The Effects of Permanent Technology Shocks on Labor Productivity and Hours in the RBC model

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Abstract

Recent work on the effects of permanent technology shocks argue that the basic RBC model cannot account for a negative correlation between hours worked and labor productivity. In this paper, I show that this conjecture is not necessarily correct. In the basic RBC model, I find that hours worked fall and labor productivity rises after a positive permanent technology shock once one allows for the possibility that the process for the permanent technology shock is slightly persistent in growth rates. A more serious limitation of the RBC model is its inability to generate a persistent rise in hours worked after a positive permanent technology shock along with a rise in labor productivity that are in line with what the data suggests. These results call for a reconsideration of the real and nominal frictions and policy response that need to be introduced in the basic RBC model in order to improve the model's ability to match the data.

Keywords: permanent technology shocks; hours worked per capita; labor productivity; real business cycle model; vector autoregressions.

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1 Introduction

Using the identifying assumption that permanent technology shocks are the only shocks that have an effect on the long-run level of productivity, Galí (1999) reports that hours worked fall while labor productivity rises after a positive permanent shock to technology. Based on this empirical result, Galí argues that the standard Real Business Cycle (RBC) model is at odds with the data for two reasons. First, he argues that the standard RBC model implies that hours worked rise after a positive permanent innovation to technology, whereas the response is negative in the data according to his estimates. Second, because he provides evidence that hours worked and labor productivity are actually strongly positively correlated in the data, while the permanent technology shock he identifies is a source of negative correlation among these variables, he concludes that some other shock(s) must be the driving source of business cycles.

Not surprisingly, the seminal work of Galí (1999) has generated considerable interest in the effects of permanent technology shocks. One set of papers, see Basu, Fernald and Kimball (2001), Francis and Ramey (2001), and Galí (2003), have obtained similar empirical results. Therefore, these papers have tried to introduce modifications to the standard RBC model in order to account for negative correlation of labor productivity and per capita hours worked induced by permanent technology shocks. Another set of papers challenges Galí's empirical results and argues that both labor productivity and hours worked actually rise after a positive permanent technology innovation, see Altig et al. (2002) and Christiano, Eichenbaum and Vigfusson (2003a, 2003b).

In this paper, I take one step back and study the effects of permanent technology shocks in the "plain-vanilla" RBC model with everything being completely standard. To gain some insight about how the permanent technology shock works in the model, I also include transitory technology shocks for comparison. As in Altig et al. (2002), I allow for the possibility that persistent processes characterize both permanent and temporary technology shocks.

In contrast to the conjectures in the above mentioned literature, I find using a completely standard calibration of the basic RBC model, that a positive permanent technology shock implies that hours worked may fall substantially while labor productivity increases. The reason why this can happen is that the process for the permanent technology is slightly persistent in growth rates, thus making it profitable to decrease the supply of hours until labor effort becomes more effective.[1] Moreover, by employing the same identifying assumption as in Galí (1999) I show

[1] It should be stressed that other papers have also emphasized the possibility that hours fall while labor productivity rise after a positive permanent technology shock, but these papers do not use the standard RBC model, or the assumption of perfect information about the shock realization as I do. An example of the former type is the recent paper by Rotemberg (2003), who use a model with N_t types of capital in service at time t where each type of capital is associated with a different technology parameter z_j. Rotemberg (2003) shows that if the permanent technology shock diffuses slowly in the economy (i.e. is correlated in growth rates and does not reach the new steady state level contemporaneously), hours fall while labor productivity rise after a positive

that the basic RBC model is able to match the estimated impulse response functions for a broad set of variables well when hours worked per capita are treated as difference stationary (as in Galí, 1999) in the data. However, when hours worked are treated as level stationary in the empirical analysis, as in Christiano, Eichenbaum and Vigfusson (2003a, 2003b), the RBC model cannot account for the resulting strong and persistent rise in hours worked along with a moderate response of labor productivity, consumption, investment and the real wage. Thus, in contrast to the conclusions in the previous literature, these results suggest that the RBC model is actually able to replicate a substantial fall in hours and a rise in labor productivity reported by Galí (1999), Basu, Fernald and Kimball (2001), and Francis and Ramey (2001), while it cannot replicate the persistent rise in hours and labor productivity reported by Christiano, Eichenbaum and Vigfusson (2003a, 2003b). Since Christiano, Eichenbaum and Vigfusson (2003a) argue convincingly that hours worked are level stationary rather than difference stationary and thus that hours rise persistently after a positive technology shock, the inability of the basic RBC model to replicate this result calls for a reconsideration of how the basic RBC model needs to be modified in order to with the data. I elaborate further on this last issue in the conclusions.[2]

The paper is structured as follows. In the next section, I briefly present the standard RBC model that is used in the paper. Sections 2 and 3 show the effects of transitory and permanent technology shocks in the RBC model. In Section 4, I investigate the model's ability to reproduce some estimated impulse response functions in the data. Finally, some conclusions are provided in Section 5.

2 The standard RBC model with temporary and permanent technology shocks

Consider a representative agent which maximizes expected utility with preferences summarized by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right), \tag{1}$$

subject to the following inter-temporal budget constraint

$$C_t + I_t = W_t H_t + R_t^K K_t. \tag{2}$$

Investment in period t produce productive capital in period $t+1$ according to

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t, \tag{3}$$

shock. Examples of the latter category are papers by Hairault, Langot and Portier (1997) and Manuelli (2000) which show that if there is imperfect information about the technology shocks, hours fall and labor productivity rise after a positive technology shock.

[2] Christiano, Eichenbaum and Vigfusson (2003a) provide evidence by means of multivariate KPSS and ADF tests (see Hansen, 1995) - which they demonstrate have substantially more power than standard univariate tests - that hours worked per capita is stationary. This empirical result is appealing because in most theoretical models, hours worked per capita is stationary.

where δ is the rate of capital depreciation.

The representative firm has access to the following production technology

$$Y_t = z_t^{1-\theta} e^{\varepsilon_t} K_t^{\theta} H_t^{1-\theta}, \tag{4}$$

where the processes for z_t and ε_t are given by

$$\begin{aligned} \ln z_t &= \ln z_{t-1} + x_t, \\ x_t &= (1-\rho_x)\,\mu_z + \rho_x x_{t-1} + \eta_t, 0 < \rho_x < 1, \eta_t \sim N(0, \sigma_\eta^2), \\ \varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + \nu_t, 0 < \rho_\nu < 1, \nu_t \sim N(0, \sigma_\nu^2). \end{aligned} \tag{5}$$

Consequently, η_t is a permanent technology shock and ν_t is a temporary technology shock. η_t is a permanent shock in the sense that it will have a permanent effect on labor productivity (Y_t/H_t) in the long run, whereas ν_t will not. Since H_t, the share of available time spent in employment, is stationary, this implies that both Y_t and Y_t/H_t will rise permanently after a positive realization of η_t, whereas Y_t and Y_t/H_t will return to their steady-state values after a positive realization of ν_t. Finally, note that the steady-state growth rate in this economy is given by μ_z.

If we assume that factor markets are characterized by perfect competition, the real pre-depreciation rental rate on capital, R_t^K, and the real wage W_t are given by

$$\begin{aligned} R_t^K &= \theta z_t^{1-\theta} e^{\varepsilon_t} \left(\frac{K_t}{H_t}\right)^{\theta-1}, \\ W_t &= (1-\theta)\, z_t^{1-\theta} e^{\varepsilon_t} \left(\frac{K_t}{H_t}\right)^{\theta}. \end{aligned} \tag{6}$$

Using (6) and (3) in (2), we have

$$C_t + K_{t+1} = z_t^{1-\theta} e^{\varepsilon_t} K_t^{\theta} H_t^{1-\theta} + (1-\delta)\, K_t. \tag{7}$$

The dynamic program for the representative individual in this economy can be written as

$$\begin{aligned} V(K_t) \equiv \; & \max_{\{K_{t+1}, H_t\}} \{u(C_t, H_t) + \mathrm{E}_t \beta V(K_{t+1})\} \\ & \text{s.t. } (7) \end{aligned} \tag{8}$$

where we notice that off-work time L_t and work time H_t have been normalized to sum to unity.

