Limited Asset Markets Participation, Monetary Policy and (Inverted) Keynesian Logic.

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Abstract. This paper incorporates limited asset markets participation in dynamic general equilibrium and develops a simple analytical framework for monetary policy analysis. Aggregate dynamics and stability properties of an otherwise standard business cycle model depend nonlinearly on the degree of asset market participation. While 'moderate' participation rates strengthen the role of monetary policy, low enough participation causes an inversion of results dictated by ('Keynesian') conventional wisdom. The slope of the 'IS' curve changes sign, the 'Taylor principle' is inverted, optimal welfare-maximizing monetary policy requires a passive policy rule and the effects and propagation of shocks are changed. The conditions for these results to hold are relatively mild compared to some existing empirical evidence. Our results may justify Fed's behavior during the 'Great Inflation' period.

Keywords: *limited asset markets participation; dynamic general equilibrium; aggregate demand; Taylor Principle; optimal monetary policy; real (in)determinacy.*

JEL codes: *E32; G11; E44; E31; E52; E58.*

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1. Introduction

A tremendous amount of research has grown recently studying monetary policy in optimizing, dynamic general equilibrium models. The importance of this research and its influence on real-life policymaking need not be stressed here further. An excellent overview of the state-of-the-art in the field is the recent book by Michael Woodford (2003)[1].

At the heart of this literature lies some form of an 'aggregate Euler equation', or 'IS' curve, derived from the households' individual Euler equations. This relationship predicts that households will substitute consumption intertemporally - for example using assets. There is thus an inverse relationship between aggregate consumption today and the expected real interest rate. Using this as a building block, the literature derives normative prescriptions, some of which are robust across a wide variety of modelling strategies. First, the central bank needs to adopt an '*active*' policy rule whereby it increases the nominal interest rate by more than inflation (i.e. increases the *real* interest rate), for policy to be consistent with a unique rational expectations equilibrium; this is labeled 'the Taylor Principle' following Woodford (2001)[2]. Secondly, welfare-based optimal policy requires minimization of inflation and output variability, and also implies that the nominal interest rate increase by more than inflation. Thirdly, when there is no trade-off between output and inflation stabilization, full stabilization of both is possible by making the policy instrument equal the '*natural rate of interest*'; however, a commitment to fulfill the Taylor principle is still required to ensure that the resulting equilibrium is unique. Relatedly, an interest rate peg[3] (and any '*passive*' policy rule) would lead to multiple equilibria and stationary sunspot fluctuations (i.e. driven by beliefs and not fundamentals). Lastly, unanticipated real interest rate increases are contractionary.

This paper shows that limited asset market participation has a non-linear effect on these predictions. If participation is restricted below a certain threshold, the predictions are **strengthened**: as more (but not 'too many') people do not hold assets, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective. However, when participation is restricted beyond a given threshold (i.e. enough agents do not participate in asset markets), the standard theoretical prescriptions or predictions are **reversed**. Namely, (i) the 'IS curve' has a positive slope, (ii) the Taylor Principle is inverted, (iii) optimal policy requires a passive rule, and (iv) effects of some shocks are overturned. In the limit (when nobody holds assets), aggregate demand ceases to be linked to interest rates and monetary policy becomes ineffective .

We derive our results analytically in a standard dynamic general equilibrium model incorporating *limited asset markets participation* (hereinafter *LAMP*). We first provide some simple intuition for our results, relating our framework to a simple Keynesian cross. We then outline the general equilibrium model and derive

[1]Earlier overviews of these issues comprise, amongst others, Clarida, Gali and Gertler (1999) and Goodfriend and King (1997).

[2]This conclusion changes under some modelling choices. For example, in continuous time, Dupor (2000) shows that merely introducing capital invalidates the Taylor principle. A non-Ricardian fiscal policy in the sense of Woodford (1996) can also require a passive policy rule for equilibrium determinacy, as noted also by Leeper (1991). In an overlapping generations framework, Benassy (2001) also shows that the Taylor principle is invalidated.

[3]As in the much celebrated paper by Sargent and Wallace (1975).

its canonical form, reducing it to two familiar equations: a 'Philips curve' and an 'IS curve'. This makes our model easily comparable to the workhorse sticky price model, which occurs as a special case[4]; since the resulting system is very simple, it might be of independent interest to some researchers. Notably, we manage to capture the influence of LAMP on aggregate dynamics through a unique parameter, the elasticity of aggregate demand to real interest rates. This parameter depends non-linearly on the degree of asset market participation and is at the core of the intuition for all our results. The degree of LAMP necessary and sufficient for our results to hold is small when compared to empirical estimates of Campbell and Mankiw (1989) or to data on asset market participation e.g. in Mulligan and Sala-i-Martin (2000) or Vissing-Jorgensen (2002).

Following an emerging literature reviewed below, we assume that some agents have zero asset holdings, being either unable (constrained) or unwilling (myopic, uninformed) to participate it asset markets[5]. Empirical support for our modelling choice comes from a variety of directions, e.g. failure of consumption smoothing as a good description of aggregate behavior, high share of wealth-poor and asset-poor households in the data, low share of households holding various assets. In a celebrated paper, Campbell and Mankiw (1989) estimated that about 40 to 50 percent of the US population merely consume their current income[6]. Data on wealth distribution (see i.a. Wolff (2000) and Wolff and Caner (2002)) shows that a high fraction of the US population is wealth-poor or asset-poor; the exact fraction depends on the particular wealth variable used, but it is striking that 50 percent of the US population have less that 5000 USD in liquid assets. It is hard to argue that these households have the means to perfectly smooth consumption. When classifying households as 'asset-poor' based on 1999 PSID data, Wolff and Caner find that 41.7 percent can be classified as such when home equity is excluded from net worth, whereas 25.9 percent are asset-poor based on net worth data. Moreover, data on asset holdings presented i.a. in Mankiw and Zeldes (1991), Vissing-Jorgensen (2002), Guiso, Haliassos and Japelli (2002) shows that few people hold assets in various forms. For instance, Vissing-Jorgensen (2002) reports based on the PSID data that of US population 21.75 percent hold stock and 31.40 percent hold bonds[7]. Data from the 1989 Survey of Consumer Finances (see e.g. Mulligan and Sala-i-Martin (2000)) shows that 59 percent of US population had no interest-bearing financial assets, while 25 percent had no checking account either.

Models trying to incorporate this insight (that not all agents behave as prescribed by standard neoclassical theory) have been used in at least two strands of the macroeconomic literature. First, some version of this assumption has been proposed by Mankiw (2000) and extended by Galí, Lopez-Salido and Valles (2003) for

[4]There is very high variance regarding a label for such a framework in the literature. This goes from 'New Keynesian' (Clarida Gali and Gertler 1999 - henceforth CGG) to 'New Neoclassical synthesis' (Goodfriend and King 1997) to 'Neomonetarist' (Kimball 1996). Woodford (2003) refers to such a framework as 'Neo-Wicksellian'.

[5]In an appendix, we outline a simple model in which high enough proportional transaction costs can rationalize limited participation. We also review some evidence concerning the magnitude of these costs necessary to generate observed non-participation levels.

[6]Other papers (Flavin (1981), Zeldes (1989)) find similar results, and study whether the failure to smooth consumption comes from liquidity constraints.

[7]She further uses this heterogeneity to obtain better micro estimates of the elasticity of intertemporal substitution and other preference parameters

fiscal policy issues. Second, it is the norm in the monetary policy literature trying to capture the 'liquidity effect' (e.g. Alvarez, Lucas and Weber (2001), Occhino (2003)). Our assumption is very close in spirit to this second approach[8]: a subset of households cannot trade assets (this is sometimes called 'market segmentation'). Introducing physical capital as in Mankiw and GLV would merely allow for more heterogeneity: this is an important extension, but is not needed for any of our points.

While having been already used to explain some puzzles in the finance-asset pricing and in the fiscal policy literature[9], this modelling choice has only recently been incorporated into the sticky-price monetary policy research. A recent paper by Galí, Lopez-Salido and Valles (2003b, henceforth GLV) indeed argues that the Taylor principle is not a good guide for policy if some 'rule-of-thumb' agents do not hold physical capital. Namely, GLV argue that if the central bank responds to current inflation via a simple Taylor rule, when the share of 'rule-of-thumb' agents is high enough the response coefficient has to be higher than that suggested by the Taylor principle. On the contrary, for a rule responding either to past or future expected inflation, GLV suggest, based on numerical simulations, that for a high share of non-asset holders the policy rule needs to violate the Taylor principle to ensure equilibrium uniqueness. One of our paper's conclusions is closest (although not identical) to this last point. Instead, we show analitically that an 'Inverted Taylor principle' holds *in general* when asset market participation is restricted enough, no matter whether the policy rule responds to contemporaneous or future expected inflation. We provide intuition and explain the economic mechanism underlying our result as part of a more general theme having to do with the aggregate demand's sensitivity to interest rates, as anticipated above. We also show how the Taylor principle can be restored by either an appropriate response to output or via distortionary redistributive taxation of dividend income.

Our results can be perhaps most relevant for analyzing (i) developing economies, in which participation in asset markets is notoriously limited; (ii) historical episodes during which even developed economies experienced exceptionally low asset market participation. Regarding the latter, many authors have argued that policy before Volcker was 'badly' conducted along one or several dimensions, which led to worse macroeconomic performance as compared to the Volcker-Greenspan era. One such argument relies upon the estimated pre-Volcker policy rule non fulfilling the 'Taylor principle', hence containing the seeds of macroeconomic instability driven by non-fundamental uncertainty (CGG (2000), Lubik and Schorfheide (2004)). In a companion paper (Bilbiie (2004)), we take a *positive* standpoint and argue that Fed policy was better managed than conventional wisdom dictates, if financial market imperfections were pervasive during the 'Great Inflation' period. The tremendous financial innovation and deregulation process in the 1979-1982 period (same time as coming to office of Paul Volcker) and the abnormally high degree of regulation

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[8]Papers in the 'liquidity effect' vein study a completely different question: whether a contractionary monetary policy shock (decrease of money supply) is indeed associated to an increase in interest rates, and has effects consistent with the data.

[9]Gali, Lopez-Salido and Valles (2002) argue that this modelling assumption can help explaining the effects of government spending shocks if this is deficit-financed, taxation is lump-sum and labor is demand-determined. See also Bilbiie and Straub (2003b) for different labor market and budgetary structures.

in the 1970's provide some support to this view. Estimation of an IS curve suggests that its slope changed sign around the same time, consistent with our theory. Hence, a passive policy rule implied a determinate equilibrium, was close to optimal policy and allows for the effects of fundamental shocks to be studied. We show that the predictions of our model are in line with stylized facts and empirical findings. The change in financial imperfections might help explain both the change in macroeconomic performance and the change in the policy response; the abrupt change in the policy rule might not be a mere coincidence, but an optimal response to the structural change.

The rest of the paper is organized as follows. In Sections 2 we present a stripped-down version of our model, useful for 'inspecting the mechanism' intuitively, while in Sections 3 and 4 we introduce the LAMP general equilibrium model and its reduced log-linear form, and discuss our core results intuitively. A discussion of the labor market equilibrium useful for further intuition is also presented. Section 5 outlines the 'Inverted Taylor Principle' and discusses ways to restore the Taylor principle by stabilizing the output gap. Section 6 analyzes optimal monetary policy, Section 7 calculates analytically the responses of the economy to cost-push, technology and sunspot shocks under various scenarios and Section 8 concludes. Most technical details are contained in the Appendices.

2. Limited Asset Market Participation and the (Non-)Keynesian Cross

We will start with an outline of the main general implications of limited asset market participation in a simple framework. The analysis is highly simplified by adopting a set of assumptions that, while making exposition simple, are not necessary for the main results to hold. This is done to isolate the core mechanism and facilitate comparison with a textbook 'Keynesian cross' framework[10]. All the simplifying assumptions are relaxed in the fully microfounded model in the next section.

Suppose aggregate expenditure consists of consumption only. There are two types of agents: asset holders indexed by S, trading state-contingent assets and shares in firms and non-asset holders indexed by H, who do not participate in any of the asset markets and simply consume their current income. The shares of these agents are $1-\lambda$ and λ respectively and are assumed to be constant. Total consumption in log-linear deviations from steady state is given by $c = \lambda c_H + [1-\lambda]\, c_S$, where c_j is consumption of group j.[11] Suppose furthermore for simplicity that labor supply of non-asset holders is inelastic $n_H = 0$, such that their consumption is equal to the real wage $c_H = w$ and total labor supply is given by $n = [1-\lambda]\, n_S$. Assume that asset holders' labor supply obeys a standard optimality condition $\varphi n_S = w - c_S$, where φ is the inverse Frisch elasticity of labor supply for type S. Total consumption will hence be: $c = \lambda w + [1-\lambda]\, c_S = \lambda \varphi_S n_S + c_S = \frac{\lambda}{1-\lambda}\varphi n + c_S$.Finally, assume that the production function for final output in log-linear form is $y = [1+\mu]\, n$, where μ represents both the steady-state net mark-up and the degree of aggregate increasing returns to scale[12]. Using this we obtain the equivalent of the 'planned

[10] I thank Jordi Galí for having suggested the 'Keynesian cross' analogy.

[11] This approximation only holds if steady-state consumption shares of the two types are equal, i.e. asset income is zero in steady-state.

[12] This insures that asset income is zero in steady-state, so that all algebra here is consistent.

expenditure' (or 'aggregate demand') equation from standard Keynesian models (see for example David Romer's textbook):

$$c = c\left(\underset{+}{y}, \underset{-}{r - \pi^e}\right) = \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu}y + c_S \tag{2.1}$$

This equation links aggregate expenditure to current income, consumption of asset holders and exogenous technology. Note that (2.1) is not a reduced-form relationship since c, y, c_S are all endogenous variables, which will be determined in general equilibrium. However, we can think of (2.1) as a schedule in the (y, c) space, for a *given* level of $c_{S,t}$. In that sense, we can say that aggregate demand (expenditure) depends positively on current income and negatively on the real interest rate. We can define the (partial) 'marginal propensity to consume' out of current income as $\partial c/\partial y = \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu} > 0$. This 'marginal propensity to consume' is in fact a *partial* marginal propensity, i.e. keeping fixed consumption of asset holders c_S. In equilibrium, of course, all output is consumed. We will loosely refer to $\partial c/\partial y$ as 'marginal propensity to consume' in the remainder. The negative impact of ex-ante real interest rates $r - \pi^e$ on aggregate demand comes from a standard Euler equation for consumption of asset-holders: $c_S = c_S^e - [r - \pi^e]$, where r is the nominal interest rate and the intertemporal elasticity of substitution in consumption is normalized to one without loss of generality.

The marginal propensity to consume is increasing in (i) the share of non-asset holders λ, for this means that a higher fraction of total population simply consumes the real wage and is insensitive to interest rate movements and (ii) the extent to which labor supply is inelastic φ, for this implies that small variations in hours (and output) are associated to large variations in real wage and hence in the consumption of non-asset holders. Hence, (2.1) is consistent with Keynes' views that the aggregate propensity to consume depends on '*the principles on which income is divided between the individuals composing [the community] - which may suffer modification as output is increased*' and further that '*we may have to make an allowance for the possible reactions of aggregate consumption to the change in the distribution of a given real income between entrepreneurs and rentiers resulting from a change in the wage-unit*' (Keynes [1935], Chapter 8, Book III).

Together with the condition that consumption equal output $c = y$, equation (2.1) leads to the Keynesian cross and the standard IS equation in case $\partial c/\partial y < 1$. A positive but low enough λ makes the economy 'more Keynesian' since the propensity to consume out of current income becomes larger than zero, its value under full participation. However, note that the marginal propensity to consume out of current income (output) $\partial c/\partial y$ can become *greater than one* for high values of λ and/or φ, namely when $\lambda > [1 + \varphi/(1+\mu)]^{-1}$. This is the case when there are enough agents who consume their wage income w, and the latter is sensitive enough to total income y (labor supply is inelastic enough).

