Empirical Analysis of Limit Order Markets*

Burton Hollifield[†]
Robert A. Miller[‡]
Patrik Sandås[§]

April, 2002

Abstract

We characterize the optimal order submission strategy in a limit order market. Traders submitting market or limit orders trade off the order price against the execution probabilities and picking off risks of alternative order submissions. The optimal order strategy is a monotone function of a trader's valuation for the asset. We develop and compute a semiparametric test of the monotonicity restriction using the order flow and limit order book for Ericsson, one of the most traded stocks on the Stockholm Stock Exchange. We fail to reject the monotonicity restriction for buy orders or sell orders separately. We reject the monotonicity restriction when we combine buy and sell orders. The expected payoffs from submitting limit orders away from the quotes are too low.

Keywords: limit orders, market orders, order submission strategy, market microstructure
JEL Codes: G10, C14, C35, C52

*We would like to thank Aydogan Alti, Dan Bernhardt, Bruno Biais, David Brown, George Deltas, Zvi Eckstein, Pierre Hillion, Ananth Madhavan, Jonas Niemeyer, Christine Parlour, John Rust, Duane Seppi, and participants at many different workshops for useful comments. We also thank the Stockholm Stock Exchange, Stockholms Fondbörs Jubileumsfond, and Dextel Findata AB for providing the sample. Hollifield gratefully acknowledges financial support from the Social Sciences Research Council of Canada and the Vancouver Stock Exchange. Part of this research was conducted while Hollifield was with the Faculty of Commerce at University of British Columbia. Sandås gratefully acknowledges financial support from the Alfred P. Sloan and the W.L. Mellon Foundations.

†Corresponding author: Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA, 15213, e-mail: burtonh@andrew.cmu.edu

‡Graduate School of Industrial Administration, Carnegie Mellon University

§The Wharton School, University of Pennsylvania, and CEPR

1 Introduction

Much of the trading in financial markets is done through limit order markets. In a limit order market, traders can submit two main types of buy and sell orders. A buy market order fills immediately at the most attractive sell price posted by previously submitted sell limit orders in the limit order book. A buy limit order specifies a particular price, but does not guarantee that the order will be filled. Unfilled buy limit orders enter the limit order book, where they are stored until they are canceled or are filled against incoming sell market orders. In most limit order markets, the traders observe a portion of the limit order book so their order submission strategies may depend on such information.

We develop the empirical restrictions of traders' optimal order submission strategies in a limit order market. We test the restrictions using the order flow and the limit order book for Ericsson, one of the most traded stocks on the Stockholm Stock Exchange. The optimal order submission depends on the trader's valuation for the asset, and the trade-offs between the order price, the risk that an order fails to fill, and the risk that order fills when the underlying value of the asset has moved against the trader. The optimal order submission strategy changes as the trade-offs change with market conditions, and is monotone in the trader's valuation for the asset. The monotonicity of the optimal order submission strategy can be tested without observing the traders' valuations for the asset. We fail to reject the monotonicity restriction for buy orders or sell orders separately. We reject the monotonicity restriction when we combine buy and sell orders. The expected payoffs from submitting limit orders with a high risk that the order fails to fill are too low.

Using the order flow and the limit order book from the Paris Bourse, Biais, Hillion, and Spatt (1995) document that traders are more likely to submit limit orders when the limit order book contains relatively few orders, or when the distance between the best buy and sell limit order prices in the book increases. In addition, order submissions are autocorrelated. For example, the conditional probability of observing a limit order at a particular price is higher when the previous order was a limit order at that same price.

A market order submitted on the New York Stock Exchange may trade with the market maker, a floor broker, or the electronic limit order book. Handa and Schwartz (1996) use a sample of New York Stock Exchange transaction prices to compute the average payoffs from submitting hypothetical market and limit orders, and show that the payoffs on hypothetical limit orders are at least as high as the payoffs on hypothetical market orders. Harris and Hasbrouck (1996), Peterson and Sirri (2001), and Lo, MacKinlay, and Zhang (2001) use actual limit and market order submissions from the New York Stock Exchange.

Harris and Hasbrouck (1996) compute the expected payoffs from submitting market or limit orders for two traders–one who is committed to trade and another who is indifferent. The expected payoffs change with the limit order book and the best available prices in the market. Traders change their order submission strategies as the limit order book and the best available prices change, tending to submit orders with the highest expected payoffs. A marketable buy limit order has a price that is equal to or higher than the best sell limit order in the book. Peterson and Sirri (2001) show that a selection bias explains why previous studies find that the payoffs are lower for marketable limit orders than for market orders. Lo, MacKinlay, and Zhang (2001) show that the time-to-fill for limit orders can be predicted using the best available prices, the limit order price and the lagged order flow.

The empirical findings are consistent with theoretical models of optimal order submission. Limit orders offer a better price than market orders, but may fail to fill. The probability that a limit order fills is called the execution probability. Cohen, Maier, Schwartz, and Whitcomb (1981) and Parlour (1998) show how optimal order submission strategies depend on the traders' preferences and their subjective beliefs about the execution probabilities for limit orders. In Parlour (1998) order submissions are autocorrelated because past order submissions have a systematic effect on equilibrium execution probabilities.

A limit order only transacts with a newly submitted market order. If traders do not continuously monitor their limit orders, then their limit orders may fill when the value of the asset has changed. Copeland and Galai (1983) point out that a commitment to trade at a fixed price is equivalent to writing a free option to the other traders in the market. The free option property of limit orders is called picking off risk. Harris (1998) and Foucault (1999) solve for the trader's optimal order submission strategies in settings where limit orders may fail to fill and face picking off risk. In Foucault (1999), the trader's optimal order submission depends on his valuation for the asset and the equilibrium execution probabilities and picking off risks associated with different orders.

Biais, Martimort, and Rochet (2000), Glosten (1994), Parlour and Seppi (2001), Rock (1996), and Seppi (1997), derive the equilibrium limit order book in single-period settings with adverse selection, where the limit order submitters trade off the price against the execution probability and the expected value of the asset, conditional upon filling. The models assume that the equilibrium limit order book satisfies a zero expected profit condition. Sandås (2001) tests and rejects the zero expected profit conditions for limit order submitters using a sample from the Stockholm Stock Exchange.

What restrictions do the trade-offs impose on order submissions? Our contribution is to provide the restrictions, and to compute a semiparametric test of them. In our model, the trader's optimal order submission depends on his valuation for the asset and subjective beliefs about execution probabilities and picking off risks. The optimal order submission strategy is a monotone function of the trader's valuation and is formed from the trader's subjective beliefs about the execution probabilities and picking off risks. We use the actual order submissions, the realized order fills, and a rational expectations assumption to form estimates of execution probabilities and picking off risks.

A buy limit order only fills after it becomes the highest-priced unfilled limit order in the book, and a sell market order is submitted by another trader. Consequently, traders must predict future traders' order submissions to determine the execution probabilities and the picking off risks associated with alternative order submissions. In a stationary environment, the execution probabilities and picking off risks associated with alternative order submissions have sample analogues.

Hotz and Miller (1993) and Manski (1993) suggest using nonparametric methods to estimate agents' conditional expectations for alternative decisions directly from the sample. The nonparametric estimates are used to identify and estimate the structural parameters in discrete choice models. Such an approach has been applied to estimate models in labor economics by Hotz and Miller (1993), Miller and Sanders (1997), Altug and Miller (1998), and to estimate models in industrial organization by Slade (1998) and Aguirregabiria (1999). Elyakime, Laffont, Loisel, and Vuong (1994) and Guerre, Perrigne, and Vuong (2000) use the empirical bid distribution to estimate bidders' conditional expectations to estimate structural auction models. We use our sample to estimate the execution probabilities and picking off risks for alternative order submissions. Our nonparametric estimates capture the rich dynamics that arise from traders adjusting their strategies to changing market conditions in the future.

Most market microstructure theory makes predictions about traders' behavior. The widespread adoption of electronic trading systems makes available, for many markets, datasets with detailed information about the orders that traders submitted and the expected payoffs on orders that traders could have submitted. Such datasets allow for tests of theories about trader behavior, using methods similar to ours.

The New York Stock Exchange recently introduced NYSE OpenBook, making information about the electronic limit order book more easily available to traders. Using data from periods prior to the introduction of OpenBook, Harris and Panchapagesan (1999), Kavajecz (1999), Ready (1999), Kavajecz and Odders-White (2001) report that the specialist uses information in the limit order book in setting his price quotes and deciding which market orders to trade with. Theoretical work by Rock (1996), Seppi (1997), Ready (1999) and Parlour and Seppi (2001) show how the specialist's ability to trade against incoming market orders is a source of picking off risk for the traders submitting limit orders. Our empirical methods can be used to estimate the response of limit order submitters to the introduction of NYSE OpenBook in the presence of a specialist.

2 Description of the Market and the Sample

In 1990 the Stockholm Stock Exchange completed the introduction of a limit order market system, the Stockholm Automated Exchange. There are no floor traders, market makers, or specialists with special quoting obligations or trading privileges. Trading is continuous from 10 a.m. to 2:30 p.m. with the opening price determined by a call auction. All order prices are required to be multiples of a fixed minimum price unit, called the tick size. When prices are below 100 SKr, the tick size is 1/2 SKr, and when prices exceed 100 SKr, the tick size is 1 SKr. During the sample period $1 was roughly equal to 6.25 SKr. The order size must be an integer multiple of a round lot, with a typical round lot size equal to 100 shares.

All trading is between market and limit orders. Unfilled limit orders are stored in the electronic limit order book and automatically fill against incoming market orders. Unfilled limit orders in the order book are prioritized first by price and then by time of submission. The prices of the sell limit orders in the book are called ask quotes, and the prices of the buy limit orders in the book are called bid quotes. If an incoming market order is for a smaller quantity than the quantity at the best quote in the book, the market order will trade in full at a price equal to the best quote. If the market order cannot be filled completely at the best quote, it will transact against multiple quotes in the book until either it is filled in full or the book is empty. Any unfilled portion of a market order converts into a limit order.

A limit order can be canceled at any time at no cost. Traders can also submit hidden limit orders, where only a portion of the order quantity is displayed in the order book. The hidden part of a limit order has lower priority than all displayed limit orders at the same order price level.

Only exchange member firms can enter orders directly into the trading system. A member firm trades both as a broker on behalf of customers, and as a dealer on its own behalf. Over our sample period, there were 24 exchange member firms, including all major Swedish banks as well as most brokerage firms that actively trade Swedish securities. We refer to the member firms as brokers.

The brokers are directly connected to the trading system and observe all quotes with the corresponding total order quantities. The brokers' information is updated almost instantaneously after order submissions or cancellations. Traders who are not directly connected to the system can obtain information about the five best bid and ask quotes and the corresponding order quantities through information vendors such as Reuters or Telerate. Commissions are negotiable. Average commissions were 0.5% of the value of the order over the sample period. There is a fixed exchange fee of less than 7 SKr and a variable exchange fee of 0.004% to submit a market order, and a fixed exchange fee of less than 6 SKr and a variable exchange fee of 0.003% to submit a limit order.

Until January 1, 1993, the Stockholm Stock Exchange was the only authorized marketplace for equity trading in Sweden. Many of the stocks listed on the exchange were also cross listed on foreign exchanges; trading in London on the international Stock Exchange Automated Quotation (SEAQ) and in the United States on the National Association of Securities Dealers Automated Quotation (NASDAQ) system accounted for a significant fraction of the trading of many Swedish stocks. Trading abroad was attractive because of the transaction tax of 0.5% levied on equity trades in Sweden until December 1, 1991. Our sample begins after the transaction tax was abolished.

Brokers can settle trades larger than 100 round lots outside the Stockholm Automated Exchange system. A trade of 100–500 round lots settled outside the system where both sides of the order are represented by the same broker is called an internal cross. An internal cross must have a transaction price between the best quotes in the limit order book. A trade of more than 500 round lots settled outside the system is called a block trade and can be settled at any price.

The Stockholm Stock Exchange provided us with the order records and the trade records directly from the Stockholm Automated Exchange system for the 59 trading days between December 3, 1991, and March 2, 1992 for Ericsson. The order records is a chronological list of order submissions, changes in the outstanding order quantities, and order cancellations. The trade records is a chronological list of transactions. Each limit order receives a unique code, and subsequent changes in the outstanding order quantity are recorded using the same code. We combine changes in the outstanding order quantity and the transactions to determine whether a change in the order quantity was caused by a trade or a cancellation. We reconstruct complete transaction and cancellation histories for limit orders and the entire history of the order book over our sample.

We have detailed information, but there are limitations. We can only identify the broker submitting the order and cannot distinguish the trades that a broker makes on his own behalf from the trades that he makes on behalf of his customers. Therefore, we cannot link orders submitted by the same traders at different times. Also, we do not observe whether or not an order includes a hidden order quantity component. We can infer that an order must have involved some hidden quantities only if the displayed proportion of the hidden order is executed in full. In our sample, there are few hidden orders whose displayed portions fully execute.

There is a censoring problem in our sample; some limit orders remain unfilled at the end of our sample period. To minimize the effects of censoring on our analysis, we do not use orders submitted during the last two days of our sample in our empirical work. Only 2.8% of the orders remain in the system for more than two trading days, and 62.3% of such orders are eventually canceled. We discard orders submitted during the first three minutes of the trading day to ensure that the sample reflects only continuous trading. The filtering rules leave us with 20,760 observations of individual market order and limit order submissions, and their realized fills.

Table 1 reports descriptive statistics on the daily trading activity for Ericsson. The tick size varies between 1/2 SKr and 1 SKr since the price fluctuates below and above 100 SKr. The average daily close-to-close return is 0.21% with a standard deviation of 3.04%. For comparison, the table also reports statistics on close-to-close returns computed using prices of Ericsson shares on NASDAQ from January 2, 1989, through to December 31, 1993, using data from the Center for Research in Security Prices. The return distribution in our sample is not unusual.

All 24 brokers trade shares in Ericsson at some point during the sample period. The fourth row of Table 1 reports that on average 19 brokers make a trade on any given day. Sorting the brokers by their trading volume, the top 3 brokers each transact 10% to 11% of the total trading volume, and the next 7 brokers each transact 5% to 9% of the total trading volume. The numbers are almost identical for order submissions.

The daily trading volume on the Stockholm Automated Exchange is reported on the fifth row of Table 1. The sixth through eight rows of Table 1 report descriptive statistics for orders crossed internally by brokers, block trades during regular trading hours, and after-hours trading. The ninth row reports the total trading volume.

The first column of Table 2 reports the number of buy and sell market and limit orders. The second column reports the average execution probabilities, equal to the average of the fraction of the limit order quantity that is filled within two trading days of order submission. The execution probabilities show the unconditional trade-off between the execution probability and the order price; limit orders at prices farther away from the quotes have lower execution probabilities than limit orders with prices closer to the quotes. The third column of Table 2 report the average time-to-fill for limit orders measured in minutes from the order submission. Limit orders at prices farther away from the quotes take longer to fill than limit orders at prices closer to the quotes. The final three columns of Table 2 report the mean, median, and standard deviation of the order quantity.

The first six rows of Table 3 provide information on the order quantities in the limit order books. An incoming buy market order transacts against the best ask quote, if the market order quantity is less than or equal to the quantity available at the best ask quote. The average quantity at the best bid or ask quote is roughly nine times the average market order quantity; only 12 market orders in our sample are for quantities that are larger than the quantities available in the order book at the best quote at the time of order submission. The order quantities in the limit order book are volatile. The standard deviations of the order quantities at the three best bid or ask quote levels are all greater than 173 round lots. The standard deviations of the cumulative order quantities are all greater than 294 round lots.

The last six rows of Table 3 provide information on the price quotes. The median distances between the quotes and the mid-quotes equal 0.5, 1.5, and 2.5 ticks for the best three bid and ask quotes. The medians are equal to the minimum values; the three best price quotes on the bid and ask sides are spaced one tick apart for at least half of the observations.

A limit order may be completely filled, may be completely canceled, or may be partially filled and then canceled. The top plot in Figure 1 is the sample survivor function for limit orders. The sample survivor function evaluated at t is the probability that a limit order remains outstanding for at least t minutes. We account for partial order fills by weighting each fill or cancellation by the fraction of the submitted order quantity filled or canceled at that time.

Most limit orders leave the book quickly; 4.26% of the limit orders last for more than one trading day (270 minutes), 2.84% last for more than two trading days, and 1.65% last for more than three trading days. The bottom two plots in Figure 1 show the cumulative distribution function for order fill and cancellation times. We use the weighting scheme described above to handle partial fills. For all limit orders that eventually fill, 90% of the fills occur within three hours. For all limit orders that eventually cancel, 70% of the cancellations occur within three hours.

Biais, Hillion, and Spatt (1995) document that traders' order submissions depend on the order book in the Paris Bourse. Griffiths, Smith, Turnbull, and White (2000) and Ranaldo (2002) use ordered probit models to show that order submissions in the Toronto Stock Exchange and the Swiss Stock Exchange can be predicted by the order book and the lagged order flow. To determine if order submissions can be predicted in our sample, we estimate ordered probits for buy order submissions and sell order submissions. Table 4 contains definitions of the conditioning variables.

We condition on two measures of the length of the order book queue, using the total number of shares offered in the order book within one tick of the mid-quote and the total number of shares offered in the order book within three ticks of the mid-quote. We do not condition on the bid-ask spread, since it is relatively constant over the sample. Biais, Hillion, and Spatt (1995) report that order submission activity in the Paris Bourse tends to be clustered in time; the execution probability may depend upon lagged activity. We measure activity by trading volume over the previous ten minutes. We condition on the standard deviation of the changes in the OMX index over the minimum of the previous sixty minutes or the number of minutes elapsed since the market's opening. We also condition on the log of the order quantity.

