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Heckscher-Ohlin Business Cycles

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Abstract

This paper introduces Heckscher-Ohlin trade features into a two-country dynamic stochastic general equilibrium model, and studies the international transmission of productivity shocks through trade in goods. This framework improves upon existing international real business cycle models in generating business cycle properties comparable with the empirical evidence concerning the terms of trade and the trade balance.

Keywords: International Trade, Heckscher-Ohlin, Business Cycles, Productivity Shocks.

JEL codes: E32, F11, F32, F41.

1 Introduction

This paper introduces comparative advantage elements into an International Real Business Cycles (IRBC) model, and studies the international transmission of productivity shocks through trade in goods. More specifically, we assume that countries have access to the same production technologies, and that comparative advantage is motivated by cross-country differences in relative factor endowments. We focus on the Heckscher-Ohlin model's factor price equalization (FPE) case, although we allow for cross-country differences in total factor productivity (TFP). This leads to a rather weak form of FPE, which has received some empirical support from Trefler [42].[1] We introduce these Heckscher-Ohlin trade features into a two-country dynamic stochastic general equilibrium model, and pay special attention to the variables through which commodity trade is supposed to propagate business cycles across countries (the terms of trade, the trade balance, and the real exchange rate).

From a theoretical perspective, we view our model as a first attempt to bridge the gap between two literatures: international macroeconomics models that emphasize the cross-country transmission of business cycles via commodity trade should aim to reproduce the observed behavior of the prices and quantities through which countries interact. At the same time, they should perform this task on the basis of international trade models that explain why countries trade and their specialization patterns.

Regarding the IRBC literature, the paper is a natural follow-up on the literature triggered by Backus *et al.* [4], who study the international propagation of business cycles through trade under an exogenously given trade structure.[2] To assess the extent to which our model improves upon the latter approach from a quantitative point of view, we carefully calibrate it on aggregate and sectoral OECD data, and compare its predictions with those of a complete specialization model in the spirit of Backus *et al.* [4], keeping the parameterization as equal as possible across models.

A first important difference between our Heckscher-Ohlin model and the standard IRBC model à la Backus *et al.* [4] concerns the terms of trade (defined as the price of imports over the price of exports). In the latter model, due to its complete specialization assumption, the terms of trade are positively correlated with income for all countries, whereas in the data the sign of this correlation varies across countries. For example, while the correlation between the terms of trade with income is positive for most OECD

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[1]Kraay and Ventura [28], [29] also study the international transmission of business cycles in a Heckscher-Ohlin framework. Their work, however, is harder to relate to the standard IRBC models than ours due to their emphasis on how business cycles differ across rich and poor countries, and to the fact that they avoid capital accumulation in their model.

[2]In Backus *et al.* [4], countries trade due to the (Armington) assumption that production of different goods is country-specific.

countries, that of the US is negative. In the Heckscher-Ohlin set-up, it is easy instead to generate correlations of different signs for different countries: commodity prices are a function of world aggregate factor endowments. Therefore the dynamics of a country's terms of trade is affected by both its comparative advantage and the dynamics of the world. For example, if the world's capital-labor ratio increases, the price of capital-intensive goods will fall relative to the price of labor-intensive goods; the terms of trade of the capital-abundant country will therefore rise, whereas the labor-abundant country's terms of trade will fall.

A second interesting result is related to the trade balance. In comparison with our Heckscher-Ohlin model, and under a similar parameterization, the model à la Backus *et al.* [4] can only reproduce long-run volumes of trade of magnitudes similar to those of the data if the Armington assumption's home bias is extremely large. Ironically enough, this implies that the international propagation of business cycles through trade is quantitatively irrelevant in the Armington assumption model: whereas the trade-balance volatility generated by the Heckscher-Ohlin model is of the same order of magnitude as that in the data, the volatility generated by the model à la Backus *et al.* [4] is more that ten times smaller.

The intuition is straightforward: in the Backus *et al.* [4] complete specialization assumption framework, the long-run volume of trade is determined by size of the "home bias" in consumption. Given that this is an exogenous feature of the model, it imposes a structural constraint on the volatility of the trade balance, too. In the Heckscher-Ohlin framework, instead, both the trade pattern and the volume of trade are endogenously determined by the dynamics of the relative factor endowments, under the assumption of identical technologies and preferences across countries, *i.e.* without assuming "home bias" effects.

A common problem to the two models is the fact that, unlike in the data, both predict that the real exchange rate is necessarily less volatile than the terms of trade, provided that productivity shocks affect the freely traded intermediate goods. When we introduce productivity shocks to the nontraded final good rather than to the traded intermediate goods, the real exchange rate becomes more volatile than the terms of trade in both models. At the same time, the Heckscher-Ohlin model introduces additional constraints on the marginal utilities of leisure via factor price equalization, and predicts that the international correlation of output is higher than the international correlation of consumption. This result is, in general, hard to deliver in the IRBC literature. Finally, the Heckscher-Ohlin model has the property that temporary shocks generate permanent effects due to the fact that FPE prevents the two countries' steady states from being uniquely determined. In a stochastic framework, this implies that all country-level simulated series are

non-stationary and cross-country cointegrated.

The rest of the paper is structured as follows. In Section 2, we present a stochastic two-country model that combines Heckscher-Ohlin driven comparative advantage with the Ramsey model. In Section 3 we subject countries to stochastic productivity shocks, and study their impulse response functions and business cycle properties. We compare the predictions of the model with those of a model à la Backus *et al.* [4]. In Section 4 we introduce shocks to the nontraded sector and once again compare the performance of both models. Section 5 concludes.

2 The Model

2.1 Households

The world consists of two countries, denoted by $j = 1, 2$. Each country is inhabited by a *continuum* of identical and infinitely lived households that can be aggregated into a representative household. The representative households' preferences over consumption and leisure flows are summarized by the following intertemporal utility function:[3]

$$U_{jt} = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{c_{js}^{1-\sigma} (1-n_{js})^{\gamma(1-\sigma)}}{1-\sigma} \right], \tag{1}$$

where $\beta \in (0,1)$ is the intertemporal discount factor; $\sigma > 0$ the rate of intertemporal substitution; $\gamma > 0$ the share of leisure in total utility; c_{jt} the per-capita consumption level in country j at date t; n_{jt} the time share devoted to labor in country j at date t; and E_t the expectation operator conditional on the information set available at date t. Households own both factors of production, capital k_{jt} and labor, and sell their services in competitive spot markets at prices r_{jt} and w_{jt}, respectively. Income is used to purchase a homogeneous final good in a competitive market at price p_{jt}. This final good can be consumed or invested. We assume that the final good is not traded internationally, and that neither capital nor labor is internationally mobile.[4]

We assume the existence of a complete set of Arrow-Debreu securities. We can then solve for the Pareto-optimal allocation, and hence for the Walrasian equilibrium, by maximizing the social welfare function $U_t \equiv \sum_{j=1}^{2} U_{jt}$ under the world-level budget constraint

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[3]For the sake of notational simplicity, we avoid to make variables state-contingent explicitly.

[4]With this assumption we are only ruling out the possibility that capital and/or labor may flow across countries instantly.

and the country-level accumulation equations:[5]

$$\sum_{j=1}^{2} p_{jt} \left(c_{jt} + i_{jt} \right) = \sum_{j=1}^{2} \left(w_{jt} n_{jt} + r_{jt} k_{jt} \right), \tag{2}$$

$$k_{jt+1} = (1 - \delta) k_{jt} + \varphi \left(\frac{i_{jt}}{k_{jt}} \right) k_{jt}. \tag{3}$$

Equation (3) describes the dynamics of capital. $\delta \in (0, 1)$ is the exogenous depreciation rate. Following Baxter and Crucini [7], we introduce a cost of adjusting capital in equation (3), such that $\varphi > 0$, $\varphi' > 0$, and $\varphi'' < 0$, and assume that it does not play any role in steady state, $i.e.$ $\varphi' \left(\frac{i}{k} \right) = 1$. We define the elasticity of the adjustment cost near the steady state as $\xi_{\varphi} \equiv -\varphi'' (i/k) (i/k)$; the parameter ξ_{φ} is the only feature of the adjustment cost function that is relevant under our solution procedure.[6]

The budget constraint (2) can be rewritten as

$$p_{1t} \left(c_{1t} + i_{1t} + \pi_t \right) = y_{1t} \equiv w_{1t} n_{1t} + r_{1t} k_{1t}, \tag{4}$$

$$p_{2t} \left(c_{2t} + i_{2t} - \frac{p_{1t}}{p_{2t}} \pi_t \right) = y_{2t} \equiv w_{2t} n_{2t} + r_{2t} k_{2t}, \tag{5}$$

where π_t is the trade balance (net exports) in country 1, expressed in units of the final good produced in the same country. The social welfare function is maximized subject to equations (3), (4), and (5), taking p_{jt}, w_{jt}, and r_{jt} as given (these prices are determined in the static trade equilibrium). Under our assumptions, the first order conditions and the transversality conditions are necessary and sufficient for the households' dynamic optimization problem.

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[5]We implicitly assume that the population size is equal across countries, and hence use identical welfare weights.

[6]Notice that the adjustment cost applies only at the aggregate level: we assume that capital moves freely across sectors. Without adjustment costs, the actual international allocation of capital in the integrated equilibrium is undetermined: once the rate of returns are equalized across countries through trade in intermediate goods, the world capital stock becomes the only relevant state variable in the planner's dynamic problem. In the two-good model developed in Backus et $al.$ [4], the adjustment cost is not necessary, since the country specificity of each intermediate good is enough to identify consumption and investment at the country level.

