Productivity Shocks and Consumption Smoothing in the International Economy

March 12, 2003

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We thank Jaewoo Lee and participants in ASSA 2003 Annual Meetings for comments, and Kit Baum for sharing his data. Ghironi gratefully acknowledges funding from the Richard W. Tresch Junior Faculty Fund of the Department of Economics, Boston College.

Abstract

This paper develops a framework that links cross-country heterogeneity in current account and consumption dynamics to structural differences. Our theoretical framework demonstrates the significance of net foreign asset accumulation for consumption smoothing in the international economy using a two-country general equilibrium model, and delivers a range of novel restrictions. We test for and then impose on the G7 data restrictions implied by the model, compare the model's predictions for consumption and net foreign asset dynamics with the estimated impulse responses, and assess the role of heterogeneity across countries in accounting for variations in consumption and net foreign assets positions.

JEL Classification: F41, C33

Keywords: net foreign assets, consumption smoothing, heterogeneity

1 Introduction

There has long been a considerable interest in current account dynamics in open economy macroeconomics. Perhaps the single most important lesson that emerges from the empirical open economy literature is that current account dynamics exhibit considerable variation across countries. For instance, the present value tests of the current account seem to do a remarkable job in explaining the current account of France (Agénor et al., 1999), but the same model performs very poorly for, say, Canada (Sheffrin and Woo, 1991; İşcan, 2002). Also, when current account dynamics are analyzed jointly with investment, (i) global productivity shocks seem to matter for the current accounts (Glick and Rogoff, 1995), and (ii) current account responses to country-specific and global productivity shocks exhibit marked variation across countries (İşcan, 2000). There is a distinctly elegant framework behind all these tests of the current account based on a small open economy assumption. However, we feel that this substantial body of (independent) evidence is suggestive of the fact that, to account for current account dynamics, one must allow for either structural differences across countries or interdependence or both.

In this paper, we first develop a framework that allows for structural heterogeneity *and* interdependence that ultimately leads to cross-country differences in net foreign asset positions, and then take a fresh look at the data.[1] Specifically, we study the role of net foreign asset distribution across countries for consumption smoothing in the international economy. The interdependence issue is by construction assumed away in the small open economy literature, and much of the macroeconomic interdependence literature focuses on cases in which net foreign assets are zero in the long-run. Yet, we argue that the *non-zero* cross-country distribution of assets has important theoretical and empirical implications, and offers novel insights.[2]

In our theoretical framework, we focus on both the steady-state and the short-run dynamics. We identify conditions under which a country would be

[1] Earlier current account models are summarized in Sen (1994). The predecessors of our theoretical approach include those described in Obstfeld and Rogoff (1996).

[2] Some of these points have been forcefully made in a series of important papers by Lane and Milesi-Ferretti (2001, 2002*a*, *b*). The very interesting paper by Masson et al. (1994) is the closest in spirit to our work, although they don't emphasize interdependence, and primarily focus on the impact of aging and public debt on net foreign assets of Germany, Japan, and the U.S. They also identify considerable heterogeneity in the data.

a debtor or creditor in the long-run. The *non-zero* steady-state net foreign asset position implies that the interest burden on previously accumulated debt is important for the *deviations* from the steady-state (or, the dynamics of the log-linearized model). Global productivity shocks lead to current account responses, and debtor and creditor countries' responses to country-specific shocks are asymmetric. A shock to world productivity matters for current account dynamics both through its impact on relative prices and the real interest burden (or income) on previously accumulated assets. This intuitive result contrasts with the earlier interpretations of the current account dynamics whereby world productivity shocks were assumed to be orthogonal to the current accounts of individual countries. Our formal analysis shows the importance of an international transmission mechanism that is overlooked in the two-country models with a symmetric, zero-assets steady state, and is altogether absent in the small open economy models.

While a range of structural factors may be responsible for non-zero steady-state net foreign asset holdings, we consider two ultimate causes: differences across countries in (i) subjective discount factors, and (ii) productivity levels. We incorporate these extensions into the overlapping-generations model of Ghironi (2000), which allows us to determine the cross-country distribution of asset holdings. The departure from the analytically convenient assumption of identical discount rates across-countries is crucial but one that we believe is realistic. Indeed, our analysis shows that even very "small" cross-country variations in discount rates lead to qualitatively different dynamics.[3] As for the second source of structural heterogeneity, we document this (not-so-surprising) fact in our empirical analysis.

We use a calibrated version of our model to analyze the short-run dynamics of consumption and net foreign assets. We consider the responses of these variables to productivity shocks in the creditor and debtor countries and to shocks to world productivity, and distinguish between transitory and permanent shocks. For instance, in response to a permanent, positive world productivity shock, we find that output and consumption increase in both countries. However, upon impact, output increases by more in the patient,

[3] As shown in Ghironi (2000), when the discount factors are identical across countries, the long-run net foreign assets are zero regardless of long-run productivity levels. In this special case, the interest rate and the terms of trade play no role in studying the deviations from the steady-state, and net foreign asset dynamics are determined simply by *relative* productivity. Hence, in a baseline model of interdependence, global shocks are indeed neutral.

less productive country and then decreases toward the new steady state level. By contrast, output in the less patient, more productive economy increases over time. On the other hand, upon impact, consumption increases by more in the less patient, more productive country, and decreases over time, whereas it increases over time in the patient, less productive economy. These dynamics are partly driven by the relative price effects, and imply an increased indebtedness in the less patient country.

In our empirical work, we test for and then impose on G7 data some of the restrictions implied by the model, and compare the model's predictions for consumption and net foreign asset dynamics with the estimated impulse responses.[4] We find that differences in net foreign asset positions across countries helps account for and interpret variations in consumption. In particular, the impulse responses to permanent world productivity shocks differ systematically across countries, and these differences have interesting counterparts in our theoretical framework. For instance, while the U.S. exhibits the behavior of a less patient, more productive economy, Japan emerges as a patient but less productive economy. Our identification scheme suggests that a positive, permanent global shock increases the foreign indebtedness of the U.S., while Japan accumulates net foreign assets. Overall, our theoretical and empirical findings seem to provide plausible explanations for why countries may respond differently to global shocks.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 discusses the model solution and illustrates some of its properties. Section 4 describes the econometric framework, and reports the empirical findings. Section 5 concludes. Technical details of the theoretical analysis and the data are contained in two appendices.

2 The Theoretical Model

2.1 Setup

The microfoundations of our model are as in Ghironi (2000), but here we allow for asymmetry in household discount factors and steady-state produc-

[4] Kraay and Ventura (2000) study the differences in the responses of the current accounts of "debtor" and "creditor" countries to transitory changes in income. However, they do not discuss why countries' net foreign asset positions may be different in the long-run, a question that is central to our analysis.

tivity levels across countries.[5]

2.1.1 The Main Assumptions

Demographics and Household Behavior—The world consists of two countries, home and foreign. In each period t, the world economy is populated by a continuum of infinitely lived households between 0 and N_t^W. (A superscript W denotes world variables.) Each household consumes, supplies labor, and holds financial assets. As in Weil (1989*a* , *b*), households are born on different dates *owning no assets* , but they own the present discounted value of their labor income. The number of households in the home economy, N_t, grows over time at the exogenous rate n, *i.e.*, $N_{t+1} = (1+n)N_t$. We normalize the size of a household to 1, so that the number of households alive at each point in time is the economy's population. Foreign population grows at the same rate as home population. We assume that the world economy has existed since the infinite past and normalize world population at time 0 so that $N_0^W = 1$.

Households at home and abroad have perfect foresight, though they can be surprised by initial, unexpected shocks. Households maximize standard intertemporal utility functions. The period utility function in both countries is logarithmic in consumption of a CES world consumption basket and in the amount of labor effort supplied by the household. Domestic households have discount factor β, $0 < \beta < 1$. Foreign households have discount factor $\alpha\beta$, $0 < \alpha \leq 1$. When $\alpha < 1$, foreign households are more impatient than domestic households.

Goods Market and Production—A continuum of goods $z \in [0, 1]$ are produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Firms produce output using labor as the only factor of production according to a linear technology that is subject to multiplicative, country-wide productivity shocks. We allow steady-state productivity levels to differ across countries. At time 0, the number of goods that are supplied in the world economy is equal to the number of households. The number of households grows over time, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads across a larger number of households. Profits are distributed to consumers via dividends, and the structure of the market for each good is

[5] Readers who are familiar with Ghironi (2000) may wish to review the main assumptions below and move directly to Section 3.

taken as given. The domestic economy produces goods in the interval $[0, a]$, which is also the size of the home population at time 0, whereas the foreign economy produces goods in the range $(a, 1]$. (Hence, world aggregates are defined as weighted averages of domestic and foreign variables with weights a and $1 - a$, respectively.)

Asset Markets—The asset menu includes a riskless real bond denominated in units of the world consumption basket and shares in firms. Private agents in both countries trade the real bond domestically and internationally. Shares in home (foreign) firms are held only by home (foreign) residents to ensure diversity of asset portfolios across agents born in the same period in different countries.

2.1.2 Households

Consumers have identical preferences over a real consumption index (C) and leisure ($1 - L$, where L is labor effort supplied in a competitive labor market, and normalize the endowment of time in each period to 1). At any time t_0, the representative home consumer j born in period $\upsilon \in [-\infty, t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{\upsilon j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\rho \log C_t^{\upsilon j} + (1-\rho) \log\left(1 - L_t^{\upsilon j}\right)\right], \tag{1}$$

with $0 < \rho < 1$.

The consumption index for the representative domestic consumer is:[6]

$$C_t^{\upsilon j} = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^{\upsilon j}\right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^{\upsilon j}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}},$$

where $\omega > 0$ is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes that aggregate individual domestic and foreign goods are, respectively:

$$C_{Ht}^{\upsilon j} = \left[(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a \left(c_t^{\upsilon j}(i)\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \text{ and } C_{Ft}^{\upsilon j} = \left[(\frac{1}{1-a})^{\frac{1}{\theta}} \int_a^1 \left(c_{*t}^{\upsilon j}(i)\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

[6]For the sake of simplicity, we will often refer to the representative member of generation υ as the "representative consumer" below. Strictly speaking, though, the model we set up is not a representative consumer one, as representative agents of different generations may behave differently.

where $c_t^{vj}(i)$ ($c_{*t}^{vj}(i)$) denotes time t consumption of good i produced in the home (foreign) country, and $\theta > 1$ is the elasticity of substitution between goods produced inside each country.[7]

The representative home consumer enters a period holding bonds and shares purchased in the previous period. He or she receives interests and dividends on these assets, may earn capital gains or incur losses on shares, earns labor income, and consumes.

Denote the date t price (in units of the world consumption basket) of a claim to the representative domestic firm i's entire future profits (starting on date $t+1$) by V_t^i, and let x_{t+1}^{vji} be the share of the representative domestic firm i owned by the representative domestic consumer j born in period v at the end of period t. d_t^i denotes the real dividends that firm i pays on date t (in units of consumption). Then, letting B_{t+1}^{vj} be the representative home consumer's holdings of bonds entering $t+1$, the period budget constraint is:

$$\begin{aligned} & B_{t+1}^{vj} + \int_0^a \left(V_t^i x_{t+1}^{vji} - V_{t-1}^i x_t^{vji} \right) di \\ = \quad & (1+r_t) B_t^{vj} + \int_0^a d_t^i x_t^{vji} di + \int_0^a \left(V_t^i - V_{t-1}^i \right) x_t^{vji} di + w_t L_t^{vj} - C_t^{vj}, \end{aligned} \tag{2}$$

where r_t is the risk-free world real interest rate between $t-1$ and t and w_t is the real wage, both in units of the consumption basket. [8]

The representative domestic consumer born in period v maximizes the intertemporal utility function (1) subject to the constraint (2). Dropping the j superscript (because symmetric agents make identical choices in equilibrium), optimal labor supply is given by:

$$L_t^v = 1 - \frac{1-\rho}{\rho} \frac{C_t^v}{w_t}, \tag{3}$$

which equates the marginal cost of supplying labor to the marginal utility of consumption generated by the corresponding increase in labor income.

[7] Foreign agents consume an identical basket of goods. Trade in goods is free. There are no transportation and transaction costs, and each individual good has an identical real price in the two economies.

[8] Given that individuals are born owning no financial wealth, because they are not linked by altruism to individuals born in previous periods, $B_v^{vj} = x_v^{vji} = 0$. As noted before, however, individuals are born owning the present discounted value of their labor income.

The first-order condition for optimal holdings of bonds yields the Euler equation:

$$C_t^v = \frac{1}{\beta(1 + r_{t+1})} C_{t+1}^v \tag{4}$$

for all $v \leq t$.

Absence of arbitrage opportunities between bonds and shares requires:

$$1 + r_{t+1} = \frac{d_{t+1}^i + V_{t+1}^i}{V_t^i}. \tag{5}$$

As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.

Foreign consumers maximize a similar intertemporal utility function and are subject to an analogous budget constraint as home consumers. The only difference is that the discount factor of foreign households is $\alpha\beta$. Otherwise, a similar labor-leisure tradeoff, Euler equation, no-arbitrage, and transversality conditions hold for foreign households.

2.1.3 Firms

Output supplied at time t by the representative domestic firm i is a linear function of labor demanded by the firm:[9]

$$Y_t^{Si} = Z_t L_t^i. \tag{6}$$

Z_t is exogenous, economy-wide productivity. Production by the representative foreign firm is a linear function of L_t^{i*}, with a productivity parameter Z_t^*.

Output demand comes from domestic and foreign consumers. The demand of home good i by the representative domestic household born in period v is:

$$c_t^v(i) = \left(RP_t^i\right)^{-\theta} (RP_t)^{\theta-\omega} C_t^v$$

obtained by maximizing C subject to a spending constraint. RP_t^i is the price of good i in units of the composite consumption basket. RP_t is the price of the sub-basket of home goods in units of consumption. Aggregating across

[9]Because all firms in the world economy are born at $t = -\infty$, after which no new goods appear, it is not necessary to index output and factor demands by the firms' date of birth.

home households alive at time t, *total demand* for home good i coming from domestic consumers is:

$$c_t(i) = \left(RP_t^i\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} a(1+n)^t c_t,$$

where

$$c_t \equiv \frac{a\left[\begin{array}{c} \dots\frac{n}{(1+n)^{t+1}}C_t^{-t} + \dots + \frac{n}{(1+n)^2}C_t^{-1} + \frac{n}{1+n}C_t^0 \\ +nC_t^1 + n(1+n)C_t^2 + \dots + n(1+n)^{t-1}C_t^t \end{array}\right]}{a(1+n)^t}$$

is *aggregate per capita* home consumption of the composite consumption basket.

Given identity of preferences across countries, total demand for home good i by foreign consumers is

$$c_t^*(i) = \left(RP_t^i\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} (1-a)(1+n)^t c_t^*,$$

where c_t^* is aggregate per capita foreign consumption, the definition of which is similar to that of c_t.

Total demand for good i produced in the home country is obtained by adding the demands for that good originating in the two countries. Using the results above, it is:

$$Y_t^{Di} = \left(RP_t^i\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} \widehat{c}_t^W, \tag{7}$$

where $\widehat{c}_t^W$ is aggregate world demand of the composite good: $\widehat{c}_t^W \equiv N_t c_t + N_t^* c_t^*$.[10]

Given the no-arbitrage condition between bonds and shares (5) and a no-speculative bubble condition, the real price of firm i's shares at time t_0 is given by the present discounted value of the real dividends paid by the firm from t_0+1 on: $V_{t_0}^i = \sum_{s=t_0+1}^{\infty} R_{t_0,s} d_s^i$, where $R_{t_0,s} \equiv \left[\prod_{u=t_0+1}^{s}(1+r_u)\right]^{-1}$, $R_{t_0,t_0} = 1$. At time t_0, firm i maximizes the present discounted value of dividends to be paid from t_0 on: $V_{t_0}^i + d_{t_0}^i = \sum_{s=t_0}^{\infty} R_{t_0,s} d_s^i$. At each point in time, dividends are given by after-tax real revenues–$(1-\tau)\, RP_t^i Y_t^i$—plus a lump-sum transfer (or tax)

[10] Where necessary for clarity, we use a "hat" to differentiate the *aggregate* level of a variable from the aggregate per capita level.

from the government–T_t^i–minus costs–$w_t L_t^i$. The firm chooses the real price of its product and the amount of labor demanded to maximize the present discounted value of its current and future profits subject to the constraints (6), (7), and the market clearing condition $Y_t^{Si} = Y_t^{Di} (= Y_t^i)$. Firm i takes the real price of the sub-basket of home goods, the wage rate, Z, the rate of taxation of revenues (τ), the transfer received from the government, and world demand of the composite good as given.

Let λ_t^i denote the Lagrange multiplier on the constraint $Y_t^{Si} = Y_t^{Di}$. λ_t^i is the shadow price of an extra unit of output in period t. The first-order condition for the optimal choice of L_t^i yields:

$$\lambda_t = \frac{w_t}{Z_t}. \tag{8}$$

At an optimum, the shadow value of output must equal the marginal cost. The i superscript has been dropped because symmetric firms make identical choices in equilibrium.

The first-order condition with respect to RP_t^i yields the pricing equation:

$$RP_t^i = RP_t = \frac{\theta}{(\theta - 1)(1 - \tau)} \lambda_t, \tag{9}$$

which equates the price charged by firm i to a markup over marginal cost. Identical equilibrium choices by symmetric firms imply that the real price of good i equals the real price of the sub-basket of home goods in equilibrium.

Using the market clearing conditions $Y_t^{Si} = Y_t^{Di}$ and $\widehat{c}_t^W = \widehat{Y}_t^{SW} = \widehat{Y}_t^{DW} (= \widehat{Y}_t^W)$, the expressions for supply and demand of good i, and recalling that symmetric firms make identical equilibrium choices, labor demand can be written as:

$$L_t^i = RP_t^{-\omega} \frac{\widehat{Y}_t^W}{Z_t}. \tag{10}$$

Ceteris paribus, firm i's labor demand is a decreasing function of real output price and productivity. It is an increasing function of world consumption demand.[11]

[11] Although all domestic firms demand the same amount of labor in equilibrium, we leave the i superscript on labor demand to differentiate labor employed by an individual firm from aggregate per capita employment, which will be denoted by dropping the superscript. Optimality conditions for foreign firms are similar.

