Technology Shocks and Job Flows*

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Abstract

We decompose the low-frequency movements in labour productivity into an investment-neutral and investment-specific technology component. We show that neutral technology shocks cause a short run increase in job creation and job destruction and leads to a reduction in aggregate employment. Investment-specific technology shocks reduce job destruction, have mild effects on job creation and are expansionary. We construct a general equilibrium search model with neutral and investment-specific technological progress. We show that the model can replicate these findings if neutral technological progress is mainly embodied into new jobs, while investment-specific technological progress benefits (almost) equally old and new jobs. This provides evidence in favor of models where old jobs can (at least partially) reap the benefits of ongoing technological progress.

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1 Introduction

Technological progress may promote the creation of new technologically advanced jobs by causing the destruction of outdated and relatively unproductive ones.[1] In practice technological adoption does not necessarily require this process of Schumpeterian *creative destruction*, since preexisting jobs may reap the benefits of recent technological advancements by upgrading their previously installed technologies. While in the long run any technological advancement leads to greater labour productivity and output, the short run response of the economy to a technology shock may differ substantially depending on whether technological adoption occurs mainly through *technological upgrading* rather than creative destruction.

Creative destruction implies that technological advancements brings about a simultaneous increase in the destruction of technologically obsolete jobs and in the creation of new highly productive units. But if reallocation is sluggish (due to labour market frictions say), this process prompts a contractionary period during which employment, output and investment fall. When instead technological adoption features the upgrading of old jobs technologies, all jobs become relatively more profitable, so job destruction falls and the economy experiences an expansionary phase characterized by greater employment and output.

To investigate the effects of technological advancements on job reallocation, we decompose the low-frequency movements in labour productivity into an investment-neutral technological component and a component associated with improvements in the quality of new capital equipment.[2] Specifically, the Solow (1960) growth model

[1] This claim is usually associated with Schumpeter (1934). Bresnahan and Raff (1991) and Jovanovic and MacDonald (1994) provide evidence, concerning the automobile industry, that the introduction of new technologies cause an increase in both job destruction and job creation

[2] Greenwood et al. (1997) have documented that both neutral and investment-specific technological change are important in accounting for productivity growth. Indeed, they show that neutral technological progress explains around 40 per cent of the trend behavior of labor productivity, while improvements in the quality of new capital explains the remaining 60 per cent.

implies that the investment-specific technology is the unique driving force of the secular trend in the relative price of equipment goods, while the neutral technology explains any remaining component in the trend of labour productivity. By imposing these long run restrictions in structural VAR models, we find that a positive shock to the neutral technology leads to a short-run increase in job creation and job destruction, and a contraction in aggregate employment, output and equipment investment. On the contrary, shocks to the quality of new equipment reduces job destruction, have mildly positive effects on job creation and are expansionary on employment, output and investment.[3]

We rationalize these findings by considering a stochastic general-equilibrium version of the vintage model by Aghion and Howitt (1994) and Mortensen and Pissarides (1998).[4] The economy is characterized by ongoing neutral and investment-specific technological progress. Due to the existence of search frictions in the labour market jobs of different productivity coexist in equilibrium. Newly created jobs embody the most advanced techniques available at the time of their creation. Preexisting jobs instead may fail to upgrade their previously installed technologies; the idea being that the adoption of new technologies requires the performance of new tasks.[5] Thus workers initially hired to operate a specific technology may not be suitable for its upgrading.

We show that the model responses to either technology shock mimic those observed in the US data when, at the yearly frequency, a fraction as high as 90 per cent of old

[3]These results suggest that technology shocks may be some of the deeper driving forces underlying the shocks identified by Davis and Haltiwanger (1999). In particular, a neutral technology shock would correspond to a positive *re-allocative* shock while advancements in the new capital quality would amount to a positive *aggregate* shock.

[4]See also Jovanovic and Lach (1989) and Caballero and Hammour (1994, 1996) for further examples of vintage models analyzed in the literature.

[5]Gordon (1990) provides examples from different industries where the adoption of new technologies requires the worker to perform a variety of new tasks. See also Brynjolfsson and Hitt (2000) for a review of the empirical evidence documenting the relation between adoption of IT technologies and transformation of organizational structure and work practices.

jobs are able to upgrade their capital equipment, while the corresponding fraction for the neutral technology is no greater than 50 per cent. Thus a neutral technology shock prompts a wave of creative destruction where job creation, job destruction and unemployment simultaneously increase, while the expansionary effects of investment specific technology shock are the result of the old jobs' capability of upgrading their capital equipment.

Our empirical results relate to the recent findings by Galí (1999). Galí assumes that there exists only one kind of technology shock that determines the long-run productivity level and he shows that this technology shock leads to a fall in the total number of hours worked, as well as in aggregate employment and output.[6] These results have cast some doubts about the possibility that technology shocks drive business cycles. We notice instead that, under fairly general balanced growth conditions, neutral and investment-specific technological change independently determine the evolution of labour productivity. Importantly, we find that neutral technology shocks are contractionary while advancements in the quality of new equipment cause an expansion in employment, output and investment –a dynamics typically associated with any expansionary phase of the business cycle.[7]

Our model falls into the labour market search tradition pioneered by Mortensen and Pissarides (1994). Several recent papers have considered labour market search within general equilibrium models with capital accumulation.[8] This paper is however first in analysing the dynamic response of the economy to technology shocks

[6]The Galí's result has been recently further scrutinized by Francis and Ramey (2001) and Altig et al. (2002). While the first paper confirms the original findings, the second argues that they may not be robust to the specification choice for the stochastic process of the variables entering the VAR. We will see in Section 2.6, that our results are robust to the Altig et al.'s favorite specification.

[7]In an independent effort complementary to ours, Fischer (2002) has also documented the expansionary effects of investment-specific technology shocks. We further relate our results to his in Section 2.6.

[8]See Merz (1995) and Andolfatto (1996) for models with exogenous job destruction rates and den-Haan et al. (2000) and Merz (1999) for versions where such a rate depends of the job idiosyncratic productivity.

in the context of a general-equilibrium search model with vintage effects in the job technology.

The rest of the paper is structured as follows. Section 2 discusses our decomposition of labour productivity and documents the response of US job flows to neutral and investment-specific technology shocks. Section 3 describes the model. Section 4 characterizes the equilibrium. Section 5 discusses calibration. The results appear in Section 6 while Section 7 further analyzes the role of the assumptions made. Section 8 concludes. The Appendix contains some technical derivations.[9]

2 Technological progress and labour productivity

We first show how different sources of technological progress affect labour productivity in the long run. This allows to identify the neutral and the investment-specific component underlying the dynamics of productivity. We then use structural VARs to analyze the economy response to advancements in either technology.

2.1 Balanced growth

To see how neutral and investment-specific technological progress determine any long run movement in labor productivity, we consider a version of the Solow (1960) growth model. The rates of saving and technological progress are exogenous, and there are three inputs, equipment, K_e, structures, K_s, and labour, L. The production function is Cobb-Douglas, so that

$$Y = Z\,(K_e)^{\alpha_e}\,(K_s)^{\alpha_s}\,(L)^{1\text{-}\alpha_e\text{-}\alpha_s}\,, \quad 0 < \alpha_e, \alpha_s, \alpha_e{+}\alpha_s < 1, \tag{1}$$

where Y is output and Z is the investment-neutral technology.[10]

[9] A Computational Appendix that thoroughly describes the procedure used to solve the model can be downloaded from the web site at http://www.cemfi.es/~michela/.

[10] The Cobb-Douglas production function is the only one that permits a balanced growth path in the presence of investment-specific technological progress.

Final output can be used for three purposes: consumption C, investment in structures I_s, and investment in equipment I_e, i.e. $Y = C + I_s + I_e$. A constant and equal fraction of output s is invested in equipment and structures, $I_e = I_s = sY$, so that the stock of equipment and structures evolves as

$$K_i' = (1-\delta)K_i + Q_i I_i, \quad \text{for } i = e, s, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate which, again for simplicity, is equal for both types of capital. The variable Q_i formalizes the notion of investment-specific technological change. An increase in Q_i implies a fall in the cost of producing a new unit of capital in terms of final output. Alternatively, it may represent an improvement in *the quality of new capital* produced with a given amount of resources. Then if the sector producing new units of a given capital good i is competitive, the inverse of its price is an exact measure of Q_i. Empirically, the price of structures has remained approximately constant while that of equipment is downward trended, so that it is convenient to think of $Q_s = 1$ while $Q_e = Q$ is upward trended.

One can easily check that the economy converges to a steady-state where the quantities $\tilde{Y}_s \equiv Y_s/(XL)$, $\tilde{K}_e \equiv K_e/(XQL)$ and $\tilde{K}_s \equiv K_s/(XL)$ are all constant and equal to $\tilde{Y}^*$, $\tilde{K}_e^*$ and $\tilde{K}_s^*$, respectively, where

$$X \equiv Z^{\frac{1}{1-\alpha_e-\alpha_s}} \, Q^{\frac{\alpha_e}{1-\alpha_e-\alpha_s}}$$

represents the (possibly stochastic) trend of the economy. After imposing the steady state conditions in (1) and (2) and after some algebra it follows that $\tilde{K}_e^* = \tilde{K}_s^* = (s/\delta)^{\frac{1}{1-\alpha_e-\alpha_s}}$ and $\tilde{Y}^* = (s/\delta)^{\frac{\alpha_e+\alpha_s}{1-\alpha_e-\alpha_s}}$.

Thus, the model predicts that the logged level of aggregate productivity, $y_n \equiv \ln Y/L$, evolves as

$$y_n = \tilde{y}^* + v + x = \frac{\alpha_e + \alpha_s}{1-\alpha_e-\alpha_s}(\ln s - \ln \delta) + v + \frac{1}{1-\alpha_e-\alpha_s} z + \frac{\alpha_e}{1-\alpha_e-\alpha_s} q \tag{3}$$

where a quantity in small letters denotes the log of the corresponding quantity in capital letters while v is a stationary term which accounts for transitional dynamics in the convergence to the steady state. Equation (3) decomposes aggregate productivity as the sum of a stationary term, which represents the steady-state and any transitional dynamics, plus a trend induced by the evolution of the neutral and the investment-specific technology, which independently determine aggregate productivity in the long run.

2.2 Empirical strategy

Given (3), and a measure of y_n and q, one can compute

$$\tilde{z} = (1 - \alpha_e - \alpha_s)y_n - \alpha_e q \tag{4}$$

which differs from the true measure of the neutral technology z only because of a stationary term which accounts for either transitional dynamics or variation in the steady state.[11] Thus changes in either the saving rate or the depreciation rate of capital as well as any shock which moves the economy away from the steady state may cause short run dynamics in $\tilde{z}$. But under our assumption that fluctuations in the saving rate and capital depreciation are stationary, only technology shocks can permanently affect the long run level of $\tilde{z}$ and q.[12] This property of technology shocks derives directly from the existence of a balanced growth path and would arise in any model which shares this feature.

To characterize the response of the economy to technology shocks, we follow, among others, Blanchard and Quah (1989) and Galí (1999) in imposing long run restrictions in a VAR model and we identify neutral and investment-specific technology shocks as the only ones that can permanently affect the level of $\tilde{z}$ and q, respectively.

[11] Given (4), our measure of neutral technological progress $\tilde{z}$ embeds any source of increase in labour productivity which is not embodied in physical capital. This includes improvements in firms organizational structure as well as rises in the level of per capita human capital due to either schooling or experience.

[12] In the empirical analysis, we formally test for the presence of a unit root in $\tilde{z}$ and q by using a Dickey Fuller test with either a linear or a quadratic trend and different lags length. The nul of a unit root is never rejected at one per cent level of significance.

We start considering two VAR models with a vector X_t of variables. In the first VAR, $X_t = (\Delta\tilde{z}_t, jc_t, jd_t)$, while in the second $X_t = (\Delta q_t, jc_t, jd_t)$ where q_t is equal to minus the logged price, in terms of consumption goods, of a new unit of equipment while jc_t and jd_t denote the job creation rate and job destruction rate, respectively. Notice that this specification allows to recover the effects of a shock on net employment growth (the difference between the job creation and the job destruction rate), the logged employment level, and the job reallocation rate (the sum of the job creation and job destruction rate).

Let $\Gamma(L)X_t = \eta_t$ denote a VAR model where $\Gamma(L)$ is a nth-order matrix of polynomials in the lag operator L, with all roots outside the unit circle, and η_t is a vector of zero-mean iid innovations with covariance matrix Σ. Then we can obtain the Wold moving average representation of X_t, $X_t = D(L)\eta_t$, by inverting $\Gamma(L)$. In general η_t is a combination of technology shocks and other (unspecified) shocks so that $\eta_t = S\varepsilon_t$ where by convention the first element of the vector ϵ_t is taken to be the technology shock. Our identifying restriction that the unit root in $\tilde{z}$ and q originates exclusively in technology shocks amounts to the requirement that the first row of the matrix of long run effects $D(1)S$ is made up of zeros apart from the first element. As Galí (1999) shows, this is enough to identify the shock and to analyze the induced impulse response of the variables in the VAR. To check for robustness we consider various specifications and we augment each of the two VARs with additional variables such as aggregate output, total number of per capita hours worked and the consumption-output and investment-output ratios.

2.3 Data

Due to data availability problems for the q and job flows series, the data are annual and the sample goes from 1948 to 1993. Real GDP is taken from FRED, (mnemonic `gdpc1`) which corresponds to seasonally adjusted billions of chained 1996 dollars.

Aggregate labor productivity is computed as the ratio of real GDP with the total number of hours worked (mnemonic `lpmhu` from DRI). The data for the job creation and job destruction rates refer to manufacturing and are from Davis and Haltiwanger (1999). The original series are quarterly and are annualized by taking one-year averages. Finally, the data on q (equal to minus the logged price of a quality-adjusted unit of new equipment) are taken from Cummins and Violante (2002) which extend the Gordon's (1990) measure of the quality of new equipment till 1999. To construct our series for the investment neutral technology, we use (4) and follow Greenwood et al. (1997) in setting $\alpha_e = 0.17$ and $\alpha_s = 0.13$. We then check that our results are robust to this choice.[13]

In Figure 1 we plot the time series for $\tilde{z}$ and q together with the NBER-dated recessions, represented by the shaded areas in the graph. The two graphs on the left-hand side of Figure 1 evidence the well known productivity slowdown in the neutral technology that starts in the mid 70's and the contemporaneous surge in the growth rate of the investment specific technology. Several authors have interpreted these two phenomena as the start of a *technological revolution* induced by the availability of new (capital embodied) IT technologies.[14] One can also notice the existence of a dramatic fall in the value of q in 1975 which Cummins and Violante (2002) attribute to a change in regulation.

The two panels on the right-hand side of Figure 1 display the dynamics of the job creation and job destruction rate. The job creation rate exhibits a fall of around one percentage point on a quarterly basis starting from the beginning of the 70s which arguably is the result of the decreasing importance of the manufacturing sector in

[13] Duménil and Lévy (1991) argue that 0.17-0.20 is a reasonable range for the output elasticity to capital equipment. Thus we checked that our results are not affected by increasing α_e to 0.2. In Section 2.6 we discuss an identification strategy that does not rely on any parametric assumption about α_e and α_s.

[14] See for example Greenwood and Yorukoglu (1997), Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001).

the US economy. Conversely, the job destruction rate has remained stationary and fluctuates around an average rate of 6 per cent per quarter. Interestingly, the growth rates of $\tilde{z}$ and q exhibit no clear cyclical pattern while recessions associates with a surge in job destruction. Table 1 reports some correlations between the variables.

2.4 Specification choice

Given this evidence, we test for the possibility of a structural break in job creation and eventually try to identify when it occurred. By using the supremum test proposed by Andrews (1993) and the average tests proposed by Andrews and Ploberger (1994) we find significant evidence of a structural break for the series and we find that 1974 is the most likely candidate year for the occurrence of the break.

Given these considerations we consider three different specifications: one which uses the raw data without any previous data treatment, one which includes a structural break in the job creation rate starting from 1974 and finally one where we allow for a change in the growth rate of $\tilde{z}$ and q starting from 1975 and a year-dummy in 1975.[15] We refer to this last specification as to the possibility that a *technological revolution* has actually started after 1974.

To ensure that our results are not affected by the Korean War, we consider two different sample periods: the first uses all available years from 1948 to 1993, the second excludes any observation previous to 1954 as in Greenwood et al. (1997). By using the likelihood ratio test proposed by Sims (1980) we find some evidence in favor of the two lags specification against the one-lag specification.[16] In brief we start considering six two-lags VARs for each technology shock.

[15] We also checked that the result of the first two specifications were not affected by the introduction of a year-dummy in 1975 for the growth rate of the $\tilde{z}$ and q series

[16] For example the VAR with the raw data with q_t ($\tilde{z}_t$) yields a value for the χ^2-statistics associated with the null that all second lag coefficients are equal to zero equal to 20.49 (29.11). In any case we investigated the robustness of the results in the VAR with just one-lag.

2.5 Evidence

Figure 2 displays the impulse response (together with the two standard deviations bands) of job creation, job destruction and employment to a neutral technology shock in the specification with raw data.[17] The shock induces a short run increases in $\tilde{z}$, that builds up till reaching a plateau after around 9 years. On impact, the shock makes job destruction and to a less extent job creation increase. As a result employment decreases and job reallocation increases. Over the transition path job creation tends to increase even further while employment recovers. Table 2 shows that this dynamics is quite robust across the different specifications and sample periods. The only exception is the impact effect on job creation: it always increases but in some cases the effect is not statistically significant. Thus we have that:

Finding 1 *On impact, a shock to the neutral technology leads to an increase in job destruction and (generally) in job creation. Thus job reallocation rises while employment falls.*

Figure 3 displays the response of the economy to an investment-specific technology shock, in the specification with raw data. The shock leads to a surge in q that tends to reach a plateau after two years. On impact job destruction falls while the effect on job creation are not statistically significant. As a result job reallocation falls and employment rises. Table 2 shows that the effects of the shock on job destruction, job reallocation and employment are very robust across the various specifications while the impact effect on job creation varies slightly. When we restrict the analysis to the post Korean War period and we allow for either a structural break in job creation or for a technological revolution, the impact effect of the q-shock on job creation is positive and statistically significant. But in all the other specifications the effect is

[17] Standard errors for impulse responses and conditional correlations (see below) are computed by using a Monte Carlo method. Reported standard errors correspond to the standard deviation of each statistics across 500 draws.

not statistically different from zero. Thus we can conclude that

Finding 2 *On impact, a shock to the investment-specific technology causes a contraction in job destruction and a mild effect on job creation. Thus job reallocation falls while employment increases.*

In order to better characterize the impulse response of the economy to a given shock it is useful to compute the correlations of specific variables conditional to the occurrence of the shock.[18] These conditional correlations are reported in Table 3 which shows that the correlation between employment and the neutral technology shock is generally close to zero while the analogous correlation with the q-shock is positive and significant. Perhaps more interestingly, one can also see that:

Finding 3 *Over the adjustment path, job creation and job destruction tend to co-move while job creation and employment are negatively correlated.*

Table 4 reports the percentage of the historical volatility of each variable in the system which is explained by z and q shocks. Notice that the sum of the percentage of the volatility of a given variable explained by the two shocks can even exceed one if the shocks are negatively correlated. Figure 4 displays the dynamics of the realized z and q shocks for some selected specifications and shows that this tends actually to be the case. Depending on the specification, the correlation between the z and q shocks ranges from minus 0.9, when the raw data are used, to minus 0.19, when a technological revolution is allowed. Interestingly, this evidence provides support to our empirical strategy of considering two separate VARs to identify the two technology shocks. Generally the structural VAR methodology rely on orthogonality restrictions

[18] Formally, the correlation between two variables x and y conditional to a given shock i is equal to the sum of the product of the coefficients of the impulse response of the two variables at each lag divided by the product of the conditional standard deviation of the two variables –i.e. the standard deviation that would arise if the only shock present in the system was the given shock i. See Galí (1999) for further details.

to identify multiple shocks in the same VAR. An assumption which, as just shown, can not be taken for granted in this specific case.

Table 4 shows that between 20 and 40 per cent of the volatility of our measure of neutral technological progress, $\tilde{z}$, is explained by z-shocks while q-shocks explain between 30 and 80 per cent of the dynamics of q. This is coherent with our theoretical model since the short run dynamics of $\tilde{z}$ is contaminated by fluctuations in both the saving and the depreciation rate as well as by transitional dynamics in the convergence to the steady state. Interestingly technology shocks explain a substantial proportion of the volatility of employment. The z-shock may account for a proportion of the employment volatility which ranges between 30 and 60 per cent while the corresponding proportion for the q-shock is in the range of 20-35 per cent. Technology shocks can also explain a substantial proportion of the volatility of job destruction and job reallocation while their contribution to explaining job creation is somewhat smaller.

2.6 Some further robustness exercises

We now investigate the robustness of our results (i) to the introduction of additional variables in the VAR and (ii) to alternative strategies to identify neutral and investment-specific technology shocks.

2.6.1 Multivariate VARs

Francis and Ramey (2001) and Altig et al. (2002) have argued that VAR models with a small number of variables may lead to conclusions affected by omitted variables bias. Furthermore one may argue that job flows series refer to the manufacturing sector and may not be representative of the whole economy. To address these concerns, we now augment our three variables VARs with the growth rate of output, the logged consumption-output ratio (mnemonic for consumption is `gcnq+gcsq` from DRI), the logged ratio between nominal output and nominal investment in equipment (the series for investment corresponds to Non Residential Investment in Equipment and Software at Historical-Cost from BEA) and the linearly detrended logged level of the total numbers of hour worked per capita (mnemonic for population is `p16` from DRI).[19]

As shown by Figure 5, the consumption and investment output ratios have substantially increased after 1974. This may be further evidence of the hypothesis that since the mid-seventies the US economy has been undergoing a technological revolution characterized by an acceleration in technological progress specific to equipment which has caused a simultaneous increase in consumption and investment in equipment.[20] To account for this phenomenon we consider two specifications for the consumption and investment output ratios: the former allows for some transitional dynamics in its dynamics to the new steady state by allowing for a linear trend, the latter assumes that after 1974 the two ratios have permanently increased to a new level. Given these considerations we also restrict our analysis to the specification with a technological revolution.

