathcal{X}}(\varphi(\overline{x})-\varphi(x))\,g(x)\,dx$$

$$+\;\int_{\mathcal{X}}\left(\int_{\mathcal{X}}(\varphi(x)-\delta\varphi(\overline{x}))\,g(x)\,dx-\int_{\underline{x}}^{\tilde{x}}(\varphi(x)-\delta\varphi(\tilde{x}))\,g(x)\,dx\right)d\Lambda(\tilde{x}),$$

where Λ is the Lagrange multiplier associated to the no shut-down condition. It is a distribution function on $\mathcal{X}$, i.e., a non-decreasing, right-continuous function such that $\Lambda(\underline{x}) = 0$. The following lemma provides a sufficient condition for $\varphi \in \Phi$ to be an optimal security (see, e.g., Luenberger (1969, §8.4, Theorem 1)).

Lemma 4 *Let $\varphi \in \Phi$, and Λ be a distribution function on $\mathcal{X}$ such that:*

$$\int_{\mathcal{X}}\left(\int_{\mathcal{X}}(\varphi(x)-\delta\varphi(\overline{x}))\,g(x)\,dx-\int_{\underline{x}}^{\tilde{x}}(\varphi(x)-\delta\varphi(\tilde{x}))\,g(x)\,dx\right)d\Lambda(\tilde{x})=0$$

and:

$$L(\varphi,\Lambda)\geq L(\tilde{\varphi},\Lambda);\qquad \tilde{\varphi}\in\Phi.$$

Then φ is an optimal security in Φ.

To prove the optimality of debt, we proceed as follows. Suppose that (9) holds. Then, by Proposition 5, the optimal debt contract from the issuer's viewpoint has face value d^m. Given this contract, the only point at which the liquidity supplier's shut-down constraint is binding is at the level f_E^m. This suggests taking as a Lagrange multiplier Λ a point-mass at f_E^m, i.e., a mapping of the form $\Lambda_\lambda(x) = \lambda\chi_{\{x\geq f_E^m\}}$ for some $\lambda > 0$. For this choice of Λ, the Lagrangian can be re-written as:

$$L(\varphi,\Lambda_\lambda) \;=\; (1-\lambda)\,\delta\int_{\mathcal{X}}(\varphi(\overline{x})-\varphi(x))\,g(x)\,dx$$

$$+\;\lambda\left(\delta\int_{\underline{x}}^{f_E^m}(\varphi(f_E^m)-\varphi(x))\,g(x)\,dx+(1-\delta)\int_{f_E^m}^{\overline{x}}\varphi(x)\,g(x)\,dx\right).$$

Proceeding as in the competitive case, this expression can be further simplified to:

$$L(\varphi,\Lambda_\lambda) \;=\; (1-\lambda)\,\delta\int_{\mathcal{X}}\dot{\varphi}(x)\,G(x)\,dx$$

$$+\;\lambda\left(\delta\int_{\underline{x}}^{f_E^m}\dot{\varphi}(x)\,G(x)\,dx+(1-\delta)\int_{f_E^m}^{\overline{x}}\varphi(x)\,g(x)\,dx\right).$$

It is clear from this expression that the second term on the right-hand side is maximized by setting $\varphi = \mathrm{Id}_{\mathcal{X}}$. The same is true for the first term if $\lambda \leq 1$. Overall, a pure equity contract maximizes the Lagrangian if $\lambda \leq 1$. But then, the no shut-down condition would not be binding, and Lemma 4 would not apply. This means that, in order to derive the optimality of debt, we must select $\lambda > 1$. Intuitively, the shadow cost of the no shut-down condition must be high enough for debt to be an optimal security.

LEMMA 5 *There exists $\lambda > 1$ such that $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d^m\}$ maximizes $L(\varphi, \Lambda_\lambda)$ with respect to $\varphi \in \Phi$.*

Since for the debt contract $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d^m\}$,

$$\int_{\mathcal{X}} (\varphi(x) - \delta\varphi(\overline{x}))\, g(x)\, dx - \int_{\underline{x}}^{f_E^m} (\varphi(x) - \delta\varphi(f_E^m))\, g(x)\, dx = 0,$$

the following result is an immediate consequence of Lemmas 4 and 5.

PROPOSITION 7 *Suppose that (9) holds. Then the debt contract with face value d^m is an optimal security from the issuer's point of view.*

Just as in the competitive case, a debt contract is optimal from the issuer's point of view. The fact that the face value of debt is smaller than in the competitive case reflects the liquidity supplier's market power. It is interesting to note that, in both cases, the optimal security is risky debt. That this optimal security is informationally sensitive, stands in stark contrast with the results of DeMarzo and Duffie (1999). In their model, if the issuer observes a perfectly informative signal about the realization of the cash-flows, there exists an optimal security whose payoff does not depend on her private information and is identically equal to the lowest possible realization of the cash-flows, $\varphi = \underline{x}$. This difference between this result of and ours reflect both the difference between our screening trading mechanism and their signaling game, and, in the monopolistic case, the fact that the liquidity supplier would be able to extract all the rent if the security was not informationally sensitive.

5. Robustness

We now investigate the robustness of our results to some of the assumptions underlying our basic model.

