Strong Contagion with Weak Spillovers*

Martin Ellison
University of Warwick and CEPR

Liam Graham
University of Warwick

Jouko Vilmunen
Bank of Finland

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Abstract

In this paper, we develop a model which explains why events in one market may trigger similar events in other markets, even though at first sight the markets appear to be only weakly related. We allow for multiple equilibria and learning dynamics in each market, and show that a jump between equilibria in one market is contagious because it more than doubles the probability of a similar jump in another market. We claim that contagion is strong since equilibrium jumps become highly synchronised across markets. Spillovers are weak because the instantaneous spillover of events from one market to another is small. To illustrate our result, we demonstrate how a currency crisis may be contagious with only weak links between countries. Other examples where weak spillovers would create strong contagion are various models of monetary policy, imperfect competition and endogenous growth.

*We are grateful for helpful conversations and comments from Tom Sargent, Noah Williams and Tony Yates. Corresponding author is Martin Ellison, Department of Economics, University of Warwick, Coventry, CV4 7AL, United Kingdom, tel: +44 2476 523472, fax: +44 2476 523030, email: martin.ellison@warwick.ac.uk. Giacomo Carboni provided excellent research assistance.

1 Introduction

On many occasions it is observed that developments in one market appear to follow those in another market, despite the fact that the markets seem to be only weakly related. One of the most obvious examples of this is the contagious nature of currency crises, since a crisis in one country is often followed by a crisis in another country, even though the two countries have only weak trade or financial linkages. Existing theories find such phenomena hard to explain and typically resort to the idea of correlated sunspots to explain the contagion of events from one market to another. However, the question then remains of why sunspots would be correlated across markets.

In this paper, we offer an explanation for why developments in separate markets may be synchronised even if there are no sunspots and spillovers between markets are weak. Our proposed explanation is based on the learning processes which determine equilibrium in each market. We characterise markets as having multiple equilibria, with jumps between equilibria occurring endogenously through learning as in Sargent (1999), and show that an equilibrium jump in one market significantly increases the probability of an equilibrium jump in the other market. The mechanism is not one in which agents observe an equilibrium jump in the other market and this directly induces an equilibrium jump in their own market, since we would interpret that as a strong spillover between markets. Instead, we restrict agents to only observe events in their own market, in which case the model is self-referential and weak spillovers are the only possible source of contagion. In other words, we demonstrate that weak spillovers create a channel by which an equilibrium jump in one market is likely to trigger an equilibrium jump in the other market.

Our preferred example to illustrate contagious equilibrium jumps is the example model of endogenous currency crises of Cho and Kasa (2003, p. 18-22), which itself is derived by simplifying and adding learning to the third generation currency crisis model of Aghion, Bacchetta and Banerjee (2000). We analyse a model of two small open economies and one large economy, in which international spillovers between the two small economies are weak. Whilst our example is drawn from the currency crisis literature, our results are applicable to a more general class of self-referential models with equilibrium jumps occurring through learning. This class includes models of monetary policy, imperfect competition, growth, and alternative models of currency crises, as discussed by Cho, Williams and Sargent (2002), Bullard and Cho (2002), Primiceri (2004), Williams (2004) and Kasa (2004). Equilibrium jumps have the potential to be contagious in all these models if there is another similar market and weak spillover of events from one market to another.

The remainder of the paper is organised as follows: Section 2 describes a version of the Cho and Kasa (2003, p. 18-22) model of endogenous currency crises in which two small economies interact with a large economy but there are no spillovers between the small economies. For this benchmark case, equilibrium jumps will not be contagious by definition. In Section 3 we introduce a weak unilateral spillover from one small economy to the other, with no spillover in the opposite direction, and show how equilibrium jumps in the first economy are likely to trigger similar jumps in the other. Section 4 presents results with weak bilateral spillovers. A final section concludes.

2 Model with no spillovers

The case of no spillovers between the small economies is a natural benchmark from which to start the analysis. Our initial model therefore consists of the following ingredients: true structural relationships linking output in each small open economy to the economy's exchange rate with respect to the large economy; a description of each central bank's perception of its own economy; a derivation of optimal exchange rate policy for each central bank; and a definition of equilibrium. In keeping with the original Cho and Kasa (2003, p. 18-22) model, there are no spillovers between the small economies in the benchmark model.

2.1 Structural relationships

The structure of each small open economy is represented by two relationships. Equation (1) is an open-economy expectations-augmented Phillips curve, in which unemployment y_t is determined by its natural rate y_0, unexpected movements in the country's own exchange rate, $s_t - E_{t-1}s_t$, and an output disturbance v_{1t}. Equation (1') is the analogous Phillips curve for the other small open economy, with the same structure and variables identified by the * superscript. As both countries are small open economies, exchange rates s_t and s_t^* are defined relative to the currency of the large economy, for example the US dollar. Unexpected depreciations in the country's currency are assumed to decrease output (θ is negative)[1].

