Structural Change in a Multi-Sector Model of Growth*

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Abstract

We study a multi-sector model of growth with differences in TFP growth rates across sectors and derive sufficient conditions for the coexistence of structural change, characterized by sectoral labor reallocation, and constant aggregate growth path. The conditions are weak restrictions on the utility and production functions commonly applied by macroeconomists. We present evidence from US two-digit industries that is consistent with our predictions about structural change and successfully calibrate the historical shift from agriculture to manufacturing and services. We show quantitatively that reasonable deviations from our conditions do not have a big impact on the properties of the model.

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1 Introduction

Economic growth takes place at uneven rates across different sectors of the economy. This paper has two objectives related to this fact, (a) to derive the implications of uneven sectoral growth for structural change, the name given to the shifts in industrial employment shares that take place over long periods of time, and (b) to show that even with different sectoral rates of total factor productivity growth, there can still be growth at constant or near-constant rate in the aggregate economy. The restrictions needed to yield structural change consistent with the facts and constant growth are weak restrictions on functional forms that are frequently imposed by economists in related contexts.

Pioneering work on the connections between growth and structural change was done by Baumol (1967; Baumol et al., 1985). Baumol divided the economy into two sectors, a "progressive" one that uses capital and new technology and grows at some constant rate and a "stagnant" one that uses labor as the only input and produces services as final output (as for example in the arts or the legal profession). He then claimed that because of factor mobility, the production costs and prices of the stagnant sector should rise indefinitely, a process known as "Baumol's cost disease." Over time, the stagnant sector should attract more labor to satisfy demand if demand is either income elastic or price inelastic, but should vanish otherwise. Baumol controversially also claimed that if the stagnant sector does not vanish, the economy's growth rate will be on a declining trend, as more weight is shifted to the stagnant sector.

We confirm Baumol's claim about structural change but also show that his conclusion about growth was overly pessimistic. Although costs rise and resources shift into low-growth sectors during structural change, the growth rate of the aggregate economy is bounded from below by a positive rate that depends on the growth rate of Baumol's progressive sector.[1] Our economy satisfies Kaldor's stylized facts of con-

[1] Ironically, we get our result because we include capital in our analysis, a factor left out of the analysis by Baumol "for ease of exposition ... that is [in]essential to the argument". We show that the inclusion of capital is essential for the more optimistic growth results, though not for structural change.

stant capital-output ratio and constant rate of return to capital, even before it gets to the limiting state of no further structural change.

We obtain our results in a baseline model by assuming that capital goods are supplied by only one sector, which we label manufacturing, and which produces also a consumption good. We show, however, that they are consistent with the existence of many capital goods and many intermediate goods under some reasonable restrictions. Production functions in our model are identical in all sectors except for their rates of TFP growth and each sector produces a differentiated good that enters a constant elasticity of substitution (CES) utility function. We show that a low (below one) elasticity of substitution across goods leads to shifts of employment shares to sectors with low TFP growth. In the limit the employment share used to produce consumption goods vanishes from all sectors except for the slowest-growing one, but the employment shares used to produce the capital good in manufacturing and any intermediate goods in other sectors converge to non-trivial stationary values.

At the aggregate level our economy exhibits constant or near-constant growth. If the utility function has unit inter-temporal elasticity of substitution, during structural change the rate of return to capital is constant and the aggregate economy is on a steady-state growth path. We calibrate the dynamic model when the intertemporal elasticity is not unity and find that with reasonable values the aggregate growth rate converges much faster to its steady state value than do the employment shares. Consequently, even without unit intertemporal elasticity, the economy exhibits nontrivial structural change and near-constant aggregate growth over long periods of time.

Our results contrast with the results of Echevarria (1997), who assumed non-homothetic preferences to derive structural change from different rates of sectoral TFP growth. In her economy balanced growth exists only in the limit, when preferences reduce to homotheticity with unit elasticity of substitution and structural change ceases. In the transition to the limiting state the aggregate growth rate first rises and then falls, in contrast to ours, which is either constant or converges fast. Our results also contrast with the results of Kongsamut et al. (2001) and Foellmi and Zweimuller (2002), who derive simultaneously constant aggregate growth and structural change. Kongsamut et al. obtain their results by imposing a restriction that maps some of the parameters of their Stone-Geary utility function on to the parameters of the production functions, violating one of the most useful conventions of modern macroeconomics, the complete independence of preferences and technologies. Foellmi and Zweimuller (2002) obtain their results by assuming endogenous growth driven by the introduction of new goods into a hierarchic utility function. Our restrictions are quantitative restrictions on a conventional CES utility function that maintains the independence of the parameters of preferences and technologies.

In the empirical literature two competing explanations (which can coexist) have been put forward for structural change. Our explanation, which is sometimes termed "technological" because it attributes structural change to different rates of sectoral TFP growth, and a utility-based explanation, which requires different income elasticities for different goods and can yield structural change even with equal TFP growth in all sectors.[2] Kravis et al. (1983) present evidence that favours the technological explanation, at least when the comparison is between manufacturing and services. Two features of their data that are satisfied by the technological explanation are (a) relative prices reflect differences in TFP growth rates and (b) real consumption shares are fairly constant. Our model has both these implications provided there is low substitutability in the CES utility function across goods. We use multi-sector data for the United States for the post-war period to show that changes in employment shares and prices are also consistent with our model's predictions under the same low substitutability requirement. Under low substitutability our model is also consistent with the historical OECD evidence presented by Kuznets (1966) and Maddison (1980), which shows a falling share of agriculture, a rising share of services and a hump-shaped share of manufacturing. In a quantitative exercise with the model's equations we evaluate the model's performance in the explanation for the long-run shifts between agriculture, manufacturing and services in the United States. We show that the model tracks the trends well, although it predicts a slower decline of agriculture than is observed in the data. This leads us to conclude that although for manufacturing and services the technological explanation may be sufficient to explain structural change, the explanation for the fast decline of agriculture may require something additional, such as a below-unity income elasticity.

Section 2 describes our model of growth with many sectors and sections 3 and 4 respectively derive the conditions for structural change and the conditions for balanced aggregate growth equilibrium. In sections 5 and 6 we study two extensions of our benchmark model, one where there are many capital goods and one where consumption goods can also be used as intermediate inputs. In section 7 we show evidence from two-digit US industries for 1977-2001 that supports our results. In sec-

[2] Some other explanations have also been forward to explain, in particular, the fast decline of agricultural employment. See section 8 for more discussion of this topic and some references.

tion 8 we focus on the long-run structural change between manufacturing, agriculture and services and show both analytically and with computations that our predictions match reasonably well the experience of the United States.

2 An economy with many sectors

The benchmark economy consists of an arbitrary number of m sectors. Sectors $i = 1, ..., m-1$ produce only consumption goods. The last sector, which is denoted by m and labeled manufacturing, produces both a final consumption good and the economy's capital stock. Manufacturing is the numeraire.[3]

We derive the equilibrium as the solution to a social planning problem. The objective function is

$$U = \int_0^\infty e^{-\rho t} v\left(c_1, .., c_m\right) dt, \tag{1}$$

where $\rho > 0$, $c_i \geq 0$ are per-capita consumption levels and the instantaneous utility function $v\left(.\right)$ is concave and satisfies the Inada conditions. The constraints of the problem are as follows.

The labor force is exogenous and growing at rate ν and the aggregate capital stock is endogenous and defines the state of the economy. Sectoral allocations are controls that satisfy

$$\sum_{i=1}^{m} n_i = 1; \qquad \sum_{i=1}^{m} n_i k_i = k, \tag{2}$$

where $n_i \geq 0$ is the employment share of sector i, $k_i \geq 0$ is the capital-labor ratio in sector i and $k \geq 0$ is the aggregate capital-labor ratio. Mobility costs are zero for both factors.

All production in sectors $i = 1, ..., m-1$ is consumed but in sector m production may be either consumed or invested. Therefore:

$$\begin{aligned} c_i &= F^i\left(n_i k_i, n_i\right) \qquad i = 1, ..., m-1 && (3) \\ \dot{k} &= F^m(n_m k_m, n_m) - c_m - \left(\delta + \nu\right) k && (4) \end{aligned}$$

where $F^i\left(.,.\right)$ is the production function of sector i and $\delta > 0$ is the depreciation rate.

[3]The label manufacturing is used for convenience. Although in the standard industrial classifications our capital-goods producing sector belongs to manufacturing, some sectors classified as manufacturing in the data (e.g. food and clothing) fall into the consumption category of our model. See below for more discussion of the empirical interpretation of our model.

The social planner chooses the allocation of factors n_i and k_i across the m sectors through a set of *static efficiency conditions,* and the allocation of output to consumption and capital through a *dynamic efficiency condition.* The static efficiency conditions are:

$$\frac{v_i}{v_m} = \frac{F_K^m}{F_K^i} = \frac{F_N^m}{F_N^i} \qquad \forall i. \tag{5}$$

and the dynamic efficiency condition is:

$$-\frac{\dot{v}_m}{v_m} = F_K^m - (\delta + \rho + \nu) . \tag{6}$$

where F_N^i and F_K^i are the marginal products of labor and capital in sector i.[4] By (5), the rates of return to capital and labor are equal across sectors.

Production functions are assumed to be Cobb-Douglas and in order to focus on the implications of different rates of TFP growth across sectors we assume that capital intensities are constant across sectors:

$$F^i = A_i(n_i k_i)^\alpha n_i^{1-\alpha}; \quad \frac{\dot{A}_i}{A_i} = \gamma_i; \quad \alpha \in (0,1), \quad \forall i. \tag{7}$$

With these production functions, static efficiency and the resource allocation constraints (2) imply

$$k_i = k; \qquad p_i = \frac{v_i}{v_m} = \frac{A_m}{A_i}; \qquad \forall i, \tag{8}$$

where p_i is the price of good i in the decentralized economy (in terms of the price of the manufacturing good, $p_m \equiv 1$).

Utility functions are assumed to have constant elasticities both across goods and over time:

$$v(c_1, ..., c_m) = \frac{\phi(.)^{1-\theta} - 1}{1-\theta}; \quad \phi(.) = \left(\sum_{i=1}^{m} \omega_i c_i^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \tag{9}$$

where $\theta, \varepsilon, \omega_i > 0$ and $\sum_{i=1}^m \omega_i = 1$. Of course, if $\theta = 1$, $v(.) = \ln \phi(.)$ and if $\varepsilon = 1$, $\ln \phi(.) = \sum_{i=1}^m \omega_i \ln c_i$. The utility function is strictly concave and satisfies the Inada conditions,[5] so if prices are finite there is a non-trivial demand for all consumption goods. In the decentralized economy demand functions have constant price elasticity $-\varepsilon$ and unit income elasticity.

[4]The corresponding transversality condition is $\lim_{t \longrightarrow \infty} k \exp\left(-\int_0^t (F_k^m - \delta - \nu)\, d\tau \right) = 0$.

[5]Note that although $\phi(.)$ does not satisfy the Inada conditions, the utility function $v(.)$ does satisfy them.

With the iso-elastic utility function, equation (8) yields:

$$\frac{p_i c_i}{c_m} = \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_i}\right)^{1-\varepsilon} \equiv x_i \qquad \forall i. \tag{10}$$

The new variable x_i is the ratio of consumption expenditure on good i to consumption expenditure on the manufacturing good. We also define consumption expenditure and output per capita in terms of the numeraire:

$$c \equiv \sum_{i=1}^{m} p_i c_i; \qquad y \equiv \sum_{i=1}^{m} p_i F^i \tag{11}$$

Following these definitions, and using static efficiency, we can rewrite per capita consumption and output as:

$$c = c_m X; \qquad y = A_m k^{\alpha} \tag{12}$$

where $X \equiv \sum_{i=1}^{m} x_i$. We note that although k is the ratio of the economy-wide capital stock to the labor force, the technology parameter for output is TFP in manufacturing and not an average of all sectors' TFP.

3 Structural change

We define structural change as the state in which at least some of the labor shares change over time, i.e., $\dot{n}_i \neq 0$ for at least some i.

We derive in Appendix 1 (Lemma 6) the time path of employment shares. For the consumption goods sectors, the employment shares satisfy:

$$n_i = \frac{x_i}{X}\left(\frac{c}{y}\right) \qquad i = 1, ..m-1, \tag{13}$$

and for the capital-producing sector:

$$n_m = \frac{x_m}{X}\left(\frac{c}{y}\right) + \left(1 - \frac{c}{y}\right). \tag{14}$$

The first term in the right side of (14) parallels the term in (13) and so represents the employment needed to satisfy the consumption demand for manufacturing goods. The second bracketed term is equal to the savings rate and represents the manufacturing employment needed to satisfy investment demand.