5.1. Menus of securities

So far, we have assumed that the choice of a security is made ex-ante. We now relax this assumption, by allowing the issuer to design ex–ante a menu of securities, from which she will select which to trade at the interim stage. A menu of securities is then a mapping $(x, \hat{x}) \mapsto \psi(x, \hat{x})$ such that $\psi(x, \hat{x}) \in [0, x]$ for all $(x, \hat{x}) \in \mathcal{X}^2$. For example, this includes the case where, if $\hat{x}$ is in a certain set, then the security is a debt contract, while if $\hat{x}$ is in the complementary set, then the security is an equity contract. Note that we do not impose any monotonicity condition on the menu of securities.

By the revelation principle, there is no loss of generality in focusing on truthful direct mechanisms $(\tau, q) : \mathcal{X} \to \mathbb{R} \times [0, 1]$ that stipulate a transfer and a trading volume as a function of the issuer's report of her type $x \in \mathcal{X}$. Incentive compatibility requires that:

$$x \in \arg\max_{\hat{x} \in \mathcal{X}} \tau(\hat{x}) - \delta\psi(x, \hat{x})q(\hat{x}); \qquad x \in \mathcal{X}. \tag{11}$$

We now characterize the second-best efficient menu of securities ψ and the associated trading structure (τ, q), that maximize the expected gains from trade. It is the solution to the following infinite-dimensional linear programming problem:

$$\sup_{\tau, q, \psi} (1 - \delta) \int_{\mathcal{X}} \psi(x, x)\, q(x)\, g(x) dx$$

subject to the incentive compatibility condition (11), the individual rationality constraint of the issuer:

$$\tau(x) - \delta\psi(x, x)q(x) \geq 0; \qquad x \in \mathcal{X},$$

and the break-even constraint of the liquidity supplier:

$$\int_{\mathcal{X}} \psi(x, x)\, q(x)\, g(x)\, dx \geq \int_{\mathcal{X}} \tau(x)\, g(x)\, dx.$$

There is no loss of generality in setting $\psi(x, \hat{x}) = x$ for $\hat{x} \neq x$, as the only impact of such a change is to relax the issuer's incentive compatibility constraint. The intuition is that it is optimal to trade equity out of the equilibrium path, as this represents the maximal possible punishment that can be inflicted to the issuer. A similar reasoning implies that there is no loss in generality in setting $q(\hat{x}) = 1$ for all $\hat{x} \in \mathcal{X}$. Indeed, one can redefine the security ψ so that $\tilde{\psi}(x, x) = \psi(x, x)q(x)$ and the incentive compatibility constraint is again relaxed by trading the maximal possible volume. It remains only to determine $\varphi(x) = \psi(x,x)$ for each $x \in \mathcal{X}$, as well as the optimal transfer τ. Let $\overline{\tau} = \sup_{x\in\mathcal{X}} \tau(x)$. The functions φ and τ solve:

$$\sup_{\tau,\varphi} (1-\delta) \int_{\mathcal{X}} \varphi(x)\, g(x) dx$$

subject to the incentive compatibility constraint:

$$\tau(x) - \delta\varphi(x) \geq \overline{\tau} - \delta x; \quad x \in \mathcal{X}, \tag{12}$$

the individual rationality constraint of the issuer:

$$\tau(x) - \delta\varphi(x) \geq 0; \quad x \in \mathcal{X}, \tag{13}$$

and the break-even constraint of the liquidity supplier:

$$\int_{\mathcal{X}} \varphi(x)\, g(x)\, dx \geq \int_{\mathcal{X}} \tau(x)\, g(x)\, dx. \tag{14}$$

It is clear from (12)-(13) that for each $x \in \mathcal{X}$, one at least of these constraints must be binding. It is immediate to check that, for each $x \in \mathcal{X}$, the incentive compatibility constraint (12) is binding if and only if $x \leq \frac{\overline{\tau}}{\delta}$, and that the individual rationality constraint (13) is binding if and only if $x \geq \frac{\overline{\tau}}{\delta}$. Moreover the only value of $x \in \mathcal{X}$ where both constraints are binding is $\frac{\overline{\tau}}{\delta}$.

We now prove that the optimal φ corresponds to the debt contract with face value d^c. The argument proceeds by showing that any pair (τ, φ) that solves the above problem is dominated by a debt contract with face value $\frac{\overline{\tau}}{\delta}$ and a constant transfer $\overline{\tau}$. The argument is twofold. Suppose first that $\varphi(x) < x$ for a set of values of $x \leq \frac{\overline{\tau}}{\delta}$ of positive measure, and consider an alternative design $\tilde{\varphi}$ that coincides with φ for $x \geq \frac{\overline{\tau}}{\delta}$, and that satisfies $\tilde{\varphi}(x) = x$ for $x \leq \frac{\overline{\tau}}{\delta}$. Modify correspondingly τ for $x \leq \frac{\overline{\tau}}{\delta}$ by setting $\tilde{\tau} = \overline{\tau}$ on this interval of values of x. If feasible, this new design clearly dominates φ. It is immediate that constraints (12)-(13) are preserved. Consider now (14). Since it is satisfied under the initial contract,

$$\begin{aligned} \int_{\mathcal{X}} \varphi(x)\, g(x)\, dx \;\geq\; & \int_{\mathcal{X}} \tau(x)\, g(x)\, dx \\ = & \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} (\overline{\tau} - \delta x + \delta\varphi(x))\, g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \tau(x)\, dx, \end{aligned}$$