$$y_t = y_0 + \theta(s_t - E_{t-1}s_t) + v_{1t} \tag{1}$$

$$y_t^* = y_0 + \theta(s_t^* - E_{t-1}s_t^*) + v_{1t}^* \tag{1'}$$

Equations (2) and (2') state that the exchange rate in each economy is a function of the level set by the respective central bank, $\hat{s}_t$ or $\hat{s}_t^*$, plus a control error v_{2t} or v_{2t}^*. We refer to $\hat{s}_t$ and $\hat{s}_t^*$ as intended exchange rates. Since private agents are assumed to have rational expectations, the expected exchange rates in equations (1) and (1') will be equal to the intended exchange rates. Unexpected exchange rate movements are caused by the control errors v_{2t} and v_{2t}^*.

$$s_t = \hat{s}_t + v_{2t} \tag{2}$$

$$s_t^* = \hat{s}_t^* + v_{2t}^* \tag{2'}$$

2.2 Central bank perceptions of the economy

Following Cho and Kasa (2003, p. 19), we assume that each central bank does not know the true structure of its economy. Instead, they have approximating models which allow for the possibility that there might be a long-run trade-off between output and the exchange rate. The approximating models are subtly misspecified because they describe a long-run relationship between output and the level of the exchange rate, when in reality it is only unexpected exchange rate movements that matter for output. Following the convention of Evans and Honkapohja (2001), we write the perceived law of motion (PLM) for each central bank as equations (3) and (3').

$$y_t = \gamma_{0t} + \gamma_{1t}s_t + \epsilon_t \tag{3}$$

$$y_t^* = \gamma_{0t}^* + \gamma_{1t}^* s_t^* + \epsilon_t^* \tag{3'}$$

ϵ_t and ϵ_t^* are approximation errors: the components of output movements that each central bank fails to explain by its model with a long-run trade-off between output and the exchange rate.

[1] The sign of θ in the Aghion, Bacchetta and Banerjee (2000) model depends on the degree to which unexpected devaluations affect balance sheets and the value of debt. We follow the lead of Cho and Kasa (2003, p. 10) and assume that balance sheet effects are strong and unexpected devaluations lead to a contraction in output.

The central banks estimate the coefficients of their perceived law of motion independently. To ensure there are no spillovers in the benchmark model, we restrict each central bank to only use data from their own country in estimation. This means that central banks are self-referential in nature and so precludes any contagion of equilibrium jumps that occurs because a central bank in one country observe jumps in the other country. Conditional on this data restriction, the central banks use discounted least squares techniques to estimate the coefficients of their perceived law of motion, as in Cho and Kasa (2003, p. 19). Equations (4) and (5) are standard recursive formulae for discounted least squares, with the matrix of regressors defined as $X_t = (1\ s_t)'$. The current estimates of the coefficients are stacked in the vector $\gamma_t = (\gamma_{0t}\ \gamma_{1t})'$, with P_t a 2×2 matrix measuring the precision of the estimates.

$$P_{t+1} = P_t + g(X_t X_t' - P_t) \tag{4}$$

$$\gamma_{t+1} = \gamma_t + g P_{t+1}^{-1} X_t (y_t - \gamma_t X_t) \tag{5}$$

In discounting past data, central banks allow for the possibility of structural breaks, even though such breaks are not explicitly present in our model. Under such circumstances, it is reasonable for the central bank to place more weight on recent data than data from the distant past. Discounting at the rate g gives a weight of $(1-g)^{n-1}$ to observations form n periods ago. Equations (4') and (5') are the estimation formulae for the other central bank, with $X_t^* = (1\ s_t^*)'$.

$$P_{t+1}^* = P_t^* + g(X_t^* X_t^{*\prime} - P_t^*) \tag{4'}$$

$$\gamma_{t+1}^* = \gamma_t^* + g P_{t+1}^{*-1} X_t^* (y_t^* - \gamma_t^* X_t^*) \tag{5'}$$

2.3 Optimal exchange rate policy

The objective of each central bank, following Cho and Kasa (2003, p. 19), is to minimise the extent to which its output and exchange rate deviate from target values $\bar{y}$ and $\bar{s}$. The central bank loss functions (6) and (6') place equal quadratic penalties on output and exchange rate deviations from target.[2]

[2] See Gerali and Lippi (2002) for a discussion of how escape dynamics change with different weights in the loss function.

$$\mathcal{L}_t = (y_t - \bar{y})^2 + (s_t - \bar{s})^2 \quad (6)$$

$$\mathcal{L}_t^* = (y_t^* - \bar{y})^2 + (s_t^* - \bar{s})^2 \quad (6')$$

Optimal policy requires a central bank to set the intended exchange rate to minimise expected losses, subject to the perceived law of motion of the economy. As in Sargent (1999) and Cho and Kasa (2003, p. 19), we assume that the central bank displays anticipated utility behaviour, following Kreps (1998). This implies that a central bank takes its best estimates of the coefficients in the perceived law of motion as being the true values, fixed now and into the indefinite future.[3] The policy problem is static and the first order conditions for expected loss minimisation under anticipated utility behaviour give policy rules (7) and (7').

