The Great Depression and the Friedman-Schwartz Hypothesis*

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Abstract

We evaluate the Friedman-Schwartz Hypothesis that a more accommodative monetary policy could have greatly reduced the severity of the Great Depression. To do this, we first estimate a dynamic, general equilibrium model using data from the 1920s and 1930s. Although the model includes 8 shocks, the story it tells about the Great Depression turns out to be a simple and familiar one. The contraction phase was primarily a consequence of a shock that induced a shift away from privately intermediated liabilities, such as demand deposits and liabilities that resemble equity, and towards currency. The slowness of the recovery from the Depression was due to a shock that increased the market power of workers.

We identify a monetary base rule which responds only to the money demand shocks in the model. We solve the model with this counterfactual monetary policy rule. We then simulate the dynamic response of this model to all the estimated shocks. Based on the model analysis, we conclude that if the counterfactual policy rule had been in place in the 1930s, the Great Depression would have been relatively mild.

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Contents

1 Introduction

The purpose of this paper is to contribute to the construction of dynamic general equilibrium models useful for the analysis of policy questions. We do this by developing a standard monetary business cycle model in three directions: we add several shocks, and we add a banking sector and financial frictions. We subject this model to what is perhaps the toughest possible test. We fit the model to United States data from the 1920s and 1930s and ask whether a different monetary policy might have moderated the output collapse experienced in the Great Depression.[1] Our analysis suggests that the answer is 'yes'. This is consistent with the hypothesis of Friedman and Schwartz (1961).

Apart from its intrinsic historical interest, there are two reasons we examine the US economy in the 1930s. First, there is a general consensus that the Great Depression was a consequence of several shocks, interacting with financial markets, labor markets and the banking system. As a result, this episode constitutes a natural laboratory for studying a model like ours. Second, there is widespread interest in understanding whether monetary policy authorities in a low interest rate environment have the power to resist deflation and output collapse. This is the situation confronted by the US monetary authorities in the 1930s.

The counterfactual monetary policy that we study temporarily expands the growth rate in the monetary base in the wake of money demand shocks. To ensure that this policy does not violate the zero lower-bound constraint on the interest rate, we consider policies which increase the monetary base in the periods *after* a shock.[2] By injecting an anticipated inflation effect into the interest rate, this delayed-response feature of our policy prevents the zero bound constraint from binding along the equilibrium paths that we consider. Of course, for the anticipated inflation effect to be operative, the public must believe that the central bank will in fact raise the future growth rate of the monetary base. In our analysis, we assume the central bank has this credibility.

However, we recognize that in a more general analysis which takes into account strategic considerations, credibility might have placed some constraints on the sort of policy that was feasible in the 1930s.[3] The early 1930s was in the era of the Gold Standard, a time

[1] There are numerous other quantitative analyses of the Great Depression, with perhaps the first being Lucas and Rapping (1972). In addition, there is the work of Bordo, Choudhri and Schwartz (1995), Bordo, Erceg and Evans (2000), Christiano (1999), Cole and Ohanian (1999, 2001, 2003, 2003a), McCallum (1990) and Sims (1999).

[2] An alternative way to take into account the zero lower bound on the interest rate is to drive the interest rate down to its lower bound for a period of time. This approach involves substantial computational challenges and we have not pursued it here. For a discussion of methods for solving models with occasionally binding contraints, see Christiano and Fisher (2000). In a simple example, Eggertsson and Woodford (2003) analyze policies that involve driving the nominal rate of interest to zero for a while.

[3] That the role of credibility might be crucial in determining the effectiveness of monetary policy at

when commitment to low inflation permeated the institutions of public policy (Eichengreen and Temin (2000).) There are two reasons to suppose that a policy shift towards at least temporarily higher inflation might have been credible to the public. First, the Gold Standard represented a commitment to a given price level, and so it was natural for the public to expect at least some inflation after the initial price level drop in the early 1930s. Second, as the Great Depression proceeded, a political constituency in favor of inflation began to take root (Kennedy (1999).) Much of this constituency was in the Democratic party, which was the party of the president after Roosevelt was elected in the fall of 1932. On this basis, we conjecture that a policy of temporary inflation, even one that somewhat exceeds what was implied by a price level target, would not have been incredible in the US in the 1930s.[4]

In our model we focus on monetary policy alone, in isolation from fiscal policy. Implicitly, we assume that the fiscal authorities accommodate any fiscal implications of monetary policy by a suitable adjustment in taxes.[5] We abstract from the distortionary effects of the latter, by assuming the adjustments are done with lump sum taxes. The notion that the fiscal authorities passively adjust the variables in their budget constraint to accommodate changes in circumstances is consistent with accounts of the 1930s.[6]

Although our estimated model has eight shocks, two of them turn out to be particularly important. A 'liquidity preference shock' plays an important role in the contraction phase of the Great Depression.[7] This shock drives households to accumulate currency, at the expense of demand deposits and other liabilities that are used to fund entrepreneurs who own the economy's stock of capital.[8] We incorporate the fractional reserve banking model in Chari, Christiano, and Eichenbaum (1995). The loss of bank reserves implied by the reduction of household demand deposits leads banks to cut back on working capital loans to firms. This

low nominal rates of interest has been emphasized by Krugman (1998), Eggertsson (2003), Eggertsson and Woodford (2003), Sims (1999) and others.

[4]Pursuading Fed officials in the 1930s to commit to increased future money growth in response to shocks would have been difficult. Orphanides (2003) documents that these officials were very worried about the possibility of inflation and were even prepared to undertake preemptive strikes to prevent any future outbreak. Our analysis suggests that this position of Fed officials was mistaken. This is consistent with the findings of Eggertsson and Woodford (2003).

[5]That the operating characteristics of monetary policy depend sensitively on the assumptions one makes about fiscal policy has been emphasized recently in the literature on the fiscal theory of the price level. See, for example, Leeper and Sims (1994), Sims (1994) and Woodford (1994) (Christiano and Fitzgerald (2000) provide a review).

[6]Concerns about fiscal solvency motivated Herbert Hoover to ask for the sizeable tax increase that became the Revenue Act of 1932 (Kennedy (1999, p. 79).) Roosevelt was also concerned about fiscal solvency. He pushed for one bill to reduce government spending and another to raise taxes because 'For three long years the Federal Government has been on the road toward bankruptcy.' (Kennedy (1999, p. 138). These bills were the Economy Act of March 20, 1933 and the Beer-Wine act of March 22, 1933.

[7]For a related discussion, see Field (1984), who argues that a money demand shock played an important role during this time.

[8]This is consistent with Sims (1999), who finds that disturbances which resulted in a flow out of bank deposits and into currency were important during the Great Depression.

has the effect of constraining economic activity. The liquidity preference shock has a number of other effects too, which we now briefly describe.

We adopt the agency cost model of financial frictions of Bernanke, Gertler and Gilchrist (1999) (BGG). In this model, entrepreneurs finance their ownership of the capital stock with a combination of their own net worth and loans from banks. Banks obtain the funds to make these loans by issuing liabilities to households. When households cut back on these liabilities in response to the liquidity preference shock, banks have less to lend to entrepreneurs who must therefore cut back on their purchases of capital. This leads to a reduction in investment activity.

The financial frictions exert a kind of 'accelerator effect' on investment, as the initial decline in investment leads to a reduction in entrepreneurial net worth, which further constrains capital purchases.[9] There are three channels through which a fall in investment leads to a decline in net worth. First, the reduction in investment results in a fall in the price of capital, which reduces the net worth of entrepreneurs by reducing the market value of their physical assets. Second, net worth is reduced by the 'debt deflation' effect emphasized by Irving Fisher (1933). In the model, the return received by households on the liabilities issued by banks to finance entrepreneurs is nominally non state contingent. In addition, a liquidity preference shock produces a surprise fall in the price level as spending declines. This surprise fall in the price level increases the real payoff to creditors (households) at the expense of debtors (entrepreneurs). Although there is generally a presumption in macroeconomics that reallocations of wealth of this type are neutral in the aggregate, in this model they are not. This is because the reallocation takes funds away from agents who have access to unique investment opportunities (i.e., the entrepreneurs) and gives them to people who do not (i.e., the households).[10] A third factor driving net worth down is the general slowdown in economic activity which, by reducing the rental rate of capital, leads to a reduction in entrepreneurial income.

Households' attempts to obtain more currency also leads them to cut back on consumption expenditures, which further depresses aggregate spending. The effects of the liquidity

[9] Cole and Ohanian (2001) cite evidence that business loans remained relatively strong during the contraction phase of the depression. They suggest that this evidence represents an embarassment for models of financial frictions. It is useful to note that it is not necessarily a problem for the model considered here. In the model, the fall in asset prices triggered by a liquidity preference shock is expected to be undone over time. The nature of the loan contract is that when this happens, the amount of loans an entrepreneur receives for a given level of net worth goes up. In effect, the anticipated rate of return on capital goes up after an asset price collapse, because there are anticipated capital gains. It is well known that during the first few years of the Great Depression, people in fact did think that the end of the output decline was near, and things would return to normal soon (see Kennedy (1999).)

[10] See Bernanke (1995) for further discussion. Also, in our model there are additional sources of non-neutrality which stem from the fact that the liquidity preference shock generates a rise in bankruptcies among entrepreneurs, which generate real resource costs.

preference shock on aggregate consumption and investment produce a fall in output and the price level. With one exception, the response to a liquidity preference shock bears a resemblance to the behavior of the US economy from late 1929 to 1933. The exception is that the mechanism implies a rise in the interest rate, whereas interest rates actually fell sharply during this period. However, it turns out that the rise in the interest rate in the model is small and transient, and is soon followed by a persistent fall. Although we have not confirmed this, we suspect the fall in the interest rate reflects the accelerator mechanism implied by the financial frictions. Our counterfactual experiment suggests that failure of the federal reserve to respond appropriately to this and other money demand shocks in the 1930s was the reason that the slump that began in 1929 utlimately became the Great Depression.[11]

A second shock plays an important role in the expansion phase of the Great Depression, according to our model. The puzzle in the expansion phase is that hours worked recovered only slightly in the period, 1933-1939. The estimated model's answer to this puzzle is that there was a rise in the market power of workers. This feature of the model accords well with the widespread notion that the policies of the New Deal had the effect of pushing up wages and reducing employment by giving workers greater bargaining power (Cole and Ohanian (2003).)

In fact, the above story oversimplifies somewhat the account of the Depression implicit in our estimated model. There are many details that the liquidity preference shock alone misses. The other 7 shocks fill in those details. Thirteen variables are used in the analysis, the Dow Jones Industrial average, aggregate output, aggregate employment, the aggregate real wage, a short term interest rate, the premium on Baa bonds, the monetary base, $M1$, inflation, investment, consumption, deposits, and bank reserves. We display our model's implications for all these variables. The large number of variables imposes substantial discipline on the analysis.

The outline of the paper is as follows. In the next section, we present an informal discussion of the data on the Great Depression, as a way to motivate key features of our model. Section 3 presents the formal model economy. Section 4 discusses the assignment of parameter values and model fit. Section 5 discusses the explanation of the Great Depression that is implicit in our estimated model. Section 6 discusses the counterfactual analysis. Section 7 concludes. Various technical details are relegated to appendices.

[11] To the extent that the Fed chose not to accommodate because it felt constrained by the gold standard, our analysis of the contraction phase of the Great Depression is consistent with the analysis of Eichengreen and others, who lay the blame for the Great Depression with the gold standard.

There is some debate over how binding a restriction the gold standard was for monetary policy (Bordo, Choudhri and Schwartz (1995).)

2 Key Macroeconomic Variables in the Interwar Period

We now briefly review the relevant data and literature to motivate the general design of our model. It is worth emphasizing both what is in the model, as well as what is not. In terms of the latter, the model we construct abstracts from international considerations. This choice is motivated by the arguments of Bordo, Choudhri and Schwartz (1995), Romer (1993) and others, that the US Depression can be understood in domestic terms only. Also, we are sympathetic to the arguments in Romer (1993) and Temin (1976), that exogenous monetary policy disturbances were probably not an importance impulse driving the Great Depression.[12] Our model abstracts completely from such shocks. We explain what it is about the data that leads us to leave them out. In addition, our model de-emphasizes somewhat the sticky wage mechanism by which spending shocks are often assumed to impact on real output and employment.[13] We explain what it is about the data that leads us to assign a relatively greater role to alternative mechanisms. As we review the data, we describe the impulse and propagation mechanisms that we think do warrant further consideration, and which are included in our model.

2.1 Aggregate Quantities

Our data are displayed in Figure 1. For convenience, the fourth quarter observations for 1929 and later are indicated by a '*'. Consider the real per capita Gross National Product (GNP) data, which are normalized to unity in 1929.[14] Note that these fall by one-third from 1929

[12] Sims (1999) uses methods based on vector autoregressions to conclude that monetary policy shocks were not an important source of variation in economic activity.

[13] See Bordo, Erceg and Evans (2001) for an formal representation of the notion that the Great Depression reflected the effects of contractionary shocks to monetary policy operating on the economy through a sticky wage mechanism.

[14] Nominal GNP data were taken from Balke and Gordon (1986). GNP was converted to 1929 dollars using the GNP deflator taken from the National Bureau of Economic Research's Macro History database at http://www.nber.org/databases/macrohistory/contents/. Real GNP was converted to per capita terms by a measure of population from Chari, Kehoe and McGrattan (2001), who derive it from Kendrick (1961) (the data were taken from Ellen McGrattan's website). Their measure of population is linearly interpolated to construct quarterly figures. Nominal investment includes household purchases of durable goods, investment in equipment, investment in residential and non-residential structures and change in inventories. It was taken from Balke and Gordon (1986). It was converted to per capita 1929 dollars using the GNP deflator and the measure of population. Nominal consumption is household consumption of non-durable goods and services and is taken from the Balke and Gordon (1986). It is converted to per capita 1929 dollars using the gnp deflator and the measure of population. Hours worked is the number of hours of all employees, plus the self employed, plus those involved in unpaid family work. It includes government hours, except for hours worked in the military. It is expressed as a ratio to the annual endowment of hours and taken from Chari, Kehoe and McGrattan (2001), who build on data from Kendrick (1961). The Dow Jones is taken from the NBER dataset. It is converted to real per capita terms using the gnp deflator and population. Monetary variables are taken from Friedman and Schwartz (1963) and are converted to per capita terms using our population measure. The short term interest rate is the three-month rate on Treasury securities

to 1933. The recovery is very slow and by 1939 output is only barely back to its 1929 level. To understand the composition of these fluctuations, consider the data on consumption and investment in Figure 1. Consumption includes household consumption of nondurables and services, while investment includes business investment plus household purchases of durable goods. The consumption and investment data have been divided by the level of output in 1929, to give an indication of their magnitude in relation to output. Note that the drop in consumption is relatively small, falling from about 65 percent of the 1929 level of GNP in 1929 to about 50 percent of the 1929 level of GNP in 1933. The behavior of investment is more dramatic. It drops by 80 percent, falling from twenty-five percent of the 1929 level of GNP in 1929 to 5 percent of that level in 1933. These are the observations that have motivated researchers to speculate that the key to understanding the start of the Great Depression lies in understanding the dynamics of investment (see Romer (1993) and Temin (1976)). Another way to see just how much investment fell, is to compare the fall in the investment to output ratio from 1929 to 1933 with the fall in that ratio in other recessions. Table 1 displays the investment to output ratio at NBER peaks and troughs for the 1929 recession and for 9 other post-war recessions. In recessions since the Great Depression, reductions in the investment to output ratio have been no greater than 3 percentage points. Results in Cole and Ohanian (2001, Table 1) indicate that the fall in the investment to output ratio in the 1920-1921 recession was also relatively small. By contrast, the drop in the investment to output ratio in the Great Depression, which was 19 percentage points, was an order of magnitude greater than what occurred in these other recessions.

Figure 1 also displays economy-wide per capita hours worked, normalized to unity in 1929. Employment dropped by roughly 25 percent from 1929 to 1933. Notably, hours worked in the 1930s never recovered much from its low level in 1933. By 1939 it settled to a level about 20 percent below its level in 1929. This observation has led many to conjecture that one set of factors at work in the recovery phase of the Great Depression may have been institutional and other changes that led to a permanent reduction in 'normal' hours worked per person. Our model is designed to accommodate such factors using a device that is in the spirit of the analysis of Cole and Ohanian (2003). In particular, we do so by adopting a

and it is taken from the NBER database. The interest rate spread is the difference between yields on BAA and AAA corporate bonds, obtained from the NBER database. The real wage is hourly compensation. It is constructed as follows. Total compensation from NIPA, available at annual frequency starting from 1929, is divided by our measure of hours worked and interpolated to obtain quarterly figures using the related series of average hourly earnings in manufacturing from Hanes (1996). For the period 1923-1928 we used average hourly earnings in manufacturing from Hanes (1996). The two series were spliced together. Finally, the series is converted to real terms using the gnp deflator. We did not use earnings in manufacturing from Hanes (1996) for the whole sample because wage in manufacturing experienced a much stronger rise than wage in the other sectors during the 1930s (see Cole and Ohanian, 1999). On the other hand, the measure of hourly compensation we constructed is not totally satisfactory, as self-employed and unpaid family workers are included in hours worked but are instead excluded from the NIPA measure of total compensation.

model specification which allows households' labor market power to fluctuate over time.

2.2 Exogenous Monetary Policy Shocks in the Great Depression

There is a general consensus that the initial phase of the contraction began with the slowdown in economic growth in the summer of 1929, just before the stock market crash. Many researchers follow Keynes' (1930, p. 196) assessment that that slowdown reflected the effects of high interest rates engineered by the Federal Reserve. Some argue that monetary policy shocks also played an important role in other phases of the Great Depression. In our analysis of the Great Depression we abstract from monetary policy shocks altogether. This subsection reviews our reasons for doing this.

Figure 1, panel F shows that real $M1$ was roughly constant during the initial phase of the contraction, 1929-1932 (and also more generally, from about 1926 to the end of 1932). In view of the substantial drop in output that occurred, this implies that $M1$ velocity fell. Models that have performed well with post war US data generally imply that an exogenous, contractionary monetary policy shock leads to a fall in real balances and a rise in velocity.[15] This is not to say that monetary policy was not tight at all during this period. For example, the real value of the monetary base (not shown in Figure 1) did fall, although only by a small amount.[16] Still, it seems unlikely that an exogenous contractionary shock to monetary policy was an important impulse for the Great Depression. To the extent that there was some tightness in monetary policy, it was relatively small, certainly by the historical standards of the time (see Romer (1990) for more discussion).

The spike in the interest rate in late 1931 is sometimes explained as reflecting a contractionary response by the Federal Reserve to the British decision to abandon the gold standard. Since British monetary policy was not in any obvious way related to economic developments in the US, it is natural in an analysis of the US Depression experience to treat it as an exogenous shock to US financial markets. But, should it be treated as a shock to money supply or money demand? The continued robust growth in the real, per capita monetary base is not consistent with the money supply interpretation. In 1931IV, the real monetary base stood 5 percent higher than its value in 1931III. Then, over the next two quarters the real, per capita monetary base grew at quarterly rates of 1 percent and 6 percent, respectively. This and other data suggests that the 1931 interest rate spike is better thought of as reflecting a shock to money demand. For example, during this period the currency to deposit ratio began to rise, and bank reserves in 1931IV were 2 percent below their level in 1931III. These observations are consistent with the notion that there was a shift in preferences away from

[15] See, for example, Christiano, Eichenbaum and Evans (2003).

[16] In 1929IV the real monetary base was 5 percent lower than it was in 1928I. Also, Bernanke (2002) reports that the Fed's portfolio of government securities fell in the period before the crash.

demand deposits. Clearly, this is one way to think of the bank runs occurring at this time. In addition, it is a way to think of the 'run on the dollar' thought to have been sparked by the British decision on gold.

There are two other episodes that are often interpreted as reflecting the operation of monetary policy shocks. These include the sharp increase in money growth in April to July of 1932, and a sharp decrease in the period January to March 1933. These monetary actions are thought to be responsible for the 'double dip' nature of the Great Depression. Although it is not evident in the data presented in Figure 1, data on manufacturing output show a dip in output in 1932, followed by a rise and then another dip in 1933.[17] This pattern closely follows the variations in money growth. In our analysis we interpret these as reflecting the interaction of non-monetary policy shocks with the monetary authority's monetary policy rule.

Finally, there is the switch to a faster money growth rate following the US departure from the gold standard in April of 1933 (see Figure 1, panel F). In our analysis we model monetary policy as a single regime, and so we abstract from the apparent change in regime that occurred in 1933. Incorporating a regime switch into the analysis would be a useful step, which we leave to future research. Addressing this properly requires taking a potentially controversial stand on the exact nature of the change in monetary policy, as well as on the nature of the public's perception of that change.

2.3 The Sticky Wage Mechanism

In our terminology, the sticky wage mechanism reflects two features. First, firms are assumed to always be on their labor demand schedule, which itself is not perturbed by shocks to aggregate spending. Second, wages are sticky.[18] As a result, the only way for a negative aggregate spending shock to reduce employment and output is for the shock to drive the price level down, and the real wage up.[19] The problem is that there is no evidence either in the US time series, or in a cross section of countries that the contraction phase of the Great Depression is associated with a sharp rise in the real wage (for a discussion of the cross-

[17] See, for example, Figure IV in Mills (1934).

[18] The two assumptions just described correspond to equations (1) and (2) of the model used by Bernanke and Carey (1996). They state that in using this model to analyze the Great Depression, they are following the lead of Eichengreen and Sachs (1985).

[19] Although the sticky wage mechanism is often attributed to Keynes, he himself changed his mind about the relationship between real wages and output after the famous critique by Dunlop and Tarshis. At that time, Keynes blamed Alfred Marshall for the idea that real wages and output necessarily move in opposite directions. He referred to Marshall's view as a 'dogma' which was in fact not consistent with the data. Upon examining data from 1880 and after, Keynes concluded that the negative relationship between real wages and output held only in the first six years of this period, 'the formative period in Marshall's thought in this matter....but has never once held good in the fifty years since [Marshall] crystallized it!' (Keynes (1939, p.38)).

section, see Cole and Ohanian (2003a).) These considerations lead us to emphasize model features which have been employed in analyses of post war data, which allow spending shocks to shift the labor demand schedule. In models with these features, it is not necessary for the real wage to rise when a spending contraction reduces output.[20]

Panel L in Figure 1, displays the economy-wide real wage. Note that there is no evidence that it surges above trend as output and employment begin their plunge after 1929. Indeed, this measure of the real wage was actually low in 1932 and 1933, when the economy hit bottom. Aggregate data for the manufacturing sector do provide some evidence of a rise in the real wage after 1929. For example, according to Cole and Ohanian (1999, Table 11) the manufacturing wage compiled by Hanes (1996), converted to real terms using the GNP deflator, is about 5 percent above trend during the period 1930-1932.[21] However, the work of Bresnahan and Raff (1991), Lebergott (1989) and Margo (1993) suggests that this rise may, for compositional reasons, overstate what happened to the typical manufacturing worker's real wage. In particular, Bresnahan and Raff (1991) and Lebergott (1989) report evidence that low wage jobs were terminated first, injecting an upward bias in standard industry-wide average estimates. Margo (1993, p. 44) concludes that eliminating the bias due to compositional factors like this '...would produce an aggregate decline in nominal wages between 1929 and 1932 as much as 48 percent larger than that measured by the Bureau of Labor Statistics.' If the bias were only a little more than one-tenth of Margo's estimate, then the rise in the measured manufacturing real wage turns into an outright fall.[22]

As emphasized by Bordo, Erceg and Evans (2001), there is another issue that must be addressed before one can draw definite conclusions from the behavior of manufacturing real wages. Most of the analyses cited in the previous paragraph convert the nominal manufacturing wage to a real wage using an aggregate measure of prices. However, the conventional theory pertains not to this, but to the real wage measured in terms of the price of the firm's product. So, it is important to consider the behavior of the manufacturing wage in relation to some measure of the manufacturing price level. According to Mills (1934, Table 5), the price index of manufactured goods was 91, 78, 70, 72 in the years, 1930-1933, with the price nor-

[20] Christiano, Eichenbaum and Evans (2003) present evidence based on minimal model assumptions (using vector autoregression techniques) to argue that a contractionary shock to monetary policy produces a small drop in the real wage. They also find that labor productivity falls somewhat after such a shock. They estimate a fully specified dynamic general equilibrium model, which is consistent with these features.

[21] Cole and Ohanian assume a trend growth rate of 1.9 percent per year for the manufacturing real wage.

[22] One way to make Margo's estimate concrete is as follows. According to Cole and Ohanian (1999, Table 11), the detrended real wage in 1932 stood at 105, with its 1929 value equal to 100. Margo's calculation suggests that if composition biases are eliminated from the data, then if the typical wage earner's wage in 1929 were 100, that worker earned only $0.52 \times 105 = 54.6$ in 1932.

Unlike the Bureau of Labor Statistics data that Margo is referring to, Hanes' (1996) data attempt to correct for some sources of composition bias. The exact magnitude of the composition bias is controversial, with at least one author (Dighe (1997)) claiming it is negligible.

malized at 100 in 1929. The measure of the GNP deflator used by Cole and Ohanian (1999, Table 8) is 97, 88, 78, 77 for the same years, and with the same normalization. Evidently, with the Mills estimate of manufacturing prices, the manufacturing real wage is higher by only 5-10 percent. This increase is still substantially smaller than Margo's estimate of the magnitude of the composition bias.[23]

The disaggregated manufacturing data also raise questions about the sticky wage mechanism. Under the mechanism, a drop in output is fundamentally driven by a fall in the price level, so that those sectors where price falls the most should have experienced the biggest output decline. Mills (1934, Figure V) presents evidence that conflicts with this prediction. He shows that in 1932-1933 production in durable and non-durable manufacturing was about 76 and 31 percent, respectively, below their 1929 levels. The magnitude of the associated price declines was the reverse of what the sticky wage mechanism predicts. Mills reports that the price of durables fell only about 22 percent, while the price of non-durables fell about 65 percent.

The US time series data on the recovery phase of the Great Depression also pose problem for the sticky wage mechanism. Note from Figure 1, panel L, that there is no evidence of a surge down in the real wage, as employment begins to recover in 1934-1936.

In sum, it does not look as though the shock or shocks that are responsible for the US Great Depression operated through the sticky wage mechanism. There just does not seem to be a tight negative relationship between the real wage one the one hand, and output and employment on the other. Accordingly, we adopt a model environment with the property that variables other than just employment and the real wage enter the labor demand curve. These variables include capacity utilization, variable markups, as well as the interest rate. The latter enters because our model takes into account that firms must borrow working capital to finance their variable costs. In our model, markups vary because of fluctuations in the monopoly power of firms, as well as because of frictions in price setting.

Incorporating variable capital utilization into an analysis of the Great Depression is quite natural. Bresnahan and Raff (1991) show that about 30 percent of the reduction in jobs in the automobile industry was associated with plant closing, i.e., idle plant and equipment. In his classic analysis of total factor productivity, Solow (1957) assumes that the rate of capital utilization is the same as the rate of utilization of the labor force, and so he measures capital utilization by 1 minus the unemployment rate. This, of course, falls dramatically during the Great Depression. Variations in the markup are also natural to consider in view of the many

[23]Sometimes, the wholesale price index is used to deflate the manufacturing real wage (see Bernanke (1996), Bordo, Erceg and Evans (2001).) However, as emphasized by Mills (1934, table 2), the prices of manufactured goods rose substantially relative to wholesale prices. So, deflating nominal manufacturing wages using the wholesale price index overstates the real cost of labor to manufacturing firms.

legislative and other changes in the 1930s, affecting the degree of competition among firms.[24]

It has long been recognized that there is a simple alternative way to reconcile the sticky wage mechanism with an absence of correlation between the real wage and output. That is to assume that there are exogenous shocks to labor supply, i.e., technology shocks.[25] As Bordo, Erceg and Evans (2001) note, the fact that the real wage did not surge above trend in 1930-1931 could be reconciled with the sticky wage mechanism, if we assume there was a negative shock to technology at the time. Nevertheless, we are skeptical that technology shocks played an important role during the Great Depression. Analysts have generally concluded that technological change continued in the 1930s at the same or higher rate than in the 1920s. For example, Field (2001) concludes, "In spite of tremendous losses due to underutilized labor and capital, the 1930s were, paradoxically, also an extraordinarily fertile period from the standpoint of technical change, one in which a disproportionately large number of key product and process innovations took place." (See also, Field (2003).) Other researchers reached similar conclusions.[26] For example, Solow (1957, p. 316) concludes from his total factor productivity measurements, that '...there is some evidence that technical change (broadly interpreted) may have accelerated after 1929'. Other researchers who similarly concluded that the pace of technical change continued without interruption after 1929 include Mills (1934) (see especially his Table 4) and Bliss (1934) (see especially his Table III). Despite this skepticism, we do include technology shocks into our model. We will let the data determine whether they are important.

