

Borrowing Limits, Housing Prices, and Multiple Equilibria

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Motivation

- Sustained increases in the housing real prices, downward trend in real interest rates, upward trend in households debt-income ratio.
- Empirical Analysis:
 - Asset Pricing (Mis-)Valuation -Irrationality, Bubble-like analysis, unsustainable-(Shiller, 2001; Ayuso-Restoy; 2003; Brunnermeier-Julliard; 2005)
 - Consumption-based/Saving Equations: Muellbauer (1997) Implications of housing wealth for consumption-saving pattern.

- Theoretical Analyses: Not many. Why? *How can an economy have a low interest rate in a high asset-price equilibrium when it requires more funding to be sustained?* [LITERATURE]

Key Ingredients and Implications

- A simple GE-Rational Expectations OLG Model.
- **Key Ingredients** : *Borrowing Limits* (Individual Heterogeneity), *Funding Mechanism*, Home-ownership bias, Supply Side Sluggishness.
- **Key Implications**
 - Individual heterogeneity in Financial Position and Housing Tenure
 - Possibility of Multiple Steady State Equilibria. Room for expectations driven/speculative paths (joint dynamics for housing prices, rents, interest rates, and debt.)

Borrowing Limits

- Borrowing limits might equally affect both home-owners and landlords, i.e. (regardless of the use of the house: self-consumption or renting). This matters for:
 - Joint behavior of renting and home-ownership markets.
 - Interaction between housing and credit markets.
- Implications for the feasibility of multiple equilibria
 - The existence of borrowing limits act as an upper bound for speculative paths (a “resting point”) - related to BF bubble like-
 - Asset valuation condition is modified (price = discounted rents)

The Baseline Environment

- The *simplest OLG* resembling that early in life households borrow to buy houses, and thus save in the form of housing. As time goes by, agents have built stocks of houses and start to increase their holding of financial assets (Villaverde and Krueger (2002), Yang (2005)).
- Downpayment is a function of Individual Characteristics (decreasing function of initial wealth, Linneman-Watchter, 89; Haurin et al. 97)
- Homeownership bias: Taxation/Technological (Slemrod-Poterba-Hend-loan.)
- Fixed supply of housing.

The Households' Problem

$$\log(c_{j,t}^y) + \beta \left[\log(c_{j,t+1}^m) + \log(h_{t+1} + s_{t+1}) \right] + \beta^2 \log(c_{j,t+2}^o) \quad (1)$$

$$c_{j,t}^y + p_t h_{j,t+1} + [p_t - (1-\tau) q_t] g_{j,t+1} + q_t s_{t+1} + a_{j,t+1}^y - b_{j,t+1} \leq z_j \quad (2)$$

$$c_{j,t+1}^m + a_{j,t+2}^m \leq p_{t+1} (h_{j,t+1} + g_{j,t+1}) + (1+r_{t+1}) (a_{j,t+1}^y - b_{j,t+1}) \quad (3)$$

$$c_{j,t+2}^o \leq (1+r_{t+2}) a_{j,t+2}^m \quad (4)$$

$$b_{j,t+1} \leq (1-\theta_j) p_t (h_{j,t+1} + g_{j,t+1}) \quad (5)$$

where $z(j) \in (0, 1]$, $\theta_j = f(\theta, z_j)$, $f_\theta > 0$, $f_{z_j} < 0$, $f(\theta, 1) > 0$.

[FIGURE]

Equilibrium

Definition : A perfect foresight competitive equilibrium is a set of allocations $\{c_t^y, c_t^m, c_t^o, a_{t+1}^y, a_{t+1}^m, b_{t+1}, h_{t+1}, s_{t+1}, g_{t+1}\}_{t=0}^{\infty}$, price and interest rate sequences $\{p_t, q_t, r_t\}_{t=0}^{\infty}$, and a sequence of taxes $\{\tau_t\}$ s.t.

1. Households maximize their utility (1) subject to constraints (2)-(5) given the sequences $\{p_t, q_t, r_t\}$ and the government policy.
2. The government satisfies: Tax Collection = Government Spending
3. All markets clear in every period.(Fixed Supply of Houses)

[Follow – up : (1) Households-Optimization (Endogenous Segmentation); (2) Aggregate Market Clearing Conditions.]