Condition (13) implies that the ratio of employment in sector i to employment in sector j is equal to the ratio x_i/x_j (for $i, j \neq m$). By differentiation we obtain that the growth rate of relative employment depends only on the difference between the sectors' TFP growth rates and the elasticity of substitution between goods:

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = (1-\varepsilon)\left(\gamma_j - \gamma_i\right) \qquad \forall i, j \neq m. \tag{15}$$

But (8) implies that the growth rate of the relative price of good i is:

$$\frac{\dot{p}_i}{p_i} = \gamma_m - \gamma_i \qquad\qquad i = 1, ..., m-1 \tag{16}$$

and so,

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = (1-\varepsilon)\left(\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}\right) \qquad \forall i, j \neq m \tag{17}$$

Proposition 1 *The rate of change of the relative price of good i to good j is equal to the difference between the TFP growth rates of sector j and sector i. In sectors producing only consumption goods, relative employment shares grow in proportion to relative prices, with the factor of proportionality given by one minus the elasticity of substitution across goods.*[6]

The dynamics of the individual employment shares satisfy:

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)\left(\bar{\gamma} - \gamma_i\right); \qquad i = 1, ...m-1 \tag{18}$$

$$\frac{\dot{n}_m}{n_m} = \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)\left(\bar{\gamma} - \gamma_m\right)\right] \frac{(c/y)\,(x_m/X)}{n_m} + \left(\frac{\dot{\overline{(1-c/y)}}}{(1-c/y)}\right)\left(\frac{1-c/y}{n_m}\right) \tag{19}$$

where $\bar{\gamma} \equiv \sum_{i=1}^{m} (x_i/X)\,\gamma_i$ is the weighted average of TFP growth rates.

Equation (18) gives the growth rate in the employment share of each consumption sector as a linear function of its own TFP growth rate. The intercept and slope of this function are common across sectors but although the slope is a constant, the intercept is in general a function of time because both c/y and $\bar{\gamma}$ are in general functions of time. Manufacturing, however, does not conform to this rule, because its employment share is made up of two components, one for the production of the consumption good (which behaves similarly to the employment share of consumption sectors) and one

[6] All derivations and proofs, unless trivial, are collected in Appendix 1.

for the production of capital goods, which behaves differently. These are key results about structural change which are compared with US data in section 7.

The properties of structural change follow immediately from (18) and (19). Consider first the case of equality in sectoral TFP growth rates, i.e., let $\gamma_i = \gamma_m \ \forall i$. Our economy in this case is one of balanced TFP growth, with relative prices remaining constant but with many differentiated goods. Because of the constancy of relative prices all consumption goods can be aggregated into one, so we effectively have a two-sector economy, one sector producing consumption goods and one producing capital goods. Structural change can still take place in this economy but only between the aggregate of the consumption sectors and the capital sector, and only if c/y changes over time. If c/y is increasing over time, the savings and investment rate are falling and labor is moving out of the manufacturing sector and into the consumption sectors. Conversely, if c/y is falling over time labor is moving out of the consumption sectors and into manufacturing. In both cases, however, the relative employment shares in consumption sectors are constant.

If c/y is constant over time, structural change requires $\varepsilon \neq 1$ and different rates of sectoral TFP growth rates. It follows immediately from (16), (18) and (19) that if $\dot{\overline{c/y}} = 0$, $\varepsilon = 1$ implies constant employment shares but changing prices. With constant employment shares faster-growing sectors produce relatively more output over time. Price changes in this case are such that consumption demands exactly absorb all the output changes that are due to the higher TFP growth rates. But if $\varepsilon \neq 1$, prices still change as before and consumption demands are either too inelastic (in the case $\varepsilon < 1$) to absorb all the output change, or are too elastic ($\varepsilon > 1$) to be satisfied merely by the increase in output due to TFP growth. So if $\varepsilon < 1$ employment has to move into the slow-growing sectors and if $\varepsilon > 1$ it has to move into the fast-growing sectors.

Proposition 2 *If $\gamma_i = \gamma_m \ \forall i = 1, ..., m-1$, a necessary and sufficient condition for structural change is $\dot{c}/c \neq \dot{y}/y$. The structural change in this case is between the aggregate of consumption sectors and the manufacturing sector. If $\dot{c}/c = \dot{y}/y$, necessary and sufficient conditions for structural change are $\varepsilon \neq 1$ and $\exists i \in \{1, .., m-1\}$ s.t. $\gamma_i \neq \gamma_m$. The structural change in this case is between all sector pairs with different TFP growth rates.*

To obtain now the behavior of output and consumption shares we use the static efficiency results in (8) and (10) to derive:

$$\frac{p_i F^i}{\sum_{i=1}^{m} p_i F^i} = n_i; \qquad \frac{p_i c_i}{\sum_{i=1}^{m} p_i c_i} = \frac{x_i}{X}; \quad \forall i. \tag{20}$$

The nominal output shares are equal to the employment shares, so the results obtained for employment shares also hold for them. Nominal consumption shares also exhibit the same dynamic behavior as employment shares, but relative real consumptions satisfy:

$$\frac{\dot{c}_i}{c_i} - \frac{\dot{c}_j}{c_j} = \varepsilon \left(\gamma_i - \gamma_j\right); \qquad \forall i, j. \tag{21}$$

A comparison of (15) with (21) reveals that a small ε can reconcile the small changes in the relative real consumption shares with the large changes in both relative nominal consumption shares and relative employment shares found by Kravis et al. (1983). This finding led the authors to conclude that the evidence favored a technological explanation for structural change. Our model shows how these changes come about and section 7 gives more empirical support for a small ε at the two-digit level.

4 Aggregate growth

With TFP in each sector growing at some rate γ_i, the aggregate economy will also grow at some rate related to the γ_is. The following Proposition derives the aggregate time paths:

Proposition 3 *Given any initial k_0, the equilibrium of the aggregate economy is defined as a path for the pair $\{c, k\}$ that satisfies the following two differential equations:*

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1} - \frac{c}{k} - (\delta + \nu), \tag{22}$$

$$\theta \frac{\dot{c}}{c} = (\theta - 1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta + \rho + \nu). \tag{23}$$

Recalling the definition of $\bar{\gamma}$ following equation (19), the key property of our equilibrium is that the contribution of each consumption sector i to aggregate equilibrium is through its weight x_i in $\bar{\gamma}$. Note that because each x_i depends on the sector's relative TFP level (A_i/A_m), the weights here are functions of time.

We characterize the aggregate equilibrium by investigating whether there is an equilibrium path that satisfies Kaldor's fact of constant capital-output ratio k/y. From the aggregation in (12) constant k/y requires $A_m k^{\alpha-1}$ to be constant, i.e., k to grow at rate $\gamma_m/(1-\alpha)$ and the rate of return to capital to be constant. The state equation (22) implies that c/k must also be a constant, so in this steady state, if it exists, aggregate output and consumption grow at the same rate as the capital-labor ratio with all aggregates defined in units of the manufacturing numeraire. We define this steady state as the balanced growth path.

We note that if all the γs are equal, relative prices are constant and the economy's average TFP growth rate is also the common γ. Our definition of aggregate consumption and output then correspond to the conventional definitions of real consumption and output, and our dynamic equations in Proposition 3 reduce to the conventional dynamic equations of the one-sector Ramsey economy. Given our results in Proposition 2, structural change takes place in the transition to the steady state of this economy, when c/y is changing, but not on the balanced growth path.

The more interesting case arises when at least some of the γs are different. In this case relative prices change and our definition of aggregate output and consumption are different from the conventional definitions, because they are deflated by the manufacturing price and not by an average of all prices. However, we can still talk of a balanced growth path defined as the state consistent with a constant rate of return to capital. We established in the preceding paragraph that on this path c/y is constant and so, by Proposition 2, structural change requires, in addition to the different γs, $\varepsilon \neq 1$. We now investigate whether such a balanced growth path exists.

It follows trivially from (23) that a necessary condition for a balanced growth path is that the expression $(\theta-1)(\gamma_m-\bar{\gamma})$ be a constant. Let for now:

$$(\theta-1)(\gamma_m-\bar{\gamma}) \equiv \psi \qquad \text{constant.} \tag{24}$$

Define aggregate consumption and the aggregate capital-labor ratio in terms of efficiency units

$$c_e \equiv c A_m^{-1/(1-\alpha)}; \qquad k_e \equiv k A_m^{-1/(1-\alpha)}.$$

The dynamic equations become

$$\frac{\dot{c}_e}{c_e} = \frac{\alpha k_e^{\alpha-1} - (\delta+\nu+\rho) + \psi}{\theta} - \frac{\gamma_m}{1-\alpha} \tag{25}$$

$$\frac{\dot{k}_e}{k_e} = k_e^{\alpha-1} - \frac{c_e}{k_e} - \left(\frac{\gamma_m}{1-\alpha} + \delta + \nu\right). \tag{26}$$

Equations (25) and (26) parallel the two differential equations in the control and state of the one-sector Ramsey model, making the aggregate equilibrium of our many-sector economy identical to the equilibrium of the one-sector Ramsey economy (when $\psi = 0$) and trivially different from it otherwise. Both models have a saddlepath equilibrium and stationary solutions $\left(\hat{c}_e, \hat{k}_e\right)$ that imply balanced growth in the three aggregates. As anticipated in the aggregate production function (12), a key result is that in our economy the rate of growth of our aggregates in the steady state is equal to the rate of growth of labor-augmenting technological progress in the sector that produces capital goods: the ratio of capital to employment in each sector and aggregate capital per worker grow at rate $\gamma_m/(1-\alpha)$. When nominal output is deflated by the price of manufacturing goods, output per worker and aggregate consumption per worker also grow at the same rate.

Proposition 2 and the results just derived give the important result:

Proposition 4 *Necessary and sufficient conditions for the existence of an aggregate balanced growth path with structural change are:*

$$\begin{aligned} \theta &= 1, \\ \varepsilon &\neq 1; \text{ and } \exists i \in \{1, .., n\} \text{ s.t. } \gamma_i \neq \gamma_m. \end{aligned} \tag{27}$$

Under the conditions of Proposition 4, $\psi = 0$, and our aggregate economy becomes formally identical to the one-sector Ramsey economy. ψ is constant under two other (alternative) conditions, which give balanced aggregate growth: $\gamma_i = \gamma_m \; \forall i$ and $\varepsilon = 1$. But as we showed in connection to Proposition 2, neither condition permits structural change.

Proposition 4 requires the utility function to be logarithmic in the consumption composite ϕ, which implies an intertemporal elasticity of substitution equal to one, but be non-logarithmic across goods, which implies non-unit price elasticities. A noteworthy implication of Proposition 4 is that balanced aggregate growth does not require constant rates of growth of TFP in any sector other than manufacturing. Because both capital and labor are perfectly mobile across sectors, changes in the TFP growth rates of consumption-producing sectors are reflected in immediate price changes and reallocations of capital and labor across sectors, without effect on the aggregate growth path.

Proposition 4 confirms Baumol's (1967) claims about structural change. When demand is price inelastic ($\varepsilon < 1$), the sectors with the low productivity growth rate attract a bigger share of labor, despite the rise in their price. The lower the elasticity of demand, the less the fall in demand that accompanies the price rise, and so the bigger the shift in employment needed to maintain high relative consumption. But in contrast to Baumol's claims, the economy's growth rate is not on an indefinitely declining trend because of the existence of capital goods. The economy-wide TFP growth rate $\bar{\gamma}$ is however falling over time when $\varepsilon < 1$.

Next, we characterize the set of expanding sectors $(\dot{n}_i \geq 0)$, denoted E_t, and the set of contracting sectors $(\dot{n}_i \leq 0)$, denoted D_t, at any time t. We establish

Proposition 5 *Both in the balanced growth path and in the transition from a low initial capital stock, the set of expanding sectors is contracting over time and the set of contracting sectors is expanding over time:*

$$E_{t'} \subseteq E_t \text{ and } D_t \subseteq D_{t'} \qquad \forall t' > t$$

Asymptotically, the economy converges to an economy with

$$n_m^* = \hat{\sigma} = \alpha \left(\frac{\delta + \nu + \gamma_m / (1-\alpha)}{\delta + \nu + \rho + \gamma_m / (1-\alpha)} \right); \qquad n_l^* = 1 - \hat{\sigma}$$

$\hat{\sigma}$ is the savings rate (equivalently, to the ratio of investment to output) along the balanced growth path and sector l denotes the sector with the smallest (largest) TFP growth rate if and only if goods are poor (good) substitutes.

In order to give some intuition for the proof (which is in the Appendix), consider the dynamics of sectors on the balanced growth path. Along this path, the set of expanding and contracting sectors satisfy:

$$\begin{aligned} E_t &= \{i \in \{1, ..., m\} : (1-\varepsilon)(\bar{\gamma} - \gamma_i) \geq 0\}; \\ D_t &= \{i \in \{1, ..., m\} : (1-\varepsilon)(\bar{\gamma} - \gamma_i) \leq 0\}. \end{aligned} \tag{28}$$

If goods are poor substitutes ($\varepsilon < 1$), sector i expands if and only if its TFP growth rate is smaller than the weighted average of all sectors' TFP growth rates, and contracts if and only if its growth rate exceeds their weighted average. But if $\varepsilon < 1$, the weighted average $\bar{\gamma}$ is decreasing over time (see Lemma 7 in the Appendix). Therefore, the set of expanding sectors is shrinking over time, as more sectors' TFP growth rates exceed $\bar{\gamma}$. If goods are good substitutes ($\varepsilon > 1$), sector i expands if and only if its TFP growth rate is greater than $\bar{\gamma}$, and contracts otherwise. But $\varepsilon > 1$ implies that

$\bar{\gamma}$ is also increasing over time, so, as before, the set of expanding sectors is shrinking over time.[7]

In contrast to each sector's employment share, once the economy is on a balanced growth path output and consumption in each consumption sector (as a ratio to the total labor force) grows according to

$$\begin{aligned} \frac{\dot{F}^i}{F^i} &= \frac{\dot{A}_i}{A_i} + \alpha\frac{\dot{k}_i}{k} + \frac{\dot{n}_i}{n_i} \\ &= \varepsilon\gamma_i + \frac{\alpha}{1-\alpha}\gamma_m + (1-\varepsilon)\,\bar{\gamma} \end{aligned} \tag{29}$$

Thus, if $\varepsilon \leqslant 1$ the rate of growth of consumption and output in each sector is positive, and so sectors never vanish, even though their employment shares in the limit may vanish. If $\varepsilon > 1$ the rate of growth of output may be negative in some low-growth sectors, and since by Lemma 7 $\bar{\gamma}$ is rising over time in this case, their rate of growth remains indefinitely negative until they vanish.