$$\hat{s}_t = \frac{\bar{s} - \gamma_{1t}(\gamma_{0t} - \bar{y})}{1 + \gamma_{1t}^2} \quad (7)$$

$$\hat{s}_t^* = \frac{\bar{s} - \gamma_{1t}^*(\gamma_{0t}^* - \bar{y})}{1 + \gamma_{1t}^{*2}} \quad (7')$$

2.4 Equilibrium

We obtain the equilibrium actual laws of motion (ALM) for output and exchange rates by substituting the policy rules (7) and (7') into the equations for the true structure of the economy (1), (1'), (2) and (2'). Since events in each country are independent, equilibrium jumps cannot be contagious and we can focus on either country to see why the jumps occur. The actual laws of motion (ALM) for output and the exchange rate in the first country are described by equations (8) and (9).

$$y_t = y_0 + \theta v_{2t} + v_{1t} \quad (8)$$

$$s_t = \frac{\bar{s} - \gamma_{1t}(\gamma_{0t} - \bar{y})}{1 + \gamma_{1t}^2} + v_{2t} \quad (9)$$

A comparison of the perceived (3) and actual (8) laws of motion for output reveals the existence of a continuum of self-confirming equilibria in the model. Abstracting from stochastic terms, beliefs γ_t in each equilibrium must satisfy

[3] Tetlow and von zur Muehlen (2003) shows that equilibrium jumps are still possible when a central bank treats its coefficient estimates as uncertain.

condition (10), obtained by equating output under the perceived and actual laws of motion.

$$y_0 = \gamma_{0t} + \gamma_{1t} \left(\frac{\bar{s} - \gamma_{1t}(\gamma_{0t} - \bar{y})}{1 + \gamma_{1t}^2} \right) \tag{10}$$

Two of the self-confirming equilibria have clear intuitive economic interpretations. One equilibrium has beliefs $\gamma_1 = \theta$ and $\gamma_0 = y_0 - \theta\bar{s} + \theta^2(y_0 - \bar{y})$, with the central bank setting an intended exchange rate of $\hat{s}_t = \bar{s} + \theta(\bar{y} - y_0)$. We term this the *Nash equilibrium*, since it is the same as the outcome that would prevail under discretionary policy if the central bank knows the true structure of the economy (1) and (2). The other equilibrium has beliefs $\gamma_1 = 0$ and $\gamma_0 = y_0$, and the central bank sets a zero intended exchange rate. We denote this the *Ramsey equilibrium*, as it is equivalent to the outcome that arises under commitment policy if the central bank knows the true structure of the economy.

2.5 Calibration

In the full stochastic economy, it is difficult to obtain further analytical results.[4] To proceed, we therefore calibrate the model and analyse its behaviour by simulation. Our calibration in Table 1 is based on the parameter values chosen by Cho, Williams and Sargent (2002) for a closed economy.

Parameter	Value
$\bar{y}$	5
y_0	0
$\bar{s}$	0
θ	-1
σ_{v1}^2	0.3
σ_{v2}^2	0.3
g	0.025

Table 1: Calibrated parameter values

In words, our calibration implies that the central bank targets output at a level 5% above a zero natural rate; the exchange rate target is zero; surprise

[4] A series of remarkable papers by Williams (2003, 2004) shows how large deviations theory can be used to obtain a numerical solution to the dominant behaviour of the economy. We are currently examining how best to integrate these methods into our analysis.

appreciations translate one for one into increased output; the variance of both shocks is 0.3; and policymakers place a weight of 0.975^{n-1} on data from n periods ago.

2.6 Simulation results

The first set of results we report are for a dynamic simulation of a single small economy for 1600 periods. The top two panels of Figure 1 show the behaviour of the exchange rate and output respectively. According to the top left panel, the exchange rate has a tendency to appreciate towards the Nash equilibrium level $s_t = -5$, but occasional large devaluations bring it back to close to the Ramsey equilibrium level $s_t = 0$. In the top right panel, output has no clear trend and fluctuates around its zero natural rate.

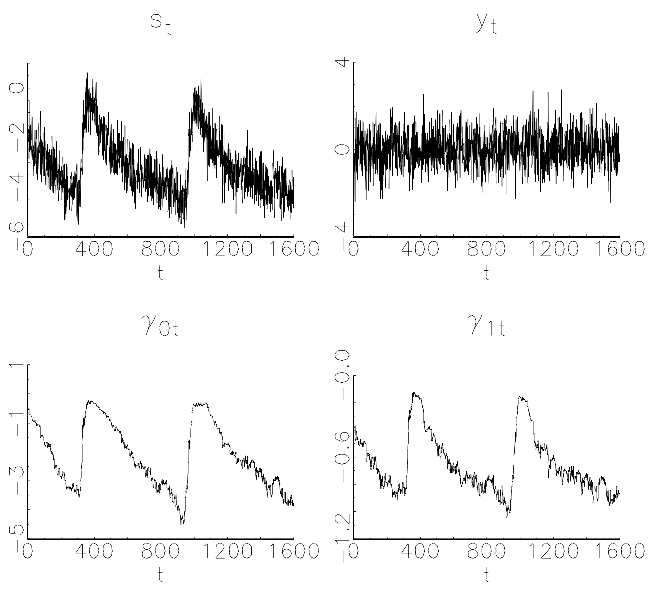

Figure 1: Simulation of the model with no spillovers

The bottom two panels of Figure 1 plot the evolution of central bank beliefs about the economy. They illustrate that large depreciations in the exchange rate are associated with a rapid realignment in beliefs $(\gamma_{0t}, \gamma_{1t})$ from close to $(-5, -1)$ to close to $(0, 0)$. In other words, the economy jumps from near the Nash self-confirming equilibrium to near the Ramsey self-confirming equilibrium. Cho and

