Efficient Use of Information and Welfare Analysis in Economies with Complementarities and Asymmetric Information*

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Abstract

This paper analyzes equilibrium and welfare for a tractable class of economies with externalities, strategic complementarity or substitutability, and incomplete information. We first characterize the *equilibrium use of information* and show how strategic payoff effects can heighten the sensitivity of equilibrium actions to noise. We next characterize the *efficient use of information*, which allows us to address whether such heightened sensitivity to noise is socially undesirable. We show how the efficient use of information trades off volatility for dispersion, and how this relates to the socially optimal degree of coordination. We finally characterize the *social value of information* in equilibrium; understanding the relation between equilibrium and efficient use of information proves instrumental for this task as well. We conclude with a few applications, including production externalities, Keynesian frictions, inefficient fluctuations, efficient market competition, and large Cournot and Bertrand games.

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1 Introduction

In many environments—including economies with production externalities, incomplete financial markets, or monopolistic competition—a coordination motive emerges: the action an agent takes depends not only on his expectations about the underlying fundamentals but also about other agents' actions. Furthermore, different agents have different information about the fundamentals and hence different beliefs about what other agents are doing. An extensive literature examines the equilibrium properties of such environments. The welfare implications, on the other hand, are far less understood. Filing this gap is the goal of this paper.

To fix ideas, consider the following example. A large number of investors are choosing how much to invest in a new sector. The profitability of this sector depends on uncertain exogenous fundamentals as well as the aggregate investment in that sector. Each investor thus tries to align his investment choice with other investors' choices. Because of this, and because public information is a better predictor of what others do relatively to private information, investment choices are highly sensitive to public information. Furthermore, more precise public information, by raising investors' reliance on it, may crowd out private information and increase aggregate volatility.

It is tempting to give a normative interpretation to these properties, but this would be premature. Is the heightened sensitivity to public information really excessive from a social perspective? And does this mean that information disseminated by central banks or the media can reduce welfare? To answer the first question, one needs to understand the *efficient use of information*; to answer the second question, one needs to understand the *social value of information.* Here we undertake these two tasks in a tractable yet flexible framework that can capture a variety of applications.

We consider a class of quadratic economies that feature rich external and strategic effects and asymmetric information. For this class, we provide a complete characterization of the equilibrium and the efficient use of information. The discrepancy—if any—between the two helps also understand the comparative statics of equilibrium welfare with respect to the information structure.

In our model, a large number of ex-ante identical small agents takes a continuous action. Individual payoffs depend not only on one's own action but also on the mean, and possibly the dispersion, of actions in the population—this is the source of external and strategic effects in the model. Agents observe noisy private and public signals about the underlying economic fundamentals—this is the source of information asymmetry. Finally, payoffs are quadratic and information is Gaussian, which makes the analysis tractable.

We first characterize the *equilibrium use of information.* The equilibrium is always unique. Strategic complementarity raises the sensitivity of actions to public information; symmetrically, strategic substitutability raises the sensitivity to private information. Noise in public information generates volatility, while noise in private information generates dispersion. It follows that complementarity contributes to higher volatility, substitutability to higher dispersion.[1]

To understand the welfare content of these equilibrium properties, we next characterize the *efficient use of information*. The efficient allocation maximizes ex-ante utility under the constraint that information cannot be centralized. It can be represented as the equilibrium of a fictitious economy where a "planner" dictates the agents what best responses to follow. The slope of these fictitious best responses with respect to the mean activity measures the extent to which agents must align their choices for efficiency to obtain; it defines what we call the *(socially) optimal degree of coordination*.

A higher degree of coordination corresponds to a higher sensitivity of actions to public information relative to private information. Since higher sensitivity to public information raises volatility, while higher sensitivity to private information raises dispersion, a higher degree of coordination trades off volatility for dispersion. The optimal degree of coordination thus increases with social aversion to dispersion, and decreases with social aversion to volatility.

Aversion to volatility and dispersion, in turn, originates in the curvature of primitive payoffs. When there are no externalities, volatility and dispersion have a symmetric effect on welfare. Complementarity, by alleviating concavity at the aggregate level, reduces aversion to volatility. Hence, as with equilibrium, complementarity contributes to a positive optimal degree of coordination [and substitutability to a negative]. But unlike equilibrium, the optimal degree of coordination also depends on external payoff effects that are irrelevant for private incentives. In the absence of such non-strategic external effects, the optimal degree of coordination—and hence the sensitivity of efficient allocations to public information—is higher than the equilibrium one when agents' actions are strategic complements [and lower when they are strategic substitutes].

This highlights the danger in extrapolating positive to normative: a heightened sensitivity to noisy public news due to a coordination motive need not be socially undesirable, even if it amplifies volatility.

We next characterize the *social value of information*. For this purpose, we parameterize the information structure by the level and the composition of noise in the agents' forecasts of the underlying fundamentals. We identify the *accuracy* of available information with the precision of these forecasts, that is, the reciprocal of total noise, and its *transparency* (or *commonality*) with the correlation of forecast error across agents, that is, the extent to which noise is common. Since in the absence of external effects welfare depends only on the level and not on the composition of noise, this parametrization seems most appropriate from a theoretical point of view.[2]

[1] The amplification effects of various sorts of complementarities are the subject of a vast literature. See Cooper (1990) for a review of complete-information applications and Morris and Shin (2002, 2003) for incomplete information.

[2] This parametrization is also appropriate for some applied questions. Think, for example, of a central banker contemplating whether to transmit information in a transparent or ambiguous way. This need not be simply a choice about the release of more or less information, but rather a choice about the extent to which individuals will adopt

The equilibrium is efficient under incomplete information if and only if the complete-information equilibrium coincides with the first best and, in addition, the equilibrium degree of coordination coincides with the optimal one. When this is the case, equilibrium welfare necessarily increases with the accuracy of information; and it increases [decreases] with the transparency of information if and only if agents' actions are strategic complements [substitutes]. Efficiency thus implies a clear relationship between the form of strategic interaction and the social value of information.

When the equilibrium is inefficient, information can affect the gap between equilibrium and efficient allocations. This gap can be mapped on two dimensions: the discrepancy between optimal and equilibrium degrees of coordination under incomplete information, and the discrepancy between first best and complete-information equilibrium. When the latter is zero, so that the inefficiency vanishes as information becomes complete, a higher [lower] than optimal degree of coordination reduces [increases] the social value of transparency relative to the efficiency benchmark; the social value of accuracy nevertheless remains positive. When, instead, inefficiency pertains even under complete information, more accurate information can reduce welfare.

We conclude the paper by illustrating how our results can help understand the potential inefficiencies in the equilibrium use of information, and the social value of information, in specific applications.

In a typical model of production spillovers like the one briefly mentioned above, where complementarities emerge in investment choices, coordination is inefficiently low. Moreover, welfare unambiguously increases with either the accuracy or the transparency of information—a case for timely provision of relevant information by the government or the media.

The same result appears to hold in standard Keynesian monetary economies, where aversion to price dispersion raises the social value of coordination. In contrast, coordination is inefficiently high and transparency can reduce welfare in economies resembling Keynes' beauty-contest metaphor for financial markets. Furthermore, in economies where equilibrium fluctuations are largely inefficient even under complete information, welfare may decrease with both accuracy and transparency—from a social perspective, ignorance could be a bless.

In a simple, competitive, two-sector production economy, the equilibrium turns out to be efficient even under incomplete information. Since individual actions are strategic substitutes, welfare increases with accuracy but decreases with transparency—perhaps a case for "constructive ambiguity" in central bank communication, despite the efficiency of the equilibrium.

These examples have a macro flavor. However, our results may also be relevant for IO applications with a large number of firms. In a Bertrand-like game, expected profits increase with both the accuracy and the transparency of information. In a Cournot-like game, instead, profits increase with accuracy but not with transparency.

idiosyncratic or common interpretations of the same piece of information.

Related literature. To the best of our knowledge, this paper is the first to conduct a complete welfare analysis for the class of economies considered here. The closest ascendants are Cooper and John (1988), who examine economies with complementarities but complete information, and Vives (1988), who shows efficiency of equilibria in a class of competitive economies that is a special case of the more general class considered here (see Section 6.4).[3]

The social value of information, on this other hand, has been the subject of a voluminous literature, going back at least to Hirshleifer (1971). More recently, and more closely related to this paper, Morris and Shin (2002) show that public information can reduce welfare in an economy that resembles a "beauty contest" and that features strategic complementarity. Angeletos and Pavan (2004) and Hellwig (2005), on the other hand, provide counterexamples where public information is socially valuable despite complementarity—a real economy with investment complementarities in the first paper, a monetary economy with pricing complementarities in the second. These works illustrate the non-triviality of the welfare effects of information within the context of specific applications, but do not explain the general principles underlying the question of interest. We fill the gap here by showing how the social value of information depends, not only on the form of strategic interaction, but also on other external effects that determine the discrepancy between equilibrium and efficient allocations.

The literature on rational expectations has emphasized how the aggregation of disperse private information in markets can improve allocative efficiency (e.g., Grossman, 1981). Laffont (1985) and Messner and Vives (2001), on the other hand, highlight how *informational* externalities can generate inefficiency in the private collection and use of information. Although the information structure here is exogenous, the paper provides an input into this line of research by studying how the welfare effects of private and public information depend on *payoff* externalities.

The paper also contributes to the debate about central-bank transparency. While earlier work focused on incentive issues (e.g., Canzoneri, 1985; Atkeson and Kehoe, 2001; Stokey, 2002), recent work emphasizes the role of coordination. Morris and Shin (2002, 2005) and Heinemann and Cornand (2004) argue that central-bank disclosures can reduce welfare if investors behave like in a "beauty contest"; Svensson (2005) and Woodford (2005) question the practical relevance of this result; Hellwig (2005) and Roca (2005) argue that disclosures improve welfare by reducing price dispersion. In Sections 6.3 and 6.4 we highlight that the welfare effects of information may depend critically on the sources of the business cycle and that an argument for constructive ambiguity is possible even if the equilibrium use of information is efficient.

The rest of the paper is organized as follows. We introduce the model in Section 2. We examine equilibrium in Section 3, efficiency in Section 4, and the social value of information in Section 5. We turn to applications in Section 6. The Appendix includes proofs omitted in the main text.

[3] Related is also Raith (1996), who examines the value of information sharing in oligopolies; see Section 6.5.

2 The model

Actions and payoffs. Consider an economy with a measure-one continuum of agents, each choosing an action $k \in \mathbb{R}$. Let Ψ denote the cumulative distribution function for k in the cross-section of the population, $K \equiv \int k d\Psi(k)$ the mean action, and $\theta = (\theta_1, ..., \theta_N) \in \mathbb{R}^N$ a vector of exogenous payoff-relevant variables (the *fundamentals*), with $N \geq 1$. Individual utility is given by

$$u = U(k, K, \theta), \tag{1}$$

where $U : \mathbb{R}^{N+2} \to \mathbb{R}$ is a strictly concave quadratic function.[4] Finally, we let $W(K, \theta) \equiv U(K, K, \theta)$ denote utility (also, aggregate welfare) when all agents choose the same action.

Externality emerges whenever $U_K \neq 0$, strategic complementarity whenever $U_{kK} \neq 0$. (We refer to U_{kK} as the complementarity even if $U_{kK} < 0$; that is, we identify substitutability with negative complementarity.) We restrict $-U_{kK}/U_{kk}$ within $(-1, +1)$. As we will see in the next section, $-U_{kK}/U_{kk}$ is the slope of best responses; restricting this slope within $(-1, +1)$ is necessary and sufficient for the existence of a unique stable equilibrium. We also impose concavity at both the individual and aggregate level in the sense that $U_{kk} < 0$ and $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK} < 0$. If U were not concave, best responses would not be well-defined; similarly, if W were not concave, the first best would not be well-defined.

Information. Before agents move, nature draws θ_n, for $n \in \bar{\mathcal{N}} \equiv \{1, ..., N\}$, from independent Normal distributions with mean μ_n and variance $\sigma^2_{\theta_n}$. The realization of $\theta = (\theta_n)$ is not observed by the agents. Instead, for each n, agents observe private signals $x^i_n = \theta_n + \xi^i_n$ and public signals $y_n = \theta_n + \varepsilon_n$, where ξ^i_n and ε_n are, respectively, idiosyncratic and common noises, independent of one another as well as of θ, with variances, respectively, $\sigma^2_{x_n}$ and $\sigma^2_{y_n}$.[5] (Throughout, we use the convenient vector notation $x = (x_n)$, $y = (y_n)$, and similarly for all other variables; we also drop the superscript i whenever it does not create confusion.)