In order to compute the steady state and the decision rules in this economy, we need to specify the functional form for $u(C_t, 1-H_t)$. As a benchmark, we use the standard functional form when consumption and leisure are non-separable and given by

$$u(C_t, H_t) = \frac{\left[C_t^{\alpha} (1-H_t)^{1-\alpha}\right]^{1-\sigma} - 1}{1-\sigma}. \tag{9}$$

This is perhaps the most commonly used specification in the RBC-literature, see e.g. Hansen and Prescott (1995).[3]

Since the technology shock z_t is permanent, we need to scale all variables except H_t with this shock in order to be able to solve for a constant steady state and to compute the decision rules for the representative agent in this economy. Let $\hat{C}_t \equiv C_t/z_t$, $\hat{Y}_t \equiv Y_t/z_t$, $\hat{I}_t \equiv I_t/z_t$ and $\hat{K}_{t+1} \equiv K_{t+1}/z_t$. As shown in Appendix A, we then have the following system of equations that characterize this economy

$$\begin{aligned}
\hat{C}_t (1-\alpha) &= \alpha (1-H_t)(1-\theta) x_t^{-\theta} e^{\varepsilon_t} \left(\frac{\hat{K}_t}{H_t}\right)^{\theta}, \qquad (10) \\
\hat{C}_t^{\alpha(1-\sigma)-1} (1-H_t)^{(1-\alpha)(1-\sigma)} &= \beta \mathrm{E}_t \left[\begin{array}{c} x_{t+1}^{\alpha(1-\sigma)-1} \left(\hat{C}_{t+1}^{\alpha(1-\sigma)-1} (1-H_{t+1})^{(1-\alpha)(1-\sigma)} \right) \\ \times \left(1 + x_{t+1}^{1-\theta} e^{\varepsilon_{t+1}} \theta \left(\frac{\hat{K}_{t+1}}{H_{t+1}} \right)^{\theta-1} - \delta \right) \end{array} \right], \\
\hat{C}_t + \hat{I}_t &= x_t^{-\theta} e^{\varepsilon_t} \hat{K}_t^{\theta} H_t^{1-\theta}, \\
\hat{K}_{t+1} &= (1-\delta) \hat{K}_t x_t^{-1} + \hat{I}_t, \\
\hat{Y}_t &= \hat{C}_t + \hat{I}_t.
\end{aligned}$$

Together with (5), the set of equations in (10) constitute the model that we want to solve. Note that we scale K_{t+1} with z_t rather than z_{t+1}, because K_{t+1} is determined in period t. Appendix A contains the details for the solution of the model.

To calibrate the model, I use standard parameters in the RBC-literature. More specifically, the model is calibrated to match quarterly data by setting $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.025$, $\alpha = 0.33$, $\sigma = 2$, and the steady-state growth rate in the model, μ_z, to $1.03^{\frac{1}{4}}$. These values are standard in the literature, see e.g. Cooley and Prescott (1995) and Christiano, Eichenbaum and Evans (2001). To calibrate the exogenous processes for the stationary technology shock (ε_t) and the unit-root technology shock ($\hat{x}_t$), I use the estimates in Altig et al. (2002) and set $\rho_x = 0.80$ and $\sigma_\eta = 0.11$.[4] In order to highlight the difference in propagation of unit-root and correlated stationary technology shocks, I use $\rho_\varepsilon = 0.80$ and $\sigma_\nu = 0.11$ as well.

3 Impulse response functions to a temporary technology shock

The impulse response functions to a temporary shock is shown in Figure 1. I include the effects on six variables in the figure; output, consumption, investment, hours worked, labor productivity

[3] I have also checked the robustness of the results reported in this paper when the utility function is separable in consumption and leisure (see e.g. Hansen, 1995), i.e. $u(C_t, H_t) = \ln C_t - AH_t$, where A is calibrated so that hours worked in steady state equals the steady-state value for H in the benchmark model (non-separable utility function). The main results of the paper are completely invariant to this specification.

[4] Note that these two parameters along with the inverse of the intertemporal elasticity of substitution (σ) are estimated in Section 5.

(defined as $\hat{y}_t + \widehat{\ln z_t} - h_t$), and the level of technology (defined as $\widehat{\ln z_t} + \varepsilon_t$).[5] As is standard in the literature, we see that hours, output and labor productivity all go up after a positive temporary technology. Thus, as can be seen from Figure 1, if this type of shock is the key driving force of business cycles, we ought to see a strong positive correlation between hours worked and labor productivity in the data.

4 Impulse response functions to a permanent technology shock

The solid line in Figure 2 shows the impulse response functions for a permanent technology shock for the baseline calibration of the shock process. Perhaps surprisingly, we see that hours worked fall for the first 5 quarters after the shock, in contrast to the conjectures made by Galí (1999) and Christiano, Eichenbaum and Vigfusson (2003a). Moreover, it can be seen that labor productivity goes up despite the fall in hours worked, inducing a negative correlation between hours worked and labor productivity, a feature of the empirical results which Galí (1999) emphasized was strong evidence against the RBC model. It is also notable that investment drop in the first quarters after the shock.

The question then arrises what the underlying mechanism is in the RBC model that enables it to generate a negative short-run response in hours worked while labor productivity goes up. To understand why, the dashed line in Figure 2 shows the impulse response functions to a zero correlated shock in growth rates (i.e. using $\rho_x = 0$) where I set the standard deviation of this shock so that the long-term impact on the level of technology shock is identical to the growth rate correlated shock. In this case, we see that the conjecture by Galí (1999) and Christiano, Eichenbaum and Vigfusson (2003a) holds; hours worked go up, and so do labor productivity. Why do the results for hours and investment differ when the permanent shock is persistent in growth rates? My intuition is that when the permanent technology shock is persistent in growth rates, it is more profitable for individuals to work and invest less as the shock hits the economy because it takes a while before labor and capital input become most productive (compare lines for technology in Figure 2), whereas in the case when the technology shock is non-permanent or non-persistent, labor and capital input rise because they become most productive immediately. Therefore, we obtain a fall in hours and investment after a positive permanent technology shock if the shock is slightly correlated in growth terms. However, due to the permanent income mechanism (i.e. the wealth effect stemming from the permanent improvement in technology), consumption rises immediately and thus labor productivity too irrespective of the shock's persistence, inducing a negative correlation between hours worked and labor productivity when the shock is correlated in growth terms.[6] Note that the results

[5] Lower case variables indicate natural logarithms. Also, note that $\hat{y}_t + \widehat{\ln z_t}$ is output in levels, whereas $\hat{y}_t$ is the "detrended" output. Same transformation have been applied for consumption, investment and the real wage.

[6] Interestingly, Basu, Fernald, and Kimball (2001) argue that the RBC model is in no case able to generate a

in Figure 2 suggest that the consumption response after a positive permanent technology is potentially a more informative way to check the consistency of the RBC model with the data compared to the response of hours worked. No matter the process for the permanent technology shock, consumption should always rise after a positive technology shock, in contrast to hours worked.

Some other aspects of the parameterization of RBC model should be emphasized. The results are not very sensitive to the choice of σ (the inverse of the inter-temporal substitution elasticity) between 1 (log-utility) and setting $\sigma = 10$ (which is a fairly high number). Moreover, as might be evident from Figure 2, a larger value for the persistence parameter ρ_x than estimated by Altig et al. (2002) only increases the fall in hours and the negative correlation between labor productivity and hours worked in the RBC model. It should be emphazied that only a very small value for the persistence parameter ρ_x is required in order to generate a substantial fall in hours worked. Finally, using the Hansen (1985) type of utility function, where consumption and hours are additive do not change the results either, such a utility function rather amplifies the mechanism since consumption and hours are addititve in the utility function.

5 Taking the basic RBC model to the data

In this section, I will examine if the basic RBC model can replicate the impulse response functions in the data by matching the impulse response functions in the model with the ones obtained in a vector autoregressive (VAR) model using the same identifying assumptions as by Galí (1999), i.e. that the permanent technology shock is the only source of fluctuations in labor productivity in the long run. We will vary the parameters σ, ρ_x, and σ_η as to minimize the following criterion

$$J = \left(\hat{\psi} - \psi\left(\sigma, \rho_x, \sigma_\eta\right)\right)' \hat{V}^{-1} \left(\hat{\psi} - \psi\left(\sigma, \rho_x, \sigma_\eta\right)\right)$$

where $\hat{\psi}$ is a $(j * k) \times 1$ vector which contains the impulse response functions j periods following the shock for the k variables of interest in the data, whereas $\psi\left(\rho_x, \sigma_\eta\right)$ is a $(j * k) \times 1$ vector with the corresponding impulse response functions in the model. $\hat{V}$ is a diagonal matrix with the estimated standard deviation of each response in the data. I set $j = 20$ in order to study the RBC-models' properties in both the short and long run. The procedure adopted is exactly the same as the one used by Christiano, Eichenbaum and Evans (2001), and Altig et al. (2002).

To generate the impulse response functions in the data, we use two VAR models. First,

drop in output after a positive permanent technology shock, and since they obtain a initial fall in output in their empirical analysis, they interpret this result as strongly contradicting basic RBC theory. This interpretation is correct if the permanent technology shock is a random walk (which is the implicit assumption in BFK), but if the permanent technology shock is correlated in growth rates, the standard RBC model can actually replicate a fall in output as can be seen in Figure 2. And the more persistent the permanent technology shock is (compared to the benchmark calibration), the more output falls initially after a positive technology shock. This finding is an important caveat to BFK's interpretation of their empirical finding.

I will estimate a bivariate VAR containing labor productivity and hours worked.[7] Second, I estimate a VAR augmented by consumption, investment and the real wage.[8] Since the recent empirical literature have stressed the importance of whether hours worked are first differenced or not, I will consider two cases. The first case will be when hours worked are first differenced in the data (as in Galí 1999, 2003), although hours worked is stationary in the model. Second, since Christiano, Eichenbaum and Vigfusson (2003a) argue that hours worked should not be differenced, I also present results for the VARs estimated with hours worked in levels. In the former case, we anticipate that hours worked will fall and labor productivity will rise whereas in the latter case we expect that both hours worked and labor productivity will rise after a positive technology shock.[9] The variables used in the VARs are depicted in Figure 3.

