Information Revelation in Auctions*

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Abstract

Auction theory has emphasized the importance of private information to the profits of bidders. However, the theory has failed to consider to what extent the bidders' information will remain private. We show that in a variety of contexts bidders will reveal their information, even if this information revelation is (ex ante) detrimental to them. Similarly, a seller may reveal her information even when this revelation lowers revenues. We also show that bidders may be harmed by private information.

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1 Introduction

An auction with interdependent values involves the sale of a good whose (expected) value to each bidder depends upon public information as well as information privately held by the bidders and the seller. For instance, the value of a painting purportedly by Hyppolite will depend on each party's estimation that the artwork is authentic. Though the idiosyncratic information the various agents possess might initially be private, much of it may be verifiable and nothing prevents the agents from revealing such information if

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they so choose.[1] Indeed, it is well known that in a symmetric auction, when the agents' signals are affiliated[2] *i*) if the seller can publicly commit to a revelation policy she will maximize ex ante revenue by committing to *always* reveal her information (Milgrom and Weber (1982*a*)) and *ii*) even if the seller cannot make a such a commitment, she will always reveal her information in a perfect Bayesian equilibrium (Milgrom (1987)). In contrast, there has been little investigation into the revelation behavior of the buyers. Perhaps this paucity stems from the belief that "it is more important to a bidder that his information be private than that it be precise." (McAfee and McMillan (1987)).

However, even if it is true that bidders profit from the privacy of their information, it does not follow that they will be able to refrain from revealing it. Suppose that signals are affiliated. Even if, say, bidder 1 favors an ex ante policy of never revealing his signal, ex post he may well prefer to conceal highly positive signals, but reveal very negative signals. This is because a negative signal has the potential to depress the bids of the other players, both because their valuations of the object have fallen and because they expect the other players to lower their bids. Thus, absent the possibility of commitment, in many cases bidder 1 will in fact reveal dismal information. But if the other players know that bidder 1 is acting thus, he will be "forced" to reveal moderately poor information as well, since this information becomes dismal relative to the possibilities the other players entertain if no disclosure is made. The argument can be reapplied iteratively, so that the bidder ends up revealing even positive signals.

Though this type of *unravelling* argument is familiar in other contexts, the fact that bidders will often deleteriously reveal their information may have escaped attention because they will not necessarily do so in the simplest models of common value auctions. However, these models are misleading in this regard. Indeed, we will argue that they are discontinuous in the sense that slight modeling changes can lead from a situation of no information revelation to one of complete revelation. Private information will also be fully revealed in very different contexts, including some pure private value auctions. These full revelation results notwithstanding, the above unravelling

[1] The standard assumption in the auction literature, when verifiability is an issue, is that signals are verifiable. We discuss the issue of verifiability more in Section 6, where we also consider a model in which information is only partially verifable.

[2] Roughly speaking, high values of one agent's estimates make high values of the other agents' estimates more likely.

argument is somewhat overstated. In fact, as we shall see, a wide range of revelation behavior is possible.

The literature on auction theory has emphasized the benefits of private information to buyers. Typical comments include “A bidder without special private information ... can never earn a positive expected payoff” (Milgrom (1981)) and “the winning bidder’s surplus is due to her private information” (Klemperer (1999)). However, this literature is at best incomplete since it has not considered whether or not bidders will be able to keep their information private, at least when it is verifiable. Moreover, in many important auctions much information is indeed verifiable. For instance, telecommunications firms bidding on licenses often hire consultants to help them estimate the value of these licenses. The consultants’ reports can easily be made public. Similarly, geological reports about oil tracts to be auctioned off can be disseminated.

There is more than just a lacuna in the theory, a bidder may actually be harmed by private information. This finding is in sharp contrast with the received theory, as illustrated by Milgrom and Weber’s (1982b) finding that a “bidder’s profits rise when he gathers extra information” (absent the possibility of information revelation).

2 No Revelation

We begin with a standard pure common value model in which it is an equilibrium for the players not to reveal any information. We then modify the game slightly to obtain a game where full information revelation is the unique outcome.

There are 2 risk-neutral bidders. The value of the good to both bidders is given by $V = v(X_1, X_2)$, where X_i is player i’s private signal. Consider, say, a simple first-price sealed-bid auction. Under weak conditions the game has a positive value to each player. Suppose the signals are verifiable and alter the game by adding a preliminary stage in which either bidder can reveal her signal. A bidder that chooses to disclose her information earns zero in the ensuing auction, regardless of the revelation policy of the other bidders (Milgrom and Weber (1982b)). Thus, neither player has an incentive to divulge any realization of a signal, favorable or unfavorable. This conclusion is misleading, however. It depends upon the fact that a player with no private information always earns zero. This fact in turn is driven by, among other things, the assumption that the value of the good is literally the same to both players.

As the name suggests, in a pure common value auction there is no private component to the bidders' valuations. With respect to mineral rights, Milgrom and Weber (1982a, p. 1093) argue that this simplification is appropriate since "To a first approximation, the values of these mineral rights to the various bidders can be regarded as equal." However, while this first approximation may be harmless for the usual analytical purposes, it is deceptive when considering the disclosure of information. In the next section, we illustrate the importance of the lack of a private component with an example that we will reconsider in greater detail in Section 3.1.1.

2.1 Full Revelation: An Example

Consider a good worth $z_1 + w$ to player 1 and w to player 2. The private component z_1 is common knowledge, but only player 1 is informed of the signal w, which is drawn from the uniform distribution on $[0, 1]$. The good is sold via a first-price sealed-bid auction in which player 1 wins the good if his bid is at least as large as player 2's bid. First suppose that $z_1 = 0$.[3] The auction has a unique equilibrium in which player 1 bids $\frac{1}{2}w$ and player 2 bids $b \in \left[0, \frac{1}{2}\right]$ uniformly. Player 1 receives an ex ante payoff of $\frac{1}{6}$. Furthermore, given any realization of $w > 0$, player 1 earns a strictly positive (expected) payoff. Now give player 1 the opportunity to disclose his signal w. If he does so, both players bid w in the ensuing auction, yielding 1 a payoff of 0. Thus, player 1 has no incentive to disclose any realized signal and there is an equilibrium in which player 1 refrains from ever making such a disclosure. Note that a policy of disclosing all his signals would earn player 1 an ex ante payoff of 0.

Now suppose that z_1 is arbitrarily small but strictly positive. The equilibrium of the first-price auction approximates the $z_1 = 0$ equilibrium, namely, player 1 bids $\max\left[z_1, \frac{1}{2}w\right]$ and player 2 bids $b \in \left[z_1, \frac{1}{2}\right]$ approximately uniformly.[4] Player 1 receives an ex ante payoff of approximately $\frac{1}{6}$. Again give player 1 the opportunity to disclose his signal w. If he does so, both players again bid w in the ensuing auction's (undominated) equilibrium, yielding player 1 a payoff of $z_1 \approx 0$. A policy of disclosing all signals again harms player 1, earning him an ex ante payoff of approximately 0. Thus far, our

[3] Engelbrecht-Wiggans et al (1983) solve the model for this $z_1 = 0$ case.

[4] A precise description is given in Section 3.1.1.

analysis of the case $z_1 \approx 0$ mirrors our analysis of the $z_1 = 0$ case. There is, however, a crucial difference.

Suppose that player 1 has received a signal $w \leq z_1$. If 1 discloses this signal, in the ensuing auction player 2 bids w instead of randomizing between $z_1 \geq w$ and higher bids. Hence, player 1 will in fact disclose any signal $w \leq z_1$, thereby earning z_1 instead of strictly less. By continuity, there exists a $\underline{w} > z_1$ such that player 1 will also disclose all $w < \underline{w}$. But then if player 1 does not disclose a signal w, player 2 knows that $w \geq \underline{w}$, and in the ensuing equilibrium 2 randomizes among bids $\underline{w}$ and above. Player 1 benefits from disclosing $\underline{w}$ and all signals $w < \underline{w}'$ for some $\underline{w}' > \underline{w}$. The argument can be reapplied, leading to the conclusion that in any equilibrium, player 1 essentially reveals all his information despite the fact that this is ex ante detrimental to him.

3 Revelation Behavior

Analyzing information revelation in the context of auctions is a bit tricky as auctions with disclosure possibilities necessarily contain asymmetric subgames, and it is often difficult to provide closed-form equilibrium characterizations in asymetric auctions. Accordingly, we will concentrate on relatively simple situations which are sufficient to demonstrate the spectrum of possibilities. We analyze both interdependent value and private value games, and first and second price auctions. In Section 7 we provide a general framework for our study.