We label this case **'non-Keynesian'** since Keynes believed a marginal propensity to consume less than unity to be '*a fundamental psychological law*'. However, it should be noted that the aggregate implications of (4.1) do not necessarily contradict Keynes' views, as argued below (the difference coming from our definition of a marginal propensity for a *given* c_S). We plot (2.1) in this case along with the $c = y$ schedule in the 'Non-Keynesian cross' in Figure1, where an increase in the real interest rate moves the (2.1) schedule rightward (by intertemporal substitution) leading to higher consumption and output.

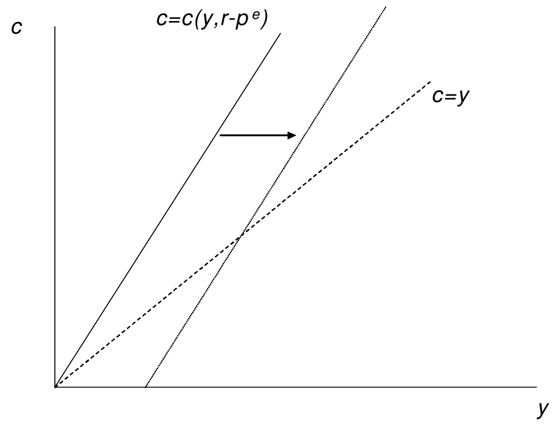

Fig.1: The Non-Keynesian Crosss

An immediate implication of the above is that the IS curve swivels (its slope *changes sign*). Consumption of asset holders is related to total output combining (2.1) with $c = y$ by $c_S = \delta y$, where $\delta \equiv 1 - \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu} = 1 - \partial c/\partial y$. Note that δ becomes negative when $\partial c/\partial y > 1$. Consumption of asset holders can be negatively related to total output since an increase in demand can only be satisfied by movements of (as opposed to movements along) the labor supply schedule when enough people hold no assets and labor supply is inelastic enough. But the necessary rightward shift of labor supply can only come from a *negative income effect* on consumption of asset holders. This negative income effect is ensured in general equilibrium by a *potential* fall in dividend income[13]. The *potential* decrease in profits is a natural result of inelastic labor supply, since the increase in marginal cost (real wage) would more than outweigh the increase in sales (hours). Therefore, this mechanism is consistent with Keynes' [op.cit.] statement that *'the consumption of the wealth-owning class may be extremely susceptible to unforeseen changes in the money-value of its wealth'.*

Substituting $c_S = \delta y$ into the Euler equation we obtain the aggregate 'IS curve', which has a positive slope when $\delta < 0$:

$$y = y^e - \delta^{-1}\left[r - \pi^e\right].$$

While we label the case where $\delta < 0$ 'non-Keynesian' (for it corresponds to a 'marginal propensity to consume' larger than one, which Keynes viewed as implausible) it should be emphasized that Keynes in fact believed that the impact of real interest rates on aggregate spending is not necessarily negative, since it depends on many contradicting factors (see next footnote). Among these, he in fact hints to

[13]This is seen clearly in the microfounded model of the next Section. We will also show that a potential fall in dividend income does not necessarily imply that *actual* profits fall in equilibrium. Intuitively, the negative income effect of a potential fall in dividends ensures a shift in labor supply and an increase in hours and a smaller equilibrium increase in wage, preventing profits from falling.

'*the appreciation or depreciation in the price of securities*', which is at the heart of our mechanism[14].

Not surprisingly, the implications of this insight on monetary policy and macroeconomic stability are dramatic. Recent research in monetary policy argues that monetary policy needs to be 'active' in order to ensure macroeconomic stability. Formally, if nominal interest rates are set as a function of expected inflation $r = \phi_\pi \pi^e$, the response coefficient needs to fulfill what Woodford (2001) has labeled 'the Taylor principle': $\phi_\pi > 1$. This ensures equilibrium determinacy (when prices are set on a forward-looking basis) or stability - when the Philips curve is backward-looking (see Taylor (1999)).

Clearly, in the non-Keynesian case an 'Inverted Taylor principle' holds: in order to ensure stability, monetary policy needs to be passive: $\phi_\pi < 1$. Otherwise, if expected inflation increases, an active policy would lead to an increase in real interest rates and a boost to consumption and output (by the non-Keynesian logic above). If a Philips curve holds such that inflation and output are positively related, inflation also increases validating the initial increase. One way to restore the Taylor Principle would be for the monetary authority to increase interest rates when output increases adopting a rule of the form $r = \phi_\pi \pi^e + \phi_x y$. Since consumption of asset holders is $c_S = c_S^e - (\phi_\pi - 1)\pi^e - \phi_x y$, the planned expenditure line becomes:

$$c = \left[\frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu} - \phi_x\right] y + c_S^e - [\phi_\pi - 1]\pi^e.$$

Hence, responding to output restores the Taylor Principle by making the slope of the PEL less than one when $\phi_x > -\delta > 0$. In this case, a non-fundamental increase in expected inflation would not result in an increase in output gap, and hence actual inflation would not increase. The rest of the paper presents the microfounded general equilibrium model leading to the 'non-Keynesian' case and studies closely its implications for aggregate dynamics, determinacy properties of interest rate rules, optimal monetary policy and the effects of shocks.

3. A General Equilibrium Model with LAMP

The model we use is a standard cashless dynamic general equilibrium model, augmented for limited asset markets participation. The latter feature is introduced by assuming that some of the households are excluded from asset markets, while

[14]'*The influence of this factor [the rate of interest] on the rate of spending out of a given income is open to a good deal of doubt. For the classical theory of the rate of interest, which was based on the idea that the rate of interest was the factor which brought the supply and demand for savings into equilibrium, it was* ***convenient to suppose*** *that expenditure on* ***consumption*** *is cet. par.* ***negatively sensitive to changes in the rate of interest****, so that any rise in the rate of interest would appreciably diminish consumption. It has long been recognised, however, that the* ***total effect of changes in the rate of interest*** *on the readiness to spend on present consumption is* ***complex and uncertain****, being dependent on* ***conflicting*** *tendencies, since some of the subjective motives towards saving will be more easily satisfied if the rate of interest rises, whilst others will be* ***weakened****.* [...] *Indirectly there may be more effects, though not all in the same direction. Perhaps the* ***most important influence****, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the* ***price of securities and other assets****. For if a man is enjoying a windfall increment in the value of his capital, it is natural that his motives towards current spending should be strengthened, even though in terms of income his capital is worth no more than before; and* ***weakened*** *if he is suffering* ***capital losses****.'* [Keynes op.cit., emphasis added]

others trade in complete markets for state-contingent securities (including a market for shares in firms). The failure to trade in asset markets could come from a variety of sources, having to do with either preferences or market frictions. We emphasize market frictions, and in Appendix 1 outline a simple asset pricing model with proportional transaction costs. We show how a distribution of proportional transaction costs can be found that rationalizes the exclusion of a given share of households from asset markets[15]. In light of this insight, in the remainder of the paper we assume the fraction of non-asset holders to be exogenous, as in most papers on market segmentation and limited participation, e.g. Alvarez, Lucas and Weber (2001). While our baseline model's reduced form is observationally equivalent to Galí, Lopez-Salido and Valles (2003), there are a few important differences (aside the difference in focus emphasized before). Firstly, we abstract from accumulation of physical capital: this allows us to obtain analytical solutions and helps understanding the mechanism behind our results[16]. Secondly, we model explicitly the asset markets, and notably the market for shares; indeed, the latter will be at the core of our results. Two other differences are: (i) an additively separable utility function, useful for emphasizing the role of labor supply; and (ii) a fixed cost in the intermediate-goods sector, which when set properly insures there are no long-run profits (and increasing returns). The differences in results are described and explained in detail below.

A role for monetary policy is introduced by assuming that prices are slow to adjust. There is a continuum of households, a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis. There is also a monetary authority setting its policy instrument, the nominal interest rate.

3.1. Households. There is a continuum of households $[0,1]$. A $1-\lambda$ share is represented by households who are forward looking and smooth consumption, being able to trade in all markets for state-contingent securities: *'asset holders'* or savers. Each asset holder (subscript S denotes the representative asset holder) chooses consumption, asset holdings and leisure solving the standard intertemporal problem: $\max E_t \sum_{i=0}^{\infty} \beta^i U_S \left(C_{S,t+i}, N_{S,t+i}\right)$, subject to the sequence of constraints:

$$B_{S,t} + \Omega_{S,t+1} V_t \leq Z_{S,t} + \Omega_{S,t} \left(V_t + P_t D_t\right) + W_t N_{S,t} - P_t C_{S,t}.$$

Asset holder's momentary felicity function $U_S \left(C_{S,t}, N_{S,t}\right) = \ln C_{S,t} - \theta_S N_{S,t}^{1+\varphi_S} / \left(1+\varphi_S\right)$ takes the form considered here to be consistent with most DSGE studies[17]. $\beta \in (0,1)$ is the discount factor, $\theta_S > 0$ indicates how leisure is valued relative to consumption, and $\varphi_S > 0$ is the coefficient of relative risk aversion to variations in leisure. $C_{S,t}, N_{S,t}$ are consumption and hours worked by saver (time endowment is normalized to unity), $B_{S,t}$ is the nominal value at end of period t of a portfolio of all state-contingent assets held, except for shares in firms. We distinguish shares

[15] Empirical estimates of lower bounds on transaction costs that rationalize observed participation rates or consumption and asset returns patterns can be found in Mulligan and Sala-i-Martin (2000) and Vising-Jorgensen (2003) or He and Modest (1995), respectively. E.g., Mulligan and Sala-i-Martin estimate the per-period cost of holding any interest-bearing asset to be 111 dolalrs per year.

[16] Note that capital accumulation in itself may overturn the Taylor principle, at least in continuous time, as emphasized by Dupor (2000). This would obscure our paper's message.

[17] This function is in the King-Plosser-Rebelo class and leads to constant steady-state hours.

from the other assets explicitly since their distribution plays a crucial role in the rest of the analysis. $Z_{S,t}$ is beginning of period wealth, not including the payoff of shares. V_t is the market value at time t of a share, D_t is its real dividend payoff and $\Omega_{S,t}$ are share holdings.

Absence of arbitrage implies that there exists a stochastic discount factor $\Lambda_{t,t+1}$ such that the price at t of a portfolio with uncertain payoff at $t+1$ is (for state-contingent assets and shares respectively):

$$B_{S,t} = E_t\left[\Lambda_{t,t+1} Z_{S,t+1}\right] \text{ and } V_t = E_t\left[\Lambda_{t,t+1}\left(V_{t+1} + P_{t+1}D_{t+1}\right)\right] \tag{3.1}$$

Note that the Euler equation for shares iterated forward gives the fundamental pricing equation: $V_t = E_t \sum_{i=t+1}^{\infty} \Lambda_{t,i} P_i D_i$.The riskless gross short-term nominal interest rate R_t is a solution to:

$$\frac{1}{R_t} = E_t \Lambda_{t,t+1} \tag{3.2}$$

Substituting the no-arbitrage conditions (3.1) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state, this will then imply the usual intertemporal budget constraint:

$$E_t \sum_{i=t}^{\infty} \Lambda_{t,i} P_i C_{S,i} \leq Z_{S,t} + V_t + E_t \sum_{i=t}^{\infty} \Lambda_{t,i} W_i N_{S,i} \tag{3.3}$$

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\begin{aligned} \beta \frac{U_C\left(C_{S,t+1}\right)}{U_C\left(C_{S,t}\right)} &= \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \\ \theta_S N_{S,t}^{\varphi_S} &= \frac{1}{C_{S,t}} \frac{W_t}{P_t} \end{aligned}$$

along with (3.3) holding with equality (or alternatively flow budget constraint hold with equality and transversality conditions ruling out overacummulation of assets and Ponzi games be satisfied: $\lim_{i\to\infty} E_t\left[\Lambda_{t,t+i} Z_{S,t+i}\right] = \lim_{i\to\infty} E_t\left[\Lambda_{t,t+i} V_{t+i}\right] = 0$). Using (3.3) and the functional form of the utility function, the short-term nominal interest rate must obey:

$$\frac{1}{R_t} = \beta E_t \left[\frac{C_{S,t}}{C_{S,t+1}} \frac{P_t}{P_{t+1}}\right].$$

The rest of the households on the $[0,\lambda]$ interval have no assets[18]: *'non-asset holders'*. Reasons for this could include constraints of participation to asset markets, myopia, extreme hyperbolic discounting, limited information (whereby current income is the most salient piece of information), etc. Assuming for instance as in Appendix 1 that these households face a common, large enough proportional cost to trade in asset markets, they will not hold any assets. The problem of the representative non-asset holder indexed by H is then equivalent to:

$$\max_{C_{H,t},N_{H,t}} \ln C_{H,t} - \theta_H \frac{N_{H,t}^{1+\varphi_H}}{1+\varphi_H} \text{ s.t. } C_{H,t} = \frac{W_t}{P_t} N_{H,t}. \tag{3.4}$$

[18]These households are labeled 'non-traders' by Alvarez, Lucas and Weber, 'rule-of-thumb' or 'non-Ricardian' by GLV, and 'spenders' by Mankiw 2000.

The first order condition is:

$$\theta_H N_{H,t}^{\varphi_H} = \frac{1}{C_{H,t}} \frac{W_t}{P_t}, \tag{3.5}$$

which further allows *reduced-form solutions* for $C_{H,t}$ and $N_{H,t}$ (functions only of W_t/P_t and exogenous processes). There is no need to keep consumption (or marginal utility of income) of H constant, as this does not depend on saving decisions or any other intertemporal feature. Note that due to the very form of the utility function, hours are constant for these agents: the utility function is chosen to obtain constant hours in steady state, and this agent is 'as if' she were in the steady state always. In this case labour supply of non-asset holders is fixed, no matter φ_H, as income and substitution effects cancel out. While this facilitates algebra, it is in no way necessary for our results (elastic labor supply will be discussed below). Hours are given by: $N_{H,t} = \theta_H^{-1/(1+\varphi_H)}$ and consumption will track the real wage to exhaust the budget constraint. In the remainder we shall assume without loss of generality that preferences are homogenous, i.e. $\varphi_S = \varphi_H = \varphi$. Certain assumptions spelled out below will imply that the relative weights of the disutility of work are also equal $\theta_S = \theta_H$.

3.2. Firms. The firms' problem is completely standard and can be skipped by some readers without loss of continuity. The **final good** is produced by a representative firm using a CES production function (with elasticity of substitution ε) to aggregate a continuum of goods intermediate indexed by i: $Y_t = \left(\int_0^1 Y_t\,(i)^{(\varepsilon-1)/\varepsilon}\, di\right)^{\varepsilon/(\varepsilon-1)}$. Final good producers behave competitively, maximizing profit $P_t Y_t - \int_0^1 P_t\,(i)\, Y_t(i) di$ each period, where P_t is the overall price index of the final good and $P_t\,(i)$ is the price of intermediate good i. The demand for each intermediate input is $Y_t(i) = (P_t\,(i)\,/P_t)^{-\varepsilon}\, Y_t$ and the price index is $P_t = \left(\int_0^1 P_t\,(i)^{1-\varepsilon}\, di\right)^{1/(1-\varepsilon)}$.

Each intermediate good is produced by a monopolist indexed by i using a technology given by:

$$Y_t(i) = A_t N_t(i) - F, \text{ if } N_t(i) > F \text{ and } 0 \text{ otherwise.}$$

F is a fixed cost assumed to be common to all firms: this will be a free parameter that can be chosen such that profits are zero in steady state and there are increasing returns to scale, consistent with evidence by Rotemberg and Woodford (1995). Alternatively, if the fixed cost is zero, there are steady-state profits (which is the case in GLV). We shall encompass both cases. Cost minimization taking the wage as given, implies that nominal marginal cost is $MC_t = W_t/A_t$. The profit function in real terms is given by: $D_t\,(i) = [P_t(i)/P_t]\, Y_t(i) - (W_t/P_t)\, N_t(i)$, which aggregated over firms gives total profits $D_t = [1 - (MC_t/P_t)\,\Delta_t]\, Y_t$. The term Δ_t is relative price dispersion defined following Woodford (2003) as $\Delta_t \equiv \int_0^1 (P_t\,(i)\,/P_t)^{-\varepsilon}\, di$ and will play a major role in the welfare analysis.

