"Minimum Quote Size, Minimum Quote and Transaction Size and Market Quality"

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ABSTRACT

Financial market regulators generally impose a minimum quote (and transaction) size for liquidity providers, but very little empirical evidence and no theoretical literature has been so far available on this issue. In this paper we build a model of bid-ask spread with asymmetric information to show how this rule affects market quality. We also provide empirical evidence using high frequency data from the Milan Stock Exchange. The model compares three regimes (No Constraint, Minimum Quote Size and Minimum Quote and Transaction Size) and shows that the market for small trades is more liquid under the unconstrained regime, which is preferred by retail traders; on the contrary, large liquidity traders and insiders are better off under the regime with minimum quote size, which is deeper for large trades. The regime with minimum quote and transaction size is Pareto dominated by the other two. Our results are confirmed by the empirical evidence both from the NASDAQ and from the Milan Stock Exchange.

Introduction

This paper evaluates the effects on the quality of financial markets of the introduction of a minimum size on quoted prices (MQS) and on traded quantities (MQTS). The relevance of this feature for the design of wholesale versus retail markets is also discussed.

A number of fact and situations help to better understand the significance of this issue: firstly, although such constraints are introduced for general convenience, i.e. in order to standardize trading lots, their implementation has an impact on market quality and agents' welfare. Secondly, the application of such rules is not homogeneous thoughout different markets: as an example, the MQS operates on the NASDAQ, whereas on most traditional European markets there is a MQTS and on European New Markets there are no MQS or MQTS. A further example is provided by EuroMTS, the leading Pan-European electronic system for Euro-denominated government benchmark bonds, where one finds both a MQS and a MQTS, while on MTSItaly only a MQTS exists. The third and most important reason is that there does not exist a theoretical model which compares the different market regimes (without any constraint, with MQS and with MQTS) and which shows the effect of a change in the MQS or MQTS. Such a model would be of extreme relevance since the markets which are regulated by a MQS or a MQTS rule see a tendency to have these sizes reduced: the most notable example took place in 1997 when the Securities and Exchange Commission introduced the Order Handling Rules on to the NASDAQ and one of these rules, the Actual Size Rule (ASR), significantly reduced the MQS. Finally, the only existing empirical evidence is on the impact of the reduction of the MQS on the NASDAQ: Barclay M., Christie W.G., Harris J.H., Kandel E. and Schultz P.H. (1999)) show that the introduction of the Actual Size Rule increased price competition and liquidity for small trades, but its effect on market depth was ambiguous. Smith (1998), Simaan, Weaver and Whitcomb (1998) and NASD (1997 and 1998) also provide similar evidence.

The model we use is based on Easley and O'Hara (1987) to incorporate three market regimes characterized respectively by the absence of any cosntraint, by a MQS and by a MQTS.

The results we obtain clearly show that the MQTS regime is Pareto dominated by the other two regimes and should not be used as a regulatory device. We also show that the MQS regime is always the most informationally efficient and it is desirable for wholesale markets, which are dominated by large institutional investors; on the contrary, the no-constraints regime is preferable for retail markets, which are dominated by the presence of small traders.

Under the MQTS regime small liquidity traders, who cannot perfectly hedge their endowment, leave the market and force liquidity providers to dilute their losses on a smaller number of uninformed traders resulting in wider bid-ask spreads. When the MQS is introduced, small liquidity traders are forced to pay the price associated to a higher trade size and, by reducing market makers' trading costs, allow them to quote more competitive prices for large trades. Under the MQS regime the equilibrium proportion of insiders is highest and insiders play pure strategies, thus maximising informational efficiency.

The empirical implications of such results are that when the MQS is reduced, the liquidity and the frequency of small trades increase, leaving retail traders better off. On the contrary, the depth and the frequency of large trades decrease making institutional traders worse off, this effect fading as the size of the institutional investors increases. Finally, informational efficiency is reduced due to the fact that being insiders worse off, the equilibrium proportion of informed traders decreases. When looking at changes in the MQTS, its reduction has the same impact on the welfare of small traders as a reduction in the MQS, whereas the impact on informational efficiency and on the liquidity of large trades and therefore on the welfare of large institutional investors is non negative.

Empirical evidence from the NASDAQ confirms the model's predictions i.e. that a reduction in the MQS increases the frequency and liquidity of small trades, and that it generates a marginal increase in the depth of large trades. The latter result is consistent with the model, considering the predominance of large institutional investors in this market.

We used the set of data from the Italian Stock Exchange to measure the effects of a reduction in the MQTS. We find that the frequency and the depth of small trades increase and that the liquidity of large trades marginally increases. Consistently with the model, we also find that the inside spread is reduced and that the frequency of large trades falls.

The plan of the paper is as follows: in Section 2 we present a model of price formation in a dealership market with asymmetric information and use it to evaluate the effects of the mandatory minimum size on market quality and traders' welfare; in Section 3 we review the existing empirical evidence on the NASDAQ and show results for a sample of two stocks, traded on the Milan Stock Exchange, which have recently experienced a reduction of the MQTS.

1 The model

We rely on a one–period model in the spirit of Glosten and Milgrom (1985) and Easley and O'Hara (1987). There are three groups of agents: risk neutral market makers quoting bid and ask prices, strategic insiders who know in advance the liquidation value of the asset and competitive uninformed liquidity traders. Nature chooses the final value of the asset ($\tilde{v}$) which can be $\overline{V} = 1$ or $\underline{V} = 0$ with equal probability and the dealers face an informed agent with probability α or an uninformed agent with the complementary probability. While the insider is risk neutral and trades to exploit her private information, liquidity traders intervene in the market to share risk. Assume that the liquidity traders have a mean variance objective:

$$E[(q+I)\tilde{v} - qp] - \frac{\gamma}{2}(q+I)^2 Var(\tilde{v})$$

where I is the endowment of the liquidity trader. When the liquidity traders can choose their order size, the first order condition yields:

$$q = \frac{E[\tilde{v}] - p}{\gamma Var(\tilde{v})} - I$$

Assuming that liquidity traders are infinitely risk averse, i.e., $\gamma \to \infty$, their trade is just the opposite of their inventory shock, $q = -I$. This is because, they desire to fully share risk, whatever the price. Liquidity traders can have negative or positive inventory shocks with equal probability and their inventory shock can be large with probability β or small with the complementary probability. We interprete uninformed traders with small shocks as retail and those with large shocks as institutional. We assume that, because of competition market makers quote prices at which they just break even.

Within the context of this model we analyze three market regimes. First we consider the case where there is no constraints on trade or quote size. In this case, market makers post quotes, equal to the expectation of the value of the asset conditional on the size and direction of the order. Second, we consider the case where there is a minimum quote size equal to two shares. In this case market makers are constrained to post prices which are for up to two shares. Consequently prices cannot differ conditionally on the size of the order and only depend on the direction of the order. Third, we consider the case where there is a minimum quote and transaction size equal to two shares. In that case, as in the previous one, market makers must quote the same price for small and large orders. In addition, liquidity traders cannot choose to place small orders. Consequently, liquidity traders with a small inventory shock choose to leave the market.

1.1 Equilibrium

When there are no minimum quote or transaction size, we are in the situation analyzed by Easley and O'Hara (1987). Informed agents a priori would like to submit large orders, to exploit their information. But such large orders might have a strong impact on the price. If the proportion of informed agents is not too large relative to the proportion of liquidity traders placing large orders, i.e., if $\beta \geq \frac{\alpha}{1-\alpha}$, they follow an aggressive strategy and always submit large orders. In this context the ask prices for one or two shares are:

$$A_1 = \frac{1}{2} \tag{1}$$

and

$$A_2 = \frac{\frac{1}{2}(1-\alpha)\beta + \alpha}{(1-\alpha)\beta + \alpha} \tag{2}$$

respectively. On the other hand, if the proportion of informed agents is large, i.e., if $\beta < \frac{\alpha}{1-\alpha}$, they mix between small and large trades. In this context the ask prices for one share and two shares are:

$$A_1 = \frac{1}{2}[(1-\beta) + \alpha(1+\beta)]. \tag{3}$$

and

$$A_2 = \frac{1}{4}[(3-\beta) + \alpha(1+\beta)] \tag{4}$$

respectively[1].