Having shown the properties of the aggregate growth path for $\theta = 1$, we now examine briefly the implications of $\theta \neq 1$. When $\theta \neq 1$ balanced growth cannot coexist with structural change, because the term $\psi = (\theta - 1)\,(\gamma_m - \bar{\gamma})$ in the Euler condition (25) is a function of time given $\bar{\gamma}$ is a function of time. But as shown in lemma 7 $\bar{\gamma}$ is monotonic. As $t \to \infty$, ψ converges to the constant $(\theta - 1)\,(\gamma_m - \gamma_l)$, where γ_l is the TFP growth rate in the limiting sector (the slowest or fastest growing consumption sector depending on whether $\varepsilon <$ or > 1). Therefore, the economy with $\theta \neq 1$ converges to an asymptotic steady state with the same growth rate as the economy with $\theta = 1$.

What characterizes the dynamic path of the aggregate economy when $\theta \neq 1$? By differentiation and a straightforward application of the result in Appendix Lemma 7, we obtain

$$\dot{\psi} = (\theta - 1)(1-\varepsilon)\sum_{i=1}^{m}(x_i/X)\,(\gamma_i - \bar{\gamma})^2 \tag{30}$$

so ψ does not change much during the transition to the asymptotic steady state if the 'variance' of the TFP growth rates is small. But from (15), employment shares can still change a lot during the transition if individual TFP growth rates differ. Moreover, $\dot{\psi}$ converges to zero over time but changes in the relative employment shares are more

[7] We can also see from (14) that on the balanced growth path, because c/y is constant, the asymptotic employment share in manufacturing is smaller than its employment share along the balanced growth path at any point in time.

persistent. We show in section 8 that observed TFP growth rates are such that for plausible θ changes in ψ are very small yet structural change is large.

5 Many capital goods

Our baseline model has only one sector producing capital goods and no intermediate inputs. We now generalize it to allow for more capital-producing sectors and (in the next section) introduce intermediate inputs. The motivation for many capital goods is obvious: more than one manufacturing sector produces capital goods and we wish to study the implications of different TFP growth rates for each of these sectors.

We suppose that there are κ different capital-producing sectors, each supplying the inputs into a production function G, which produces a capital aggregate that can be either consumed or used as an input in all production functions F^i. Thus, the model is the same as before, except that now the capital input k_i is not the output of a single sector but of the production function G. Appendix 1 derives the equilibrium for the case of a CES function with elasticity μ, i.e., when

$$G = \left[\sum_{j=1}^{\kappa} \xi_{m_j} \left(F^{m_j} \right)^{(\mu-1)/\mu} \right]^{\mu/(\mu-1)} \tag{31}$$

where $\mu > 0$, $\xi_{m_j} \geq 0$ and F^{m_j} is the output of each capital goods sector m_j. G now replaces the output of the "manufacturing" sector in our baseline model, F^m.

It follows immediately that the structural change results derived for the $m-1$ consumption sectors remain intact, as we have made no changes to that part of the model. But there are new results to derive concerning structural change within the capital-producing sectors. The relative employment shares across the capital-producing sectors satisfy:

$$\begin{aligned} \frac{n_{m_j}}{n_{m_i}} &= \left(\frac{\xi_{m_j}}{\xi_{m_i}} \right)^{\mu} \left(\frac{A_{m_i}}{A_{m_j}} \right)^{1-\mu}; \quad \forall i, j = 1, .., \kappa \\ \frac{\dot{\overline{n_{m_j}/n_{m_i}}}}{n_{m_j}/n_{m_i}} &= (1-\mu) \left(\gamma_{m_i} - \gamma_{m_j} \right); \quad \forall i, j = 1, .., \kappa \end{aligned} \tag{32}$$

If $\mu = 1$ (G is Cobb-Douglas), then the relative employment shares across capital-producing sectors remain constant over time. If $\mu > 1\,(< 1)$, then more productive capital-producing sectors increase (decrease) their employment share over time.

Comparing the new results to the results derived for consumption sectors in the baseline model, the A_m of the baseline model is replaced by $G_{m_j} A_{m_j}$, where G_{m_j} denotes the marginal product and A_{m_j} denotes TFP of capital good m_j. This term measures the rate of return to capital in the jth capital-producing sector, which is equal across all κ sectors because of the free mobility of capital. Defining $A_m \equiv G_{m_1} A_{m_1}$ we derive the growth rate:

$$\gamma_m = \sum_{j=1}^{\kappa} \zeta_j \gamma_{m_j}; \quad \zeta_j \equiv \xi_{m_j}^{\mu} A_{m_j}^{(\mu-1)} / \left(\sum_{j=1}^{\kappa} \xi_{m_j}^{\mu} A_{m_j}^{(\mu-1)} \right), \tag{33}$$

which is a weighted average of TFP growth rates in all capital-producing sectors. The dynamic equations for c and k are the same as in the baseline model.

If TFP growth rates are equal across all capital-producing sectors, c and k grow at a common rate in the steady state. But then all capital producing sectors can be aggregated into one, and the model reduces to one with a single capital-producing sector.

If TFP growth rates are different across the capital-producing sectors and $\mu \neq 1$, there is structural change within the capital-producing sectors along the transition to the asymptotic state. Asymptotically, only one capital-producing sector remains. In the asymptotic state, c and k again grow at common rate, so there exists an asymptotic balanced growth path with only one capital-producing sector.

A necessary and sufficient condition for the coexistence of a balanced growth path and multiple capital-producing sectors with different TFP growth rates is $\mu = 1$. The aggregate growth rate in this case is $\gamma_m / (1 - \alpha)$ and (33) implies $\gamma_m = \sum_{j=1}^{\kappa} \xi_{m_j} \gamma_{m_j}$. Using (32), the relative employment shares across capital-producing sectors are equal to their relative input shares in G. There is no structural change within the capital producing sectors, their relative employment shares remaining constant independently of their TFP growth rates.

The extended model with $\varepsilon < 1$ and $\mu = 1$ predicts that along the balanced growth path there is reallocation from high TFP growth consumption sectors to low TFP growth sectors but no relation between TFP growth rates and changes in employment shares across the capital-producing sectors.

6 Intermediate goods

In our second extension we allow all sectors to produce intermediate goods which can be used as an input in production by other sectors. The key difference between intermediate goods and capital goods is that capital goods are re-usable while intermediate goods depreciate fully after one usage. The motivation for the introduction of intermediate inputs is that many of the sectors that may be classified as consumption sectors produce in fact for businesses. Business services is one obvious example. Input-output tables show that a large fraction of output in virtually all sectors of the economy is sold to businesses.[8]

As in the baseline model, sectors are of two types. The first type, which consists of sectors such as food and services, produces perishable goods that are either consumed by households or used as intermediate inputs by firms. We continue referring to these sectors as consumption sectors for short. The second type of sector consists of sectors such as engineering and metals and produces goods that can be used as capital. For generality's sake, we assume that the outputs of capital-producing sectors can also be processed into both consumption goods and intermediate inputs. As before, we assume that there are $i = 1, ..., m-1$ consumption sectors and there is only one capital-goods sector.

As in the case of many capital goods, we assume the existence of an aggregate intermediate production function $\Phi(h_1, ..., h_m)$ that produces a single intermediate good Φ. The output of consumption sector i is $c_i + h_i$, where h_i is the output that is used as an input in the production of the intermediate good. Manufacturing output can be consumed, c_m, used as an intermediate input, h_m, or used as new capital, $\dot{k}$. The production functions are modified to $F^i \equiv A_i n_i k_i^{\alpha} q_i^{\beta}$, $\forall i$, where q_i is the ratio of the intermediate good to employment in sector i and β is its input share, with $\alpha, \beta > 0$ and $\alpha + \beta < 1$. When $\beta = 0$, we return to our baseline model. Restricting $\Phi(.)$ to the CES class with elasticity $\eta > 0$, we show in Appendix 1 that a necessary and sufficient condition for a balanced growth path with structural change requires $\eta = 1$, i.e. $\Phi(.)$ to be Cobb-Douglas.[9] When $\Phi(.)$ is Cobb-Douglas, all our results

[8] According to input-output tables for the United States, in 1990 the percentage distribution of the output of two-digit sectors across three types of usage, final consumption demand, intermediate goods and capital goods was 43, 48 and 9 respectively. In virtually all sectors, however, a large fraction of the intermediate goods produced are consumed by the same sector.

[9] This result is related to the result in Oulton (2001). Oulton claimed that if the slow-growing sectors that attract employment produce intermediate goods, and if the elasticity of substitution

from the baseline model remain essentially the same, except for the results for relative employment shares, (13) and (14), which require modification.

The aggregate equilibrium is similar to the baseline model:

$$\frac{\dot{c}}{c} = \alpha A k^{(\alpha+\beta-1)/(1-\beta)} - (\delta+\rho+\nu), \tag{34}$$

$$\frac{\dot{k}}{k} = (1-\beta) A k^{(\alpha+\beta-1)/(1-\beta)} - \frac{c}{k} - (\delta+\nu) \tag{35}$$

where $A \equiv \left[A_m (\beta\Phi_m)^\beta\right]^{1/(1-\beta)}$ and Φ_m is the marginal product of the manufacturing good in Φ. The growth rate of A is constant and equal to $\gamma = \gamma_m + (\beta \sum_{i=1}^m \varphi_i\gamma_i) / (1-\beta)$, where φ_i is the input share of sector i in Φ. Therefore, we can define aggregate consumption and the aggregate capital-labor ratio in terms of efficiency units and obtain a balanced growth path where the common growth rate is $(\gamma_m + \beta \sum_{i=1}^m \varphi_i\gamma_i) / (1-\alpha-\beta)$. Recall the aggregate growth rate in the baseline model depended only on the TFP growth rate in manufacturing. In the extended model with intermediate goods, the TFP growth rates in all sectors contribute to aggregate growth. Note that if $\beta = 0$ the model collapses to the baseline case.

The employment shares are now:

$$n_i = \left(\frac{c}{y}\right)\left(\frac{x_i}{X}\right) + \varphi_i\beta; \qquad i = 1, ..., m-1 \tag{36}$$

$$n_m = \left[\left(\frac{c}{y}\right)\left(\frac{x_m}{X}\right) + \varphi_m\beta\right] + \left[1 - \beta - \frac{c}{y}\right] \tag{37}$$

which are intuitive compared to (13) and (14). For the consumption sectors, the extra term in (36) is $\varphi_i\beta$, which captures the employment required for producing intermediate goods. φ_i is the share of sector i's output used for intermediate purposes and β is the share of the aggregate intermediate input in aggregate output. For the manufacturing sector, the terms in the first bracket parallel those of the consumption sectors. The second term captures the employment share for investment purposes. The relative employment shares across consumption sectors are no longer equal to x_i/x_j (as in the baseline model) because of the presence of intermediate goods. Therefore, Proposition 1 only holds for relative prices, but not for relative

between the intermediate goods and labor in the sectors that lose labor is bigger than 1, the growth rate of the latter sectors rises over time and Baumol's stagnationist results could be overturned (as in Baumol, capital goods are absent from Oulton's model). No such possibility arises with Cobb-Douglas production functions.

employment. The modification, however, is straightforward because $\varphi_i\beta$ is constant, and the results about the direction of structural change hold as in the baseline model. The asymptotic results in Proposition 5 are, however, modified. Asymptotically, the employment share used for the production of consumption goods still vanishes in all sectors except for the slowest growing one (when $\varepsilon < 1$), but the employment share used to produce intermediate goods, $\varphi_i\beta$, survives in all sectors.