3.1 One Signal

We will begin with a variant of the following simple two-player pure common value first-price sealed bid auction.[5]

1. Player 1 receives a verifiable signal $w \in [w_m, w_M]$ drawn from the atomless distribution function $F(w)$.

2. Player 1 submits a bid b_1 and player 2 submits a bid b_2. Player 1 wins the good if and only if $b_1 \geq b_2$. The payoff to player i is $w - b_i$ if he wins the object and 0 otherwise.

[5] This model was introduced by Wilson (1967) and studied later by Wilson (1975), Weverburgh (1979) and Engelbrecht-Wiggans et al. (1983).

Note that player 1, and only player 1, is perfectly informed of the value of the object. Henceforth we refer to this game as the *one-sided common value game*. Engelbrecht-Wiggans et al. (1983) show that this game has a unique equilibrium in which player 1 bids

$$b_1(w, F) = E_F[W \mid W \leq w]$$

and player 2 draws a signal x from $[w_m, w_M]$ with the distribution F, and bids $b_1(x, F)$ .Given w, equilibrium payoffs are

$$\begin{array}{l} \hat{u}_1(w, F) = F(w)(w - b_1(w, F)) \\ \hat{u}_2(F) = 0 \end{array} \tag{1}$$

We note that expression (1) remains valid even if F is not atomless.

We now give player 1 the opportunity to disclose his information. Since the signal is verifiable, the disclosure must be truthful. The players engage in the following game:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function $F(w)$.

2. Player 1 chooses whether or not to disclose his signal w.

3. Each player submits a bid b_i. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (w - b_1, 0) & \text{if } b_1 \geq b_2 \\ (0, w - b_2) & \text{if } b_1 < b_2 \end{cases}$$

We will refer to this game as the *one-sided common value disclosure game*. Player 1 never has an incentive to reveal his information in this game, since if he does so both players bid w, resulting in a payoff of 0 to him. Thus, there is a perfect Bayesian equilibrium in which no disclosure takes place and the addition of the revelation stage 2 is irrelevant. However, as discussed in Section 2, this pure common value model is misleadingly restrictive. The next subsection addresses this issue.

3.1.1 Private Component

In the one-sided common value disclosure game, player 1 always ends up with a profit of zero when he discloses his signal. Crucial to this result is the (extreme) assumption that the value of the good is exactly the same to both players. In this section, we show that a continuous departure from this assumption can have a discontinuous impact upon equilibrium behavior. Specifically, we add a (possibly small) private component to 1's valuation of the good. We assume that this private component is common knowledge so that no new informational considerations are introduced. The one-sided common value game is usually considered to be a reasonable model of the auction of an oil tract in which player 1 has a neighboring tract, and thus superior information. The added private component can be thought of as an independent benefit player 1 would obtain from owning adjacent land, say from reduced clean-up costs.

In the modified game, when player 1 wins the good with a bid of $b_1 \geq b_2$ his payoff is $z_1 + w - b_1$; when player 2 wins the good with a bid of $b_2 > b_1$ her payoff is (still) $w - b_2$. As before, player 1's only private information is w, which is drawn from the distribution F; the parameter $z_1 > 0$ is common knowledge. When z_1 is small this game "approximates" the one-sided common value game; in particular, revealing w yields player 1 about zero. Nonetheless, adding this component has drastic consequences when player 1 is given the option of disclosing his signal.

Consider the following game:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function F.

2. Player 1 chooses whether or not to disclose his signal w.

3. Each player submits a bid b_i. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (z_1 + w - b_1, 0) & \text{if } b_1 \geq b_2 \\ (0, w - b_2) & \text{if } b_1 < b_2 \end{cases}$$

We say that player 1's signal is *almost surely known* if *i*) player 1 discloses almost all signals in stage 2, or *ii*) the set of undisclosed signals with positive measure is at most a singleton.

Proposition 1 *In any undominated perfect Bayesian equilibrium of the above game, player* 1*'s signal is almost surely known.*

All propositions are proved in the appendix. In essence, player 1 always strictly wants to disclose a signal at the bottom of the support of his signals and an unravelling ensues. One subtlety is worth observing. When player 1 discloses a signal at the bottom of the support, player 2's bids are lowered in the first order stochastic domination sense whenever $z_1 \geq 0$. However, if $z_1 = 0$, when 1 reveals signals close to the bottom, while 2's bids are "mostly" lowered, they are not lowered in the same first order sense. As a result, when z_1 is equal to 0, all types except the bottom type are strictly harmed by revelation, while the bottom type is unaffected. On the other hand, when, $z_1 > 0$ player 2's bids *are* first order stochastically lowered when low types reveal. Furthermore, the bottom type then strictly prefers to reveal. Hence, the role of z_1.

Note the discontinuity. For all $z_1 > 0$, player 1's signal is almost surely known in any equilibrium and as z_1 tends to 0, so do player 1's profits. On the other hand, when $z_1 = 0$ there is an equilibrium in which player 1 conceals all signals and earns a positive profit.

Let us return to the specific example of Section 2.1, where $w \sim U[0, 1]$ and $0 < z_1 < \frac{1}{2}$. In the standard sealed-bid auction in which player 1 is not given the disclosure option, the unique equilibrium is that player 1 bids

$$\max \left[z_1, \frac{1}{2} w \right],$$

while player 2 bids $b \in \left[z_1, \frac{1}{2} \right]$ with cumulative distribution

$$\frac{2z_1}{2z_1 + 1} + \frac{2}{2z_1 + 1} b.$$

Player 1's ex ante payoff is

$$\frac{1}{6} \frac{-8z_1^3 + 12z_1^2 + 6z_1 + 1}{2z_1 + 1}.$$

On the other hand, when given the possibility to reveal his signal, player 1 does so, yielding him a payoff of z_1. Note that

$$\frac{1}{6} \frac{-8z_1^3 + 12z_1^2 + 6z_1 + 1}{2z_1 + 1} > z_1$$

In particular, when $z_1 = 0$ the left hand side is $\frac{1}{6}$. This is consistent with the general belief in the literature that a player is harmed by relinquishing his private information. Nonetheless, he relinquishes it.

In Section 5, we consider another departure from the one-sided common value disclosure game that also yields full revelation.

Second-Price Auction Suppose that the good is auctioned off using a second-price auction instead of a first-price auction. Without disclosure possibilities, player 1's dominant strategy is to bid $z_1 + w$ and, given this, player 2's best strategy is to bid 0. Allowing for disclosure has no effect; when dominated strategies are iteratively removed, bidding behavior is unchanged and player 1 does not reveal any signal (except, possibly $w = 0$). As we shall see, however, this lack of disclosure is not a general feature of second-price auctions.

3.1.2 Additional Equilibria

Consider again the first-price auction without a private component ($z_1 = 0$). Suppose that w is drawn uniformly from $[0, 1]$. As we know, there is a perfect Bayesian equilibrium in which player 1 never discloses his signal. Given a realization w', he earns $\frac{1}{2}w'^2$. A disclosure of w' would have earned player 1 zero. Although disclosing his signal is never beneficial to 1, there is also a perfect Bayesian equilibrium of this game in which player 1 always discloses his signal, thereby earning 0. This equilibrium is supported by an out-of-equilibrium belief of player 2 that $w = 1$ if no disclosure is made.[6] In fact, there are a continuum of equilibria, where for each $\hat{w}$ player 1 discloses his signal w if and only if $w \leq \hat{w}$, and if he does not disclose, player 2 believes that $w \geq \hat{w}$. Thus, the discontinuity we noted in the previous section is a failure of lower hemicontinuity in the equilibrium correspondence, as a function of z_1.

It seems to us that the no-disclosure equilibrium is the "reasonable" equilibrium of this game, but there is not enough structure for refinements such as sequential equilibrium to be of any use and we do not insist upon this selection. At any rate, to the extent that the equilibria with some revelation are viewed as equally reasonable the main point of this paper is only reinforced, since even in pure common value games information revelation cannot be ignored.

[6] As this example suggests, sufficient conditions for full disclosure to obtain in *some* equilibrium are quite weak. See Theorem 2 in Section 7.1.

3.2 Two Signals

In the previous sections, only player 1 receives a signal. In this section, we consider a game in which both players receive signals. At the same time, again in contrast to the previous models, either player might have the greater valuation for the good.[7] The game displays new behavior: there must be a positive measure of revelation, but this revelation may be partial or full.

Consider a second-price auction in which player 1 wins a good iff $b_1 \geq b_2$. The player's valuations for the good are

$$\begin{aligned} v_1 &= x_1 + x_2 + z_1 \\ v_2 &= 2x_2 + 2x_1 \end{aligned} \tag{2}$$

where $z_1 > 0$ is common knowledge and each x_i is drawn from an atomless f_i on $[0, X]$, with $E(x_1) > z_1$. Player i is informed of x_i.