Figures 6-7 show the effect of a neutral technology shock in our seven variables VAR in the specification with a break in the series of the ratio between consumption and investment to output. Figure 6 shows that the results in terms of job destruction, job reallocation and employment are qualitatively the same as those of the three variables VAR. Now, however, job creation slightly falls on impact but it immediately recovers and it is above normal levels just one year after the shock. Interestingly, Figure 7 evidences that a z-shock leads to a contraction not only in employment but also in output, total number of hours worked and investment in equipment. Importantly, labour productivity increases both on impact and in the long run, so that the net effect of the shock on output, after employment has recovered, is positive. Table 5 shows that these results are quite robust across different sample periods and

[19] By using the likelihood ratio we find clear evidence in favor of the one-lag specification against the two-lags specification. So, in this case, we estimate VARs with just one lag.

[20] Hobijn and Jovanovic (2001) also attribute the jump in the consumption-output ratio to the IT revolution.

different specifications.

Figures 8-9 display the impulse response of the economy to a q-shock in the specification with a break in the consumption and investment output ratio. Again Figure 8 shows that the results of the three variables VAR are robust except for the impact effect on job creation which tends to be slightly positive. Interestingly, both Figure 9 and Table 6 show that not only employment but also output, hours worked and equipment investment increase in response to a q-shock. Thus we can safely conclude that:

Finding 4 *In the short run, a neutral technology shock is contractionary while an improvement in the quality of new capital leads to an expansion in economic activity. In the long run, either shock brings about an increase of output and labour productivity.*

2.6.2 An alternative identification strategy

The identification of technology shocks used so far does not impose any assumption on the correlation between z and q shocks but required a choice for the output elasticities with respect to structure, α_s, and equipment, α_e. Alternatively, we could follow Fischer (2002) in assuming that the two shocks are uncorrelated and identify their effects regardless of the value of α_s and α_e. To do so, we consider a four variables VAR including the growth rates of labour productivity, y_n, the growth rate of q_t and the job creation and job destruction rates. We then identify the z-shock as the only shock with zero long-run effects on the level of q_t and non-nil long-run effects on productivity. Conversely, q-shocks are those which could simultaneously affect the long-run level of labour productivity and q_t.

Table 7 shows that the responses of job destruction, job reallocation and employment to either technology shock remain qualitatively unchanged under this alternative identification strategy. For example, the left-hand side of the table shows that any z-shock that leads to a long run increase in labour productivity causes, in the short run, a rise in job creation, job destruction and a fall in employment. Similarly the short-run expansionary effects of a q-shock on job destruction and employment remains broadly unaffected, even if, in some specifications, their statistical significance is reduced. Similarly to Fischer (2002), we also find that q-shocks have sometimes either negative or not significant long-run effects on labour productivity. This however may be the result of the negative correlation between z and q shocks documented in Figure 4.

3 The model

To rationalize the previous findings, we consider an economy in discrete time where there is just one consumption good, which is the numeraire.

3.1 Job output and technologies

A job consists of a firm-worker pair. A worker can be employed in at most one job where he supplies one unit of labour at an effort cost (in utility terms) c_w. A job with *neutral technology* z and capital stock $\tilde{k}$ produces an amount of output equal to $e^z \tilde{k}^\alpha$.

Newly created jobs always embody leading-edge technologies while *old jobs* may be incapable of upgrading their previously installed technologies. The idea is that the adoption of new technologies requires the performance of new tasks. Hence workers initially hired to operate a specific technology may not be suitable for its upgrading.

3.1.1 Job neutral technology

Specifically, a job which starts producing at time t operates with a neutral technology z_{it} equal to the economy *leading technology* z_t of that time, while old jobs are capable of adopting the current leading technology only with probability $a_z \in [0, 1]$.[21] For-

[21] Following Mortensen and Pissarides (1998) and Jovanovic and Nyarko (1996), we could endogenize the adoption probabilities a_z and a_q (see below) by assuming that firms face idiosyncratic time-varying adoption costs which may depend on worker's versatility and the complexity of the

mally, with probability $1 - a_z$ the current period job neutral technology, z_{it}, remains in expected value unchanged, so that

$$z_{it} = z_{it-1} + \epsilon_{it}$$

where ϵ_{it} is an idiosyncratic shock which is iid normal with zero mean and standard deviation σ_ϵ, while, with probability a_z, a job catches up with the leading technology in the economy and

$$z_{it} = z_t + \epsilon_{it}$$

so that the job technology equals (in expected value) the leading technology of that time, z_t. Hereafter, we will refer to the difference between the leading technology z_t and the job neutral technology z_{it} as the job *technological gap*.

3.1.2 Job investment-specific technology

As in Solow (1960) and Greenwood et al. (1997), the sector producing capital is perfectly competitive and at time t can produce one unit of quality adjusted capital at marginal cost e^{-q_t}, which will also be the price of a capital unit at that time. We refer to q_t as the *quality of new capital*. A newly created job installs its desired capital level acquired at the price of the time when it starts production. Conversely, an old job in operation at time t can adapt its capital stock to reap the benefits of the most recent advancements in capital quality only with probability $a_q \in [0, 1]$. In that case, new capital can be installed at marginal cost e^{-q_t}. Otherwise, the job makes use of the capital stock inherited from the previous period. Capital (stochastically) depreciates by a factor $e^{-\delta}$ where δ is iid normal with mean μ_δ and standard deviation σ_δ.[22]

new tasks to be learned. In equilibrium only a fraction of old jobs will switch onto the technological frontier.

[22] The introduction of the idiosyncratic shocks ϵ and δ guarantees that the cross-sectional distribution of job technology and capital has no mass points. In turn, this property ensures a smooth transitional dynamics by ruling out the possibility that persistent oscillations occur over the tran-

A value of a_z and a_q both equal to zero corresponds to the case of a standard vintage model where technological progress is entirely embodied in new jobs, while a_z and a_q equal to one means that technological progress is *new-jobs disembodied*.[23] Generally, the parameters a_z and a_q quantify over the unit interval the extent to which old jobs can upgrade their neutral and investment-specific technology, respectively.

Jobs are destroyed when their technology and/or capital stock become too obsolete relative to the current leading technology and the quality of new capital. In case of destruction, the capital stock of the job is recovered while the worker can be employed in another job.

3.2 Technology frontier

The leading technology, z_t, and the quality of new capital, q_t, grows at an expected rate of μ_z and μ_q, respectively. Specifically, the stochastic process that governs the evolution of z_t is given by

$$z_t = z_{t-1} + g_{zt} \tag{5}$$

where g_{zt} is iid Normal with mean μ_z and standard deviation σ_z, while q_t evolves according to

$$q_t = q_{t-1} + g_{qt} \tag{6}$$

where g_{qt} is iid Normal with mean μ_q, standard deviation σ_q and covariance σ_{qz} with g_{zt}.

sition path –the "echo effects" emphasized by, among others, Benhabib and Rustichini (1991) and Boucekkine et al. (1997).

[23] See Jovanovic and Lach (1989), Aghion and Howitt (1994) and Caballero and Hammour (1994, 1996) for examples of vintage models where technological progress is assumed to be entirely embodied in new jobs. Mortensen and Pissarides (1998) consider instead a model where technology adoption costs determine whether jobs upgrade their technology.

3.3 Search frictions

The labour market for workers is subject to search frictions. The matching process within a period takes place at the same time as production for that period. Workers and firms whose matches are severed can enter their respective matching pools and be re-matched within the same period. All separated workers are assumed to reenter the unemployment pool (i.e. we abstract from workers' labour force participation decisions). Workers and firms that are matched in period t begin active relationships at the start of period $t+1$, while unmatched workers remain in the unemployment pool.

Following Pissarides (2000), we model the flow of viable matches using a matching function $m(u,v)$ whose arguments denote the masses of unemployed workers and vacancies, respectively. This function is homogeneous of degree one, increasing in each of its arguments, concave, continuously differentiable and satisfies $m(u,v) \leq \min(u,v)$. Its homogeneity implies that a vacancy gets filled with probability

$$q(\theta) = \frac{m(u,v)}{v} = m(1, \frac{1}{\theta}),$$

which is decreasing in the degree of labour market *tightness* $\theta \equiv v/u$. Analogously, an unemployed worker finds a job, with probability $p(\theta) \equiv \theta q(\theta)$, which is increasing in θ.

Free entry by firms determines the size of the vacancy pools. Processing the applications for a vacancy requires the services of a *recruiter* which can be hired in a perfectly competitive labour market. In providing these services the recruiter incurs a utility cost r_t per vacancy equal to

$$r_t = \bar{r}\,(v_t)^{\nu}$$

where v_t denotes the total number of posted vacancies and $\nu \geq 0$ captures the possibility that the representative recruiter has decreasing marginal utility in leisure.[24]

[24] This formulation for the job creation costs encompasses others already proposed in the literature.

3.4 Wages

If a firm and a worker who have met separated, both would loose the opportunity of producing and each would have to go through a time-consuming process of search before meeting a new suitable partner. Hence, there is a surplus from a job. We assume that at each point in time the worker and the firm split such surplus by using a generalized Nash bargaining solution in which the bargaining powers of the worker and the firm are β and $1-\beta$, respectively. Division of the surplus is accomplished via wage payments. Nash bargaining also determines the conditions upon which a job is destroyed.

3.5 Representative household

The economy is populated by a continuum of identical infinitely-lived households of measure one. Each household is thought of as a large extended family which contains a continuum of workers and one recruiter. For simplicity, the population of workers in the economy is assumed to be constant and normalized to one. We follow, among others, Merz (1995), Andolfatto (1996) and den Haan et al. (2000) in assuming that workers and recruiters pool their income at the end of the period and choose consumption and effort costs to maximize the sum of the expected utility of the household's members; thus a representative household exists. Specifically, let $\tilde{C}_t$ denote aggregate consumption, then we assume that the representative household maximizes the expected discounted value of its instantaneous utility given by $\ln \tilde{C}_t$ minus the utility costs incurred by workers and recruiters, respectively.[25] The household's discount

The standard search model with linear utility, posited by Mortensen and Pissarides (1994, 1998), arises when ν is equal to zero. When instead ν is strictly positive, job creation costs tend to be increasing in the aggregate number of jobs created at that time, as emphasized by Caballero and Hammour (1994, 1996). We will see below that this helps the model in replicating the small and sluggish response of job creation to shocks observed in the data.

[25] We could allow for a more general utility function, provided that the elasticity of substitution between effort and consumption is maintained equal to one, so as to guarantee the existence of a steady state path with constant employment.

factor is ρ.

We assume that the claims on the profit streams of firms are traded. In equilibrium the household owns a diversified portfolio of all such claims, implying that the discount factor used by firms to discount future profits from time $t+j$ to t is consistent with the household's intertemporal decisions and therefore equal to the expected discounted value of the ratio of the marginal utility of consumption at time $t+j$ to its value at time t.

4 Equilibrium conditions

We characterize the balanced growth path of the economy and derive the equilibrium conditions of the model. In doing so we first characterize firms' decisions in terms of capital choice, job destruction and vacancy creation and then we turn to the determination of market clearing conditions. We conclude by defining the equilibrium for the economy. To guide the reader, Table 8 provides a legend for the main symbols used throughout the analysis.

4.1 Stochastic trend

Our economy fluctuates around the stochastic trend given by $X_t \equiv e^{x_t}$, where

$$x_t = \frac{1}{1-\alpha} z_t + \frac{\alpha}{1-\alpha} q_t$$

is a composite index of the neutral and investment-specific technology. Then, to make the environment stationary, we scale all quantities, unless otherwise specified, by $X_t \equiv e^{x_t}$. Notice that, given (5) and (6), x_t evolves as

$$x_t = x_{t-1} + g_{xt} \tag{7}$$

where g_{xt} is iid Normal with mean $\mu_x = \frac{\mu_z}{1-\alpha} + \frac{\alpha\mu_q}{1-\alpha}$ and variance $\sigma_x^2 = \frac{\sigma_z^2+\alpha^2\sigma_q^2+2\alpha\sigma_{qz}}{(1-\alpha)^2}$.

4.2 Job net surplus

Hereafter we keep the convention that a suffix t added to a given quantity implies that this is a function of the aggregate state variables of that time. Let $k_t \equiv \tilde{k}e^{-(q_t+x_t)}$ denote the (scaled) time-t *capital* value of a job whose unscaled capital stock is $\tilde{k}$. Then the time-t net surplus of a job with capital k and technological gap τ, $S_t(k,\tau)$, solves the following asset equation:

$$
\begin{aligned}
S_t(k,\tau)e^{x_t} &= e^{z_t-\tau}\left(ke^{q_t+x_t}\right)^{\alpha} - ke^{x_t} - c_w\tilde{C}_t - H_te^{x_t} \\
&\quad + E_t\left\{\frac{\rho\tilde{C}_t}{\tilde{C}_{t+1}}\left[H_{t+1}e^{x_t+1} + \int_R e^{-i-q_{t+1}+q_t+x_t}kdG_\delta(i)\right]\right\} + J_t(k,\tau)e^{x_t},
\end{aligned}
$$

where $G_\varkappa$ denotes the distribution function of the random variable $\varkappa$ while H_t denotes the value to the worker of staying at home at time t. To understand the expression, notice that the terms in the first row of the right-hand side computes the instantaneous net surplus of the job as the difference between job output and three terms which account for the capital value, the effort cost of working (measured in consumption units by dividing c_w by the marginal utility of consumption) and the worker's outside option, respectively. The terms in the second row represent instead the discounted future job value which is equal to the sum of the future outside options of the firm and the worker and $J_t(k,\tau)$ which denotes the expected present value of the job future net surplus.

After dividing the left and right hand side of the previous equation by e^{x_t}, it follows that

$$
S_t(k,\tau) = e^{-\tau}k^\alpha - k - c_wC_t - H_t + E_t\left\{\frac{\rho C_t}{C_{t+1}}\left[H_{t+1} + \int_R \Delta_{t+1}(i)kdG_\delta(i)\right]\right\} + J_t(k,\tau) \tag{8}
$$

where $C_t \equiv \frac{\tilde{C}_t}{X_t}$ represents aggregate *consumption* while

$$
\Delta_{t+1}(\delta) = e^{-\delta-g_{qt+1}-g_{xt+1}}
$$

is the (actual) *depreciation factor* of capital between time t and time $t+1$.

4.3 Optimal capital choice

Given (8), the net surplus of a job with technological gap τ that can upgrade its capital level is

$$S_t(\tau) = \max_k S_t(k, \tau), \tag{9}$$

while its optimal capital choice $k_t^*(\tau)$ solves

$$e^{-\tau}\alpha\,[k_t^*(\tau)]^{\alpha-1} + \frac{\partial J_t(k_t^*(\tau), \tau)}{\partial k} = 1 - E_t\left[\frac{\rho C_t}{C_{t+1}}\int_R \Delta_{t+1}(i)dG_\delta(i)\right], \tag{10}$$

which says that the optimal capital level must be such that its current marginal productivity plus its future marginal value is equal to the user cost of capital.

4.4 Job destruction

A job is kept in operation only if it yields a positive net surplus. Thus, it exists a critical technological gap $\bar{\tau}_t$ which solves

$$S_t(\bar{\tau}_t) = 0, \tag{11}$$

such that a job which can deploy its optimal capital level remains in operation only if its technological gap is smaller than $\bar{\tau}_t$. Similarly a job with a given capital k, whose level cannot be upgraded, is destroyed whenever its technological gap is greater than the threshold $\tau_t^*(k)$ that solves

$$S_t(k, \tau_t^*(k)) = 0. \tag{12}$$

Given (9) and the fact that $S_t(k, \tau)$ is decreasing in τ, these expressions immediately imply that jobs whose capital can be upgraded remain in operation for greater technological gaps, $\tau_t^*(k) \leq \bar{\tau}_t$, $\forall k$.

4.5 Future job net surplus

The present value of the future job net surplus $J_t(k, \tau)$ solves the asset equation

$$J_t(k,\tau) = E_t\left\{\frac{\rho C_t}{C_{t+1}}(1-a_q)a_z\int_{R^2}\max\left(0, S_{t+1}(\Delta_{t+1}(i)k, j)\right)dG_\delta(i)dG_\epsilon(j)\right\}$$
$$+E_t\left\{\frac{\rho C_t}{C_{t+1}}(1-a_q)(1-a_z)\int_{R^2}\max\left(0, S_{t+1}(\Delta_{t+1}(i)k, \tau+g_{zt+1}+j)\right)dG_\delta(i)dG_\epsilon(j)\right\}$$
$$+E_t\left\{\frac{\rho C_t}{C_{t+1}}a_q\int_R\left[a_z\max\left(0, S_{t+1}(j)\right)+(1-a_z)\max\left(0, S_{t+1}(\tau+g_{zt+1}+j)\right)\right]dG_\epsilon(j)\right\} \tag{13}$$

where, in writing the expression, we made use of the fact that the density function of the idiosyncratic shock ϵ is symmetric around zero. To understand the expression, notice that jobs are destroyed whenever they would yield a negative surplus. Thus the first term in the right-hand-side accounts for the net surplus generated by a job which tomorrow (in expected value) will use today depreciated capital and the next period leading technology, the second for the net surplus of a job which will use the today depreciated capital and the today technology while the third (which is independent of the current value of k) accounts for the net surplus generated by a job which will update its capital level and will use either the leading technology of next period or the same technology as the current one.

4.6 Free entry

Once bargaining with a firm, the worker always receives his outside option (the value of staying at home) plus a fraction β of the job net surplus. As newly created jobs install their optimal capital level and operate the leading technology of the time when they start producing, $\tau = 0$, the worker's value of staying at home, H_t, solves the asset type equation

$$H_t = E_t\left\{\rho\frac{C_t}{C_{t+1}}\left[H_{t+1}+p(\theta_t)\beta S_{t+1}(0)\right]\right\} \tag{14}$$

where H_{t+1} is next period worker's outside option, while $\beta S_{t+1}(0)$ is the amount of net surplus appropriated by the worker in case he finds a job, which occurs with probability $p(\theta_t)$.

Analogously, a firm which bargains with a worker receives his outside option (the value of a vacancy) plus a fraction $1-\beta$ of the job net surplus. Thus the value of a vacancy at time t, V_t, satisfies the asset equation

$$V_t = -r_t C_t + E_t \left\{ \rho \frac{C_t}{C_{t+1}} \left[V_{t+1} + q(\theta_t)\left(1-\beta\right) S_{t+1}(0) \right] \right\},$$

where $r_t C_t$ accounts for today cost of hiring the recruiter to process the applications for the vacancy while the last term in the right-hand side represents the expected future present value of searching for a worker today. Since vacancies are posted till the exhaustion of any rents, in equilibrium $V_t = V_{t+1} = 0$, so that the *free entry condition*

$$\frac{r_t C_t}{q(\theta_t)} = E_t \left\{ \rho \frac{C_t}{C_{t+1}} \left(1-\beta\right) S_{t+1}(0) \right\} \tag{15}$$

holds at any point in time.

4.7 Employment and job creation

Let $f_t(k,\tau)$ denote the time t measure of old jobs which inherits a depreciated capital level k from the previous period and that, in case they are kept in operation, would produce with technological gap τ. In other words, f_t describes the *beginning-of-period distribution* of old jobs previous to any investment and destruction decision at time t. It then follows from the definition of the two critical technological gaps $\bar{\tau}_t$ and $\tau_t^*(k)$, that time t employment is equal to

$$N_t = a_q \int_R \left[\int_{-\infty}^{\bar{\tau}_t} f_t(k,\tau) d\tau \right] dk + (1-a_q) \int_R \left[\int_{-\infty}^{\tau_t^*(k)} f_t(k,\tau) d\tau \right] dk + m_{t-1} \tag{16}$$

since any job which can (not) upgrade its capital stock k is kept in operation only if its technological gap is no greater than $\bar{\tau}_t$ $(\tau_t^*(k))$ while all newly created jobs, m_{t-1}, are productive. Notice that, given aggregate employment and the degree of labour market tightness, m_{t-1} can be expressed as

$$m_{t-1} = p(\theta_{t-1})(1 - N_{t-1}). \tag{17}$$

4.8 Aggregate budget constraint

The time-t (scaled) value of aggregate *output* is equal to

$$\begin{aligned} Y_t = & \int_R \left[a_q \int_{-\infty}^{\bar{\tau}_t} e^{-\tau} \left[k_t^*(\tau)\right]^\alpha f_t(k,\tau) d\tau + (1-a_q) \int_{-\infty}^{\tau_t^*(k)} e^{-\tau} k^\alpha f_t(k,\tau) d\tau \right] dk \\ & + m_{t-1} \left[k_t^*(0)\right]^\alpha , \end{aligned} \tag{18}$$

where the first integral accounts for the output produced by the old jobs which adjust their capital stock, the second for those which produce with the capital level they inherit from the previous period while the term in the second row is the output produced by new jobs. Then the aggregate budget constraint can be conveniently expressed as

$$Y_t = C_t + I_t \tag{19}$$

where I_t denotes aggregate *investment* expenditures. By definition, I_t is equal to

$$I_t \equiv I_t^u - D_t^d$$

where I_t^u denotes the investment expenditures of those firms which are kept in operation and upgrade the capital stock, while D_t^d is the value of the disinvestment triggered by job destruction.

More formally, it follows from the definition of f_t that the component of investment due to capital upgrading is given by

$$I_t^u = a_q \int_{R \times [-\infty, \bar{\tau}_t]} \left[k_t^*(\tau) - k\right] f_t(k,\tau) dk d\tau + m_{t-1} k_t^*(0)$$

since, provided they are not destroyed, a fraction a_q of old jobs upgrade their capital while all new jobs acquire a capital level $k_t^*(0)$. By similar logic, the disinvestment due to job destruction is equal to

$$D_t^d = \int_R \left[a_q \int_{\bar{\tau}_t}^{\infty} k f_t(k,\tau) d\tau + (1-a_q) \int_{\tau_t^*(k)}^{\infty} k f_t(k,\tau) d\tau \right] dk$$

since jobs are destroyed whenever the technological gap is too large relative to the capital stock which could be used in case of production.