Now turn to the case where there is a mandatory quote size. In this case, as the price is the same for one or two units, it is not attractive for insiders to follow a mixed strategy. They always place large orders. Hence the frequency of large trades increases. In this market regime, as shown in the appendix, the ask price is:

$$A_Q = \frac{1}{2}\frac{\beta(\alpha-1) - 3\alpha - 1}{\beta(\alpha-1) - \alpha - 1} \tag{5}$$

Straightforward manipulations show that, irrespective of whether the informed agents follow a mixed strategy or not in the unconstrained market, $A_1 < A_Q < A_2$. Finally, consider the market with minimum quote and transaction size. In this context there are only large trades, because the liquidity traders with small endowment shocks exit from the market, and the insiders mimic the trades of the liquidity traders. In this market regime, as shown in the appendix, the ask price is:

$$A_{QT} = \frac{\frac{1}{2}(1-\alpha)\beta + \alpha}{(1-\alpha)\beta + \alpha}$$

Comparing this price and those obtained above, we get the following results:

$$B_{QT} \leq B_2 < B_Q < B_1 < A_1 < A_Q < A_2 \leq A_{QT}$$

where the equality holds for the equilibrium with insiders playing pure strategies. Figures 1 and 2 report the ask prices for the two equilibria with insiders playing pure strategies and mixing between 1 and 2 respectively. Building on the above analysis, we obtain our first proposition:

Proposition 1 *The inside spread is the tightest in the unconstrained market, and the widest with minimum quote and transaction size. Depth for large trades is the greatest in the market with minimum quote size and the smallest with minimum quote and trade size. The frequency of large trades is greater with minimum quote size than without. Proof: see the appendix.*

Our analysis shows that imposing a minimum quote and transaction size reduces the liquidity of the market. In addition it reduces the risk sharing possibilities of small liquidity traders. This suggests it is not desirable.

1.2 Welfare

Comparing traders' welfare under the three regimes, we obtain the following proposition:

Proposition 2 *Insiders and large institutional investors are better off under the regime with minimum quote size and they are worse off with minimum quote and transaction size. Small-retail traders are better off under the unconstrained regime and are worse off with minimum quote and transaction size. Proof: see the appendix.*

Large institutional investors and insiders[2] trade large quantities and are better off under the regime with the lowest ask (highest bid) price associated to large trades. Retail traders instead are better off under the unconstrained regime where they can perfectly hedge their endowment and minimize trading costs; under the regime with minimum quote size, they can still hedge their endowment, but pay a higher price. Finally, with minimum quote and transaction size they are worse off being prevented from perfect risk sharing.

1.3 General framework

Previous results can be generalized to the case with two trade sizes equal to Q and q, respectively, with Q>q. Under the unconstrained regime we have:

$$A(Q) = \frac{(Q-q)(1-\beta+\beta\alpha)+Q+\alpha q}{2Q} \tag{6}$$

$$A(q) = \frac{(Q-q)\beta(\alpha-1)+q(\alpha+1)}{2q} \tag{7}$$

With minimum quote (and transaction) size we obtain

$$A_{QS} = \frac{1}{2}(1+\frac{Q\alpha}{q+(Q-q)(\alpha+\beta-\alpha\beta)}) \tag{8}$$

$$A_{QTS} = \frac{\frac{1}{2}(1-\alpha)\beta+\alpha}{(1-\alpha)\beta+\alpha} \tag{9}$$

respectively. Now assume liquidity traders' endowment, I, is uniformly distributed over the interval $[-(\Omega+K),(\Omega+K)]$, being Ω the value of I which makes liquidity traders indifferent between trading Q or q. Since endowment is uniformly distributed around zero, we need to modify the extensive form of the game by allowing liquidity traders to choose, if they wish, not to trade. The following proposition shows how an increase of the endowment shock equal to K changes liquidity traders' strategies.

Proposition 3 *The larger the institutional investors' size, the higher the frequency of large trades and the lower the ask price: markets with large* *institutional investors are deeper. However, when the size of liquidity traders increases, the advantage of the regime with minimum quote size decreases. Proof: see the appendix.*

This result explains the empirical evidence on the NASDAQ (NASD 1997 and 1998) where the reduction of the MQS had no relevant effects on market depth.

1.4 Informational efficiency

We use the following measure of informational efficiency:

$$IE = \frac{1}{E[Var(\widetilde{v}|q_i)]}$$

with i=1,2 for small and large trade and find the following results.

Proposition 4 *Informational efficiency is highest under the regime with minimum quote size. Proof: see the appendix.*

The degree of informational efficiency depends on both the equilibrium proportion of insiders and on the type of strategies they play. Our results have shown that under the regime with minimum quote size, the equilibrium proportion of insiders is highest and insiders play pure strategies: it follows that under this regime informational efficiency is maximized. The difference between the degree of informational efficiency under the other two regimes depends on the values of the parameters. With separating equilibrium both the proportion of insiders and the type of their strategies are the same; thus, informational efficiency is the same; otherwise, with pooling equilibrium the proportion of insiders is higher under the regime without constraints, but insiders play pure strategies only under the regime with MQTS; this explains why the sign of the difference between the degree of informational effciency under the two regimes is ambiguous.

1.5 Market performance and policy implications

Table 1 summarizes the results obtained by comparing market quality under the regimes with no constraint (NC), minimum quote size (MQS) and minimum quote and transaction size (MQTS) respectively.

The unconstrained regime is advisable for small retail investors, since the inside spread is narrowest, the price impact is smallest and retail traders' welfare highest. On the contrary, for large institutional investors we recommend the regime with MQS where they enjoy a deeper market and a higher welfare. Nevertheless, as the institutional investors' size increases, the advantage of the MQS compared with the unconstrained regime reduces. Insiders prefer the regime with MQS, due to the high frequency of large trades; this is the reason why this regime is the most informationally efficient. The regime with MQTS is Pareto dominated by the other two and it should therefore not be taken into consideration as a viable alternative. Markets with a large proportion of institutional investors or large liquidity suppliers like EUROMTS could enhance liquidity by switching from the MQTS to the MQS regime.

1.6 Empirical evidence

In 1997 the Order Handling Rules (OHR) were introduced on the NASDAQ: available empirical evidence concerns the effects of the introduction of one of such rules, the Actual Size Rule (ASR), which reduced the MQS. Barclay M., Christie W.G., Harris J.H., Kandel E. and Schultz P.H. (1999) analyse the effects of the introduction of the OHR by looking at two groups of shares: the first is subject to all of the new rules; the second is subject to all but the Actual Size Rule. Among the results of their studies, of particular relevance is the evidence of the effect of the ASR on the distribution of quote and trade sizes: only for the first group of shares do they find a dramatic decline in the percentage of 1000 share quotes and a corresponding increase in the proportion of 100 share quotes[3]. It is also of interest their finding that the reduction in the minimum quote size is accompanied by a steep reduction in the number of 1000 share trades and by an increase in the frequency of small trades. Smith (1998) analyses the effects of the OHR on market depth by using a procedure which controls differences in share prices: he finds that the average total depth declines by 4% after the introduction of the new rules for all of the NASDAQ listed shares included in his sample. Simaan, Weaver and Whitcomb (1998) find evidence on the impact of the ASR on liquidity during a period of market downturn (October 1997). When comparing the results from two groups of stocks (only one of which subject to the ASR), they find that in both groups ask sizes increase while bid sizes decrease. Since bid sizes decrease by a statistically significant larger percentage for the ASR group of stocks, they conclude that a higher minimum quote size does effectively force market makers to provide more liquidity, especially at times of market stress. Finally, NASD (1997 and 1998) do not find relevant consequences on NASDAQ market quality following the removal of the 1000 Share Quote Size Rule, both during normal times and during periods of market stress. As a matter of fact, they do find that the introduction of the ASR generates a significant reduction in the quoted depth associated to large trades, but this effect fades when controlling for volume variantions.

Having considered the empirical evidence derived from the NASDAQ and the conclusions reached by the above mentioned works, we can conclude that a reduction of the MQS has a significant impact on the distribution of both trade and quote sizes, a weak effect on the inside spread and a negligible effect on market depth.

The remainder of this Section is devoted to an empirical analysis based on a set of data from the Italian Stock Market, where the minimum quote and transaction size for a number of stocks included in the MIB30[4] has been recently reduced. This variation was introduced as it often happens for those stocks whose price level increases over time in order to standardize trading lots. We evaluate the effects of such a reduction by making use of indicators of liquidity: to this end, we selected tick by tick data from a sample of stocks included in the MIB30, for every Wednesday from March 1999 to April 2000. We chose Wednesday in order to control for most of the day of the week effects (Corielli and Rindi (1998)). This data set includes transaction prices, bid and ask prices, quantities and time stamps. Table 2 shows the stocks included in the MIB30 which, during the period under analysis, were subject to inimum quote and transaction size reductions. Consequently, we have been able to check for changes in market quality after such rule modifications by averaging data relative to transactions on days before and after the events.

Taking AEM and SEAT stocks as an example[5], Table 2 shows that these stocks have been subject to two minimum quote size revisions: on 20 December 1999 and on 20 March 2000 for AEM; on 19 July 1999 and on 20 March 2000 for SEAT. Accordingly, we have been able to analyze three different sample periods corresponding to three different minimum quote and trade size regimes (Table 3).