Without disclosure possibilities, one equilibrium of this game is

$$\begin{aligned} b_1(x_1) &= 2x_1 + z_1 \\ b_2(x_2) &= \begin{cases} 2x_2 & x_2 < \frac{z_1}{2} \\ 2X + z_1 & x_2 \geq \frac{z_1}{2} \end{cases} \end{aligned} \tag{3}$$

Now suppose that the two players are free to reveal their information. How should this be modeled? A standard approach in the literature is to have the players simultaneously make a disclosure decision. While this tack is plausible, there seems to be no reason to preclude a player who has not yet revealed his information from doing so once the other player has revealed. Accordingly, we allow for two rounds of disclosure[8]:

1. Player i receives a signal x_i drawn from f_i.
2. Player i discloses x_i if he so chooses.
3. Given the other player's choice in round 2, i is given another chance to reveal x_i.

[7] A more parsimonious approach would be to first consider a game in which both players receive signals and it is common knowledge who values the good more. The present approach accomplishes our aim of displaying the various revelation possibilities more swiftly.

[8] Obviously, there is no need for precisely two rounds, but this is the minimal amount which allows for a response by the players. We note that Proposition 2 below remains valid even if there are fewer or more rounds of revelation.

4. Each player submits a bid b_i. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (x_1 + x_2 + z_1 - b_2, 0) & \text{if } b_1 \geq b_2 \\ (0, 2x_2 + 2x_1 - b_1) & \text{if } b_1 < b_2 \end{cases}$$

Proposition 2 *In any undominated Perfect Bayesian equilibrium of the above game, there must be (a positive measure of) revelation.*

As an illustration of Proposition 2, in one equilibrium:

i) In the first round of revelation player 1 always discloses his signal.

ii) In the second round of revelation, player 2 discloses her signal if $x_1 + x_2 < z_1$.

iii) In the auction, player 1 bids $x_1 + x_2 + z_1$ if player 2 has disclosed, otherwise player 1 bids $2z_1$; player 2 bids $2x_2 + 2x_1$ if player 1 has disclosed, otherwise player 2 bids $2x_2 + 2X$ (believing that $x_1 = X$).

In another equilibrium:

i) In the first round of revelation, player 2 discloses her signal.

ii) In the second round of revelation, player 1 discloses his signal if $x_1 + x_2 < z_1$.

iii) In the auction, player 1 bids $x_1 + x_2 + z_1$ if player 2 has disclosed, otherwise player 1 bids $x_1 + X + z_1$ (believing that $x_2 = X$); player 2 bids $2x_2 + 2x_1$ if player 1 has disclosed, otherwise player 2 bids $2x_2 + 2X$.

Let us now modify player 2's valuation of the good so that we have

$$\begin{aligned} v_1 &= x_1 + x_2 + z_1 \\ v_2 &= 2x_2 + x_1 \end{aligned} \tag{4}$$

On the face of it, the game described by these values does not seem very different than the game described by the values of (2). Nonetheless, in this modified game information revelation is no longer necessary.[9] This illustrates the subtlety involved in trying to obtain a general result on information revelation.[10]

[9] The following is a no-revelation equilibrium of the game with valuations as in 4:

i) Neither player ever reveals any signal.

ii) If player 2 does not reveal, player 1 bids $x_1 + 2a$, otherwise player 1 bids $x_1 + x_2 + a$. If player 1 does not reveal player 2 bids $2x_2$ when $a > x_2$, and bids $2x_2 + X$ when $a \leq x_2$. If player 1 does reveal player 2 bids $2x_2 + x_1$.

[10] Note that the equilibrium (3) without disclosure possibilities of the original game is inefficient, whereas the no-revelation equilibrium of the revised game is efficient. A conjecture is that there must be information revelation if the no-revelation equilibrium is inefficient.

3.3 Pure Private Values

One reason for believing that a player benefits from having private information in an interdependent value auction is that winner's curse fears depress rivals' bids. In this section we analyze a pure private values setting in which there are no such fears. We find that while the players fully reveal their information, they are indeed not harmed by this revelation.

Consider a pure private values second-price auction where, in addition to information about his own valuation, each player has private information about the other player's valuation. Specifically, $v_1 = z_1 + x_2$ and $v_2 = z_2 + x_1$, where player i observes (z_i, x_i). Following the disclosure decisions, the players engage in a second-price sealed-bid auction, where it is a dominant strategy for each player to bid his (expected) value.

We define a disclosure game in which each player receives his expected payoff from a second-price auction. For ease of exposition (in the appendix), we set $z_1 = z_2 = 0$.

1. Nature chooses $x_i \in [x_m, x_M]$ according to the strictly increasing atomless distribution function F_i, for $i = 1, 2$; player i is informed only of x_i.

2. Player i chooses whether or not to disclose x_i.

3. Each player submits a bid b_i. Payoffs are:

$$u(b_1, b_2) = \begin{cases} (x_2 - b_2, 0) & \text{if } b_1 \geq b_2 \\ (0, x_1 - b_1) & \text{if } b_1 < b_2 \end{cases}$$

Proposition 3 *In any perfect Bayesian equilibrium of this game, the players' signals are almost surely known. Furthermore, each player is better off fully disclosing his signals than not disclosing any signals (regardless of the disclosure policy of the other player).*

While the players are not harmed by the disclosure of their information in this pure private values game, it is not always true that with pure private values full disclosure does not harm a player. For instance, consider a first-price sealed-bid auction of a good worth x_i to player $i = 1, 2$, where the x_i's are i.i.d $U[0, 1]$. If the valuations remain private, player 1's payoff to this game is $\frac{1}{6}$. On the other hand, if player 1's value is always revealed he has a payoff of $\frac{1}{8}$ (Vickrey (1961) analyzes the game in which player 1's signal is known). We note, however, that in this game player 1 is not "forced" to reveal his signal. That is, there is an equilibrium in the game with disclosure possibilities in which no disclosure is made.[11]

3.4 Seller Revelation

Recall that when the seller receives a signal in a symmetric affiliated signals auction, then *i*) she maximizes ex ante revenues by committing to reveal the signal and *ii*) even if she cannot make a such a commitment, she always reveals the signal when given the opportunity. At this point the reader may suspect that, despite appearances, these two statements are essentially unrelated. The following asymmetric example confirms this suspicion.

There are two bidders who receive unverifiable private signals x_1 and x_2 and a seller who receives a verifiable private signal s; all signals are drawn from distributions with support $[0,1]$. The valuations of the bidders are,

$$\begin{aligned} v_1(x_1, x_2, s) &= x_1 + \alpha(x_2 + s) \\ v_2(x_1, x_2, s) &= x_2, \end{aligned}$$

where $\alpha \in \left(0, \frac{1}{2}\right)$. Suppose the good is sold using a second-price sealed-bid auction. Krishna (2002) shows[12] that if S is *never* disclosed the equilibrium price is

$$P^N = \min\left\{\frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}E[S], X_2\right\},$$

whereas when S is disclosed the equilibrium price is

$$P^S = \min\left\{\frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}S, X_2\right\}.$$

As Krishna observes, $E\left[P^S\right] < E\left[P^N\right]$ so that here full disclosure is detrimental to the seller. Nevertheless, as we now show, absent commitment possibilities the seller still fully discloses.

Consider the following game:

[11] This follows from the fact that player 1 is ex post harmed by the revelation of any signal $x_i > 0$.

[12] Krisha (2002) makes further distributional assumptions on the game, but these are not necessary.

1. Bidder i receives a signal x_i, and the seller receives a signal s. The signals are independently drawn from distributions with support $[0,1]$.

2. The seller chooses whether or not to disclose his signal s.

3. Each bidder submits a bid b_i. Payoffs to the bidders are

$$u_b(b_1,b_2,s) = \begin{cases} (x_1 + \alpha(x_2+s) - b_2, 0) & \text{if } b_1 \geq b_2 \\ (0, x_2 - b_1) & \text{if } b_1 < b_2 \end{cases}$$

while the payoff to the seller is

$$u_s(b_1,b_2,s) = \min(b_1,b_2)$$

Proposition 4 *In any perfect Bayesian equilibrium of this game, the seller's signal is almost surely known.*

4 Inducing Disclosure

In a pure common value auction, a player with no private information always earns zero profit; disclosure is not inevitable, since it is never (strictly) beneficial for a player to reveal his signal. Nonetheless, we now show that the seller may be able to "force" full revelation by providing an arbitrarily small payment.

Consider a good worth $v(x_1,x_2)$ to both players, where $x_1,x_2 \in [0,X]$ and v is increasing and continuous. The good is sold using a second price auction. The seller offers to pay $\varepsilon > 0$ to any player who reveals his information. Formally, we have

1. Nature chooses the signal x_i according to the distribution F_i. Player i is informed of x_i.

2. Player i chooses whether or not to disclose his signal. Specifically, i chooses $t_i \in \{x_i, \emptyset\}$, where $t_i = \emptyset$ indicates that i makes no disclosure.