4.9 Dynamics of the beginning-of-period distribution

Figure 10 describes the sequence of events that characterize the evolution of the beginning-of-period distribution between time $t-1$ and t. At time $t-1$, and depending on whether capital can be upgraded, some old jobs are destroyed while some others that remain in operation upgrade their capital level. The result of these decisions is the "end-of-period" distribution of old jobs at time $t-1$ which determines employment and aggregate output at that time.[26] Then to obtain the beginning-of-period distribution of old jobs at time t one has to take account (i) of the (aggregate and idiosyncratic) shocks to the job neutral technology that determine the job technological gap, (ii) of the shocks to capital depreciation that affect the value of job capital at the beginning of time t and, finally, (iii) of the inflow of newly created jobs at time $t-1$, m_{t-2}, that will belong to the pool of old jobs at time t.

Thus, the law of motion of f_t can be described through an operator $\mathbf{\Phi}$, that maps f_{t-1}, the capital-adjustment and job destruction decisions, the aggregate shocks and m_{t-2} into f_t so that

$$f_t = \mathbf{\Phi}(f_{t-1}, k_{t-1}^*, \tau_{t-1}^*, \bar{\tau}_{t-1}, g_{zt}, g_{qt}, m_{t-2}), \tag{20}$$

where f_{t-1}, k_{t-1}^* and τ_{t-1}^* are functions while the remaining quantities are scalars. The exact form of the relation between these quantities is described by (23) in the Appendix.

[26] Notice that, differently from f_t, this "end-of-period" distribution exhibits mass points at the optimal capital level associated with a given technological gap.

4.10 Equilibrium

An equilibrium consists of a stationary tuple

$$\left(k_t^*(\tau), \bar{\tau}_t, \tau_t^*(k), \theta_t, N_t, m_t, C_t, f_t(k,\tau)\right)$$

which satisfies the condition for the optimal capital choice (10), the two job destruction conditions (11) and (12), the free entry condition for vacancy creation (15), the constraint on the number of employees (16), the constraint on job creation (17), the aggregate budget constraint (19) and the law of motion of the beginning-of-period distribution (20).

We solve the model by log-linearizing the first order conditions around the steady state of the model without aggregate shocks, $g_{zt} = \mu_z$ and $g_{qt} = \mu_q$. This yields a system of linear stochastic difference equation that can be solved, for example, with the method proposed by Sims (2002).[27] To characterize the beginning-of-period distribution, f_t, we follow Campbell (1998) and Merz (1999) in considering its values at a fixed grid of technological gaps and capital levels. A Computational Appendix describes in more details the procedure used.[28]

5 Calibration

We start defining some useful statistics. Job destruction at time t is equal to

$$JD_t = \int_R \left[a_q \int_{\bar{\tau}_t}^{\infty} f_t(k,\tau) d\tau + (1-a_q) \int_{\tau_t^*(k)}^{\infty} f_t(k,\tau) d\tau \right] dk \tag{21}$$

since jobs are destroyed whenever their technological gap is too large relative to the capital that can be installed. Given this definition, the law of motion of employment satisfies

$$N_t = N_{t-1} + JC_t - JD_t \tag{22}$$

[27] Following Blanchard and Kahn (1980), several methods are now available to solve systems of linear stochastic difference equations. See, among others, Anderson and Moore (1985) and Christiano (2002).

[28] The Appendix is available on the web site at http://www.cemfi.es/~michela.

where $JC_t = m_{t-1}$ denotes *job creation* at time t.[29] Finally, and given Davis and Haltiwanger (1992), we define the time-t job destruction and job creation rate as equal to

$$jd_t = \frac{2JD_t}{N_{t-1} + N_t} \text{ and } jc_t = \frac{2m_{t-1}}{N_{t-1} + N_t},$$

respectively.

We follow den Haan et al. (2000) in positing the following matching function:

$$m(u_t, v_t) = \frac{u_t v_t}{[(u_t)^\eta + (v_t)^\eta]^{\frac{1}{\eta}}},$$

where u_t and v_t denote the pool of searching workers and firms, respectively.[30]

The parameters' value used in our baseline specification are summarized in Table 9. The choice for ρ and β is standard at the quarterly level. The absence of capital structures in the model implies that matching the long run response of labour productivity to a unitary z and q shock would require setting a different value of α, which, given the values of α_s and α_e used in the empirical part, would be equal to 0.3 and 0.19, respectively. Hence we compromise by setting α to 0.24. The parameter μ_q is set to yield a yearly growth rate of 4.5 per cent in the quality of new equipment which is the average in our data over the 1948-1993 period. Given the choices for α and μ_q, the average growth rate of the leading technology, μ_z, is chosen, by using (7), to yield a yearly growth rate of 2 per cent in labour productivity, which approximates its US historical trend.

The five parameters η, c_w, σ_ϵ, μ_δ and σ_δ are set to match, in steady state, the moments reported in Table 9. First, the average depreciation rate, $\mu_\delta - \sigma_\delta^2/2$, is 12.4 per cent on a yearly basis, as in Greenwood et al. (1997). Secondly, we follow den Haan et al. (2000) in requiring that $p(\theta)$ and $q(\theta)$ are equal to 0.45 and 0.71,

[29] We define time-t employment as given by all workers producing at that time. Alternatively, one could also include in the pool of employed workers those who find a job at time t and will start producing at time $t+1$. In this case the definition of job creation and job detsruction should be modified accordingly so as to satisfy a law of motion for employment analogous to (22).

[30] Den Haan et al. (2000) provide some microfoundations for this matching technology.

respectively. Thirdly, the job destruction rate is set equal to 6 per cent which is the quarterly average in our 1948-1993 sample.[31]

Our fifth moment condition comes from Cooper and Haltiwanger (2002) that study the pattern of capital adjustment at the plant level. They document (see their Table 1) that the fraction of plants adjusting capital with positive investment is approximately equal to 89 per cent. Since in our model each 'plant' consists of a single worker, we match this statistics by considering as 'plants adjusting capital' both those jobs that upgrade their capital level and those that are destroyed.

Finally, we require that in steady state 30 per cent of the existing jobs are more productive than a newly created one. This is in line with the findings by Baily et al. (1992) and Bartelsman and Dhrymes (1998). They show that the mode of the distribution of newly created plants corresponds to the top quintile of the distribution of productivity of existing plants.[32] This would suggest that at most twenty per cent of existing plants are more productive than newly created ones. But, due to the substantial dispersion in the productivity of newly created plants, their average productivity corresponds to the third rather than the first quintile of the distribution of productivity of existing plants.[33] This would imply that on average 40 per cent of existing plants are more productive than newly created ones. Our choice comes as a compromise between these two extremes.

The value for the jobs' probability of capital upgrading, $a_q = 0.45$, yields a 91

[31] These numbers imply a steady state unemployment rate of 11.7 per cent, which is reasonably close to the level of 11 per cent which is obtained by using the data reported by Blanchard and Diamond (1990) once one includes in the pool of unemployed also those people formally classified as out of the labour force but who declare that "want a job". Specifically they report that over the period 1968-1986 the average number of employed, unemployed and out-of-the labour force people who want a job is equal to 93.2, 6.5 and 4.7 millions, respectively.

[32] See Table 4 in Baily et al. (1992) and Table A3 in Bartelsman and Dhrymes (1998). Both Baily et al. (1992) and Bartelsman and Dhrymes (1998) analyse total factor rather than labour productivity. Since they do not adjust capital for differences in quality, we associate their numbers to a measure of labour productivity.

[33] This number is obtained by using information contained in Table 4 in Baily et al. (1992) and Table A3 in Bartelsman and Dhrymes (1998).

per cent probability of capital adjustment at the yearly frequency, which corresponds to the fraction of plants adjusting capital equipment as reported by Cooper and Haltiwanger (2002) (see their Table one). Given this parameters' choice, we observe that a value of a_z strictly greater than 15 per cent generates a positive response of employment to a z-shock. Thus we set a_z equal to 10 per cent in our baseline specification, but any smaller value tends to yield similar dynamics and may be equally reasonable. Finally, we set ν equal to one and we discuss below the effect of such a choice.

6 Results

We next analyze the response of the economy to a neutral and investment specific technology shock in our baseline specification.

6.1 A neutral technology shock

Figure 11 and Figure 12 plot the response of the economy to a one-per-cent increase in the leading technology, z_t. As z_t increases, old jobs tend to become more obsolete relative to the technological frontier so the marginal distribution of the beginning-of-period technological gaps, $\int f_t(k,\tau)dk$, shifts to the right on impact –see the dotted line in the first panel of Figure 11. The initial rise in job destruction is then the result of two opposite forces affecting the jobs net surplus. On the one hand, old jobs which fail to upgrade their neutral technology become technologically more obsolete and therefore less profitable. On the other, the slow adoption of the new technology requires time and investment in capital. Thus consumption, C_t, falls below its long-run steady state value which reduces the value of the effort cost of working, $c_w C_t$, and thereby the incentive to destroy jobs. To see why the first effect dominates and job destruction increases, notice that the initial horizontal shift of the marginal distribution of τ is inversely related to the old-jobs probability of upgrading their neutral technology, a_z, since only jobs which fail to upgrade their neutral technology tend to experience an increase in their technological gap. Thus, when a_z is low enough, a sufficiently large number of jobs becomes technologically obsolete and the economy experiences a surge in job destruction.

The initial cleansing of technologically outdated jobs prompts a reduction in employment as well as in (unscaled) output and investment. Conversely, the impact effect on (unscaled) labour productivity is positive due to both the destruction of relatively unproductive jobs and the productivity gains of those which successfully upgrade their technology.

In the quarter immediately after the shock, the job creation rate, jc_t, rises sharply both because new jobs are now more profitable and because the pool of searching workers has increased. Thus, the initial upsurge in unemployment is gradually absorbed and, as unemployed workers starts operating the more advanced technology, output, investment and labour productivity reach their permanently increased new steady-state value. Interestingly, employment in the first year of the shock falls, on average, by approximately a half per cent and it takes around four quarters to go back to normal levels. As emphasized by den Haan et al. (2000), the dynamics of the endogenous job destruction rate (at least partly) explains these persistent effects. Indeed, since consumption slowly reaches its steady state value, the interest rate is above normal levels over the whole transition path, thereby causing a persistently greater job destruction rate which slows the recovery down.

6.2 An investment-specific technology shock

Figure 11 and Figure 13 plot the response of the economy to a one-per-cent improvement in the quality of new capital, q_t. As q_t rises, the value of the previously installed capital gets reduced. Thus the marginal distribution of the beginning-of-period capital values, $\int f_t(k, \tau) d\tau$, shifts to the left on impact –see the dotted line in the second panel of Figure 11. The initial fall in job destruction is again the equilibrium outcome of two opposite forces. On the one hand, old jobs which fail to upgrade their capital equipment tends to operate with capital of relatively worse quality. On the other, the costly (in terms of time and resources) adoption of the new capital quality prompts a fall in consumption, C_t, which reduces the value of the effort cost of working, $c_w C_t$. When the old jobs probability of upgrading their capital is high enough, this last effect dominates and job destruction falls. As a result, employment rises by approximately one per cent during the first year after the shock while (unscaled) output and investment overshoot their permanently increased new steady state value. The impact effect on labour productivity is instead quite small –and actually negative in our specification–, since the fall in job destruction implies that relatively unproductive jobs remain in operation.

In the quarters following the shock, the job creation rate, jc_t, falls due to the reduction in the pool of searching workers. Thus, the initial increase in employment, output and investment is gradually absorbed and, after around two years, employment has returned to its pre-shock level while output and investment have reached their new steady state value.

7 Further discussion

We next discuss the response to technology shocks that would arise under some alternative specifications.

7.1 Embodied and disembodied technological progress

In standard vintage models, technological progress is assumed to be entirely embodied into new jobs, so that no technological upgrading is contemplated, $a_z = a_q = 0$.[34]

[34] See for example, Jovanovic and Lach (1989), Aghion and Howitt (1994) and Caballero and Hammour (1994, 1996).

In this case, both neutral and investment-specific technology shocks prompt a wave of creative destruction where job destruction, job creation and unemployment simultaneously rise while workers are reallocated from outdated, technologically obsolete units to new more productive ones.

Conversely, when technological progress benefits equally old and new jobs, $a_z = a_q = 1$, a technology shock (either neutral or investment-specific) makes the net surplus of any job increase, since consumption and, consequently, the effort cost of working drop below their steady state value. But then, job destruction falls and the economy experiences an expansionary phase with greater employment, output and investment.

7.2 Risk neutrality

In the original vintage model by Aghion and Howitt (1994) and Mortensen and Pissarides (1998) agents are risk neutral and all quantities are (exogenously) scaled by the economy's technology level, X_t.[35] Under this assumption the marginal utility of consumption and, thus, the interest rate are unaffected by shocks. To study the response of the economy that would arise in this set-up, we impose, that, in our baseline specification, C_t remains constant and equal to its steady state value during the whole transition path. When so, any technology shock necessarily leads to a wave of creation destruction where unemployment rises. Indeed, old jobs tend to become technologically more obsolete as in our baseline model, but now, since the marginal utility of consumption fails to increase, there is no compensating effect on the job net surplus that arises from the fall in the value of the effort cost of working. Thus old jobs become on average less profitable and job destruction increases.

[35] See also Hornstein et al. (2002) for an application of the model to the study of wage inequality.

7.3 Job creation costs

In our baseline specification, we assumed that recruitment costs increase when more vacancies are posted, $\nu > 0$. This assumption does not affect the impact response of the economy but it renders the effects of technology shocks more persistent by smoothing job creation. For example, a strictly positive ν implies that, when unemployment rises, vacancies can not increase too abruptly, so the increase in job creation spreads over time and the recovery slows down. Quantitatively, setting ν equal to one increases the persistence of shocks between one and two quarters relative to the case where ν is equal to zero.

8 Conclusions

We relied on the Solow (1960) vintage model to decompose the low frequency movements in labour productivity into a neutral and an investment-specific technology component. By using long-run restrictions in structural VAR models we found that any advancement in the neutral technology leads to an increase in job destruction, job creation and unemployment while output and investment fall before they permanently increase. Conversely improvements in the quality of new capital equipment are expansionary on employment, output and investment.

We have shown that these findings are coherent with the idea that neutral technology shocks prompt a wave of creative destruction where outdated, technologically obsolete units are pruned out of the productive system, while investment-specific technology shocks are expansionary since a substantial proportion of old jobs upgrade their capital equipment and thereby reap the benefits of the most recent advancements in the quality of new equipment. Specifically we calculate that, on a yearly basis, 90 per cent of old jobs upgrade their capital equipment while the corresponding fraction for the neutral technology is no greater than 50 per cent.

In practice several reasons may explain why worker reallocation increases only in response to advancements in the neutral technology. For example, if *any* worker can learn how to operate a more powerful computer or more efficient means of telecommunication and robotization of assembly lines –which are sources of investment-specific technological change–, firms can upgrade their capital equipment without displacing their employees. Conversely, if only *some* specific workers can get accustomed to the new routines and discipline associated with a given change in firm's organization, the adoption of this neutral technology requires (at least partly) replacing the current employees with new, more suitable ones. Furthermore, our measure of the neutral technology embeds any form of human capital. Thus, an improvement in the quality of the *new* labour force is a neutral technology shock which, by definition, has to be embodied into new jobs and, thereby, causes creative destruction.

Appendix

The operator $\mathbf{\Phi}$ Consider Figure 10 that describes the sequence of events that characterize the evolution of f_t between time $t-1$ and t. Then after taking into account all these events one can see that the operator $\mathbf{\Phi}$ in (20) is implicitly defined by the following equation that relates $f_t(k,\tau)$ to f_{t-1} and the jobs destruction and investment decisions at time $t-1$:

$$\begin{aligned}
f_t(k,\tau) &= (1-a_q)a_z \int_R \left[\int_{-\infty}^{\tau^*_{t-1}(j)} g_\delta(\ln j - \ln k - g_{qt} - g_{xt}) g_\epsilon(\tau) f_{t-1}(j,i)di \right] dj \\
&+ (1-a_q)(1-a_z) \int_R \left[\int_{-\infty}^{\tau^*_{t-1}(j)} g_\delta(\ln j - \ln k - g_{qt} - g_{xt}) g_\epsilon(i + g_{zt} - \tau) f_{t-1}(j,i)di \right] dj \\
&+ a_q a_z \int_R \left[\int_{-\infty}^{\bar{\tau}_{t-1}} g_\delta(\ln k^*_{t-1}(i) - \ln k - g_{qt} - g_{xt}) g_\epsilon(\tau) f_{t-1}(j,i)di \right] dj \\
&+ a_q (1-a_z) \int_R \left[\int_{-\infty}^{\bar{\tau}_{t-1}} g_\delta(\ln k^*_{t-1}(i) - \ln k - g_{qt} - g_{xt}) g_\epsilon(i + g_{zt} - \tau) f_{t-1}(j,i)di \right] dj \\
&+ g_\delta(\ln k^*_{t-1}(0) - \ln k - g_{qt} - g_{xt}) \left[a_z g_\epsilon(\tau) + (1-a_z) g_\epsilon(g_{zt} - \tau) \right] m_{t-2},
\end{aligned} \tag{23}$$

where in writing the expression we made use of the fact that the distribution of ϵ is symmetric around zero. To get familiarized with the expression focus on the first term in the right-hand side which deals with old jobs that fail to upgrade their capital level at time $t-1$ and that, at time t, catch up with the leading technology, which occurs with probability $1-a_q$ and a_z, respectively. These units are kept in operation at time $t-1$ provided that the technological gap is not too high relative to the capital that they inherit from time $t-2$. Then consider a job, which, at time $t-1$, produces with a capital stock j and technological gap i. Then, this job will end up with capital stock k and technological gap τ at the beginning of time t only if the following two events occur. First, it must be that the realization of the idiosyncratic shock ϵ is equal to $-\tau$, which has probability $g_\epsilon(\tau)$. Secondly, the capital stock must depreciate at a rate such that the beginning of period capital stock at time t is exactly equal to

k, which occurs with probability $g_\delta(\ln j - \ln k - g_{qt} - g_{xt})$. The term in the first row then integrates overall all possible values of the capital stock j and technological gap i, which do not lead to job destruction at time $t-1$.

The terms in the remaining rows are obtained analogously. The one in the second row corresponds to old jobs that fail both to upgrade their capital level at time $t-1$ and to catch up with the leading technology at time t. The third row corresponds to old jobs that upgrade both their capital level at time $t-1$ and their neutral technology at time t. The fourth row to old jobs that upgrade their capital level at time $t-1$ but fail to catch up with the leading technology at time t. Finally, the last row accounts for the inflow of newly created jobs at time $t-1$, m_{t-2}, that enters at the leading technology of that time and with an optimal capital level.

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Sample	(jc, jd)	$(\Delta\widetilde{z}, \Delta n)$	$(\Delta q, \Delta n)$
		Raw Data	
48-93	0.06	-0.02	0.27
	(0.85)	(0.12)	(0.14)
54-93	0.20	-0.07	0.29
	(0.16)	(0.10)	(0.16)
		Structural Change in Job Creation	
48-93	-0.17	-0.02	0.27
	(0.56)	(0.13)	(0.14)
54-93	0.20	-0.08	0.29
	(0.16)	(0.10)	(0.16)
		Technological Revolution (74-break)	
48-93	0.06	0.03	0.29
	(0.85)	(0.12)	(0.13)
54-93	0.20	0.03	0.24
	(0.16)	(0.10)	(0.23)

Table 1: Unconditional bivariate correlations. Δn denotes net-employment change defined as the difference between the job creation and job destruction rate. The specification with a structural change in the job creation rate includes a break in the job creation series starting from 1974. The "technological revolution" specification includes a break in the growth rate of $\tilde{z}$ and q starting from 1975 and a year-dummy in 1975 for the two series.

	Effect of a z-shock in the					Effect of a q-shock in the				
	Short Run:				*Long Run:*	*Short Run:*				*Long Run:*
Sample	jc	jd	n	jr	n	jc	jd	n	jr	n
					Raw data					
48-93	+	+	-	+	0	0	-	+	-	0
54-93	+	+	-	+	-	0	-	+	-	+
					Structural Change in Job Creation					
48-93	0	+	-	+	-	0	-	+	-	+
54-93	0	+	-	+	-	+	-	+	-	+
					Technological Revolution (74-break)					
48-93	0	+	-	+	0	0	-	+	-	+
54-93	0	+	-	+	-	+	-	+	-	+

Table 2: Sign of the short and long run effecs of technology shocks. All VARs have two lags. The short run effect is the response of the corresponding variable in the same year as that of the technology shock. The long-run effect corresponds to the effect after 10 years. jr and n denotes the job reallocation rate and the logged employment level, respectively. A '0' means that the effect is not significant at a five per cent significance level. A positive or a minus sign means that the effect is significant at a five per cent level and has the corresponding sign.

Sample	z-Shock (jc,jd)	$(\Delta\widetilde{z},\Delta n)$	(jc,n)	(jd,n)	q-Shock (jc,jd)	$(\Delta q,\Delta n)$	(jc,n)	(jd,n)
				Raw data				
48-93	0.56 (0.11)	0.24 (0.04)	-0.01 (0.02)	-0.10 (0.03)	0.52 (0.15)	0.65 (0.16)	-0.05 (0.02)	-0.15 (0.04)
54-93	0.62 (0.12)	0.07 (0.01)	-0.75 (0.15)	-0.62 (0.12)	0.40 (0.10)	0.90 (0.26)	-0.50 (0.10)	-0.42 (0.08)
				Structural Change in Job Creation				
48-93	0.22 (0.04)	-0.37 (0.06)	-0.28 (0.08)	-0.21 (0.03)	0.47 (0.11)	0.88 (0.09)	-0.29 (0.14)	-0.22 (0.12)
54-93	0.28 (0.07)	-0.27 (0.06)	-0.23 (0.05)	-0.17 (0.04)	0.20 (0.04)	0.96 (0.24)	-0.19 (0.08)	-0.18 (0.09)
				Technological Revolution (74-break)				
48-93	0.27 (0.03)	0.01 (0.02)	-0.21 (0.04)	-0.30 (0.05)	0.38 (0.05)	0.81 (0.16)	-0.78 (0.10)	-0.69 (0.09)
54-93	0.16 (0.01)	0.05 (0.02)	-0.45 (0.11)	-0.36 (0.09)	-0.73 (0.05)	0.96 (0.11)	0.21 (0.04)	-0.02 (0.02)

Table 3: Bivariate correlations conditional to a z and a q shock. All VARs have two lags. The specification with a structural change in the job creation rate includes a break in the job creation series starting from 1974. The "technological revolution" specification includes a break in the growth rate of $\tilde{z}$ and q starting from 1975 and a year-dummy in 1975 for the two series.