2.2 Firms

2.2.1 Final goods

The final good is produced in each country by a continuum of competitive firms that use a number m of intermediate goods with the following Cobb-Douglas production function:

$$Y_{jt} = \prod_{i=1}^{m} x_{ijt}^{\alpha_i}, \tag{6}$$

where $\alpha_i \in (0,1)$, $\sum_{i=1}^{m} \alpha_i = 1$; Y_{jt} is the per-capita output level of the final good; and x_{ijt} denotes the amounts of intermediate good i used in the production of Y_{jt}.

2.2.2 Intermediate goods

Intermediate goods are freely traded. We assume that the markets for intermediates are also competitive, and that firms in both countries have access to the same technologies to produce them:

$$y_{ijt} = a_{jt} k_{ijt}^{\phi_i} n_{ijt}^{1-\phi_i}, \tag{7}$$

where $\phi_i \in (0,1)$. y_{ijt} denotes the amounts of intermediate good i produced in country j at date t, while k_{ijt} and n_{ijt} are respectively the amounts of capital and labor employed in the production of good i; a_{jt} denotes the total factor productivity (TFP) level.[7] We assume that TFP follows an exogenous stationary stochastic Markov process. In particular, we assume that the logarithm of $a_t \equiv [a_{1t}, a_{2t}]'$ is governed by a VAR(1):[8]

$$\ln a_{t+1} = B \ln a_t + \varepsilon_t, \tag{8}$$

where B is the persistence matrix, and $\varepsilon_t \sim N(0, \Sigma)$ is an *iid* vector of innovations. Note that the current levels of TFP are known at date t.

2.3 Trade Equilibrium

We choose the final good produced in country 2 as the numeraire, *i.e.* $p_{2t} = 1$; given free trade, $p_{1t} = p_{2t} = 1$. We assume that countries are similar enough in their relative factor endowments for the trade equilibrium to yield factor price equalization. The FPE theorem implies that international trade in intermediate goods acts as a substitute for trade in factors, equalizing wage and rental rates across countries.[9] Trefler [42] argues that

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[7]Our FPE setup does not allow for country-sector specific productivity shocks.

[8]The steady-state TFP level is normalized to one.

[9]In the Heckscher-Ohlin model, each country can produce the m intermediate goods if the relative factor endowments of countries are similar enough. In this case, if there is at least as many goods as

FPE cannot be rejected by the data once one controls for cross-country factor-augmenting productivity differences. In other words, factor prices seem to be equal across countries when production factors are made comparable across nations. When FPE holds, the trade equilibrium yields the same resource allocation and prices as the world's integrated equilibrium, in which both goods and factors are perfectly mobile internationally. Since the integrated equilibrium behaves like a closed economy, factor prices only depend on world aggregates.[10] The wage rate w and the rate of return to capital r depend, respectively, positively and negatively on the world's capital-labor ratio expressed in efficiency units:

$$w_t = \Gamma \left(\frac{s_N}{s_K} \right)^{s_K} \left(\frac{K_t}{N_t} \right)^{s_K}, \tag{9}$$

$$r_t = \Gamma \left(\frac{s_K}{s_N} \right)^{s_N} \left(\frac{K_t}{N_t} \right)^{s_N}, \tag{10}$$

where $\frac{K_t}{N_t} \equiv \frac{a_{1t}k_{1t} + a_{2t}k_{2t}}{a_{1t}n_{1t} + a_{2t}n_{2t}}$. $\Gamma \equiv \prod_i \left[\alpha_i \phi_i^{\phi_i} (1 - \phi_i)^{1 - \phi_i} \right]^{\alpha_i}$, $s_K \equiv \frac{rK}{wN + rK} = \sum_i \alpha_i \phi_i$, and $s_N \equiv 1 - s_K$ are positive constants. Factor prices per "raw" unit are simply $w_{jt} = w_t a_{jt}$ and $r_{jt} = r_t a_{jt}$. Concerning the trade pattern, the capital-abundant (labor-abundant) country will export capital-intensive (labor-intensive) goods on average.

2.4 Dynamic Equilibrium

A dynamic recursive equilibrium under FPE can be summarized by equations (3), (4), (5), (9), (10), and:

$$c_{1t}^{-\sigma} (1 - n_{1t})^{\gamma(1-\sigma)} = c_{2t}^{-\sigma} (1 - n_{2t})^{\gamma(1-\sigma)}, \tag{11}$$

$$c_{1t}^{1-\sigma} (1 - n_{1t})^{\gamma(1-\sigma)-1} \frac{a_{2t}}{a_{1t}} = c_{2t}^{1-\sigma} (1 - n_{2t})^{\gamma(1-\sigma)-1}, \tag{12}$$

$$E_t \left\{ \lambda_{jt+1} \left[\varphi' \left(\frac{i_{jt+1}}{k_{jt+1}} \right) r_{jt+1} + 1 - \delta + \Phi \left(\frac{i_{jt+1}}{k_{jt+1}} \right) \right] \right\} = \frac{\lambda_{jt}}{\beta}, \tag{13}$$

where $\Phi \left(\frac{i_t}{k_t} \right) \equiv \varphi' \left(\frac{i_t}{k_t} \right) \left(\frac{i_t}{k_t} \right) - \varphi \left(\frac{i_t}{k_t} \right)$, and the costate variable $\lambda_{jt} = c_{jt}^{-\sigma} (1 - n_{jt})^{\gamma(1-\sigma)} / \varphi'$ represents the shadow value of installed capital. The previous system of equations is valid if and only if the FPE condition is satisfied *ex-post* at all dates $t \in [0, \infty)$.[11]

Notice that under FPE and complete markets both the marginal utility of consumption

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production factors, the equilibrium factor prices of countries depend only on goods prices through the competitive price conditions. Given free trade and identical technologies across countries, factor prices are equalized internationally. (See Dixit and Norman [19].)

[10]Given homotheticity in the production function of Y, world demand for intermediates does not depend on the way income is distributed across countries.

[11]More precisely, the first order conditions are valid only if the agents consider the possibility of leaving FPE as an event with probability zero.

and the marginal utility of leisure are equalized across countries. Combining equations (11) and (12) yields

$$\frac{1 - n_{1t}}{1 - n_{2t}} = \left(\frac{a_{1t}}{a_{2t}}\right)^{\kappa - 1}, \tag{14}$$

$$\frac{c_{1t}}{c_{2t}} = \left(\frac{a_{1t}}{a_{2t}}\right)^{\kappa}, \tag{15}$$

where $\kappa \equiv [\gamma(1 - \sigma) - 1] / [\gamma(1 - \sigma) - \sigma]$.

2.5 Steady State

The system is in steady state if and only if the following condition holds:

$$r = \frac{1}{\beta} - 1 + \delta. \tag{16}$$

Consider equation (10) evaluated at the steady state:

$$r = \Gamma \left(\frac{s_N}{s_K}\right)^{-s_N} \left(\frac{K}{N}\right)^{-s_N}. \tag{17}$$

It is easy to show that equations (16) and (17), together with the other first order conditions and resource constraints, characterize the integrated economy's steady state.

Under complete markets, the levels of consumption and leisure can be different across countries only as long as the TFP levels are also different: the fact that the world is a stationary system is enough to pin down the steady-state levels of consumption and worked hours at the country level. However, this is not enough to fully pin down the country-level steady state: any combination of k_1 and k_2 such that FPE holds and

$$\frac{k_1 + k_2}{2n} = \frac{K}{N} \tag{18}$$

is compatible with the steady state. The unique world-level steady state is therefore compatible with a multiplicity of steady states at the country level. These steady states are fully characterized by the cross-country distribution of capital stocks.[12]

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[12]Giavazzi and Wyplosz [20] show that a similar result holds in a two-country model under rational expectations and perfect capital mobility. In our set-up, international trade in goods is a substitute for perfect capital mobility. Becker [9] and Krusell and Ríos-Rull [30] obtain this result in a heterogenous-agents closed economy, due basically to the same economic mechanism: if all agents face the same rate or return, and the latter depends on the aggregate capital stock only, the steady-state wealth and income distributions are indeterminate. Lucas and Stokey [34] discuss the issue in detail, and suggest that the multiplicity of stationary equilibria can be avoided by introducing the hypothesis of increasing marginal impatience. See also Stiglitz [40] and Baxter [6].

The multiplicity of steady states does not imply the indeterminacy of the model's solution: once the initial conditions k_{j0} are exogenously given, the transitional dynamics leads the system to a unique and non-degenerate steady state. This can be understood as follows: (*i*) the world as a whole is a standard stationary Ramsey economy with a well specified steady state, characterized by a unique value of K/N; (*ii*) given the initial conditions and our assumptions on the functional forms, the adjustment paths for all country-level variables are uniquely determined; (*iii*) equation (18) and the FPE condition imply that $(k/n)_{\min} \leq k_j/n \leq (k/n)_{\max}$ for some $(k/n)_{\max} > (k/n)_{\min} > 0$. In other words, the world reaches a steady state in which equations (16) and (17) hold, and both k_1 and k_2 are strictly positive. Such a steady state may be characterized by different values of income, investment and capital across countries.

Combining equations (3), (4) and (5), evaluated at the steady state, with equation (16) leads to

$$\pi = \frac{1-\beta}{2\beta}\left(k_1 - k_2\right). \tag{19}$$

The steady-state trade balance can be zero only if the steady state itself is symmetric. Otherwise, country 1 runs a permanent trade balance surplus if $k_1 > k_2$, or a permanent deficit if $k_1 < k_2$. Consumption and leisure levels have to be equalized in steady state, but capital stocks do not. The country with a higher capital stock therefore has to transfer a positive share of its income to the poorer country permanently.[13]

2.6 Calibration

To solve and simulate the model numerically, we need to calibrate a large set of parameters. We set the elasticity of intertemporal substitution to $\sigma = 2$ and the intertemporal discount factor to $\beta = 0.99$. In the integrated equilibrium, α_i equals the share of sector i in the world's total value added. The OECD publishes estimates of sectoral value added for 24 countries and 28 sectors in its *Annual National Accounts Detailed Tables*. Internationally comparable estimates for all sectors and all countries are available only for two years: 1995 and 1996. Given that figures do not change significantly over this period, we focus on 1995. After converting all observations in US$ with the OECD PPP exchange rates, we aggregate sectors across countries and calculate the share of each sector in total OECD value added. We calibrate the α_i's to match these observed shares.