In Figure 4, I show the results when matching the models impulse response functions with the ones obtained in the estimated bivariate VAR with hours in first differences. The parameter estimates of σ, ρ_x, and σ_η for this VAR are reported in Table 1, along with the estimates for the other VAR models. We see that although the model is able to generate a substantial drop in hours worked per capita (-0.2 percent) and rise in labor productivity, the response of hours worked is still slightly outside the grey area (which indicates the 95-percent confidence interval) in the first period.[10] It should be pointed out that if σ is restricted to 2 (i.e. the intertemporal elasticity of substitution is considerably higher than estimated), then the RBC model still produces a sizeable initial fall in H_t of -0.12 percent. So as noted by Rotemberg and Woodford (1996), the estimated low intertemporal elasticity of substitution is an important ingredient why the model is able to generate a substantial fall in H_t. However, allowing the permanent technology shock to be correlated in growth rates is a more important channel. If I restrict the permanent technology shock to a random walk (i.e. non-correlated in growth terms),

[7] See Altig et al. (2002) for details about the data, the sample period is $1959Q1 - 2001Q4$. 4 lags are used in the estimations. When estimating the VAR, I impose the restriction that the permanent technology shock is the only shock that influence labor productivity in the long run. Moreover, in the VARs estimated with hours in first differences, I estimate the hours equation in double differences for hours (using first differences of labor productivity and hours as instruments) in order to impose the restriction that hours return to nil after a permanent technology shock. The standard errors in $\hat{V}$ are bootstrapped (using 1000 repetitions).

[8] I follow Altig et al. (2002) and include $\ln(C_t/Y_t)$, $\ln(I_t/Y_t)$, and $(\ln W_t/(Y_t/H_t))$ as variables and convert the ratios to levels when computing the impulse response functions. These variables are stationary in the model and also appear to be stationary in the data (see Figure 3).

[9] Since the modulus of the largest root of the characteristic polynomial was very close to unity (0.993) for the VAR with hours in levels (implying that the bootstrapped standard errors bands for the impulse response functions become very wide), I expanded the sample for the VAR with hours in levels by starting $1948Q1$ instead of $1960Q1$. By this procedure, the modulus of the largest root was considerably lower (0.94). As an alternative, I also detrended hours worked with a linear-quadratic trend prior to estimation of the VARs with hours in levels for the benchmark sample period. For the 5-variables VAR, I then obtained very similar results as the ones reported in Figure 7. However, the response of hours in a bivariate VAR is very sensitive to whether hours worked are detrended or not. If hours are detrended, hours fall initially consistent with the findings by Galí (1999, 2003), but if hours are not detrended hours rise strongly as can be seen in Figure 6. Thus, the empirical finding that hours fall even when it is detrended with a linear-quadratic trend appear to be sensitive to the specification of the VAR model. This mechanism is also evident by comparing the empirical impulse response functions in Figures 4 and 5. In the bivariate VAR for hours in first differences hours fall strongly, whereas in the 5-variables VAR with hours in first differences, there is only a minor initial fall followed by an increase in hours.

[10] It should be pointed out that the model is able to match the impulse response functions of H_t only very well, but then the response of labor productivity is too strong relative to the data.

the initial fall in hours is only −0.07 percent.

However, for the VAR with hours in first differences with the larger set of variables, the RBC model fits quite nicely as can be seen in Figure 5. In this case, the estimated initial drop in the data for hours worked is not as pronounced as in the bivariate VAR and the model responses are well inside the grey area and close to the empirical point estimates for all variables in both the short and long run. The parameter estimates of σ, ρ_x, and σ_η for this VAR are all very reasonable. For σ, the number could easily have been the outcome of an ordinary calibration procedure, and for ρ_x it is interesting to note that we do not need much persistence in the permanent shock in growth rates in order to line up the model with the data.

Table 1: Parameter estimates when matching the RBC model with US data.

	Bivariate VAR model		5-variables VAR model	
Parameter	H_t in first differences	H_t in levels	H_t in first differences	H_t in levels
ρ_x	0.43	0.00	0.32	0.01
σ_η	0.55	0.77	0.43	0.57
σ	7.28	0.75	1.73	0.01

Turning to the bivariate VAR with hours worked per capita in levels in Figure 6, we first notice that we obtain a "hump-shaped" rise in hours worked instead of a fall as in Figure 4. The short-run response of labor productivity is also somewhat stronger than in Figure 4, but the long-run response of labor productivity is about the same. However, as was the case in Figure 4, the RBC model is not able to replicate the estimated joint effects of the technology shock in the data, the initial effect on labor productivity is too weak and the hours response is also too weak compared to the data although it is inside the gray area. The parameter estimates of σ, ρ_x and σ_η for this VAR model are quite different than for the VAR model with hours in first differences. Given that we obtain a rise in hours worked, it comes as no surprise that the estimated value of ρ_x is zero, whereas the estimate of σ is very low. Figure 7 presents the results for the 5-variables VAR with hours in levels. As can be seen from the figure, the RBC model cannot match the obtained impulse response functions in the data for hours worked with this specification because hours worked rise strongly after the shock in line with the results in Christiano, Eichenbaum and Vigfusson (2003a). The parameter estimates of σ, ρ_x, and σ_η for this VAR are 0.01 (corner solution), 0.01, and 0.57, respectively. The estimate of σ is implausibly low, implying that the utility function is almost linear. Imposing $\sigma = 2$ instead, we obtain an even lower response of hours.

To sum up, it is clear by comparison of Figures 5 and 7 reveal that the RBC model can easier accommodate a fall in hours worked along with a rise in labor productivity after a positive permanent technology shock than vice versa, in contrast to the conjectures in the empirical literature, see e.g. Galí (1999), Basu, Fernald and Kimball (2001), Christiano, Eichenbaum and Vigfusson (2003a), and Francis and Ramey (2001). Moreover, the estimated persistence of the technology shock in growth rates to achieve these results is very low, around $0.3 - 0.4$ at the quarterly frequency and thus less than 0.05 at an annual frequency.

6 Conclusions

In this paper, I have shown that the standard RBC model (Cooley and Prescott, 1995) can produce a substantial fall in hours worked along with a reasonable rise in labor productivity after a positive permanent technology shock, once one allows for the possibility that the technology shock is slightly correlated over time in growth terms.

This finding implies that the evidence presented by e.g. Galí (1999), i.e. that hours worked drop while labor productivity rises after a permanent technology shock, cannot be taken as evidence against the RBC model of US business cycles. Moreover, and very importantly, this also implies that it is harder to argue that the empirical results provide indirect evidence that we need sticky prices along with non-accommodative monetary policy (Basu, Fernald and Kimball, 2001, and Galí, 1999) or real frictions (Francis and Ramey, 2001) in the model in order to generate a fall in hours worked.

However, I have also shown that the basic RBC model cannot accommodate the persistent rise in hours worked following a permanent technology shock reported by Altig et al. (2002) and Christiano, Eichenbaum and Vigfusson (2003a) when hours worked per capita are treated as level stationary in the data. According to their estimates (similar results were obtained in this paper in a 5-variables VAR), the response of hours is "hump-shaped" with a peak effect after $1 - 2$ years.[11] In order to account for this "hump-shaped" rise, some real and nominal frictions along with monetary policy accommodation need to be incorporated in the basic RBC model. Altig et al. (2002) present an example of one such model economy. The important difference in this model compared to the modifications of the RBC model suggested by Galí (1999), Basu, Fernald and Kimball (2001) and Francis and Ramey (2001) being that it is accommodative monetary policy along with sticky wages and real frictions which enables the model to replicate

[11] Fernald (2003) argues using bivariate VARs that by allowing for breaks in mean labor productivity growth in 1973 and 1995, the empirical findings in Galí/Francis-Ramey and Christiano, Eichenbaum and Vigfusson (2003a) can be reconciled and hours fall about -0.2 percent on impact after a positive technology shock irrespectively if hours are included in levels or growth rates in the VAR. Although further analysis is needed before we may draw further firmer conclusions regarding this finding, we notice here that if Fernald is correct, then the conclusions made in this paper regarding the frictions and monetary policy response that we need to bring in to the RBC model in order to make it consistent with the dynamic effects of technology shocks will be altered.

the rise in hours worked and labor productivity after a positive permanent technology shock.[12]

Moreover, the observation that the identified neutral permanent technology shock need not be the major source of business cycle fluctuations in the data cannot be used as evidence against the notion of technology driven business cycles in a modified version of the RBC model.[13] Although the data suggests that the identified permanent neutral technology shock is perhaps not the most important shock for business cycles, it is possible that other types of technology shocks can account for most of the cyclical variation. For example, we have seen in this paper (see Figure 1) that a temporary technology shock induces a strong positive correlation between hours worked and labor productivity.[14] Another possibility is that investment-specific shocks account for more of the business cycle fluctuations. Using identified VARs, Fisher (2002) reports that permanent investment-specific technology shocks seem to be an important source of US business cycles.

Finally, one word of caution is that some recent papers question the ability of the identified VAR approach employed in this literature to correctly identify the shocks in the short run on sample sizes that are empirically relevant, see e.g. Erceg, Guerrieri and Gust (2003), Rotemberg (2003) and Uhlig (2003). The message from these papers is that the inability of the RBC model to correctly reproduce the estimated impulse response functions in the data could in some instances be related to an inappropriate identification of the shocks in the data.