7 Multi-sector evidence

A full empirical test of our model will need to take into account barriers to factor mobility, which slow down the adjustment to the balanced growth equilibrium. We postpone this topic for future work. Here we show some facts about structural change in the United States, as the country least likely to suffer from barriers to inter-sectoral allocations. Our objective is to pick up underlying trends characterizing structural change, so we take five-year averages of the relevant variables to smooth temporary fluctuations.[10]

Because of the difficulties of measuring TFP for individual sectors, we focus on the implications of our model for prices and employment shares across sectors. Relative prices in our model change according to differences in TFP growth rates, as in (16). The employment share of consumption sector i is given either by (36) if the sector produces intermediate goods or by (13) if it does not. We differentiate the more general (36) to obtain

$$\frac{\dot{n}_i}{n_i - \phi_i\beta} = \left(\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)\,\bar{\gamma} \right) - (1-\varepsilon)\gamma_i; \qquad i = 1, ...m-1. \tag{38}$$

We note that the right-hand side is made up of a term that is a function of time but is common to all sectors and a second term that is proportional to the sector's own TFP growth rate. When the sector's share of intermediate good production is small the left-hand side is approximately equal to the percentage rate of growth of the sector's employment share. Combining (16) and (38) we obtain the following relation

[10] Nickell et al. (2004) recently estimated the pattern of "deindustrialization" across the OECD by using annual data since 1970 and concluded, consistent with our model, that TFP differences are a major source of differences in the speed of deindustrialization observed in OECD countries.

between employment growth and prices

$$\frac{\dot{n}_i}{n_i - \phi_i \beta} = \left(\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)\,\bar{\gamma} - \gamma_m \right) + (1-\varepsilon)\frac{\dot{p}_i}{p_i}; \qquad i = 1, ...m-1. \tag{39}$$

We plot five-year averages of employment growth (measured in hours or number of persons employed) against prices for three sets of two-digit sectors, on the basis of the type of output that they produce (the construction of the data is described in Appendix 3). There are three sectors that produce large amounts of capital goods (more than 25 percent of total output) and two other sectors that produce only intermediate goods and sell their output to a capital producing sector. We call these five sectors the capital sectors. Next, we group together all the remaining sectors for which the production of goods for final consumption is at least 50 percent of output (12 sectors) and agriculture, which, although it produces mainly intermediate goods it supplies mostly itself and the foods industry, which is itself a consumption sector. The remaining 9 sectors produce a range of intermediate goods and supply a large number of sectors, and they are grouped together into an intermediate category.

The 13 consumption sectors have small ϕ_i in equation (39). Our model says that the relation between their percent employment change and percent price change should be approximately linear but the intercept may change from one five-year period to the next. Our sample covers 1977-2001, so there are five five-year periods. Figure 1 shows the scattergram of the growth in the sector share of hours against the growth in the sector's value-added price for the five five-year periods. A positive relationship is clearly discernible. However, our model says that the 13 points for each five-year period should approximately lie on parallel lines; i.e., although the slope of the line through each set of 13 points should be positive and unchanging across periods, the intercept might change. The five lines are drawn in the second panel of figure 1, and their slope is remarkable similar across periods. The average slope is 0.67. Had this being an unbiased estimate of the cross-section regression coefficient it would suggest a value of 0.33 for ε, but note that since both employment and prices are endogenous variables and there are no other controls or instruments in the regression, the average slope is only indicative of the magnitude of ε. As we pointed out in preceding sections, a value of $\varepsilon < 1$ is needed for predictions consistent with the empirical evidence.

In figure 2 we repeat the same exercise for the intermediate and for the capital sectors. Our model says that for intermediate sectors the relationship may also hold, but with more variance. For capital sectors the relationship is not likely to hold. The results are as suggested by our model. For the intermediate sectors three of the five lines are flat and two have smaller positive slopes (the average of the two upward-sloping ones is 0.52) whereas for the capital sectors one line has negative slope, two are flat and the other two have positive slopes that are different from each other.

We repeated the exercise by replacing hours of work by persons employed. The results are very close to the previous results with hours and do not need further discussion (see figure 3). For the 13 consumption sectors the five lines are all positively-sloped with mean slope 0.66, virtually identical to the 0.67 found with hours. The multi-sector data appear to support both our technological reason for changes in employment shares and a small ε, which is crucial if our model is to explain the co-existence of large changes in employment shares with small changes in consumption shares.

8 Historical evidence

Next we make use of historical data, going back to 1870, to compare the predictions of our model with structural change between the three sectors documented in historical statistics, agriculture, manufacturing and services. But first, we compare our prediction of constant growth for the economy's aggregates deflated by the manufacturing price with the 'stylized fact' of constant growth of the aggregates normally studied by macroeconomists, which use either fixed weights or a chain-weighted series. We show that at the level of 'stylized facts' there is not much to differentiate growth in our aggregate economy from growth in the more commonly studied one-sector economy.

Our aggregate per capita income in (11) is, in nominal terms, $p_m y$, with the normalization $p_m \equiv 1$. So, if national statistics report real incomes deflated by some other implicit or explicit index $\tilde{p}$, reported real income in our notation is $p_m y/\tilde{p}$. The difference between our aggregate y and the reported one is the ratio of the price of our manufacturing good to the deflator, $p_m/\tilde{p}$.When Kaldor and others concluded that a constant rate of growth of per capita GDP is a "stylized fact" that could be imposed on aggregative models, they were looking at the rate of growth of $p_m y/\tilde{p}$.[11] In our model, the average relative manufacturing price does not grow at constant rate, even on our balanced growth path, because the relative sector shares that are used to calculate $\tilde{p}$ are changing during structural change. So it is not possible to have a

[11] More specifically, Kaldor was looking at a "steady trend rate" of growth of the "aggregate volume of production." See Kaldor (1961, p.178)

precisely constant rate of growth of both our y and another aggregate $p_m y$ deflated by a weighted average of sector prices. But because sector shares do not change rapidly over time, at least visually, there is nothing to distinguish the "stylized fact" of constant growth in the chain-weighted (or fixed-weights) per capita GDP and in our per capita output variable. The two series for the United States are shown in Figure 4 for 1929-2000 (data before 1929 are not available for the chain-weighted series). The growth rates of the chain-weighted and our series are, respectively, 2.46 and 2.44 percent, and at least at the level of 'stylized facts' they look comparable.

Turning now to the long-term shifts between agriculture, manufacturing and services, we note that if empirically the relative price of services in terms of manufacturing goods is rising while the relative price of agriculture goods is falling, the model implies the ranking of their TFP growth rates is such that $\gamma_a > \gamma_m > \gamma_s$. Then the TFP growth rate for agriculture is always above the weighted average of TFP growth rates while the TFP growth rate for services is always below it, i.e. $\gamma_a > \bar{\gamma}_t > \gamma_s$ for all t. Therefore, the model predicts that if the three goods are poor substitutes, the agricultural employment share should decline indefinitely and the service sector employment share should rise. The manufacturing employment share may rise before it starts to decline if its TFP growth rate is lower than the initial economy-wide weighted average of TFP growth rates. But even if the share of manufacturing increases at first, eventually it should decline, as the weighted average of the TFP growth rates falls over time. Asymptotically, employment shares in the three-sector economy converge to manufacturing and services, with the employment share of manufacturing equal to the investment to output ratio.

From (13), the employment shares at any time t obey

$$\begin{aligned} n_{it} &= (1-\hat{\sigma})\frac{x_{it}}{X_t} \qquad i = a, s \\ n_{mt} &= 1 - n_{at} - n_{st}, \end{aligned} \tag{40}$$

with the notation $i = a$ for agriculture, $i = m$ for manufacturing and $i = s$ for services. Therefore, given any initial distribution of employment shares (n_{a0}, n_{s0}, n_{m0}), we have $x_{a0} = n_{a0}/(n_{m0} - \hat{\sigma})$ and $x_{s0} = n_{s0}/(n_{m0} - \hat{\sigma})$. With information on the parameter ε and the growth rate of relative prices (or differences in their TFP growth rates), the model implies that the growth rate of x_{it} is equal to $(1-\varepsilon)(\dot{p}_i/p_i)$ (or $(1-\varepsilon)(\gamma_m - \gamma_s)$). The distribution of employment shares over time can then be derived from (40). As shown previously, our model implies that the time path of employment in manufacturing is hump-shape if its TFP growth rate is less than the initial weighted average of all TFP growth rates. Therefore, by matching the initial employment distribution and growth rates of relative prices, this condition reduces to $n_{s0}\left(\dot{p}_s/p_s\right) < n_{a0}\left(-\dot{p}_a/p_a\right)$ or equivalently $n_{s0}\left(\gamma_m - \gamma_s\right) < n_{a0}\left(\gamma_a - \gamma_m\right)$.

To evaluate the quantitative implications of our model for the long-term historical shifts, we calibrate our balanced growth path to the US economy from 1869 to 1998. We describe how we conducted the calibration in Appendix 2. Our model makes predictions about the aggregate economy, relative prices and employment shares. The strategy is to choose parameters to match the first two and let the model determine the dynamics of employment shares. In brief, we set $\hat{\sigma}$ to match the aggregate investment rate and $(\gamma_m - \gamma_a, \gamma_m - \gamma_s)$ to match the average growth rate for the relative prices of agriculture and services in terms of manufacturing. The calibrated parameters are: $\hat{\sigma} = 0.2$, $\gamma_m - \gamma_s = 0.01$ and $\gamma_a - \gamma_m = 0.01$.We use two values for ε, a baseline one of 0.3 and a smaller one of 0.1. We then match the employment shares in 1869, and examine how the predictions of the model compare with the employment shares in the data, on the assumption the economy has been on our balanced growth path throughout the period.

Figure 5, panel (a), reports the results for $\varepsilon = 0.3$ and panel (b) for $\varepsilon = 0.1$. The model captures the general features of the data, especially for the lower value of ε. The hump shape for manufacturing employment is a feature of the data in both the US and other OECD countries and as a prediction is, we believe, unique to our model.[12]. For $\varepsilon = 0.1$ the model predicts a decline in the share of agriculture of 38 percentage points between 1970 and 2000 (for $\varepsilon = 0.3$ the decline is 32 points). The actual fall was 46 points. This suggests productivity growth alone may not be sufficient to account for the decline in agriculture, but the model predicts well the allocations of non-agricultural employment between manufacturing and services. If the surplus predicted share in agriculture is redistributed to manufacturing and services according to their existing share proportions, we obtain a share of manufacturing and services which are very close to their respective actual shares.[13]

[12] Kuznets (1966) documented structural change for 13 OECD countries and the USSR between 1800 and 1960 and Maddison (1980) documented the same pattern for 16 OECD countries from 1870 to 1987. They both found a pattern with the same general features as our predictions. The "shallow bell shape" for manufacturing was found by Maddison (1980, p. 48) for each of the 16 OECD countries.

[13] A number of authors have made alternative assumptions to match the decline of agricultural employment. Laitner (2000) and Gollin et al. (2002) use a utility function which implies both

In the calibration we imposed the restriction $\theta = 1$ and examined steady-state growth only, on the premise that growth in the US economy has been approximately constant. It is interesting to check, however, the quantitative implications of a different value of θ, when growth is exactly constant only in the asymptotic steady state, but (as we argued in section 4) it may be near-constant on the transition to the asymptotic state. We compute the equilibrium for the commonly-used value of $\theta = 2$ by making use of the initial capital and calibrated parameters that we used for $\theta = 1$. As previously shown, the economy will converge to an asymptotic balanced growth path where both the investment rate and the growth rate are constant. The growth rate is the same as in the economy with $\theta = 1$.

The results of the calibration are shown in figure 6. The most noticeable feature of figure 6 is that although the growth rate converges quickly to its asymptotic steady-state value of 2 percent, structural change is taking place at about the same rate as for $\theta = 1$. The most striking new feature of structural change is that now the "bell shape" predicted for manufacturing employment is less shallow, which is a feature of the data. with $\theta = 2$ agricultural employment declines slightly faster and service employment rises more slowly but the differences are small.

9 Conclusion

We have shown that predicted sectoral change that is consistent with the facts requires low substitutability between the final goods produced by each sector. Balanced growth requires in addition a logarithmic intertemporal utility function. Underlying the balanced aggregate growth there is a shift of employment away from sectors with high rate of technological progress towards sectors with low growth, and eventually, in the limit, all employment converges to only two sectors, the sector producing capital goods and the sector with the lowest rate of productivity growth. If the economy also produces intermediate goods the sectors that produce these goods also retain some employment in the limit, for similar reasons. Reasonable deviations form the restriction required for balanced aggregate growth have only a small impact on structural change and the aggregate economy converges fast to a state with near-constant

an income elasticity less than one and a zero elasticity of substitution after a subsistence level of agricultural consumption has been reached. In contrast Caselli and Coleman (2002), assume a unit elasticity of substitution but match the fast decline of agricultural employment by arguing that the cost of moving out of agriculture fell because of the increase of educational attainment in rural areas.

growth rate.

An examination of two-digit industrial data for the United States has shown that our predictions are consistent with the facts. In an examination of historical evidence since 1869 we found that focusing on uneven sectoral growth and abstracting from all other causes of structural change (such as different capital intensities and non-unit income elasticities) can explain a large fraction of the observed long-term employment shifts. More specifically, it can explain large parts of the shift of employment from agriculture to manufacturing and services and subsequently from manufacturing to services, albeit at a somewhat lower rate than is observed in the data. Of course, enriching the model with different capital intensities and non-unit income elasticities may improve the predictions. Future empirical work also needs to deal with intermediate goods and frictions in factor mobility. Intermediate goods alter some of our conclusions although not the important ones about structural change.

Finally, our model has implications for studies that take structural change as a fact and calculate its contribution to overall growth (Broadberry, 1998, Temple, 2001). For example, Broadberry and others calculate an economy's growth rate under the counterfactual of no structural change. They then attribute the difference between the actual growth rate and their hypothetical rate to structural change. Our model shows that structural change is a necessary part of aggregate growth and should be taken into account when designing accounting exercises of this kind.