The common posterior for θ_n given public information alone is Normal with mean $z_n \equiv \mathbb{E}[\theta_n | y] = \lambda_n y_n + (1 - \lambda_n)\mu_n$ and variance $\sigma^2_{z_n}$, where $\lambda_n \equiv \sigma^{-2}_{y_n}/\sigma^{-2}_{z_n}$ and $\sigma_{z_n} \equiv (\sigma^{-2}_{y_n} + \sigma^{-2}_{\theta_n})^{-1/2}$. In what follows we often identify public information with z rather than y. Private posteriors, on the other hand, are Normal with mean $\mathbb{E}[\theta_n | x^i, y] = (1 - \delta_n)x^i_n + \delta_n z_n$ and variance σ^2_n, where $\delta_n \equiv \sigma^{-2}_{x_n}/\sigma^{-2}_n$ and $\sigma_n \equiv (\sigma^{-2}_{x_n} + \sigma^{-2}_{z_n})^{-1/2}$.

[4] That is, $U(k, K, \theta) = v\mathbf{U}v'$ where $\mathbf{U}$ is a $(n+3) \times (n+3)$ negative-definite matrix and $v = (1, k, K, \theta)$. Also note that U depends on Ψ only through its first moment (mean activity); we extend the model to incorporate an external effect from the second moment (cross-sectional dispersion) at the end of Section 4.

[5] The analysis is simplest when $N = 1$, but $N > 1$ allows us capture the possibility that there are fundamentals that are relevant for equilibrium but not for efficient allocations, and vice versa. The assumption then that (θ_n) are orthogonal to each other is only a normalization: if (θ_n) are correlated, there is a linear one-to-one transformation $(\theta_n) \mapsto (\theta'_n)$ such that (θ'_n) are orthogonal. The orthogonality in the errors (ξ_n, ε_n) across n, on the other hand, permit us to interpret (x_n, y_n) as signals solely about θ_n: if the errors were correlated across n, then (x_n, y_n) would include information also for $n' \neq n$.

If we let $\omega_n^i \equiv \theta_n - \mathbb{E}[\theta_n|x^i, y]$ denote agent i's forecast error about θ_n, then

$$\sigma_n^2 = Var\left(\omega_n^i\right) \qquad \text{and} \qquad \delta_n = Corr(\omega_n^i, \omega_n^j),\ i \neq j.$$

Hence, σ_n measures the total noise in agents' forecasts about the fundamentals and δ_n the extent to which noise is common across agents. We accordingly identify the *accuracy* of information with σ_n^{-2} and its *transparency* (or *commonality*) with δ_n.

We prefer to parametrize the information structure by (δ_n, σ_n) rather than $(\sigma_{x_n}, \sigma_{y_n})$ for two reasons. First, this is without any loss of generality since, given the prior, there is a one-to-one mapping between $(\sigma_{x_n}, \sigma_{y_n})$ and (δ_n, σ_n) :

$$\sigma_n^{-2} = \sigma_{x_n}^{-2} + \sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2} > 0 \quad \text{and} \quad \delta_n = \frac{\sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2}}{\sigma_{x_n}^{-2} + \sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2}} \in (0,1). \tag{2}$$

Second, a change in σ_{x_n} or σ_{y_n} combines a change in the level of noise, σ_n, with a change in its composition, δ_n. If there were no externalities and strategic interactions, welfare would depend only on σ_n, not δ_n. With strategic interactions, instead, the extent to which information is public plays an important role since it affects the structure of higher order beliefs. From a theoretical point of view, it thus seems most interesting to separate these two effects.[6]

3 Equilibrium use of information

Each agent chooses k so as to maximize his expected utility, $\mathbb{E}[U(k, K, \theta)|x, y]$. The solution to this optimization problem gives the best response for the individual. The fixed point is the equilibrium.

Definition 1 *An equilibrium allocation is any function $k : \mathbb{R}^{2N} \to \mathbb{R}$ such that, for all (x, y),*

$$k\,(x, y) = \arg\max_{k'}\ \mathbb{E}[\ U(k', K, \theta)\ |\ x, y\ ],$$

where $K(\theta, \varepsilon) = \mathbb{E}[\ k\,(x, y)\ \ |\ \theta, \varepsilon\ ]$ for all (θ, ε).[7]

It is useful to consider first the complete-information benchmark. When θ is known, the (unique) equilibrium is $k = \kappa\,(\theta)$, where $\kappa\,(\theta)$ is the unique solution to $U_k\,(\kappa, \kappa, \theta) = 0$. Since U is quadratic, κ is linear: $\kappa\,(\theta) = \kappa_0 + \kappa_1\theta_1 + ... + \kappa_N\theta_N$ for some constants $\kappa_n \in \mathbb{R}$, $n \in \bar{\mathcal{N}} \equiv \{0, 1, ..., N\}$.[8] The incomplete-information equilibrium is then characterized as follows.

[6] In the context of specific applications, however, it is also interesting to translate the results in terms of comparative statics with respect to (σ_x, σ_z) . See Section 6 for some examples.

[7] A state of the world is given by the realizations of θ, ε, and $\{\xi^i\}_{i\in[0,1]}$. However, since ξ is i.i.d. across agents, K and other aggregates are functions of (θ, ε) alone.

[8] In particular, $\kappa_0 = -U_k\,(0,0,0)\,/(U_{kk} + U_{kK})$ and $\kappa_n = -U_{k\theta_n}/(U_{kk} + U_{kK})$, $n \in \bar{\mathcal{N}}$. It follows that $\kappa_n \neq 0$ if and only if $U_{k\theta} \neq 0$.

Proposition 1 *Let* $\kappa(\theta) = \kappa_0 + \kappa_1\theta_1 + ... + \kappa_N\theta_N$ *denote the complete-information equilibrium allocation,* $\mathcal{N} \equiv \{n \in \bar{\mathcal{N}} : \kappa_n \neq 0\} \neq \varnothing$,[9] *and*

$$\alpha \equiv \frac{U_{kK}}{|U_{kk}|}. \tag{3}$$

(**i**) *An allocation* $k : \mathbb{R}^{2N} \to \mathbb{R}$ *is an equilibrium if and only if, for all* (x, y),

$$k(x, y) \;=\; \mathbb{E}[\ (1-\alpha)\kappa + \alpha K \mid x, y\ ] \tag{4}$$

where $K(\theta, \varepsilon) = \mathbb{E}[\ k(x, y) \mid \theta, \varepsilon\ ]$ *for all* (θ, ε).

(**ii**) *The equilibrium exists, is unique, and is given by*

$$k(x, y) = \kappa_0 + \sum_{n \in \mathcal{N}} \kappa_n \left[(1-\gamma_n)x_n + \gamma_n z_n\right], \tag{5}$$

$$\gamma_n = \delta_n + \frac{\alpha\delta_n(1-\delta_n)}{1-\alpha(1-\delta_n)} \quad \textit{for all } n \in \mathcal{N}. \tag{6}$$

Proof. *Part (i).* Take any strategy $k : \mathbb{R}^{2N} \to \mathbb{R}$ and let $K(\theta, \varepsilon) = \mathbb{E}[k(x, y)|\theta, \varepsilon]$. A best-response is a strategy $k'(x, y)$ that solves, for all (x, y), the first-order condition

$$\mathbb{E}[U_k(k', K, \theta)|x, y] = 0.$$

Since U is quadratic, $U_k(k', K, \theta) = U_k(\kappa, \kappa, \theta) + U_{kK} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa)$; and since $U_k(\kappa, \kappa, \theta) = 0$ for all θ, the above reduces to

$$\mathbb{E}[U_k(\kappa, \kappa, \theta) + U_{kk}(k' - \kappa) + U_{kK}(K - \kappa)|x, y] = 0,$$

or equivalently $k'(x, y) = \mathbb{E}[(1-\alpha)\kappa + \alpha K|x, y]$. In equilibrium, $k'(x, y) = k(x, y)$, which gives (4).

Part (ii). Since $\mathbb{E}[\kappa|x, y]$ is linear in (x, z), it is natural to look for a fixed point that is linear in x and z. Thus suppose[10]

$$k(x, y) = a + b \cdot x + c \cdot z \tag{7}$$

for some $a \in \mathbb{R}$ and $b, c \in \mathbb{R}^N$. Then $K(\theta, \varepsilon) = a + b \cdot \theta + c \cdot z$ and (4) reduces to

$$k(x, y) = (1-\alpha)\kappa_0 + \alpha a + ((1-\alpha)\kappa + \alpha b) \cdot \mathbb{E}[\theta|x, y] + \alpha c \cdot z$$

where $\kappa = (\kappa_1, ..., \kappa_n)$. Substituting $\mathbb{E}[\theta|x, y] = (\mathbf{I} - \mathbf{\Delta})x + \mathbf{\Delta}z$, where $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{\Delta}$ is the $N \times N$ diagonal matrix with n-th element equal to δ_n, we conclude that (7) is an equilibrium if and only if a, b and c solve

$$a = (1-\alpha)\kappa_0 + \alpha a, \quad b = (\mathbf{I} - \mathbf{\Delta})\left[(1-\alpha)\kappa + \alpha b\right], \quad \text{and} \quad c = \mathbf{\Delta}\left[(1-\alpha)\kappa + \alpha b\right] + \alpha c.$$

[9] The assumption $N \neq \varnothing$ avoids the trivial case that the fundamentals are irrelevant for equilibrium.

[10] A dot between two vectors denotes inner product.

Equivalently $a = \kappa_0$, $b_n = \kappa_n(1-\alpha)(1-\delta_n)/[1-\alpha(1-\delta_n)]$, and $c_n = \kappa_n\delta_n/[1-\alpha(1-\delta_n)]$, $n \in \{1, ..., N\}$. Note that $b_n + c_n = \kappa_n$ always; $b_n = c_n = 0$ whenever $\kappa_n = 0$; and $b_n \in (0, \kappa_n)$ and $c_n \in (0, \kappa_n)$ otherwise. Letting $\gamma_n \equiv c_n/\kappa_n \in (0,1)$ for any $n \in \mathcal{N}$ gives (5)-(6). Clearly, this is the unique linear equilibrium. Furthermore, since best responses are linear in $\mathbb{E}[\theta|x,y]$ and $\mathbb{E}[K|x,y]$, there do not exist equilibria other than this one. This last part follows from the same argument as in Morris and Shin (2002); our payoffs are more general but the structure of beliefs and best responses is essentially the same. *QED* ■

Condition (4) has a simple interpretation: an agent's best response is an affine combination of his expectation of some given "target" and his expectation of aggregate activity. The target is simply the complete-information equilibrium. The slope of best responses with respect to aggregate activity, α, is what we identify with the *equilibrium degree of coordination.*

As evident in condition (6), the sensitivity of the equilibrium allocation to private and public information depends, not only on the relative precision of the two, but also on the degree of coordination.When $\alpha = 0$, the weights on signals x_n and z_n are simply the Bayesian weights, so that $\gamma_n = \delta_n$ and (5) reduces to

$$k(x,y) = \mathbb{E}[\kappa(\theta)|x,y].$$

That is, when $\alpha = 0$, the incomplete-information equilibrium strategy is simply the best predictor of the complete-information equilibrium strategy. When, instead, $\alpha \neq 0$, equilibrium behavior is tilted towards public or private information depending on whether agents' actions are strategic complements or substitutes: $\gamma_n > \delta_n$ if $\alpha > 0$ and $\gamma_n < \delta_n$ if $\alpha < 0$.

The term $[\alpha\delta_n(1-\delta_n)]/[1-\alpha(1-\delta_n)]$ in condition (6) thus measures the excess sensitivity of equilibrium allocations to public information as compared to the case where there are no complementarities. This term is increasing in α : stronger complementarity leads to a higher relative sensitivity to public information. This is a direct implication of the fact that, in equilibrium, public information is a relatively better predictor of aggregate behavior than private information. In other words, public information has also a coordinating role.

If information were complete ($\sigma_n = 0$ for all n, or at least for all $n \in \mathcal{N}$), all agents would choose $k = K = \kappa$. Incomplete information affects equilibrium behavior in two ways. First, common noise generates *(non-fundamental) volatility*, that is, variation in aggregate activity around the complete-information level. Second, idiosyncratic noise generates *dispersion*, that is, variation in the cross-section of the population. The first is measured by $Var(K-\kappa)$, the second by $Var(k-K)$. Their dependence on the degree of coordination and the information structure is characterized below.[11]

[11] In the following, whenever we say "volatility" we mean volatility of aggregate activity around its complete-information counterpart.