First of all we show how the reductions of the MQTS had an impact on the frequency distribution of trade and quote sizes: changes in these frequency distributions influence both liquidity and depth, since they affect the spread and the waiting time associated to different trade sizes. Tables 4.1-4.2 and figures 5 and 6 show the results for AEM and SEAT stocks. Table 4.1 and 4.2 show the distribution of trade sizes before and after the two reductions of the MQTS: from 2500 to 1000 and to 500 for AEM and from 5000 to 2500 and to 500 for SEAT. Trades are divided into size categories up to 10000 shares. The frequency of all trades falling into each category being then calculated. The results for AEM from Table 4.1 indicate an impressive decline of the proportion of trades of 2500/3000 size from 62.5 % to 4.8 % and 4.5% respectively for the three periods under analysis. In contrast, the proportion of trades of smaller sizes sharply increased after the first reduction showing that the MQTS was binding, while it spread towards the reduced minimum quote size during the third period. Similarly for SEAT,

Table 4.2 shows that the proportion of trades of 3000/5000 shares sharply declined from 56.5% to 17.8% and 9.5% when the MQTS was reduced respectively to 5000, 2500 and 500 shares. Tables 4.1 and 4.2 also show that, consistently with the predictions of the model, the frequency of large trades falls with the reduction of the MQTS. It is interesting to notice that the proportion of trades of the minimum size slightly increased after the first reduction, while it consistently decreased after the second one (from 62.5%, to 66.0%, to 41.5% for AEM and from 56.5%, to 58.9%, to 35.3% for SEAT). This result raises the issue of what is the optimal MQTS, which would be an interesting extension of the present work. By looking at the cumulative distributions of volumes reported in figures 5 and 6, it is evident how the cumulative functions associated to the larger MQTS (2500 for AEM and 5000 for SEAT) lay below the cumulative functions associated to smaller MQTS (1000 and 500 for AEM and 2500 and 500 for SEAT). If we interprete the reduction of the MQTS as a switch to the unconstrained regime, these empirical findings are consistent with the results from our model (Table 1). Barclay M., Christie W.G., Harris J.H., Kandel E. and Schultz P.H. (1999) find similar results for the NASDAQ, which are also consistent with our theoretical findings.

We use as a measure of market tightness the mid-point weighted value of the bid-ask spread associated to different trade sizes ($\sum \frac{1}{N}|\frac{A_t - B_t}{\frac{A_t + B_t}{2}}|$): figures 7-10 show that when the MQTS is reduced, the inside spread narrows for both AEM and SEAT stocks; moreover, the spread associated to both small and large quantities diminishes following a reduction of the MQTS. These findings confirm the results from our model which predicts a narrower inside spread and a greater market depth following a reduction of the MQTS.

We replicate the analysis by looking at the waiting time between consecutive transactions as a measure of market depth and we calculate the probability density function of different durations associated to the ranges of trading volumes surrounding the MQTS. Figures 11-14 show the frequencies of the durations associated to 1/10000 and 1/1000 of daily trading volume respectively. The reduction of the MQTS from 2500, to 1000 and to 500 for AEM and from 5000, to 2500 and to 500 for SEAT moderately increased the frequencies associated to short durations especially for small trades (1/10000) and confirms previous results on the bid-ask spread. Moreover, by looking at the sequence of histograms related to the small and large classes of trading volumes we find a peculiar result: independently from the class of trading volume, the frequencies of the durations for the period with a MQTS of 2500 shares for AEM and 5000 shares for SEAT are clustered around either the smallest values or the largest ones. This means that under the regime with the highest MQTS, trades of the size considered are executed either with a very short waiting time or after a long period (longer that 120 seconds). It would be interesting to explain this result.

2 Conclusions

Financial market regulators are currently reducing the mandatory minimum size both on the NASDAQ and on most European Stock Exchanges. Very little empirical evidence and no theoretical literature has been so far available on this issue. We built a model of bid-offer spread with asymmetric information to show how the minimum quote (and transaction) size affects market quality and traders' welfare under three regimes. Using high frequency data from the Milan Stock Exchange we evaluated the effects of the minimum quote and transaction size.

The model shows that the inside spread is narrowest under the unconstrained regime; this is confirmed by the evidence from the Italian Stock Exchange and the NASDAQ. The model also shows that market depth and the frequency of large trades are highest under the regime with MQS which is also the most informationally efficient. Small investors benefit from the regime without constraints, while large institutional investors and insiders prefer the MQS regime which is deeper. Nevertheless, as liquidity traders' size increases, the advantage of the MQS regime decreases. This explains the empirical evidence from the NASDAQ, where the reduction of the MQS did not have relevant effects on market depth. The third regime, with MQTS, is Pareto dominated by the other two and should therefore not be taken into consideration as a viable alternative. The mandatory minimum size also affects price discovery by modifying agents' orders. Limit orders and large orders, for example, are more informative than market or small orders; consequently, if a reduction of the minimum quote size increases only small orders, we expect a change in informational efficiency to take place. A transition model of the order flow which jointly explains the duration and the relative aggressiveness of different orders (Bisière and Kamionka (1998) and Engle and Russel (1998)) could be used to analyse this issue. This is an interesting topic for future research.

3 Appendix

With $\overline{V} = 1$ and $\underline{V} = 0$ and minimum quote size, the equilibrium ask price which drives market makers expected profits to zero must satisfy the following condition:

$$E[\pi_{Ask}] = \Pr(\overline{V})E[\pi_{Ask}|\overline{V}] + \Pr(\underline{V})E[\pi_{Ask}|\underline{V}] = 0$$

i.e.,

$$\Pr(\overline{V})[1(A_Q - 1)\Pr(1|\overline{V}) + 2(A_Q - 1)\Pr(2|\overline{V})] +$$

$$+\Pr(\underline{V})[1(A_Q - 0)\Pr(1|\underline{V}) + 2(A_Q - 0)\Pr(2|\underline{V})] = 0$$

Substituting for the conditional probabilities and solving for A_Q, we obtain equation 5. The equilibrium ask price with minimum quote and transaction size is equal to:

$$A_{QT} = P(\overline{V}|2)(+1) + (1 - P(\overline{V}|2)(0) =$$

$$= \frac{\frac{1}{2}(1-\alpha)\beta + \alpha}{(1-\alpha)\beta + \alpha}$$

Given symmetry, straightforward algebra allows for the bid prices.

Proof of Proposition 1:

$$A_2 - A_Q = \frac{\frac{1}{4}(1-\alpha)^2(\beta^2-1)}{(-1+\alpha)\beta-\alpha-1} > 0$$

When insiders play pure strategies ($\mu = 1$), A_{QT} =A_2. Being,

$$\frac{\partial A_2}{\partial \mu} = -\frac{1}{2}\frac{\alpha\beta(\alpha-1)}{(-\beta+\alpha\beta-\alpha\mu)^2} > 0$$

we obtain:

$$A_{QT} - A_2 > 0$$

Moreover we get:

$$A_Q - A_1 = -\frac{1}{2}\frac{\alpha(\beta-1)(\alpha-1)}{(\beta-1)\alpha-\beta-1} + \frac{1}{2}(1-\alpha)\beta > 0$$

$\forall\ \alpha, \beta \in (0,1)$. Measuring market depth by the price impact of a trade, we obtain:

$$\frac{A_1 - E(v)}{1-0} < \frac{A_Q - E(v)}{1-0} \tag{10}$$

for small trades and

$$\frac{A_{QT} - 1/2}{2-0} > \frac{A_2 - 1/2}{2-0} > \frac{A_Q - 1/2}{2-0}$$

for large trades ($E(v) = \frac{1}{2}$). With minimum quote size the ask price for large trades is lowest and their frequency should increase. However, since the ask price is a function of the proportion of insiders, we need to make the equilibrium number of insiders endogenous. Assuming market makers face 1 potentially informed trader and one uninformed, the former will buy information with probability λ paying a cost equal to c and with probability $1-\lambda$ he will decide to remain uninformed. If he does not buy the information, he does not trade. The probability of market makers facing an informed trader is therefore equal to $\alpha = \frac{\lambda}{1+\lambda}$. The potentially informed trader will buy information up to the point where the expected profits associated to a trade of size 2 are equal to the expected costs of buying information.

$$\alpha_i 2(\overline{V} - A_i) = \lambda_i \ c$$

for $i = \{2, Q, QT\}$. Substituting for $\alpha_i = \frac{\lambda_i}{1+\lambda_i}$ and solving for λ_i, with $\overline{V} = 1$,we get:

$$\lambda_i = \frac{2(1 - A_i)}{c} - 1$$

and

$$\alpha_i = 1 - \frac{c}{2(1 - A_i)}$$

Since $A_Q < A_2 \leq A_{QT}$, $\forall \alpha, \beta$, we obtain: $\alpha_Q > \alpha_2 \geq \alpha_{QT}$. Since under the unconstrained regime the proportion of insiders trading large is equal to $\mu\alpha_2$, in equilibrium we obtain, $\alpha_Q > \alpha_{QT} \geq \mu\alpha_2$. Within this model with risk neutral market makers a higher ask price ($A_{QT} > A_2$) can only be justified by a larger proportion of insiders trading large quantities; this explains the latter inequality, while the former is due to small liquidity traders bearing part of the adverse selection costs.