2.1.4 The Government

We simplify the government sector and reduce its role to taxing firm revenues and rebating tax income to firms via lump-sum transfers. We assume that the government taxes revenues at a rate that compensates for monopoly power and removes the markup charged by firms over marginal costs in equilibrium.[12] The tax rate is determined by $1-\tau = \frac{\theta}{\theta-1}$, which yields $\tau = -\frac{1}{\theta-1}$. Because the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes determined by:

$$T_t^i = \tau R P_t^i Y_t^i. \tag{11}$$

2.2 Aggregation

2.2.1 Households

Aggregate per capita labor supply equations are obtained by aggregating labor-leisure tradeoff equations across generations and dividing by total population at each point in time. The aggregate per capita labor-leisure tradeoffs in the two economies are:

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{c_t}{w_t}, \quad L_t^* = 1 - \frac{1-\rho}{\rho}\frac{c_t^*}{w_t^*}. \tag{12}$$

Aggregate labor supply rises with the real wage and decreases with consumption.

Consumption Euler equations in aggregate per capita terms contain an adjustment for consumption by the newborn generation at time $t+1$:

$$c_t = \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1} - \frac{n}{1+n}C_{t+1}^{t+1}\right), \quad c_t^* = \frac{1+n}{\alpha\beta(1+r_{t+1})}\left(c_{t+1}^* - \frac{n}{1+n}C_{t+1}^{t+1*}\right). \tag{13}$$

Along with playing a crucial role in ensuring steady-state determinacy (see Ghironi, 2000), the presence of the adjustments for consumption of newborn generations at $t+1$ in the Euler equations for aggregate per capita consumption provides the degree of freedom that is necessary for existence of a well defined, non-degenerate steady state when discount factors differ across countries.

[12] The presence of the subsidy greatly simplifies the solution for the steady state. It has no impact on the log-linear model that we solve later, because firms charge a constant markup, the effect of which would disappear in the linearized version.

Newborn households hold no assets, but they own the present discounted value of their labor income. Using the Euler equation (4) and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of its human wealth, h:

$$C_t^t = \rho\left(1-\beta\right)h_t, \quad C_t^{t*} = \rho\left(1-\alpha\beta\right)h_t^*. \tag{14}$$

h and h^* are defined as the present discounted values of the households' lifetime endowments of time in terms of the real wages:

$$h_t \equiv \sum_{s=t}^{+\infty} R_{t,s}w_s, \quad h_t^* \equiv \sum_{s=t}^{+\infty} R_{t,s}w_s^*. \tag{15}$$

The dynamics of h and h^* are described by the following forward-looking difference equations:

$$h_t = \frac{h_{t+1}}{1+r_{t+1}} + w_t, \quad h_t^* = \frac{h_{t+1}^*}{1+r_{t+1}} + w_t^*. \tag{16}$$

The law of motion of aggregate per capita assets *held by domestic consumers* is obtained by aggregating the budget constraint (2) across generations alive at each point in time. Using the no-arbitrage condition (5) and recalling that newborn agents hold no assets, aggregate per capita assets of domestic and foreign consumers obey, respectively:

$$\begin{aligned}
\left(1+n\right)\left(B_{t+1}+V_t\right) &= \left(1+r_t\right)\left(B_t+V_{t-1}\right)+w_tL_t-c_t, \\
\left(1+n\right)\left(B_{t+1}^*+V_t^*\right) &= \left(1+r_t\right)\left(B_t^*+V_{t-1}^*\right)+w_t^*L_t^*-c_t^*,
\end{aligned} \tag{17}$$

where V_t and V_t^* denote the aggregate per capita equity value of the home and foreign economy entering period $t+1$, respectively ($V_t \equiv \frac{aV_t^i}{N_{t+1}}$, $V_t^* \equiv \frac{aV_t^{*i}}{N_{t+1}^*}$).[13]

2.2.2 Firms

Aggregate per capita output in each economy is obtained by expressing production of each differentiated good in units of the composite basket, multiplying by the number of firms, and dividing by population. It is:

$$y_t = RP_tZ_tL_t, \quad y_t^* = RP_t^*Z_t^*L_t^*. \tag{18}$$

[13]These equations hold in all periods following the initial one. The no arbitrage condition may be violated between time t_0-1 and t_0 if an unexpected shock surprises agents at the beginning of period t_0. See Ghironi (2000) for details.

For given employment and productivity, each country's real GDP rises with the relative price of the representative good produced in that country, as this is worth more units of the consumption basket.

Aggregate per capita labor demand is:

$$L_t = RP_t^{-\omega} \frac{y_t^W}{Z_t}, \quad L_t^* = RP_t^{*-\omega} \frac{y_t^W}{Z_t^*}, \tag{19}$$

where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption, c_t^W. It is $y_t^W = ay_t + (1-a)\, y_t^*$ and $c_t^W = ac_t + (1-a)\, c_t^*$, $y_t^W = c_t^W$ to ensure market clearing.

Domestic and foreign relative prices are equal to marginal costs, because government subsidies remove the effect of the monopolistic distortion on pricing in equilibrium:

$$RP_t = \frac{w_t}{Z_t}, \quad RP_t^* = \frac{w_t^*}{Z_t^*} \tag{20}$$

Shares are a liability in the balance sheets of firms. In the absence of arbitrage opportunities between bonds and shares, the aggregate per capita equity value of the home and foreign economies entering period $t+1$ must evolve according to:

$$V_t = \frac{1+n}{1+r_{t+1}} V_{t+1} + \frac{d_{t+1}}{1+r_{t+1}}, \quad V_t^* = \frac{1+n}{1+r_{t+1}} V_{t+1}^* + \frac{d_{t+1}^*}{1+r_{t+1}}. \tag{21}$$

where d_t and d_t^* denote aggregate per capita dividends, equal to $(1-\tau)\, y_t + T_t - w_t L_t$ and $(1-\tau^*)\, y_t^* + T_t^* - w_t^* L_t^*$, respectively. In equilibrium, $\tau = \tau^* = -\frac{1}{\theta-1}$ implies $d_t = d_t^* = 0$ and $V_t = V_t^* = 0$ $\forall t$ in the absence of speculative bubbles.

2.2.3 Government

The government budget constraint in aggregate per capita terms is:

$$T_t = \tau y_t, \quad T_t^* = \tau^* y_t^*. \tag{22}$$

2.2.4 Net Foreign Asset Accumulation

Each country's accumulation of net foreign assets is described by an equation that combines the budget constraints of households, the fact that shares are liabilities of firms towards consumers in the respective economies, and the government budget constraints. In aggregate per capita terms, it is:

$$(1+n)\,B_{t+1} = (1+r_t)\,B_t + y_t - c_t, \quad (1+n)\,B^*_{t+1} = (1+r_t)\,B^*_t + y^*_t - c^*_t. \tag{23}$$

For asset markets to be in equilibrium, aggregate home assets (liabilities) must equal aggregate foreign liabilities (assets), *i.e.*, it must be $\widehat{B}_t + \widehat{B}^*_t = 0$. In aggregate per capita terms, it must be:

$$aB_t + (1-a)\,B^*_t = 0. \tag{24}$$

Using (24), equations (23) reduce to $y^W_t = c^W_t$: consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

3 Steady State and Dynamics

3.1 The Steady State

It is known, at least since Becker (1980), that a standard representative agent model with identical discount factors across agents (*i.e.*, $n = 0$, $\alpha = 1$) results in indeterminacy of the steady-state distribution of net foreign assets. If discount factors differ across agents with no other modification to the standard model ($n = 0$, $\alpha < 1$), the distribution of wealth across agents ends up collapsing into one in which the most patient household owns all the wealth. Buiter (1981) and Weil (1989*b*) demonstrated that models with overlapping generations in which households are not linked by intergenerational altruism can deliver a non-degenerate distribution of asset holdings across countries. Our model achieves precisely the same goal by assuming $n > 0$ and absence of intergenerational linkages in the form of altruism or government transfers. Ghironi (2000) shows that, when $\alpha = 1$, this delivers a determinate steady state and stationary dynamics of prices and aggregate per capita quantities following non-permanent shocks. Appendix A contains the details of the solution for the steady state of our extended model ($\alpha \leq 1$, $\frac{\overline{Z}}{\overline{Z}^*}$ possibly different from 1), and here we summarize its main characteristics. In what follows, we denote steady-state levels of variables with overbars.

To demonstrate the influence of structural asymmetry on our analysis, we start with the special case in which all preference parameters are identical across domestic and foreign households. When $\alpha = 1$, steady-state levels of labor effort are identical across countries ($\frac{\overline{L}}{\overline{L}^*} = 1$), and net foreign assets are zero ($\overline{B} = \overline{B}^* = 0$), *regardless of* relative productivity ($\frac{\overline{Z}}{\overline{Z}^*}$). This happens because, when consumers' intertemporal preferences are identical at home and abroad, given a common world interest rate, households in the two countries have identical incentives to borrow or lend. (The desired slope of the consumption profile is the same for each domestic and foreign household.) In this case, the only possible steady-state equilibrium in the setup of this paper is one in which $\overline{r} = \frac{1-\beta}{\beta}$ and net foreign assets are zero even if $\frac{\overline{Z}}{\overline{Z}^*} \neq 1$. Domestic and foreign GDPs in units of consumption differ ($\overline{y} \neq \overline{y}^*$), and so do consumption levels ($\overline{c} \neq \overline{c}^*$). But consumption equals GDP in each country, so that net foreign assets are zero. Since $\overline{y} = \overline{w}\overline{L}$ and $\overline{y}^* = \overline{w}^*\overline{L}^*$ in equilibrium (because revenue subsidies τ offset monopoly power in pricing), $\overline{L} = \overline{L}^*$ when $\alpha = 1$ implies that the different GDP levels generated by different productivity levels translate into different real wages and labor incomes across countries. The more productive country has a higher steady-state real wage and consumption and a lower relative price for the same labor effort as the less productive country.[14]

In the general case $\alpha \leq 1$, Appendix A proves that we can write the solution for $\overline{r}$,$\overline{B}$, and cross-country ratios of any pair of other endogenous variables $\overline{x}$ and $\overline{x}^*$ as functions of the steady-state productivity ratio $\frac{\overline{Z}}{\overline{Z}^*}$. The characteristics of these functions depend on the values of structural parameters, and the steady-state levels of $\overline{r}$,$\overline{B}$, and other endogenous variables can be obtained numerically given assumptions on $\overline{Z}$ and $\overline{Z}^*$.[15]

Consider the following examples:

1.–If $\alpha < 1$ and $\overline{Z} = \overline{Z}^* = 1$, plausible parameter values yield $\overline{B} > 0$ ($\overline{B}^* < 0$), $\overline{c} > \overline{c}^*$, $\overline{L} < \overline{L}^*$, $\overline{w} > \overline{w}^*$, $\overline{RP} > \overline{RP}^*$, $\overline{y} < \overline{y}^*$.[16] If domestic agents are more patient than foreign, they accumulate steady-state assets, which make it possible to sustain relatively higher consumption with a smaller

[14] If $\alpha = 1$ and $\overline{Z} = \overline{Z}^* = 1$, the steady state is the same as in Ghironi (2000): $\overline{B} = 0$, and the steady state is symmetric in all respects: $r = \frac{1-\beta}{\beta}$, $\overline{B} = \overline{B}^* = 0$, $\overline{c} = \overline{c}^* = \overline{L} = \overline{L}^* = \overline{y} = \overline{y}^* = \rho$, $\overline{w} = \overline{w}^* = \overline{RP} = \overline{RP}^* = 1$. See Ghironi (2000) for the details of the solution in this case.

[15] For the reasons discussed above, the functions defined in the appendix are such that, if $\alpha = 1$, it is $\overline{B} = 0$, $\frac{\overline{L}}{\overline{L}^*} = 1$, and $\frac{\overline{c}}{\overline{c}^*} \gtrless 1$ if $\overline{Z} \gtrless \overline{Z}^*$.

[16] These results arise with the benchmark parameterization we discuss below ($\beta = .99$, $\omega = 3$, $a = .5$, $\rho = .33$, $\alpha = .9999$, $n = .01$) as well as under a number of other plausible parameterizations.

labor effort. Lower labor supply generates a higher equilibrium real wage and relative price.[17] The labor effort differential prevails on the relative price differential in generating lower GDP at home than abroad, where higher GDP is required to pay interest on the accumulated debt.

2.–If $\alpha < 1$ and $\overline{Z} = 1 < \overline{Z}^*$, plausible parameter values yield $\overline{B} > 0$ ($\overline{B}^* < 0$), $\overline{c} < \overline{c}^*$, $\overline{L} < \overline{L}^*$, $\overline{w} < \overline{w}^*$, $\overline{RP} > \overline{RP}^*$, $\overline{y} < \overline{y}^*$.[18] Higher productivity in the more impatient country causes the steady-state real wage differential to switch sign, so that the real wage is now higher in the foreign economy. This induces foreign agents to consume more, and their consumption rises above that of domestic agents, with an increase in the size of the foreign economy's debt.

3.2 The Log-Linear Solution

The aggregate model of Section 2.2 can be safely log-linearized around the steady state. The assumptions that $n > 0$ and newborn households enter the economy with no assets pin down the steady state endogenously and generate stationary model dynamics following non-permanent shocks.[19] We present the log-linear equations in Appendix A. The log-linear model can then be solved with the method of undetermined coefficients following Campbell (1994). In what follows we use sans serif fonts to denote percentage deviations from the steady state, and focus on the model solution in terms of the minimum state vector, which at time t consists of the predetermined levels of net foreign assets and the risk-free real interest rate (the endogenous states) and the current levels of domestic and foreign productivity, *i.e.*,

[17] We assume that labor does not move across countries. Given a steady-state real wage differential, we motivate absence of long-run labor flows by appealing to the presence of unspecified costs of relocating abroad that more than offset the welfare differential implied by small differences in real wages.

[18] The same parameter values as in the previous example and $\overline{Z} = 1$, $\overline{Z}^* = 1.29$ yield these results. (See below on the choice of $\overline{Z}^*$.)

[19] In the representative agent model with $n = 0$, the consumption differential across countries is a random walk. All shocks have permanent consequences via wealth redistribution regardless of their nature. In a stochastic setting, the unconditional variance of endogenous variables is infinite, even if exogenous shocks are bounded. Log-linearization is not a reliable solution technique in this case.

$[\mathsf{B}_t, \mathsf{r}_t, \mathsf{Z}_t, \mathsf{Z}_t^*]'$. [20] The solution of the model can then be written as:

$$
\begin{aligned}
\mathsf{B}_{t+1} &= \eta_{BB}\mathsf{B}_t + \eta_{Br}\mathsf{r}_t + \eta_{BZ}\mathsf{Z}_t + \eta_{BZ^*}\mathsf{Z}_t^*, \\
\mathsf{r}_{t+1} &= \eta_{rB}\mathsf{B}_t + \eta_{rr}\mathsf{r}_t + \eta_{rZ}\mathsf{Z}_t + \eta_{rZ^*}\mathsf{Z}_t^*, \\
\mathsf{x}_t &= \eta_{xB}\mathsf{B}_t + \eta_{xr}\mathsf{r}_t + \eta_{xZ}\mathsf{Z}_t + \eta_{xZ^*}\mathsf{Z}_t^*, \\
\mathsf{x}_t^* &= \eta_{x^*B}\mathsf{B}_t + \eta_{x^*r}\mathsf{r}_t + \eta_{x^*Z}\mathsf{Z}_t + \eta_{x^*Z^*}\mathsf{Z}_t^*,
\end{aligned} \tag{25}
$$

where x_t and x_t^* are any pair of endogenous variables other than net foreign assets and the interest rate, and the η's are elasticities of endogenous variables to the endogenous and exogenous components of the state vector. We assume that productivities at home and abroad obey the following processes in all periods after the time of an initial impulse ($t = 0$ in the impulse responses below):

$$\mathsf{Z}_t = \phi\mathsf{Z}_{t-1}, \quad \mathsf{Z}_t^* = \phi\mathsf{Z}_{t-1}^*, \quad 0 \leq \phi \leq 1.$$

Non-zero steady-state net foreign assets introduce an additional channel through which the past history of the economy matters for current dynamics relative to the model with zero steady-state assets of Ghironi (2000). The predetermined, risk-free interest rate is an additional state variable in the solution. The intuition is simple. If steady-state net foreign assets are zero (if $\alpha = 1$), the effect of the interest burden on previously accumulated debt is lost in the log-linearization of the laws of motion for domestic and foreign net foreign assets in (23). This is no longer the case when steady-state assets differ from zero, as forcefully argued by Lane and Milesi-Ferretti (2001, 2002*a*, *b*). This implies that the effect of net foreign asset accumulation on cross-country differences in the levels of other endogenous variables is amplified relative to a model with zero steady-state net foreign assets.

A second, important implication of an asymmetric steady state is that the solution for net foreign assets in (25) can no longer be written as a function of the cross-country productivity differential. A consequence of this is that worldwide productivity shocks, which have no impact on the current account in Ghironi's (2000) version of the model, affect net foreign asset accumulation

[20] Ghironi (2000)shows that the log-linear model has a unique solution when $\alpha = 1$ and steady-state productivities are equal across countries. (In that case, percentage deviations of net foreign assets are defined around the steady-state level of consumption, and the international asset market equilibrium condition is $a\mathsf{B}_t + (1-a)\,\mathsf{B}_t^* = 0$.) While we cannot verify determinacy analytically when the steady state is asymmetric, we conjecture that determinacy of the solution is preserved for α close to 1 and steady-state productivities that are not too far from each other.

both through their impact on the world interest rate and the interest rate burden (or income) on previously accumulated asset balances and through terms of trade effects (see below on this). Several tests of the intertemporal model of the current account are based on the premise that global shocks should have no impact on the current account of an open economy (see, e.g., Glick and Rogoff, 1995). However, this hypothesis is frequently rejected by the data, and is viewed as a "puzzle." Our analysis suggests that variable interest rate effects on outstanding debt and terms of trade dynamics may at least partly explain these findings.