Kasa (2003, p. 3) propose that such equilibrium jumps may be an explanation for currency crises, since jumps coincide with the type of dramatic exchange rate depreciations typically observed in a currency crisis. More generally, the equilibrium jumps are an example of the type of escape dynamics studied by Williams (2004).

To understand the behaviour of the economy, it is necessary to explain why the exchange rate tends to appreciate towards the Nash level and then occasionally jumps back and depreciates to the Ramsey level. A full technical analysis appears in Cho, Williams and Sargent (2002), so we focus on giving an intuitive explanation.

The tendency of the exchange rate to appreciate arises because of the effect that control errors have on the incentive for a central bank to use policy to achieve output and exchange rate targets. Control errors v_{2t} cause unexpected movements in the exchange rate, which affect output through the open economy Phillips curve (1). As the slope of the Phillips curve is negative, the resulting changes in exchange rate and output are negatively correlated. The central bank (incorrectly because of its belief in a misspecified model) interprets this as evidence of a long-run negative relationship between the two variables, creating an incentive for the central bank to appreciate the exchange rate to stimulate output. The appreciation continues until the exchange rate reaches its Nash level.

The jumps in the exchange rate occur because of rare combinations of shocks that lead the central bank to think (correctly in this case) that there is no long-run relationship between output and the exchange rate. With no apparent trade-off to exploit, the central bank sets the intended exchange rate equal to its target, at which point the actual exchange rate depreciates rapidly to its Ramsey level. An insight into the nature of the rare combinations that trigger jumps can be obtained by looking at the shocks hitting the economy around the time of large exchange rate devaluations.[5] Figure 2 summarises the distribution of output shocks v_1 and exchange rate control errors v_2 in the period immediately preceding an exchange rate devaluation and for the next three periods. In each panel, the mostly likely combinations of v_1 and v_2 are marked with a dot and

[5] An alternative approach favoured by Cho, Williams and Sargent (2002) is to numerically solve for the dominant escape route from Nash to Ramsey equilibrium. In the limit, the most likely combinations of shocks from our simulations should converge to the shocks occurring along the dominant escape route.

surrounded by a one standard deviation confidence region. The top left panel is for the shocks immediately preceding a devaluation; in the top right panel the devaluation has already begun. The bottom two panels are for shocks occurring as the devaluation progresses.

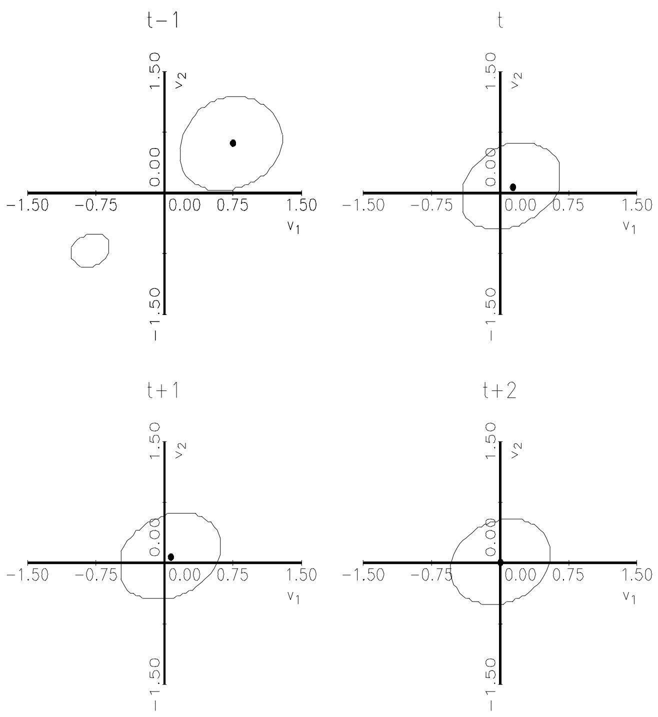

Figure 2: Distributions of shocks at the time of large devaluations

The panels in Figure 2 show considerable regularities in the distribution of shocks around the time of a large devaluation. If the shocks v_1 and v_2 were completely random then the most likely combination in each period would be $(0, 0)$ and the confidence region would be a perfect circle cutting the axes at ± 0.548, one standard deviation of each shock. Instead, the shocks appear to be both positive and positively correlated in each period. This is reflected in the most likely combinations of shocks lying in the positive-positive quadrant and the confidence regions being skewed towards the positive-positive and negative-negative quadrants. The pattern is particularly clear in the period immediately preceding a devaluation. The salient features of the shocks are summarised in

Proposition 1.