The estimated coefficients and associated standard errors are reported in Table 5. We reject the null hypotheses that all coefficients are jointly equal to zero at the 1% level in both probits. The ordered probits show that in our sample, buy and sell order submissions are predictable using information in the order book and lagged order flow.

3 Model

We focus on a representative trader's order submission decision in a limit order market. At time t, one trader has the opportunity to submit an order. The trader is risk neutral, characterized by his valuation for the asset, v_t, and the quantity that he wishes to trade, q_t.

We decompose v_t into two components,

$$v_t = y_t + u_t. \tag{1}$$

The random variable y_t is the common value of the asset at time t; one interpretation is that it is equal to the traders' expectations of the liquidation value of asset. The common value changes as the traders learn new information, with

$$y_{t+1} = y_t + \delta_{t+1}. \tag{2}$$

Innovations in the common value δ_{t+1} satisfy

$$E_t[\delta_{t+1}] = 0, \tag{3}$$

where the subscript t denotes conditioning on information available after the common value is known at time t, but before the trader arrives at time t. The distribution of common value innovations has bounded support. Common value innovations are drawn from a stationary process, and the distribution of the innovations is conditioned upon the history of common value innovations through a finite dimensional set of sufficient statistics. Conditioning on a set of sufficient statistics allows for persistence in the higher-order moments of the innovations. For example, the current volatility of the common value innovations may predict the future volatility of the common value innovations.

The random variable u_t is the trader's private value for the asset. The private value is drawn from a continuous distribution

$$\Pr_t(u_t \le u) \quad \equiv \quad G_t(u), \tag{4}$$

where, as above, the subscript t denotes conditioning on information available after the common value is known at time t, but before the trader arrives at time t. The distribution of the private value has continuous density and bounded support. The private value is drawn from a stationary process. The conditional distribution of the private value depends on the same finite dimensional vector of sufficient statistics as the conditional distribution of common value innovations. Once a trader arrives at the market, his private value remains constant while he has an order outstanding.

At a random time, $t + \tau_{cancel}$, after the trader arrives at the market, the payoff from any unfilled limit orders placed by the trader will go to zero, causing the trader to cancel any unfilled limit orders. The trader does not know the cancellation time at time t. The conditional distribution of the cancellation time depends upon the same finite dimensional vector of sufficient statistics as the common value innovations. Let $\Upsilon < \infty$ be the maximum possible life of the order,

$$\Pr_t(\tau_{cancel} \le \Upsilon < \infty) = 1. \tag{5}$$

Realizations of the cancellation time are independent of all other random variables in the model.

The trader's desired order quantity, q_t, is independent of the trader's valuation, and is drawn from a distribution with bounded support. The conditional distribution of the order quantity depends upon the same set of sufficient statistics as the common value innovations.

At time t, the trader has a single opportunity to submit either a market order or a limit order. The trader observes the queue of orders in the limit order book, the current common value, and the history of common value innovations.

We use the decision indicators $d_{t,s}^{sell}$ for $s = 0, 1, \ldots, S$, and $d_{t,b}^{buy}$ for $b = 0, 1, \ldots, B$ to denote the trader's order submission at time t. If the trader submits a sell market order, then $d_{t,0}^{sell} = 1$, and the order price is equal to the best bid quote. If the trader submits a buy market order, then $d_{t,0}^{buy} = 1$, and the order price equals the best ask quote. If the trader submits a sell limit order at the price s ticks above the current best bid quote, then $d_{t,s}^{sell} = 1$. If the trader submits a buy limit order at the price b ticks below the current best ask quote, then $d_{t,b}^{buy} = 1$. If the trader does not submit any order at time t, then $d_{t,s}^{sell} = 0$ for all s and $d_{t,b}^{buy} = 0$ for all b. Both $S < \infty$ and $B < \infty$, so that the trader chooses from a finite set of order prices.

The trader pays a cost of c per share to submit an order. The cost is the same for all types of orders submitted. We can also allow for a fixed cost for order submissions and fixed and variable costs for filling an order.

Suppose that at time t a trader with valuation $v_t = y_t + u_t$ submits a buy order of quantity q_t, at a price $p_{t,b}$, b ticks below the current best ask quote, so that $d_{t,b}^{buy} = 1$. Define $dQ_{t,t+\tau}$ as the number of shares of the order submitted at t that transact at $t+\tau$. If the realized cancellation time is $t + \tau_{cancel}$, then $dQ_{t,t+\tau} = 0$ for all $\tau \geq \tau_{cancel}$.

The payoff that the trader receives from a transaction of $dQ_{t,t+\tau}$ shares of the security in τ periods at price $p_{t,b}$ is equal to

$$dQ_{t,t+\tau} \left(y_{t+\tau} + u_t - p_{t,b} \right) = dQ_{t,t+\tau} \left(v_t - p_{t,b} \right) + dQ_{t,t+\tau} \left(y_{t+\tau} - y_t \right), \tag{6}$$

where $y_{t+\tau}$ is the common value of the security in τ periods. The term $dQ_{t,t+\tau} \left(v_t - p_{t,b} \right)$ is the surplus that a transaction of $dQ_{t,t+\tau}$ would earn upon immediate execution at price $p_{t,b}$. The term $dQ_{t,t+\tau} \left(y_{t+\tau} - y_t \right)$ is equal to the number of shares transacted in τ periods multiplied by the change in the common value. Summing over all possible fill times for the order and including the cost of submitting the order, the realized payoff is equal to

$$U_{t,t+\Upsilon} = \sum_{\tau=0}^{\Upsilon} dQ_{t,t+\tau} \left(v_t - p_{t,b} \right) + \sum_{\tau=0}^{\Upsilon} dQ_{t,t+\tau} \left(y_{t+\tau} - y_t \right) - q_t c. \tag{7}$$

We define the execution probability as

$$\psi_t^{buy}(b, q_t) \equiv E_t \left[\sum_{\tau=0}^{\Upsilon} \frac{dQ_{t,t+\tau}}{q_t} \middle| d_{t,b}^{buy} = 1, q_t \right] \tag{8}$$

and the picking off risk as

$$\xi_t^{buy}(b, q_t) \equiv E_t \left[\sum_{\tau=0}^{\Upsilon} \frac{dQ_{t,t+\tau}}{q_t} \left(y_{t+\tau} - y_t \right) \middle| d_{t,b}^{buy} = 1, q_t \right]. \tag{9}$$

The conditional expectations in equations (8) and (9) do not depend upon the trader's private value. If the order is a market order, then the execution probability is one and the picking off risk is zero.

The trader's expected payoff is equal to the expected value of equation (7), conditional on the trader's information set, which includes the current limit order book, the current common value, the history of common value innovations, and the trader's private value, order quantity, and decision,

$$E_t\left[U_{t,t+\Upsilon}\left|d_{t,b}^{buy}=1,u_t,q_t\right.\right] \quad = \quad q_t\psi_t^{buy}(b,q_t)\left(v_t-p_b\right)+q_t\xi_t^{buy}(b,q_t)-q_tc. \tag{10}$$

The first term in the trader's expected payoff is equal to the expected number of shares that will eventually transact times the current surplus per share for a certain fill at price $p_{t,b}$. The second term in the trader's expected payoff equals the covariance of changes in the common value with the quantity of the order that transacts. The final term in the trader's expected payoff is the cost of submitting the order. The expected payoff to a trader submitting a sell order for q_t shares at a price s ticks above the current best bid quote is defined similarly.

The trader submits the order that maximizes his expected payoff, conditional on his information, private value, and order quantity q_t,

$$\max_{\left\{d_{t,s}^{sell}\right\}_{s=0}^{S},\left\{d_{t,b}^{buy}\right\}_{b=0}^{B}} \sum_{s=0}^{S} d_{t,s}^{sell}E_t\left[U_{t,t+\Upsilon}\left|d_{t,s}^{sell}=1,u_t,q_t\right.\right]+\sum_{b=0}^{B} d_{t,b}^{buy}E_t\left[U_{t,t+\Upsilon}\left|d_{t,b}^{buy}=1,u_t,q_t\right.\right], \tag{11}$$

subject to:

$$d_{t,s}^{sell}\in\{0,1\},\ \text{for } s=0,1,\ldots,S,\ d_{t,b}^{buy}\in\{0,1\},\ \text{for } b=0,1,\ldots,B, \tag{12}$$

$$\sum_{s=0}^{S} d_{t,s}^{sell}+\sum_{b=0}^{B} d_{t,b}^{buy}\leq 1. \tag{13}$$

Equation (13) imposes the constraint that at most one order is submitted. Let $d_t^{sell*}(s,u_t,q_t)$ and $d_t^{buy*}(b,u_t,q_t)$ be the optimal strategy, detailing the trader's submission as a function of his beliefs and information, private value, and order quantity.

Lemma 1 *Suppose that a trader with private value u and quantity q optimally submits a buy order at price $b\geq 0$ ticks below the ask quote, so that $d_t^{buy*}(b,u,q)=1$.*

1. *A trader with private value $u'>u$ and quantity q submits a buy order at a price b' ticks below the ask quote such that the execution probability is higher at b' than at b:*

$$\psi_t^{buy}(b',q)\geq\psi_t^{buy}(b,q). \tag{14}$$

2. *Suppose that the execution probabilities are strictly decreasing in the distance between the limit order price and the best ask quote, $\psi_t^{buy}(b+1,q) < \psi_t^{buy}(b,q)$, for all $b = 0, 1, \ldots, B-1$. A trader with private value $u' > u$ for q shares submits a buy order at a price b' ticks below the ask quote with $b' \leq b$.*

Similar results hold on the sell side.

The optimal order submission depends upon the trader's valuation. The common value is fixed at t so that the only source of heterogeneity in the decision at t is the trader's private value. If the trader buys, the higher the trader's private value, the higher the execution probability is for the trader's optimal buy order. If the trader sells, the lower the trader's private value, the higher the execution probability is for the trader's optimal sell order.

Lemma 1 and the discrete price grid imply that we can partition the set of valuations into intervals. All traders wishing to trade the same quantity whose valuations lie within the same interval make the same order submission. Define threshold valuation $\theta_t^{buy}(b, b', q)$ as the valuation of a trader who is indifferent between submitting a buy order at price $p_{t,b}$ and a buy order at price $p_{t,b'}$,

$$\theta_t^{buy}\left(b, b', q\right) = p_{t,b} + \frac{\left(p_{t,b} - p_{t,b'}\right)\psi_t^{buy}(b,q) + \left(\xi_t^{buy}(b',q) - \xi_t^{buy}(b,q)\right)}{\psi_t^{buy}(b,q) - \psi_t^{buy}(b',q)}. \tag{15}$$

The threshold valuation for a buy order at price $p_{t,b}$ and not submitting an order is

$$\theta_t^{buy}\left(b, \mathrm{NO}, q\right) = p_{t,b} + \frac{\xi_t^{buy}(b,q) + c}{\psi_t^{buy}(b,q)}. \tag{16}$$

The threshold valuation for a sell order at price $p_{t,s}$ and a sell order price $p_{t,s'}$ is

$$\theta_t^{sell}\left(s, s', q\right) = p_{t,s} - \frac{\left(p_{t,s'} - p_{t,s}\right)\psi_t^{sell}(s',q) + \left(\xi_t^{sell}(s,q) - \xi_t^{sell}(s',q)\right)}{\psi_t^{sell}(s,q) - \psi_t^{sell}(s',q)}. \tag{17}$$

The threshold valuation for a limit sell order at price $p_{t,s}$ and not submitting any order is

$$\theta_t^{sell}\left(s, \mathrm{NO}, q\right) = p_{t,s} - \frac{\xi_t^{sell}(s,q) + c}{\psi_t^{sell}(s,q)}. \tag{18}$$

The threshold valuation for a sell order at price $p_{t,s}$ and a buy order at price $p_{t,b}$ is

$$\theta_t\left(s, b, q\right) = p_{t,s} + \frac{\left(p_{t,b} - p_{t,s}\right)\psi_t^{sell}(s,q) - \left(\xi_t^{sell}(s,q) - \xi_t^{buy}(b,q)\right)}{\psi_t^{sell}(s,q) + \psi_t^{buy}(b,q)}. \tag{19}$$

Let $\mathcal{B}_t^*(q)$ index the set of buy order prices that are optimal for some trader who wishes to trade q shares at time t,

$$\mathcal{B}_t^*(q) \equiv \left\{ b \,\middle|\, d_t^{buy*}(b,u,q) = 1 \text{ for some } u \right\}, \tag{20}$$

with elements $b_{i,t}^*(q)$, for $i = 1, \ldots, I$, ordered by the execution probabilities.

$$\psi_t^{buy}(b_{i,t}^*(q), q) > \psi_t^{buy}(b_{i+1,t}^*(q), q). \tag{21}$$

Let $\mathcal{S}_t^*(q)$ index the set of sell order prices that are optimal for some trader who wishes to trade q shares at time t, with elements $s_{j,t}^*(q)$, for $j = 1, \ldots, J$, ordered by the execution probabilities.

Lemma 2

$$\theta_t^{buy}\left(b_{1,t}^*(q), b_{2,t}^*(q), q\right) > \theta_t^{buy}\left(b_{2,t}^*(q), b_{3,t}^*(q), q\right) > \ldots > \theta_t^{buy}\left(b_{I-1,t}^*(q), b_{I,t}^*(q), q\right), \tag{22}$$

$$\theta_t^{sell}\left(s_{J-1,t}^*(q), s_{J,t}^*(q), q\right) > \theta_t^{sell}\left(s_{J-2,t}^*(q), s_{J-1,t}^*(q), q\right) > \ldots > \theta_t^{sell}\left(s_{1,t}^*(q), s_{2,t}^*(q), q\right), \tag{23}$$

$$\theta_t^{buy}\left(b_{I-1,t}^*(q), b_{I,t}^*(q), q\right) > \theta_t\left(s_{J,t}^*(q), b_{I,t}^*(q), q\right) > \theta_t^{sell}\left(s_{J-1,t}^*(q), s_{J,t}^*(q), q\right). \tag{24}$$

To describe the optimal decision rule, define the marginal thresholds for sellers and buyers as

$$\begin{aligned} \theta_t^{buy}\left(Marginal_t(q), q\right) &= \max\left(\theta_t\left(s_{J,t}^*(q), b_{I,t}^*(q), q\right), \theta_t^{buy}\left(b_{I,t}^*(q), \text{NO}, q\right)\right), \\ \theta_t^{sell}\left(Marginal_t(q), q\right) &= \min\left(\theta_t\left(s_{J,t}^*(q), b_{I,t}^*(q), q\right), \theta_t^{sell}\left(s_{J,t}^*(q), \text{NO}, q\right)\right). \end{aligned} \tag{25}$$

If the marginal threshold for the buyers is equal to the marginal threshold for the sellers then all traders find it optimal to submit an order. Otherwise, there are traders who find it optimal not to submit any order.

Lemma 3 *The optimal order submission strategy is*

$$d_t^{buy*}(b,u,q) = 1, \;\; \textit{if} \; \begin{cases} b = b_{1,t}^*(q) \;\; \textit{and} \\ \theta_t^{buy}(b_{1,t}^*(q), b_{2,t}^*(q), q) \leq y_t + u < \infty, \\ \\ b = b_{i,t}^*(q) \;\; \textit{for} \; i = 2, ..., I-1 \;\; \textit{and} \\ \theta_t^{buy}(b_{i+1,t}^*(q), b_{i,t}^*(q), q) \leq y_t + u < \theta_t^{buy}(b_i^*(q), b_{i-1,t}^*(q), q), \\ \\ b = b_{I,t}^*(q) \;\; \textit{and} \\ \theta_t^{buy}(Marginal_t(q), q) \leq y_t + u < \theta_t^{buy}(b_{I,t}^*(q), b_{I-1,t}^*(q), q), \end{cases} \tag{26}$$

$$
d_t^{sell*}(s,u,q) = 1, \;\; if \begin{cases} s = s_{1,t}^*(q), \;\; and \\ -\infty \le y_t + u < \theta_t^{sell}(s_{1,t}^*(q), s_{2,t}^*(q), q), \\ \\ s = s_{j,t}^*(q), for\ some\ j = 2,\ldots,J-1 \;\; and \\ \theta_t^{sell}(s_{j-1,t}^*(q), s_{j,t}^*(q), q) \le y_t + u < \theta_t^{sell}(s_{j,t}^*(q), s_{j+1,t}^*(q), q), \\ \\ s = s_{J,t}^*(q), \;\; and \\ \theta_t^{sell}(s_{J-1,t}^*(q), s_{J,t}^*(q), q) \le y_t + u < \theta_t^{sell}(Marginal_t(q), q), \end{cases} \tag{27}
$$

otherwise,

$$
d_t^{buy*}(b,u,q) = d_t^{sell*}(s,u,q) = 0. \tag{28}
$$

The proof of the lemma provides a construction of $\mathcal{B}_t^*(q)$ and $\mathcal{S}_t^*(q)$.

Define $V_t(y_t + u, q)$ to be the indirect utility function for a trader at t with valuation $y_t + u$ and quantity q. The indirect utility function is computed by substituting the optimal decision rule in equations (26) through (28) into the trader's objective function, equation (11).