Sample	by z-shocks $\Delta\widetilde{z}$	jc	jd	n	jr	by q-shocks Δq	jc	jd	n	jr
					Raw data					
48-93	17	18	39	24	37	31	15	36	24	30
54-93	19	22	47	32	43	42	20	46	42	33
					Structural Change in Job Creation					
48-93	38	24	63	37	72	66	18	36	18	33
54-93	36	28	81	61	76	69	23	48	37	48
					Technological Revolution (74-break)					
48-93	19	22	43	32	37	77	7	19	13	16
54-93	32	22	61	63	37	65	5	8	15	2

Table 4: Percentage of variance explained by technology shocks in the business fluctuations of the post-World-War II period. See Table 3 for further details.

Sample	*Short Run*: jc	jd	n	jr	y	h	y–h	c–y	i	*Long Run*: n	y	h	y–h	i
(Linear) trend in c–y and i–y														
48-93	0	+	-	+	-	-	+	-	-	0	+	0	+	0
54-93	-	+	-	+	-	-	+	0	-	-	+	0	+	+
74-break in c–y and i–y														
48-93	0	+	-	+	-	-	+	-	-	0	+	0	+	0
54-93	-	+	-	+	-	-	+	0	-	-	+	0	+	+

Table 5: Short run and long run sign-effects of a z-shock in a seven-variables VAR when a technological revolution is allowed. Each VAR includes the growth rate of $\tilde{z}$, the growth rate of labour productivity, the job creation and job destruction rate, hours per capita and the logged consumption-output and (equipment) investment-output ratios. All VARs have one lag.

Sample	*Short Run*: jc	jd	n	jr	y	h	y–h	c–y	i	*Long Run*: n	y	h	y–h	i
(Linear) trend in c–y and i–y														
48-93	0	-	+	-	+	+	+	0	0	0	0	0	0	+
54-93	+	-	+	-	+	+	+	0	-	+	+	0	+	+
74-break in c–y and i–y														
48-93	+	-	+	-	+	+	+	+	+	0	+	0	+	+
54-93	+	-	+	-	+	+	+	+	+	+	+	0	+	+

Table 6: Short run and long run sign-effects of a q-shock in a seven-variables VAR when a technological revolution is allowed. Each VAR includes the growth rate of q, the growth rate of labour productivity, the job creation and job destruction rate, hours per capita and the logged consumption-output and (equipment) investment-output ratios. All VARs have one lag.

	Effect of a z-shock in the							Effect of a q-shock in the						
	Short Run :					*Long Run* :		*Short Run* :					*Long Run* :	
Sample	jc	jd	n	jr	y_n	n	y_n	jc	jd	n	jr	y_n	n	y_n
							Raw data							
48-93	+	+	-	+	+	+	+	-	-	+	-	+	-	0
54-93	+	+	-	+	+	+	+	-	-	+	-	+	+	0
							Structural Change in Job Creation							
48-93	+	+	-	+	-	+	+	-	-	0	-	+	-	0
54-93	-	-	+	-	+	-	-	0	-	+	-	+	+	+
							Technological Revolution (74-break)							
48-93	+	+	-	+	-	+	+	-	-	0	-	+	-	+
54-93	-	-	+	-	+	+	-	+	0	+	+	0	+	+

Table 7: Sign of the short and long run effecs of technology shocks by using the same identification strategy as in Fischer (2002). Each VAR includes the job creation rate, jc, the job destruction rate, jd, the growth rate of labour productivity, y_n, and the growth rate of q and has two lags. The short run effect is the response of the corresponding variable in the same year as that of the technology shock. The long-run effect corresponds to the effect after 10 years. jr and n denotes the job reallocation rate and the employment level, respectively. A '0' means that the effect is not significant at a five per cent significance level. A positive or a minus sign means that the effect is significant at a five per cent level and has the corresponding sign.

Parameters		
ρ	:	household's discount factor
β	:	workers' bargaining power
$\bar{r}$	:	intercept of the recruitment effort function
ν	:	elasticity of the marginal utility of leisure of the recruiter
c_w	:	effort cost of working
σ_ϵ	:	sd of job idiosyncratic productivity
μ_δ (σ_δ)	:	average (sd of) depreciation rate
μ_z (σ_z)	:	average (sd of) growth rate of leading technology
μ_q (σ_q)	:	average (sd of) growth rate of new capital quality
a_z	:	probability of upgrading the neutral technology
a_q	:	probability of upgrading the investment-specific technology
α	:	output elasticity wrt capital

Asset Values (scaled by $X_t \equiv e^{x_t}$)		
S_t	:	job net surplus
V_t	:	value of a vacancy (to a firm)
J_t	:	future job net surplus
H_t	:	value of staying at home (to a household)

Equilibrium tuple		
$k_t^*(\tau)$	:	optimal capital choice for a job with technological gap τ
$\bar{\tau}_t$	:	maximum technological gap after capital adjustment
$\tau_t^*(k)$	:	maximum technological gap for a job with capital k
θ_t	:	labour market tightness
m_t	:	number of new matches
N_t	:	aggregate employment
C_t	:	(scaled) aggregate consumption
$f_t(k,\tau)$	:	measure of old jobs with capital k and technological gap τ previous to any investment and destruction decision

Table 8: Legend

Parameter Values

ρ	:	0.99	β	:	0.5	α	:	0.24
μ_z	:	0.11%	μ_q	:	1.125%	c_w	:	1.052
$\bar{r}$	:	0.638	ν	:	1	μ_δ	:	3.12%
σ_δ	:	2.12%	σ_ϵ	:	0.98%	η	:	1.289
a_z	:	0.1	a_q	:	0.45			

Moment Conditions

		Data	Model
Average depreciation rate, $\mu_\delta - \sigma_\delta^2/2$	:	0.031	0.031
Workers' employment probability, $p(\theta)$	:	0.45	0.45
Firms' hiring probability, $q(\theta)$	:	0.71	0.71
Job destruction rate, jd	:	0.06	0.06
Fraction of jobs more productive than a newly created one	:	0.30	0.30
Fraction of jobs adjusting capital with positive investment	:	0.89	0.88

Table 9: Moment conditions and parameters values used in the baseline specification.

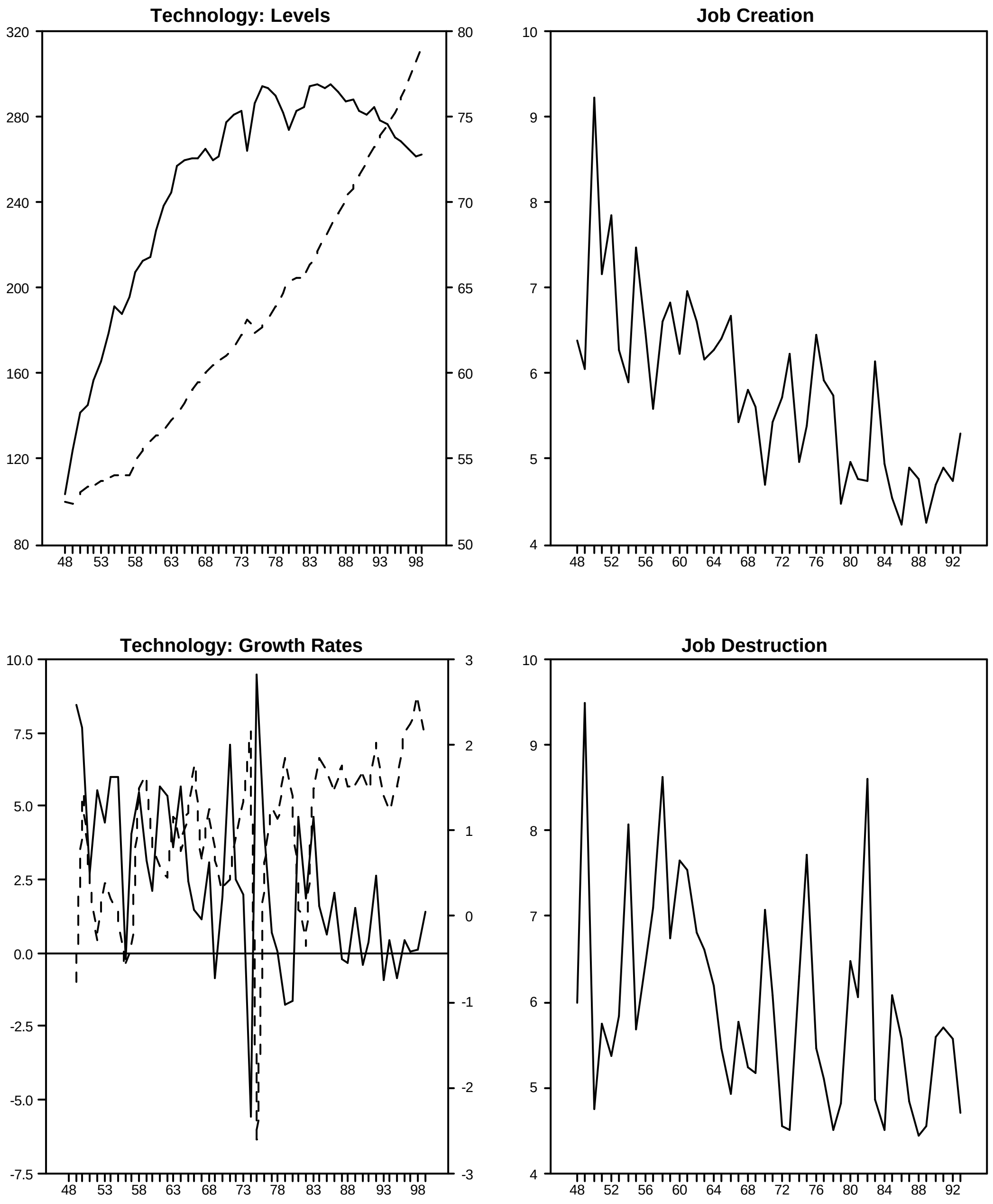

Figure 1: Left panels: continuous lines correspond to variable $\tilde{z}$, dotted lines correspond to variable q. Right panels: the series for job creation and job destruction correspond to the rate of the corresponding variable. All variables are multiplied by 100.

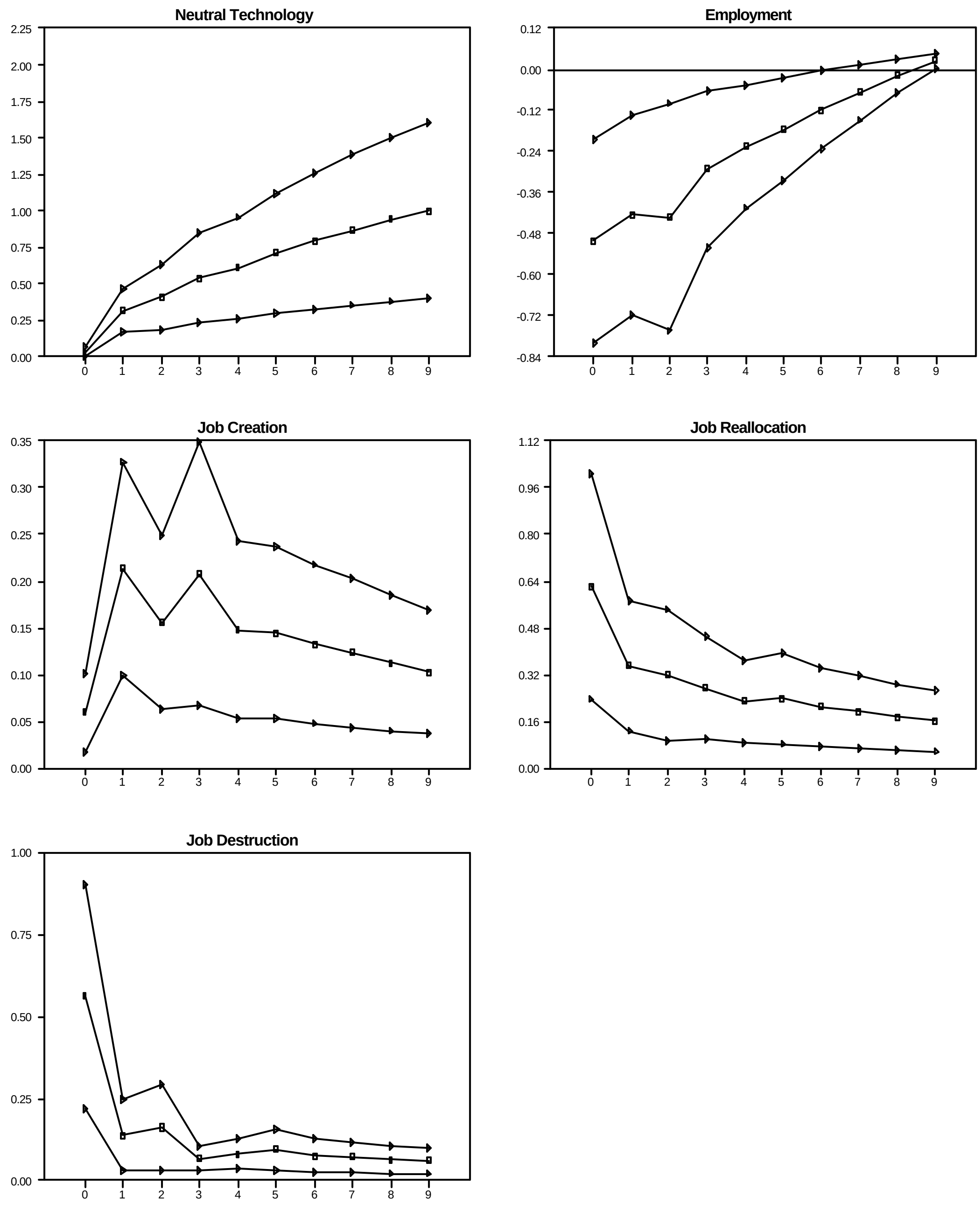

Figure 2: Impulse responses to a one-per-cent z–shock. Three-variables VAR with raw data and sample period 1948-1993. The VAR includes the rate of growth of $\tilde{z}$, the job creation and job destruction rate, and has two lags. Years after the shock in horizontal axes. The effect of the shock on z after ten years is normalized to one.

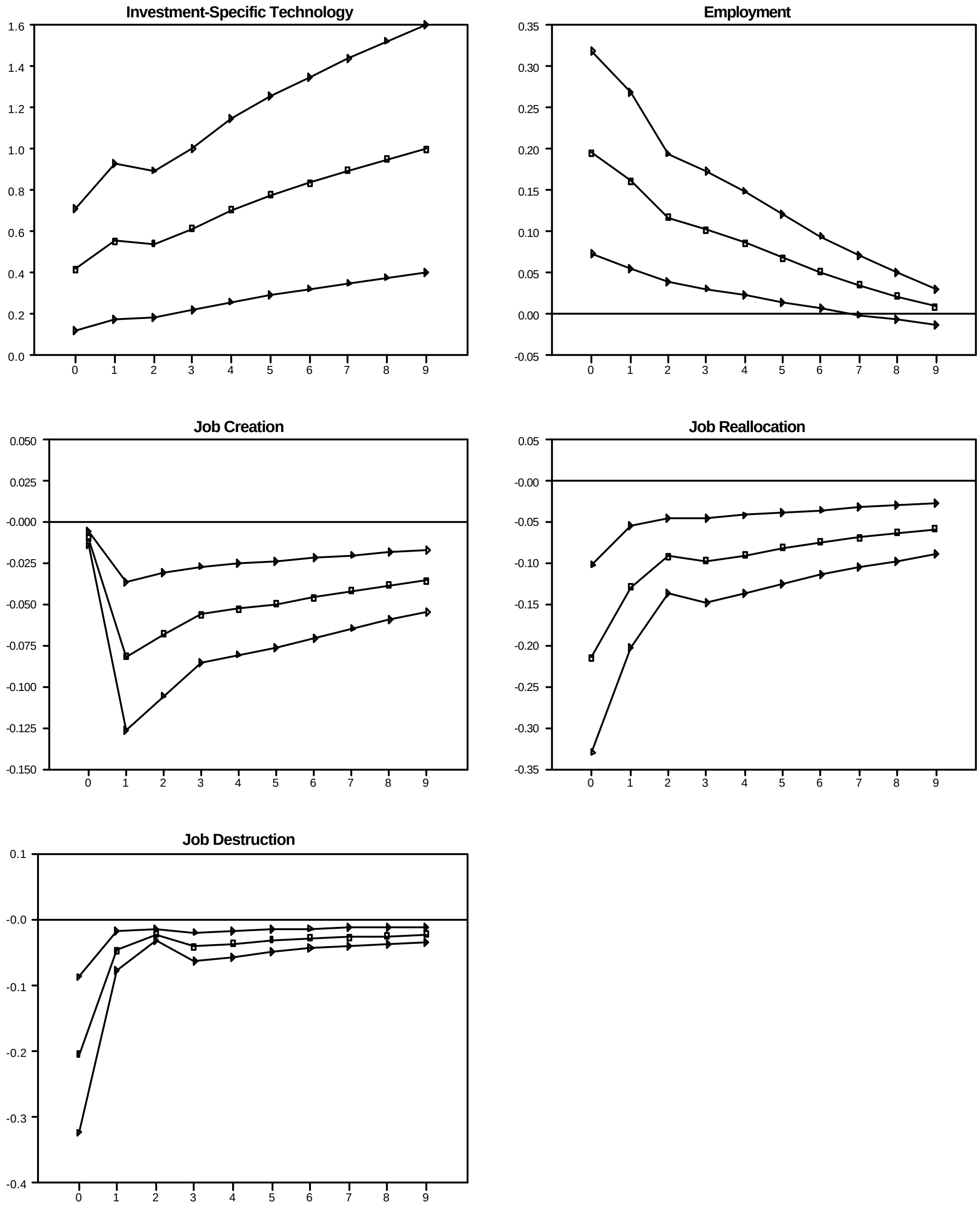

Figure 3: Impulse responses to a one-per-cent q–shock. Three-variables VAR with raw data and sample period 1948-1993. The VAR includes the rate of growth of q, the job creation and job destruction rate, and has two lags. Years after the shock in horizontal axes. The effect of the shock on q after ten years is normalized to one.

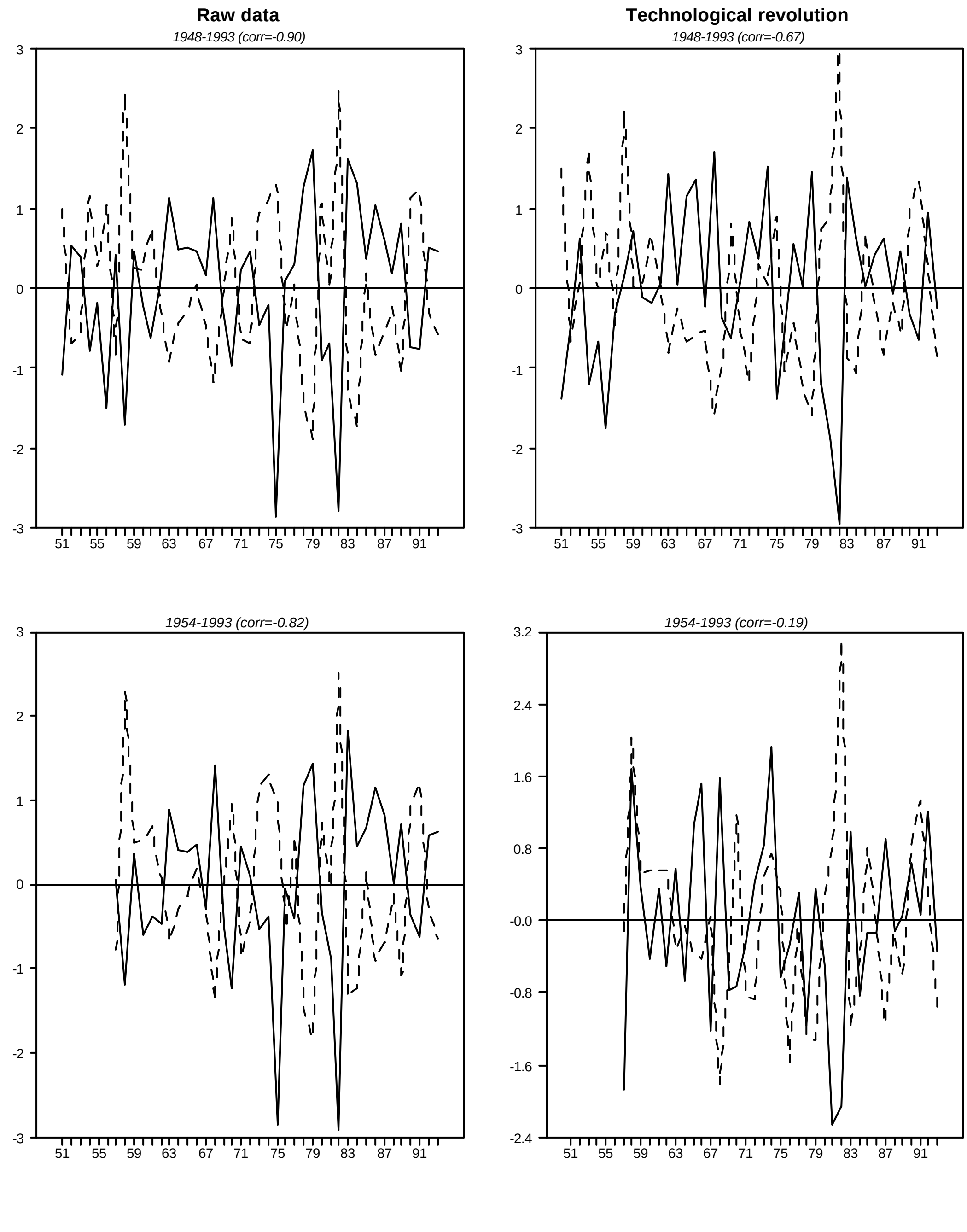

Figure 4: Estimated structural shocks. Left panels: specification with raw data and different sample periods. Right panels: specification with a technological revolution and different sample periods. Continuous lines corresponds to z-shock, dotted lines correspond to q-shock.