Proposition 2 ***(i)*** *Volatility, $Var(K-\kappa)$, necessarily increases with α and σ_n, and increases with δ_n if and only if $\alpha < 0$ or $\delta_n < \frac{1-\alpha}{\alpha}$. Moreover, the impact of noise on volatility increases with α (i.e., $\frac{\partial^2 Var(K-\kappa)}{\partial\sigma_n\partial\alpha} > 0$).*

(ii) *Dispersion, $Var(k-K)$, necessarily decreases with α and δ_n and increases with σ_n. Moreover, the impact of noise on dispersion decreases with α (i.e., $\frac{\partial^2 Var(k-K)}{\partial\sigma_n\partial\alpha} < 0$).*

Higher complementarity thus mitigates the impact of noise on dispersion, and obtains a better alignment of individual choices; but it amplifies aggregate volatility. Higher transparency also reduces dispersion possibly at the expense of higher volatility. Higher accuracy, on the other hand, reduces both volatility and dispersion. We will examine in more detail the welfare effects of information later. In the next section, we turn to the characterization of the efficient use of information and show how this relates to the socially optimal degree of coordination.

4 Efficient use of information

The property that complementarity generates high sensitivity to common noise, and thereby amplifies volatility, is interesting on its own. But this is only a *positive* property. To address the *normative* question of whether these effects are socially undesirable, one needs to understand what is the efficient use of information. We define *efficient allocations* as those that maximize ex-ante utility among the ones that are measurable in the agents' decentralized information.

Definition 2 *An efficient allocation is a function $k : \mathbb{R}^{2N} \to \mathbb{R}$ that maximizes ex-ante utility*

$$\mathbb{E}u = \int_{(\theta,\varepsilon)} \int_x U(k(x,y), K(\theta,\varepsilon), \theta) dP(x|\theta,\varepsilon) dP(\theta,\varepsilon)$$

subject to

$$K(\theta,\varepsilon) = \int_x k\,(x,y)\, dP(x|\theta,\varepsilon), \text{ for all } (\theta,\varepsilon),$$

where $P(\theta,\varepsilon)$ stands for the c.d.f. of the joint distribution of (θ,ε) and $P(x|\theta,\varepsilon)$ for the conditional distribution of x given θ and ε.

We believe that this notion of efficiency is appropriate for the purposes of this paper. The allocation defined above is the solution to the "team problem" where agents choose a strategy cooperatively and commit to it. It thus answers exactly the question of interest for this paper, namely how allocations and welfare would change if agents were to internalize their payoff interdependencies and appropriately adjust their use of available information.[12] What is more, as we

[12] Our efficiency concept is the same as in Radner (1962) or Vives (1988) and shares with Hayek (1945) the idea that information is disperse and can not be communicated to a "center". Clearly, this is different from efficiency concepts that assume costless communication and focus on incentive constraints (e.g., Mirrlees, 1971; Holmstrom and Myerson, 1983).

will see in Section 5, it is precisely this notion of efficiency that helps understand the social value of information in equilibrium.

We start by deriving a necessary and sufficient condition for efficient allocations.

Lemma 1 *An allocation* $k : \mathbb{R}^{2N} \to \mathbb{R}$ *is efficient if and only if, for almost all* (x, y),

$$\mathbb{E}[\ U_k(k(x,y), K, \theta) + U_K(K, K, \theta) \mid x, y\ ] \ = \ 0, \tag{8}$$

where $K(\theta, \varepsilon) = \mathbb{E}[\ k(x,y) \mid \theta, \varepsilon\ ]$ *for all* (θ, ε).

This result has a simple interpretation. The first-best allocation, which corresponds to the case where θ is commonly known and is henceforth denoted by $\kappa^*(\theta)$, maximizes $W(K, \theta) \equiv U(K, K, \theta)$. It thus solves the the first-order condition $W_K(K, \theta) = 0$, or equivalently $U_k(K, K, \theta) + U_K(K, K, \theta) = 0$.[13] The incomplete-information counterpart of this condition is (8).

We can then expand this condition to characterize the efficient allocation under incomplete information in a similar fashion as with equilibrium.

Proposition 3 *Let* $\kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta_1 + ... + \kappa_N^*\theta_N$ *denote the first-best allocation,* $\mathcal{N}^* \equiv \{n \in \bar{\mathcal{N}} : \kappa_n^* \neq 0\} \neq \varnothing$, *and*

$$\alpha^* \equiv \frac{2U_{kK} + U_{KK}}{|U_{kk}|} = 2\alpha + \frac{U_{KK}}{|U_{kk}|}. \tag{9}$$

(**i**) *An allocation* $k : \mathbb{R}^{2N} \to \mathbb{R}$ *is efficient if and only if, for almost all* (x, y),

$$k(x,y) \ = \ \mathbb{E}[\ (1-\alpha^*)\kappa^* + \alpha^* K \mid x, y\ ] \tag{10}$$

where $K(\theta, \varepsilon) = \mathbb{E}[\ k(x,y) \mid \theta, \varepsilon\ ]$ *for all* (θ, ε).

(**ii**) *The efficient allocation exists, is essentially unique, and is given by*

$$k(x,y) = \kappa_0^* + \sum_{n \in \mathcal{N}^*} \kappa_n^* \left[(1-\gamma_n^*)x_n + \gamma_n^* z_n\right], \tag{11}$$

$$\gamma_n^* = \delta_n + \frac{\alpha^*\delta_n(1-\delta_n)}{1-\alpha^*(1-\delta_n)} \qquad \text{for all } n \in \mathcal{N}^*. \tag{12}$$

Proof. *Part* (*i*). Since U is quadratic, condition (8) can be rewritten as

$$\mathbb{E}[\ U_k(\kappa^*, \kappa^*, \theta) + U_{kk} \cdot (k(x,y) - \kappa^*) + U_{kK} \cdot (K - \kappa^*) + \\ + U_K(\kappa^*, \kappa^*, \theta) + (U_{kK} + U_{KK}) \cdot (K - \kappa^*) \mid x, y\ ] \ = \ 0.$$

Using $W_K(\kappa^*, \theta) = U_k(\kappa^*, \kappa^*, \theta) + U_K(\kappa^*, \kappa^*, \theta) = 0$, the above reduces to

$$\mathbb{E}[\ U_{kk}\,(k(x,y) - \kappa^*) + (2U_{kK} + U_{KK})(K - \kappa^*) \mid x, y\ ] = 0,$$

[13] Since U and hence W is quadratic, $\kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta_1 + ... + \kappa_N^*\theta_N$, where $\kappa_0^* = -W_K(0,0)/W_{KK}$ and $\kappa_n^* = -W_{K\theta_n}/W_{KK}$, $n \in \bar{\mathcal{N}}$. It follows that $\kappa_n^* \neq 0$ if and only if $W_{K\theta_n} \equiv U_{k\theta_n} + U_{K\theta_n} \neq 0$.

which together with $W_{KK} = U_{kk} + 2U_{kK} + U_{KK}$ gives (10).

Part (ii). This follows from the same steps as in the proof of Proposition (1) replacing α with α^* and $\kappa(\cdot)$ with $\kappa^*(\cdot)$. *QED* ■

In equilibrium, each agent's action was an affine combination of his expectation of κ, the complete-information equilibrium action, and of his expectation of aggregate activity. The same is true here for the efficient allocation if we replace κ with κ^*, the first-best action, and α with α^*. In this sense, condition (10) is the analogue for efficiency of the best response for equilibrium. This idea is formalized by the following.

Proposition 4 *Given an economy* $\mathbf{e} = (U; \sigma, \delta, \mu, \sigma_\theta) \in \mathcal{E}$, *let* $\mathcal{U}(\mathbf{e})$ *be the set of functions* U' *such that, if agents perceive their payoffs to be* U' *rather than* U, *the equilibrium coincides with the efficient allocation for* $\mathbf{e}$.

(**i**) *For every* $\mathbf{e}$, $\mathcal{U}(\mathbf{e})$ *is non-empty.*

(**ii**) *For every* $\mathbf{e}$, $U' \in \mathcal{U}(\mathbf{e})$ *only if* $\alpha' \equiv -U'_{kK}/U'_{kk} = \alpha^*$.

Part (i) states that the efficient allocation can be represented as the equilibrium of a fictitious game where individual incentives coincide with social incentives in the actual economy.[14] Part (ii), on the other hand, explains why we identify α^* with the *optimal degree of coordination*: α^* describes the level of complementarity that agents should perceive if the efficient allocation were to obtain as an equilibrium outcome, that is, if all externalities were to be internalized.

The counterpart of optimal coordination is the efficient use of information: by condition (12), the higher the optimal degree of coordination, the higher the sensitivity of efficient allocations to public information.

Corollary 1 *The sensitivity of the equilibrium allocations to public noise is inefficiently high if and only if the equilibrium degree of coordination is higher than the optimal one, which in turn is the case if and only if the complementarity is low enough relative to second-order non-strategic effects:*

$$\forall n \in \mathcal{N} \cap \mathcal{N}^*, \quad \gamma_n \geq \gamma_n^* \iff \alpha \geq \alpha^* \iff U_{kK} \leq -U_{KK}. \tag{13}$$

Proposition 3 and Corollary 1 show how the efficient use of information depends on the primitives of the environment and how it compares to the equilibrium one. As with equilibrium, the optimal degree of coordination is increasing in U_{kK}, the level of complementarity. But unlike equilibrium, the optimal degree of coordination depends also on U_{KK}, a second-order external effect that does not affect private incentives. In the absence of such an effect, $\alpha^* = 2\alpha$: the optimal degree

[14] In some cases, this may also suggest a way to implement the efficient allocation. For example, the government may be able to use taxes and subsidies to fashion individual best-responses.

of coordination is higher (in absolute value) than the equilibrium one, reflecting the internalization of the externality associated with the complementarity. In this case, the heightened sensitivity of equilibrium allocations to noise is anything but excessive.

To understand better the forces behind the determination of the optimal degree of coordination, an alternative representation is useful. Welfare (ex-ante utility) at the efficient allocation can be expressed as $\mathbb{E}u = \mathbb{E}W(\kappa^*, \theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \frac{|W_{KK}|}{2} Var(K - \kappa^*) + \frac{|U_{kk}|}{2} Var(k - K). \tag{14}$$

Note that $\mathbb{E}W(\kappa^*, \theta)$ is welfare in the first-best allocation, while $\mathcal{L}^*$ captures the welfare losses associated with incomplete information, namely those due to aggregate volatility and cross-sectional dispersion.[15]

That volatility and dispersion generate welfare losses follows directly from concavity of preferences. Naturally, the weight on volatility is given by W_{KK}, the curvature of welfare with respect to aggregate activity, while the weight on dispersion is given by U_{kk}, the curvature of utility with respect to individual activity. Note that $W_{KK} = U_{kk} + 2U_{kK} + U_{KK}$. When there are no strategic and second-order external effects (in the sense that $U_{kK} = U_{KK} = 0$), aggregate welfare inherits the curvature of individual utility ($W_{KK} = U_{kk}$), so that volatility and dispersion contribute equally to welfare losses. Complementarity ($U_{kK} > 0$) alleviates aggregate concavity by offsetting the diminishing returns faced at the individual level, and therefore lowers social aversion to volatility. The converse is true for substitutability ($U_{kK} < 0$) or external concavity ($U_{KK} < 0$).

Volatility is generated by noise in public information, dispersion by noise in private information. Increasing the relative sensitivity of allocations to public information—equivalently, raising the degree of coordination—dampens dispersion at the expense of higher volatility. The efficient use of information reflects the resolution of this trade-off.

Corollary 2 *The optimal degree of coordination equals one minus the weight that welfare assigns to volatility relative to dispersion:*

$$\alpha^* = 1 - \frac{W_{KK}}{U_{kk}}. \tag{15}$$

Extension. In some applications of interest, cross-sectional dispersion has a direct external effect on individual utility. For example, price dispersion has a negative effect on individual utility in New-Keynesian monetary models (see Hellwig, 2005). In the beauty contest of Morris and Shin (2002), on the other hand, dispersion has positive external effect (see Section 6.2).