Lemma 4 *$V_t(y_t + u, q)$ has the following properties.*

1. *$V_t(y_t + u, q)$ is a positive, convex function of $y_t + u$.*
2. *Suppose that $d_t^{buy*}(b,u,q) = 1$ for some (b,u,q). Then for $u' > u$, $V_t(y_t+u', q) > V_t(y_t+u, q)$.*
3. *Suppose that $d_t^{sell*}(s,u,q) = 1$ for some (s,u,q). Then for $u' < u$, $V_t(y_t+u', q) > V_t(y_t+u, q)$.*

Figure 2 plots $V_t(y_t + u, q)$ computed using a set of execution probabilities and picking off risks. Here, $q = 1$, $\mathcal{S}_t^*(1) = \{0, 1, 2\}$ and $\mathcal{B}_t^*(1) = \{0, 1, 2\}$. Market buy and sell orders, one tick and two tick limit buy and sell orders are optimal for a trader with some valuation, with order quantity equal to one share. The expected payoffs as a function of the trader's valuation from submitting different sell orders are plotted with dashed lines and the expected payoffs from submitting different buy orders are plotted with dashed-dotted lines. From equation (10), the trader's expected payoff from submitting any particular order is a linear function of his valuation, with slope equal to the execution probability for that order.

A change in the order entry cost, c, leads to a parallel shift in the expected payoff from submitting any particular order. A change in the picking off risk for any particular order leads to a parallel shift in the expected payoff from submitting that particular order, while keeping unchanged the expected payoff from submitting any other order. A change in the execution probability for any particular order leads to a shift in the slope in the expected payoff from submitting that particular order, while keeping unchanged the expected payoff from submitting any other order.

Geometrically, the thresholds are the valuations for which the expected payoffs intersect. For example, the threshold for a market sell and a limit sell at one tick from the best bid quote is $\theta_t^{sell}(0,1,1)$; a trader with a valuation less than $\theta_t^{sell}(0,1,1)$ submits a market sell order. The thresholds associated with submitting any particular order and submitting no order are the valuations where the expected payoffs cross the horizontal axis. Here, $\theta_t^{sell}(2,\text{NO},1) < \theta_t(2,2,1)$, and $\theta_t^{buy}(2,\text{NO},1) > \theta_t(2,2,1)$, so that if the trader's valuation is between $\theta_t^{sell}(2,\text{NO},1)$ and $\theta_t^{buy}(2,\text{NO},1)$, the trader does not submit any order.

Consider increasing the order entry cost, c. The expected payoffs for submitting any order decrease, with all payoff curves shifting down by the same amount. As a consequence, only the thresholds associated with submitting an order and submitting no order change. The thresholds associated with submitting any order and submitting no order will increase on the buy side and decrease on the sell side, as a consequence of increasing the order entry cost.

Consider increasing the picking off risk for the one tick sell limit order. The expected payoffs for submitting a one tick sell order decreases, and the expected payoffs for any other order submissions do not change. The expected payoffs for a one tick sell order make a parallel outward shift, implying that the threshold for the one tick sell order and market order increases, and the threshold for the one tick sell order and the two tick sell order decreases. The payoff curve for a market order is steeper than the payoff curve for the two tick sell order; the threshold associated with the market order increases by less than the threshold associated with the two tick order decreases as a consequence of increasing the picking off risk.

Consider increasing the execution probability for the one tick sell order. The expected payoffs for submitting a one tick sell order increase, and the expected payoffs for any other order submissions do not change, implying that the threshold for the one tick sell order and market order decreases, and the threshold for the one tick sell limit order and the two tick sell limit order increases.

The indirect utility function $V_t(y_t + u, 1)$ is plotted with a thick solid line in the figure. The indirect utility function is the upper envelope of the payoffs associated with the different order submissions. The indirect utility function is higher for traders with more extreme private values than for traders with less extreme private values.

Using the optimal submission submission strategy in equation (28), the probability of a trader submitting a sell order at a price $s^*_{1,t}(q)$ ticks above the current best bid quote, conditional on the arrival of a trader who wishes to trade q shares, is

$$\begin{aligned}
\Pr_t\left(d^{sell*}\left(s^*_{1,t}(q), u_t, q\right) = 1 \middle| q\right) &= \Pr_t\left(y_t + u_t \leq \theta^{sell}_t\left(s^*_{1,t}(q), s^*_{2,t}(q), q\right) \middle| q\right) \\
&= G_t\left(\theta^{sell}_t\left(s^*_{1,t}(q), s^*_{2,t}(q), q\right) - y_t\right). \qquad (29)
\end{aligned}$$

The last line follows from the definition of G_t in equation (4) and because the quantity and the private value are independent random variables. Similarly, for $j = 2, \ldots, J-1$

$$\begin{aligned}
\Pr_t\left(d^{sell*}_t\left(s^*_{j,t}(q), u_t, q\right) = 1 \middle| q\right) &= G_t\left(\theta^{sell}_t\left(s^*_{j,t}(q), s^*_{j+1,t}(q), q\right) - y_t\right) \\
&\quad - G_t\left(\theta^{sell}_t\left(s^*_{j-1,t}(q), s^*_{j,t}(q), q\right) - y_t\right), \qquad (30)
\end{aligned}$$

and

$$\begin{aligned}
\Pr_t\left(d^{sell*}_t\left(s^*_{J,t}(q), u_t, q\right) = 1 \middle| q\right) &= G_t\left(\theta^{sell}_t\left(Marginal_t(q), q\right) - y_t\right) \\
&\quad - G_t\left(\theta^{sell}_t\left(s^*_{J-1,t}(q), s^*_{J,t}(q), q\right) - y_t\right). \qquad (31)
\end{aligned}$$

Similar expressions to equations (29) through (31) hold for buy orders.

Equations (29) through (31) show that there are two reasons for order submissions to be autocorrelated. First, the threshold valuations may be autocorrelated through autocorrelation in the expected payoffs from different order submissions. Second, innovations in the common value y_t are correlated to current order submissions, and have a permanent affect on the level of the common value.

Equations (29) through (31) show that the conditional probabilities of observing different order submission conditional on the arrival of a trader form an ordered qualitative response model, as defined by Amemiya (1985). A standard ordered qualitative response model is the ordered probit. An ordered probit formulation of the order submission decision is

$$Pr_t\left(d^{sell*}_t\left(s^*_{j,t}(q), u_t, q\right) = 1 \middle| q\right) = \Phi\left(\Gamma^{sell}_j - x'_t\Gamma\right) - \Phi\left(\Gamma^{sell}_{j-1} - x'_t\Gamma\right), \qquad (32)$$

where $\Phi(\cdot)$ is the cumulative normal distribution, x_t an observable vector of conditioning variables, Γ is a coefficient vector and Γ^{sell}_j and Γ^{sell}_{j-1} are fixed constants, similarly for any buy order. From equations (29) through (31), our model is consistent with an ordered probit formulation for order submissions only if the private value is normally distributed, and the thresholds are of the form

$$\theta_t^{sell}\left(s_{j,t}^*(q), s_{j+1,t}^*(q), q\right) - y_t = \Gamma_j^{sell} - x_t'\Gamma. \tag{33}$$

In the ordered probit formulation, a change in the conditioning variables affects all the threshold valuations in the same direction and by the same amount.

Consider a shock to the conditioning variables that leads to a change in the picking off risk for a one tick sell limit order. Either the the shock does not change the picking off risks or execution probabilities of other orders, or the shock changes the picking off risk or execution probability for at least one other order. In the first case, the shock will cause the threshold associated with the market order to increase and the threshold associated with the two tick sell limit order to decrease. A probit formulation requires all the thresholds to move by the same amount when the conditioning variables change. A probit formulation is therefore inconsistent with the first case. For the ordered probit formulation to be consistent with the second case, the shock must change the picking off risks and execution probabilities for all orders so that all thresholds change by same amount.

Figure 3 plots the optimal order submission strategy corresponding to the indirect utility function in Figure 2. The distribution of private values G_t is a mixture of three normal distributions. The horizontal axis in the figure is the trader's private value and the vertical axis is the cumulative probability distribution of the private values. The probability of various order submissions is determined by the thresholds, the common value, and the distribution of private values G_t by equations (29) through (31).

For example, the probability of a trader submitting a sell market order is $G_t\left(\theta_t^{sell}(0,1,1) - y_t\right)$. A trader with valuation between $\theta_t^{sell}(1,2,1) - y_t$ and $\theta_t^{sell}(0,1,1) - y_t$ places a sell limit order at one tick from the bid quote. The probability of a trader submitting a sell limit order one tick from the bid quote is $G_t\left(\theta_t^{sell}(1,2,1) - y_t\right) - G_t\left(\theta_t^{sell}(0,1,1) - y_t\right)$. A trader with a private value between $\theta_t^{sell}(2, \mathrm{NO}, 1)$ and $\theta_t^{buy}(2, \mathrm{NO}, 1)$ does not submit any order, and the probability of such an event is

$$Pr_t\left(\mathrm{NO}|q\right) = G_t\left(\theta_t^{buy}(2, \mathrm{NO}, 1) - y_t\right) - G_t\left(\theta_t^{sell}(2, \mathrm{NO}, 1) - y_t\right). \tag{34}$$

Given that a trader may find it optimal to submit no order, the probability of observing a sell market order, conditional on observing any order submission, is equal to

$$Pr_t\left(d_t^{sell*}\left(s_{0,t}^*(q), u_t, q\right) = 1 \middle| q, \text{order submission}\right) = \frac{G_t\left(\theta_t^{sell}(0,1,1) - y_t\right)}{1 - Pr_t\left(\text{NO } |q\right)}. \tag{35}$$

The model can be tested without estimating $G_t(u)$. Lemma 2 provides a basis for an empirical test of the model, as illustrated by the following example. Conditional on observing an order submission, the probability of observing each of the following buy orders is strictly positive: a buy market order, a one tick buy limit order, and a two tick buy limit order. The best ask quote is equal to 100, and the tick size is 1. The execution probabilities for buy orders are $\psi_t^{buy}(0,1) = 1$, $\psi_t^{buy}(1,1) = 0.7$ and $\psi_t^{buy}(2,1) = 0.6$. For simplicity, the picking off risk for all buy limit orders is zero. The order quantity is equal to one. Observing such data implies that the model is false.

Figure 4 plots the payoffs for a trader submitting the three different buy orders against the trader's valuation. The dashed line is the expected payoff from submitting a buy market order, the light solid line is the expected payoff from submitting a one tick buy limit order, and the dashed-dotted line is the expected payoff from submitting a two tick buy limit order. There is no valuation for which the one tick buy limit order is optimal; the expected payoff from submitting a one tick buy limit order is always lower than the payoffs from submitting a market order or a two tick limit order. The threshold valuations are equal to

$$\begin{aligned}
\theta_t^{buy}(0,1,1) &= 100 + \frac{(1)(0.7)}{(1.0 - 0.7)} = 102.33, \\
\theta_t^{buy}(1,2,1) &= 99 + \frac{(1)(0.6)}{(0.7 - 0.6)} = 105.00.
\end{aligned} \tag{36}$$

In the example, $\theta_t^{buy}(1,2,1) > \theta_t^{buy}(0,1,1)$; the threshold valuations violate the monotonicity restriction. Since traders submit one tick limit orders with positive probability, the observed order submissions are not the outcome of the optimization in equations (11) through (13). Computing the threshold valuations requires only the execution probabilities for the orders that are actually submitted. It does not require knowledge of the traders' valuations, nor knowledge of the execution probabilities and picking off risks of orders not submitted by the traders.

The example is not a knife-edge case. Hold the execution probability for a buy market order equal to one and the execution probability for a two tick buy limit order equal to 0.6. With execution probabilities for a one tick buy limit order between 0.6 and 0.75, the model is inconsistent with traders submitting a one tick limit order with strictly positive probability. With execution probabilities for a one tick buy limit order between 0.75 and 1.00, the model is consistent with traders submitting a one tick buy limit order with strictly positive probability.

4 Empirical Evidence

We test the monotonicity restrictions in Lemma 2 in our sample. To form a test, we need to establish two auxiliary empirical results. First, we need to identify orders chosen with strictly positive probability. Second, we need to rank the orders chosen with strictly positive probability according to their execution probabilities. Once we establish the auxiliary results, we form a test of the monotonicity restrictions in Lemma 2.

We assume that the traders have rational expectations, and that their conditioning information can be captured by a vector of conditioning variables. Given that the model imposes weak functional form restrictions on the execution probabilities and picking off risks, we estimate the traders' expectations using nonparametric regressions of the realized fill history of each of the orders onto a set of conditioning variables. Ideally, we would include the entire limit order book and the variables that predict the distribution of the common value innovations, the distribution of private values and the cancellation times in the conditioning set, as well as the order quantity. It is impractical to do so. As such, we limit ourselves to the same conditioning variables used in the probits reported earlier. Let X_t denote the conditioning information. Table 4 contains definitions of the conditioning variables.

4.1 Test of Strictly Positive Conditional Choice Probabilities

Let

$$\ddot{B}(X_t) = \left\{ b \middle| \Pr\left(d_{b,t}^{buy} = 1 \middle| X_t \right) > 0 \right\} \tag{37}$$

index buy order prices that are chosen with strictly positive probability in our sample conditional upon X_t, with a similar definition for $\ddot{S}(X_t)$. Order $\ddot{B}(X_t)$ by the distance from the best ask quote and order $\ddot{S}(X_t)$ by the distance from the best bid quote.

An order is in $\ddot{B}(X_t)$ or $\ddot{S}(X_t)$ for all X_t, if it has strictly positive conditional choice probability for all X_t. Suppose that buy orders at $n, n+1, \ldots, N$ and sell orders at $o, o+1, \ldots, O$ all have conditional choice probabilities greater than or equal to LB, where LB is strictly greater than zero. Let z_t^{++} be a vector of strictly positive measurable functions of the vector X_t, and let $\otimes$ be the Kronecker product. Define

$$PC = E\left[\begin{pmatrix} d_{t,n}^{buy} - LB \\ d_{t,n+1}^{buy} - LB \\ \vdots \\ d_{t,N}^{buy} - LB \\ d_{t,o}^{sell} - LB \\ d_{t,o+1}^{sell} - LB \\ \vdots \\ d_{t,O}^{sell} - LB \end{pmatrix} \otimes z_t^{++}\right]. \tag{38}$$

Using the law of iterated expectations, the conditional choice probabilities greater than or equal to LB, implies the null hypothesis

$$H_0 : PC > 0. \tag{39}$$

We use the sample moment analogue $\widehat{PC}_T$ to form an an estimator for PC. Under standard conditions, $\sqrt{T}\left(\widehat{PC}_T - PC\right)$ converges in distribution to a normal random variable, with asymptotic variance-covariance matrix, $\mathcal{A}_{PC}$. The asymptotic covariance matrix is estimated using the Newey and West (1987) procedure.

Wolak (1989) derives a test statistic for a local test of H_0,

$$M_{PC} = \min_{\{a|a\geq 0\}} T(\widehat{PC}_T - a)\mathcal{A}_{PC}^{-1}(\widehat{PC}_T - a)', \tag{40}$$

and shows that under H_0, M_{PC} converges in distribution to the weighted sum of χ^2 variables,

$$Pr(M_{PC} \geq r) = \sum_{k=0}^{dim(\mathcal{A}_{PC})} \Pr[\chi_k^2 \geq r]\mathrm{w}(dim(\mathcal{A}_{PC}), dim(\mathcal{A}_{PC}) - k, \mathcal{A}_{PC}), \tag{41}$$

where χ_k^2 is a χ^2 variable with k degrees of freedom, $dim(\mathcal{A}_{PC})$ is the rank of the asymptotic variance covariance matrix and $\mathrm{w}(dim(\mathcal{A}_{PC}), dim(\mathcal{A}_{PC}) - k, \mathcal{A}_{PC})$, $k = 0, \ldots, dim(\mathcal{A}_{PC})$ are a set of weights that depend on the asymptotic variance-covariance matrix. Wolak (1989) describes a Monte Carlo method for calculating the weights.

Table 6 reports the results for the tests that the conditional choice probabilities are greater than 0.02. The tests are computed for the one tick limit order, the two tick limit orders, and the three tick limit orders, for both the buy and sell sides. Table 2 reports that in our sample, approximately 50% of the orders submitted are market orders, and so we do not include market orders in the test.

Each row reports the point estimates of the unconditional differences in decision indicators and 0.02 multiplied by positive instruments, standard errors, and p-values for the null of monotonicity of the execution probabilities for different order submissions.[1] Each column corresponds to a different positive instrument. The final row of the table reports the M_{PC} test described above for each instrument and all submissions, and the final column of the table reports the test statistic across the instruments. All of the point estimates are strictly positive and none of the tests reject the null hypothesis of monotonicity of the execution probabilities; the p-values are all greater than 0.98.

4.2 Test of Monotonicity of the Execution Probabilities

The execution probabilities are computed as a nonparametric regression of realized fills on information known at the time of order submission. Let $\mathcal{K}$ be a multidimensional kernel function and h_T a bandwidth associated with each argument. The nonparametric estimate of $\psi^{sell}(\ddot{s}, X_t)$ is

$$\widehat{\psi}^{sell}(\ddot{s}, X_t) \equiv \frac{\sum_{t' \neq t}^{T} \left(d_{\ddot{s},t'}^{sell} \sum_{\tau=1}^{\Upsilon} \frac{dQ_{t',t'+\tau}}{q_{t'}} \right) \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}{\sum_{t' \neq t}^{T} \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}, \tag{42}$$

for $\ddot{s} \in \ddot{S}(X_t)$, with a similar definition on the buy side. From the definition of $\ddot{S}(X_t)$, $\widehat{\psi}^{sell}(\ddot{s}, X_t)$ is well defined. Since almost all limit orders remain in the limit order book for less than two days in our sample, we set the maximum life of the order, Υ in equation (42), equal to two days.

Elements of $\ddot{B}(X_t)$ are ordered by the distance from the best quotes. If the execution probabilities are monotone in the distance from the best quotes, then ordering the thresholds by the distance from the best quotes is equivalent to ordering by the execution probabilities.