These novel mechanisms demonstrate the advantages of our theoretical framework and its empirical relevance. We should also note that, while we view our analysis as an extension of the intertemporal model of the current account, the mechanisms that we have identified also call for a different empirical approach. For instance, the ultimate causes of differences in net foreign asset positions do have implications for how we interpret the potentially different responses of consumption levels of debtor and creditor countries to disturbances.[21] In what follows, we discuss this heterogeneity in responses to shocks using the impulse responses implied by a plausible parameterization of the model. This helps us build some intuition to interpret the empirical counterparts of these responses in the second part of the paper.

3.3 Impulse Responses

We interpret periods as quarters and choose the following benchmark parameter values: $\beta = .99$ (a standard choice), $\alpha = .9999$ (so that the foreign discount factor is .9899), $\omega = 3$, $\rho = .33$, $a = .5$ (countries have equal size), $n = .01$, $\overline{Z} = 1$, and $\overline{Z}^* = 1.29$. The benchmark value of α is very close to 1. The reason is that even small differences between the foreign and home discount factors result in very large steady-state net foreign asset positions in the model of this paper. For instance, given the other parameter values, $\alpha = .995$ (a foreign discount factor equal to .985) would result in a long-run debt/GDP ratio for the foreign economy equal to 52.74 (or $5,274$

[21] One could argue that heterogeneity is an inherent feature of small open economy models. However, (i) these models are also frequently used to estimate the familiar present value models of the current account for "large" economies such as Japan, Germany, and the U.S., and (ii) they should be best viewed as " short-cuts" in relation to a more general model. As we discussed above, our analysis highlights a number of issues that are overlooked by these small open-economy models.

percent)! This would arguably lead us to overstate the effect of interest rate changes. To avoid this problem, we choose a value of α such that the long-run debt/GDP ratio for the foreign economy is 1.37 (137 percent): high, but not unreasonable. We discuss the consequences of lower values of α below. The value of ω is in (the lower portion of) the range of estimation results from the trade literature on the U.S. and OECD countries (Feenstra 1994, Harrigan 1993, Shiells, Stern, and Deardorff 1986, Trefler and Lai, 1999).[22] The choice of ρ implies that households in both countries spend one third of their time working in the symmetric steady-state world. $\alpha < 1$ and $\overline{Z} \neq \overline{Z}^*$ yield a steady-state employment differential. The choice of n is higher than realistic, at least if one has developed economies in mind and n is interpreted strictly as the rate of growth of population.[23] Extending the model to incorporate probability of death as in Blanchard (1985) would make it possible to reproduce the dynamics generated by $n = .01$ with a lower rate of entry of new households by choosing the proper value of the probability of death. The choice of $n = .01$ thus mimics the behavior of a more complicated, yet largely isomorphic setup. Our choice of parameter values is plausible if we think of the more impatient economy as the U.S., consistent with the evidence in favor of a lower propensity to save for U.S. households relative to European and Asian ones. As for the steady-state productivity differential, our data suggest that, on average, U.S. productivity has been 29 percent higher than in the rest of the world.

The parameter values above result in the steady-state configuration of Example 2 above: $\overline{B} > 0$ ($\overline{B}^* < 0$), $\overline{c} < \overline{c}^*$, $\overline{L} < \overline{L}^*$, $\overline{w} < \overline{w}^*$, $\overline{RP} > \overline{RP}^*$, $\overline{y} < \overline{y}^*$. Relative consumer impatience causes the U.S. economy to accumulate a steady-state debt against the rest of the world. Nevertheless, higher productivity results in higher real wage, GDP, consumption, and labor effort (the latter is higher than abroad for the need to pay interest on the accumulated debt). Larger U.S. GDP comes with a lower price of U.S. goods relative to the patient economy (home). Numerical values for the steady-state levels of variables are in Table 1, which displays the values of the elasticities of endogenous variables to the state vector in the model solution.

[22] Ghironi (2000) shows that lower (higher but finite) values of ω reduce (amplify) the elasticities of cross country differentials to net foreign asset accumulation in the symmetric version of the model. Consistent with Cole and Obstfeld (1991) and Corsetti and Pesenti (2001), there is no role for asset accumulation if $\omega = 1$ and steady-state assets are zero.

[23] The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been .0025.

We consider three shocks: a shock to home productivity, a shock to foreign productivity, and an aggregate shock to world productivity. (In all cases, we consider one percent initial impulses.) For reasons of empirical plausibility, we focus on the following values of the persistence parameter ϕ: .9 for country-specific shocks and 1 for the world-wide shock. In the case of country-specific shocks, persistence .9 is at the lower end of the range that is usually considered by the international real business cycle literature (e.g., Baxter and Crucini, 1995).

3.3.1 A Productivity Shock in the Creditor Country

Figure 1 shows the impulse responses to a 1 percent increase in productivity in the more patient, less productive country (home)–the country that has positive net foreign assets in steady state. The shock causes home agents to accumulate more assets to smooth the effect of temporarily higher income on consumption. Symmetrically, the foreign country's debt increases as foreign agents borrow to share the beneficial effect of higher home productivity and sustain higher consumption with unchanged foreign productivity.[24] Net foreign asset accumulation peaks approximately 6 years after the shock. After that time, net assets return to the steady state gradually. We show in the appendix that, if steady-state home labor effort is not very far from foreign (a condition that is satisfied in our example) the solution for the risk-free, world interest rate can also be written as:

$$\begin{aligned} \mathsf{r}_{t+1} &\approx a\frac{\overline{w}}{\overline{w}^W}\left(\mathsf{Z}_{t+1}-\mathsf{Z}_t\right)+(1-a)\frac{\overline{w}^*}{\overline{w}^W}\left(\mathsf{Z}^*_{t+1}-\mathsf{Z}^*_t\right) \\ &= -a\frac{\overline{w}}{\overline{w}^W}(1-\phi)\,\mathsf{Z}_t+(1-a)\frac{\overline{w}^*}{\overline{w}^W}(1-\phi)\,\mathsf{Z}^*_t, \end{aligned} \tag{26}$$

which reduces to $\mathsf{r}_{t+1} = -a\frac{\overline{w}}{\overline{w}^W}(1-\phi)\,\mathsf{Z}_t$ in the case of a home shock.[25] Since home productivity is expected to decrease and return to the steady state over time after the initial shock, the real interest rate falls on impact and returns to the steady state monotonically as the productivity shock dies

[24] $\mathsf{B}^*_t = \mathsf{B}_t$, where B^*_t is the percentage change in the foreign country's debt. Hence, we omit the response of B^*_t from the figures. Figures 1 and 2 show net foreign assets *at the end* of the corresponding period. For this reason, home net foreign assets are denoted with B1 in the figure.

[25] The approximation is accurate to the fifth decimal point for the parameterization in our example.

out. Increased supply of home goods at any given level of labor effort results in a lower (higher) relative price of home (foreign) goods, *i.e.*, a deterioration of home's terms of trade, on impact. This expands (lowers) the demand for home (foreign) labor and causes the home real wage to increase. The foreign real wage increases as a consequence of optimal pricing by foreign firms: In the absence of changes in foreign productivity, the relative price of foreign goods and the foreign real wage are tied to each other.

As the shock dies out, the effect of wealth accumulation in the home economy is that agents can sustain a higher level of consumption with lower labor effort than in steady state.[26] For this reason, labor effort at home falls below the steady state approximately 6 and a half years after the shock. Eventually, lower labor supply translates into less supply of home goods and an increase of their relative price above the steady state. The home labor effort (relative price) returns to the steady state from below (above) as net foreign assets return to their long-run level. The opposite dynamics take place in the foreign economy: More debt eventually forces foreign agents to increase their supply of labor above the steady state in order to sustain consumption. In turn, this lowers the relative price of foreign goods (and the foreign real wage) below the steady state. The foreign labor effort (relative price and real wage) then return to the steady state from above (below). Consistent with empirical evidence on the " transfer problem" in Lane and Milesi-Ferretti (2000), *ceteris paribus*, the effect of asset accumulation is to appreciate the terms of trade of the home economy.

Home GDP rises above the steady state on impact: Higher productivity and labor effort more than offset the effect of a lower relative price on the consumption-value of home production. Foreign GDP falls, because labor supply falls by more than the increase in the relative price of foreign goods. Eventually, the wealth effects described before cause home (foreign) GDP to fall (rise) below (above) the steady state, from where it returns to its long-run level. As briefly mentioned above, consumption rises on impact in both countries–though, of course, it rises by more in the home economy. Even if home GDP returns from the steady state from below, home consumption does not fall below the steady state during the transition dynamics. The reason is that increased net foreign assets allow home agents to sustain higher consumption directly and indirectly (by keeping the real wage above the steady

[26] The effect of a larger stock of assets prevails on that of a lower interest rate on those assets.

state for the length of the transition through lower supply of home labor) even if the consumption-value of home output is below the steady state. Instead, foreign consumption falls below the steady state approximately 8 years after the initial shock and returns to the steady state from below. A larger debt eventually causes lower consumption and more supply of labor in the foreign economy until the steady state is reached. The consumption-value of foreign output rises as foreign agents supply more effort to smooth the decrease in consumption caused by the debt burden and to drive the latter back to the steady state.

3.3.2 A Productivity Shock in the Debtor Country

Figure 2 shows the impulse responses to a 1 percent increase in productivity in the more impatient, more productive country (foreign)–the country that has a steady-state debt. Foreign debt decreases as foreign agents smooth the effect of temporarily higher income on consumption. The real interest rate falls and returns to the steady state as the shock dies out. The relative price of foreign goods falls, which leads to more demand of labor effort in the foreign economy and a higher real wage. Dynamics in the home economy mirrors those of foreign variables: The relative price of home goods rises, with a contractionary effect on labor demand. (The real wage increases at home as it is tied to the home relative price.) Foreign GDP increases, whereas home GDP falls, and consumption rises above the steady state in both countries (of course, it does so by more in the foreign country).

Wealth effects dominate the dynamics as the shock dies out. Foreign labor supply decreases below the steady state, because debt is smaller. This results in a higher relative price of foreign goods, combined with a lower consumption-value of foreign production as the economy returns to the steady state. A smaller debt implies that, foreign consumption returns to the steady state from above. A smaller asset stock is the source of mirroring dynamics at home, where labor supply increases above the steady state, resulting in a lower relative price of home goods but a higher consumption-value of production, with consumption that returns to the steady state from above.

A result that emerges from figures 1 and 2 is that the dynamics after productivity shocks in creditor and debtor countries are not different on *qualitative* grounds. The main difference is *quantitative*. As Table 1 and figures 1 and 2 show, a 1 percent productivity shock in the creditor country (home) causes home assets (foreign debt) to increase by more than a 1 percent productivity shock in the debtor country (foreign) causes foreign debt (home assets) to decrease (.3737 vs. $-.3661$, respectively). Intuitively, foreign agents are more impatient. Hence, they have a smaller incentive to save a portion of the increase in income in the form of lower debt. For the same reason (a stronger incentive to save in the home economy), a home shock causes home consumption to increase by less than a foreign shock causes foreign consumption to increase (.4807 vs. .5678, respectively), and the home shock causes foreign consumption to increase by more than the foreign shock causes home consumption to increase (.5122 vs. .4382, respectively). These differences in elasticities would not exist if it were $\alpha = 1$ and $\overline{Z}^* = \overline{Z}$. The home shock would cause home (foreign) consumption to increase by the same amount as the foreign shock causes the foreign (home) consumption to increase. Heterogeneity in the structural characteristics of the two economies that results in an asymmetric steady state also implies quantitative, if not qualitative, heterogeneity in the short-run responses of countries to shocks.

Asymmetry of the steady state is responsible for the small quantitative difference in the response of the world interest rate to domestic and foreign productivity shocks ($-.0458$ vs. $-.0542$, respectively; recall equation (26)). A home productivity shock causes the home relative price to decrease by more than a foreign shock does to the foreign price ($-.3174$ vs. $-.2654$, respectively). The home shock causes the foreign price to increase by less than the foreign shock does to the home price (.2678 vs .3146, respectively).[27] As a consequence of asymmetric relative price effects, a home shock expands home employment and GDP by more than a foreign shock does for foreign employment and GDP (.4099 vs. .3386 and 1.0925 vs. 1.0732, respectively), and the home shock lowers foreign employment and GDP by less than the foreign shock lowers home employment and GDP ($-.3458$ vs. $-.4014$ and $-.0780$ vs. $-.0868$, respectively).

3.3.3 A Permanent World Productivity Shock

In the familiar case of a symmetric steady state with zero net foreign assets and equal productivities at home and abroad, a permanent increase in world

[27] In the symmetric steady-state case, the elasticity of RP to Z is identical to the elasticity of RP^* to Z^*, and the same is true of the elasticity of RP^* to Z and RP to Z^*. Not only, in the symmetric case, all these elasticities have the same absolute value: A home (foreign) productivity shock lowers the home (foreign) relative price exactly by the same amount as it increases the foreign (home) price.

productivity would result in no movement in net foreign assets. GDP, the real wage, and consumption in both countries would increase immediately by the full amount of the shock. There would be no change in labor effort and relative prices. Anticipating the permanent consequences of the shock, agents in both countries would simply find it optimal to consume the entire consumption value of the increase in productivity in all periods without adjusting their labor effort. Symmetry of the shock across countries would imply that no terms of trade movement is necessary to deliver equilibrium. [28]

In contrast to the symmetric case, asymmetry of the steady state results in interesting dynamics following a permanent shock to world productivity. This is illustrated in Figure 3, which shows the impulse responses to a 1 percent, permanent increase in productivity at home and abroad. A permanent productivity shock has no effect on the risk-free interest rate. Equation (26) implies that the interest rate does not move in response to permanent shocks. Therefore, the dynamics in Figure 3 do not originate in the effect of changes in the interest rate on the burden of (income from) the initial steady-state debt (assets). We shall see that terms of trade adjustment is the driving force of the responses in Figure 3.

The home economy accumulates assets over time in response to the shock, the foreign economy accumulates an increasing debt. Eventually, the increase in home assets (foreign debt) converges to an amount equal to the increase in world productivity. The relative price of home goods falls, the relative price of foreign goods rises, yielding a deterioration of home's terms of trade. Relative prices return to the steady state over time. Equilibrium labor effort increases at home, decreases abroad, reflecting the expansionary (contractionary) effect of a lower (higher) relative price on labor demand. After the initial jump, labor effort slowly returns to the original steady state in both countries. The real wage increases at home and abroad. It increases by more in the foreign economy, which explains the increase in the foreign relative price, and the decrease in equilibrium employment in the foreign country. Both the domestic and the foreign real wages converge over time to a higher

[28] The ratio $\frac{RP_t}{RP_t^*}$ is home's terms of trade. In the case of a permanent asymmetric shocks–say, to home productivity–net foreign assets would not move, as home agents would still find it optimal to consume the entire value of the shock in all periods without changing their labor effort. However, consumption and GDP would increase by less than the shock, because the terms of trade of the home economy would deteriorate due to the relative increase in the supply of home goods. See Ghironi, 2000, for details.

steady-state level that reflects the full amount of the world productivity shock. The domestic real wage increases over time, the foreign real wage decreases. GDP and consumption increase in both countries. In the long run, the increase reflects the full amount of the world shock. In the short run, GDP (consumption) increases by more (less) in the patient, less productive country (home) and then decreases (increases) toward the new steady state level. Foreign GDP (consumption) increases (decreases) over time. At the time of the shock, home (foreign) agents know that the long-run, permanent increase in home GDP will be smaller (larger) than the short-run increase, as the terms of trade and labor effort return to the steady state in the long run. Optimal consumption smoothing dictates that home agents save part of the short-run increase in GDP in the form of net foreign asset accumulation. Similarly, optimal consumption smoothing by foreign agents causes them to borrow in the anticipation of a long-run level of GDP that is higher than the short-run expansion. In the long run, the foreign economy has a permanently larger debt–and its new long-run consumption and GDP levels remain higher than those at home, as in the initial steady state. The key for the dynamics in Figure 3 is the slow adjustment of relative prices and the terms of trade. When the steady-state level of relative prices (and the terms of trade) differs from 1 (as in the case of an asymmetric steady state), long-run consumption differs from long-run labor income in each country, and even symmetric, permanent productivity shocks end up redistributing demand across countries in a way that induces agents to adjust their labor effort over time rather than keeping it unchanged. Consumption smoothing then results in accumulation of increasing assets (or debt) during the transition dynamics.

Kraay and Ventura (2000) argue that a favorable productivity shock should cause surplus (deficit) in a creditor (debtor) country. We do not reach the same conclusion in the case of country-specific shocks. Regardless of whether the shock takes place in the creditor or in the debtor country, the country that experiences higher productivity responds by improving its foreign asset position (either by accumulating more assets or by reducing its debt) to smooth the effect of the shock on consumption. We attribute this difference to the absence of physical capital accumulation from our model. Including capital in the production function would make it possible to generate current account deficits in response to favorable productivity shocks through the resulting increase in investment. However, we do replicate Kraay and Ventura's pattern in the case of a permanent increase in world productivity: As Figure 3 shows, optimal consumption smoothing behavior leads the creditor country to respond by accumulating assets, whereas the debtor country responds by running an increasing debt. Thus, a consumption-driven, intertemporal approach to the current account that explicitly accounts for structural, cross-country heterogeneity can explain the regularity documented by Kraay and Ventura at least for the case of permanent world-wide shocks.

3.3.4 Relative Steady-State Productivity vs. Impatience

How are the results above affected by changes in relative steady-state productivity and/or in the relative degree of impatience in the foreign economy? For example, given $\alpha < 1$, does it matter for the responses to productivity shocks whether $\frac{\overline{Z}}{\overline{Z}^*}$ is equal to or different from 1? As we observed above, whether $\overline{Z} = \overline{Z}^*$ or $\overline{Z} < \overline{Z}^*$ matters for the sign of some cross-country steady-state differentials. (Recall examples 1 and 2 in our discussion of the steady state.) If $\overline{Z} = \overline{Z}^*$, it is no longer the case that $\overline{c} < \overline{c}^*$ and $\overline{w} < \overline{w}^*$. Steady-state home consumption and real wage are higher than abroad if the two countries are equally productive. To investigate the effect of this change on impulse responses, we re-calculated the responses under the assumption $\overline{Z} = \overline{Z}^* = 1$ keeping the values of the structural parameters unchanged. Although the exercise resulted in some quantitative differences in the responses to productivity shocks in the creditor or debtor country, no qualitative difference emerged. The responses looked very much like those in figures 1 and 2.[29] Therefore, we conclude that the patterns in figures 1 and 2 are robust to changes in the steady-state productivity ratio.