To introduce a role for monetary policy in affecting the real allocation in this simple cashless model we follow Calvo (1983) and Yun (1996) and introduce sticky prices. We assume that intermediate good firms adjust their prices infrequently, where θ is the probability of keeping the price constant. This exogenous probability is independent of history. Each period the fraction of firms that keep their prices unchanged is also equal to θ by the law of large numbers. Shareholders maximize the value of the firm, i.e. the discounted sum of future nominal profits, using the relevant stochastic discount factor $\Lambda_{t,t+i}$ used as the pricing kernel for nominal payoffs:

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\theta^s \Lambda_{t,t+s} \left[P_t(i) Y_{t,t+s}(i) - MC_{t+i} Y_{t,t+s}(i) \right],$$

subject to the demand equation (at $t+s$, conditional upon price set s periods in advance) $Y_{t,t+s}(i) = \left(P_t(i)/P_{t+s}\right)^{-\varepsilon} Y_{t+s}$.The optimal price of the firm is then found as a markup over a weighted average of expected future nominal marginal costs:

$$\begin{aligned} P_t^{opt}(z) &= (1+\mu) E_t \sum_{s=0}^{\infty} \varpi_{t,t+s} MC_{t+s} \\ \varpi_{t,t+s} &= \frac{\theta^s \Lambda_{t,t+s} \left(P_{t+s}\right)^{\varepsilon-1} Y_{t+s}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left(P_{t+k}\right)^{\varepsilon-1} Y_{t+k}} \end{aligned} \tag{3.6}$$

In equilibrium each producer that chooses a new price $P_t(i)$ in period t will choose the same price and the same level output. Then the dynamics of the price index given the aggregator above is: $P_t = \left((1-\theta) P_t^{opt}(i)^{1-\varepsilon} + \theta P_{t-1}(i)^{1-\varepsilon} \right)^{1/(1-\varepsilon)}$. The combination of these two conditions leads in the log-linearized equilibrium to the well known New Keynesian Phillips curve given below.

3.3. Monetary policy. We consider two policy frameworks prominent in the literature. First, we study *instrument rules* in the sense of a feedback rule for the instrument (short-term nominal interest rate) as a function of macro variables, mainly inflation. We focus on rules within the family (where variables with a star denote variables calculated under flexible prices, defined below):

$$R_t = \left(R_t^*\right)^{\phi^*} R \left(E_t \frac{P_{t+1}}{P_t} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_x} e^{\varepsilon_t}. \tag{3.7}$$

We shall also consider targeting rules under discretionary policymaking, whereby the path of the nominal rates is found by optimization by the central bank - this is described in detail in Section 6 below. Such a framework will also imply a behavioral relationship for the instrument rule, but this is only an *implicit instrument rule.*

3.4. Market clearing, aggregation and accounting. Labor and goods market clearing imply respectively $N_t = \lambda N_{H,t} + (1-\lambda) N_{S,t}$ and $Y_t = C_t \equiv \lambda C_{H,t} + (1-\lambda) C_{S,t}$, where C_t is aggregate consumption. State-contingent assets are in zero net supply, as is the case since markets are complete and agents trading in them are identical. By Walras' Law, the equity market clears and share holdings of each asset holder are then given by:

$$\Omega_{S,t+1} = \Omega_{S,t} = \Omega = \frac{1}{1-\lambda}.$$

4. Dynamics in the linear[ized] aggregate model

We can loglinearize the equilibrium conditions of the above model around the non-stochastic steady-state and express dynamics in terms of aggregate variables only, following similar steps as in Section 2 (see Appendix B for details). This makes our model readily comparable with the standard full-participation framework (see CGG (1999), Woodford (2003)) and amenable to policy exercises. We first express consumption of asset holders as a function of aggregate output y_t and log exogenous technology a_t:

$$c_{S,t} = \delta y_t + (1 + F_Y)(1 - \delta) a_t, \text{ where } \delta \equiv 1 - \varphi \frac{\lambda}{1-\lambda} \frac{1}{1+\mu},$$

$\mu \equiv (\varepsilon - 1)^{-1}$ is the steady-state net mark-up and F_Y the share of the fixed cost in total output in steady state (and the degree of increasing returns to scale). As noted before, consumption of asset holders can be related negatively to aggregate output ($\delta < 0$) when participation is low or labor supply is inelastic enough, more precisely when:

$$\lambda > \lambda^* = \frac{1}{1 + \varphi / (1 + \mu)}. \tag{4.1}$$

Consumption of asset holders can be negatively related to total output since an increase in demand can only be satisfied by movements of (as opposed to movements along) the labor supply schedule when enough people hold no assets and labor supply is inelastic enough. But the necessary rightward shift of labor supply can only come from a negative income effect on consumption of asset holders. This negative income effect is ensured in general equilibrium by a *potential* fall in dividend income. Note that asset holders have in their portfolio $(1 - \lambda)^{-1}$ shares: if total profits fell by one unit, dividend income of one asset holder would fall by $(1 - \lambda)^{-1} > 1$ units[19]. The *potential* decrease in profits is a natural result of inelastic labor supply, since the increase in marginal cost (real wage) would more than outweigh the increase in sales (hours).

However, note that *actual* profits may not fall, precisely due to the negative income effect making asset holders willing to work more; for as a result of this effect hours will increase by more and marginal cost by less. In fact, for certain combinations of parameters, shocks or policies our model would *not imply countercyclical profits* in equilibrium (or at least implies more procyclical profits than a standard full-participation model with countercyclical markups). This is an important point, since it is widely believed that profits are procyclical (see Section 7 for further discussion). It is also important to note that the negative income effect does not mean that for a given increase in output, the consumption of asset holders will necessarily *decrease*. In fact, if the increase in output is due to technology, c_S will *increase* in most cases (i.e. when the equilibrium elasticity of output to technology is less than $(1 + \mu)(1 - \delta^{-1})$).

We call '*non-Keynesian*' an economy in which participation in asset markets is limited enough such that $\delta < 0$, since as noted before in Section 2 increases in real interest rates are expansionary (see Figure 1). It is obvious that the only way for

[19] In the standard model all agents hold assets, so this mechanism is completely irrelevant. Any increase in wage exactly compensates the decrease in dividends, since all output is consumed by asset holders.

δ to be independent of λ is for φ to be zero, i.e. labor supply of asset holders be infinitely elastic. In this case, consumption of all agents is independent of wealth, making the heterogeneity introduced in this paper irrelevant.

Having expressed consumption of asset holders as a function of aggregate output, we can now substitute it back into the Euler equation and find an aggregate Euler equation, or 'IS curve':

$$x_t = E_t x_{t+1} - \delta^{-1} [r_t - E_t \pi_{t+1} - r_t^*] \quad (4.2)$$

In (4.2) we used the output gap $x_t \equiv y_t - y_t^*$ and 'natural' levels of output y_t^* and interest rates r_t^*, calculated under flexible prices as functions of technology:

$$\begin{aligned} y_t^* &= \left[1 + F_Y \left(1 - \chi^{-1}\right)\right] a_t;\ r_t^* \equiv [1 + F_Y (1 - \delta/\chi)] [E_t a_{t+1} - a_t], \\ \text{where } \chi &\equiv 1 + \varphi \left(1 + \frac{\lambda}{1-\lambda} D_Y\right) / (1 + F_Y) \geq 1 \geq \delta, \end{aligned}$$

and $D_Y \equiv (\mu - F_Y) / (1 + \mu)$ is the share of profits in steady state output. Note that permanent technology shocks have permanent effects on natural output and positive temporary effects on the natural interest rate since $\delta < \chi$ (whereas temporary technology shocks cause a fall in r_t^*). Following the discussion above, note that under flexible prices, consumption of asset holders will always increase in response to technology shocks, despite its partial elasticity to total output δ being negative. In fact, consumption of asset-holders under flexible prices is solved as $c_{S,t}^* = [1 + F_Y (1 - \delta/\chi)] a_t$ and is procyclical. Real profits under flexible prices are given by $d_t^* = [\mu / (1 + \mu)] y_t^*$, and are also procyclical.

Direct inspection of (4.2) suggests the impact that LAMP has on the dynamics of a standard business cycle model through modifying the elasticity of aggregate demand to real interest rates $-\delta^{-1}$ in a non-linear way. For high enough participation rates $\lambda < \lambda^*$ (where the latter is given by (4.1)) we are in a 'Keynesian' region, whereby real interest rates restrain aggregate demand. As λ increases towards λ^*, the sensitivity to interest rates increases in absolute value, making policy more effective in containing demand. However, once λ is above the threshold λ^* we move to the 'non-Keynesian' region where increases in real interest rates become expansionary (see also Figure 1). As λ tends to its upper bound of 1, $-\delta^{-1}$ decreases towards zero - policy is ineffective when nobody holds assets.

Log-linearization of the pricing equations leads to a Philips curve relating inflation to expected inflation and output gap variations:

$$\pi_t = \beta E_t \pi_{t+1} + \psi \chi x_t + u_t, \text{where } \psi \equiv (1 - \theta)(1 - \theta\beta)/\theta \quad (4.3)$$

Following e.g. Clarida, Galí and Gertler (1999) we introduce 'cost-push' shocks u_t, i.e. variations in marginal cost not due to variations in excess demand[20]. The aggregate supply side in (4.3) differs from the standard framework only insofar as the presence of non-asset holders modifies χ (the elasticity of marginal cost to movements in the output gap) and hence the response of inflation to aggregate demand variations. However, with increasing returns to scale of a degree making steady-state profits zero $\mu = F_Y$, the Philips curve is not influenced by the presence of LAMP. Rotemberg and Woodford (1995) make a strong case for such a degree of

[20]These could come from the existence of sticky wages creating a time-varying wage markup, a time-varying elasticity of substitution among intermediate goods or other sources creating this inefficiency wedge between the efficient and natural levels of output (e.g. distortionary taxation). For details as to what these time-varying wedges could be, see Woodford (2003, Ch. 6).

increasing returns. We assume that technology growth ($\Delta a_t \equiv a_t - a_{t-1}$) is given by an AR(1) process $\Delta a_t = \rho^a \Delta a_{t-1} + \varepsilon_t^a$, which implies shocks to technology have permanent effects (see Galí (1999)).

Parameterization We shall now have a first glance at the magnitude of λ required for our results to hold quantitatively. To that end we parameterize the model at quarterly frequency; the baseline case follows GLV (except for the mentioned differences) and most monetary policy studies. Namely, we set the discount factor β such that $r = 0.01$, the steady state markup $\mu = 0.2$ corresponding to an elasticity of substitution of intermediate goods of 6. The fixed cost parameter (and degree of increasing returns to scale) is set to either 0 (steady-state profits) or $F_Y = \mu = 0.2$. The average price duration is one year, implying $\theta = 0.75$. As to parameterizing labour, this is somehow more delicate, for there is no data to the best of our knowledge disentagling various preferences for leisure, or equivalently hours worked, as a function of wealth. Since we have no priors for assuming otherwise, we assume that both types work the same hours in steady state $N = N_S = N_H = 1/3$ (as commonly assumed in the literature). Recall that the elasticity of the marginal disutility of labor to labor φ is homogenous across groups. Then, if steady-state asset income is zero (profits are zero), consumption shares are equal across groups $C_H = C_S = C$ and hence preferences for leisure are also homogeneous $\theta_H = \theta_S$[21] Apart from φ, we also consider different values for the share of non-asset holders λ since this is probably our most controversial parameter - empirical evidence by Campbell and Mankiw (1989) suggests this is around 0.4-0.5 for the US economy.

In Figure 2 we plot the threshold λ as a function of φ, such that values under the curve give the $\delta > 0$ case, whereas above the curve we have the non-Keynesian economy with $\delta < 0$.

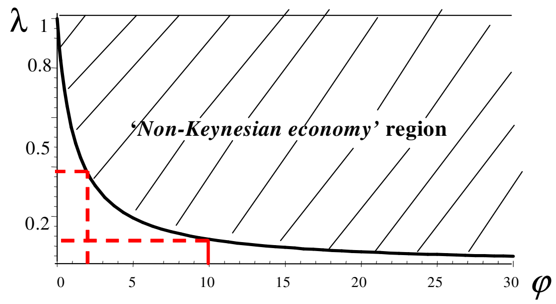

Fig. 2: Threshold share of non-asset holders as a function of inverse labor supply elasticity. Above the threshold we have the 'LAMP economy' where the Inverted Taylor Principle applies.

For Keynesian logic to work, the Frisch elasticity of labor supply (and of intertemporal substitution in labor supply), should be high, and the higher, the higher the share of non-asset holders λ. For a range of φ between 1 (unit elasticity) and 10 (0.1 elasticity) the threshold share of non-asset holders should be lower than 0.5 to as low as around 0.1 respectively. This shows that the required share of non-asset holders to end up in the non-Keynesian case is not that large.

[21]If instead steady-state asset income is positive (when for instance there are non-zero profits in steady-state), steady-state consumption shares are different. In that case, preferences for leisure will be different across households $\theta_H < \theta_S$: asset holders need to dislike labor less in order to work the same hours as non-asset holders in steady-state.

4.1. Further intuition - the labor market. The key to understanding the results obtained here is the labor market equilibrium. In system (4.4) we outline the labor supply and the equilibrium wage-hours locus. The labor supply schedule LS represents the locus of wages and hours for a given level of consumption of asset holders (all the intertemporal substitution in labor supply comes naturally from asset holders). The equilibrium wage-hours locus labeled WN is derived taking into account all equilibrium conditions, most notably how consumption is related to real wage in equilibrium. This schedule is invariant to endogenous forces in equilibrium (in fact, it will be shifted by technology shocks only).

$$\begin{aligned} LS &: \quad w_t = \varphi \frac{1}{1-\lambda} n_t + c_{S,t} \\ WN &: \quad w_t = \left[(1+F_Y)\,\delta + \varphi \frac{1}{1-\lambda} \right] n_t + (1+F_Y)\,a_t \end{aligned} \tag{4.4}$$

A 'non-Keynesian economy' ($\delta < 0$) has an intuitive interpretation in labor market terms, for it implies that the equilibrium wage-hours locus is less upward sloping than (and hence cuts from above) the labor supply curve. Intuitively, the presence of non-asset holders generates overall a 'negative income effect', which cannot be obtained ceteris paribus when $\lambda = 0$ and $\delta = 1$. In the latter, standard case, the wage-hours locus is more upward sloping than LS. The difference between the two is the intertemporal elasticity of substitution in consumption, normalized to 1 in our case (multiplied by returns to scale $1+F_Y$). Ceteris paribus, if the labor demand shifts out, labor supply shifts leftward due to the usual income effect, since agents anticipate higher income and higher consumption. If labor supply shifts up due to a positive income effect, same effect makes labor demand shift out. This gives a WN locus more upward sloping than the labor supply curve LS. The threshold value for λ for this insight to change is the same as that making $\delta < 0$ and given in (4.1) above. When the share of non-asset holders is higher than this threshold (or equivalently for a given share, labor supply of asset holders is inelastic enough), the wage-hours locus becomes less upward sloping than the labor supply. An intuition for that follows, as illustrated in Figure 3 where we assume that the real interest rate is kept constant for simplicity[22].

Take first an exogenous outward *shift in labor demand.* Keeping supply fixed, there would be an increase in real wage and an increase in hours. The increase in the real wage would boost consumption of non-asset holders, amplifying the initial demand effect. When labor supply is relatively inelastic, this increase in wage is large and the increase in hours is small compared to that necessary to generate the extra output demanded; note that the effect induced on demand is larger, the higher the share of non-asset holders. The only way for supply to meet demand is for labor supply to shift right. This is insured in equilibrium by the potential fall in profits resulting from: (i) increasing marginal cost (since wage increases) and (ii) the weak increase in hours and hence in output and sales. This is like and indirect negative income effect induced on asset holders by the presence of non-asset holders. Next consider a *shift in labor supply,* for example leftward as would be the case if consumption of asset holders increased. Keeping demand fixed, wage increases and hours fall. The increase in wage (and the increase in consumption of asset holders

[22] How the nominal interest rate reacts to inflation, generated here by variations in demand, will be crucial in the further analysis.

itself) has a demand effect due to sticky prices. As labor demand shifts right, the real wage would increase by even more; hours would increase, but by little due to the relatively inelastic labor supply (the overall effect would again depend on the relative slopes of the two curves). The increase in the real wage means extra demand through non-asset holders' consumption[23]. To meet this demand, only way for increasing output is an increase in labor supply, which instead obtains only if labor supply shifts right, which is insured as before by the potential fall in profits. This explains why in a '*non-Keynesian economy*' the wage-hours locus cuts the labor supply curve from above. This instead will help our intuition in explaining the further results[24]. Note that such a wage-hours locus implies that the model generates a higher partial elasticity of hours to the real wage, and more so more negative δ is.