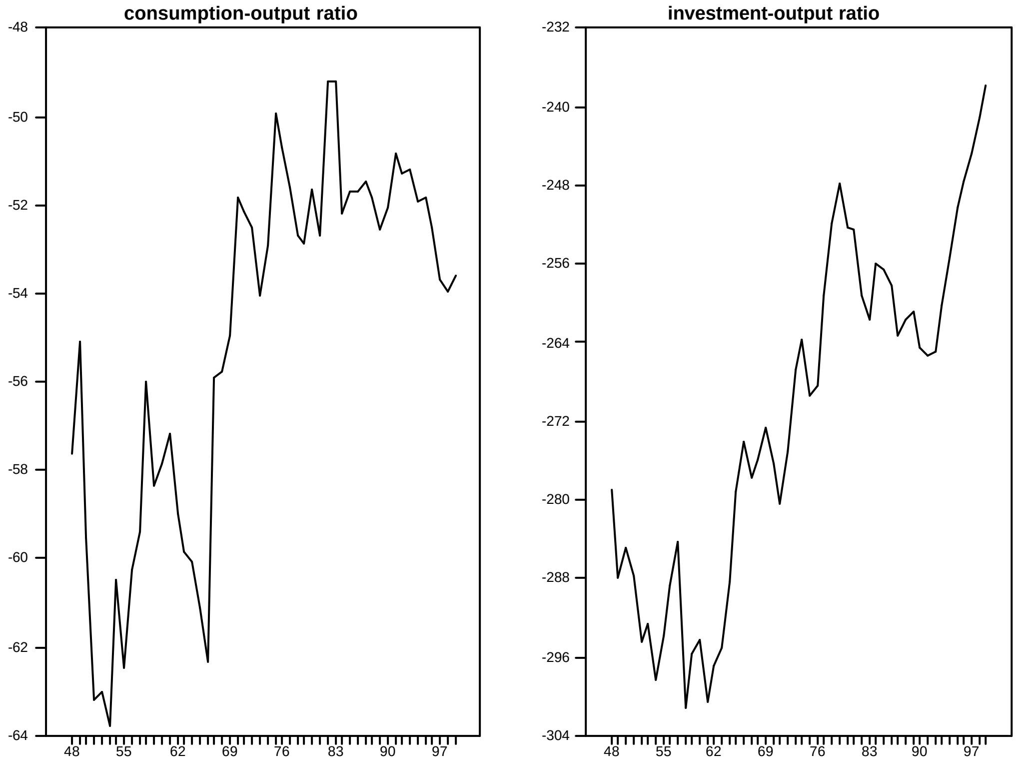

Figure 5: Logged consumption-output and (equipment) investment-output ratio. All variables are multiplied by 100.

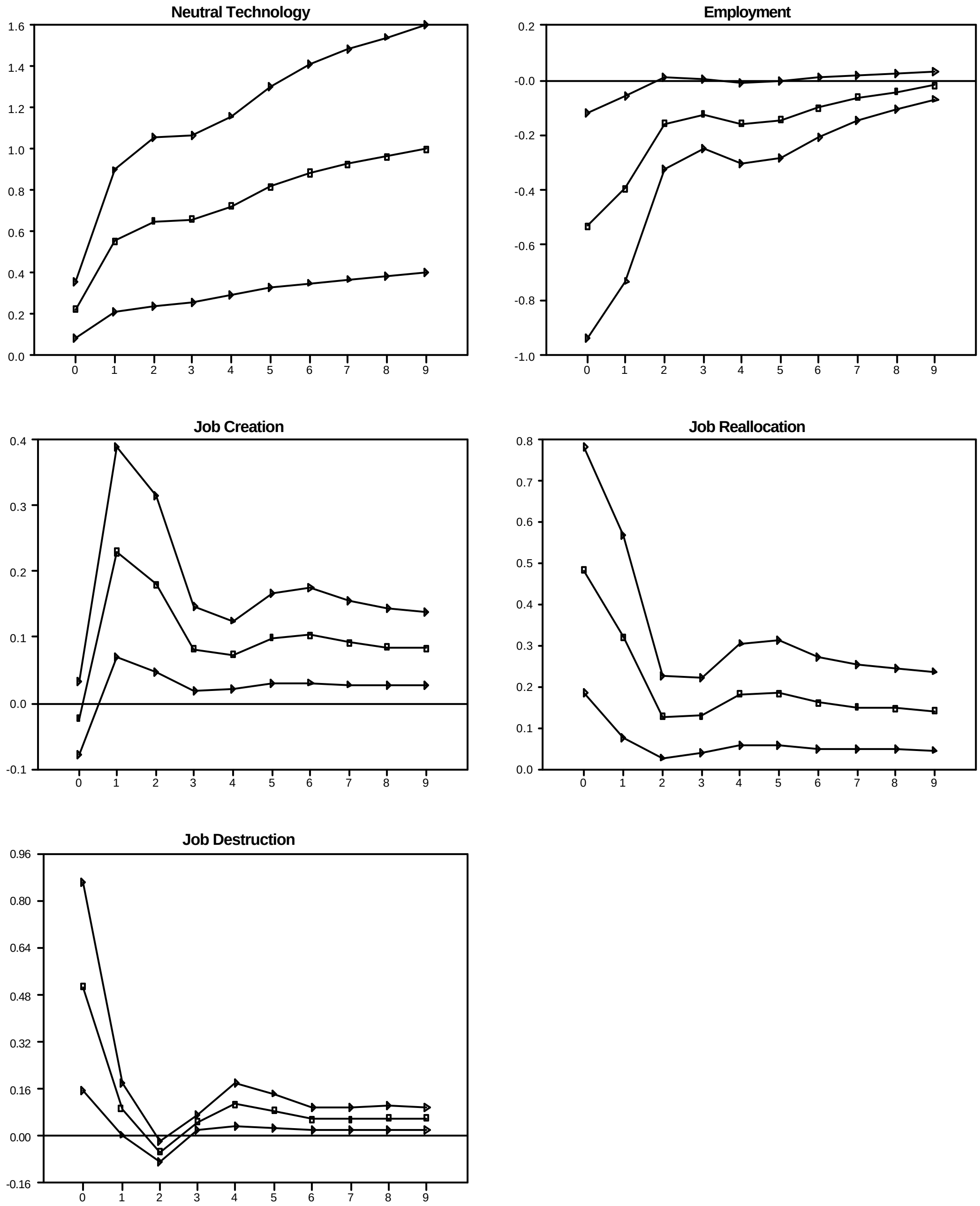

Figure 6: Impulse responses to a one-per-cent z–shock (old variables). Seven-variables VAR with break in the consumption-output and investment-output ratios and sample period 1948-1993. The VAR includes the growth rate of $\tilde{z}$, the growth rate of labour productivity, the job creation and job destruction rate, hours per capita and the logged consumption-output and (equipment) investment-output ratios. The VAR allows for a break in the growth rate of $\tilde{z}$ after 1974 and a year dummy in 1975 and has one lag. Years after the shock in horizontal axes. The effect of the shock on z after ten years is normalized to one.

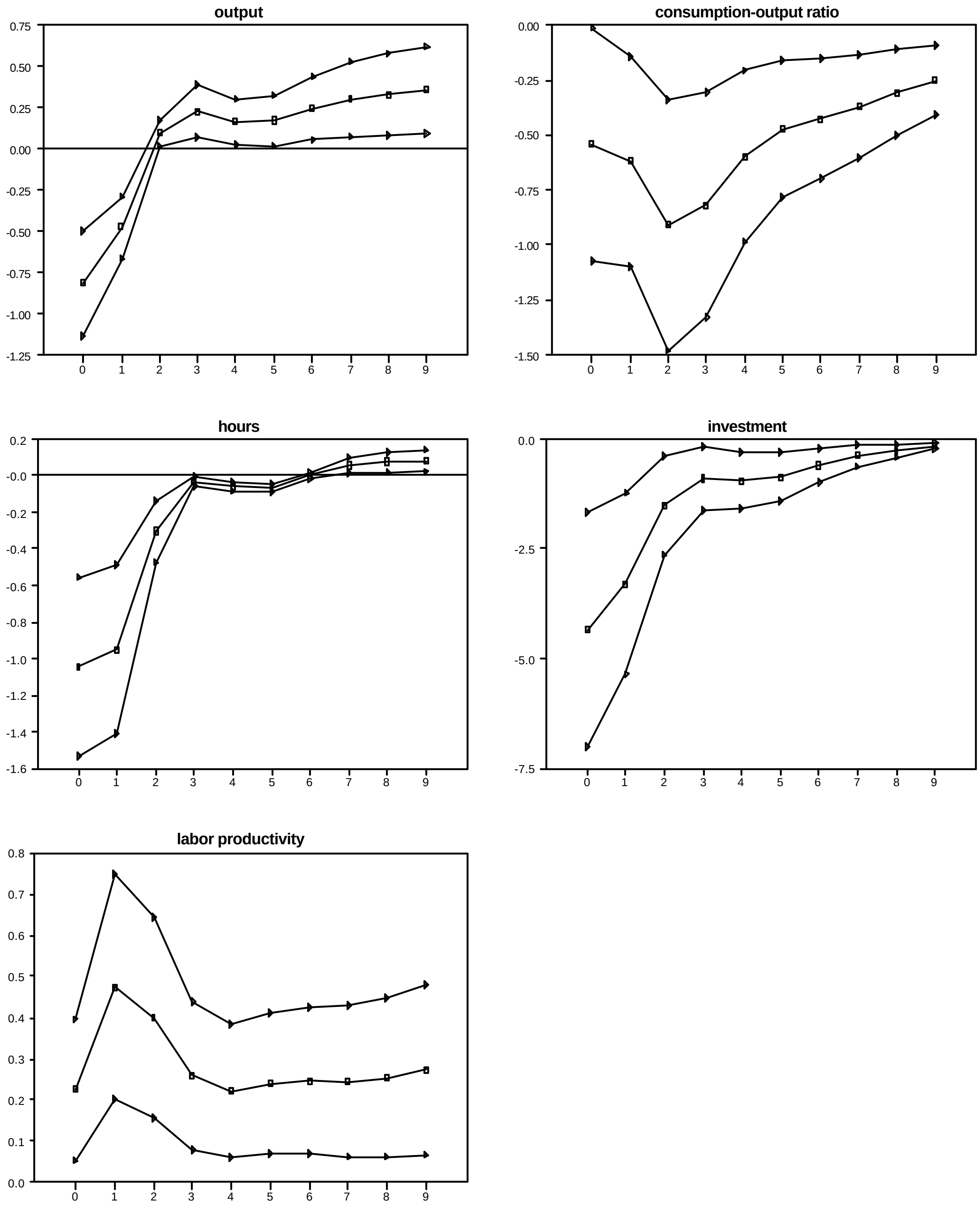

Figure 7: Impulse responses to a one-per-cent z–shock (added variables). Seven-variables VAR with break in the consumption-output and investment-output ratios and sample period 1948-1993. See Figure 6 for further details.

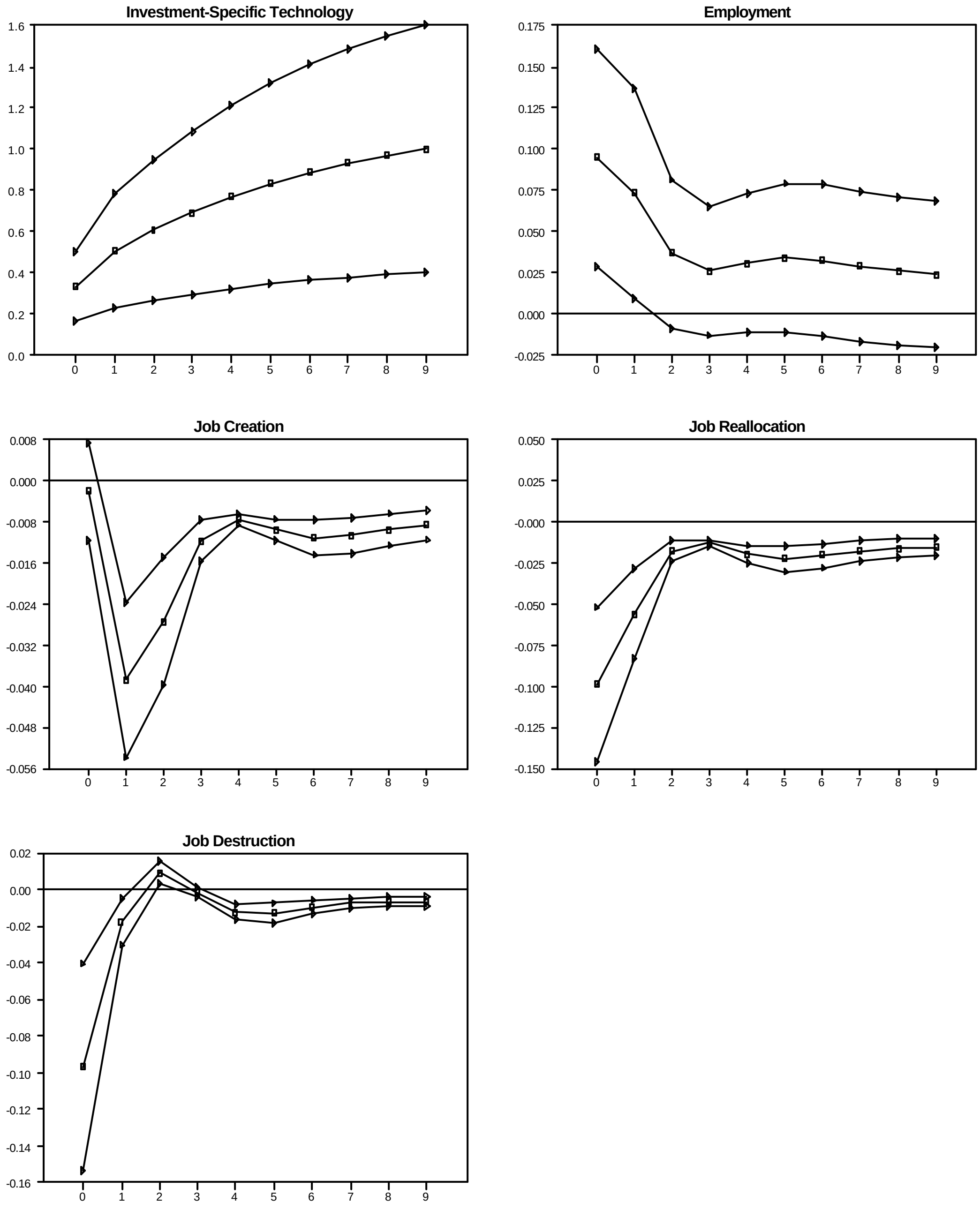

Figure 8: Impulse responses to a one-per-cent q–shock (old variables). Seven-variables VAR with break in the consumption-output and investment-output ratios and sample period 1948-1993. The VAR includes the growth rate of q, the growth rate of labour productivity, the job creation and job destruction rate, hours per capita and the logged consumption-output and (equipment) investment-output ratios. The VAR allows for a break in the growth rate of q after 1974 and a year dummy in 1975 and has one lag. Years after the shock in horizontal axes. The effect of the shock on q after ten years is normalized to one.

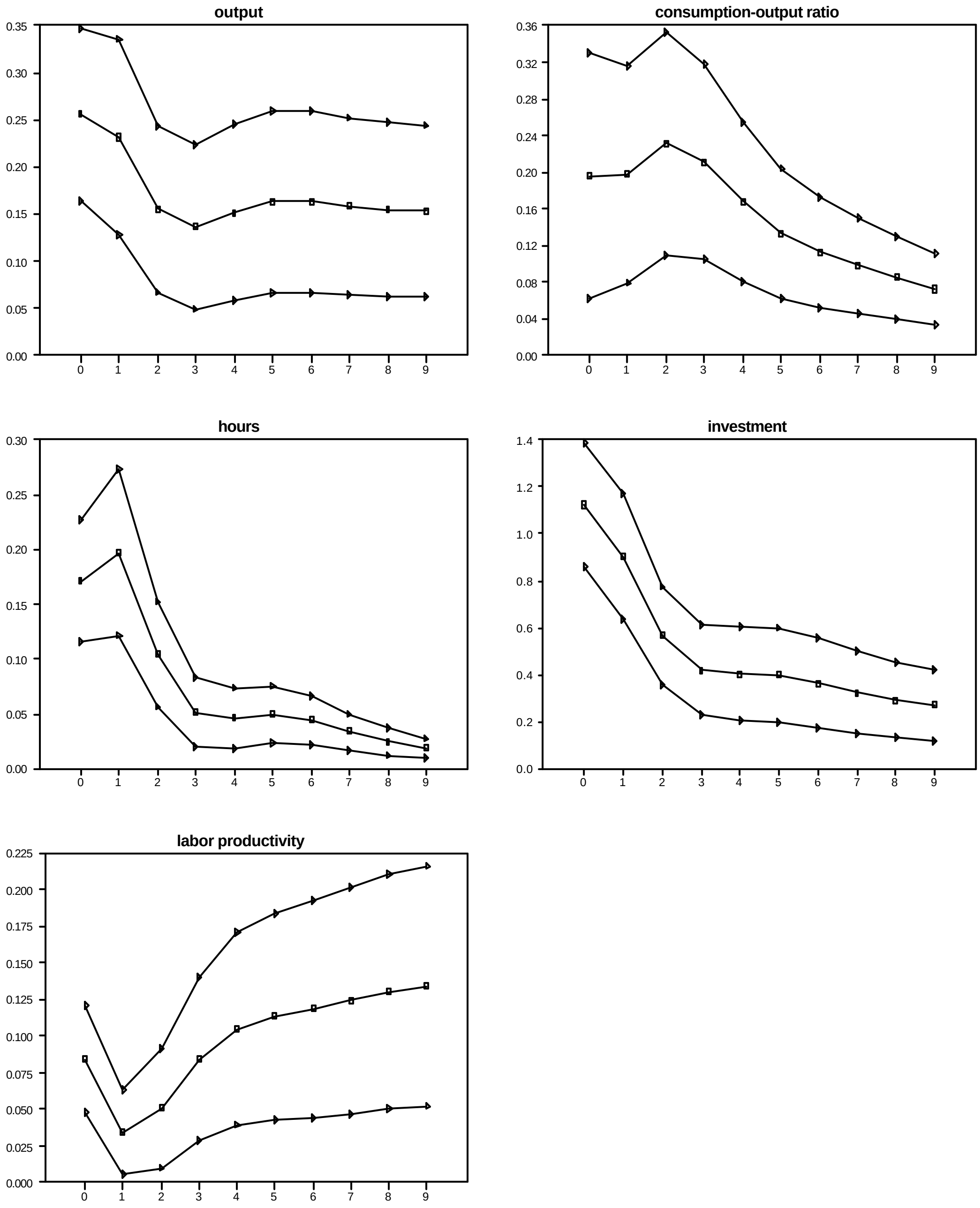

Figure 9: Impulse responses to a one-per-cent q–shock (added variables). Seven-variables VAR with break in the consumption-output and investment-output ratios and sample period 1948-1993. See Figure 8 for further details.