In equilibrium, ϕ_i equals sector i's capital share in value added. The OECD *Annual National Accounts* provide data on compensations of employees (wages and salaries), total

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[13]We assume that international consumption risk sharing agreements are enforceable. This is a particularly strong assumption here, since permanent international income transfers can arise in equilibrium. Kehoe and Perri [26] discuss how endogenous incomplete markets affect the international transmission of productivity shocks.

employment, and the share of employees in total employment at the sectoral level for a smaller set of countries. Under the assumption of identical technologies, we focus on the US and, for comparability, on year 1995. As a first approximation, the sectoral labor share can be obtained as the share of compensation of employees in value added. With this approach, the share of income perceived by the self-employed as a remuneration of their own work is recorded as capital income rather than labor income.[14] We follow Gollin [22], and adjust the sectoral labor share using data on the share of employees over total employment: we assume that self-employed workers earn a wage that is comparable to the competitive market wage earned by employees, and correct the sectoral labor shares by dividing them by the sectoral share of employees in total employment. This correction changes the labor share significantly in a few cases only. We compute the sectoral capital share as one minus the corresponding labor share.[15] Table 1 summarizes these results: we report the unadjusted and adjusted sectoral labor shares, as well as the sectoral shares in world value added. These estimates imply a world capital share equal to $s_K = 0.37$.

In the Heckscher-Ohlin model, the trade pattern is indeterminate in the presence of more goods than factors. To avoid this indeterminacy, we aggregate our 28 sectors in only two larger sectors: a labor-intensive sector and a capital-intensive one. Any such aggregation seems in principle arbitrary. However, in the integrated equilibrium, the dynamics of the system is influenced exclusively by the world capital share s_K. Therefore any aggregation that leaves s_K unaffected will not bias the results in any way (apart from pinning down the trade pattern).[16] Note that:

$$s_K = \sum_{i=1}^{m} \alpha_i \phi_i = \sum_{i=1}^{z} \alpha_i \frac{\sum_{i=1}^{z} \alpha_i \phi_i}{\sum_{i=1}^{z} \alpha_i} + \sum_{i=z+1}^{m} \alpha_i \frac{\sum_{i=z+1}^{m} \alpha_i \phi_i}{\sum_{i=z+1}^{m} \alpha_i} = \alpha \psi + (1 - \alpha) \phi \qquad (20)$$

for any z and adequately defined α, ψ, and ϕ. For the sake of symmetry, we order the sectors according to their labor intensities and choose $z = 14$; the resulting parameterization is $\alpha = 0.61$, $\phi = 0.17$, and $\psi = 0.49$. Abusing notation, we denote these "aggregate sectors" with ϕ and ψ.

We are left with the preference parameter γ and the depreciation rate δ. The usual procedure consists in calibrating these parameters in order to match the observed long-run time share devoted to labor and the long-run capital-income ratio. In our framework, however, the country-level capital-income ratio depends directly on the country-level capital-

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[14]The OECD does not report proprietors income separately. Therefore the procedure outlined in Cooley and Prescott [17] to correct for this problem cannot be applied here.

[15]We consider land as a component of the capital stock.

[16]The only key variable that depends on the particular aggregation we use is the terms of trade. (See equation (22) below.) We have performed robustness checks that show the results reported in the paper for the terms of trade do not change significantly for other aggregations.

Sector	$(1-\phi_j)$ (N.Adj.)	$(1-\phi_j)$ (Adj.)	α_j
Electricity, gas and water supply	0.26	0.26	0.026
Man. of coke, refined petroleum prod. and nuclear fuel	0.34	0.34	0.006
Mining and quarrying	0.34	0.35	0.010
Real estate, renting and business activities	0.31	0.37	0.175
Man. of chemicals, chem. prod. and man-made fibres	0.44	0.44	0.021
Man. of food products, beverages and tobacco	0.47	0.47	0.024
Man. of leather and leather products	0.53	0.55	0.002
Financial intermediation	0.55	0.57	0.061
Transport, storage and communication	0.55	0.59	0.069
Hotels and restaurants	0.59	0.62	0.016
Wholesale and retail trade; repair services	0.57	0.62	0.143
Man. of electrical and optical equipment	0.63	0.63	0.027
Agriculture, hunting, forestry, and fishing	0.34	0.65	0.023
Manufacturing n.e.c.	0.60	0.66	0.008
Man. of basic metals and fabricated metal products	0.67	0.68	0.024
Man. of wood and wood products	0.63	0.69	0.006
Man. of pulp, paper and paper prod.; pub. and printing	0.66	0.70	0.019
Man. of other non-metallic mineral products	0.68	0.70	0.008
Man. of machinery and equipment n.e.c.	0.75	0.76	0.020
Man. of rubber and plastic products	0.77	0.77	0.007
Man. of textiles and textile products	0.76	0.78	0.008
Man. of transport equipment	0.78	0.78	0.021
Other community, social and pers. services	0.70	0.83	0.061
Construction	0.67	0.85	0.055
Health and social work	0.81	0.88	0.050
Public admin. and defence; comp. social security	0.89	0.89	0.083
Education	0.94	0.95	0.026
Private households with employed persons	1.00	1.00	0.002

Table 1: Sectoral labor shares and shares in value added at the world level.

labor ratios. Since the steady state of our model is *ex-ante* indeterminate, we need to specify a starting point explicitly (the initial steady state) to pin down the dynamic equilibrium. We assume that a share $1/2 < \omega < 1$ of the world's capital stock is in country 1. In this initial asymmetric steady state, capital-abundant country 1 will be a net exporter of the capital-intensive good ψ, while labor-abundant country 2 will be a net exporter of the labor-intensive good ϕ. The value of ω influences the trade volume directly: the farther the capital-labor ratios, the larger the volume of trade, defined as the ratio between imports plus exports over income.

We calibrate γ, δ, and ω jointly in order to make country 1 reproduce the U.S. values for the time share devoted to labor (0.31), the yearly capital/income ratio (3.32), and the average U.S. trade volume's GDP share (0.15).[17] The implied values are $\gamma = 1.86$, $\delta = 0.019$, and $\omega = 0.553$. This parameterization implies a capital share equal to 0.39 in country 1. Finally, the elasticity of the adjustment cost ξ_φ is calibrated so as to have country 1 display the observed U.S. relative volatility of investment. The corresponding value is $\xi_\varphi = 1/35.4$.

In the IRBC literature it is customary to approximate TFP with the standard Solow residual, defined as $\ln s_{jt} \equiv \ln y_{jt} - s_N \ln n_{jt} - s_K \ln k_{jt}$.[18] The joint stochastic properties of TFP are usually estimated by running a VAR(1) on the country-level proxies for the Solow residuals, as in Backus *et al.* [3] and others. We aggregate the EU15 European countries, Canada, and Japan into a "Rest of the World" country. We take quarterly constant-price series for GDP and investment from the OECD *Quarterly National Accounts* for all countries over the 1961:I-2001:IV period. All series are then transformed in constant US\$ using the appropriate OECD PPP exchange rates. We build series for the physical capital stock using the permanent inventory method. As a proxy for the labor input, we take Total Civilian Employment, again from the *Quarterly National Accounts*, for the 1973:I-2001:IV period.[19] Finally, the "Rest of the World" is obtained by summing up the country-level variables. The joint stochastic properties of TFP are estimated by running a symmetric VAR(1) on the Solow residuals over the 1973:I-2001:IV period using the Full Information Maximum Likelihood approach. The estimation results are the following (p

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[17]The values for the U.S. time share and capital/income ratio have been taken from Cooley and Prescott [17]. The U.S. volume of trade's GDP share is computed as the average of the value of imports plus exports over GDP (all variables in nominal terms) over the 1960-2001 period.

[18]In our framework, the Solow residuals remains a good empirical proxy for TFP: $\ln s_{jt} = \ln a_{jt}$ up to a constant.

[19]The OECD does not report Total Civilian Employment at the quarterly frequency for a few minor European countries: we use all available data in our computations.

-values for standard t-tests in parenthesis):

$$
B = \begin{bmatrix} 0.94 & 0.014 \\ (0.00) & (0.61) \\ 0.014 & 0.94 \\ (0.61) & (0.00) \end{bmatrix}, \quad \Sigma = 1^{-6} \begin{bmatrix} 9.28 & 2.05 \\ 2.05 & 4.43 \end{bmatrix} \tag{21}
$$

The implied shocks' standard deviations are 0.003 in country 1 and 0.002 in country 2, while the shocks' international correlation is 0.32. Given that technological spillovers are clearly not significantly different from zero, we set the out-of-the-diagonal elements in the persistence matrix to zero.

To summarize, our benchmark parameterization is the following:

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\sigma = 2, \quad \beta = 0.99, \quad \gamma = 1.86,
$$
$$
\delta = 0.019, \quad \xi_\varphi = 1/35.4,
$$
$$
\phi = 0.17, \quad \psi = 0.49, \quad \alpha = 0.61,
$$
$$
B_{jj} = 0.94, \quad \sigma_{1\varepsilon} = 0.003, \quad \sigma_{2\varepsilon} = 0.002, \quad Cor = 0.32.
$$

3 Results

The model is log-linearized around the initial asymmetric steady state, and solved with the standard King et $al.$ [27] procedure. We study the impulse response functions and the stochastic properties of the approximated model. The Appendix discusses the accuracy of the solution method.[20]

To clarify the contribution of our Heckscher-Ohlin (HO) trade structure, we compare our impulse-response functions with their counterparts for a standard Complete Specialization (CS) model developed along the Backus et $al.$ [4] tradition, and formally described in the Appendix. For comparability reasons, we keep the same parameterization but for α, the Cobb-Douglas exponent in the final good production function. (The need for this modification is discussed in the Appendix). This exponent is calibrated to make country 1 match the long-run U.S. trade volume's GDP share under complete specialization.