Endogenous Segmentation (I)

<i>Financial Position</i>	<i>Housing Tenure</i>			
	Pure Renters $s > 0, h = 0$	Renters Buyers $s > 0, h > 0$	C. Buyers $s = 0, g = 0$ $h > 0$	Land Lords $g > 0, h > 0$
Constrained	\emptyset	\checkmark	\checkmark	\checkmark
Unconstrained	\emptyset	\emptyset	—	\checkmark

Endogenous Segmentation (II)

Lemma 1. All Landlords are either constrained or unconstrained.

Non arbitrage argument (houses as only investment.)

Return of buying to put on rent and selling tomorrow: $\mathfrak{R}_{t+1}^c = \frac{p_{t+1}}{p_t - (1-\tau)q_t}$.

If there is an unconstrained land-lord then $\mathfrak{R}_{t+1} = 1 + r_{t+1}$

But a constrained landlord requires a higher return, i.e. $\mathfrak{R}_{t+1}^c > 1 + r_{t+1}$;
which leads to contradiction.

Endogenous Segmentation (III)

Lemma 2(a). When $\tau > 0$ then:

(a) $h_{t+1} > 0$, (From viewpoint of asset holdings housing is preferred to renting)

(b) $g_{t+1} \cdot s_{t+1} = 0$, (No one will buy a house to self-renting)

(c) $s_{t+1} = 0$, *if the borrowing constraint is not binding.*

If the guy is not constrained will never get services in the market being taxed, since she could produce those services himself (*by exploiting her borrowing capacity to buy*) so avoiding taxes. Formally, [Cost for a tenant of renting: $q_t = \frac{1}{(1-\tau)} \left[p_t - \frac{p_{t+1}}{1+r_{t+1}} \right] > \left[p_t - \frac{p_{t+1}}{1+r_{t+1}} \right]$ Cost of buying the same unit].

Endogenous Segmentation (IIIbis)

Lemma 2(b) When $\tau > 0$ and $\theta_j = f(\theta, z_j)$, $f_{z_j} < 0$ there exist a non-empty set of households such that $g_{t+1} = s_{t+1} = 0$. (A group of households do not participate in the rental mkt.)

- Limiting Renter-Cons.Buyer j such that $s_{jt+1} = 0$ (notice that by $f_{z_j} < 0$ every guy i such that $z_i < z_j$ optimally chooses $s_{it+1} > 0$). Then guy j is uniquely identified by: $\theta_{jt} \equiv \theta_{1,t} = (2 + \beta) \frac{q_t}{p_t}$
- Limiting Cons.Buyer-Cons.Landlord k such that $g_{kt+1} = 0$ (notice that by $f_{z_k} < 0$ every guy i such that $z_i > z_k$ optimally chooses $g_{it+1} > 0$). Then guy k is uniquely identified by: $\theta_{kt} \equiv \theta_{2,t} = (2 + \beta) (1 - \tau) \frac{q_t}{p_t}$

- Limiting Cons. Buyer-Landlord Unconstrained (supply of g is fully elastic). We pin down the guy k such that the return is equal to $1 + r_{t+1}$. Then the guy k is uniquely identified by: $\theta_{kt} \equiv \theta_{2,t} = (2 + \beta)(1 - \tau) \frac{q_t}{p_t}$.
- Thus, $\tau > 0 \Rightarrow \theta_{2,t} < \theta_{1,t}$.

Endogenous Segmentation (IV): Returns and Downpayment

Lemma 3. For any constrained household the following holds:

$$\frac{\lambda_t}{\beta\lambda_{t+1}} \equiv \mathfrak{R}_{t+1} \propto \frac{p_{t+1} - (1 + r_{t+1})(1-\theta)p_t}{\theta p_t - \mathbb{k}_t} > 1 + r_{t+1},$$

$$\mathbb{k}_t = \begin{cases} q_t & \text{if } s_{t+1} > 0 \\ 0 & \text{if } g_{t+1} = s_{t+1} = 0 \\ (1-\tau)q_t & \text{if } g_{t+1} > 0 \end{cases}$$

Net (of financial costs) profits over invested internal funds, i.e. return on internal funds.