where the equality follows from the fact that (12) is binding for $x \leq \frac{\overline{\tau}}{\delta}$. So, in particular, as $x \geq \varphi(x)$, we obtain that:

$$
\begin{aligned}
\int_{\mathcal{X}} \tilde{\varphi}(x)\, g(x)\, dx &= \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x\, g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x)\, g(x)\, dx \\
&\geq (1-\delta) \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} \varphi(x)\, g(x)\, dx + \delta \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x\, g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x)\, g(x)\, dx \\
&\geq \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \tau(x)\, g(x)\, dx \\
&= \int_{\mathcal{X}} \tilde{\tau}(x)\, g(x)\, dx,
\end{aligned}
$$

which implies that (14) holds under the new contract. Hence an optimal contract must have $\varphi(x) = x$ for all $x \leq \frac{\overline{\tau}}{\delta}$. Consider now the values of $x \geq \frac{\overline{\tau}}{\delta}$. We know that the individual rationality constraint (13) is binding for such x. Therefore one must have $\varphi(x) \leq \frac{\overline{\tau}}{\delta}$ by definition of $\overline{\tau}$. Suppose that $\varphi(x) < \frac{\overline{\tau}}{\delta}$ for a set of values of $x \geq \frac{\overline{\tau}}{\delta}$ of positive measure. Consider an alternative design that coincides with φ for $x \leq \frac{\overline{\tau}}{\delta}$ and that satisfies $\tilde{\varphi}(x) = \frac{\overline{\tau}}{\delta}$ otherwise. Modify correspondingly τ for $x \geq \frac{\overline{\tau}}{\delta}$ by setting $\tilde{\tau} = \overline{\tau}$ on this interval of values of x. If feasible, this new design clearly dominates φ. It is immediate that constraints (12)-(13) are preserved. Consider now (14). Since it is satisfied under the initial contract, and since $\tau(x) = \overline{\tau}$ for all $x \leq \frac{\overline{\tau}}{\delta}$,

$$
\begin{aligned}
\int_{\mathcal{X}} \varphi(x)\, g(x)\, dx &\geq \int_{\mathcal{X}} \tau(x)\, g(x)\, dx \\
&= \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \delta \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x)\, g(x)\, dx,
\end{aligned}
$$

where the equality follows from the fact that (13) is binding for $x \geq \frac{\overline{\tau}}{\delta}$. So, particular, as $\frac{\overline{\tau}}{\delta} \geq \varphi(x)$ and $\varphi(x) = x$ for $x \leq \frac{\overline{\tau}}{\delta}$, we obtain that:

$$
\begin{aligned}
\int_{\mathcal{X}} \tilde{\varphi}(x)\, g(x)\, dx &= \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x\, g(x)\, dx + \left(1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right) \frac{\overline{\tau}}{\delta} \\
&\geq \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x\, g(x)\, dx + (1-\delta) \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x)\, g(x)\, dx + \left(1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right) \overline{\tau}
\end{aligned}
$$

$$\geq \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \left(1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right)\overline{\tau}$$

$$= \overline{\tau},$$

which implies that (14) holds under the new contract. Combining this with the earlier result, we obtain that φ is a debt contract with face value $\frac{\overline{\tau}}{\delta}$. At an optimum, we must have $\frac{\overline{\tau}}{\delta} = d^c$ by Proposition 8. Thus we can state the following proposition:

PROPOSITION 8 *If the issuer can design a menu of securities from which she selects which security to trade at time t, with a competitive liquidity supplier the equilibrium allocations and the traded security are the same as in the basic model.*

Furthermore, this shows that the monotonicity is imposed without loss of generality in the basic model. In particular, "live or die" contracts in the spirit of Innes (1990) are never optimal.

5.2. Oligopolistic Screening

So far, we have focused on the case where the issuer designs the trading mechanism, or where there is a single liquidity supplier who acts as a monopolist. We now turn to the case where $N > 1$ liquidity suppliers offer simultaneously non–exclusive price-quantity schedules $\{T_i\}_{i=1,...,N}$. Consistently with previous models of multiprincipal mechanism design (see, e.g., Stole (1990), Martimort (1992), or Biais, Martimort and Rochet (2000)), this can be seen as a situation of competition between trading mechanisms in which each principal cannot contract on the quantitities that are sold to his competitors. This corresponds to many situations observed in practice in financial markets where individual trades cannot be made contingent on the quotes or trades made by others. In this context, we show that when the issuer designs a debt contract with face value d^c, there exists an equilibrium of this game that decentralizes the competitive allocation described in Section 4.

The Trading Game. The extensive form of the game is similar to that described in Section 2, except that i) the N liquidity suppliers simultaneously post trading mechanisms $T_i : [0, 1] \to \mathbb{R}$, $i \in \{1..., N\}$, for the sale of any fraction $q_i \in [0, 1]$ of the securitized asset, and ii) if the issuer accepts the trading mechanisms $\{T_j\}_{j\in J}$, $J \subset \{1, ..., N\}$, she trades volumes $\{q_j\}_{j\in J}$ of the security, $\sum_{j\in J} q_j \leq 1$, for which she obtains transfers $\{T_j(q_j)\}_{j\in J}$.

For a given security F, we focus on perfect Bayesian equilibria of this screening game. In these equilibria, liquidity suppliers post transfer schedules that are best response to the strategies of the other liquidity suppliers, given the behavior of the issuer in the subsequent stage of the game.