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[20]Giavazzi and Wyplosz [21] show that linear dynamic systems characterized by the indeterminacy of the stationary equilibrium ($i.e.$ singularity of the transition matrix) converge to a unique stationary equilibrium for any set of initial conditions, if the stability conditions are satisfied.

3.1 Impulse-Response Functions

In this section we discuss the dynamic response of our HO model to an unexpected positive shock to a_1. We assume that a_1 suddenly increases by 1% at date 1, and solve for the corresponding impulse-response functions over a 60-quarter time horizon. The impulse responses for the country-level main variables are plotted in Figures 1 and 2. The simulated series are expressed in percentage deviations from the initial steady state (the trade balance is the only variable expressed in absolute deviations from its steady-state value.)

The world reacts to the productivity shock as in a closed-economy RBC model: income, consumption, investment, and the time share devoted to labor increase on impact and then converge slowly to their initial steady state values. Figures 1-2 tell quite a different story as far as the country-level variables are concerned. To understand the properties of our model, we need to study how prices and quantities react to the productivity shock on impact and during the transition to the final steady state.

Impact We focus on each of the two transmission channels that are at work in our framework separately: international trade in goods and international risk sharing. We analyze them in turn, starting with the reaction of factor prices under our trade regime and their effect on the main aggregate quantities.

For the sake of the discussion, let us assume for the moment that n_j does not vary on impact, and consider the effect of the shock on prices. For constant n_j's, we can show that if $k_1/n_1 > k_2/n_2$, $\partial w/\partial a_1 > 0$ and $\partial r/\partial a_1 < 0$. An increase in a_1 raises country 1's endowment of both capital and labor in efficiency units in the same proportions. Given that country 1 is capital-abundant, this implies an increase in the world's capital-labor ratio in efficiency units. Changes in a_1 also have a direct effect on country 1 factor prices in raw units: $\partial w_1/\partial a_1 > 0$ and $\partial r_1/\partial a_1 > 0$. In country 2, however, factor prices in raw units equal factor prices in efficiency units, since a_2 is not affected by a_1.

The increase in the wage and rental rates has the following effects in country 1: (i) the rise in the wage raises labor supply through the intertemporal labor/leisure substitution effect; (ii) the rise in the rental rate tends to an increase in the slope of the consumption path, and - *ceteris paribus* - leads consumption to fall and investment to rise; (iii) the rise in the time share allocated to labor reduces the amount of leisure enjoyed by the representative household, and this tends - *ceteris paribus* - to reduce consumption; (iv) the increase in factor prices raises income, and therefore stimulates both consumption and investment. As a result, income, consumption, investment and the labor input in country 1 react positively on impact; consumption and investment react proportionally less and more than income, respectively. The change in country 2's factor prices has the following

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effects on impact: (i) the rise in the wage rate increases labor supply; (ii) the fall in the rental rate tends - *ceteris paribus* - to increase consumption and decrease investment; (iii) the fall in leisure tends - *ceteris paribus* - to decrease consumption; (iv) the rise in labor income increases total income, and therefore stimulates both consumption and investment.

The joint increase in n_1 and n_2 raises world labor supply. For a given world capital stock, this reduces the wage rate and raises the rental rate. In country 1, the increase in the wage rate generated directly by the productivity shock is partially dampened, while the rental rate increases even further. In country 2, the wage rate falls, while the rental rate rises. It turns out that this "labor supply" effect dominates and yields a reduction of the wage rate and an increase of the rental rate on impact.[21]

The overall change in country 2's factor prices has the following effects on impact: (i) the fall in the wage rate reduces labor supply; (ii) the rise in the rental rate tends - *ceteris paribus* - to decrease consumption and increase investment; (iii) the rise in leisure tends - *ceteris paribus* - to increase consumption; (iv) the fall in labor income decreases total income, and therefore depresses both consumption and investment. As a result, income, investment and the labor input in country 2 react negatively on impact, while consumption increases slightly.

Under complete markets, international consumption risk sharing is an additional channel of propagation. Given that the final good is non-tradable, this transmission mechanism works through trade in intermediate goods: a positive, persistent, and transitory shock in one of the two countries generates (i) a positive wealth effect in both countries, increasing - *ceteris paribus* - consumption and leisure; (ii) an incentive to transfer resources to the temporarily more productive country, that has strong effects on the dynamics of investment. The second mechanism dominates on impact: investment falls dramatically in country 2 and rises sharply in country 1.

Transition   During the transition towards the final steady state, three main forces are at work: (i) the stochastic properties of TFP drive a_1 slowly back to its long-run value; (ii) the process of capital accumulation raises the capital stock in country 1 and reduces the capital stock in country 2; (iii) international consumption risk sharing drives the consumption levels and the number of hours worked back to their common steady-state values rapidly, as predicted by equations (14) and (15).

Initially, the positive growth of capital in country 1 is higher in modulus than the negative growth rate of capital in country 2, and therefore the world capital stock increases. This reverses the impact variations of factor prices gradually, and leads to a decrease in

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[21]The intuition is clear: a_{1t} affects both the numerator and the denominator of the world capital-labor ratio in efficiency units, while the n_j's affect the denominator only.

the labor supply in country 1, while the opposite happens in country 2. The negative growth rate of labor in country 1 dominates, and the world labor input converges to its initial steady state value. After a while, the world capital stock reverses its dynamics, and starts to converge to its initial steady state value. As a consequence, factor prices tend to converge to their original steady-state values. Factor prices in raw units are different across countries only as long as the productivity levels a_1 and a_2 differ. The convergence of a_1 equalizes factor prices in raw units across countries, while the process of capital accumulation drives the world to its initial steady state.

Long-run effects   As soon as both countries share the same rental rate, their consumption paths become similar enough to prevent country-level capital stocks from converging to their initial steady-state levels. The capital stock k_1 remains permanently higher than before the shock, while k_2 remains permanently lower. Since the world capital stock must reach its initial steady-state level, the increase in k_1 exactly offsets the decrease in k_2. This permanent difference in the capital stocks implies permanent symmetric differences in income and investment. Since GDP in country 2 ends up being permanently lower than in the initial steady state, a permanent inflow of resources from country 1 is needed to finance the optimal consumption level. In other words, country 1 runs a trade balance surplus in the new steady state: households in country 2 will only transfer resources to country 1 on impact if these resources will be paid back in the long run with a permanent wealth transfer.

The labor share in country 1 converges to a permanently lower value, while the opposite happens in country 2. The investment share, the capital-income ratio, and the capital-labor ratio converge to a higher value in country 1 and to a lower one in country 2. The joint dynamics of capital and labor also have a permanent effect on the average productivity of labor, defined as the ratio between total income and the labor input. The transitory shock to productivity raises the labor productivity in country 1 and lowers it in country 2 permanently, since countries 1 and 2 become capital-abundant and capital-scarce, respectively.

Although the world as a whole is a stationary system, some country-level variables are non-stationary from a stochastic point of view. Aggregate income, investment, and capital in each country are unit-root processes, and are cointegrated with the corresponding variables in the other country: they are individually non-stationary, but their sum is actually stationary. This implies that the country-level steady states to which the system tends after a shock are different from the initial ones, but endogenously determined by the adjustment process itself.

In our model, investing in physical capital is the only way to accumulate wealth over

time. Hence, productivity shocks have permanent effects on the cross-country wealth distribution, and indirectly on the income distribution. Baxter and Crucini [8] show that in a standard IRBC model with restricted asset markets, in which only riskless bonds are internationally traded, the steady-state level of asset holdings - a sufficient statistic for the cross-country wealth distribution - is not invariant to productivity shocks. In a different framework, Obstfeld and Rogoff [37] show that demand shocks can have permanent effects on the cross-country consumption differential and wealth distribution when again asset markets are restricted to riskless bonds. In both these contributions, the restricted financial markets introduce a more or less direct link between consumption growth rates, via the common interest rate. Similarly, in our model the consumption/leisure paths are eventually driven in both countries by the same interest rate, but this international link is generated by trade in goods rather than trade in bonds.[22]

Terms of trade   On impact, the productivity shock raises the labor supply in country 1 and reduces it in country 2, leaving the capital stocks unaltered. The model's Heckscher-Ohlin trade structure implies that in country 1 resources flow from the capital-intensive sector to the labor-intensive sector while the opposite happens in country 2. The productivity shock in country 1 simply exacerbates this trade pattern in the new steady state. As a result, during the transition country 1 gradually reallocates both factors to the capital-intensive sector, while the opposite happens in country 2.

In Figure 4 we plot country 1's terms of trade, defined as the price of imports over the price of exports. The pattern of trade is affected permanently by transitory shocks to productivity. For small deviations from the initial asymmetric steady state, country 1 remains a net exporter of the capital intensive good and a net importer of the labor intensive one. The p_ϕ/p_ψ price ratio corresponds therefore to country 1's terms of trade. In the Technical Appendix we show that

$$\frac{p_{\phi t}}{p_{\psi t}} = \Xi \left(\frac{K_t}{N_t} \right)^{\psi - \phi}, \tag{22}$$

where Ξ is a positive constant. Given $\psi > \phi$, the initial increase in n_{jt} has on impact a negative effect on the terms of trade in country 1; the process of capital accumulation reverses this effect soon, raising the terms of trade. The latter deviate positively from the steady state during the remaining part of the transition. Since the steady-state capital-

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[22]Hall [23] shows that in a partial equilibrium model of the life-cycle permanent income hypothesis the marginal utility of consumption follows a martingale - and the consumption level is therefore a unit root process - if the real interest rate is exogenous. Similar results emerge in a small open economy framework, as pointed out by Sen and Turnovsky [39] and Correia *et al.* [18]. In our set-up, the equilibrium real interest rate is endogenous at the world level, but partially exogenous at the country level. Hence, Hall's result applies to a limited extent at the country level.

labor ratio is uniquely pinned down at the world level, so should be the steady-state value of the terms of trade. Figure 4 confirms that the terms of trade worsen on impact, but then improve quite remarkably during the transition to the steady state. In the long run, the terms of trade are not affected by the productivity shock permanently.