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10 Appendix 1: Proofs

Lemma 6 *The employment shares satisfy:*

$$\begin{aligned} n_i &= \left(\frac{x_i}{X}\right)\left(\frac{c}{y}\right), \qquad \frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_i); \qquad i = 1, ...m-1, \\ n_m &= \left(\frac{c/y}{X}\right) + 1 - \frac{c}{y}, \quad \dot{n}_m = \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_m)\right]\left(\frac{c/y}{X}\right) - \dot{\overline{c/y}} \end{aligned}$$

where $\bar{\gamma} \equiv \sum_{i=1}^{m} (x_i/X)\gamma_i$ *is the weighted average of TFP growth rates.*

Proof. n_i follows from substituting F^i into (10), and n_m is derived from (2). Given $\dot{x}_i/x_i = (1-\varepsilon)(\gamma_m - \gamma_i)$ and $\dot{X} = \sum_{i=1}^{m} \dot{x}_i = (1-\varepsilon)(\gamma_m - \bar{\gamma}) X$, we have

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + \frac{\dot{\overline{x_i/X}}}{x_i/X} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_i) \quad i = 1, ..m-1$$

and by (2),

$$\begin{aligned} \dot{n}_m &= -\sum_{i=1}^{m-1} \dot{n}_i = -\frac{\dot{\overline{c/y}}}{c/y}(1-n_m) - (1-\varepsilon)\left(\frac{c/y}{X}\right)\sum_{i=1}^{m-1} x_i(\bar{\gamma}-\gamma_i) \\ &= \frac{\dot{\overline{c/y}}}{c/y}\left(\frac{c/y}{X} - \frac{c}{y}\right) + (1-\varepsilon)\left(\frac{c/y}{X}\right)(\bar{\gamma}-\gamma_m) \\ &= \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_m)\right]\left(\frac{c/y}{X}\right) - \dot{\overline{c/y}}. \end{aligned}$$

■

Proof of Proposition 3. Use (2) and (8) to rewrite (4) as:

$$\dot{k}/k = A_m k^{\alpha-1}(1 - \textstyle\sum_{i=1}^{m-1} n_i) - c_m/k - (\delta+\nu).$$

But $p_i = A_m/A_i$ and by the definition of c, this is equivalent to:

$$\dot{k}/k = A_m k^{\alpha-1} - c/k - (\delta+\nu).$$

Next, ϕ is homogenous of degree one: $\phi = \sum_{i=1}^{m} \phi_i c_i = \sum_{i=1}^{m} p_i c_i \phi_m = \phi_m c$. But $\phi_m = \omega_m (\phi/c_m)^{1/\varepsilon}$ and $c = c_m X$, thus $\phi_m = \omega_m^{\varepsilon/(\varepsilon-1)} X^{1/(\varepsilon-1)}$ and $v_m = \phi^{-\theta}\phi_m = \left(\omega_m^{\varepsilon/(\varepsilon-1)} X^{1/(\varepsilon-1)}\right)^{1-\theta} c^{-\theta}$, so (6) becomes

$$\theta \dot{c}/c = (\theta-1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta+\rho+\nu).$$

Lemma 7 $d\bar{\gamma}/dt \lesseqgtr 0 \Leftrightarrow \varepsilon \lesseqgtr 1$.

Proof. Totally differentiating $\bar{\gamma}$ as defined in Proposition 3 we obtain

$$\begin{aligned} d\bar{\gamma}/dt &= \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i\, (\dot{x}_i/x_i - \sum_{i=1}^{m} \dot{x}_j/X) \\ &= (1-\varepsilon) \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i \left(\gamma_m - \gamma_i - \sum_{i=1}^{m} (x_i/X)(\gamma_m - \gamma_j)\right) \\ &= (1-\varepsilon)\left(\bar{\gamma}^2 - \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i^2\right) = -(1-\varepsilon) \textstyle\sum_{i=1}^{m} (x_i/X)(\gamma_i - \bar{\gamma})^2 \end{aligned}$$

Since the summation term is always positive the result follows. ■

Proof of Proposition 5

Lemma 8 *Along the balanced growth path, if* $\varepsilon \lesseqgtr 1$, $\forall i = 1,..,m-1$, n_i *is non-monotonic if and only if* $\bar{\gamma}_0 \gtreqless \gamma_i$. *The non-monotonic* n_i *first increases at a decreasing rate for* $t < t_i$, *then decreases and converges to constant* n_i^* *asymptotically, where* t_i *is such that* $\bar{\gamma}_{t_i} = \gamma_i$. *The monotonic* n_i *are decreasing and converge to zero asymptotically.*

Proof. $\forall i = 1,..,m-1$, Lemma 6 implies that along the balanced growth path, $\dot{n}_i/n_i = (1-\varepsilon)(\bar{\gamma}_t - \gamma_i) > 0 \Leftrightarrow \bar{\gamma}_t > \gamma_i$. Lemma 7 implies n_i eventually decreases. Therefore, n_i is non-monotonic if and only if $\bar{\gamma}_0 > \gamma_i$. ■

Corollary 9 *If* $\varepsilon < 1$, $t_s \to \infty$ *where* s *is such that* $\gamma_s = \min\{\gamma_i\}_{i=1,.,m}$. *If* $\varepsilon > 1$, $t_f \to \infty$ *where* f *is such that* $\gamma_f = \max\{\gamma_i\}_{i=1,..,m}$.

To establish now the claims in Proposition 5, assume, without loss of generality, $\varepsilon < 1$, $\gamma_1 > ... > \gamma_{m-1}$ and $\gamma_m < \gamma_h = \bar{\gamma}_0$ where $1 < h < m-1$. Then, Lemma 8 implies $t_i = 0$ $\forall i \leq h$, and $i \in E_0$ $\forall i \geq h$, moreover, $E_{t_{h+1}} \cup \{h+1\} = E_0$ and $D_{t_h+1} = D_0 \cup \{h+1\}$, thus $E_{t_{h+1}} \subseteq E_0$ and $D_0 \subseteq D_{t_{h+1}}$.The result follows for any arbitrary $t > 0$. Next, we prove that the economy converges to a two-sector economy. Without loss of generality, consider $\varepsilon < 1$. Given $X/x_i = \sum_{i=1}^{m} (\omega_j/\omega_i)^{\varepsilon} (A_i/A_j)^{1-\varepsilon}$, and $A_i/A_j \to 0 \Leftrightarrow \gamma_i < \gamma_j$, we have $X/x_i \to 1 \Leftrightarrow \gamma_i = \min\{\gamma_j\}_{j=1,.,m}$. Therefore, asymptotically, $n_l^* = \hat{c}_e \hat{k}_e^{-\alpha}$ and $n_m^* = 1 - n_l^*$, where $\gamma_l = \min\{\gamma_i\}_{i=1,.,m}$.

We now prove these results hold also in the transition to the steady state from any small k_0. Let $z \equiv c_e/k_e$, (25) and (26) (with $\psi = 0$ and $\theta = 1$) imply:

$$\dot{z}/z = (\alpha-1)\, k_e^{\alpha-1} + z - \rho, \qquad \dot{k}_e/k_e = k_e^{\alpha-1} - z - [\gamma_m/(1-\alpha) + \delta + \nu].$$

A phase diagram can be drawn with $\dot{z} < 0$ along the transition. For c/y, we have:

$$\frac{\dot{\overline{c/y}}}{c/y} = \frac{\dot{c}_e}{c_e} - \alpha\frac{\dot{k}_e}{k_e} = \alpha z - \rho - (1-\alpha)\left(\frac{\gamma_m}{1-\alpha} + \delta + \nu\right).$$

Since $\dot{\overline{c/y}} = 0$ in the steady state but $\dot{z} < 0$ in the transition, thus $\dot{\overline{c/y}} > 0$ and $\ddot{\overline{c/y}} < 0$ along the transition. Also, $\forall t, \forall i = 1, .., m-1$, we have:

$$\dot{n}_i/n_i = \alpha z - \rho - (1-\alpha)\left[\gamma_m/(1-\alpha) + \delta + \nu\right] + (1-\varepsilon)(\bar{\gamma} - \gamma_i),$$

which decreases along the transition given lemma 7 and $\dot{z} < 0$. Thus, starting from any small k_0, if $i \in E_0$ then $\dot{n}_i > 0$, $\ddot{n}_i < 0$, and if $i \neq l$, $i \in E_t$ $\forall t < t_i$, and $i \in D_t$ $\forall t \geq t_i$, where t_i is defined in Lemma 8. If $i \in D_0$, then $i \in D_t$ $\forall t$. Therefore, Lemma 8 holds along the transition.

Many capital-producing sectors $\forall j = 1, .., \kappa$, $F^{m_j} \equiv A_{m_j} n_{m_j} k_{m_j}^{\alpha}$, which together produce good m through $G = \left[\sum_{j=1}^{\kappa} \xi_{m_j} \left(F^{m_j}\right)^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}$, $\xi_{m_j} \geqslant 0, \mu > 0$, and $\sum_{j=1}^{\kappa} \xi_{m_j} = 1$. The planner's problem is similar to before with $\dot{k} = G - c_m - (\delta+\nu)k$ replacing (4), and $\left(k_{m_j}, n_{m_j}\right)_{j=1,.,\kappa}$ as additional control variables.

The static efficiency conditions are $F_K^i/F_N^i = F_K^{m_j}/F_N^{m_j}$, $\forall i = 1, ..m-1$, $\forall j = 1, ., \kappa$, so $k_i = k_{m_j} = k$. Also $G_{m_j}/G_{m_i} = F_K^{m_i}/F_K^{m_j} = A_{m_i}/A_{m_j}$, $\forall i, j = 1, ..\kappa$, which implies $n_{m_j}/n_{m_i} = \left(\xi_{m_j}/\xi_{m_i}\right)^{\mu}\left(A_{m_i}/A_{m_j}\right)^{1-\mu}$ and grows at $(1-\mu)\left(\gamma_{m_i} - \gamma_{mj}\right)$. Let $n_m \equiv \sum_{j=1}^{\kappa} n_{m_j}$, we have $n_m = n_{m_1}\sum_{j=1}^{\kappa}\left(\xi_{m_j}/\xi_{m_1}\right)^{\mu}\left(A_{m_1}/A_{m_j}\right)^{1-\mu}$. Next, $\forall i = 1, ..m-1$, $p_i = v_i/v_m = A_m/A_i$, where $A_m \equiv G_{m_1}A_{m_1}$. Thus, n_i/n_j and p_i/p_j are the same as in the baseline model. For the aggregate equilibrium, note $G = \sum_{j=1}^{\kappa} F^{m_j} G_{m_j} = A_m k^{\alpha} n_m$, so $\dot{c}/c$ and $\dot{k}/k$ become the same as in the baseline model. Thus, we obtain the same equilibrium if $\dot{A}_m/A_m$ is constant. Note $G_{m_1} = \xi_{m_1}\left(G/F^{m_1}\right)^{1/\mu}$ and $G/F^{m_1} = \left[\sum_{j=1}^{\kappa} \xi_{m_j}\left(A_{m_j}n_{m_j}/\left(A_{m_1}n_{m_1}\right)\right)^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}$, then use the result on n_{m_j}/n_{m_1}, we have $G/F^{m_1} = \left[\sum_{j=1}^{\kappa} \xi_{m_j}^{\mu}\left(\xi_{m_1}A_{m_1}\right)^{1-\mu} A_{m_j}^{(\mu-1)}\right]^{\mu/(\mu-1)}$, thus we have

$$A_m = G_{m_1}A_{m_1} = \left[\sum_{j=1}^{\kappa} \xi_{m_j}^{\mu} A_{m_j}^{(\mu-1)}\right]^{1/(\mu-1)}$$

and its growth rate is

$$\gamma_m = \sum_{j=1}^{\kappa} \zeta_j \gamma_{m_j}; \quad \zeta_j \equiv \xi_{m_j}^{\mu} A_{m_j}^{(\mu-1)} / \left(\sum_{j=1}^{\kappa} \xi_{m_j}^{\mu} A_{m_j}^{(\mu-1)}\right).$$

So γ_m is constant if $(\mu-1)\sum_{j=1}^{\kappa} \zeta_j \left(\gamma_{m_j} - \gamma_m\right)^2 = 0$, i.e. if (1) $\gamma_{m_i} = \gamma_{m_j}$ $\forall i, j = 1, ., \kappa$ or (2) $\mu = 1$. If (1) is true, the capital-producing sectors can be aggregated into one and the model reduces to one with only one capital-producing sector. Thus, coexistence of multiple capital-producing sectors with different TFP growth rates and a balanced growth path requires (2), i.e., $G = \prod_{j=1}^{\kappa} \left(F^{m_j}\right)^{\xi_j}$ and $\gamma_m = \sum_{j=1}^{\kappa} \xi_{m_j} \gamma_{m_j}$.

Intermediate goods The production functions are, $F^i \equiv A_i n_i k_i{}^{\alpha} q_i^{\beta}$, $\forall i$, $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$. For $i = 1,..,m-1$, F^i is either bought by consumers (c_i) or by businesses (h_i). But F^m can also be used as investment. Intermediate goods are produced by $\Phi(h_1,..,h_m)$, which satisfies $\Phi_i > 0, \Phi_{ii} < 0$, and constant return to scale. The planner's problem is similar to before with $\dot{k} = F^m - h_m - c_m - (\delta+\nu) k$ replacing (4), $\sum_{i=1}^{m} n_i q_i = \Phi$ as an additional resource constraint and the additional controls are $\{h_i, c_i, q_i\}_{i=1,..,m}$.