What about the effect of a lower value of α? We know that this will increase the size of foreign steady-state debt. To verify the effect of this change, we re-calculated the responses to country-specific shocks under the assumption $\overline{Z} = 1 < \overline{Z}^* = 1.29$ and the same parameter values as above, but with $\alpha = .999$. In the case of a productivity shock in the debtor country, no qualitative change in the impulse responses is observed and the same intuitions as for Figure 2 apply. The main difference relative to Figure 2 is that the foreign relative price rises above the steady state and the foreign labor effort and GDP fall below earlier than in the benchmark case. (Similarly, the home relative price falls and home labor effort and GDP rise above

[29]The responses for the case $\overline{Z} = \overline{Z}^* = 1$ are available on request. Similarly for other impulse responses that we do not include.

the steady state earlier than in Figure 2.) The consumption differential is somewhat amplified.

The case of a productivity shock in the creditor country is more interesting and is shown in Figure 4. The following main differences emerge relative to Figure 1. Initially, the home economy accumulates assets to smooth the effect of the shock on consumption, as in Figure 1. However, in Figure 4, home assets fall below the steady state less than two years after time 0 and return to the steady state from below. RP now converges to the steady state from below, L and y from above. Similarly, RP^* now converges to the steady state from above, L^* and y^* from below. The foreign real wage returns to the steady state from above. Foreign consumption now rises above home consumption on impact, and both c and c^* return to the long-run level from above. The intuition for these changes is as follows. Other things given, y^* below and c^* above the steady state would cause foreign debt to increase. However, the effect of deviations of GDP and consumption from the steady state on debt dynamics must be weighed by the ratios $\frac{\overline{y}^*}{\overline{B}^*}$ and $\frac{\overline{c}^*}{\overline{B}^*}$, respectively.[30] When α is .999 rather than .9999, the implied increase in steady-state foreign debt ($\overline{B}^*$) causes both these ratios to be extremely small. As a consequence, debt dynamics after time 0 end up mirroring the dynamics of the real interest rate. Put differently, when steady-state debt is very large, the interest burden on previously accumulated debt becomes the main determinant of debt dynamics. As the interest rate falls and is below the steady state throughout the transition to the long run, foreign debt decreases after the initial increase, and it returns to the steady state from below. A lower interest rate burden allows more impatient foreign households (who anticipate that both the interest rate and debt will be below the steady state for the longer portion of the transition) to increase their consumption above that of home households by borrowing more in the initial periods. In turn, foreign debt dynamics are responsible for the dynamics of other endogenous variables once the productivity shocks has died out.[31]

The analysis of this subsection leads us to conclude that the discount factor differential is a more important determinant of model dynamics than

[30] See Appendix A.2. $\frac{\overline{y}^*}{\overline{B}^*}$ and $\frac{\overline{c}^*}{\overline{B}^*}$ are now approximately equal to $-.0764$ and $-.0757$, respectively.

[31] As for the case $\alpha = .9999$, setting $\overline{Z} = \overline{Z}^* = 1$ does not generate any qualitative change in impulse responses to country-specific shocks relative to the situation in which $\overline{Z} = 1 < \overline{Z}^* = 1.29$.

relative steady-state productivity. This is so because changes in the degree of relative impatience have a large impact on steady-state net foreign assets, which can amplify the role of the interest burden on previously accumulated debt in the determination of future asset holdings. When α is as "low" as .999, the importance of interest payments becomes paramount, leading to the dynamics in Figure 4. In reality, the importance of changes in the interest burden on debt in the determination of foreign asset positions and consumption dynamics is of course an issue for empirical investigation, to which we turn next.

4 Empirical Evidence

In this section we first describe the econometric methodology we use to interpret the G7 data, and then discuss our empirical findings.

4.1 The Empirical Methodology

The econometric methodology is based on cointegrated structural VARs (SVARs) with a common trend representation developed and applied by King et al. (1991) and Mallender et al. (1992). Consistent with our theoretical framework, we focus on the minimal state vector, pertaining to the log-linear model solution, and estimate the dynamic responses to productivity shocks of the endogenous variables included in this vector. We then compare the qualitative results from the estimated impulse response functions with those that emerge from our calibrated model, and assess whether the model can appropriately account for the dynamics in the data.

The minimal state vector consists of four variables (all in natural logarithms): home and foreign productivity, net foreign assets, and the risk-free real interest rate. Since much of the literature has focused on the influence of consumption smoothing on the current account dynamics, we augment the minimal state vector by home and foreign consumption. A lag length of two for all variables considered was selected based on standard selection criteria. To obtain normal reduced form residuals, we also augment the minimal state vector with a set of seasonal dummy variables, and an impulse dummy for the German unification (January 1991).

For bilateral comparisons, we focus on Germany, Japan, and the U.S. (G3). Each of these countries are considered against the aggregate of the remaining G7 which represents the foreign economy or "rest-of-the-world" in our framework. Estimation and hypothesis testing are conducted on a bilateral basis, which allow us to compare our empirical results across countries to assess the extent of heterogeneity in our data.

The identification of a common, permanent productivity shock and a country-specific, temporary shock is achieved in several steps. First, we assume that foreign and home productivity are cointegrated (hence share a common stochastic trend), with cointegration vector given by the long-run relation observed in the data during our sample period. In particular, we specify the following long-run relationship (in log-levels) between domestic and foreign productivity:

$$\log \overline{Z} = \gamma_1^{\overline{Z}} + \gamma_2^{\overline{Z}} \log \overline{Z}^*, \tag{27}$$

where $\gamma_1^{\overline{Z}} > 0$ and $\gamma_2^{\overline{Z}} \geq 0$.

Second, as discussed in Appendix A.1, the theoretical model (equations A.19–A.22) delivers three *non-linear* steady-state relations for domestic consumption relative to foreign ($\overline{c}/\overline{c}^*$), home net foreign assets ($\overline{B}$), and the risk-free real interest rate ($\overline{r}$) as functions of the steady-state productivity ratio ($\overline{Z}/\overline{Z}^*$). Since there are no analytic solutions for these functions, we assume that the empirical counterparts of $\overline{r}$ and $\overline{B}$ consist of the following log-linear relations:

$$\log \overline{r} = \gamma_1^{\overline{r}} + \gamma_2^{\overline{r}} \left(\log \overline{Z} - \log \overline{Z}^* \right), \tag{28}$$

$$\log \overline{B} = \gamma_1^{\overline{B}} + \gamma_2^{\overline{B}} \left(\log \overline{Z} - \log \overline{Z}^* \right). \tag{29}$$

As shown in Appendix A.4, the relative steady-state consumption can be rewritten as:

$$\begin{aligned} \log \overline{c} &= \gamma_1^{\overline{c}} + \gamma_2^{\overline{c}} \log \overline{Z} + \gamma_3^{\overline{c}} \log \overline{Z}^*, \\ \log \overline{c}^* &= \gamma_1^{\overline{c}^*} + \gamma_2^{\overline{c}^*} \log \overline{Z} + \gamma_3^{\overline{c}^*} \log \overline{Z}^*. \end{aligned}$$

Thus, after using (27), we obtain:

$$\begin{aligned} \log \overline{Z} &= \gamma_1^{\overline{Z}} + \gamma_2^{\overline{Z}} \log \overline{Z}^*, \\ \log \overline{B} &= \gamma_1^{\overline{B}\prime} + \gamma_2^{\overline{B}'} \log \overline{Z}, \\ \log (1 + \overline{r}) &= \gamma_1^{\overline{r}\prime} + \gamma_2^{\overline{r}\prime} \log \overline{Z}, \\ \log \overline{c} &= \gamma_1^{\overline{c}'} + \gamma_2^{\overline{c}'} \log \overline{Z}, \\ \log \overline{c}^* &= \gamma_1^{\overline{c}^{*\prime}} + \gamma_3^{\overline{c}^{*\prime}} \log \overline{Z}^*. \end{aligned}$$

This system contains five linear relations in six variables. If the six variables considered are I(1) and the five linear combinations are I(0), these represent a set of long-run, cointegration relations and the six variables must share a single common stochastic trend. We interpret innovations to this common stochastic trend as a *common, permanent* productivity shocks. *Temporary, country specific* shocks are then identified simply assuming that home and foreign consumption, net foreign assets and the real interest rate do not affect productivity contemporaneously; a restriction that is fully consistent with the block-recursive structure of our model. The remaining four shocks are left unidentified. [32]

In practice, both common/permanent shocks and temporary/country-specific productivity shocks are identified by using the method described by Mallender et al. (1992).[33] We extract the common trend from the series for foreign productivity, while the innovation in the cointegration vector between LZ and $LZEX$ is interpreted as the temporary, country specific shock.

4.2 Data

We construct a quarterly data set for the G7 using primarily two sources: (i) OECD, *Analytical Database*, which provides comparable data on *business sector* output, consumption, employment, and hours worked, and (ii) net foreign assets (kindly provided to us by Kit Baum) which are largely based on annual series constructed by Lane and Milesi-Ferretti (2001).

Details of how we construct our variables are provided in Appendix B. Here we review them briefly. Labor productivity is business sector real output per employee hour worked. Net foreign asset data are vis-à-vis the rest-of-the-world (not the remaining G7). The real interest rate is *ex-post*, and a country-specific measure. Average labor productivity and consumption series for the "rest-of-the-world" are constructed by *summing* over the variables of interest, after converting them into US$ at constant PPP exchange rates. Consumption and net foreign assets are in per capita terms. Average labor productivity is in hours worked. We restrict the available full sample pe-

[32] To retain flexibility on the short-run dynamics, we do not impose restrictions on the lags of the empirical VAR model. Note also that as the econometric model includes four other shocks, we can carry out our dynamic analysis without ruling out the presence of other structural disturbances.

[33] The econometric model is estimated and simulated using the RATS code written by Andrew Warne.

riod (1977:Q1-1997:Q4) to a shorter sub-sample (1980:Q1-1994:Q4) to avoid introducing too many dummy variables in the VAR (especially for the U.S.).

4.3 Cointegration Results

Our identification assumptions depend on the existence of five cointegration relations. Table 1a and 1b report the results of the application of the Johansen's procedure to VARs in levels specified as above. These results are broadly consistent with our theoretical framework and show that there are striking differences across the countries considered in terms of their responses to changes in the level of productivity.

Specifically, for the U.S., the cointegration rank test does not reject the hypothesis that the rank is less than or equal to three. However, only one eigenvalue is clearly close to zero, suggesting the presence of five stationary components. In addition, when productivity is entered exogenously, we do reject the hypothesis that there are less than or equal to four stationary components. Further, in a bivariate system we are unable to accept the hypothesis that there is cointegration relation between LZ and $LZEX$ (results not reported but available on request). These evidence suggests that a six variable system comprised of five cointegration vectors is a plausible specification for the U.S., when foreign and domestic productivity are modelled as a cointegration relation. Indeed, in this case, the over-identifying restrictions implied by our approximation of the theoretical long-run relations are not rejected by the data (and with a wide margin). Further, the estimated coefficients suggest that the risk-free real interest rate may be stationary, that productivity in the rest-of-the-world was growing faster than in the U.S. during our sample period, and that net foreign assets' response to movements in productivity are large and significant.

For Japan, there is only one eigenvalue close to zero. The test on the system cointegration rank with productivity entered exogenously rejects the hypothesis of less than four cointegration vectors, again suggesting that five stationary components is a plausible specification. However, the over-identifying restrictions are marginally rejected for Japan.

For Germany, the empirical results are more mixed. It appears that there are less than five stationary components, even after imposing cointegration between foreign and domestic productivity. In fact, there are at least two eigenvalues very close to zero. The over-identifying restrictions are rejected at the five percent significance level—although all estimated coefficients have plausible magnitudes and expected signs. In any event, the results for Germany should be interpreted with caution because the reduced form VAR has an explosive root (results not reported).[34]

In summary, the cointegration results suggest that, for the US and Japan, the (approximate) long-run relations implied by the theoretical model provide empirically appropriate specifications that allow us to identify a common, permanent productivity shock. These results also point to striking qualitative and quantitative differences across countries. For example, in Japan, increased productivity is associated with an accumulation of net foreign assets, while in the U.S. it is associated with a sharp increase in foreign liabilities. Consumption responses to fluctuations productivity are also markedly different across countries: U.S. consumption response is almost twice as large as that of Japan.

4.4 Empirical Impulse Responses

Figures 5-7 report, for the U.S., Japan, and Germany the responses of the log-levels of the variables to one-standard deviation shock to the common stochastic trend.[35]

Consider the net foreign assets' response (labelled as "LBT1") in Japan and the U.S. The asymmetry is striking. While the U.S. responds by increasing its foreign debt, Japan accumulates substantial foreign assets. These findings are consistent with the two observations that have been made in the previous literature: (i) the current account responses exhibit marked heterogeneity across countries, and (ii) the current accounts respond to global productivity shocks.

Consumption responses are also interesting. On impact, consumption (variables labelled as "LC") increases in all countries. Then, in the case of the U.S., own consumption reaches its new steady state from *above*, whereas, in the case of Japan, own consumption reaches the new steady-state from *below*. The rest-of-the-world consumption patterns (variables labelled as "LCEX")

[34] Masson et al. (1994) also document the econometric difficulties associated with the German data.

[35] We have considered also responses to temporary country-specific shocks. Our (preliminary) empirical results suggest that responses to country-specific shocks display considerably more asymmetry than our theoretical framework and the benchmark parameter values can account for. Therefore, in the rest of this section, we focus only on the responses to common, permanent shocks.

are consistent with our equilibrium model: consumption excluding Japan reaches its new steady state from above, and consumption excluding the U.S. is reaches its steady-state from below, albeit less clearly than in the other cases.

How can we interpret these impulse responses and the underlying structural asymmetries in light of our theoretical framework? Our theoretical impulse responses demonstrated that, upon impact, less patient, more productive economy would accumulate foreign debt and its own consumption would reach to the new steady state from above. These results appear to be consistent with a view of the world whereby the U.S. can be interpreted as the less patient (and more productive given our estimates) country. Japan would be the counterpart of this hypothesis, and its consumption would reach its new steady-state from below.

Our theoretical model also predicts that net foreign asset and consumption dynamics are largely driven by variations in relative prices, and consumption based real interest rate, with no dynamic response from the risk-free real interest rate. Although the point estimates indicates a positive initial intrest rate response, the standard errors suggest that this is not statistically different from zero.

We have much less confidence in our estimates for Germany, because of the explosive root in the reduced form VAR. What emerges from our estimates, however, suggests that the current account response to a permanent shock is negligible; net foreign assets hardly move. One interpretation of this finding is that households in Germany and the rest of the world face very similar intertemporal tradeoffs; perhaps due to a patience parameter that is representative of the G7.

Our SVAR model and findings regarding permanent global productivity shocks are difficult to compare with existing studies. Glick and Rogoff (1995) only report pooled estimates of current account response to a permanent global shock. Gregory and Head (1999) use dynamic factory analysis to estimate individual country current account responses to a common *transitory* technology shock, whereas Kraay and Ventura (2000) study country-specific shocks.

5 Conclusions

In this paper, we proposed a framework to study international consumption smoothing by allowing for a non-degenerate distribution of net foreign assets. We developed a two-country model of interdependence and considered two sources of structural heterogeneity: (i) differences in degree of patience; and (ii) average labor productivity levels. Our calibration results show that even mild and empirically plausible differences in these structural parameters can impart considerable heterogeneity in net foreign asset and consumption dynamics following productivity shocks.

Our empirical results suggest that (i) the dynamic responses of the net foreign assets and consumption indeed vary considerably across Germany, Japan, and the U.S., and (ii) the U.S. data systematically matches the behavior of a less patient, more productive economy, with Japan emerging as a counterpart. As well, our framework goes some way towards reconciling the attractive notion of international consumption smoothing with empirical evidence. One particularly significant aspect of the analysis is its ability to account for asymmetric responses to global shocks. This asymmetry may provide an explanation for the well documented "puzzle" (for standard intertemporal models) that current account should not respond to world shocks.

Nonetheless, several issues remain. First, our theoretical framework is admittedly stylized. Second, aside from the familiar cointegration tests, most of our comparisons of the calibrated and estimated dynamic responses were qualitative. Given the complexity of allowing for *any* cross-country heterogeneity in an equilibrium model, we confined our analysis to essentially a single fundamental source of asymmetry. Clearly, there may be other sources of heterogeneity that can potentially account for empirically relevant consumption smoothing behavior and the current account dynamics. However, we believe that the resulting differences in steady state and transition dynamics documented in this paper are significant, both theoretically and empirically.