2.4 Other Variables and Mechanisms

There are several variables that exhibit dramatic fluctuations during the Great Depression. One suspects that they point to important sources of shocks, or sources of propagation for other shocks. For example, Panel H in Figure 1 displays the Dow Jones Industrial Average (DOW), deflated by the GNP deflator. Note how dramatically the stock market fell from 1929 to 1933. The magnitude of the drop is similar to that of investment. It is therefore not surprising that several researchers have argued that financial asset markets must be

[24] For an excellent review of these, see Kennedy (1999).

[25] This approach has been advocated in Cole and Ohanian (1999, 2003).

[26] An example is Bliss (1934, p. 6): 'A period of depression is conducive to improvement in labor productivity. Faced with narrowing profit margins, businessmen strive for cheaper, more direct, more efficient methods of production. With overhead costs per unit increasing special effort is made to reduce direct costs per unit, largely by laying off the less efficient workers and by improving management.' Commenting on his estimates that the productivity of employed labor exhibited strong growth in the period 1929-1932, Bliss (1934, p. 7) remarks, 'Taken together, these estimates indicate an increase in output per man-hour of approximately 25 percent per in four years, an amazing advance indeed.' Mills attributed some, but not all, the advance in productivity to temporary factors.

part of any explanation of the Great Depression.[27] We integrate financial markets into our environment using the modeling strategy of Bernanke, Gertler and Gilchrist (1999).

Friedman and Schwartz (1963) draw attention to the very large movements in the currency to deposit ratio and the reserves to deposit ratio (see Panels I and J, respectively). As discussed above, to some extent the movements in these variables may be a reaction to the British decision to abandon the gold standard. However, they are probably primarily related to the bank runs that occurred in waves starting in late 1930. We follow Friedman and Schwartz (1963) and others in pursuing the idea that these movements have much to do with the dynamics of the Great Depression. We do not pursue a 'deep' theory of bank runs in this paper. Instead, we model the movements in the currency to deposit ratio and in the reserves to deposit ratios as reflecting various types of money demand shocks.

Discussions of risk play a role in analyses of the Great Depression (see, for example, Romer (1990)). According to evidence in Harrison and Weder (2003), concerns about risk were probably not an important factor at the very beginning, in late 1929 or early 1930. For example, the premium on Baa over Aaa corporate bonds does not exhibit a sharp rise until the first wave of bank runs, in late 1930 (see Figure 1, panel K). However, the premium is so large in the mid-1930s by comparison to what it was throughout the 1920s, that it may have played a role in prolonging the Great Depression, perhaps by keeping investment low.[28] Our way of introducing financial asset markets allows us to consider the impact of increased risk on investment.

The dramatic fall in employment, combined with the small fall in the real wage, raises a question concerning the household: Is it consistent with household optimization and clearing in the labor market for the real wage to drop hardly at all, while employment fell a great deal? Why did employment plummet in the initial phases of the Great Depression, even though the real wage continued on its trend set in the 1920s? Of course, this is a classic puzzle that confronts equilibrium analyses of business cycles generally. Hall (1996) gave the puzzle a quantitative expression, by showing that an equilibrium approach to business fluctuations leads to the implication that households are hit by a shock that resembles an increased preference for leisure in a recession (see also Ingram, Kocherlakota and Savin (1997)). Applying the approach of Hall, who works with post war data, Chari, Kehoe and McGrattan (2003) and Mulligan (2002) find that the early phase of the Great Depression is also characterized by a shift up in the preference for leisure. This shift has various interpretations.

[27] See, for example, Bernanke (2002), Cecchetti (1998), Fisher (1933), Mishkin (1978), Romer (1993) and Temin (1976).

[28] A related possibility is raised by the analysis in Bernanke (1983). This suggests that the rise in the risk premium and the fall in investment may both have been a consequence of the damage done to the banking system by the banking panics. See Gertler (2001) for a more extensive discussion of the premium in Baa over Aaa bonds. He expresses skepticism that the premium reflects risk alone.

For example, it can be interpreted as reflecting an increase in the labor market power of households, which leads them to restrict the supply of their labor services. Following Gali, Gertler and Lopez-Salido (2002), it can also reflect that households are, in effect, 'off their labor supply schedules', because of the presence of sticky wages. These considerations lead us to incorporate household labor market power and frictions in wage setting into our model.

An unusual feature of the Great Depression is that, in the estimation of Chari, Kehoe and McGrattan (2003) and Mulligan (2002), the preference for leisure does not fall again in the recovery phase, as Hall shows it does in post war recessions. This is a manifestation of the fact that employment rose so little (see Panel D, Figure 1) in the recovery phase, even though the real wage was quite high. The standard explanation is that this reflects the effects of New Deal programs designed to prop up the real wage.[29] Following the spirit of approach of Cole and Ohanian, we will attempt to capture this by adopting a model specification which allows the degree of labor market power of households to vary over time.

The preceding considerations suggest to us that a model which captures the key forces in play during the Great Depression must have several features:

- it must capture the determinants of investment behavior. To be interesting, these determinants should include a possible role for changes in the value of financial assets and in perceptions of risk.
- the real side of the economy should incorporate disturbances to the labor demand schedule, including time varying markups, capacity utilization and other variables.
- the model should include frictions in the setting of wages, and should allow for fluctuations in the monopoly power of households.
- the model should incorporate banking system that is sufficiently rich that one can consider the interactions between real economic activity and various monetary aggregates such as currency, bank reserves, and demand deposits.

A model that integrates all these features in a coherent way would constitute a credible laboratory for assessing whether an improved monetary policy might have ameliorated the Great Depression. In this paper we take a step towards constructing such a model by combining the models of Christiano, Eichenbaum and Evans (2003), BGG and Chari, Christiano and Eichenbaum (1995).

[29] See Bordo, Erceg and Evans (2000), Cole and Ohanian (2003), Temin (1989, 1990) and Weinstein (1980).

3 The Model Economy

In this section we describe our model economy and display the optimization problems solved by intermediate and final good firms, entrepreneurs, producers of physical capital, banks and households. Final output is produced using the usual Dixit-Stiglitz aggregator of intermediate inputs. Intermediate inputs are produced by monopolists who set prices using a variant of the approach described in Calvo (1983). These firms hire the services of capital and labor, and we assume that a fraction of these costs ('working capital') must be financed in advance through banks. Labor services are an aggregate of specialized services, each of which is supplied by a monopolist household. Households set wages subject to the type of frictions modeled in Calvo (1983).[30]

Capital services are supplied by entrepreneurs who own the physical stock of capital and choose how intensely to utilize it. Our model of the entrepreneurs follows BGG. In particular, the entrepreneurs have their own financial wealth, but not enough to finance the full amount of capital they purchase. A part of their purchase must be financed with loans. Lending to entrepreneurs involves agency costs, because the capital purchased by entrepreneurs is subject to an idiosyncratic productivity shock. The only way the lender can observe this shock is by expending valuable resources in monitoring, so that it is efficient to adopt a lending contract which economizes on monitoring costs. We assume the borrower receives a standard debt contract, which specifies a loan amount and an amount to be repaid in the event that the borrower is solvent. A borrower who cannot repay is said to be bankrupt, and turns over everything to the lender, after being monitored. The rate of return paid by solvent entrepreneurs must be high enough to cover the cost of funds to the bank, as well as monitoring costs net of whatever the bank can salvage from bankrupt entrepreneurs. The excess of this rate of return over the cost of funds to the bank is the external finance premium. Being an endogenous variable, the magnitude of the premium is a function of all the shocks in the model. Among these, one that plays a notable role is the variance of the idiosyncratic shock to entrepreneurial productivity. We assume that this is the realization of a stochastic process. The bank obtains the funds that it lends to entrepreneurs by issuing time deposit contracts to households. Because these contracts pay a nominal non state contingent return, the model can at least qualitatively account for the debt deflation phenomenon emphasized by Fisher (1933), as discussed in the introduction.

As in most models of credit market frictions, there is a tendency for entrepreneurs to 'grow' away from the financial constraint by accumulating enough wealth. To ensure that the credit market restrictions remain binding, we follow BGG in assuming that a randomly

[30] This aspect of the model follows Christiano, Eichenbaum and Evans (2003), who in turn build on Erceg, Henderson and Levin (2000).

selected fraction of entrepreneurial financial wealth is destroyed exogenously each period. The fraction is itself subject to stochastic fluctuations. A jump in the rate of destruction of entrepreneurial financial wealth resembles in some respects the bursting of a stock market bubble.

The frictions in the entrepreneurial sector have the effect of amplifying the output effects of certain types of shocks. This is the 'accelerator effect' emphasized by BGG. To see this, it is useful to understand the evolution of entrepreneurial net worth. In a given period, net worth is equal to what it was in the previous period, plus earnings from renting capital, plus the current market value of the stock of capital, minus obligations to banks arising from past loans. As a result, a shock which reduces the rental rate of capital, or its market value, produces a fall in investment by reducing entrepreneurial net worth. Similarly, as noted above, a shock which reduces the aggregate price level reduces net worth by raising the real value of entrepreneurial debt payments. Thus, the financial frictions amplify the effects of shocks which reduce output and either the rental rate of capital or the aggregate price level.

In addition to amplifying the effects of certain shocks originating outside the entrepreneurial sector, the model also posits new shocks that originate within the sector itself, and which can be useful for understanding macroeconomic dynamics. A 'bursting stock market bubble' has a depressive effect on investment, because the destruction of entrepreneurial wealth inhibits the ability of entrepreneurs to buy capital. Also, an increase in the riskiness of entrepreneurs leads to a fall in investment because the rise in interest payments to banks cuts into their net worth.

The actual production of physical capital is carried out by capital producing firms, which combine old capital and investment goods to produce new, installed, capital. The production of new capital involves adjustment costs, so that the price of capital increases with the amount of capital sold. There are no financing problems or agency costs in the capital producing sector. Inputs in this sector are financed out of contemporaneous receipts from production.

All financial intermediation activities occur in a representative, competitive 'bank'. The bank's activities involve two separable components. One pertains to the financing to entrepreneurs, and is financed with time deposit liabilities issued to households. The other, which follows Chari, Christiano and Eichenbaum (1995), pertains to the provision of working capital loans to intermediate good producers. These loans are financed with demand deposits issued to households. Total demand deposits are composed of these household demand deposits, plus firm demand deposits created as a by-product of making working capital loans. Banks are required to set aside a fraction of total demand deposit liabilities in the form of reserves. Associated with demand deposits there is a flow of transactions services. The bank produces these using a neoclassical production function involving labor, capital and reserves

in excess of what is required. The presence of excess reserves in the production function is meant to capture, in a reduced form way, the liquidity needs that arise in a banking system where banks clear demand deposit checks among each other. The bank's assets and liabilities match in their maturity structure and are risk free.

We now discuss the different sectors of the model in more detail.

3.1 Firm Sector

A final good, Y_t, is produced by a perfectly competitive, representative firm. It does so by combining a continuum of intermediate goods, indexed by $j \in [0,1]$, using the technology

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_{f,t}}} dj\right]^{\lambda_{f,t}}. \tag{1}$$

Here, $1 \leq \lambda_{f,t} < \infty$, and Y_{jt} denotes the time t input of intermediate good j. Let P_t and P_{jt} denote the time t price of the consumption good and intermediate good j, respectively. The firm chooses Y_{jt} and Y_t to maximize profits, taking prices as given. The parameter, $\lambda_{f,t}$, is a realization of a stochastic process, to be discussed below. Intermediate good firms have more market power, the higher is $\lambda_{f,t}$.

The j^{th} intermediate good is produced by a monopolist who sets its price, P_{jt}, subject to Calvo-style frictions that will be described shortly. The intermediate good producer is required to satisfy whatever demand materializes at its posted price. Given quantity demanded, the intermediate good producer chooses inputs to minimize costs. The production function of the j^{th} intermediate good firm is:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt}\right)^{1-\alpha} - \Phi z_t & \text{if } \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt}\right)^{1-\alpha} > \Phi z_t \\ 0, & \text{otherwise} \end{cases}, \; 0 < \alpha < 1, \tag{2}$$

where Φ is a fixed cost and K_{jt} and l_{jt} denote the services of capital and labor. The variable, z_t, is the trend growth rate in technology, with

$$z_t = \mu_z z_{t-1}.$$

The variable, ϵ_t, is a stationary shock to technology. The time series representation of ϵ_t is discussed below.

Intermediate good firms are competitive in factor markets, where they confront a rental rate, Pr_t^k, on capital services and a wage rate, W_t, on labor services. Each of these is expressed in units of money. Also, each firm must finance a fraction, ψ_k, of its capital services expenses in advance. Similarly, it must finance a fraction, ψ_l, of its labor services in advance. The interest rate it faces for this type of working capital loans is R_t.

We adopt the variant of Calvo pricing proposed in Christiano, Eichenbaum and Evans (2003). In each period, t, a fraction of intermediate good firms, $1 - \xi_p$, can reoptimize its

price. The complementary fraction must set its price equal to what it was in period $t-1$, scaled up by the inflation rate from $t-2$ to $t-1$, π_{t-1}.

3.2 Capital Producers

There is a large, fixed, number of identical capital producers. They are competitive and take prices as given. They are owned by households, who receive any profits or losses in the form of lump-sum transfers. Capital producers purchase previously installed capital, x, and investment goods, I_t, and combine these to produce new installed capital. Investment goods are purchased in the goods market, at price P_t. The time t price of previously installed capital is denoted $Q_{\bar{K}',t}$. New capital, x', is produced using the following technology:

$$x' = x + F(I_t, I_{t-1}).$$

The presence of lagged investment reflects that there are costs to changing the flow of investment. Since the marginal rate of transformation from previously installed capital into new capital is unity, the price of new capital is also $Q_{\bar{K}',t}$. The firm's time t profits are:

$$\Pi_t^k = Q_{\bar{K}',t}\left[x + F(I_t, I_{t-1})\right] - Q_{\bar{K}',t}x - P_t I_t.$$

The capital producer's problem is dynamic because of the adjustment costs. It solves:

$$\max_{\{I_{t+j}, x_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \Pi_{t+j}^k \right\},$$

where E_t is the expectation conditional on the time t information set, which includes all time t shocks.

Let $\bar{K}_{t+j}$ denote the beginning-of-period $t+j$ physical stock of capital in the economy and let δ denote its rate of depreciation. From the capital producer's problem it is evident that any value of x_{t+j} whatsoever is profit maximizing. Thus, setting $x_{t+j} = (1-\delta)\bar{K}_{t+j}$ is consistent with profit maximization and market clearing. The stock of capital evolves as follows

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + F(I_t, I_{t-1}).$$

3.3 Entrepreneurs

The details are presented in BGG, so our discussion of the entrepreneurs can be brief. Because of linearity assumptions, aggregate decisions can be represented as functions of aggregates only. This greatly simplifies the computational analysis. In addition, we exploit this property of the model in the following presentation. At the end of period t, the state of an entrepreneur is summarized by its net worth, N_{t+1} (see Figure 2). The net worth, in

combination with a bank loan, is used to purchase the period t stock of installed capital, $\bar{K}_{t+1}$. After the purchase, each entrepreneur draws an idiosyncratic shock which changes $\bar{K}_{t+1}$ to $\omega\bar{K}_{t+1}$. Here, ω is a unit mean, log-normal random variable distributed independently over time and across entrepreneurs. The standard deviation of $\log(\omega)$ at date t, σ_t, is itself a stochastic process. Although the realization of ω is not known at the time the entrepreneur makes its capital decision, the value of σ_t is known. The properties of this random variable are described below. In a slight abuse of previous notation, we write the distribution function of ω as F_t :

$$\Pr\left[\omega \leq x\right] = F_t(x).$$

After observing the period $t+1$ aggregate shocks, the entrepreneur decides on the period $t+1$ level of capital utilization, u_{t+1}, and then rents out capital services, $K_{t+1} = u_{t+1}\bar{K}_{t+1}$. High capital utilization gives rise to high costs in terms of goods, according to the following convex function:

$$a(u_{t+1})\omega\bar{K}_{t+1},\ a', a'' > 0.$$

The entrepreneur chooses u_{t+1} to solve:

$$\max_{u_{t+1}} \left[u_{t+1}r^k_{t+1} - a(u_{t+1})\right]\omega\bar{K}_{t+1}P_{t+1}.$$

For an entrepreneur who receives idiosyncratic productivity shock, ω, the rate of return on capital purchased in period t is:

$$\begin{aligned} 1 + R^{k,\omega}_{t+1} &= \left\{\frac{\left[u_{t+1}r^k_{t+1} - a(u_{t+1})\right] + (1-\delta)q_{t+1}}{q_t}\frac{P_{t+1}}{P_t}\right\}\omega \\ &= (1 + R^k_{t+1})\omega, \end{aligned}$$

say, where q_t is Tobin's q :

$$q_t = \frac{Q_{\bar{K}',t}}{P_t}.$$

Here, R^k_{t+1} is the average rate of return on capital across all entrepreneurs.

We suppose that $N_{t+1} < Q_{\bar{K}',t}\bar{K}_{t+1}$, where $Q_{\bar{K}',t}\bar{K}_{t+1}$ is the cost of the capital purchased by entrepreneurs with net worth, N_{t+1}. Since the entrepreneur does not have enough net worth to pay for its capital, he must borrow the rest:

$$B_{t+1} = Q_{\bar{K}',t}\bar{K}_{t+1} - N_{t+1} \geq 0. \tag{3}$$

We suppose that the entrepreneur receives a standard debt contract from the bank. This specifies a loan amount, B_{t+1}, and a gross rate of interest, Z_{t+1}, to be paid if ω is high enough. Entrepreneurs who draw ω below a cutoff level, $\bar{\omega}_{t+1}$, cannot pay this interest rate and must give everything they have to the bank. The cutoff is defined as follows:

$$\bar{\omega}_{t+1}\left(1 + R^k_{t+1}\right)Q_{\bar{K}',t}\bar{K}_{t+1} = Z_{t+1}B_{t+1}. \tag{4}$$

The bank finances its period t loans to entrepreneurs, B_{t+1}, by borrowing from households. We assume the bank pays households a rate of return, R^e_{t+1}, that is not contingent upon the realization of $t+1$ shocks. In the usual way, the parameters of the entrepreneur's debt contract are chosen to maximize entrepreneurial utility, subject to zero profits in each state of nature for the bank and to the requirement that R^e_{t+1} be uncontingent upon period $t+1$ shocks.[31] This implies that Z_{t+1} and $\bar{\omega}_{t+1}$ are both functions of period $t+1$ aggregate shocks. A feature of the loan contract is that

$$\frac{Q_{\bar{K}',t}\bar{K}_{t+1}}{N_{t+1}}$$

is independent of the entrepreneur's net worth. Aggregation in the model is trivial because of the fact that borrowing and capital purchases are proportional to an entrepreneur's level of net worth.

The law of motion for aggregate N_{t+1} is

$$\begin{aligned} N_{t+1} &= \gamma_t\{\left(1+R^k_t\right)Q_{\bar{K}',t-1}\bar{K}_t \\ &\quad - \left[1+R^e_t+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R^k_t\right)Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t-\bar{N}_t}\right](Q_{\bar{K}',t-1}\bar{K}_t-N_t)\} + W^e_t. \end{aligned} \tag{5}$$

Here, γ_t reflects that at the end of the period, after the entrepreneur has sold his capital, paid off his debt and earned rental income, he exits the economy with probability $1-\gamma_t$. At the same time a fraction, $1-\gamma_t$, of entrepreneurs enter. The fraction, γ_t, who survive and the fraction, $1-\gamma_t$, who enter both receive a transfer, W^e_t. Without this transfer, entering entrepreneurs would have no net worth, and so they would not be able to buy any capital, ever. Also, among the γ_t entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they too would never again be able to buy capital.

The first term in braces in (5) represents the revenues from selling capital, plus the rental income of capital, net of the costs of utilization, averaged across all entrepreneurs. The object in square brackets is the average gross rate of return paid by all entrepreneurs on period $t-1$ loans, $(Q_{\bar{K}',t-1}\bar{K}_t-\bar{N}_t)$. This aggregates over payments received from entrepreneurs who are

[31] Given our setup of the model, the restriction that R^e_{t+1} is not a function of time $t+1$ shocks is likely to be binding. We are indebted to V. V. Chari for having pointed out to us that in a world with full competition in contracts, risk neutral entrepreneurs would in effect shoulder some of households' consumption risk. In such a world, households' rate of interest, R^e_{t+1}, would covary positively with the marginal utility of consumption. As in all other aspects of the model of entrepreneurs, however, we follow BGG in assuming R^e_{t+1} is state independent. In a private communication, Mark Gertler has conjectured that if R^e_{t+1} were permitted to covary positively with the marginal utility of consumption, then the accelerator effect associated with net worth constraints emphasized by BGG would be amplified. This is because in states of the world when output is low, R^e_{t+1} would be high, requiring a high payment from non-brankrupt entrepreneurs. This higher payment would reduce net worth, forcing a cut back in entrepreneurial capital expenditures and, hence, in investment and output.

bankrupt, as well as those who are not. The $(1-\gamma_t)$ entrepreneurs who are selected for death, consume:

$$P_t C_t^e = \Theta(1-\gamma_t)V_t,$$

where V_t is their net worth. In practice, we set $\Theta = 0$.

Following BGG, we define the 'external finance premium' as the term involving μ in square brackets in (5). It is the difference between the 'internal cost of funds', $1+R_t^e$, and the expected cost of borrowing to an entrepreneur. The reason for calling $1+R_t^e$ the internal cost of funds is that in principle one could imagine the entrepreneur using its net worth to acquire time deposits, instead of physical capital (the model does not formally allow this). In this sense, the opportunity cost of the entrepreneur's own funds is $1+R_t^e$.

3.4 Banks

We assume that there is a continuum of identical, competitive banks. Banks issue deposit liabilities, D_t^h, to households. Part of this is set aside in the form of reserves, and the other is used to finance working capital loans. Working capital loans are extended to firms in the form of demand deposits, D_t^f. The management of total deposit liabilities, D_t, requires capital services, K_t^b, labor, l_t^b, and excess reserves, E_t^r according to the following technology:

$$\frac{D_t}{P_t} \leq x^b \left(\left(K_t^b\right)^\alpha \left(z_t l_t^b\right)^{1-\alpha} \right)^{\xi_t} \left(\frac{E_t^r}{P_t}\right)^{1-\xi_t}, \tag{6}$$

where

$$D_t = D_t^h + D_t^f.$$

In (6), $0<\alpha<1$ and x^b is a constant. In addition, $\xi_t \in (0,1)$ is a shock to the relative value of excess reserves, E_t^r. The stochastic process governing this shock will be discussed later. As noted above, we include excess reserves as an input to the production of demand deposit services as a reduced form way to capture the precautionary motive of a bank concerned about the possibility of unexpected withdrawals.

Demand deposits pay interest, R_{at}. We denote the interest rate on working capital loans, net of interest on the associated demand deposits, by R_t. Since firms receive interest on the deposit associated with their loan, the gross interest payment on loans is $R_t + R_{at}$. The maturity of period t working capital loans and the associated demand deposit liabilities coincide. A period t working capital loan is extended prior to production in period t, and pays off after production in period t. The household deposits funds into the bank before production in period t and liquidates the deposit after production occurs.

Turning to time deposits, we assume the bank faces no costs for maintaining this type of liability. The maturity structure of time deposits coincides with that of the standard debt

contract offered to entrepreneurs. Thus, time deposits and entrepreneurial loans are created at the end of a given period's goods market. This is when newly constructed capital is sold by capital producers to entrepreneurs. Time deposits and entrepreneurial loans pay off near the end of next period's goods market, when the entrepreneurs sell their undepreciated capital to capital producers (who use it as raw material in the production of new capital). The timing of the payoff on the entrepreneurial loan coincides with the timing of the payoff on time deposits. The maturity structure of the two types of bank liabilities can be seen in Figure 3.

The entrepreneur/time-deposit and the working capital/demand-deposit sides of the firm can be considered separately. The former were considered in the previous subsection. The profit maximization problem arising from the latter is:

$$\max_{A_t, S_t^w, K_t^b, l_t^b} \{R_t S_t^w - R_{at} D_t^h - R_t^b F_t - \left[(1+\psi_k R_t) P_t r_t^k K_t^b\right] - \left[(1+\psi_l R_t) W_t l_t^b\right]\}$$

subject to (6), where

$$E_t^r = D_t^h + F_t - \tau\left(D_t^h + S_t^w\right).$$

Here, K_t^b and l_t^b denote capital and labor services hired by the banking sector. Note our assumption that a fraction of these must be paid in advance with working capital. Also, S_t^w represents working capital loans, so that $S_t^w = D_t^f$. The term, F_t, denotes loans of reserves between banks. Since banks are all identical, equilibrium requires $F_t = 0$. Still, the presence of F_t allows us to define the interest rate on interbank loans, R_t^b.

The clearing condition in the market for working capital loans is:

$$S_t^w = \psi_l W_t l_t + \psi_k P_t r_t^k K_t, \tag{7}$$

where l_t and K_t denote economy-wide aggregate labor and capital services. Here, S_t^w represents the supply of loans, and the terms on the right of the equality in (7) represent total demand.

3.5 Households

There is a continuum of households, each indexed by $j \in (0,1)$. Households consume, decide how to allocate their wealth between demand deposits, currency and time deposits, and supply a specialized labor input, h_{jt}. Since the household is a monopoly supplier of its labor service, it can set its wage rate. We assume that it faces Calvo (1983)-type frictions in the setting of this wage. Since this uncertainty is idiosyncratic in nature, different households work different amounts and earn different wage rates. So, in principle they are also heterogeneous with respect to consumption and asset holdings. A straightforward extension of arguments in Erceg, Henderson and Levin (2000) establishes that the existence of state

contingent securities ensures that in equilibrium households are homogeneous with respect to consumption and asset holdings. We assume, though we do not discuss, the presence of these securities. Given these considerations, our notation assumes that households are homogeneous with respect to consumption and asset holdings, and heterogeneous with respect to the wage rate that they earn and hours worked.

The preferences of the j^{th} household are given by:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left\{ \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} \frac{\psi_L}{2} h_{j,t+l}^2 - \upsilon_t \frac{\left[\left(\frac{P_{t+l} C_{t+l}}{M_{t+l}} \right)^{\theta_{t+l}} \left(\frac{P_{t+l} C_{t+l}}{D_{t+l}^h} \right)^{1-\theta_{t+l}} \right]^{1-\sigma_q}}{1-\sigma_q} \right\}, \tag{8}$$

where C_t denotes time t consumption; h_{jt} denotes time t supply of a specialized labor service; υ_t is a unit-mean liquidity preference shock; and ζ_t is a shock with mean unity to the preference for leisure. This shock is isomorphic to a shock to the household's degree of monopoly power in the supply of h_{jt}. When $b > 0$, (8) allows for habit formation in consumption.[32] The term in square brackets captures the notion that currency, M_t, and demand deposits, D_t^h, contribute to utility by facilitating transactions. It is because υ_t affects the magnitude of these non-pecuniary liquidity services that we refer to it as a liquidity preference shock.