Proof of Proposition 2: insiders' expected profits with no constraint (NC) are equal to:

$$
\begin{aligned}
E(\pi^I_{Ask})_{NC} &= E(\pi^I_{Ask}|\overline{V}) \Pr(\overline{V}) + E(\pi^I_{Ask}|\underline{V}) \Pr(\underline{V}) = \\
&= [\mu 2(1 - A_2) + (1 - \mu)(1 - A_1)] \Pr(\overline{V}) = \\
&= (1 - A_2)
\end{aligned}
$$

and with minimum quote (and transaction size) they are:

$$E(\pi^I_{Ask})_{Q,QT} = (1 - A_{Q,QT})$$

Since $A_{QT} \geq A_2 > A_Q$, we have $E(\pi^I_{Ask})_{QT} \leq E(\pi^I_{Ask})_{NC} < E(\pi^I_{Ask})_Q$.

Large liquidity traders' expected profits are equal to:

$$\begin{aligned} E(\pi^U_{Ask}) &= E(\pi^U_{Ask}|\overline{V})\Pr(\overline{V}) + E(\pi^U_{Ask}|\underline{V})\Pr(\underline{V}) = \\ &= [2(1 - A_{2,Q,QT})]\Pr(\overline{V}) + [2(0 - A_{2,Q,QT})]\Pr(\underline{V}) = \\ &= 1 - 2A_{2,Q,QT} \end{aligned}$$

Since $A_{QT} \geq A_2 > A_Q$, it follows that $E(\pi^U_{Ask})_{QT} \leq E(\pi^U_{Ask})_{NC} < E(\pi^U_{Ask})_Q$.

Proof of Proposition 3: under the unconstrained regime and the regime with minimum quote size, general conditions for hedgers' mixed strategies between Q and q are:

$$E[U(\pi(I,Q))] = E[U(\pi(I,q))]$$

$$(Q+I)\frac{1}{2} - QA(Q) - \gamma(Q+I)^2\frac{1}{4} = (q+I)\frac{1}{2} - qA(q) - \gamma(q+I)^2\frac{1}{4}$$

which must hold together with previous condition on insiders' profits:

$$Q(1 - A(Q)) = q(1 - A(q))$$

In order for these two conditions to be satisfied, it is necessary that:

$$I = -\frac{1}{2}(Q + \frac{2}{\gamma} + q) = -(\frac{Q+q}{2} + \frac{1}{\gamma}) = -\Omega \ for \ \gamma \to \infty$$

Similarly, conditions for hedgers mixed strategies between 0 and q are:

$$I = -\frac{q}{2} + \frac{(2\beta\alpha - 2\beta)}{2\gamma}(1 - \frac{Q}{q}) - \frac{\alpha}{\gamma} = -\frac{q}{2} \ for \ \gamma \to \infty$$

With I uniformly distributed, the probability of $I = -\Omega$ is almost zero, which excludes equilibrium mixed strategies. Figure 3 shows that the probability of uninformed traders playing Q is equal to:

$$\beta_1 = \Pr(|I_n| > |\Omega|) = \frac{K}{\Omega + K}$$

Similarly, the probability of large liquidity traders playing 0 and q are equal to:

$$\beta_2 = \Pr(|I_n| < |\frac{q}{2}|) = \frac{q}{2(\Omega + K)}$$

$$1 - \beta_1 - \beta_2 = 1 - \frac{K}{\Omega + K} - \frac{q}{2(\Omega + K)} = \frac{Q}{2(\Omega + K)}$$

respectively. The above results show that the larger are liquidity traders' endowment shocks, the higher the probability they trade large. Equilibrium ask prices are decreasing functions of β and, the higher is β, the narrower the distance between $A(Q)$ and A_{QS}. Using equations 6 and 8, we obtain:

$$\begin{aligned} g_1 &= \frac{\partial A(Q)}{\partial \beta} = -\frac{(Q-q)(1-\alpha)}{2Q} < 0 \\ g_2 &= \frac{\partial A_{QS}}{\partial \beta} = -\frac{(Q-q)(1-\alpha)Q\alpha}{2[(Q+q)(\beta\alpha-1)+\beta(q-Q)-q]^2} < 0 \end{aligned}$$

For small values of $\frac{Q}{q}$ and $\frac{\beta}{\alpha}$ we obtain:

$$g_1 - g_2 < 0 \quad \forall \quad \alpha, \beta$$

Proof of Proposition 4:

$$E[Var(\widetilde{v}|q_i)] = \sum_{i=1}^{2}[Var(\widetilde{v}|q_i^A)\Pr(q_i^A) + Var(\widetilde{v}|q_i^B)\Pr(q_i^B)] =$$

$$= 2\sum_{i=1}^{2} Var(\widetilde{v}|q_i^A)\Pr(q_i^A) =$$

$$= 2\sum_{i=1}^{2}[(\overline{V} - E(\widetilde{v}|q_i^A))^2 \Pr(\overline{V}|q_i^A) + (\underline{V} - E(\widetilde{v}|q_i^A))^2 \Pr(\underline{V}|q_i^A)]\Pr(q_i^A)$$

Comparing the informational efficiency under the three regimes, we obtain:

$$IE_{NC} = \frac{1}{E[Var(\widetilde{v}|q_i)]} = \frac{1}{\frac{1}{8}[(1-\alpha)\beta(1-\beta) - 2(1+\alpha)](\alpha-1)}$$

$$IE_j = \frac{1}{E[Var(\widetilde{v}|q_i)]} = \frac{4[\beta(\alpha_j - 1) - \alpha_j]}{[(\alpha_j - 1)(\beta + \alpha_j)]}$$

with j=Q,QT. Moreover, since for a given value of α,

$$IE_{NC} - IE_j =$$

$$= -4\frac{-\alpha\beta - \beta^2 + 3\alpha\beta^2 + \beta^3 - 2\alpha\beta^3 - \alpha^2\beta + 2\alpha^2 + \alpha^2\beta^3 - 2\alpha^2\beta^2}{(-2+\beta-\beta^3 - 2\alpha + \alpha\beta^2 - \alpha\beta)(-1+\alpha)(\beta+\alpha)}$$

and Figure 4 shows that this difference is negative, we could conclude that informational efficiency is higher under the regime with minimum quote (and transaction) size than under the unconstrained regime. However if we allow for endogenous entry of insiders, the results may change. Since $\alpha_Q > \alpha_2 \geq \alpha_{QT}$, and $\partial IE_z/\partial \alpha_z > 0$ with $z = \{NC, MQS, MQTS\}$, we obtain that:

$$IE_{MQS} > IE_{NC,MQTS}$$

while the comparison between the unconstrained and the MQTS regimes remains ambiguous.

Footnotes

1. The proof of these results is available from the author upon request.

2. Under the unconstrained regime each insider can play mixed strategies, but according to the indifference condition, her expected profits are equal to: $2(1 - A_2)$.

3. According to the ASR, the minimum quote size was reduced from 1000 to 100 units of shares.

4. MIB30 is the price index for the largest thirty stocks listed on the Milan Stock Exchange.

5. AEM and SEAT are highly traded stocks; we are currently working to a wider sample of stocks and preliminary results conferm our findings.

References

[1] Barclay M., Christie W.G., Harris J.H., Kandel E. and Schultz P.H. (1999), "Effects of market reform on the trading costs and depth of NASDAQ stocks", *Journal of Finance*, p.1-34.

[2] Bisière C. and Kamionka T. (2000), "Timing of Orders, Orders Aggressiveness and the Order Book in the Paris Bourse"- *Annales d'Economie et des Statistique.Working Paper*, no. 60, Oct/Dec.

[3] Corielli F.and Rindi B. (1992), "Calendar regularities in stock returns: the Italian case", *Ricerche Economiche,* XLVI, N.3-4, p. 303-326.

[4] Easley D. and O'Hara M. (1987), "Price, trade size and information in securities markets", *Journal of Financial Economics*, 19, pp. 69-90.A

[5] Engle and Russel (1998), "Autoregressive Conditional Duration: a New Model for Irregularly Spaced Transaction Data", *Econometrica.*

[6] NASD Economic Research (1997), "Effects of the removal of minimum sizes for proprietary quotes in the NASDAQ stock market", *Staff Paper, NASD Economic Research department.*

[7] Glosten L.R. and Milgrom P.R. (1985), "Bid-Ask and transaction prices in a specialist market with heterogeneously informed traders", *Journal of Financial Economics*, 14, pp. 71-100.