Proposition 1 *Large devaluations in the exchange rate tend to be preceded by output shocks and exchange rate control errors that are (i) positively correlated and (ii) positive.*

Property (i) that shocks tend to be positively correlated just before an equilibrium jump is explained by Cho, Williams and Sargent (2002). The basic intuition can be understood by noting that, under normal circumstances, control errors v_2 lead to unexpected exchange rate changes and observable movements in output, which reinforce the Nash equilibrium. However, if v_2 is positively correlated with the output shock v_1 then the movement in output will be offset by an output shock, and output does not appear to react to the exchange rate change. For example, a positive control error causes an unexpected depreciation of the exchange rate and a contraction of output, but a positive output shock would offset the output contraction and make it appear that output has not reacted to the depreciation in the exchange rate. Similarly, the expansion in output caused by an unexpected appreciation of the exchange rate will be hard to observe if it coincides with a negative output shock. In such circumstances, the central bank starts to discount the possibility that there is a long-run relationship between output and the exchange rate, the exchange rate depreciates and an equilibrium jump to Ramsey is likely.

Property (ii) that shocks tend to be positive just before an equilibrium jump has received less attention in the literature. An intuitive explanation for why positive shocks precede the jump is that they create a strong signal that there is no relationship between output and the exchange rate. Figure 3 shows how the distribution of shocks v_1 and v_2 at time $t-1$ maps into the values of output y and the exchange rate s observed in the period immediately before the jump, along with the perceived laws of motion associated with the Nash and Ramsey equilibria. The dot and one standard deviation confidence region now denote the most likely combinations of output and the exchange rate. In the figure, it is clear that having both positive and positively correlated shocks creates outcomes that appear more consistent with Ramsey than Nash equilibrium beliefs. Furthermore, the predominance of positive output shocks guarantees that output tends to be above its natural rate, which strengthens the signal that there is no long-run relationship between output and the exchange rate.

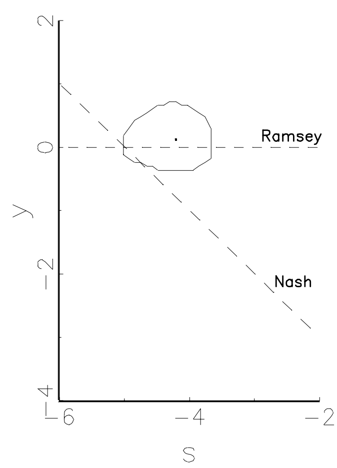

Figure 3: Distribution of exchange rate and output before large devaluations

Taken together, the two properties of Proposition 1 imply significant regularities in the distributions of shocks hitting the economy at the time of large devaluations. The fact that equilibrium jumps are associated with particular rare combinations of shocks will play an important role in explaining why the jumps become contagious, even if there are only weak spillovers between markets.

3 Model with weak unilateral spillovers

The essence of a spillover is that events in one market influence events in a second market. We therefore proceed by introducing a mechanism whereby developments in the first small country spillover to the second small country. Our assumption is that conditions in the first small economy can be summarised by its exchange rate with respect to the large country, s_t, and that these conditions have a linear effect on output y_t^* in the second small economy. Equations (11) and (11') show how output is determined in each small country in the presence of the spillover. As the direction of the spillover is from the first to the second small country and not *vice versa*, determination of output in the first small country (11) is identical to that in the no spillovers case (1).

$$y_t = y_0 + \theta(s_t - E_{t-1}s_t) + v_{1t} \tag{11}$$

$$y_t^* = y_0 + \theta(s_t^* - E_{t-1}s_t^*) + v_{1t}^* + \delta^* s_t \tag{11'}$$

The spillover mechanism is assumed to operate via financial markets through the effect of the exchange rate on balance sheets and the value of debt. A depreciation in the currency of the first small economy with respect to the large economy (an increase in s_t) improves the balance sheet position of the second small economy and therefore boosts output (δ^* is positive).[6] The coefficient δ^* should be sufficiently small to ensure that spillovers are weak. We set $\delta^* = 0.1$ so spillovers contribute only weakly to the determination of output in the second small economy.

The central banks are assumed to be unaware of the presence of spillovers: they maintain their belief in the long-run relationship between output and the exchange rate summarised by the perceived laws of motion (3) and (3'). This introduces another subtle misspecification in the perceived laws of motion because a central bank interprets output movements induced by spillovers as simple approximation errors rather than due to events in the other small country.

Equilibrium in the model with weak unilateral spillovers is described by Phillips curve equations (11) and (11'), exchange rate control equations (2) and (2'), perceived laws of motion (3) and (3'), and exchange rate policy rules (7) and (7'). As the first country is not influenced by the second, its behaviour in equilibrium is given by equations (8) and (9) as before, the actual laws of motion for output and the exchange rate in the no spillovers case. For the second country, the behaviour of the economy is shown by equations (12) and (13), with the spillover entering the actual law of motion for output.