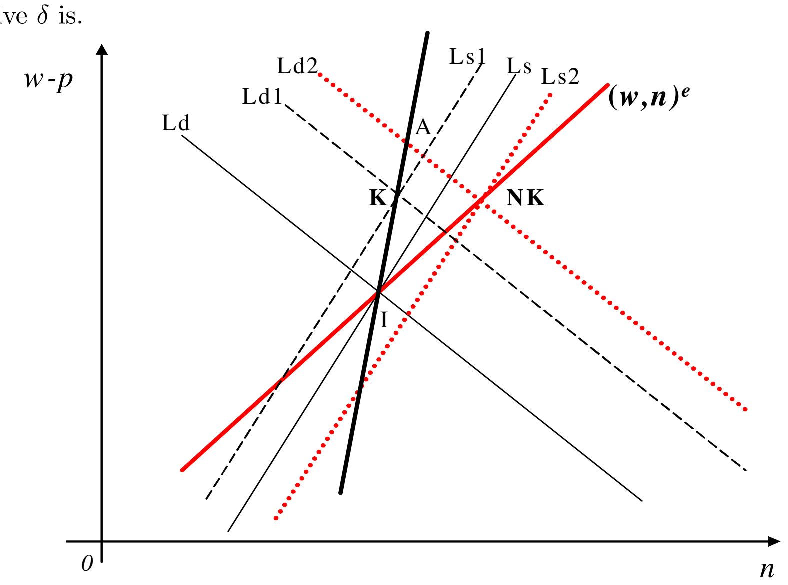

Fig. 3: The equilibrium wage-hours locus and labor supply curve with LAMP.

Having derived the equilibrium wage-hours locus gives us a simple way of thinking intuitively about the effects of shocks and of monetary policy in general; monetary policy, by changing nominal interest rates, modifies real interest rates and hence shifts the labor supply curve (by changing the intertemporal consumption profile of asset holders). But this has no effect on the wage-hours locus by construction, since this describes a relationship that holds in equilibrium always and is shifted only by technology shocks.

4.2. Robustness. One might rightly wonder whether the mere theoretical possibility of a change in the sign of δ is entirely dependent upon the specification of preferences. It turns out this possibility is robust to two obvious candidates:

[23]The assumptions on preferences ensuring constant steady-state hours are less crucial than it might seem. Below we consider alternative preference specifications.

[24]Note that the intuition for real indeterminacy to obtain in standard models (see e.g. Benhabib and Farmer 1994) requires the wage hours locus be upward sloping but cut the labor supply curve from *below*. This is also the case in standard sticky-price models, and gives rise to a certain requirement for the monetary policy rule to result into real determinacy - see below. Our intuition will be that having the wage-hours locus cut the labor suply from *above*, changes determinacy properties in a certain way.

an elastic labor supply of non-asset holders, and a non-unitary elasticity of intertemporal substitution in consumption. For completion, we briefly study these extensions jointly. Consider preferences given by a general CRRA utility function for both agents j (γ is relative risk aversion for both agents and also inverse of intertemporal elasticity of substitution in consumption for asset holders):

$$U_j(.,.) = \frac{C_{j,t}^{1-\gamma}}{1-\gamma} - \theta_j \frac{N_{j,t}^{1+\varphi}}{1+\varphi} \tag{4.5}$$

Following the same method as before one can show that the solution to non-asset holders' problem will be (in log-linearized terms, where elasticity of hours to wage $\eta \equiv (1-\gamma)/(\gamma+\varphi)$ is positive iff $\gamma < 1$):

$$n_{H,t} = \eta w_t; \quad c_{H,t} = (1+\eta) w_t \tag{4.6}$$

For asset holders, the new Euler equation and intratemporal optimality in log-linearized form are:

$$E_t c_{S,t+1} - c_{S,t} = \gamma^{-1} (r_t - E_t \pi_{t+1}) \tag{4.7}$$

$$\varphi n_{S,t} = w_t - \gamma c_{S,t} \tag{4.8}$$

Using the same method as previously, one finds that the new condition to be fulfilled in order for δ to become negative and hence end up in a 'non-Keynesian economy':

$$\lambda > \frac{1}{1+\varphi(1-\eta\mu)/(1+\mu)} \tag{4.9}$$

By comparing (4.9) with (4.1) one immediately notices that under the more general preferences (4.5) the threshold value of λ is lower (higher) than under log utility if $\eta > 0$ (< 0). The intuition is that while making aggregate labor supply more elastic, a positive η also makes *equilibrium* hours more elastic to wage changes since it makes consumption of non-asset holders more responsive to the wage. In general however, the difference induced by having $\gamma \neq 1$ on the threshold value of λ is quantitatively negligible[25]. In view of the relative innocuousness of these assumptions, we shall continue using the log-CRRA utility function in the remainder, since it preserves constant steady-state hours and hence allows analyzing permanent technology shocks.

5. The Inverted Taylor Principle: Determinacy properties of interest rate rules

In this Section we study determinacy properties of simple interest rate rules[26]. We first consider rules involving a response to expected inflation, as done for example by CGG (2000). This specification provides simpler (sharper) determinacy conditions, and captures the idea that the central bank responds to a larger set of information than merely the current inflation rate:

$$r_t = \phi_\pi E_t \pi_{t+1} + \varepsilon_t \tag{5.1}$$

where ε_t is the non-systematic part of policy-induced variations in the nominal rate. The dynamic system for the $z_t \equiv (x_t, \pi_t)'$ vector of endogenous variables and the

[25]Even evaluating the difference between the threshold values corresponding to $\gamma = 0$ and 100 respectively, one obtains under the baseline parameterization (for values of $\varphi = 0.5; 1; 5; 10$ respectively): 0.13;0.09; 0.03; 0.01.

[26]For analytical simplicity we abstract from inertia (interest rate smoothing) but this extension should be straightforward.

$\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$ vector of disturbances is obtained by replacing (5.1) into (4.2) and (4.3) as:

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi \nu_t,$$

where coefficient matrices are given by:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 - \beta^{-1}\delta^{-1}\kappa(\phi_\pi - 1) & \delta^{-1}\beta^{-1}(\phi_\pi - 1) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix}, \quad (5.2)$$

$$\Psi = \begin{bmatrix} \delta^{-1} & -\beta^{-1}\delta^{-1}(\phi_\pi - 1) \\ 0 & -\beta^{-1} \end{bmatrix}.$$

Since both inflation and output gap are forward-looking variables, determinacy requires that both eigenvalues of $\mathbf{\Gamma}$ be outside the unit circle. The determinacy properties of rule (5.1) are emphasized in Proposition 2 (the proof is in Appendix C).

Proposition 1. ***The Inverted Taylor Principle**: Under policy rule (5.1) there exists a locally unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:*

Case I: When $\delta > 0: \ \phi_\pi \in \left(1, 1 + \delta\frac{2(1+\beta)}{\kappa}\right);$

Case II: When $\delta < 0: \ \phi_\pi \in \left(1 + \delta\frac{2(1+\beta)}{\kappa}, 1\right) \cap [0, \infty).$

Case I corresponds to the standard 'Keynesian' case: the Taylor principle (Woodford (2001)) is at work: as noted in the previous literature the central bank should respond more than one-to-one to increases in inflation[27]. Case II is the 'non-Keynesian economy'. In this case, the Central Bank should follow an *Inverted Taylor Principle*: only passive policy is consistent with a unique rational expectations equilibrium. Obviously, the condition for the *Inverted Taylor Principle* to hold is the same as the one causing a change in the sign of δ, as in (4.1).

5.1. Intuition: sunspot equilibria with LAMP. An intuitive explanation of Proposition 1 is in order. We will discuss intuition as to why in cases covered by the Proposition sunspot shocks have no real effect, whereas in the opposite cases they lead to self-fulfilling expectations. In Section 7 below we will compute sunspot equilibria formally. Note that by substituting the rule (5.1) into (4.2) we obtain aggregate demand as a function of expected inflation:

$$x_t - E_t x_{t+1} = -\delta^{-1}(\phi_\pi - 1) E_t \pi_{t+1}. \quad (5.3)$$

Suppose for simplicity and without losing generality that a sunspot shock hits inflationary expectations. In a *Non-Keynesian economy* ($\delta < 0$) a non-fundamental increase in expected inflation generates an increase in the output gap today if the policy rule is active ($\phi_\pi > 1$) as can be seen from (5.3). By the Philips curve inflation today increases, validating the initial non-fundamental expectation. This is not the case in the Keynesian economy ($\delta > 0$), since an active rule generates a fall in output gap and (by Philips curve logic) actual inflation, contradicting initial expectations.

How does a passive policy rule ensure equilibrium determinacy when $\delta < 0$? A non-fundamental increase in expected inflation causes a fall in the real interest rate, a fall in the output gap today by (5.3) and deflation, contradicting the initial

[27]It should also not respond 'too much', which is a well-established result first noted by Bernake and Woodford (1997).

expectation that are hence not self-fulfilling. At a more micro level, the transmission is as follows. The fall in the real rate leads to an increase in consumption of asset holders, and an increase in the demand for goods; but note that these are now partial effects. To work out the overall effects one needs to look at the component of aggregate demand coming from non-asset holders and hence at the labor market. The partial effects identified above would cause an increase in the real wage (and a further boost to consumption of non-asset holders) and a fall in hours. Increased demand, however, means that (i) some firms adjust prices upwards, bringing about a further fall in the real rate (as policy is passive); (ii) the rest of firms increase labor demand, due to sticky prices. Note that the real rate will be falling along the entire adjustment path, amplifying these effects. But since this would translate into a high increase in the real wage (and marginal cost) and a low increase in hours, it would lead to a fall in profits, and hence a negative income effect on labor supply. The latter will then not move, and no inflation will result, ruling out the effects of sunspots. This happens when asset markets participation is limited 'enough' in a way made explicit by (4.1).

If the policy rule is instead active $(\phi_\pi > 1)$ sunspot equilibria can be constructed. The shock to inflationary expectations leads to an increase in the real rate and in aggregate demand by (5.3). This generates inflation and makes the initial expectations self-fulfilling. At a micro level, transmission is as follows: consumption of asset holders increases due to the real rate increase, which implies a rightward shift of labor supply, and hence a fall in wage and increase in hours. Consumption of non-asset holders also falls one-to-one with the wage, and hence aggregate demand falls by more than it would in a full-participation economy. Firms who can adjust prices will adjust them downwards, causing deflation, and a further fall in the real rate. Firms who cannot adjust prices will cut demand, causing a further fall in the real wage and a small fall in hours (since labor supply is inelastic). But this will mean higher profits (since marginal cost is falling), and eventually a positive income effect on labor supply of asset holders. As labor supply starts moving leftward, demand starts increasing, its increase being amplified by the sensitivity of non-asset holders to wage increases. The economy will establish at a point on the wage-hours locus consistent with the overall negative income effect on labor supply of asset holders, i.e. with higher inflation and real activity. Hence, the initial inflationary expectations become self-fulfilling.

5.2. Output stabilization restores the Taylor Principle. Following the intuition at the end of Section 2, we now study how a policy rule incorporating an output stabilization motive can make the Taylor principle a good policy prescription even in a 'non-Keynesian economy' where $\delta < 0$. In contrast to the simple framework considered earlier, there are non-trivial interactions among parameters due to the nature of the price-adjustment equation. Consider a rule of the form:

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_x x_t, \tag{5.4}$$

whereϕ_x is the response to output gap. Replacing this into (4.2) and (4.3), the $\mathbf{\Gamma}$ matrix becomes:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 - \delta^{-1}\left[\beta^{-1}\kappa\left(\phi_\pi - 1\right) - \phi_x\right] & \delta^{-1}\beta^{-1}\left(\phi_\pi - 1\right) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix}$$

Applying exactly the same method as in the proof of Proposition 1 it can be shown that the determinacy conditions are as follows.

Proposition 2. *(a) Under (5.4), there exists a locally unique rational expectations equilibrium if and only if:*

Case I: When $\delta > 0:$ $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ *and* $\phi_\pi < 1 + \frac{1+\beta}{\kappa}(\phi_x + 2\delta)$ *(the 'Taylor Principle')*

Case II: When $\delta < 0:$ *EITHER*

$$\text{II.A: } \phi_x < -\delta(1-\beta) \text{ and } \phi_\pi + \frac{1-\beta}{\kappa}\phi_x < 1 \text{ and } \phi_\pi > 1 + \frac{1+\beta}{\kappa}(\phi_x + 2\delta)$$

OR

$$\text{II.B: } \phi_x > -\delta(1+\beta) \text{ and } \phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1 \text{ and } \phi_\pi < 1 + \frac{1+\beta}{\kappa}(\phi_x + 2\delta)$$

(b) The equilibrium is indeterminate regardless of ϕ_π *if* $\delta < 0$ *and* $\phi_x \in (-\delta(1-\beta); -\delta(1+\beta))$.

Part (a) studies equilibrium uniqueness. Case I is the standard Taylor principle for an economy where $\delta > 0$. In contrast to Proposition 1, in Case II the inversion of the Taylor Principle is now *not* granted. If either δ is very large in absolute value (a high degree of limited participation λ) or the response to output is low, we end up in case II.A and an instance of the Inverted Taylor Principle is observed. However, for moderate values of λ and/or a high enough response to the output gap, the Taylor Principle is restored. Another way to put this is that for a given share of non-asset holders, the Taylor Principle is a good guide for policy only insofar as the response to output is high enough. The response to output, however, can generate perverse effects if it is not high enough and participation to asset markets is very limited. As part (b) of the Proposition shows, the equilibrium is indeterminate if ϕ_x is in a certain range, regardless of the magnitude of the inflation response. This region is increasing with the share of non-asset holders.

To assess the magnitude of the policy coefficients needed for restoring the Taylor principle, consider the otherwise baseline parameterization for a 'non-Keynesian economy' with $\lambda = 0.4$ and $\varphi = 2$ giving $\delta = -0.11$ and $\kappa = 0.228$. The conditions for Case II.B are $\phi_x > 0.21$; $\phi_\pi > 1 - 0.043\phi_x$; $\phi_\pi < 8.728\,\phi_x - 0.92016$. The figure below shows that as soon as the Central Bank responds to output, the Taylor principle is restored under the baseline parameterization for a large parameter region. However, this result should be taken with care, for the very dangers associated with responding to output might outweigh potential benefits. As soon as the share of non-asset holders increases or labor supply becomes more inelastic, equilibrium is more likely to become indeterminate for any inflation response. In that case, it seems advisable not to respond to the output gap.

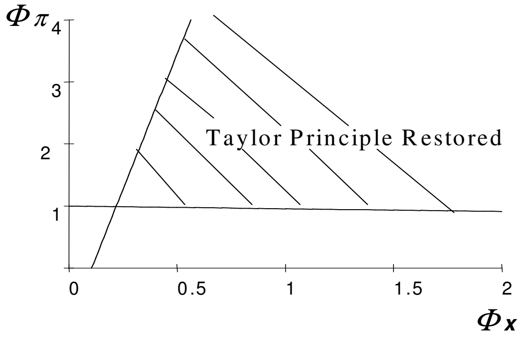

Fig. 4 : Policy parameter region whereby Taylor Principle is restored in a 'LAMP economy' by responding to output under baseline parameterization.

