Monetary and fiscal policy coordination and macroeconomic stabilization. A theoretical analysis

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Abstract

We examine the relations between monetary and fiscal policies in the process of macroeconomic stabilization. Our model suggests that each policy maker prefers to be the second mover in a "Stackelberg" situation, i.e. where one policy makers precommits its policy choice. At the same time, both Stackelberg solutions are preferable, for each policymaker, to the Nash solution. We argue that there is a natural way to choose among the two Stackelberg games. This solution implies that the government acts as a leader and sets fiscal policy according to the minimization of a Social Welfare Function (which fully internalizes also the objective of price stability).

We interpret our results in relation to the debate on monetary-fiscal coordination in EMU and on the role of the Stability and Growth Pact. We argue that the Pact is welfare improving and that a coordination mechanism analogous to the Broad Economic Policy Guidelines is desirable.

Keywords: EMU, Monetary Policy, Fiscal Policy, Policy Coordination, Stability and Growth Pact.

JEL codes: E520, E610, E630, H700.

1 Introduction

After taking charge for conducting the single monetary policy of the European Economic and Monetary Union in January 1999, the European Central Bank has quickly built a reputation for its committment to maintaining price stability. To reinforce this committment, the Treaty of Maastricht[1] clearly stated the independence of the ECB from all Community institutions and bodies and from any government of a member state, and also formulated certain criteria for fiscal discipline[2], designed to prevent the occurrence of unsustainable fiscal policies, which might otherwise cause pressures to adopt an inflationary monetary policy.

In addition, the Treaty also states the aim for a "closer coordination of economic policies and sustained convergence of the economic performances". To this purpose, the European Council is requested to formulate "broad guidelines of the economic policies of the Member States", to monitor economic developments in each of the Member States and, where necessary, to adopt recommendations to ensure consistency of national economic policies with the Community guidelines (Art. 99). The importance and implications of these additional provisions has not been immediately clear. Both EU governments and independent commentators initially discounted their importance. However, with the subsequent adoption of the Stability and Growth Pact (SGP), it became apparent that the Broad Economic Policy Guidelines (BEPG) could in principle become quite important, especially in view of the additional goal, stated in the SGP, that national fiscal policies should aim for "the medium-term budgetary objective of close to balance or in surplus".

But, what does a "closer coordination of economic policies" imply in practice? When provoked on this issue, many economists would tend to react in one of two ways. To some, this is just cheap talk, introduced perhaps only to please or silence incompetent advocates of a more federalist approach to European integration. Others instead react with a sense of fastidiousness: maybe fiscal policies need to be coordinated across countries, but certainly as far as monetary policy is concerned there is no need for such "coordination". In their view, to advocate the coordination between monetary and fiscal

[1] Now consolidated into the Treaty Establishing the European Community

[2] Protocols No. 20 on the excessive deficit procedure, and No. 21 on the convergence criteria referred to in Article 121.

policy is an indirect way to suggest a revival of old-fashioned Keynesianism, when monetary policy was not primarily concerned with price stability.

In this paper, we offer a different interpretation: we suggest that coordination of fiscal and monetary policy is beneficial to ensure a smooth performance of monetary policy - that is, to minimize the costs of price stability. Our view complements a recent but now well-established opinion in monetary economics, which has forcefully argued in favor of central bank independence (CBI). Within this tradition, the only challenge from fiscal policy to CBI appears when an "indisciplined" fiscal policy, possibly unsustainable in the long run, forces the central bank to give up its independence and monetize the fiscal debt[3]. Our argument is that even when fiscal policy is perfectly sustainable (in the long run) it may still undermine the policy stance adopted by the monetary authority. A simple example may provide an intuitive introduction to this reasoning. Let us compare two situations, which differ only by the stance of fiscal policy. We assume a setup such that, *ceteris paribus*, a more expansive fiscal policy will result in a higher equilibrium real rate of interest. This leads to one of two policy scenarios. If *monetary policy is set after* fiscal policy, then the outcome of the policy rule according to which the central bank sets interest rates will differ in the two countries. Moreover, if the central bank also aims to minimize the variability of nominal interest rates, then also the inflation rates may differ. If alternatively *fiscal policy is set after* monetary policy, then a (relatively more) expansive fiscal policy (even within the 3% reference value adopted in the Maastricht convergence criteria!) will have an expansionary effect on aggregate demand and consequently also on the rate of inflation. Thus in this second case a more expansive (but perfectly sustainable and "disciplined") fiscal policy could potentially undermine the stance of monetary policy in the pursuit of price stability. While it may be possible to suggest that in equilibrium (that is, in the average of the business cycle) this objection is not empirically relevant, we think that it might become quite relevant during the different phases of a business cycle. In fact we believe that it is precisely to pre-empt the occurence of these potentially negative implications that the

[3]There are two rather different versions of this argument: the unpleasant monetarist arithmetics of Sargent and Wallace (1981), and the fiscal theory of the price level of Leeper (1991) and Woodford (1995, 1998). Iin the first approach, CBI is challenged by fiscal authorities, which play the role of a Stackelberg leader against the central bank. In the second approach, instead, CBI becomes irrelevant, since the price level adjusts independently of the policy actions decided by the central bank.

drafters of the SGP added "the medium-term budgetary objective of close to balance or in surplus".

The aim of this paper is to propose a framework, where these questions can be formally addressed. The paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 sets up the basic model of the economy. Section 4 analyzes the interaction of fiscal and monetary policy in a Nash equilibrium. Sections 5 and 6 analyze Stackelberg equilibria, with the central bank acting respectively as a follower and as a leader. Naturally, the outcome of the game will be different, in each of these three sections. In section 7 we propose a simple and natural criterion to choose among the different equilibria. Section 8 concludes.

Before concluding this section, it is useful to anticipate that in setting up the model we deliberately allow for a duplicity of motivations on the side of the government. We assume that either the government is concerned only with output stabilization (we call this the Treasury view) or it is concerned both about output *and* inflation stabilization (we call this the Government view, implying that the government adopts a full "social welfare function" approach to macroeconomic policies). This duplicity allows us to reproduce in the context of a one-country model the same issue which arises in EMU: namely the conflict between national governments (only interested in the Treasury view, that is domestic output stabilization) and the EC authorities (which want to enforce the SGP, and thus take into account stabilization of both output and inflation[4]).

[4]The following two quotations are useful to exemplify the way in which the EC authorities adopt the "social welfare function" approach in their policy recommendations. The first is from the Issue paper on the Broad Economic Policy Guidelines 2001(March 2001):*"Budgetary policies should continue to be geared to the achievement of public finances close to balance or in surplus, so as to support the price-stability orientation of monetary policy, and thereby to foster continued economic growth and employment creation. "* . The second is from the European Council Recommendation of 26/02/2001 (on fiscal policy in Ireland): *"The budget for 2001 will give a further substantial boost to demand in Ireland and its possible supply effects are likely to be small in the short term. It will therefore aggravate overheating and inflationary pressures and widen the positive output gap."* (http://ue.eu.int/emu/convergence/irl/IR-RECOMMENDATION2001.pdf) .

Incidentally, also note that the ECB shares this view of the interaction of fiscal and monetary policies as joint determinants of inflation: *"The expansionary fiscal policies planned for this year [2001] in a number of euro area countries are not conducive to containing aggregate demand and inflationary pressures. Particularly in the countries experiencing high economic growth rates, inflationary pressures will receive an additional stimulus from expansionary fiscal policies"* (ECB Annual Report 2000, May 2001, p. 47).

2 Review of the literature

If monetary policy is committed to price stability, why should a need for coordination between monetary and fiscal policies arise? In this paper we suggest a straight answer to this question. However, our approach is somewhat innovative in the context of the recent literature. Analyzing this literature, we may identify at least five different approaches[5].

First, several papers in this area do not share our main assumption - that monetary policy is effectively committed. In particular, Beetsma and Uhlig (1999) observe that, assuming that the central bank would like to stabilize output around its natural level, if distortionary fiscal policy induces a wedge between actual and natual output, then the central bank will be tempted into adopting (time inconsistent) inflationary policies[6].