[12] Note that the "hump-shaped" rise in hours worked cannot be replicated with real and nominal frictions unless monetary policy is accommodative. The reason for this is that (*i*) real frictions (e.g. investment adjustment costs and habit persistence in consumer preferences) will tend to decrease hours worked even more initially (see e.g. Francis and Ramey, 2001), and (*ii*) nominal frictions (e.g. sticky nominal prices and wages) will also tend to drive down the hours response even more because prices and wages are expected to rise more in the future than initially. Therefore, monetary policy must be strongly accommodative in order to generate a substantial persistent rise in hours worked after a positive technology shock.

[13] It should be emphasized that the share of output fluctuations due to the identified technology shock are dependent on (*i*) whether hours worked are treated as difference or level stationary in the VARs, and (*ii*) the set of variables that are included in the VARs. If hours worked are treated as level stationary, then the identified permanent technology shock account for a substantial share of the business cycle fluctuations (around 40 percent in the 5-variables VAR), whereas if hours worked are treated as difference stationary it accounts for a small share of the business cycle fluctuations.

[14] Basu, Fernald and Kimball (2001), henceforth BFK, compute a direct measure of technology that builds on Solow residual accounting and report that hours worked fall after a positive innovation in technology. Since their technology series is a mixture of permanent and transitory technology shocks, this evidence suggest that neither permanent nor temporary disembodied technology shocks are an important source of US business cycles. However, Christiano, Eichenbaum and Vigfusson (2003b), using the BFK technology series, show that the BFK results can be reversed (i.e. CEV obtain a strong rise in hours worked) by allowing for measurement errors and including hours in levels instead of first differenced hours in the analysis. Thus, we cannot rule out the possibility that transitory and/or permanent disembodied technology shocks are an important source of business cycles.

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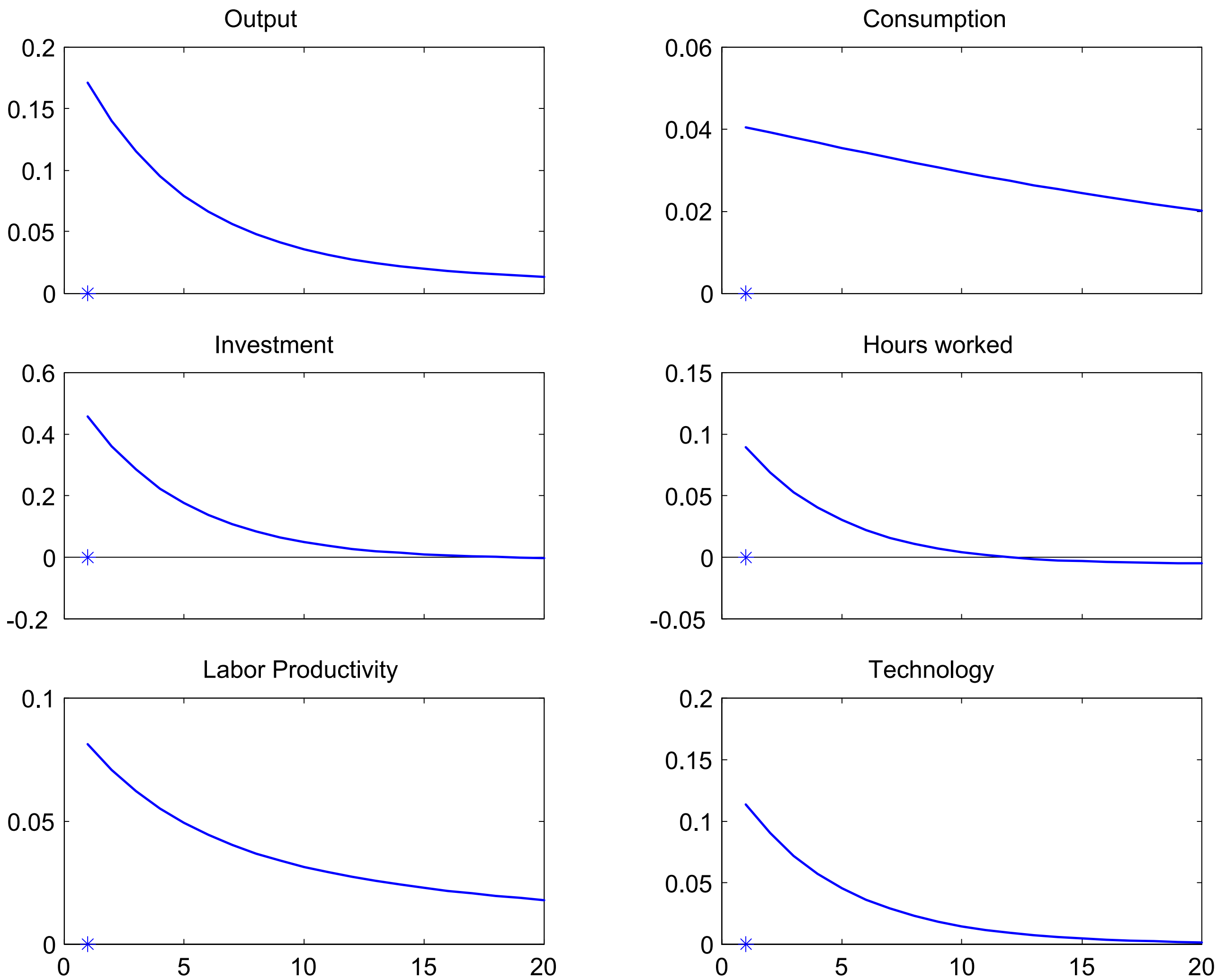

Figure 1: Impulse response functions to a temporary technology shock. Basline parameterization.

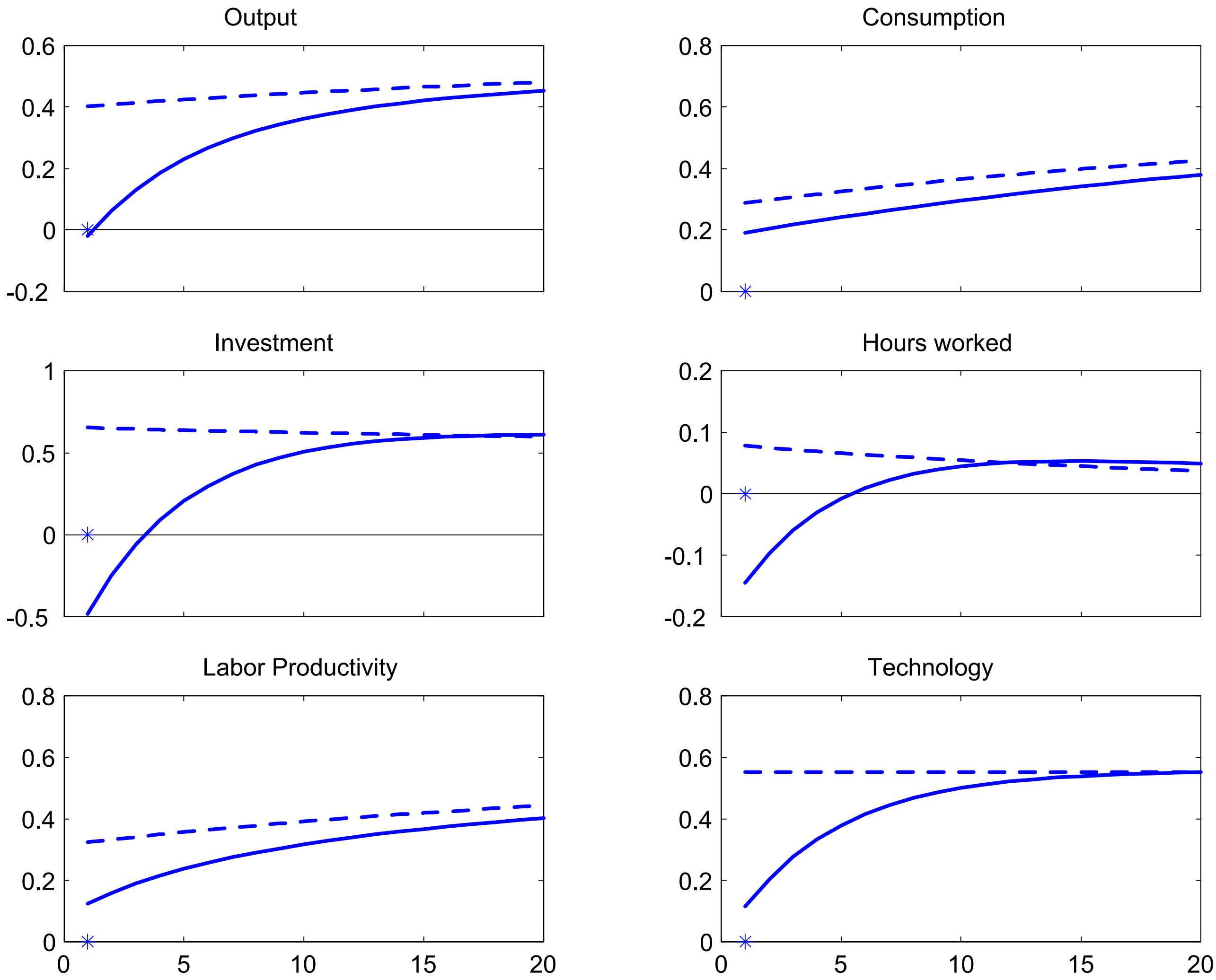

Figure 2: Impulse response functions to a permanent technology shock. Baseline paramerization (solid line) and when the persistence parameter (ρ_x) equals 0 (dashed line).