Finally, in Appendix C we show that a version of the *Inverted Taylor Principle* holds for a contemporary rule also. This is done to further illustrate the differences of our determinacy results from GLV (2003*b*), where there is a dramatic distinction between forward-looking and contemporaneous rules. GLV do note (relying upon numerical simulations and not as a general result) a result similar to our Proposition 2: namely, the Taylor principle may need to be violated for a forward-looking rule, and only for a high share of non-asset holders. But for a contemporaneous rule to be compatible with a unique equilibrium, they argue that the central bank should respond to inflation more strongly than in the full-participation economy (and indeed very strongly under some parameter constellations). The message of our paper in what regards determinacy properties of policy rules is different: we provide analytical conditions for an inverted Taylor principle to hold generically, independently on the policy rule followed. Our results for a simple Taylor rule have the same flavor as for a forward-looking rule: in the 'non-Keynesian economy' the inverted Taylor principle holds 'generically' (i.e. if we exclude some extreme values for some of the parameters) for a somewhat larger share of non-asset holders than was the case under a forward-looking rule. It is also the case, as in GLV, that a policy rule responding to current inflation very strongly would insure equilibrium uniqueness[28]. But the implied response ($\phi_\pi = 35$ under the baseline parameterization): (i) is much larger than any plausible empirical estimate; (ii) would imply that zero bound on nominal interest rates be violated for even small deflations; (iii) would have little credibility. This is in contrast with GLV, who do not consider a possible inversion of the Taylor principle in their numerical analysis of such rules, but instead argue that for a large share of non-asset holders making the required policy response too strong under a Taylor rule, the central bank should switch to a passive forward-looking rule.

6. Optimal monetary policy (when the steady-state is efficient).

The above analysis suggests that in an economy with limited participation in asset markets, the central bank following an active rule would leave room for

[28]This is not the case under a forward looking rule, since there, even in a standard full-participation economy too strong a response leads to indeterminacy - see Bernake and Wodford 1997.

sunspot-driven real fluctuations. The size of these fluctuations would depend upon the size of the sunspot shocks (something impossible to quantify in practice), but this would unambiguously increase the variances of real variables such as output and inflation. If such variance is welfare-damaging, it is clear that such policies would be suboptimal since sunspot fluctuations themselves would be welfare-reducing. In contrast, in the same 'non-Keynesian economy', a passive rule would rule out such fluctuations and would be closer to optimal policy. But well beyond ruling out sunspot fluctuations, the presence of limited asset markets participation is likely to modify the optimal response to fundamental shocks too. Our next task is to characterize optimal policy rules in the presence of non-asset holders.

The objective function is calculated as follows. Following Woodford (2003) we use a second-order approximation to a convex combination of households' utilities, described in detail in Appendix D. We make a series of assumptions that allow us to use this second-order approximation techniques. Firstly, we assume that efficiency of the steady state is obtained by appropriate fiscal instruments inducing marginal cost pricing in steady state (subsidies for sales at a rate equal to the stead-state net mark-up financed by lump-sum taxes on firms). Since this policy makes steady-state profit income zero, the steady-state is also *equitable*: steady-state consumption shares of the two agents are equal, making aggregation much simpler. This ensures consistency with the model outlined above[29]. Secondly, we assume that the social planner maximizes a convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U_H(C_{H,t}, N_{H,t}) + [1-\lambda] U_S(C_{S,t}, N_{S,t})$. We will assume that the central bank maximizes the future discounted value of this objective function. This is consistent with our view that limited participation to asset markets comes from *constraints* and not preferences, since in the latter case maximizing intertemporally the utility of non-asset holders would be hard to justify on welfare grounds. However, note that for the discretionary Markov equilibrium studied here, this choice makes no difference since terms from time $t+1$ onwards are treated parametrically in the maximization and the time-t objective function is identical. The following Proposition shows how the objective function can be represented (up to second order) as a discounted sum of squared output gap and inflation (the proof is in Appendix D).

PROPOSITION 3. *If the steady state of the model in Section 3 is* ***efficient*** *the aggregate welfare function can be approximated by (ignoring terms independent of policy and terms of order higher than 2):*

$$\mathbf{U}_t = -\frac{U_C C}{2}\frac{\varepsilon}{\psi} E_t \sum_{i=t}^{\infty} \left\{ \alpha x_{t+i}^2 + \pi_{t+i}^2 \right\}, \tag{6.1}$$

$$\alpha \equiv \frac{\varphi+\gamma}{1-\lambda}\left[1-\lambda(1-\gamma)(1+\varphi)\right]\frac{\psi}{\varepsilon}.$$

Note that when $\lambda = 0$ we are back to the standard case $\alpha = (\varphi+\gamma)\frac{\psi}{\varepsilon}$. In the case studied extensively in the rest of this paper ($\gamma = 1$), the relative weight on output gap is $\alpha = \frac{1+\varphi}{1-\lambda}\frac{\psi}{\varepsilon}$ and is increasing in the share of non-asset holders. In

[29] Note, however, that since steady-state consumption shares are equal we do not need to assume increasing returns. Under these assumptions, the reduced-form coefficients simply modifiy as follows: $\chi^o = 1+\varphi$ and $\delta^o = 1-\varphi\lambda/(1-\lambda)$.

general, an increase in the share of non-asset holders leads to an increase in the relative weight on output (if $\gamma \geq \frac{\varphi}{1+\varphi}$, which is empirically plausible)[30]. When λ tends to one, the implicit relative weight on output stabilization tends to infinity (for $\varphi > 0$). Hence, the presence of non-asset holders modifies the trade-off faced by the monetary authority. The intuition for this result is simple. Since aggregate real profits can be written as $D_t = [1 - (MC_t/P_t)\,\Delta_t]\,Y_t$, relative price dispersion Δ_t (related here linearly to squared inflation) erodes aggregate profit income for given levels of output and marginal cost. Given that only a fraction of $(1-\lambda)$ receives profit income, when this fraction falls the welfare-based relative weight on inflation (price dispersion) also falls. Inflation becomes completely irrelevant for welfare purposes when $\lambda \to 1$: since nobody holds assets, asset income need not be stabilized.

The optimal discretionary rule $\{r_t^o\}_0^\infty$ is found by minimizing $-\mathbf{U}_t$ taking as a constraint the system given by (4.3) and (4.2) and re-optimizing every period[31]. Note that by usual arguments this equilibrium will be time-consistent. This is, up to interpretation of the solution, isomorphic to the standard problem in CGG (1999). Hence, for brevity, we skip solution details available elsewhere and go to the result:

(6.2)
$$x_t = -\frac{\kappa}{\alpha}\pi_t = -\frac{\chi\varepsilon}{\Theta}\pi_t$$

When inflation increases (decreases) the central bank has to act in order to contract (expand) demand. Assuming an AR(1) process for the cost-push shock $E_t u_{t+1} = \rho_u u_t$ for simplicity, we obtain the following reduced forms for inflation and output from the aggregate supply curve:

(6.3)
$$\begin{aligned} \pi_t &= \alpha \frac{1}{\kappa^2 + \alpha\left(1 - \beta\rho_u\right)} u_t \\ x_t &= -\kappa \frac{1}{\kappa^2 + \alpha\left(1 - \beta\rho_u\right)} u_t \end{aligned}$$

Since α is generally increasing in λ, in an economy with limited asset market participation optimal policy results in greater inflation volatility and lower output gap volatility than in a full participation economy ($\lambda > 0$). Optimal policy in this case requires more output stabilization at the cost of accommodating inflationary pressures.

Substituting the expressions given by (6.3) into the IS curve, we obtain the *implicit instrument rule* consistent with optimality[32]:

(6.4)
$$\begin{aligned} r_t^o &= r_t^* + \phi_\pi^o E_t \pi_{t+1}, \\ \phi_\pi^o &= \left[1 + \frac{\delta\kappa}{\alpha}\frac{1-\rho_u}{\rho_u}\right]. \end{aligned}$$

Some of the results obtained in a full-participation economy carry over: from the existence of a trade-off between inflation and output stabilization, to convergence

[30] When this condition is not fulfilled, so $\gamma < \varphi/\left(1+\varphi\right)$, the relative weight on output gap is decreasing in λ and can even become negative when $\lambda > \left[\left(1-\gamma\right)\left(1+\varphi\right)\right]^{-1}$. We exclude this parameter region on grounds of its being empirically irrelevant.

[31] To keep things simple, we focus on the discretionary, and not fully optimal (commitment) solution to the central banker's problem. This case can be argued to be more realistic in practice, as do CGG (1999).

[32] A positive policy response to inflation requires $\alpha \geq -\delta\kappa\frac{1-\rho_u}{\rho_u}$.

of inflation to its target under the optimal policy (e.g. CGG (1999)). Also, real disturbances affect nominal rates only insofar as they affect the Wicksellian interest rate, as discussed for example by Woodford (2003 p.250). There is one important exception however, emphasized in the following Proposition.

PROPOSITION 4. *In a non-Keynesian economy ($\delta < 0$) the implied instrument rule for optimal policy is passive $\phi_\pi^o < 1$. The optimal response to inflation is decreasing in the share of non-asset holders $\frac{\partial \phi_\pi^o}{\partial \lambda} < 0$ and changes from passive to active as δ changes sign.*

The above Proposition shows the exact way in which the central bank has to change its instrument in order to meet the targeting rule (6.2): contract demand when inflation increases, but move nominal rates such that the real rate *decreases* when δ is negative. This happens because, as explained previously, real interest rate cuts are associated with a fall in current aggregate demand when the slope of the IS curve is positive ($\delta < 0$).

Finally, when cost-push shocks are absent (and there is no inflation-output stabilization trade-off), the flexible-price allocation can be achieved by having the nominal rate equal the Wicksellian rate at all times $r_t^o = r_t^*$, as in the standard model (e.g. Woodford, Ch. 4). However, note an important difference with respect to the baseline model: when $\delta < 0$ this policy can also be consistent with a unique rational expectations equilibrium[33]. To see this note that such a policy is equivalent from an equilibrium-determinacy standpoint to an interest rate peg, i.e. an interest rate rule with $\phi_\pi = 0$. From Proposition 1, one can easily see that such a policy rule leads to equilibrium determinacy if $1 + \delta \frac{2(1+\beta)}{\kappa} \leq 0$[34].However, the ability of the central bank to achieve full price stability as the *unique* equilibrium applies to the simple model assumed here and relies upon the ability/willingness of the bank to monitor the natural rate of interest and match its movement one-to-one by movements in the nominal rate. Moreover, the natural interest rate can sometimes be negative. All these caveats suggest that this result is unlikely to have much practical relevance.

7. The effects of shocks and cyclical implications

In this section we go back to the simple instrument rule and compute analytically the effects of fundamental and sunspot shocks under determinacy and indeterminacy, allowing for the change in sign of δ due to LAMP. Our interest in this exercise is twofold. First, it might be of interest in itself to understand the effects of shocks in a *determinate* non-Keynesian economy. One obvious historical candidate for such a case is the pre-Volcker period; it is fairly well established (see e.g. CGG (2000), Taylor (1999), Lubik and Schorfheide (2004)) that the response of monetary policy in that period implied a (long-run) response to inflation of less than one. But if we allow for the possibility that participation to asset markets was so limited that $\delta < 0$, this would **not** imply that policy was inconsistent with a unique equilibrium. Hence, we will be able to assess the effects of fundamental

[33] In the baseline model, the bank needs to commit to respond to inflation by fulfilling the Taylor principle $r_t^o = r_t^* + \phi_\pi \pi_t, \phi_\pi > 1$ in order to pin down a unique equilibrium.

[34] In terms of deep parameters, this condition translates into $\lambda \geq \left[1 + \frac{1}{1+F_Y}\varphi \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}\right] / \left[1 + \frac{1}{1+\mu}\varphi\right]$.

shocks, an impossible task under indeterminacy. Indeed, in a companion paper (Bilbiie (2004)) we argue that this was the case, and fundamental shocks can explain stylized facts of the pre-Volcker period (impulse responses to shocks and moments) quite well. Secondly, there is the mirror image of the above argument. Estimates of policy rule coefficients in the Volcker-Greenspan era for the US and other industrialized countries indicate a response of nominal rates to inflation larger than one (see e.g. CGG (2000)). Coupled with the possibility of a non-Keynesian economy ($\delta < 0$), this would instead imply indeterminacy. Hence, it might be of interest to assess the effects of various (fundamental and sunspot) shocks in an indeterminate equilibrium. This might be relevant for countries with underdeveloped financial markets that nevertheless pursue an active policy.

We follow the new method proposed by Lubik and Schorfheide (2003) to compute sunspot equilibria by decomposing expectation errors. The (4.2)-(4.3) system can be written, in terms of newly defined variables (for $k = x, \pi$) $\xi_t^k \equiv E_t k_{t+1}$ and expectation errors $\eta_t^k \equiv k_t - E_{t-1} k_t$:

$$\xi_t = \mathbf{\Gamma}\xi_{t-1} + \mathbf{\Psi}\nu_t + \mathbf{\Gamma}\eta_t$$

where $\xi_t \equiv (\xi_t^x, \xi_t^\pi)'$ and $\eta_t \equiv (\eta_t^x, \eta_t^\pi)'$. The coefficient matrices $\mathbf{\Gamma}, \mathbf{\Psi}$ are still the same given in (5.2). We replace $\mathbf{\Gamma}$ by its Jordan decomposition $\mathbf{\Gamma} = JQJ^{-1}$, define the auxiliary variables $z_t = J^{-1}\xi_t$ and rewrite the above model as

$$z_t = Qz_{t-1} + J^{-1}\Psi\varepsilon_t + J^{-1}\Gamma\eta_t \tag{7.1}$$

The eigenvalues of $\mathbf{\Gamma}$ are:

$$q_\pm = \frac{1}{2}\left[tr\Gamma \pm \sqrt{(tr\Gamma)^2 - 4\det\Gamma}\right],$$

(where the determinant and trace are $\det \mathbf{\Gamma} = \beta^{-1} > 1, tr\mathbf{\Gamma}{=}1{+}\beta^{-1}{-}\beta^{-1}\delta^{-1}\kappa\left(\phi_\pi - 1\right)$). The corresponding eigenvectors are stacked in the J matrix:

$$J = \begin{bmatrix} \frac{1}{\kappa}(1-\beta q_-) & \frac{1}{\kappa}(1-\beta q_+) \\ 1 & 1 \end{bmatrix}$$

7.1. Determinacy. The equilibrium under determinacy is particularly easy to calculate when shocks have zero persistence, since the only stable solution is $\xi_t = 0$, obtained for:

$$\mathbf{\Psi}\nu_t + \mathbf{\Gamma}\eta_t = 0$$

Hence, the expectation errors are determined exclusively by fundamental shocks (and sunspot shocks would have no effect on dynamics) by $\eta_t = -\mathbf{\Gamma}^{-1}\mathbf{\Psi}\nu_t$, namely:

$$\eta_t = -\delta^{-1}\begin{bmatrix} 1 \\ \kappa \end{bmatrix}(\varepsilon_t - r_t^*) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_t \tag{7.2}$$

The initial impact on output and inflation is also given by the same expression. Since both roots are eliminated under determinacy, there is no persistence. Note the sharp differences for the two sub-cases identified above, showing asymmetric effects of some shocks depending on the sign of δ.

Case I: $\delta > 0, \phi_\pi \in \left(1, 1 + \delta\frac{2(1+\beta)}{\kappa}\right)$: The effects at work are of the usual sign, but of different magnitude. A policy-induced interest rate cut or an increase in the natural rate of interest (coming here only from shocks to technology growth) increase both the output gap and inflation. These effects are stronger, the higher the share of non-asset holders (and the higher δ^{-1}). One-time cost-push shocks have no effect on the output gap, and increase inflation one-to-one; this is only because the interest rate rule responds to expected future inflation, whereas a one-time shock increases only inflation today.

Case II: *non-Keynesian economy,* $\delta < 0, \phi_\pi \in \left(1 + \delta\frac{2(1+\beta)}{\kappa}, 1\right)$. In contrast to the standard case, a monetary contraction (positive ε_t) has expansionary effects, and causes inflation. This follows directly by the mechanism discussed in detail above. An increase in the natural rate of interest driven by technology results in a recession and deflation. It is clear then that a policy response increasing the nominal rate by more than the natural rate $\varepsilon_t > r_t^*$ increases both output and inflation, whereas when it falls short of doing so, it has deflationary effects, and causes a fall in output. These effects diminish as λ tends to 1, since δ^{-1} tends to zero. Cost-push shocks have the same effects regardless of the sign of δ due to the zero-persistence assumption. However, in the presence of persistence the *magnitude* of the responses to these shocks will depend on δ, since in that case the roots of the system matter for dynamics (see Bilbiie (2004) for some simulations).

Cyclical implications.