3. Each player submits a bid b_i. Payoffs are:

$$\begin{aligned} u(b_1,b_2) &= \begin{cases} (v(x_1,x_2) - b_2 + \varepsilon(t_1), \varepsilon(t_2)) & \text{if } b_1 \geq b_2 \\ (\varepsilon(t_1), v(x_1,x_2) - b_1 + \varepsilon(t_2)) & \text{if } b_1 < b_2 \end{cases} \\ \text{where } \varepsilon(t_i) &= \begin{cases} \varepsilon & \text{if } t_i = x_i \\ 0 & \text{if } t_i = \emptyset \end{cases} \end{aligned}$$

Proposition 5 *When dominated strategies are iteratively removed, both players' signals are almost surely known.*

As an example, suppose that $v_1(x_1, x_2) = v_2(x_1, x_2) = v(x_1, x_2)$. If the seller does not offer a payment ($\varepsilon = 0$), there is a perfect Bayesian equilibrium in which no signals are disclosed. The symmetric equilibrium strategies in the auction phase call for i to bid $v(x_i, x_i)$. For any realization of signals, the seller's profit is $\min\{v(x_1, x_1), v(x_2, x_2)\}$. On the other hand, if the seller offers $\varepsilon > 0$, (essentially) all signals are known. The buyers bid $v(x_1, x_2)$ and the seller's profit is $v(x_1, x_2) - \varepsilon \approx v(x_1, x_2) > \min v(x_i, x_i)$ for small ε and $x_1 \neq x_2$. Thus, in this pure common value case, offering a payment of ε results in a "virtually optimal" auction (the seller extracts virtually all the surplus). Furthermore, offering a small payment can be used to design virtually optimal auctions in many common value settings, of which the one-sided common value model of Engelbrecht-Wiggans et al. is one example. We note that existing results on optimal auctions do not cover this model.

5 Harmful Information

Consider a good worth w to two players, where w is drawn from $[0, 1]$ with strictly positive density f (with cumulative F), and only Player 1 will receive more information than this. Would he prefer to receive one signal about the good's value or two signals ?

Milgrom and Weber (1982b) show that Player 1 unambiguously prefers to receive two signals – more information cannot harm him. However, their analysis presumes that 1 does not have the option of disclosing his information. In this section we show that when disclosure possibilities are recognized, Player 1 may be harmed by additional information.

Consider the following two games:

i) Player 1 receives an estimate x, which indicates in which one of n equal intervals the value of the object, w, lies. He receives no further information. He has the option of disclosing x before the object is auctioned off in a first-price sealed-bid auction.

ii) Player 1 receives the estimate x, which he again has the option of disclosing. Following his disclosure decision, he is informed of the exact value w and the good is auctioned off. Formally, this game is:

1. Player 1 receives a signal $x \in \{0, 1, \ldots, n-1\}$ with probability $F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)$.

2. Player 1 chooses whether or not to reveal x.

3. Player 1 receives a signal $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$ according to F.

4. A first-price auction is played.

Note that the game of *i*) differs from the above game in that stage 3 is absent.

Proposition 6 *In the game of i) there is a perfect Bayesian equilibrium in which player* 1*'s signal* x *is never known by player* 2*. In the game of ii), in any perfect Bayesian equilibrium player* 1*'s signal* x *is always known by player* 2*.*

For concreteness, suppose that $w \sim U[0,1]$. When player 1 receives two signals, he always reveals his initial estimate x.[13] As the number of intervals $n \to \infty$, this estimate becomes increasingly accurate, leaving player 1 with almost no private information and an equilibrium payoff of 0. On the other hand, when player 1 receives only the signal x, he is not "forced" to disclose it. As $n \to \infty$, his expected equilibrium payoff in the no-revelation equilibrium approaches $\frac{1}{6}$. Thus, when revelation is allowed for, additional information may harm a player.

Milgrom and Weber (1982*b*) also argue that a bidder would rather gather information on the value of an item overtly than covertly. Their intuition is that overt information gathering induces a fear of the winner's curse, which causes the other players to bid timidly. Hence they would expect a specialist to loudly proclaim his presence at an auction. Our intuition is quite different. The other players will not fear the winner's curse as they know that the specialist will end up revealing his information. Our specialist would prefer to send an anonymous proxy to do his bidding.

In the present context, suppose that both players know that player 1 knows x. With no disclosure possibilities, if player 1 is to receive the signal w as well, he wants player 2 to be aware of this. With disclosure possibilities, he prefers that player 2 be unaware that he has the extra information. In the

[13] Actually, player 1 discloses all his signals with the possible exception of $x = n - 1$. Obviously, when he does not disclose $x = n - 1$, player 2 can infer x's value.

next section we consider a model in which player 2 is uncertain as to whether or not player 1 is informed.

We note that the game of *ii*) indicates another discontinuity in the one-sided common value disclosure game. As $n \to \infty$ the game of *ii*) approaches this game, yet it always has full disclosure.

5.1 Possibly Uninformed Agent

We have thus far followed the standard approach of assuming that, while an agent's signal is (initially) private, the fact that he has received a signal is common knowledge. A realistic alternative is to assume that one agent may not be certain whether or not another one has even received a signal. In this section we consider such a situation.

Consider an independent private value setting in which the good is worth x_1 to player 1 and x_2 to player 2. The x_i's are independently drawn from $U[0,1]$. Player 1 observes x_1 and with probability $\frac{1}{2}$ observes x_2 as well. He has the option of disclosing x_2. Player 2 makes no observation. Following player 1's disclosure decision, a second-price sealed-bid auction takes place.

Proposition 7 *In any perfect Bayesian equilibrium of the above game, there is a positive measure of revelation. However, this revelation is always less than almost full.*

To see why this proposition is true, first note that player 1 must disclose some (positive measure of) signals whenever $x_1 > 0$. Otherwise, player 2 would bid $\frac{1}{2}$ in the auction – her expected value – and player 1 would prefer to disclose any $x_2 < \frac{1}{2}$ whenever $x_1 > x_2$. On the other hand, player 1 will not always disclose his signals. If he did, following no revelation player 2 would bid $\frac{1}{2}$ on the presumption that player 1 had not observed x_2, and so high types of player 1 would prefer to conceal all $x_2 > \frac{1}{2}$.

The game has many partial revelation equilibria. In one perfect Bayesian equilibrium, player 1 reveals all $x_2 \leq 2 - \sqrt{2}$ and conceals higher signals.

6 Verifiability

We have assumed that information is verifiable, an assumption which will be met in some cases but not others. For instance, estimating the value of a recently discovered artifact may involve extensive research in objective sources pertaining to the site where the discovery was made, the material used in the artifact, the style of the artifact, etc.... The results of such research are clearly verifiable. On the other hand, a bidder's estimate could depend upon her own unverifiable knowledge as a specialist.

Even when information is not verifiable, our analysis may apply since it depends less upon the verifiability of disclosed information than its veracity, and in many circumstances even nonverifiable information may be presumed accurate. Thus, bidders in auctions have been known to rely on the expertise of outside firms (for example, phone companies bidding for licenses often hire well established consulting firms to estimate the value of each license). These firms may have reputational incentives to truthfully reveal their reports, if asked to do so, as well as legal incentives not to make fraudulent statements.

An example combining various of the above elements, comes from sculptures dredged from the Fosso Reale in Livorno, in 1984. Experts agreed that the carvings were the work of Modigliani. Their initial belief came from biographical accounts claiming that the artist had thrown statues into the canal, and from stylistic details of the recovered pieces. Further confirmation came from scientific tests on the stone material of the statues.[14]

A more general, and perhaps more realistic assumption than perfect verifiability/veracity, is that within a single auction some information a bidder possesses is verifiable, or may be presumed to be true, and some is not. Alternatively, it may be that all the bidder's information is only imperfectly verifiable. Our results are amenable to either modification. In this section, we describe a model in which information is imperfectly verifiable.

We modify the model of Section 3.1.1. Again, there is a single good worth $z_1 + w$ to player 1 and w to player 2, and (only) player 1 is informed of w. Now, however, when player 1 "divulges" his signal w, player 2 can only verify that the signal lies in an interval around w. At one extreme, if the interval is of length zero the signal is perfectly verifiable. At the other extreme, if the interval is infinite, and the distribution is diffuse enough, the signal is essentially unverifiable. Clearly, for revelation to have a meaning, the interval should not be too large. Consider the following game:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the atomless distribution function F.

[14]The sculptures were in the public domain, so that infomation revelation was not an issue. Despite expert agreement on their authenticity, the sculptures were in fact faked by university students as a prank.

2. Player 1 chooses whether or not to divulge his signal w. If 1 divulges his signal, both players observe $w + x$, where x is drawn from an atomless distribution on $[-a, a]$ with full support.