Our goal is not to give a full characterization of the perfect Bayesian equilibria of the trading game. Rather, our aim is to construct an equilibrium that implements the same allocation and transfers than in the competitive case, i.e., $\sum_i q_i = 1$ and $\sum_i T_i = \delta f_F^c$.[7] To do so, let us introduce the following candidate equilibrium strategies:

$$T_i(q_i) = \delta f_F^c q_i; \qquad q_i \in [0, 1] \tag{15}$$

for each of the liquidity suppliers. That is, each liquidity supplier offers the issuer to buy an arbitrary volume of her securities at the competitive price δf_F^c. To check that these strategies form an equilibrium, suppose that all liquidity suppliers $i \in \{2, \ldots, N\}$ offer this schedule, while the first liquidity supplier offers an alternative schedule $\tilde{T}_1$. Now consider an issuer with type $f \in [\underline{f}, f_F^c]$. Then, whatever the volume q_1 of her securities that she sells to the first liquidity supplier, she will sell the remaining fraction $1 - q_1$ to the other liquidity suppliers, since the price δf_F^c at which she can sell to them is higher than her retention cost δf. Hence, her problem can be written as:

$$U_F(f) = \sup_{q_1 \in [0,1]} \tilde{T}_1(q_1) + \delta f_F^c (1 - q_1) - \delta f. \tag{16}$$

The remarkable fact about (16) is that the set of its solutions does not depend on f. This follows from the fact that all types of the issuer stands ready to sell their securities at the competitive price. Therefore we can assume that, out of the equilibrium path, the deviating liquidity supplier will face the same issuer distribution than the non-deviating ones, and that all types of the issuer sell the same volume of securities q_1 to him. If this is so, then the only way that the deviating liquidity supplier can attract the issuer is by offering a transfer $\tilde{T}_1(q_1) \geq \delta f_F^c q_1$, thereby obtaining a profit:

$$\left(\int_{\underline{f}}^{f_F^c} f g(f)\, df + (1 - G(d^c)) f_F^c - \frac{\tilde{T}_1(q_1)}{q_1} \right) q_1$$

which is less or equal than what he would get by offering the competitive schedule, i.e., zero. It follows that the candidate strategies form an equilibrium. In this equilibrium,

[7] For simplicity we neglect the case where a constant transfer τ_0^c is allocated to the issuer even if she does not trade.

the issuer is able to sell all of her securities to the liquidity suppliers. Hence, we have proved the following decentralization result.

Proposition 9 *There exists a perfect Bayesian equilibrium of the non-exclusive trading game that implements the competitive allocation.*

The basic logic of this result is that of Bertrand competition: competition among liquidity suppliers allows to implement a constrained efficient allocation. We stress that the non-exclusivity clause plays a crucial role in the above argument, in that it ensures that no cream-skimming deviation is possible from the competitive schedule: any deviation that would attract some type of the issuer would also attract the other types. As a result, the transfer schedule (15) is entry-proof in the sense of Rothschild and Stiglitz (1976): no liquidity supplier can deviate by offering an alternative schedule without losing money. It should be noted that, in contrast with other models of competition in trading mechanisms such as Biais, Martimort and Rochet (2000), this result does not require a large number of liquidity suppliers. This follows from linearity of the issuer's preferences.

6. Conclusion

This paper analyzes the links between the characteristics of securities and their liquidity. Our theoretical analysis is in the line of the insightful recent paper by DeMarzo and Duffie (1999). One difference is that we compare strategic and competitive liquidity supply. Furthermore, we take a mechanism design approach to characterize both the optimal security and the optimal trading mechanism.

Similarly to Duffie and DeMarzo (1999), and in line with Myers and Majluf (1984), we find that the optimal security is debt. In the case where liquidity suppliers are competitive, the intuition is that this is the less information sensitive security, and thus minimizes the consequences of adverse selection. Moreover, debt optimally copes with the market power of the liquidity supplier. In contrast with the signalling equilibrium analyzed by Duffie and DeMarzo (1999), in the optimal mechanism we characterize, the optimal debt contract is perfectly liquid: all issuer types sell 100% of their holdings of this security. Consequently, debt issuance does not convey a negative signal to the market, and, correspondingly, has no price impact. This is in line with the results of several empirical studies (see, e.g., Dann and Mikkelson (1984), Eckbo (1986), and Mikkelson and Partch (1986)).

Our main results are robust to relaxing two assumptions of our basic model (which also were made by Duffie and DeMarzo, 1999). In the competitive liquidity supply case, the optimal security and trades are unchanged if one allows the issuer to design a *menu* of securities, instead of a single security. Furthermore, for this result to hold, no monotonicity assumptions need to be imposed. This points at the qualitative difference between our analysis of the optimality of debt – which does not rely upon monotonicity restrictions – and that of Innes (1990) whereby, without these restrictions, the optimal contract allocates all the cash flows to the outside financier up to a given threshold, and all the cash flow to the inside manager above that threshold.

Finally, the constrained efficient allocation reached in the competitive liquidity supply case, where the issuer designs the trading mechanism, can be decentralized in a very natural way by allowing multiple liquidity suppliers to offer non-exclusive transfer schedules – as in financial markets where liquidity suppliers compete in limit orders to buy, or bid prices.