The static efficiency conditions are:

$$\frac{v_i}{v_m} = \frac{F_K^m}{F_N^i} = \frac{F_N^m}{F_N^i} = \frac{F_Q^m}{F_Q^i} = \frac{\Phi_i}{\Phi_m}; \qquad \forall i,$$

which implies $k_i = k$, $q_i = \Phi$, $p_i = A_m/A_i$, $\forall i$, and $y = A_m k^{\alpha} \Phi^{\beta}$. Define aggregate intermediate inputs $h \equiv \sum_{i=1}^{m} p_i h_i$. To solve for h, use the planner's optimal conditions for h_m and q_m to obtain $1 = \beta \Phi_m A_m k^{\alpha} \Phi^{\beta-1}$. But Φ is homogenous of degree one: $\Phi = \sum_{i=1}^{m} \Phi_i h_i = \sum_{i=1}^{m} \Phi_m p_i h_i = \Phi_m h$, we have $h = \beta y$, together with static efficiency,

$$\dot{k} = A_m k^{\alpha} \Phi^{\beta} \left(1 - \sum\nolimits_{i=1}^{m-1} n_i\right) - h_m - c_m - (\delta+\nu) k = h(1-\beta)/\beta - c - (\delta+\nu) k.$$

The dynamic efficiency condition is $-\dot{v}_m/v_m = \alpha A_m k^{\alpha-1} \Phi^{\beta} - (\delta+\rho+\nu)$, so

$$\dot{c}/c = \alpha h/(\beta k) - (\delta+\rho+\nu), \quad \dot{k}/k = (1-\beta) h/(\beta k) - c/k - (\delta+\nu).$$

Constant $\dot{c}/c$ requires constant h/k and constant $\dot{k}/k$ requires constant c/k. Thus, $\dot{h}/h$ must be constant, i.e. Φ/Φ_m must be growing at a constant rate. Suppose Φ is a CES function, $\Phi = \left(\sum_{i=1}^{m} \varphi_i h_i^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}$, then the static efficiency conditions imply $\forall i$, $p_i h_i/h_m = (\varphi_i/\varphi_m)^{\eta} (A_m/A_i)^{1-\eta} \equiv z_i$, $h = Z h_m$, where $Z \equiv \sum_{i=1}^{m} z_i$, so $\Phi_m = \varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)}$ and $\Phi = \left(\beta A_m k^{\alpha} \varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)}\right)^{1/(1-\beta)}$. Hence,

$$\begin{aligned} h &= \Phi/\Phi_m = (\beta A_m k^{\alpha})^{1/(1-\beta)} \left(\varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)}\right)^{\beta/(1-\beta)} \\ &\implies (1-\beta)\dot{h}/h = \left(\gamma_m + \alpha \dot{k}/k\right) + \beta\left(\sum\nolimits_{i=1}^{m} (z_i/Z)\gamma_i - \gamma_m\right) \end{aligned}$$

which is constant only if $\sum_{i=1}^{m} z_i \gamma_i$ is constant. From the definition of z_i and given that the γ_i are not the same across all i, constancy requires $\eta = 1$, and so $\Phi = \prod_{i=1}^{m} h_i^{\varphi_i}$, $Z = 1/\varphi_m$, and $z_i = \varphi_i/\varphi_m$, $\forall i$. The static efficiency conditions imply $\Phi = h_m \prod_{i=1}^m (z_i A_i/A_m)^{\varphi_i}$ and so $\Phi_m = \varphi_m \Phi / h_m = \prod_{i=1}^m (\varphi_i A_i/A_m)^{\varphi_i}$. But $\Phi = [\beta A_m k^\alpha \Phi_m]^{1/(1-\beta)}$, so $h = \Phi/\Phi_m = (\beta A_m k^\alpha)^{1/(1-\beta)} \Phi_m^{\beta/(1-\beta)}$. The dynamic equations become:

$$\begin{aligned}
\frac{\dot{c}}{c} + \delta + \rho + \nu &= \frac{\alpha}{\beta k} \left[(\beta A_m k^\alpha) \Phi_m^\beta \right]^{\frac{1}{1-\beta}} = \alpha \left[k^{\alpha+\beta-1} A_m (\beta \Phi_m)^\beta \right]^{\frac{1}{1-\beta}}, \\
\frac{\dot{k}}{k} + \frac{c}{k} + \delta + \nu &= \frac{(1-\beta)}{\beta k} \left[(\beta A_m k^\alpha) \Phi_m^\beta \right]^{\frac{1}{1-\beta}} = (1-\beta) \left[k^{\alpha+\beta-1} A_m (\beta \Phi_m)^\beta \right]^{\frac{1}{1-\beta}}.
\end{aligned}$$

Define $c_e \equiv c A^{-(1-\beta)/(1-\alpha-\beta)}$ and $k_e \equiv k A^{-(1-\beta)/(1-\alpha-\beta)}$, where $A \equiv \left[A_m (\beta \Phi_m)^\beta \right]^{1/(1-\beta)}$, and $\gamma \equiv \dot{A}/A = \left[\gamma_m + \beta \sum_{i=1}^m \varphi_i (\gamma_i - \gamma_m) \right] / (1-\beta) = \gamma_m + \left(\beta \sum_{i=1}^m \varphi_i \gamma_i \right) / (1-\beta)$,

$$\begin{aligned}
\dot{c}_e / c_e &= \alpha k_e^{(\alpha+\beta-1)/(1-\beta)} - \left[\delta + \rho + \nu + (1-\beta) \gamma / (1-\alpha-\beta) \right], \\
\dot{k}_e / k_e &= (1-\beta) k_e^{(\alpha+\beta-1)/(1-\beta)} - c_e/k_e - \left[\delta + \nu + (1-\beta) \gamma / (1-\alpha-\beta) \right],
\end{aligned}$$

which imply existence and uniqueness of a balanced growth path. $\forall i = 1, ..m-1$, we obtain n_i using $F^i = c_i + h_i$, i.e. $A_i n_i k^\alpha \Phi^\beta p_i = p_i (c_i + h_i) = x_i c_m + z_i h_m = c x_i / X + \varphi_i h$. Substitute p_i and h to obtain $n_i y = c x_i / X + \varphi_i \beta y$, finally

$$n_i = (c/y)(x_i/X) + \varphi_i \beta; \quad \forall i, \qquad n_m = \left[(c/y)(x_m/X) + \varphi_m \beta \right] + \left[1 - \beta - c/y \right].$$

11 Appendix 2: Calibration and sources of historical evidence

Sources of historical evidence (1) Historical Statistic of the United States: Colonial Times to 1970, Part 1 and 2: for employment (series F250-258), for prices (series E17, E23-25, E42, E52-E53) and index of manufacturing production (series P13-17); (2) Economic Report of the President: for prices and index of manufacturing production; and (3) Bureau of Economic Analysis (BEA): for investment rate and capital-output ratio.

Model in discrete time The Euler condition (25) and the feasibility condition (26) are:

$$\frac{c_e(t+1)}{c_e(t)} = \left[\left(\frac{1 - \delta + \alpha k_e^{\alpha-1}(t+1)}{(1+g)^\theta (1+\nu)/\beta} \right) \left(\frac{X(t+1)}{X(t)} \right)^{(1-\theta)/(\varepsilon-1)} \right]^{1/\theta} \tag{41}$$

$$\frac{k_e(t+1)}{k_e(t)} = \frac{[k_e(t)]^{\alpha-1} - c_e/k_e + (1-\delta)}{(1+g)(1+\nu)} \tag{42}$$

where $g = (1+\gamma_m)^{1/(1-\alpha)} - 1$ is the aggregate growth rate. The variables in our model are matched to US annual data.

Calibration to aggregate balanced growth path ($\theta = 1$) The parameters are the preference parameters $(\omega_a, \omega_m, \omega_s, \rho, \varepsilon)$, the technology parameters $(\gamma_m, \gamma_s, \gamma_a, A_{a0}, A_{m0}, A_{s0}, \alpha, \delta)$ and the labor force growth rate ν.

We first calibrate the parameters that determine employment shares. The contribution of parameters $(\gamma_m, \alpha, \delta, \nu, \rho)$ is captured by $\hat{\sigma}$, while the contribution of $(\omega_a, \omega_m, \omega_s)$ and (A_{a0}, A_{m0}, A_{s0}) is captured through the initial weights (x_{a0}, x_{s0}) . Therefore, we only need to know $\varepsilon, \hat{\sigma}, x_{a0}, x_{s0}, (\gamma_m - \gamma_s)$ and $(\gamma_m - \gamma_a)$ to compute the employment shares. Ideally, we want an estimate for the elasticity of substitution for the period 1869-1998. Without this measure, we use $\varepsilon = 0.3$, which is consistent with the lines in figure 1 for the period 1977-2001, and a lower value of $\varepsilon = 0.1$. The remaining five parameters are set to match five targets: the average investment rate, the agriculture and services employment shares in 1869 and the growth rate of relative prices in services and agriculture. Maddison (1992) shows that the investment rate is constant for the period 1870-1988 except for the great depression and the war periods. This is also consistent with the data from BEA for the period 1929-1998. Hence, we set $\hat{\sigma} = 0.2$ to match the average investment rate for this period. The agriculture employment share is 0.5 and the service employment share is 0.25 in 1869. We compute $x_{a0} = n_{a0}/(n_{m0} - \hat{\sigma})$ and $x_{s0} = n_{s0}/(n_{m0} - \hat{\sigma})$. The price data for agriculture and manufacturing start from 1869. However, the price data for services start in 1929. The average annual growth rate for the relative price of services in terms of manufacturing is 0.98% for the period 1929-1998. For the same period, the relative price of agriculture (in terms of manufacturing price) is falling at an average rate of 1.03%. Thus, $\gamma_m - \gamma_a = -0.01$ and $\gamma_m - \gamma_s = 0.01$.

We next calibrate the rest of the parameters to match the growth rate of total employment, the aggregate growth rate, and the aggregate capital-output ratio. The average annual growth rate of total employment for the period 1869-1998 is 1.9%, i.e. $\nu = 0.02$. The average annual growth rate of labor productivity in manufacturing was about 2% between 1869 and 1998, which is consistent with the aggregate growth rate. We set $g = 0.02$. Given the capital-output ratio (which is about 3 for the period 1929-1998), the feasibility condition implies $1 - \delta = (1 + g)(1 + \nu) - \hat{\sigma}(y/k)$. We set the capital share $\alpha = 0.3$, but our results are robust to the range 0.3-0.4. Finally, the Euler condition is used to derive $\beta = (1 + \nu)(1 + g)/(1 - \delta + \alpha y/k)$. To summarize, the baseline parameters are:

ε	$\gamma_m - \gamma_s$	$\gamma_a - \gamma_m$	ν	g	δ	α	β
0.3	0.01	0.01	0.02	0.02	0.03	0.3	0.97

Computation for economy with $\theta \neq 1$ We use the baseline parameters to compute the transition path for the economy with $\theta \neq 1$. We start with the same initial income and initial employment distribution. The initial income is matched by choosing the same $k_e(1)$ as in the benchmark. To match the initial employment we need to adjust x_{a0} and x_{s0}. These two parameters were previously calibrated to match the initial employment shares under the assumption that the investment rate is $\hat{\sigma}$. When $\theta \neq 1$, the initial employment shares depend on the initial investment rate, which is different from $\hat{\sigma}$.

We rewrite (41) and (42) in terms of $z \equiv c/k$:

$$\begin{aligned}
\frac{z(t+1)}{z(t)} &= \left(\frac{k_e(t)}{k_e(t+1)}\right)\left[\left(\frac{1-\delta+\alpha k_e^{\alpha-1}(t+1)}{(1+g)^{\theta}(1+\nu)/\beta}\right)\left(\frac{X(t+1)}{X(t)}\right)^{(1-\theta)/(\varepsilon-1)}\right]^{1/\theta} \\
\frac{k_e(t+1)}{k_e(t)} &= \frac{[k_e(t)]^{\alpha-1}-z(t)+(1-\delta)}{(1+g)(1+\nu)}.
\end{aligned}$$

Given $k_e(1)$, if we know $z(1)$, the equilibrium is derived by iteration. We use a shooting algorithm to look for $z(1)$. As $t \longrightarrow \infty$, $X(t+1)/X(t)$ converges to a constant, so this economy converges to an asymptotic steady state with $\alpha k_e^{\alpha-1} - \delta - \nu = \rho - \psi + \theta g$ and $\dot{k}/k = g$. First note that the transversality condition holds if $\rho > (1-\theta)(g - (\gamma_m - \gamma_s))$, which is satisfied under the calibrated parameters at all positive θ. Let z^* be the asymptotic steady state value. Thus, for sufficiently large T, $z(T) \approx z^*$. The shooting algorithm is as follows: given an initial $k_e(1)$, guess a $z(1)$. For any t, if $k_e(t) < 0$, decrease $z(1)$. Since X is a function independent of z and k_e, $z(T)$ is increasing in $z(1)$. Therefore, if $z(T) < (>)\, z^*$, increase (decrease) $z(1)$.

12 Appendix 3: Sectorial Data sources and construction

We make use of two data sources, both of which are available online at www.oecd.org:

1. OECD Structural Analysis Database for Industrial Analysis (STAN), which is entirely based on national US sources, and covers the period 1977-2001.