We can easily accommodate such an effect—and we do so for the rest of the paper—provided that dispersion enters linearly in the utility function:

$$u = U(k, K, \theta, \sigma_k^2),$$

[15] Condition (14) follows from a Taylor expansion around $k = K = \kappa^*(\theta)$; see the Appendix.

where $\sigma_k^2 \equiv \int (k-K)^2 d\Psi(k)$ and $U_{\sigma_k^2} \in \mathbb{R}$. In analogy to $W_{KK} < 0$, we impose $U_{kk} + 2U_{\sigma_k^2} < 0$; this is necessary and sufficient for higher dispersion to reduce welfare. Then all our results go through once we replace the welfare weight on dispersion with $U_{kk} + 2U_{\sigma_k^2}$. In particular, welfare is now given by $\mathbb{E}u = \mathbb{E}W(\kappa^*, \theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \frac{|W_{KK}|}{2} Var(K - \kappa^*) + \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} Var(k - K). \tag{16}$$

The optimal degree of coordination is

$$\alpha^* = 1 - \frac{W_{KK}}{U_{kk} + 2U_{\sigma_k^2}} = 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{kk} + 2U_{\sigma_k^2}}.$$

Finally, condition (13) becomes

$$\forall n \in \mathcal{N} \cap \mathcal{N}^*, \quad \gamma_n^* \geq \gamma_n \quad \Longleftrightarrow \quad \alpha^* \geq \alpha \quad \Longleftrightarrow \quad U_{kK} \geq -U_{KK} + 2U_{\sigma_k^2}.$$

Note that α^* is increasing in U_{KK} and decreasing in $U_{\sigma_k^2}$. This is intuitive. A higher U_{KK} decreases the social cost of volatility, while a higher $U_{\sigma_k^2}$ decreases the social cost of dispersion. Both these effects are external and non-strategic—they affect the social value of coordination without affecting private incentives. The former contributes to a higher optimal degree of coordination, the latter to a lower.

Efficient economies. We conclude this section with necessary and sufficient conditions for the equilibrium to be efficient under incomplete information.

Proposition 5 *The equilibrium is efficient if and only if*

$$\kappa(\cdot) = \kappa^*(\cdot) \quad \textit{and} \quad \alpha = \alpha^*.$$

This in turn is the case if and only if

$$U_{kK} + U_{KK} - 2U_{\sigma_k^2} = 0, \quad U_K(0,0,0) = \frac{U_{kK}}{U_{kk}} U_k(0,0,0), \quad \textit{and} \quad U_{K\theta_n} = \frac{U_{kK}}{U_{kk}} U_{k\theta_n} \ \forall n.$$

The condition $\kappa(\cdot) = \kappa^*(\cdot)$ means that the equilibrium is efficient under *complete* information. But efficiency under complete information alone does not guarantee efficiency under *incomplete* information. What is also needed is efficiency in the use of information which obtains when in addition the equilibrium and the optimal degrees of coordination coincide. Clearly, strategic effects alone do not imply inefficiency: the equilibrium can be efficient despite $\alpha \neq 0$.[16]

[16] Note that α and α^* depend on U but not on (σ, δ). This explains why efficiency can be checked on the basis of the payoff structure alone, as shown in Proposition 5 above.

5 Social value of information

We now examine the comparative statics of equilibrium welfare with respect to the information structure, starting with economies where the equilibrium is efficient. This provides a useful benchmark, not only because efficiency is always an excellent starting point, but also because in our class of economies efficiency implies a clear relation between the form of strategic interaction and the social value of information.

Proposition 6 *Suppose the equilibrium is efficient. For any $n \in \mathcal{N}$, welfare necessarily decreases with σ_n, and increases [decreases] with δ_n if and only if agents' actions are strategic complements [substitutes].*

As highlighted in the previous section, the impact of information on welfare at the efficient allocation is summarized in the impact of noise on volatility and dispersion (see condition (16)). An increase in σ_n for given δ_n raises both volatility and dispersion and therefore necessarily reduces welfare. An increase in δ_n for given σ_n, on the other hand, is equivalent to a reduction in dispersion, possibly at the expenses of volatility. Such a substitution is welfare-improving if and only if the social cost of dispersion is higher than that of volatility, which is the case if and only if $\alpha^*(= \alpha)$ is positive.

Note that, when the equilibrium allocation is efficient, it maximizes ex-ante expected utility. That accuracy is beneficial can then be obtained also an implication of Blackwell's theorem. Indeed, the same observation implies that, when the equilibrium is efficient, welfare necessarily decreases with either σ_{x_n} or σ_{y_n}, for any $n \in \mathcal{N}$.

Corollary 3 *Suppose the equilibrium is efficient. Welfare necessarily increases with the precision of either private or public information.*

In economies where the equilibrium is inefficient, the welfare effects of information are more complicated for two reasons. First, the equilibrium degree of coordination need not coincide with the optimal one ($\alpha \neq \alpha^*$), thus introducing inefficiency in the way the trade-off between volatility and dispersion is resolved. Second, the equilibrium level of activity may differ from the socially optimal one even under complete information ($\kappa \neq \kappa^*$), thus introducing first-order welfare losses in addition to those associated with volatility and dispersion.

Consider first the role of $\alpha \neq \alpha^*$, maintaining for a moment $\kappa = \kappa^*$. The welfare losses associated with incomplete information continue to be the weighted sum of volatility and dispersion, as in (16).[17] For given α, a higher α^* means a lower relative weight on volatility and hence a lower

[17] As obvious from the derivation of (14) in the Appendix, (14) and similarly (16) extend to $\alpha \neq \alpha^*$ as long as $\kappa = \kappa^*$. This can also be seen from (17) below noting that $W_K(\kappa, \theta) = 0$ when $\kappa = \kappa^*$.

cost associated with an increase in δ_n.[18] It follows that, relatively to the efficiency benchmark (Proposition 6), inefficiently low coordination ($\alpha < \alpha^*$) increases the social value of transparency, while inefficiently high coordination ($\alpha > \alpha^*$) reduces it. On the other hand, the possibility that $\alpha \neq \alpha^*$ does not affect the value of accuracy: a lower σ_n reduces both volatility and dispersion and therefore necessarily increases welfare.

Consider next the role of $\kappa \neq \kappa^*$, in which case the equilibrium is inefficient even under complete information. In equilibrium, welfare is given by $\mathbb{E}u = \mathbb{E}W(\kappa, \theta) - \mathcal{L}$, where

$$\mathcal{L} = -Cov\left(K - \kappa, W_K(\kappa, \theta)\right) + \frac{|W_{KK}|}{2} \cdot Var(K - \kappa) + \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \cdot Var(k - K) \qquad (17)$$

are the welfare losses due to incomplete information.[19] The last two terms in $\mathcal{L}$ are the familiar welfare losses associated with volatility and dispersion (second-order effects). The covariance term, on the other hand, captures a novel first-order effect. When the complete-information equilibrium is efficient ($\kappa = \kappa^*$ and hence $W_K(\kappa, \theta) = 0$), the covariance term is zero; this is merely an implication of the fact that small deviations around a maximum have zero first-order effects. But when the complete-information equilibrium is inefficient due to externalities ($W_K(\kappa, \theta) \neq 0$), the covariance term contributes to a welfare loss or gain; this is because a positive [negative] correlation between $K - \kappa$, the "error" in aggregate activity due to incomplete information, and $W_K(\kappa, \theta)$, the social return to activity, mitigates [exacerbates] the first-order losses associated with externalities.

As shown in the Appendix (Proof of Proposition 7), this covariance term can be expressed as

$$Cov\left(K - \kappa, W_K(\kappa, \theta)\right) \;=\; |W_{KK}| Cov\left(K - \kappa, \kappa^* - \kappa\right) \;=\; |W_{KK}| \sum_{n \in \mathcal{N}} \phi_n v_n \qquad (18)$$

where, for all $n \in \mathcal{N}$,

$$\begin{aligned} \phi_n &\equiv \frac{\kappa_n^* - \kappa_n}{\kappa_n} = \frac{Cov\left(\, \kappa^* - \kappa, \kappa \mid \theta_{-n} \,\right)}{Var\left(\, \kappa \mid \theta_{-n} \,\right)}, \\ v_n &\equiv -\frac{1}{1 - \alpha + \alpha\delta_n} \kappa_n^2 \sigma_n^2 = Cov\left(\, K - \kappa, \kappa \mid \theta_{-n} \,\right), \end{aligned}$$

with θ_{-n} standing for $(\theta_j)_{j \neq n}$. The coefficients v_n capture the covariation between $K - \kappa$, the aggregate "error" due to incomplete information, and κ, the complete-information equilibrium, while the coefficients ϕ_n capture the covariation between the latter and $\kappa^* - \kappa$, the efficiency gap under complete information.

A lower σ_n always implies a v_n closer to zero, since less noise brings K closer to κ for any given θ. But how this affects welfare depends on whether getting K closer to κ also means getting K closer to κ^*; this in turn depends on the correlation between complete-information equilibrium and

[18] Recall from Proposition 2 that volatility increases with δ_n if and only if $\alpha < 0$ or $\delta < (1 - \alpha)/\alpha$, which we assume here in order to simplify the discussion. In the alternative case, welfare necessarily increases with δ_n (when $\kappa = \kappa^*$).

[19] Condition (17) follows from a Taylor expansion around $K = \kappa(\theta)$; see Appendix.

first best. Intuitively, less noise brings K closer to κ^* when $\phi_n > 0$ but further away when $\phi_n < 0$. As a result, the welfare contribution of a lower σ_n through the covariance term in (17) is positive when $\phi_n > 0$ but negative when $\phi_n < 0$. Combining this with the effect of σ_n on volatility and dispersion, we conclude that higher accuracy necessarily increases welfare when $\phi_n > 0$ (i.e., when the correlation between equilibrium and first best is positive) but can reduce welfare when ϕ_n is sufficiently negative.

The impact of δ_n on v_n, on the other hand, depends on the sign of the complementarity: higher transparency increases the covariance between K and κ when $\alpha > 0$ but decreases it when $\alpha < 0$. How this in turn affects welfare depends again on the sign of ϕ_n. Hence, as evident from (18), the sign of the effect of δ_n on first-order welfare losses depends on the sign of the product of α and ϕ_n. Combining this with the effects of δ_n on volatility and dispersion, and noting that the covariance term dominates for ϕ_n sufficiently away from zero, we conclude that ϕ_n sufficiently high [low] suffices for the welfare effect of δ to have the same [opposite] sign as α.

These insights are verified in the following complete characterization of the welfare effects of information.

Proposition 7 *There exist functions* $\underline{\phi}, \underline{\phi}', \bar{\phi}, \bar{\phi}' : (-1,1) \times (-\infty, 1) \rightarrow \mathbb{R}$, *with* $\underline{\phi} \leq \bar{\phi}$ *and* $\underline{\phi}' \leq \bar{\phi}' < 0$, *such that the following are true for any* $n \in \mathcal{N}$:

Strategic Independence. *When* $\alpha = 0$, *welfare increases* [*decreases*] *with* δ_n *for all* (σ_n, δ_n) *if and only if* $\alpha^* > 0$ [$\alpha^* < 0$].

Strategic Complementarity. *When* $\alpha \in (0,1)$, *welfare increases* [*decreases*] *with* δ_n *for all* (σ_n, δ_n) *if and only if* $\phi_n > \bar{\phi}(\alpha, \alpha^*)$ [$\phi_n < \underline{\phi}(\alpha, \alpha^*)$].

Strategic Substitutability. *When* $\alpha \in (-1,0)$, *welfare increases* [*decreases*] *with* δ_n *for all* (σ_n, δ_n) *if and only if* $\phi_n < \underline{\phi}(\alpha, \alpha^*)$ [$\phi_n > \bar{\phi}(\alpha, \alpha^*)$].

Accuracy. *Welfare decreases* [*increases*] *with* σ_n *for all* (σ_n, δ_n) *if and only if* $\phi > \bar{\phi}'(\alpha, \alpha^*)$ [$\phi < \underline{\phi}'(\alpha, \alpha^*)$]

The functions $\underline{\phi}, \underline{\phi}', \bar{\phi}, \bar{\phi}'$ *are invariant with respect to* $\mathcal{E}$ *and satisfy the following properties:* (*i*) $\underline{\phi} = \underline{\phi}' = \bar{\phi} = \bar{\phi}' = -\frac{1}{2}$ *when* $\alpha = \alpha^*$; (*ii*) *for* $\alpha \in (0,1)$, $\underline{\phi} < 0$ *if and only if* $\alpha > 1/2$ *or* $\alpha^* > -\alpha^2/(1-2\alpha)$, *while* $\bar{\phi} < 0$ *if and only if* $\alpha^* > \alpha^2$; (*iii*) *for* $\alpha \in (-1,0)$, $\underline{\phi} < 0$ *if and only if* $\alpha^* < \alpha^2$, *while* $\bar{\phi} < 0$ *if and only if* $\alpha^* < -\alpha^2/(1-2\alpha)$.