Appendix

A Steady State and Log-Linear Model

A.1 The Steady State

The steady state can be obtained as follows. Using steady-state versions of the consumption functions for domestic and foreign newborn households (14) and of the definition of a household's human wealth (15), steady-state domestic and foreign Euler equations for aggregate per capita consumption at home and abroad are:

$$\overline{c} = \frac{n\rho\left(1-\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\beta\left(1+\overline{r}\right)\right]}\overline{w}, \quad \overline{c}^{*} = \frac{n\rho\left(1-\alpha\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}\overline{w}^{*}. \tag{A.1}$$

Steady-state labor-leisure tradeoffs in aggregate per capita terms imply:

$$\overline{L} = 1 - \frac{1-\rho}{\rho}\frac{\overline{c}}{\overline{w}}, \quad \overline{L}^{*} = 1 - \frac{1-\rho}{\rho}\frac{\overline{c}^{*}}{\overline{w}^{*}}. \tag{A.2}$$

Now, international equilibrium requires that total production of the consumption basket be equal to total consumption:

$$a\overline{y} + \left(1-a\right)\overline{y}^{*} = a\overline{c} + \left(1-a\right)\overline{c}^{*}. \tag{A.3}$$

Steady-state domestic and foreign GDPs in units of the composite consumption basket are:

$$\overline{y} = \overline{RPZL}, \quad \overline{y}^{*} = \overline{RP}^{*}\overline{Z}^{*}\overline{L}^{*}. \tag{A.4}$$

Optimal price setting is such that $\overline{RP} = \frac{\overline{w}}{\overline{Z}}$, $\overline{RP}^{*} = \frac{\overline{w}^{*}}{\overline{Z}^{*}}$. Hence:

$$\overline{y} = \overline{w}\overline{L}, \quad \overline{y}^{*} = \overline{w}^{*}\overline{L}^{*}. \tag{A.5}$$

Substituting these equations into (A.3) yields:

$$a\overline{w}\overline{L} + \left(1-a\right)\overline{w}^{*}\overline{L}^{*} = a\overline{c} + \left(1-a\right)\overline{c}^{*}. \tag{A.6}$$

Use equations (A.2) to substitute for $\overline{L}$ and $\overline{L}^{*}$ into (A.6) and rearrange the resulting equation to obtain:

$$\frac{\overline{c}^{*}}{\rho\overline{w}^{*}}\left(a\frac{\overline{c}}{\overline{c}^{*}} + 1 - a\right) = a\frac{\overline{w}}{\overline{w}^{*}} + 1 - a. \tag{A.7}$$

The steady-state Euler equations in (A.1) imply:

$$\frac{\overline{c}}{\overline{c}^*} = \frac{(1-\beta)\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}{(1-\alpha\beta)\left[1+n-\beta\left(1+\overline{r}\right)\right]}\frac{\overline{w}}{\overline{w}^*}. \tag{A.8}$$

Also, the equation for foreign consumption implies:

$$\frac{\overline{c}^*}{\overline{w}^*} = \frac{n\rho\left(1-\alpha\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}. \tag{A.9}$$

Substituting (A.8) and (A.9) into (A.7) yields an equation that relates the world interest rate $\overline{r}$ to the real wage ratio $\frac{\overline{w}}{\overline{w}^*}$:

$$\begin{aligned}&\frac{n\left(1-\alpha\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}\left\{a\frac{(1-\beta)\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}{(1-\alpha\beta)\left[1+n-\beta\left(1+\overline{r}\right)\right]}\frac{\overline{w}}{\overline{w}^*}+1-a\right\}\\ = \; & a\frac{\overline{w}}{\overline{w}^*}+1-a.\end{aligned} \tag{A.10}$$

Steady-state labor demand equations, optimal pricing, and the definition of world demand imply:

$$\overline{L} = \left(\frac{\overline{w}}{\overline{Z}}\right)^{-\omega}\frac{a\overline{y}+(1-a)\,\overline{y}^*}{\overline{Z}}, \quad \overline{L}^* = \left(\frac{\overline{w}^*}{\overline{Z}^*}\right)^{-\omega}\frac{a\overline{y}+(1-a)\,\overline{y}^*}{\overline{Z}^*}. \tag{A.11}$$

Hence,

$$\frac{\overline{L}}{\overline{L}^*} = \left(\frac{\overline{w}}{\overline{w}^*}\right)^{-\omega}\left(\frac{\overline{Z}}{\overline{Z}^*}\right)^{\omega-1}, \text{ or } \frac{\overline{w}}{\overline{w}^*} = \left(\frac{\overline{L}}{\overline{L}^*}\right)^{-\frac{1}{\omega}}\left(\frac{\overline{Z}}{\overline{Z}^*}\right)^{\frac{\omega-1}{\omega}}. \tag{A.12}$$

Substitute for the wage ratio from (A.12) into (A.10) and rearrange:

$$\begin{aligned}&a\left(\frac{\overline{L}}{\overline{L}^*}\right)^{-\frac{1}{\omega}}\left(\frac{\overline{Z}}{\overline{Z}^*}\right)^{\frac{\omega-1}{\omega}}\left\{\frac{n\left(1-\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\beta\left(1+\overline{r}\right)\right]}-1\right\}\\ = \; & (1-a)\left\{1-\frac{n\left(1-\alpha\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}\right\}.\end{aligned} \tag{A.13}$$

The labor-leisure tradeoffs (A.2) and equation (A.8) imply:

$$1-\overline{L} = \frac{(1-\beta)\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}{(1-\alpha\beta)\left[1+n-\beta\left(1+\overline{r}\right)\right]}\left(1-\overline{L}^*\right). \tag{A.14}$$

Rearrange equation (A.14) as:

$$\frac{\overline{L}}{\overline{L}^*} = \frac{1}{\overline{L}^*} - \frac{(1-\beta)\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}{(1-\alpha\beta)\left[1+n-\beta\left(1+\overline{r}\right)\right]}\left(\frac{1}{\overline{L}^*}-1\right). \tag{A.15}$$

The foreign labor-leisure tradeoff and Euler equation imply:

$$\overline{L}^* = 1 - \frac{1-\rho}{\rho}\frac{n\rho\left(1-\alpha\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}. \tag{A.16}$$

Substituting (A.16) into the right hand side of (A.15) and rearranging yields:

$$\frac{\overline{L}}{\overline{L}^*} = \frac{\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]\left\{\left(\overline{r}-n\right)\left[1-\beta\left(1+\overline{r}\right)\right]+n\rho\left(1-\beta\right)\left(1+\overline{r}\right)\right\}}{\left[1+n-\beta\left(1+\overline{r}\right)\right]\left\{\left(\overline{r}-n\right)\left[1-\alpha\beta\left(1+\overline{r}\right)\right]+n\rho\left(1-\alpha\beta\right)\left(1+\overline{r}\right)\right\}}. \tag{A.17}$$

Equations (A.13) and (A.17) constitute a system of two equations in two unknowns, the steady-state world interest rate $\overline{r}$ and the labor effort ratio $\frac{\overline{L}}{\overline{L}^*}$ as functions of parameters and the productivity ratio $\frac{\overline{Z}}{\overline{Z}^*}$.

Note that setting $\alpha = 1$ in (A.17) yields $\frac{\overline{L}}{\overline{L}^*} = 1$ regardless of $\frac{\overline{Z}}{\overline{Z}^*}$. If agents' intertemporal preferences are identical at home and abroad, the only possible equilibrium is one in which $\overline{r} = \frac{1-\beta}{\beta}$ and net foreign assets are zero even if $\frac{\overline{Z}}{\overline{Z}^*} \neq 1$. To see this, observe that, if $\alpha = 1$, equation (A.13) can be rewritten as:

$$\left\{\frac{n\left(1-\beta\right)\left(1+\overline{r}\right)}{\overline{r}\left[1+n-\beta\left(1+\overline{r}\right)\right]}-1\right\}\left[a\left(\frac{\overline{Z}}{\overline{Z}^*}\right)^{\frac{\omega-1}{\omega}}+1-a\right] = 0. \tag{A.18}$$

This has solutions $\frac{1-\beta}{\beta}$ and n, but the latter is not admissible, as it would imply that steady-state net foreign assets would not be defined.[36] Thus, if

[36] Using results in this appendix and the steady-state version of the law of motion for domestic net foreign assets, it is easy to verify that steady-state net foreign assets are equal to:

$$\overline{B} = \frac{1}{\overline{r}-n}\left\{\frac{n\left(1-\beta\right)\left(1+\overline{r}\right)-\overline{r}\left[1+n-\beta\left(1+\overline{r}\right)\right]}{\overline{r}\left[1+n-\beta\left(1+\overline{r}\right)\right]}\right\}\overline{w}.$$

Similarly,

$$\overline{B}^* = \frac{1}{\overline{r}-n}\left\{\frac{n\left(1-\alpha\beta\right)\left(1+\overline{r}\right)-\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}{\overline{r}\left[1+n-\alpha\beta\left(1+\overline{r}\right)\right]}\right\}\overline{w}^*,$$

and international equilibrium requires $a\overline{B} + (1-a)\overline{B}^* = 0$, which is equivalent to the condition used in the text ($\overline{y}^W = \overline{c}^W$) by Walras' Law.

$\alpha = 1$ but $\frac{\overline{Z}}{\overline{Z}^*} \neq 1$, domestic and foreign GDPs in units of consumption differ, and so do consumption levels. But consumption equals GDP in each country, so that net foreign assets are zero. Since $\overline{y} = \overline{w}\overline{L}$ and $\overline{y}^* = \overline{w}^*\overline{L}^*$ in equilibrium, $\overline{L} = \overline{L}^*$ when $\alpha = 1$ implies that the different GDP levels generated by different productivity levels translate into different real wages and labor incomes across countries. The more productive country has a higher steady-state real wage and consumption and a lower relative price for the same labor effort as the less productive country.

If $\alpha = 1$ and $\overline{Z} = \overline{Z}^* = 1$, the steady state is the same as in Ghironi (2000): $\overline{B} = 0$, and the steady state is symmetric in all respects: $r = \frac{1-\beta}{\beta}$, $\overline{B} = \overline{B}^* = 0$, $\overline{c} = \overline{c}^* = \overline{L} = \overline{L}^* = \overline{y} = \overline{y}^* = \rho$, $\overline{w} = \overline{w}^* = \overline{RP} = \overline{RP}^* = 1$.

In the general case $\alpha \leq 1$, let us write the solution for $\overline{r}$ as a function of $\frac{\overline{Z}}{\overline{Z}^*}$, which we obtain numerically, as:

$$\overline{r} = \overline{r}\left(\frac{\overline{Z}}{\overline{Z}^*}\right). \tag{A.19}$$

Substituting (A.19) into (A.17) yields an equation that can be solved for the steady-state labor effort ratio as a function of relative productivity. We write the solution as:

$$\frac{\overline{L}}{\overline{L}^*} = \overline{L}^R\left(\frac{\overline{Z}}{\overline{Z}^*}\right), \tag{A.20}$$

where $\overline{L}^R\left(\frac{\overline{Z}}{\overline{Z}^*}\right)$ is a function of relative productivity, the characteristics of which depend on structural parameter values, and $\overline{L}^R\left(\frac{\overline{Z}}{\overline{Z}^*}\right) = 1$ if $\alpha = 1$.

Given (A.19), we can obtain solutions for steady-state consumption, wage, and GDP ratios, as well as net foreign assets. In particular:

$$\frac{\overline{c}}{\overline{c}^*} = \overline{c}^R\left(\frac{\overline{Z}}{\overline{Z}^*}\right), \tag{A.21}$$

$$\overline{B} = \overline{B}\left(\frac{\overline{Z}}{\overline{Z}^*}\right). \tag{A.22}$$

For the reasons discussed above, if $\alpha = 1$, it is $\overline{B}\left(\frac{\overline{Z}}{\overline{Z}^*}\right) = 0$, and $\overline{c}^R\left(\frac{\overline{Z}}{\overline{Z}^*}\right) \gtrless 1$ if $\overline{Z} \gtrless \overline{Z}^*$. If $\alpha < 1$ and $\overline{Z} = \overline{Z}^* = 1$, plausible parameter values yield $\overline{B} > 0$ ($\overline{B}^* < 0$), $\overline{c} > \overline{c}^*$, $\overline{L} < \overline{L}^*$, $\overline{w} > \overline{w}^*$, $\overline{RP} > \overline{RP}^*$, $\overline{y} < \overline{y}^*$. If $\alpha < 1$ and $\overline{Z} = 1 < \overline{Z}^*$, plausible parameter values yield $\overline{B} > 0$ ($\overline{B}^* < 0$), $\overline{c} < \overline{c}^*$, $\overline{L} < \overline{L}^*$, $\overline{w} < \overline{w}^*$, $\overline{RP} > \overline{RP}^*$, $\overline{y} < \overline{y}^*$.

A.2 The Log-Linear Model

The main log-linear equations of the model are as follows. (Sans serif fonts denote percentage deviations of variables from the respective steady-state levels. In the case of the interest rate, we consider the percentage deviation of the gross interest rate from the steady state.)

The laws of motion for aggregate per capita home and foreign net foreign assets are:

$$(1+n)\,\mathsf{B}_{t+1} = (1+\overline{r})\,(\mathsf{r}_t + \mathsf{B}_t) + \frac{\overline{y}}{\overline{B}}\mathsf{y}_t - \frac{\overline{c}}{\overline{B}}\mathsf{c}_t, \tag{A.23}$$

$$(1+n)\,\mathsf{B}^*_{t+1} = (1+\overline{r})\,(\mathsf{r}_t + \mathsf{B}^*_t) + \frac{\overline{y}^*}{\overline{B}^*}\mathsf{y}^*_t - \frac{\overline{c}^*}{\overline{B}^*}\mathsf{c}^*_t. \tag{A.24}$$

International asset markets equilibrium requires that a country's asset accumulation must be mirrored by the other country's debt:

$$\mathsf{B}_t - \mathsf{B}^*_t = 0. \tag{A.25}$$

Home and foreign GDPs are, respectively:

$$\mathsf{y}_t = \mathsf{RP}_t + \mathsf{L}_t + \mathsf{Z}_t, \tag{A.26}$$

$$\mathsf{y}^*_t = \mathsf{RP}^*_t + \mathsf{L}^*_t + \mathsf{Z}^*_t. \tag{A.27}$$

Labor demand at home and abroad is a function of the relative price of the goods a country produces, of world demand of the consumption basket, and of productivity:

$$\mathsf{L}_t = -\omega\mathsf{RP}_t + \mathsf{y}^W_t - \mathsf{Z}_t, \tag{A.28}$$

$$\mathsf{L}^*_t = -\omega\mathsf{RP}^*_t + \mathsf{y}^W_t - \mathsf{Z}^*_t, \tag{A.29}$$

where $\omega > 0$ is the elasticity of substitution between home and foreign goods in consumption and $\mathsf{y}^W_t = \mathsf{c}^W_t = \frac{a\overline{c}}{a\overline{c}+(1-a)\overline{c}^*}\mathsf{c}_t + \frac{(1-a)\overline{c}^*}{a\overline{c}+(1-a)\overline{c}^*}\mathsf{c}^*_t$.

Prices are equal to marginal costs:

$$\mathsf{RP}_t = \mathsf{w}_t - \mathsf{Z}_t, \tag{A.30}$$

$$\mathsf{RP}^*_t = \mathsf{w}^*_t - \mathsf{Z}^*_t. \tag{A.31}$$

Labor supply in each country is such that the marginal disutility of an extra unit of labor effort equals the value of the real wage in terms of marginal utility of consumption:

$$\mathsf{L}_t = -\frac{1-\rho}{\rho}\frac{\bar{c}}{\overline{w}\overline{L}}\left(\mathsf{c}_t - \mathsf{w}_t\right), \tag{A.32}$$

$$\mathsf{L}_t^* = -\frac{1-\rho}{\rho}\frac{\bar{c}^*}{\overline{w}^*\overline{L}^*}\left(\mathsf{c}_t^* - \mathsf{w}_t^*\right). \tag{A.33}$$

Euler equations for aggregate per capita domestic and foreign consumption include an additional term that depends on consumption by newborn households at time $t+1$. In turn, newborn households' consumption is a function of the households' human wealth, defined as the present discounted value of the households infinite lifetime in terms of the real wage. It is:

$$\mathsf{c}_t = -\mathsf{r}_{t+1} + \frac{1+n}{\beta\left(1+\bar{r}\right)}\mathsf{c}_{t+1} - \frac{n\rho\left(1-\beta\right)}{\beta\left(1+\bar{r}\right)}\frac{\overline{h}}{\bar{c}}\mathsf{h}_{t+1}, \tag{A.34}$$

$$\mathsf{c}_t^* = -\mathsf{r}_{t+1} + \frac{1+n}{\alpha\beta\left(1+\bar{r}\right)}\mathsf{c}_{t+1}^* - \frac{n\rho\left(1-\alpha\beta\right)}{\alpha\beta\left(1+\bar{r}\right)}\frac{\overline{h}^*}{\bar{c}^*}\mathsf{h}_{t+1}^*. \tag{A.35}$$

Human wealth at home and abroad is such that:

$$\mathsf{h}_t = \frac{1}{1+\bar{r}}\left(\mathsf{h}_{t+1} - \mathsf{r}_{t+1}\right) + \frac{\overline{w}}{\overline{h}}\mathsf{w}_t, \tag{A.36}$$

$$\mathsf{h}_t^* = \frac{1}{1+\bar{r}}\left(\mathsf{h}_{t+1}^* - \mathsf{r}_{t+1}\right) + \frac{\overline{w}^*}{\overline{h}^*}\mathsf{w}_t^*. \tag{A.37}$$

Finally, to close the model, productivities at home and abroad are described by the processes assumed in the text in all periods after the time of an initial impulse ($t = 0$):

$$\mathsf{Z}_t = \phi\mathsf{Z}_{t-1}, \quad \mathsf{Z}_t^* = \phi\mathsf{Z}_{t-1}^*, \quad 0 \leq \phi \leq 1. \tag{A.38}$$

A.3 The World Interest Rate

As shown in Ghironi (2000), the risk-free, real interest rate is determined by the expected rate of world-wide real wage growth:

$$1 + r_{t+1} = \frac{w_{t+1}^W}{\beta w_t^W},$$

where $w_t^W \equiv aw_t + (1-a)\, w_t^*$. In log-linear terms:

$$\mathsf{r}_{t+1} = \mathsf{w}_{t+1}^W - \mathsf{w}_t^W, \tag{A.39}$$

with $\mathsf{w}_t^W = a\frac{\overline{w}}{\overline{w}^W}\mathsf{w}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{w}_t^*$.