Appendix

Proof of Lemma 1. Part (i) follows from the fact that U_F is the maximum of a family of affine functions, as is easily seen from (3). As a convex function, U_F is a.e. differentiable (see, e.g., Rockafellar (1970, Theorem 25.5)). Part (ii) then immediately follows from the envelope theorem. ■

Proof of Proposition 1. Fix a security F. Using the incentive constraint for types $f, \tilde{f} \in \mathcal{F}$ of the issuer, it is easy to check that:

$$-\delta(\tilde{f} - f)q(f) \leq U_F(\tilde{f}) - U_F(f) \leq -\delta(\tilde{f} - f)q(\tilde{f}); \qquad (f, \tilde{f}) \in \mathcal{F}^2.$$

Since q is bounded, this implies that U_F is Lipschitzian, hence absolutely continuous on $\mathcal{F}$. As $\dot{U} = -\delta q$ a.e. on $\mathcal{F}$, this implies that for all $f \in \mathcal{F}$, $U_F(f) = \delta \int_f^{\overline{f}} q(\phi)\, d\phi + U_F(\overline{f})$. In the competitive case, we may substitute this in the binding participation constraint of the liquidity supplier:

$$U_F(\overline{f}) = \int_{\mathcal{F}} \left((1-\delta) f q(f) - \delta \int_f^{\overline{f}} q(\phi)\, d\phi \right) dG^{\varphi}(f).$$

Using the fact that G^{φ} is a right-continuous function of bounded variation, we may integrate by parts (see, e.g., Dellacherie and Meyer (1982, Theorem VI.90)) to obtain:

$$\int_{\mathcal{F}} \int_f^{\overline{f}} q(\phi)\, d\phi\, dG^{\varphi}(f) = \int_{\mathcal{F}} G^{\varphi}(f^-) q(f)\, df,$$

where $f \mapsto G^{\varphi}(f^-)$ is the left-continuous regularization of G^{φ}, which satisfies $G^{\varphi}(\underline{x}^-) = 0$ by convention. Hence the issuer's problem is to maximize:

$$(1-\delta) \int_{\mathcal{F}} f q(f)\, dG^{\varphi}(f)$$

with respect to q non-increasing and taking its values in $[0, 1]$, and subject to:

$$\int_{\mathcal{F}} (1-\delta) f q(f)\, dG^{\varphi}(f) - \delta \int_{\mathcal{F}} G^{\varphi}(f^-) q(f)\, df \geq 0.$$

The Lagrangian for this problem is:

$$\int_{\mathcal{F}} (1+\lambda)(1-\delta) f q(f)\, dG^{\varphi}(f) - \lambda\delta \int_{\mathcal{F}} G^{\varphi}(f^-) q(f)\, df = \int_{\mathcal{F}} q(f)\, dH_{\lambda}(f),$$

where, for any $\lambda \geq 0$, $H_{\lambda} : f \mapsto \int_{\underline{f}}^{f} (1+\lambda)(1-\delta)\phi\, dG^{\varphi}(\phi) - \int_{\underline{f}}^{f} G(\phi^-)\, d\phi$ is a right-continuous function of bounded variation. It is immediate to see that an optimal q must be left-continuous. Hence, we may integrate by parts to obtain:

$$\int_{\mathcal{F}} q(f)\, dH_{\lambda}(f) = \int_{\mathcal{F}} (H_{\lambda}(f) - H_{\lambda}(\overline{f}))\, d(1-q)(f^+),$$

where $f \mapsto q(f^+)$ is the right-continuous regularization of q. For any fixed $\lambda \geq 0$, the maximum of the Lagrangian is obtained by putting all the weight of the measure with distribution $f \mapsto 1 - q(f^+)$ on a maximum f_λ of the function H_λ. This implies that the optimal quantity schedule q^c has the required bang-bang property. When $\lambda > 0$, the threshold f_F^c is determined as the largest solution in f_λ to:

$$\int_{\underline{f}}^{f_\lambda} (1-\delta) f \, dG^\varphi(f) - \int_{\underline{f}}^{f_\lambda} G^\varphi(f) \, df = 0,$$

i.e., the complementary slackness condition for the Lagrangian at the optimum. In the monopolistic case, we have $U_F(\overline{f}) = 0$, and the liquidity supplier's objective becomes:

$$\int_{\mathcal{F}} \left((1-\delta) f q(f) - \delta \int_f^{\overline{f}} q(\phi) \, d\phi \right) dG^\varphi(f).$$

Along the same lines as above, we find that the liquidity supplier's problem is to maximize:

$$\int_{\mathcal{F}} (1-\delta) f q(f) \, dG^\varphi(f) - \delta \int_{\mathcal{F}} G^\varphi(f^-) q(f) \, df$$

with respect to q non-increasing and taking its values in $[0,1]$. The result follows again from the linearity of this objective with respect to q and the constraint that q be non-increasing. That $f_F^c \geq f_F^m$ follows from a direct comparison of the isssuer's and the liquidity supplier's objectives. Finally, suppose that $\tau_0^c > 0$ while $f_F^c < \overline{f}$. Then the liquidity supplier's break-even constraint yields:

$$\tau_0^c = \int_{\underline{f}}^{f_F^c} (f - \delta f_F^c) \, dG^\varphi(f),$$

while the issuer's expected rent is given by:

$$(1-\delta) \int_{\underline{f}}^{f_F^c} f \, dG^\varphi(f).$$

It is then clear that the issuer could strictly gain by decreasing slightly the lump-sum tax τ_0^c and increasing the threshold f_F^c while still preserving the break-even constraint, a contradiction. ■