[8] NASD Economic Research (1998), "The effect of the Actual Size Rule on the NASDAQ stock market", *Staff Paper, NASD Economic Research department.*.

[9] Simaan Y., Weaver D.G. and Whitcomb D. (1998), "The quotation behaviour of ECNs and NASDAQ market makers", *mimeo.*

[10] Smith J. (1998), "The Effects of Order Handling Rules and 16ths on NASDAQ: a cross-sectional analysis", *NASD* Working Paper 98-02.

Table 1 Market quality under the three regimes: NC, MQS and MQTS.

Liquidity	NC>	MQS>	MQTS
Depth for small trades	NC>	MQS>	MQTS
Depth for large trades	MQS>	NC≥	MQTS
Freq small trades	NC≥	MQS>	MQTS
Freq large trades	MQS>	MQTS≥	NC
Welfare retail-small traders	NC>	MQS>	MQTS
Welfare institutional-large traders	MQS>	NC≥	MQTS
Welfare insiders	MQS>	NC≥	MQTS
Informational efficiency	MQS>	NC ?	MQTS ?

Table 2.1 MIB30 stocks: MQTS changes

Stock	3/3/99	6/21/99	7/19/99	8/23/99	12/20/99	1/5/00	3/20/00	4/26/00
Aem	2500	2500	2500	2500	1000	1000	500	500
Alleanza	250	250	250	250	250	250	250	250
Autostrade	1000	1000	500	500	500	500	500	500
B.Fideuram	1000	1000	500	500	500	500	250	250
B.Intesa	1000	1000	1000	1000	500	500	500	500
B.Roma	2500	2500	2500	2500	2500	2500	2500	2500
B.Carire	250	250	100	100	100	100	25	25
Bnl	1000	1000	1000	1000	1000	1000	1000	1000
Comit	1000	1000	500	500	500	500	500	500
Edison	500	500	500	500	500	500	250	250
Enel	-	-	-	500	500	500	500	500
Eni	500	500	500	500	500	500	500	500
Fiat	1000	1000	1000	100	100	100	100	100
Finmec.	5000	5000	5000	5000	2500	2500	2500	2500
Generali	100	100	100	100	100	100	100	100

Table 2.2 MIB30 stocks: MQTS changes								
Stock	3/3/99	6/21/99	7/19/99	8/23/99	12/20/99	1/5/00	3/20/00	4/26/00
Mediaset	500	500	500	500	250	250	100	100
Mediobanca	250	250	250	250	250	250	250	250
Mediolanum	500	500	500	500	500	500	250	250
Montedison	2500	1300	2000	2000	1000	1000	1000	1000
MPS	-	1000	1000	1000	1000	1000	1000	1000
Olivetti	2500	2500	2500	2500	1000	1000	1000	1000
Pirelli	1000	1000	1000	1000	1000	1000	1000	1000
Ras	250	250	250	250	250	250	250	250
Rolo Banca	100	100	100	100	100	100	100	100
Sanpaolo-Imi	250	250	250	250	250	250	250	250
Seat P.Gialle	5000	5000	2500	2500	2500	2500	500	500
Tecnost	1000	300	300	300	300	300	300	300
Telecom	500	500	500	500	250	250	250	250
Tim	500	500	500	500	500	500	250	250
Unicredit	1000	1000	1000	1000	500	500	500	500

Table 3. Sample Description				
Stock	Sample Period	N° Days	N°Observations	MQTS
AEM	3/3/1999 – 12/20/1999	42	35682	2500
	12/22/1999 – 3/15/2000	13	53685	1000
	3/22/2000 – 4/26/2000	6	36997	500
SEAT	3/3/1999 – 7/14/1999	20	23793	5000
	7/21/1999 – 3/15/2000	35	139035	2500
	3/22/2000 – 4/26/2000	6	49686	500

Table 4.1- AEM: Distribution of trade sizes surrounding the reduction of the MQTS

N. of days*	42	13	6
MQTS	2500	1000	500
500	0.00%	0.00%	41.50%
1000	0.00%	66.00%	31.50%
1500-2000	0.00%	15.40%	10.90%
2500-3000	62.50%	4.80%	4.50%
3500-4000	0.00%	2.60%	2.10%
4500-5000	16.90%	3.70%	3.10%
5500-10000	11.70%	4.80%	4.30%
>10000	9.00%	2.60%	2.20%

*Number of Wednesdays before the reduction of the MQTS

Table 4.2- SEAT: Distribution of trade sizes surrounding the reduction of the MQTS			
N. of days*	20	24	6
MQTS	5000	2500	500
500	0.00%	0.00%	35.30%
1000	0.00%	0.00%	23.00%
1500-2000	0.00%	0.00%	11.90%
2500	0.00%	58.90%	7.30%
3000-5000	56.50%	17.80%	9.50%
5500-10000	18.40%	10.70%	6.40%
>10000	25.10%	12.70%	6.70%

*Number of Wednesdays before the reduction of the MQTS

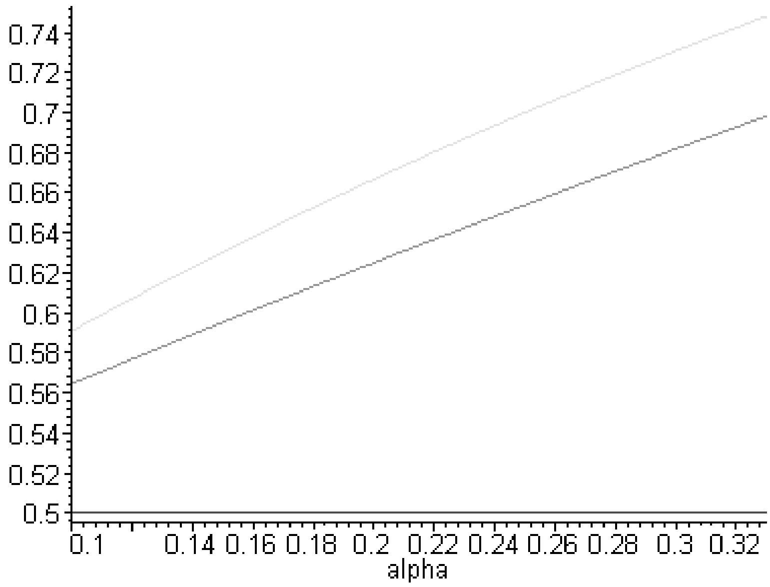


Figure 1: Equilibrium pure strategies($\beta = \frac{1}{2}$). On the vertical axis, from below:A_1,A_Q,A_2=A_{QT}

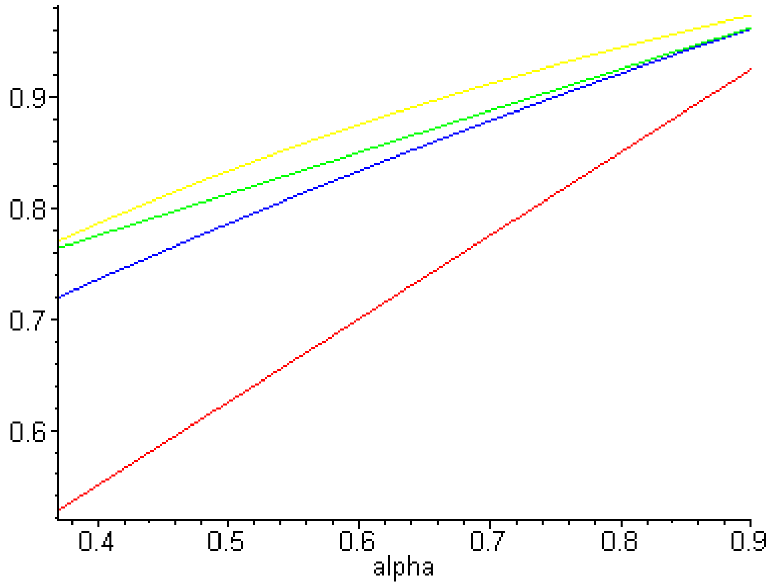


Figure 2: Equilibrium mixed strategies. On the vertical axis, from below A_1,A_Q,A_2 and A_{QT}.

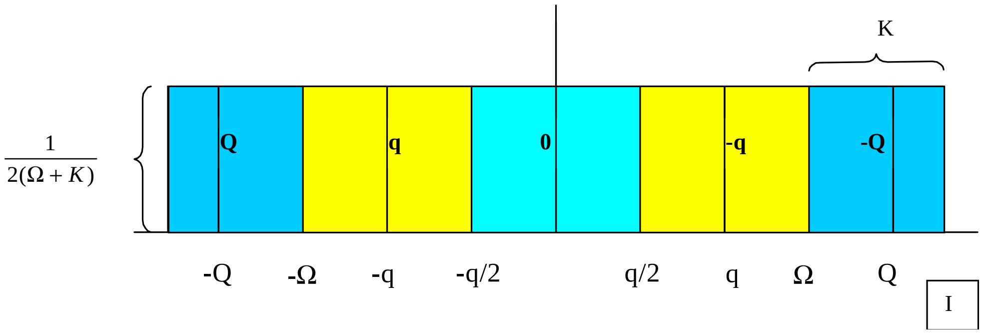


Figure 3: Endowment shock distribution function.