We now discuss the household's period t sources and uses of funds. At the beginning of the period, the household is in possession of the economy's stock of high-powered money, M_t^b, which it splits into currency, M_t, and deposits with the bank, A_t, subject to the following liquidity constraint:

$$M_t^b \geq M_t + A_t. \tag{9}$$

The central bank credits the household's bank deposit, D_t^h, with X_t units of high powered money, so that

$$D_t^h = A_t + X_t. \tag{10}$$

As already mentioned, the household receives interest, R_{at}, on these deposits. Additional sources of funds include profits from producers of capital, from banks, from intermediate good firms and the net payoff on the state contingent securities mentioned above. Households also receive lump-sum transfers corresponding to the net worth of the entrepreneurs which exit the economy in the current period. Finally, the households pay a lump-sum tax to finance transfer payments to surviving entrepreneurs and to the newly born entrepreneurs.

The household can use its funds to purchase consumption goods, $P_t C_t$, or accumulate high powered money, M_{t+1}^b In addition, it can use its funds to acquire time deposits, T_t.

[32] Various authors, such as Fuhrer (2000), McCallum and Nelson (1998), and Christiano, Eichenbaum and Evans (2003), have argued that $b > 0$ is important for understanding the monetary transmission mechanism in the postwar period. In addition, habit formation is useful for understanding other aspects of the economy, including the size of the premium on equity (see, for example, Boldrin, Christiano and Fisher (2001)).

These pay a rate of return, R^e_{t+1}, at the end of the period $t+1$ goods market. The rate of return, R^e_{t+1}, is known in period t.

These observations are summarized in the following asset accumulation equation:

$$\begin{aligned} & (1+R_{at})\, D^h_t + (1+R^e_t)\, T_{t-1} + \left(1-\tau^l\right) W_{j,t} h_{j,t} + M_t + Lump_t \\ \geq \;& M^b_{t+1} + T_t + (1+\tau^c)\, P_t C_t, \end{aligned} \tag{11}$$

where $Lump_t$ summarizes the lump sum transfers. The household's problem is to maximize (8) subject to (9), (10) and (11).

We now turn to the way the j^{th} household sets its period t wage, $W_{j,t}$. We follow closely the setup in Erceg, Henderson and Levin (2000). In period t, the household can with probability, $1-\xi_w$, choose its wage rate optimally. It turns out that each household that sets its wage optimally, sets it to the same value, which we denote, $\tilde{W}_t$. With probability ξ_w, the household must follow a rule of thumb by setting its time t wage to what it was in the previous period, scaled up by $\pi_{t-1}\mu_z$. That is, for these households, $W_{j,t} = \pi_{t-1}\mu_z W_{j,t-1}$. The household must in each period satisfy its demand curve:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} l^s_t, \tag{12}$$

where W_t is the aggregate wage index, which turns out to be:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\pi_{t-1}\mu_z W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}. \tag{13}$$

Also, l^s in (12) represents employment services, which are related to the differentiated labor services of households according to the following technology:

$$l^s = \left[\int_0^1 (h_j)^{\frac{1}{\lambda_w}}\, dj\right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty.$$

The household takes the aggregate wage and employment index as given. The household that reoptimizes its wage, $\tilde{W}_t$, does so by optimizing (8) subject to (11), and the various frictions discussed above. In the linear approximation of our models solution, λ_w and the preference parameter, ζ, are observationally equivalent. From here on, we treat ζ as the realization of a stochastic process and refer to it as a measure of household labor market power. For further details on the sticky wage part of our model, see Christiano, Eichenbaum and Evans (2003) or Erceg, Henderson and Levin (2000).

3.6 Final Goods Market Clearing

Here, we develop the aggregate resource constraint, relating the quantity of final goods produced to the quantity of aggregate labor and capital services, as well as to the distribution

of production among intermediate goods firms and to the distribution of employment among households. Our approach follows Yun (1996). In particular,

$$Y_t = (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z_t^{1-\alpha} \epsilon_t \left(\nu_t K_t\right)^{\alpha} \left(\nu_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} l_t \right)^{1-\alpha} - z_t \Phi \right], \; w_t^* = \frac{W_t^*}{W_t}, \; p_t^* = \frac{P_t^*}{P_t}.$$

Here, K_t and l_t are the unweighted integral of all labor and capital in the economy:

$$K_t = \int_0^1 K_{jt} dj, \; l_t = \int_0^1 h_{jt} dj.$$

The endogenous variable, ν_t, represents the fraction of aggregate labor and capital services used in the goods producing sector. The objects, W_t^* and W_t, represent different weighted integrals of W_{jt} over all j, and similarly for P_t^* and P_t. The scalars, w_t^* and p_t^*, capture the loss of final output that occurs when resources are not evenly distributed across sectors, as efficiency requires. There is no efficiency loss when all wages and intermediate good prices are equal, so that $p_t^* = w_t^* = 1$. The price and wage frictions that we assume imply that $p_t^* = w_t^* = 1$ only holds in a nonstochastic steady state. The reasoning in Yun (1996) can be used to show that in the type of linear approximation about steady that we study here, we can set $p_t^* = w_t^* = 1$.We do this from here on.

To complete our discussion, final goods are allocated to monitoring for banks, utilization costs of capital, government consumption, household consumption and investment:

$$\begin{aligned} & \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \left(1 + R_t^k\right) Q_{\bar{K}',t-1} \bar{K}_t + a(u_t) \bar{K}_t + G_t + C_t + I_t \qquad (14) \\ \leq \; & \left[z_t^{1-\alpha} \epsilon_t \left(\nu_t K_t\right)^{\alpha} \left(\nu_t l_t\right)^{1-\alpha} - z_t \phi \right], \end{aligned}$$

Here, government consumption is modeled as follows:

$$G_t = z_t g,$$

where g is a constant.

3.7 Exogenous Shocks

There are eight exogenous shocks in the model. These are the monopoly power parameter, $\lambda_{f,t}$, corresponding to intermediate good firms; the parameter controlling bank demand for excess reserves, ξ_t; the parameter controlling household preferences for currency versus demand deposits, θ_t; the monopoly power parameter for household labor supply, ζ_t; the parameter governing household preference for liquidity, υ_t; the productivity shock to intermediate good firms, ϵ_t; and the shock to the riskiness of entrepreneurs, σ_t; and the parameter governing the survival probability of entrepreneurs, γ_t.

Three of our variables, γ_t, ξ_t and θ_t, must lie inside the unit interval. Let y_t denote one of these variables. We ensure that y_t lies inside the unit interval by assuming that it is generated by a stochastic process, x_t, via the following transformation:

$$y_t = \frac{1}{1+\exp(-x_t)}. \tag{15}$$

Note that $x_t \in (-\infty, \infty)$ maps y_t into the unit interval. If we let dx_t denote a small perturbation of x_t about its nonstochastic steady state value, and let $\hat{y}_t = dy_t/y$, where y is the nonstochastic steady state of y_t, then

$$dy_t = y(1-y)dx_t. \tag{16}$$

For the case when y_t is γ_t, ξ_t or θ_t, we model dx_t as having first order autoregressive, moving average (ARMA(1,1)) representations. We also model $\hat{\lambda}_{f,t}$, $\hat{\upsilon}_t$, $\hat{\zeta}_t$ and $\hat{\epsilon}_t$ as having ARMA(1,1) representations. Consider, for example, $\hat{\lambda}_{f,t}$. The joint evolution of this variable and its monetary response, $x_{f,t}$, are given by:

$$\begin{pmatrix} \hat{\lambda}_{f,t} \\ \hat{\varphi}_{ft} \\ x_{f,t} \end{pmatrix} = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & \theta_f^1 & \theta_f^2 \end{bmatrix} \begin{pmatrix} \hat{\lambda}_{f,t-1} \\ \hat{\varphi}_{f,t-1} \\ x_{f,t-1} \end{pmatrix} + \begin{pmatrix} \hat{\varphi}_{ft} \\ \hat{\varphi}_{ft} \\ \theta_f^0 \hat{\varphi}_{ft} \end{pmatrix}.$$

Because at time t, $\hat{\sigma}_{t-1}$, enters the model (see (5)), and because of the nature of the computational methods we use to solve the model, we find it convenient to handle $\hat{\sigma}_t$ somewhat differently. In particular,

$$\begin{pmatrix} \hat{\sigma}_t \\ \hat{\sigma}_{t-1} \\ x_{\sigma,t} \end{pmatrix} = \begin{bmatrix} \rho_\sigma & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \theta_\sigma^2 \end{bmatrix} \begin{pmatrix} \hat{\sigma}_{t-1} \\ \hat{\sigma}_{t-2} \\ x_{\sigma,t-1} \end{pmatrix} + \begin{pmatrix} \hat{\varphi}_{\sigma,t} \\ 0 \\ \theta_\sigma^0 \hat{\varphi}_{\sigma,t} \end{pmatrix}.$$

We stack all our random variables into the 24 by 1 vector, Ψ_t, which evolves as follows:

$$\Psi_t = \rho \Psi_{t-1} + D\varphi_t, \tag{17}$$

where ρ is 24 by 24 and D is 24 by 8.

3.8 Monetary Policy

The law of motion for the monetary base is:

$$M_{t+1}^b = M_t^b(1+x_t),$$

where x_t is the net growth rate of the monetary base. Let $\hat{x}_t$ denote the percent deviation of x_t from its mean value of x, so that $\hat{x}_t = (x_t - x)/x$. Monetary policy is a feedback rule from the innovations, φ_t, in the shocks to the economy to $\hat{x}_t$:

$$\hat{x}_t = \sum_{i=0}^{p} x_{it},$$

where x_{it} is the component of money growth reflecting the i^{th} element in φ_t and:

$$x_{it} = \theta_i^2 x_{i,t-1} + \theta_i^0 \varphi_{i,t}, \tag{18}$$

for $i = 0, 1, ..., p$, with $\theta_0^0 \equiv 1$. In our analysis, $p = 8$ and we suppose that for $i = 0$, $x_{0,t}$, represents an exogenous component to monetary policy. Although we set this to zero in the estimation of our model, monetary policy shocks will nevertheless be useful for interpreting the results of our counterfactual analysis.

It is useful to spell out in more detail the response of the monetary base to a shock. Iterating on the law of motion for the monetary base:

$$\log\left(\frac{M_{t+1}^b}{M_1^b}\right) = \sum_{j=1}^{t} \log(1 + x_j) \approx \sum_{j=1}^{t} x_j = x \sum_{j=1}^{t} (\hat{x}_j + 1).$$

Suppose there is a perturbation in the i^{th} economic shock only, so that $\hat{x}_j = x_{i,j}$ and:

$$\log\left(\frac{M_{t+1}^b}{M_1^b}\right) = x \sum_{j=1}^{t} (x_{i,j} + 1).$$

Let $\bar{M}_{t+1}^b$ denote the value of the monetary base in the event that there is no shock, with $M_1^b = \bar{M}_1^b$. Straightforward algebra implies:

$$\log\left(\frac{M_{t+1}^b}{\bar{M}_{t+1}^b}\right) = x \sum_{j=1}^{t} x_{i,j}.$$

Making use of (18), we conclude that if there is a one-standard deviation perturbation, σ_i, in $\varphi_{i,1}$ in period 1, the impact on the period $t+1$ money stock is:

$$\log\left(\frac{M_{t+1}^b}{\bar{M}_{t+1}^b}\right) = \frac{1 - \left(\theta_i^2\right)^t}{1 - \theta_i^2} x\theta_i^0 \sigma_i = \begin{cases} \frac{x\theta_i^0}{1-\theta_i^2}\sigma_i & t \to \infty \\ x\theta_i^0 \sigma_i & t = 1 \end{cases}.$$

3.9 Equilibrium and Model Solution

We adopt a standard sequence of markets equilibrium concept, and we use the method in Christiano (2003), described in the appendix, to develop a linear approximation to the equilibrium quantities and prices. The solution is a set of matrices, A, B and a core set of 23 endogenous variables contained in the vector, $\tilde{z}_t$, satisfying

$$\tilde{z}_t = A\tilde{z}_{t-1} + B\Psi_t. \tag{19}$$

Here, A is 23 by 23 and B is 23 by 24 for $i = 1, 2$. The vector, $\tilde{z}_t$, is defined in the appendix. Each element in $\tilde{z}_t$ is expressed as a percent deviation from a steady state value, so that, in

nonstochastic steady state, $\tilde{z}_t = 0$. From the variables in $\tilde{z}_t$ and the various equilibrium relationships in the model, it is possible to compute any desired equilibrium variable. Suppose these are contained in the vector, X_t. After linearization, let the relationship of X_t to $\tilde{z}_t$ and Ψ_t be expressed as follows:

$$X_t = \alpha + \tau z_t + \bar{\tau} z_{t-1} + \tau^s \Psi_t, \tag{20}$$

where α, τ, $\bar{\tau}$, and τ^s are functions of the model parameter values and its steady state. The set of variables of interest in our analysis is:

$$X_t = \begin{pmatrix} \log\left(\frac{N_{t+1}}{P_t Y_t}\right) \\ \log(\pi_t) \\ \log(l_t) \\ R_t^b \\ \Delta \log(Y_t) \\ \log\left(\frac{W_t}{P_t Y_t}\right) \\ \log(\frac{I_t}{Y_t}) \\ \log(V_t^1) \\ \log(\frac{C_t}{Y_t}) \\ P_t^e \\ \log(d_t^c) \\ \log(V_t^b) \\ \log(d_t^r) \end{pmatrix}. \tag{21}$$

Here, V_t^1 and V_t^b are the time t velocity of $M1$ and the monetary base, respectively. Also, d_t^c and d_t^r represent the currency to demand deposit ratio and the bank reserves to demand deposit ratio, respectively. Finally, P_t^e is the external finance premium.

4 Model Estimation and Fit

Our aim in choosing parameter values is to produce a quantitative model that resembles key aspects of the US economy in the 1920s and 1930s. This is what we need in order to have a credible laboratory for evaluating the consequences of alternative, counterfactual, monetary policies. The model parameters are divided into two sets: (i) those that govern the evolution of the exogenous shocks and the monetary response to them, and (ii) the rest. We begin with the latter. We then estimate the stochastic parameters in (i) by a maximum likelihood procedure. Ideally, we would have estimated all parameters simultaneously. However, computational challenges make this strategy less than straightforward, and we leave such an exercise for future work. After discussing the model parameter estimates, we discuss the quality of model fit.

4.1 Parameters of Nonstochastic Part of Model

The non-stochastic model parameters are listed in Table 2, and various properties of the model's steady state are reported in Tables 3-4. In many cases, the corresponding sample averages for both US data from the 1920s and for the post war period are also reported. Our procedure for computing the steady state is discussed in the Appendix.

To compute the steady state, we found it convenient to proceed by specifying some of the economically endogenous variables to be exogenous. In particular, we set the steady state ratio of currency to monetary base, m, the steady state rental rate of capital, r^k, the steady state share of capital and labor in goods production, ν, and the steady share of government consumption of goods, G/Y. These were set to $m = 0.70$, $r^k = 0.043$, $\nu = 0.99$, $G/Y = 0.07$, respectively. The currency to base ratio is a little high relative to the data from the 1920s. The value of r^k may also be a little high because the model implies a value of the capital output ratio that is a little low, compared with our data on the 1920s (see Table 3). To make these four variables exogenous for purposes of computing the steady state required making four model parameters endogenous. For this purpose, we chose ψ_L, x^b, ξ and g.

The model parameters in Table 2 are organized by sector. Turning to the household sector, values for β, λ_w, ξ_w, σ_L, and b were taken from Altig, et. al. (2003). The parameters, θ, υ and σ_q were set exogenously. Regarding the goods-producing sector, all but one of the parameters were taken from Altig, et. al. (2003). The exception is ψ_k, which does not appear in Altig, et. al. (2003). We set this exogenously.

The Calvo price and wage stickiness parameters, ξ_p and ξ_w, imply that the amount of time between reoptimization for prices and wages is 1/2 year and 1 year, respectively. As noted in Altig, et. al. (2003), these values are consistent with recent survey evidence on price frictions.

Our selection of parameter values for the entrepreneurial sector is based on the calibration discussion in BGG. Following them, we assume that the idiosyncratic shock to entrepreneurs, ω, has a log-normal distribution. We impose on our calibration that the number of bankruptcies corresponds roughly to the number reported by BGG for the post world war II period. In particular, we set $F(\bar{\omega})$ is 0.008, so that on average 0.8 percent of firms fail to meet their debt obligations in a given quarter. To understand how we were able to specify $F(\bar{\omega})$ exogenously, recall that the log-normal distribution has two parameters - the mean and variance of $\log \omega$. We set the mean of ω to unity. We are left with one degree of freedom, the variance of $\log \omega$. Conditional on the other parameters of the model, this can be set to ensure the exogenously set value of $F(\bar{\omega})$. The value of this variance is reported in Table 2. Finally, as noted above, the two parameters of the banking sector are an output of the steady state calculations.

The steady state implications of the model can be compared with the corresponding

empirical quantities in Tables 3 - 5. There are five things worth emphasizing about Table 3. First, as noted above, the capital output ratio in the model is a little low. Corresponding to this, the investment to output ratio is low, and the consumption to output ratio is high. Second, note that $N/(\bar{K}-N)$ is roughly unity. This corresponds well with the data if we follow BGG in identifying N with equity and $N-\bar{K}$ with debt. This requires some interpretation, since there is no equity market in the model. Instead, our entrepreneurs resemble small, wholy owned proprietorships such as gasoline stations and restaurants. From this perspective, we treat the DOW simply as an indirect indicator of the value of physical assets. Third, the relative size of the banking sector in the model, which is quite small, conforms roughly with the size of the actual banking sector in the 1920s. Fourth, although we have not obtained data on the fraction of GDP used up in bankruptcy costs, we suspect that the relatively low number of 0.84 percent is not be far from the mark. Finally, note that while inflation in the model is low by postwar standards, it is somewhat higher than what it was in the 1920s.

Table 4 reports the consolidated asset and liability accounts for the banks in the model. Several things are worth noting here. First, in the model most demand deposits are created in the process of extending working capital loans. These deposits are what we call 'firm demand deposits', and they are roughly 7 times as large as the quantity of demand deposits created when households deposits their financial assets with banks (i.e., 'household demand deposits'). We are not aware of data that would allow us to evaluate this implication of the model. Second, the results in the table indicate that the amount of bank reserves in the model matches reasonably well with the corresponding quantity in the data. For example, in the model excess reserves are 2.2 percent of the sum of bank reserves and working capital loans. Our estimate is that the analogous number for the US economy in the 1920s is only slightly larger, around 3.4 percent.

Table 5 reports various monetary and interest rate statistics. The left set of columns shows that the basic orders of magnitude are on track: base velocity and $M1$ velocity in the model and the data match up reasonably well with the data. The ratio of currency to demand deposits is also reasonable. However, the fraction of currency in the monetary base is high, as noted above. The interest rate implications of the model accord reasonably well with the data. However, the interbank loan rate is a little high.

Taken together, these results suggest to us that the steady state implications of the model correspond reasonably well with the data.

4.2 Parameters of Exogenous Stochastic Processes

We estimate the stochastic parameters of the model using quarterly observations covering the period 1923I-1939IV on the data in (21). We adopt a standard state-observer setup in

supposing that the measured data corresponds to X_t plus an error that is independently distributed over time and across variables. We follow convention in referring to this error as 'measurement error', although we actually think of it as some combination of literal measurement error and model specification error. We then estimate the unknown parameters using a standard maximum likelihood procedure. This part of our estimation procedure focuses only on fluctuations. We abstract from means at both the parameter estimation and model diagnostic stages. At the estimation stage we do so by subtracting the sample means from the data and setting the model mean to zero. When we simulate the model to determine how well it matches the data, we impose that the mean in the model coincides with the mean in the data.

4.2.1 Methodology

For convenience, we describe our system using the notation in Hamilton (1994, chapter 13). Let the state vector, ξ_t, be:

$$\xi_t = \begin{pmatrix} \tilde{z}_t \\ \tilde{z}_{t-1} \\ \Psi_t \end{pmatrix}.$$

Then, the state equation, which summarizes (17) and (19), is

$$\begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{z}_t \\ \Psi_{t+1} \end{pmatrix} = \begin{bmatrix} A & 0 & B\rho \\ I & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{z}_{t-1} \\ \Psi_t \end{pmatrix} + \begin{pmatrix} B_1 D \\ 0 \\ D \end{pmatrix} \varphi_{t+1},$$

or, in obvious, compact notation:

$$\xi_{t+1} = F\xi_t + v_{t+1}. \tag{22}$$

Here

$$v_{t+1} = \begin{pmatrix} B_1 D \\ 0 \\ D \end{pmatrix} \varphi_{t+1}. \tag{23}$$

The observation equation is

$$y_t = H\xi_t + w_t, \tag{24}$$

where w_t is a vector of measurement errors and

$$H = \begin{bmatrix} \tau & \bar{\tau} & \hat{\tau}^s \end{bmatrix}.$$

Note from (20) that $H\xi_t = X_t$, apart from the constant vector, α. We interpret the 13 variables on which we have observations as y_t in (24).

To complete the description of the state space system, we must also specify the variance covariance matrix of v_t and of the measurement error, w_t. We suppose that both these objects

are *iid* and diagonal. In addition, we suppose that w_t is orthogonal to y_t and ξ_t at all leads and lags.[33] The variance covariance matrix of w_t is R. The variance covariance matrix of v_t has some structure in our setting:

$$Ev_tv_t' = E\begin{pmatrix} B_1D\hat{\varphi}_{t+1} \\ 0 \\ D\hat{\varphi}_{t+1} \end{pmatrix}\begin{pmatrix} \hat{\varphi}_{t+1}'D'B_1' & 0 & \hat{\varphi}_{t+1}'D' \end{pmatrix} = \begin{bmatrix} B_1DV_\varphi D'B_1' & 0 & B_1DV_\varphi D' \\ 0 & 0 & 0 \\ DV_\varphi D'B_1' & 0 & DV_\varphi D' \end{bmatrix}.$$

Our system is completely characterized by (F, H, R, V_φ). For our purposes here, the free parameters are the diagonal elements of V_φ and R, as well as the parameters of the time series representations of the exogenous shocks, which are contained in ρ and D. There are four parameters per shock: an autoregressive and variance term for the time series representation of the shock, as well as two parameters governing the monetary policy response to each shock (we model the shocks and monetary responses as first order autoregressions). In addition, for each of the 13 variables in y_t there is one measurement error variance. Since there are 8 shocks, there is a total of 37 free parameters for the estimation. Denote these by the vector, Υ. Given the values assigned to the other parameters of the model, (F, H, R, V_φ) can be constructed once values are assigned to Υ. We choose these values to maximize the Gaussian density function, as discussed in Hamilton (1994, section 13.4). In this analysis we ignore the levels of variables by removing the sample mean from the data and replacing the constant term in (20) by a vector of zeros (this was done implicitly in the definition of y_t in (24).

4.2.2 Results

Tables 6 and 7 report our estimation results. Table 6 displays the estimates of the time series representations of the exogenous shocks, and the associated monetary policy responses. Table 7 presents the estimated standard deviation of the measurement errors, w_t. We now discuss these in turn.

Exogenous Shocks and Monetary Policy Response

There are three things worth noting in Table 6. First, in several cases, the autoregressive root of a shock was driven very nearly to unity (numbers in the table are rounded). This is how the model captures the substantial, trend-like, persistence in the data. Second, to understand the magnitude of the innovation standard deviations, it is useful to express them in terms of their impact on the level of the shock itself.[34] Thus, Table 6 displays the steady

[33] These assumptions make it hard to interpret the shock, w_t, literally as measurement error. For data such as aggregate output it seems likely that statistical agencies apply some optimal smoothing before release, and this has the consequence that measurement error in the published data are correlated with true values.

[34] Suppose the shock of interest is y_t. In the case of shocks with hats, a one-standard deviation innovation represents a perturbation to $\hat{y}_t = (y_t - y)/y$, which translates into a $\sigma \times y$ shock to y_t. So, for a variable with a hat, Table 6 displays $(y, y + \sigma y)$.For a variable with a d a one standard deviation innovation represents a $y(1-y)\sigma$ shock to y_t. So, for a variable with a d, Table 6 displays $(y, y(1-y)\sigma)$.

state value of each shock, as well as its position after a positive one standard deviation innovation. For example, the steady state value of the firm markup, λ_f, is 1.20, and its value after a one standard deviation innovation is 1.21. In almost all cases, the displacement is relatively small. An exception is the shock to γ_t where a one standard deviation positive innovation drives it a little above its upper limit of unity. (The fact that it exceeds its upper bound reflects a break down in the accuracy of (16) as an approximation to (15).) Overall, we were frankly somewhat surprised that these innovations seem to be of reasonable magnitude, despite the enormous variation in the data of the 1930s. Third, the response of monetary policy to the various shocks is relatively weak. The strongest response appears to be to the technology shock. And, as we show below, this response is not very strong either.

Because there is a large literature focused on the estimation and analysis of aggregate technology shocks, it is of interest to discuss the parameters pertaining to the technology shock, ϵ_t, in greater detail. For comparability with other studies, it is useful to consider the impact of ϵ_t on GNP in a steady state, holding fixed the utilization of capital and labor resources, and holding fixed the amount of resources absorbed by bankruptcy. After scaling by z_t, GNP is:

$$Y_t^z = \epsilon_t F(u_t, \bar{k}_t, \nu_t^k, l_t) - \phi - d_t - a(u_t)\frac{1}{\mu_z}\bar{k}_t,$$

where d_t is the resources used up in bankruptcy (see the Appendix for an explicit representation of d_t). Here, the function F is the production function of the typical intermediate good producer, (2), after scaling. The fact that aggregate output is a function of aggregate quantities alone reflects that in a steady state, the nature of our price and wage updating rules leads to symmetric behavior across all households and intermediate good firms. Totally differentiating the above expression with respect to Y_t^z and ϵ_t and holding all other variables fixed at their steady state values, we obtain $Y^z\hat{Y}_t^z = \hat{\epsilon}_t F$, where $\hat{Y}_t^z \equiv dY_t^z/Y^z$ and $\hat{\epsilon}_t = d\epsilon_t$ (recall, $\epsilon = 1$). We obtain a simple expression for F/Y^z by noting first, that $Y^z = F - \phi - d$. Second, our assumption that intermediate good profits are zero in steady state implies that firm revenues equal firm costs, so that $F - \phi = F/\lambda_f$, where λ_f is the reciprocal of real marginal cost (i.e., it is the markup). Combining the previous two expressions we obtain $Y^z = F/\lambda_f - d$, so that:

$$\hat{Y}_t^z = \frac{F}{Y^z}\hat{\epsilon}_t = \frac{\lambda_f\left(Y^z + d\right)}{Y^z}\hat{\epsilon}_t \approx \lambda_f\hat{\epsilon}_t,$$

since d/Y^z is small (see Table 3). So, a one percent change in ϵ_t results, approximately, in a 1.2 percent change in gross national product (recall, from Table 2, $\lambda_f = 1.2$). According to Table 6, an innovation in ϵ_t has standard deviation, 0.0031, which translates into an innovation in GNP of 0.0037. This is about one-half of Prescott's (1986) estimate of 0.00763 for the standard deviation of the innovation to the aggregate technology shock. That Prescott's

estimate should be larger than ours is perhaps not surprising, since he attributes the entire Solow residual to exogenous technology, whereas our Solow residual includes sources of endogenous variation, in part because of the presence of variable capital utilization in our model. Our estimate of the autocorrelation of the technology resembles Prescott's in that both are high.