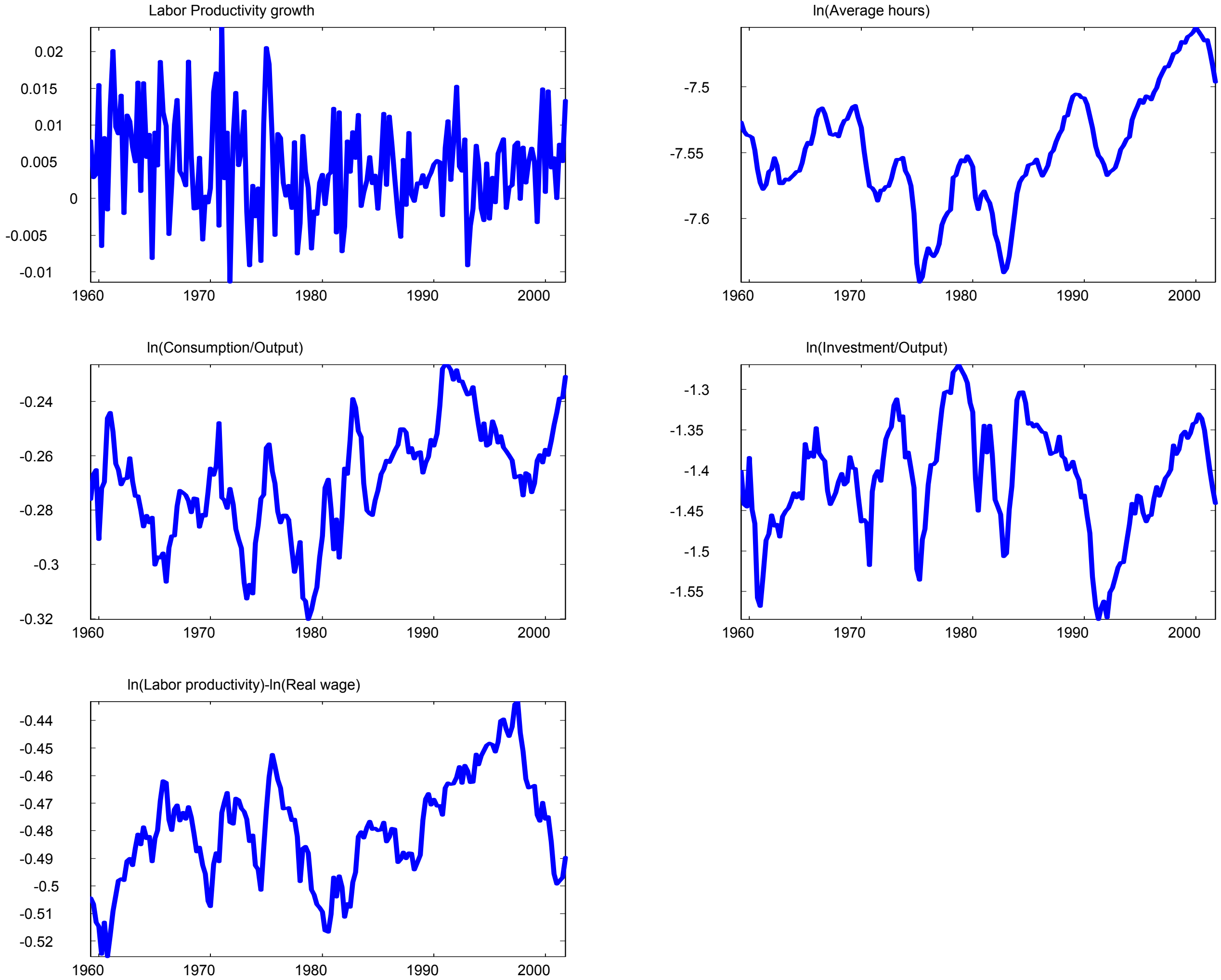

Figure 3: A plot of the U.S. data used in the VARs.

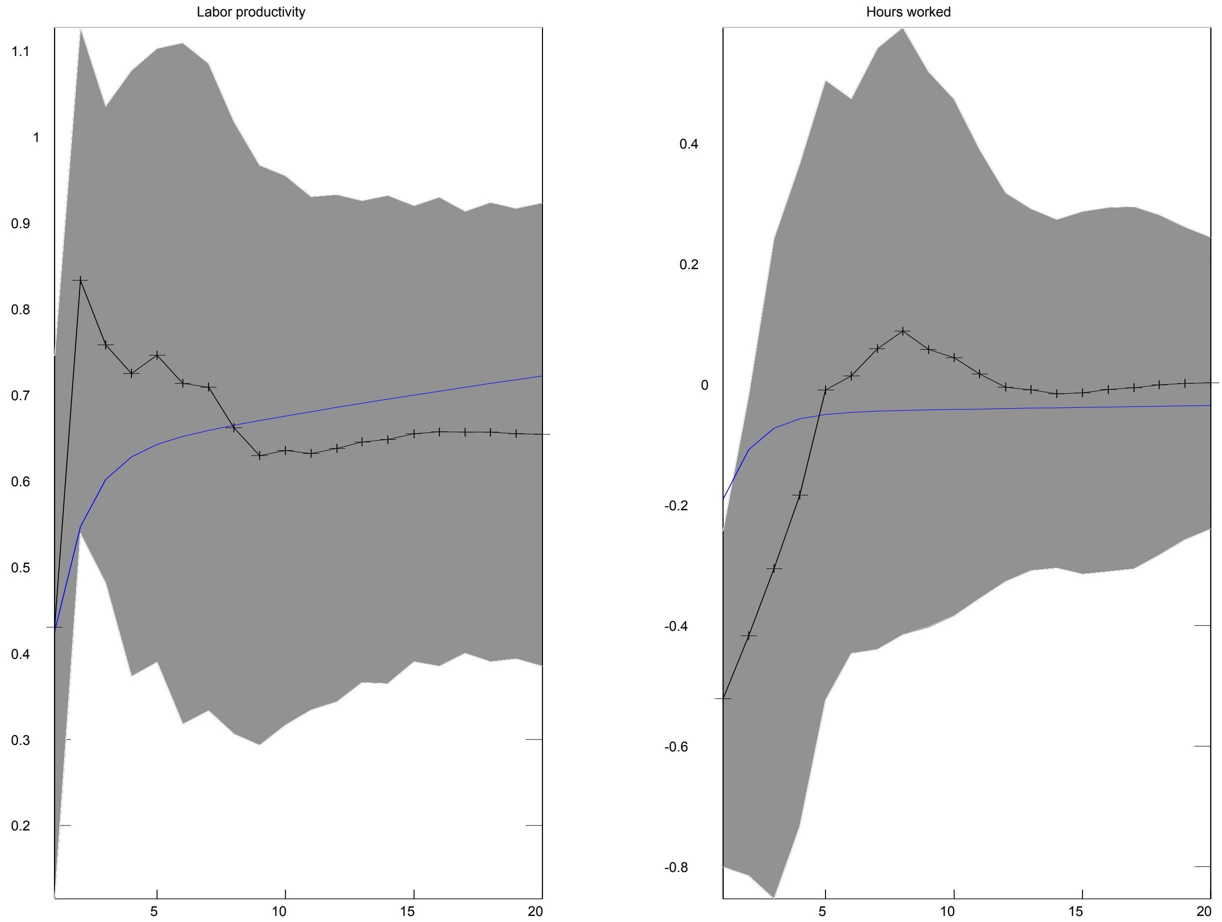

Figure 4: Impulse response functions in the RBC-model (solid line) and the data (line with +) to a permanent technology shock. Bivariate VAR estimated for hours in first differences. Grey area indicates 95-percent confidence interval.