By Proposition 5, the equilibrium is efficient if and only if $\alpha = \alpha^*$ and $\kappa = \kappa^*$, in which case the welfare effects of information are given by 6. If the only inefficiency is either that $\kappa_0 \neq \kappa_0^*$ or that $\kappa_n^* \neq 0 (= \kappa_n)$ for some $n \notin \mathcal{N}$, then this inefficiency does not affect the comparative statics of equilibrium welfare with respect to (δ_n, σ_n) for $n \in \mathcal{N}$. That is, Proposition 6 continues to hold as long as $\alpha = \alpha^*$ and $\phi_n = 0$ for all $n \in \mathcal{N}$. Otherwise, Proposition 5 implies that the social value of information can still be understood as a function of α, α^*, and $(\phi_n)_{n \in \mathcal{N}}$.

The following sufficient conditions are then immediate for the case where $\kappa^* - \kappa$, the complete-information efficiency gap, is either constant or positively correlated with κ.

Corollary 4 *Suppose $\phi_n \geq 0$ for all $n \in \mathcal{N}$, in which case $Cov(\kappa^* - \kappa, \kappa) \geq 0$. Then, welfare always increases with the accuracy of information, whereas it increases with its transparency if $\alpha^* \geq \alpha > 0$, and decreases with it if $\alpha^* \leq \alpha < 0$.*

The following case is also interesting, as it contrasts with the Blackwell-like result we encountered earlier for efficient economies.

Corollary 5 *Suppose $\phi_n < -1/2$ and $\alpha = \alpha^* = 0$. Welfare decreases with the precision of either private or public information about θ_n.*

To recap, characterizing the efficient use of information proved to instrumental for understanding the social value of information. For cases where the equilibrium is efficient, Proposition 6 provided a sharp answer: the welfare effect of accuracy is then unambiguously positive and that of transparency is pinned down by the form of strategic interaction alone. For all other cases, Proposition 5 provides a complete taxonomy, which can be useful in applications.

6 Applications

In this section, we show how our results can guide welfare analysis in specific contexts of interest. For simplicity, most examples assume a single fundamental variable ($N = 1$ and $\theta = \theta_1$).

6.1 Investment complementarities

The canonical model of production externalities can be nested by interpreting k as investment and defining individual payoffs as follows:

$$U(k, K, \theta) = A(K, \theta)k - C(k), \tag{19}$$

where $A(K, \theta) = (1 - a)\theta + aK$ represents the private return to investment, with $a \in (0, 1/2)$ and $\theta \in \mathbb{R}$, and $C(k) = k^2/2$ the private cost of investment. Variants of this specification appear in Bryant (1983), Romer (1986), Matsuyama (1992), Acemoglu (1993), and Benhabib and Farmer (1994), as well as models of network externalities and spillovers in technology adoption.[20] The

[20] This is also the example we examined in Angeletos and Pavan (2004, Section 2), although there we computed welfare conditional on θ, thus omitting the effect of $\phi \neq 0$ on welfare losses.

important ingredient is that the private return to investment increases with the aggregate level of investment—the source of both complementarity and externality in this class of models.

The equilibrium level of investment under complete information is $\kappa(\theta) = \theta$, whereas the first best is $\kappa^*(\theta) = \frac{1-a}{1-2a}\theta$, and hence $\phi = \frac{a}{1-2a} > 0$. This is simply because the private return to investment is lower than the social one for all $\theta > 0$, and the more so the higher θ. Furthermore, there is a positive complementarity but no other second-order external effect: $U_{kK} = a > 0$ and $U_{KK} = U_{\sigma_k^2} = 0$ (and $U_{kk} = -1$). It follows that $\alpha = a > 0$ and $\alpha^* = 2\alpha > \alpha$. That is, in this economy the agents' private incentives to coordinate, and the consequent amplification of volatility featured in equilibrium, are anything but excessive. Finally, by Corollary 4 we immediately have that welfare here decreases with σ and increases with δ.[21]

Corollary 6 *In the investment example described above, coordination is inefficiently low and welfare unambiguously increases with both the accuracy and the transparency of information.*

Economies with frictions in financial markets—where complementarities emerge through collateral constraints, missing assets, or other types of market incompleteness—are often related to economies with investment complementarities like the one considered here. Although this is appropriate for many positive questions, it need not be so for normative purposes. As the examples we study in the next two sections highlight, the result here depends on the absence of certain second-order external effects and on a sufficiently strong correlation between equilibrium and first-best activity. Whether these properties are shared by mainstream incomplete-market models is an open question.

6.2 "Beauty contests" versus other Keynesian frictions

Keynes contended that financial markets often behave like "beauty contests" in the sense that traders try to forecast and outbid one another's forecasts, but this motive is (presumably) not warranted from a social perspective because it is due to some (unspecified) market imperfection. Capturing this idea with proper microfoundations is an open question, but one possible shortcut, following Morris and Shin (2002), is to define a *beauty-contest economy* as an economy in which $\alpha > 0 = \alpha^*$ and $\kappa(\cdot) = \kappa^*(\cdot)$. The first condition means that the private motive to coordinate is not warranted from a social perspective; the second means that the inefficiency of equilibrium vanishes as information becomes complete. By Proposition 7 we then have the following.

[21] Translating these results in terms of σ_x and σ_y, it is easy to show that welfare unambiguously increases with a reduction in either σ_x or σ_y. Hence, both public and private information are beneficial in this example. However, a higher α, by increasing the value of transparency, increases the welfare gain of public information and decreases that of private information.

Corollary 7 *In beauty-contest economies, welfare is increasing in accuracy but non-monotonic in transparency.*

The specific payoff structure assumed by Morris and Shin (2002) is given by

$$u_i = -(1-r)\cdot(k_i-\theta)^2 - r\cdot(L_i-\bar{L})$$

where $\theta \in \mathbb{R}$ is the underlying fundamental, $L_i = L(k_i) \equiv \int (k'-k_i)^2 d\Psi(k')$ is the mean square-distance of agent i's action from other agents' actions, $\bar{L} = \int L(k) d\Psi(k)$ is the cross-sectional mean of L_i, and $r \in (0,1)$.[22] This example is nested in our framework with[23]

$$U(k, K, \theta, \sigma_k^2) = -(1-r)\cdot(k-\theta)^2 - r\cdot(k-K)^2 + r\cdot\sigma_k^2.$$

It follows that $\kappa^*(\theta) = \kappa(\theta) = \theta$, $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{\sigma_k^2} = r$, and hence $\phi = 0$ and $\alpha = r > 0 = \alpha^*$.

Note how this example features two external effects that tilt the trade-off between volatility and dispersion in the opposite direction than the complementarity. In particular, $U_{KK} < 0$ increases the social cost of volatility, while $U_{\sigma_k^2} > 0$ decreases the social cost of dispersion. Both effects are non-strategic, in the sense that they do not affect private incentives, and both contribute to reducing the social value of coordination. In the specific example considered by Morris and Shin (2002), these effects perfectly offset the impact of the complementarity, so that the optimal level of coordination is zero—which explains why transparency, and thereby public information, can be welfare-reducing.

Keynesian frictions such as monopolistic competition or incomplete markets are in the heart of various macroeconomic complementarities (a.k.a. "multipliers" or "accelerators"). These frictions share with beauty-contest economies the idea that complementarity originates in some market imperfection. However, the normative properties of beauty contests need not be shared by other Keynesian frictions.

Consider, for example, monetary models with incomplete information (e.g., Woodford, 2002; Hellwig, 2005; Lorenzoni, 2005; Roca, 2005). In these models, complementarity emerges in monopolistic competition—a market friction, or a "real rigidity" as some authors have called it. However, imperfect substitutability across goods implies that cross-sectional dispersion in prices imposes a negative externality ($U_{\sigma_k^2} < 0$). This in turn contributes to a *higher* optimal degree of coordination—the opposite of what happens in the beauty-contest economy above. Hellwig (2005) provides an

[22] The first term in u_i captures the value of taking an action close to a fundamental "target" θ. The L_i term introduces a private value for taking an action close to others' actions, whereas the $\bar{L}$ term ensures that there is no social value in doing so. Indeed, aggregating across agents gives $w = -(1-r)\int(k-\theta)^2 d\Psi(k)$, so that, from a social perspective, it is as if utility were simply $u = -(k-\theta)^2$, in which case there is of course no social value to coordination.

[23] Note that $L_i = \int ((k'-K)-(k_i-K))^2 d\Psi(k') = (k_i-K)^2 + \sigma_k^2$, $\bar{L} = 2\cdot\sigma_k^2$, and $L_i - \bar{L} = (k_i-K)^2 - \sigma_k^2$.

excellent analysis of this class of models. He shows that the efficient sensitivity to public information is higher than the equilibrium one. Moreover, the business cycle is efficient under complete information (in the sense that the gap between first best and equilibrium does not vary with the business cycle). Translating these properties in our framework gives $\alpha^* > \alpha > 0$ and $\phi = 0$, in which case, by Corollary 4, welfare increases with both accuracy and transparency. This helps understand why, unlike in Morris and Shin (2002), public information is welfare improving in Hellwig (2005) and Roca (2005).

6.3 Inefficient fluctuations

The focus in the previous section was on how the complementarity and second-order effects tilt the trade-off between volatility and dispersion. We now turn focus to first-order effects. In particular, we consider economies where the efficiency gap $\kappa^* - \kappa$ covaries negatively with κ, that is, economies where recessions are inefficiently deep.

To isolate the impact of first-order effects ($\phi \neq 0$), we abstract from strategic and second-order external effects ($U_{kK} = U_{KK} = U_{\sigma_k^2} = 0$), so that $\alpha^* = \alpha = 0$. From Proposition 7 then $\phi < -1/2$ is necessary and sufficient for welfare to decrease with accuracy and be independent of transparency.

Corollary 8 *Suppose that $\alpha^* = \alpha = 0$ and that equilibrium fluctuations are sufficiently inefficient in the sense that $Cov(\kappa, \kappa^*) < \frac{1}{2}Var(\kappa)$. Then welfare decreases with the precision of either private or public information.*

As an example, consider an economy where $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$ and where agents engage in an investment activity without complementarity but for which private and social returns differ:

$$U(k, K, \theta, \sigma_k^2) = \theta_1 k - k^2/2 + \lambda(\theta_2 - \theta_1)K,$$

for some $\lambda \in (0, 1)$. The private return to investment is θ_1, while the social return is $\theta_2 \neq \theta_1$. It follows that $\kappa(\theta) = \theta_1$, while $\kappa^*(\theta) = (1 - \lambda)\theta_1 + \lambda\theta_2$, and hence $\phi_1 = -\lambda$. If $\lambda < 1/2$, meaning that the discrepancy between private and social returns is small enough, then $\phi_1 > -1/2$ and welfare increases with either private or public information about θ_1. But if $\lambda > 1/2$, meaning that the correlation between private and social returns is close to zero, then $\phi < -1/2$ and welfare *decreases* with either private or public information about θ_1. A special case of this is when $\lambda = 1$ and $\sigma_{\theta_2} = 0$, so that κ^* is constant and the entire fluctuation in investment is inefficient.[24]

The recent debate on the merits of transparency in central bank communication has focused on the role of complementarities in new-Keynesian models (e.g., Morris and Shin, 2002; Svensson,

[24] If we maintain that the correlation between κ and κ^* is low enough but let $\alpha^* = \alpha > 0$, then welfare continues to decrease with accuracy but now it also decreases with transparency—which strengthens particularly the case against public information.

2005; Woodford, 2005; Hellwig, 2005; Roca, 2005). The example here suggests that this debate might be missing a critical element—the potential inefficiency of equilibrium fluctuations under complete information.

For example, we conjecture that the result in Hellwig (2005) and Roca (2005) that public information has a positive effect on welfare relies on the property that, under complete information, the business cycle is *efficient* in these models. In standard new-Keynesian models, the monopolistic mark-up introduces an efficiency gap. But as long as this mark-up is constant over the business cycle—which is the case in the models of Hellwig and Roca—the efficiency gap also remains constant. If, instead, business cycles are driven primarily by shocks in mark-ups or "labor wedges," it seems possible that providing markets with information that helps predict these shocks can reduce welfare. This is an interesting question that we leave open for future research.