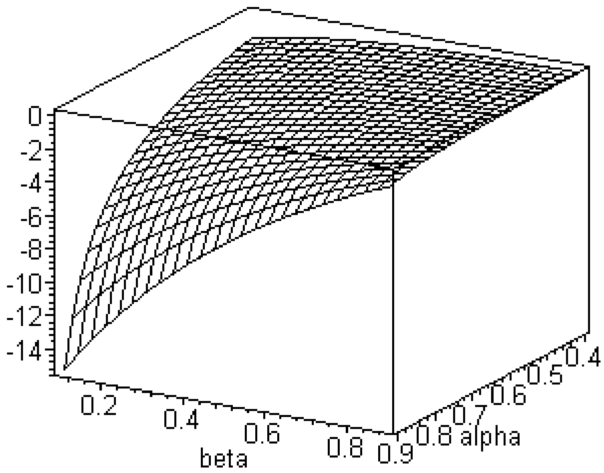


Figure 4: $IE_{NC} - IE_j$ where j=MQS, MQTS

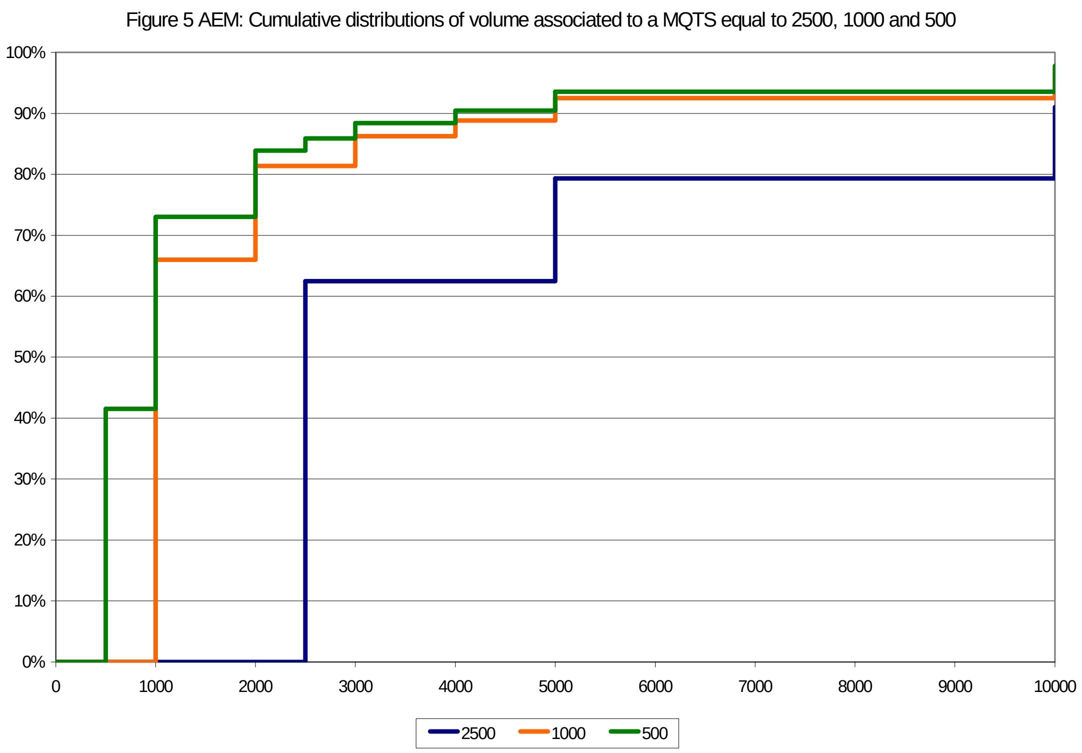


Figure 5 AEM: Cumulative distributions of volume associated to a MQTS equal to 2500, 1000 and 500
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
2500
1000
500

Figure 5:

Figure 6   SEAT: Cumulative distributions of volume associated to a MQTS equal to 5000, 2500 and 500

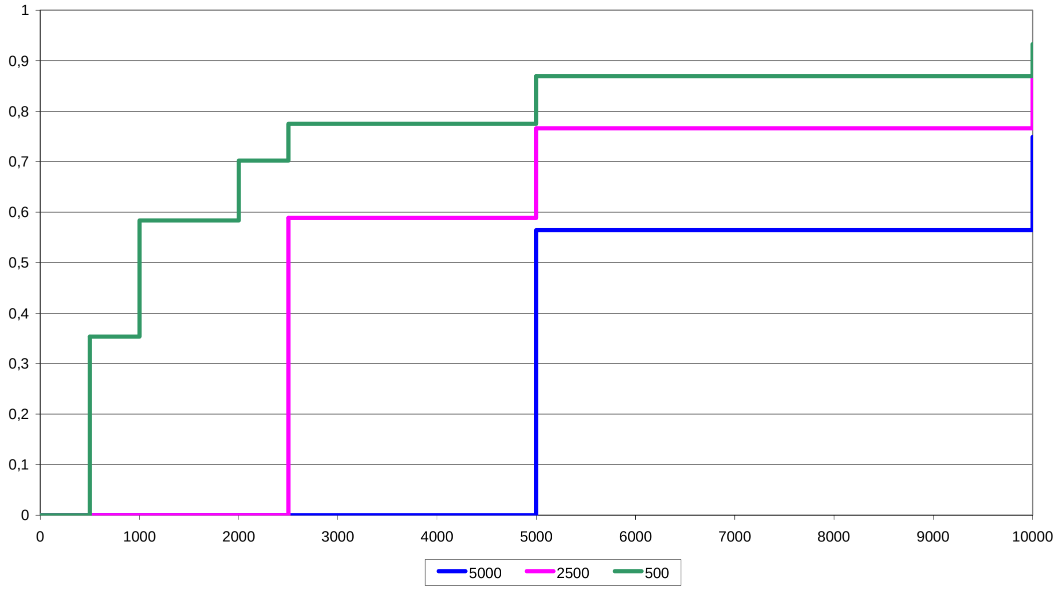


Figure 6:

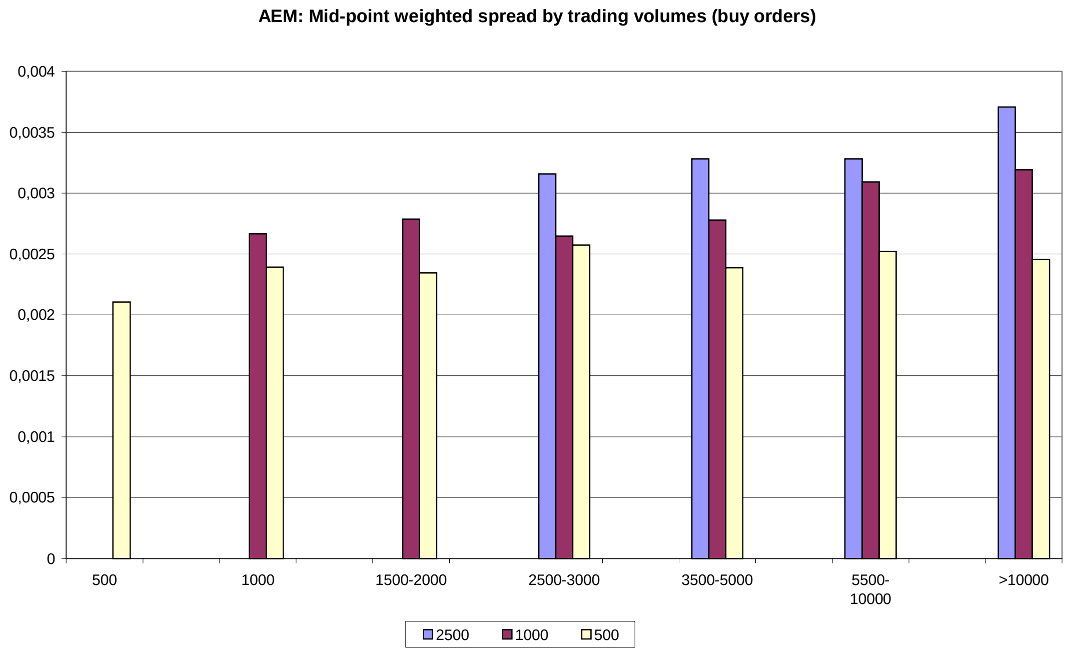

AEM: Mid-point weighted spread by trading volumes (buy orders)
0,004
0,0035
0,003
0,0025
0,002
0,0015
0,001
0,0005
0
500
1000
1500-2000
2500-3000
3500-5000
5500-
10000
>10000
2500
1000
500