Proof of Proposition 2. We can assume w.l.o.g. that $f_F^c < \overline{x}$. In that case, we have $\tau_0^c = 0$ and the price at which equity is traded is δf_F^c. The result then follows from saturating the liquidity supplier's break-even constraint. ■

Proof of Proposition 3. Ignoring first the convexity constraint on U_E, and proceeding as in the proof of Proposition 1, the liquidity supplier's objective can be expressed, after an integration by parts, as:

$$\int_{\mathcal{F}} \left((1-\delta) f q(f) - \delta \int_f^{\overline{x}} q(\phi) \, d\phi \right) g(f) \, df = \int_{\mathcal{F}} \left((1-\delta) f - \delta \, \frac{G(f)}{g(f)} \right) q(f) \, g(f) \, df.$$

Pointwise maximization with respect to q implies that $q = 1$ on the set of $f \in \mathcal{X}$ such that $\frac{1-\delta}{\delta} f \geq \frac{G(f)}{g(f)}$ and $q = 0$ elsewhere. Assumption (9) ensures that this set is an interval. Hence the associated rent $U_E(f) = \delta(f_E^m - f)\chi_{\{f \leq f_E^m\}}$ is convex in f, which implies the result. ■

Proof of Proposition 4. Suppose w.l.o.g. that $f_E^c < \overline{x}$, so that an equity contract would imply some shut-down on the part of issuer. If $d < f_E^c$, then the issuer does not want to shut-down any type $f \in [\underline{x}, d]$, for otherwise she would like to lower the shut-down threshold from f_E^c when she offers equity. Suppose now that $d = f_E^c$. By the above reasoning, if the issuer shut-downs some types below f_E^c, she obtains at most:

$$(1-\delta)\int_{\underline{f}}^{f_E^c} fg(f)\,df.$$

This is clearly less than what she obtains if she does not shut-down any type and saturate the liquidity supplier's break-even constraint, i.e.,

$$(1-\delta)\left(\int_{\underline{f}}^{f_E^c} fg(f)\,df + (1 - G(f_E^c))f_E^c\right).$$

By continuity, it is clear that for any $d \in [f_E^c, d^c]$, the issuer will not shut-down any type when the traded contract is a standard debt contract with face value d, while still preserving the liquidity supplier's break-even constraint. ■

Proof of Proposition 5. Suppose w.l.o.g. that $f_E^m < \overline{x}$, so that an equity contract would imply some shut-down on the part of the liquidity supplier. If $d < f_E^m$, then the liquidity supplier does not want to shut-down any type $f \in [\underline{x}, d]$, for otherwise he would like to lower the shut-down threshold from f_E^m when the issuer offers an equity contract. Suppose now that $d = f_E^m$. By the above reasoning, if the liquidity supplier shut-downs some types below f_E^m, he obtains at most:

$$\int_{\underline{x}}^{f_E^m} fg(f)\,df - \delta f_E^m G(f_E^m)$$

since (9) holds. If $f_E^m < \overline{x}$ so that $G(f_E^m) < 1$, then, since $\delta < 1$, this is clearly less than what he obtains if he does not shut-down any type, i.e.,

$$\int_{\underline{x}}^{f_E^m} fg(f)\,df + f_E^m(1 - G(f_E^m)) - \delta f_E^m.$$

By continuity, it is clear that for any $d \in [f_E^m, d^m]$, the liquidity supplier will not shut-down any type of the issuer when the traded contract is a standard debt contract with face value d. ■

Proof of Lemma 3. In the competitive case, this follows at once from the fact that the issuer maximizes the gains from trade. In the monopolistic case, let us suppose that F is an optimal design such that $\varphi(\underline{x}) < \underline{x}$. Then, given (MR), there exists $\varepsilon > 0$ such that $\varphi(x) < x - \varepsilon$ for all $x \in \mathcal{X}$. Consider the design F_ε defined by $\varphi_\varepsilon = \varphi + \varepsilon$. By Proposition 1, given this new design, the liquidity supplier chooses a shut-down threshold $\tilde{f}$ so as to maximize his expected profit:

$$\int_{\underline{f}+\varepsilon}^{\tilde{f}} (f - \delta\tilde{f})\, dG^{\varphi_\varepsilon}(f) = \int_{\underline{f}}^{\tilde{f}-\varepsilon} (f - \delta(\tilde{f} - \varepsilon))\, dG^{\varphi}(f) + (1-\delta)\, \varepsilon\, G^{\varphi}(\tilde{f} - \varepsilon).$$

The first term on the right-hand side of this equation is maximized by setting $\tilde{f} = f_F^m + \varepsilon$, where f_F^m is the liquidity supplier's optimal shut-down threshold for security F. Since the second term is non-decreasing in $\tilde{f}$, this implies that the optimal shut-down threshold $f_{F_\varepsilon}^m$ for the new design is greater or equal than $f_F^m + \varepsilon$. It is then easy to check that the issuer's expected rent under the design F_ε is at least as large as under F. ■