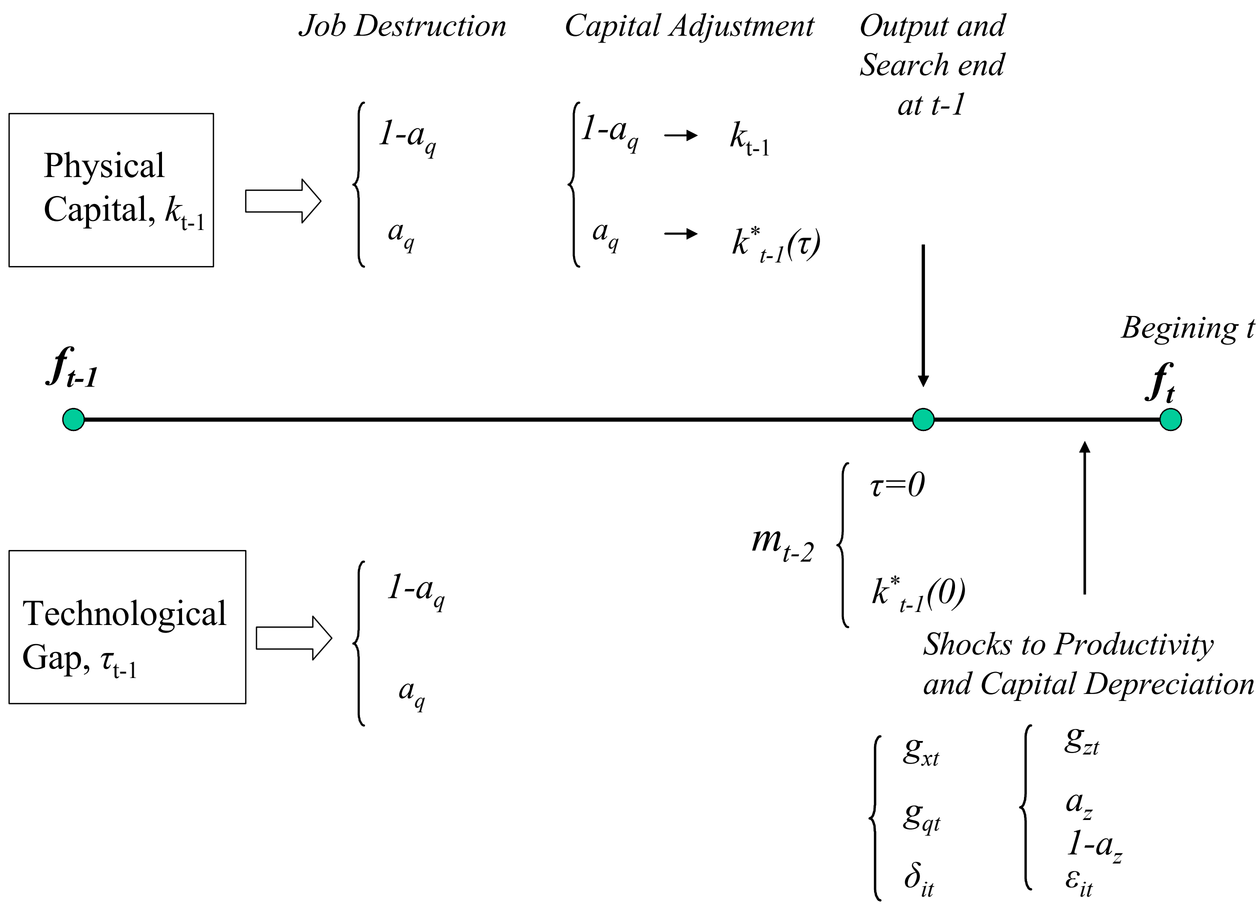

Figure 10: Evolution of the beginning-of-period distribution

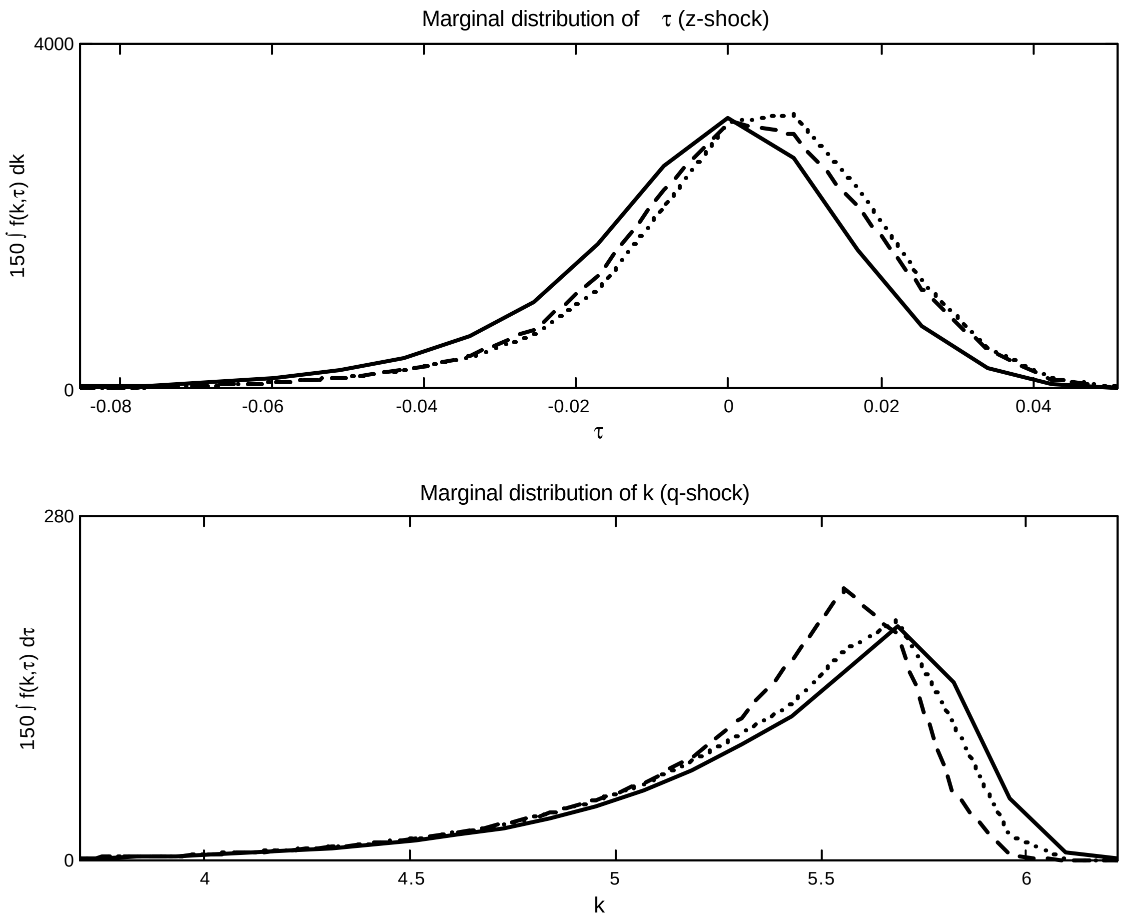

Figure 11: Upper panel refers to the marginal distribution of τ, $\int f_t(k,\tau)dk$: bold line corresponds to steady state, dotted line to the distribution in the same year of the z-shock, dashed line to the distribution in the following year. Lower panel refers to the marginal distribution of k, $\int f_t(k,\tau)d\tau$: bold line corresponds to steady state, dotted line to the distribution in the same year of the q-shock, dashed line to the distribution in the following year.

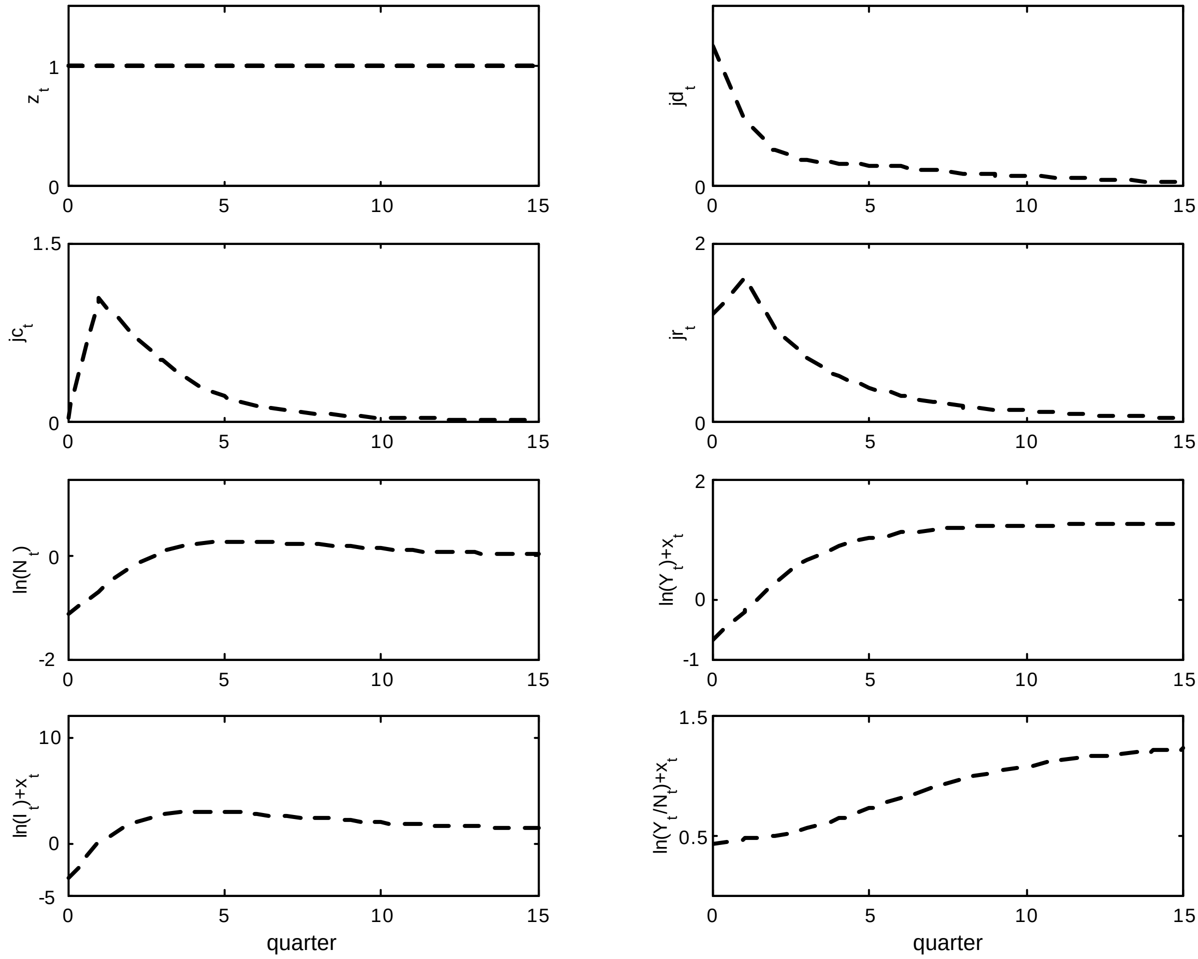

Figure 12: Impulse responses to a one-per-cent neutral technology shock. From left to right, top to bottom: neutral technology, z_t, job destruction rate, jd_t, job creation rate, jc_t, job reallocation rate, jr_t, (logged) employment, $\ln N_t$, unscaled (logged) output, $\ln Y_t + x_t$, unscaled (logged) investment, $\ln I_t + x_t$, and unscaled (logged) labour productivity, $\ln(Y_t/N_t) + x_t$. All impulse responses are multiplied by 100.

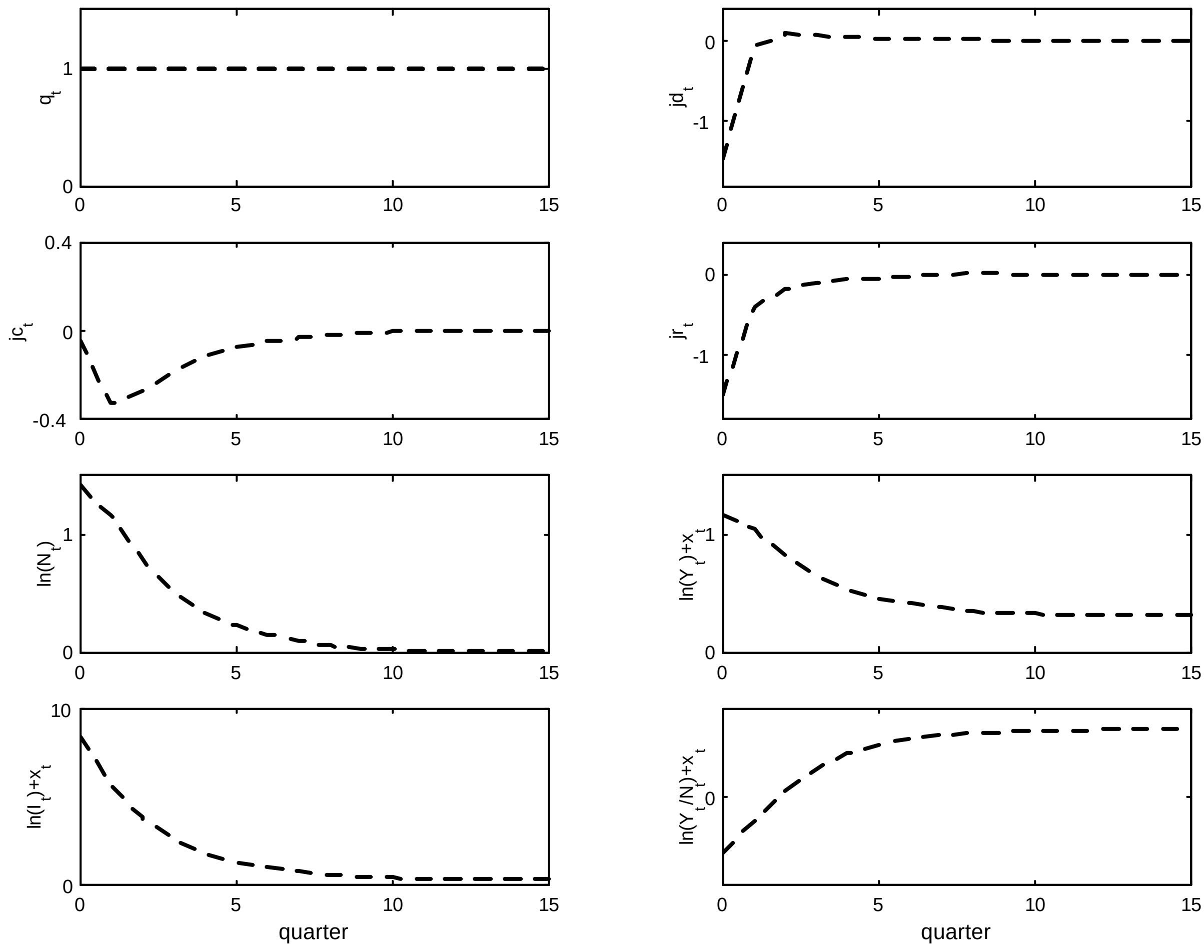

Figure 13: Impulse responses to a one-per-cent improvement in the quality of new capital. From left to right, top to bottom: investment-specific technology, z_t, job destruction rate, jd_t, job creation rate, jc_t, job reallocation rate, jr_t, (logged) employment, $\ln N_t$, unscaled (logged) output, $\ln Y_t + x_t$, unscaled (logged) investment, $\ln I_t + x_t$, and unscaled (logged) labour productivity, $\ln(Y_t/N_t) + x_t$. All impulse responses are multiplied by 100.

Computational Appendix

In this appendix, we first discuss how to solve for the steady state of the model without aggregate shocks, $g_{zt} = \mu_z$ and $g_{qt} = \mu_q$. Then we explicitly linearize the equilibrium conditions of the model with aggregate shocks. This yields a system of linear stochastic difference equation that we solve by using the method proposed by Sims (2002).

A Solving for the steady state

We next characterize the equilibrium conditions of the model in steady state and then discuss the algorithm used to compute the equilibrium.

A.1 Equilibrium conditions

The equilibrium conditions of the economy in steady state can be obtained by dropping the time subscripts from the corresponding expressions in Section 4 and equation (23).

Let

$$\Delta(\delta) = e^{-\delta-\mu_q-\mu_x}$$

and $\bar{\Delta} = e^{-\mu_\delta-\mu_q-\mu_x+\sigma_\delta^2/2}$ denote the depreciation factor of capital and its expected value, respectively. Then the steady-state net surplus for a job with capital k and technological gap τ is equal to

$$S(k,\tau) = e^{-\tau}k^\alpha - (1-\rho\bar{\Delta})k - c_w C - (1-\rho)H + J(k,\tau), \tag{24}$$

while the net surplus for a job with technological gap τ that deploys its desired capital level, $k^*(\tau)$, is

$$S(\tau) = e^{-\tau}\left[k^*(\tau)\right]^\alpha - (1-\rho\bar{\Delta})k^*(\tau) - c_w C - (1-\rho)H + J(k^*(\tau),\tau). \tag{25}$$

The present value of the future job net-surplus is equal to

$$\begin{aligned}
J(k,\tau) &= \rho(1-a_q)a_z \int_{R^2} \max\left(0, S(\Delta(i)k, j)\right) g_\delta(i) g_\epsilon(j)\, di\, dj \\
&+ \rho(1-a_q)(1-a_z) \int_{R^2} \max\left(0, S(\Delta(i)k, \tau+\mu_z+j)\right) g_\delta(i) g_\epsilon(j)\, di\, dj \\
&+ \rho a_q \int_R \left[a_z \max\left(0, S(j)\right) + (1-a_z)\max\left(0, S(\tau+\mu_z+j)\right)\right] g_\epsilon(j)\, dj
\end{aligned}$$

which, after transforming some of the variables of integration, can be expressed as

$$
\begin{aligned}
J(k,\tau) &= \rho(1-a_q)a_z \int_{R^2} \max(0, S(i,j)) g_\delta(\ln k - \ln i - \mu_q - \mu_x) g_\epsilon(j)\, didj \\
&+ \rho(1-a_q)(1-a_z) \int_{R^2} \max(0, S(i,j)) g_\delta(\ln k - \ln i - \mu_q - \mu_x) g_\epsilon(j - \tau - \mu_z)\, didj \\
&+ \rho a_q \left[a_z \int_R \max\left(0, S(j)\right) g_\epsilon(j) dj + (1-a_z) \int_R \max\left(0, S(j)\right) g_\epsilon(j-\tau-\mu_z) dj \right],
\end{aligned} \tag{26}
$$

which will be used below. Finally the worker's value of staying at home can be expressed as

$$
H = \frac{rC\beta\theta}{(1-\rho)(1-\beta)} \tag{27}
$$

where

$$
r = \bar{r}\left[\theta\left(1-N\right)\right]^{\nu}
$$

denotes the steady-state recruitment effort cost at the equilibrium value of labour market tightness, θ, and unemployment, $1-N$.

Given these definitions a steady state equilibrium is characterized by a tuple

$$
(k^*(\tau), \bar{\tau}, \tau^*(k), \theta, N, m, f\left(k,\tau\right), C)
$$

which solves the following eight equilibrium conditions.

1. The condition for the optimal capital choice:

$$
e^{-\tau}\alpha\left[k^*(\tau)\right]^{\alpha-1} + \frac{\partial J(k^*(\tau),\tau)}{\partial k} = 1 - \rho\bar{\Delta}, \quad \forall\tau. \tag{28}
$$

2. The conditions for the maximum technological gap (when capital can be upgraded):

$$
S(\bar{\tau}) = 0. \tag{29}
$$

3. The conditions for the maximum technological gap (in the absence of capital upgrading):

$$
S(k, \tau^*(k)) = 0, \quad \forall k. \tag{30}
$$

4. The free-entry condition for vacancy creation:

$$
\frac{rC}{q(\theta)} = \rho\left(1-\beta\right) S(0). \tag{31}
$$

5. The constraint on aggregate employment:

$$N = \int_R \left[a_q \int_{-\infty}^{\bar{\tau}} f(k,\tau) d\tau + (1 - a_q) \int_{-\infty}^{\tau^*(k)} f(k,\tau) d\tau \right] dk + m. \tag{32}$$

6. The definition of job creation:

$$m = p(\theta)(1 - N). \tag{33}$$

7. The law of motion of the beginning-of-period distribution of old jobs:

$$\begin{aligned} f(k,\tau) &= (1-a_q)a_z \int_R \left[\int_{-\infty}^{\tau^*(j)} g_\delta(\ln j - \ln k - \mu_q - \mu_x) g_\epsilon(\tau) f(j,i) di \right] dj \\ &+ (1-a_q)(1-a_z) \int_R \left[\int_{-\infty}^{\tau^*(j)} g_\delta(\ln j - \ln k - \mu_q - \mu_x) g_\epsilon(i + \mu_z - \tau) f(j,i) di \right] dj \\ &+ a_q a_z \int_R \left[\int_{-\infty}^{\bar{\tau}} g_\delta(\ln k^*(i) - \ln k - \mu_q - \mu_x) g_\epsilon(\tau) f(j,i) di \right] dj \\ &+ a_q (1-a_z) \int_R \left[\int_{-\infty}^{\bar{\tau}} g_\delta(\ln k^*(i) - \ln k - \mu_q - \mu_x) g_\epsilon(i + \mu_z - \tau) f(j,i) di \right] dj \\ &+ g_\delta(\ln k^*(0) - \ln k - \mu_q - \mu_x) \left[a_z g_\epsilon(\tau) + (1-a_z) g_\epsilon(\mu_z - \tau) \right] m, \qquad \forall k, \forall \tau. \end{aligned} \tag{34}$$

8. The aggregate budget constraint:

$$Y + D^d - I^u - C = 0 \tag{35}$$

where output, Y, disinvestment triggered by job destruction, D^d, and the investment of firms upgrading their capital stock, I^u, are equal to

$$\begin{aligned} Y &= \int_R \left[a_q \int_{-\infty}^{\bar{\tau}} e^{-\tau} \left[k^*(\tau)\right]^\alpha f(k,\tau) d\tau + (1-a_q) \int_{-\infty}^{\tau^*(k)} e^{-\tau} k^\alpha f(k,\tau) d\tau \right] dk \\ &+ m \left[k^*(0)\right]^\alpha, \\ D^d &= \int_R [a_q \int_{\bar{\tau}}^{\infty} k f(k,\tau) d\tau + (1-a_q) \int_{\tau^*(k)}^{\infty} k f(k,\tau) d\tau] dk, \\ I^u &= a_q \int_{R \times [-\infty, \bar{\tau}]} \left[k^*(\tau) - k\right] f(k,\tau) dk d\tau + m k^*(0), \end{aligned}$$

respectively.

A.2 Rearranging and simplification

In computing the steady-state equilibrium of the model is convenient to focus on the distribution of log capital and technological gap $h(l, \tau)$ so as to justify a discretization of the state space in equally spaced intervals. By the law of transformation of random variables we know that f and h are linked through

$$f(k, \tau) = \frac{1}{k} h(\ln k, \tau).$$

By the same reasoning that allowed us to write (23), we obtain that in steady state $h(l, \tau)$ must satisfy

$$\begin{aligned}
h(l,\tau) &= (1-a_q)a_z \int_R \left[\int_{-\infty}^{\tau^*(e^j)} g_\delta(j - l - \mu_q - \mu_x) g_\epsilon(\tau) h(j,i) di \right] dj \\
&+ (1-a_q)(1-a_z) \int_R \left[\int_{-\infty}^{\tau^*(e^j)} g_\delta(j - l - \mu_q - \mu_x) g_\epsilon(i + \mu_z - \tau) h(j,i) di \right] dj \\
&+ a_q a_z \int_R \left[\int_{-\infty}^{\bar\tau} g_\delta(\ln k^*(i) - l - \mu_q - \mu_x) g_\epsilon(\tau) h(j,i) di \right] dj \\
&+ a_q (1-a_z) \int_R \left[\int_{-\infty}^{\bar\tau} g_\delta(\ln k^*(i) - l - \mu_q - \mu_x) g_\epsilon(i + \mu_z - \tau) h(j,i) di \right] dj \\
&+ g_\delta(\ln k^*(0) - l - \mu_q - \mu_x) \left[a_z g_\epsilon(\tau) + (1-a_z) g_\epsilon(\mu_z - \tau) \right] m, \qquad \forall k, \forall \tau.
\end{aligned} \tag{36}$$

Also notice that by using (32) and (33) to solve for m we obtain that

$$m = \frac{p(\theta)}{1+p(\theta)} \left\{ 1 - \int_R \left[a_q \int_{-\infty}^{\bar\tau} h(j,i) di + (1-a_q) \int_{-\infty}^{\tau^*(e^j)} h(j,i) di \right] dj \right\} \tag{37}$$

which will be used in the algorithm that we now detail.

A.3 Algorithm

Our algorithm is just a version of value function iteration, which consists of a "guessing", a "checking" and an "updating" part.