Corollary (a) If either s_{t+1} or $g_{t+1} > 0$, then $\frac{\partial \mathfrak{R}_{t+1}}{\partial z} > 0$; (b) if $g_{t+1} = s_{t+1} = 0$, then $\frac{\partial \mathfrak{R}_{t+1}}{\partial z} < 0$. [FIGURES]

Steady State Analysis: Multiple (interior) Equilibria

Definition. A HVE (LVE) is such that landlords are constrained (unconstrained).

Low Valuation Equilibrium (LVE)

- It follows from FOC (g): $\Rightarrow p = \frac{1+r}{r} (1 - \tau) q$
- Thus, (fully-) *elastic supply of houses for renting*. Equilibrium quantity in rental market is driven by demand. But, only up to the limit of landlords' credit capacity.
- That is, a NEC for LVE is $\eta \equiv \frac{p}{q} = (1 - \tau) \frac{1+r}{r}$ **and** $\{S \leq G^{\max}\}$, where S is the aggregate *demand for renting* and G^{\max} is the *supply of renting* such that for every landlord is at her borrowing limit, i.e. $b_j = (1 - \theta_j) p (h_j + g_j^{\max})$ and $a^y = 0$.

High Valuation Equilibrium (HVE)

- Everyone is constrained (all landlords are constrained.) The critical difference is the equilibrium in the rental market:
 - The supply of houses for renting *is not longer fully elastic* (This is an alternative to the NAC. Given that the landlord is constrained, in order for a constrained landlord to be willing to supply an additional unit or renting (instead of self-consuming housing), the relative price η should be lower, i.e. higher relative rents.
 - More formally: Next

High Valuation Equilibrium (HVE)

- $g_j = \frac{\gamma\beta}{p} \left[\frac{2+\beta-\frac{\eta}{(1-\tau)}\theta_j}{\theta_j-\frac{(1-\tau)}{\eta}} \right] z_j, \frac{\partial g_j}{\partial \eta} < 0$, where $\gamma = (1 + \beta)^{-2}$

- $p < \frac{1+r}{r} (1 - \tau) q$, i.e. $\eta < \frac{1+r}{r} (1 - \tau)$.

– This does not mean that η is lower than in the LVE, on the contrary $\eta^{HVE} > \eta^{LVE}$ (key point: the interest rates are lower to support the *HVE*).

Proposition 1. If both types of equilibria coexist then in the LVE (HVE)
 (i) the ratio of housing prices to rents is lower (higher), (ii) the interest
 rate is higher (lower), (iii) housing prices and the amount of renting are
 lower (higher), (iv) finally the volume of debt is lower (higher).

Proof (a detour)

(i) The excess capacity, $G^{\max} - S$, in the *LVE* satisfies: $\frac{\partial[G^{\max} - S]}{\partial\eta} < 0$
 ($\frac{\partial G^{\max}}{\partial\eta} < 0$, $\frac{\partial S}{\partial\eta} > 0$). Thus, $\exists \eta^*$ such that $\eta^{LVE} \leq \eta^*$, i.e. using η
 $= \frac{1+r}{r} (1 - \tau)$, then $\exists r^*$ such that $r^{LVE} \geq r^*$.

(ii) Likewise, $\eta^{HVE} > \eta^*$ and using $\eta < \frac{1+r}{r} (1 - \tau)$, then $r^{HVE} < r^*$.

[FIGURES]

(iii) Housing prices across equilibria

Using market clearing conditions *in both* the renting and the housing markets (in either equilibrium: $H=H^R+H^B+H^L+G(=S)$), it follows :

$$\frac{\partial p}{\partial \eta} = \gamma \beta \left[\underbrace{\int_0^{z_1} z dF(z)}_{\text{RENTERS}} + \frac{1}{1-\tau} \underbrace{\int_{z_2}^1 z dF(z)}_{\text{LANDLORDS}} \right] > 0$$

where $z_j = f^{-1}(\theta, z_j)$, $j = 1, 2$. What is behind? (NB: C.buyers don't participate in rental mkt.)

- Renters (demand-shift) The total amount of housing services (self-consumption plus renting) is an increasing function of η . $\frac{\partial ps}{\partial \eta} > -\frac{\partial ph}{\partial \eta} > 0$ (overall higher demand, notice $\eta > 1 - p > q$ -, durability).
- Landlords (either constrained or not: i.e. the same activity buy-rent-sell). An increase in η implies that it is less attractive to buy for renting (even if the return is $1+r$ in LVE), hence increasing the amount of housing for self-consumption.