3. A first-price sealed-bid auction takes place.

For simplicity, we allow only *cutoff* strategies, which are of the form "disclose if and only if $w \leq y$." Player 1 reveals no signals if $y = w_m$, while he fully reveals if $y = w_M$.

Proposition 8 *If $a > 0$ is small enough, then in any perfect Bayesian equilibrium with cutoff strategies player 1 fully reveals his signals.*

7 A General Framework

In this section we develop a general framework for analyzing information revelation in auctions. For ease of exposition, we allow for only one round of revelation. Theorems 2.1 and 2 are amenable to several rounds.

In a fairly general auction setting, there are n players each of whom receives a verifiable private signal X_i drawn from the joint distribution F. A good whose value to player i is $v_i(X_1, X_2, \ldots, X_n)$ is to be auctioned off. At the interim stage in which i has seen a signal x_i, but before the auction takes place, i has an expected equilibrium payoff which we can write as $\hat{u}_i(x_i, F)$ (if the auction has multiple equilibria, assume that some selection has been made).

Now suppose that each player is given the option of disclosing her signal before playing the auction. Since the signal is verifiable, its disclosure must be truthful. We have the following game:

1. Nature chooses $(x_1, x_2, ..., x_n)$ from F; player i is informed only of x_i.

2. Each player i reports $t_i \in \{x_i, \emptyset\}$.

3. The good is auctioned off.

In effect, the disclosure option in stage 2 changes the joint distribution from which the signals are drawn for the auction in stage 3. In the overall equilibrium of this new game, following the reports the bidders play to an equilibrium of the auction using an updated conditional joint distribution function. Let $r_i : \mathbb{X}_i \to \{\mathbb{X}_i, \emptyset\}$ be a reporting strategy for player i. Given a (presumed) reporting strategy combination r and the (actual) reports t, let $F(\cdot \mid t, r)$ be the joint distribution of x conditional on t and r. In the auction of stage 3, a player i with signal x_i gets a payoff of $\hat{u}_i(x_i, F(\cdot \mid t, r))$, which is simply i's equilibrium payoff in a standard setting where the types are drawn from the distribution $F(\cdot \mid t, r)$.

Now consider stage 2 where player i has seen his own signal x_i, but before the reports of the other players are made public. If the other players follow r, while player i reports t_i, then i has an expected payoff of

$$E_{x_{-i}} \hat{u}_i \left(x_i, F\left(\cdot \mid \left(t_i, r_{-i}\left(x_{-i}\right)\right), r\right)\right), \tag{5}$$

which is derived by taking an expectation over x_{-i} given x_i, and where $(t_i, r_{-i}(x_{-i})) \equiv (r_1(x_1), ..., t_i, ..., r_n(x_n))$. For instance, suppose $n = 3$ and that the conditional distribution function has an associated density function. Then, player 1 has an expected payoff of

$$\int\int \hat{u}_1 \left(x_1, F\left(\cdot \mid \left(t_1, r_2\left(z_2\right), r_3\left(z_3\right)\right), r\right)\right) f\left(z_2, z_3 \mid x_1\right) dz_2 dz_3.$$

A perfect Bayesian equilibrium of this game is a reporting strategy combination r^* in which $t_i = r_i^*(x_i)$ maximizes (5) for all i and all $x_i \in \mathbb{X}_i$.

As a preliminary to the general result of the next section define

$$u_i(x_i, t_i, r) \equiv E_{x_{-i}} \hat{u}_i \left(x_i, F\left(\cdot \mid \left(t_i, r_{-i}\left(x_{-i}\right)\right), r\right)\right). \tag{6}$$

Then a perfect Bayesian equilibrium is an r^* such that:

$$\begin{aligned} u_i\left(x_i, r_i^*\left(x_i\right), r^*\right) &\geq u_i\left(x_i, x_i, r^*\right) \\ u_i\left(x_i, r_i^*\left(x_i\right), r^*\right) &\geq u_i\left(x_i, \emptyset, r^*\right) \\ \forall i \forall x_i &\in \mathbb{X}_i \end{aligned}$$

7.1 The Disclosure Game

We now derive a general unravelling result which enables us to avoid duplicating unravelling arguments in various auction applications.

We first define a generic n-person game in which each player i receives a private verifiable signal X_i drawn from a metric space $(\mathbb{X}_i, d_i)$, and is given the option of (truthfully) disclosing it. A *disclosure game* is the following three stage game:

1. Nature chooses $(x_1, x_2, ..., x_n)$ according to the probability measure F on $\mathbb{X} = \prod \mathbb{X}_i$; player i is informed only of x_i.

2. Each player i chooses a report $t_i \in \{x_i, \emptyset\}$.

3. Each player i receives a payoff $u_i(x_i, t_i, r)$, where $r_i : \mathbb{X}_i \to \{\mathbb{X}_i, \emptyset\}$ such that $\forall x_i \in \mathbb{X}_i \; r_i(x_i) \in \{x_i, \emptyset\}$.

We can think of r_i as a reporting strategy for player i.

A *disclosure game equilibrium* is an r^* such that

$$
\begin{aligned}
u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, x_i, r^*) \\
u_i(x_i, r_i^*(x_i), r^*) &\geq u_i(x_i, \emptyset, r^*) \\
\forall i \forall x_i &\in \mathbb{X}_i
\end{aligned}
$$

Thus, a disclosure game equilibrium is a reporting strategy combination such that each type of each player maximizes by following the reporting strategy.[15] When u is an auction payoff, as in (6) of the previous section, a disclosure game equilibrium gives a perfect Bayesian equilibrium of the auction preceded by the possibility of disclosure, and vice-versa.

Given a probability measure F, let F_i be the marginal probability measure over $\mathbb{X}_i$. When $\mathbb{X}_i \subseteq \mathbf{R}$, we will (abusively) use F and F_i to denote distributions as well. Thus, $F_i(X_i > x_i) \equiv 1 - F_i(x_i)$, while $F_i\{X_i = x_i\} = \Pr_{F_i}(X_i = x_i)$. As usual, $F(\cdot \mid t, r)$ denotes the conditional probability given t and r. Correspondingly, $F_i(\cdot \mid t_i, r_i)$ is the marginal probability conditional on i's report and reporting strategy. Observe that since x_i is verifiable, $F_i(X_i = x_i \mid x_i, r_i) \equiv \Pr_{F_i(\cdot|x_i, r_i)}(X_i = x_i) = 1$ regardless of r_i.

Given a strategy profile r, we say that player i's signal is *almost surely known* if either $r_i(x_i) = x_i$ for almost all $x_i \in X_i$, or $F_i(X_i = x_i \mid \emptyset, r_i) = 1$ for some $x_i \in \mathbb{X}_i$. That is, the set of undisclosed signals with positive measure is at most a singleton.

We now give a sufficient condition for player i to essentially disclose all her information. Theorem 1 says that if player i always wants to disclose some signal in the support of the types that are not revealing for the proposed strategy combination, then she will essentially reveal (almost) all her signals. In auctions, it is typically easiest to verify that a player wants to reveal a signal at the bottom of the support of her signals.

[15] Note that a disclosure game equilibrium is not equivalent to a Nash equilibrium of the disclosure game.

Theorem 1 *Assume that for all* r, $u_i(x_i, x_i, r)$ *and* $u_i(x_i, \emptyset, r)$ *are continuous in* x_i*, and that* $\forall r_i$ *with non degenerate* $F_i(\cdot \mid \emptyset, r_i), \exists \widetilde{x}_i \in Support F_i(\cdot \mid \emptyset, r_i)$ *for which* $u_i(\widetilde{x}_i, \widetilde{x}_i, r) > u_i(\widetilde{x}_i, \emptyset, r)$*. Then player* i*'s signal is almost surely known in any disclosure game equilibrium.*

The above theorem is a general result about unravelling. In contrast to most results in the literature about unravelling (for instance, Grossman (1981) and Milgrom and Roberts (1986)), it covers the case of many informed parties. The previous result which is most similar to Theorem 1 is in Okuno-Fujiwara et al. (1990), where several informed parties play a revelation stage, and then a game amongst themselves. Both our result and theirs show that strategic considerations do not alter the standard result that full revelation obtains. Our result, however, does not assume that the signals are independent – a particularly poor assumption in an interdependent value auction – or that they are drawn from a finite space.

Theorem 1 concerns full disclosure in *all* equilibria. The conditions necessary for full disclosure to obtain in *some* equilibrium are quite weak. In an auction with affiliated signals it is typically bad for player i if the other players think that he has received a high signal, since this tends to increase their bids. Bearing this in mind, suppose the signals are drawn from compact intervals and that each player's payoffs are minimized when the other players believe that he has received his highest signal. Then there is a Perfect Bayesian equilibrium in which all players fully disclose, and silence by a player is interpreted to mean that he has received his highest signal. In disclosure game terms, we have the following theorem.