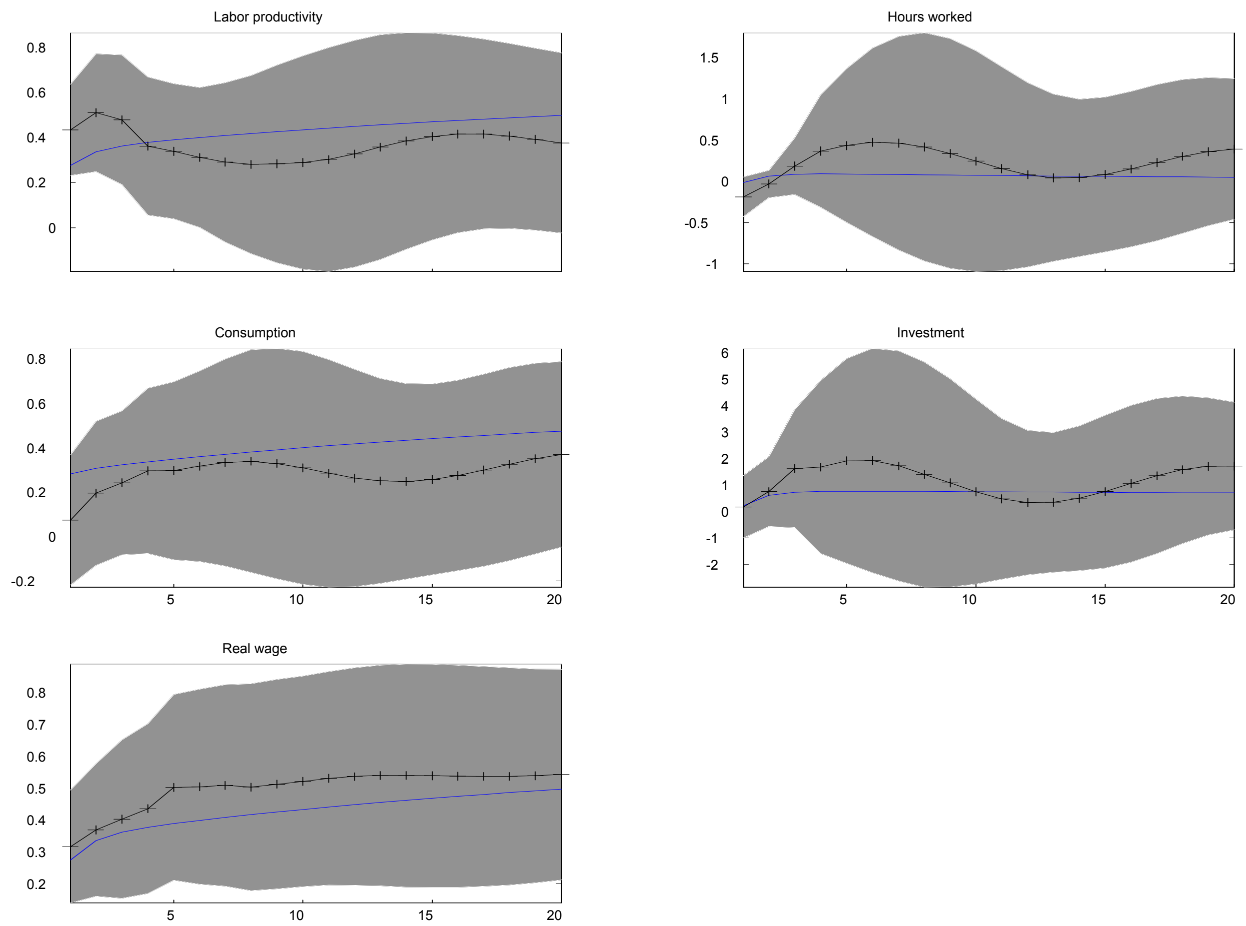

Figure 5: Impulse response functions in the RBC-model (solid line) and the data (line with +) to a permanent technology shock. 5-variables VAR estimated for hours in first differences. Grey area indicates 95-percent confidence interval.

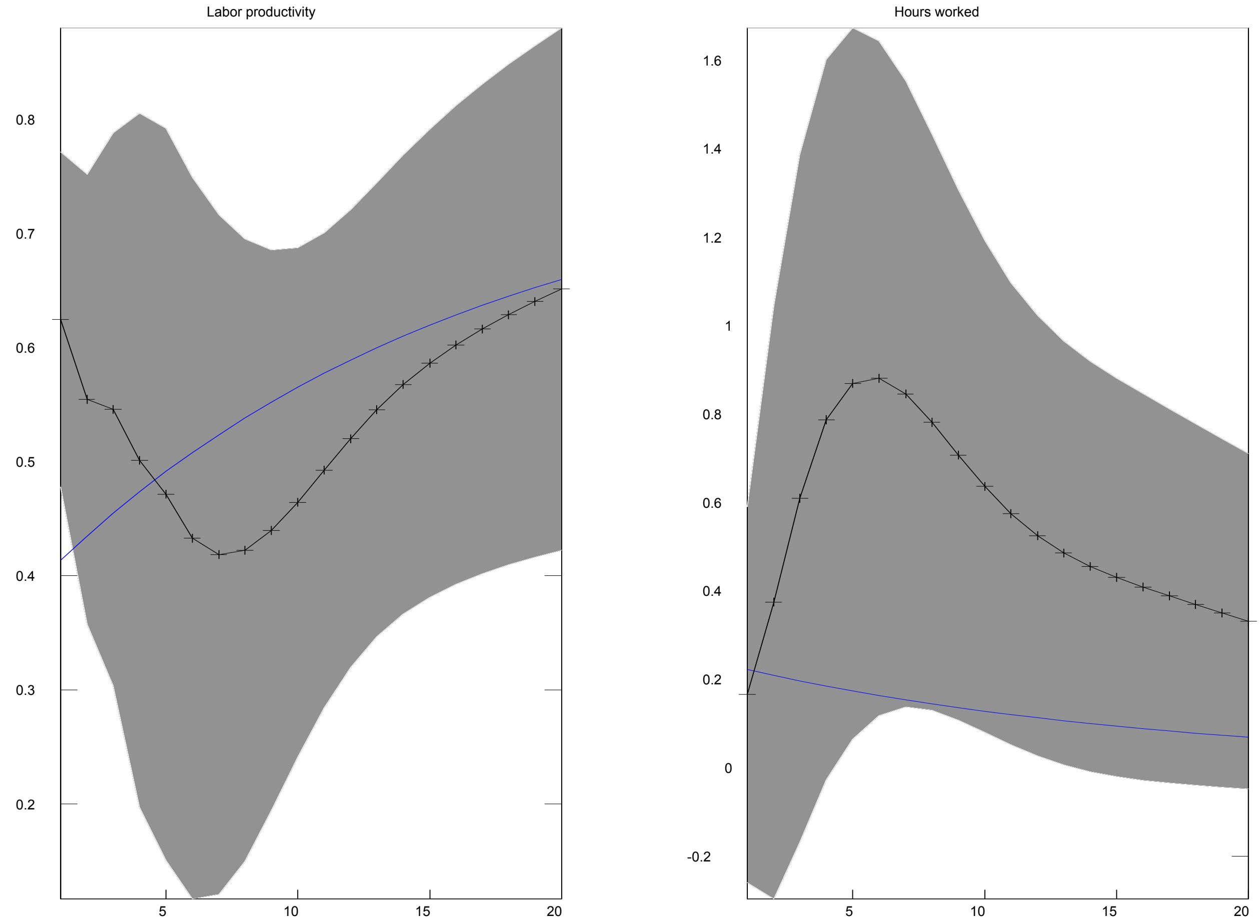

Figure 6: Impulse response functions in the RBC-model (solid line) and the data (line with +) to a permanent technology shock. Bivariate VAR estimated with hours in levels. Grey area indicates 95-percent confidence interval.

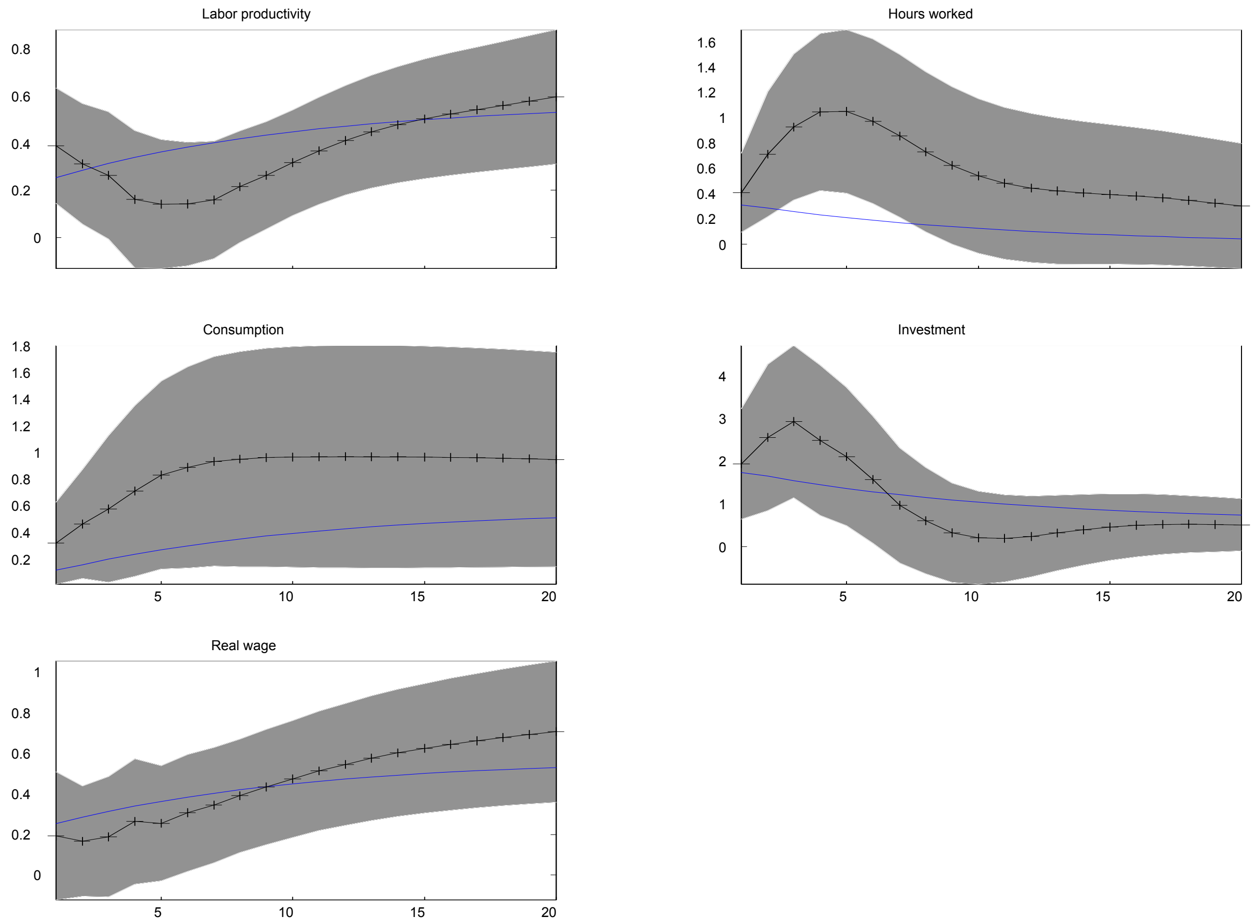

Figure 7: Impulse response functions in the RBC-model (solid line) and the data (line with +) to a permanent technology shock. 5-variables VAR estimated with hours in levels. Grey area indicates 95-percent confidence interval.