Second, the literature on the monetary implications of fiscal (in)discipline, which originates with Sargent and Wallace (1981), emphasizes that, to the extent that the path of a government's fiscal deficit is predetermined and unsustainable, then monetary policy and the price level are no longer exogenous to it. A similar point arises in the context of the 'Fiscal Theory of the Price Level' (Leeper, 1991, and Woodford, 1995). However, in these frameworks the goals of fiscal policy are not explicitly discussed, and in particular they do not include macro stabilization. Nevertheless, the scenario analyzed by Sargent and Wallace has surely been influential in motivating the emphasis on fiscal discipline as a pre-requisite for monetary stability, which has been placed in the Treaty of Maastricht and, in particular, on the design of the criteria for admission to the third phase of EMU.

A different question is examined in a third strand of literature, where a more inflation-averse central bank is shown to have a perverse effect on the incentive of fiscal authorities to reduce debt levels (Beetsma and Bovenberg, 1999 and 2001).

Fourth, Dixit and Luisa Lambertini(2001; 2002) assume that both fiscal and monetary authorities act according to time- and mutually-inconsistent rules, and discuss how different coordination mechanisms may or may not alleviate the undesirable consequences of non-coordinated behavior. Notice

[5]In this review, we benefitted from several comments and suggestions by Alessandro Missale, who also gave us access to his unpublished review notes on the literature. We thank him warmly, although we have not entirely followed his suggestions.

[6]A similar issue was raised by Debelle and Fischer (1995) in the context of a model where the central bank did *not* accept the natural level of output as its ultimate goal.

that in their model fiscal policies affect output and inflation in different directions, contrary to the standard result generally assumed in the literature (and also in practice, witness the evidence in Fatas and Mihov (2001) In particular Dixit and Luisa Lambertini (2002) model a race between fiscal policy, which tries to achieve output and inflation levels beyond the central bank's ideal level, and monetary policy, which tries to achieve output and inflation below the fiscal authority's ideal level. At the end of the game, monetary policy is too contractionary and fiscal policy not expansionary enough. In this case, with (discretionary) fiscal policy being chosen strategically, the reaction function of the fiscal authority acts as a constraint on the monetary rule, and this effectively negates the advantage of monetary commitment. The conclusion of their analysis is that, without a time-consistency problem, both the monetary and the fiscal authority should have identical output and price goals that coincide with the socially optimal ones. With a time-consistency problem, the inflation obiective should be lowered, but it is crucial that the output and inflation objectives of the two authorities should be the same.

The issue of decentralized fiscal authorities is analyzed in a fifth group of papers. The main differences between these papers is in hte assumptions about the objective functions of the different authorities. Beetsma and Bovenberg (2001) analyze the case when both monetary and fiscal authorities are unable to commit to their policy targets and nominal wages are predetermined. They analyze under which conditions this leads to a "wasteful strategic accumulation of government debt". In particular they argue that, in the absence of an explicit commitment by fiscal authorities, ex-post coordination at the fiscal level may actually be harmful. A similar result emerges in a related paper by Beetsma, Debrun and Klaassen (2001). Andersen (2002) finds that the costs of non-cooperative fiscal policies tend to be large in the case of aggregate (symmetric) shocks, and increase with the number of policy actors; on the contrary, these costs are small in the case of idiosyncratic shocks, and decreasing in the number of actors. Uhlig (2002) assumes that the central bank is motivated by the desire to minimize deviations of output from its natural level and of inflation from its target. In this case, adding a number of fiscal authorities essentially concerned only with ouput may result in inefficiencies, as there will be an aggregate pressure to stabilize output, which will have inflationary implications and thus induce the cetnral bank to raise interest rates. In his model, differently from ours (see next section) the central bank attaches no cost to the level of interest rates. Hence in equilibrium an excessive fiscal expansion will have no conse-

quences either on output or on inflation. In this case, however, coordination by fiscal authorities would be beneficial, in the limited sense that it will help to keep interest rates down.

Finally, a sixth approach assumes that the main source of interaction between the two policies originates from the fact that they both similarly affect aggregate demand and inflation. Hence fiscal policy matters for its impact on aggregate demand, while the issue of debt accumulation if neglected (as fiscal deficits average to zero over time, consistently with the requirements of EMU under the SGP). Also, in this strand of literature, the central bank is assumed to act in a time-consistent fashion, following an objective function formalized in accord to the mandate assigned to the ECB. Buti, Roeger and in't Veld (2001) analyze in this vein the interaction of monetary and fiscal policies. Assuming that fiscal authorities do not care for inflation, they find that cooperation is desirable, in particular when the economies are hit by a supply shock. In a related, more general framework van Aarle, Engwerda and Plasmans (2001) analyze two countries, with decentralized fiscal authorities and a centralized monetary authority. Their basic framework is similar to the one which we adopt below. They analyze - by numerical simulations - the equilibrium strategies which arise in continuous time over an infinite horizon. The cases they consider include: non cooperation between the three authorities; full cooperation; coalition between the two fiscal authorities only; coalition between one fiscal authority and the monetary authority. These setups are examined under both assumptions of symmetry and asymmetry between the two countries involved. Their main finding is that cooperation is efficient for fiscal authorities in that a common stance against the ECB produces a Pareto improvement. This may not hold at the equilibrium of the fully cooperative (that is, including the ECB) game. In Luca Lambertini and Rovelli (2002) we also show analytically that there are gains if the two authorities cooperate, in the sense that fiscal policy should be set taking into account a welfare function defined over both output and inflation stabilization, the latter defined consistently with the ECB. In that paper, we also interpret the model in the light of the interaction between national fiscal authorities and the European Commission, and conclude that enforcement of the Stability and Growt Pact is desirable.

3 The model

We analyze a simple model of a closed economy, which is the static equivalent of a conventional aggregate demand / aggregate suppy model, with short run price rigidity.[7] In the short run, we may observe a positive value of the output gap ($y > y^*$) following either an expansionary monetary policy (which temporarily lowers the short run real rate of interest $(i - \pi^*)$ below the long run equilibrium value, $\overline{r}$, or an expansionary fiscal policy ($f > 0$) or an unexpected positive demand shock, ε_1.

$$\text{AD} \ : y = y^* - \alpha \left(i - \pi^* - \overline{r} \right) + \eta f + \varepsilon_1 \tag{1}$$

Inflation, π , will increase/decrease relative to the target level, π^*(which, in the absence of shocks, is also its expected value), in response to positive/negative values of the output gap, and also to unexpected supply shocks ε_2:

$$\text{AS} \ : \pi = \pi^* + \beta \left(y - y^* \right) + \varepsilon_2 \tag{2}$$

Also note that α, β, η are positive parameters and that the shocks $\varepsilon_i, i = 1, 2$ are i.i.d, and we assume below that both fiscal and monetary policy can be set optimally with no lag, in response to realized values of the two shocks.

We define the following policy problem. We assume that, given the available resources, social welfare is maximum when, in the absence of shocks, $y = y^*$ and $\pi = \pi^*$. In this cases it is then optimal to adopt a neutral policy stance, that is $i = \overline{r} + \pi^*$and $f = 0$. When shocks occur, then the economy is temporarily driven away from the social optimum, and both fiscal and monetary policy may adopt a non-neutral stance.

We also assume that there is positive, convex social cost associated to the use of either policy instrument. The existence of a cost associated to changes in the real interest rate is conventionally embodied in many models of monetary policy [8], and also documented empirically (Favero and Rovelli, 2002). As regards fiscal policy, this assumption reflects two facts: (i) a higher level of fiscal expansion implies a higher crowding out of private expenditures, and this is perceived to be costly; (ii) in the EU, the Stability Pact requires

[7] See e.g. Svensson (1997)

[8] For instance, Walsh (1998, ch.10) argues why the central bank might attach a positive value to interest rate smoothing. Empirically, this choice is motivated by the desire to account for the observed persistence or graduality in the setting of the Federal Funds rate.

that the fiscal stance is on average neutral ($f = 0$), so that departures from a balanced budget should only be small and temporary[9]. We thus postulate the following quadratic social loss function, which defines the preferences of society, and hence also of the government:

$$L_S \equiv (\pi - \pi^*)^2 + \mu (r - \overline{r})^2 + (y - y^*)^2 + \gamma f^2 \tag{3}$$

Note that this formulation of the social loss function assumes that the output and inflation terms share the same weight. While this is arbitrary (but not irrealistic), it avoids introducing an additional weighting parameter. It will become clear below that no qualitative result depends critically on this assumption. In addition, we assume that μ, γ are positive parameters.