6.4 Efficient competitive economies

The examples considered so far feature either positive complementarity or some form of inefficiency. We now turn to competitive economies where agents' choices are strategic substitutes and where the equilibrium is efficient under both complete and incomplete information.[25]

There is a continuum of households, each consisting of a consumer and a producer, and two commodities. Let q_{1i} and q_{2i} denote the respective quantities purchased by consumer i (the consumer living in household i). His preferences are given by

$$u_i = v(q_{1i}, \theta) + q_{2i}, \tag{20}$$

where $v(q, \theta) = \theta q - bq^2/2$, $\theta \in \mathbb{R}$, and $b > 0$. His budget is

$$pq_{1i} + q_{2i} = e + \pi_i, \tag{21}$$

where p is the price of good 1 relative to good 2, e is an exogenous endowment of good 2, and π_i are the profits of producer i (the producer living in household i), which are also denominated in terms of good 2. Profits in turn are given by

$$\pi_i = pk_i - C(k_i) \tag{22}$$

where k_i denotes the quantity of good 1 produced by household i and $C(k)$ the cost in terms of good 2, with $C(k) = k^2/2$.[26]

[25] We constructed this example of quadratic competitive economies independently but then found out that Vives (1988) had proved efficiency of equilibria for essentially the same example long before us. Hence, for this particular case, only the welfare effects of information are novel here.

[26] Implicit behind this cost function is a quadratic production frontier. The resource constraints are therefore given by $\int_i q_{1i} = \int_i k_i$ and $\int_i q_{2i} = e - \frac{1}{2}\int_i k_i^2$ for good 1 and 2, respectively.

The random variable θ represents a shock in the relative demand for the two goods. Exchange and consumption take place once θ has become common knowledge. On the contrary, production takes place at an earlier stage, when information is still incomplete.

Consumer i chooses (q_{1i}, q_{2i}) so as to maximize (20) subject to (21), which gives $p = \theta - bq_{1i}$. Clearly, all households consume the same quantity of good 1, which together with market clearing gives $q_{1i} = K$ for all i and $p = \theta - bK$, where $K = \int k d\Psi(k)$. It follows that i's utility can be restated as $u_i = v(K, \theta) - pK + e + \pi_i = bK^2/2 + e + \pi_i$, where $\pi_i = pk_i - C(k_i) = (\theta - bK)k_i - k_i^2/2$. This example is thus nested in our model with

$$U(k, K, \theta, \sigma_k^2) = (\theta - bK)k - k^2/2 + bK^2/2 + e,$$

in which case $\kappa^*(\theta) = \kappa(\theta) = \theta/(1+b)$, $U_{kk} = -1$, $U_{kK} = -b$, $U_{KK} = b$, $U_{\sigma_k^2} = 0$, and therefore $\phi = 0$ and $\alpha^* = \alpha = -b < 0$.

That the complete-information equilibrium is efficient $(\kappa = \kappa^*)$ should not be a surprise. Under complete information, the economy is merely an example of a complete-markets competitive economy in which the first welfare theorem applies. What is interesting is that the equilibrium remains efficient under incomplete information and despite the absence of ex-ante complete markets. This is because the strategic substitutability perceived by the agents coincides with the one that the planner would have liked them to perceive $(\alpha^* = \alpha)$. The following is then a direct implication of Proposition 6.

Corollary 9 *In the competitive economy described above, the equilibrium is efficient and welfare unambiguously increases with the accuracy of information but decreases with its transparency.*

If we interpret "transparent" central bank disclosures as information that admits a single common interpretation and "ambiguous" disclosures as information that admits multiple idiosyncratic interpretations, then the result above makes a case for constructive ambiguity. This is reminiscent of the related result for beauty-contest economies (Section 6.2), but is different. Whereas the result there is driven by inefficiently high coordination ($\alpha^* = 0 < \alpha$), here it is due to efficient substitutability ($\alpha^* = \alpha < 0$). It is interesting that a case for constructive ambiguity—if interpreted as above—can be made even for economies where the use of information is efficient.[27]

6.5 Cournot versus Bertrand

We next turn to two IO applications with a large number of firms: a Cournot-like game, where firms compete in quantities and actions are strategic substitutes; and a Bertrand-like game, where firms compete in prices and actions are strategic complements.

[27] Of course, if more "transparency" central-bank communication means a decrease in σ_y for given σ_x, then this can not reduce welfare.

Consider first Cournot. The demand faced by a firm is given by $p = a_0 + a_1\theta - a_2 q - a_3 Q$, where p denotes the price at which the firm sells each unit of its product, q the quantity it produces, Q the average quantity in the market, and θ an exogenous demand shifter $(a_0, a_1, a_2, a_3 > 0)$. Individual profits are $\pi = pq - C(q)$, with a quadratic cost function, $C(q) = c_1 q + c_2 q^2$ $(c_1, c_2 > 0)$.

This is nested in our framework with $k \equiv q$, $K \equiv Q$ (actions are quantities), and

$$U\left(k, K, \theta, \sigma_k^2\right) = (a_0 - c_1 + a_1\theta - a_3 K)\, k - (a_2 + c_2)k^2,$$

It follows that $U_{kK} = -a_3 < 0$, $U_{kk} = -a_2 - c_2 < 0$, and $U_{KK} = U_{\sigma_k^2} = 0$, so that the equilibrium degree of coordination is $\alpha = -\frac{a_3}{a_2+c_2} < 0$, while the optimal one is $\alpha^* = 2\alpha < a < 0$ (as usual, we restrict $\alpha > -1$). Moreover, under complete information the joint profit-maximizing (i.e., monopoly) quantity is $\kappa^*(\theta) = \frac{a_0 - c_1 + a_1\theta}{2(a_2+c_2+a_3)}$, while the equilibrium (i.e., Cournot) quantity is $\kappa(\theta) = \frac{a_0 - c_1 + a_1\theta}{2(a_2+c_2)+a_3}$, so that $\phi = \frac{\alpha}{2(1-\alpha)} < 0$. That is, both the monopoly and the Cournot quantity increase with the demand intercept, but the monopoly one less so than the Cournot one. Using $\alpha^* = 2\alpha < \alpha = 0$ with the formulas for the bounds $\bar{\phi}'$ and $\underline{\phi}$ derived in the proof of Proposition 7 (see Appendix), we have that $\bar{\phi}' = \frac{-1+\alpha}{2(1-2\alpha)} > \frac{-2+\alpha}{2(1-2\alpha)} = \bar{\phi}$. Together with $\phi = \frac{\alpha}{2(1-\alpha)}$, this ensures that $\phi > \bar{\phi}'$ and $\phi > \bar{\phi}$. By Proposition 7 we then have the following.

Corollary 10 *In the Cournot game described above, the equilibrium degree of coordination is inefficiently high; expected profits increase with the accuracy of information but decrease with its transparency.*

Next, consider Bertrand. Demand is now given by $q = b_0 + b_1\theta' - b_2 p + b_3 P$, where q denotes the quantity sold by the firm, p the price the firm sets, P the average price in the market, and θ' again an exogenous demand shifter $(b_0, b_1, b_2, b_3 > 0)$; we assume $b_3 < b_2$ so that an equal increase in p and P reduces q. Individual profits are $\pi = pq - C(q)$, with $C(q) = c_1 q + c_2 q^2$ $(c_1, c_2 > 0)$.

This is nested in our framework with $k \equiv p - c_1$, $K \equiv P - c_1$ (actions are now prices), and

$$U\left(k, K, \theta, \sigma_k^2\right) = b_2[(\theta - k + bK)\, k - c\,(\theta - k + bK)^2],$$

where $\theta \equiv b_0/b_2 + b_1/b_2\theta' - c_1(1-b)$, $b \equiv b_3/b_2 \in (0,1)$, and $c \equiv c_2 b_2 > 0$; without loss of generality, we let $b_2 = 1$. It follows that $U_{kK} = (1+2c)\, b > 0$, $U_{kk} = -2(1+c) < 0$, $U_{\sigma_k^2} = 0$, and $U_{KK} = -2cb^2$, so that the equilibrium degree of coordination is $\alpha = \frac{1+2c}{2+2c} b \in (0,1)$ and the optimal one is $\alpha^* = \frac{2+2c(2-b)}{2+2c} b > \alpha$ (as usual, we restrict $\alpha^* < 1$). Moreover, under complete information the monopoly price is $\kappa^*(\theta) = \frac{1+2c(1-b)}{2(1-b)[1+c(1-b)]}\theta$, while the equilibrium price is $\kappa(\theta) = \frac{1+2c}{1+(1+2c)(1-b)}\theta$, so that $\phi > 0$. That is, the Betrand price reacts too little to θ as compared to the monopoly price. The following is then immediate by Corollary 4.

Corollary 11 *In the Bertrand game described above, the equilibrium degree of coordination is inefficiently low; expected profits increase with both the accuracy and the transparency of information.*

These results are related to Raith (1996), who examines the impact of information-sharing in Cournot and Bertrand oligopolies with a finite number of firms. The payoff structure in that paper is nested in the ones examined in this section; the difference is that Raith (1996) restricts $U_{KK} = 0$ and hence $\alpha^* = 2\alpha$ in both the Cournot and the Bertrand case. Proposition 4.2 in that paper shows that expected profits necessarily increase with a uniform increase in the precision of the signals that firms receive about demand (or costs), and that they increase with the correlation of noise across firms in the Bertrand case but decrease in the Cournot case. Since Raith's specification features $\alpha^* = 2\alpha$ and $\phi > \max\{\bar{\phi}, \bar{\phi}'\}$, this result could be read off our Proposition 7 if it were not for the difference in the number of players and the information structure. Yet, this coincidence confirms that the logic behind our results is not unduly sensitive to the details of the assumed environment.

7 Concluding remarks

This paper examined an abstract class of economies with externalities, strategic complementarity or substitutability, and asymmetric information. For this class, we provided a complete characterization of the equilibrium and efficient use of information, and a complete taxonomy of the welfare effects of information. We then showed how these results can give guidance for welfare analysis in concrete applications.

The need for tractability imposed certain modeling choices about payoffs and information. Our analysis may be a good benchmark for more general concave environments with a unique equilibrium. The implications of payoff convexities and multiple equilibria, on the other hand, are clearly beyond the scope of this paper.[28]

Inefficiencies in the *use* of information are likely to propagate to inefficiencies in the *collection* of information. For example, in economies with a high social value for coordination, the private collection of information can reduce welfare by decreasing the correlation of beliefs across agents and thereby hampering coordination. Symmetrically, in environments where substitutability is important, the *aggregation* of information through prices or other channels could reduce welfare by increasing correlation in beliefs.[29] Clearly, the model in this paper did not endogenize either the collection or the aggregation of information. Extending the analysis in these directions seems a promising line for future research.

[28] In coordination environments with multiple equilibria the information structure affects the determinacy of equilibria (Morris and Shin, 2003). Aggregate convexities, on the other hand, may introduce a value for lotteries.

[29] The role of prices in coordination environments is also the subject of Angeletos and Werning (2004), but the focus there is on how information aggregation can lead to multiplicity.