2. OECD input-output database (I/O database).

We extract from STAN data for total hours of work, total employment and prices (obtained as the ratio of the sector's value added at current prices to the volume measure of value added). There is a complete annual data set for 1977-2001 for 27 groupings of two-digit industries, shown in Table 1. Our sectors cover about 98 percent of employment in the Unites States. The table also shows the sample mean of the share of each sector in total hours and also whether the sector is classified as consumption, intermediate or capital.

In order to classify sectors we used information from the 1990 input-output tables. The tables can be used to calculate the share of output of each sector that is capital formation (column headed "Private GFCF"), intermediate inputs (column headed "Total") and private consumption (column headed "Private Domestic Consumption"). We defined a sector's output as the sum of the three columns and calculated the shares of each. There are 3 sectors where production of capital goods amount to more than 28% of total output. The next highest capital-producing sector is wholesale and retail trade whose production of capital goods amount to 6% of total output. We classify the first three sectors as capital (or manufacturing) sectors.[14] In addition, we include two other sectors which respectively produced 94% and 100% intermediate goods but supplied almost exclusively one capital sector each: 93% of the output of non-mineral manufacturing consisted of intermediate goods, and 70% of it was bought either by itself or by the construction sector; and 80% of the output of the combined basic metals and fabricated metals sectors (which produce only intermediate goods) is bought by the machinery and equipment sector.

Of the remaining sectors, those that supply more than 50% of output to final consumption use are classified as consumption sectors. In addition, agriculture produces almost exclusively intermediate goods for the food industries, which are a consumption sector, and is classified as a consumption sector. The remaining sectors produce more than 50% of their output for a range of other sectors and are classified as intermediate sectors.

[14] The 1990 tables appear to have an aberration that we ignored: 9.7% of the output of mining is reported as capital goods. In 1977,1982 and 1985 (all the other years that tables are available) capital goods are less than 1% of the output of this sector. The percentage distribution for other sectors is virtually unchanged between years.

Table 1: Sectors and Employment Shares

consumption sectors			intermediate sectors		
sector	code	share	sector	code	share
agriculture	01-05	1.85	mining/energy	10-12	0.67
food	15-16	1.75	mining/other	13-14	1.19
textiles	17-18	1.76	paper	21-22	2.12
leather	19	0.16	petrol./fuel	23	0.17
trade	50-52	22.10	chemical	24	1.07
hotels/rest	55	1.28	rubber/plastic	25	0.88
finance	65-67	4.66	utilities	40-41	0.89
real est.	70-74	8.75	transport	60-63	3.34
public admin	75	15.71	telecom	64	2.18
education	80	1.46			
health/social	85	7.93	Capital sectors		
comm serv	90-93	3.59	non metallic	26	0.62
households	95	0.83	metals	27-28	2.40
			machinery	29-33	4.88
			trans. equip.	34-35	1.93
			construction	45	5.15

Notes. The time period is 1977-01. Code refers to ISIC Rev 3 and share to the sample mean share of the sector in total hours of work

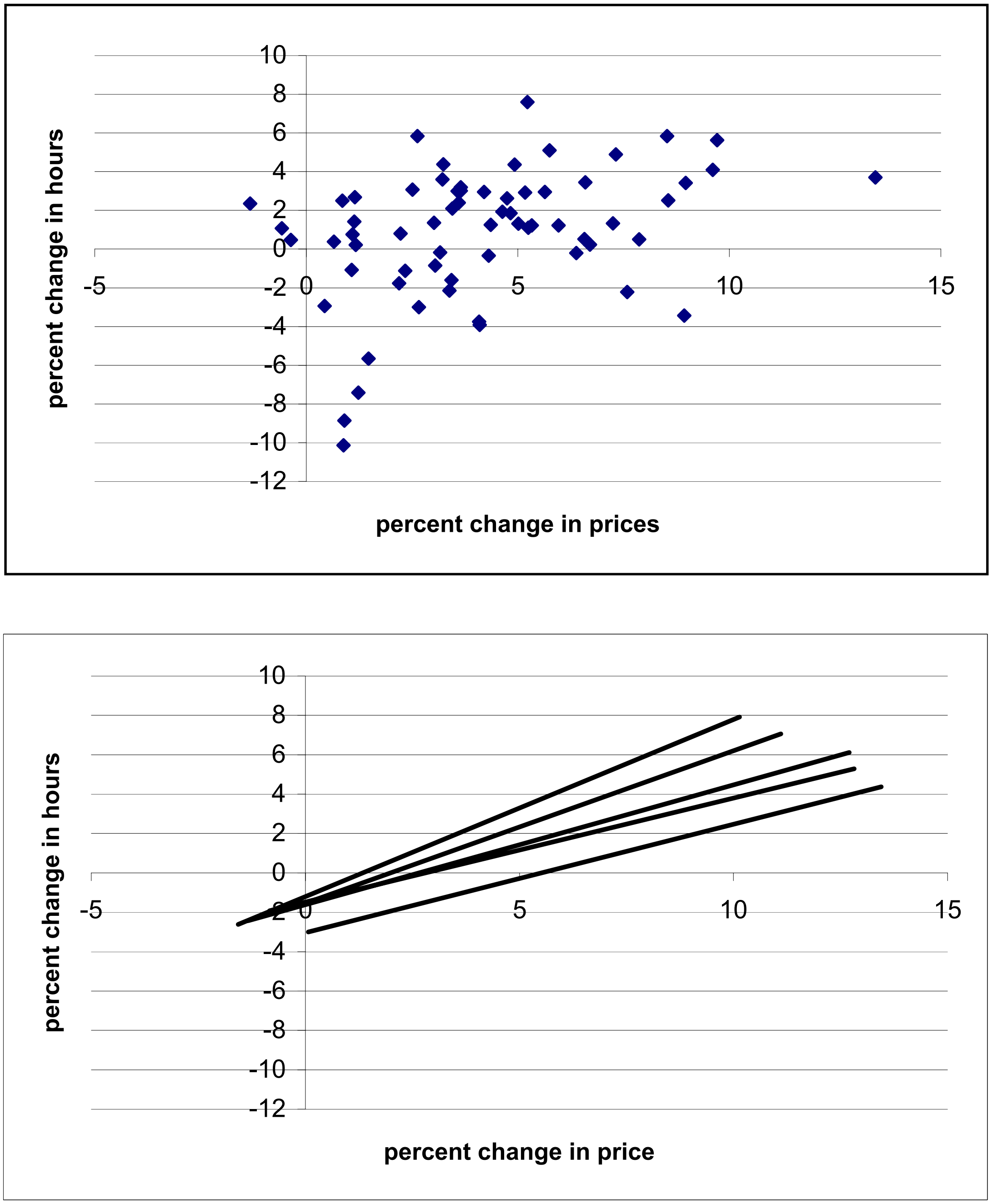


Figure 1
Changes in hours of work and prices, 13 consumption sectors
five-year averages 1977-2001

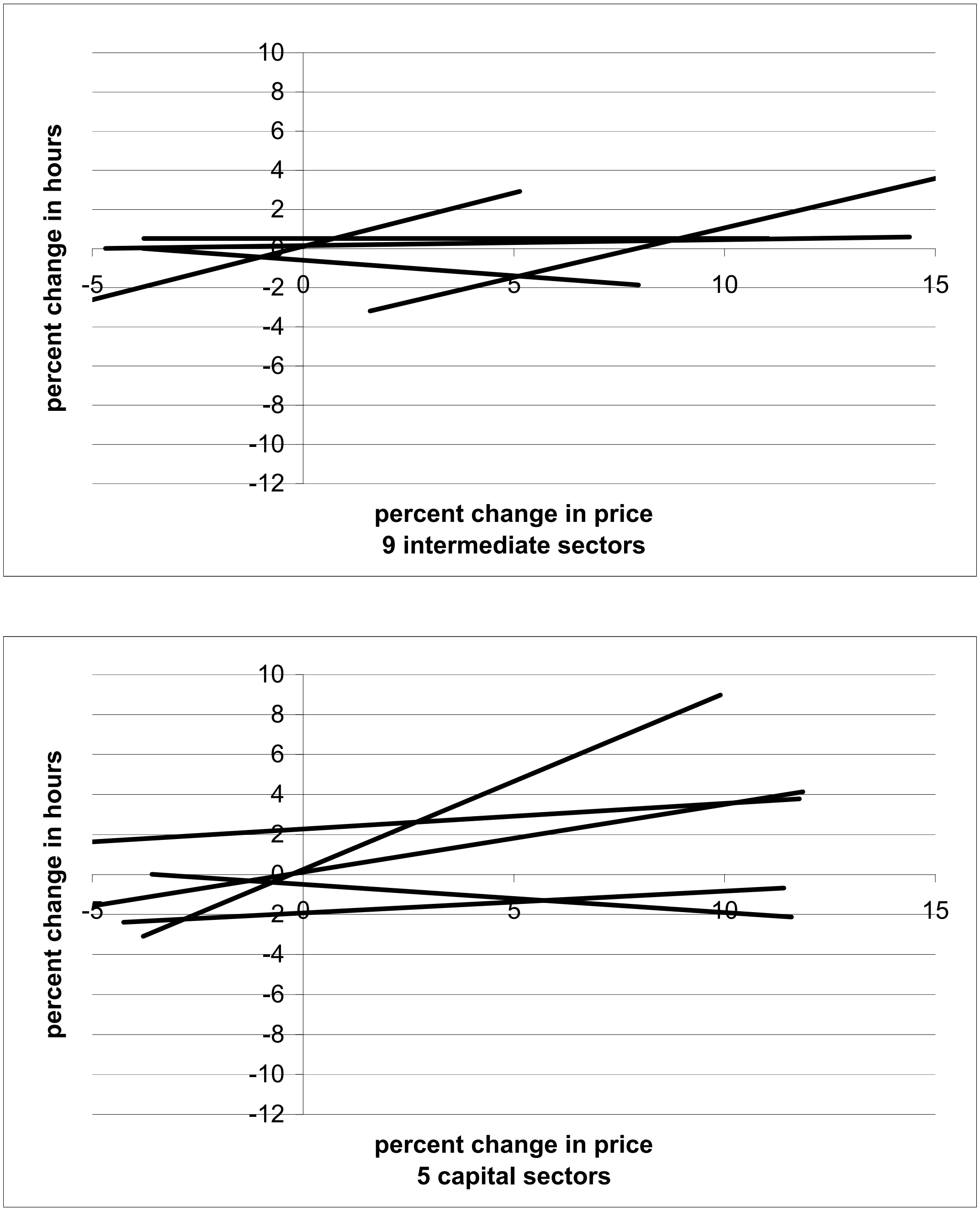
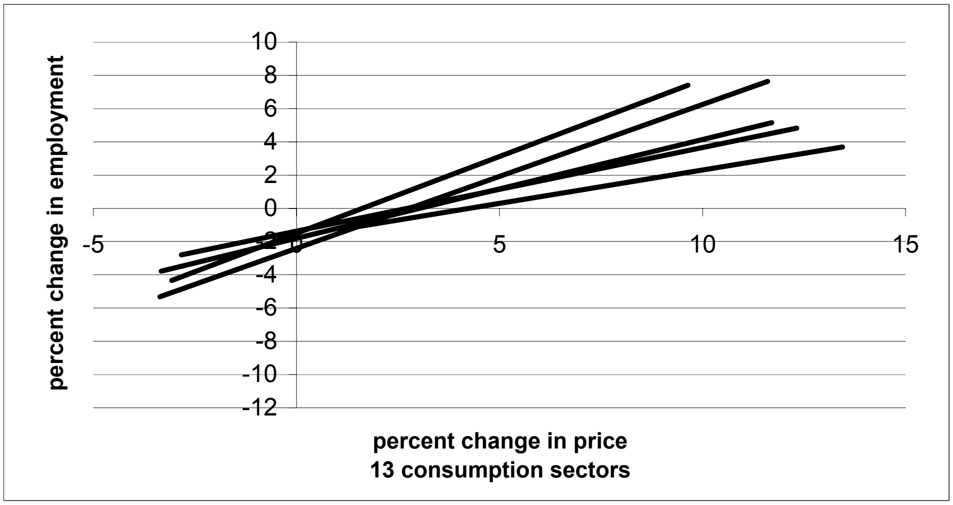
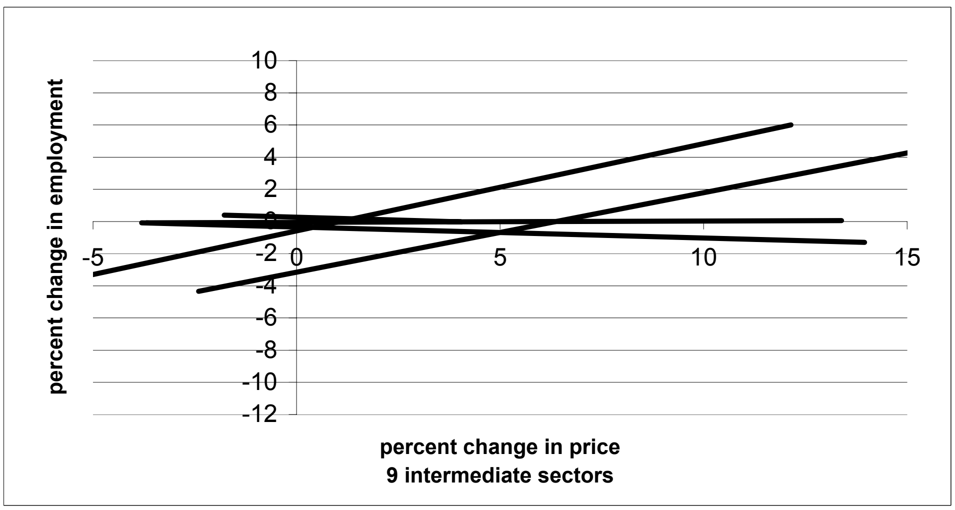