Now we need to define the institutional setup for policy decisions, which mimics the one adopted in the Euro-area (and also in all countries which have set up and independent central bank) We assume that the central bank is *independent* of the government and commited to inflation stabilization. Hence, the government delegates to it the following subset of L_S.

$$L_M \equiv (\pi - \pi^*)^2 + \mu (r - \overline{r})^2 \tag{4}$$

As regards fiscal policy, the government delegates it to an authority (the Treasury minister) who is empowered with the setting of the fiscal stance, but is *not independent* of the government. How should the objective function of the Treasury be optimally specified? A priori, the answer is not clear, and surprisingly this question does not seem to have been previously addressed. In principle, there are two alternatives, or benchmark solutions: either, given that the control of inflation has been assigned to the central bank, the Treasury is assigned responsibility only for the management of AD via the fiscal stance, or the government imposes to the Treasury to act in accord with its general objective function. To the extent that bureacracies tend to live a life of their own, the Treasury will be posing pressure to focus only on the management of AD (also, perhaps, in response to public or trade union pressures). In any case, if the Treasury looks only at AD and at the cost of setting its fiscal stance differet from zero, then it will minimize:

$$L_F \equiv (y - y^*)^2 + \gamma f^2 \tag{5}$$

[9] This assumption also implies that there will not be a sequence of government deficits, potentially generating an excessive accumulation of government debt, such as to pose a threat to the independence of monetary policy, as in Sargent and Wallace (1981).

Finally, note that , given $(4-5)$, the social loss function may be simply defined as the sum of the two sub-functions:

$$L_S \equiv L_F + L_M \tag{6}$$

In the following sections, we shall be explicitly concerned with the following two questions: (1) given that the central bank has been set up with an independent mandate, which is the desirable sequence of decisions between the setting of monetary and fiscal policies? (2) given that the government is concerned with the minimization of L_S, subject to the independence of the central bank, should it run fiscal policy according to the minimization of L_S (the *"Government view"*) or should it instead run it according to the *"Treasury view"*, that is to the minimization of L_F only?

(discutere ancora: queste domande hanno interesse specifico in relazione a EMU with decentralized fiscal authorities)

Now we derive the optimal policy functions of the two authorities. Given our static setup, we cannot write them down as Taylor rules, since this could be done only if we assumed that y and π were predetermined. We thus write each authority's best reply function, obtained by optimizing their respective loss functions subject to the constraints $(1-2)$ and assuming as given the choice of the other authority:

$$\text{Central bank:} \quad i^{br} = \overline{r} + \pi^* + \frac{\alpha\beta}{\alpha^2\beta^2 + \mu}\left[\beta\left(\eta f + \varepsilon_1\right) + \varepsilon_2\right] \tag{7}$$

$$\text{Fiscal authority using } L_F\text{:} \quad f^{br} = \frac{\eta}{\eta^2 + \gamma}\left[\alpha\left(i - \pi^* - \overline{r}\right) - \varepsilon_1\right] \tag{8}$$

$$\text{Fiscal authority using } L_S\text{:} \quad f^{br} = \frac{\alpha\eta\left(1+\beta^2\right)\left(i - \pi^* - \overline{r}\right) - \eta\left[\left(1+\beta^2\right)\varepsilon_1 + \beta\varepsilon_2\right]}{\gamma + \eta^2\left(1+\beta^2\right)} \tag{9}$$

where superscript *br* stands for *best reply* function.

Note that, as it should be expected, each authority manouvres its policy instrument in a restrictive way (higher i, lower f) in response to any expansionary (>0) shock to aggregate demand (AD: ε_1) or supply (AS: ε_2). Moreover, best replies are everywhere increasing, irrespective of the values taken by the vector of shocks. In particular, the slope of the reaction function of the central bank w.r.t. the fiscal stance f is:

$$\frac{\partial i^{br}}{\partial f} = \frac{\alpha\beta^2\eta}{\alpha^2\beta^2 + \mu} \tag{10}$$

If the treasury sets f so as to minimise L_F, the slope of the best reply function is:

$$\frac{\partial f^{br}}{\partial i} = \frac{\alpha\eta}{\eta^2 + \gamma} \tag{11}$$

while if the government sets f so as to minimise the social loss L_F, the slope is:[10]

$$\frac{\partial f^{br}}{\partial i} = \frac{\alpha\eta\left(1+\beta^2\right)}{\gamma + \eta^2\left(1+\beta^2\right)} \tag{12}$$

with

$$\frac{\alpha\eta\left(1+\beta^2\right)}{\gamma + \eta^2\left(1+\beta^2\right)} > \frac{\alpha\eta}{\eta^2 + \gamma} \tag{13}$$

for all $\beta > 0$. Therefore:

Lemma 1 *Provided that the weight of the output gap in the AS curve is positive, the best reply function of the government is steeper than the best reply function of the treasury.*

When the government directly manages the budget with the aim of maximising social welfare, the optimal reaction to a given change in the interest rate is, in general, higher than it would be if the treasury controlled the fiscal instrument, as in the former case the government gets a further boost by internalising the target of price stability.

Mapping the indifference curves of the fiscal and monetary authorities and drawing accordingly their best reply functions in the space $\{f, i\}$, we can fully characterise their preferences over the timing of moves. That is, we can tell whether they will non-cooperatively choose to play a Nash or a Stackelberg equilibrium, according to the rules of an 'extended game' as defined by d'Aspremont and Gérard-Varet (1980) and Hamilton and Slutsky (1990).[11] The structure of the extended game is as follows. Before choosing optimally the levels of f and i, the fiscal authority (i.e., the government or the treasury) and the central bank have to play a preliminary stage (the so-called *preplay* stage) where they have to decide non-cooperatively and

[10] Therefore, the nominal interest rate and the fiscal stance are strategic complements (Bulow, Geanakoplos and Klemperer, 1985).

[11] The same toolbox is applied to the problem of the international coordination of monetary policies in Lambertini (1999), using the same model as in Hamada (1976) and Canzoneri and Henderson (1991), *inter alia*.

simultaneously the order of moves which will be adhered to at the following stage. Assume that there are two instants, t_1 and t_2, at which the two authorities can move. These instants are purely logical entities, and do not belong to calendar time; they represent the pure strategies available to players at the first (preplay) stage. Accordingly, there is no discounting. If both players declare that they want to move at the same instant, either t_1 or t_2, then the relevant equilibrium concept for solution of the second stage is the Nash equilibrium. Otherwise, if one player chooses t_1 while the other chooses t_2, the relevant solution concept for the ensuing stage is going to be a Stackelberg equilibrium, with the player that has chosen t_1 as the leader.

Hence, the extended game is a two-stage game where the first stage concerns the choice of timing, while the second stage is the proper policy game where policy instruments are to be set according to the sequence selected at the previous stage. The solution concept for the tw-stage game is the subgame perfect equilibrium by backward induction.

Now observe figure 1, where, as an example, we have drawn the isoloss curves and the best reply functions of the central bank and the government (the graph would be qualitatively equivalent in the case where the treasury controls f). As highlighted above, the reaction functions are everywhere increasing. Moreover, the loss suffered by each player decreases as we depart from the origin, with the bliss points of both authorities locating to the North-East of the intersection of best replies, which identifies the Nash equilibrium (point N).[12] For the sake of simplicity, figure 1 describes only one of the two possible Stackelberg equilibria, namely, that where the leader's role is assumed by the government (point g). In such equilibrium, both players are better off than in the Nash equilibrium. The same property would emerge in the Stackelberg equilibrium where the bank leads. This entails that, given rational expectations concerning the shocks, both Stackelberg equilibria Pareto-dominate the Nash equilibrium. That is, if players have to non-cooperatively choose the timing of moves on the basis of the expected values of their respective loss functions, then they will surely desire to avoid playing simultaneously. Once they have declared to be willing to move sequentially, neither of the players has any incentive to renege *ex post*, and therefore sequential play is part of a subgame perfect equilibrium.