Heckscher-Ohlin vs. Complete Specialization Figures 3-4 report the impulse-response functions for the CS model, which tell quite a different story. From a qualitative point of view, the reaction of country 1 is very similar in both models, except for the fact that the CS model is completely stationary. That is, no long-run effect of productivity shocks can emerge in that framework. However, there are some striking quantitative differences. First, the reaction of investment on impact is much less vigorous. Second, and more importantly, the trade balance in country 1 decreases on impact as in the HO model, but the size of its decrease is almost six times smaller.

As far as country 1 is concerned, the only relevant qualitative difference between the two frameworks regards the behavior of the terms of trade: in the HO model, the terms of trade react negatively on impact, while exactly the opposite happens in the CS model. In the latter model, a positive shock to productivity in country j raises output, reducing the price of country j's intermediate good: this implies that the terms of trade react positively on impact. In contrast, in the HO model productivity shocks have no direct effect on the terms of trade, since intermediate good prices depend only on the capital-labor ratio (in efficiency units) at the world level.

If we move to country 2, instead, many more striking differences emerge: aggregate variables in that country seem hardly affected by the productivity shock in country 1. Income, consumption, hours worked, and factor prices hardly react on impact: when the CS model is calibrated to replicate the observed U.S. long-run trade volume's GDP share and technological spillovers are ruled out, international trade becomes irrelevant as a transmission mechanism. This clearly contrasts with the behavior of the HO model, in which international trade plays a much more important role.

This striking result can be understood as follows: under a complete specialization assumption, and assuming a unitary elasticity of substitution among intermediate goods, the import-export shares in income will depend directly on the "home bias" parameter α. In order to match the observed long-run U.S. trade volume share in income, the parameter α has to be set to a very high value, equal to 0.966 (see the Appendix for more details). This implies that the import shares in income in both countries will be equal to a tiny and roughly constant fraction of income, leaving a very limited role to trade as a transmission mechanism. Given that the final good is not traded, trade in intermediate goods remains the only way to smooth consumption internationally, and the need to assume a high

"home bias" limits this channel enormously.[23]

In the HO framework, instead, the trade pattern and the trade volume are both endogenously determined by comparative advantage considerations, and no "ad hoc" exogenous assumption is needed to reproduce the long-run trade volume apart from fixing the (*ex-ante* undetermined) initial conditions. In particular, both countries share the same final good production function, with no distinction between "home" and "foreign" intermediate goods: *i.e.* the "home bias," so essential in the complete specialization framework to limit the extent of trade, plays absolutely no role in the HO framework, and therefore does not introduce an exogenous constraint on the dynamics of the trade balance.

3.2   Stochastic Properties

To study the stochastic properties of our model, we simulate it for 10.000 times over a 116 quarter horizon, drawing the shocks from a multivariate normal distribution. At each round we check whether the FPE condition holds and country 1 remains the capital-abundant country over the whole simulation horizon.[24] Rounds that do not satisfy these requirements are dropped from the experiment. We then apply the Hodrick and Prescott [25] filter[25] to the simulated series, and compute the standard business cycles statistics for the cyclical components in country 1: the relative volatility of all main aggregate variables, measured as the ratio between the standard deviation of each variable and the standard deviation of income, their correlation with income and net trade, and their international correlations. Table 2 reports the averages and standard deviations of these statistics over the whole sample. We also report the observed counterparts for the U.S. based on quarterly OECD data over the 1973:I-2001:IV period.[26] The average and standard deviation of the observed international correlations are taken from Maffezzoli ([36], Tab. A.7).[27]   Finally, Table 2 also reports the simulated statistics for the Complete

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[23]In the Appendix we remind the reader that in the CS model countries do not need to be identical in steady state, and will not be if the factor shares are different across countries. Let us stress here that the very low volatility of the trade balance does not depend on this difference in country size: similar results occur in the symmetric case too.

[24]We performed several simulations of 10,000 rounds each and the model *nevel* left the FPE region.

[25]We Hodrick-Prescott filter the simulated series for two main reasons: (*i*) for comparison purposes with the existing literature; (*ii*) to extract the unit roots and obtain stationary cyclical components. Canova [12] and others, however, show that applying the Hodrick-Prescott filter to integrated series is likely to induce spurious results. Furthermore, being the Hodrick-Prescott a univariate procedure, it ignores by construction the cointegrating relationships that link different variables in the same country and the same variables across countries. Note, however, that the same filtering procedure is used to extract the cyclical component from the data: in this sense, the model is compared to the data under the same conditions.

[26]The trade balance has been computed as the ratio between net exports (exports minus imports) over GDP, both at current prices. We Hodrick-Prescott filtered the series without logging it.

[27]The data set regards ten OECD countries (Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, UK and USA). The sample period is 1970:1-1997:4 (for Australia, Germany, and

	Rel. Std.			Cor. y			Cor. π/y			Int. Cor.		
	US	HO	CS	US	HO	CS	US	HO	CS	Data	HO	CS
y	-	-	-	-	-	-	-	-	-	0.42	0.07	0.36
	-	-	-	-	-	-	-	-	-	*0.20*	*0.17*	*0.15*
c	0.80	0.37	0.43	0.87	0.98	0.99	-	-	-	0.23	0.48	0.40
	-	*0.01*	*0.00*	-	*0.01*	*0.00*	-	-	-	*0.22*	*0.14*	*0.14*
i	3.83	3.83	2.50	0.94	0.98	1.00	-	-	-	0.31	-0.51	0.28
	-	*0.14*	*0.03*	-	*0.01*	*0.00*	-	-	-	*0.21*	*0.13*	*0.16*
n	0.64	0.55	0.40	0.88	0.97	1.00	-	-	-	0.35	-0.44	0.33
	-	*0.02*	*0.00*	-	*0.01*	*0.00*	-	-	-	*0.21*	*0.14*	*0.15*
$\frac{\pi}{y}$	0.63	0.32	0.06	-0.46	-0.78	-0.80	-	-	-	-	-	-
	-	*0.06*	*0.01*	-	*0.07*	*0.06*	-	-	-	-	-	-
p	1.71	0.12	0.69	-0.18	-0.62	0.42	0.10	0.19	-0.74	-	-	-
	-	*0.02*	*0.11*	-	*0.10*	*0.14*	-	*0.17*	*0.07*	-	-	-

Table 2: Business cycle properties (shocks to the traded sectors).

Specialization counterpart, obtained under the same conditions.

As far as the national business cycle properties are concerned, the HO model generates a relative volatility of consumption lower than the observed one, as is usual with this class of models. The relative volatilities of both the trade balance and the terms of trade are also below the observed ones. Similar results obtain in the CS framework: in particular, the relative volatility of the trade balance is particularly small (less than one tenth of the observed one).[28]

In the HO framework, the correlations of the trade balance and the terms of trade with income have the right signs, although they are larger in absolute terms than the observed values. The CS model delivers instead a *positive* correlation of the terms of trade with income. Heathcote and Perri [24] show that the Backus *et al.* [4] model generates a significantly positive correlation between the terms of trade and income due to the complete specialization assumption. In that model, a positive shock to productivity in country j raises output, reducing the price of country j's intermediate good. This implies that the terms of trade react positively on impact, being therefore highly correlated with income. Exactly the same mechanism is at work in our CS framework. In contrast,

in the HO model productivity shocks have no direct effect on the terms of trade, since intermediate good prices depend only on the capital-labor ratio at the world level.

Backus *et al.* [4] and our CS model, yields a significantly negative correlation between the terms of trade and the trade balance: the terms of trade react positively to the shock, while the trade balance response is negative. In the HO model, both the terms of trade and the trade balance react negatively on impact to a positive productivity shock in the capital-abundant country, and are therefore positively correlated; the simulated statistic is furthermore not very different from the observed one. The intuition is clear: a positive shock to a_1 leads country 1 to borrow from abroad, and it causes n_1 and subsequently N to rise on impact. This triggers a fall in both K/N and country 1's terms of trade p_x/p_z. The correlation between the trade balance and the terms of trade is negative instead for the labor-abundant country: a positive shock to a_2 leads country 2 to borrow from abroad and to a fall in K/N, but to a rise in country 2's terms of trade p_z/p_x. This implies a positive correlation between income and the terms of trade, and a negative correlation between the trade balance and the terms of trade. In this respect, it is interesting to note that Backus *et al.* [4] report a positive correlation between net exports and the terms of trade for the U.S., and negative correlations for most of the other countries in their sample. In fact, in their sample, most of the countries exhibiting a negative correlation between the terms of trade and the trade balance also exhibit a positive correlation between the terms of trade and income.

A look at the international business cycle properties suggests that the HO model is unable to solve the quantity puzzle: even if the international correlation of income is slightly positive, the international correlations of investment and hours worked are significantly negative. Furthermore, the international correlation of consumption is significantly higher than the correlation of income. The CS model seems to perform clearly better, since it reproduces positive international correlations for all variables. However, this first impression is misleading: the CS model generates positive international correlations by simply reducing the importance of trade as a transmission mechanism. These positive correlations simply reproduce the international correlation of productivity shocks, equal to 0.32.