Appendix

Proof of Proposition 2. From condition (5),

$$\begin{aligned} k(x,y) &= \kappa_0 + \sum_{n\in\mathcal{N}} \kappa_n \left[(1-\gamma_n)x_n + \gamma_n z_n\right], \\ K(\theta,\varepsilon) &= \kappa_0 + \sum_{n\in\mathcal{N}} \kappa_n \left[(1-\gamma_n)\theta_n + \gamma_n z_n\right]. \end{aligned}$$

Hence $k - K = \sum_{n\in\mathcal{N}} \kappa_n \left[(1-\gamma_n)(x_n-\theta_n)\right]$ and $K - \kappa = \sum_{n\in\mathcal{N}} \kappa_n\gamma_n(z_n - \theta_n)$. Using $Var(x_n-\theta_n) = \sigma_{x_n}^2$, $Var(z_n-\theta_n) = \sigma_{z_n}^2 = \left(\sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2}\right)^{-1}$ and $\delta_n = \sigma_{z_n}^{-2}/\sigma_n^{-2}$, together with (6), we have

$$\begin{aligned} Var(k-K) &= \sum_{n\in\mathcal{N}} \kappa_n^2 \left[(1-\gamma_n)^2\sigma_{x_n}^2\right] = \sum_{n\in\mathcal{N}} \kappa_n^2 \frac{(1-\alpha)^2(1-\delta_n)}{(1-\alpha+\alpha\delta_n)^2}\sigma_n^2, \\ Var(K-\kappa) &= \sum_{n\in\mathcal{N}} \kappa_n^2\gamma_n^2\sigma_{z_n}^2 = \sum_{n\in\mathcal{N}} \kappa_n^2 \left[\frac{\delta_n}{(1-\alpha+\alpha\delta_n)^2}\sigma_n^2\right], \end{aligned}$$

which gives the result. ■

Proof of Lemma 1. The Lagrangian of the problem in Definition 2 can be written as

$$\begin{aligned} \Lambda = &\int_{(\theta,\varepsilon)} \int_x U(k(x,y), K(\theta,\varepsilon), \theta) dP(x|\theta,\varepsilon) dP(\theta,\varepsilon) + \\ &+ \int_{(\theta,\varepsilon)} \lambda(\theta,\varepsilon) \left[K(\theta,\varepsilon) - \int_x k\left(x,y\right) dP(x|\theta,\varepsilon)\right] dP(\theta,\varepsilon). \end{aligned}$$

The first order conditions for $K(\theta,\varepsilon)$ and $k(x,y)$ are therefore given by

$$\int_x U_K(k(x,y), K(\theta,\varepsilon), \theta) dP(x|\theta,\varepsilon) + \lambda(\theta,\varepsilon) = 0 \qquad \text{for almost all } (\theta,\varepsilon) \tag{23}$$

$$\int_{(\theta,\varepsilon)} [U_k(k(x,y), K(\theta,\varepsilon), \theta) - \lambda(\theta,\varepsilon)] dP(\theta,\varepsilon|x,y) = 0 \qquad \text{for almost all } (x,y) \tag{24}$$

Noting that U_K is linear in its arguments and that $K(\theta,\varepsilon) = \int_x k\left(x,y\right) dP(x|\theta,\varepsilon)$, condition (23) can be rewritten as $-\lambda(\theta,\varepsilon) = U_K(K(\theta,\varepsilon), K(\theta,\varepsilon), \theta)$. Replacing this into (24) gives (8). Since U is strictly concave and the constraint is linear, (8) is both necessary and sufficient, which completes the proof. *QED* ■

Proof of Proposition 4. Consider first part (ii). When agents perceive payoffs to be U', the equilibrium is the unique function $k : \mathbb{R}^{2N} \to \mathbb{R}$ that solves

$$k(x,y) = \mathbb{E}[\ (1-\alpha')\kappa' + \alpha' K \mid x,y\ ], \tag{25}$$

for all $(x,y) \in \mathbb{R}^{2N}$, where $K(\theta,\varepsilon) = \mathbb{E}[k(x,y)|\theta,\varepsilon]$, $\alpha' \equiv -U'_{kK}/U'_{kk}$ and $\kappa'(\theta) = \kappa'_0 + \kappa'_1\theta_1 + ... + \kappa'_N\theta_N$ is the unique solution to $U'_k(\kappa',\kappa',\theta) = 0$. From the same arguments as in the proof of Proposition 1, the unique solution to (25) is given by

$$k(x,y) = \kappa'_0 + \sum_{n\in\mathcal{N}'} \kappa'_n \left[(1-\gamma'_n)x_n + \gamma'_n z_n\right],$$

where

$$\gamma'_n = \delta_n + \frac{\alpha'\delta_n(1-\delta_n)}{1-\alpha'(1-\delta_n)} \quad \forall n \in \mathcal{N}' \equiv \{n \geq 1 : \kappa'_n \neq 0\} \neq \varnothing$$

For this to coincide with the efficient allocation for all $(x,y) \in \mathbb{R}^{2N}$, it is necessary and sufficient that $\kappa'(\cdot) = \kappa^*(\cdot)$ and that $\alpha' = \alpha^*$, which proves part (ii).

For part (i) it suffices to let $U'(k,K,\theta) = U(k,K,\theta) + U_K(K,K,\theta)k$, in which case it is immediate that $\kappa'(\cdot) = \kappa^*(\cdot)$, and $\alpha' = \alpha^*$. ■

Proof of Condition (14). Since U is quadratic, a second-order Taylor expansion around $k = K$ is exact:

$$U(k,K,\theta) = U(K,K,\theta) + U_k(K,K,\theta)\cdot(k-K) + \frac{U_{kk}}{2}\cdot(k-K)^2.$$

It follows that ex-ante utility is given by

$$\mathbb{E}u = \mathbb{E}[W(K,\theta)] + \frac{U_{kk}}{2}\mathbb{E}[(k-K)^2],$$

where $k = k(x,y)$ and $K = K(\theta,\varepsilon)$ are shortcuts for the efficient allocation and $W(K,\theta) \equiv U(K,K,\theta)$. A quadratic expansion of $W(K,\theta)$ around κ^*, which is exact since U and thus W are quadratic, gives

$$W(K,\theta) = W(\kappa^*,\theta) + W_K(\kappa^*,\theta)\cdot(K-\kappa^*) + \frac{W_{KK}}{2}\cdot(K-\kappa^*)^2.$$

By definition of κ^*, $W_K(\kappa^*,\theta) = 0$. It follows that

$$\mathbb{E}u = \mathbb{E}W(\kappa^*,\theta) + \frac{W_{KK}}{2}\cdot\mathbb{E}[(K-\kappa^*)^2] + \frac{U_{kk}}{2}\cdot\mathbb{E}[(k-K)^2].$$

At the efficient allocation, $k - \kappa^* = \sum_{n\in\mathcal{N}^*}\kappa^*_n\left[(1-\gamma^*_n)(x_n-\theta_n) + \gamma^*_n(z_n-\theta_n)\right]$ implying that $\mathbb{E}k = \mathbb{E}K = \mathbb{E}\kappa^*$ and therefore $\mathbb{E}[(K-\kappa^*)^2] = Var(K-\kappa^*)$ and $\mathbb{E}[(k-K)^2] = Var(k-K)$, which gives the result. ■

Proof of Proposition 5. The result follows directly from the proof of Proposition 4 together with the definitions of $\kappa(\cdot)$, $\kappa^*(\cdot)$, α and α^*. ■

Proof of Proposition 6. Suppose $\kappa(\cdot) = \kappa^*(\cdot)$ and $\alpha = \alpha^*$ and consider the set $\mathcal{K}$ of allocations that satisfy

$$k(x,y) = \mathbb{E}[(1-\alpha')\kappa + \alpha' K|x,y]$$

for some $\alpha' < 1$, or equivalently $k(x,y) = \kappa_0 + \sum_{n\in\mathcal{N}} \kappa_n \left[(1-\gamma_n')x_n + \gamma_n' z_n\right]$, where

$$\gamma_n' = \delta_n + \frac{\alpha'\delta_n(1-\delta_n)}{1-\alpha'(1-\delta_n)} \quad \text{for all } n \in \mathcal{N}.$$

Clearly, the equilibrium (and efficient) allocation is nested with $\alpha' = \alpha(= \alpha^*)$. Since for any any allocation in $\mathcal{K}$ $\mathbb{E}K = \mathbb{E}k = \mathbb{E}\kappa$, ex-ante welfare can be written as $\mathbb{E}u = \mathbb{E}W(\kappa,\theta) - \mathcal{L}$, where

$$\mathcal{L} \;=\; \frac{|W_{KK}|}{2} Var\left(K-\kappa\right) + \frac{|U_{kk}+2U_{\sigma_k^2}|}{2} Var(k-K) \;=\; \frac{|U_{kk}+2U_{\sigma_k^2}|}{2}\Omega,$$

with

$$\Omega \;\equiv\; (1-\alpha^*)Var\left(K-\kappa\right) + Var(k-K).$$

Using

$$\begin{aligned} Var\left(K-\kappa\right) &= \sum_{n\in\mathcal{N}} \kappa_n^2 \gamma_n'^2 \sigma_{z_n}^2 = \sum_{n\in\mathcal{N}} \kappa_n^2 \gamma_n'^2 \left(\sigma_n^2/\delta_n\right), \\ Var(k-K) &= \sum_{n\in\mathcal{N}} \kappa_n^2 \left[(1-\gamma_n')^2 \sigma_{x_n}^2\right] = \sum_{n\in\mathcal{N}} \kappa_n^2 \left[(1-\gamma_n')^2 \left(\sigma_n^2/(1-\delta_n)\right)\right], \end{aligned}$$

we have that

$$\Omega = \sum_{n\in\mathcal{N}} \kappa_n^2 \left\{ \frac{(1-\alpha^*)\gamma_n'^2}{\delta_n} + \frac{(1-\gamma_n')^2}{1-\delta_n} \right\} \sigma_n^2.$$

Note that $\mathbb{E}u$ depends on α' and (δ_n, σ_n), for $n \in \mathcal{N}$, only through Ω. Since the efficient allocation is nested with $\alpha' = \alpha^*$, it must be that $\alpha' = \alpha^*$ maximizes $\mathbb{E}u$, or equivalently that $\gamma_n' = \gamma_n^*$ solves $\partial\Omega/\partial\gamma_n' = 0$; that is,

$$(1-\alpha^*)\frac{\gamma_n^*}{\delta_n} = \frac{1-\gamma_n^*}{1-\delta_n}. \tag{26}$$

Next note that Ω increases, and hence $\mathbb{E}u$ decreases, with any σ_n. Finally, consider the effect of δ_n. By the envelope theorem,

$$\frac{d\Omega}{d\delta_n} = \left.\frac{\partial\Omega}{\partial\delta_n}\right|_{\gamma_n'=\gamma_n^*} = \kappa_n^2 \left\{ -\frac{(1-\alpha^*)\gamma_n^{*2}}{\delta_n^2} + \frac{(1-\gamma_n^*)^2}{(1-\delta_n)^2} \right\} \sigma_n^2$$

Using (26), we thus have that $d\mathbb{E}u/d\delta_n > [<] 0$ if and only if $\gamma_n^*/(1-\gamma_n^*) > [<]\ \delta_n/(1-\delta_n)$, which is the case if and only if $\alpha^* > [<] 0$. Using $\alpha = \alpha^*$ (by efficiency) then gives the result. ■

Proof of Condition (17). Since $U\left(k, K, \theta, \sigma_k^2\right)$ is quadratic in k and linear in σ_k^2,

$$U\left(k, K, \theta, \sigma_k^2\right) = U(K, K, \theta, 0) + U_k(K, K, \theta, 0)(k-K) + \frac{U_{kk}}{2}(k-K)^2 + U_{\sigma_k^2}\sigma_k^2.$$

Using the fact that $\sigma_k^2 = \mathbb{E}[(k-K)^2|\theta, \varepsilon]$ and hence $\mathbb{E}\sigma_k^2 = \mathbb{E}[(k-K)^2]$, we have that ex-ante utility is given

$$\mathbb{E}u = \mathbb{E}W(K,\theta) + \frac{U_{kk}+2U_{\sigma_k^2}}{2} \cdot \mathbb{E}[(k-K)^2].$$

A Taylor expansion of $W(K,\theta)$ around $K=\kappa$ then gives

$$W(K,\theta)=W(\kappa,\theta)+W_K(\kappa,\theta)(K-\kappa)+\frac{W_{KK}}{2}(K-\kappa)^2$$

and hence

$$\mathbb{E}u=\mathbb{E}W(\kappa,\theta)+\mathbb{E}[W_K(\kappa,\theta)\cdot(K-\kappa)]+\frac{W_{KK}}{2}\cdot\mathbb{E}[(K-\kappa)^2]+\frac{U_{kk}+2U_{\sigma_k^2}}{2}\cdot\mathbb{E}[(k-K)^2].$$

In equilibrium, $\mathbb{E}k=\mathbb{E}K=\mathbb{E}\kappa$ and therefore, $\mathbb{E}[W_K(\kappa,\theta)\cdot(K-\kappa)]=Cov[W_K(\kappa,\theta),(K-\kappa)]$, $\mathbb{E}[(K-\kappa)]^2=Var(K-\kappa)$ and $\mathbb{E}[(k-K)^2]=Var(k-K)$, which gives the result. ■

Proof of Proposition 7. We prove the result in three steps. Step 1 computes the welfare losses due to incomplete information. Step 2 derives the comparative statics. Step 3 characterizes the bounds $\underline{\phi}$, $\bar{\phi}$, $\underline{\phi}'$, $\bar{\phi}'$.