Appendix A Solving the models

In this appendix, I describe in detail how I have solved for the steady state and the decision rules in the models.

The first-order conditions for dynamic programming problem given by (8) are (introducing the notation $u_X(t) \equiv \frac{\partial}{\partial X} u(X_t, Y_t)$)

$$\begin{aligned} -u_C(t) + \beta \mathrm{E}_t V_K(t+1) &= 0, \text{ (w.r.t. } K_{t+1}) \\ -u_L(t) + u_C(t) W_t &= 0, \text{ (w.r.t. } H_t) \end{aligned} \tag{A.1}$$

By applying the envelope theorem, we obtain

$$V_K(t) = u_C(t)\left(1 + R_t^K - \delta\right), \text{ (w.r.t. } K_t) \tag{A.2}$$

and if we insert (A.2) in (A.1), we finally derive the first-order conditions

$$\begin{aligned} u_L(t) &= u_C(t)(1-\theta) z_t^{1-\theta} e^{\varepsilon_t} \left(\frac{K_t}{H_t}\right)^{\theta}, \\ u_C(t) &= \beta \mathrm{E}_t u_C(t+1)\left(1 + z_{t+1}^{1-\theta} e^{\varepsilon_{t+1}} \theta \left(\frac{K_{t+1}}{H_{t+1}}\right)^{\theta-1} - \delta\right), \end{aligned} \tag{A.3}$$

or equivalently, using (6),

$$\begin{aligned} u_L(t) &= u_C(t) W_t, \\ u_C(t) &= \beta \mathrm{E}_t u_C(t+1)\left(1 + R_{t+1}^K - \delta\right). \end{aligned}$$

The latter equations have standard interpretations, the first being the condition for intra-temporal optimality, and the latter is the condition for inter-temporal optimality.

To sum up, the economy is characterized by the equations (A.3), (7), (3), (4), (5) and the aggregate resource constraint

$$Y_t = C_t + I_t. \tag{A.4}$$

A.1 Case 1: Cobb-Douglas utility

Using (9) in (A.3), we obtain

$$\begin{aligned}
\left[C_t^{\alpha}\left(1-H_t\right)^{1-\alpha}\right]^{-\sigma} C_t^{\alpha}\left(1-\alpha\right)\left(1-H_t\right)^{-\alpha} &= \left[C_t^{\alpha}\left(1-H_t\right)^{\alpha}\right]^{-\sigma} \alpha C_t^{\alpha-1}\left(1-H_t\right)^{1-\alpha}\left(1-\theta\right) z_t^{1-\theta} e^{\varepsilon_t}\left(\frac{K_t}{H_t}\right)^{\theta} \\
&\Leftrightarrow \qquad\qquad (A.5) \\
C_t\left(1-\alpha\right) &= \alpha\left(1-H_t\right)\left(1-\theta\right) z_t^{1-\theta} e^{\varepsilon_t}\left(\frac{K_t}{H_t}\right)^{\theta},
\end{aligned}$$

and

$$\begin{aligned}
\left[C_t^{\alpha}\left(1-H_t\right)^{1-\alpha}\right]^{-\sigma} \alpha C_t^{\alpha-1}\left(1-H_t\right)^{1-\alpha} &= \beta \mathrm{E}_t\left(\left[C_{t+1}^{\alpha}\left(1-H_{t+1}\right)^{1-\alpha}\right]^{-\sigma} \alpha C_{t+1}^{\alpha-1}\left(1-H_{t+1}\right)^{1-\alpha}\right)\left(1+z_{t+1}^{1-\theta} e^{\varepsilon_{t+1}} \theta\left(\frac{K_{t+1}}{H_{t+1}}\right)^{\theta-1}-\delta\right) \\
&\Leftrightarrow \qquad\qquad (A.6) \\
C_t^{\alpha(1-\sigma)-1}\left(1-H_t\right)^{(1-\alpha)(1-\sigma)} &= \beta \mathrm{E}_t\left[\left(C_{t+1}^{\alpha(1-\sigma)-1}\left(1-H_{t+1}\right)^{(1-\alpha)(1-\sigma)}\right)\left(1+z_{t+1}^{1-\theta} e^{\varepsilon_{t+1}} \theta\left(\frac{K_{t+1}}{H_{t+1}}\right)^{\theta-1}-\delta\right)\right].
\end{aligned}$$

A.1.1 Computation of steady state

Since all variables in (10) are stationary, we can compute a constant steady state (dropping time subscripts).

The equations in (10) become

$$
\begin{aligned}
\hat{C}(1-\alpha) &= \alpha(1-H)(1-\theta)\mu_z^{-\theta}\left(\frac{\hat{K}}{H}\right)^{\theta}, \\
1 &= \beta\left[\mu_z^{\alpha(1-\sigma)-1}\left(1+\mu_z^{1-\theta}\theta\left(\frac{\hat{K}}{H}\right)^{\theta-1}-\delta\right)\right], \\
\hat{C}+\hat{K} &= \mu_z^{-\theta}\hat{K}^{\theta}H^{1-\theta}+(1-\delta)\hat{K}\mu_z^{-1}, \\
\hat{K} &= (1-\delta)\hat{K}\mu_z^{-1}+\hat{I}, \\
\hat{Y} &= \hat{C}+\hat{I}.
\end{aligned}
\tag{A.7}
$$

To solve this system, it is convenient to define $\tilde{\beta}=\beta\mu_z^{\alpha(1-\sigma)-1}$. From the second equation in (A.7), we have that

$$
\begin{aligned}
1+\mu_z^{1-\theta}\theta\left(\frac{\hat{K}}{H}\right)^{\theta-1}-\delta &= \frac{1}{\tilde{\beta}} \\
&\Leftrightarrow \\
\frac{\hat{K}}{H} &= \left(\frac{1-\tilde{\beta}(1-\delta)}{\theta\tilde{\beta}\mu_z^{1-\theta}}\right)^{\frac{1}{\theta-1}} \\
&\Leftrightarrow \\
\hat{K} &= \left(\frac{1-\tilde{\beta}(1-\delta)}{\theta\tilde{\beta}\mu_z^{1-\theta}}\right)^{\frac{1}{\theta-1}}H.
\end{aligned}
\tag{A.8}
$$

(A.8) can then be used in (A.7) to reduce out $\hat{K}$ and $\hat{K}/H$ from the first and third equation in (A.7) to solve for H. After some tedious algebra, it can be shown that this solution is given by

$$H=\frac{\frac{\alpha}{1-\alpha}\frac{(1-\theta)}{\theta}R^K}{\frac{\alpha}{1-\alpha}\frac{(1-\theta)}{\theta}R^K+\frac{R^K}{\theta}+1-\delta-\mu_z}$$

where$R^K\equiv\frac{1-\tilde{\beta}(1-\delta)}{\tilde{\beta}}$. Given the solution for H, it is trivial to solve for the other variables $\hat{C}$, $\hat{K}$, $\hat{I}$ and $\hat{Y}$.

A.1.2 Computation of equilibrium

We will solve the model given by (10) and (5) by the standard method of log-linearizing the first-order conditions and resource constraints around the computed steady state. Log-linearizing the equations in (10), introducing the notation $\hat{x}_{t+j} \equiv \frac{d\hat{X}_{t+j}}{X}$, we obtain the following log-linearized equations

$$\begin{aligned}
\hat{c}_t &= -\frac{H}{1-H}\hat{h}_t - \theta\hat{x}_t + \varepsilon_t + \theta\hat{k}_t - \theta\hat{h}_t, && \text{(A.9)}\\
\left(\alpha\left(1-\sigma\right)-1\right)\hat{c}_t - \frac{\frac{(1-\alpha)(1-\sigma)}{}H}{1-H}\hat{h}_t &= \mathrm{E}_t\left[\begin{array}{c} \left(\alpha\left(1-\sigma\right)-1\right)\hat{x}_{t+1} + \left(\alpha\left(1-\sigma\right)-1\right)\hat{c}_{t+1} - \frac{(1-\alpha)(1-\sigma)H}{1-H}\hat{h}_{t+1} + \\ \beta\mu_z^{(\alpha(1-\sigma)-1)}\mu_z^{1-\theta}\theta\left(\frac{\hat{K}}{H}\right)^{\theta-1}\left(\left(1-\theta\right)\hat{x}_{t+1} + \varepsilon_{t+1} + \left(\theta-1\right)\hat{k}_{t+1} - \left(\theta-1\right)\hat{h}_{t+1}\right)\end{array}\right],\\
\frac{\hat{C}}{\hat{Y}}\hat{c}_t + \frac{\hat{I}}{\hat{Y}}\hat{\imath}_t &= -\theta\hat{x}_t + \varepsilon_t + \theta\hat{k}_t + \left(1-\theta\right)\hat{h}_t,\\
\frac{\hat{K}}{\hat{Y}}\hat{k}_{t+1} &= \frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}}\hat{k}_t - \frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}}\hat{x}_t + \frac{\hat{I}}{\hat{Y}}\hat{\imath}_t\\
\hat{y}_t &= \frac{\hat{C}}{\hat{Y}}\hat{c}_t + \frac{\hat{I}}{\hat{Y}}\hat{\imath}_t,\\
\hat{x}_t &= \rho_x\hat{x}_{t-1} + \eta_t,\\
\varepsilon_t &= \rho_\varepsilon\varepsilon_{t-1} + \nu_t
\end{aligned}$$