[6] An alternative specification for the open economy context would be to assume that spillovers act through the bilateral exchange rate between the two small countries, in which case the final term in equation (11') would be $\delta^*(s_t - s_t^*)$. The results with this alternative specification are very close to the results we report. Another possibility is to assume that spillovers act through unexpected movements in the bilateral exchange rate, i.e. the spillover term should be $\delta^*[E_t(s_t - s_t^*) - (s_t - s_t^*)]$. In this case the spillover proves too weak and it is not possible to obtain strong contagion.

$$y_t^* = y_0 + \theta v_{2t}^* + v_{1t}^* + \delta^* s_t \tag{12}$$

$$s_t^* = \frac{\bar{s} - \gamma_{1t}^*(\gamma_{0t}^* - \bar{y})}{1 + \gamma_{1t}^{*2}} + v_{2t}^* \tag{13}$$

To study the implications of introducing weak unilateral spillovers, we analyse long numerical simulations of the model and compare them to alternative simulations with no spillovers. Figure 4 shows one such comparison.

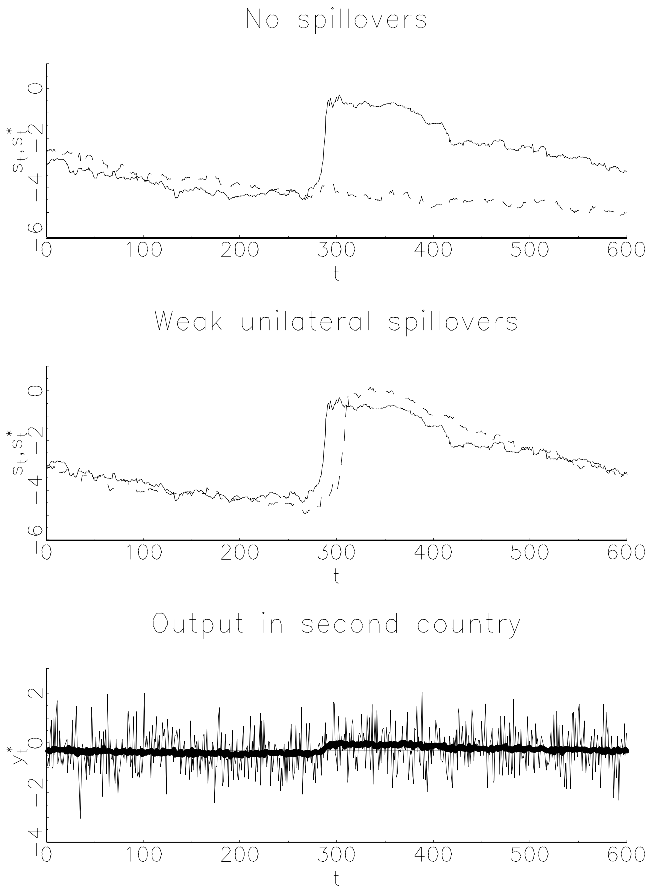

Figure 4: Simulation of the model with unilateral spillovers

The top panel of Figure 4 shows the behaviour of the intended exchange rates in the no spillovers case: the solid line is the intended exchange rate $\hat{s}_t$ for the first country; the dashed line is the intended exchange rate $\hat{s}_t^*$ for the second country. With no spillovers, the intended exchange rates are independent and there is no interaction between the countries. Even though $\hat{s}_t$ jumps around period $t = 290$, there is no effect on $\hat{s}_t^*$. In sufficiently long simulations, the correlation between $\hat{s}_t$ and $\hat{s}_t^*$ is zero.

The middle panel of Figure 4 shows simulated paths for intended exchange rates when there are weak unilateral spillovers from the first country to the second. As the spillover is unidirectional, it has no effect on the first country and the solid line in the middle panel is the same as in the upper panel for the no spillovers case. In contrast, the dashed line for the intended exchange rate in the second country is very different. Rather than the gradual appreciation seen with no spillovers, $\hat{s}_t^*$ jumps soon after the jump in $\hat{s}_t$. The large devaluation in the first country appears to have triggered a similar large devaluation in the second country. We interpret this as evidence that equilibrium jumps are strongly contagious in the model. The contagion leads to a positive correlation between the intended exchange rates in the two countries. In long simulations the correlation coefficient is approximately 0.19.

To gauge the importance of spillovers, the bottom panel of Figure 4 plots as a thin line the simulated path of output in the second country. There are three factors in the model which can cause output to deviate from its natural rate: output shocks v_{1t}^*, exchange rate control errors v_{2t}^*, and spillovers from the first country $\delta^* s_t$. The thick line in the bottom panel shows the deviation in output that is due to the third factor. The role of spillovers is apparently small, with deviations due to spillovers barely discernible amongst the much larger deviations caused by output shocks and unexpected exchange rate movements. In variance decomposition terms, only 3% of the variance in output in the second country is attributable to spillovers from the first country. It is in this sense that we claim to have only weak spillovers in the model.