While evidence overwhelmingly suggests that profits are procyclical (see Rotemberg and Woodford (1999)), the mechanism underlying our results might seem to rely on countercyclical profits. In this subsection we briefly show that this is not necessarily the case. First, note that the condition for profits to be procyclical $dd/dy > 0$ is (see Appendix): $dx < \frac{\mu-(1+F_Y)du}{\chi(1+F_Y)+(1+F_Y)du-\mu}dy^*$, where we dropped time subscripts, and the right-hand side term is exogenous and depends on technology. When shocks have zero persistence, we know the solution for output gap from (7.2) as: $dx = -\delta^{-1}d\left(\varepsilon - r^*\right) < \frac{\mu-(1+F_Y)du}{\chi(1+F_Y)+(1+F_Y)du-\mu}dy^*$. Without a cost-push shock, this condition becomes $-\delta^{-1}d\left(\varepsilon - r^*\right) < \left[\mu/\left(\chi\left(1+F_Y\right)-\mu\right)\right]dy^*$. We can hence see an example whereby $\delta^{-1} < 0$ satisfies this condition and leads to procyclical profits. Namely, if the shock to technology is such that $dy^* > 0$, but the policy response is such that $d\varepsilon < dr^*$ profits are always procyclical in the non-Keynesian economy, and countercyclical otherwise. Alternatively, the same is true if there is no shock to technology $(dy^* = 0)$ and $d\varepsilon < 0$.

Moreover, we can assess how relative profit cyclicality depends on the degree of asset market participation[35]. We take two economies with different participation rates and ask under what conditions does one have more procyclical profits than the other (take the case where $F_Y = \mu$ since χ is independent on λ and so y^* is also independent of λ), namely:

$\frac{dd}{dy} > \frac{dd^0}{dy^0} \Leftrightarrow \frac{dmc}{dy} < \frac{dmc^0}{dy^0} \Leftrightarrow \frac{dy^*}{dy} > \frac{dy^*}{dy^0} \Leftrightarrow dx < dx^0$. For the zero-persistence case again this becomes: $\delta^{-1}d\left(\varepsilon - r^*\right) > \delta_0^{-1}d\left(\varepsilon - r^*\right)$. Hence, if $d\left(\varepsilon - r^*\right) > 0$, the condition for $dd/dy > dd^0/dy^0$ is $\delta^{-1} > \delta_0^{-1}$. This means that in case the shocks configuration is such that the policy response exceeds the natural rate, a Keynesian economy has more procyclical profits the higher its λ (δ^{-1} is increasing in λ). A non-Keynesian economy $\delta^{-1} < 0$ has always less procyclical profits than a Keynesian one (but getting more and more procyclical as λ increases). However, when the

[35]This is especially important, since an economy with limited participation can imply *more* procyclical profits than an economy with full participation without necessarily implying procyclical profits. That is because in a standard sticky-price model profits can be strongly counter-cyclical, unless one introduces labor hoarding, variable utilization, or other features meant to break the link between markup and profits (see Rotemberg and Woodford 1999).

policy response falls short of the natural rate, the opposite holds: $\delta^{-1} < \delta_0^{-1}$. A non-Keynesian economy will hence always have more procyclical profits than a Keynesian one. Finally, note that if the source of fluctuations is only a cost-push shock the condition is $\frac{du}{dy} < \frac{du}{dy^0}$ so $dy > dy^0$. This can easily be the case when the shock persistence is different than zero (note that with zero persistence the response of output is zero). For instance, under the optimal policy solution calculated in (6.3), the response of output to a cost shock is always larger (less negative) when λ is larger; this implies that profits are more procyclical the larger is λ.Similar reasoning applies to the cyclicality of real wage, noting that more procyclical profits imply less procyclical real wage.

7.2. Indeterminacy. In this case one of the roots $q_{\pm}$ will be inside the unit circle. Sunspot shocks have real effects, and the responses to fundamental shocks change too in a way made explicit below. We confine ourselves to the case whereby the smaller root is inside the unit circle and the larger one is greater than one, i.e. $q_- \in (-1, 1)$ and $q_+ > 1$. This can be shown to be the case if either (i) $\delta > 0, \phi_\pi < 1$ or (ii) $\delta < 0, \phi_\pi > 1$[36]. Since in this case there is one-dimensional indeterminacy, the stability condition for (7.1) modifies: expectation errors are not spanned by fundamental shocks, but by both fundamental and sunspot shocks.

We can apply the results in Proposition 1 in Lubik and Schorfheide (2003) to solve for the full solution set for the expectation errors. This is described in some detail in the Appendix, and the solution is:

$$\begin{aligned}\eta_t &= -\frac{\kappa\delta^{-1}}{d^2}\begin{bmatrix}\kappa q_+ \\ 1-q_+\end{bmatrix}(\varepsilon_t - r_t^*) + \frac{1}{d^2}\begin{bmatrix}\kappa\beta^{-1}(1-q_+) \\ (1-q_+)\left(q_- - \beta^{-1}\right)\end{bmatrix}u_t + \\ &\quad + \frac{1}{d}\begin{bmatrix}q_+ - 1 \\ \kappa q_+\end{bmatrix}\left(M_1\nu_t + \varsigma_t^*\right),\end{aligned} \tag{7.3}$$

where M_1 is an arbitrary 2×2 matrix and ς_t^* is a reduced-form sunspot shock, which can be interpreted as a belief-induced increase in output and/or inflation of undetermined size. First thing to note is that a positive realization of this shock will increase output and inflation no matter whether $\delta \lesseqgtr 0$ since $q_+ > 1$ as established above. This conforms our intuitive construction of sunspot equilibria when discussing determinacy properties of interest-rate rules.

On the other hand, the effects of fundamental shocks become ambiguous, and depend crucially upon the choice of the M_1 matrix.Unfortunately, there is nothing to pin down a choice for this matrix, which captures a well-known problem of indeterminate equilibria - the effects of fundamental shocks cannot be studied without further restrictions. Two leading possibilities to restrict the M_1 matrix are suggested by Lubik and Schorfheide.

7.2.1. *Orthogonality.* The two sets of shocks are orthogonal in their contribution to the forecast error, and hence $M_1 = 0$ in (7.3). The effect of a cost-push shock is of the same sign under either scenario, as is independent of δ. A positive realization of this shock would increase inflation (since $(1-q_+)\left(q_- - \beta^{-1}\right) > 0$) and decrease output ($q_+ > 1$). The effects of policy shocks, and of shocks to the natural rate of interest, are again different depending on which case we consider:

[36]For the rest of the parameter regions where there is indeterminacy we would have $q_+ \in (-1, 1)$ and $q_- < -1$, but this can be shown to imply very restrictive conditions on the deep parameters and the policy rule coefficient.

I. Standard case, $\delta > 0$: An interest rate increase keeping constant the natural rate decreases output under its natural level but causes inflation as $1 - q_+ < 0$ (this is also found by Lubik and Schorfheide for a contemporaneous rule). An increase in the natural rate without a discretionary policy response increases the output gap and causes deflation.

II. 'Non-Keynesian economy' $\delta < 0$: A policy-induced interest rate increase increases output and causes deflation. An increase in the natural rate not matched by policy depresses output and causes inflation. In either case, the overall effect on inflation and the output gap depends on whether the policy response is stronger or weaker than the variation in the natural rate.

7.2.2. *Continuity.* In order to preserve continuity of the impulse responses to the fundamental shock when passing from determinacy to indeterminacy, M_1 can be chosen such that it implies that the response to the fundamental shock is the same, i.e.:

$$\eta_t = -\delta^{-1} \begin{bmatrix} 1 \\ \kappa \end{bmatrix} (\varepsilon_t - r_t^*) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + \frac{1}{d} \begin{bmatrix} q_+ - 1 \\ \kappa q_+ \end{bmatrix} \varsigma_t^*$$

This happens for a very particular M_1 matrix and implies that the effects of fundamental shocks are as under determinacy, namely in the 'non-Keynesian economy' case a contractionary policy shock increases both output and inflation. While continuity is an attractive feature, there is nothing to insure that the M_1 takes exactly the form necessary to get this result.

8. Distortionary redistributive taxation restores Keynesian logic.

The mechanism of the previous results relies on income effects on labor supply from the return on shares. This hints to an obvious way to restore Keynesian logic relying on a specific fiscal policy rule: tax dividend income and distribute proceedings as transfers to non-asset holders. We focus on the non-Keynesian case whereby in the absence of fiscal policy $\delta < 0$. To make this point, consider the following simplified fiscal rule: profits are taxed at rate τ_t and the budget is balanced period-by-period, with total tax income $\tau_t D_t$ being distributed lump-sum to all non-asset holders. We focus on the case where profits are zero in steady-state. The balanced-budget rule then is $\tau_t D_t = \lambda L_{H,t}$ which in log-linearized form (both profits and transfers are shares of steady-state GDP) is: $\lambda l_{H,t} = \tau d_t$. Replacing this into the new budget constraint of non-asset holders we get $c_{H,t} = w_t + \frac{\tau}{\lambda} d_t$. Asset holders' consumption will then be given by (substituting the expression for profits): $c_{S,t} = (1-\lambda)^{-1} [1 - \tau\mu / (1+\mu)] y_t + (\tau - \lambda) / (1-\lambda) w_t$. Setting shocks to zero for simplicity, the wage-hours locus is then obtained following the same method as before: $w_t = (1-\tau)^{-1} [1 + \varphi + \mu (1-\tau)] n_t$. Finally, consumption of asset holders as a function of total output is:

$$c_{S,t} = \delta_\tau y_t, \text{ where } \delta_\tau = \frac{1}{1-\tau} \left[1 - \tau \frac{\mu}{1+\mu} + \frac{\tau - \lambda}{1-\lambda} \frac{\varphi}{1+\mu} \right]$$

For a given λ there exists a minimum threshold for the tax rate such that $\delta_\tau > 0$ when in the absence of such fiscal policy $\delta < 0$. This threshold is (note that $\varphi / (1-\lambda) - \mu > 0$ where $\delta < 0$):

$$\tau > 1 - \frac{1+\varphi}{\frac{1}{1-\lambda}\varphi - \mu} < 1$$

The necessary tax rate is higher, the more inelastic is labor supply and the higher the share of agents with no assets. The intuition for this result is straightforward: a higher tax rate on asset income eliminates some of the income effect of dividend variation on asset holders' labor supply.

9. Conclusions

Interest rate changes modify the intertemporal consumption and labor supply profile of *asset holders*, agents who smooth consumption by trading in asset markets. This affects the real wage, and the demand thereby of agents who have no asset holdings, are oversensitive to real wage changes, and insensitive directly to interest rate changes. Variations in the real wage (marginal cost) lead to variations in profits and hence in the dividend income of asset holders. These variations can either reinforce (if participation is not 'too' limited) or *overturn* the initial impact of interest rates on aggregate demand. The latter case occurs if the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough, for the potential variations in profit income offset the interest rate effects on the demand of asset holders. This is the main mechanism identified by this paper to change dramatically the effects of monetary policy as compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders. The required share of non-asset holders for these results to hold is relatively mild compared to empirical estimates of Campbell and Mankiw (1989) or direct data on asset holding (see Mulligan and Sala-i-Martin (2000) and Vissing-Jorgensen (2002)).

This paper develops an analytical framework (building on the existing literature) and uses it to study in detail the monetary policy implications of the foregoing insight. Our results have clear normative implications. In a nutshell, central bank policy should be pursued with an eye to the aggregate demand side of the economy: the extent to which agents participate it asset markets would become an important part of the policy input. While the degree of development of financial markets may well make this not a concern in present times in the developed economies, central banks in developing countries with low participation in financial markets might find this of practical interest. The theoretical results hinting to such policy prescriptions are that limited participation beyond a certain threshold makes the economy behave in a 'non-Keynesian' way. Namely, the IS curve changes sign, and an 'Inverted Taylor Principle' applies[37]: the central bank needs to adopt a passive policy rule to ensure equilibrium uniqueness and rule out the possibility of self-fulfilling, sunspot-driven fluctuations. Moreover, optimal time-consistent monetary policy also requires that the central bank move nominal rates such that real rates decline (thereby containing aggregate demand). The effects and transmission of shocks are also dramatically modified.

The modest scope of this study is to make a contribution to the literature emphasizing the role of LAMP in shaping macroeconomic policy and helping towards

[37]This result depends only to a small extent on whether the rule is specified in terms of current or expected future inflation. As discussed in text in more detail, this is in contrast to Gali, Lopez-Salido and Valles (2003) who, while having noted the possibility to violate the Taylor principle for a forward-looking rule, also argue that a strengthening of the Taylor principle is required for a contemporaneous rule to result in equilibrium uniqueness. A very strong response to current inflation would also insure determinacy in our model, but we find the implied coefficient is higher than any plausible estimates, makes policy non-credible and qould lead to violation of the zero lower bound in case of small deflations.

a better understanding of the economy. In that respect, we just seek to add to a new developing literature analyzing the role of non-asset holders in macroeconomic dynamic general equilibrium models (see Mankiw (2000), Galí, Lopez-Salido and Valles (2002) or Alvarez, Lucas and Weber (2001)). Our model has the advantage of simplicity: we studied economies with limited asset markets participation analytically in the same type of framework used in standard, full-participation analyses. This simplicity (shared with the rest of the literature), while justified on tractability grounds, can potentially also be a shortcoming for it implies many realistic features have been left out. It is important in our view to model the decision to participate in asset markets explicitly, allowing for household heterogeneity and an endogenous share of asset holders. Lastly, an empirical assessment of limited participation, its dynamics and implications at the macroeconomic aggregate level, is in our view a necessary step for understanding business cycles, which we pursue in current work.

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Appendix A. Limited participation due to transaction costs: an example.

The purpose of this Appendix is to show that our assumption on limited participation in asset markets can be supported by the presence of heterogenous transaction costs. Our model draws on a large literature on asset-pricing with transaction costs, and its purpose is merely to show how for a given level of the non-participation rate λ, there exists a structure of costs that supports it. Such costs have been emphasized by many as one important explanation for the observed participation structures (see Vissing-Jorgensen [2003] and Paiella [2004] for empirical estimates). This literature originates with Cochrane [1989] , who showed that the foregone utility gains from consuming one's income as opposed to following a permanent-income decision rule are likely to be very small (10 cents to 1 dollar per quarter); consequently, the lower bound on transaction costs preventing an agent from following a permanent-income rule (e.g. participating in asset markets) is likely to be small. He and Modest [1995] study the role of market frictions, including proportional transaction costs, in reconciling asset market equilibrium with data on consumption and asset returns. Using data from the Survey of Consumer Finances, Mulligan and Sala-i-Martin (2002) estimate that the median per-period transaction cost for *any interest-bearing asset* is 111 dollars per year.

Suppose that each time the household goes to the asset market it has to pay a proportional transaction cost which is household-specific k^j. The optimality condition under proportional transaction costs is (where the second inequality holds for situation in which the household sells the asset short, i.e. brings consumption forward - see also He and Modest):

$$\left(1+k^j\right) \geqslant E_t\left[\Lambda_{t,t+1}^{j} R_{t+1}^{a}\right] \geqslant \frac{1}{\left(1+k^j\right)}$$

where $\Lambda_{t,t+1}^{j} \equiv \beta P_t U_C\left(C_{j,t+1}\right) / \left[U_C\left(C_{j,t}\right) P_{t+1}\right]$ is the stochastic discount factor of household j and R_{t+1}^{a} is the expected gross return of asset a. Suppose for simplicity that there are two types of agents: one type indexed by S faces no transaction costs $k^S = 0$ while the other indexed by H has to pay a proportional cost $k^H = k$ for each transaction. Such an extreme, bimodal distribution of costs is not necessary, but makes aggregation simpler and is enough for our point. Notably, in equilibrium no cost will be paid since agents who participate face a cost of zero, and agents who would have to pay the cost choose not to pay it. As in the model of Section 3, consumption of type-S agents obeys a standard Euler equation, which for the riskless bond reads:

$$\frac{1}{R_t} = E_t\left[\Lambda_{t,t+1}^{S}\right] \tag{A.1}$$

Type-H agents face the following optimality condition:

$$(1+k)\frac{1}{R_t} \geqslant E_t\left[\Lambda_{t,t+1}^{H}\right] \geqslant \frac{1}{(1+k)}\frac{1}{R_t}$$

Substituting for R_t from $(A.1)$ we have: $1+k \geqslant E_t\left[\Lambda_{t,t+1}^{H}\right] / E_t\left[\Lambda_{t,t+1}^{S}\right] \geqslant (1+k)^{-1}$, which with log utility becomes:

$$1+k \geqslant \frac{C_{H,t} E_t\left[C_{H,t+1}^{-1}\right]}{C_{S,t} E_t\left[C_{S,t+1}^{-1}\right]} \geqslant \frac{1}{1+k}$$

We will look for a minimum level of the proportional transaction cost $\bar{k}$ that makes the measure of households at a corner solution holding no assets (for which the above is a strict inequality) be precisely λ. Assuming log-normality and homoskedasticity, we can approximate to second order this lower bound on costs by

$$\bar{k} \approx \left| E_t \Delta c_{S,t+1} - E_t \Delta c_{H,t+1} + \frac{1}{2}\left(\sigma_H^2 - \sigma_S^2\right) \right| \geqslant 0, \tag{A.2}$$

where $\sigma_j^2 = var\left(c_{j,t+1} - E_t c_{j,t+1}\right)$ and lowercase letters denote logs. Note that since consumption growth is stationary, $\bar{k}$ is bounded above (applying the triangle inequality to $(A.2)$).