Real wages at home and abroad are tied to relative prices and productivity by optimal pricing in the two economies and the markup-offsetting subsidies: $RP_t = \frac{w_t}{Z_t}$ and $RP_t^* = \frac{w_t^*}{Z_t^*}$. Put differently, competitive labor markets and markup-offsetting subsidies imply that workers in the two countries are paid the consumption-value of their marginal products: $w_t = RP_t Z_t$ and $w_t^* = RP_t^* Z_t^*$. Therefore, $w_t^W = aRP_t Z_t + (1-a)\, RP_t^* Z_t^*$, or, in log-linear terms:

$$\mathsf{w}_t^W = a\frac{\overline{w}}{\overline{w}^W}\left(\mathsf{RP}_t + \mathsf{Z}_t\right) + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\left(\mathsf{RP}_t^* + \mathsf{Z}_t^*\right), \tag{A.40}$$

and

$$\begin{aligned} \mathsf{r}_{t+1} &= a\frac{\overline{w}}{\overline{w}^W}\left(\mathsf{RP}_{t+1} - \mathsf{RP}_t + \mathsf{Z}_{t+1} - \mathsf{Z}_t\right) \\ &\quad + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\left(\mathsf{RP}_{t+1}^* - \mathsf{RP}_t^* + \mathsf{Z}_{t+1}^* - \mathsf{Z}_t^*\right). \end{aligned} \tag{A.41}$$

Now, equation (A.40) can be rewritten as:

$$a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^* = \mathsf{w}_t^W - \left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{Z}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{Z}_t^*\right].$$

When the steady state is symmetric, $\overline{w} = \overline{w}^* = \overline{w}^W$, $\mathsf{w}_t^W = \mathsf{Z}_t^W \equiv a\mathsf{Z}_t + (1-a)\,\mathsf{Z}_t^*$, and $a\mathsf{RP}_t + (1-a)\,\mathsf{RP}_t^* = 0$. In this case, there is no relative price effect on the real interest rate, which is simply equal to expected world-wide productivity growth: $\mathsf{r}_{t+1} = \mathsf{Z}_{t+1}^W - \mathsf{Z}_t^W$ (Ghironi, 2000). We show below that, when the steady state is asymmetric, but steady-state home labor effort is close to foreign, it is $\mathsf{w}_t^W \approx a\frac{\overline{w}}{\overline{w}^W}\mathsf{Z}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{Z}_t^*$, or $a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^* \approx 0$. In this case, relative price effects on the world interest rate are negligible (home and foreign relative price effects cancel each other out), and equation (A.41) is well approximated by:

$$\mathsf{r}_{t+1} \approx a\frac{\overline{w}}{\overline{w}^W}\left(\mathsf{Z}_{t+1} - \mathsf{Z}_t\right) + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\left(\mathsf{Z}_{t+1}^* - \mathsf{Z}_t^*\right). \tag{A.42}$$

The interest rate is no longer equal to world productivity growth, but it is approximately equal to a weighted average of productivity growth in the two countries, where the weights are adjusted to reflect the share of a country's wage in the world real wage.

To prove this result, take a weighted average of equations (A.26) and (A.27) with weights $a\frac{\overline{w}}{\overline{w}^W}$ and $(1-a)\frac{\overline{w}^*}{\overline{w}^W}$, respectively. It is:

$$\begin{aligned} & a\frac{\overline{w}}{\overline{w}^W}\mathsf{y}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{y}_t^* \\ = \; & a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^* + a\frac{\overline{w}}{\overline{w}^W}\mathsf{Z}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{Z}_t^* \\ & + a\frac{\overline{w}}{\overline{w}^W}\mathsf{L}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{L}_t^*. \end{aligned} \tag{A.43}$$

Take the same weighted average of (A.28) and (A.29), use the fact that $a\frac{\overline{w}}{\overline{w}^W} + (1-a)\frac{\overline{w}^*}{\overline{w}^W} = 1$, and rearrange:

$$\begin{aligned} \mathsf{y}_t^W = \; & \omega\left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^*\right] + a\frac{\overline{w}}{\overline{w}^W}\mathsf{Z}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{Z}_t^* \\ & + a\frac{\overline{w}}{\overline{w}^W}\mathsf{L}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{L}_t^*. \end{aligned} \tag{A.44}$$

Subtract equation (A.43) from (A.44):

$$\mathsf{y}_t^W - \left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{y}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{y}_t^*\right] = (\omega-1)\left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^*\right]. \tag{A.45}$$

Repeat the exercise of equations (A.43)–(A.45) with weights $a\frac{\overline{y}}{\overline{y}^W}$ and $(1-a)\frac{\overline{y}^*}{\overline{y}^W}$ (of course, $a\frac{\overline{y}}{\overline{y}^W} + (1-a)\frac{\overline{y}^*}{\overline{y}^W} = 1$). Because $\mathsf{y}_t^W = a\frac{\overline{y}}{\overline{y}^W}\mathsf{y}_t + (1-a)\frac{\overline{y}^*}{\overline{y}^W}\mathsf{y}_t^*$, it must be:

$$a\frac{\overline{y}}{\overline{y}^W}\mathsf{RP}_t + (1-a)\frac{\overline{y}^*}{\overline{y}^W}\mathsf{RP}_t^* = 0. \tag{A.46}$$

Solve (A.46) for RP_t $(= -\frac{1-a}{a}\frac{\overline{y}^*}{\overline{y}^W}\mathsf{RP}_t^*)$ and substitute into (A.45), recalling that $\overline{y} = \overline{w}\overline{L}$ and $\overline{y}^* = \overline{w}^*\overline{L}^*$. It is:

$$\mathsf{y}_t^W - \left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{y}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{y}_t^*\right] = -(\omega-1)(1-a)\frac{\overline{w}^*}{\overline{w}^W}\left(\frac{\overline{L}^*}{\overline{L}} - 1\right)\mathsf{RP}_t^*. \tag{A.47}$$

If $\alpha = 1$, $\overline{L} = \overline{L}^*$ regardless of $\overline{Z}$ vs. $\overline{Z}^*$. In that case, $\mathsf{y}_t^W = \left[a\frac{\overline{w}}{\overline{w}^W}\mathsf{y}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{y}_t^*\right]$ and $a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^* = 0$. (In fact, $\frac{\overline{y}}{\overline{y}^W} = \frac{\overline{w}\overline{L}}{a\overline{w}\overline{L} + (1-a)\overline{w}^*\overline{L}^*} = \frac{\overline{w}}{a\overline{w} + (1-a)\overline{w}^*\frac{\overline{L}^*}{\overline{L}}} \neq$

$\frac{\overline{w}}{\overline{w}^W}$, unless $\overline{L} = \overline{L}^*$. Similarly for $\frac{\overline{y}^*}{\overline{y}^W}$ and $\frac{\overline{w}^*}{\overline{w}^W}$.) If $\alpha < 1$, equations (A.45) and (A.47) show that the difference between $a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^*$ and 0 is proportional to the difference between $\frac{\overline{L}^*}{\overline{L}}$ and 1. Therefore, if steady-state home labor effort is close to foreign, $a\frac{\overline{w}}{\overline{w}^W}\mathsf{RP}_t + (1-a)\frac{\overline{w}^*}{\overline{w}^W}\mathsf{RP}_t^*$ is close to zero, and equation (A.42) ((26) in the main text) approximates the solution for the world interest rate accurately.

A.4 From Theory to the Long-Run Empirical Restrictions

The functions $\overline{r}(\cdot)$, $\overline{L}^R(\cdot)$, $\overline{c}^R(\cdot)$, and $\overline{B}(\cdot)$ in equations (A.19)–(A.22) are non-linear and depend on the structural parameters. We assume that steady-state domestic and foreign productivity levels are such that:

$$\overline{Z} = \Gamma_1^{\overline{Z}}\left(\overline{Z}^*\right)^{\gamma_2^{\overline{Z}}}, \tag{A.48}$$

where $\Gamma_1^{\overline{Z}}$ and $\gamma_2^{\overline{Z}}$ are coefficients such that $\Gamma_1^{\overline{Z}} > 0$ and $\gamma_2^{\overline{Z}} \geq 0$. In logs:

$$\log\overline{Z} = \gamma_1^{\overline{Z}} + \gamma_2^{\overline{Z}}\log\overline{Z}^*,$$

where $\gamma_1^{\overline{Z}} = \log\Gamma_1^{\overline{Z}}$.

We assume that the empirical counterpart to the functions $\overline{r}(\cdot)$, $\overline{c}^R(\cdot)$, and $\overline{B}(\cdot)$ consists of the following log-linear relations:

$$\begin{aligned}
\log\overline{r} &= \gamma_1^{\overline{r}} + \gamma_2^{\overline{r}}\left(\log\overline{Z} - \log\overline{Z}^*\right), \\
\log\overline{c} - \log\overline{c}^* &= \gamma_1^{\overline{c}^R} + \gamma_2^{\overline{c}^R}\left(\log\overline{Z} - \log\overline{Z}^*\right), \\
\log\overline{B} &= \gamma_1^{\overline{B}} + \gamma_2^{\overline{B}}\left(\log\overline{Z} - \log\overline{Z}^*\right).
\end{aligned}$$

Now, for any pair of individual country variables x_t and x_t^*, define the following variables: $x_t^A \equiv (x_t)^a (x_t^*)^{1-a}$ and $x_t^R \equiv \frac{x_t}{x_t^*}$.[37] Given these definitions, the following equalities hold:

$$x_t = x_t^A\left(x_t^R\right)^{1-a}, \quad x_t^* = x_t^A\left(x_t^R\right)^{-a}. \tag{A.49}$$

[37] x_t^A differs from the definition of world aggregate x_t^W ($\equiv ax_t + (1-a)x_t^*$) used in the text. For this reasons, we use a different superscript.

In logs:

$$\log x_t = \log x_t^A + (1-a)\log x_t^R, \quad \log x_t^* = \log x_t^A - a\log x_t^R, \tag{A.50}$$

with $\log x_t^R = \log x_t - \log x_t^*$.

In steady state:

$$\log \overline{c} = \log \overline{c}^A + (1-a)\left(\log \overline{c} - \log \overline{c}^*\right), \quad \log \overline{c}^* = \log \overline{c}^A - a\left(\log \overline{c} - \log \overline{c}^*\right). \tag{A.51}$$

Now, use $\log \overline{c} - \log \overline{c}^* = \gamma_1^{\overline{c}^R} + \gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right)$. We have:

$$\log \overline{c} = \log \overline{c}^A + (1-a)\gamma_1^{\overline{c}^R} + (1-a)\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right), \tag{A.52}$$

$$\log \overline{c}^* = \log \overline{c}^A - a\gamma_1^{\overline{c}^R} - a\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right). \tag{A.53}$$

We need the solution for $\overline{c}^A$. Asymmetry of the steady state complicates matters greatly. However, $\overline{c}^A$ must ultimately be a function of steady-state productivity at home and abroad: $\overline{c}^A = \overline{c}^A\left(\overline{Z}, \overline{Z}^*\right)$. We approximate this function with a log-linear relation, so that:

$$\log \overline{c}^A = \gamma_1^{\overline{c}^A} + \gamma_2^{\overline{c}^A}\log \overline{Z} + \gamma_3^{\overline{c}^A}\log \overline{Z}^*. \tag{A.54}$$

Then, we can write:

$$\begin{aligned}\log \overline{c} &= \gamma_1^{\overline{c}^A} + (1-a)\gamma_1^{\overline{c}^R} + \gamma_2^{\overline{c}^A}\log \overline{Z} + \gamma_3^{\overline{c}^A}\log \overline{Z}^* + (1-a)\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right) \\ &= \gamma_1^{\overline{c}} + \gamma_2^{\overline{c}^A}\log \overline{Z} + \gamma_3^{\overline{c}^A}\log \overline{Z}^* + (1-a)\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right), \quad \text{(A.55)} \\ \log \overline{c}^* &= \gamma_1^{\overline{c}^A} - a\gamma_1^{\overline{c}^R} + \gamma_2^{\overline{c}^A}\log \overline{Z} + \gamma_3^{\overline{c}^A}\log \overline{Z}^* - a\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right) \\ &= \gamma_1^{\overline{c}^*} + \gamma_2^{\overline{c}^A}\log \overline{Z} + \gamma_3^{\overline{c}^A}\log \overline{Z}^* - a\gamma_2^{\overline{c}^R}\left(\log \overline{Z} - \log \overline{Z}^*\right), \quad \text{(A.56)}\end{aligned}$$

where $\gamma_1^{\overline{c}} \equiv \gamma_1^{\overline{c}^A} + (1-a)\gamma_1^{\overline{c}^R}$ and $\gamma_1^{\overline{c}^*} \equiv \gamma_1^{\overline{c}^A} - a\gamma_1^{\overline{c}^R}$. Steady-state domestic and foreign consumption levels are functions of steady-state productivity in each country and of the cross-country productivity differential. Rearranging terms and defining $\gamma_2^{\overline{c}} \equiv \gamma_2^{\overline{c}^A} + (1-a)\gamma_2^{\overline{c}^R}$, $\gamma_3^{\overline{c}} \equiv \gamma_3^{\overline{c}^A} - (1-a)\gamma_2^{\overline{c}^R}$, $\gamma_2^{\overline{c}^*} \equiv \gamma_2^{\overline{c}^A} - a\gamma_2^{\overline{c}^R}$, and $\gamma_3^{\overline{c}^*} \equiv \gamma_3^{\overline{c}^A} + a\gamma_2^{\overline{c}^R}$ gives the long-run restrictions on the behavior of home and foreign consumption stated in the text.

A.5 Growth Rates

Assume exogenous long-run productivity growth such that:

$$\log \overline{Z}_t^* - \log \overline{Z}_{t-1}^* = g^*.$$

Then, (27) implies:

$$\log \overline{Z}_t - \log \overline{Z}_{t-1} = g = \gamma_2^{\overline{Z}} g^*.$$

First-differencing (28) yields:

$$\Delta \log \overline{B}_t = \gamma_2^{\overline{B}} (g - g^*) = \gamma_2^{\overline{B}} \left(\gamma_2^{\overline{Z}} - 1 \right) g^*.$$

First-differencing equation (A.55) yields:

$$\begin{aligned} \Delta \log \overline{c}_t &= \gamma_2^{\overline{c}^A} g + \gamma_3^{\overline{c}^A} g^* + (1-a)\, \gamma_2^{\overline{c}^R} (g - g^*) \\ &= \gamma_2^{\overline{c}^A} \gamma_2^{\overline{Z}} g^* + \gamma_3^{\overline{c}^A} g^* + (1-a)\, \gamma_2^{\overline{c}^R} g^* \left(\gamma_2^{\overline{Z}} - 1 \right) \\ &= \left[\gamma_2^{\overline{c}^A} \gamma_2^{\overline{Z}} + \gamma_3^{\overline{c}^A} + (1-a)\, \gamma_2^{\overline{c}^R} \left(\gamma_2^{\overline{Z}} - 1 \right) \right] g^*. \end{aligned}$$

Finally, first-differencing equation (A.56) yields:

$$\Delta \log \overline{c}_t^* = \left[\gamma_2^{\overline{c}^A} \gamma_2^{\overline{Z}} + \gamma_3^{\overline{c}^A} - a \gamma_2^{\overline{c}^R} \left(\gamma_2^{\overline{Z}} - 1 \right) \right] g^*.$$

B Data

B.1 Sources and Coverage

We use quarterly data for the G7 countries. These data are, to the best of our ability, comparable across countries. We primarily use three data sources. (i) OECD, *Analytical Database* (AD), which provides quarterly comparable data on *business sector* output, consumption, employment (retrieved on 18 February 2002); (ii) Quarterly net foreign assets (NFA) data which were graciously provided by Kit Baum, who builds on the annual series constructed by Lane and Milesi-Ferretti (2001); and (iii) IMF, *International Financial Statistics* (IFS).

Country abbreviations are:

CAN	Canada	JPN	Japan
FRA	France	UK	United Kingdom
DEU	Germany	US	United States
ITA	Italy		

While some series go back to 1960, available OECD data for the business sector are mostly limited to the period from 1970Q1 to 1999Q4. The sample period for NFA is from 1977Q1 to 1997Q4, the first year we have NFA data for Japan. This is the limiting variable. In empirical analysis, when needed, we re-scaled net foreign asset data to ensure strictly positive series.

B.2 Variables

Output.—Gross domestic product (GDP), business sector, volume, factor cost, in millions of local currency units. For Canada and the US, the base year is 1997 and 1996, respectively. For the rest of the G7, the base year is 1995. We re-based the Canadian and US business sector GDP so that 1995 is the common base year. We used the GDP business sector deflator for this purpose.
Employment.—Employment of the business sector, millions of persons.
Hours worked.—Actual hours worked per employee in the business sector. The US series is and index. We back-casted it starting from 1989 by using the annual average hours actually worked obtained from the OECD, *Employment Outlook*, 2001 Edition, Table F.
Exchange rate.—Purchasing power parity (PPP), local currency per US dollar, from the AD; annual interpolated to quarterly by means of a cubic spline.

Consumption.—Business sector private final consumption deflated by using the (business sector) GDP deflator, except in the case of UK for which we lacked the series on GDP deflator, and we used the private final consumption deflator.
Population.—Annual series from AD were interpolated by using a cubic spline.
NFA.—Deflated with the CPI and expressed in per capita. Net foreign asset are deflated by national CPI's from the IFS. These series are then converted into current US dollars using market exchange rates (from the IFS) and deflated by the US GDP business sector deflator (from the AD).
Real interest rate.—Nominal interest rate (i) adjusted for annualized quarterly inflation in the average consumer price index (CPI):

$$1 + r_t = \log \frac{1 + i_t/100}{(\mathrm{CPI}_t/\mathrm{CPI}_{t-1})^4}.$$

The nominal interest rate is a quarterly average of 3-month T-bill rates on an annual basis, except for Japan. For Japan we used the call money market rate because we lacked comparable data. Sources: IFS.

B.3 Labor Productivity

To obtain "per unit of labor service", we first calculated business sector GDP per hour worked in local currency units as GDP per employee hour worked:

$$gdp = GDP/(\text{employment} \times \text{hours worked}).$$

We then converted this variable to a common currency. To do this, we pursue two alternative strategies. However, the two methods yield very similar results (see Table B1).

The first method uses national price deflators. In this case, we deflated GDP per employee hour worked by national business sector GDP deflators (P),

$$Rgdp(t) = gdp(t)/P(t),$$

and then calculated *Rgdp* per hour worked in US dollars by using 1995 PPP (PPP_{1995}):

$$Z(t) = Rgdp(t)/PPP_{1995}.$$

This is our basic labor productivity measure.

The second method uses US price deflator for all countries. In this case, we first converted GDP per employee hour worked into US dollars using interpolated PPP series, and then deflated all series by the US business sector GDP deflator ($\mathrm{P_{us}}$):

$$Z^{'}(t) = [gdp(t)/PPP(t)]/P_{\mathrm{us}}(t).$$

The only exception is the UK for which we do not have the GDP business sector deflator data prior to 1987Q1. In this case, we used the consumption deflator to convert real GDP business sector values into nominal GDP, and then proceeded as above.

Per capita consumption is private final consumption expenditures divided by population. Our measure of consumption is real per capita consumption in PPP US dollars. We computed per capita consumption data using the two methodologies discussed above for labor productivity.