Theorem 2 *Suppose that for each* i *there is a signal* $\bar{x}_i \in X_i$ *such that for all* $x_i \in X_i$ *and reporting strategies* r, $u_i(x_i, x_i, r) \geq u_i(x_i, \emptyset, r)$ *whenever* $F_i(X_i = \bar{x}_i \mid \emptyset, r_i) = 1$*. Then there exists a disclosure game equilibrium in which all signals are disclosed.*

8 Conclusion

There is a consensus in auction theory that bidders derive their profits from private information. In this paper we have argued that this consensus has been reached prematurely. The literature has considered the question of the value of information to a bidder under the implicit assumption that she will be able to keep her information private. However, this ability needs to be demonstrated. In fact, as we have shown, in a variety of contexts the bidders' information will be revealed. This revelation may be complete, or partial. In any case, there is little justification for the presumption that no information will be revealed. On the seller's side, the well-known result that a seller will reveal her information is unrelated to the ex ante profitability of this revelation.

Bidding on ebay for vintage guitar pickups, e.g. PAFs, presents an intriguing, albeit imperfect, example of information revelation. These pickups may or may not be authentic, and bidders, as well as others, discuss the merits of the items in "chat rooms," such as lespaulforum. While only some of the information discussed is verifiable, many of the bidders are repeat players who have reputations and appear credible. Indeed, when general agreement is reached that an item is the "real deal," bidding is typically vigorous.[16]

Some theorists have emphasized the importance of preventing communication among bidders, due to fears of collusion (see for example Klemperer (2002)). However, the danger of collusion should be balanced against the potential benefits from information sharing. At an empirical level, more work into the revelation behavior of bidders needs to be done. At a theoretical level, analyses which presume that the bidders' information is not revealed should explicitly assume that none of the information is verifiable, or that releasing information is impractical for some reason, or else provide an argument that in the relevant equilibrium no information is disclosed.

If information acquisition is costly, and only private information is valuable, why would an agent acquire information only to disclose it? One possible answer is that agents only acquire information in settings where revelation is partial.

9 Appendix

In this section we provide proofs of the theorems and propositions.

Proof of Proposition 1. We apply Theorem 1. In order to do this, we must define the appropriate disclosure game. Let $\hat{u}_i(w, z_1, F)$ be player i's equilibrium payoff in the first-price sealed-bid auction once player 1 has seen w. If the auction has several equilibria then we choose an equilibrium

[16] We thank Mehmet Barlo for this example.

in undominated strategies. If there are several such equilibria then some selection is made. The disclosure game is:

1. Player 1 receives a signal $w \in [w_m, w_M]$ according to the distribution function F.

2. Player 1 chooses $t \in \{w, \emptyset\}$.

3. Player 1 receives

$$u_1(w, t, r) = \hat{u}_1(w, z_1, F(\cdot \mid t, r))$$

where $\hat{u}_1$ is as defined in 1. If the auction in which 1's signal is drawn from $F(\cdot \mid t, r)$ has no equilibrium, we set $\hat{u}_1(w, z_1, F(\cdot \mid t, r)) = 0$.

First consider the first-price sealed-bid auction. Since the object is worth less to player 2 than player 1, and player 2 has no private information, 2 earns 0 in any equilibrium (see Theorem 2, in Engelbrecht-Wiggans et al. (1983)). That is, $E_G\hat{u}_2(w, z_1, G) = 0$ for any distribution function G over w.

In the unique undominated equilibrium of the sealed-bid auction where w is common knowledge, player 1 wins the good for w. Thus, $u_1(w, w, r) = z_1$ for all w, which is continuous.

Now consider the sealed-bid auction where the signals are drawn from a non-degenerate $F(\cdot \mid \emptyset, r)$ and $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = \underline{w}$. This $\underline{w}$ will play the role of $\widetilde{x}_i$ in Theorem 1. Given the signal $\underline{w}$, if player 1 bids b_1 he earns

$$p(b_1)[z_1 + \underline{w} - b_1],$$

where $p(b_1)$ is the probability that a bid of b_1 wins the object. We now show that $p(b_1)[z_1 + \underline{w} - b_1] < z_1$.

Clearly, $p(b_1)[z_1 + \underline{w} - b_1] \geq z_1$ only if $b_1 \leq \underline{w}$. If $b_1 = \underline{w}$ then it must be that $p(\underline{w}) = 1$. Therefore all of 2's bids are at most $\underline{w}$ and $b_1(w) \leq \underline{w}$ for all w. But this cannot be an equilibrium, since 2 could earn a positive profit with a bid of $\underline{w} + \varepsilon$, for small enough ε.

Therefore, all of 1's winning bids must be strictly below $\underline{w}$. But this cannot be the case either since then player 2 could earn a positive profit with a bid of $\underline{w} - \varepsilon$, for small enough ε.

Hence, $p(b_1)[z_1 + \underline{w} - b_1] < z_1$ so that $u_1(\underline{w}, \emptyset, r) = \hat{u}_1(\underline{w}, z_1, F(\cdot \mid \emptyset, r)) < z_1$ whenever $F(\cdot \mid \emptyset, r)$ is non degenerate and $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = \underline{w}$. Also $u_1(w, \emptyset, r)$ is clearly continuous in w.

The conditions of Theorem 1 are met, establishing the proposition. ■

Proof of Proposition 2. Fact 1. Equilibrium bids are weakly increasing in the type.

Pf. It will suffice to show that payoffs satisfy the single crossing property (if bidding high is better than low for a low type, the same is true for a high type) for any given bidding strategy of the opponent. Let $x_h > x_l$ be high and low types of player 1, and $b_h > b_l$ be high and low bids. Let $b_2(x_2)$ be any bidding function of player 2. Assume that

$$\begin{aligned} u_1(b_h; x_l) &\geq u_1(b_l; x_l) \Leftrightarrow \\ \int_{x_2 : b_h \geq b_2(x_2)} (x_l + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) &\geq \int_{x_2 : b_l \geq b_2(x_2)} (x_l + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) \Leftrightarrow \\ \int_{x_2 : b_h \geq b_2(x_2) > b_l} (x_l + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) &\geq 0. \end{aligned}$$

Then, since

$$\int_{x_2 : b_h \geq b_2(x_2) > b_l} (x_h + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) \geq \int_{x_2 : b_h \geq b_2(x_2) > b_l} (x_l + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) \geq 0$$

we obtain

$$\int_{x_2 : b_h \geq b_2(x_2) > b_l} (x_h + x_2 + z_1 - b_2(x_2))\, dF_2(x_2) \geq 0 \Leftrightarrow u_1(b_h; x_h) \geq u_1(b_l; x_h)$$

as was to be shown. Similarly for player 2.

Fact 2. If revelation is allowed, in every equilibrium without revelation, payoffs of player 1 are continuous at 0.

Pf. Suppose not, and let $u_1(x_1) \geq u_1(0) + \varepsilon$ for all $x_1 > 0$ and some $\varepsilon > 0$ (this is the only possible type of discontinuity, since equilibrium payoffs are weakly increasing in the type, because every type could bid as a lower type and earn at least as much). Let $b_i(x_i)$ be player i's equilibrium bid function, for some arbitrary equilibrium. Since every type $x_1 > 0$ must have a strictly positive chance of winning in order to get at least ε, let $b_2^{-1} : \mathbf{R}_+ \to [0, X]$ be

$$b_2^{-1}(b_1) \equiv \sup \{x_2 : b_2(x_2) \leq b_1\}.$$

For small enough x_1, we have

$$\begin{aligned} u_1(x_1) - \varepsilon &= \int_0^{b_2^{-1}(b_1(x_1))} (x_1 + x_2 + z_1 - b_2(x_2))\, dF_2 - \varepsilon < \int_0^{b_2^{-1}(b_1(x_1))} (x_2 + z_1 - b_2(x_2))\, dF_2 \\ &\leq \int_0^{b_2^{-1}(b_1(0))} (x_2 + z_1 - b_2(x_2))\, dF_2 = u_1(0), \end{aligned}$$

a contradiction.

Fact 3. If revelation is allowed and $b_2(x_2)$ is an equilibrium bid fuction of an equilibrium without revelation, then $b_2(x_2) \leq z_1$ for all $x_2 < \frac{z_1}{2}$.

Pf. Suppose not, so that for some $x_2 < \frac{z_1}{2}$, $b_2(x_2) > z_1$.