To test monotonicity of the execution probabilities, define

$$DF \equiv E\left[I(X_t \in \bar{X}) \begin{pmatrix} \psi^{buy}(\ddot{b}_1, X_t) - \psi^{buy}(\ddot{b}_2, X_t) \\ \psi^{buy}(\ddot{b}_2, X_t) - \psi^{buy}(\ddot{b}_3, X_t) \\ \vdots \\ \psi^{sell}(\ddot{s}_1, X_t) - \psi^{buy}(\ddot{s}_2, X_t) \\ \psi^{sell}(\ddot{s}_2, X_t) - \psi^{sell}(\ddot{s}_3, X_t) \\ \vdots \end{pmatrix} \otimes z_t^{++} \right], \tag{43}$$

where $I(X_t \in \bar{X})$ is a trimming indicator for the set $\bar{X}$ in the interior of the support of X_t. The trimming indicator is used to simplify the asymptotic distribution. See Ahn and Manski (1993)

[1] The standard errors are computed with 50 lags with the Newey and West (1987) procedure. The empirical results are robust to changes in the lag length. The asymptotic p-values for the monotonicity tests are computed using 10,000 Monte Carlo simulation trials.

for details. Applying the law of iterated expectations, monotonicity of the execution probabilities implies the null hypothesis

$$H_1: \ DF > 0. \tag{44}$$

We use the sample moment analogue of DF to form the estimator $\widehat{DF}_T$, using the nonparametric estimators of the execution probabilities. In Appendix C, we provide regularity conditions under which $\sqrt{T}\left(\widehat{DF}_T - DF\right)$ converges in distribution to a normal random variable, and we provide the asymptotic variance-covariance matrix, $\mathcal{A}_{DF}$. We form a similar test statistic to M_{PC} in equation (40) above as a test of H_1.

Table 7 reports the results of the monotonicity tests of the execution probabilities. The tests are computed using the execution probabilities for market and one tick limit order, the one and two tick limit orders, and the two and three tick limit orders, for both the buy and sell orders.

Each row reports the point estimates of the unconditional differences in execution probabilities multiplied by positive instruments, standard errors, and p-values for the null of monotonicity of the execution probabilities for different order submissions.[2] Each column corresponds to a different positive instrument. The final row of the table reports the M_{DF} test described above for each instrument and all submissions, and the final column of the table reports the test statistic across each choice. All of the point estimates are strictly positive and none of the tests reject the null hypothesis of monotonicity of the execution probabilities; the p-values are all greater than 0.98.

Together, the test statistics reported in Table 6 and Table 7 fail to reject that market orders, one tick, two tick and three tick limit buy orders are in $\ddot{B}(X_t)$, and similarly for the sell orders, and that ordering the orders by the distance from the quotes is that same as ordering them by the execution probabilities. As such, we compute the associated threshold valuations, and a test for monotonicity of them.

4.3 Test of Lemma 2

Computing the threshold valuations requires estimates of the picking off risks. Estimates of the picking off risks requires estimates of changes in the common value. From equation (3), the common

[2] The standard errors are computed with 50 lags using the method described in Appendix C to capture the overlap in the errors in the execution probabilities between orders submitted at different times. The empirical results are robust to changes in the lag length. The asymptotic p-values for the monotonicity tests are computed using 10,000 Monte Carlo simulation trials.

value is integrated of order one, or $I(1)$. We assume that there is an $I(1)$ vector of factors, f_t, such that

$$y_t = \beta f_t, \tag{45}$$

with β a parameter.

The best bid quote is observed when there are buy limit orders outstanding in the order book. Accordingly, denote by t' a time period where there are outstanding buy limit orders in the book. We provide conditions in Appendix B so that the best bid quote is is co-integrated with the common value,

$$\begin{aligned} p^{sell}_{0,t'} &= y_{t'} + \varepsilon_{t'} \\ &= \beta f_{t'} + \varepsilon_{t'}, \end{aligned} \tag{46}$$

where $p^{sell}_{0,t'}$ is the best bid quote at time t' and $\varepsilon_{t'}$ is $I(0)$. Let $\widehat{\beta}_{T'}$ denote the least squares estimate of β obtained by regressing $p^{sell}_{0,t'}$ on $f_{t'}$. We form an estimate of the common value as

$$\widehat{y}_t = \widehat{\beta}_{T'} f_t. \tag{47}$$

We used minute-by-minute observations of the value of the OMX index as our factor series. The OMX index is a value-weighted index of the 30 most traded companies on the Stockholm Stock Exchange. We also experimented with including the daily sampled US/SKr exchange rate and daily sampled Swedish interest rates as factors. The first column of Table 8 reports a Dickey-Fuller test statistics for the null hypothesis of unit root in the OMX index, the bid quote, and the ask quote; the test fails to reject the null. The final two columns of Table 8 report the results from estimating a cointegrating regression between the OMX index and the bid, and the ask. We do not reject the null hypotheses that the bid and the ask are cointegrated with the OMX index.

Our estimator for $\xi^{sell}(\ddot{s}, X_t)$ is

$$\widehat{\xi}^{sell}(\ddot{s}, X_t) = \frac{\sum_{t' \neq t}^{T} \left(d^{sell}_{\ddot{s},t'} \sum_{\tau=0}^{\Upsilon} \frac{dQ_{t',t'+\tau}}{q_{t'}} \left(\widehat{y}_{t'+\tau} - \widehat{y}_{t'} \right) \right) \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}{\sum_{t' \neq t}^{T} \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}, \tag{48}$$

where $\widehat{y}_t$ is the estimator of the common value in (47), and $\ddot{s} \in \ddot{S}(X_t)$. We form a similar estimator for buy order picking off risks.

We estimate the threshold valuations by

$$\widehat{\theta}^{sell}(\ddot{s},\ddot{s}',X_t) = p_{\ddot{s},t} - \frac{\left(p_{\ddot{s}',t} - p_{\ddot{s},t}\right)\widehat{\psi}^{sell}(\ddot{s}',X_t) + \left(\widehat{\xi}^{sell}(\ddot{s},X_t) - \widehat{\xi}^{sell}(\ddot{s}',X_t)\right)}{\widehat{\psi}^{sell}(\ddot{s},X_t) - \widehat{\psi}^{sell}(\ddot{s}',X_t)}, \tag{49}$$

with a similar estimator for the buy side. If $\psi^{sell}(\ddot{s},X_t) - \psi^{sell}(\ddot{s}',X_t) > 0$, then $\theta^{sell}(\ddot{s},\ddot{s}',X_t)$ is a continuous function of the execution probabilities and picking off risks. Consistency of the estimators for the execution probabilities and picking off risks then implies that $\widehat{\theta}^{sell}(\ddot{s},\ddot{s}',X_t)$ is a consistent estimator.

We use our estimators for the threshold valuations, equation (49), to form a test statistic for the monotonicity restrictions in equation (22) of Lemma 2. If

$$\{0,1,2,3\} \subset \mathcal{S}^*(X_t), \text{ and } \{0,1,2,3\} \subset \mathcal{B}^*(X_t), \tag{50}$$

then Lemma 2 implies that

$$\theta^{buy}(0,1,X_t) > \theta^{buy}(1,2,X_t) > \theta^{buy}(2,3,X_t), \tag{51}$$

$$\theta^{sell}(2,3,X_t) > \theta^{sell}(1,2,X_t) > \theta^{sell}(0,1,X_t), \tag{52}$$

and

$$\theta^{buy}(2,3,X_t) > \theta^{sell}(2,3,X_t). \tag{53}$$

Define

$$D\theta \equiv E\left[I(X_t \in \bar{X})\begin{pmatrix} \theta^{buy}(0,1,X_t) - \theta^{buy}(1,2,X_t) \\ \theta^{buy}(1,2,X_t) - \theta^{buy}(2,3,X_t) \\ \theta^{sell}(1,2,X_t) - \theta^{sell}(0,1,X_t) \\ \theta^{sell}(2,3,X_t) - \theta^{sell}(1,2,X_t) \\ \theta^{buy}(2,3,X_t) - \theta^{sell}(2,3,X_t) \end{pmatrix} \otimes z_t^{++}\right]. \tag{54}$$

Inequalities (51) through (53) and the law of iterated expectations imply the null hypothesis,

$$H_2: \ D\theta > 0. \tag{55}$$

We use the sample moment analogue of $D\theta$ to form the estimator $\widehat{D\theta_T}$, using the nonparametric estimators for the threshold valuations. In Appendix C, we provide conditions under which $\sqrt{T}\left(\widehat{D\theta_T} - D\theta\right)$ converges in distribution to a normal random variable and provide the asymptotic variance-covariance matrix. We form a similar test statistic to M_{PC} in equation (40) above as a test of H_2.

Table 9 reports estimates of the average threshold valuation differences. The first panel reports the average of the differences for buy orders multiplied by positive instruments; reported below each estimate are associated asymptotic standard errors and p-values for the null that the differences are positive.[3] Each column uses a different positive instrument. The final column reports the $M_{D\theta}$ statistic for each difference for all the instruments jointly, with asymptotic p-values reported in parentheses.

The point estimates of the threshold valuation differences are positive for all buy order thresholds and the tests do not reject the null hypothesis of monotonicity, either individually for each pair of threshold valuations and instrument, or jointly across all instruments.

The second panel reports estimates of the differences for sell orders. The point estimates of the differences between the threshold valuation for a two tick and a three tick sell limit order and the threshold valuation for a one tick and a two tick sell limit order are negative for all the instruments, except the close depth. The formal tests, however, fail to reject the null hypothesis of monotonicity, with the lowest p-value being 0.47. The point estimates of the differences between the threshold valuations for one tick and a two tick sell limit order and the threshold valuations for a sell market order and one tick sell limit order are strictly positive. The test fails to reject the null hypothesis of monotonicity, with all p-values being close to one.

The third panel reports estimates of the differences between the threshold valuation for a two tick and a three tick buy limit order and the threshold valuation for a two and a three tick sell limit order. The point estimates are negative for all instruments. The associated tests all reject the null hypothesis of monotonicity at the 5% level. The joint test across all instruments reported in the last column rejects the null hypothesis at the 1% level.

The bottom panel of Table 9 reports the joint tests for the buy threshold valuation differences, the sell threshold valuation differences, and the buy and sell threshold valuations together with asymptotic p-values reported below the point estimates. For all instruments, we fail to reject the null hypothesis of monotonicity for the buy and the sell thresholds separately. The final two rows of the table test the monotonicity of all thresholds simultaneously, and the tests all reject the null

[3]The standard errors are computed as described in Appendix C using the Newey and West (1987) procedure with 50 lags, and the asymptotic p-value for the $M_{D\theta}$ statistic is computed using the simulation method given in Wolak (1989) with 10,000 Monte Carlo simulation trials. The results are robust to changes in the lag length.

hypothesis at the 1% level.

5 Interpretation of the Evidence

Figure 5 plots the estimated payoffs for buy and sell market, one, two and three tick limit orders, evaluated at the observation in the sample where the conditioning variables are closest to their sample averages. The estimated payoffs for traders with valuations equal to the threshold valuations are computed by substituting estimates of the threshold valuations, the execution probabilities, and picking off risks, and the order quantity into equation (10), and dividing by the order quantity. The order entry cost per share, c, is set equal to zero. The estimated payoffs for traders with valuations between the threshold valuations lie on the linear segment between the estimated payoffs at the threshold valuations. The top plot is the expected payoffs for the sell orders and the bottom plot is the expected payoffs for the buy orders. The horizontal axis is the trader's valuation and the vertical axis is the expected payoff. The thick solid line is the maximum obtainable payoffs, if the traders were constrained to submit sell orders or buy orders only.

On the sell side, the threshold valuations do not satisfy the monotonicity restriction; there is no valuation for which a two tick sell order is optimal. On the buy side, the threshold valuations do satisfy the monotonicity restrictions. The threshold valuations do not satisfy the monotonicity restrictions for the buy and sell orders jointly.

Suppose that traders were restricted to submit buy orders only. The optimal buy order for a trader with a valuation equal to 115 would be to submit a two tick buy limit order. But if such a trader were allowed to submit a sell order, he would obtain a higher expected payoff from submitting a one tick sell limit order. Suppose that traders were restricted to submit sell orders only. The optimal sell order for a trader with valuation equal to 116 would be to submit a three tick sell limit order. But if such a trader were allow to submit a buy order, he would obtain a higher expected payoff from submitting a one tick buy limit order. The situations illustrated in Figure 5 are common enough in our sample for the the model to be rejected.

As a further diagnostic, Table 10 reports the sample averages of the execution probabilities, picking off risks, and the average estimated payoffs per share for traders with valuations equal to the six threshold valuations used in computing the monotonicity test. The second column of Table 10 reports the average of the estimated picking off risks. On average, buy orders away from best bid quote and sell orders away from the best ask quote face a larger picking off risk than orders closer to the quotes. The exception is the three tick sell limit order, which has smaller a picking off risk than the picking off risk for the two tick sell limit order.

The third column of Table 10 reports the average estimated payoffs for traders with valuations equal to the threshold valuations. For buy orders, the estimated payoffs are increasing the closer the order submission is to the quotes. There are two reasons that the estimated payoffs change across order submissions. First, the price, the execution probabilities, and the picking off risks change across order submissions. Second, the estimated valuations of the trader submitting the order changes across order submissions. The monotonicity of the estimated payoffs is consistent with the monotonicity of the indirect utility function in Lemma 4. The average estimated payoffs for three tick buy limit orders is negative. The average estimated payoffs for two tick sell limit orders is less than that for three tick sell limit orders, although the difference is small. The estimated payoffs further illustrate how the model fails.

By construction, the threshold valuations for two order submissions involve a comparison of the payoffs from submitting the orders; costs that enter both expected payoffs do not affect the threshold valuations. Fixed or variable costs of submitting orders that are the same across order types do not change the threshold valuations, and so such costs cannot explain the rejections. Fixed or variables costs of filling an order that are common across order types enter the threshold valuations additively and do not affect the difference between two threshold valuations, and so such costs also cannot explain the rejections.

The threshold valuations are the valuations where the expected payoffs from submitting two orders are equal. We reject the model because the thresholds for the two versus three tick buy limit order is lower than the thresholds for the two versus three tick sell limit order. How might we modify the model so that it is not rejected?

A decrease in the expected payoffs of the two tick buy limit order, an increase in the expected payoffs of the three tick buy limit order, or a combination of the two would lead to an increase in the threshold for the two versus three tick buy limit order. Similar changes for sell orders would lead to a decrease in the threshold for the two versus three tick sell limit order. Such changes could pull the thresholds apart.

We also find the two versus three tick sell limit threshold is approximately equal to the two versus one tick sell limit threshold. To pull the thresholds apart, the expected payoffs from submitting two tick sell limit orders must increase relative to the one tick sell limit orders.

Adding costs of monitoring outstanding limit orders would lower the payoffs of all limit orders, and lower the payoffs more for orders with lower execution probabilities. Orders with lower execution probabilities remain in the limit order book for a longer time than orders with higher execution probabilities. As a consequence, the monitoring costs will be higher for orders with lower execution probabilities; monitoring costs alone would not pull the thresholds apart. Similarly, risk aversion is likely to reduce the expected payoffs more for limit orders with lower execution probabilities than for orders with higher execution probabilities. As a consequence, risk aversion would not pull the thresholds apart.

Consider allowing the traders to resubmit orders if their initial limit orders fail to fill. The expected payoff from resubmitting a order is only important when the original order is canceled and the trader chooses to resubmit. The value of the option to resubmit would likely make limit orders with lower execution probabilities relatively more attractive than limit orders with higher execution probabilities. Allowing the traders to resubmit may pull the thresholds apart.

Three tick sell limit orders lead to higher payoffs than two tick or three tick buy limit orders. Traders submit both two tick and three tick buy limit orders in our sample. Perhaps there is a higher cost for submitting sell orders than for submitting buy orders. Such a cost could reduce the payoffs for submitting sell orders relative to the payoffs for submitting buy orders. A natural candidate is a short selling cost.

6 Conclusions

We characterize the optimal order submission strategy for traders in a limit order market facing the trade-off between order price, execution probabilities, and picking off risks. The optimal order strategy is a monotone function of a trader's unobserved valuation for the asset. We develop and compute a semiparametric test of the monotonicity restriction. We fail to reject the monotonicity restriction for buy orders or sell orders separately. We reject the monotonicity restriction when we combine buy and sell orders. We find that the expected payoffs from submitting buy and sell limit orders away from the quotes are too low.

We consider a one-shot order submission problem, without allowing for the possibility of resubmissions and cancellations in the traders' order submission strategies. Extending our model and empirical approach to allow for cancellations and resubmissions is a useful direction for future work.

References

Aguirregabiria, V. (1999), "The Dynamics of Markups and Inventories in Retailing Firms," *Review of Economic Studies*, 66, 275–308.

Ahn, H. and C. Manski (1993), "Distribution Theory of the Analysis of Binary Choice under Uncertainty with Nonparametric Estimation of Expectations," *Journal of Econometrics*, 56, 291–321.

Altug, S. and R.A. Miller (1998), "The Effect of Work Experience on Female Wages and Labor Supply," *Review of Economic Studies*, 65, 45–85.

Amemiya, T. (1985), *Advanced Econometrics*, (Cambridge: Harvard University Press).

Biais, B., P. Hillion, and C. Spatt (1995), "An Empirical Analysis of the Limit Order Book and the Order Flow in the Paris Bourse," *Journal of Finance*, 50, 1655–1689.

Biais, B., D. Martimort, and J-C. Rochet (2000), "Competing Mechanisms in a Common Value Environment," *Econometrica*, 68, 799–837.

Cohen, K., S. Maier, R. Schwartz, and D. Whitcomb (1981), "Transactions Costs, Order Placement Strategy and Existence of the Bid-Ask Spread," *Journal of Political Economy*, 89, 287–305.