Figure 7:

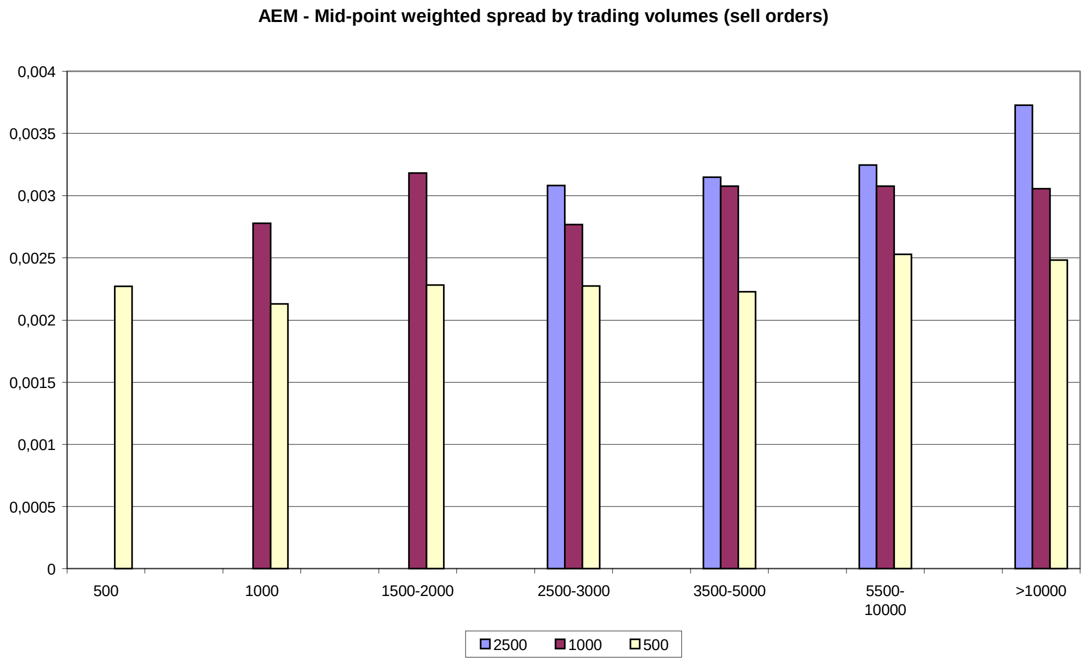

AEM - Mid-point weighted spread by trading volumes (sell orders)
0,004
0,0035
0,003
0,0025
0,002
0,0015
0,001
0,0005
0
500
1000
1500-2000
2500-3000
3500-5000
5500-10000
>10000
2500
1000
500

Figure 8:

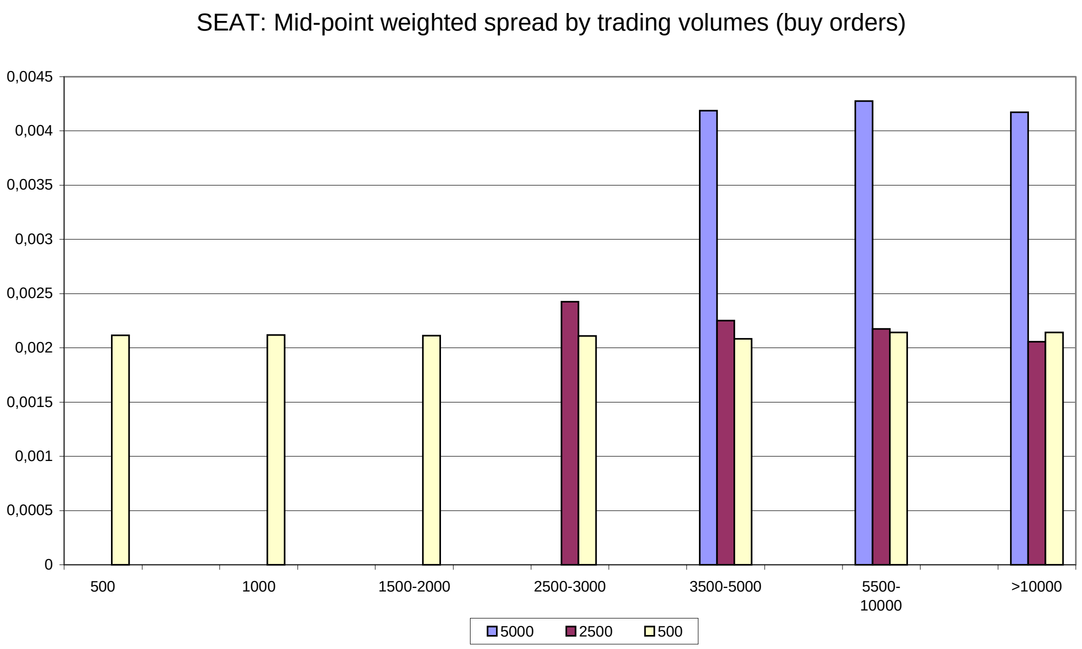

SEAT: Mid-point weighted spread by trading volumes (buy orders)
0,0045
0,004
0,0035
0,003
0,0025
0,002
0,0015
0,001
0,0005
0
500
1000
1500-2000
2500-3000
3500-5000
5500-10000
>10000
5000
2500
500

Figure 9:

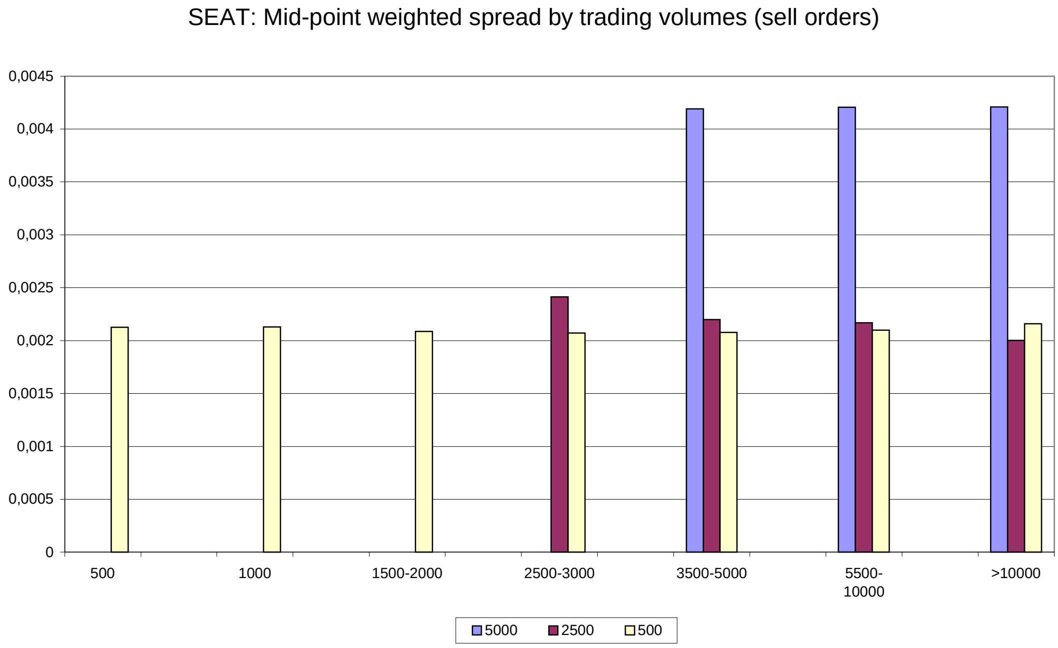
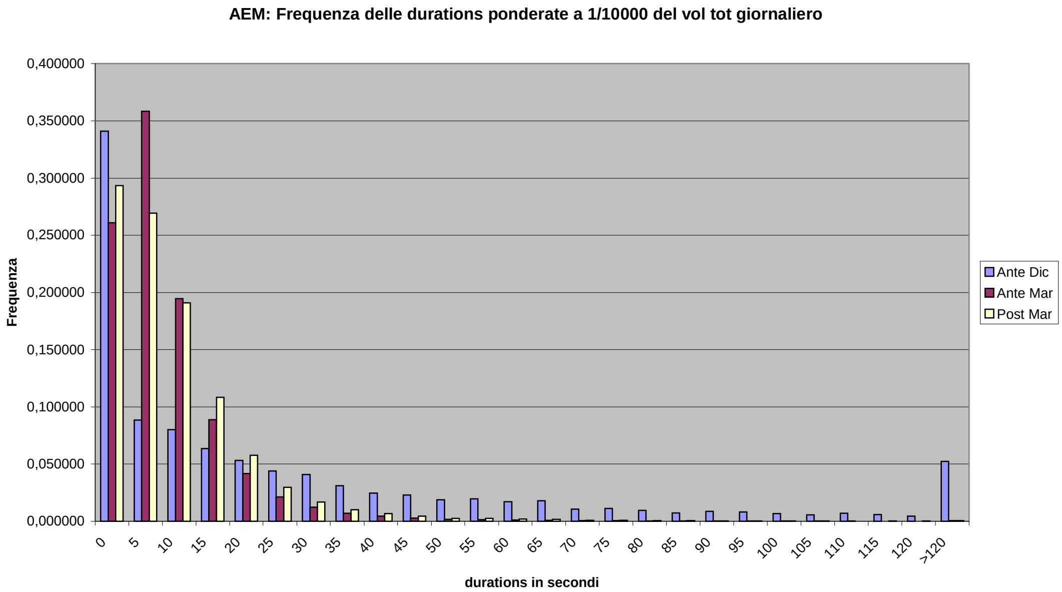
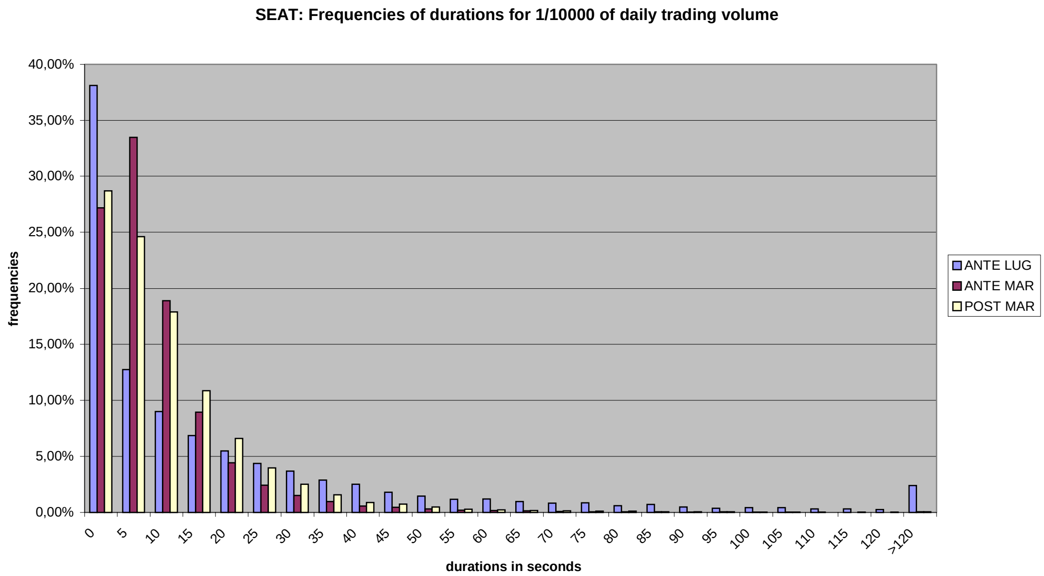