4 Shocks to the non-traded sector

In our set-up, preferences and technologies - apart from productivity levels - are identical across countries. Under free trade, our model is therefore unable to produce any dynamics of the real exchange rate, since the price levels in both countries always remain the same. Backus *et al.* [4] and our CS model are able to bypass the problem by assuming a cross-

	Rel. Std.			Cor. y			Cor. π/y			Int. Cor.		
	US	HO	CS	US	HO	CS	US	HO	CS	Data	HO	CS
y	-	-	-	-	-	-	-	-	-	0.42	0.25	0.30
	-	-	-	-	-	-	-	-	-	*0.20*	*0.16*	*0.16*
c	0.80	0.52	0.50	0.87	1.00	1.00	-	-	-	0.23	0.20	0.35
	-	*0.01*	*0.00*	-	*0.00*	*0.00*	-	-	-	*0.22*	*0.17*	*0.15*
i	3.83	3.83	2.30	0.94	0.96	1.00	-	-	-	0.31	-0.53	0.27
	-	*0.20*	*0.01*	-	*0.02*	*0.00*	-	-	-	*0.21*	*0.13*	*0.16*
n	0.64	0.38	0.34	0.88	0.99	1.00	-	-	-	0.35	0.02	0.27
	-	*0.01*	*0.00*	-	*0.00*	*0.00*	-	-	-	*0.21*	*0.17*	*0.16*
$\frac{\pi}{y}$	0.63	0.49	0.04	-0.46	-0.71	-0.95	-	-	-	-	-	-
	-	*0.09*	*0.00*	-	*0.08*	*0.02*	-	-	-	-	-	-
e	5.79	0.83	0.89	0.09	0.70	0.50	0.16	-0.95	-0.53	-	-	-
	-	*0.10*	*0.14*	-	*0.09*	*0.13*	-	*0.03*	*0.12*	-	-	-
p	1.71	0.10	0.27	-0.18	-0.74	-0.65	0.10	0.27	0.72	-	-	-
	-	*0.01*	*0.03*	-	*0.07*	*0.08*	-	*0.16*	*0.07*	-	-	-

Table 3: Business cycle properties (shocks to the non-traded sector).

country asymmetry - a "home bias" - in the final good's production function.

To study the predictions of our model as far as the real exchange rate is concerned, we assume that productivity shocks affect the non-traded final good sector instead of the traded intermediate good sectors. In other words, the production function for the final good becomes

$$Y_{jt} = a_{jt} \prod_{i=1}^{m} x_{ijt}^{\alpha_i}. \tag{23}$$

We interpret the stochastic components a_{jt} literally as TFP levels, and in fact they represent the total productivity of factors used in the final good sector. However, productivity shocks in the consumption good sector can be interpreted also as demand shocks for the intermediate goods.

The relative price of the final good in country 1 in terms of the numeraire becomes $p_{1t} = a_{2t}/a_{1t}$. We define the real exchange rate as the ratio between the price of consumption in country 2 over the price of consumption in country 1, *i.e.* $e_t \equiv p_{2t}/p_{1t} = a_{1t}/a_{2t}$. Hence, productivity shocks in our framework can be interpreted also as shocks to the real exchange rate.

Table 3 reports the simulated statistics for our models, as well as their estimated empirical counterparts for the U.S.[29] The relative volatilities of both the terms of trade and the real exchange rate are much lower than their estimated values, but we replicate

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[29]We define the real exchange rate as the inverse of the price-adjusted and trade-weighted Broad Index of the foreign exchange value of the U.S. dollar. The quarterly data for the 1973:1-2001:4 period come from the Board of Governors and have been logged and Hodrick-Prescott filtered.

their relative rank: the terms of trade are less volatile than the real exchange rate. Notice that in our model the volatility of the real exchange rate has a one-to-one relationship with the volatility of a_1/a_2, whereas the volatility of p_ϕ/p_ψ has a one-to-one relationship with the volatility of K/N. That is, the volatility of productivity shocks affects the volatility of a country's real exchange rate directly, while its effect on the volatility of a country's terms of trade is indirect and dampened by the fact that intermediate goods prices are formed at the world level.

The relative volatilities of worked hours and the trade balance are slightly smaller than in the data. The correlations with income of the trade balance, the real exchange rate, and the terms of trade have the right signs, although they are larger in absolute terms than the observed value; in particular, the correlation of the real exchange rate with income largely exceed its observed value. Our model generates a significantly negative correlation of the trade balance with the real exchange rate, which is at odds with the positive correlation in the U.S. data. The terms of trade, instead, remain positively correlated with the trade balance, as in the data.

The qualitative performance of the HO and CS models is quite similar as far as the national stochastic properties are concerned: all statistics share the same sign. In particular, when the shocks hit the non-traded sector, the CS model generates a negative correlation of the terms of trade with income, and therefore a positive correlation of the terms of trade with the trade balance. However, there are some important quantitative differences. The most striking one regards the dynamics of the trade balance: when the CS model is calibrated to reproduce the actual long-run trade volume's GDP share, the trade balance's relative volatility is dramatically small (in our particular case, it is 16 times lower than the observed one). This result confirms that in the CS model international trade in intermediate goods seems not to be a quantitatively relevant transmission mechanism.

Concerning the international business cycle properties, the CS model still generates an international correlation of consumption that is higher that the correlation of income, but these positive international correlations mainly reflect the exogenous positive international correlation of productivity shocks, given the negligible dynamics of the trade balance. The HO model's international business cycle properties exhibit striking changes: the international correlation of GDP is now significantly positive, and *greater* than the correlation of consumption. The international correlation of employment is positive, although not significantly. However, the international correlation of investment is significantly negative.

Canova and Ravn [15] show that some testable restrictions implied by international consumption risk sharing in the standard IRBC framework are strongly rejected by the data. Lewis [32] suggests that financial market restrictions can help explain the apparent

lack of international consumption risk sharing in the data. In our model, the further restrictions introduced by FPE - in particular the cross-country equality of the marginal utility of leisure - can help explain this empirical puzzle, once we focus on shocks to the non-traded sector.[30] The key relationship to understand this result is equation (15): we can rewrite it in terms of deviations from the steady state as $\hat{c}_1 - \hat{c}_2 = \kappa \left(\hat{a}_1 - \hat{a}_2 \right)$.[31] Note that, since $\gamma > 0$ and $\sigma > 0$ by assumption, $\kappa < 0$ only if $\left(\gamma - 1 \right) / \gamma < \sigma < \gamma / \left(\gamma + 1 \right)$ or $\gamma / \left(\gamma + 1 \right) < \sigma < \left(\gamma - 1 \right) / \gamma$. Given that $\mathrm{std}\left(\hat{a}_1 \right) = \mathrm{std}\left(\hat{a}_2 \right)$ by assumption, and $\mathrm{std}\left(\hat{c}_1 \right) = \mathrm{std}\left(\hat{c}_2 \right)$ due to the symmetry of our set-up, we can show that

$$\mathrm{cor}\left(\hat{c}_j, \hat{c}_{-j} \right) = 1 - \kappa \left[\mathrm{cor}\left(\hat{c}_j, \hat{a}_j \right) - \mathrm{cor}\left(\hat{c}_j, \hat{a}_{-j} \right) \right] \frac{\mathrm{std}\left(\hat{a}_j \right)}{\mathrm{std}\left(\hat{c}_j \right)} \qquad (24)$$

The international correlation of consumption is smaller than one as long as: (i) if $\kappa > 0$, $\mathrm{cor}\left(\hat{c}_j, \hat{a}_j \right) > \mathrm{cor}\left(\hat{c}_j, \hat{a}_{-j} \right)$; (ii) if $\kappa < 0$, $\mathrm{cor}\left(\hat{c}_j, \hat{a}_j \right) < \mathrm{cor}\left(\hat{c}_j, \hat{a}_{-j} \right)$. It decreases - *ceteris paribus* - if: (i) κ increases; (ii) the correlation of consumption with TFP in the same country increases; (iii) the correlation of consumption with TFP in the other country decreases; (iv) the volatility of consumption decreases with respect to the volatility of TFP. Hence, since consumption levels are usually highly correlated with TFP in the same country and less volatile than TFP, the international correlation of consumption is likely to be significantly lower than one. Under our benchmark parameterization, it turns out to be lower than the correlation of GDP.

5   Concluding Remarks

Understanding the international transmission of business cycles through commodity trade requires modelling how the production structures of countries, their factor prices and trade patterns evolve over the cycle. This paper shows that combining standard international trade and macro models may help us in this direction, improving upon a whole generation of IRBC models that have neglected the endogenous reaction of trade patterns to the cycle.

Although FPE captures the idea that countries' factor prices and production structures depend on world fundamentals, it might be too strong an assumption from an empirical perspective. Current research in international trade endeavors to uncover more realistic

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[30]Our findings are related to those in Stockman and Tesar [41]. In a different framework, they show that taste shocks help explaining the observed properties of the data, while productivity shocks hitting the traded and non-traded sectors do not.

[31]Equation (15) implies also that $\ln \left(c_j / c_{-j} \right) = \kappa \ln e_j$, where e_j is the real exchange rate. In other words, the model predicts a close relationship between consumption ratios and bilateral real exchange rates as long as $\kappa \gg 0$. Backus and Smith [5] find little empirical evidence for this relationship in OECD time series. Under our benchmark parameterization, κ is strictly positive; hence we are left with a puzzle that cannot be addressed in our framework.

trade models that might bring new insights into international macroeconomics.