Collomb, G. and W. Härdle (1986), "Strong Uniform Convergence Rate in Robust Nonparametric Time Series Analysis and Prediction: Kernel Regression Estimation from Dependent Observations," *Stochastic Processes and Their Applications*, 23, 77–89.

Copeland, T. and D. Galai (1983), "Information Effects on the Bid-Ask Spread," *Journal of Finance*, 38, 1457–1469.

De Jong, R. (2001), "Nonlinear Estimation Using Estimated Co-integrating Relations," *Journal of Econometrics*, 101, 109–122.

Elyakime, B., J-J. Laffont, P. Loisel, and Q. Vuong (1994), "First-Price Sealed-Bid Auctions with Secret Reservation Prices," *Annales D'Économie et de Statistique*, 34, 115–141.

Foucault, T. (1999), "Order Flow Composition and Trading Costs in a Dynamic Limit Order Market," *Journal of Financial Markets*, 2, 99–134.

Glosten, L. (1994), "Is the Electronic Limit Order Book Inevitable?," *Journal of Finance*, 49, 1127–1161.

Griffiths, M., B. Smith, D. Turnbull and R. White (2000), "The Costs and the Determinants of Order Aggressiveness," *Journal of Financial Economics*, 56, 65–88.

Guerre, E., I. Perrigne, and Q. Vuong (2000), "Optimal Nonparametric Estimation of First–Price Auctions," *Econometrica*, 68, 525–574.

Handa, V. and R. Schwartz (1996), "Limit Order Trading," *Journal of Finance*, 51, 1835–1861.

Harris, L. (1998), "Optimal Order Submission Strategies in Some Stylized Trading Problems," *Financial Markets, Institutions and Instruments*, 7, no. 2.

Harris, L. and J. Hasbrouck (1996), "Market vs. Limit Orders: The SuperDOT Evidence on Order Submission," *Journal of Financial and Quantitative Analysis*, 31, 213–231.

Harris, L. and V. Panchapagesan (1999), "Information Content of the Limit Order Book: Evidence from NYSE Specialist Actions," Working Paper, Washington University.

Hotz, V.J. and R.A. Miller (1993), "Conditional Choice Probabilities and the Estimation of Dynamic Models," *Review of Economic Studies*, 60, 497–529.

Kavajecz, K. (1999), "A Specialist's Quoted Depth and the Limit Order Book," *Journal of Finance*, 54, 747–771.

Kavajecz, K. and E. Odders-White (2001), "An Examination of Changes in Specialists' Posted Price Schedules, *Review of Financial Studies,* 14, 681–704.

Lo, A., A.C. MacKinlay, and J. Zhang (2001), "Econometric Models of Limit Order Executions," Rodney L. White Center Working Paper #12-99; forthcoming in *Journal of Financial Economics.*

Manski, C. (1993), "Dynamic Choice in Social Settings: Learning from the Experience of Others," *Journal of Econometrics,* 58, 121–136.

Miller, R.A. and S. Sanders (1997), "Human Capital Development and Welfare Participation," *Carnegie Rochester Conference Series on Public Policy*, 46, 1–45.

Newey, W. and K. West (1987), "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–708.

Parlour, C. (1998), "Price Dynamics in Limit Order Markets," *Review of Financial Studies*, 11, 789–816.

Parlour, C. and D. Seppi (2001), "Liquidity-Based Competition for Order Flow," Working Paper, Carnegie Mellon University; forthcoming in *Review of Financial Studies.*

Peterson, M. and E. Sirri (2001), "Order Submission Strategy and the Curious Case of Marketable Limit Orders," Working Paper, Babson College; forthcoming in *Journal of Financial and Quantitative Analysis.*

Ranaldo, A. (2002), "Order Aggressiveness," Working Paper, Olsen and Associates (Zurich).

Ready, M. (1999), "The Specialist's Discretion: Stopped Orders and Price Improvement," *Review of Financial Studies*, 12, 1075–1112.

Robinson, P. (1989), "Hypothesis Testing in Semiparametric and Nonparametric Models for Economic Time Series," *Review of Economic Studies*, 56, 511–536.

Rock, K. (1996), "The Specialist's Order Book and Price Anomalies," Working Paper, Harvard University.

Sandås, P. (2001), "Adverse Selection and Competitive Market Making: Evidence from a Limit Order Market," *Review of Financial Studies*, 14, 705–734.

Seppi, D. (1997), "Liquidity Provision with Limit Orders and a Strategic Specialist," *Review of Financial Studies*, 10, 103–150.

Slade, M. (1998), "Optimal Pricing with Costly Adjustment: Evidence from Retail-Grocery Prices," *Review of Economics Studies*, 65, 87–107.

Stokey, N. and R. Lucas, with E. Prescott (1989), *Recursive Methods in Economic Dynamics,* (Cambridge: Harvard University Press).

Wolak, F. (1989), "Local and Global Testing of Linear and Nonlinear Inequality Constraints in Nonlinear Econometric Models," *Econometric Theory*, 5, 1–35.

A Proofs

Proof of Lemma 1. Assume that $u' > u$. Given $d_t^{buy*}(b, u, q) = 1$,

$$q\psi_t^{buy}(b,q)\left(y_t + u - p_{b,t}\right) + q\xi_t^{buy}(b,q) - qc \geq q\psi_t^{buy}(b',q)\left(y_t + u - p_{b',t}\right) + q\xi_t^{buy}(b',q) - qc, \quad \text{(A1)}$$

and $d_t^{buy*}(b', u', q) = 1$,

$$q\psi_t^{buy}(b',q)\left(y_t + u' - p_{b',t}\right) + q\xi_t^{buy}(b',q) - qc \geq q\psi_t^{buy}(b,q)\left(y_t + u' - p_{b,t}\right) + q\xi_t^{buy}(b,q) - qc. \quad \text{(A2)}$$

Subtracting inequality (A2) from inequality (A1) and dividing by q,

$$\left(\psi_t^{buy}(b,q) - \psi_t^{buy}(b',q)\right)\left(u - u'\right) \geq 0. \quad \text{(A3)}$$

Since $u' > u$, equation (A3) implies that $\psi_t^{buy}(b',q) > \psi_t^{buy}(b,q)$. If the execution probability is monotone in distance from the best ask quote, then equation (A3) implies that $b' \leq b$. The proof for the sell side is symmetric. ■

Proof of Lemma 2. If it is optimal for a trader with private value u to submit a buy order, then it is also optimal for traders with private values $u' > u$ to submit buy orders. Let s be an arbitrary sell order, and suppose that $d_t^{buy}(b, u, q) = 1$. After dividing by q we have

$$\begin{aligned}
\psi_t^{buy}(b,q)\left(y_t + u' - p_{b,t}\right) + \xi_t^{buy}(b,q) - c &> \psi_t^{buy}(b,q)\left(y_t + u - p_{b,t}\right) + \xi_t^{buy}(b,q) - c \\
&\geq \psi_t^{sell}(s,q)\left(p_{s,t} - y_t - u\right) - \xi_t^{sell}(s,q) - c \\
&\geq \psi_t^{sell}(s,q)\left(p_{s,t} - y_t - u'\right) - \xi_t^{sell}(s,q) - c. \quad \text{(A4)}
\end{aligned}$$

The first line follows because $u' > u$; the second line follows because it is optimal for a trader with private value u to submit a buy order at b; the third line follows because $u' > u$. Symmetric arguments hold for sellers. Thus, there exists $\overline{u}_t \geq \underline{u}_t$ such that all traders with private values $u > \overline{u}_t$ find it optimal to submit buy orders, and all trader with private values $u < \underline{u}_t$ find it optimal to submit sell orders. Monotonicity of the associated thresholds follows from Lemma 1. ■

Proof of Lemma 3. We start with a construction of $\mathcal{B}_t^*(q)$. Let $\breve{b}$ be an arbitrary buy order, and define the functions,

$$\bar{b}_t(\breve{b}, q) = \arg\max_b \theta_t^{buy}(\breve{b}, b, q), \text{ subject to } \psi_t^{buy}(b,q) \leq \psi_t^{buy}(\breve{b}, q), \quad \text{(A5)}$$

$$\bar{\theta}_t^{buy}(\breve{b}, q) = \theta_t^{buy}(\breve{b}, \bar{b}_t(\breve{b}, q), q), \quad \text{(A6)}$$

$$\bar{\theta}_t^{sell}(\breve{b}, q) = \max_s \theta_t(s, \breve{b}, q), \quad \text{(A7)}$$

and

$$\bar{\theta}_t(\breve{b}, q) = \max\left(\bar{\theta}_t^{buy}(\breve{b}, q), \theta_t^{buy}(\breve{b}, \text{NO}, q), \bar{\theta}_t^{sell}(\breve{b}, q)\right). \quad \text{(A8)}$$

Here, $\bar{\theta}_t^{buy}(\breve{b}, q)$ is the highest possible threshold valuation for a buy order at $\breve{b}$ and a buy order with a lower execution probability with $\bar{b}_t(\breve{b}, q)$ the associated buy order, and $\bar{\theta}_t^{sell}(\breve{b}, q)$ is the highest possible threshold valuation for a buy order at $\breve{b}$ and any sell order. Finally, $\bar{\theta}_t(\breve{b}, q)$ is the highest possible threshold for a buy order at $\breve{b}$, any buy order with lower execution probability, no order, or a sell order.

Let $\dot{b}_t(q)$ be the buy order with the highest possible expected execution probability.

$$\dot{b}_t(q) \equiv \arg\max_b \psi_t^{buy}(b,q). \tag{A9}$$

Compute $\bar{\theta}(\dot{b}_t(q), q)$. Let $\bar{u}$ be the highest possible value for the private value. If

$$\bar{\theta}(\dot{b}_t(q), q) - y_t \leq \bar{u}, \tag{A10}$$

then set

$$b_{1,t}^*(q) = \dot{b}_t(q). \tag{A11}$$

If inequality (A10) does not hold, then $\dot{b}_t(q)$ is not an optimal order for a trader with any possible private value. If inequality (A10) does not hold, and

$$\bar{\theta}_t^{buy}(\dot{b}_t(q), q) \neq \bar{\theta}_t(\dot{b}_t(q), q), \tag{A12}$$

then either no order or a sell order leads to the highest threshold valuation associated with $\dot{b}_t(q)$. In this case,

$$\mathcal{B}_t^*(q) = \emptyset. \tag{A13}$$

If inequality (A10) does not hold, and

$$\bar{\theta}_t^{buy}(\dot{b}_t(q), q) = \bar{\theta}_t(\dot{b}_t(q), q), \tag{A14}$$

then a buy order leads to the highest threshold valuation associated with $\dot{b}_t(q)$. In this case, set $\dot{b}_t(q)$ equal to $\bar{b}_t(\dot{b}_t(q), q)$, and perform the same computations described above, until either $b_{1,t}^*(q)$ is found or $\mathcal{B}_t^*(q) = \emptyset$. By assumption, there is a finite number of potential buy prices; one of the two conditions will be met.

Once $b_1^*(q)$ is found, the following recursive procedure is used to find the remaining optimal prices. Let $b_{j,t}^*(q)$ be given. Compute $\bar{\theta}_t^{buy}(b_{j,t}^*(q), q)$, $\bar{\theta}_t^{sell}(b_{j,t}^*(q), q)$ and $\bar{\theta}_t(b_{j,t}^*(q), q)$. If

$$\bar{\theta}_t^{buy}(b_{j,t}^*(q), q) = \bar{\theta}_t(b_{j,t}^*(q), q), \tag{A15}$$

then

$$b_{j+1,t}^*(q) = \bar{b}_t(b_{j,t}^*(q), q). \tag{A16}$$

If

$$\bar{\theta}_t^{buy}(b_{j,t}^*(q), q) \neq \bar{\theta}_t(b_{j,t}^*(q), q), \tag{A17}$$

then there are no more elements in $\mathcal{B}_t^*(q)$.

The set of optimal sell prices is constructed similarly.

We now show that equations (26) through (28) in Lemma 3 provide the optimal submission strategy. Let u_t satisfy

$$\theta_t^{buy}(b_{j+1,t}^*(q), b_{j,t}^*(q), q) \leq y_t + u_t < \theta_t^{buy}(b_j^*(q), b_{j-1,t}^*(q), q), \tag{A18}$$

so that according to equation (26), $b_{j,t}^*(q)$ is the optimal order submission.

Suppose that a buy order at b leads to a higher payoff for the trader with private value u_t than an order at $b_{j,t}(q)$. Then after dividing by q,

$$\psi^{buy}(b_{j,t}^*(q), q)(y_t + u_t - p_{b_{j,t}^*(q)}) + \xi^{buy}(b_{j,t}^*(q), q) - c < \psi^{buy}(b, q)(y_t + u_t - p_b) + \xi^{buy}(b, q) - c. \tag{A19}$$

Assume that the execution probability for an order at b is higher than the execution probability at $b_{j,t}^*(q)$. Then, inequality (A19) implies that for $u > u_t$,

$$\psi^{buy}(b_{j,t}^*(q), q)(y_t + u_t - p_{b_{j,t}^*(q)}) + \xi^{buy}(b_{j,t}^*(q), q) - c < \psi^{buy}(b, q)(y_t + u_t - p_b) + \xi^{buy}(b, q) - c, \tag{A20}$$

Evaluating inequality (A20) at $\theta_t^{buy}(b_j^*(q), b_{j+1}^*(q), q)$ contradicts equation (A16).

Assume that the execution probability for an order at b is lower than the execution probability at $b_{j,t}^*(q)$. Then, inequality (A19) implies that for $u > u_t$,

$$\psi^{buy}(b_{j,t}^*(q), q)(y_t + u_t - p_{b_{j,t}^*(q)}) + \xi^{buy}(b_{j,t}^*(q), q) - c - \Big(\psi^{buy}(b, q)(y_t + u_t - p_b) + \xi^{buy}(b, q) - c\Big), \tag{A21}$$

is decreasing in u. Therefore,

$$\theta^{buy}(b, b_{j,t}^*(q), q) > \theta^{buy}(b_{j-1,t}^*(q), b_{j,t}^*(q), q), \tag{A22}$$

contradicting the definition of $b_{j-1,t}^*$ in equation (A16). Similar arguments hold if either a sell order or no order leads to higher payoffs for the trader with valuation u_t than $b_{j,t}^*$.

Similar arguments hold for sell orders. ■

Proof of Lemma 4.

1. $V_t(u_t, q) \geq 0$, since the trader can always submit no order and earn a zero payoff. To show convexity, consider two private values, u and u'. Let $0 < \lambda < 1$ and let $u_\lambda = \lambda u + (1-\lambda)u'$. Suppose $d_t^{buy*}(b, u_\lambda, q) = 1$,

$$V_t(y_t + u, q) \geq q\psi_t^{buy}(b, q)(y_t + u - p_{b,t}) + q\xi^{buy}(b, q) - qc, \tag{A23}$$

and

$$V_t(y_t + u', q) \geq q\psi_t^{buy}(b, q)(y_t + u' - p_{b,t}) + q\xi^{buy}(b, q) - qc. \tag{A24}$$

Taking the convex combination of inequalities (A23) and (A24) and using the definition of u_λ,

$$\begin{aligned} \lambda V_t(y_t + u, q) + (1-\lambda)V_t(u_t + u', q) &\geq q\psi_t^{buy}(b, q)(y_t + u_\lambda - p_{b,t}) + q\xi^{buy}(b, q) - qc \\ &= V_t(y_t + u_\lambda, q). \end{aligned} \tag{A25}$$

The proof is similar if it is optimal for the trader with private value u_λ to submit a sell order or to submit no order.

2. Let $d_t^{buy}(b, u, q) = 1$ and $u' > u$.

$$\begin{aligned} V_t(y_t + u', q) &\geq q\psi_t^{buy}(b, q)(y_t + u' - p_{b,t}) + q\xi_t^{buy}(b, q) - qc \\ &> q\psi_t^{buy}(b, q)(y_t + u - p_{b,t}) + q\xi_t^{buy}(b, q) - qc \\ &= V_t(y_t + u, q). \end{aligned} \tag{A26}$$

3. The proof for sellers is symmetric to that in part 2 above.

■

B Cointegration Results

We assume that there is a strictly positive minimum tick size. Therefore, feasible order prices are elements of a countable set, with a lower bound of zero. Order the set of feasible prices from lowest to highest, so that $P_i < P_{i+1}$. Let $Q_{ij,t}$ be the order quantity outstanding at the i^{th} price at time t, submitted at time $t-j$, with $Q_{ij,t} > 0$ denoting buy quantities, $Q_{ij,t} < 0$ denoting sell quantities, and $Q_{ij,t} = 0$ denoting that no order quantity is outstanding.

The rules of the trading mechanism imply that there cannot be both buy and sell orders outstanding at the same price at the same time.

Let the common value at time t equal y_t. Define the feasible relative prices at time t as the elements of the set of feasible prices minus y_t. The relative order book at time t is

$$H_t = (P_i - y_t, Q_{ij,t}) \text{ for } i = 1, 2, 3, \ldots, \infty, \text{ and } j = 1, 2, 3, \ldots, \infty. \tag{B1}$$

We make the following assumptions.

CI1 The maximum life of each limit order is some finite integer $\Upsilon < \infty$.

CI2 Suppose a limit order is submitted to the limit book at time t. The conditional probability that the order is canceled at time $t+\tau$ for $\tau < \Upsilon$ depends on a finite dimensional vector of variables, $R_{t+\tau}$. The conditional probabilities are uniformly bounded below by a strictly positive constant.

CI3 The process (R_t, δ_t) is a Markov process and satisfies Condition M of Stokey and Lucas (page 348, 1989).

CI4 The conditional distribution of δ_t, u_t, q_t only depends on R_t and δ_t, u_t, q_t are conditionally independent.