Turning to the monetary response to shocks, note from Table 6 that this is quite small in most cases. We briefly discuss the policy response to an innovation in technology, because that has been the subject of a literature. Table 6 indicates that a one standard deviation innovation in ϵ_t produces an immediate increase of 0.13 percent in the monetary base. Thereafter, the monetary base continues to rise until eventually it is roughly 0.96 percent higher than it would have been in the absence of a shock. That monetary policy accommodates aggregate technology is consistent with the findings for the postwar period in Altig, et. al. (2003) and Gali, Lopez-Sadilo and Valles (2003). However, our estimated response is very weak. In particular, the dynamic response of aggregate variables such as output, consumption, employment, capital utilization and investment are roughly the same, whether monetary policy responds or not. Either way, all these variables respond positively, in a hump-shaped pattern to a shock. In the absence of monetary accommodation, the response is slightly weaker, and in the period of the shock the response of capital utilization and employment are actually slightly negative.

Measurement Error

To understand the estimates in Table 7, it is useful to also consider the results in Figure 4. The thirteen panels in that figure display the raw data, y_t, $t = 1, ..., T$, where $t = 1$ corresponds to 1924QI and $t = T$ corresponds to 1939QIV. The relatively smooth line in the figures displays the projection of X_t, for each t, on the entire data set, $\Omega = y_1, ..., y_T$. To compute this projection, $E(\xi_t|\Omega)$, we used the two-sided Kalman smoothing algorithm (see, e.g., Hamilton (1994).) We then formed $E(X_t|\ \Omega) = H \times E(\xi_t|\Omega)$. Prior to graphing the model simulated data, we adjusted the data so that the sample mean in the simulated data coincide with the sample mean of the actual data. This is consistent with our overall strategy for estimating the stochastic part of the model, which ignores the difference between the mean in the data and in the model.

In addition to the projection interpretation, there is another interpretation $E(X_t|\ \Omega)$ that is more convenient for our purposes. To see this, note first that one can obtain an estimate of the economic shocks, $E(\varphi_t|\Omega)$, by projecting both sides of (22) onto Ω :

$$E(v_t|\Omega) = E(\xi_t|\Omega) - F \times E(\xi_{t-1}|\Omega).$$

and making use of (23).[35] The smooth data in Figure 4 can also be interpreted as the dynamic response of our model economy to the time series of estimated shocks, $E(\varphi_t|\Omega)$, $t = 1, ..., T$. This is the interpretation that we adopt in the remainder of this paper.

The vertical distance between the line depicting the raw data and the model simulation represents an estimate of the measurement error, w_t, in the state-observer system in section 4.2.1. The size of the measurement error varies substantially across variables. For example, in three cases - $\log\left(N_{t+1}/\left(P_tY_t\right)\right)$, $\log(l_t)$, $R_t^b - w_t$ is so small that the two lines in the graph essentially coincide. According to Table 7, the maximum likelihood estimate of the standard deviation of the measurement error in these variables is 0.017, 0.00014 and 0.0000034, respectively. (The actual standard errors of the vertical distances in Figure 4 are displayed in parentheses in Table 7, and these correspond roughly to the maximum likelihood estimates.) At the other extreme lie the variables from the national income and product accounts: the growth rate of GNP, the investment to output ratio and the consumption to output ratio. Measurement error is estimated to account for 73, 15 and 46 percent of the variance in these variables, respectively. To the extent that we interpret these errors as model specification error, they may at first seem large enough to be a source of concern about the quality of model fit. We discuss model fit in the next subsection.

4.3 General Observations About Model Fit

To assess model fit, we converted the variables that appear as ratios and growth rates in Figure 4 into levels. The results are shown in Figure 5, where the variables in Figure 4 that were not transformed are reproduced for convenience. Note how the basic simulation results resemble the actual data remarkably closely. The failures of the model captured by the large measurement errors in Table 7 appear to be concentrated in the high frequency components of the data.

There are some weaknesses in the model. For example, consumption does not fall enough. In addition, the fall in output and investment are also not quite large enough. Interestingly, the model's real wage exhibits excessive growth during the contraction phase of the Great Depression. Although the degree of wage rigidity in the model is by some measures quite modest, this evidence suggests the model might perform better with somewhat less rigidity.

[35] Note from (23) that there is considerable structure on v_t. The Kalman filter algorithm produces $E(v_t|\Omega)$ that respects this structure. Thus, although there are many ways to recover $E(\varphi_t|\Omega)$ from $E(v_t|\Omega)$, they all produce the same result.

5 The US Great Depression From the Perspective of the Model

This section studies the estimated economic shocks and considers their role in the dynamics of the Great Depression. The time series of the 8 shocks are displayed in Figure 6. In discussing these shocks, it is useful to organize them into three groups: real shocks, financial shocks, and monetary shocks.

The real shocks include the technology shock, and the variables that control firm and household market power, $\lambda_{f,t}$ and ζ_t. Of these, only ζ_t plays an important role. Its persistent rise helps the model account for the weakness in employment during the recovery phase of the Great Depression. Our analysis is consistent with the conclusion of Prescott (1999) and Cole and Ohanian (1999, 2003) that the failure of hours worked to rise in the late 1930s reflected changes in the institutions of labor markets which had the effect of increasing the market power of workers. At the same time, ζ_t seems to have had little to do with contraction phase of the Great Depression. This is true also for our two financial market shocks, σ_t and γ_t.

Finally, we consider three monetary shocks: bank demand for reserves, ξ_t, household money demand, θ_t, and the household liquidity preference shock, υ_t. The last shock stands out above all our other shocks in terms of its role in accounting for the contraction phase of the Great Depression. We provide a detailed analysis of the transmission of this shock.

A crude summary of our model's account of the Great Depression is as follows. Much of the contraction phase is due to a rise in the liquidity preference shock, υ_t. A notable feature of the expansion phase is the absence of a substantial recovery in employment. According to the model, this is due to a persistent increase in the labor market power of households. We find both of these implications reasonable, and they build confidence that this is a useful model for analysis of counterfactual monetary policy.

5.1 Real shocks

Our analysis indicates that the role of fluctuations in technology in the Great Depression is very small. The technology shock, ϵ_t, drops only about 1 percent from 1929 to 1933. The third row of graphs in Figure 7 displays the dynamic behavior of output, employment, the price level and investment when the only shock is our estimated technology shock. The figures show that the technology shock contributes almost nothing to variation about trend in output, the price level and investment.[36]

Consider firm market power next. Our estimated $\lambda_{f,t}$ is high before 1929 and then drops

[36] The analysis of TFP in a previous section suggests that if TFP had been included in the data used for estimation, the role assigned to technology might have been greater.

roughly four percent to a lower level, where it is also more volatile (see Figure 6). As discussed in section 4.3, the firm wedge drops about 10 percent from its position in late 1929 to its position in late 1930. The results here suggest that a little less than half the drop is due to the fall in λ_{ft}. The gross nominal rate of interest contributes only about one-half of one percent to the fall in the firm wedge. So, a little over half that fall reflects the interaction of sticky prices with other shocks, and the effects of fixed costs in our production function.

In results not shown here, we found that our estimated $\lambda_{f,t}$'s play only a minor role in the dynamics of the Great Depression. Although a drop in this variable helps somewhat towards explaining the drop in the price level, the implied increase in competition has the counterfactual implication that output and employment are strong. We presume this is the reason our econometric procedure chose to assign a small role to $\lambda_{f,t}$.

According to Figure 6, our estimate of households' labor market power, ζ_t, increased sharply by 50-60 percent in the period, 1929 to 1933. After this, the estimated ζ_t stabilizes. It is a challenge to find an economically interesting interpretation of the rise in ζ_t from 1929 to 1933. Recall that we have described two interpretations of ζ_t. Under one, ζ_t is a measure of the market power of workers. But, to interpret the low level of employment in the early 1930s, for the given real wage, as reflecting increased market power of workers seems implausible. For example, Goldin (2000, Figure 9) shows that union membership remained stable from the mid 1920s until well into the 1930s. As noted above, ζ_t can alternatively be interpreted as reflecting an increased aversion in utility for work. At best, this interpretation needs greater elaboration than is provided in our model. Although the rise in ζ_t during the period 1929 to 1933 is hard to interpret, the fact that it is high in the late 1930s is easy to interpret. This is the time when the New Deal legislation was passed to give greater bargaining power to workers (Kennedy (1999)).

It is easy to verify that the 60 percent rise in ζ_t and the 4 percent drop in $\lambda_{f,t}$ can roughly account for the persistent 20 percent drop in employment in a model like ours. For example, consider the version of our model with no fixed costs, money, variable capital utilization, investment adjustment costs, working capital, agency costs, growth or banking system. The resource constraint and utility function for this model are:

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t &= k_t^{\alpha} l_t^{1-\alpha}, \\ u(c,l) &= \log(c - bc_{-1}) - \zeta l^2/2. \end{aligned}$$

It is easy to verify that, in steady state:

$$\begin{aligned} \frac{d\log l}{d\log \zeta} &= -\frac{1}{2} \\ \frac{d\log l}{d\log \lambda_f} &= \frac{\lambda_f}{2\alpha\delta}\left[\frac{1}{\beta} - 1 + \delta\right] \approx 2.3. \end{aligned}$$

The second relationship uses $\beta = 1.03^{-.25}$, $\lambda_f = 1.2$, $\alpha = 1/3$, $\delta = 0.10/4$. Evidently, a one percent change in λ_f has a four times larger impact on employment than a one percent change in ζ. Still, the magnitude of the change in ζ is so much greater that it dominates. In particular, the above expression suggests that the 60 percent rise in ζ alone drives down employment by 30 percent, while the 4 percent fall in λ_f stimulates employment by about 8 percent. The net effect, 22 percent, corresponds well with the observation that employment in 1939 was about 20 percent lower than it was in 1929.

Figure 7 displays the behavior of our model in response to the ζ_t shock. The first row of figures displays the dynamic simulation of output, employment, investment and the price level in response to our estimated time series of ζ_t. (The dotted line reports the model's simulation in response to all shocks, for convenience.) Note that ζ_t is the only shock that can account for the fact that employment is substantially lower in 1939 than in 1929 (see the solid line). According to the figures, the ζ_t shock is not particularly useful for explaining the contraction phase of the Great Depression. For example, it fails to explain the fall in the price level. Somewhat surprisingly, it also fails to account for the fall in investment. Analysis of the model's impulse response function (not displayed) reveals that investment rises in response to a positive shock to ζ_t. The rise lasts for a number of periods, after which it eventually falls as a steady state analysis suggests. We found that this initial increase in investment is in part a function of the high estimated autocorrelation of ζ_t. When we replace the estimated autocorrelation of nearly unity with 0.90, then investment falls immediately in response to a positive shock to ζ_t. We suspect that the explanation for the transient rise in investment has to do with our entrepreneurial sector. However, this is something that is still under investigation. For our present purposes, the message is that the ζ_t shock is not an important shock for understanding the contraction phase of the Great Depression.

5.2 Financial Market Shocks

We have two financial market shocks. The first, γ_t, measures variations in the rate of destruction of financial wealth. The second, σ_t, measures the riskiness of entrepreneurs. Figure 7 displays the dynamic response of output, employment, investment and the price level to these two shocks.

Consider the shock, γ_t. In results not displayed, we found that this shock helps the model account well for the fluctuations in the value of the stock market. It also has a noticeable effect on the risk premium and, according to row 2 of Figure 7, on investment. Still according to the results in Figure 7 this shock plays essentially no role explaining the fluctuations in output and employment. We suspect that the small role accorded to this shock by the estimation strategy reflects that a drop in γ_t has a tendency to generate counterfactually strong consumption. This is a consequence of the fact that in our model, when an entrepreneur

dies their net worth is transferred to households, who proceed to consume a large part of it. Indeed, we suspect that this shock is responsible for the counterfactual jump in consumption evident in Figure 5. A possible improvement to the model that would allow γ_t to play a greater role is if some fraction of the entrepreneur's physical stock of capital is destroyed when the entrepreneur dies.

Turning to σ_t, we found that this shock has a substantial impact on the risk premium and on investment. According to the fourth row of Figure 7, this shock also has a quantitatively noticeable impact on output and employment. However, the model-simulated movements in these variables are not well correlated with the corresponding US data. Finally, a difficulty that the σ_t shock shares with γ_t is that by reducing the flow of resources into investment, it stimulates consumption.[37] Presumably, this is why it did not have a larger role assigned to it.

5.3 Monetary shocks

Consider the shock to bank reserve demand, ξ_t. According to Figure 6, this displays a trend fall beginning in 1930. In results not displayed here, we found that the primary effect of this is to enable the model to explain the trend rise in the reserves to deposit ratio (see Figure 5). According to the model, this is not a key shock underlying the Great Depression. The same is true for θ_t.

Consider now the liquidity preference shock, υ_t. Figure 6 shows that this shock rises rapidly beginning in 1929. After 1933 its growth rate falls, though the shock continues to rise for the rest of the decade. Figure 8 displays the result of simulating our model in response to the estimated time series of υ_t shocks alone. For comparison, the dotted lines display the actual data, reproduced from the solid line in Figure 5.

The key thing to notice in Figure 8 is that the υ_t shock accounts for several key features of the Great Depression. Notice in particular, that it explains part of the loss of value of the stock market, the fall in the price level, the fall in employment, and part of the fall in output and investment. It captures part of the rise in the premium and a major part of the fall in $M1$. Since this shock appears to be particularly important in our model's account of the Great Depression, we now explore the economics of how a rise in υ_t initially affects financial markets, and then how it is transmitted to the rest of the economy.

To understand the chain of events that an innovation in υ_t initiates, it is useful to understand how the household reacts to a tightening in its liquidity constraint, (9). Let μ_t denote the multiplier on (9). Recall that the household starts the period in possession of the economy's stock of base money, M_t^b, and allocates this between currency, M_t, and deposits, A_t. So, μ_t represents the marginal value to the household of additional beginning of period

[37] This mechanism has also been emphasized in Romer (1990).

t base money. Let λ_t denote the multiplier on the household's asset accumulation equation, (11). This multiplier represents the marginal value of end of period t base money. The first order condition associated with M_{t+1}^b in the Lagrangian representation of the household's problem implies $\lambda_t = \beta\mu_{t+1}$. (We ignore uncertainty in this discussion.) That is, the shadow value of end of period base money is equal to the discounted shadow of base money at the beginning of next period.

Using these multipliers, we can write out the first order conditions associated with the household's currency and deposit decision:

$$u_{M,t} + \beta\mu_{t+1} = \mu_t \tag{25}$$

$$u_{D^h,t} + (1 + R_{at})\beta\mu_{t+1} = \mu_t. \tag{26}$$

Here, $u_{M,t}$ and $u_{D^h,t}$ denotes the period t marginal utility of currency and household deposits, respectively. In (25) and (26), the left side measures the benefit of the given liquid asset, and the right side, the cost. In the case of currency, the benefit of an additional unit of currency is the marginal utility of that currency. There is also a pecuniary benefit, $\beta\mu_{t+1}$, because at the end of the period t, currency held during period t adds to the household's beginning of period $t+1$ holdings of base money. The right side of (25) has μ_t, the shadow value of a unit of current period base. This captures the fact that a marginal unit of currency requires giving up a unit of monetary base. Expression (26) is the analogous condition, applied to the household's deposit decision. The principle substantive difference between the two expressions is that the pecuniary benefit associated with deposits is greater, since they pay interest.

Since the marginal cost of an extra unit of deposits is the same as the marginal cost of an extra unit of currency, the marginal benefits of the two assets must be the same. Since deposits have a greater pecuniary payoff, their non-pecuniary payoff is necessarily smaller:

$$u_{D^h,t} < u_{M,t}.$$

Now, suppose something happens to raise the shadow value, μ_t, of current period base, while leaving R_{at} and μ_{t+1} unaffected. Then, the total marginal benefit of both assets must rise. Since the non-pecuniary part of the return to deposits is relatively small, it must be that $u_{D^h,t}$ rises by a greater percentage than $u_{M,t}$. Now, our functional form assumptions imply

$$u_{M,t} = \upsilon_t u_{L,t}\theta\frac{1}{M_t}, \tag{27}$$

$$u_{D^h,t} = \upsilon_t u_{L,t}\left(1-\theta\right)\frac{1}{D_t^h}, \tag{28}$$

where

$$u_{L,t} = \left[\frac{P_t C_t}{\left(M_t\right)^{\theta}\left(D_t^h\right)^{1-\theta}}\right]^{1-\sigma_q}.$$

Then,

$$\frac{u_{M,t}}{u_{D^h,t}} = \frac{\theta}{1-\theta}\frac{D_t^h}{M_t}.$$

It follows that when the shadow value of current period monetary base rises, then households allocate more base to currency.

With these considerations in mind, we can proceed to discuss the effects of a jump in υ_t. According to our estimates (see Table 6), the monetary authority kept the monetary base roughly unchanged in response to a liquidity preference shock. At the same time, a jump in υ_t drives up the marginal utility of deposits and currency (see (27)-(28)). With the desire for liquidity up, and the total supply unchanged, the shadow value of liquidity, μ_t, rises. By the argument in the previous paragraph, this induces households to reduce D_t^h/M_t. Since monetary policy keeps the sum of these two variables fixed, this requires reducing D_t^h and raising M_t. Figure 9 displays the response of these variables, expressed as a ratio to their unshocked steady state growth paths, in response to a one-standard deviation positive innovation in υ_t (for the magnitude of this shock, see Table 6 and the associated discussion.) Note how currency rises to a peak of about 1 percent above steady state, while deposits fall by about 2 percent. In the quest for additional liquidity, households cut back on consumption and on the acquisition of time deposits. Figure 9 shows how these variables drop relative to their unshocked steady state growth paths. Time deposits eventually drop by 2 percent, and consumption eventually drops a little over 1 percent.

The reduced supply of reserves to banks leads to a drop in $M1$. These effects show up in interest rates. Figure 9 shows the interbank loan rate rises, at least initially.

Entrepreneurs feel the effects of these developments. The interest rate paid by non-bankrupt entrepreneurs rises and this together with the lost income due to the fall in capital rent in the slowing economy, drives up the bankruptcy rate (these are not shown). The latter rises to over one percent per quarter above the steady state value of 0.8 percent. Entrepreneurs cut back on purchases of new capital, and this leads capital producers to reduce investment. This leads to a fall in the price of capital which exacerbates the fall in net worth. This fall further constrains the ability of entrepreneurs to purchase capital, leading to an additional fall in investment. In effect, the credit market restrictions have the effect of amplifying the negative output effects associated with the initial reductions in spending. Output ultimately falls over 1.5 percent in response to the one-standard deviation jump in the liquidity preference shock. Investment falls over 4 percent.

Qualitatively, at least, all these effects resemble what actually happened to the economy after 1929. One exception is the interest rate, which rises initially. However, this effect is transient, and rates fall very soon after their initial rise. We suspect that this reflects the fall in money demand as the amount of economic activity declines.

6 Counterfactual Analysis

We identify a monetary policy rule which, if implemented starting in 1929IV, would have resulted in a mild recession in the 1930s rather than the Great Depression. The rule feeds back on the monetary shocks only.[38] The policy calls for a temporary increase in money growth in the periods after the shock. We pursue this type of policy in order to be consistent with the fact that there was essentially no scope for short term interest rates to fall after 1932. We illustrate the importance of 'backloading' the money response in this way by comparing it to a policy in which the full money base response is concentrated in the period of the shock ('frontloading'). This policy would have made the zero bound restriction on the interest rate bind.

To describe the policy, it is convenient to reproduce the monetary policy rule in section 3.8:

$$\log\left(\frac{M_{t+1}^b}{M_t^b}\right) = x\sum_i \left(x_{i,t}+1\right),$$

where x is the steady state net growth rate in the monetary base. Also, $x_{i,t}$ is the component of monetary policy that reacts to the i^{th} shock:

$$x_{i,t} = \theta_i^0\varphi_{i,t} + \theta_i^1\varphi_{i,t-1} + \theta_i^2 x_{i,t-1}$$

When we backload policy, we set $\theta_i^0 = 0$ and $\theta_i^1 \neq 0$, indicating that the monetary policy response does not begin until the period after the shock. In addition, we set $\theta_i^2 > 0$, so that part of the policy response occurs more than one period after the shock. We considered only those i's corresponding to the monetary shocks, ξ_t, θ_t and υ_t. The policy we computed is one of 'leaning against the wind'. That is, if in the absence of a monetary response an innovation would lead to a fall in output, then our counterfactual policy calls for an increase in the base. Otherwise, the policy calls for a decrease.

In the case of the bank demand for excess reserves shocks, ξ_t, $\theta_i^1 = -7$ and $\theta_i^2 = 0.85$. With this parameterization, the monetary base falls 0.55 percent in the period after a one-standard deviation innovation in ξ_t. Because $\theta_i^1 > 0$, the monetary base continues to fall in later periods and it eventually stabilizes at a level 3.7 percent below where it would have been in the absence of a shock.[39] This is a policy of 'leaning against the wind because a rise

[38] In some models an appropriately designed feedback rule can exactly neutralize the impact of money demand shocks on the economy. In our model, this is not the case, even though we consider monetary base rules that react to individual shocks. This can be seen by studying the equations that characterize equilibrium in our model (these are listed in the appendix). A given money demand shock and the money base growth rate appear asymmetrically across more than one equation. As a result, it is not possible to set the monetary base to exactly extinguish a shock.

[39] The contemporaneous response to an innovation of σ in a shock is computed as $\sigma \times x \times \theta_i^1 \times 100$, where σ is found in Table 6. The eventual response is $\sigma \times x \times \theta_i^1 \times 100/(1-\theta_i^2)$.

in ξ_t signifies a fall in bank demand for excess reserves, which would lead to an expansion in output in the absence of a monetary response. The policy response is substantially greater than what we estimated actually occurred (see Table 6).

In the case of θ_t, $\theta_i^1 = -1$ and $\theta_i^2 = 0.85$. Thus, the drop in the monetary base in the period after a one-standard deviation innovation in θ_t is 0.08 percent. Eventually, the base falls by 0.55 percent. In the absence of any monetary policy response, an innovation in θ_t has the effect of stimulating output in our model.[40] The response of the counterfactual policy to θ_t is somewhat weaker than what we estimated to have occurred. Finally, in the case of υ_t, $\theta_i^1 = 2$ and $\theta_i^2 = 0.6$. As a result, the monetary base rises by 0.29 percent in the period after a one-standard deviation innovation in υ_t. Eventually, the monetary base rises by 0.72 percent in response to such a shock. As discussed above, an innovation in υ_t would reduce output, in the absence of a monetary policy response. The response of policy to υ_t

The dotted line in Figure 10 displays the dynamic behavior of the variables in our model, in response to all the estimated shocks and the counterfactual policy rule for the period 1929-1939. The solid line indicates the behavior of the same variables under the estimated monetary policy rule and shocks. With one caveat, it corresponds to the dotted line in Figure 5. The caveat reflects our estimation strategy, which focusses on model means at the calibration stage, but abstracts from them at the stage where we estimate the parameters of the stochastic processes. Thus, the model variables in Figure 5 are adjusted by an additive constant so that the model means coincide with those in the data, while the model variables in Figure 10 are untransformed. One difference between the figures that deserves emphasis is that the short term interest rate (the 'policy rate') is nearly 4 percentage points higher in Figure 10 than in Figure 5. Because of this, the interest rate in our model has a longer way to fall before hitting zero than the interest rate in the US economy in the 1930s did. To make sure that we do not give ourselves any flexibility that policy makers in the 1930s did not have, we only considered counterfactual policies in which the interest rate never falls below the lowest value taken on by the interest rate in the baseline simulation.

Note first the dynamic behavior of real output. The magnitude of the contraction in the period 1929-1933 is cut substantially under the counterfactual policy (see Table 8). In the version of our model with the estimated policy rule, output falls by 26 percent (i.e., 4 times the 6.4 percent annual rate), while it falls only 6 percent under the counterfactual policy rule. Over the whole decade, output rises 8 percent under the counterfactual rule, versus the 10 percent fall recorded under the estimated policy rule. Output growth over the entire

[40] To see why, note from (8) that, since the currency to household deposit ratio exceeds unity, one of the effects of an increase in θ_t is to raise the marginal utility of consumption. See Table 5 for the currency to household deposit ratio. (Note that the currency to deposit ratio is well below unity.) Given our parameterization, this is the dominant effect of an increase in θ_t, and this is why a positive innovation in this shock stimulates output.

decade of the 1930s is 0.8 percent per year, a little over one-half its trend value of 1.5 percent. Initially, the interest rate falls relative to the baseline, but in the later part of the 1930s, it is higher than the baseline.

Turning to the other variables in the counterfactual simulation, consider first the monetary base. The rise in this variable over the full decade is 12 percent per year. The rise in the price level is roughly 3.2 percent per year. The counterfactual policy has a very substantial effect on asset values, driving up the real value of net worth by as much as 40 percent more after 1932. The impact on asset values before this is less, and perhaps this is the reason that the counterfactual policy does not have a large impact on investment until later. The counterfactual policy introduces some volatility in consumption and investment, which appears to cancel in output. We do not, as yet, understand the reason for this volatility.

It is interesting that the counterfactual policy has such a substantial stimulative impact on output without driving the interest rate down unduly. The reason for this is that the monetary policy response to shocks occurs primarily in the periods after the shock. As a result, the policy injects a substantial anticipated inflation effect into the interest rate. To illustrate this, we considered a monetary policy in which θ_i^0 is replaced with $\theta_i^0 = \theta_i^1/(1 - \theta_i^2)$ and θ_i^1, θ_i^2 are both set to zero. With this policy the response of money to a shock is completely frontloaded. This reduces by a substantial amount the magnitude of the anticipated inflation effect on the interest rate. In addition, because the monetary response is completely unanticipated when it occurs, it has a relatively large impact on real allocations. To make the policy comparable to our counterfactual, we scaled the money responses to a shock by 0.69 so that the growth rate of output is 0.8 percent per year over the 1930s, as in the counterfactual policy in Table 8.[41] Not surprisingly, with this alternative counterfactual, the rise in the price level is much less, only 0.8 percent per year in the 1930s, versus 3.2 percent per year in the counterfactual (see Table 8 for the latter). In addition, $M1$ rises by less as well. The other variables in Table 8 are roughly unchanged. However, the interest rate response, displayed in Figure 11, is quite different. Under the alternative counterfactual, the interest rate drops substantially below the baseline simulation results. Such a large reduction in the interest rate could not have been implemented in the 1930s because it would have violated the zero lower bound constraint.

It is worth emphasizing that if the policy reaction function were stronger than it is in the counterfactual, then the output response would have been stronger as well. However, this would have come at the cost of substantially higher inflation. For example, when we set θ_i^1 to 7, rather than its value of 2 in the counterfactual, we found that average output growth in the 1930s is close to trend, 1.6 percent. However, this comes at the cost of substantially higher inflation. The annual inflation over the period 1929-1939 is 7.3 percent and over the

[41] Thus, we replaced θ_i^0 with $\theta_i^0 = 0.69 \times \theta_i^1/(1 - \theta_i^2)$.

period 1933-1939 it is 12.5 percent.

We conclude this section by pointing out that the effectiveness of monetary policy in reducing the severity of the Great Depression depends very much on the state contingent nature of the policy. To establish this, we considered a second alternative counterfactual experiment in which we instead adopted a completely deterministic monetary policy in 1929IV. In this policy, the money growth rate is roughly equal to the realized money growth in our counterfactual experiment. The impact on real variables of this policy was minor. This reflects that in the long-run, monetary policy is roughly neutral in our model. Unexpected monetary policy, whether driven by exogenous random shocks, or disturbances to the economy's fundamentals do have a substantial effect. To document this, we simulated a version of our model in which the only shock driving the monetary base is an iid monetary policy shock. We picked the sequence of shocks to force the monetary base to grow as in our counterfactual experiment. We found that the impact on output and employment of this policy were roughly comparable to what we obtained in the counterfactual experiment.