Rest-of-the-world labor productivity is calculated as follows: we first computed real GDP in national currency units (RGDP), then converted the RGDP of those countries that make up the rest-of-the-world into US dollars using $\mathrm{PPP_{1995}}$, we summed over countries, and finally divide the summed by the rest-of-the-world total employee hours. The steps are similar for per capita consumption.

Table B1: Correlations Between Alternative Measures

	CAN	FRA	GER	ITA	JPN	UK	US
Z	0.99093	0.99437	0.99963	0.99565	0.99697	0.99329	1.00000
C	0.99337	0.99913	0.97749	0.99644	0.99666	0.99616	1.00000

Notes: Z is labor productivity, and C is per capita consumption. Alternative measures use national and U.S. price deflators.

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Table 1. The benchmark solution

Steady-state levels

$$\begin{aligned} \overline{r} &= .01015554120 \quad \overline{B} = .5588806661 \\ \overline{L} &= .3299443011 \quad \overline{L}^{*} = .3300470075 \\ \overline{w} &= 1.045291364 \quad \overline{w}^{*} = 1.238564933 \\ \overline{y} &= .3448879285 \quad \overline{y}^{*} = .4087846498 \\ \overline{c} &= .3449748265 \quad \overline{c}^{*} = .4086977516 \end{aligned}$$

Elasticities

	B_t	r_t	Z_t $(\phi = .9)$	Z_t^* $(\phi = .9)$	Z_t $(\phi = 1)$	Z_t^* $(\phi = 1)$
B_{t+1}	.9924	.9924	.3737	−.3661	.0039	.0037
r_{t+1}	0	0	−.0458	−.0542	0	0
RP_t	.0028	.0028	−.3174	.3146	−.1823	.1794
RP_t^*	−.0024	−.0024	.2678	−.2654	.1538	−.1514
L_t	−.0085	−.0085	.4099	−.4014	.0044	.0041
L_t^*	.0072	.0072	−.3458	.3386	−.0037	−.0035
w_t	.0028	.0028	.6826	.3146	.8177	.1794
w_t^*	−.0024	−.0024	.2678	.7346	.1538	.8486
h_t	.0017	.0017	.4833	.5150	.8200	.1784
h_t^*	−.0013	−.0013	.4362	.5652	.1549	.8465
y_t	−.0057	−.0057	1.0925	−.0868	.8222	.1835
y_t^*	.0048	.0048	−.0780	1.0732	.1501	.8451
c_t	.0070	.0070	.4807	.5122	.8156	.1774
c_t^*	−.0059	−.0059	.4382	.5678	.1556	.8504

Table 2a. Cointegration results (Johansen procedure)

USA

eigenvalue	loglik for	rank
	1298.406	0
0.51379	1320.040	1
0.42826	1336.812	2
0.34351	1349.438	3
0.26254	1358.574	4
0.11075	1362.095	5
4.2240e-005	1362.096	6

H0:rank<=	Trace test	[Prob]	
0	127.38	[0.000]	**
1	84.114	[0.002]	**
2	50.569	[0.026]	*
3	25.318	[0.155]	
4	7.0449	[0.579]	
5	0.0025345	[0.960]	

beta

LZ_US	1.0000	-1.8527	0.00000	5.6363	0.00000
LZEX_US	-0.60000	0.00000	-1.0000	0.00000	0.00000
LC_US	0.00000	1.0000	0.00000	0.00000	0.00000
LCEX_US	0.00000	0.00000	1.0000	0.00000	0.00000
LBT1S_US	0.00000	0.00000	0.00000	1.0000	0.00000
LR1_US1	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: Chi^2(3) = 2.1357 [0.5447]

Japan

eigenvalue	loglik for	rank
	1265.169	0
0.57377	1290.753	1
0.52972	1313.385	2
0.24911	1321.980	3
0.15328	1326.972	4
0.13512	1331.327	5
0.017610	1331.860	6

H0:rank<=	Trace test	[Prob]	
0	133.38	[0.000]	**
1	82.214	[0.003]	**
2	36.949	[0.354]	
3	19.759	[0.450]	
4	9.7761	[0.304]	
5	1.0660	[0.302]	

beta

LZ_JPN	1.0000	-0.94173	0.00000	-1.7095	0.00000
LZEX_JPN	-1.7106	0.00000	-1.0724	0.00000	0.00000
LC_JPN	0.00000	1.0000	0.00000	0.00000	0.00000
LCEX_JPN	0.00000	0.00000	1.0000	0.00000	0.00000
LBT1S_JPN	0.00000	0.00000	0.00000	1.0000	0.00000
LR_JPN1	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: Chi^2(1) = 4.0522 [0.0441]*

Table 2b. Cointegration results (Johansen procedure)

==

Germany

==

I(1) cointegration analysis, 1980 (1) to 1994 (4)

eigenvalue	loglik for	rank
	1270.652	0
0.63002	1300.481	1
0.43938	1317.842	2
0.29438	1328.303	3
0.20248	1335.090	4
0.072443	1337.346	5
0.017149	1337.865	6

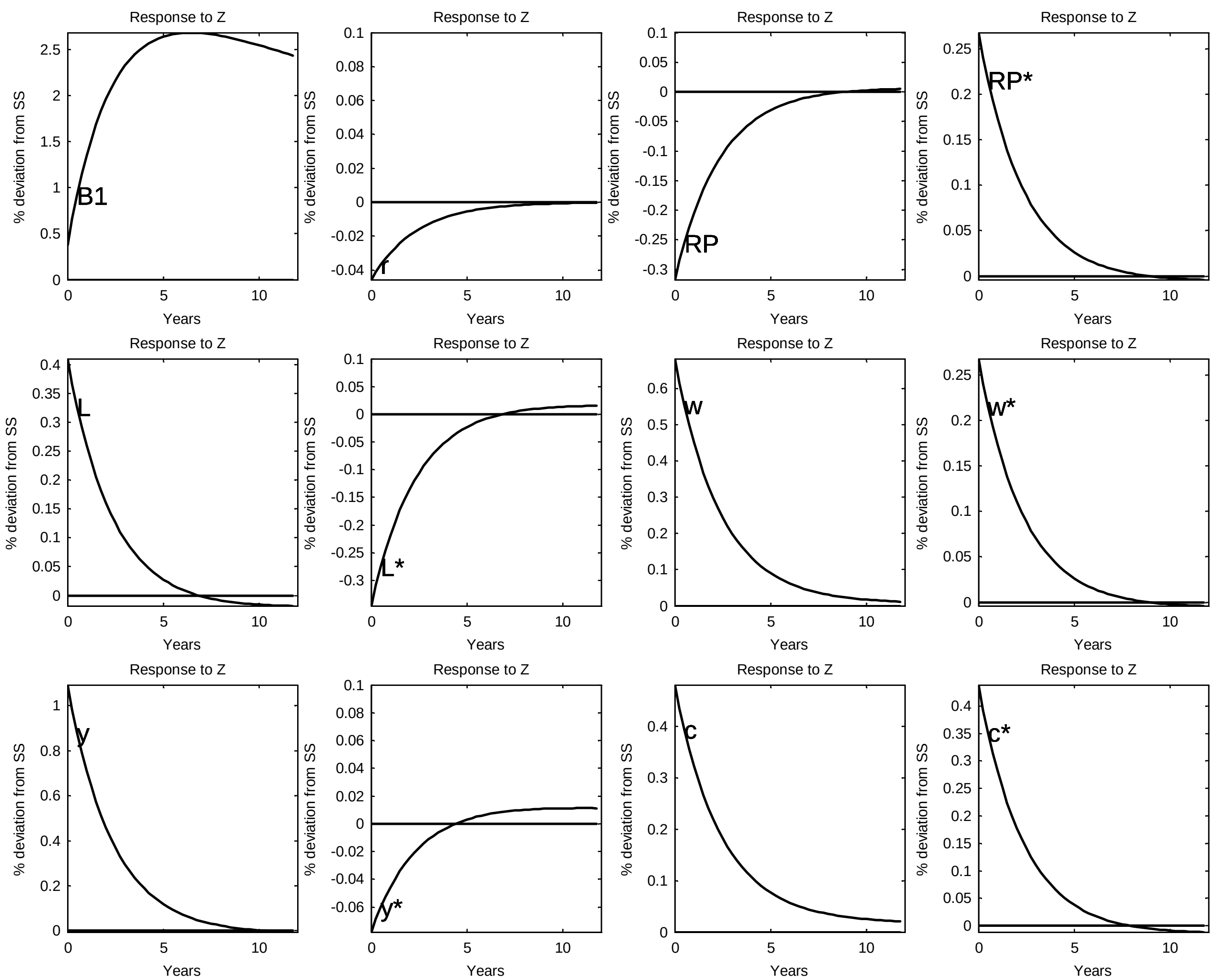
H0:rank<=	Trace test	[Prob]	
0	134.43	[0.000]	**
1	74.768	[0.018]	*
2	40.046	[0.223]	
3	19.125	[0.495]	
4	5.5499	[0.749]	
5	1.0378	[0.308]	

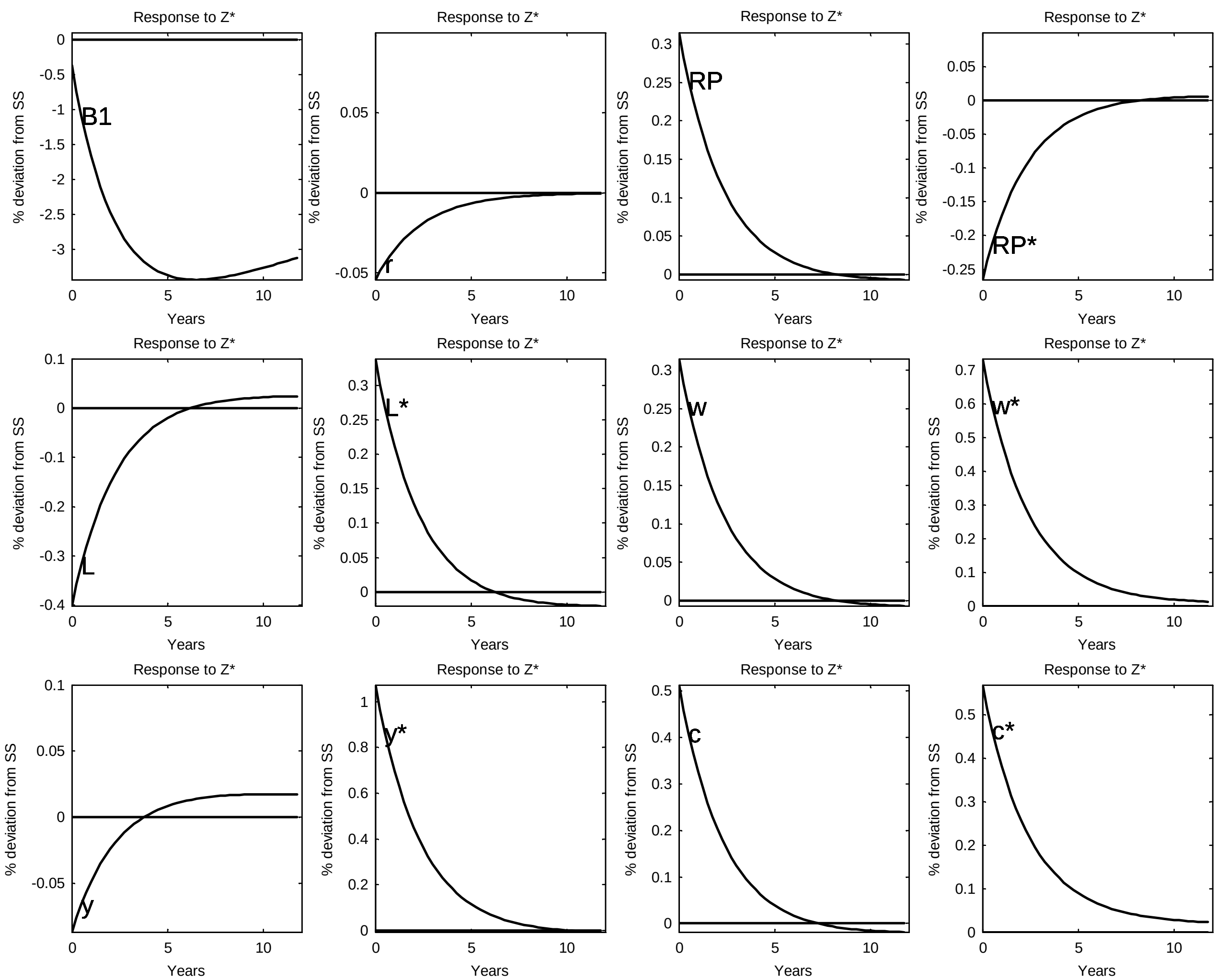
beta

LZ_GER	1.0000	-0.29742	0.00000	-0.94476	0.00000
LZEX_GER	-1.0000	0.00000	-1.3148	0.00000	0.00000
LC_GER	0.00000	1.0000	0.00000	0.00000	0.00000
LCEX_GER	0.00000	0.00000	1.0000	0.00000	0.00000
LBT1S_GER	0.00000	0.00000	0.00000	1.0000	0.00000
LR_GER1	0.00000	0.00000	0.00000	0.00000	1.0000

LR test of restrictions: Chi^2(2) = 6.4482 [0.0398]*

Figure 1-2

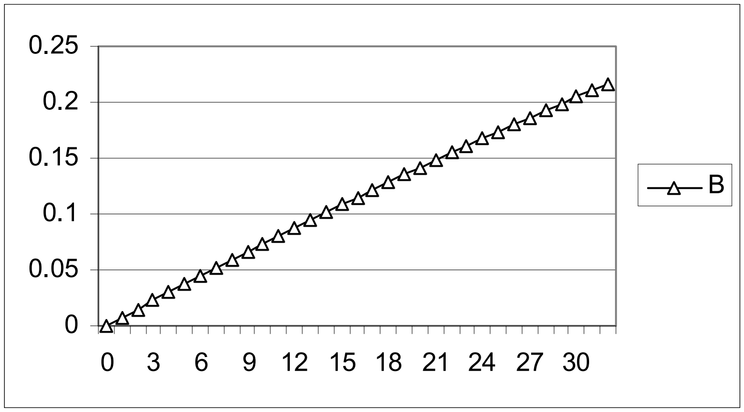

Response to Z
% deviation from SS
Years
B1
r
RP
RP*
L
L*
w
w*
y
y*
c
c*



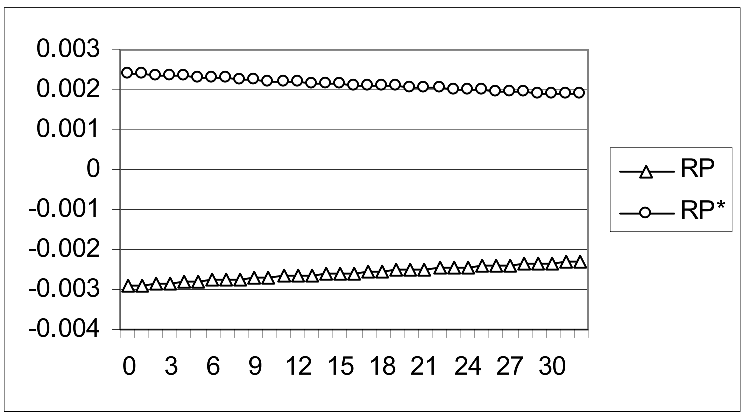
Response to Z*
% deviation from SS
Years
B1
r
RP
RP*
L
L*
w
w*
y
y*
c
c*

Figure 3. Permanent world productivity shock

i.



ii.



iii.

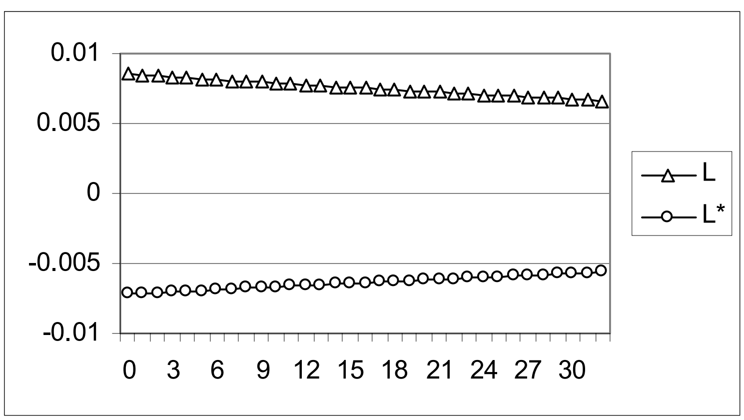
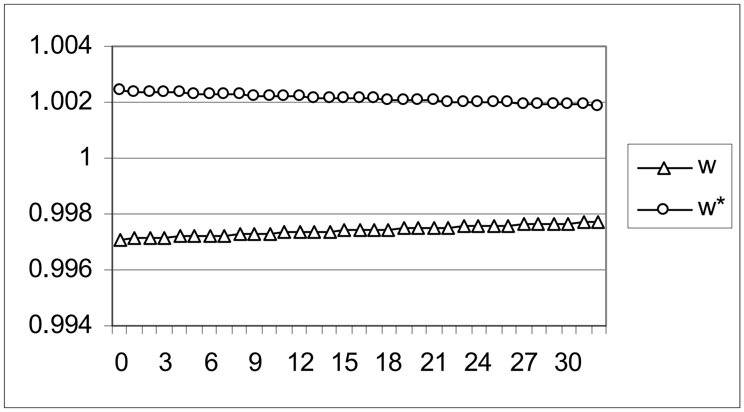
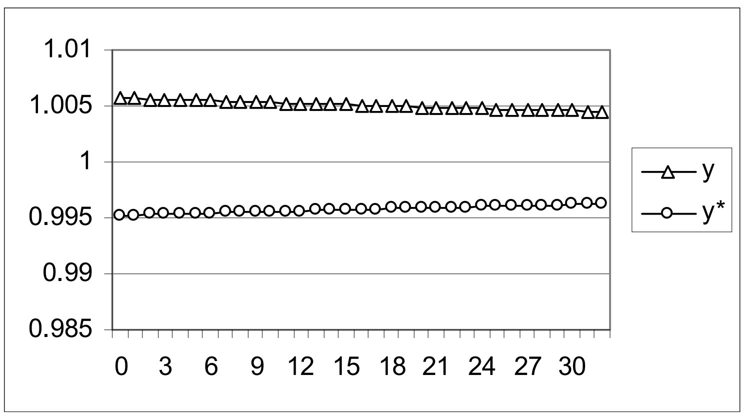


Figure 3, continued

iv.



v.



vi.