[12] The fact that best reply functions intersect in the positive quadrant is obviously arbitrary, since there are infinitely many values of the two shocks such that the optimal fiscal stance may take negative values.

Equivalently, announcements are strictly adhered to by both players, and the resulting equilibrium behaviour is time consistent.

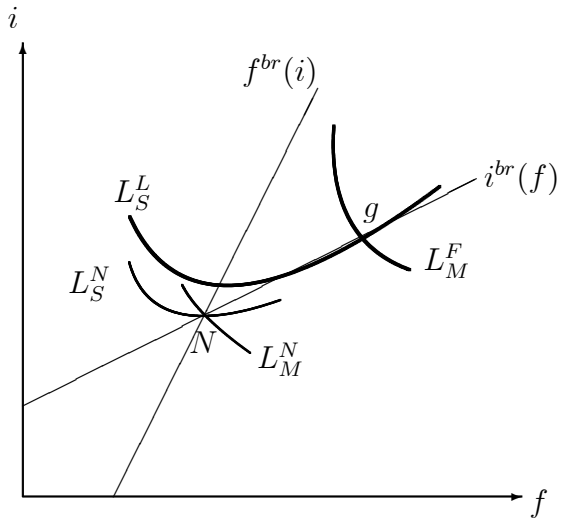


Figure 1 : Isoloss curves and reaction functions

In line of principle, the Pareto-dominance of both Stackelberg equilibria (i) does not necessarily entail that the policy game will be played sequentially instead of simultaneously, and (ii) does not solve the coordination problem. Issue (i) arises precisely because players must choose simultaneously and non-cooperatively the respective timing, involving a positive probability that simultaneous play be observed at the mixed strategy equilibrium. Issue (ii) relates to the fact that there are two Pareto-dominant Stackelberg equilibria, so that a problem of multiplicity exists *a priori*. As we shall prove in the remainder of the paper, the present model allows us to answer both questions by selecting one of the two Stackelberg equilibria, as the 'natural' solution of the game.

4 Monetary and fiscal policy coordination as a Nash equilibrium

Now we examine the case where the two authorities set their respective instruments (i, f) simultaneously and compute the resulting Nash equilibria.In this section, we consider a single country, and evaluate the government's incentive to use L_S or L_F. Notice that in all what follows, in order to economize on notation, we redefine:

$$\overline{r} + \pi^* = \pi^*$$

4.1 Case I: The treasury sets f to minimise L_F

Here, we consider the equilibrium policy setting obtained at the intersection of (7-8):[13]

$$f^{NF} = \frac{\eta\left(\alpha^2\beta\varepsilon_2 - \mu\varepsilon_1\right)}{\alpha^2\beta^2\gamma + \left(\eta^2+\gamma\right)\mu} \tag{14}$$

$$i^{NF} = \pi^* + \frac{\alpha\beta\left(\left(\eta^2+\gamma\right)\varepsilon_2 + \beta\gamma\varepsilon_1\right)}{\alpha^2\beta^2\gamma + \left(\eta^2+\gamma\right)\mu} \tag{15}$$

yielding the following equilibrium losses:

$$L_F^F = \frac{\gamma\left(\gamma+\eta^2\right)\left(\mu\varepsilon_1 - \alpha^2\beta\varepsilon_2\right)}{\left[\alpha^2\beta^2\gamma + \left(\eta^2+\gamma\right)\mu\right]^2} \tag{16}$$

$$L_M^F = \frac{\mu\left[\gamma\left(\beta\varepsilon_1+\varepsilon_2\right) + \eta^2\varepsilon_2\right]^2\left(\alpha^2\beta^2+\mu\right)}{\left[\alpha^2\beta^2\gamma + \left(\eta^2+\gamma\right)\mu\right]^2} \tag{17}$$

$$L_S^F = \frac{\mu\left[\gamma\left(\beta\varepsilon_1+\varepsilon_2\right) + \eta^2\varepsilon_2\right]^2\left(\alpha^2\beta^2+\mu\right) + \gamma\left(\gamma+\eta^2\right)\left(\mu\varepsilon_1 - \alpha^2\beta\varepsilon_2\right)}{\left[\alpha^2\beta^2\gamma + \left(\eta^2+\gamma\right)\mu\right]^2} \tag{18}$$

[13] For sufficiently negative values of both shocks (or a combination thereof), the expression in (15) becomes negative. However, it would be natural to impose a non-negativity constraint on the nominal interest rate.

4.2 Case II: The government sets f to minimise L_S

Now the government is in charge of fiscal policy, and his behavior is described by (9). At the Nash equilibrium of the game, we have:

$$f^{NS} = \frac{\eta\left[\beta\left(\alpha^2-\mu\right)\varepsilon_2 - \mu\left(1+\beta^2\right)\varepsilon_1\right]}{\alpha^2\beta^2\gamma + \left[\eta^2\left(1+\beta^2\right)+\gamma\right]\mu} \tag{19}$$

$$i^{NS} = \pi^* + \frac{\alpha\beta\left[\left(\eta^2+\gamma\right)\varepsilon_2 + \beta\gamma\varepsilon_1\right]}{\alpha^2\beta^2\gamma + \left[\eta^2\left(1+\beta^2\right)+\gamma\right]\mu} \tag{20}$$

yielding the following equilibrium losses:

$$L_M^S = \frac{\mu\left[\gamma\left(\beta\varepsilon_1+\varepsilon_2\right)+\eta^2\varepsilon_2\right]^2\left(\alpha^2\beta^2+\mu\right)}{\left[\alpha^2\beta^2\gamma + \left(\eta^2\left(1+\beta^2\right)+\gamma\right)\mu\right]^2} \tag{21}$$

$$\begin{aligned} L_F^S \;=\; & \frac{\alpha^2\beta\gamma\varepsilon_2\left[\alpha^2\beta\varepsilon_2\left(\eta^2+\gamma\right) - 2\varepsilon_1\mu\left(\eta^2\left(1+\beta^2\right)+\gamma\right)\right]}{\left[\alpha^2\beta^2\gamma + \left(\eta^2\left(1+\beta^2\right)+\gamma\right)\mu\right]^2} + \\ & \frac{\mu^2\left\{\eta^2\left[\beta^2\left(\eta^2\varepsilon_2^2 + \left(2\varepsilon_1^2+\beta^2\varepsilon_1^2+2\beta\varepsilon_1\varepsilon_2+\varepsilon_2^2\right)\gamma\right)+\varepsilon_1^2\gamma\right]+\varepsilon_1^2\gamma^2\right\}}{\left[\alpha^2\beta^2\gamma + \left(\eta^2\left(1+\beta^2\right)+\gamma\right)\mu\right]^2} \end{aligned} \tag{22}$$

$$L_S^S = L_F^S + L_M^S \tag{23}$$

In both cases, we note that both authorities react with a more restrictive policy setting (higher i, lower f) to a positive AD shock (ε_1), whereas the response is different to positive AS shocks: again restrictive for the central bank, but instead expansive for the fiscal authority. Also note that the loss function of the central bank collapses to zero (bliss point) whenever $\mu = 0$.

Now we compare the equilibrium loss which each authority obtains, when the fiscal authority is setting its instrument to minimize either L_F or L_S. For the central bank, the comparison of eqs.(17) and (21) yields:

$$L_M^F - L_M^S = \frac{\mu^2\beta^2\eta^2\left[\gamma\left(\beta\varepsilon_1+\varepsilon_2\right)+\eta^2\varepsilon_2\right]^2\left(\alpha^2\beta^2+\mu\right)\left[2\alpha^2\beta^2\gamma + \mu\left(\eta^2\left(2+\beta^2\right)+2\gamma\right)\right]}{\left[\alpha^2\beta^2\gamma+\mu\left(\eta^2+\gamma\right)\right]^2\left[\alpha^2\beta^2\gamma + \left(\eta^2\left(1+\beta^2\right)+\gamma\right)\mu\right]^2} \tag{24}$$

Expression (24) suffices to prove the following result:

Lemma 2 *Since* $L_M^F - L_M^S > 0$ *for all* $\mu > 0$, *the central bank prefers the government to set its policy according to the minimization of* L_S.