CI5 The random variables δ_t, u_t, q_t each have uniformly bounded support.

CI6 Assume that traders only condition on the order book relative to the common value, H_t and R_t when making order submissions at t.

CI7 The cost per share of entering orders, c, is strictly positive.

Lemma 5 *Under assumptions* **CI1-CI7***, H_t is characterized by a finite number of elements, each of which is a bounded random variable.*

Proof of Lemma 5. We show that at most a finite number of relative prices have order quantities at any time.

Let $[\underline{u}, \overline{u}]$ be the support for the private value, let $[\underline{\delta}, \overline{\delta}]$ be the support for common value innovations, let $[0, \underline{q}]$ be the support for sell order quantities and let $[0, \overline{q}]$ be the support for buy order quantities.

Let p_t be the price of the order submission at time t. A buyer never submits an order than leads to a negative surplus with probability one, and The highest possible valuation that a buyer could have is his valuation at the time of filling, which is bounded by assumption. Therefore,

$$\begin{aligned} p_t &\leq y_{t+\Upsilon} + \overline{u} \\ &\leq y_t + \Upsilon\overline{\delta} + \overline{u}. \end{aligned} \tag{B2}$$

Similarly for sellers,

$$p_t \geq y_t + \Upsilon\underline{\delta} + \underline{u}. \tag{B3}$$

With a cost per share for order entry, no buyer would every submit an order that has a zero execution probability. Since a limit buy order only transacts with a future sell order and the longest that a limit order lasts is Υ, for a buy order to have a positive probability of being filled, it must satisfy

$$\begin{aligned} p_t &\geq y_{t+\Upsilon} + \Upsilon\underline{\delta} + \underline{u} \\ &\geq y_t + \Upsilon\underline{\delta} + \Upsilon\underline{\delta} + \underline{u} \\ &= y_t + 2\Upsilon\underline{\delta} + \underline{u}. \end{aligned} \tag{B4}$$

Combining inequalities (B2) and (B4),

$$y_t + 2\Upsilon\underline{\delta} + \underline{u} \leq p_t \leq y_t + \Upsilon\delta + \overline{u}, \tag{B5}$$

or

$$2\Upsilon\underline{\delta} + \underline{u} \leq p_t - y_t \leq \Upsilon\delta + \overline{u}. \tag{B6}$$

A similar result holds for sell orders. Therefore, the relative prices in the relative order book are all bounded at the time of entry,

$$2\Upsilon\underline{\delta} + \underline{u} \leq p_t - y_t \leq 2\Upsilon\delta + \overline{u}. \tag{B7}$$

Since orders last for up to Υ periods, there can be at most Υ orders outstanding at any time, and so the relative prices of all orders in the relative order book are bounded. Since there is a finite price grid, there are a finite number of relative prices satisfying the bound.

By assumption, order quantity is bounded. ■

From Lemma 5, there are finite number of relative prices that could have outstanding orders at any time. Let that finite number be $\mathcal{M}$. Therefore, the relative order book can be represented by

$$H_t = (P_{i,t} - y_t, Q_{ij,t}) \text{ for } i = 1, 2, 3, \ldots, \mathcal{M}, \text{ and } j = 1, 2, 3, \ldots, \Upsilon. \tag{B8}$$

Here, $P_{1,t}$ is the lowest price satisfying the bound in equation (B7) and $P_{\mathcal{M},t}$ is the highest.

Lemma 6 *Under assumptions* **CI1- CI7**, (H_t, R_t, δ_t) *is a stationary Markov process, with a unique ergodic set.*

Proof of Lemma 6. By assumption, new order submissions depend upon the relative order book R_t and the trader's private value. The conditional distribution of H_{t+1} depends upon H_t, new order submissions, cancellations and innovations in the common value. By assumption, (R_t, δ_t) is Markov, and the distribution of q_t depends only on R_t. Therefore, the process (H_t, R_t, δ_t) is also Markov.

The hazard rates for cancellation are bounded below by a strictly positive number; there is a strictly positive probability that all orders will cancel from any state, leaving all the quantities in the book zero. The conditional distribution of relative prices depends δ_t. By assumption, (R_t, δ_t) satisfies Condition M. Therefore, (H_t, R_t, δ_t) also satisfies Condition M. Theorem 11.12 of Stokey and Lucas (1989) then applies and so (H_t, R_t, δ_t) is a stationary process with unique ergodic set. ■

Define the random variable

$$\dot{\varepsilon}_t = \begin{cases} \max_{i=1,\ldots,\mathcal{M}} \left\{ P_{i,t} - y_t | \sum_j Q_{ij,t} > 0 \right\} & \text{if some } Q_{ij,t} > 0, \\ 0 & \text{else.} \end{cases} \tag{B9}$$

The random variable $\dot{\varepsilon}_t$ is the difference between the best bid quote and the common value, if there are buy orders in the relative book, or zero if there are no buy orders in the book

Lemma 7 *Under assumptions* **CI1 – CI7**, $\dot{\varepsilon}_t$ *is stationary.*

Proof of Lemma 7. The random variable $\dot{\varepsilon}_t$ is a mapping of (H_t, R_t, δ_t). Under assumptions **CI1-CI7** (H_t, R_t, δ_t) is a stationary Markov process with a unique ergodic set. ∎

The best bid quote does not exist if there are no outstanding buy order in the relative book. The ergodic set for the relative order book contains the states where there are no orders in the book. The next assumption guarantees that the ergodic set also contains books with limit buy orders in the book.

CI8 Suppose that there are no orders in the book at time t. Then, the probability that a buy order is submitted is uniformly strictly positive, for all possible value of R_t.

Lemma 8 *Under assumptions* **CI1-CI8***, the best bid price and the common value are cointegrated.*

Proof of Lemma 8. By assumption, y_t is $I(1)$. From Lemma 7, $\dot{\varepsilon}_t$ is stationary and ergodic and is equal the difference between the best bid price and y_t when there are buy orders on the relative book. The ergodic set contains books with no orders. Assumption **CI8** implies that books with buy orders also in the ergodic set, and the random variable equal to the indicator function equal to one when buy orders are in the book and zero otherwise is stationary and ergodic. Therefore, the process $\varepsilon_{t'}$ formed by sampling the process $\dot{\varepsilon}_t$ when there are buy orders on the book is also stationary. ∎

C Econometrics Appendix

In this appendix, we briefly describe the asymptotic properties for the estimators used in the monotonicity tests. Our data consists of observations of the vector of M conditioning variables, X_t, the decision indicators, $d^{sell}_{\ddot{s},t}$, for $\ddot{s} \in \ddot{S}(X_t)$, $d^{buy}_{\ddot{b},t}$, for $\ddot{b} \in \ddot{B}(X_t)$, the realized fills for each order, and the realized product of the fills and the changes in the estimated common value for each order. Let w_t be the vector of variables whose conditional expectations we compute. Define the conditional expectation functions

$$C^{sell}_{\ddot{s}}(X_t) \equiv E[w|d^{sell}_{\ddot{s},t} = 1, X_t], \tag{C1}$$

$$C^{buy}_{\ddot{b}}(X_t) \equiv E[w|d^{buy}_{\ddot{b},t} = 1, X_t] \tag{C2}$$

for $\forall \ddot{s} \in \ddot{S}$ and $\forall \ddot{b} \in \ddot{B}$, and let

$$C(X_t) \equiv \left(C^{sell}_{\ddot{s}_1}(X_t), C^{sell}_{\ddot{s}_2}(X_t), \ldots, C^{buy}_{\ddot{b}_2}(X_t), C^{buy}_{\ddot{b}_1}(X_t) \right) \tag{C3}$$

be the vector of conditional expectations. The object to be estimated depends on the vector valued function $\rho(C(X_t), X_t)$. Define

$$\varrho \equiv E\left[I(X_t \in \bar{X}) \rho(C(X_t), X_t) \right], \tag{C4}$$

where $I(X_t \in \bar{X})$ is a trimming indicator for the set $\bar{X}$ in the interior of the support of X_t. Our estimator for ϱ is

$$\hat{\varrho}_T \equiv \frac{1}{T} \sum_{t=1}^{T} I(X_t \in \bar{X}) \rho(\hat{C}(X_t), X_t), \tag{C5}$$

where $\hat{C}(X_t)$ is estimated using a nonparametric kernel regression. For example,

$$\hat{C}^{sell}_{\ddot{s}}(X_t) \equiv \frac{\sum_{t' \neq t}^{T} w_{t'} d^{sell}_{\ddot{s},t'} \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}{\sum_{t' \neq t}^{T} \mathcal{K}\left(h_T^{-1}(X_{t'} - X_t) \right)}, \tag{C6}$$

where h_T is a bandwidth and $\mathcal{K}$ is a multi-dimensional kernel function.

For the tests described in the text, the vector of conditional expectations is equal to

$$C(X_t) \equiv \left(\psi^{sell}(\ddot{s}_1, X_t), \xi^{sell}(\ddot{s}_1, X_t), \ldots, \psi^{buy}(\ddot{b}_1, X_t), \xi^{buy}(\ddot{b}_1, X_t)\right). \tag{C7}$$

For testing monotonicity of the execution probabilities,

$$\rho(C(X_t), X_t) \equiv \begin{pmatrix} \psi^{buy}(\ddot{b}_1, X_t) & - & \psi^{buy}(\ddot{b}_2, X_t), \\ \psi^{buy}(\ddot{b}_2, X_t) & - & \psi^{buy}(\ddot{b}_3, X_t), \\ & \vdots & \\ \psi^{sell}(\ddot{s}_2, X_t) & - & \psi^{sell}(\ddot{s}_3, X_t) \\ \psi^{sell}(\ddot{s}_1, X_t) & - & \psi^{sell}(\ddot{s}_2, X_t) \end{pmatrix} \otimes z_t^{++}, \tag{C8}$$

where z_t^{++} are strictly positive measurable functions of the vector X_t, and $\otimes$ is the Kronecker product. For testing monotonicity of the thresholds, we define the composite function

$$\rho(\theta(C(X_t), X_t), X_t) \equiv \begin{pmatrix} \theta^{buy}(\ddot{b}_1, \ddot{b}_2, X_t) & - & \theta^{buy}(\ddot{b}_2, \ddot{b}_3, X_t) \\ \theta^{buy}(\ddot{b}_2, \ddot{b}_3, X_t) & - & \theta^{buy}(\ddot{b}_3, \ddot{b}_4, X_t) \\ & \vdots & \\ \theta^{sell}(\ddot{s}_1, \ddot{s}_2, X_t) & - & \theta^{sell}(\ddot{s}_2, \ddot{s}_3, X_t) \\ \theta^{sell}(\ddot{s}_2, \ddot{s}_3, X_t) & - & \theta^{sell}(\ddot{s}_1, \ddot{s}_2, X_t) \\ & \vdots & \end{pmatrix} \otimes z_t^{++}, \tag{C9}$$

and

$$\theta(C(X_t), X_t) \equiv \left(\theta^{buy}(\ddot{b}_1, \ddot{b}_2, X_t; C(X_t)), \ldots, \theta^{sell}(\ddot{s}_1, \ddot{s}_2, X_t; C(X_t))\right), \tag{C10}$$

with

$$\theta^{sell}(\ddot{s}_1, \ddot{s}_2, X_t; C(X_t)) = p_{\ddot{s}_1,t} - \frac{(p_{\ddot{s}_2,t} - p_{\ddot{s}_1,t})\psi^{sell}(\ddot{s}_2, X_t) + (\xi^{sell}(\ddot{s}_1, X_t) - \xi^{sell}(\ddot{s}_2, X_t))}{\psi^{sell}(\ddot{s}_1, X_t) - \psi^{sell}(\ddot{s}_2, X_t)}. \tag{C11}$$

Under the regularity conditions provided below, the results in Robinson (1989) and Ahn and Manski (1993)[4] imply that $\sqrt{T}\,(\hat{\varrho}_T - \varrho)$ converges in distribution to a normal random vector with covariance matrix defined in equation (C24) below.

EC1 $X_t, w_t, d^{sell}_{\ddot{s},t}, d^{buy}_{\ddot{b},t}$, are absolutely regular and the beta-mixing coefficient is $o(j^{-\nu})$. Also,

$$\sup_{X \in \bar{X}} \|\rho(C(X), X)\|^{\varphi} < \infty, \tag{C12}$$

where $\nu > 1 + \frac{2}{\varphi - 2}$. For a definition of absolute regularity and the beta-mixing coefficient, see Robinson (1989).

EC2 (a) The distribution of X_t has Lebesgue density $\pi(\cdot)$ that is bounded and at least M+1 times differentiable, with the first M+1 derivatives bounded.

(b) The realized fills and picking off risks have bounded support.

(c) $C^{sell}_{\ddot{s}}(X_t)$ and $C^{buy}_{\ddot{b}}(X_t)$ are M+1 times differentiable with bounded derivatives.

[4] Ahn and Manski (1993) consider an environment with *i.i.d.* data. The uniform consistency results from Collomb and Härdle (1986) regarding the kernel estimators applied in Ahn and Manski (1993) continue to apply in our time-series environment.

(d) The conditional choice probabilities,

$$\alpha_{\ddot{s}}^{sell}(X_t) = \text{Prob}(d_{\ddot{s}}^{sell} = 1|X_t) \tag{C13}$$

are M+1 times differentiable with bounded derivatives. A similar restriction holds on the buy side. The function $\pi(X)\alpha_{\ddot{s}}^{sell}(X)$ satisfies

$$\inf_{X \in \bar{X}} \pi(X)\alpha_{\ddot{s}}^{sell}(X) > 0 \tag{C14}$$

for $\forall \ddot{s} \in \ddot{S}$, similarly for the buy side.

EC3 (a) The partial derivatives satisfy

$$\sup_{X_t \in \bar{X}} \left\| \frac{\partial \rho(C(X_t), X_t)}{\partial C(X_t)} \right\| < \infty. \tag{C15}$$

(b) There is an $F < \infty$ such that the cross partial derivatives satisfy

$$\sup_{X_t \in \bar{X}} \left\| \frac{\partial^2 \rho(C(X_t), X_t)}{\partial C(X_t) \partial C(X_t)'} \right\| < F. \tag{C16}$$

EC4 Define the matrix of expected derivatives as

$$\mu(X) \equiv E\left[\frac{\partial \rho(C(X), X)}{\partial C(X)} \middle| X \right], \tag{C17}$$

with generic element $\mu_{ij}(X)$, that satisfies

$$\frac{\mu_{ij}(X)}{\alpha_{\ddot{s}}^{sell}(X)} < \infty, \tag{C18}$$

and is M+1 times differentiable with bounded derivatives.

EC5 Define the vector of error terms

$$\epsilon_{\ddot{s},t}^{sell} = d_{\ddot{s},t}^{sell}[w_t - C_{\ddot{s}}^{sell}(X_t)], \tag{C19}$$

with a similar definition for $\epsilon_{\ddot{b},t}^{buy}$, and define

$$\epsilon_t = \sum_{\ddot{s} \in \ddot{S}} \epsilon_{\ddot{s},t}^{sell} + \sum_{\ddot{b} \in \ddot{B}} \epsilon_{\ddot{b},t}^{buy}. \tag{C20}$$

There exists a positive semi-definite matrix $\mathcal{C}$ such that

$$\sup_{X_t \in \bar{X}} \lim_{LL \to \infty} \sum_{ll=-LL}^{LL} E[\epsilon_{t-ll}\epsilon_{t+ll}'|X_t] < \mathcal{C}. \tag{C21}$$

EC6 (a) The bandwidth sequence is such that $Th_T^{2(M+1)} \to \infty$, $T^{1-2\kappa}h_T^{2M} \to 0$ as $T \to \infty$ for some $\kappa > 0$.

(b) The kernel function $\mathcal{K}$ is bounded and symmetric around zero, $\int \mathcal{K}(z)dz = 1$, and $\int |z|^{2(M+1)}\mathcal{K}(z)dz < \infty$. There exists $\gamma > 0$ and $c < \infty$ such that $\mathcal{K}$ satisfies the Lipschitz condition that $|\mathcal{K}(z) - \mathcal{K}(z')| \leq c|z - z'|^{\gamma}$ for all $z, z' \in \mathcal{R}^M$.

(c) The first M moments of $\mathcal{K}(\cdot)$ are zero.

Define

$$\eta_t = \rho(C(X_t), X_t) - \varrho \tag{C22}$$

and the vector

$$e_t \equiv \left(\frac{\epsilon^{sell}_{\ddot{s}_1,t}}{\alpha^{sell}_{\ddot{s}_1,t}(X_t)}, \ldots, \frac{\epsilon^{sell}_{\ddot{s}_K}, t}{\alpha^{sell}_{\ddot{s}_K,t}(X_t)}, \frac{\epsilon^{buy}_{\ddot{b}_1,t}}{\alpha^{buy}_{\ddot{b},t}(X_t)}, \ldots, \frac{\epsilon^{buy}_{\ddot{b}_L,t}}{\alpha^{buy}_{\ddot{b}_L,t}(X_t)} \right). \tag{C23}$$

Then,

$$\mathcal{A} = \lim_{LL \to \infty} \sum_{ll=-LL}^{LL} E\left[(\eta_{t-ll} + \mu(X_{t-ll})' e_{t-ll})(\eta_{t+ll} + \mu(X_{t+ll})' e_{t+ll})' \right]. \tag{C24}$$

We estimate η_t with $\rho(\hat{C}(X_t), X_t) - \hat{\varrho}_T$. We estimate e_t using the kernel estimators in equations (C19) and (C20) and using kernel estimators for the conditional choice probabilities in equation (C13). We use a Newey and West (1987) procedure to form an estimator for $\mathcal{A}$.