An intuitive understanding of why weak spillovers are sufficient to cause strong contagion can be obtained by recalling the pattern of shocks that typically precedes an equilibrium jump. According to Proposition 1 in Section 3, jumps tend to be triggered by a series of output shocks and exchange rate control errors that are (i) positively correlated and (ii) positive. Weak spillovers cause strong contagion by creating a mechanism whereby an equilibrium jump in the first country increases the probability that output shocks in the second country are positive, which in turn makes it more likely that there will be an equilibrium jump in the second country. During the initial jump, the exchange rate in the first country depreciates rapidly, which spills over via $\delta^{*\rightarrow} > 0$ into higher than expected output in the second country. The spillover therefore creates conditions in the second country that are analogous to a run of positive v_{2t}^* output shocks. Condition (ii) is more likely to be satisfied and the probability of an equilibrium jump in the second country increases in the immediate aftermath of a jump in the first country.[7]

The intuition is confirmed by Table 2, which reports how the presence of weak unilateral spillovers increases the probability of an equilibrium jump in the second country occurring within a given number of periods of a jump in the first country. The increase in probability is heavily dependent on the level of the exchange rate in the second country at the time the first country jumps, ranging from no change when $s_t^* > -2$ to more than doubling when $s_t^* < -4$. The dependency arises because conditions have to be ripe for an equilibrium jump in the first country to trigger a jump in the second country. An exchange rate in the second country close to the Nash level $s_t^* = -5$ puts the country in the "danger zone" and makes it more susceptible to jumps and contagion. Conversely, if the exchange rate in the second country is already close to its Ramsey level $s_t^* = 0$ then jumps and contagion are highly unlikely.

[7] A similar mechanism operates in the model of McGough (2004), where a permanent but unobservable increase in the natural rate of output creates a series of positive output shocks that increases the probability of an equilibrium jump.

	Probability of jump in second country within k periods of jump in first country			
	$k = 5$	$k = 10$	$k = 20$	$k = 50$
No spillovers				
$s_t^* > -2$	0.000	0.000	0.000	0.000
$-2 > s_t^* > -4$	0.001	0.011	0.017	0.018
$-4 > s_t^*$	0.010	0.022	0.049	0.125
Unilateral spillovers				
$s_t^* > -2$	0.000	0.000	0.000	0.000
$-2 > s_t^* > -4$	0.003	0.008	0.013	0.018
$-4 > s_t^*$	0.024	0.067	0.128	0.258

Table 2: Probability of equilibrium jumps in second country

Further evidence that contagion is strong is provided in Table 3, which gives summary statistics for the relationship between the exchange rates of the two countries in the frequency domain. Coherence measures the correlation between the exchange rates at a given frequency and group delay can be interpreted as the extent to which the exchange rate in the first country leads or lags that in the second country at a given frequency.[8] If there are no spillovers then coherence and group delay are zero by definition. According to Table 3, weak unilateral spillovers create significant coherence between exchange rates at very low frequencies, with the group delay statistic indicating that the first country leads the second country by just short of 20 periods. There is very little coherence at frequencies higher than 0.08. The results are consistent with contagion acting through jumps in the first country triggering jumps in the second: coherence at low frequency matches the long period between equilibrium jumps in Figure 1; a group delay of about 20 periods confirms the lead of the first country over the second seen in the simulation of Figure 4.

[8] See Hannan and Thompson (1971) for more details on the interpretation of coherence and group delay.

Frequency	Coherence	Group delay
0.005	0.213	19.2
0.010	0.175	18.6
0.020	0.104	16.9
0.040	0.044	12.8
0.080	0.018	7.1

Table 3: Frequency domain properties of exchange rates with weak unilateral spillovers

Our contention that spillovers are weak is based on the observation that they only make a small contribution to the variance of output in the second country. The robustness of this result is examined in Figure 5, which shows how the share of output variance attributed to spillovers depends on δ^*, the coefficient on the unilateral spillover. As a comparison, we also show the degree of contagion for each δ^* by plotting the corresponding coherence between exchange rates at low frequency (0.005Hz). At low levels of δ^*, the share of variance is low and spillovers are weak. In contrast, coherence is much higher and contagion is strong. Figure 5 provides the evidence for the central claim of our paper: weak spillovers are sufficient to cause strong contagion.

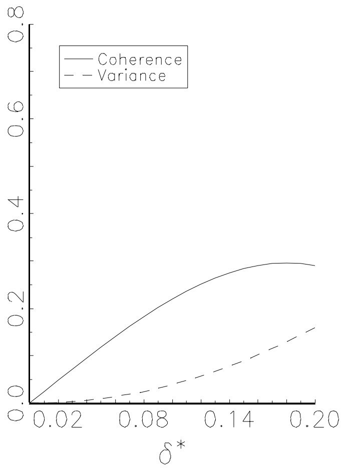

Figure 5: Coherence and variance share of spillovers with unilateral spillovers

4 Model with weak bilateral spillovers

In a more general setting it is reasonable to believe that equilibrium jumps may be mutually contagious from one small country to another. To allow for this possibility, we introduce bilateral spillovers and rewrite the Phillips curve equations as (14) and (14'). In this case, output in each small country is influenced by the exchange rate of the other small country. We define $\delta = \delta^* = 0.1$ so spillovers are weak and symmetric.