This result is intuitive, since if the government controls f, it shares part of the same burden of the central bank, and thus the central bank may use less intensely its policy instrument. In fact, we may also note from equation (24) that when the monetary policy instrument is costless ($\mu > 0$), then the central bank is indifferent to what the fiscal authority does.

For fiscal authorities, results are not quite as clear-cut. In general, we want to answer the following question: if the government wants to minimize L_S, should it then set its policy instrument, f, directly, or delegate its control to the treasury?

To proceed, let us first compare the two cases of eqs. (16) and (22). Consider the preferences of the treasury as to who must be in charge of settin f so as to obtain the minimum value of L_F :

$$L_F^F - L_F^S < 0 \text{ for all } \varepsilon_2 \in (\min\{\varepsilon_{2a}, \varepsilon_{2b}\}, \max\{\varepsilon_{2a}, \varepsilon_{2b}\}) \tag{25}$$

and conversely it is instead preferable to set f according to the minimization of L_S outside the range: $(\min\{\varepsilon_{2a}, \varepsilon_{2b}\}, \max\{\varepsilon_{2a}, \varepsilon_{2b}\})$, where:

$$\varepsilon_{2a} = -\frac{\beta\gamma\varepsilon_1}{\eta^2 + \gamma}$$

$$\varepsilon_{2b} = \frac{\beta\gamma\mu\varepsilon_1\left[\alpha^2\beta^2\left(\alpha^2\gamma\left(2+\beta^2\right) + (\eta^2+1)\,2\mu\right)\right] + \mu\left[(2\alpha^2+\mu)(\eta^2+\gamma)\right]}{\alpha^2\beta^2\left[\alpha^2\gamma\left(2\alpha^2\beta^2\gamma + 2\mu(\eta^2+\gamma) + \beta^2\mu(\eta^2-\gamma)\right) - 2\gamma^2\mu^2(1+\eta^2)\right]}$$

Second, comparing the two cases of eqs.(18)and (23) reveals that there exists a range of parameter values for which, *assuming instead that the government controls* f *to minimize* L_S, it is preferable to delegate the control of f to the treasury who will then minimise L_F:

$$L_S^F - L_S^S < 0 \text{ for all } \varepsilon_2 \in (\min\{\varepsilon_{2c}, \varepsilon_{2d}\}, \max\{\varepsilon_{2c}, \varepsilon_{2d}\}) \tag{26}$$

and conversely it is instead preferable to set f according to the minimization of L_S outside the range $(\min\{\varepsilon_{2c}, \varepsilon_{2d}\}, \max\{\varepsilon_{2c}, \varepsilon_{2d}\})$, where:

$$\varepsilon_{2c} = \varepsilon_{2a} = -\frac{\beta\gamma\varepsilon_1}{\eta^2 + \gamma}$$

$$\varepsilon_{2d} = \frac{\beta\gamma\mu\varepsilon_1 \left[\alpha^2 \left(\alpha^2\beta^2\gamma \left(2+\beta^2\right) + \left(\left(2-\beta^2\right)\eta^2 + 2\gamma\left(1-\beta^2\right)\right)\mu\right) - \mu^2\left(\gamma + \eta^2\left(1+\beta^2\right)\right)\right]}{\Psi},$$

where:

$$\begin{aligned}\Psi &= \left(\eta^2+\gamma\right)\left[\gamma+\eta^2\left(1+\beta^2\right)\right]\mu^3 + \alpha^2\beta^2\{2\alpha^4\beta^2\gamma^2 + \mu[\eta^2(\alpha^2\gamma(2+3\beta^2) + \\ &\quad \mu((2+\beta^2)\eta^2 + \gamma(4+\beta^2))) + \gamma^2(2\alpha^2 + \alpha^2\beta^2 + 2\mu)]\}\end{aligned}$$

The comparison between ε_{2b} and ε_{2d} reveals that

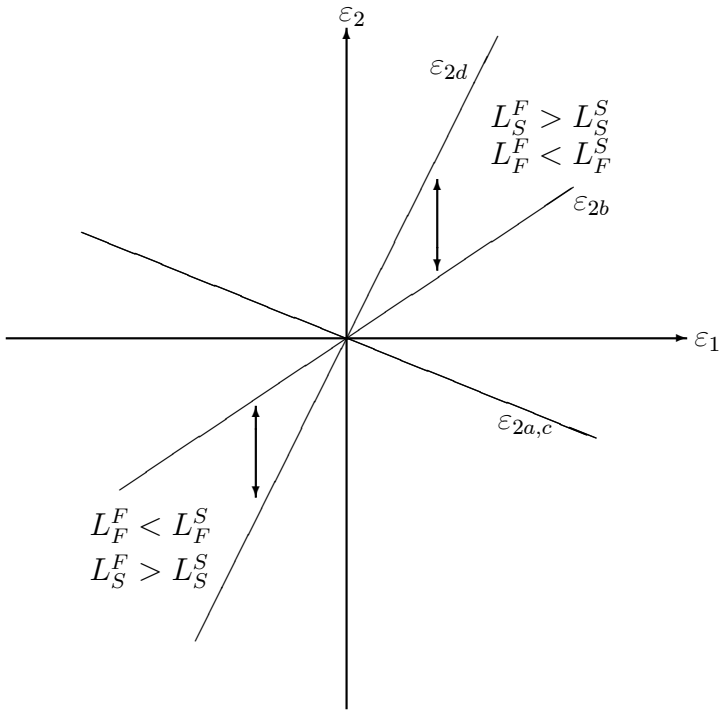
$$\varepsilon_{2d} > \varepsilon_{2b} \text{ for all } \varepsilon_1 > 0$$

and conversely. This holds for all $\mu > 0$. When $\mu = 0$, $\varepsilon_{2d} = \varepsilon_{2b} = 0$. We have thus proved the following:

Proposition 1 *Assume $\mu > 0$. Then for all $\varepsilon_2 \in (\min\{\varepsilon_{2b}, \varepsilon_{2d}\}, \max\{\varepsilon_{2b}, \varepsilon_{2d}\})$, if the government prefers to delegate the control of fiscal policy to the treasury.*

Notice that $\min\{\varepsilon_{2b}, \varepsilon_{2d}\} = \varepsilon_{2b}$ for all $\varepsilon_1 > 0$, and conversely. This implies that (taking as an example a situation when there is a positive shock to AD, i.e. $\varepsilon_1 > 0$), Proposition 1 identifies a range of AS (ε_2) shocks for which the government is better off having delegated the control over fiscal policy, even if, in fact, it cares about the loss function L_S. The reason for this puzzling result is that the government must take into account also the policy choice of the central bank, which is also optimising $L_M \subset L_S$. However, this incentive to delegate disappears when $\mu = 0$, since in this case the government is always better off by fully internalising the behaviour of the central bank. Graphically, as μ decreases towards zero, both ε_{2b} and ε_{2d} rotates downwards at different speeds. They coincide at $\mu = 0$. This situation is shown in Figure 2, where we assume $\mu > 0$, so that $\varepsilon_{2d} \gtreqless \varepsilon_{2b}$ for all $\varepsilon_1 \gtreqless 0$.

Figure 2 : Socially harmful fiscal deviations



5 The Stackelberg equilibrium with the bank as the follower

Here we consider the case where the fiscal authority minimises her loss function under the constraint given by the central bank's best reply function (7). We analyse first the case where the government delegates fiscal policy to the treasury minister.