Aggregate Market Clearing Conditions: Asymmetries

HVE

LVE

Renting

$$\eta : S(\eta) = G(\eta)$$

$$\eta = \frac{(1-\tau)(1+r)}{r}$$

Housing

$$p : H^S = H^D(\eta, p)$$

$$p : H^S = H^D(\eta(r), p)$$

Credit

$$r : B(\eta) = A(\eta, r)$$

$$r : B(\eta(r)) = A(\eta(r), r)$$

Segmentation

$$z_j : f^{-1}(\theta_j(\eta))$$

The Funding Mechanism

- Negatively sloped aggregate supply of funding [the case of OLG models with capital accumulation, (see e.g. Diamond (1965) and more recently Caballero et al. (2005))]
- At the heart of this mechanism is the existence of a sufficiently strong negative income (wealth) effect.
- In our model, reversal of the slope of aggregate supply of funding across equilibria. Heterogeneity: $SAVING_j = RETURN_j * z$
- Important: we look at the effects of changes in r on demand and supply of funds assuming the rental market is in equilibrium.

Supply of Funds: Savings and Interest Rates

	Returns	Debt Cost	Rel. Price		Total	
			<i>LVE</i>	<i>HVE</i>	<i>LVE</i>	<i>HVE</i>
Renter	$\frac{1-(1+r)(1-\theta)}{\theta-1/\eta}$	-	+	0	+	-
Buyer	$\frac{1-(1+r)(1-\theta)}{\theta}$	-	0	0	-	-
Land.U	$1+r = \frac{1}{1-(1-\tau)/\eta}$	$i?^*$	+	NA	+	NA
Land.C	$\frac{1-(1+r)(1-\theta)}{\theta-(1-\tau)/\eta}$	-	NA	0	NA	-
TOTAL					$(+)^{**}$	-

(*) Net borrower? (**) *if* $\tau < \tau^*$ (stronger than necessary)

Aggregate Debt and Interest Rates (LVE)

$$\begin{aligned}
 B^{LVE} = & \underbrace{\int_0^{z_1} (1+\beta) \frac{1-\theta_j}{\theta_j - \frac{1}{\eta}} z dF(z)}_{\text{RENTERS } \left(\frac{\partial}{\partial \eta} < 0\right)} + \underbrace{\int_{z_1}^{z_2} (2+\beta) \frac{1-\theta_j}{\theta_j} z dF(z)}_{\text{BUYERS } \left(\frac{\partial}{\partial \eta} = 0\right)} \\
 & + \underbrace{\left(1 - \frac{1-\tau}{\eta}\right) \int_0^{z_1} \left(\eta - \frac{1+\beta}{\theta_j - \frac{1}{\eta}}\right) z dF(z) + \int_{z_2}^1 \left(\frac{\eta}{1-\tau} - (2+\beta)\right) z dF(z)}_{\text{LANDLORDS } \left(\frac{\partial}{\partial \eta} > 0\right)}
 \end{aligned}$$

* Extensive Margins cancel out

* Intensive Margins are important: $\frac{\partial B^{LVE}}{\partial \eta} > 0$, i.e. in the LVE, $\frac{\partial B^{LVE}}{\partial r} < 0$

Aggregate Debt and Interest Rates (HVE)

$$\begin{aligned}
 B^{HVE}(\eta) = & \underbrace{\int_0^{z_1} (1+\beta) \frac{1-\theta_j}{\theta_j - \frac{1}{\eta}} z dF(z)}_{\text{RENTERS } \left(\frac{\partial}{\partial \eta} < 0\right)} + \underbrace{\int_{z_1}^{z_2} (2+\beta) \frac{1-\theta_j}{\theta_j} z dF(z)}_{\text{BUYERS } \left(\frac{\partial}{\partial \eta} = 0\right)} \\
 & + \underbrace{\int_0^{z_1} \left(\eta - \frac{1+\beta}{\theta_j - \frac{1}{\eta}} \right) z dF(z) + \int_{z_2}^1 \left(\frac{\eta}{1-\tau} - \frac{(1+\beta)\theta_j}{\theta_j - \frac{1-\tau}{\eta}} \right) z dF(z)}_{\text{LANDLORDS } \left(\frac{\partial}{\partial \eta} > 0\right)}
 \end{aligned}$$