Step 1. The property that W is quadratic, along with $W_K(\kappa^*,\theta)=0$ (by definition of the first best), and $W_{KK}<0$, imply that

$$W_K(\kappa,\theta)=W_K(\kappa^*,\theta)+W_{KK}\cdot(\kappa-\kappa^*)=|W_{KK}|\cdot(\kappa^*-\kappa)\,.$$

It follows that

$$Cov(K-\kappa,W_K(\kappa,\theta))=|W_{KK}|\cdot Cov\left(K-\kappa,\kappa^*-\kappa\right). \tag{27}$$

Since $K-\kappa=\sum_{n\in N}\kappa_n\gamma_n(z_n-\theta_n)$, $z_n-\theta_n=\left[\lambda_n(\varepsilon_n)+(1-\lambda_n)(\mu_{\theta_n}-\theta_n)\right]$, and $(\varepsilon_n,\varepsilon_j,\theta_n,\theta_j)$ are mutually orthogonal whenever $n\neq j$, we have

$$\begin{aligned}Cov\left(K-\kappa,\kappa^*-\kappa\right) &= Cov\Big(\sum\kappa_n\gamma_n(z_n-\theta_n)\ ,\ \sum\left(\kappa_n^*-\kappa_n\right)\theta_n\Big)=\\ &= \sum\left(\kappa_n^*-\kappa_n\right)\kappa_n\gamma_n Cov\left(\theta_n,z_n-\theta_n\right)=\\ &= \sum_{n\in\mathcal{N}}\left(\frac{\kappa_n^*-\kappa_n}{\kappa_n}\right)\kappa_n^2\gamma_n\left[-\left(1-\lambda_n\right)Var\left(\theta_n\right)\right]\end{aligned}$$

Using $\phi_n\equiv(\kappa_n^*-\kappa_n)/\kappa_n$, $\gamma_n=\delta_n/(1-\alpha+\alpha\delta_n)$, and $(1-\lambda_n)\,Var\left(\theta_n\right)=(\sigma_{\theta_n}^{-2}/\sigma_{z_n}^{-2})\sigma_{\theta_n}^2=\sigma_{z_n}^2=\sigma_n^2/\delta_n$, we have that

$$Cov\left(K-\kappa,\kappa^*-\kappa\right)=\sum_{n\in\mathcal{N}}\phi_n\left\{-\frac{1}{1-\alpha+\alpha\delta_n}\kappa_n^2\sigma_n^2\right\} \tag{28}$$

while

$$Cov\left(\ K-\kappa,\kappa\mid\theta_{-n}\ \right)=\kappa_n^2\gamma_n Cov\left(z_n-\theta_n,\theta_n\right)=-\frac{1}{1-\alpha+\alpha\delta_n}\kappa_n^2\sigma_n^2.$$

Next, as in the proof of Proposition 2,

$$Var(K-\kappa)=\sum_{n\in\mathcal{N}}\frac{\delta_n}{(1-\alpha+\alpha\delta_n)^2}\kappa_n^2\sigma_n^2 \tag{29}$$

$$Var(k-K)=\sum_{n\in\mathcal{N}}\frac{(1-\alpha)^2(1-\delta_n)}{(1-\alpha+\alpha\delta_n)^2}\kappa_n^2\sigma_n^2. \tag{30}$$

Substituting (27)-(30) into (17), using $v = (1-\alpha^*)|U_{kk} + 2U_{\sigma_k^2}|$, and rearranging, we get

$$\mathcal{L} = \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \sum_{n \in \mathcal{N}} \Lambda\left(\alpha, \alpha^*, \phi_n, \delta_n\right) \kappa_n^2 \sigma_n^2$$

where

$$\Lambda\left(\alpha, \alpha^*, \phi, \delta\right) \equiv \frac{(1-\alpha^*)\left[2\phi(1-\alpha+\alpha\delta)+\delta\right] + (1-\alpha)^2(1-\delta)}{(1-\alpha+\alpha\delta)^2}. \tag{31}$$

Step 2. Note that $\mathbb{E}W(\kappa, \theta)$ is independent of (δ_n, σ_n) and hence the comparative statics of welfare with respect to (δ_n, σ_n) coincide with the opposite of those of $\mathcal{L}$. Note that

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \sigma_n^2} &= \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \kappa_n^2 \Lambda\left(\alpha, \alpha^*, \phi_n, \delta_n\right) \\
\frac{\partial \mathcal{L}}{\partial \delta_n} &= \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \kappa_n^2 \sigma_n^2 \frac{\partial \Lambda\left(\alpha, \alpha^*, \phi_n, \delta_n\right)}{\partial \delta_n}
\end{aligned}$$

We thus need to understand the sign of Λ and $\partial\Lambda/\partial\delta_n$.

By condition (31),

$$\frac{\partial \Lambda}{\partial \delta_n} = \frac{\alpha^2[(1-\delta_n)(1-\alpha)-\delta_n] - \alpha^*(1-\alpha-\alpha\delta_n) - 2\alpha\phi(1-\alpha^*)(1-\alpha+\alpha\delta_n)}{(1-\alpha+\alpha\delta_n)^3}$$

When $\alpha = 0$, this reduces to

$$\frac{\partial \Lambda}{\partial \delta_n} = -\alpha^*$$

and hence, for any $n \in \mathcal{N}$, $\partial\mathcal{L}/\partial\delta_n > [<] 0$ if and only if $\alpha^* < [>] 0$.

When instead $\alpha \neq 0$,

$$\frac{\partial \Lambda}{\partial \delta_n} = \frac{2(1-\alpha^*)}{[1-\alpha+\alpha\delta_n]^2} \alpha[f(\alpha, \alpha^*, \delta_n) - \phi_n],$$

where

$$f(\alpha, \alpha^*, \delta) \equiv \frac{\alpha^2[(1-\delta)(1-\alpha)-\delta] - \alpha^*(1-\alpha-\alpha\delta)}{2\alpha(1-\alpha+\alpha\delta)(1-\alpha^*)}.$$

Since $\alpha^* < 1$, $sign[\partial\mathcal{L}/\partial\delta_n] = sign[\alpha] \cdot sign[f(\alpha, \alpha^*, \delta_n) - \phi_n]$. Let

$$\underline{\phi}(\alpha, \alpha^*) \equiv \min_{\delta \in [0,1]} f(\alpha, \alpha^*, \delta) \quad \text{and} \quad \bar{\phi}(\alpha, \alpha^*) \equiv \max_{\delta \in [0,1]} f(\alpha, \alpha^*, \delta).$$

If $\phi_n \in (\underline{\phi}, \bar{\phi})$, then $\partial\mathcal{L}/\partial\delta_n$ alternates sign as δ_n varies between 0 and 1, no matter whether $\alpha > 0$ or $\alpha < 0$. Hence, $\phi_n < \underline{\phi}$ is necessary and sufficient for $\partial\mathcal{L}/\partial\delta_n > 0\ \forall\delta_n$ when $\alpha > 0$ and for $\partial\mathcal{L}/\partial\delta_n < 0\ \forall\delta_n$ when $\alpha < 0$, whereas $\phi_n > \bar{\phi}$ is necessary and sufficient for $\partial\mathcal{L}/\partial\delta_n < 0\ \forall\delta_n$ when $\alpha > 0$ and for $\partial\mathcal{L}/\partial\delta_n > 0\ \forall\delta_n$ when $\alpha < 0$.

Finally, note that $\partial\mathcal{L}/\partial\sigma_n^2 > [<]\ 0$ if and only if $\phi_n > [<]\ g(\alpha, \alpha^*, \delta_n)$, where

$$g(\alpha, \alpha^*, \delta) = -\frac{(1-\alpha)^2(1-\delta) + \delta(1-\alpha^*)}{2(1-\alpha^*)(1-\alpha+\alpha\delta)} < 0.$$

Letting

$$\underline{\phi}'(\alpha, \alpha^*) \equiv \min_{\delta \in [0,1]} g(\alpha, \alpha^*, \delta) \quad \text{and} \quad \bar{\phi}'(\alpha, \alpha^*) \equiv \max_{\delta \in [0,1]} g(\alpha, \alpha^*, \delta),$$

we get that $\partial \mathcal{L}/\partial \sigma_n^2 > 0$ [< 0] for all $\delta_n \in [0,1]$ if $\phi_n > \bar{\phi}'$ [$< \underline{\phi}'$], whereas $\partial \mathcal{L}/\partial \sigma_n^2$ alternates sign as δ_n varies if $\phi_n \in (\underline{\phi}', \bar{\phi}')$.

Step 3. Note that both f and g are monotonic in δ, with

$$\frac{\partial f}{\partial \delta} = -2\frac{\partial g}{\partial \delta} = \frac{(1-\alpha)}{(1-\alpha^*)(1-\alpha+\alpha\delta)^2}(\alpha^* - \alpha)$$

When $\alpha^* = \alpha$, both f and g are independent of δ, and

$$\underline{\phi}'(\alpha, \alpha^*) = \underline{\phi}(\alpha, \alpha^*) = \bar{\phi}(\alpha, \alpha^*) = \bar{\phi}'(\alpha, \alpha^*) = -\frac{1}{2} < 0.$$

When instead $\alpha^* > \alpha$, f is strictly increasing (and g strictly decreasing) in δ, so that

$$\begin{aligned} \underline{\phi}(\alpha, \alpha^*) &= f(\alpha, \alpha^*, 0) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1), \\ \underline{\phi}'(\alpha, \alpha^*) &= g(\alpha, \alpha^*, 1) < \bar{\phi}'(\alpha, \alpha^*) = g(\alpha, \alpha^*, 0), \end{aligned}$$

and when $\alpha^* < \alpha$, f is strictly decreasing (and g strictly increasing) in δ, so that

$$\begin{aligned} \underline{\phi}(\alpha, \alpha^*) &= f(\alpha, \alpha^*, 1) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) \\ \underline{\phi}'(\alpha, \alpha^*) &= g(\alpha, \alpha^*, 0) < \bar{\phi}'(\alpha, \alpha^*) = g(\alpha, \alpha^*, 1). \end{aligned}$$

Consider first the case $\alpha \in (0,1)$. If $\alpha^* > \alpha$, then $\alpha^2 + (1-2\alpha)\alpha^* > 0$ (using the fact that $\alpha^* < 1$) and therefore

$$\underline{\phi}(\alpha, \alpha^*) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = -\frac{\alpha^2 + (1-2\alpha)\alpha^*}{2\alpha(1-\alpha^*)} < 0.$$

If instead $\alpha^* < \alpha$, then

$$\underline{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = -\frac{\alpha^2 + (1-2\alpha)\alpha^*}{2\alpha(1-\alpha^*)} < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = -\frac{\alpha^* - \alpha^2}{2\alpha(1-\alpha^*)}$$

and therefore $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1-2\alpha)$, while $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$. Since $-\alpha^2/(1-2\alpha) < 0$ whenever $\alpha < 1/2$, we conclude that, for $\alpha \in (0,1)$, $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1-2\alpha)$, and $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$.

Next, consider the case $\alpha \in (-1, 0)$. If $\alpha^* > \alpha$, then

$$\underline{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1-\alpha^*)} < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 1) = \frac{\alpha^2 + (1-2\alpha)\alpha^*}{(-2\alpha)(1-\alpha^*)}$$

and hence $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, while $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1-2\alpha)$. If instead $\alpha^* < \alpha$, then $\alpha^* < 0 < \alpha^2$ and hence

$$\underline{\phi}(\alpha, \alpha^*) < \bar{\phi}(\alpha, \alpha^*) = f(\alpha, \alpha^*, 0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1-\alpha^*)} < 0.$$

We conclude that, for $\alpha \in (0,1)$, $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, and $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1-2\alpha)$.

Finally, note that

$$g(\alpha, \alpha^*, 0) = -\frac{(1-\alpha)}{2(1-\alpha^*)} < 0 \quad \text{and} \quad g(\alpha, \alpha^*, 1) = -\frac{1}{2} < 0.$$

Hence, $\underline{\phi}' = -\frac{(1-\alpha)}{2(1-\alpha^*)} < -1/2 = \bar{\phi}'$ for $\alpha < \alpha^*$, $\underline{\phi}' = \bar{\phi}' = -\frac{1}{2}$ for $\alpha = \alpha^*$, and $\underline{\phi}' = -1/2 < \bar{\phi}' = -\frac{(1-\alpha)}{2(1-\alpha^*)} < 0$ for $\alpha > \alpha^*$. ■

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