7 Conclusion

In this paper we have constructed a model of the US economy in the 1920s and 1930s. We argue that that model deserves to be taken seriously as a laboratory for monetary policy because it reproduces key features of the data of the period. Our analysis raises the possibility that a liquidity preference shock played a key role, especially in the contraction phase of the Great Depression. Using the model, we found that a monetary policy that reacts to the innovations in the monetary shocks would have substantially reduced the magnitude of the Great Depression. This is consistent with the Friedman-Schwartz hypothesis.

The analysis of this paper suggests several avenues for future research. First, we have emphasized the importance of a liquidity preference shock for the dynamics of the Great Depression. It would be of interest to see whether this shock plays an important role in other economies and times such as the post war US, Japanese or Euro area economies.[42] Second, it would be of interest to explore at a more foundational level, the nature and sources of our liquidity preference shock. Third, in our analysis we have identified a particular feedback rule which reacts to the fundamental shocks, which would have mitigated the severity of the Great Depression. Discussions about monetary policy often focus on rules that relate variables such as the interest rate or the monetary base, to inflation, and the output gap. We conjecture that there are monetary policy rules like this that would in practice work like the one that we study, and it would be of interest to explore this further. Fourth,

[42] Richard Portes has emphasized to us the possibility that the liquidity preference shock may have played an important role in the US in the fall of 1998, at the time of the financial uncertainties surrounding the Russian debt default and other events.

it would be of interest to explore optimal monetary policy in our model. Presumably, this requires a solution method which can accommodate an occasionally binding zero lower bound constraint on the interest rate. We suspect that such rules would involve accommodating money demand shocks, as our counterfactual rule does.

A Solving the Model

The model solution strategy was described in section 3.9. That involves a set of core endogenous variables, $\tilde{z}_t$. The variables in this 23 by 1 vector are:

$$\tilde{z}_t = \begin{pmatrix} \hat{\pi}_t \\ \hat{s}_t \\ \hat{r}_t^k \\ \hat{\imath}_t \\ \hat{u}_t \\ \widehat{\bar{\omega}}_t \\ \hat{R}_t^k \\ \hat{n}_{t+1} \\ \hat{q}_t \\ \hat{\nu}_t \\ \hat{e}_{\nu,t} \\ \hat{m}_t^b \\ \hat{R}_t \\ \hat{u}_{c,t}^z \\ \hat{\lambda}_{z,t} \\ \hat{m}_t \\ \hat{R}_{a,t} \\ \hat{c}_t \\ \hat{w}_t \\ \hat{l}_t \\ \widehat{\bar{k}}_{t+1} \\ \hat{R}_{t+1}^e \\ \hat{x}_t \end{pmatrix}$$

Here, and throughout this paper, a hat '^' over a variable indicates percent deviation from its nonstochatic steady state. That is, if x is the steady state value of x_t, then $\hat{x}_t = (x_t - x)/x$. Most of the variables in $\tilde{z}_t$ have been defined before. One exception is real marginal cost for intermediate good producers:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(r_t^k \left[1+\psi_k R_t\right]\right)^{\alpha} \left(w_t \left[1+\psi_l R_t\right]\right)^{1-\alpha}}{\epsilon_t}. \tag{29}$$

In addition, we adopt the following scaling of variables:

$$\begin{aligned} w_t &= \frac{W_t}{z_t P_t},\ q_t = \frac{Q_{\bar{K}',t}}{P_t},\ n_{t+1} = \frac{N_{t+1}}{P_t z_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t},\ m_t^b = \frac{M_t^b}{P_t z_t}, \\ m_t &= \frac{M_t}{M_t^b},\ c_t = \frac{C_t}{z_t},\ i_t = \frac{I_t}{z_t}, \end{aligned}$$

Finally, $e_{v,t}$ is the ratio of real excess reserves to value-added in the banking sector:

$$e_{v,t} = \frac{\frac{A_t + X_t - \tau_t(A_t + X_t + S_t^w)}{P_t}}{\left(z_t\left(1-\nu_t\right)u_t\bar{k}_t/\mu_z\right)^{\alpha}\left(z_t\left(1-\nu_t\right)l_t\right)^{1-\alpha}} \tag{30}$$

To solve for the 23 variables in $\tilde{z}_t$, we linearize 23 equations of our model. We express these in matrix form as follows:

$$E_t\left[\alpha_0\tilde{z}_{t+1} + \alpha_1\tilde{z}_t + \alpha_2\tilde{z}_{t-1} + \beta_0\Psi_{t+1} + \beta_1\Psi_t\right] = 0. \tag{31}$$

We seek a solution of the form, (19), which ensures that the above expression is satisfied for all possible $\tilde{z}_{t-1}$ and Ψ_t, and which is covariance stationary. This requires an A and B that satisfy:

$$\begin{aligned} \alpha_0 A^2 + \alpha_1 A + \alpha_2 I &= 0 \\ \alpha_0(AB + B\rho) + \alpha_1 B + \beta_0\rho + \beta_1 &= 0. \end{aligned}$$

We found that there is a unique A with eigenvalues inside the unit circle that satisfies the first equation. Conditional on this value of A, the second set of equations is linear in B.

To complete our discussion of the solution method requires explaining how α_0, α_1, α_2, β_0, β_1 were computed. This requires a discussion of the steady state of the model, and of the actual equations in (31).

A.1 Model Steady State

As explained in the appendix, solving for the steady state involves solving 25 equilibrium conditions for 25 unknowns. Unlike the problem of solving for a steady state in, say, a real business cycle model, we were not able to find a simple recursive structure in these equilibrium conditions. We were able to simplify the system somewhat by using 22 of the equilibrium conditions to define 22 of the variables in terms of three, r^k, R and ν^k. In this way, we were able to reduce the problem of finding the steady state to one of solving three equations in the three unknowns, r^k, R and ν^k. In particular, let $\varsigma = (r^k, R, \nu^k)$ and let $f(\cdot;\Psi) : \Omega \to R^3$ denote the three equations, where $\Omega \subseteq R^3$ denotes the set of feasible values of ς and Ψ denotes a set of values for the model parameters. The set, Ω, denotes the values of ς for which there exists admissible values (i.e., values that are real and satisfy various

non-negativity and other inequality conditions) for the other 22 steady state variables that solve the remaining 22 steady state equilibrium conditions. The condition, $f(\zeta;\Psi)=0$, implicity defines a function, $\zeta=g(\Psi)$. Then,

***The heart of our calibration strategy involves solving for the model's steady state. This requires finding the values of 25 variables that satisfy 25 equilibrium conditions (see the Appendix). These equations do not appear to have a simple structure, and so solving them turned out to be a substantial challenge. As a result, we did not do calibration in the usual way, by choosing model parameters so that the model's steady state coincides precisely with a given set of sample averages computed in the data. We instead proceeded informally, identifying a set of model parameters for which the steady state 'looks like' sample averages in the data.]

This section describes our strategy for computing the steady state for our model. In many models, solving for the steady state is straightforward, because one can find a recursive ordering among the equations. We were not able to do so for our modcl, and so the computation of the steady state represents a considerable challenge. We developed two strategies. In the first one we exogenously set some of the endogenous variables of the model to values that seem reasonable based on empirical evidence. We moved an equal number of exogenous variables into the list of endogenous variables. This approach allowed us to reduce the problem of computing the steady state to the relatively manageable one of solving a single equation in a single unknown. This approach works well for solving our baseline model. However, it is impractical when we considered a counterfactual experiment in which it was desired to change the value of one (and only one) exogenous variable. A second strategy was developed for this. We now describe the two strategies.

For the first strategy, the set of exogenously set variables are:

$$\begin{aligned}&\tau^l,\ \tau^c,\ \beta,\ F(\bar{\omega}),\ \mu,\ x,\ \mu_z,\ \lambda_f,\ \lambda_w,\ \alpha,\ \psi_k,\ \psi_l,\ \delta,\ \upsilon,\\ &\tau^k,\ \gamma,\ \tau,\ \tau^T,\ \tau^D,\ \sigma_L,\ \zeta,\ \sigma_q,\ \theta,\ \upsilon,\ w^e,\ \nu^l,\ \nu^k,\ m,\ \eta_g,\ r^k.\end{aligned}$$

The variables to be solved for are:

$$\begin{aligned}&q,\ \pi,\ R^e,\ R_a,\ h_{e^r},\ R,\ R^k,\ \bar{\omega},\ k,\ n,\ i,\ w,\ l,\ c,\ u_c^z,\ m^b,\\ &\lambda_z,\ \psi_L,\ e_z^r,\ e_\upsilon,\ x^b,\ \xi,\ h_{K^b},\ y,\ g,\ \sigma.\end{aligned}$$

The equations available for solving for these unknowns are summarized below. The first three variables are trivial functions of the structural parameters, and from here on we treat them as known. In addition, R_a can be solved using (51). There remain 22 unknowns. Below, we have 22 equations that can be used to solve for them.

The algorithm involves finding a value of R that solves (40) below. To evaluate (40) requires first solving for the other 'endogenous' variables. For a given R, we proceed as

follows. Solve for h_{e^r} using (46). Solve for R^k using (36); solve for $\bar{\omega}$ using (37); solve for σ using the given values of $F(\bar{\omega})$ and $\bar{\omega}$, as well as the condition, $E\omega = 1$; solve for k and n using (38) and (39); solve for i using (35); solve for w using (32); solve for l using (33); solve for c using (55); solve (56) and (54) for g and y; solve for u_c^z using (52); solve for m^b and λ_z using (49) and (50); solve (53) for ψ_L; solve for e_z^r using (48); solve ξ from (47); solve e_v from (43); solve x^b from (42); h_{K^b} from (41). Vary R until (40) is satisfied. In these calculations, all variables must be positive, and:

$$0 \leq m \leq 1 + x,\ 0 \leq \xi \leq 1,\ \lambda_z > 0,\ k > n > 0.$$

Our second strategy solves the steady state when the 'exogenous variables' are the economically exogenous ones, and the 'endogenous variables' are the economically endogenous ones. In particular, consider the situation in which the exogenous variables are:

$$\tau^l,\ \tau^c,\ \beta,\ \mu,\ x,\ \mu_z,\ \lambda_f,\ \lambda_w,\ \alpha,\ \psi_k,\ \psi_l,\ \delta,\ \upsilon,\ x^b,\ \xi,\ \sigma,$$
$$\tau^k,\ \gamma,\ \tau,\ \tau^T,\ \tau^D,\ \sigma_L,\ \zeta,\ \sigma_q,\ \theta,\ \upsilon,\ w^e,\ \nu^l,\ \eta_g,\ \psi_L.$$

and the variables to be solved for are:

$$q,\ \pi,\ R^e,\ R_a,\ h_{e^r},\ r^k,\ R^k,\ \bar{\omega},\ F(\bar{\omega}),\ k,\ n,\ i,\ w,\ l,\ c,\ u_c^z,$$
$$m^b,\ R,\ \lambda_z,\ e_z^r,\ e_v,\ h_{K^b},\ y,\ g,\ \nu^k,\ m.$$

We solve for the above 26 variables as follows. The first three are solved in the same way as before. The remainder are solved by solving three equations, (40), (42) and (46), in the three unknowns, r^k, ν^k and R. Ideally, we start in a neighborhood of the solution obtained in the previous calculations. Fix a set of values for r^k, ν^k and R. The basic sequence of calculations is the same as above. Solve for R^k using (36), and then $\bar{\omega}$, $F(\bar{\omega})$ using (37) and the value of σ. Then, solve for k, n, and i using (38), (39) and (35), in that order. Next, we obtain w from (32) and l from (33). The resource constraint, (55) can be used to obtain c, and (56), (54) can be used to compute y, g. Then, obtain λ_z and u_c^z from (52) and (53). Solve (49), (50) and (51) for R_a, m, and m^b. This can be made into a one-dimensional search in m. In particular, for a given m, solve for R_a from (51) and for m^b from (49). Vary m until (50) is satisfied. In this search, the lower bound on m is zero, while the upper bound is $\theta(1 + x)$, and is dictated by $R_a \geq 0$. Compute h_{e^r}, h_{K^b}, e_ν, e_z^r, using (41), (43), (47), and (48). We can now evaluate (40), (42) and (46). Vary r^k, ν^k and R until these equations are satisfied.

The equations that r^k, ν^k and R solve are not well behaved. They are not well defined for all possible values of r^k, ν^k and R. Typically, this happened because there was no m that satisfied (50). We found that if the equations are defined for a particular set of values of r^k, ν^k and R, then they are not defined for what seems like a tiny perturbation. Equations

of this type are hard to solve with standard Gauss-Newton algorithm, unless one has an extremely good idea of the exact solution. To find such a solution, we applied a random search method. We made a guess of the true solution, and then constructed an interval about that solution. We drew randomly from that interval, and recorded the parameter constellation which produced the outcome closest to zero for the equations of interest. After the algorithm ran for 5 minutes or so, it had found a solution close enough that a Gauss-Newton method could take over and productively drive into the exact solution for r^k, ν^k and R.

We now describe the equations of the steady state.

A.1.1 Firm Sector

From the firm sector, and the assumption that there are no price distortions in a steady state, we have

$$s = \frac{1}{\lambda_f}.$$

Also, evaluating (29) in steady state:

$$\frac{1}{\lambda_f} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(r^k\left[1+\psi_k R\right]\right)^{\alpha}\left(w\left[1+\psi_l R\right]\right)^{1-\alpha}, \tag{32}$$

Combining (29) and (30):

$$\frac{r^k\left[1+\psi_k R\right]}{w\left[1+\psi_l R\right]} = \frac{\alpha}{1-\alpha}\frac{\mu_z l}{\bar{k}} \tag{33}$$

A.1.2 Capital Producers

The first order necessary condition for maximization of I_t for the capital producers is

$$E_t\left[\lambda_t P_t q_t F_{1,t} - \lambda_t P_t + \beta\lambda_{t+1}P_{t+1}q_{t+1}F_{2,t+1}\right] = 0.$$

Here, F_{1t} and F_{2t} denote the derivatives of the adjustment cost function with respect to its first and second arguments. Multiplying the first order condition by z_t/P_t, we obtain, in steady state:

$$\lambda_z q F_1 - \lambda_z + \frac{\beta}{\mu_z}\lambda_z q F_2 \pi = 0. \tag{34}$$

Since $F_1 = 1$ and $F_2 = 0$,

$$q = 1.$$

Also,

$$\bar{k}_{t+1} = (1-\delta)\frac{1}{\mu_{z,t}}\bar{k}_t + \left[1 - S\left(\frac{i_t\mu_{z,t}}{i_{t-1}}\right)\right]i_t,$$

so that in steady state, when $S = 0$,

$$\frac{i}{\bar{k}} = 1 - \frac{1-\delta}{\mu_z}. \tag{35}$$

A.1.3 Entrepreneurs

From the entrepreneurs:

$$r^k = a'.$$

Also,

$$u = 1.$$

The after tax rate of return on capital, in steady state, is:

$$R^k = \left[(1-\tau^k)r^k + (1-\delta)\right]\pi + \tau^k\delta - 1 \tag{36}$$

Conditional on a value for R^k, R^e, the steady state value for $\bar{\omega}$ may be found using the following equation:

$$\left[1-\Gamma(\bar{\omega})\right]\frac{1+R^k}{1+R^e} + \frac{1}{1-\mu\bar{\omega}h(\bar{\omega})}\left[\frac{1+R^k}{1+R^e}\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right) - 1\right] = 0, \tag{37}$$

where the hazard rate, h, is defined as follows:

$$h(\omega) = \frac{F'(\omega)}{1-F(\omega)}.$$

This equation has two additional parameters, the two parameters of the lognormal distribution, F. These two parameters, however, are pinned down by the assumption, $E\omega = 1$, and the fact that we specify $F(\bar{\omega})$ exogenously. With these conditions, the above equation forms a basis for computing $\bar{\omega}$. Note here that when $\mu = 0$,(37) reduces to $R^k = R^e$. Then, combining (36) with the first order condition for time deposits, we end up with the conclusion that r^k is determined as it is in the neoclassical growth model.

From the single first order condition for k in the costly-state-verification problem, and conditional on $F(\bar{\omega})$ and $\bar{\omega}$, we may solve for k, thus obtaining:[43]

$$\frac{\bar{k}}{n} = \frac{1}{1-\frac{1+R^k}{1+R^e}\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right)}. \tag{38}$$

The law of motion for net worth implies the following relation in steady state:

$$n = \frac{\frac{\gamma}{\pi\mu_z}\left[R^k - R^e - \mu G(\bar{\omega})\left(1+R^k\right)\right]\bar{k} + w^e}{1-\gamma\left(\frac{1+R^e}{\pi}\right)\frac{1}{\mu_z}} \tag{39}$$

[43] See BGG for details.

A.1.4 Banks

The first order condition associated with the bank's capital decision is:

$$\left(1+\psi_k R\right) r^k = \frac{R h_{K^b}}{1+\tau h_{e^r}}. \tag{40}$$

The first order condition for labor is redundant given (32), (33), and (40), and so we do not list it here. In the preceding equations,

$$h_{K^b} = \alpha\xi a^b x^b \left(e_v\right)^{1-\xi} \left(\frac{\mu_z l}{k}\right)^{1-\alpha}, \tag{41}$$

$$h_{e^r} = \left(1-\xi\right) a^b x^b \left(e_v\right)^{-\xi}, \tag{42}$$

and

$$e_v = \frac{\left(1-\tau\right) m^b \left(1-m+x\right) - \tau\left(\psi_l w l + \frac{1}{\mu_z}\psi_k r^k \bar{k}\right)}{\left(\frac{1}{\mu_z}(1-\nu^k)\bar{k}\right)^{\alpha} \left((1-\nu^l)l\right)^{1-\alpha}}. \tag{43}$$

The first order conditions associated with the bank's supply of deposits to household, A_t, and working capital loans, S_t^w, are, respectively,

$$-R_{at} + \lambda_t^b \frac{1}{P_t}\left[\left(1-\tau_t\right) h_{e^r,t} - 1\right] = 0, \tag{44}$$

$$R_t - \lambda_t^b \frac{1}{P_t}\left[\tau_t h_{e^r,t} + 1\right] = 0. \tag{45}$$

where λ_t^b is the multiplier on the technology associated with the provision of bank deposits. Hence, another efficiency condition for the banks can be obtained by taking the ratio of (45) to (44). Rewriting that ratio, we obtain:

$$1 + \frac{R}{R_a} = h_{e^r}\left[\left(1-\tau\right)\frac{R}{R_a} - \tau\right] \tag{46}$$

Substituting out for $a^b x^b \left(e_v\right)^{-\xi}$ from (42) into the scaled production function, we obtain:

$$\frac{h_{e^r}}{\left(1-\xi\right)} e_z^r = m^b \left(1-m+x\right) + \psi_l w l + \psi_k r^k \frac{\bar{k}}{\mu_z}, \tag{47}$$

where

$$e_z^r = \left(1-\tau\right) m^b \left(1-m+x\right) - \tau\left(\psi_l w l + \psi_k r^k \frac{\bar{k}}{\mu_z}\right). \tag{48}$$

A.1.5 Households

The first order condition for T :

$$1+R^e=\frac{\mu_z\pi}{\beta}$$

The first order condition for M :

$$\upsilon\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}[\frac{\theta}{m}-\frac{1-\theta}{1-m+x}]\left(m^b\right)^{\sigma_q-2} \tag{49}$$
$$-\lambda_z R_a=0$$

The first order condition for M^b

$$\upsilon\left(1-\theta\right)\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}\left(\frac{1}{m^b}\right)^{2-\sigma_q}\left(\frac{1}{1-m+x}\right)$$
$$=\pi\lambda_z\left\{\frac{\mu_z}{\beta}-\frac{(1+R_a)}{\pi}\right\}$$

Under the ACEL specification of preferences, c in the previous two expressions are replaced by unity. The first order condition for consumption corresponds to:

$$u_c^z-\left(1+\tau^c\right)\lambda_z=\upsilon c^{-1}\left(m^b\right)^{\sigma_q-1}\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}, \tag{50}$$

Under the ACEL specification, the expression to the right of the equality in (50) is replaced by zero.

Taking the ratio of (49) and the first order conditions for m^b, and rearranging, we obtain:

$$\begin{aligned} R_a &= \frac{\frac{(1-m+x)}{m}\theta-(1-\theta)}{\frac{(1-m+x)}{m}\theta}\left(\frac{\pi\mu_z}{\beta}-1\right) \\ &= \left[1-\frac{m}{1-m+x}\frac{(1-\theta)}{\theta}\right]R^e \end{aligned} \tag{51}$$

Note that non-negativity of R_a requires $m\leq\theta(1+x)$.

$$\upsilon\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}[\frac{\theta}{m}-\frac{1-\theta}{1-m+x}]\left(m^b\right)^{\sigma_q-2}$$
$$= \lambda_z\left(1-\tau^D\right)\frac{m}{\theta}\left[\frac{\theta}{m}-\frac{(1-\theta)}{1-m+x}\right]\frac{1-\tau^T}{1-\tau^D}R^e$$

or

$$\upsilon c\frac{u_c^z-\left(1+\tau^c\right)\lambda_z}{\upsilon m^b}=\lambda_z\left(1-\tau^D\right)\frac{m}{\theta}\frac{1-\tau^T}{1-\tau^D}R^e$$
$$\frac{m}{\theta}\left[\frac{\theta}{m}-\frac{(1-\theta)}{1-m+x}\right]\frac{1-\tau^T}{1-\tau^D}R^e$$

The marginal utility of consumption is:

$$\frac{M_{t+1}-\bar{M}_{t+1}}{\bar{M}_{t+1}}=\frac{1+x_t-1-x}{1+x}=\frac{x_t-x}{1+x}$$

$$cu_c^z=\frac{\mu_{z,}}{\mu_z-b}-b\beta\frac{1}{\mu_z-b}=\frac{\mu_z-b\beta}{\mu_z-b} \tag{52}$$

The first order condition for households setting wages is:

$$w\frac{\lambda_z(1-\tau^l)}{\lambda_w}=\zeta\psi_L l^{\sigma_L} \tag{53}$$

A.1.6 Monetary Authority

$$\pi=\frac{(1+x)}{\mu_z}.$$

A.1.7 Resource Constraint and Zero Profits

After substituting out for the fixed cost in the resource constraint using the restriction that firm profits are zero in steady state and using

$$g=\eta_g y, \tag{54}$$

we obtain:

$$c=\left(1-\eta_g\right)\left[\frac{1}{\lambda_f}\left(\frac{1}{\mu_z}\nu^k k\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha}-\mu G(\bar{\omega})(1+R^k)\frac{k}{\mu_z\pi}\right]-i. \tag{55}$$

The object in square brackets is gross national product, y, so that (55) corresponds to $c+i+g=y$. To see that the object in square brackets indeed is y, consider:

$$y=\left(\frac{1}{\mu_z}\nu^k k\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha}-\phi-\mu G(\bar{\omega})(1+R^k)\frac{k}{\mu_z\pi}, \tag{56}$$

Write the first two expressions after the equality as $F-\phi$. These are the revenues, in units of goods, paid to the typical intermediate good producer. The total cost of producing these goods is sF, where s denotes marginal cost. In steady state, $s=1/\lambda_f$. Zero profits requires that revenues equal costs, so that $F/\lambda_f=F-\phi$, or, $\phi=F(1-1/\lambda_f)$, or

$$\phi=\left(\frac{\nu^k k}{\mu_z}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha}(1-\frac{1}{\lambda_f}). \tag{57}$$

We obtain the expression for gross national product in the square brackets in (55) by substituting out for ϕ in (56) from (57).

A.2 Linearizing the Model Economy

This section describes the 23 equations that characterize the equilibrium for our model economy. In each case, we present the log-linear expansion of the equation about the non-stochastic steady state. With a few exceptions, we also present the non-linear representation of the equation.

A.2.1 Firms

The log-linearized expression for inflation in our model is taken directly from Christiano, Eichenbaum and Evans (2003):

$$E_t\left[\hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}\hat{\pi}_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}\left(\hat{s}_t + \hat{\lambda}_{f,t}\right)\right] = 0. \tag{58}$$

Linearizing the expression for the real marginal cost for a firm producing intermediate goods, (29), we obtain:

$$\alpha\hat{r}_t^k + (1-\alpha)\,\hat{w}_t + \left[\frac{\alpha\psi_k R}{1+\psi_k R} + \frac{(1-\alpha)\,\psi_l R}{1+\psi_l R}\right]\hat{R}_t - \hat{\epsilon}_t - \hat{s}_t = 0 \tag{59}$$

Marginal cost must also satisfy another condition:

$$s_t = \frac{r_t^k\left[1+\psi_k R_t\right]}{\alpha\epsilon_t\left(\frac{\mu_z l_t}{u_t\bar{k}_t}\right)^{1-\alpha}}.$$

Linearizing this,

$$\hat{r}_t^k + \frac{\psi_k R}{1+\psi_k R}\hat{R}_t - \hat{\epsilon}_t - (1-\alpha)\left(\hat{l}_t - \left[\hat{\bar{k}}_t + \hat{u}_t\right]\right) - \hat{s}_t = 0 \tag{60}$$

A.2.2 Capital Producers

Following Christiano, Eichenbaum and Evans (2003), we suppose that $F(I_t, I_{t-1})$ has the following form:

$$F(I_t, I_{t-1}) = \left[1 - S(I_t/I_{t-1})\right] I_t,$$

where S'' denotes the second derivative of S, evaluated at the steady state value of I_t/I_{t-1}. In addition, we suppose that $S = S' = 0$ in steady state. Recalling that the the first order necessary condition for maximization of I_t for the capital producers can be written:

$$E_t\left[\lambda_{zt}q_t F_{1,t} - \lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}}\lambda_{z,t+1}q_{t+1}F_{2,t+1}\pi_{t+1}\right] = 0,$$

we linearize this expression as,

$$E_t\left[\hat{q}_t - S''\mu_z^2(1+\pi\beta)\hat{\imath}_t + S''\mu_z^2\hat{\imath}_{t-1} + \beta S''\pi\mu_z^2\hat{\imath}_{t+1}\right] = 0. \tag{61}$$

A.2.3 Entrepreneurs

There are five equations pertaining to entrepreneurs. The first is the optimality condition associated with the entrepreneur's capital utilization decision:

$$r_t^k - a'(u_t) = 0,$$

After linearization, this reduces to:

$$E\hat{r}_t^k - \sigma_a \hat{u}_t = 0, \tag{62}$$

where $\sigma_a = a''/a'$ and a'', a' denote the second and first derivatives of a, respectively, evaluated in steady state.

Of the other four equations corresponding to the entrepreneurial sector, two characterize the loan contracts received from banks, and the third defines the law of motion of entrepreneurial net worth. Finally, there is an equation which defines the rate of return on capital earned by entrepreneurs.

The necessary condition associated with optimality of the loan contract received by entrepreneurs from banks is:

$$E_t\left\{\left[1-\Gamma_t(\bar{\omega}_{t+1})\right]\frac{1+R_{t+1}^k}{1+R_{t+1}^e} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})}\left[\frac{1+R_{t+1}^k}{1+R_{t+1}^e}\left(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right) - 1\right]\right\} = 0.$$

Here, R_{t+1}^e is the rate of return received by households, which is constrained not to be a function of the realization of date $t+1$ random variables. The functions, G_t and Γ_t, are defined as follows:

$$\begin{aligned} G_t(\bar{\omega}_{t+1}) &= \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega). \\ \Gamma_t(\bar{\omega}_{t+1}) &= \bar{\omega}_{t+1}\left[1 - F_t(\bar{\omega}_{t+1})\right] + G_t(\bar{\omega}_{t+1}), \end{aligned}$$

where here F_t denotes the cumulative distribution function for ω_{t+1}. Here, a prime indicates derivative with respect to $\bar{\omega}_{t+1}$. We assume that F_t corresponds to a log-normal distribution with mean zero and standard deviation parameter, σ_t. The second equation that characterizes the optimal contract corresponds to the condition that bank profits are zero in each state of nature:

$$\frac{Q_{\bar{K}',t}\bar{K}_{t+1}}{N_{t+1}}\frac{1+R_{t+1}^k}{1+R_{t+1}^e}\left(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right) - \frac{Q_{\bar{K}',t}\bar{K}_{t+1}}{N_{t+1}} + 1 = 0.$$

After scaling, this reduces to:

$$\frac{q_t\bar{k}_{t+1}}{n_{t+1}}\frac{1+R_{t+1}^k}{1+R_{t+1}^e}\left(\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right) - \frac{q_t\bar{k}_{t+1}}{n_{t+1}} + 1 = 0.$$

For a detailed discussion of the two equations that characterize the optimal contract, see Bernanke, Gertler and Gilchrist (1999).