The thresholds are linear in β, and so the super-consistency of the cointegrating regression implies that the asymptotic distribution is unaffected by pre-estimating β. See De Jong (2001) for details.

Following Altug and Miller (1998) and Hotz and Miller (1993), and the simulation evidence in Robinson (1989, pp. 521-522) we use independent Gaussian product kernels in forming estimates of the conditional expectations. We use bandwidths equal to

$$4 \times 1.06 \times \hat{\sigma}(X_{it}) T^{\frac{1}{2\times 5+2}}. \tag{C25}$$

Here, $X_t = (X_{1t}, \ldots, X_{5t})$ are the conditioning variables, with $\hat{\sigma}(X_{it})$ the associated sample standard deviations. For the monotonicity tests discussed in the text, we trim the outer 5% of the observations according to

$$(X_t - \bar{X}) cov(X_t)^{-1} (X_t - \bar{X})', \tag{C26}$$

where $cov(X_t)$ is the covariance matrix of the conditioning information and $\bar{X}$ is the sample mean, leaving us with 19,732 observations.

Table 1: **Daily Trading Activity**

	Mean	Std. Dev.	Min.	Max.
Daily closing mid-quote (Skr)	110.15	9.19	89.00	127.00
Daily close-to-close return (percent)	0.21	3.04	-6.31	11.44
Daily close-to-close return (percent) on NASDAQ, January 2, 1989–December 31, 1993	0.10	2.45	-16.95	13.69
Number of active brokers per day	19.28	2.28	14.00	23.00
Daily trading volume in millions of SKr				
Stockholm Automated Exchange	38.77	19.53	11.58	114.88
Internal crosses	12.20	9.04	1.48	57.51
Block trades	0.39	0.92	0.00	5.45
After-hours trades	4.66	6.15	0.00	28.26
Total trading volume	56.02	29.52	13.06	201.28
Daily number of Stockholm Automated Exchange orders, 10:03 a.m.–2:30 p.m.				
All orders	364.23	141.13	128.00	733.00
Limit orders	212.18	77.58	73.00	408.00
Market orders	152.04	67.14	55.00	330.00

The table reports summary statistics on the daily trading activity of Ericsson. The daily close-to-close returns are calculated using the mid-quotes. The daily close-to-close returns for Ericsson shares traded on NASDAQ are calculated using daily data from the Center for Research in Security Prices. The number of active brokers per day is defined as the number of brokers who made at least one trade on a given trading day.

Table 2: **Order Flow**

Order	Number of submissions	Execution probability	Time-to-fill	Order quantity Mean	Median	Std. Dev.
			Buy orders			
market	6031	1.00	0.00	19.72	6.00	39.21
1 tick limit	4225	0.68	24.55	25.14	10.00	37.60
2 tick limit	992	0.33	73.94	29.63	10.00	44.58
3 tick limit	893	0.12	172.33	14.65	4.00	32.38
			Sell orders			
market	4044	1.00	0.00	30.73	11.00	48.90
1 tick limit	3212	0.63	18.31	36.08	20.00	49.48
2 tick limit	800	0.28	84.21	37.08	20.00	44.87
3 tick limit	563	0.13	170.95	23.73	10.00	91.29

The table reports descriptive statistics for the order flow in Ericsson by order. The execution probability is defined as the fraction of the order quantity that fills within two trading days of the order submission. The time-to-fill is the number of minutes elapsed from the order submission until the order filling. When there are multiple fills we weight each fill according to the fraction of the order quantity that is filled. Fills that occur later than two trading days after the order was submitted are ignored. The order quantity is measured in 100's of shares. There are a total of 20,760 orders.

Table 3: **Order Books**

	Average	Median	Std.Dev.	Min.	Max.
Order quantities (100's of shares)					
3rd Ask	169.8	105.0	175.9	1.0	1061.1
2nd Ask	260.8	214.0	203.0	1.0	1161.0
1st Ask	200.9	147.0	194.1	1.0	1314.0
1st Bid	185.9	143.0	173.8	1.0	1504.2
2nd Bid	242.9	190.0	217.7	1.0	1504.2
3rd Bid	167.9	115.0	190.9	1.0	1355.2
Cumulative order quantities (100's of shares)					
1st+2nd+3rd Ask	631.5	564.0	368.8	13.0	1935.6
1st+2nd Ask	461.7	401.0	294.9	2.0	1809.0
1st+2nd Bid	428.8	359.0	310.2	5.0	2176.9
1st+2nd+3rd Bid	596.8	505.5	396.0	15.0	2730.4
Distance between quotes and the mid-quote (absolute value of the distance measured in ticks)					
3rd Ask	2.7	2.5	0.6	2.5	9.5
2nd Ask	1.6	1.5	0.3	1.5	6.5
1st Ask	0.5	0.5	0.1	0.5	3.0
1st Bid	0.5	0.5	0.1	0.5	3.0
2nd Bid	1.6	1.5	0.3	1.5	9.0
3rd Bid	2.7	2.5	0.7	2.5	19.0

Descriptive statistics for the order books. The statistics in the table are computed for each order book observed in the market immediately prior to an order submission. There are a total of 20,760 observations.

Table 4: **Description of the Conditioning Variables**

Conditioning variable	Description
Order Quantity	The logarithm of the number of shares of the order submitted at t.
Close Depth	The logarithm of the total number of shares offered in the order book within one tick of the mid-quote.
Far Depth	The logarithm of the total number of shares offered in the order book within three ticks of the mid-quote.
Lagged Volume	The logarithm of the cumulative number of shares transacted during the time interval $[t - 10 \text{ minutes}, t)$.
Index Volatility	The logarithm of one plus the standard deviation of the OMX index returns over the minimum of 60 minutes and the number of minutes since the opening. The standard deviation is normalized by multiplying by the square root of the number of minutes per trading day.

Table 5: **Ordered Probit Models**

Order Quantity	Close Depth	Far Depth	Lagged Volume	Index Volatility
Buy orders (12,141 observations)				
-0.04	0.07	0.05	0.03	-0.09
(0.00)	(0.00)	(0.02)	(0.01)	(0.03)
$\chi^2_5 = 569.88$, p-value = 0.00				
Sell orders (8,619 observations)				
0.02	-0.08	0.12	-0.02	0.18
(0.01)	(0.00)	(0.03)	(0.01)	(0.03)
$\chi^2_5 = 386.09$, p-value = 0.00				

Estimation results from an ordered probit model for buy and sell order submissions. The estimated coefficients and standard errors are reported on the first two rows of each panel. The third row reports a χ^2_5 test of the null hypothesis that all coefficients are jointly equal to zero, with the corresponding p-value.

Table 6: **Test of Strictly Positive Conditional Choice Probabilities**

Order		Instruments					Joint
	Constant	Order Quantity	Close Depth	Far Depth	Lagged Volume	Index Volatility	M_{PC} statistic
			Buy limit orders				
1 tick	0.18	1.27	1.77	2.12	1.63	0.12	0.00
	(0.01)	(0.03)	(0.05)	(0.06)	(0.04)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
2 tick	0.03	0.20	0.21	0.31	0.25	0.02	0.00
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.99
3 tick	0.02	0.13	0.20	0.25	0.20	0.02	0.00
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
			Sell limit orders				
1 tick	0.13	1.00	1.31	1.57	1.21	0.09	0.00
	(0.01)	(0.03)	(0.04)	(0.05)	(0.03)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
2 tick	0.02	0.15	0.13	0.21	0.18	0.01	0.00
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.99
3 tick	0.01	0.04	0.05	0.08	0.07	0.01	0.00
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
			Joint M_{PC} statistic				
All limit orders	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.99	0.99	0.99	0.99	0.99	0.99	1.00

The top panels of the table report the average differences of the order choices and 0.02, multiplied by positive instruments. Below each point estimate are the asymptotic standard errors in parentheses and the p-values. The rightmost column and the bottom part of the table report joint M_{PC} test statistics across the instruments, the order prices, and across instruments and order prices, with p-values reported below each test statistic. We ensure that all instruments are strictly positive by replacing them with 0.00001 if they are zero.

Table 7: **Test of Monotonicity of the Execution Probabilities**

Execution probability difference	Constant	Instruments: Order Quantity	Close Depth	Far Depth	Lagged Volume	Index Volatility	Joint M_{DF} statistic
			Buy orders				
market	0.32	2.24	3.21	3.67	2.90	0.21	0.00
- 1 tick limit	(0.01)	(0.10)	(0.15)	(0.16)	(0.13)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
1 tick limit	0.36	2.48	3.59	4.14	3.28	0.23	0.00
- 2 tick limit	(0.02)	(0.15)	(0.23)	(0.26)	(0.20)	(0.02)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.99
2 tick limit	0.21	1.38	2.02	2.38	1.92	0.14	0.00
- 3 tick limit	(0.02)	(0.16)	(0.25)	(0.28)	(0.22)	(0.02)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
			Sell orders				
market	0.36	2.55	3.66	4.16	3.27	0.23	0.00
- 1 tick limit	(0.01)	(0.10)	(0.15)	(0.17)	(0.13)	(0.01)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
1 tick limit	0.42	2.84	4.22	4.88	3.88	0.28	0.00
- 2 tick limit	(0.02)	(0.14)	(0.21)	(0.23)	(0.18)	(0.02)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.99
2 tick limit	0.11	0.78	1.13	1.32	1.06	0.08	0.00
- 3 tick limit	(0.02)	(0.15)	(0.22)	(0.24)	(0.20)	(0.02)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.98
			Joint M_{DF} statistic				
All orders	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.98	0.98	0.98	0.98	0.98	0.98	1.00

The top panels of the table report the average differences of the execution probability for different order prices multiplied by positive instruments. Below each point estimate are reported the asymptotic standard errors in parentheses and the p-values. The rightmost column and the bottom part of the table report joint M_{DF} test statistics across the instruments, the order prices, and across instruments and order prices, with p-values reported below each test statistic. We ensure that all instruments are strictly positive by replacing them with 0.00001 if they are zero.

Table 8: **Common Value Estimation**

	Unit root test	Cointegrating regression	
		Coefficient	Cointegration test
OMX index	-0.89		
	(0.79)		
Bid quote	-1.11	0.37	-3.67
	(0.71)	(0.00)	(<0.03)
Ask quote	-1.08	0.36	-3.62
	(0.72)	(0.00)	(<0.03)

The first column reports unit root tests for the best bid quote, best ask quote, and the OMX index. All series are demeaned. There are 20,760 observations. The unit root test is an augmented Dickey-Fuller t-test with 10 lags, and p-values are reported below each t-statistic in parentheses. The cointegrating regression results are reported for the best bid quote and the best ask quote in the second column. The estimated coefficient on the demeaned OMX index is reported for each quote series with the standard error in parentheses. The lag length is 10. An augmented Engle-Granger test for cointegration is reported in the third column with p-values in parentheses.

Table 9: **Test of Monotonicity of the Threshold Valuations**

Threshold valuation difference	Constant	Order Quantity	Close Depth	Far Depth	Lagged Volume	Index Volatility	Joint $M_{D\theta}$ statistic
			Buy threshold valuations				
$\theta^{buy}(0,1,X_t)$	2.45	2.98	2.48	1.07	2.71	0.79	0.00
$-\theta^{buy}(1,2,X_t)$	(0.16)	(0.21)	(0.29)	(0.11)	(0.25)	(0.08)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.81
$\theta^{buy}(1,2,X_t)$	1.17	1.42	1.10	0.51	1.42	0.37	0.00
$-\theta^{buy}(2,3,X_t)$	(0.17)	(0.20)	(0.16)	(0.10)	(0.31)	(0.07)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.79
			Sell threshold valuations				
$\theta^{sell}(1,2,X_t)$	2.39	2.90	2.86	1.08	2.65	0.77	0.00
$-\theta^{sell}(0,1,X_t)$	(0.14)	(0.19)	(0.37)	(0.09)	(0.20)	(0.07)	
	1.00	1.00	1.00	1.00	1.00	1.00	0.83
$\theta^{sell}(2,3,X_t)$	-0.03	-0.04	0.10	-0.01	-0.01	-0.02	0.01
$-\theta^{sell}(1,2,X_t)$	(0.55)	(0.71)	(0.68)	(0.36)	(0.75)	(0.22)	
	0.48	0.48	0.56	0.50	0.49	0.47	0.76
			Buy and sell threshold valuations				
$\theta^{buy}(2,3,X_t)$	-1.37	-1.66	-1.15	-0.59	-1.64	-0.42	7.84
$-\theta^{sell}(2,3,X_t)$	(0.50)	(0.65)	(0.64)	(0.34)	(0.70)	(0.21)	
	0.01	0.01	0.04	0.03	0.01	0.02	0.01
			Joint $M_{D\theta}$ statistic				
Buy thresholds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.81	0.79	0.78	0.81	0.79	0.81	0.98
Sell thresholds	0.00	0.00	0.00	0.00	0.00	0.01	0.01
	0.78	0.80	0.77	0.81	0.81	0.75	0.98
Buy and sell	86.84	80.69	60.82	61.09	39.03	52.10	96.17
thresholds	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The top panels of the table report the average differences of threshold valuations for different order prices multiplied by positive instruments. Below each point estimate are reported the asymptotic standard errors in parentheses and the p-values. The rightmost column and the bottom part of the table report joint $M_{D\theta}$ test statistics across the instruments, the order prices, and across instruments and order prices, with p-values reported below each test statistic. We ensure that all instruments are strictly positive by replacing them with 0.00001 if they are zero.

Table 10: **Average Estimated Payoffs for Different Order Submissions**

Order	Execution probability	Picking off risk	Estimated payoff
	Buy orders		
1 tick limit	0.68	0.01	1.98
	(0.01)	(0.02)	(0.12)
2 tick limit	0.33	-0.11	0.33
	(0.02)	(0.05)	(0.06)
3 tick limit	0.11	-0.18	-0.05
	(0.01)	(0.08)	(0.04)
	Sell orders		
1 tick limit	0.63	0.01	1.61
	(0.01)	(0.02)	(0.10)
2 tick limit	0.23	0.14	0.10
	(0.02)	(0.06)	(0.06)
3 tick limit	0.11	0.10	0.11
	(0.02)	(0.06)	(0.14)

The table reports unconditional averages of the execution probabilities, the picking off risks, and estimated payoffs for traders with valuations equal to threshold valuations for the market and one tick, one tick and two tick, and two tick and three tick buy and sell orders. The estimated payoffs for traders with valuations equal to the threshold valuations are computed by substituting estimates of the threshold valuations, the execution probabilities, and the picking off risks, and the order quantity into equation (10), and dividing by the order quantity. The order entry cost of c per share is set equal to zero. Asymptotic standard errors are reported in parentheses, and are computed using 50 lags.

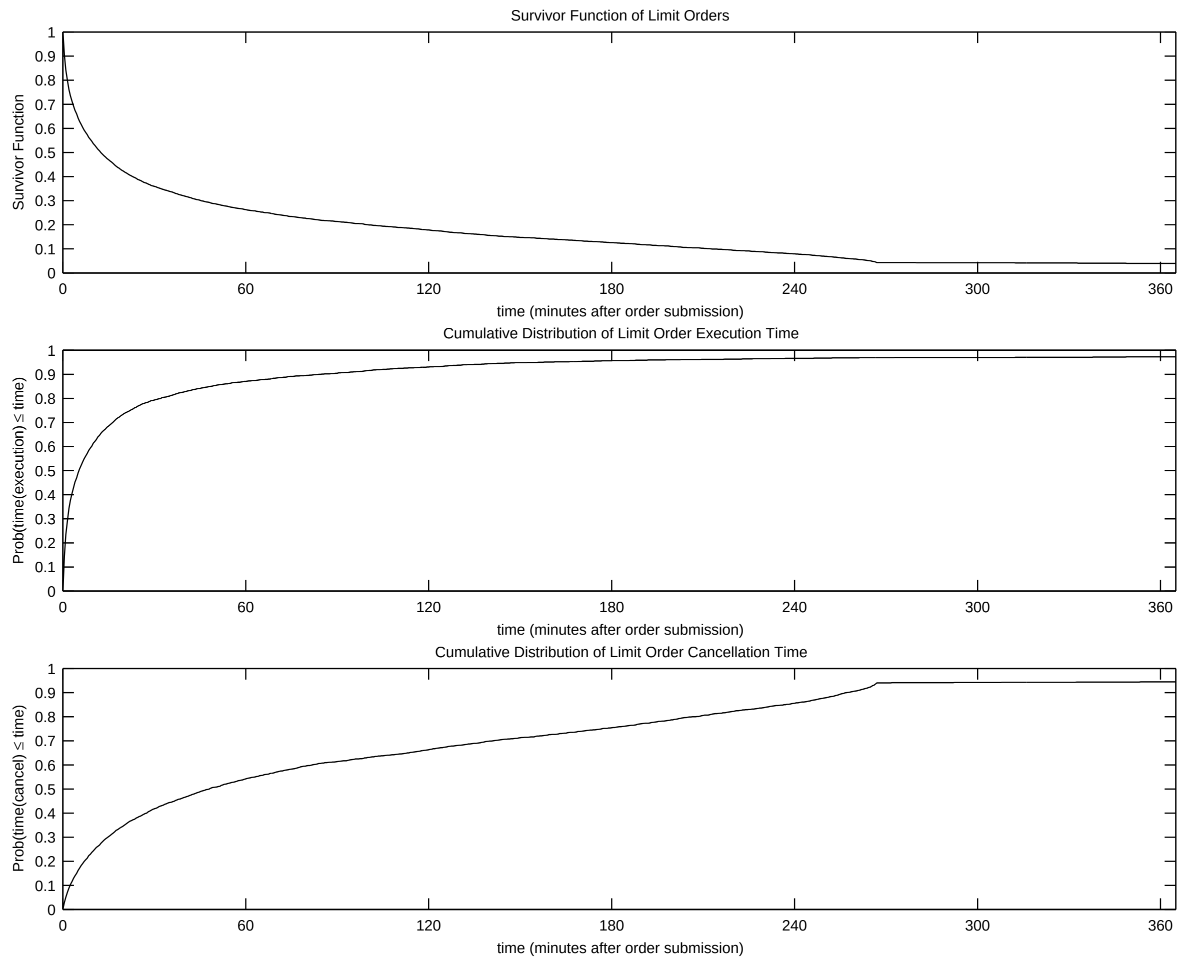

Figure 1: The top graph plots the survivor function for limit orders. The survivor function evaluated at t is defined as the probability that a limit order is outstanding t minutes after it was submitted. The middle and the bottom graphs plot the cumulative distribution functions for the limit order fill and cancellation times. All three functions are computed for orders submitted between 10:03 a.m. and 2:30 p.m. There are a total of 11,760 limit orders submitted. The survivor and distribution functions are calculated by assigning a weight to each observation equal to the fraction of the order quantity filled or canceled. Limit orders submitted during the last two trading days in our sample are not used in these calculations.