Proof of Proposition 7. We first consider the competitive case. Suppose that F is a security sold at a price δf_F^c for which $f_E^c < \overline{f}$. To show that this cannot be optimal, we show that there exists an alternative security and transfer which Pareto dominates F. Consider the design $\overline{F}$ defined by $\overline{\varphi} = \min\{\varphi, f_F^c\}$ with price $\delta f_F^c + \varepsilon$ for some $\varepsilon > 0$. By construction, issuers with $\overline{\varphi}(x) < f_F^c$ are still willing to trade. Issuers with $\overline{\varphi}(x) = f_F^c$ are ready to trade since by doing so their informational rent is ε instead of zero. Consider now the liquidity suppliers. Their expected profit under F is:

$$\int_{\underline{f}}^{f_F^c} (f - \delta f_F^c)\, dG^{\varphi}(f),$$

while their expected profit under $\overline{F}$ is:

$$\int_{\underline{f}}^{f_F^c} (f - \delta f_F^c)\, dG^{\varphi}(f) + (1-\delta)(1 - G^{\varphi}(f_F^c)) - \varepsilon.$$

Since $G^{\varphi}(f_F^c) < 1$, they are strictly better off under the new design for ε small enough. Consider now the monopolistic case. Suppose by way of contradiction that F is an optimal security such that the liquidity supplier shut-downs the types above $f_F^m < \overline{f}$, thereby obtaining a profit:

$$\int_{\underline{f}}^{f_F^m} (f - \delta f_F^m)\, dG^{\varphi}(f).$$

Consider the design F_ε defined by $\varphi_\varepsilon = \min\{\varphi, f_F + \varepsilon\}$ for $\varepsilon > 0$. Since $f_F^m < \overline{f}$, φ is continuous, and the density g is positive on $\mathcal{X}$, one can choose ε such that $G^{\varphi}(f_F^m + \varepsilon^-) < 1$. If the liquidity supplier decides to shut-down some types below $f_F^m + \varepsilon$ given this new design, the optimal way to do so is to set $f_{F_\varepsilon}^m = f_F^m$, for a profit equal to that which obtains under F. However, if he decides not to shut-down any type, the liquidity supplier obtains:

$$\int_{\underline{f}}^{f_F^m+\varepsilon} (f - \delta(f_F^m + \varepsilon))\, dG^{\varphi_\varepsilon}(f),$$

which can be rewritten as:

$$\int_{\underline{f}}^{f_F^m} (f - \delta(f_F^m + \varepsilon))\, dG^{\varphi}(f) + \int_{f_F^m}^{f_F^m + \varepsilon^-} (f - \delta(f_F^m + \varepsilon))\, dG^{\varphi}(f) + (1-\delta)(f_F^m + \varepsilon)(1 - G^{\varphi}(f_F^m + \varepsilon^-)).$$

The first term converges to $\int_{\underline{f}}^{f_F^m} (f - \delta f_F^m)\, dG^{\varphi}(f)$ as ε goes to 0. Moreover, for ε small enough, the second term in this expression is positive. Since the last term is positive and bounded away from zero for ε small enough, the liquidity supplier obtains strictly more by not shutting down any type. It is then immediate to check that the issuer's expected rent under the design F_ε,

$$\delta \int_{\underline{f}}^{f_F^m + \varepsilon} (f_F^m + \varepsilon - f)\, dG^{\varphi_\varepsilon}(f) = \delta \int_{\underline{f}}^{f_F^m + \varepsilon^-} (f_F^m + \varepsilon - f)\, dG^{\varphi}(f),$$

is strictly larger than under F. Hence F cannot be an optimal security, which implies the result. ■

Proof of Proposition 8. For any fixed $\lambda > 0$, we study the problem of maximizing $L(\varphi, \lambda)$ with respect to $\varphi \in \Phi$. We treat this as an optimal control problem with state variable φ and control variable $\dot{\varphi}$, with the additional restriction that $0 \leq \dot{\varphi} \leq 1$. The Hamiltonian can be written as:

$$H(x, \varphi(x), p(x), v) = -(1 + \lambda - \delta) v G(x) + p(x) v$$

where v is the control variable and p the co-state variable. By Pontryagin's maximum principle, a necessary condition for (φ, v) to be optimal is that v maximizes pointwise the Hamiltonian for some p that satisfies the Hamilton-Jacobi equation:

$$\dot{p}(x) = -\frac{\partial H}{\partial \varphi}(x, \varphi(x), p(x), v(x)) = 0$$

at all points of continuity of v. Since the boundary $\overline{x}$ is free, the transversality condition yields $p(\overline{x}) = (1-\delta)(1+\lambda)$ so that p is constant and equal to $(1-\delta)(1+\lambda)$. Substituting this back into the Hamiltonian, we find that an optimal control is:

$$v(x) = \chi_{\{-(1+\lambda-\delta)G(x) + (1-\delta)(1+\lambda) \geq 0\}}.$$

Since the Hamiltonian is linear in (φ, v), Mangasarian's sufficiency conditions are satisfied, so v is indeed optimal. Note that since the mapping $x \mapsto -(1+\lambda-\delta)G(x) + (1-\delta)(1+\lambda)$ is decreasing, the corresponding φ is a debt contract with face value d satisfying $-(1+\lambda-\delta)G(d) + (1-\delta)(1+\lambda) = 0$. Thus provided that $G(d^c) > 1-\delta$, $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d^c\}$ maximizes $L(\varphi, \lambda)$ whenever:

$$\lambda = \frac{(1-\delta)\,(1 - G(d^c))}{G(d^c) - (1-\delta)}.$$

It is easy to check that the optimal debt contract from the liquidity supplier's point of view satisfies $G(d) = 1-\delta$. Hence $G(d^c) > 1-\delta$ as required. Since for this contract we have $\int_{\mathcal{X}} \varphi(x)\, g(x)\, dx = \delta \varphi(\overline{x})$, the conclusion follows directly from Luenberger (1969, §8.3, Theorem 1). ■