SEAT: Mid-point weighted spread by trading volumes (sell orders)
0,0045
0,004
0,0035
0,003
0,0025
0,002
0,0015
0,001
0,0005
0
500
1000
1500-2000
2500-3000
3500-5000
5500-10000
>10000
5000
2500
500

Figure 10:



AEM: Frequenza delle durations ponderate a 1/10000 del vol tot giornaliero
Frequenza
0,400000
0,350000
0,300000
0,250000
0,200000
0,150000
0,100000
0,050000
0,000000
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
>120
durations in secondi
Ante Dic
Ante Mar
Post Mar

Figure 11:



SEAT: Frequencies of durations for 1/10000 of daily trading volume
frequencies
40,00%
35,00%
30,00%
25,00%
20,00%
15,00%
10,00%
5,00%
0,00%
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
>120
durations in seconds
ANTE LUG
ANTE MAR
POST MAR

Figure 12:

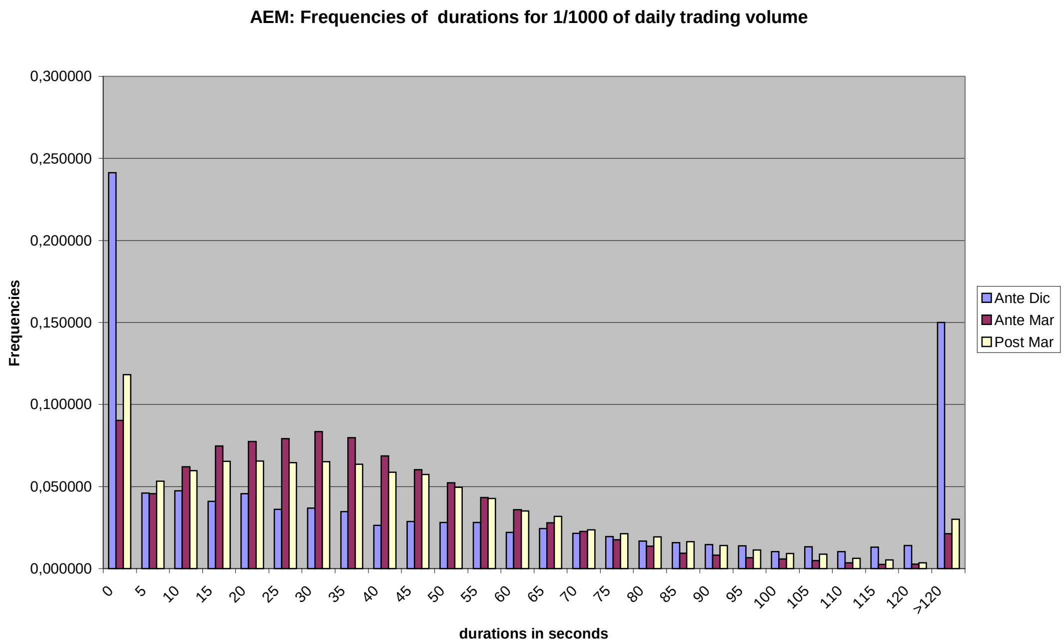
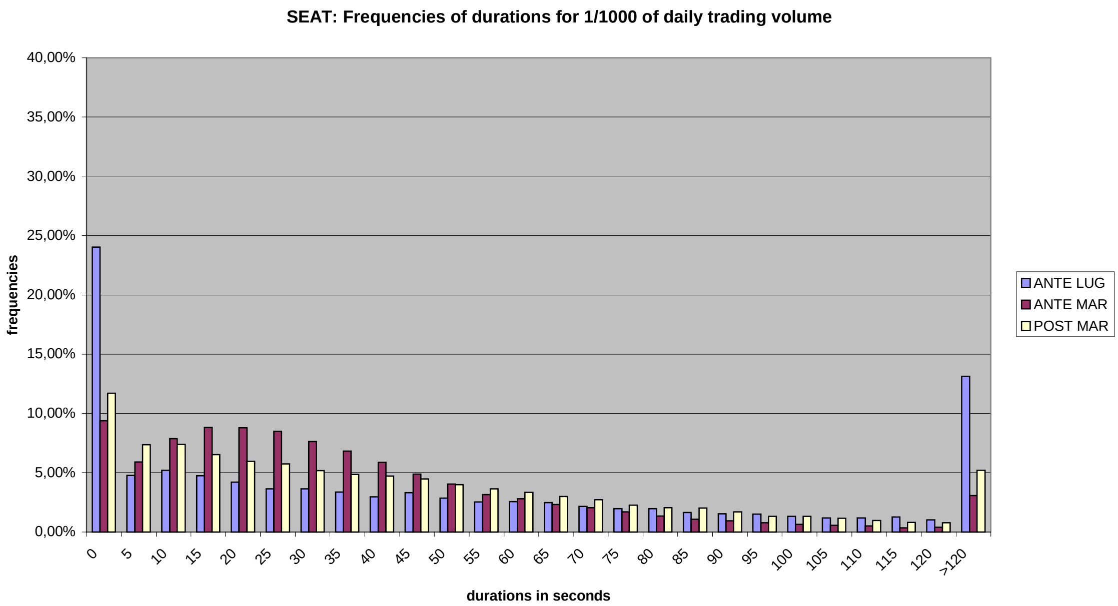

AEM: Frequencies of durations for 1/1000 of daily trading volume
Frequencies
0,300000
0,250000
0,200000
0,150000
0,100000
0,050000
0,000000
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
>120
durations in seconds
Ante Dic
Ante Mar
Post Mar

Figure 13:



SEAT: Frequencies of durations for 1/1000 of daily trading volume
40,00%
35,00%
30,00%
25,00%
20,00%
15,00%
10,00%
5,00%
0,00%
frequencies
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
>120
durations in seconds
ANTE LUG
ANTE MAR
POST MAR

Figure 14: