

Massachusetts Institute of Technology  
Department of Economics  
Working Paper Series

**HORIZONTAL MERGERS, ENTRY, AND  
EFFICIENCY DEFENCES**

David Spector

Working Paper 01-18  
May 2001

Room E52-251  
50 Memorial Drive  
Cambridge, MA 02142

This paper can be downloaded without charge from the  
Social Science Research Network Paper Collection at  
[http://papers.ssrn.com/paper.taf?abstract\\_id=272181](http://papers.ssrn.com/paper.taf?abstract_id=272181)

# Horizontal mergers, entry, and efficiency defences\*

David Spector (MIT)<sup>†</sup>

May, 2001 (first draft: April, 2001)

## Abstract

This paper addresses the effect of horizontal mergers on prices. It is shown that if firms compete in quantities and marginal costs are nondecreasing, any profitable merger failing to generate technological synergies must harm consumers through higher prices, irrespective of entry conditions in the industry. However this result does not hold if products are differentiated and firms compete in prices. The implications for merger policy are discussed.

## 1 Introduction

This paper provides a theoretical contribution to the analysis of horizontal mergers. The main result establishes that in industries in which firms compete in quantities and there are no scale economies beyond those induced by the presence of fixed costs, any profitable merger failing to generate technological synergies raises prices and harms consumers, irrespective of entry conditions. This is not true, however, if products are differentiated and firms compete in prices: in such cases, even without synergies, a merger may be profitable despite triggering sufficient entry to make consumers better off.

---

\*I am grateful to Peter Davis, Luke Froeb, David Gilo, Massimo Motta, Patrick Rey, Gregory Werden and Robert Willig for their helpful comments. *JEL* classification: D43, K21, L13, L41.

<sup>†</sup>MIT Economics Department, 50 Memorial Drive, Cambridge MA 02142. Email: [spector@mit.edu](mailto:spector@mit.edu)

Since ease of entry and the presence or absence of merger-specific synergies are among the most important criteria used by antitrust authorities when assessing whether an attempted merger should be challenged, an improved understanding of the relationship between these two elements is of paramount importance for policy.<sup>1</sup> The result established in this paper implies that if competition policy aims to protect consumers, then in markets where marginal costs are nondecreasing and the Cournot model is applicable, any merger inducing no technological synergies should be prevented, even if entry appears to be relatively easy.

### *Relation to the literature*

Farrell and Shapiro [1990] address the effect of mergers on prices under the assumption that entry is impossible. They ask how the antitrust authority ought to respond to the claim that a merger will allow firms to reap efficiency gains, and find a very striking result. They note that a merger may yield two kinds of efficiency gains. First, even if a merger does not affect the technologies available to firms, it may induce a cost-saving reorganization of production: for example it may allow firms to shift production from high-cost facilities to low-cost ones. Second, a merger may create *synergies*, through learning for example, and expand the post-merger production set beyond the sum of the pre-merger production sets. Farrell and Shapiro [1990] show (Proposition 2, p. 112) that if firms compete in quantities and there are no scale economies, the first type of efficiency gain is not enough to make a merger beneficial for consumers. Although a merger not accompanied by synergies may raise aggregate welfare by improving productive efficiency, it necessarily causes prices to rise and thus consumers to be harmed. Deneckere and Davidson [1985] prove a similar result in the case of differentiated products and price competition.

The policy consequence seems quite clear: in the absence of entry and of any economies of scale, the antitrust authority should reject all mergers

---

<sup>1</sup>For example, as explained in the Merger Guidelines issued in 1992, the U.S. Department of Justice and the Federal Trade Commission consider three elements when a merger is likely to significantly increase market concentration: "[They] assess whether entry would be timely, likely and sufficient either to deter or to counteract the competitive effects of concern. [Then, they] assess any efficiency gains that reasonably cannot be achieved by the parties through other means. Finally [they] assess whether, but for the merger, either party to the transaction would be likely to fail, causing its assets to exit the market."

which do not generate technological synergies<sup>2</sup>.

But these assumptions are very demanding, to the point that this policy recommendation appears to be of little help. Even though ease of entry varies across industries, some entry is usually possible at least at some cost, and it is important to understand how concerns about entry and merger-specific synergies should be articulated. Also, the industries in which mergers warrant scrutiny are usually highly concentrated, and high concentration is likely to reflect the presence of significant scale economies, often resulting from large fixed costs. This paper addresses these two issues by analyzing a model in which fixed costs and potential entry are present and thus allows us to discuss how efficiency defences and entry conditions should be jointly dealt with.

The paper most closely related to the present one is Werden and Froeb [1998]. Considering both the case of quantity competition and that of price competition under very specific assumptions about costs and demand, they find that in the absence of synergies, a merger followed by entry is necessarily unprofitable for the merging firms, and accordingly that any profitable merger must raise price.

This paper significantly extends their result in the case of quantity competition, proving that it holds under very general conditions. However, in the case of differentiated products and price competition, the example presented in this paper sheds light on the limits of their result, which relies on specific functional forms for demand and costs.

#### *Limits of this paper*

The approach followed in this paper has been subject to two main criticisms.

First of all, as Hay and Werden [1991] write, "the Cournot model has endured a century of criticism". Since the main result of this paper is proved for the case of quantity competition, these criticisms should certainly be addressed. While this paper does not contribute any new ideas to the debate about the relevance of the Cournot model, a large body of theoretical and empirical research suggests that the observation that firms often seem to set prices rather than quantities does not suffice to discard the Cournot model.<sup>3</sup>

---

<sup>2</sup>See Farrell and Shapiro [2001] for a policy-oriented survey on the question of efficiency defences arising in horizontal mergers.

<sup>3</sup>Kreps and Scheinkman [1983] show that the Cournot model can be seen as the reduced form of a game where firms first set capacities and then compete in prices. On the empirical

In any case, it must be applicable at least in *some* sectors, as the Merger Guidelines recognize implicitly by dividing industries between those in which products are differentiated and firms compete primarily in prices (section 2.21), and those in which products are close to each other, and firms compete by setting capacities (section 2.22).

The second possible limitation of our analysis is the focus on noncooperative equilibria, rather than on the risk of collusion. But antitrust authorities' increased emphasis on "unilateral effects", mirroring economists' skepticism as to the pervasiveness of collusion and the possibility of designing workable criteria to address it, may justify this approach.

The paper is organized as follows. First, I present an example showing that merger-induced entry can suffice to make a merger beneficial for consumers even without synergies (section 2.1). However, if attention is restricted to profitable mergers, this is no longer true: after a general model of quantity competition is developed (section 2.2), the main result of the paper, stated in section 2.3, establishes indeed that such mergers necessarily raise price if they do not generate synergies. In Section 3, a specific example is used to show that this result does not carry over to the case of price competition with differentiated products. The concluding remarks in Section 4 relate these results to actual merger policy.

## 2 The case of quantity competition

### 2.1 An example

In a market where firms compete in quantities, can merger-induced entry offset the adverse effects of mergers on consumers' welfare? The example presented hereafter shows that the answer is affirmative: if entry is possible, a merger may cause prices to fall even without synergies or economies of scale.

---

side, the diversity of competition forms across industries notwithstanding, much evidence suggests that the Cournot model is roughly right on average (Froeb and Werden [1991], Schmalensee [1989]).

## The model

There are three firms. Demand is linear, and given by

$$p(X) = A - X. \quad (1)$$

Firm  $i$  has a constant marginal cost of production equal to  $c_i$ , and there exists  $\varepsilon > 0$  sufficiently small and  $c < A$  such that

$$\begin{aligned} c_1 &= c \\ c_2 &= c \\ c_3 &= c - \varepsilon. \end{aligned}$$

Also, firms 1 and 2 can costlessly enter the market, while firm 3 must pay a sunk cost  $F > 0$ . I assume that

$$\left(\frac{A-c}{4}\right)^2 < F < \left(\frac{A-c}{3}\right)^2. \quad (2)$$

It is well known that if demand is given by (1) and  $n$  firms have a constant marginal cost of  $c$ , the Cournot outcome has each firm earning profits equal to  $\left(\frac{A-c}{n+1}\right)^2$ . Therefore, (2) implies that if  $\varepsilon$  is sufficiently small, firm 3 does not find it profitable to enter the market if both firms 1 and 2 are present, while finding entry profitable in the presence of a single incumbent<sup>4</sup>. Since firms 1 and 2 can enter without paying any entry cost, this implies that the only equilibrium is one in which both firms 1 and 2 enter. It can be shown that the price corresponding to this duopoly is

$$p_{1,2} = \frac{A + c_1 + c_2}{3} = \frac{A + 2c}{3}.$$

## Effect of a merger

Consider now the impact of a merger between firms 1 and 2, without synergies. The merged entity has constant marginal cost  $c$ , as did firms 1 and 2 pre-merger. As explained above, (2) ensures that, if  $\varepsilon$  is sufficiently close to zero, firm 3's profit in the equilibrium of a Cournot duopoly involving

---

<sup>4</sup>For Cournot profits and firm 3's entry cost  $F$  to be compared in this way, one must either assume that the oligopolistic interaction takes place only once, or, more realistically, that it is repeated, and that the entry cost is equivalent to a flow of  $F$  per period.

firms 1 and 3 only is greater than  $F$ . Since the merger of firms 1 and 2 yields a firm that is equivalent to firm 1 alone (or firm 2 alone), it necessarily triggers firm 3's entry. The post-merger equilibrium is therefore the duopoly outcome involving firms 1 and 3 (or, equivalently, firms 2 and 3). The corresponding price is

$$p_{1,3} = \frac{A + c_1 + c_3}{3} = \frac{A + 2c}{3} - \frac{\varepsilon}{3}.$$

This price is less than the pre-merger price, proving the result: Farrell and Shapiro's [1990] conclusion that mergers generating no synergies necessarily harm consumers does not hold once the possibility of entry is considered.

However, this example has little relevance, because the merger it considers is not profitable, as it causes the sum of the profits of firms 1 and 2 to fall from  $2\left(\frac{A-c}{3}\right)^2$  pre-merger to  $\left(\frac{A-c-\varepsilon}{3}\right)^2$  post-merger. Since one may expect that only profitable mergers are attempted, analysis can justifiably be restricted to such mergers. The following two subsections study this subject in further detail.

## 2.2 Profitable mergers and entry: the model

I present below a general model in order to analyze the price effects of profitable mergers in markets where firms compete in quantities. Its assumptions are very standard. In particular, they coincide with those made by Farrell and Shapiro [1990], with the exception that I allow for entry.

### Demand

There is only one good, and demand is given by the equation  $p=p(X)$ , where  $X$  is industry output and  $p$  is price. It satisfies the conditions

$$p'(X) < 0 \tag{3}$$

and

$$p'(X) + Xp''(X) < 0. \tag{4}$$

The first inequality simply means that demand is downward-sloping, while the second ensures that each firm's reaction function is downward-sloping.

## Technology

All firms' cost functions are of the form

$$C_i(y) = \begin{cases} F_i + c_i(y) & \text{if } y > 0 \\ 0 & \text{if } y = 0 \end{cases},$$

( $F_i$  and the function  $c_i$  vary by firm), where  $F_i \geq 0$  and the variable cost function  $c_i$  is increasing and (weakly) convex. In order to enter the market, a potential entrant must pay an additional entry cost, which may vary across firms. Exit costs may also be present.

## Comparative statics

The scenario considered in this section is as follows. Prior to the merger, firms compete in quantities. After a merger is announced, each firm decides whether to exit or to enter the market (possibly at a cost). After the merger takes place and non-merging firms' entry and/or exit decisions are implemented, the firms present in the market compete in quantities. The merger is profitable if the merged entity's total profit in this last stage is greater than the sum of all merging firms' pre-merger profits.

In the presence of fixed and sunk costs, this game may have no pure strategy equilibrium, or may have more than one. For the sake of simplicity, I choose not to solve the game explicitly. Rather, I assume that it has at least one pure strategy equilibrium, and that all firms coordinate on a single one.<sup>5</sup>

## 2.3 The central result

The assumption that attempted mergers should be profitable allows us to draw inferences about their price effects, as the following result - the main contribution of this paper - shows.

**Proposition 1** *A profitable merger which does not generate technological synergies causes prices to rise, even if entry is possible and fixed costs generate scale economies.*

---

<sup>5</sup>For a dynamic model where mergers, entry and exit are endogenous, see Gowrisankaran [1999].



The formal proof is in the appendix, but its logic can be explained in a few words. For a merger to be profitable, at least one of the merging firms, say firm 1, must earn a greater profit post-merger than pre-merger. Firm 1's profit post-merger is less than the profit it would earn if it maximized its profit alone (rather than the whole post-merger's firm profit) given all other firms' output levels. But, since firm 1's maximal profit is a decreasing function of the sum of all other firms' outputs, the sum of all other firms' output levels must be lower post-merger than pre-merger. If firm 1 maximized its own profit both pre- and post-merger, a decrease in all other firms' output levels would cause firm 1 to increase its output, but not by an amount sufficient to offset other firms' output fall, and total output would fall. The fact that firm 1 maximizes the merged firm's profit rather than its own only reinforces this conclusion, because taking into account the adverse effect of any price increase on the other merging parties causes firm 1 to produce less than if it cared only about its own profit. Therefore aggregate output falls, and the price rises<sup>6</sup>.

This result implies that if the assumptions made in this section (quantity competition and nondecreasing marginal costs) apply, then one can be certain that a proposed merger generating no synergies will raise prices, without any need to ponder entry conditions. The point is simply that if large-scale entry were very easy, causing the price to fall below its pre-merger level, then the merger would be unprofitable, and hence not undertaken. This is true even if the merger allows the merging firms to avoid wasteful duplication of fixed costs: such savings cannot be sufficient to make a merger profitable if merger-induced entry causes the price to fall. The policy consequences of this result are discussed in more detail below (in Section 4). In order to assess the generality of the result, I examine in the next section the other polar case of oligopolistic interaction, namely the case of price competition in differentiated product markets.

### **3 Price competition: a counter-example**

In many markets, price competition between firms selling differentiated products is considered a more accurate description of reality than quantity com-

---

<sup>6</sup>If a merger allows firms to reallocate capital across plants, the result does not hold in the short-run, but still holds in the long-run if the merger is assumed to raise profits in the long-run (the argument is the same as in Farrell and Shapiro [2000], p. 112).

petition among producers of a homogeneous good. This section examines whether Proposition 1, proved in the case of quantity competition, extends to that of price competition. I show that it does not, by presenting a counterexample: if entry is possible, a merger may benefit both the merging firms and consumers, even without synergies. This stands in contrast with a result found by Werden and Froeb [1998] under restrictive demand assumptions.

The intuition for the possibility that a profitable merger might benefit consumers is straightforward. Because differentiated products involve many different prices, a merger may allow the merging firms to raise their prices while triggering entry by other firms, forcing some non-merging firms' prices down and making consumers better off on average. Given the arbitrariness with which differentiated product markets can be modelled, little can be done beyond illustrating this possibility through an *ad hoc* example, such as the one presented below<sup>7</sup>.

### Demand

- There are four goods, indexed 1 to 4. Firm  $i$  can produce good  $i$  only. The four firms are located on a straight line, and the distance between two consecutive firms is 1.
- A mass 1 of agents is uniformly distributed between any pair of firms.
- All consumers have unit demand.
- Consumers located between firms 1 and 2 (resp. 3 and 4) only derive utility from good 1 or good 2 (resp. 3 or 4). Their valuation from the good they consume is  $v_{12}$  (resp.  $v_{34}$ ).
- Consumers located between firms 2 and 3 can derive utility from any good. Their valuation is  $v_{23}$ .
- All consumers face a transportation cost  $t$  per unit of distance.  $t$  is much smaller than any of the  $v_{ij}$ .

---

<sup>7</sup>It is well-known that a merger of firms producing goods which complement each other may benefit consumers, without any entry. In contrast, this section considers a merger of firms producing substitutes.

## Technology, ownership structure, and the nature of competition

- Each firm can produce costlessly (marginal costs are zero), but firm 3 must decide whether to enter the market and pay an entry cost  $F$ , with

$$t < F < \frac{49}{18}t. \quad (5)$$

- Initially, all firms have separate ownership. We then consider the effect of a merger of firms 1 and 2.
- Firms compete in prices.

## Pre-merger equilibrium

If all four firms produce a positive amount then in equilibrium  $p_1 = p_2 = p_3 = p_4 = t$ , and firm 3's profit is  $t$ , which is not sufficient to cover the entry cost  $F$ .<sup>8</sup>

Therefore in equilibrium, firm 3 chooses not to enter. One can easily show that the equilibrium prices are

$$\begin{cases} p_1 = \frac{5}{3}t \\ p_2 = \frac{7}{3}t \\ p_4 = v_{34} - t. \end{cases} \quad (6)$$

The intuition is as follows: in the absence of firm 3, the best strategy for firm 4 is to set a very high price, serving the "captive" market of consumers located between firms 3 and 4. Therefore firm 4 sets the highest price allowing it to serve this entire set of consumers, i.e.  $v_{34} - t$ . This leaves firms 1 and 2 competing for all remaining consumers (those located to the left of firm 3), leading to prices  $\frac{5}{3}t$  and  $\frac{7}{3}t$ .

## Post-merger equilibrium

I consider first the equilibrium after a merger of firms 1 and 2, assuming firm 3 decided to enter. The situation of the merged entity is analogous to the situation of firm 4 before the merger: it is better off setting a very high

---

<sup>8</sup>This statement and the characterization of equilibria below result from simple calculations, available from the author upon request.

price and selling only to the "captive" consumers (located between firms 1 and 2) than trying to compete with firm 3 in order to attract consumers located between firms 2 and 3. This leaves firms 3 and 4 competing for consumers located to the right of firm 2. By symmetry, the prices set by firms 3 and 4 post-merger are the same as those set by firms 2 and 1 pre-merger:  $p_3 = \frac{7}{3}t$  and  $p_4 = \frac{5}{3}t$ . Firms 1 and 2 set the highest price such that the consumer located at the midpoint between them chooses to buy. Since the transportation cost for this consumer is  $\frac{t}{2}$ , this leads to  $p_1 = p_2 = v_{12} - \frac{t}{2}$ . To recapitulate,

$$\begin{cases} p_1 = v_{12} - \frac{t}{2} \\ p_2 = v_{12} - \frac{t}{2} \\ p_3 = \frac{7}{3}t \\ p_4 = \frac{5}{3}t, \end{cases}$$

yielding firm 3 a profit equal to  $p_3 \left( \frac{3}{2} + \frac{p_4 - p_3}{2t} \right) = \frac{49}{18}t$ , which is greater than the entry cost  $F$ . Therefore, the merger of firms 1 and 2 induces firm 3 to enter.

### Effect on consumers' welfare

The effect of the merger can be summarized as follows: it causes the market to shift from a situation in which consumers located between firms 3 and 4 face a monopoly, to one in which consumers located between firms 1 and 2 face a monopoly. If the monopoly rent is significantly greater in the market located between firms 3 and 4 than for the market located between firms 1 and 2, this shift should benefit consumers. Indeed, one can easily check that aggregate consumers' welfare rises by  $v_{34} - v_{12} - \frac{t}{4}$ , which is positive for  $v_{34} - v_{12}$  sufficiently large.

## 4 Conclusion

The general result proved in this paper in the case of quantity competition has two striking consequences.

First, an efficiency defence relying on the idea that a merger allows firms to exploit scale economies is not convincing if these scale economies result only from the presence of fixed costs. While the elimination of fixed cost duplication may cause a merger to increase aggregate welfare, such a merger

necessarily harms consumers if it does not also generate synergies. This remark is of some importance, both because such scale economies are very frequent in the literature (many models assume fixed costs in addition to constant or increasing marginal costs), and because they may be even more likely in merger cases, as they provide a rationale for merging. The result, however, is not surprising: because fixed costs do not affect the first-order conditions determining equilibrium prices, the price effect of mergers should not depend on their presence or absence.

Second, and perhaps more importantly, in the absence of synergies or economies of scale (other than those induced by fixed costs), a profitable merger necessarily harms consumers through higher prices, irrespective of entry conditions.

Taking this result too literally, one might be tempted to argue that antitrust authorities should not ponder entry conditions and should instead focus on whether there are merger-specific synergies<sup>9</sup>. Stated as such, this view seems quite extreme: should ease and scale of potential entry really be disregarded? This would be at odds with the observation that if potential entrants' supply is infinitely elastic at pre-merger prices, no merger can possibly raise the market price, and thus that all mergers proposed in such an environment should be authorized to move forward.

How can this remark be reconciled with our result? The answer is that in an industry where any price increase would trigger massive entry, a merger is necessarily unprofitable unless it induces synergies. Symmetrically, in the absence of synergies, the mere fact that a merger is attempted implies that entry prospects are limited, at least according to the merging firms' expectations, and that the merger will raise price.

In other words, our result does not imply that entry conditions are of no importance. Rather, it sheds light on how merger policy should articulate the assessment of merger-specific synergies with that of entry conditions. If, with a sufficient degree of certainty, entry is found to be very easy at (or slightly above) current prices, then it is not necessary to challenge a merger. However, in situations where the absence of merger-specific synergies is a near-certain fact, while entry conditions either are unclear, or at least do not

---

<sup>9</sup>This is Werden and Froeb's [1998] conclusion: "the entry issue in merger cases collapses into efficiency considerations, and in the absence of strong evidence [of merger-specific efficiency gains], there is a sound basis for presuming that entry obstacles will prevent entry in response. Thus, the best way for courts to treat entry in many merger cases may be not to consider it at all."

guarantee that a very modest price rise will trigger massive entry, our result implies that a merger should be prevented, without any need for a precise assessment of entry conditions.

To summarize, this paper identifies circumstances in which antitrust authorities do not need to perform a detailed analysis of *both* entry conditions and merger-specific synergies: if entry is very easy, then, as the Merger Guidelines recognize, there is no need to challenge a merger further. But if a merger clearly generates no synergies, it can safely be prevented without delving deeply into the question of entry<sup>10</sup>.

Therefore, the assessment of entry conditions should not be given any priority over the assessment of alleged merger-specific efficiency gains. Only if large-scale entry is likely near pre-merger prices is it legitimate not to challenge a merger. Otherwise, the likelihood and magnitude of merger-specific synergies should be addressed. If these synergies are found to be unlikely or very small, a merger should be prevented even if some entry appears likely. In such a situation, the evidence (or rather the lack of evidence) concerning synergies should take priority over the analysis of entry conditions. In all other cases, the antitrust authorities' decision must reflect a combination of considerations with respect to both entry conditions and synergies.

Several caveats limit the relevance of this paper. The proposition applies to mergers which are profitable even after the entry of new firms. But in industries where entry takes time, it is possible that the profitability of a merger results only from the temporary high profits earned prior to other firms entering, after which prices and profits may fall below their pre-merger levels. When handling such a merger, the antitrust authority must strike a delicate balance between the short-run harm to consumers and the long-run benefits.

As discussed in the introduction, a more serious limitation is the assumption of Cournot competition and homogeneous products<sup>11</sup>, because the

---

<sup>10</sup>The reader may wonder what antitrust authorities should do if these two statements lead to contradictory policy prescriptions. The answer is simply that, in situations where the Cournot model applies, this cannot happen: the point of this paper is that a merger cannot be profitable if entry at near-current prices is easy and there are no synergies. Any such situation would mean either that entry conditions or the presence merger-specific synergies are not assessed properly, or that the nature of the oligopolistic interaction is not Cournot, or that there are economies of scale beyond those induced by fixed costs.

<sup>11</sup>More generally, the assumption of any particular form of competition (Cournot, Bertrand, collusive, or intermediate between these polar cases) is problematic. As merger policy recognized long ago with its focus on the risk that mergers could foster collusion,

Bertrand model is considered more appropriate than the Cournot model for many industries. As the example presented in Section 3 shows, the result does not carry over to the case of differentiated products and price competition, although it is hard to assess whether the possibility it highlights corresponds to a frequent real-world situation. While this paper establishes a general result in the case of quantity competition, which has relevance for merger policy, finding economically meaningful conditions under which a similar result applies to differentiated product markets and price competition should be the goal of future research.<sup>12</sup>

## REFERENCES

**Bresnahan, T., and P. Reiss** [1991], "Entry and Competition in Concentrated Markets", *Journal of Political Economy*, vol. 99 (5), 977-1009.

**Deneckere, R., and C. Davidson** [1985], "Incentives to form coalitions with Bertrand competition", *Rand Journal of Economics*, vol. 16 (4), 473-486.

**Dixit, A.** [1986], "Comparative Statics for Oligopoly", *International Economic Review*, vol. 27, 107-122.

**Farrell, J., and C. Shapiro** [1990], "Horizontal Mergers: An Equilibrium Analysis", *American Economic Review*, vol. 80 (1), 107-126.

**Farrell, J., and C. Shapiro** [2001], "Scale economies and synergies in horizontal merger analysis", *Antitrust Law Journal*, vol. 68 (3), 685-710.

**Froeb, L., and G. Werden** [1991], "Endogeneity in the Concentration-Price Relationship: Causes and Consequences", *U.S. Department of Justice Economic Analysis Group Discussion Paper No. 91-7*.

---

changes in the degree of market concentration may cause the interaction between firms to shift from noncooperative to cooperative (or conversely). In fact, according to Bresnahan and Reiss [1991], the empirical relationship between concentration and the nature of competition is much more complex than a simple Bertrand / collusion dichotomy would suggest.

<sup>12</sup>The literature on endogenous product differentiation may provide a starting point (see chapter 7 of Tirole [1988], pp. 276-295).

**Gowrisankaran, G.** [1999], "A Dynamic Model of Endogenous Horizontal Mergers", *Rand Journal of Economics*, vol. 30 (1), 56-83.

**Hay, G., and G. Werden** [1993], "Horizontal Mergers: Law, Policy, and Economics", *American Economic Review*, vol. 83 (2), 173-177 (Papers and Proceedings of the Hundred and Fifth Annual Meeting of the American Economic Association).

**Kreps, D., and J. Scheinkman** [1983], "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics*, vol. 14 (2), 326-337.

**Schmalensee, R.** [1989], "Inter-Industry Studies of Structure and Performance", ch. 16 in R. Schmalensee and R. Willig, ed., *The Handbook of Industrial Organization*, vol.2, Amsterdam: North Holland.

**Seade, J.** [1980], "The Stability of Cournot Revisited", *Journal of Economic Theory*, vol. 23 (1), 15-27.

**Tirole, J.** [1988], *The Theory of Industrial Organization*, Cambridge: MIT Press.

**United States Merger Guidelines** [1992, revised 1997], available at <http://www.ftc.gov/bc/docs/horizmer.htm>.

**Werden, G., and L. Froeb** [1998], "The Entry-Inducing Effects of Horizontal Mergers," *Journal of Industrial Economics*, vol. 46 (4), 525-543

## APPENDIX

### Proof of Proposition 1

Step 1. Consider the  $r$  merging firms, labeled, without loss of generality, firm 1 to firm  $r$ . For every firm  $i$  (irrespective of whether it is one of the merging firms), let  $q_i$  and  $q'_i$  denote, respectively, firm  $i$ 's output level prior to the merger of firms 1 through  $r$ , and firm  $i$ 's output level subsequent to the merger (if firm  $i$  is one of the merging firms, then  $q'_i$  denotes the output



produced using firm  $i$ 's facilities). Let  $Q$  and  $Q'$  denote aggregate output pre- and post-merger, respectively.

Since the merger is profitable, the merging firms' total post-merger profit,  $\sum_{1 \leq i \leq r} [q'_i p(Q') - C_i(q'_i)]$  is strictly greater than the sum of these firms' pre-merger profits  $\sum_{1 \leq i \leq r} [q_i p(Q) - C_i(q_i)]$ . This implies that for at least one merging firm, say firm 1, profit is greater post-merger than pre-merger:

$$q'_1 p(Q') - C_1(q'_1) > q_1 p(Q) - C_1(q_1). \quad (7)$$

This implies that  $q'_1 > 0$  (firm 1's initial profit is nonnegative because firm 1 always had the option of earning a zero profit by producing nothing). Also, since all active firms produce a strictly positive amount pre-merger,  $q_1 > 0$ .