Linearizing the efficiency condition associated with the optimal contract, we obtain:

$$\begin{aligned} E_t\{\lambda\left(\frac{R^k\hat{R}^k_{t+1}}{1+R^k}-\frac{R^e\hat{R}^e_{t+1}}{1+R^e}\right) \\ -\left[1-\Gamma\right]\frac{1+R^k}{1+R^e}\left[\frac{\Gamma_{\omega\omega}\bar{\omega}}{\Gamma_\omega}-\frac{\lambda\left[\Gamma_{\omega\omega}-\mu G_{\omega\omega}\right]\bar{\omega}}{\Gamma_\omega}\right]\widehat{\bar{\omega}}_{t+1} \\ +\frac{1+R^k}{1+R^e}\left(\sigma\left[-\Gamma_\sigma+\lambda\left(\Gamma_\sigma-\mu G_\sigma\right)\right]-\left[1-\Gamma\right]\left[\frac{\Gamma_{\omega\sigma}\sigma}{\Gamma_\omega}-\frac{\lambda\left[\Gamma_{\omega\sigma}-\mu G_{\omega\sigma}\right]\sigma}{\Gamma_\omega}\right]\right)\hat{\sigma}_t\} &= 0. \end{aligned} \tag{63}$$

Here, $\Gamma_{\omega\omega}$ and $G_{\omega\omega}$ denote the second derivatives of Γ and G with respect to $\bar{\omega}$, evaluated in steady state. Also, Γ_σ and G_σ denote the derivatives with respect to σ in steady state, and $\Gamma_{\omega\sigma}$, $G_{\omega\sigma}$ represent the corresponding cross derivatives. Finally, λ denotes the steady state value of the multiplier on the bank zero profit condition in the Lagrangian representation of the problem solved by the optimal contract. It is:

$$\lambda=\frac{\Gamma_\omega}{\Gamma_\omega-\mu G_\omega}.$$

The linearized zero profit condition is:

$$\begin{aligned} \left(\frac{\bar{k}}{n}-1\right)\frac{R^k}{1+R^k}\hat{R}^k_t-\left(\frac{\bar{k}}{n}-1\right)\frac{R^e}{1+R^e}\hat{R}^e_t+\left(\frac{\bar{k}}{n}-1\right)\frac{\left(\Gamma_\omega-\mu G_\omega\right)}{\left(\Gamma-\mu G\right)}\bar{\omega}\widehat{\bar{\omega}}_t \\ +\left(\frac{\bar{k}}{n}-1\right)\frac{\left(\Gamma_\sigma-\mu G_\sigma\right)}{\left(\Gamma-\mu G\right)}\sigma\hat{\sigma}_{t-1}-\left(\hat{q}_{t-1}+\widehat{\bar{k}}_t-\hat{n}_t\right)=0. \end{aligned} \tag{64}$$

The law of motion governing the evolution of aggregate net worth is, after scaling:

$$n_{t+1}=\frac{\gamma_t}{\pi_t\mu_z}\left\{R^k_t-R^e_t-\mu\left[\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right]\left(1+R^k_t\right)\right\}\bar{k}_tq_{t-1}+w^e_t+\gamma_t\left(\frac{1+R^e_t}{\pi_t}\right)\frac{1}{\mu_z}n_t.$$

Here, γ_t is the probability that an entrepreneur survives from period t to period $t+1$. Also, $w^e_t=W^e_t/\left(z_tP_t\right)$ and W^e_t is a transfer made to each entrepreneur in existence in period $t+1$. This includes the γ_t who were alive in period t and survive into period $t+1$, as well as the $1-\gamma_t$ new entrepreneurs born in period t. The linearized law of motion for scaled net worth is:

$$\begin{aligned} -\hat{n}_{t+1}+a_0\hat{R}^k_t+a_1\hat{R}^e_t+a_2\left(\widehat{\bar{k}}_t+\hat{q}_{t-1}\right)+a_4\left(\hat{\gamma}_t-\hat{\pi}_t\right) \\ +a_8\widehat{\bar{\omega}}_t+a_9\hat{n}_t+a_{10}\hat{\sigma}_{t-1}=0. \end{aligned} \tag{65}$$

where

$$\begin{aligned}
a_0 &= \frac{\gamma}{\pi\mu_z}\left(1-\mu G\right)\frac{\bar{k}q}{n}R^k \\
a_1 &= \left(1-\frac{\bar{k}q}{n}\right)\frac{\gamma R^e}{\pi\mu_z} \\
a_2 &= \frac{\gamma}{\pi\mu_z}\left\{R^k - R^e - \mu G\left(1+R^k\right)\right\}\frac{\bar{k}q}{n} \\
a_4 &= a_2 + \frac{\gamma\left(1+R^e\right)}{\mu_z\pi} \\
a_8 &= -\frac{\gamma}{\pi\mu_z}\mu G_\omega\left(1+R^k\right)\frac{\bar{k}q}{n}\bar{\omega} \\
a_9 &= \gamma\left(\frac{1+R^e}{\pi}\right)\frac{1}{\mu_z} \\
a_{10} &= -\frac{\gamma}{\pi\mu_z}\mu G_\sigma\left(1+R^k\right)\frac{\bar{k}q}{n}\sigma
\end{aligned}$$

We define the rate of return on capital as follows:

$$R^k_{t+1} = \frac{\left(1-\tau^k\right)\left[u_{t+1}r^k_{t+1} - a(u_{t+1})\right] + (1-\delta)q_{t+1}}{q_t}\pi_{t+1} + \tau^k\delta - 1,$$

where τ^k is the tax rate on capital. Linearizing this expression, we obtain:

$$\hat{R}^k_{t+1} - \frac{(1-\tau^k)r^k + (1-\delta)q}{R^k q}\pi\left[\frac{\left(1-\tau^k\right)r^k\hat{r}^k_{t+1} + (1-\delta)q\hat{q}_{t+1}}{(1-\tau^k)r^k + (1-\delta)q} + \hat{\pi}_{t+1} - \hat{q}_t\right]. \tag{66}$$

A.2.4 Banking Sector

Following is a discussion of the four equations corresponding to the banking sector. For a detailed discussion of these equations, see Chari, Christiano and Eichenbaum (1995). It is useful to begin by developing an expression for the ratio of excess reserves to value added. After imposing the definition of working capital loans, S^w_t, and imposing our scaling convention, (30) reduces to:

$$e_{v,t} = \frac{\left(1-\tau\right)m^b_t\left(1-m_t+x_t\right) - \tau\left(\psi_l w_t l_t + \frac{1}{\mu_z}\psi_k r^k_t k_t\right)}{\left(\frac{1}{\mu_z}(1-\nu^k_t)k_t\right)^\alpha\left((1-\nu^l_t)l_t\right)^{1-\alpha}}.$$

Linearizing this expression about steady state:

$$\begin{aligned}
&-\hat{e}_{v,t} + n_{m^b}\hat{m}^b_t + n_m\hat{m}_t + n_x\hat{x}_t \\
&+\left(n_k - d_k\right)\left[\hat{\bar{k}}_t + \hat{u}_t\right] + n_{r^k}\hat{r}^k_t + n_w\hat{w}_t \\
&+\left(n_l - d_l\right)\hat{l}_t - d_{\nu^k}\hat{\nu}^k_t - d_{\nu l}\hat{\nu}^l_t = 0.
\end{aligned} \tag{67}$$

Here, we have used the condition $k_t = \bar{k}_t u_t$. Also,

$$\begin{aligned}
n_{m^b} &= (1-\tau)\, m^b\, (1-m+x)\, /\bar{n} \\
n_m &= -(1-\tau)\, m^b m/\bar{n} \\
n_x &= (1-\tau)\, m^b x/\bar{n} \\
n_w &= n_l = -\tau \psi_l w l/\bar{n} \\
n_{r^k} &= n_k = -\tau \frac{1}{\mu_z} \psi_k r^k k/\bar{n} \\
n_{\mu_z} &= \tau \frac{1}{\mu_z} \psi_k r^k k/\bar{n},
\end{aligned}$$

where

$$\bar{n} = (1-\tau)\, m^b\, (1-m+x) - \tau \left(\psi_l w l + \frac{1}{\mu_z} \psi_k r^k k \right).$$

Also,

$$\begin{aligned}
d_{\mu_z} &= \frac{-\alpha \left(\frac{1}{\mu_z}(1-\nu^k)k \right)^{\alpha} \left((1-\nu^l)l \right)^{1-\alpha}}{\left(\frac{1}{\mu_z}(1-\nu^k)k \right)^{\alpha} \left((1-\nu^l)l \right)^{1-\alpha}} = -\alpha \\
d_k &= \alpha \\
d_{\nu^k} &= -\alpha \frac{\nu^k}{1-\nu^k} \\
d_l &= 1-\alpha \\
d_{\nu^l} &= -(1-\alpha) \frac{\nu^l}{1-\nu^l}
\end{aligned}$$

where

$$d = \left(\frac{1}{\mu_z}(1-\nu^k)k \right)^{\alpha} \left((1-\nu^l)l \right)^{1-\alpha}.$$

The technology associated with the provision of bank deposits is:

$$x^b\, (e_{v,t})^{-\xi_t}\, e_t^r = \frac{M_t^b - M_t + X_t + S_t^w}{P_t},$$

where the term on the right corresponds to the real value of deposits and e_t^r represents the real value of excess reserves. It is useful to develop expressions for the partial derivative of the function on the left of the equality, which we denote by h, with respect to the excess reserves and labor. The derivative of h with respect to real excess reserves is:

$$h_{e^r,t} = (1-\xi_t)\, x^b\, (e_{v,t})^{-\xi_t},$$

which, in linearized form, is:

$$\hat{h}_{e^r,t} = -\left[\frac{1}{1-\xi} + \log(e_v) \right] \xi \hat{\xi}_t - \xi \hat{e}_{v,t}.$$

The derivative of h with respect to labor is:

$$h_{l^b,t} = (1-\alpha)\,\xi_t x^b\,(e_{v,t})^{1-\xi_t}\left(\frac{\mu_z(1-\nu_t^l)l_t}{(1-\nu_t^k)k_t}\right)^{-\alpha} z_t.$$

Defining $h_{z,l^b,t} = h_{l^b,t}/z_t$ and linearizing $h_{z,l^b,t}$:

$$\hat{h}_{z,l^b,t} = [1-\log(e_v)\,\xi]\,\hat{\xi}_t + (1-\xi)\,\hat{e}_{v,t} - \alpha\left[-\frac{\nu^l\hat{\nu}_t^l}{1-\nu^l} + \hat{l}_t + \frac{\nu^k\hat{\nu}_t^k}{1-\nu^k} - \hat{k}_t\right].$$

The first order conditions in the Lagrangian representation of the bank problem associated with A_t and S_t^w are given in (44) from (45) above. The first order condition for l_t^b is:

$$-\left(1+\psi_{l,t}R_t\right)W_t + \lambda_t^b h_{l^b,t} = 0. \tag{68}$$

Substituting for λ_t^b in (68) from (45), and taking into account our scaling convention, we obtain:

$$0 = \frac{R_t h_{z,l^b,t}}{1+\tau h_{e^r,t}} - (1+\psi_l R_t)\,w_t.$$

Linearizing this, and taking into account the expressions for $\hat{h}_{e^r,t}$ and $\hat{h}_{z,l^b,t}$, we obtain:

$$\begin{aligned} 0 &= l_R\hat{R}_t + l_\xi\hat{\xi}_t - \hat{w}_t + l_e\hat{e}_{v,t} \\ &\quad + l_{\nu^l}\hat{\nu}_t^l + l_{\nu^k}\hat{\nu}_t^k + l_l\hat{l}_t + l_k\hat{k}_t, \end{aligned} \tag{69}$$

where

$$\begin{aligned} l_i &= k_i \text{ for all } i, \text{ except} \\ l_R &= \left[1-\frac{\psi_l R}{1+\psi_l R}\right],\ l_{\psi l} = -\frac{\psi_l R}{1+\psi_l R} \\ l_\mu &= k_\mu - 1,\ l_{\nu^l} = k_{\nu^l} + \frac{\nu^l}{1-\nu^l},\ l_l = k_l - 1, \\ l_{\nu^k} &= k_{\nu^k} - \frac{\nu^k}{1-\nu^k},\ l_k = k_k + 1, \end{aligned}$$

and

$$\begin{aligned} k_R &= \left[1-\frac{\psi_k R}{1+\psi_k R}\right],\ k_\xi = 1-\log(e_v)\,\xi + \frac{\tau h_{e^r}\left[\frac{1}{1-\xi}+\log(e_v)\right]\xi}{1+\tau h_{e^r}} \\ k_e &= 1-\xi+\frac{\tau h_{e^r}\xi}{1+\tau h_{e^r}}, \\ k_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l},\ k_{\nu^k} = (1-\alpha)\frac{\nu^k}{1-\nu^k},\ k_l = (1-\alpha),\ k_k = -(1-\alpha). \end{aligned}$$

Recalling that the ratio of (45) to (44) is:

$$R_{at} = \frac{(1-\tau)\,h_{e^r,t}-1}{\tau h_{e^r,t}+1}R_t.$$

we can linearize this expression. Taking into account the expressions for $\hat{h}_{e^r,t}$ and $\hat{h}_{z,l^b,t}$, we obtain:

$$\begin{aligned}&\hat{R}_{at}-h_{e^r}\left[\frac{1-\tau}{(1-\tau)\,h_{e^r}-1}-\frac{\tau}{\tau h_{e^r}+1}\right]\left[-\left(\frac{1}{1-\xi}+\log\left(e_v\right)\right)\xi\hat{\xi}_t-\xi\hat{e}_{v,t}\right] \\ &-\hat{R}_t=0\end{aligned} \tag{70}$$

Then we scale the production function for real deposits:

$$\begin{aligned}x^b\left(e_{v,t}\right)^{-\xi_t}e^r_{z,t} &= \frac{M^b_t-M_t+X_t+\left(\psi_{l,t}W_tl_t+\psi_{k,t}P_tr^k_tK_t\right)}{z_tP_t} \\ &= m_{1t}+m_{2t},\end{aligned}$$

where

$$\begin{aligned}m_{1t} &= m^b_t\left(1-m_t+x_t\right) \\ m_{2t} &= \psi_lw_tl_t+\psi_kr^k_tk_t/\mu_z \\ e^r_{z,t} &= \frac{e^r_t}{z_t}.\end{aligned}$$

Linearizing this, we obtain:

$$\begin{aligned}&0=\xi\hat{e}_{v,t}+\log\left(e_v\right)\xi\hat{\xi}_t \\ &+\left[\frac{m_1}{m_1+m_2}-\frac{(1-\tau)m_1}{(1-\tau)m_1-\tau m_2}\right]\left[\hat{m}^b_t+\frac{-m\hat{m}_t}{1-m+x}\right] \\ &+\left[\frac{m_2}{m_1+m_2}+\frac{\tau m_2}{(1-\tau)m_1-\tau m_2}\right] \\ &\times[\frac{\psi_lwl}{\psi_lwl+\psi_kr^kk/\mu_z}\left(\hat{w}_t+\hat{l}_t\right)+\frac{\psi_kr^kk/\mu_z}{\psi_lwl+\psi_kr^kk/\mu_z}\left(\hat{r}^k_t+\hat{k}_t\right)].\end{aligned} \tag{71}$$

It is of interest to note that monetary policy cannot exactly neutralize the $\hat{\xi}_t$ shock. From (66), we see that the impact of $\hat{x}_t$ on $\hat{e}_{v,t}$ is determined by n_x. So, according to (70), neutralizing the $\hat{\xi}_t$ shock requires setting the component in $\hat{x}_t$ pertaining to $\hat{\xi}_t$, $\hat{x}_{\hat{\xi}_t,t}$ as follows:

$$\hat{x}_{\hat{\xi}_t,t}=-\frac{1}{n_x}\left(\frac{1}{1-\xi}+\log\left(e_v\right)\right)\hat{\xi}_t.$$

The same reasoning applied to (71) implies:

$$\hat{x}_{\hat{\xi}_t,t}=-\frac{1}{n_x}\log\left(e_v\right)\hat{\xi}_t$$

Evidently, these two equations cannot both be satisfied at the same time.

A.2.5 Household Sector

We now turn to the equations associated with the household sector. It is useful to define $u_{c,t}$ as the derivative of the present discounted value of utility with respect to C_t :

$$E_t\left\{u_{c,t} - u'(C_t - bC_{t-1}) + b\beta u'(C_{t+1} - bC_t)\right\} = 0.$$

Using the definition, $u^z_{c,t} = z_t u_{c,t}$, and our functional form assumption, this reduces to:

$$E_t\left[u^z_{c,t} - \frac{\mu_z}{c_t\mu_z - bc_{t-1}} + b\beta\frac{1}{c_{t+1}\mu_z - bc_t}\right] = 0.$$

Linearizing this expression:

$$\begin{aligned} E_t\{u^z_c\hat{u}^z_{c,t} + \frac{\mu_z^2 + \beta b^2}{c^2\left(\mu_z - b\right)^2}c\hat{c}_t \\ -\frac{b\beta\mu_z}{c^2\left(\mu_z - b\right)^2}c\hat{c}_{t+1} - \frac{b\mu_z}{c^2\left(\mu_z - b\right)^2}c\hat{c}_{t-1}\} \quad &= \quad 0. \end{aligned} \tag{72}$$

The household's first order condition with respect to time deposits is:

$$E_t\left\{-\lambda_t + \beta\lambda_{t+1}\left[1 + R^e_{t+1}\right]\right\} = 0$$

To scale this, we multiply by z_tP_t :

$$E_t\left\{-\lambda_{z,t} + \frac{\beta}{\mu_z\pi_{t+1}}\lambda_{z,t+1}\left[1 + R^e_{t+1}\right]\right\} = 0,$$

where $\lambda_{z,t} = \lambda_t z_t P_t$. Linearizing this expression:

$$E\left\{-\hat{\lambda}_{z,t} + \hat{\lambda}_{z,t+1} - \hat{\pi}_{t+1} + \frac{R^e}{1+R^e}\hat{R}^e_{t+1}\right\} = 0. \tag{73}$$

The household's first order condition with respect to currency, M_t, is:

$$\begin{aligned} E_t\{\upsilon_t\left[\left(\frac{P_tC_t}{M_t}\right)^{\theta_t}\left(\frac{P_tC_t}{M^b_t - M_t + X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}[\frac{\theta_t}{M_t} - \frac{(1-\theta_t)}{M^b_t - M_t + X_t}] \\ -\lambda_t R_{at}\} = 0 \end{aligned}$$

We scale this by multiplying both sides by z_tP_t :

$$\begin{aligned} \upsilon_t\left[c_t\left(\frac{1}{m_t}\right)^{\theta_t}\left(\frac{1}{1 - m_t + x_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}[\frac{\theta_t}{m_t} - \frac{1-\theta_t}{1 - m_t + x_t}]\left(\frac{1}{m^b_t}\right)^{2-\sigma_q} \\ -\lambda_{zt}R_{at} = 0 \end{aligned}$$

Linearizing this expression:

$$\begin{aligned}\{\ \hat{\upsilon}_t + (1-\sigma_q)\,\hat{c}_t + \left[-(1-\sigma_q)\left(\theta-(1-\theta)\frac{m}{1-m+x}\right) - \frac{\frac{\theta}{m}+\frac{1-\theta}{(1-m+x)^2}m}{\frac{\theta}{m}-\frac{1-\theta}{1-m+x}}\right]\hat{m}_t \\ - \left[\frac{(1-\sigma_q)\,(1-\theta)\,x}{1-m+x} - \frac{\frac{1-\theta}{(1-m+x)^2}x}{\frac{\theta}{m}-\frac{1-\theta}{1-m+x}}\right]\hat{x}_t \\ + \left[-(1-\sigma_q)\,(\log(m)-\log(1-m+x)) + \frac{1+x}{\theta\,(1+x)-m}\right]\theta\hat{\theta}_t \\ - (2-\sigma_q)\,\hat{m}^b_t - \left(\hat{\lambda}_{z,t}+\hat{R}_{a,t}\right)\} &= 0\end{aligned}$$

The first order condition with respect to M^b_{t+1} is:

$$\begin{aligned}&E_t\{\beta\upsilon_{t+1}\,(1-\theta_{t+1})\left[P_{t+1}C_{t+1}\left(\frac{1}{M_{t+1}}\right)^{\theta_{t+1}}\left(\frac{1}{M^b_{t+1}-M_{t+1}+X_{t+1}}\right)^{(1-\theta_{t+1})}\right]^{1-\sigma_q}\times \\ &\times\frac{1}{M^b_{t+1}-M_{t+1}+X_{t+1}}+\beta\lambda_{t+1}\,[1+R_{a,t+1}]-\lambda_t\} \\ =\ &0.\end{aligned}$$

Scaling this by multiplying by $P_t z_t$:

$$\begin{aligned}E_t\{\beta\upsilon_{t+1}\,(1-\theta_{t+1})\left[c_{t+1}\left(\frac{1}{m_{t+1}}\right)^{\theta_{t+1}}\left(\frac{1}{1-m_{t+1}+x_{t+1}}\right)^{(1-\theta_{t+1})}\right]^{1-\sigma_q} \\ \times\left(\frac{1}{m^b_{t+1}}\right)^{2-\sigma_q}\frac{1}{\pi_{t+1}\mu_z}\frac{1}{1-m_{t+1}+x_{t+1}} \\ +\beta\frac{1}{\pi_{t+1}\mu_z}\lambda_{z,t+1}\,[1+R_{a,t+1}]-\lambda_{z,t}\}=0 \quad .\end{aligned}$$

Linearizing:

$$\begin{aligned}&E_t\{\frac{\beta}{\pi\mu_z}\upsilon\,(1-\theta)\left[c\left(\frac{1}{m}\right)^{\theta}\right]^{1-\sigma_q}\left(\frac{1}{1-m+x}\right)^{(1-\theta)(1-\sigma_q)+1}\left(\frac{1}{m^b}\right)^{2-\sigma_q} \\ &\times\{\hat{\upsilon}_{t+1}-\frac{\theta\hat{\theta}_{t+1}}{1-\theta}+(1-\sigma_q)\hat{c}_{t+1}-(1-\sigma_q)\log(m)\,\theta\hat{\theta}_{t+1}-\theta(1-\sigma_q)\hat{m}_{t+1} \\ &-[(1-\theta)\,(1-\sigma_q)+1]\left(\frac{1}{1-m+x}\right)[x\hat{x}_{t+1}-m\hat{m}_{t+1}] \qquad (74) \\ &+(1-\sigma_q)\log(1-m+x)\,\theta\hat{\theta}_{t+1}-(2-\sigma_q)\,\hat{m}^b_{t+1}\} \\ &+\frac{\beta}{\pi\mu_z}\lambda_z\,[1+R_a]\,\hat{\lambda}_{z,t+1} \\ &+\frac{\beta}{\pi\mu_z}\lambda_z R_a\hat{R}_{a,t+1}-\lambda_z\left[\hat{\lambda}_{zt}+\hat{\pi}_{t+1}\right]\}\end{aligned}$$

$$= 0.$$

The household's first order condition for consumption is:

$$E_t\left\{u_{c,t} - \upsilon_t C_t^{-\sigma_q}\left[\left(\frac{P_t}{M_t}\right)^{\theta_t}\left(\frac{P_t}{M_t^b - M_t + X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q} - \left(1+\tau_t^c\right)P_t\lambda_t\right\} = 0.$$

Scaling this by multiplying by z_t, we obtain:

$$E_t\left\{u_{c,t}^z - \upsilon_t c_t^{-\sigma_q}\left[\frac{1}{m_t^b}\left(\frac{1}{m_t}\right)^{\theta_t}\left(\frac{1}{1-m_t+x_t}\right)^{1-\theta_t}\right]^{1-\sigma_q} - \left(1+\tau^c\right)\lambda_{z,t}\right\} = 0.$$

Linearizing this expression,

$$\begin{aligned}
&E_t\{u_c^z\hat{u}_{c,t}^z - \upsilon c^{-\sigma_q}\left[\frac{1}{m^b}\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q} \\
&\times[\hat{\upsilon}_t - \sigma_q\hat{c}_t + (1-\sigma_q)\left(-\hat{m}_t^b - \theta_t\hat{m}_t - (1-\theta_t)\left(\frac{-m}{1-m+x}\hat{m}_t + \frac{x}{1-m+x}\hat{x}_t\right)\right) \\
&+ (1-\sigma_q)\left[\log\left(\frac{1}{m}\right) - \log\left(\frac{1}{1-m+x}\right)\right]\theta\hat{\theta}_t] \\
&- \left(1+\tau^c\right)\lambda_z\hat{\lambda}_{z,t}\} = 0
\end{aligned} \tag{75}$$

The linearized expression for wages is taken directly from Christiano, Eichenbaum and Evans (2003):

$$E_t\left\{\eta_0\hat{w}_{t-1} + \eta_1\hat{w}_t + \eta_2\hat{w}_{t+1} + \eta_{\bar{3}}\hat{\pi}_{t-1} + \eta_3\hat{\pi}_t + \eta_4\hat{\pi}_{t+1} + \eta_5\hat{l}_t + \eta_6\hat{\lambda}_{z,t} + \eta_7\hat{\zeta}_t\right\} = 0 \tag{76}$$

where

$$\eta = \begin{pmatrix} b_w\xi_w \\ -b_w\left(1+\beta\xi_w^2\right) + \sigma_L\lambda_w \\ \beta\xi_w b_w \\ b_w\xi_w \\ -\xi_w b_w\left(1+\beta\right) \\ b_w\beta\xi_w \\ -\sigma_L\left(1-\lambda_w\right) \\ 1-\lambda_w \\ -\left(1-\lambda_w\right) \end{pmatrix} = \begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_{\bar{3}} \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \end{pmatrix}.$$

A.2.6 Aggregate Restrictions

The condition that the use of final goods equals the supply of final goods implies:

$$\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\frac{Q_{\bar{K}',t-1}}{P_t}\bar{K}_t + a(u_t)\bar{K}_t + G_t + C_t + I_t \leq z_t^{1-\alpha}\epsilon\left(\nu_t^k K_t\right)^{\alpha}\left(\nu_t^l L_t\right)^{1-\alpha} - z_t\phi,$$

Here we have ignored additional terms in the aggregate resource constraint which appear when labor and capital are misallocated across the intermediate goods firms. For a justification, see the argument in Christiano, Eichenbaum and Evans (2003), which builds on the important work of Yun (1996). Scaling the goods constraint by z_t:

$$d_t + a(u_t)\frac{1}{\mu_{z,t}}\bar{k}_t + g_t + c_t + i_t \leq \epsilon_t \left(u_t \frac{1}{\mu_{z,t}}\nu_t^k \bar{k}_t\right)^{\alpha} \left(\nu_t^l L_t\right)^{1-\alpha} - \phi,$$

where,

$$d_t = \mu G(\bar{\omega}_t, \sigma_{t-1}) \left(1 + R_t^k\right) q_{t-1} \frac{1}{\mu_{z,t}} \frac{1}{\pi_t} \bar{k}_t.$$

Linearizing the goods constraint:

$$\begin{aligned} 0 = d_y \left[\frac{G_\omega}{G}\bar{\omega}\widehat{\bar{\omega}}_t + \frac{G_\sigma}{G}\sigma\hat{\sigma}_{t-1} + \frac{R^k}{1+R^k}\hat{R}_t^k + \hat{q}_{t-1} + \widehat{\bar{k}}_t - \hat{\pi}_t\right] + u_y\hat{u}_t + c_y\hat{c}_t + \bar{k}_y\frac{i}{\bar{k}}\hat{\imath}_t \quad (77) \\ -\alpha\left(\hat{u}_t + \widehat{\bar{k}}_t + \hat{\nu}_t^k\right) - (1-\alpha)\left(\hat{l}_t + \hat{\nu}_t^l\right) - \hat{\epsilon}_t \end{aligned}$$

where

$$\begin{aligned} c_y &= \frac{c}{y+\phi+d},\ d_y = \frac{d}{y+\phi+d} \\ v_y &= \frac{v}{y+\phi+d},\ u_y = \frac{a'\frac{1}{\mu_z}\bar{k}}{y+\phi+d}, \\ \bar{k}_y &= \frac{\bar{k}}{y+\phi+d}. \end{aligned}$$

The capital accumulation equation is:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + F(I_t, I_{t-1}).$$

Scaling, by dividing both sides by z_t :

$$\bar{k}_{t+1} = (1-\delta)\frac{1}{\mu_z}\bar{k}_t + \frac{F(I_t, I_{t-1})}{z_t}.$$

Linearizing and taking into account the restrictions on F discussed above,

$$\widehat{\bar{k}}_{t+1} - \frac{1-\delta}{\mu_z}\left(\widehat{\bar{k}}_t - \hat{\mu}_{z,t}\right) - \frac{i}{\bar{k}}\hat{\imath}_t = 0. \quad (78)$$

A.2.7 Monetary Policy

The monetary base evolves as follows:

$$M_{t+1}^b = M_t^b(1 + x_t),$$

where x_t is the net growth rate of the monetary base. The law of motion of the scaled monetary base, $m_t^b = M_t^b/(P_t z_t)$, is:

$$\frac{M_{t+1}^b}{P_{t+1}z_{t+1}} = \frac{P_t z_t}{P_{t+1}z_{t+1}} \frac{M_t^b}{P_t z_t}(1+x_t),$$

or,

$$m_{t+1}^b = \frac{1}{\pi_{t+1}\mu_z} m_t^b (1+x_t).$$

Linearizing this expression,

$$\hat{m}_{t-1}^b + \frac{x}{1+x}\hat{x}_{t-1} - \hat{\pi}_t - \hat{m}_t^b = 0. \tag{79}$$

Monetary policy has the following representation:

$$\hat{x}_t = \sum_{i=1}^{p} x_{it}, \tag{80}$$

where the x_{it}'s are functions of the underlying shocks, as discussed in section 3.7.