Guessing: Guess a function for $J(k, \tau)$. In practice the function $J(k, \tau)$ is discretized so that the guess is a matrix of order $M_k \times M_\tau$.

We then solve the model for a given value of C. We start by guessing a value of unemployment u. The we use (28) to determine the value of $k^*(\tau)$ at the M_τ points of τ at which J is evaluated, including $\tau = 0$. Then, given $k^*(0)$, use (25), (27) and

(31) to solve for θ, $S(0)$ and H. Given (24) and (30) determine $\tau^*(k)$ at the M_k points of k at which J is evaluated. Proxy $\bar{\tau}$ with $\max_k \tau^*(k)$. Finally, and given a quadrature approximation (see below), use the previous quantities to determine the value of $h(l,\tau)$ at $I_k \times I_\tau$ point as a solution of the linear system implied by (36) after substituting m by using (37). We then iterate over u to find the value that solves the resource constraint $1 - N - u = 0$ by using a bisecant method where N is given by (32). Given this equilibrium value of u we iterate over consumption to find the value of C that solves (35) by using a bisecant method.
Updating: Given $\tau^*(k), \bar{\tau}$ and the new values of $S(k,\tau)$ and $S(\tau)$ use (26) to update $J(k,\tau)$.
Checking: If the updated expression for $J(k,\tau)$ is equal to the initial guess the algorithm has converged, otherwise construct a new guess for $J(k,\tau)$ by using the new updated expression.

A.4 Quadrature approximation

To compute integrals with infinite bounds of integration such as $\int_{-\infty}^{\infty} f(x)dx$, we first truncate the range of integration to (x_1, x_M). The abscissas $x_1 < x_2 <....< x_M$ are uniformly distributed over the interval (x_1, x_M) so that each interval has size $(x_M - x_1)/\,(M-1)$. Then the integral is approximated by the weighted sum $\sum_{j=1}^{M} f(x_j)w_j$ where $w_j = (x_M - x_1)/\,(M-1)$ are the integration weights.

In several cases either the lower or the upper bound of the range of integration is truncated and equal to either $\bar{\tau}$ or one of the $\tau^*(e^j)$. We accommodate this by taking into account whether a given point x_j is inside the range of integration and we follows Press et al. (1989) in using a trapezoidal rule to improve the quality of the approximation close to the endpoints. Specifically, we approximate integrals of the type $\int_{-\infty}^{\bar{\tau}} f(x)dx$ with $\sum_{j=1}^{\bar{M}+1} f(x_j)w_j$ where $\bar{M} = \max\{i : x_i \leq \bar{\tau}\}$, $w_j = (x_M - x_1)/\,(M-1)\,, \forall j < \bar{M}$, $w_{\bar{M}} = (1+\bar{w}) \cdot 1/2 \cdot (x_M - x_1)/\,(M-1)$, while $w_{\bar{M}} = \bar{w}/2 \cdot (x_M - x_1)/\,(M-1)$ where $\bar{w} = S(x_{\bar{M}})/\,|S(x_{\bar{M}+1}) - S(x_{\bar{M}})|$. The weight $\bar{w}$ intends to better approximate the integral by linearly interpolating $x_{\bar{M}}$ and $x_{\bar{M}+1}$ to obtain a more accurate expression for $\bar{\tau}$. The case where the upper bound is one of the $\tau^*(e^j)$ or where the lower bound of the range of integration is truncated is treated analogously. The grid of points used to approximate the integrals is always the finest as available.

A.5 Computational details

In our baseline specification the support for τ and $\ln k$ is given by $[-.085, .058]$ and $[1.280, 1.831]$, respectively. In solving for the steady state we characterize the beginning-of-period distribution at a grid of 17 technological gaps, $I_\tau = 17$, and 24 capital levels, $I_k = 24$, equally spaced on the support for τ and $\ln k$, respectively. The function J is instead evaluated at a more accurate grid of 102 technological gaps, $M_\tau = 102$, and 120 capital levels, $M_k = 120$.

B Linearization Procedure

We next discuss how we linearize the equilibrium conditions of the model. In what follows we linearize with respect to the log of a given variable when this is defined only for positive values, while we will linearize with respect to any variable which is defined over the whole real line. Before formally discussing our linearization procedure we simplify some equilibrium conditions of the model and introduce some useful new objects.

B.1 New objects

Like in the analysis of the steady state we focus on the dynamics of the distribution of log capital and technological gap $h(l,\tau)$. By the same reasoning that allowed us to write (23), we obtain

$$\begin{aligned}
h_t(l,\tau) = (1-a_q)a_z \int_R \left[\int_{-\infty}^{\tau^*_{t-1}(e^j)} g_\delta \left(j - l - \frac{g_{qt}+g_{zt}}{1-\alpha} \right) g_\epsilon(\tau) h_{t-1}(j,i) di \right] dj \\
+(1-a_q)(1-a_z) \int_R \left[\int_{-\infty}^{\tau^*_{t-1}(e^j)} g_\delta \left(j - l - \frac{g_{qt}+g_{zt}}{1-\alpha} \right) g_\epsilon(i + g_{zt} - \tau) h_{t-1}(j,i) di \right] dj \\
+a_q a_z \int_R \left[\int_{-\infty}^{\bar{\tau}_{t-1}} g_\delta \left(\ln k^*_{t-1}(i) - l - \frac{g_{qt}+g_{zt}}{1-\alpha} \right) g_\epsilon(\tau) h_{t-1}(j,i) di \right] dj \\
+a_q (1-a_z) \int_R \left[\int_{-\infty}^{\bar{\tau}_{t-1}} g_\delta \left(\ln k^*_{t-1}(i) - l - \frac{g_{qt}+g_{zt}}{1-\alpha} \right) g_\epsilon(i + g_{zt} - \tau) h_{t-1}(j,i) di \right] dj \\
+g_\delta \left(\ln k^*_{t-1}(0) - l - \frac{g_{qt}+g_{zt}}{1-\alpha} \right) \left[a_z g_\epsilon(\tau) + (1-a_z) g_\epsilon(g_{zt} - \tau) \right] m_{t-2},
\end{aligned} \tag{38}$$

To log-linearize the model, we use the following more explicit expression for

$J_t(k,\tau)$:

$$J_t(k,\tau) = E_t\{\frac{\rho C_t}{C_{t+1}}(1\text{-}a_q)a_z\int_R[\int_{-\infty}^{\tau^*_{t+1}(\Delta_t(i)k)} S_{t+1}(\Delta_{t+1}(i)k,j)dG_\epsilon(j)]dG_\delta(i)\}$$

$$+E_t\{\frac{\rho C_t}{C_{t+1}}(1\text{-}a_q)(1\text{-}a_z)\int_R[\int_{-\infty}^{\tau^*_{t+1}(\Delta_{t+1}(i)k)\text{-}\tau\text{-}g_{zt+1}} S_{t+1}(\Delta_{t+1}(i)k,\tau+g_{zt+1}+j)dG_\epsilon(j)]dG_\delta(i)\}$$

$$+E_t\{\frac{\rho C_t}{C_{t+1}}a_q[a_z\int_{\text{-}\infty}^{\bar{\tau}_{t+1}} S_{t+1}(j)dG_\epsilon(j) + (1\text{-}a_z)\int_{\text{-}\infty}^{\bar{\tau}_{t+1}\text{-}\tau\text{-}g_{zt+1}} S_{t+1}(\tau+g_{zt+1}+j)dG_\epsilon(j)]\},$$

which after some changes of variables and after remembering that

$$g_{qt} + g_{xt} = \frac{g_{qt} + g_{zt}}{1-\alpha}$$

can be written as

$$J_t(k,\tau) = E_t\{\frac{\rho C_t}{C_{t+1}}(1\text{-}a_q)a_z\int_R[\int_{\text{-}\infty}^{\tau^*_{t+1}(i)} S_{t+1}(i,j)g_\epsilon(j)dj]g_\delta(\ln k\text{-}\ln i\text{-}\ \frac{g_{qt+1}+g_{zt+1}}{1-\alpha})\}$$
$$+E_t\{\frac{\rho C_t}{C_{t+1}}(1\text{-}a_q)(1\text{-}a_z)\int_R[\int_{\text{-}\infty}^{\tau^*_{t+1}(i)} S_{t+1}(i,j)g_\epsilon(j\text{-}\tau\text{-}g_{zt+1})dj]g_\delta(\ln k\text{-}\ln i\text{-}\ \frac{g_{qt+1}+g_{zt+1}}{1-\alpha})di\}$$
$$+E_t\{\frac{\rho C_t}{C_{t+1}}a_q[a_z\int_{\text{-}\infty}^{\bar{\tau}_{t+1}} S_{t+1}(j)g_\epsilon(j)dj + (1-a_z)\int_{\text{-}\infty}^{\bar{\tau}_{t+1}} S_{t+1}(j)g_\epsilon(j\text{-}\tau\text{-}g_{zt+1})dj]\}. \tag{39}$$

where in writing the integrals we made use of the fact that the distribution of ϵ is symmetric around zero.

By deriving with respect to k in (39) we also obtain that

$$\frac{\partial J_t(k_t^*(\tau),\tau)}{\partial k} = \frac{\rho C_t(1-a_q)}{k_t^*(\tau)}\cdot$$
$$\left(E_t\left\{\frac{a_z}{C_{t+1}}\int_R\left[\int_{\text{-}\infty}^{\tau^*_{t+1}(i)} S_{t+1}(i,j)g_\epsilon(j)dj\right]g'_\delta(\ln k_t^*(\tau)\text{-}\ln i\text{-}\ \frac{g_{qt+1}+g_{zt+1}}{1-\alpha})di\right\}\right.$$
$$\left.+E_t\left\{\frac{(1\text{-}a_z)}{C_{t+1}}\int_R\left[\int_{\text{-}\infty}^{\tau^*_{t+1}(i)} S_{t+1}(i,j)g_\epsilon(j\text{-}\tau\text{-}g_{zt+1})dj\right]g'_\delta(\ln k_t^*(\tau)\text{-}\ln i\text{-}\ \frac{g_{qt+1}+g_{zt+1}}{1-\alpha})di\right\}\right) \tag{40}$$

that will be used below.

B.2 Choice of variables

Let $\hat{S}_t(i,j) \equiv S_t(i,j) - S(i,j)$ and $\hat{S}_t(j) \equiv S_t(j) - S(j)$ denote the deviation from its steady state value of the surplus and the maximized-with-respect-to capital surplus, respectively. It then follows from the envelope theorem and (9) that

$$\hat{S}_t(\tau) = \hat{S}_t(k^*(\tau), \tau).$$

Similarly let $\hat{\tau}^*_t(i) \equiv \tau^*_t(e^i) - \tau^*(e^i)$ and let $\hat{\bar{\tau}}_t \equiv \bar{\tau}_t - \bar{\tau}$, denote the deviation from its steady state value of the critical technological gap at a given logged capital i and of the critical technological gap when the firm can upgrade its capital quality, respectively. Then the envelope theorem and (11) imply that

$$\hat{\bar{\tau}}_t = \hat{\tau}^*_t(\ln(k^*(\bar{\tau})))).$$

Given these considerations, we will linearize the equilibrium conditions of the model (8), (10)-(15), (17)-(19) and (38) with respect to the following variables:

- The log of $h_t(i, j)$ at $I_k \times I_\tau$ combinations of (logged) capital stock and technological gap, $\hat{h}_t(i, j) \equiv \ln h_t(i, j) - \ln h(i, j)$.
- The log of $k^*_t(\tau)$ at I_τ technological gaps, $\hat{k}^*_t(\tau) \equiv \ln k^*_t(\tau) - \ln k^*(\tau)$.
- The log of θ_t, $\hat{\theta}_t \equiv \ln \theta_t - \ln \theta$.
- The log of m_t, $\hat{m}_t \equiv \ln m_t - \ln m$.
- The log of m_{t-1}, $\hat{m}_{t-1} \equiv \ln m_{t-1} - \ln m$.
- The quantity $S_t(k, \tau)$ at $I_k \times I_\tau$ combinations of capital stock and technological gap, $\hat{S}_t(i, j) \equiv S_t(i, j) - S(i, j)$.
- The quantity $\tau^*_t(k)$ at I_k capital levels, $\hat{\tau}^*_t(i) \equiv \tau^*_t(e^i) - \tau^*(e^i)$.
- The log of C_t, $\hat{C}_t \equiv \ln C_t - \ln C$.
- The log of H_t, $\hat{H}_t \equiv \ln H_t - \ln H$.
- The aggregate shocks g_{zt} and g_{qt}, $\hat{g}_{zt} \equiv g_{zt} - \mu_z$ and $\hat{g}_{qt} \equiv g_{qt} - \mu_q$.

B.3 Linearized equations

We now proceed to linearize the equilibrium conditions of the model around the steady state.

B.3.1 Linearization of (38)

By using (38), it follows that

$$
\begin{aligned}
h(l,\tau)\hat{h}_t(l,\tau) &= (1-a_q)a_z \int_R \left[\int_{-\infty}^{\tau^*(e^j)} g_\delta(j-l-\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(\tau)h(j,i)\hat{h}_{t\text{-}1}(j,i)di\right]dj \\
&+ (1-a_q)(1-a_z)\int_R \left[\int_{-\infty}^{\tau^*(e^j)} g_\delta(j-l-\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(i+\mu_z-\tau)h(j,i)\hat{h}_{t\text{-}1}(j,i)di\right]dj \\
&+ a_q a_z \int_R \left[\int_{-\infty}^{\bar{\tau}} g_\delta\left(\ln k^*(i) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(\tau)h(j,i)\hat{h}_{t\text{-}1}(j,i)di\right]dj \\
&+ a_q(1-a_z)\int_R \left[\int_{-\infty}^{\bar{\tau}} g_\delta\left(\ln k^*(i) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(i+\mu_z-\tau)h(j,i)\hat{h}_{t\text{-}1}(j,i)di\right]dj \\
&+ a_q a_z \int_{-\infty}^{\bar{\tau}}\left[\int_R h(j,i)dj\right] g'_\delta\left(\ln k^*(i) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(\tau)\hat{k}^*_{t-1}(i)di \\
&+ a_q(1-a_z)\int_{-\infty}^{\bar{\tau}}\left[\int_R h(j,i)dj\right] g'_\delta\left(\ln k^*(i) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(i+\mu_z-\tau)\hat{k}^*_{t-1}(i)di \\
&+ g'_\delta\left(\ln k^*(0) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)\left[a_z g_\epsilon(\tau)+(1-a_z)g_\epsilon(\mu_z-\tau)\right]m\hat{k}^*_{t-1}(0), \\
&+ (1-a_q)a_z\int_R g_\delta(j-l-\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(\tau)h(j,\tau^*(e^j))\hat{\tau}^*_{t-1}(j)dj \\
&+ (1-a_q)(1-a_z)\int_R g_\delta(j-l-\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(\tau^*(e^j)+\mu_z-\tau)h(j,\tau^*(e^j))\hat{\tau}^*_{t-1}(j)dj \\
&+ a_q a_z\left[\int_R g_\delta\left(\ln k^*(\bar{\tau}) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(\tau)h(j,\bar{\tau})dj\right]\widehat{\bar{\tau}}_{t-1} \\
&+ a_q(1-a_z)\left[\int_R g_\delta\left(\ln k^*(\bar{\tau}) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(\bar{\tau}+\mu_z-\tau)h(j,\bar{\tau})dj\right]\widehat{\bar{\tau}}_{t-1} \\
&+ g_\delta\left(\ln k^*(0) - l - \frac{\mu_q+\mu_z}{1-\alpha}\right)\left[a_z g_\epsilon(\tau)+(1-a_z)g_\epsilon(\mu_z-\tau)\right]m\hat{m}_{t-2} \\
&- \frac{(1\text{-}a_q)a_z}{1-\alpha}\left\{\int_R\left[\int_{-\infty}^{\tau^*(e^j)} g'_\delta(j\text{-}l\text{-}\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(\tau)h(j,i)di\right]dj\right\}(\hat{g}_{qt}+\hat{g}_{zt}) \\
&- \frac{(1\text{-}a_q)(1\text{-}a_z)}{1-\alpha}\left\{\int_R\left[\int_{-\infty}^{\tau^*(e^j)} g'_\delta(j\text{-}l\text{-}\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(i+\mu_z\text{-}\tau)h(j,i)di\right]dj\right\}(\hat{g}_{qt}+\hat{g}_{zt}) \\
&- \frac{a_q a_z}{1-\alpha}\left\{\int_R\left[\int_{-\infty}^{\bar{\tau}} g'_\delta\left(\ln k^*(i)\text{ -}l\text{-}\frac{\mu_q+\mu_z}{1-\alpha}\right)g_\epsilon(\tau)h(j,i)di\right]dj\right\}(\hat{g}_{qt}+\hat{g}_{zt})
\end{aligned}
$$

$$-\frac{a_q(1\text{-}a_z)}{1-\alpha}\left\{\int_R\left[\int_{-\infty}^{\bar{\tau}} g'_\delta\left(\ln k^*(i) \text{-}l\text{-} \frac{\mu_q+\mu_z}{1-\alpha}\right) g_\epsilon(i+\mu_z \text{-}\tau)h(j,i)di\right]dj\right\}(\hat{g}_{qt}+\hat{g}_{zt})$$
$$-\frac{1}{1-\alpha}\left\{g'_\delta\left(\ln k^*(0)-l-\frac{\mu_q+\mu_z}{1-\alpha}\right)\left[a_z g_\epsilon(\tau)+(1-a_z)\,g_\epsilon(\mu_z-\tau)\right]m\right\}(\hat{g}_{qt}+\hat{g}_{zt})$$
$$+(1\text{-}a_q)(1\text{-}a_z)\left\{\int_R\left[\int_{-\infty}^{\tau^*(e^j)} g_\delta(j-l-\frac{\mu_q+\mu_z}{1-\alpha})g'_\epsilon(i+\mu_z\text{-}\tau)h(j,i)di\right]dj\right\}\hat{g}_{zt}$$
$$+a_q(1\text{-}a_z)\left\{\int_R\left[\int_{-\infty}^{\bar{\tau}} g_\delta\left(\ln k^*(i)\text{-}l-\frac{\mu_q+\mu_z}{1-\alpha}\right)g'_\epsilon(i+\mu_z\text{-}\tau)h(j,i)di\right]dj\right\}\hat{g}_{zt}$$
$$+\left\{g_\delta\left(\ln k^*(0)-l-\frac{\mu_q+\mu_z}{1-\alpha}\right)(1-a_z)\,g'_\epsilon(\mu_z-\tau)m\right\}\hat{g}_{zt}.$$

B.3.2 Linearization of (10)

Let

$$\frac{\partial J(k^*(\tau),\tau)}{\partial k}=\frac{\rho(1\text{-}a_q)a_z}{k^*(\tau)}\int_R\left[\int_{-\infty}^{\tau^*(i)} S(i,j)g_\epsilon(j)dj\right]g'_\delta(\ln k^*(\tau)\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha})di$$
$$+\frac{\rho(1\text{-}a_q)(1\text{-}a_z)}{k^*(\tau)}\int_R\left[\int_{-\infty}^{\tau^*(i)} S(i,j)g_\epsilon(j\text{-}\tau\text{-}\mu_z)dj\right]g'_\delta(\ln k^*(\tau)\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha})di,$$

denote the steady state marginal value of capital at the optimal capital choice for the given technological gap τ. Then by using (10) and (40), it follows that

$$e^{-\tau}\alpha(\alpha-1)\left[k^*(\tau)\right]^{\alpha-1}\hat{k}^*_t(\tau)-\frac{\partial J(k^*(\tau),\tau)}{\partial k}\hat{k}^*_t(\tau)$$
$$+\frac{\rho(1\text{-}a_q)a_z}{k^*(\tau)}\left\{\int_R\left[\int_{-\infty}^{\tau^*(i)} S(i,j)g_\epsilon(j)dj\right]g''_\delta(\ln k^*(\tau)\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha})di\right\}\hat{k}^*_t(\tau)$$
$$+\frac{\rho(1\text{-}a_q)(1\text{-}a_z)}{k^*(\tau)}\left\{\int_R\left[\int_{-\infty}^{\tau^*(i)} S(i,j)g_\epsilon(j\text{-}\tau\text{-}\mu_z)dj\right]g''_\delta(\ln k^*(\tau)\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha})di\right\}\hat{k}^*_t(\tau)$$
$$+\left[\frac{\partial J(k^*(\tau),\tau)}{\partial k}+\rho\bar{\Delta}\right]\hat{C}_t$$
$$+\frac{\rho(1\text{-}a_q)a_z}{k^*(\tau)}\int_R\left[\int_{-\infty}^{\tau^*(i)} g'_\delta(\ln k^*(\tau)\text{-}\ln i-\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(j)E_t\left(\hat{S}_{t+1}(i,j)\right)dj\right]di$$
$$+\frac{\rho(1\text{-}a_q)(1\text{-}a_z)}{k^*(\tau)}\int_R\left[\int_{-\infty}^{\tau^*(i)} g'_\delta(\ln k^*(\tau)\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha})g_\epsilon(j\text{-}\tau\text{-}\mu_z)E_t\left(\hat{S}_{t+1}(i,j)\right)dj\right]di$$
$$-\left[\frac{\partial J(k^*(\tau),\tau)}{\partial k}+\rho\bar{\Delta}\right]E_t\left(\hat{C}_{t+1}\right)=0,$$

where we made use of the fact that $E_t(\hat{g}_{zt+1}) = E_t(\hat{g}_{qt+1}) = 0$.