5.1 The treasury takes the lead

The leader's problem is:

$$\begin{aligned} \max\ L_F &= (y - y^*)^2 + \gamma f^2 \\ s.t. &: i = \overline{r} + \pi^* + \frac{\alpha\beta}{\alpha^2\beta^2 + \mu}\left[\beta\left(\eta f + \varepsilon_1\right) + \varepsilon_2\right] \end{aligned} \tag{27}$$

Proceeding by substitution, the objective function (27) can be differentiated with respect to f, to obtain the leader's first order condition:

$$\frac{\partial L_F}{\partial f} = \frac{2\eta\mu\left[\mu\left(\eta f + \varepsilon_1\right) - \alpha^2\beta\varepsilon_2\right]}{\left[\alpha^2\beta^2 + \mu\right]^2} + 2\gamma f = 0 \tag{28}$$

yielding:

$$f^t = \frac{\eta\mu\left(\alpha^2\beta\varepsilon_2 - \mu\varepsilon_1\right)}{\alpha^2\beta^2\gamma\left(\alpha^2\beta^2 + 2\mu\right) + \mu^2\left(\eta^2 + \gamma\right)} \tag{29}$$

where superscript t indicates that the Treasury is leading. Loss functions at equilibrium are:

$$L_F^t = \frac{\gamma\left(\mu\varepsilon_1 - \alpha^2\beta\varepsilon_2\right)^2}{\alpha^2\beta^2\gamma\left(\alpha^2\beta^2 + 2\mu\right) + \mu^2\left(\eta^2 + \gamma\right)} \tag{30}$$

$$L_M^t = \frac{\mu\left(\alpha^2\beta^2 + \mu\right)\left[\alpha^2\beta^2\gamma\left(\beta\varepsilon_1 + \varepsilon_2\right) + \mu\left(\gamma\left(\beta\varepsilon_1 + \varepsilon_2\right) + \eta^2\varepsilon_2\right)\right]}{\left[\alpha^2\beta^2\gamma\left(\alpha^2\beta^2 + 2\mu\right) + \mu^2\left(\eta^2 + \gamma\right)\right]^2} \tag{31}$$

Accordingly, the loss of the government is $L_S^t = L_F^t + L_M^t$.

5.2 The government takes the lead

If the government sets fiscal policy so as to maximise social welfare in a Stackelberg setting where the bank follows, the leader's problem is:

$$\begin{aligned} \max\ L_S &= L_F + L_M \\ s.t. &: i = \overline{r} + \pi^* + \frac{\alpha\beta}{\alpha^2\beta^2 + \mu}\left[\beta\left(\eta f + \varepsilon_1\right) + \varepsilon_2\right] \end{aligned} \tag{32}$$

The optimal behaviour of the government is described by:

$$\frac{\partial L_S}{\partial f} = \frac{2\eta\mu\left[\beta\left(\alpha^2\beta + \mu\right)\left(\beta\left(\eta f + \varepsilon_1\right) + \varepsilon_2\right) + \mu\left(\eta f + \varepsilon_1\right) - \alpha^2\beta\varepsilon_2\right]}{\left(\alpha^2\beta^2 + \mu\right)^2} + 2\gamma f = 0 \tag{33}$$

which entails:

$$f^g = -\frac{\eta\mu\left[\alpha^2\beta\left(\beta^3\varepsilon_1 + \varepsilon_2\left(\beta^2 - 1\right)\right) + \mu\left(\varepsilon_1\left(1 + \beta^2\right) + \beta\varepsilon_2\right)\right]}{\alpha^2\beta^2\left[\gamma\left(\alpha^2\beta^2 + 2\mu\right) + \beta^2\eta^2\mu\right] + \mu^2\left[\eta^2\left(1 + \beta^2\right) + \gamma\right]} \tag{34}$$

where superscript g indicates that the government is leading. The loss of the treasury is:

$$L_F^g = \frac{\eta^2\gamma\mu^2\left[\alpha^2\beta\left(\beta^3\varepsilon_1+\varepsilon_2\left(\beta^2-1\right)\right)+\mu\left(\varepsilon_1\left(1+\beta^2\right)+\beta\varepsilon_2\right)\right]^2}{\left\{\alpha^2\beta^2\left[\gamma\left(\alpha^2\beta^2+2\mu\right)+\beta^2\eta^2\mu\right]+\mu^2\left[\eta^2\left(1+\beta^2\right)+\gamma\right]\right\}^2} + \frac{\left(\alpha^2\beta^2+\mu\right)^2\left[\gamma\mu\varepsilon_1-\beta\varepsilon_2\left(\alpha^2\gamma+\eta^2\mu\right)\right]^2}{\left\{\alpha^2\beta^2\left[\gamma\left(\alpha^2\beta^2+2\mu\right)+\beta^2\eta^2\mu\right]+\mu^2\left[\eta^2\left(1+\beta^2\right)+\gamma\right]\right\}^2} \tag{35}$$

while the loss of the central bank amounts to:

$$L_M^g = \frac{\mu\left(\alpha^2\beta^2+\mu\right)\left[\left(\alpha^2\beta^2+\mu\right)\gamma\left(\beta\varepsilon_1+\varepsilon_2\right)+\mu\eta^2\varepsilon_2\right]}{\left\{\alpha^2\beta^2\left[\gamma\left(\alpha^2\beta^2+2\mu\right)+\beta^2\eta^2\mu\right]+\mu^2\left[\eta^2\left(1+\beta^2\right)+\gamma\right]\right\}^2} \tag{36}$$

and the social loss is $L_S^g = L_F^g + L_M^g$.

Now, comparing L_S^g against L_S^t, we obtain the following expression:

$$L_S^g - L_S^t = -\frac{\beta^2\eta^2\mu^2\left(\alpha^2\beta^2+\mu\right)^2\left[\alpha^2\beta^2\gamma\left(\beta\varepsilon_1+\varepsilon_2\right)+\mu\left(\varepsilon_2\left(\eta^2+\gamma\right)+\beta\gamma\varepsilon_1\right)\right]^2}{\Omega} \tag{37}$$

where:

$$\Omega \equiv \left[\alpha^2\beta^2\gamma\left(\alpha^2\beta^2+2\mu\right)+\mu^2\left(\eta^2+\gamma\right)\right]^2\left[\alpha^2\beta^2\left(\alpha^2\beta^2\gamma+2\mu\gamma+\beta^2\eta^2\mu\right)+\mu^2\left(\gamma+\eta^2\left(1+\beta^2\right)\right)\right]$$

which reveals that $L_S^g < L_S^t$ always. Therefore, the following result holds:

Lemma 3 *Since $L_S^g < L_S^t$ holds for all admissible values of parameters and shocks, the government always prefers to set the fiscal policy according to the maximisation of social welfare.*

Now, evaluating L_M^g and L_M^t, we have $L_M^g < L_M^t$ always. Hence, we can state:

Lemma 4 *Since $L_M^g < L_M^t$ holds for all admissible values of parameters and shocks, the central bank always prefers when the government sets fiscal policy according to the social welfare function.*

Lemmata 3-4 entail the following Proposition:

Proposition 2 *When the government sets fiscal policy as the leader, the incentives of the fiscal authority and the monetary authority are reciprocally compatible.*

When the government sets fiscal policy as the leader, it is in the position of fully internalising the monetary target. This obviously goes some way towards reducing the burden of stabilization left for the central bank. To check this, it suffices to verify that $i^g < i^t$ over the whole admissible space of parameters.

6 The central bank takes the lead

Here, we consider the case where the central bank chooses the interest rate before the fiscal authority decides the size of f. Again, we have two cases: one is the situation where fiscal policy is set by the treasury, taking into account its loss function L_F; the other is the situation where the government sets fiscal policy so as to maximise social welfare $L_S = L_F + L_M$.