* $\frac{\partial B^{HVE}}{\partial \eta} > 0$, as before, but now there is no link from r to equilibrium η (the opposite is not true, because everybody is constrained.). Thus, $\frac{\partial B^{LVE}}{\partial r} = 0$.
 [FIGURE]

Rental Market

$$\text{HVE: } \underbrace{\int_0^{z_1} \left(\eta - \frac{1+\beta}{\theta_j - \frac{1}{\eta}} \right) z f(z) dz}_{\frac{\partial}{\partial \eta} > 0, \frac{\partial}{\partial \theta} > 0} = \underbrace{\int_{z_2}^1 \left(\frac{1+\beta}{\theta_j - \frac{1-\tau}{\eta}} - \frac{\eta}{1-\tau} \right) z f(z) dz}_{\frac{\partial}{\partial \eta} < 0, \frac{\partial}{\partial \theta} < 0}$$

LVE: $\int_0^{z_1} \left(\eta - \frac{1+\beta}{\theta_j - \frac{1}{\eta}} \right) z f(z) dz = \text{Supply}$, s.t. for a given r , supply is fully elastic: $\eta = \frac{1+r}{r} (1-\tau)$ up to G^{\max} .

[FIGURE]

Existence and Coexistence (Small Tax Distortion $\tau \searrow 0$)

Proposition 2. Existence. (i) Iff $0 < \theta \leq \theta^*$, there exists a unique LVE. (ii) Iff $\theta^* > \theta \geq \theta_* > 0$ there exists a unique HVE. Coexistence. There exist a non-empty set of θ 's for which both equilibria co-exist, i.e. $\theta^* > \theta_*$.

Proof (a detour)

1. (LVE) If $\theta > \theta^* \rightarrow$ excess capacity constraint is violated $\frac{\partial[G^{\max}]}{\partial\theta} < 0$, $\frac{\partial S}{\partial\theta} > 0$, i.e. $\frac{\partial[G^{\max}-S]}{\partial\eta} < 0$, (given NAC). That is θ^* is such that $[G^{\max}-S]=0$. Uniqueness of r follows, given a negligible measure of constrained buyers ($A(\bar{r})=B(\bar{r})$).

2. a) (HVE) If $\theta < \theta_* \rightarrow B > A \forall r \geq 0$. (notice that $r < 0$ is not possible since $\eta < \frac{1+r}{r}(1-\tau)$). b) If $\theta > \theta^* \rightarrow \nexists \eta$ such that rental and credit markets clear, and $\eta < \frac{1+r}{r}(1-\tau)$ simultaneously.

Proposition 3. Inside the range of coexistence the distance between both equilibria (as measured by the differences in housing prices) is decreasing in θ . (Looser borrowing limits/Easier access to credit widen the gap between equilibria.)

Proof (a detour)

1. (LVE) $\frac{dr}{d\theta} = 0$ (using $\eta = \frac{1+r}{r} (1 - \tau) \rightarrow \frac{d\eta}{d\theta} = 0$). Key: $\frac{\partial A}{\partial \theta} = 0$ (return of renters $\searrow (1+r)$), $\frac{\partial B}{\partial \theta} = 0$ (Renters: $\uparrow \theta \rightarrow \uparrow s \downarrow h$ ($\Delta p(h + s) = 0$) $\rightarrow \downarrow b$, but landlords $\uparrow b$ to meet $\uparrow s$)

2. (HVE) Rental market equilibrium $\rightarrow \frac{d\eta}{d\theta} < 0$ ($\uparrow \theta$ higher demand and lower supply). Then, using the credit market equilibrium condition $\frac{dr}{d\theta} > 0$ ($\uparrow \theta \rightarrow \downarrow b$, $\uparrow \theta \rightarrow \downarrow$ returns (saving), but the excess demand falls $B - A$, given the backward slope supply of funds, hence r goes up) [FIGURE]

Existence and Coexistence (Large Tax Distortions)