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Table 1: Behavior of Investment to GNP Ratio in Recessions

Peak to Trough	Peak I/Y	Trough I/Y
1929III-1933I	0.25	0.06
1948IV-1949IV	0.26	0.23
1953II-1954II	0.24	0.22
1957III-1958II	0.24	0.21
1960II-1961I	0.23	0.21
1969IV-1970IV	0.24	0.22
1973IV-1975I	0.26	0.22
1980I-1982IV	0.27	0.22
1990III-1991I	0.23	0.21
2001I-2001IV	0.26	0.26

Notes: Source for post-war business cycle data: Bureau of Economic Analysis Website. I - Nominal Household Purchases of Durable Goods, plus Gross Private Domestic Investment, Y - Nominal Gross Domestic Product

Table 2: Model Parameters (Time unit of Model: quarterly)		
Panel A: Household Sector		
β	Discount rate	$1.03^{-0.25}$
ψ_L	Weight on Disutility of Labor	145.32
σ_L	Curvature on Disutility of Labor	1.00
υ	Weight on Utility of Money	2e-008
σ_q	Curvature on Utility of money	-10.00
θ	Power on Currency in Utility of money	0.75
b	Habit persistence parameter	0.63
ξ_w	Fraction of households that cannot reoptimize wage within a quarter	0.70
λ_w	Steady state markup, suppliers of labor	1.05
Panel B: Goods Producing Sector		
μ_z	Growth Rate of Technology (APR)	1.50
S''	Curvature on Investment Adjustment Cost	7.69
σ_a	Curvature on capital utilization cost function	0.01
ξ_p	Fraction of intermediate good firms that cannot reoptimize price within a quarter	0.50
ψ_k	Fraction of capital rental costs that must be financed	0.70
ψ_l	Fraction of wage bill that must be financed	1.00
δ	Depreciation rate on capital.	0.02
α	Power on capital in production function	0.36
λ_f	Steady state markup, intermediate good firms	1.20
Φ	Fixed cost, intermediate goods	0.036
Panel C: Entrepreneurs		
γ	Percent of Entrepreneurs Who Survive From One Quarter to the Next	97.00
μ	Fraction of Realized Profits Lost in Bankruptcy	0.120
$F(\bar{\omega})$	Percent of Businesses that go into Bankruptcy in a Quarter	0.80
$Var(log(\omega))$	Variance of (Normally distributed) log of idiosyncratic productivity parameter	0.07
Panel D: Banking Sector		
ξ	Power on Excess Reserves in Deposit Services Technology	0.9690
x^b	Constant In Front of Deposit Services Technology	82.1902
Panel E: Policy		
τ	Bank Reserve Requirement	0.100
τ^c	Tax Rate on Consumption	0.00
τ^k	Tax Rate on Capital Income	0.29
τ^l	Tax Rate on Labor Income	0.04
x	Growth Rate of Monetary Base (APR)	1.610

Table 3: Steady State Properties of the Model, Versus US Data			
Variable	Model	US, 1921-29	US, 1964-2001
$\frac{k}{y}$	8.35	10.8[1]	9.79
$\frac{i}{y}$	0.20	0.24	0.25
$\frac{c}{y}$	0.73	0.67	0.57
$\frac{g}{y}$	0.07	0.07	0.19
r^k	0.043		
$\frac{N}{K-N}$ ('Equity to Debt')	0.999	1-1.25[2]	1-1.25[2]
$\frac{W^e}{py}$	0.055		
Percent of Goods Output Lost to Bankruptcy	0.371%		
Percent of Aggregate Labor and Capital in Banking	1.00%	1%[3]	2.5%[5]
Inflation (APR)	0.11%	-0.6%[4]	4.27%[6]

Note: [1]End of 1929 stock of capital, divided by 1929 GNP, obtained from CKM. [2]Masoulis (1988) reports that the debt to equity ratio for US corporations averaged 0.5 - 0.75 in the period 1937-1984. [3]Share of value-added in the banking sector, according to Kuznets (1941), 1919-1938. [4] Average annual inflation, measured using the GNP deflator, over the period 1922-1929. [5]Based on analysis of data on the finance, insurance and real estate sectors [6] Average annual inflation measured using GNP deflator.

Table 4: Consolidated Banking Sector Balance Sheet, Model versus US Data							
Variable	Model	1921-1929	1995-2001	Variable	Model	1921-1929	1995-2001
Assets (Fraction of Annual GNP)	1.296	0.722	0.604	Liabilities (Fraction of Annual GNP)	1.296		0.604
Total Reserves	0.122	0.152	0.081	Total Demand Deposits	1.000	1.0	1.0
∘ Required Reserves	0.100	0.118	0.052	∘ Firm Demand Deposits	0.878		0.523
∘ Excess Reserves	0.022	0.034	0.029	∘ Household Demand Deposits	0.122		0.477
Working Capital Loans	0.878	0.848	0.919				
∘ Capital Rental Expenses	0.249						
∘ Wage Bill Expenses	0.629						
Entrepreneurial Loans	0.803	0.525	0.828	Time Deposits	0.803	0.525	0.828

Notes on Table 4: Total assets consists of reserves plus working capital loans plus loans to entrepreneurs. The first line shows the ratio of these to annual goods output. With the exception of the bottom row of numbers, remaining entries in the table are expressed as a fraction of bank reserves plus working capital loans. The bottom row of numbers is expressed as a fraction of total assets.

Data for the period 1995-2001: we define working-capital loans as total demand deposits minus total reserves. This number is the same order of magnitude as the sum of short-term bank loans with maturity 24 months or less (taken from the Fed's 'Banking and Monetary Statistics') and commerical paper (Table L101 in Flow of Funds) Long-term entrepreneurial loans are defined as the total liabilities of the non-financial business sector (non-farm non-financial corporate business plus non-farm non-corporate business plus farm business) net of municipal securities, trade payables, taxes payables, 'miscellaneous liabilites' and the working capital loans. Source: With exception of required and excess reserves, the source is the Federal Reserves' Flow of Funds' data. Required and excess reserves are obtained from Federal Reserve Bank of St. Louis.

Data for the period 1921-1929: we define working-capital loans as total demand deposits minus total reserves for all banks. Entrepreneurial loans are constructed on the basis of all bank loans minus working capital loans plus outstanding bonds issued by all industries. Source: Banking and Monetary Statistics, Board of Governorns, September 1943, and NBER Historical Database.

Table 5: Money and Interest Rates, Model versus US Data							
Money	Model	1921-1929	1964-2002	Interest Rates (APR)	Model	1921-1929	1964-2002
Monetary Base Velocity	9.77	12	16.6	Demand Deposits	1.07		3.21
M1 Velocity	3.92	3.5	6.5	Time Deposits	4.66		6.96
				Rate of Return on Capital	6.91		17.33
Currency / Demand Deposits	0.28	0.2	0.3	Entrepeneurial Standard Debt Contract	5.31	5.74	8.95
Currency / Monetary Base	0.70	0.55	0.73	Interest Rate on Working Capital Loans	4.76	4.72	7.10
Curr. / Household D. Deposit	2.30			Interbank Loan Rate	5.87	3.90	6.86

Notes to Table 5:

Data for 1921-1929: (1) 'Federal Funds Rate' is the average of Bankers' Acceptances Rate. (2) Interest rate on working capital loans is the commerical paper rate. (3) Rate on loans to entrepreneurs is the average between AAA and BAA corporate bonds. (4) Rate on time deposits is available only from 1933 onwards. Reported data in Board of Governors (1943) only cite the administrative rate (maximum rate) set by the Fed. The average of this rate was 2.7% over the period 1933-41. (5) There are no data available on the rate paid on demand deposits (to our knowledge).

Data for 1964-2002:

(1) The Federal Funds Rate is over the period 1964.3-2002.3. Source: Federal Reserve Board of Governors. (2) The rate on demand deposits is the 'Money Zero Maturity Own Rate' (1964.3-2002.3). Source: Federal Reserve Bank of Saint Louis. (3) The rate on loans to entrepreneurs is the average between AAA and BAA corporate bonds (1964.3-2002.3). Source: Federal Reserve Board of Governors. (4) The rate on time deposit is the rate on 3-month CDs (1964.3-2002.3). Source: Federal Reserve Board of Governors. (5) The rate of return on capital is the rate of profit on stockholders' equity for the manufacturing sector (1980.1-2001.4). Source: Bureau of the Census (2002), Table I. (6) The rate on Working Capital Loans is the rate on Commercial paper (dealer-placed unsecured short-term negotiable promissory note issued by companies with Aa bond ratings and sold to investors). Average over 1971.2-2002.3. Source: Federal Reserve Board of Governors. (7) The Currency to M1 ratio is an average over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis. (8) The Currency to Monetary Base ratio is the average over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis. (9) The Monetary Base and M1 velocities are averages over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis.

Table 6: Parameters of Exogenous Shocks and Monetary Policy Response

	Shock, $\lambda_t : \lambda_t = \rho\lambda_{t-1} + \varphi_t$, $E(\varphi_t)^2 = \sigma^2$						
	Monetary Policy, $x_t : x_t = \theta^2 x_{t-1} + \theta^0 \varphi_t$						
Shock	ρ	σ	(steady state, steady state plus one-σ innovation)$^{(i)}$	θ^2	θ^0	% contemporaneous impact on base$^{(ii)}$	% long-run impact on base$^{(iii)}$
firm markup, $\hat{\lambda}_{f,t}$	0.932	0.0090	$(1.20, 1.21)$	0.499	12.813	0.046	0.09
bank demand for reserves, $d\xi_t$	0.999	0.1962	$(0.969, 0.975)$	0.956	-0.380	-0.030	-0.68
money demand, $d\theta_t$	1.000	0.2067	$(0.750, 0.789)$	0.941	-4.648	-0.38	-6.51
household market power, $\hat{\zeta}_t$	1.000	0.0602	$(1.00, 1.06)$	0.792	-10.120	-0.24	-1.17
liquidity demand, $\hat{\upsilon}_t$	0.981	0.3586	$(2.00, 2.72) \times 10^{-8}$	0.265	-0.078	-0.011	-0.015
rate of survival of entrepreneurs, $d\gamma_t$	0.588	1.2354	$(0.970, 1.006)$	0.529	-0.840	-0.42	-0.88
shock to technology of goods-producing sector, $\hat{\epsilon}_t$	0.935	0.0031	$(1.000, 1.003)$	0.868	101.896	0.13	0.96
shock to riskiness of entrepreneurs, $\hat{\sigma}_t$	1.000	0.0890	$(0.27, 0.29)$	0.235	5.994	0.21	0.28

Notes - (i) (y, z), y ~steady state value of shock, $z = y + y \times \sigma$ for variables with hat, $z = y + y \times (1 - y) \times \sigma$ for variables with a d,

(ii) $100 \times x \times \theta^0 \times \sigma$, (iii) $100 \times x \times \theta^0 \times \sigma/(1 - \theta^2)$, where x is the steady state net growth rate of the monetary base. See text for elaboration.

Table 7: Estimated Standard Error of Measurement Error in Indicated Variables

Variable	σ_w	$\frac{\sigma_w^2}{\sigma_X^2}$	Variable	σ_w	$\frac{\sigma_w^2}{\sigma_X^2}$
$\log\left(\frac{\tilde{N}_{t+1}}{P_t Y_t}\right)$	1.7×10^{-2} (3.7×10^{-3})	0.000	$\log(\frac{I_t}{Y_t})$	1.4×10^{-1} (1.4×10^{-1})	0.149
$\log(\pi_t)$	1.3×10^{-2} (1.3×10^{-2})	0.473	$\log(V_t^1)$	4.1×10^{-2} (4.4×10^{-2})	0.106
$\log(l_t)$	1.4×10^{-4} (3.4×10^{-6})	0.000	$\log(\frac{C_t}{Y_t})$	5.0×10^{-2} (4.3×10^{-2})	0.463
R_t^b	3.4×10^{-6} (9.5×10^{-9})	0.000	P_t^e	5.9×10^{-4} (4.6×10^{-4})	0.035
$\Delta\log(Y_t)$	3.1×10^{-2} (3.2×10^{-2})	0.730	$\log(d_t^c)$	6.1×10^{-2} (6.0×10^{-2})	0.079
$\log\left(\frac{W_t}{P_t Y_t}\right)$	3.9×10^{-2} (3.7×10^{-2})	0.052	$\log(V_t^b)$	5.3×10^{-2} (6.8×10^{-2})	0.034
			$\log(d_t^r)$	2.4×10^{-2} (1.8×10^{-2})	0.002

Notes: (i) σ_w not in parentheses - point estimate of standard deviation of measurement error from maximum likelihood estimation; (ii) σ_w in parentheses - standard deviation of vertical distance between actual and fitted data in Figure 4; (iii) ratio of σ_w^2 from (ii) to σ_X^2, the sample variance of raw data on variable in first column

Table 8: Annualized Percent Growth Rates, Data Versus Alternative Models

	1929-1933				1933-1939				1929-1939			
	(1)[(i)]	(2)	(3)	(4)[(ii)]	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)[(ii)]
Output	-9.2	-6.4	-3.8	-1.6	5.1	2.6	1.9	2.4	-0.6	-1.0	-0.4	0.8
Investment	-35.3	-28.5	-17.3	-15.7	16.9	13.2	4.1	13.8	-4.0	-3.5	-4.5	2.0
Hours Worked	-7.3	-7.3	-5.0	-2.1	1.4	1.4	1.5	0.2	-2.1	-2.1	-1.1	-0.7
Price Level	-7.6	-9.2	-7.8	-6.5	1.7	2.1	-1.6	9.7	-2.0	-2.4	-4.1	3.2
M1	-9.1	-8.5	-7.3	-1.7	8.1	6.2	1.4	12.8	1.2	0.3	-2.1	7.0
Real Wage	0.4	2.8	1.3	2.6	4.0	2.2	1.4	2.1	2.6	2.4	1.4	2.3
TFP	-2.3	-1.1	-0.4	-0.8	3.2	1.8	1.2	1.6	1.0	0.6	0.6	0.6
Firm Wedge	-9.0	-7.3	-0.5	-8.3	-2.0	-6.1	-5.6	0.5	-11.1	-13.3	-6.1	-7.8
Household Wedge[(iii)]	54.9	43.8	30.7	23.7[(iv)]	1.3	7.0	-8.1	42.3	56.2	50.8	22.6	65.9[(iv)]

Note: statistics denote $100\times \log(x_j/x_i)$, where x_j and x_i denote average values of x in the indicated years

(except where noted) except in the last two rows, numbers are converted to annualized, percent terms

(i) (1) data; (2) estimated model with all shocks; (3) estimated model with only υ_t shock;

(4) results for policy counterfactual (ii) results for 1929 correspond to 1929IV

(iii) results for 1939 represent average over 1939I-1939III, (iv) results for 1929 are actually average for 1930

Figure 1: Key Macroeconomic Data for the 1920s and 1930s (fourth Quarters after 1928 Indicated by *)

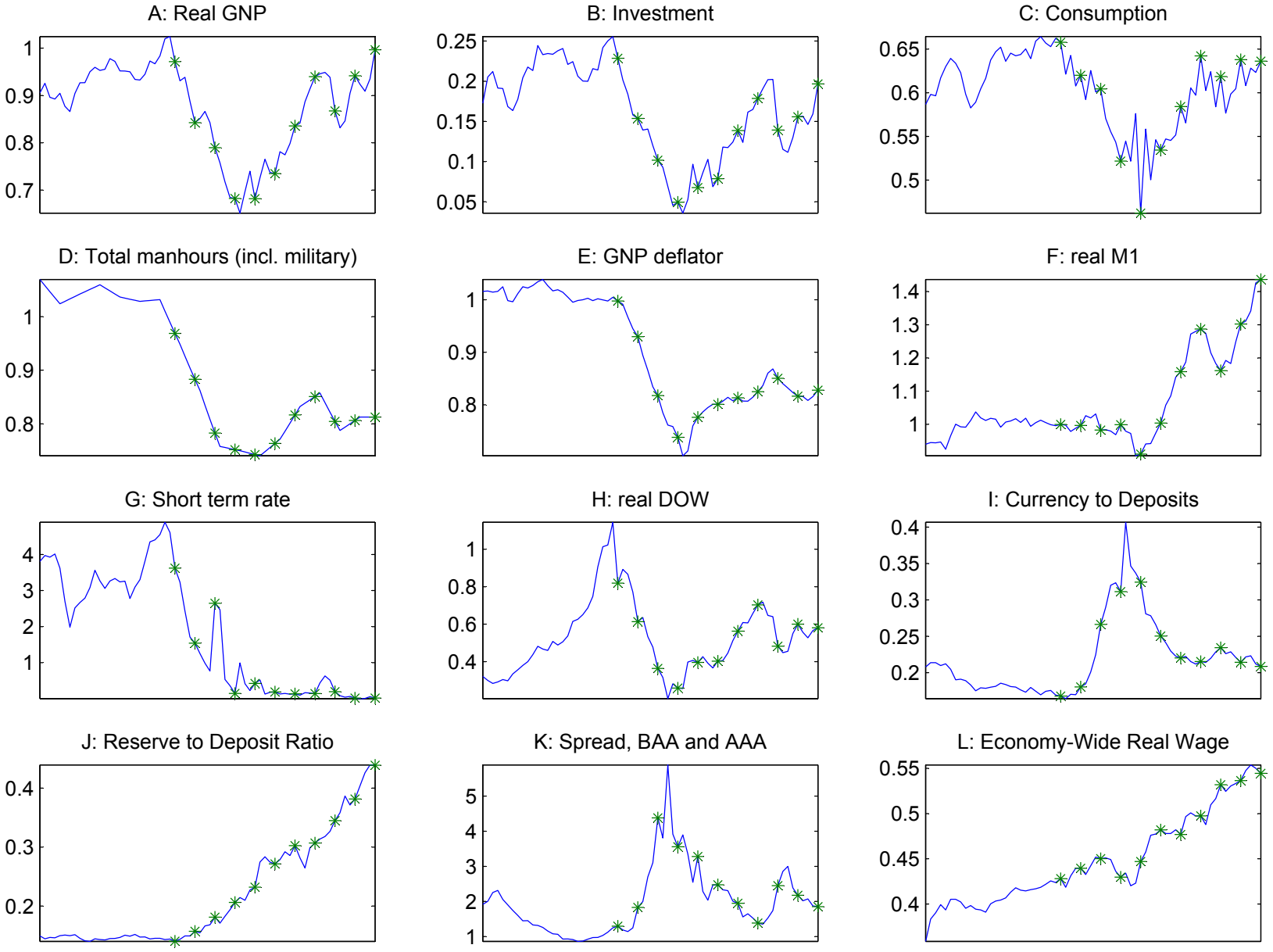
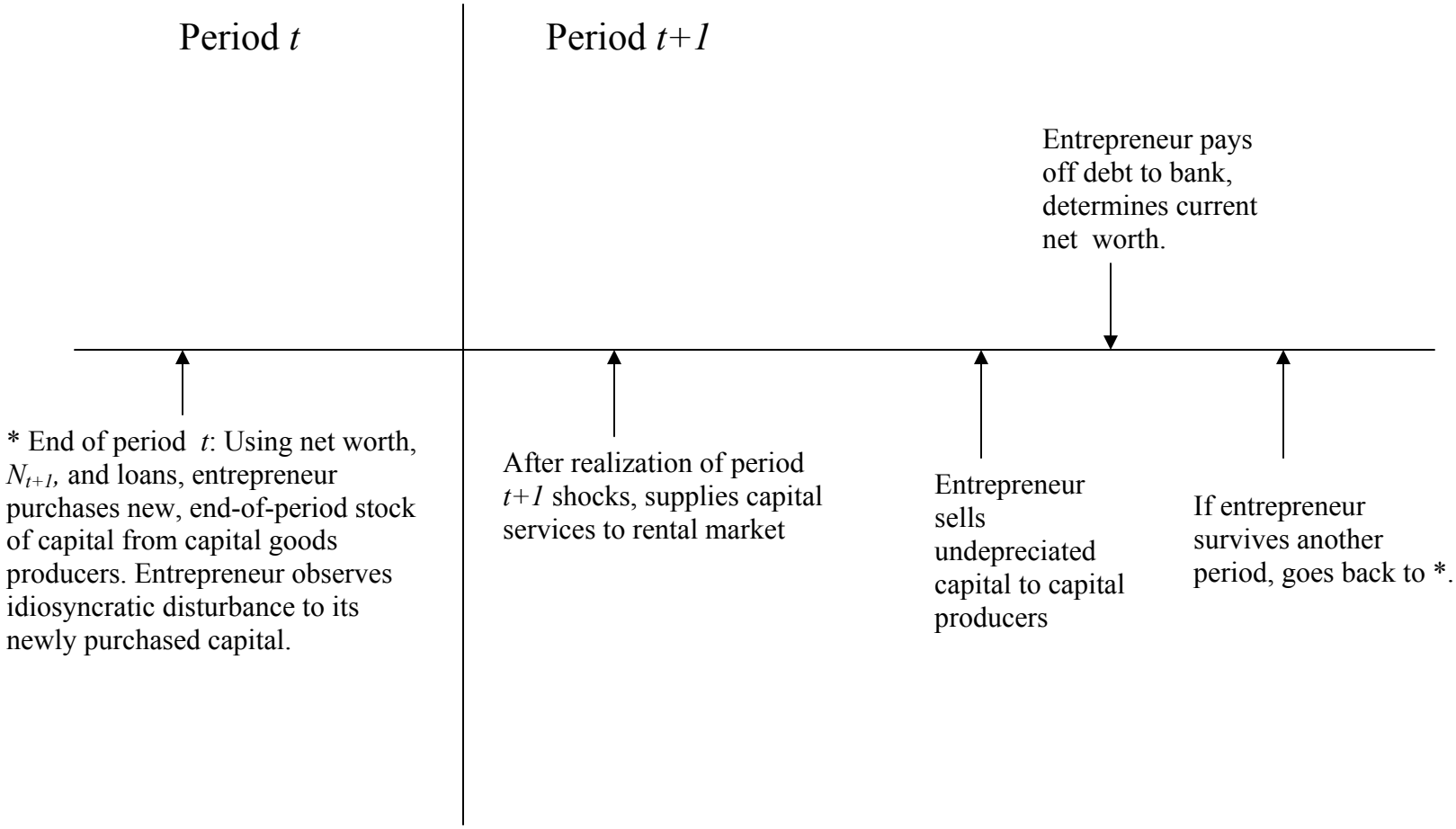