6   Appendix

6.1   The Complete Specialization Model

Following Backus *et al.* [4], we assume that, for technological reasons,[32] countries specialize in the production of a single good, labeled x for country 1 and z for country 2. Hence, households in both countries face the following resource constraints:

$$x_{1t} + x_{2t} = a_{1t} k_{1t}^{\psi} n_{1t}^{1-\psi} \tag{25}$$

$$z_{1t} + z_{2t} = a_{2t} k_{2t}^{\phi} n_{2t}^{1-\phi} \tag{26}$$

where x_{jt} and z_{jt} are the quantities of the two goods consumed in country j at date t. A final good, used for consumption and investment, is produced in each country using foreign and domestic goods via Cobb-Douglas production functions. We introduce a "home bias" by assuming that production functions are not symmetric across countries: domestic goods are relatively more important than foreign goods. Formally, households face the following additional resource constraints:

$$c_{1t} + i_{1t} = x_{1t}^{\alpha} z_{1t}^{1-\alpha} \tag{27}$$

$$c_{2t} + i_{2t} = x_{2t}^{1-\alpha} z_{2t}^{\alpha} \tag{28}$$

where α is the share of domestic goods in the final good production.[33] The capital stocks evolve according to equation (3).

We can easily show that:

$$\pi_t = \frac{p_{a,1t} a_{2t} - p_{b,1t} b_{1t}}{y_{1t}} = \frac{1-\alpha}{1-2\alpha} \left(1 - e_t \frac{y_{2t}}{y_{1t}} \right) \tag{29}$$

where $e_t = p_{a,1t}/p_{a,2t}$. Note that in steady state $y_1 \neq y_2$, $\pi \neq 0$, and $e \neq 1$ as long as $\psi \neq \phi$: the corresponding values can be found by numerically solving the system of first

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[32]More precisely, we have to assume that the TFP level in the labor-intensive (capital-intensive) sector drops to zero in country 1 (country 2). This is the only way to make sure that each country produces only one of the two goods. See Baxter [6].

[33]Backus *et al.* [4] assume a CES production function for the final consumption good, and favour a calibration characterized by a high elasticity of substitution between intermediate goods. For comparability reasons, we stick to the Cobb-Douglas case. Besides, Heathcote and Perri [24] estimate the elasticity of substitution in the Backus *et al.* [4] framework, and obtain a value of 0.9. Hence, our Cobb-Douglas seems not completely at odds with the empirical evidence.

order conditions evaluated at the steady state.[34]

The different role of α in the final good production function is the only structural difference between our Heckscher-Ohlin setup and the Complete Specialization framework. This difference is unavoidable: with Cobb-Douglas functional forms and under complete specialization, cross-country symmetry implies that the GDP of each country is simply a constant share of world GDP. While the Heckscher-Ohlin and the Complete Specialization frameworks share all the remaining parameters, α has to be calibrated separately. Following again Backus *et al.* [4], we calibrate α to make country 1's steady-state trade volume share in GDP match the U.S. long-run trade volume share (0.15). The corresponding value for α is 0.966.

This approach guarantees that, in both models, the key feature that drives the trade volume in steady state, i.e. the difference in the relative factor endowments for the HO model and the "home bias" for the CS one, is calibrated to make country 1's steady-state trade volume match its observed counterpart for the U.S.

6.2 The FPE condition

This Appendix discusses the implementation of the FPE condition, which is based on the integrated equilibrium's resource allocation. See Dixit and Norman [19] for a theoretical discussion of the FPE condition.

6.2.1 Integrated Equilibrium's Resource Allocation

Employment and capital are allocated as follows:

$$n_{\phi t} = \frac{(1-\phi)(1-\alpha)Y_t}{w_t}, \tag{30}$$

$$n_{\psi t} = \frac{(1-\psi)\alpha Y_t}{w_t}, \tag{31}$$

$$k_{\phi t} = \frac{\phi(1-\alpha)Y_t}{r_t}, \tag{32}$$

$$k_{\psi t} = \frac{\psi\alpha Y_t}{r_t}, \tag{33}$$

where $n_{\phi t}$ ($k_{\phi t}$) and $n_{\psi t}$ ($k_{\psi t}$) denote the amount of labor (capital) allocated at the world level to sector ϕ and ψ, respectively, and $Y_t \equiv \sum_{j=1}^{2} p_{jt} y_{jt}$ denotes the integrated equilib-

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[34]A detailed Technical Appendix is available from the authors on request.

rium's world income. Capital-labour intensities are therefore given by:

$$\frac{k_{\phi t}}{n_{\phi t}} = \frac{\phi}{1-\phi}\frac{w_t}{r_t}, \tag{34}$$

$$\frac{k_{\psi t}}{n_{\psi t}} = \frac{\psi}{1-\psi}\frac{w_t}{r_t}. \tag{35}$$

6.2.2 Trade Equilibrium's Resource Allocation

Under FPE, countries allocate resources according to the trade equilibrium's capital-labor intensities and their own factor endowments:

$$n_{\phi,1t} = \frac{\left(\frac{k_{\psi t}}{n_{\psi t}} - \frac{k_{1t}}{n_{1t}}\right)}{\left(\frac{k_{\psi t}}{n_{\psi t}} - \frac{k_{\phi t}}{n_{\phi t}}\right)}n_{1t}, \tag{36}$$

$$n_{\psi,1t} = n_{1t} - n_{\phi,1t}, \tag{37}$$

$$k_{\phi,1t} = \frac{k_{\phi t}}{n_{\phi t}}n_{\phi,1t}, \tag{38}$$

$$k_{\psi,1t} = k_{1t} - k_{\phi,1t}, \tag{39}$$

$$n_{\phi,2t} = n_{\phi t} - n_{\phi,1t}, \tag{40}$$

$$n_{\psi,2t} = n_{\psi t} - n_{\psi,1t}, \tag{41}$$

$$k_{\phi,2t} = k_{\phi t} - k_{\phi,1t}, \tag{42}$$

$$k_{\psi,2t} = k_{\psi t} - k_{\psi,1t}. \tag{43}$$

6.2.3 The Factor Price Equalization Condition

In the implementation of the FPE condition we need to distinguish several cases.

Case 1: $n_\psi < n_\phi$

1. $n_1 \in (0, n_\psi]$:
$$\frac{k_\phi}{n_\phi} \leq \frac{k_1}{n_1} \leq \frac{k_\psi}{n_\psi}. \tag{44}$$

2. $n_1 \in (n_\psi, n_\phi]$:
$$\frac{k_\phi}{n_\phi} \leq \frac{k_1}{n_1} \leq \frac{\left[k_\psi + \frac{k_\phi}{n_\phi}(n_1 - n_\psi)\right]}{n_1}. \tag{45}$$

3. $n_1 \in (n_\phi, N)$:
$$\frac{\left[k_\phi + \frac{k_\psi}{n_\psi}(n_1 - n_\phi)\right]}{n_1} \leq \frac{k_1}{n_1} \leq \frac{\left[k_z + \frac{k_\phi}{n_\phi}(n_1 - n_\psi)\right]}{n_1}. \tag{46}$$

Case 2: $n_\phi < n_\psi$

1. $n_1 \in (0, n_\phi]$:

$$\frac{k_\phi}{n_\phi} \leq \frac{k_1}{n_1} \leq \frac{k_\psi}{n_\psi}. \tag{47}$$

2. $n_1 \in (n_\phi, n_\psi]$:

$$\frac{\left[k_\phi + \frac{k_\psi}{n_\psi}(n_1 - n_\phi)\right]}{n_1} \leq \frac{k_1}{n_1} \leq \frac{k_\psi}{n_\psi}. \tag{48}$$

3. $n_1 \in (n_\psi, N)$:

$$\frac{\left[k_\phi + \frac{k_\psi}{n_\psi}(n_1 - n_\phi)\right]}{n_1} \leq \frac{k_1}{n_1} \leq \frac{\left[k_\psi + \frac{k_\phi}{n_\phi}(n_1 - n_\psi)\right]}{n_1}. \tag{49}$$

Case 3: $n_\phi = n_\psi$

1. $n_1 \in (0, n_\phi]$:

$$\frac{k_\phi}{n_\phi} \leq \frac{k_1}{n_1} \leq \frac{k_\psi}{n_\psi}. \tag{50}$$

2. $n_1 \in (n_\phi, N)$:

$$\frac{\left[k_\phi + \frac{k_\psi}{n_\psi}(n_1 - n_\phi)\right]}{n_1} \leq \frac{k_1}{n_1} \leq \frac{\left[k_\psi + \frac{k_\phi}{n_\phi}(n_1 - n_\psi)\right]}{n_1}. \tag{51}$$