Equation $(A.2)$ can be compared to data as follows. Given an observed non-participation rate λ and time series on consumption of asset-holders and non-asset holders $c_{S,t}, c_{H,t}$, one can compute the value of the cost that would explain this participation structure. This value can then be compared to actual transaction costs. However, measurement issues abound related both to classifying households relative to their asset-holding status and to finding an appropriate measure of the cost. Indeed, most costs that prevent an agent hold any asset at all are likely to be non-pecuniary and related to information imperfections, time spent understanding the way asset markets work, etc. An alternative route is to solve for the moments involved in $(A.2)$ in a dynamic general equilibrium framework as functions of fundamental shocks, for a given level of no-participation $\hat{\lambda}$. Then, for the assumed $\hat{\lambda}$ there exists a minimum level of the cost $\bar{k}$ rationalizing it that can be found by solving $(A.2)$.

Finally, note that as regards shareholding, the cost that prevents households participate from the stock market needs to be larger than $\bar{k}$, due to the existence of an equity premium. In the framework of Section 3, the Euler equation for shares of agents who face a zero cost is $1 = E_t\left[R_{t+1}^A \Lambda_{t,t+1}^S\right]$, where $R_{t+1}^A \equiv \left(V_{t+1} + P_{t+1} D_{t+1}\right)/V_t$ is the gross return on shares. Agents facing a cost k^A choose not to hold shares $\Omega_{H,t} = 0$ iff $1 + k^A > E_t\left[R_{t+1}^A \Lambda_{t,t+1}^H\right] > \left(1 + k^A\right)^{-1}$. Taking second-order approximations under assumptions of joint conditional lognormality and homoskedasticity as above yields a lower bound for the transaction cost in the stock market:

$$k^A \approx \left| E_t \Delta c_{S,t+1} - E_t \Delta c_{H,t+1} + \frac{1}{2}\left(\sigma_H^2 - \sigma_S^2\right) + \sigma_{AS} - \sigma_{AH} \right|$$

where $\sigma_{Aj} = cov\left(r_{t+1}^A - E_t r_{t+1}^A, c_{j,t+1} - E_t c_{j,t+1}\right)$ can be computed from the general equilibrium model, as before. The bottomline is that a certain level of non-participation λ can be rationalized by proportional transaction costs $\bar{k}, k^A$.

Appendix B. Log-linearized equilibrium

For asset holders, we have the Euler equation, intratemporal and budget constraint (d_t are real profits as a share of steady-state GDP, $d_t \equiv (D_t - D)/Y$ and we already imposed that the equilibrium value of share holdings Ω is $\frac{1}{1-\lambda}$):

$$E_t c_{S,t+1} - c_{S,t} = r_t - E_t \pi_{t+1}, \tag{B.1}$$

$$\varphi n_{S,t} = w_t - c_{S,t}\ , \tag{B.2}$$

$$\frac{C_S}{Y} c_{S,t} = \frac{W}{P}\frac{N_S}{Y}\left(w_t + n_{S,t}\right) + \frac{1}{1-\lambda} d_t. \tag{B.3}$$

For non-asset holders, we have the intratemporal optimality condition and budget constraint:

$$n_{H,t} = 0, \tag{B.4}$$
$$c_{H,t} = w_t. \tag{B.5}$$

For firms:

$$y_t = (1+F_Y)\, n_t + (1+F_Y)\, a_t, \tag{B.6}$$
$$mc_t = w_t - a_t, \tag{B.7}$$
$$d_t = -\frac{1+F_Y}{1+\mu} mc_t + \frac{\mu}{1+\mu} y_t, \tag{B.8}$$
$$\pi_t = \beta E_t \pi_{t+1} + \psi mc_t,\ \psi = \frac{(1-\theta)(1-\theta\beta)}{\theta}. \tag{B.9}$$

Labour market clearing implies (n =labour demand by firms):

$$n_t = \frac{(1-\lambda)\, N_S}{N} n_{S,t}. \tag{B.10}$$

Aggregate consumption is:

$$c_t = \frac{\lambda C_H}{C} c_{H,t} + \frac{(1-\lambda)\, C_S}{C} c_{S,t}. \tag{B.11}$$

Equilibrium in goods market holds by Walras' law (and is redundant once equilibrium in all other markets has been imposed).

$$y_t = c_t. \tag{B.12}$$

Monetary policy rule is:

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_x x_t + \varepsilon_t. \tag{B.13}$$

where ε_t are policy shocks, i.e. movements in nominal rates coming form anything else than systematic response to inflation or output gap.

B.1. Steady state.

$$R = \frac{1}{\beta} \text{where } R \equiv 1 + r$$
$$\frac{W}{P} = \frac{Y+F}{N}\frac{MC}{P} = \frac{Y}{N}\frac{1+\frac{F}{Y}}{1+\mu}$$
$$\frac{D}{Y} = \frac{\mu - F_Y}{1+\mu}$$

We assume hours are the same for the two groups in steady state only, $N_H = N_S = N$.Then, for the log-linear budget constraints of both agents the coefficients are fully determined:

$$\frac{W}{P}\frac{N_S}{Y} = \frac{1+F_Y}{1+\mu};\ \frac{C_S}{Y} = \frac{1+F_Y}{1+\mu} + \frac{\mu - F_Y}{1+\mu}\frac{1}{1-\lambda} = \frac{1}{1-\lambda}\left(1 - \lambda\frac{1+F_Y}{1+\mu}\right)$$
$$\frac{WN_H}{PY} = \frac{C_H}{Y} = \frac{1+F_Y}{1+\mu}$$

B.2. Deriving the IS-AS system. We seek to express everything in terms of aggregate variables, and then use the two dynamic equations to get dynamics only in terms of output, inflation and interest rate. First, try to express consumption of asset holders as function of aggregate variables, from $(B.5)$, $(B.10)$,$(B.11)$ using the steady state coefficients just calculated:

$$c_{S,t} = \frac{1}{1-\lambda}\frac{C}{C_S}y_t - \frac{\lambda}{1-\lambda}\frac{C_H}{C_S}w_t \tag{B.14}$$

Substituting this, together with$(B.10)$ into $(B.2)$ and using the production function we get:

$$\begin{aligned} w_t &= \chi y_t - (1+F_Y)(\chi - 1)a_t, \\ \text{where } \chi &\equiv \left[1+\varphi\frac{C_S}{C}\frac{1}{1+F_Y}\right] = \left[1+\varphi\frac{1}{1-\lambda}\frac{1}{1+F_Y}\left(1-\lambda\frac{1+F_Y}{1+\mu}\right)\right] \geq 1. \end{aligned} \tag{B.15}$$

Substituting back into $(B.14)$ and using the steady state consumption shares we get consumption of asset-holders as a function of output:

$$c_{S,t} = \delta y_t + (1+F_Y)(1-\delta)a_t,\ \delta \equiv 1-\varphi\frac{\lambda}{1-\lambda}\frac{1}{1+\mu}$$

Note $\chi = \delta + \varphi\frac{1}{1-\lambda}\frac{1}{1+F_Y}$. We just need to replace these last two equations in the Euler and New Keynesian Philips curve to obtain a system in output and inflation. We will write the whole system in terms of the output gap (difference of actual output from output under flexible prices) as is usually done in the literature. Real marginal cost is given by $mc_t = w_t - a_t$; since in the flexible-price equilibrium this is constant (and so is the markup) we see directly from $(B.15)$ that natural output is: $y_t^* = \left[1+F_Y\left(1-\frac{1}{\chi}\right)\right]a_t$, so marginal cost is related to the output gap $x_t \equiv y_t - y_t^*$ by:

$$mc_t = \chi(y_t - y_t^*) + \psi^{-1}u_t = \chi x_t + \psi^{-1}u_t. \tag{B.16}$$

where, following Clarida, Galí and Gertler (1999) or Galí (2002) we also introduce cost-push shocks $\tilde{u}_t = \psi^{-1}u_t$, i.e. variations in marginal cost not due to variations in excess demand. These could come from the existence of sticky wages creating a time-varying wage markup, or other sources creating this inefficiency wedge although we do not model this explicitly here. Substituting consumption of asset holders in the Euler equation, we can write

(B.17)
$$\delta E_t x_{t+1} = \delta x_t + [r_t - E_t\pi_{t+1}] + (1+F_Y)(1-\delta)[a_t - E_t a_{t+1}] - \delta\left[E_t y_{t+1}^* - y_t^*\right]$$

We define the natural rate of interest (Wicksellian interest rate) r_t^* as the level of the interest rate consistent with output being at its natural level (and hence with zero inflation), as in Woodford (2003). Solving from $(B.17)$ we obtain:

$$r_t^* = \left[1+F_Y\left(1-\frac{\delta}{\chi}\right)\right][E_t a_{t+1} - a_t] \tag{B.18}$$

Using$(B.16)$ and $(B.18)$ into the New Keynesian Philips curve and the $(B.17)$ we get the reduced system:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + u_t, \text{where } \kappa \equiv \psi\chi \tag{B.19}$$

$$E_t x_{t+1} = x_t + \delta^{-1}[r_t - E_t\pi_{t+1} - r_t^*] \tag{B.20}$$

The model is closed by the policy rule $(B.13)$.

Appendix C. Proof of Proposition 1

Necessary and sufficient conditions for determinacy are as follows (given in Woodford Appendix to Chapter 4). Either **Case A**: $(Aa)\det\mathbf{\Gamma}>1$; $(\mathbf{A}b)\det\mathbf{\Gamma}-tr\mathbf{\Gamma}>-1$ and $(Ac)\det\mathbf{\Gamma}+tr\mathbf{\Gamma}>-1$ or **Case B**: $(Ba)\det\mathbf{\Gamma}-tr\mathbf{\Gamma}<-1$ and $(Bb)\det\mathbf{\Gamma}+tr\mathbf{\Gamma}<-1$. For our forward -looking rule case, the determinant and trace are:

$$\text{(C.1)}\qquad \begin{aligned}\det\mathbf{\Gamma} &= \beta^{-1}>1\\ tr\mathbf{\Gamma} &= 1+\beta^{-1}-\beta^{-1}\delta^{-1}\kappa\left(\phi_\pi-1\right)\end{aligned}$$

Imposing the determinacy conditions in Case A above (where Case B can be ruled out due to sign restrictions), we obtain the requirement for equilibrium uniqueness:

$$\delta^{-1}\left(\phi_\pi-1\right)\in\left(0,\frac{2\left(1+\beta\right)}{\kappa}\right)$$

This implies the two cases in Proposition 1: Case I: $\delta>0,\phi_\pi\in\left(1,1+\delta\frac{2(1+\beta)}{\kappa}\right)$, which is a non-empty interval; Case II: $\delta<0,\phi_\pi\in\left(1+\delta\frac{2(1+\beta)}{\kappa},1\right)$. Notice that (i) $1+2\delta\left(1+\beta\right)/\kappa<1$ so the interval is non-empty; (ii) $1+2\delta\left(1+\beta\right)/\kappa>0$ implies instead that we can rule out an interest rate peg, whereas a peg is consistent with a unique REE for $1+\delta\frac{2(1+\beta)}{\kappa}<0$. The last condition instead holds if and only if $\lambda\geq\left(1+\frac{1}{1+F_Y}\varphi\frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}\right)/\left(1+\frac{1}{1+\mu}\varphi\right)\geq\left(1+\frac{1}{1+\mu}\varphi\right)^{-1}$.

When this condition is not fulfilled, we have $0<1+\delta\frac{2(1+\beta)}{k}<1$, so there still exist policy rules $\phi_\pi\in\left(1+\delta\frac{2(1+\beta)}{k},1\right)$ bringing about a unique rational expectations equilibrium. But in this case an interest rate peg, and any policy rule with too weak a response $\phi_\pi\in\left[0,1+\delta\frac{2(1+\beta)}{k}\right)$ is not compatible with a unique equilibrium.

C.1. Determinacy properties of a simple Taylor rule. We consider rules of the form:

$$\text{(C.2)}\qquad r_t=\phi_\pi\pi_t+\varepsilon_t$$

Replacing this in the IS equation (B.20) and using the same method as previously we obtain the following Proposition.

PROPOSITION 5. *An interest rate rule such as* $(C.2)$ *delivers a unique rational expectations equilibrium if and only if:*

Case I: If $\delta>0,\phi_\pi>1$ *(the 'Taylor Principle')*
Case II: If $\delta<0$,

$$\phi_\pi\in\left[0,\min\left\{1,\delta\frac{\beta-1}{\kappa},\delta\frac{-2\left(1+\beta\right)}{\kappa}-1\right\}\right)\cup\left(\max\left\{1,\delta\frac{-2\left(1+\beta\right)}{\kappa}-1\right\},\infty\right)$$

It turns out that the 'inverted Taylor principle' holds in Case II for a somewhat larger share of non-asset holders than was the case under a forward-looking rule.

PROOF. Substituting the Taylor rule in the IS equation and writing the dynamic system in the usual way for the $z_t\equiv\left(y_t,\pi_t\right)'$ vector of endogenous variables and the $\nu_t\equiv\left(\varepsilon_t-r_t^*,u_t\right)'$ vector of disturbances :

$$E_tz_{t+1}=\mathbf{\Gamma}z_t+\Psi\nu_t$$

The coefficient matrices are given by:

$$\mathbf{\Gamma} = \begin{bmatrix} 1+\beta^{-1}\delta^{-1}\kappa & \delta^{-1}\left(\phi_\pi - \beta^{-1}\right) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \text{ and } \Psi = \begin{bmatrix} \delta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix}$$

Determinacy requires that both eigenvalues of $\mathbf{\Gamma}$ be outside the unit circle. Note that:

$$\det \mathbf{\Gamma} = \beta^{-1}\left(1+\delta^{-1}\kappa\phi_\pi\right) \text{ and } tr\mathbf{\Gamma}{=}1+\beta^{-1}\left(1+\delta^{-1}\kappa\right)$$

For Case A we have: (Aa) implies:

$$\delta^{-1}\phi_\pi > \frac{\beta-1}{\kappa}$$

(Ab) implies

$$\delta^{-1}\left(\phi_\pi - 1\right) > 0$$

(Ac) implies

$$\delta^{-1}\left(1+\phi_\pi\right) > \frac{-2\left(1+\beta\right)}{\kappa}$$

The determinacy requirements are as follows. First, note that (Ab) merely requires that δ^{-1} and $(\phi_\pi - 1)$ have the same sign. Hence, we can distinguish two cases:

Case I: $\delta^{-1} > 0, \phi_\pi > 1$. The standard case is encompassed here and the Taylor principle is at work as one would expect. The other conditions are automatically satisfied, since both $\delta^{-1}\phi_\pi$ and $\delta^{-1}\left(1+\phi_\pi\right)$ are positive, and $\frac{\beta-1}{\kappa}, \frac{-2(1+\beta)}{\kappa} < 0$.