Proof of Lemma 4. Suppose by way of contradiction that $\tilde{\varphi}$ guarantees the issuer a higher payoff than φ,

$$\delta \int_{\mathcal{X}} (\tilde{\varphi}(\overline{x}) - \tilde{\varphi}(x))\, g(x)\, dx > \delta \int_{\mathcal{X}} (\varphi(\overline{x}) - \varphi(x))\, g(x)\, dx,$$

while satisfying the no shut-down condition:

$$\int_{\mathcal{X}} (\tilde{\varphi}(x) - \delta\tilde{\varphi}(\overline{x}))\, g(x)\, dx \geq \int_{\underline{x}}^{\tilde{x}} (\tilde{\varphi}(x) - \delta\tilde{\varphi}(\tilde{x}))\, g(x)\, dx; \qquad \tilde{x} \in \mathcal{X}.$$

The no shut-down condition on $\tilde{\varphi}$, together with the fact that Λ defines a positive measure on $\mathcal{X}$, implies:

$$\int_{\mathcal{X}} \left(\int_{\mathcal{X}} (\tilde{\varphi}(x) - \delta\tilde{\varphi}(\overline{x}))\, g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} (\tilde{\varphi}(x) - \delta\tilde{\varphi}(\tilde{x}))\, g(x)\, dx \right) d\Lambda(\tilde{x}) \geq 0.$$

But then, since:

$$\int_{\mathcal{X}} \left(\int_{\mathcal{X}} (\varphi(x) - \delta\varphi(\overline{x}))\, g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} (\varphi(x) - \delta\varphi(\tilde{x}))\, g(x)\, dx \right) d\Lambda(\tilde{x}) = 0,$$

we would get that $L(\tilde{\varphi}, \Lambda) > L(\varphi, \Lambda)$, a contradiction. ∎

Proof of Lemma 5. For any fixed $\lambda > 1$, we study the problem of maximizing $L(\varphi, \Lambda_\lambda)$ with respect to $\varphi \in \Phi$. Re-arranging the expression of $L(\varphi, \Lambda_\lambda)$, we obtain:

$$L(\varphi, \Lambda_\lambda) = \delta \int_{\underline{x}}^{f_E^m} \dot{\varphi}(x)\, G(x)\, dx + \int_{f_E^m}^{\overline{x}} ((1-\lambda)\delta\dot{\varphi}(x)G(x) + \lambda(1-\delta)\varphi(x)g(x))\, dx.$$

Since $\lambda > 0$, it is clear that it is optimal to set $\varphi(x) = x$ on $[\underline{x}, f_E^m]$. We are thus left with the problem of maximizing:

$$\int_{f_E^m}^{\overline{x}} ((1-\lambda)\delta\dot{\varphi}(x)G(x) + \lambda(1-\delta)\varphi(x)g(x))\, dx$$

with respect to functions φ on $[f_E^m, \overline{x}]$ satisfying (LL)–(M)–(MR). We treat this as an optimal control problem with state variable φ and control variable $\dot{\varphi}$, with the additional restriction that $0 \leq \dot{\varphi} \leq 1$. The Hamiltonian can be written as:

$$H(x, \varphi(x), p(x), v) = (1-\lambda)\delta v G(x) + \lambda(1-\delta)\varphi(x)g(x) + p(x)v$$

where v is the control variable and p the co-state variable. By Pontryagin's maximum principle, a necessary condition for (φ, v) to be optimal is that v maximizes pointwise the Hamiltonian for some p that satisfy the Hamilton-Jacobi equation:

$$\dot{p}(x) = -\frac{\partial H}{\partial \varphi}(x, \varphi(x), p(x), v(x)) = -\lambda(1-\delta)g(x)$$

at all points of continuity of v. Since the boundary $\overline{x}$ is free, the transversality condition yields $p(\overline{x}) = 0$, so that $p = \lambda(1-\delta)(1-G)$. Substituting this back into the Hamiltonian, we find that a candidate optimal control is:

$$v(x) = \chi_{\{(\delta-\lambda)G(x)+\lambda(1-\delta)\geq 0\}}.$$

Since the Hamiltonian is linear in (φ, v), Mangasarian's sufficiency conditions are satisfied, so v is indeed optimal. Note that since the mapping $x \mapsto (\delta-\lambda)G(x) + \lambda(1-\delta)$ is decreasing as $\lambda > 1 > \delta$, the corresponding φ is to a debt contract with face value d satisfying $(\delta-\lambda)G(d) + \lambda(1-\delta) = 0$. Thus, provided that $G(d^m) > 1-\delta$, $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d^m\}$ maximizes $L(\varphi, \Lambda_\lambda)$ whenever:

$$\lambda = \frac{\delta G(d^m)}{G(d^m) - (1-\delta)}.$$

It is easy to check that the optimal debt contract from the liquidity supplier's point of view satisfies $G(d) = 1-\delta$. Hence $G(d^m) > 1-\delta$, which concludes the proof as $\lambda > 1$ whenever $d^m < \overline{x}$. ■

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