$$y_t = y_0 + \theta(s_t - E_{t-1}s_t) + v_{1t} + \delta s_t^* \quad (14)$$

$$y_t^* = y_0 + \theta(s_t^* - E_{t-1}s_t^*) + v_{1t}^* + \delta^* s_t \quad (14')$$

Equilibrium in the model with weak bilateral spillovers is determined by the system of equations (15), (15'), (16) and (16').

$$y_t = y_0 + \theta v_{2t} + v_{1t} + \delta s_t^* \quad (15)$$

$$y_t^* = y_0 + \theta v_{2t}^* + v_{1t}^* + \delta^* s_t \quad (15')$$

$$s_t = \frac{\bar{s} - \gamma_{1t}(\gamma_{0t} - \bar{y})}{1 + \gamma_{1t}^2} + v_{2t} \quad (16)$$

$$s_t^* = \frac{\bar{s} - \gamma_{1t}^*(\gamma_{0t}^* - \bar{y})}{1 + \gamma_{1t}^{*2}} + v_{2t}^* \quad (16')$$

The frequency domain properties of exchange rates are reported in Table 4. The group delay statistics are zero because the model is symmetric, with neither country having a systematic lead or lag over the other. Coherence values are high, especially at low frequencies where they are more than double the values observed under weak unilateral spillovers.

Frequency	Coherence	Group delay
0.005	0.527	0
0.010	0.429	0
0.020	0.246	0
0.040	0.089	0
0.080	0.023	0

Table 4: Frequency domain properties of exchange rates with weak bilateral spillovers

The fact that coherence with bilateral spillovers is more than double that observed with unilateral spillovers suggests there is a strategic complimentarity in the model. The complimentarity arises because of the way in which exchange rates become more synchronised once an equilibrium jump in the first country triggers a jump in the second. With greater synchronisation, both exchange rates tend to re-enter the danger zone around the Nash level at about the same time and conditions are then highly conducive to a jump in the second country triggering a jump in the first. The contagion of a jump from the first to second country therefore increases the probability of experiencing contagion in the opposite direction in the future. Overall, coherence is reinforced.

The final piece of evidence that weak spillovers create strong contagion is Figure 6, which shows the share of output variance due to spillovers and the coherency of exchange rates at low frequency (0.005Hz) for different bilateral spillover coefficients $\delta = \delta^*$. In all cases, variance share is small relative to coherence. For the baseline calibration, $\delta = \delta^* = 0.1$, coherence is more than ten times higher than the share of variance. We therefore conclude that contagion is an order of magnitude stronger than the spillovers that create it.

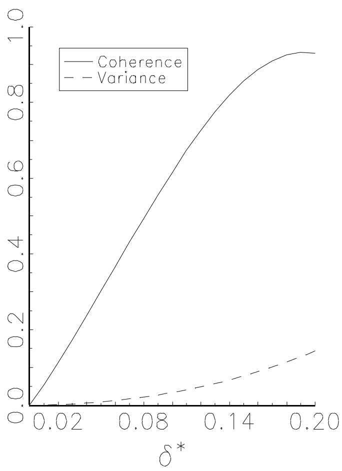

Figure 6: Coherence and variance share of spillovers with bilateral spillovers

5 Conclusions

The central claim of this paper is that developments in one market may have a profound effect on other markets, even though at first sight the other markets appear to be only weakly related. To obtain our result, we constructed a simple model of two markets with weak spillovers from one market to another. Following Sargent (1999), individual markets were characterised by multiple equilibria and learning dynamics that occasionally caused them to jump between equilibria. Our analysis showed that weak spillovers are sufficient to make a jump in one market significantly increase the probability of a similar jump in the other market. We therefore concluded that equilibrium jumps are strongly contagious in the model. Contagion occurs because a jump in one market spills over and creates conditions that are conducive to a jump in the other market.

The claim that weak spillovers create strong contagion applies to a class of models with weakly related markets and endogenous equilibrium jumps. In our preferred example, we extended the model of Cho and Kasa (2003, p. 18-22) to show that currency crises may be contagious with only weak financial links between countries. In the model, endogenous equilibrium jumps were equated to currency crises and spillovers were weak because they only accounted for a small proportion of output fluctuations in each country. Our simulations indicated that a currency crisis in one country more than doubles the probability of a currency crisis in another country. Currency crises are therefore strongly contagious, with a crisis in one country highly likely to trigger crises in other countries.

Other situations in which we expect to observe strong contagion with weak spillovers are suggested by the models of Cho, Williams and Sargent (2002) and Williams (2004). In monetary policy, a rapid disinflation in one country may trigger a rapid disinflation in another country. With imperfect competition, an outbreak of collusion in one market makes collusion more likely in another market, even when the cross-price elasticity between the two markets is low. For endogenous growth models, a growth spurt in one country may create growth spurts in other countries.

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