These equations are casted in matrix form as

$$\mathrm{E}_t\left\{\alpha_0\tilde{z}_{t+1} + \alpha_1\tilde{z}_t + \alpha_2\tilde{z}_{t-1} + \beta_0\theta_{t+1} + \beta_1\theta_t\right\} = 0 \qquad \text{(A.10)}$$

where $\tilde{z}_t$ and θ_t are column vectors with all the endogenous and exogenous variables, respectively. θ_t follows

$$\theta_t = \rho\theta_{t-1} + u_t. \qquad \text{(A.11)}$$

If we define

$$\tilde{z}_t \equiv \begin{bmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{k}_{t+1} \\ \hat{\imath}_t \\ \hat{y}_t \end{bmatrix}, \theta_t = \begin{bmatrix} \varepsilon_t \\ \hat{x}_t \end{bmatrix}, \rho = \begin{bmatrix} \rho_\varepsilon & 0 \\ 0 & \rho_x \end{bmatrix}, u_t = \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}$$

then the fundamental difference equation in (A.10) for this model is given by

$$\mathrm{E}_t \left\{ \begin{array}{l} \left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & -A_p & B_p \frac{H}{1-H} + C_p\left(\theta - 1\right) & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{bmatrix} \hat{c}_{t+1} \\ \hat{h}_{t+1} \\ \hat{k}_{t+2} \\ \hat{\imath}_{t+1} \\ \hat{y}_{t+1} \end{bmatrix} + \left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & \frac{H}{1-H} + \theta & 0 & 0 & 0 \\ 2 & A_p & -B_p \frac{H}{1-H} & -C_p\left(\theta - 1\right) & 0 & 0 \\ 3 & \frac{\hat{C}}{\hat{Y}} & -\left(1-\theta\right) & 0 & \frac{\hat{I}}{\hat{Y}} & 0 \\ 4 & 0 & 0 & \frac{\hat{K}}{\hat{Y}} & -\frac{\hat{I}}{\hat{Y}} & 0 \\ 5 & -\frac{\hat{C}}{\hat{Y}} & 0 & 0 & -\frac{\hat{I}}{\hat{Y}} & 1 \end{array} \right] \begin{bmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{k}_{t+1} \\ \hat{\imath}_t \\ \hat{y}_t \end{bmatrix} + \\ \left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & -\theta & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & -\theta & 0 & 0 \\ 4 & 0 & 0 & -\frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}} & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{bmatrix} \hat{c}_{t-1} \\ \hat{h}_{t-1} \\ \hat{k}_t \\ \hat{\imath}_{t-1} \\ \hat{y}_{t-1} \end{bmatrix} + \left[\begin{array}{ccc} & 1 & 2 \\ 1 & 0 & 0 \\ 2 & -C_p & -A_p - C_p\left(1-\theta\right) \\ 3 & 0 & 0 \\ 4 & 0 & 0 \\ 5 & 0 & 0 \end{array} \right] \theta_{t+1} + \left[\begin{array}{ccc} & 1 & 2 \\ 1 & -1 & \theta \\ 2 & 0 & 0 \\ 3 & -1 & \theta \\ 4 & 0 & \frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}} \\ 5 & 0 & 0 \end{array} \right] \theta_t \end{array} \right\} = 0$$

where the composite parameters A_p, B_p, and C_p are defined as

$$\begin{aligned} A_p &\equiv \left(\alpha\left(1-\sigma\right) - 1\right), \\ B_p &\equiv \left(1-\alpha\right)\left(1-\sigma\right), \\ C_p &\equiv \beta\mu_z^{\left(\alpha\left(1-\sigma\right)-1\right)}\mu_z^{1-\theta}\theta\left(\frac{\hat{K}}{H}\right)^{\theta-1}. \end{aligned}$$

Note that $\hat{k}_{t+1}$ is included in $\tilde{z}_t$ because it is determined in period t in the model.

To solve this system, I use the Anderson and Moore algorithm well described in Anderson (1999).

A.2 Case 2: Hansen utility

In this case, we use a version of the Hansen (1985) utility function, i.e.

$$u(C_t, H_t) = \ln C_t - AH_t.$$

With this utility function, the first-order conditions in (10) change to

$$\begin{aligned}
\hat{C}_t &= \frac{1}{A}(1-\theta)\, x_t^{-\theta} e^{\varepsilon_t} \left(\frac{\hat{K}_t}{H_t}\right)^{\theta}, \\
\hat{C}_t &= \beta \mathrm{E}_t \left[x_{t+1}^{-1} \hat{C}_{t+1}^{-1} \left(1 + x_{t+1}^{1-\theta} e^{\varepsilon_{t+1}} \theta \left(\frac{\hat{K}_{t+1}}{H_{t+1}}\right)^{\theta-1} - \delta\right)\right].
\end{aligned}$$

A.2.1 Computation of steady state

The new steady-state expression for H is

$$H = \frac{\frac{1-\theta}{A} D_0}{D_0 + 1 - \delta - \mu_z}$$

where $D_0 \equiv \frac{1-\tilde{\beta}(1-\delta)}{\theta\tilde{\beta}}$, and $\tilde{\beta} \equiv \beta\mu_z^{-1}$.

A.2.2 Computation of equilibrium

The log-linearized versions of the modified first-order equations are

$$\begin{aligned}
\hat{c}_t &= -\theta\hat{x}_t + \varepsilon_t + \theta\hat{k}_t - \theta\hat{h}_t, \\
-\hat{c}_t &= \mathrm{E}_t \left[-\hat{c}_{t+1} - \hat{x}_{t+1} + \beta\mu_z^{-\theta}\theta \left(\frac{\hat{K}}{H}\right)^{\theta-1} \Big((1-\theta)\,\hat{x}_{t+1} + \varepsilon_{t+1} + (\theta-1)\,\hat{k}_{t+1} - (\theta-1)\,\hat{h}_{t+1}\Big)\right].
\end{aligned}$$

In all other respects, this model is identical to the baseline model. The fundamental difference equation now reads

$$
\mathrm{E}_t \left\{ \begin{array}{l}
\left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & C_p\left(\theta-1\right) & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}\right]
\left[\begin{array}{c} \hat{c}_{t+1} \\ \hat{h}_{t+1} \\ \hat{k}_{t+2} \\ \hat{\imath}_{t+1} \\ \hat{y}_{t+1} \end{array}\right]
+
\left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & \theta & 0 & 0 & 0 \\ 2 & -1 & 0 & -C_p\left(\theta-1\right) & 0 & 0 \\ 3 & \frac{\hat{C}}{\hat{Y}} & -\left(1-\theta\right) & 0 & \frac{\hat{I}}{\hat{Y}} & 0 \\ 4 & 0 & 0 & \frac{\hat{K}}{\hat{Y}} & -\frac{\hat{I}}{\hat{Y}} & 0 \\ 5 & -\frac{\hat{C}}{\hat{Y}} & 0 & 0 & -\frac{\hat{I}}{\hat{Y}} & 1 \end{array}\right]
\left[\begin{array}{c} \hat{c}_{t} \\ \hat{h}_{t} \\ \hat{k}_{t+1} \\ \hat{\imath}_{t} \\ \hat{y}_{t} \end{array}\right]
+ \\
\left[\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & -\theta & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & -\theta & 0 & 0 \\ 4 & 0 & 0 & -\frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}} & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}\right]
\left[\begin{array}{c} \hat{c}_{t-1} \\ \hat{h}_{t-1} \\ \hat{k}_{t} \\ \hat{\imath}_{t-1} \\ \hat{y}_{t-1} \end{array}\right]
+
\left[\begin{array}{ccc} & 1 & 2 \\ 1 & 0 & 0 \\ 2 & -C_p & 1-C_p\left(1-\theta\right) \\ 3 & 0 & 0 \\ 4 & 0 & 0 \\ 5 & 0 & 0 \end{array}\right] \theta_{t+1}
+
\left[\begin{array}{ccc} & 1 & 2 \\ 1 & -1 & \theta \\ 2 & 0 & 0 \\ 3 & -1 & \theta \\ 4 & 0 & \frac{1-\delta}{\mu_z}\frac{\hat{K}}{\hat{Y}} \\ 5 & 0 & 0 \end{array}\right] \theta_t
\end{array} \right\} = 0
$$

where $C_p \equiv \beta\mu_z^{-\theta}\theta\left(\frac{\hat{K}}{H}\right)^{\theta-1}$.