Step i. Consider type $x_1 = 0$. We will show that $x_1 = 0$ strictly prefers to reveal its type. Consider 0's equilibrium bid $b_1(0) \geq z_1$. Let

$$\begin{aligned} L &= \left\{ x_2 < \frac{z_1}{2} : b_2(x_2) \leq z_1 \right\} \\ M &= \left\{ x_2 < \frac{z_1}{2} : b_1(0) \geq b_2(x_2) > z_1 \right\} \\ H &= \left\{ x_2 < \frac{z_1}{2} : b_2(x_2) > b_1(0) \geq z_1 \right\} \end{aligned}$$

It must be the case that for some x_2, $b_2(x_2) \leq b_1(0)$, since otherwise $x_1 = 0$ would reveal his type, and earn a strictly positive payoff. Let

$$\overline{x_2} \equiv \sup \{ x_2 : b_2(x_2) \leq b_1(0) \}.$$

Notice that since equilibrium bids are weakly increasing, the types of player 2 that lose are all those below $\overline{x_2}$. Consider now the following deviation by type $x_1 = 0$: "reveal type and bid $\max\{2\overline{x_2}, z_1\}$". After revelation, player 2 bids $2x_2$.

Case 1: $\overline{x_2} \geq \frac{z_1}{2}$. In this case, player 1 bids $2\overline{x_2}$ and the payoff for the deviation is $\int_0^{\overline{x_2}} (x_2 + z_1 - 2x_2)\, dF_2$, which equals

$$\begin{aligned}
\int_L (x_2 + z_1 - 2x_2)\, dF_2 + \int_{M \cup H} (x_2 + z_1 - 2x_2)\, dF_2 + \int_{\frac{z_1}{2}}^{\overline{x_2}} (x_2 + z_1 - 2x_2)\, dF_2 &\geq \\
\int_L (x_2 + z_1 - b_2(x_2))\, dF_2 + \int_{M \cup H} (x_2 + z_1 - 2x_2)\, dF_2 + \int_{\frac{z_1}{2}}^{\overline{x_2}} (x_2 + z_1 - b_2(x_2))\, dF_2 &> \\
\int_L (x_2 + z_1 - b_2(x_2))\, dF_2 + \int_{M \cup H} (x_2 + z_1 - b_2(x_2))\, dF_2 + \int_{\frac{z_1}{2}}^{\overline{x_2}} (x_2 + z_1 - b_2(x_2))\, dF_2 &=
\end{aligned}$$

$\int_0^{\overline{x_2}} (x_2 + z_1 - b_2(x_2))\, dF_2$, which is the payoff of not revealing and bidding $b_1(0)$. Therefore, type $x_1 = 0$ is strictly better off revealing, a contradiction.

Case 2: $\overline{x_2} < \frac{z_1}{2}$. In this case, player 1 bids z_1 and the payoff for the deviation is

$$\begin{aligned}
\int_0^{\frac{z_1}{2}} (x_2 + z_1 - 2x_2)\, dF_2 &= \int_{L \cup M} (x_2 + z_1 - 2x_2)\, dF_2 + \int_H (x_2 + z_1 - 2x_2)\, dF_2 \\
&= \int_{L \cup M} (x_2 + z_1 - 2x_2)\, dF_2 + \int_H (z_1 - x_2)\, dF_2 \\
\text{(since } x_2 \text{is less than } \frac{z_1}{2}\text{)} &> \int_{L \cup M} (x_2 + z_1 - 2x_2)\, dF_2 \geq \int_{L \cup M} (x_2 + z_1 - b_2(x_2))\, dF_2 \\
&= \int_0^{\overline{x_2}} (x_2 + z_1 - b_2(x_2))\, dF_2
\end{aligned}$$

which is the payoff of not revealing and bidding $b_1(0)$. Therefore, type $x_1 = 0$ is strictly better off revealing, a contradiction.

Note to Case 2: The only weak inequality above could be strict if M were nonempty, since types in M are strictly lowering their bids).

Note to Sept i: While it is obvious that all types will weakly lower their bids, and some strictly (those in M and H), it is not obvious that player 1 would be strictly better off by revealing for all bids. It could happen that for a fixed bid of 1, before revealing 1 was losing, and after revealing 2 loses but 1 makes a negative gain when winning.

Step ii. We will now show that for small x_1, x_1 also strictly prefers to reveal its type. After revelation, the equilibrium payoff for a type x_1 is

$$u_1^r(x_1) = \arg\max_b \int_0^{\frac{b}{2}-x_1} (x_1 + x_2 + z_1 - 2x_1 - 2x_2)\, dF_2 = \arg\max_b \int_0^{\frac{b}{2}-x_1} (z_1 - x_1 - x_2)\, dF_2$$

which is continuous in x_1 by the theorem of the maximum (here we use that F_2 is atomless). By Fact 2, $u_1(x_1)$ is continuous at 0, so $u_1^r(x_1) - u_1(x_1)$ is continuous at 0, and since $u_1^r(0) - u_1(0) > 0$ by Step i, for small enough x_1, x_1 wants to reveal.

Fact 4. If revelation is allowed, there is no equilibrium without revelation.

Pf. Suppose there was an equilibrium without revelation. Then, by Fact 3, 1 wins with probability 1 when $x_2 < \frac{z_1}{2}$. But for such x_2, player 2 is better off revealing and bidding $2x_2 + 2X$. Given x_2 the expected payoff to such a strategy is

$$\begin{aligned} & 2x_2 + E2x_1 - (Ex_1 + x_2 + z_1) \\ = \;& x_2 + Ex_1 - z_1 \geq Ex_1 - z_1 > 0. \end{aligned}$$

■

Proof of Proposition 3. We first note that in the unique undominated equilibrium of a second-price auction, each player bids the conditional expected value of the object.

Let r^* be a reporting equilibrium strategy and suppose that, say, player 1 does not reveal a positive measure of signals. Define

$$\underline{x}_i = \min \text{Support} F_i\left(\cdot \mid \emptyset, r_i^*\right).$$

By continuity, player 1 weakly prefers not to reveal $\underline{x}_1$. If 1 discloses $\underline{x}_1$ then 2 bids $\underline{x}_1$, whereas if 1 does not disclose $\underline{x}_1$, 2 bids $E(x_1 \mid \emptyset, r_1^*) > \underline{x}_1$. Since 1 does not benefit from revealing $\underline{x}_1$, he must win the object with probability zero. This implies that for almost every revelation of player 2,player 1 bids weakly less than $\underline{x}_1$, so that $r_2^*(x_2) = \emptyset$ for almost all $x_2 > x_1$. Hence, $\underline{x}_1 \geq \underline{x}_2$. Symmetric reasoning establishes that $\underline{x}_1 = \underline{x}_2$. But then, $i)$ there is a positive probability that player 2 does not disclose and $ii)$ $E(x_2 \mid \emptyset, r_2^*) >$

$\underline{x}_2 = \underline{x}_1$, so that 1 wins the object with positive probability if he discloses $\underline{x}_1$; a contradiction.

We now show that the ex ante payoff to 1 from full disclosure is larger than the payoff from not revealing, regardless of 2's revelation strategy. Let $E_2 \equiv E(X_2 \mid \emptyset)$, $NR \equiv r_2^{-1}(\emptyset)$ and let R be the set of types of 2 that reveals, that is, $R \equiv [x_m, x_M] - r_2^{-1}(\emptyset)$. We have that the payoff to 1 of always revealing is

$$F_2(R) \int_0^1 \int_R \max\{x_2 - x_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2 dF_1 + F_2(NR) \int_0^1 \max\{E_2 - x_1, 0\} dF_1 \tag{7}$$

and the payoff to 1 of never revealing is

$$F_2(R) \int_R \max\{x_2 - E_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2 + F_2(NR) \max\{E_2 - E_1, 0\} \tag{8}$$

Note that

$$\int_R \max\{x_2 - x_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2$$

in (7) is a convex function of x_1, so that the expected value with respect to x_1 is higher than

$$\int_R \max\{x_2 - E_1, 0\} \frac{f_2(x_2)}{F_2(R)} dx_2$$

by Jensen's inequality. Therefore, the first term in (7) is larger than the first term in (8). Similarly, since $\max\{E_2 - x_1, 0\}$ is a convex function of x_1, its expectation is larger than $\max\{E_2 - E_1, 0\}$, and so the second term in (7) is larger than the second term in (8). ■

Proof of Proposition 4. We apply Theorem 1. In order to do this, we first define the appropriate disclosure game.