FIGURE 2: A Day in the Life of an Entrepreneur



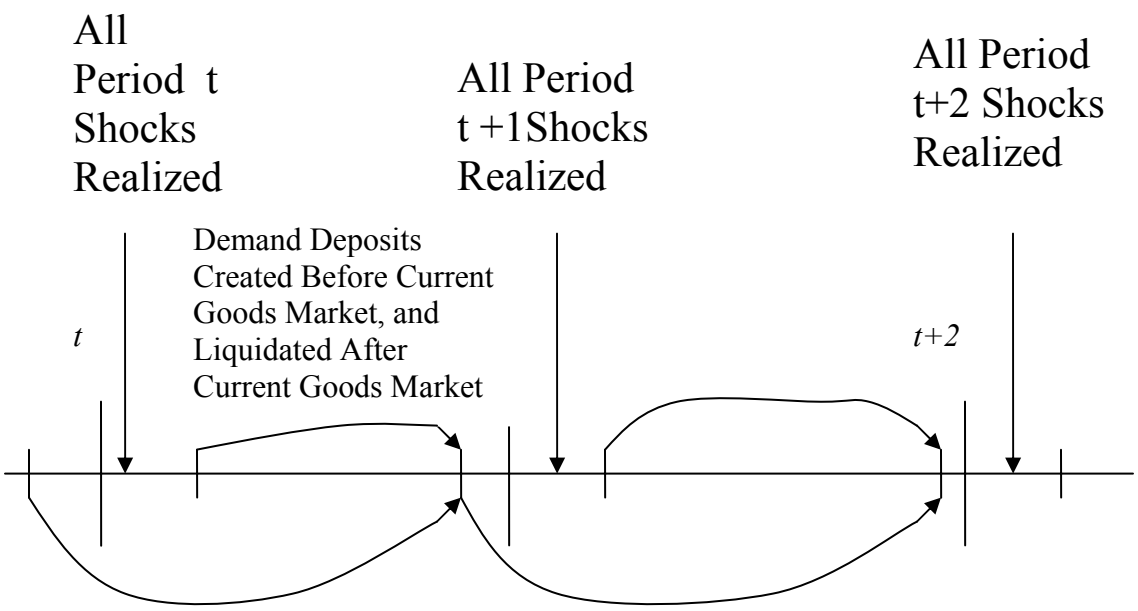
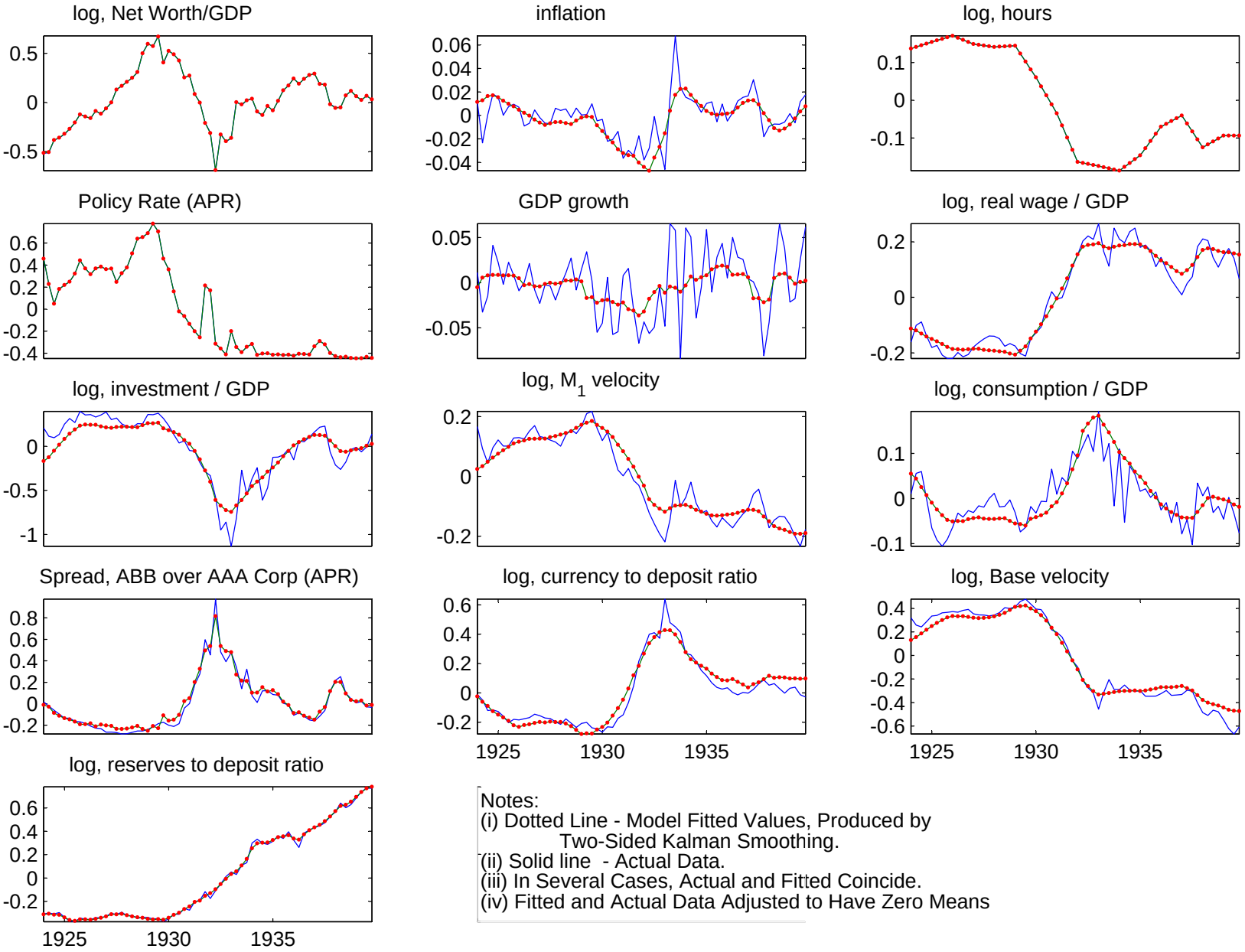


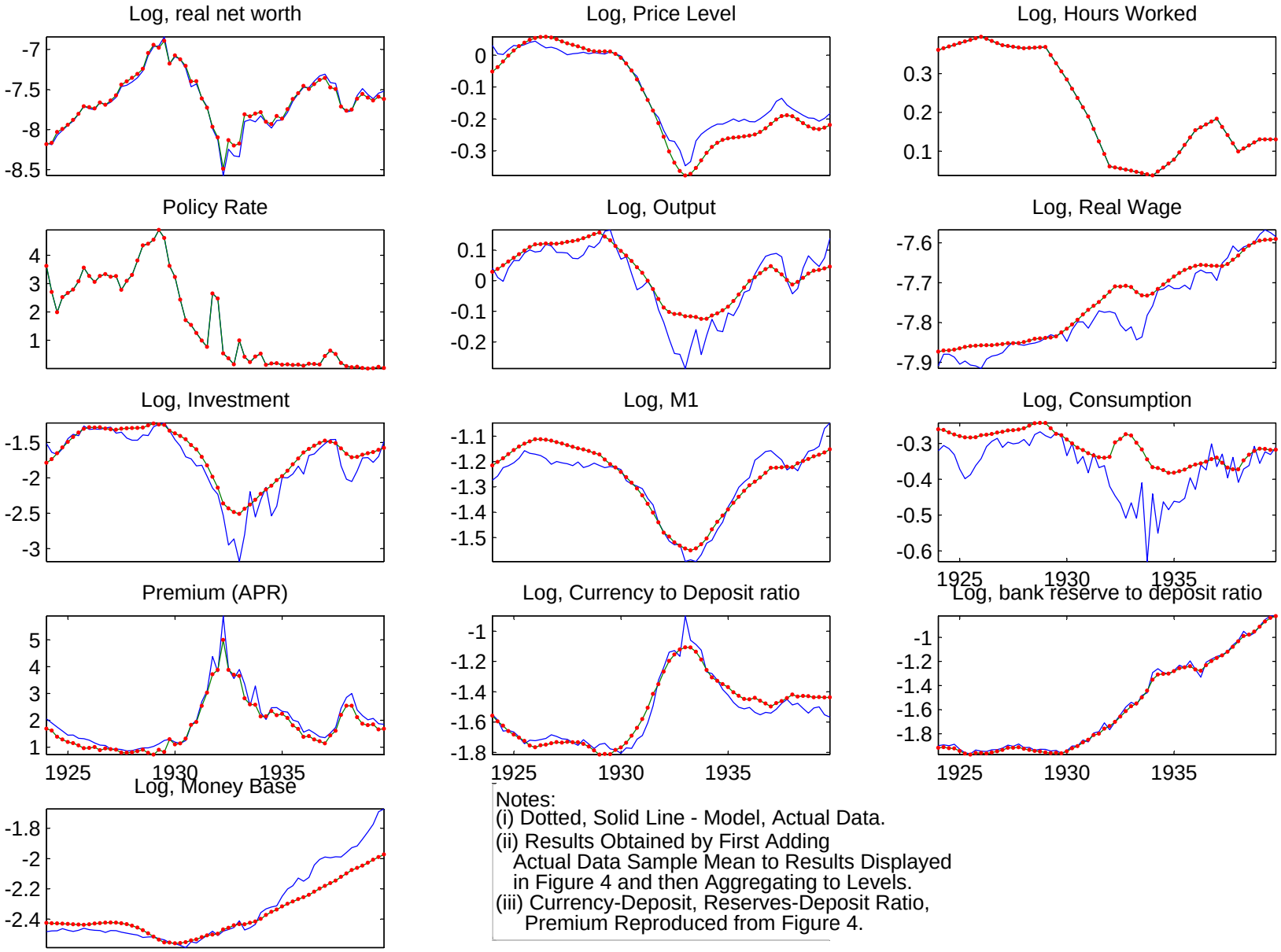
Figure 3: Maturity Structure of Time and Demand Deposits

Figure 4: Actual and Fitted Data, Growth Rates and Ratios



Notes:
(i) Dotted Line - Model Fitted Values, Produced by
Two-Sided Kalman Smoothing.
(ii) Solid line - Actual Data.
(iii) In Several Cases, Actual and Fitted Coincide.
(iv) Fitted and Actual Data Adjusted to Have Zero Means

Figure 5: Actual and Fitted Data, Converted to Levels



Notes:
(i) Dotted, Solid Line - Model, Actual Data.
(ii) Results Obtained by First Adding
Actual Data Sample Mean to Results Displayed
in Figure 4 and then Aggregating to Levels.
(iii) Currency-Deposit, Reserves-Deposit Ratio,
Premium Reproduced from Figure 4.

Figure 6: Estimated Economic Shocks

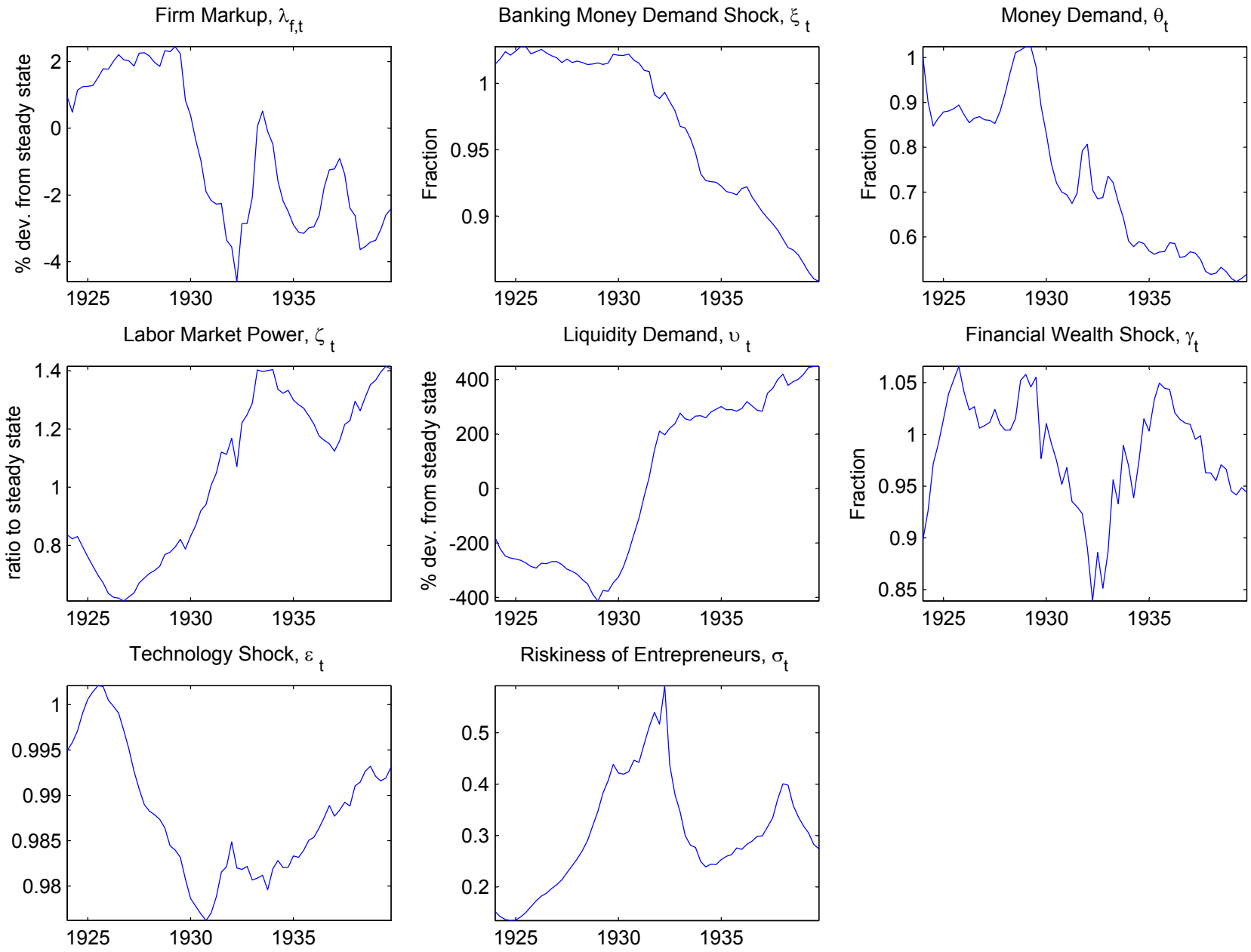
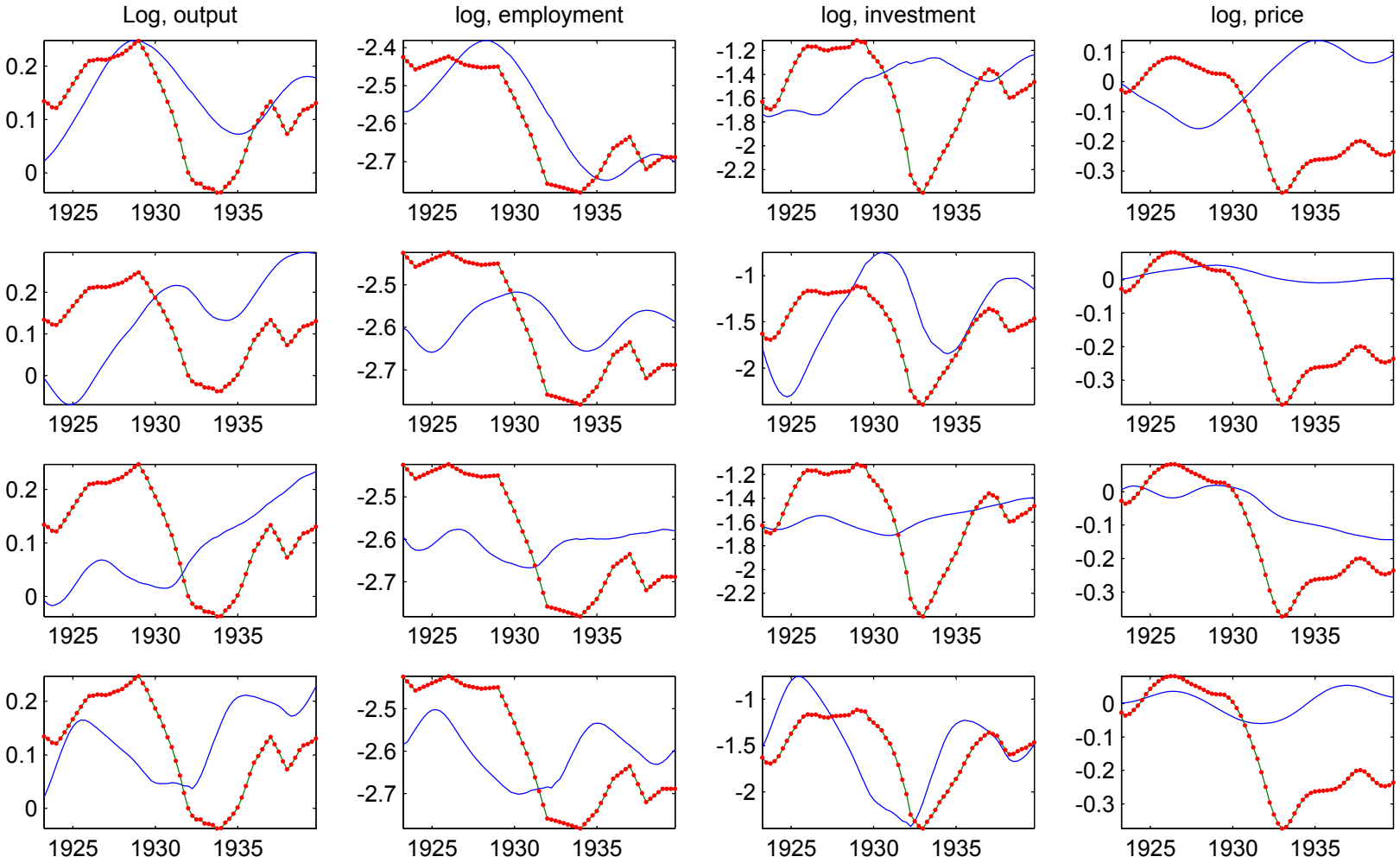


Figure 7: Output, Employment, Investment and Price Response to Indicated Single Shock, and all Shocks



Notes:
(i) Dotted Line: Response of Model to All Estimated Shocks
(ii) Solid Line: Response of Model to One of the Estimated Shocks
(iii) All Simulations Assume Economy is in Nonstochastic Steady State in 1923Q1
(iv) Row 1: Labor Market Power, ζ_t, Row 2: Financial Wealth Shock, γ_t, Row 3: Technology Shock, ε_t, Row 4: Riskiness of Entrepreneurs, σ_t

Figure 8: Model Response with Only υ_t Shocks, and Data

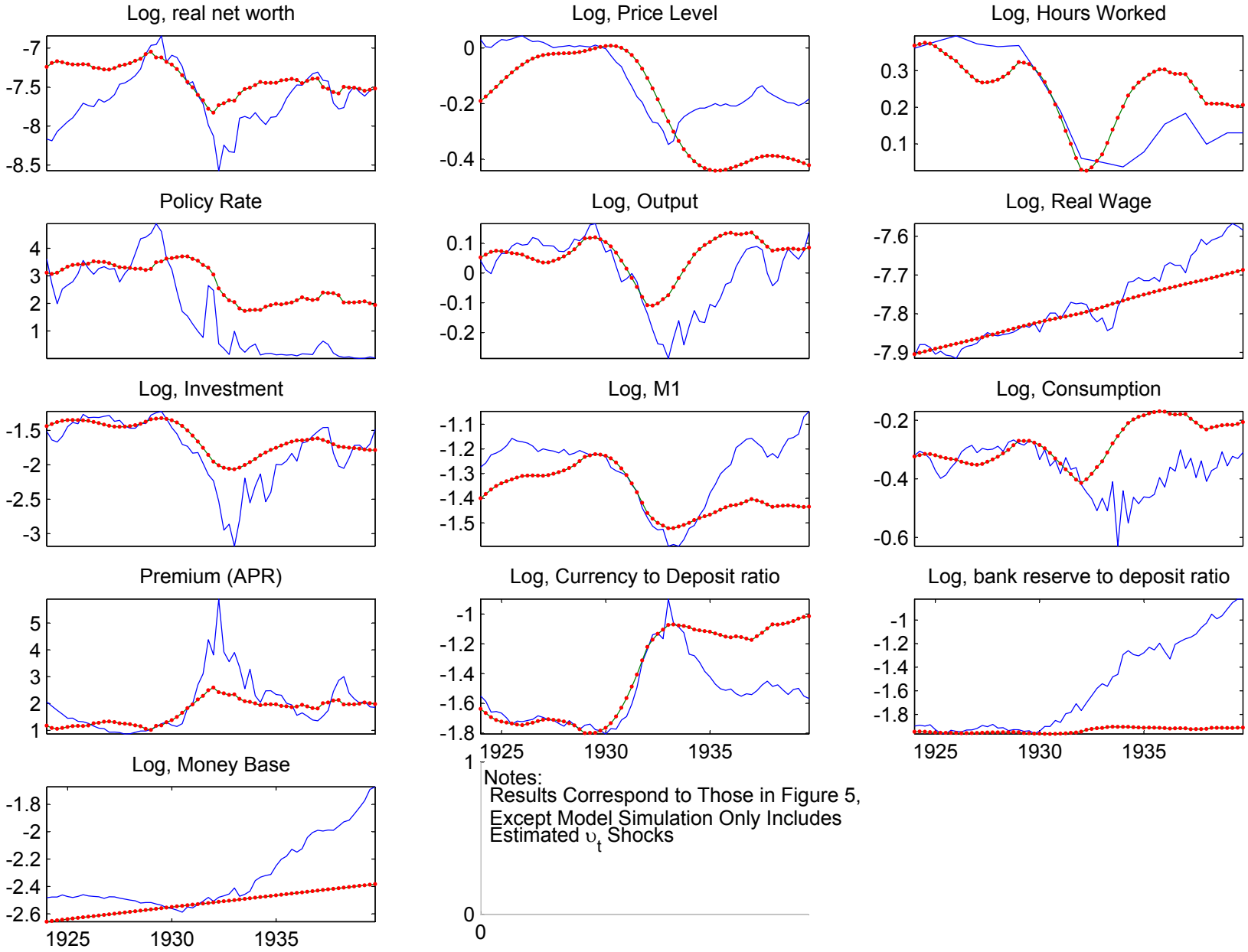


Figure 9: Response to One-Standard Deviation Innovation to Liquidity Preference, υ_t

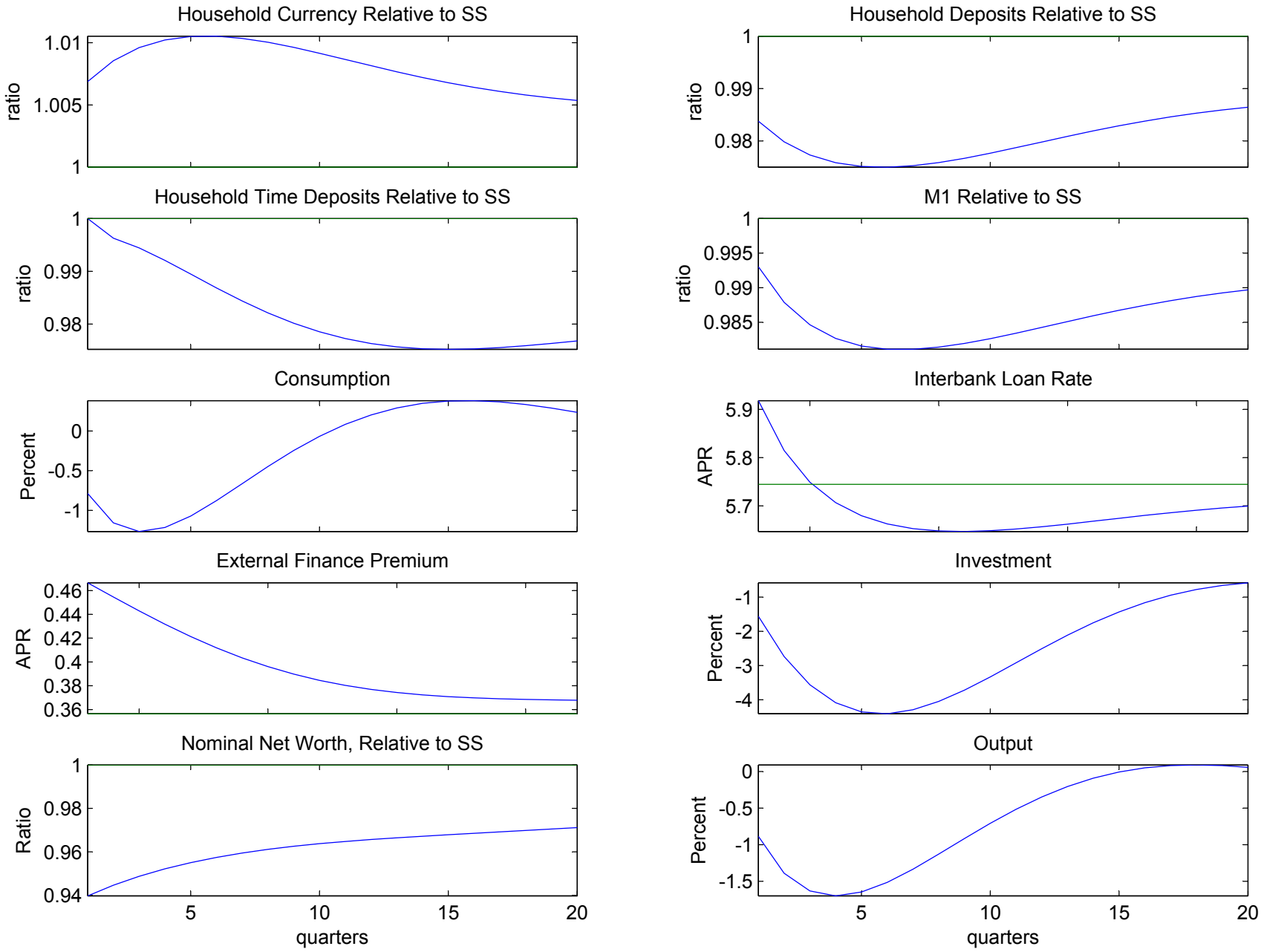


Figure 10: Baseline Estimated Policy (Solid Line) and Counterfactual Policy (Dotted Line)

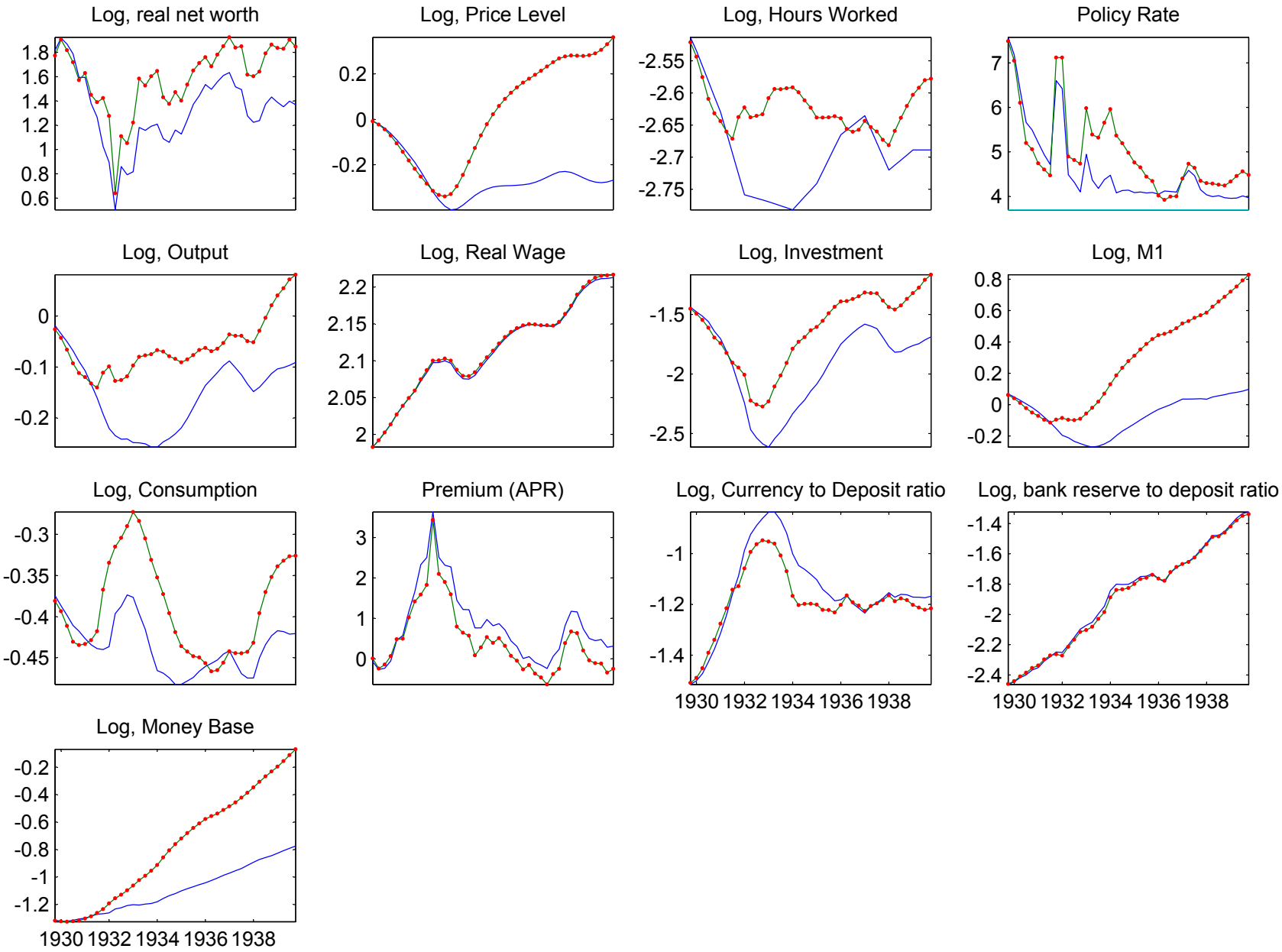


Figure 11: Policy Rate Under Baseline Simulation (Solid Line) and Counterfactual with Front-Loading of Monetary Response to Shocks (Dotted)