1. HVE. Unimportant qualitative changes.

2. LVE. a) Home-Ownership bias (buyers show up) tends to reduce the *slope of the saving* (even change the sign).

b) $\frac{\partial A}{\partial \theta}$? (two effects: (i) const.buyers $\frac{\partial a}{\partial \theta} > 0$, (ii) renters $\frac{\partial a}{\partial \theta} < 0$).

c) $\frac{\partial B}{\partial \theta}$? (two effects: (i) const.buyers $\frac{\partial b}{\partial \theta} < 0$, (ii) renters-landlords: $\frac{\partial b^R}{\partial \theta} < 0$, $\frac{\partial b^L}{\partial \theta} > 0$.(why?: Remember renters the total amount of housing services (self-consumption plus renting) is an increasing function of η . Here an increase of θ reduce the demand of h (lower debt) but increase s (more), the landlords will meet the increase in s increasing b by more than the reduction of debt by renters).

d) Hence $\frac{dr}{d\theta} = \frac{d\eta}{d\theta} = 0$ does not apply. Two cases. [FIGURE]

Proposition 4. (Home-ownership bias.) For given θ , there exist τ^* such that: (i) for $\tau < \tau^*$, then $\theta < \theta_*$ (i.e. only a LVE exists), and (ii) for $\tau > \tau^*$, then $\theta_* \leq \theta < \theta^*$ (i.e. both equilibria co-exist).

(A rise in taxes -stronger home-ownership bias- may give rise to multiple steady state to coexist) [FIGURE].

A Word on Dynamics

- $\{p, q, r\}$ joint behavior, but the link at work may vary (focus on steady-state):

- The “common wisdom” Price-to-Rents (PR) ratio:

price = discounted sum of (net) rental price

In our model, this corresponds to the LVE: $\frac{p}{(1-\tau)q\frac{1+r}{r}}=1$

- But if frictions are present (borrowing limits), one may have

price < discounted sum of (net) rental price

This happens in the HVE: $\frac{p}{(1-\tau)q\frac{1+r}{r}} < 1$

$\{p, q, r\}$: Steady-state vs. dynamics

- Thus, along a speculative path: *initial* $PR(\approx 1) > \textit{final} PR(< 1)$
(see figure)
- But we observe that PR is well above 1 along the boom
- Is the model at odds with this key empirical fact? NO
- Key: capital gains, $p_{t+1} > p_t$, do not matter in steady state, but important along transition.

Along the transition (provided capital gains), while in the vicinity of LVE, the following FOC holds:

$$\frac{p_{t+1}}{1+r_{t+1}} = p_t - (1-\tau)q_t, \rightarrow PR_t \equiv \frac{p_t}{(1-\tau)q_t \frac{1+r_{t+1}}{r_{t+1}}} = \frac{r_{t+1}}{1+r_{t+1} - \frac{p_{t+1}}{p_t}}.$$

Hence if $p_{t+1} > p_t$, then $PR_t > 1$ [FIGURE]

$\{p,q,r\}$: A bottom line

- How important is *initial PR* (≈ 1) $>$ *final PR* (< 1) here?
- Critical*ij*
 - What makes a speculative path possible is the expectation of a future scenario in which buying houses for investment purposes (buy-rent-sell) is more attractive than holding deposits (formally, *final (HVE) PER* (< 1)).

Conclusions and Further Research

- A simple model with a few key ingredients (borrowing limits, housing-supply rigidities and income & access to credit heterogeneity) gives some hints on current episodes of rapid increases in housing prices
- Expectations might drive house price fluctuations as opposed to fluctuations originated by 'exogenous shocks'. Allows for a well-structured analysis of the relative role fundamentals and expectations
- By retaining the Rational Expectations assumption gives a tool suitable for policy analysis.

Related Literature

- *Benchmark literature I: Housing.* Stein 95, *QJE*; and Ortalo-Magné and Rady, 05, *RES*)
 - Partial Equilibrium: No role for interest rates, No treatment of the credit-market and funding mechanisms, Secondary role for the rental market.
- *Benchmark literature II: Macro models with Multiple Equilibria* (MK, 91 *QJE*; BER, 01 *JPE*; Caballero et al., 05 *AER*). [History and Fundamentals vs. Expectations and Funding Mechanism Feedback to sustain investment booms and asset high valuation] But no for the housing markets. [BACK]