1. The seller receives a signal s from the distribution $F(s)$ for $s \in [0, 1]$.
2. The seller reports $t \in \{s, \emptyset\}$
3. The seller receives

$$u(s, t, r) = E \min\left\{\frac{1}{1-\alpha} X_1 + \frac{\alpha}{1-\alpha} E[S \mid t, r], X_2\right\}$$

Suppose $F(\cdot \mid \emptyset, r)$ is non degenerate and $\max\{\text{Support } F(\cdot \mid \emptyset, r)\} = s$, so that $E[S \mid s, r] > E[S \mid \emptyset, r]$. For large enough X_2 and small enough X_1

$$\begin{aligned} \frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}E[S \mid \emptyset, r] &< \frac{1}{1-\alpha}X_1 + \frac{\alpha}{1-\alpha}E[S \mid s, r] < X_2 \\ &\Rightarrow u(s, s, r) > u(s, \emptyset, r) \end{aligned}$$

Theorem 1 implies the proposition. ■

Proof of Proposition 5. A player's strategy consists of a revelation policy, and a bid as a function of his information (and revelation policy, in principle). We now iteratively remove dominated strategies, but first define $f : [0, X] \times [0, X] \to \mathbf{R}$ by

$$f(w, z) = \max_{x_2} [v(w, x_2) - v(z, x_2)]$$

Note that by the theorem of the maximum, f is continuous, and $f(x, x) = 0$ for all x.

1.a Round 1.a. Player 1: Dominant strategy – Bid $v(x_1, x_2)$ if player 2 reveals x_2. Also, it is dominated to bid less than $v(x_1, 0)$ regardless of what is revealed. Similarly for player 2.

1.b Round 1.b. Player 1: Given Round 1.a, it is dominant to reveal x if

$$f(x, 0) = \max_{x_2} [v(x, x_2) - v(0, x_2)] < \varepsilon \tag{9}$$

since the left hand side is the greatest surplus 1 can earn without receiving a payment. Let X_1^1 be the set of x's for which equation (9) is satisfied (the subscript denotes the player, the superscript the iteration number). By continuity in x, X_1^1 is nonempty, and since v is increasing, it is an interval of the form $[0, a_1^1)$. Symmetrically for 2.

2.a Round 2.a. Player 1: Since it is dominant to reveal $x < a_2^1$, it is iteratively dominated to bid less than $v(x_1, a_2^1)$. Similarly for player 2.

2.b Round 2.b. Player 1: Given the previous rounds, it is dominant to reveal x if

$$f\left(x, a_1^1\right) = \max_{x_2} \left[v(x, x_2) - v\left(a_1^1, x_2\right)\right] < \varepsilon \tag{10}$$

since the left hand side is the most player 1 can earn without revealing. Let X_1^2 be the set of xs for which equation (10) is satisfied. By continuity, and v increasing, $X_1^2 = [0, a_1^2)$ for $a_1^2 > a_1^1$.

Continuing in this manner, we obtain that it is dominant to reveal all x's in X. Note that it cannot happen that the process is repeated infinitely many times, and that $a_1^n \to \overline{x} < X$. This is because, for N sufficiently large, a_1^N is close to $\overline{x}$ and we would therefore have that since f is continuous and $f(\overline{x}, \overline{x}) = 0$

$$f\left(\overline{x}, a_1^N\right) = \max_{x_2}\left[v\left(\overline{x}, x_2\right) - v\left(a_1^N, x_2\right)\right] < \varepsilon$$

so that $\overline{x} \in \left[0, a_1^{N+1}\right)$ contradicting that $a_1^N \to \overline{x}$ (recall that a_1^n is an increasing sequence). ■

Proof of Proposition 6. When player 1 receives only signal x, if he discloses, he earns 0. Therefore, there is an equilibrium in which 1 does not disclose.

We now establish that if player 1 receives both x and w, then in any equilibrium 1 always discloses x. We apply Theorem 1. In order to do this, we first define the appropriate disclosure game.

1. Player 1 receives a signal $x \in \{0, 1, \ldots, n-1\}$ with probability $F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)$.
2. Player 1 chooses $t \in \{x, \emptyset\}$.
3. Player 1 receives

$$u_1(x, t, r) = \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid t, r)\left(w - b_1\left(w, F(\cdot \mid t, r)\right)\right) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw$$

In particular, if x is revealed, letting $G^x(w) = \frac{F(w) - F\left(\frac{x}{n}\right)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}$ and g^x its density, we have

$$u_1(x, x, r) = \int_{\frac{x}{n}}^{\frac{x+1}{n}} G^x(w)\left(w - b_1\left(w, G^x\right)\right) g^x(w)\, dw,$$

Suppose that $F(\cdot \mid \emptyset, r)$ is non degenerate and $\min\{\text{Support } F(\cdot \mid \emptyset, r)\} = x$. Then $F(\cdot \mid \emptyset, r)$ is the posterior of F conditional on

$$w \in \left[\frac{x}{n}, \frac{x+1}{n}\right] \cup \left[\frac{k_1}{n}, \frac{k_1+1}{n}\right] \cup \ldots \cup \left[\frac{k_t}{n}, \frac{k_t+1}{n}\right] \equiv K,$$

for some $x < k_1 < k_2, < \cdots < k_t$. We have that

$$f(w \mid \emptyset, r) = \begin{cases} \frac{f(w)}{F(K.)} & w \in K. \\ 0 & \text{otherwise} \end{cases}$$

and for $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$

$$F(w \mid \emptyset, r) = \frac{F(w) - F\left(\frac{x}{n}\right)}{F(K)}.$$

We first note that $b_1(w, F(\cdot \mid \emptyset, r)) = b_1(w, F(\cdot \mid x, r))$ for all $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$. This is so because for all $w \in \left[\frac{x}{n}, \frac{x+1}{n}\right]$,

$$\begin{aligned} b_1(w, F(\cdot \mid x, r)) &= E_{F(\cdot|x,r)}[W \mid W \leq w] = \int_{\frac{x}{n}}^{w} W \frac{f(W \mid x, r)}{F(w \mid x, r)} dW \\ &= \int_{\frac{x}{n}}^{w} W \frac{\frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}}{\frac{F(w) - F\left(\frac{x}{n}\right)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)}} dW = \int_{\frac{x}{n}}^{w} W \frac{f(w)}{F(w) - F\left(\frac{x}{n}\right)} dW \\ &= \int_{\frac{x}{n}}^{w} W \frac{\frac{f(w)}{F(K)}}{\frac{F(w) - F\left(\frac{x}{n}\right)}{F(K)}} dW = E_{F(\cdot|\emptyset,r)}[W \mid W \leq w] \\ &= b_1(w, F(\cdot \mid \emptyset, r)) \end{aligned}$$

We have:

$$\begin{aligned} u_1(x, \emptyset, r) &= \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid \emptyset, r)(w - b_1(w, F(\cdot \mid \emptyset, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &= \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid \emptyset, r)(w - b_1(w, F(\cdot \mid x, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &< \int_{\frac{x}{n}}^{\frac{x+1}{n}} F(w \mid x, r)(w - b_1(w, F(\cdot \mid x, r))) \frac{f(w)}{F\left(\frac{x+1}{n}\right) - F\left(\frac{x}{n}\right)} dw \\ &= u_1(x, x, r) \end{aligned}$$

where the inequality follows from the fact that $F(w \mid \emptyset, r) < F(w \mid x, r)$ for all $w > \frac{x}{n}$.

To complete the proof, we note that continuity is trivially satisfied. ■

Proof of Theorem 1. Suppose player i's signal is not almost surely known. Then r_i^*, player i's strategy in a disclosure game equilibrium, is such that a positive mass of x_i's, with not all the mass concentrated on one x_i, are concealing their types. Therefore, $F_i(\cdot \mid \emptyset, r_i^*)$ is non degenerate, so by hypothesis, $\exists \widetilde{x}_i \in \mathit{Support} F_i(\cdot \mid \emptyset, r_i^*)$ for which $u_i(\widetilde{x}_i, \widetilde{x}_i, r^*) > u_i(\widetilde{x}_i, \emptyset, r^*)$. Since r^* is an equilibrium, we must have $r_i^*(\widetilde{x}_i) = \widetilde{x}_i$. By continuity of $u_i(x_i, x_i, r)$ and $u_i(x_i, \emptyset, r)$, there exists an ε such that if $d(x_i, \widetilde{x}_i) < \varepsilon$, then $u_i(x_i, x_i, r^*) > u_i(x_i, \emptyset, r^*)$ and hence $r_i^*(x_i) = x_i$. We obtain that $\mathbb{X}_i \backslash \{x_i : d(x_i, \widetilde{x}_i) < \varepsilon\}$ is closed and has probability 1 according to $F_i(\cdot \mid \emptyset, r_i^*)$, and so $\widetilde{x}_i$ can not belong to $\mathit{Support} F_i(\cdot \mid \emptyset, r_i^*)$, a contradiction. ■

Proof of Theorem 2. For all i, let r_i^* be such that $r_i^*(x_i) = x_i$ for all $x_i \in X_i$, and set $F_i(X_i = \bar{x}_i \mid \emptyset, r_i^*) = 1$. The hypothesis of the theorem implies that $u_i(x_i, x_i, r^*) \geq u_i(x_i, \emptyset, r^*)$ for all x_i, so that r^* is a disclosure game equilibrium. ■

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