Step 2. Consider firm 1's reaction and profit functions  $R(q)$  and  $\pi(q)$ , defined as

$$R(q) = \underset{q_1 \geq 0}{\text{ArgMax}} [q_1 p(q + q_1) - C_1(q_1)]$$

and

$$\pi(q) = \underset{q_1 \geq 0}{\text{Max}} [q_1 p(q + q_1) - C_1(q_1)].$$

The downward-sloping demand curve implies that the function  $\pi(\cdot)$  is decreasing, and assumption (4) implies that the function  $R(\cdot)$  is as well. While the fixed cost  $F_1$  can induce discontinuities in  $R(\cdot)$  (production may discontinuously fall to zero in response to a small increase in other firms' output), on the range of values of  $q$  ensuring that  $R(q) > 0$ , the assumptions made with respect to demand and technology ensure that  $R(\cdot)$  is differentiable and that

$$-1 < R'(q) < 0 \quad (8)$$

(this was already noted in Dixit [1986]; see also Seade [1980]). The inequality  $q_1 > 0$  is equivalent to  $R(Q - q_1) > 0$ , so  $R$  is continuous and satisfies (8) on the interval  $[0, Q - q_1]$ .

Step 3. The definition of the function  $\pi(\cdot)$  implies that  $q'_1 p(Q') - c_1(q'_1) \leq \pi(Q' - q'_1)$ . This inequality and (7) together imply

$$\pi(Q' - q'_1) \geq q'_1 p(Q') - c_1(q'_1) > q_1 p(Q) - c_1(q_1) = \pi(Q - q_1).$$

Therefore,

$$Q' - q'_1 < Q - q_1. \quad (9)$$

Step 4. I show now that  $q'_1 \leq R(Q' - q'_1)$ . The reason is simple: when firm 1's response to all other firms' output levels is set to maximize the merged firm's profit, it produces less than it would if it maximized only its own profit, that is, less than  $R(Q' - q'_1)$ . Indeed, if we write  $Q'_{2,r}$  for  $\sum_{2 \leq i \leq r} q'_i$  and  $Q'_{-1}$  for  $Q' - q'_1$ ,  $q'_1$  satisfies the first-order condition

$$\frac{\partial}{\partial q} ((q + Q'_{2,r})p(q + Q'_{-1}) - c_1(q)) = 0,$$

while  $R(Q' - q'_1)$  ( $= R(Q'_{-1})$ ) satisfies the first-order condition

$$\frac{\partial}{\partial q} (qp(q + Q'_{-1}) - c_1(q)) = 0.$$

Defining  $f(q, x)$  by

$$\begin{aligned} f(q, x) &= \frac{\partial}{\partial q} ((q + x)p(q + Q'_{-1}) - c_1(q)) \\ &= p(q + Q'_{-1}) + (q + x)p'(q + Q'_{-1}) - c'_1(q), \end{aligned}$$

the above first-order conditions can be written as

$$f(q'_1, Q'_{2,r}) = 0 \quad (10)$$

and

$$f(R(Q'_{-1}), 0) = 0. \quad (11)$$

If  $x \leq Q'_{-1}$ , then (4) implies that

$$\begin{aligned} \frac{\partial}{\partial q}(f(q, x)) &= 2p'(q + Q'_{-1}) + (q + x)p''(q + Q'_{-1}) - c''_1(q) \\ &< p'(q + Q'_{-1}) + (q + x)p''(q + Q'_{-1}) - c''_1(q) \\ &= \frac{q + x}{q + Q'_{-1}} (p'(q + Q'_{-1}) + (q + Q'_{-1})p''(q + Q'_{-1}) - c''_1(q)) \\ &\quad + \frac{Q'_{-1} - x}{q + Q'_{-1}} (p'(q + Q'_{-1}) - c''_1(q)) \\ &< 0. \end{aligned}$$

Similarly,

$$\frac{\partial}{\partial x}(f(q, x)) = p'(q + Q'_{-1}) < 0.$$

Since both 0 and  $Q'_{2,r}$  are less than  $Q'_1$ , the last two inequalities hold in the interval  $[0, Q'_{2,r}]$ . Together with the first-order conditions (10) and (11), they imply that

$$q'_1 \leq R(Q'_{-1}). \tag{12}$$

Step 5. Inequalities (8), (9), and (12) together imply

$$Q' = Q'_{-1} + q'_1 \leq Q'_{-1} + R(Q'_{-1}) < Q - q_1 + R(Q - q_1) = Q.$$

(The second inequality results from the fact that (8) holds on the interval  $[Q'_{-1}, Q - q_1]$  where the function  $R$  takes strictly positive values.) Therefore, output is lower post-merger than pre-merger, and price is greater. **QED**