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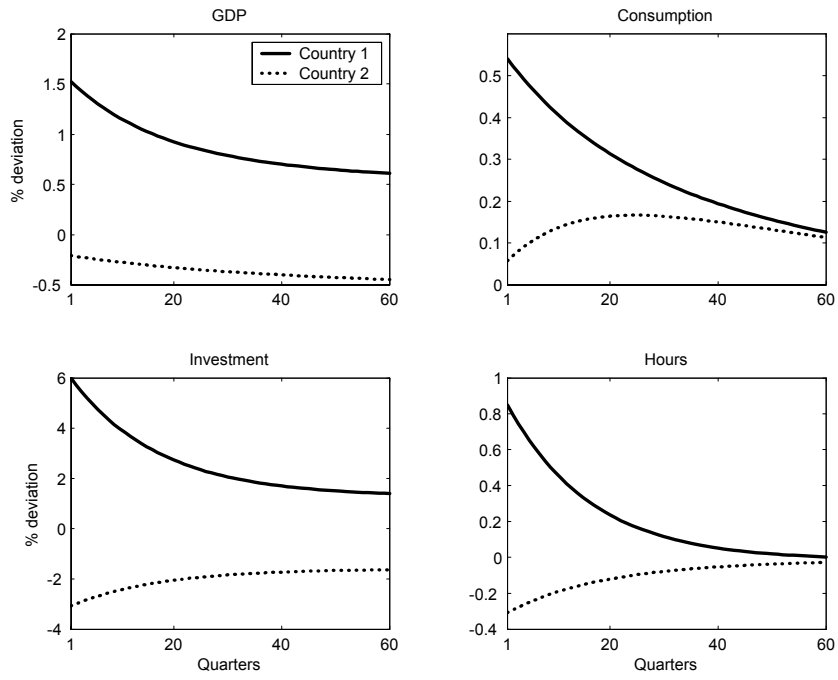
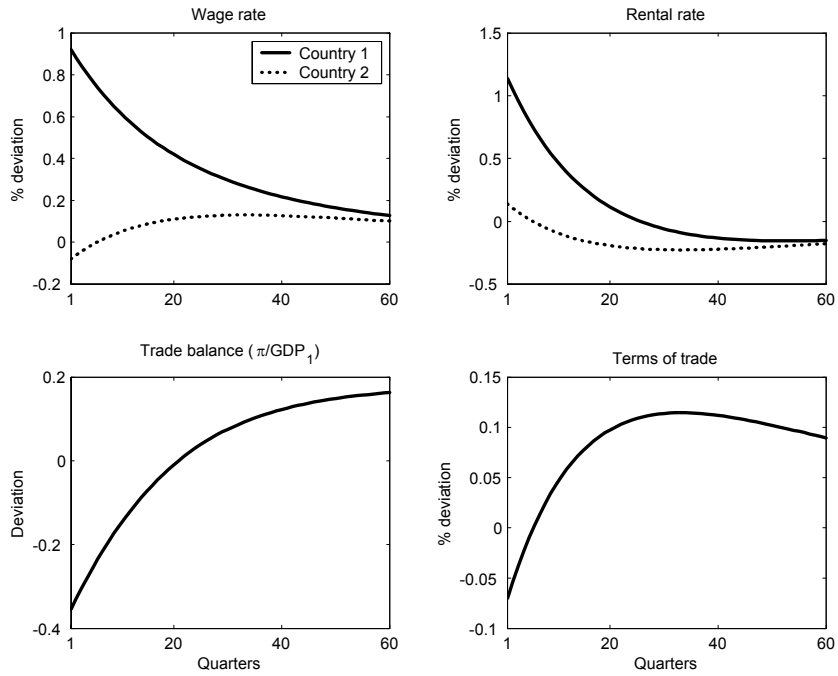Figure 1: Income and its components (HO - shock to traded sectors).

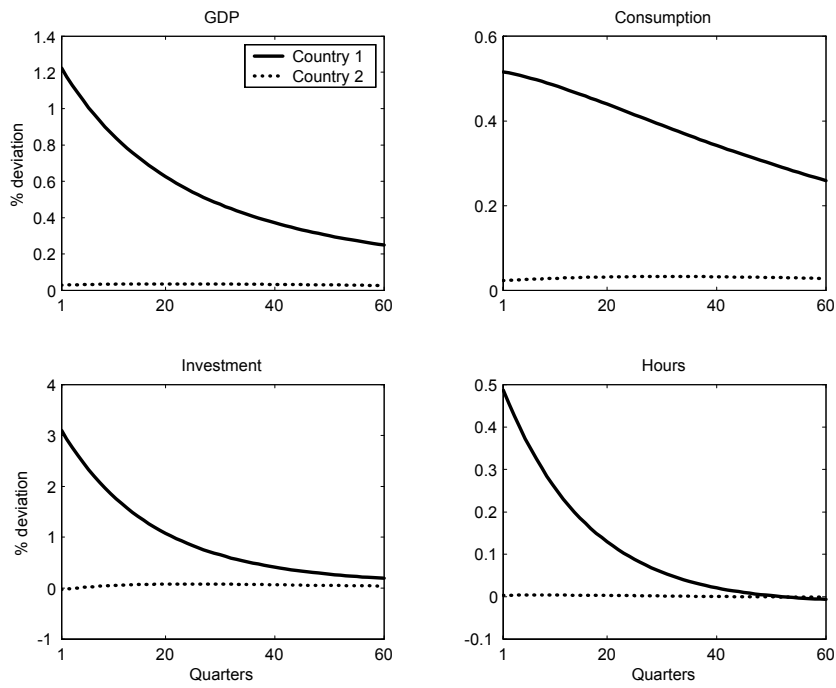Figure 2: Wages and rental rates (HO - shock to traded sectors).

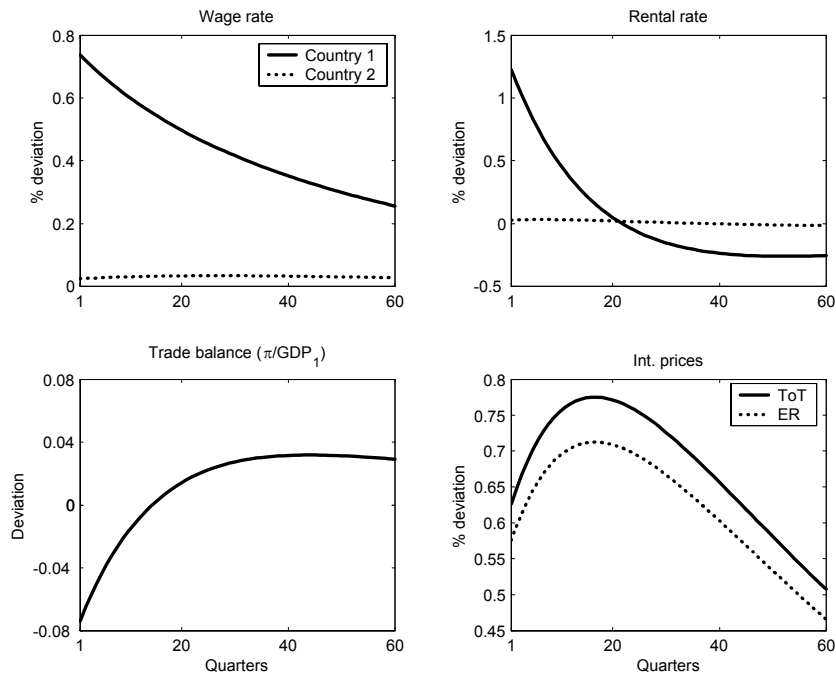Figure 3: Income and its components (CS - shock to traded sector).

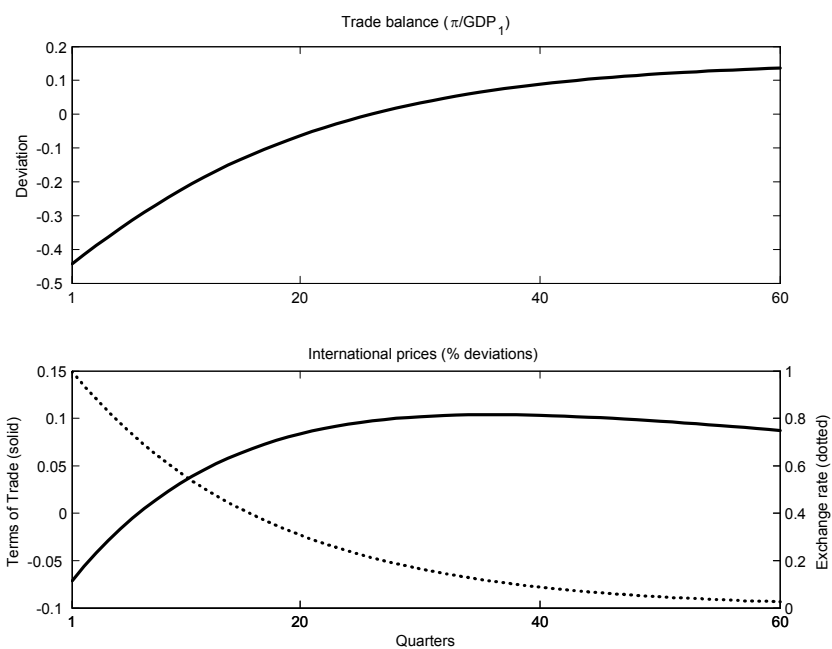Figure 4: Wages, rental rates, international variables (CS - shock to traded sector).

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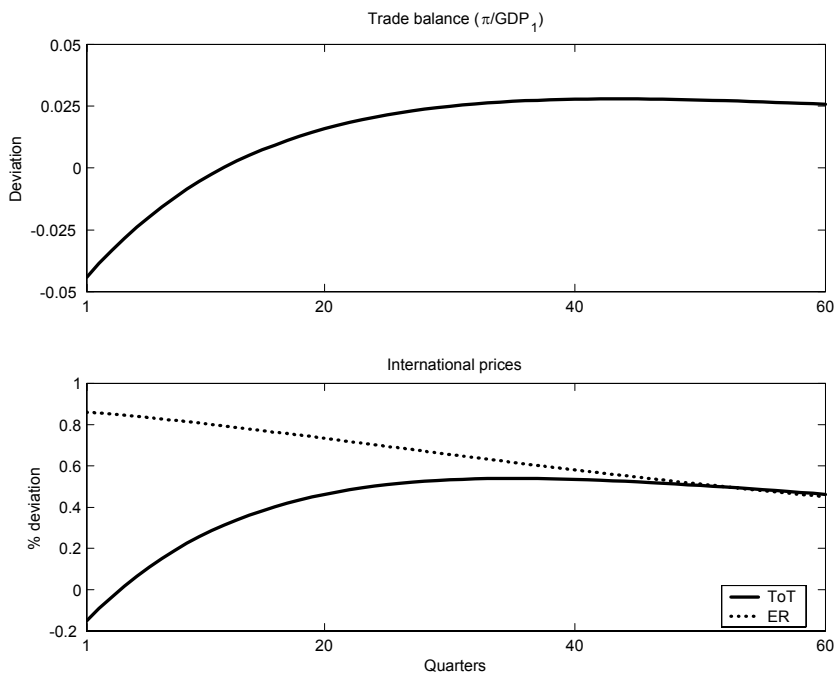Figure 5: International variables (HO - shock to non-traded sector).

Figure 6: Imternational variables (CS - shock to non-traded sector).