6.1 Case I: The treasury controls fiscal policy

The treasury plays the follower's role by minimising L_F w.r.t. f, taking the interest rate as given. This produces the following reaction function:

$$\frac{\partial L_F}{\partial f} = \left(\eta^2 + \gamma\right) f + \eta \left[\varepsilon_1 - \alpha \left(i - \pi^*\right)\right] = 0. \tag{38}$$

The problem of the central bank is:

$$\begin{aligned} \max_i L_M &= \left[\varepsilon_2 + \beta \left(\varepsilon_1 + \eta f - \alpha \left(i - \pi^*\right)\right)\right]^2 + \mu \left(i - \pi^*\right)^2 \\ s.t. &: f = -\frac{\eta \left[\varepsilon_1 - \alpha \left(i - \pi^*\right)\right]}{\eta^2 + \gamma} \end{aligned} \tag{39}$$

The first order condition is:

$$\frac{\partial L_M}{\partial i} = \frac{2\alpha\beta\gamma \left[\alpha\beta\gamma \left(i - \pi^*\right) - \varepsilon_2 \left(\eta^2 + \gamma\right) - \beta\gamma\varepsilon_1\right]}{\left(\eta^2 + \gamma\right)^2} + 2\mu \left(i - \pi^*\right)^2 = 0 \tag{40}$$

yielding the following optimal interest rate:

$$i_T^b = \frac{\alpha\beta\gamma \left[\varepsilon_2 \left(\eta^2 + \gamma\right) + \beta\gamma\varepsilon_1\right]}{\alpha^2\beta^2\gamma^2 + \mu \left(\eta^2 + \gamma\right)^2} + \pi^*, \tag{41}$$

where superscript b indicates that the bank is leading, and subscript T indicates that fiscal policy is set by the treasury. The resulting loss functions are:

$$L_{MT}^{b} = \frac{\mu\left[\gamma\left(\beta\varepsilon_1 + \varepsilon_2\right) + \eta^2\varepsilon_2\right]^2}{\alpha^2\beta^2\gamma^2 + \mu\left(\eta^2 + \gamma\right)^2} \tag{42}$$

$$L_{FT}^{b} = \frac{\gamma\left(\eta^2 + \gamma\right)\left[\mu\varepsilon_1\left(\eta^2 + \gamma\right) - \alpha^2\beta\gamma\varepsilon_2\right]^2}{\left[\alpha^2\beta^2\gamma^2 + \mu\left(\eta^2 + \gamma\right)^2\right]^2} \tag{43}$$

and social welfare $L_{ST}^{b} = L_{FT}^{b} + L_{MT}^{b}$.

6.2 Case II: The government controls fiscal policy

Here, the government plays the follower's role by minimising L_S w.r.t. f, taking the interest rate as given. This produces the following reaction function:

$$\frac{\partial L_F}{\partial f} = \left(\eta^2 + \gamma\right) f + \eta\left[\varepsilon_1 - \alpha\left(i - \pi^*\right)\right] + 2\beta\eta\left[\varepsilon_2 + \beta\left(\varepsilon_1 + \eta f - \alpha\left(i - \pi^*\right)\right)\right] = 0. \tag{44}$$

The problem of the central bank is:

$$\begin{aligned} \max_i L_M &= \left[\varepsilon_2 + \beta\left(\varepsilon_1 + \eta f - \alpha\left(i - \pi^*\right)\right)\right]^2 + \mu\left(i - \pi^*\right)^2 \\ s.t. &: f = -\frac{\eta\left[\varepsilon_1\left(1 + \beta^2\right) + \beta\varepsilon_2 - \alpha\left(1 + \beta^2\right)\left(i - \pi^*\right)\right]}{\eta^2\left(1 + \beta^2\right) + \gamma} \end{aligned} \tag{45}$$

From the first order condition we obtain the optimal interest rate:

$$i_G^{b} = \frac{\alpha\beta\gamma\left[\varepsilon_2\left(\eta^2 + \gamma\right) + \beta\gamma\varepsilon_1\right]}{\alpha^2\beta^2\gamma^2 + \mu\left[\eta^2\left(1 + \beta^2\right)\left(\eta^2\left(1 + \beta^2\right) + 2\gamma\right) + \gamma^2\right]} + \pi^*, \tag{46}$$

where subscript G indicates that fiscal policy is set by the government. The associated loss of the central bank is:

$$L_{MG}^{b} = \frac{\mu\left[\varepsilon_2\left(\eta^2 + \gamma\right) + \beta\gamma\varepsilon_1\right]^2}{\alpha^2\beta^2\gamma^2 + \mu\left[\eta^2\left(1 + \beta^2\right)\left(\eta^2\left(1 + \beta^2\right) + 2\gamma\right) + \gamma^2\right]}. \tag{47}$$

The loss of the treasury is:

$$L_{FG}^{b} = \frac{\eta^2\gamma\left[\mu\left(1 + \beta^2\right)\left(\varepsilon_1\left(\eta^2\left(1 + \beta^2\right) + \gamma\right) + \beta\eta^2\varepsilon_2\right) - \alpha^2\beta\gamma\varepsilon_2\right]^2}{\Gamma} + \tag{48}$$

$$\frac{\left[\mu\left(\eta^2\left(\gamma\varepsilon_1-\beta\eta^2\varepsilon_2\right)\left(1+\beta^2\right)+\gamma\left(\gamma\varepsilon_1-\beta\eta^2\varepsilon_2\right)\right)-\alpha^2\beta\gamma^2\varepsilon_2\right]^2}{\Gamma}$$

where

$$\Gamma=\left[\alpha^2\beta^2\gamma^2+\mu\left[\eta^2\left(1+\beta^2\right)\left(\eta^2\left(1+\beta^2\right)+2\gamma\right)+\gamma^2\right]\right]^2. \tag{49}$$

The resulting social loss is $L^b_{SG}=L^b_{FG}+L^b_{MG}$.

6.3 The follower's choice

Here, we want to assess whether the government finds it convenient to set fiscal policy so as to maximise social welfare or allowing the treasury to manoeuvre the budget.

First, comparing the levels of L_M in the two cases, we have:

Lemma 5 *Since $L^b_{MG}<L^b_{MT}$ for all values of parameters and shocks, the central bank prefers the government to set fiscal policy so as to maximise social welfare.*

Then, note that

$$i^b_G-i^b_T\propto-\left[\beta\gamma\varepsilon_1+\varepsilon_2\left(\eta^2+\gamma\right)\right] \tag{50}$$

i.e., if both shocks are positive, $i^b_G<i^b_T$; if both shocks are negative, $i^b_G>i^b_T$. If instead shocks have opposite signs, then things can go either way. This entails that, when the bank leads, the government will not necessarily choose to support or contrast its policy stance. The ultimate implication of the above inequality is that the government will surely prefer the treasury to set f in some region of $\{\varepsilon_1\,,\,\varepsilon_2\}$. For future reference, note that

$$i^b_G=i^b_T \text{ at } \varepsilon_2=-\frac{\beta\gamma\varepsilon_1}{\eta^2+\gamma}\,. \tag{51}$$

Now compare L^b_{SG} and L^b_{ST}. They coincide at

$$\varepsilon_{2m}=-\frac{\beta\gamma\varepsilon_1}{\eta^2+\gamma}\,; \tag{52}$$

$$\begin{aligned}\varepsilon_{2n} \;=\;& \gamma\mu\varepsilon_1\left[\gamma^3\left(3\gamma^2+14\gamma+15\right)+\gamma\mu\left(2\gamma^4+18\gamma^3+50\gamma^2+57\gamma+24\right)+\right.\\ & -\mu^2\left(8+\gamma\left(\gamma^4+8\gamma^3+25\gamma^2+38\gamma+28\right)\right)\left.\right]/\left[2\gamma^5\left(3+2\gamma\right)+\gamma^2\mu\left(15\gamma+\right.\right.\\ & 27\gamma^2+19\gamma^3+5\gamma^4+\left(7+19\gamma+20\gamma^2+10\gamma^3+2\gamma^4\right)\mu\left.\right)+\mu^3\left(8+\right.\\ & \left.\left.\gamma\left(36+66\gamma+63\gamma^2+33\gamma^3+9\gamma^4+\gamma^5\right)\right)\right]\,.\end{aligned} \tag{53}$$

Examining the sign of $L^b_{SG}-L^b_{ST}$, we have the following:

Lemma 6 $L^b_{SG} - L^b_{ST} > 0$ *for all* $\varepsilon_2 \in (\min\{\varepsilon_{2m}, \varepsilon_{2n}\}, \max\{\varepsilon_{2m}, \varepsilon_{2n}\})$, *and conversely outside this interval.*

Then, examine the preferences of the treasury. We have $L^b_{FG} = L^b_{FT}$ at:

$$\varepsilon_{2o} = -\frac{\beta\gamma\varepsilon_1}{\eta^2 + \gamma}\ ;\ \varepsilon_{2p} = \overline{\varepsilon}_{2p} \tag{54}$$

where the expression of $\overline{\varepsilon}_{2p}$ is too long to be printed and, in general, for positive values of ε_1, $\overline{\varepsilon}_{2p} > \varepsilon_{2n}$ for sufficiently high values of the ratio γ/μ, while $\overline{\varepsilon}_{2p} < \varepsilon_{2n}$ for sufficiently low values of the ratio γ/μ. The opposite holds for negative values of ε_1.