Figure 2
Changes in hours of work and prices,
five-year averages 1977-2001





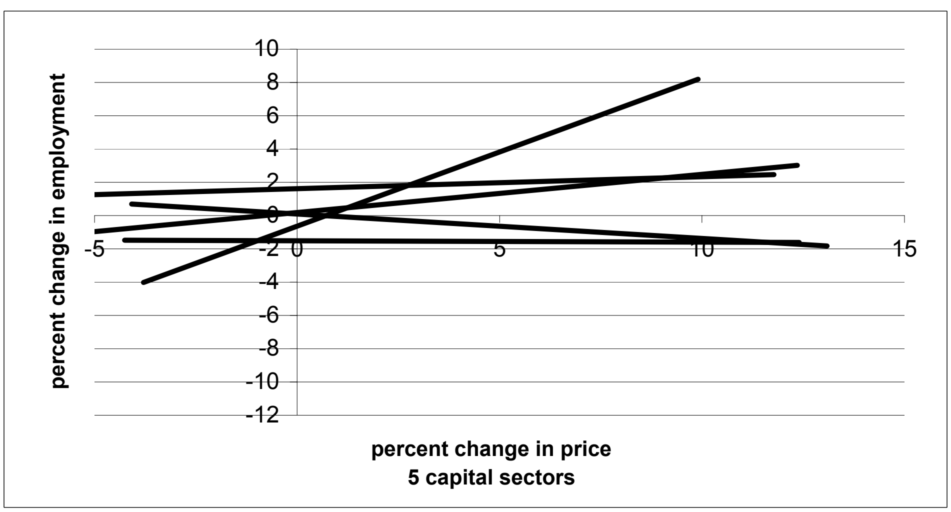
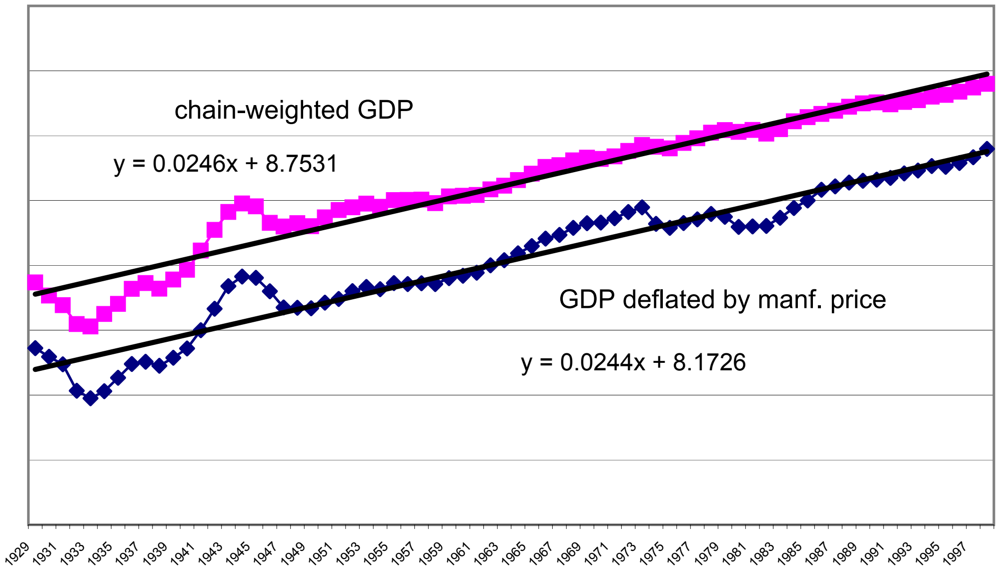
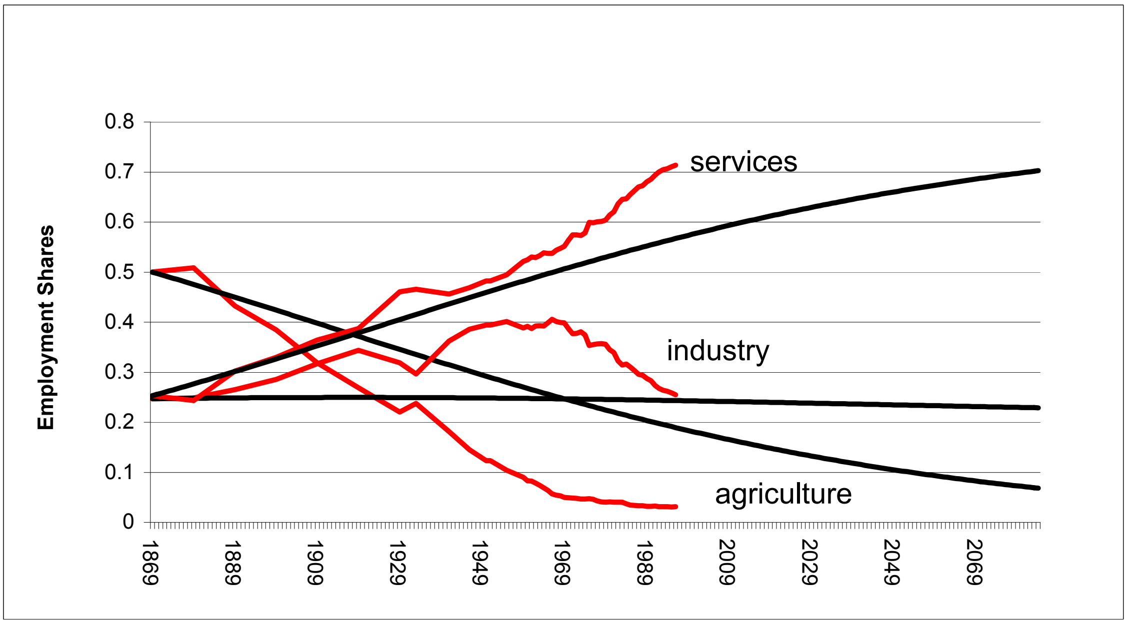


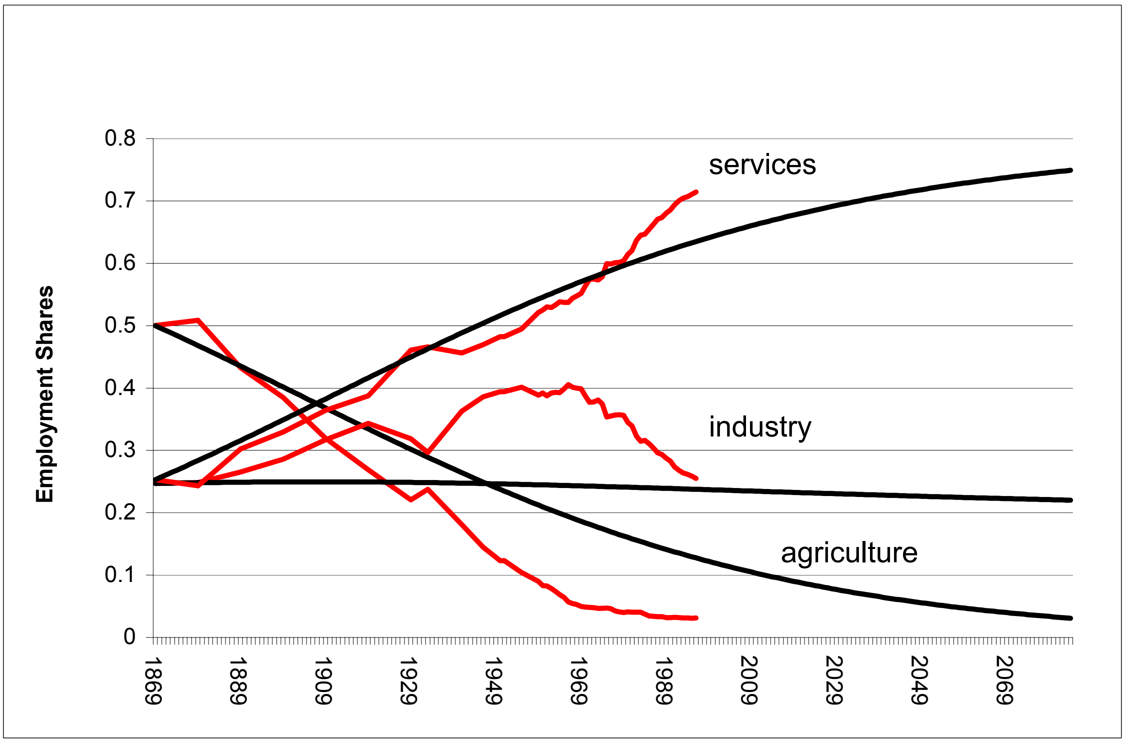
Figure 3
Changes in employment and prices, five-year averages 1977-2001

Figure 4
Chain-weighted GDP and GDP deflated by manufacturing price, US, per capita, log scale
(base years differ)





(a) epsilon = 0.3



(b) epsilon = 0.1

Figure 5

Structural transformation in the US economy
model calibration data

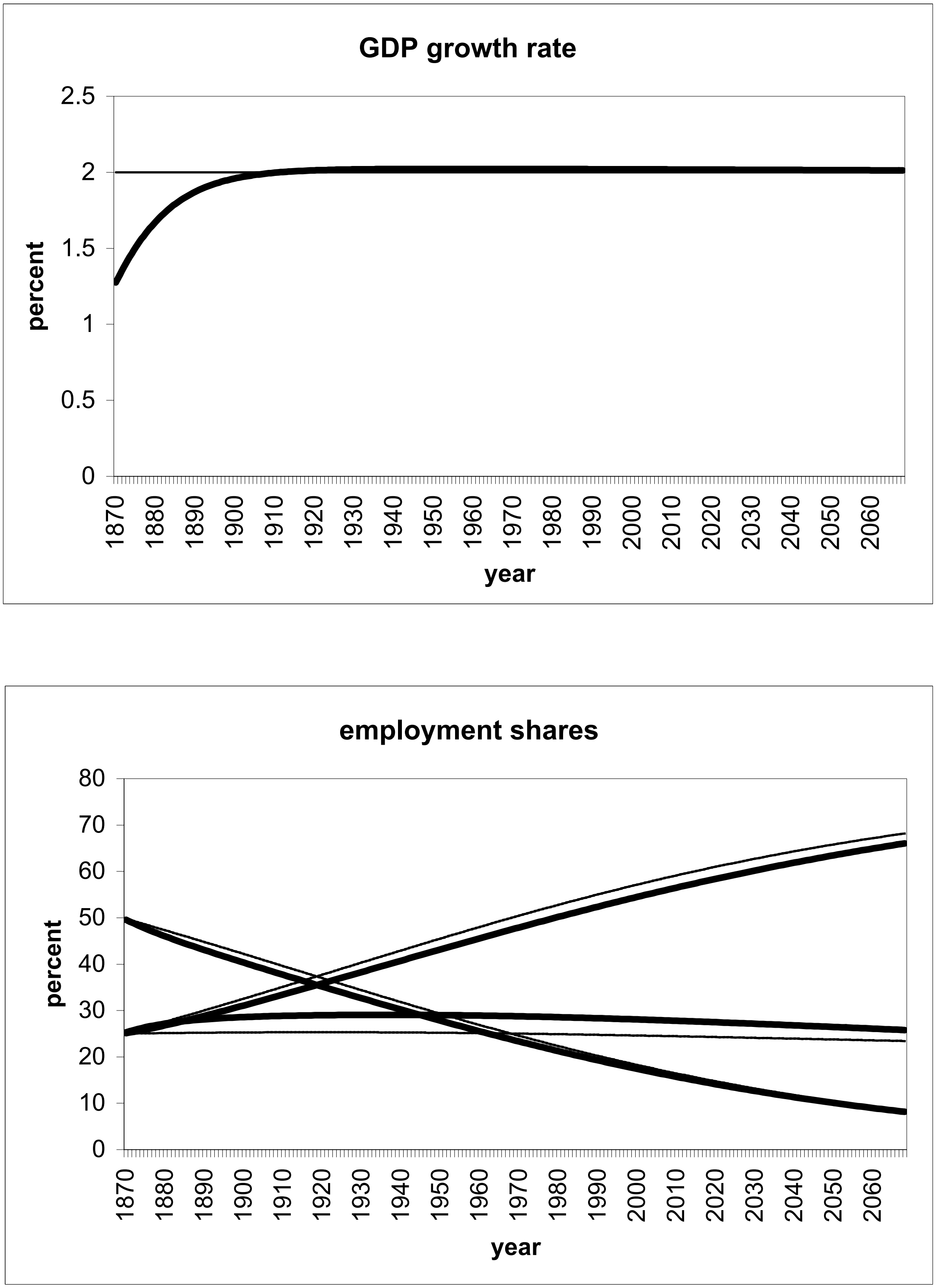


Figure 6

growth and employment shares for theta=2, compared with theta=1

theta=2

theta=1