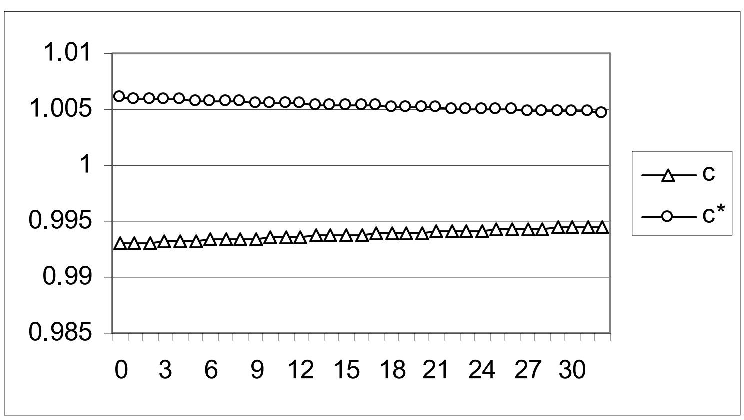
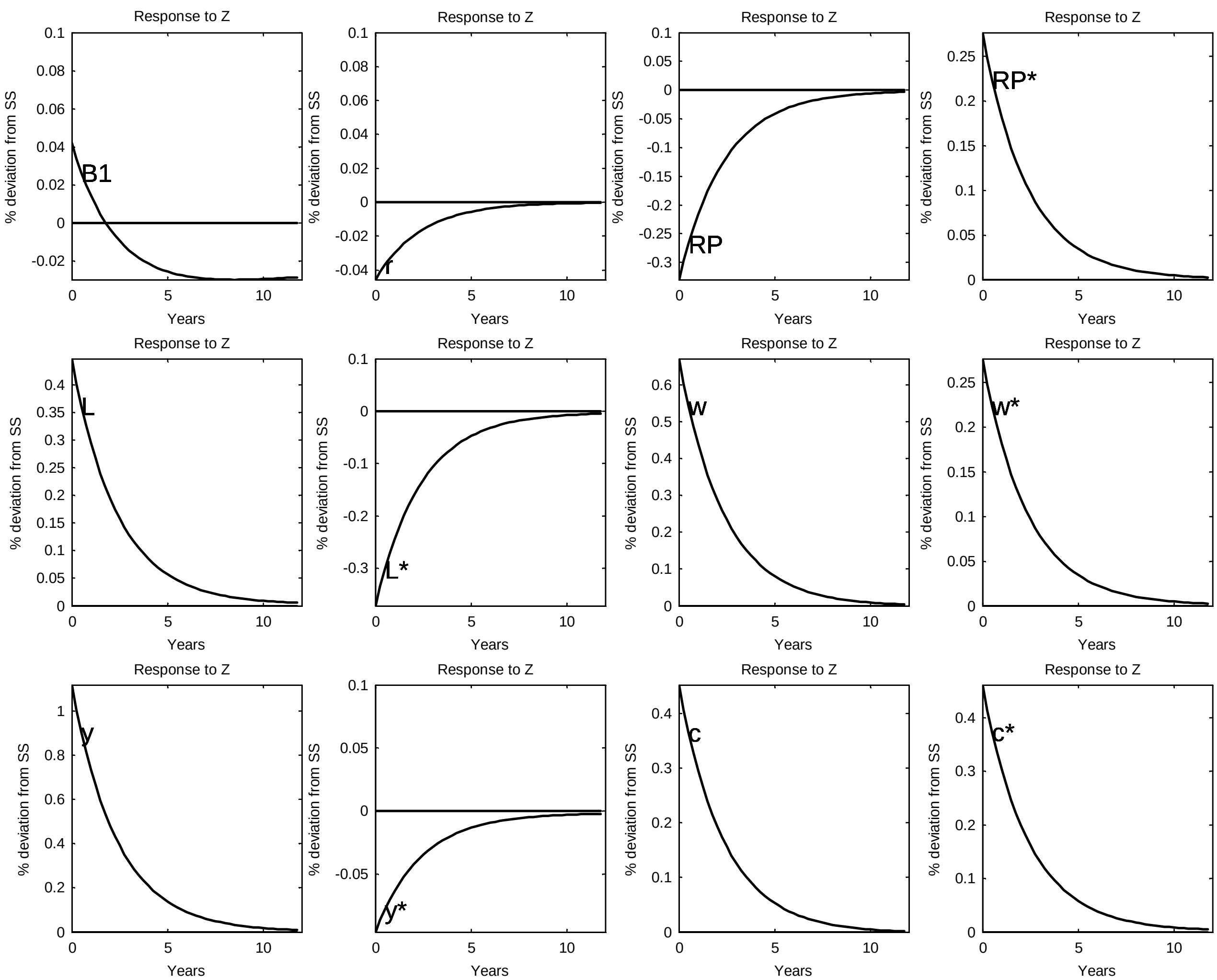
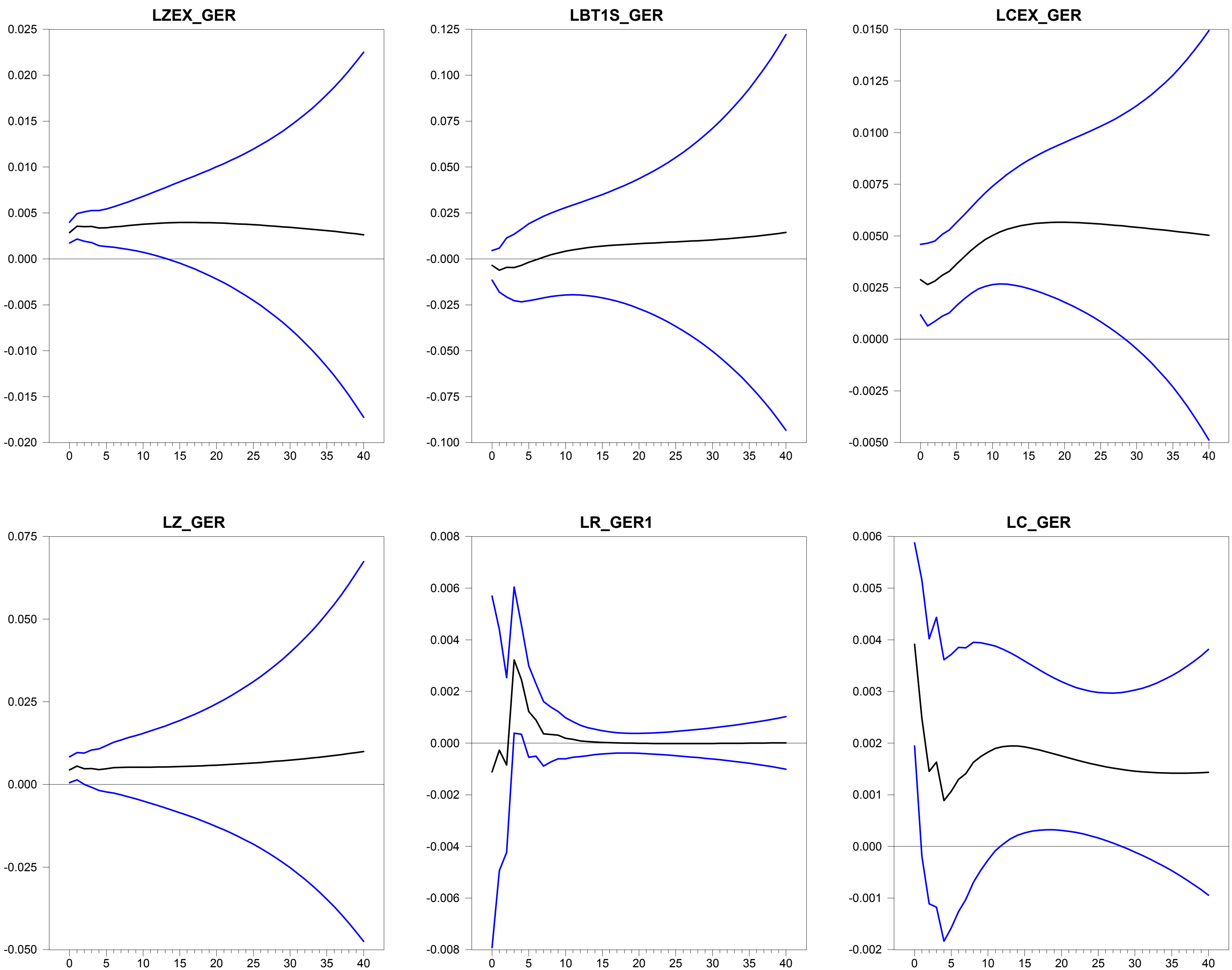


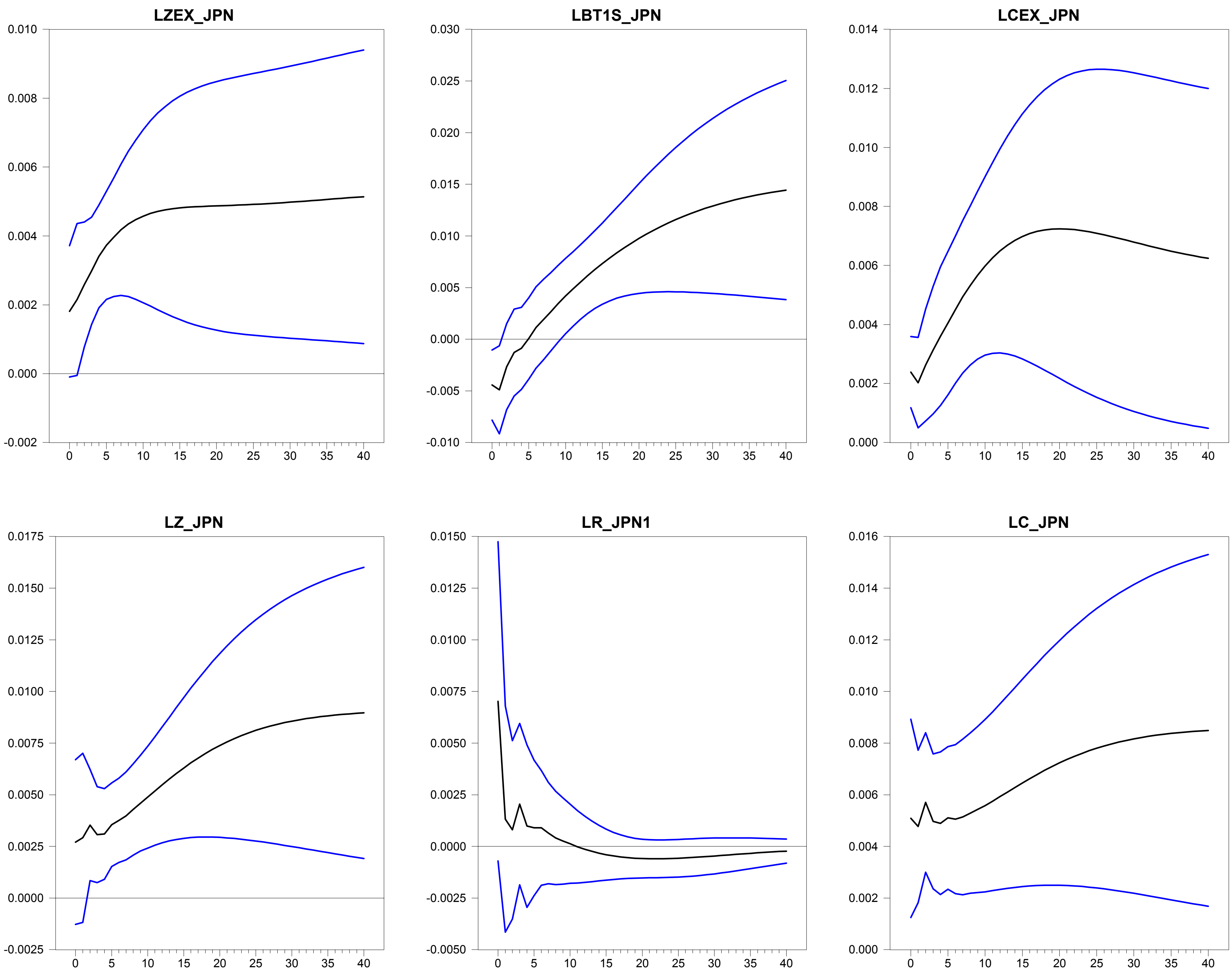
Figure 4



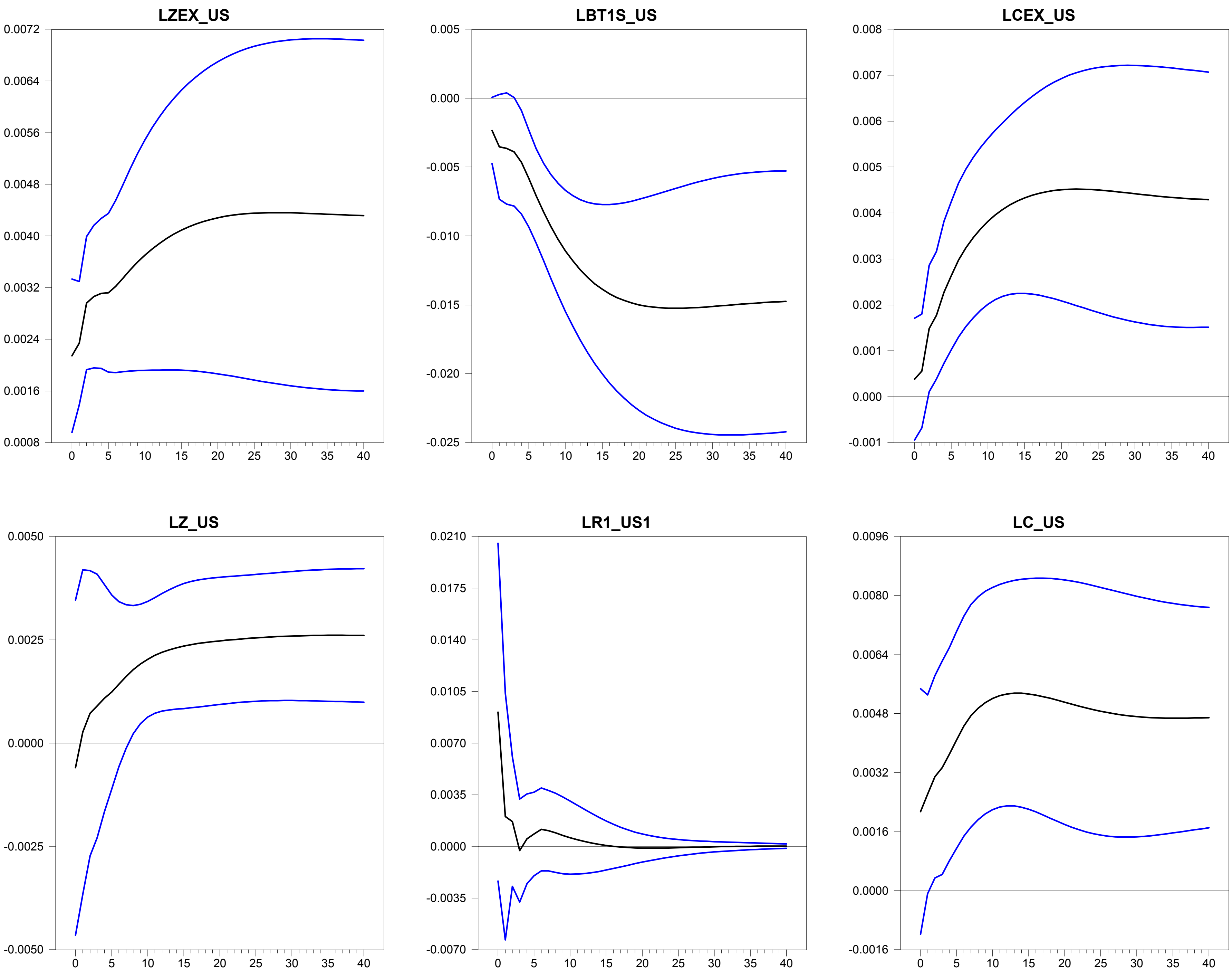
Response to Z
% deviation from SS
Years
B1
r
RP
RP*
L
L*
w
w*
y
y*
c
c*

Figure 5-7


LZEX_GER
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
-0.020
0 5 10 15 20 25 30 35 40
LBT1S_GER
0.125
0.100
0.075
0.050
0.025
-0.000
-0.025
-0.050
-0.075
-0.100
0 5 10 15 20 25 30 35 40
LCEX_GER
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
-0.0025
-0.0050
0 5 10 15 20 25 30 35 40
LZ_GER
0.075
0.050
0.025
0.000
-0.025
-0.050
0 5 10 15 20 25 30 35 40
LR_GER1
0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
-0.006
-0.008
0 5 10 15 20 25 30 35 40
LC_GER
0.006
0.005
0.004
0.003
0.002
0.001
0.000
-0.001
-0.002
0 5 10 15 20 25 30 35 40



LZEX_JPN
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
0 5 10 15 20 25 30 35 40
LBT1S_JPN
0.030
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
0 5 10 15 20 25 30 35 40
LCEX_JPN
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0 5 10 15 20 25 30 35 40
LZ_JPN
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
-0.0025
0 5 10 15 20 25 30 35 40
LR_JPN1
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
-0.0025
-0.0050
0 5 10 15 20 25 30 35 40
LC_JPN
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0 5 10 15 20 25 30 35 40



LZEX_US
0.0072
0.0064
0.0056
0.0048
0.0040
0.0032
0.0024
0.0016
0.0008
LBT1S_US
0.005
0.000
-0.005
-0.010
-0.015
-0.020
-0.025
LCEX_US
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
-0.001
LZ_US
0.0050
0.0025
0.0000
-0.0025
-0.0050
LR1_US1
0.0210
0.0175
0.0140
0.0105
0.0070
0.0035
0.0000
-0.0035
-0.0070
LC_US
0.0096
0.0080
0.0064
0.0048
0.0032
0.0016
0.0000
-0.0016
0 5 10 15 20 25 30 35 40