B.3.3 Linearization of (12)

By using (12) and (39) evaluated at $\tau = \tau^*(k)$ it follows that

$$
\begin{aligned}
&- e^{-\tau^*(k)} k^{\alpha} \hat{\tau}_t^*(\ln k) \\
&- \rho(1\text{-}a_q)(1\text{-}a_z) \left\{ \int_R \left[\int_{-\infty}^{\tau^*(i)} S(i,j) g'_{\epsilon}(j\text{-}\tau^*(k)\text{-}\mu_z) dj \right] g_{\delta}(\ln k\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha}) di \right\} \hat{\tau}_t^*(\ln k) \\
&- \rho a_q(1-a_z) \left[\int_{-\infty}^{\bar{\tau}} S(j) g'_{\epsilon}(j\text{-}\tau^*(k)\text{-}\mu_z) dj \right] \hat{\tau}_t^*(\ln k) \\
&+ \left[\rho \left(H + \bar{\Delta} \right) + J(k, \tau^*(k)) - c_w C \right] \hat{C}_t - H \hat{H}_t \\
&+ \rho(1\text{-}a_q) a_z \int_R \left[\int_{-\infty}^{\tau^*(i)} g_{\delta}(\ln k\text{-} \ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha}) g_{\epsilon}(j) E_t \left(\hat{S}_{t+1}(i,j) \right) dj \right] di \\
&+\rho(1\text{-}a_q)(1\text{-}a_z) \int_R \left[\int_{-\infty}^{\tau^*(i)} g_{\delta}(\ln k\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1-\alpha}) g_{\epsilon}(j\text{-}\tau^*(k)\text{-}\mu_z) E_t \left(\hat{S}_{t+1}(i,j) \right) dj \right] di \\
&+\rho a_q \left[a_z \int_{-\infty}^{\bar{\tau}} g_{\epsilon}(j) E_t \left(\hat{S}_{t+1}(j) \right) dj + (1-a_z) \int_{-\infty}^{\bar{\tau}} g_{\epsilon}(j\text{-}\tau^*(k)\text{-}\mu_z) E_t \left(\hat{S}_{t+1}(j) \right) dj \right] \\
&- \left[\rho \left(H + \bar{\Delta} \right) + J(k, \tau^*(k)) \right] E_t \left(\hat{C}_{t+1} \right) + \rho H E_t(\hat{H}_{t+1}) = 0,
\end{aligned}
$$

where we made use of the fact that $E_t(\hat{g}_{zt+1}) = E_t(\hat{g}_{qt+1}) = 0$.

B.3.4 Linearization of (15)

Notice that

$$r_t = \bar{r} \left(m_t \right)^{\nu} \left[q(\theta_t) \right]^{-\nu}.$$

Then, by linearizing (15), and after using (16) and the fact that $q'(\theta) = -\eta(\theta) q(\theta)/\theta$ where $\eta(\theta)$ is the elasticity of the matching function with respect to unemployment it follows that

$$
\begin{aligned}
&(1+\nu) \bar{r} m^{\nu} \left[q(\theta) \right]^{-\nu-1} \eta(\theta) \hat{\theta}_t + \nu \bar{r} m^{\nu} \left[q(\theta) \right]^{-\nu-1} \hat{m}_t - \frac{\rho(1\text{-}\beta)}{C} E_t \left(\hat{S}_{t+1}(0) \right) \\
&+ \frac{\rho \, (1\text{-}\beta) \, S(0)}{C} E_t \left(\hat{C}_{t+1} \right) = 0.
\end{aligned}
$$

B.3.5 Linearization of (17)

By evaluating (17) at time t, and after using (16) and the fact that $p'(\theta) = (1 - \eta(\theta))q(\theta)$ it follows that

$$p(\theta)\left\{a_q \int_R \left[\int_{-\infty}^{\bar{\tau}} h(j,\tau)\hat{h}_t(j,\tau)d\tau\right]dj + (1\text{-}a_q)\int_R \left[\int_{-\infty}^{\tau^*(e^j)} h(j,\tau)\hat{h}_t(j,\tau)d\tau\right]dj\right\}$$
$$+ p(\theta)\left\{a_q\left[\int_R h(j,\bar{\tau})dj\right]\hat{\bar{\tau}}_t + (1\text{-}a_q)\int_R h(j,\tau^*(e^j)\hat{\tau}_t^*(j)dj\right\}$$
$$-(1-\eta(\theta))p(\theta)(1-N)\hat{\theta}_t + m\hat{m}_t + p(\theta)m\hat{m}_{t-1} = 0.$$

B.3.6 Linearization of (17) at $t-1$

The previous equation evaluated at time $t-1$ can be conveniently expressed as

$$\hat{m}_{t-1} = \hat{m}_{t-1}.$$

B.3.7 Linearization of (8)

By using (8) and (39) one obtains

$$-\hat{S}_t(k,\tau) + \left[\rho\left(H+\bar{\Delta}\right) + J(k,\tau) - c_w C\right]\hat{C}_t - H\hat{H}_t$$
$$+\rho(1\text{-}a_q)a_z\int_R\left[\int_{\text{-}\infty}^{\tau^*(i)} g_\delta\left(\ln k\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1\text{–}\alpha}\right)g_\epsilon(j)E_t\left(\hat{S}_{t+1}(i,j)\right)dj\right]di$$
$$+\rho(1\text{-}a_q)(1\text{-}a_z)\int_R\left[\int_{\text{-}\infty}^{\tau^*(i)} g_\delta(\ln k\text{-}\ln i\text{-}\frac{\mu_q+\mu_z}{1\text{–}\alpha})g_\epsilon(j\text{-}\tau\text{-}\mu_z)E_t\left(\hat{S}_{t+1}(i,j)\right)dj\right]di$$
$$+\rho a_q\left[a_z\int_{\text{-}\infty}^{\bar{\tau}} g_\epsilon(j)E_t\left(\hat{S}_{t+1}(j)\right)dj + (1\text{–}a_z)\int_{\text{-}\infty}^{\bar{\tau}} g_\epsilon(j\text{-}\tau\text{-}\mu_z)E_t\left(\hat{S}_{t+1}(j)\right)dj\right]$$
$$-\left[\rho\left(H+\bar{\Delta}\right) + J(k,\tau)\right]E_t(\hat{C}_{t+1}) + \rho H E_t(\hat{H}_{t+1}) = 0$$

where we made use of the fact that $E_t(\hat{g}_{zt+1}) = E_t(\hat{g}_{qt+1}) = 0$.

B.3.8 Linearization of (19)

By using (19), and after taking into account the definition of Y_t and I_t it follows that

$$
\begin{aligned}
&\int_R [a_q \int_{-\infty}^{\bar{\tau}} e^{-\tau} \left[k^*(\tau)\right]^{\alpha} h(j,\tau)\hat{h}_t(j,\tau)d\tau + (1\text{-}a_q) \int_{-\infty}^{\tau^*(e^j)} e^{-\tau} \exp(\alpha j) h(j,\tau)\hat{h}_t(j,\tau)d\tau] dj \\
&- a_q \int_{R\times[-\infty,\bar{\tau}]} k^*(\tau)h(j,\tau)\hat{h}_t(j,\tau)dj\,d\tau + \int_{R^2} \exp(j)h(j,\tau)\hat{h}_t(j,\tau)dj\,d\tau \\
&- (1\text{-}a_q) \int_R \left[\int_{-\infty}^{\tau^*(e^j)} \exp(j)h(j,\tau)\hat{h}_t(j,\tau)d\tau\right] dj \\
&+ a_q\alpha \int_{-\infty}^{\bar{\tau}} \left[\int_R h(j,\tau)dj\right] e^{-\tau} \left[k^*(\tau)\right]^{\alpha}\ \hat{k}^*_t(\tau)d\tau \\
&- a_q \int_{-\infty}^{\bar{\tau}} \left[\int_R h(j,\tau)dj\right] k^*(\tau)\hat{k}^*_t(\tau)d\tau + m\alpha \left[k^*(0)\right]^{\alpha} \hat{k}^*_t(0) - mk^*(0)\hat{k}^*_t(0) \\
&+ (1\text{–}a_q) \int_R \left[e^{-\tau^*(e^j)} \exp(\alpha j) - \exp(j)\right] h(j,\tau^*(e^j))\hat{\tau}^*_t(j)dj \\
&+ a_q \left\{\int_R e^{-\bar{\tau}} \left[k^*(\bar{\tau})\right]^{\alpha} h(j,\bar{\tau})dj - \int_R k^*(\bar{\tau})h(j,\bar{\tau})dj\right\} \hat{\bar{\tau}}_t \\
&+ \left\{\left[k^*(0)\right]^{\alpha} - k^*(0)\right\} m\hat{m}_{t-1} - C\hat{C}_t = 0.
\end{aligned}
$$

B.3.9 Linearization of (14)

By using (14), it follows that

$$
\begin{aligned}
&\rho(1-\eta(\theta))p(\theta)\beta S(0)\hat{\theta}_t + \rho\left[H + p(\theta)\beta S(0)\right]\hat{C}_t - H\hat{H}_t \\
&+ \rho p(\theta)\beta E_t\left(\hat{S}_{t+1}(0)\right) - \rho\left[H + p(\theta)\beta S(0)\right] E_t\left(\hat{C}_{t+1}\right) + \rho H E_t\left(\hat{H}_{t+1}\right) = 0.
\end{aligned}
$$

B.4 Implementation of Sims' method

Sims (2002) consider linear rational expectations models written in the form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + Co + \Psi z_t + \Pi\eta_t$$

where y_t is the set of variables determined at time t, Co is a vector of constants, z_t is a vector of exogenous shocks while η_t is a vector of expectational errors, $E_{t-1}(\eta_t) = 0$, $\forall t$.

In our case $Co = 0$ since we are linearizing around the steady state, while y_t is given by the following vector of dimension $n = 3(I_k \cdot I_\tau) + 7 + I_\tau + I_k$:

$$y_t = \begin{bmatrix} \underset{I_k \cdot I_\tau \times 1}{\hat{h}_t(j,\tau)} \\ \cdots\cdots\cdots \\ \underset{I_\tau \times 1}{\hat{k}^*_t(\tau)} \\ \cdots\cdots\cdots \\ \underset{I_k \times 1}{\hat{\tau}^*_t(j)} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{\hat{\theta}_t} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{\hat{m}_t} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{\hat{m}_{t-1}} \\ \cdots\cdots\cdots \\ \underset{I_k \cdot I_\tau \times 1}{\hat{S}_t(k,\tau)} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{\hat{C}_t} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{\hat{H}_t} \\ \cdots\cdots\cdots \\ \underset{I_k \cdot I_\tau \times 1}{E_t(\hat{S}_{t+1}(k,\tau))} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{E_t(\hat{C}_{t+1})} \\ \cdots\cdots\cdots \\ \underset{1\times 1}{E_t(\hat{H}_{t+1})} \end{bmatrix} \tag{41}$$

so that the number of expectational errors is given by $r = I_k \cdot I_\tau + 2$. Thereafter, we approximate all integrals with quadrature methods by following the procedure detailed in section A.4. The fixed grid of points corresponds to the grid used to characterize the beginning-of-period distribution in the computation of the steady state equilibrium, i.e. $I_\tau = 17$, $I_k = 24$. One can easily see that $\hat{h}_t$ is a predetermined variable, $\hat{S}_t(k,\tau)$, $\hat{C}_t$ and $\hat{H}_t$ are jump variables while $\hat{k}^*_t(\tau)$, $\hat{\tau}^*_t(j)$, $\hat{\theta}_t$, $\hat{m}_t$ and $\hat{m}_{t-1}$ are redundant in the sense that in principle they could expressed as a function of the remaining variables contained in the vectors y_t and y_{t-1}. This also implies that Γ_0 has no full rank and therefore is not invertible.

B.4.1 The matrix Γ_0

The matrix Γ_0 of dimension $n \times n$ has the following form:

$$
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
\hline
\underset{I_kI_\tau \times I_kI_\tau}{\mathbf{A}^0} & \underset{I_kI_\tau \times I_\tau}{0} & \underset{I_kI_\tau \times I_k}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times I_kI_\tau}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times I_kI_\tau}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} \\
\hline
\underset{I_\tau \times I_kI_\tau}{0} & \underset{I_\tau \times I_\tau}{\mathbf{B}^0} & \underset{I_\tau \times I_k}{0} & \underset{I_\tau \times 1}{0} & \underset{I_\tau \times 1}{0} & \underset{I_\tau \times 1}{0} & \underset{I_\tau \times I_kI_\tau}{0} & \underset{I_\tau \times 1}{\mathbf{C}^0} & \underset{I_\tau \times 1}{0} & \underset{I_\tau \times I_kI_\tau}{\mathbf{D}^0} & \underset{I_\tau \times 1}{\mathbf{E}^0} & \underset{I_\tau \times 1}{0} \\
\hline
\underset{I_k \times I_kI_\tau}{0} & \underset{I_k \times I_\tau}{0} & \underset{I_k \times I_k}{\mathbf{F}^0} & \underset{I_k \times 1}{0} & \underset{I_k \times 1}{0} & \underset{I_k \times 1}{0} & \underset{I_k \times I_kI_\tau}{0} & \underset{I_k \times 1}{\mathbf{G}^0} & \underset{I_k \times 1}{\mathbf{H}^0} & \underset{I_k \times I_kI_\tau}{\mathbf{L}^0} & \underset{I_k \times 1}{\mathbf{O}^0} & \underset{I_k \times 1}{\mathbf{P}^0} \\
\hline
\underset{1 \times I_kI_\tau}{0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{0} & \underset{1 \times 1}{\mathbf{Q}^0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{\mathbf{R}^0} & \underset{1 \times 1}{\mathbf{S}^0} & \underset{1 \times 1}{0} \\
\hline
\underset{1 \times I_kI_\tau}{\mathbf{T}^0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{\mathbf{U}^0} & \underset{1 \times 1}{\mathbf{V}^0} & \underset{1 \times 1}{\mathbf{Z}^0} & \underset{1 \times 1}{\mathbf{AA}^0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} \\
\hline
\underset{1 \times I_kI_\tau}{0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{\mathbf{1}} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} \\
\hline
\underset{I_kI_\tau \times I_kI_\tau}{0} & \underset{I_kI_\tau \times I_\tau}{0} & \underset{I_kI_\tau \times I_k}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times I_kI_\tau}{-\mathbf{I}} & \underset{I_kI_\tau \times 1}{\mathbf{BB}^0} & \underset{I_kI_\tau \times 1}{\mathbf{CC}^0} & \underset{I_kI_\tau \times I_kI_\tau}{\mathbf{DD}^0} & \underset{I_kI_\tau \times 1}{\mathbf{EE}^0} & \underset{I_kI_\tau \times 1}{\mathbf{FF}^0} \\
\hline
\underset{1 \times I_kI_\tau}{\mathbf{GG}^0} & \underset{1 \times I_\tau}{\mathbf{HH}^0} & \underset{1 \times I_k}{\mathbf{LL}^0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{\mathbf{MM}^0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{\mathbf{NN}^0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} \\
\hline
\underset{1 \times I_kI_\tau}{0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{0} & \underset{1 \times 1}{\mathbf{OO}^0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{\mathbf{PP}^0} & \underset{1 \times 1}{\mathbf{QQ}^0} & \underset{1 \times I_kI_\tau}{\mathbf{RR}^0} & \underset{1 \times 1}{\mathbf{SS}^0} & \underset{1 \times 1}{\mathbf{TT}^0} \\
\hline
\underset{I_kI_\tau \times I_kI_\tau}{0} & \underset{I_kI_\tau \times I_\tau}{0} & \underset{I_kI_\tau \times I_k}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} & \multicolumn{3}{c|}{} & \underset{I_kI_\tau \times I_kI_\tau}{0} & \underset{I_kI_\tau \times 1}{0} & \underset{I_kI_\tau \times 1}{0} \\
\underset{1 \times I_kI_\tau}{0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \multicolumn{3}{c|}{\underset{r \times r}{\mathbf{I}}} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} \\
\underset{1 \times I_kI_\tau}{0} & \underset{1 \times I_\tau}{0} & \underset{1 \times I_k}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} & \multicolumn{3}{c|}{} & \underset{1 \times I_kI_\tau}{0} & \underset{1 \times 1}{0} & \underset{1 \times 1}{0} \\
\end{array}
$$

where the sub-matrices denoted with alphabetical letters and a suffix "0" can be recovered from the previous equation while I denotes an identity matrix. Notice that the first 9 blocks of rows correspond to equations (38), (10), (12), (15), (17), (17) evaluated at time $t-1$, (8), (19) and (14), respectively. The remaining three blocks of rows correspond instead to the equations that define the expectational errors.

B.4.2 The matrix Γ_1

The matrix Γ_1 of dimension $n \times n$ has the following form:

$\underset{I_kI_\tau\times I_kI_\tau}{\mathbf{A}^1}$	$\underset{I_kI_\tau\times I_\tau}{\mathbf{B}^1}$	$\underset{I_kI_\tau\times I_k}{\mathbf{C}^1}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{\mathbf{D}^1}$	$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$
$\underset{I_\tau\times I_kI_\tau}{0}$	$\underset{I_\tau\times I_\tau}{0}$	$\underset{I_\tau\times I_k}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times I_kI_\tau}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times I_kI_\tau}{0}$	$\underset{I_\tau\times 1}{0}$	$\underset{I_\tau\times 1}{0}$
$\underset{I_k\times I_kI_\tau}{0}$	$\underset{I_k\times I_\tau}{0}$	$\underset{I_k\times I_k}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times I_kI_\tau}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times I_kI_\tau}{0}$	$\underset{I_k\times 1}{0}$	$\underset{I_k\times 1}{0}$
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{\mathbf{1}}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$
$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times I_\tau}{0}$	$\underset{I_kI_\tau\times I_k}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$
$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times I_\tau}{0}$	$\underset{I_kI_\tau\times I_k}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times I_kI_\tau}{0}$	$\underset{I_kI_\tau\times 1}{0}$	$\underset{I_kI_\tau\times 1}{0}$			
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$		$\underset{r\times r}{\mathbf{I}}$	
$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times I_\tau}{0}$	$\underset{1\times I_k}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times I_kI_\tau}{0}$	$\underset{1\times 1}{0}$	$\underset{1\times 1}{0}$			

where the sub-matrices denoted with alphabetical letters and a suffix "1" can be recovered from the previous equations .

B.4.3 The matrix Ψ

The matrix Ψ of dimension $n \times 2$ has the following form:

$$\begin{bmatrix} \underset{I_kI_\tau\times 2}{A^{\boldsymbol{\Psi}}} \\ \cdots\cdots\cdots \\ \underset{(n-I_kI_\tau)\times 2}{0} \end{bmatrix}$$

where the sub-matrix $A^{\boldsymbol{\Psi}}$ can be recovered from the previous equations. We keep the convention that the first column refers to the z-shock while the second to the q-shock.

B.4.4 The matrix Π

The matrix Π of dimension $n \times r$ has the following form:

$$\begin{bmatrix} \underset{(n-r)\times r}{0} \\ \cdots\cdots \\ \underset{r\times r}{\mathbf{I}} \end{bmatrix}.$$

B.5 Implied impulse responses

In this section we discuss how to recover the impulse responses of employment, N_t, output, Y_t, and job destruction, JD_t, given the impulse responses of the variables included in the vector y_t in (41).

Let $\tilde{h}_t(j,\tau)$, $\tilde{k}_t^*(\tau)$, $\tilde{\tau}_t^*(j)$, $\tilde{\theta}_t$, $\tilde{m}_t$ and $\tilde{C}_t$ denote the impulse responses of $\hat{h}_t(j,\tau)$, $\hat{k}_t^*(\tau)$, $\hat{\tau}_t^*(j)$, $\hat{\theta}_t$, $\hat{m}_t$, and $\hat{C}_t$, respectively. Then, and given (18), the impulse response of $\hat{Y}_t = Y_t - Y$ is equal to

$$\begin{aligned}
\tilde{Y}_t = & \int_R [a_q \int_{-\infty}^{\bar{\tau}} e^{-\tau} \left[k^*(\tau)\right]^\alpha h(j,\tau)\tilde{h}_t(j,\tau)d\tau + (1\text{-}a_q)\int_{-\infty}^{\tau^*(e^j)} e^{-\tau}\exp(\alpha j)h(j,\tau)\tilde{h}_t(j,\tau)d\tau]dj \\
& + a_q\alpha \int_{-\infty}^{\bar{\tau}} \left[\int_R h(j,\tau)dj\right] e^{-\tau} \left[k^*(\tau)\right]^\alpha \ \tilde{k}_t^*(\tau)d\tau + m\alpha \left[k^*(0)\right]^\alpha \tilde{k}_t^*(0) \\
& + (1 - a_q)\int_R e^{-\tau^*(e^j)}\exp(\alpha j)h(j,\tau^*(e^j))\tilde{\tau}_t^*(j)dj \\
& + a_q \left\{\int_R e^{-\bar{\tau}} \left[k^*(\bar{\tau})\right]^\alpha h(j,\bar{\tau})\, dj\right\} \tilde{\bar{\tau}}_t + \left[k^*(0)\right]^\alpha m\tilde{m}_{t-1}.
\end{aligned}$$

Analogously, and given (16) the impulse response of $\hat{N}_t = N_t - N$ is given by

$$\begin{aligned}
\tilde{N}_t = & \, a_q \int_R \left[\int_{-\infty}^{\bar{\tau}} h(j,\tau)\tilde{h}_t(j,\tau)d\tau\right] dj + (1\text{-}a_q)\int_R \left[\int_{-\infty}^{\tau^*(e^j)} h(j,\tau)\tilde{h}_t(j,\tau)d\tau\right] dj \\
& + a_q \left[\int_R h(j,\bar{\tau})dj\right] \tilde{\bar{\tau}}_t + (1\text{-}a_q)\int_R h(j,\tau^*(e^j)\tilde{\tau}_t^*(j)dj + m\tilde{m}_{t-1}.
\end{aligned}$$

Finally, by using the relation $N_t = N_{t-1} + JC_t - JD_t$ and after remembering that $JC_t = m_{t-1}$, it follows that the impulse response of $\widetilde{JD}_t = JD_t - JD$ solves

$$\widetilde{JD}_t = \tilde{m}_{t-1} + \tilde{N}_{t-1} - \tilde{N}_t.$$

Given these results, the impulse responses of the job destruction rate, $\widetilde{jd}_t$, and job creation rate, $\widetilde{jc}_t$, can be recovered by noticing that

$$\widetilde{jd}_t = \frac{1}{N}\widetilde{JD}_t - \frac{JD}{2N^2}\tilde{N}_{t-1} - \frac{JD}{2N^2}\tilde{N}_t,$$

and

$$\widetilde{jc}_t = \frac{m}{N}\tilde{m}_{t-1} - \frac{JD}{2N^2}\tilde{N}_{t-1} - \frac{JD}{2N^2}\tilde{N}_t,$$

respectively.