The preferences of the treasury on the objective of fiscal policy are described by the sign of $L^b_{FG} - L^b_{FT}$. Given the roots ε_{2o} and ε_{2p}, its preferences are summarised by the following Lemma:

Lemma 7 $L^b_{FG} - L^b_{FT} > 0$ *for all* $\varepsilon_2 \in (\min\{\varepsilon_{2o}, \varepsilon_{2p}\}, \max\{\varepsilon_{2o}, \varepsilon_{2p}\})$, *and conversely outside this interval.*

On the basis of Lemmata 6-7, we can claim the following:

Proposition 3 *When the central bank leads, there are shock configurations such that the government may want to delegate control over fiscal policy to the treasury.*

We can outline graphically the region of shocks where there exists a conflict between the treasury and the government as to who has to be in charge of fiscal policy.

When γ/μ is large enough, $\overline{\varepsilon}_{2p} > \varepsilon_{2n}$ for positive ε_1 and $\overline{\varepsilon}_{2p} < \varepsilon_{2n}$ for negative ε_1. Therefore:

$$\text{for all } \varepsilon_2 \in (\min\{\varepsilon_{2n}, \varepsilon_{2p}\}, \max\{\varepsilon_{2n}, \varepsilon_{2p}\}), \tag{55}$$

both authorities would like to set the fiscal policy.

Conversely, if γ/μ is low enough, $\overline{\varepsilon}_{2p} < \varepsilon_{2n}$ for positive ε_1 and $\overline{\varepsilon}_{2p} > \varepsilon_{2n}$ for negative ε_1. Therefore, for all $\varepsilon_2 \in (\min\{\varepsilon_{2n}, \varepsilon_{2p}\}, \max\{\varepsilon_{2n}, \varepsilon_{2p}\})$, both authorities would like that the other sets fiscal policy.

7 Equilibrium selection

As we have illustrated in section 3, relying upon d'Aspremont and Gérard-Varet (1980) and Hamilton and Slutsky (1990), we can select between Nash and Stackelberg equilibria. The selection mechanism is based on the slope of reaction functions, which are everywhere increasing. Therefore, both Stackelberg equilibria Pareto-dominate the Nash equilibrium. Moreover, if there exists a preplay stage where players non-cooperatively and simultaneously choose the timing of their moves, both Stackelberg equilibria are Nash equilibria of such preplay stage.

The next question is how to identify which will be selected, among the multiplicity of Stackelberg equilibria (two in pure strategies and one in mixed strategy). While formally they are all plausible outcomes, in practice two independent factors point to the fiscal authority becoming the Stackelberg leader. First, this is the outcome which minimizes frictions within the government and between the central bank and the government. As we noted above *(Proposition 3)*, if the central bank leads, then there are shock configurations where the government might want to set fiscal policy according to the minimization of L_F, without taking into account the inflation objective. This would create confusion between the different levels of government (why is the government switching between different obiective functions at different times?), and also in the face of the central bank and of course of the public opinion. This confusion would be further aggravated in a monetary union such as EMU, because it would imply switching the decision level of fiscal policy back and forth between the EC and the national authorities.

The second reason for the fiscal authority to become the Stackelberg leader is inherent to the institutional process. Since the leader commits to the first move, it would be highly implausible if this were the central bank, as the decision to fix *ex-ante* and once-and-for-all the interest rate level would be unprecedented! Quite to the contrary, in practice we observe that fiscal policy is generally set prior to monetary policy, and revised much less frequently. Typically, fiscal policy is set once a year, whereas monetary policy is usually revised, both in EMU and in the US, every two weeks. This situation is interpretable as one where the fiscal authority is the first mover, i.e. the Stackelberg leader. As we noted in the previous section (*Lemma 3*), in this case there would also be no doubt as to the choice of the appropriate objective function, nor would the central bank ever want to question the stance adopted by the fiscal policy (*Lemma 4*).

Hence, on the basis of the above reasoning, we conclude that it is both prefarable and quite probable that the fiscal authority will in practice emerge as the Stackelberg leader in the macroeconomic policy game.

8 Conclusions

In this paper we have examined the interrelations between monetary and fiscal policies, in a game situation where both policies are set consistenly over time and with each other, and both policy instruments are costly to operate. This implies that there is a cost associated to changes in the (real) interest rate and one associated to non-zero budget levels. The latter cost is modelled after the requirement of the Stability and Growth Pact, that budget levels should be balanced over the medium term.

We also assume that the government is motivated by minimization of a welfare function defined in terms of output and inflation deviations from their natural or target levels, and delegates the "inflation subset" of this function to the central bank. This situation is modelled after the constitutional mandate of the ECB towards maintaining price stability. However, this also leads to an interesting question: having delegated to an independent agent (the ECB) part of its welfare function, should the government then set its own policy instrument (the fiscal stance) according to the overall welfare criterion or only to that part which has *not* been delegated to the ECB? In the paper we refer to these alternatives respectively as the "government view" (whereby fiscal policy is set to minimize L_S) and the "treasury view" (whereby fiscal policy is set to minimize L_F). This distinction becomes even more interesting and to the point if we additionally interpret it in the context of a monetary union with decentralized fiscal policies (as is the case of EMU), since the treasury view can naturally be attributed to national governments (which do not internalize the consequences of their action for the union-wide rate of inflation) while the government view can naturally be attributed to the European Commission. In fact the Commission is required to monitor the application of the Stability and Growth Pact, hence it must also take into account the possible inflationary implications of the fiscal policies adopted by the national governments (Formally this takes place through the adoption of the Broad Economic Policy Guidelines). Hence, while we do not model explicitly the interaction between national authorities and the EC, our results can easily be referred also to this case.

Turning to the formal analysis of policy interactions, as it was pointed out in the previous section, our results suggest that both fiscal and monetary authorities prefer the outcome of a Stackelberg to that of a Nash game,independently of whom is the leader. However, the nature of the game is such that, if asked, each player would leave to the other the *disadvantage* of the first move. In our context, this is quite intuitive: since the two authorities place different but non-conflicting weights on the welfare goals, each one would prefer to be the last one to move, to turn the overall result in the preferred direction. This raises the question of which particular Stackelberg solution will then emerge in practice.

In this respect we argued in the previous section that in practice two independent factors point to the fiscal authority becoming the Stackelberg leader. The first reason is that, if this were not the case, then depending on specific shock configurations the government might want to set fiscal policy either according to the minimization of L_F (that is, without taking into account the inflation objective) or of L_S. This would create confusion between the different levels of government (why is the government switching between different obiective functions at different times?), and also in the face of the central bank and of course of the public opinion. This confusion would be further aggravated in a monetary union such as EMU, because it would imply switching the decision level of fiscal policy back and forth between the EC and the national authorities.

The second reason for the fiscal authority to become the Stackelberg leader is imbedded into the institutional process. In practice we always observe that fiscal policy is set prior to monetary policy, and revised much less frequently. Typically, fiscal policy is set once a year, whereas monetary policy is usually revised, both in EMU and in the US, every two weeks. This situation is interpretable as one where the fiscal authority is the first mover, i.e. the Stackelberg leader.

Hence we may conclude that the fiscal authority will in practice emerge as the Stackelberg leader in the macroeconomic policy game, and that this situation is indeed desirable. Notice also that in this case, according to *Lemma 2* the government will always want to set fiscal policy according to the minimization of L_S (the all-inclusive social welfare function). In the context of EMU, this implies that the fiscal stance should *not* be decided looking only to the the stabilization of national output levels. In other words, this result points to the desirability of a coordination process of national fiscal policies, whereby they are set taking into account also the goal of monetary

policy. In practice, this yields support to the goals stated in the Stability and Growth Pact and to the adoption of the Broad Economic Policy Guidelines between members of the monetary union.

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