Case II: $\delta^{-1} < 0, \phi_\pi < 1$. Condition (Aa) implies (note that since $\delta < 0$ the right-hand quantity will be positive): $\phi_\pi < \delta\frac{\beta-1}{\kappa}$. The third requirement for uniqueness (Ac) implies: $\phi_\pi < \delta\frac{-2(1+\beta)}{\kappa} - 1$. Since $\phi_\pi \geq 0$, this last requirement implies a further condition on the parameter space, namely $\delta\frac{-2(1+\beta)}{\kappa} - 1 \geq 0$. Overall, the requirement for determinacy when $\delta^{-1} < 0$ is hence:

$$\text{(C.3)} \qquad 0 \leq \phi_\pi < \min\left\{1, \delta\frac{\beta-1}{\kappa}, \delta\frac{-2\left(1+\beta\right)}{\kappa} - 1\right\}$$

Case B, instead, involves fulfilment of the following conditions: (Ba) implies $\delta^{-1}\left(\phi_\pi - 1\right) < 0$ and (Bb) implies $\delta^{-1}\left(1+\phi_\pi\right) < \frac{-2(1+\beta)}{\kappa}$. Note that in Case I, whereby $\delta^{-1} > 0$, these conditions cannot be fulfilled due to sign restrictions (this is the case in a standard economy as in Woodford (2003), e.g.). In Case II however, the two conditions imply:

$$\text{(C.4)} \qquad \phi_\pi > \max\left\{1, \delta\frac{-2\left(1+\beta\right)}{\kappa} - 1\right\}$$

C.3 and C.4 together imply the following overall determinacy condition for the policy parameter:

$$\phi_\pi \in \left[0, \min\left\{1, \delta\frac{\beta-1}{\kappa}, \delta\frac{-2\left(1+\beta\right)}{\kappa} - 1\right\}\right) \cup \left(\max\left\{1, \delta\frac{-2\left(1+\beta\right)}{\kappa} - 1\right\}, \infty\right)$$

□

To assess the magnitude of policy responses needed for determinacy as a function of deep parameters, we can distinguish a few cases for different parameter regions (note that we are always looking at the subspace whereby $\delta^{-1} < 0$):

Share of non-asset holders	Determinacy condition
$\lambda < \bar{\lambda}_1$	$\phi_\pi > 1$
$\lambda \in [\bar{\lambda}_1, \bar{\lambda}_2)$	$\phi_\pi \in \left[0, \delta\frac{-2(1+\beta)}{\kappa} - 1\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_2, \bar{\lambda}_3)$	$\phi_\pi \in \left[0, \delta\frac{\beta-1}{\kappa}\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_3, \bar{\lambda}_4)$	$\phi_\pi \in \left[0, \delta\frac{\beta-1}{\kappa}\right) \cup \left(\delta\frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$
$\lambda \in [\bar{\lambda}_4, 1)$	$\phi_\pi \in [0, 1) \cup \left(\delta\frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$

where

$$\bar{\lambda}_i = \left(1 + \frac{1}{1+F_Y}\varphi\frac{(1-\theta)(1-\beta\theta)}{h_i(\theta)}\right) / \left(1 + \frac{1}{1+\mu}\varphi\right)$$

$h_1(\theta) = (1+\theta)(1+\beta\theta)$; $h_2(\theta) = 1 + \beta\theta^2 + 2\beta\theta$; $h_3(\theta) = 1 + \beta\theta^2$; $h_4(\theta) = 1 - \beta\theta^2$

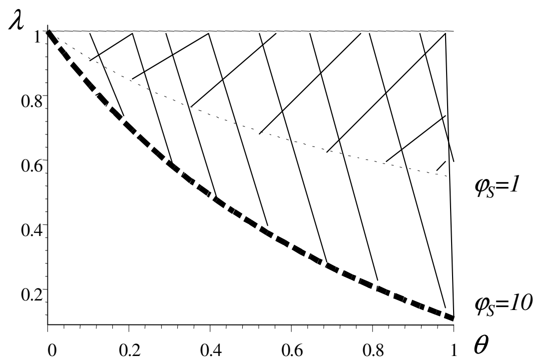

Fig.4: Threshold value for share of non-asset holders making determinacy conditions closest to Inverted Taylor Principle.

We plot the last case $\lambda \in [\bar{\lambda}_4, 1)$; $\phi_\pi \in [0, 1) \cup \left(\delta\frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$ in Figure 4 above, where the region above the curve and below the horizontal line gives parameter combinations compatible with the above condition. The different curves correspond to different labor supply elasticities ($\varphi = 1$ dotted line and $\varphi = 10$ thick line). In view of usual estimates of λ in the literature (e.g. 0.4-0.5 Campbell and Mankiw (1989)) we shall consider this case as the most plausible. Whenever these parameter restrictions are met, determinacy is insured by either a violation of the Taylor principle, or for a strong response to inflation. However, note that the lower bound on the inflation coefficient then becomes very large (35.433 under the baseline calibration), which is far from any empirical estimates. Indeed, the threshold inflation coefficient is sharply increasing in the share of non-asset holders and inverse elasticity of labor supply, as can be seen by merely differentiating $\delta\frac{-2(1+\beta)}{\kappa} - 1$ with respect to these parameters.

Appendix D. Proof of Proposition 3: derivation of aggregate welfare function

Assume the steady-state is efficient and equitable, in the sense that consumption shares across agents are equalized. This is ensured by having a fiscal authority tax/subsidize sales at the constant rate τ and redistribute the proceedings in a lump-sum fashion T such that in steady-state there is marginal cost pricing, and profits are zero. The profit function becomes $D_t(i) = (1-\tau)[P_t(i)/P_t]Y_t(i) - (MC_t/P_t)N_t(i) + T_t$, where by balanced budget $T_t = \tau P_t(i)Y_t(i)$. Efficiency requires $\tau = -\mu$, such that under flexible prices $P_t^*(i) = MC_t^*$ and hence profits are $D_t^*(i) = 0$. Note that under sticky prices profits will not be zero since the mark-up is not constant. Under this assumption we have that in steady-state:

$$\frac{V'(N_H)}{U'(C_H)} = \frac{V'(N_S)}{U'(C_S)} = \frac{W}{P} = 1 = \frac{Y}{N},$$

where $N_H = N_S = N = Y$ and $C_H = C_S = C = Y$.

Suppose further that the social planner maximizes a convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U_H(C_{H,t}, N_{H,t}) + [1-\lambda]U_S(C_{S,t}, N_{S,t})$. A second-order approximation to type $j's$ utility around the efficient flex-price equilibrium delivers:

$$\begin{aligned}\hat{U}_{j,t} &\equiv U_j(C_{j,t}, N_{j,t}) - U_j\left(C_{j,t}^*, N_{j,t}^*\right) = \\ &= U_C C_j\left[\hat{C}_{j,t} + \frac{1-\gamma}{2}\hat{C}_{j,t}^2 + (1-\gamma)c_{j,t}^*\hat{C}_{j,t}\right] - \\ &\quad -V_N N_j\left[\hat{N}_{j,t} + \frac{1+\varphi}{2}\hat{N}_{j,t}^2 + (1+\varphi)n_{j,t}^*\hat{N}_{j,t}\right] + t.i.p + O\left(\| \zeta \|^3\right),\end{aligned}$$

where variables with a hat denote log-deviations from the flex-price level (or 'gaps') $\hat{C}_t \equiv \log\frac{C_t}{C_t^*} = c_t - c_t^*$, and variables with a star flex-price values as above $c_t^* \equiv \log\frac{C_t^*}{C}$. Note that since $U_C C_H = U_C C_S$ and $V_N N_H = V_N N_S$ and using $c_{H,t}^* = c_{S,t}^* = c_t^*$ (which holds since asset income in the flex-price equilibrium is zero, as profits are zero) we can aggregate the above into:

$$\begin{aligned}\hat{U}_t &= U_C C\left[\hat{C}_t + (1-\gamma)c_t^*\hat{C}_t + \frac{1-\gamma}{2}\left(\lambda\hat{C}_{H,t}^2 + (1-\lambda)\hat{C}_{S,t}^2\right)\right] - \\ &\quad -V_N N\left[\hat{N}_t + (1+\varphi)n_t^*\hat{N}_t + \frac{1+\varphi}{2}\left(\lambda\hat{N}_{H,t}^2 + (1-\lambda)\hat{N}_{S,t}^2\right)\right] + t.i.p + O\left(\| \zeta \|^3\right)\end{aligned}$$

Note that $\hat{C}_t = \hat{Y}_t$ and $\hat{N}_t = \hat{Y}_t + \Delta_t$, where Δ_t is price dispersion as in Woodford (2003), $\Delta_t = \log\int_0^1 (P_t(i)/P_t)^{-\varepsilon} di$. Since $U_C C = V_N N$ we can show that the linear term boils down to:

$$\begin{aligned}&U_C C\left[\hat{C}_t + (1-\gamma)c_t^*\hat{C}_t\right] - V_N N\left[\hat{N}_t + (1+\varphi)n_t^*\hat{N}_t\right] \\ &= -U_C C\left[-\hat{Y}_t + (\gamma-1)c_t^*\hat{Y}_t + \hat{Y}_t + \Delta_t + (1+\varphi)n_t^*\hat{Y}_t\right] + O\left(\| \zeta \|^3\right) \\ &= -U_C C[\Delta_t] + O\left(\| \zeta \|^3\right)\end{aligned}$$

Quadratic terms can be expressed as a function of aggregate output. For that purpose, note that in evaluating quadratic terms we can use first-order approximations of the optimality conditions (higher order terms would imply terms of order higher than 2, irrelevant for a second-order approximation). Up to first order, we have that $\hat{C}_{H,t} = (1+\eta)\hat{W}_t$, $\hat{N}_{H,t} = \eta\hat{W}_t$ and $\hat{W}_t = \varphi\hat{N}_t + (1-\gamma)\hat{C}_t = (\varphi+\gamma)\hat{Y}_t + \varphi\Delta_t$

so:

$$\begin{aligned}
\hat{C}^2_{H,t} &= (1+\varphi)^2 \hat{Y}^2_t + O\left(\parallel \zeta \parallel^3\right) \\
\hat{N}^2_{H,t} &= (1-\gamma)^2 \hat{Y}^2_t + O\left(\parallel \zeta \parallel^3\right) \\
\hat{C}^2_{S,t} &= \frac{1}{(1-\lambda)^2}\left[1-\lambda(1+\varphi)\right]^2 \hat{Y}^2_t + O\left(\parallel \zeta \parallel^3\right) \\
\hat{N}^2_{S,t} &= \frac{1}{(1-\lambda)^2}\left[1-\lambda(1-\gamma)\right]^2 \hat{Y}^2_t + O\left(\parallel \zeta \parallel^3\right)
\end{aligned}$$

The aggregate *per-period* welfare function is thence, up to second order (ignoring terms independent of policy and of order larger than 2):

$$\begin{aligned}
\hat{U}_t &= -U_C C\left[\frac{\gamma-1}{2}\left(\lambda \hat{C}^2_{H,t} + (1-\lambda)\hat{C}^2_{S,t}\right) + \frac{1+\varphi}{2}\left(\lambda \hat{N}^2_{H,t} + (1-\lambda)\hat{N}^2_{S,t}\right) + \Delta_t\right] \\
&= -U_C C\left\{\frac{1}{2}\frac{\varphi+\gamma}{1-\lambda}\left[1-\lambda(1-\gamma)(1+\varphi)\right]\hat{Y}^2_t + \Delta_t\right\}
\end{aligned}$$

The intertemporal objective function of the planner will hence be $\mathbf{U}_t = \sum_{i=t}^{\infty} U_{t+i}$. This is consistent with our view that limited participation to asset markets comes from constraints and not preferences. In the latter case, maximizing intertemporally the utility of non-asset holders would be hard to justify on welfare grounds. However, note that for the discretionary (Markov) equilibrium studied here, this choice makes no difference since terms from time $t+1$ onwards are treated parametrically in the maximization and the time-t objective function is identical and equal to $\hat{U}_t$. By usual arguments readily available elsewhere (see Woodford (2003), Galí and Monacelli (2004)) we can express price dispersion as a function of the cross-section variance of relative prices: $\Delta_t = (\varepsilon/2)\, Var_i\left(P_t(i)/P_t\right)$ and the (present discounted value of the) cross-variance of relative prices as a function of the inflation rate $\sum_{t=0}^{\infty}\beta^t Var_i\left(P_t(i)/P_t\right) = \psi^{-1}\sum_{t=0}^{\infty}\beta^t \pi_t^2$. Hence, the present discounted values of price dispersion and the inflation rate are related by $\sum_{t=0}^{\infty}\beta^t\Delta_t = (\varepsilon/2\psi)\sum_{t=0}^{\infty}\beta^t\pi_t^2$ and the intertemporal objective function becomes (6.1) in Proposition 3 (reintroducing the notation $\hat{Y}_t = x_t$).

Appendix E. Cyclical implications and sunspot equilibria

The condition for profits cyclicality used in text is derived as follows, using the expression for profits $(B.8)$ and the relationship between marginal cost and output gap: $\frac{dd}{dy} > 0 \Leftrightarrow \frac{\mu}{1+\mu} - \frac{1+F_Y}{1+\mu}\frac{dmc}{dy} > 0 \Leftrightarrow \chi\frac{dx}{dy} + \frac{du}{dy} < \frac{\mu}{1+F_Y} \Leftrightarrow \chi\left(1-\frac{dy^*}{dy}\right) + \frac{du}{dy} < \frac{\mu}{1+F_Y} \Leftrightarrow \chi\frac{dy^*}{dy} > \chi - \frac{\mu}{1+F_Y} + \frac{du}{dy} \overset{dy>0}{\Leftrightarrow} dy < \frac{\chi(1+F_Y)}{\chi(1+F_Y)+(1+F_Y)du-\mu}dy^*$. In terms of output gap: $dx < \frac{\mu-(1+F_Y)du}{\chi(1+F_Y)+(1+F_Y)du-\mu}dy^*$ where the right-hand side term is exogenous and depends on technology.

The stability condition in the case of indeterminacy is - see Lubik and Schorfheide (2003), p. 278 (where $[A]_{2.}$ denotes the second row of the A matrix, attached to the explosive component):

$$\left[J^{-1}\Psi\right]_{2.}\nu_t + \left[J^{-1}\Gamma\right]_{2.}\eta_t = 0$$

Straightforward algebra to calculate

$$\begin{aligned}\left[J^{-1}\Psi\right]_{2.} &= \frac{1}{q_{+}-q_{-}}\left[\begin{array}{cc} -\kappa\delta^{-1} & \beta^{-1}-q_{-}\end{array}\right]\\ \left[J^{-1}\Gamma\right]_{2.} &= \frac{1}{q_{+}-q_{-}}\left[\begin{array}{cc} -\kappa q_{+} & q_{+}-1\end{array}\right]\end{aligned}$$

delivers the stability condition as:

$$-\kappa\delta^{-1}\left(\varepsilon_t - r_t^{*}\right) + \left(\beta^{-1}-q_{-}\right)u_t - \kappa q_{+}\eta_t^{y} + \left(q_{+}-1\right)\eta_t^{\pi} = 0$$

Since only one root is suppressed, there is endogenous persistency of the effects of shock (which was not the case under determinacy).

Following Lubik and Schorfheide (2003) we compute a singular value decomposition of $\left(q_{+}-q_{-}\right)\left[J^{-1}\Gamma\right]_{2.}$:

$$\begin{aligned}\left[J^{-1}\Gamma\right]_{2.} &= 1\cdot\left[\begin{array}{cc} d & 0\end{array}\right]\cdot\left[\begin{array}{cc} \frac{-\kappa q_{+}}{d} & \frac{q_{+}-1}{d}\\ \frac{q_{+}-1}{d} & \frac{\kappa q_{+}}{d}\end{array}\right]\\ d &= \sqrt{\left(\kappa q_{+}\right)^2 + \left(q_{+}-1\right)^2}\end{aligned}$$

Using also $\beta\left(q_{+}-q\right)\left[J^{-1}\Psi\right]_{2.} = \left[\begin{array}{cc} -\kappa\delta^{-1} & \beta^{-1}-q_{-}\end{array}\right]$ we get the full set of stable solutions as described in text.