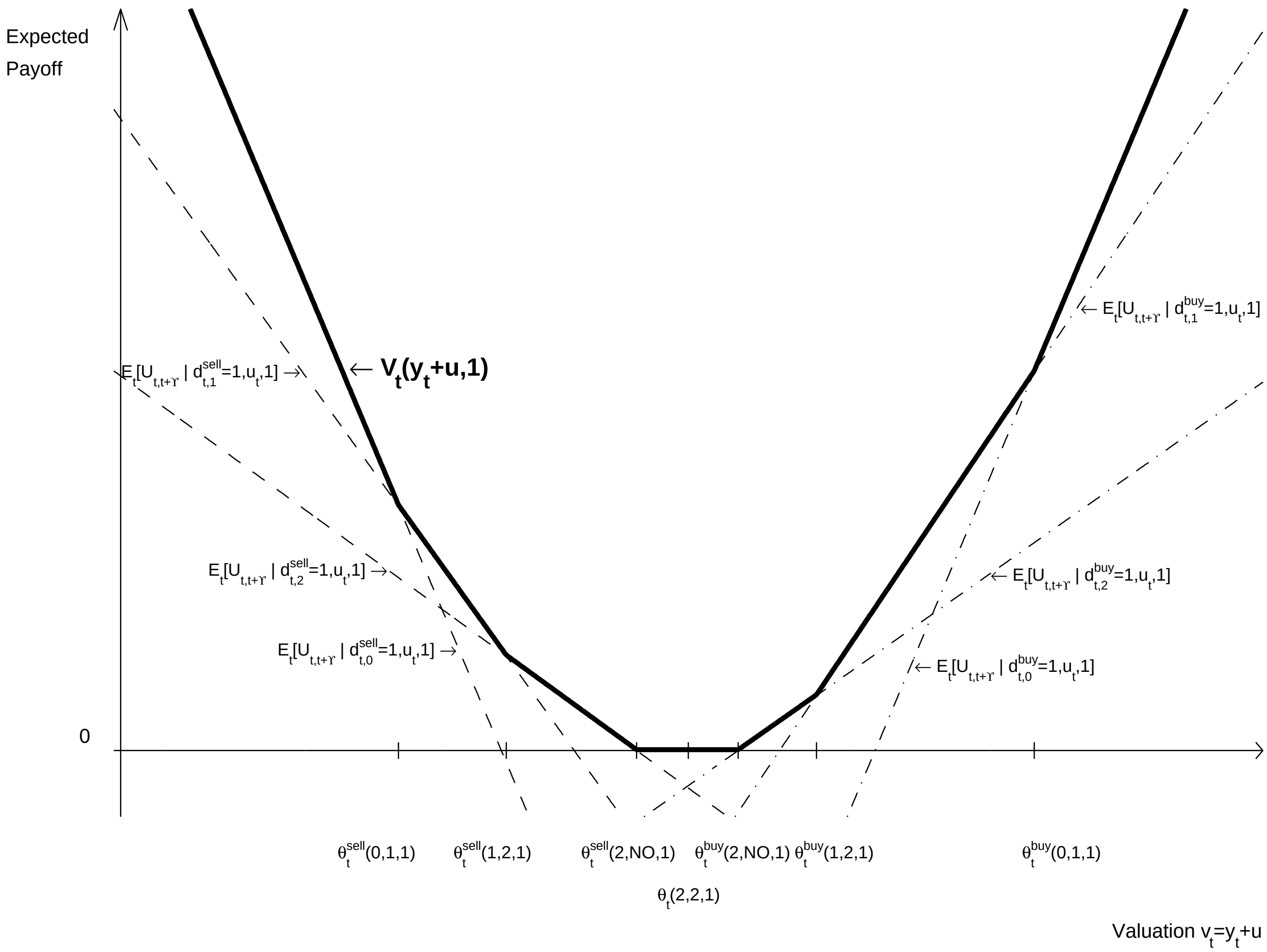

Figure 2: The graph provides an example of the indirect utility function. The order quantity is set equal to one. The horizontal axis is the trader's valuation, and the vertical axis is the expected payoff from alternative order submissions. Sell orders are plotted with dashed lines (- - -) and buy orders are plotted with dashed-dotted lines (-.-.). The indirect utility function is plotted with the dark solid line (—).

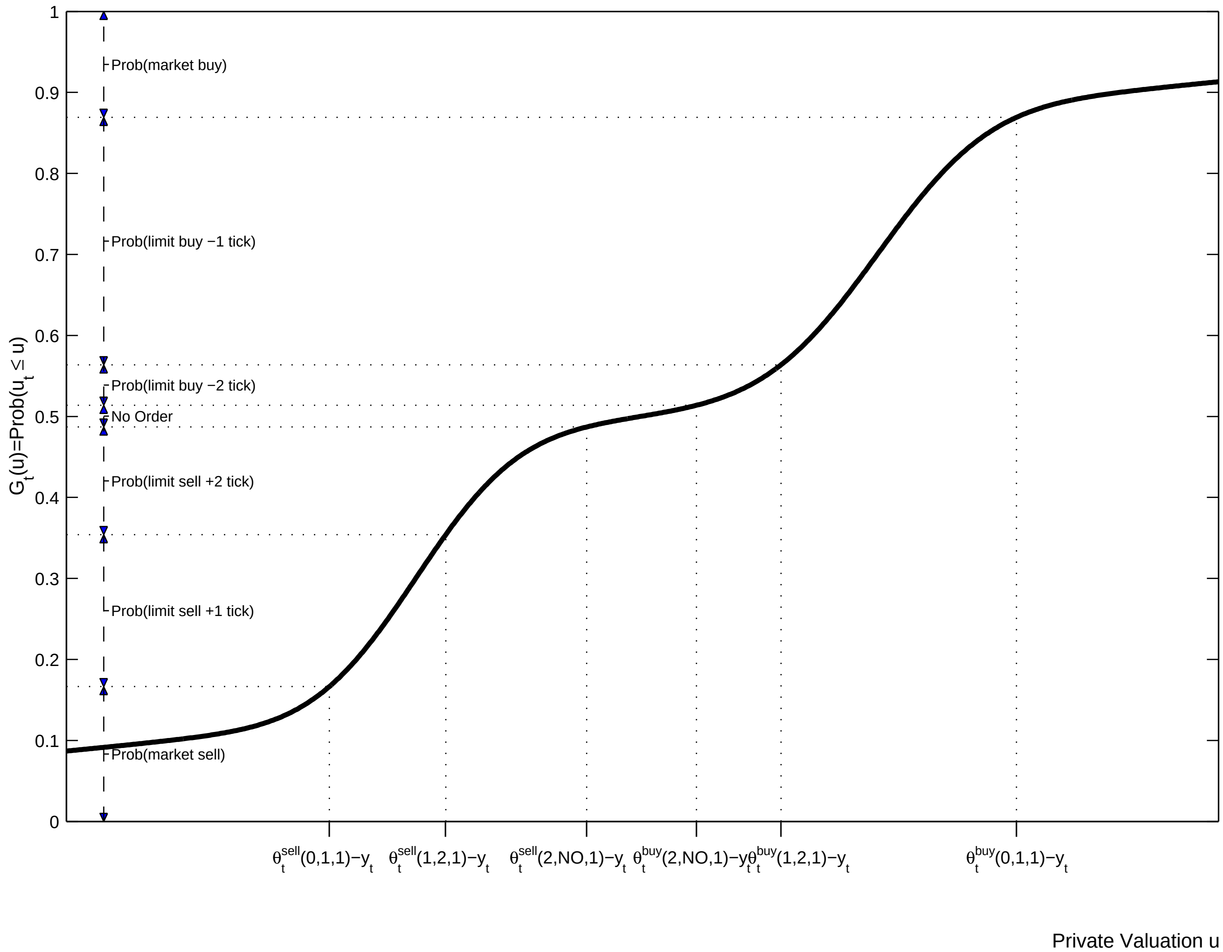

Figure 3: The graph illustrates the optimal order submission strategy. The probabilities of observing different order submissions are determined by the threshold valuations and the distribution of private valuations. The threshold valuations are computed using equations (15) through (18). The distribution of the private valuations G_t is a mixture of three normal distributions.

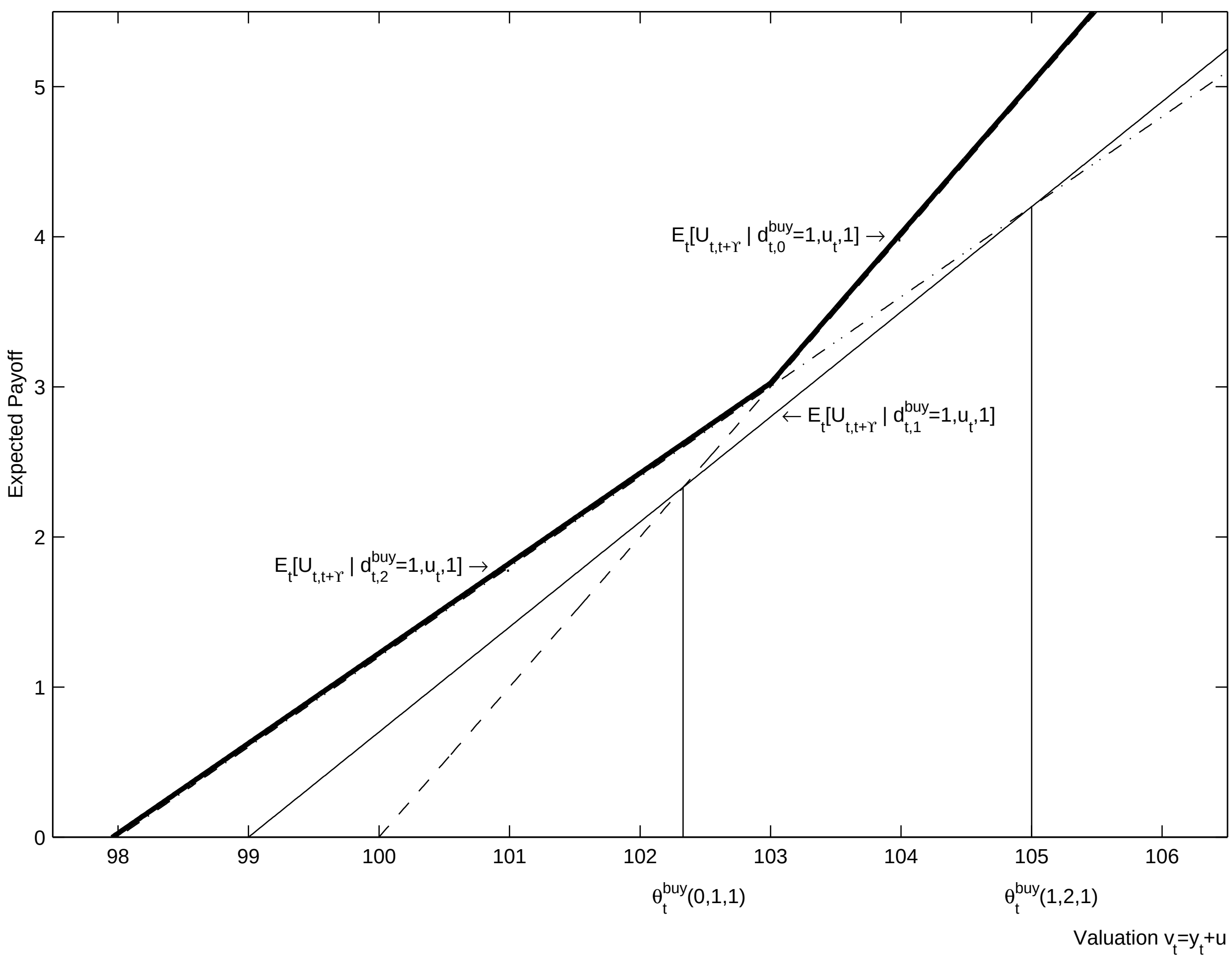

Figure 4: The figure illustrates a situation in which the threshold valuations do not satisfy the monotonicity condition: $\theta_t^{buy}(0,1,1) < \theta_t^{buy}(1,2,1)$. The execution probabilities for limit orders are monotonically decreasing in the distance between the limit order price and the best quote. The execution probabilities are $\psi_t^{buy}(0,1) = 1$, $\psi_t^{buy}(1,1) = 0.7$, $\psi_t^{buy}(2,1) = 0.6$; the tick size is 1; the best ask quote is 100; and the picking off risks are equal to zero. The indirect utilities for a trader submitting a buy market order (- - -), a one tick buy limit order (—), and a two tick buy limit order(-.-.) are plotted as a function of the trader's valuation.

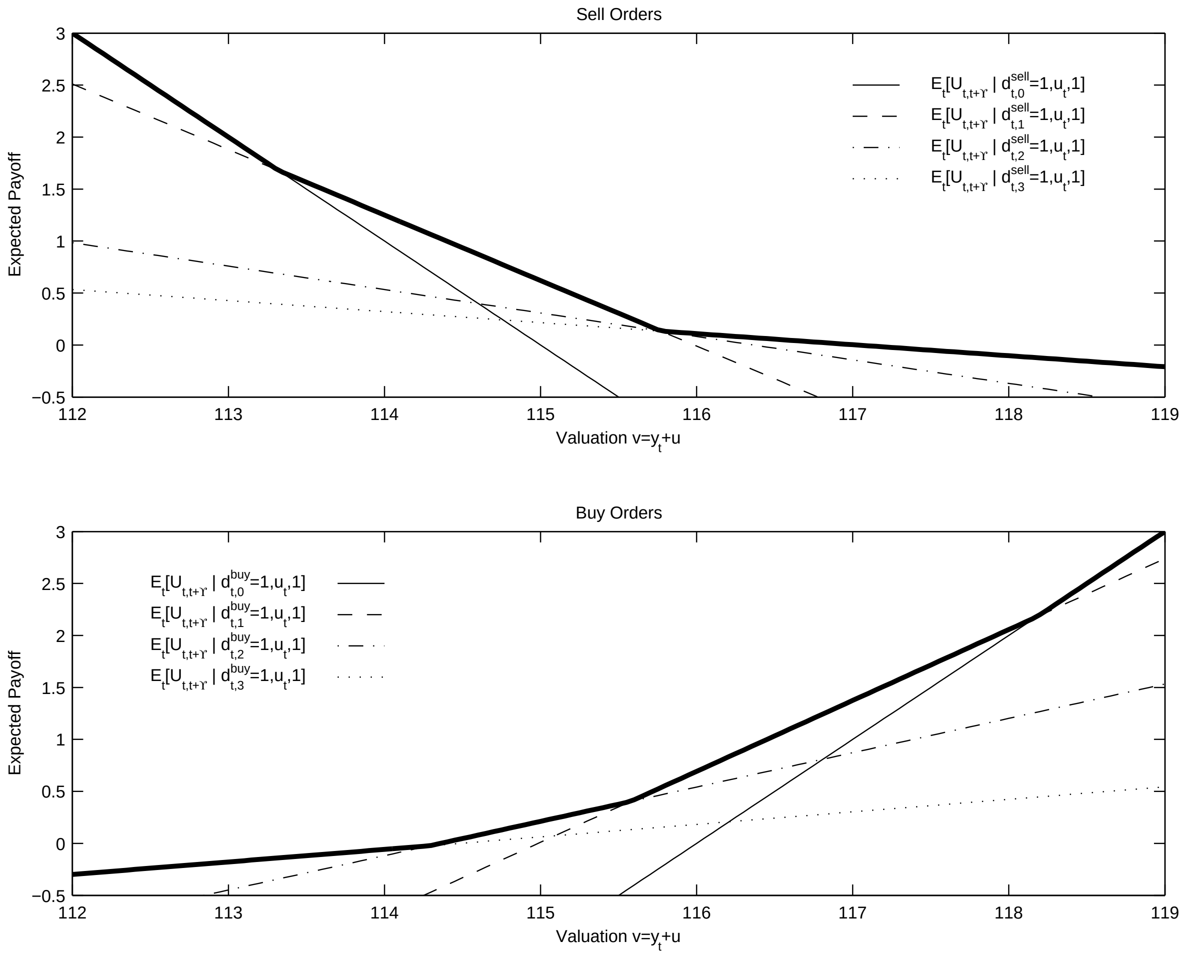

Figure 5: The figure plots the estimated payoffs as a function of the trader's valuation. The estimated payoffs are evaluated at the sample observation with conditioning variables closest to their sample averages. The horizontal axis gives the trader's valuation and the vertical axis the payoffs for alternative order submissions. The top plot is for sell orders and the bottom plot is for buy orders.