Nominal Debt as a Burden on Monetary Policy*

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Abstract

This paper explores the roles played by indexed debt and nominal debt when monetary policy is designed sequentially. In our model economy, when the outstanding stock of government debt is indexed, the optimal monetary policy is time consistent and it results in constant interest rates and debt levels. In contrast, when the stock of debt is nominal, that is, not indexed, the incentive to reduce the stock of debt partially through an unanticipated inflation creates the standard time-inconsistency problem. In this article we study the optimal sequential choice of monetary policy when the stock of debt is nominal and there is no commitment technology. In this case, the incentive to generate unanticipated inflations increases the cost of the outstanding debt even no unanticipated inflation episodes occur in equilibrium. The optimal sequential policy is to deplete the outstanding stock of debt progressively until these extra costs disappear. Nominal debt is therefore a burden on monetary policy, not only because it must be serviced, but also because it creates a time inconsistency problem that distorts interest rates. The introduction of alternative forms of taxation may lessen this burden. If there is full commitment to an optimal fiscal policy, then the resulting monetary policy is the Friedman rule of zero nominal interest rates, independently of whether debt is indexed or nominal and of the degree of commitment of monetary authorities.

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1 Introduction

Fiscal discipline has often been seen as a precondition to sustain price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. More precisely, it is understood that an economy with a large stock of nominally denominated government debt can benefit from inflation surprises that reduce the need for distortionary taxation in the future. This means that optimal monetary policy under full commitment (the Ramsey policy) can be time inconsistent. In other words, if a government with the ability to honor its commitments were to re-optimize at a later date, it may choose to deviate from the policy originally announced. In this context, a constraint on the level of debt may reduce the impact of such time-inconsistency distortions.

However, while this argument is known, what is less well understood is how severe the time-inconsistency problem is and, in particular, what is the optimal monetary policy when there is an outstanding stock of goverment debt and the commitment possibilities of the government are limited. The purpose of this paper is to address these issues and, more specifically, to study the effects of nominal debt on the optimal sequential choice of monetary policy. To this aim, we identify the mechanisms at work in a simple general equilibrium monetary model. By pursuing model simplicity, we not only can provide a sharp characterization of the potential effects of nominal debt on monetary policy and prices, but we also gain powerful insights on the characterization of optimal sequential policies in recursive equilibria.

Our benchmark model economy is a cash-in-advance economy with indexed debt. We characterize the optimal monetary policy in this economy and we compare it with the optimal policies that obtain when the debt is nominal (i) under full commitment and (ii) when the government is unable to fully commit to its announced policy. In this case, we restrict our attention to the Markov perfect equilibrium.

The structure of the optimal taxation problems that we solve is the following: first, we assume that the government has to finance a given constant flow of expenditures with revenues levied using only seigniorage. To solve these optimal taxation problems, the government chooses the paths on seigniorage that maximize the household's utility subject to the implementability and budget constraints. Unexpected inflation is costly, because we assume that the consumption good must be purchased with cash carried over from the previous period, as in Svensson (1985). This timing of the cash-in-advance constraint implies that, if the government decided to surprise the household with an unexpected increase in inflation in any given period, the household's consumption would be smaller than planned because its predetermined cash balances would be insufficient to purchase the intended amount of consumption. When considering whether or not to carry out such a surprise inflation, the government compares the reduction in the household's current utility that results from this lower level of consumption with the increase in the household's future utility that results from the reduction in future seigniorage.

After describing our model economy in Section 2, in Section 3 we characterize this time-inconsistency problem of the optimal policy with full commitment, when the outstanding stock of government debt is indexed. Our results build up on those of Nicolini (1998), who shows that, when the utility function is logarithmic in consumption and linear in leisure and the stock of government debt is indexed, the optimal monetary policy —in an economy similar to ours— is to abstain from inflation surprises. This result follows from applying optimal taxation principles and it implies that, in this model economy, the solution to the Ramsey problem is time consistent. Furthermore, as we show below, the solution to this problem is stationary, and there is a unique interest rate that balances the government budget.

Next we study the optimal monetary policies that obtain when the outstanding stock of government debt is nominal. In Section 4, we assume that there is full commitment to monetary policy. We show that interest rates are kept constant from period one onwards, but that the initial interest rate is higher, since it is optimal to cancel part of the inherited stock of nominal debt. In this case, after period zero the interest rate is lower than the one that obtains in the equilibrium with indexed debt, since in this latter case the government cannot reduce of the inherited stock of debt by increasing the initial price. We characterize the Ramsey equilibrium in which rational expectations are satisfied even in period zero (i.e., as expectations of ancestrors). This equilibrium has the property that initial real liabilities are the same as in the case of indexed debt. Since there is no 'free lunch' surprise inflation, the equilibrium that obtains with nominal debt and full commitment is less efficient than the time consistent equilibrium that obtains with indexed debt.

With these regimes as reference, in Section 5 we present the main results of the paper. We study the optimal policy that obtains in the absence of commitment. In this case, we restrict our attention to the Markov perfect equilibrium. We call this equilibrium recursive as in Cole and Kehoe (1996) and Obstfeld (1997). Two interesting features of the optimal policy that obtains under this recursive equilibrium are that the optimal inflation tax is non-stationary and that it converges to the inflation tax that obtains when there is no government debt. This result arises because, in the recursive equilibrium, it is optimal for the government to asymptotically deplete the stock of nominal government debt. An implication of this result is that, in this economy, the optimal nominal interest is initially higher than the one prevailing when debt is indexed but, in the limit, it is lower. This decreasing path for the nominal interest rate is another indication that nominal debt is indeed a burden for monetary policy. Not only it has to be serviced, but it also distorts interest rates. In fact, it is because the optimal policy endogenizes these distortions that it asymptotically monetizes the debt as the way to eliminate them. In Section 6 we carry out a numerical example and we describe our findings which allow us to compare the different regimes numerically.

In Section 7 we ask whether these results are robust to the introduction of additional taxes. This is important since, in advanced economies, seignorage is a marginal tax and we would like to know if our results still hold when government outlays are financed with other taxes. Specifically, we study the case of consumption taxes. We impose the natural assumption that taxes are chosen before the monetary policy decisions are made. We find that the same equilibria arise when there is both seignorage and consumption taxes and when there is only seignorage, provided that the resulting optimal monetary policy distortions result in non negative interest rates. However, the fiscal authority can constraint the monetary authority to follow the Friedman rule, of zero nominal rates, from the outset. In this case, since monetary distortions resulting in negative interest rates are not equilibrium rates, the monetary authority has no incentive to monetarize the debt and, as a result, implements the optimal equilibrium that obtains with indexed debt.

The relationship between fiscal and monetary policy has been adressed in the unpleasant monetarist arithmetic literature of Sargent and Wallace (1981), and in the fiscal theory of the price level of Sims (1994) and Woodford (1996). In these approaches, however, policies are taken to be exogenous. This is not the case both in our analysis, and in the related work of Chari and Kehoe (1999), Rankin (2002) and Obstfeld (1997). These last two papers are the closest to ours. Both, however, assume that debt is real and they only focus on monetary policy. They aim at characterizing the Markov perfect equilibrium when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. This source of time inconsistency is ambiguous: while in Lucas and Stokey (1983) the government would want to completely deplete the outstanding money balances, in Svensson (1985)'s set up, as was shown in Nicolini (1998), under certain elasticity conditions, the government problem would be time consistent. This ambiguity led Obstfeld (1997) to consider an ad-hoc cost of a surprise inflation. Our analysis differs from Obstfeld's both because we consider nominal debt and because, in our model economy, the cost of unanticipated inflation arises from the timing of the cash-in-advance constraint rather than being imposed ad-hoc. In a similar framework, Rankin (2002) shows that the size of the initial debt matters for the direction of the time inconsistency problem, but he does not provide a full characterization of the resulting dynamic equilibrium. He shows that, for general preferences, there can be a value of debt where the elasticity is unitary and, therefore, there exists a steady state with positive debt.

Finally, an additional contribution of this paper is the full characterization and the computation of the optimal policy in a recursive equilibrium with a state variable. In this respect, our work is closely related to the recent work of Krusell, Martín and Ríos-Rull (2003) who characterize recursive equilibria in the context of an optimal labor taxation problem.

2 The model economy

The economy is made up of a government sector and a private sector. We assume that the government in this economy issues currency, M^g, and nominal debt, B^g, to finance an exogenous and constant level of public consumption, g. We abstract from all other sources of public revenues. In each period $t \geq 0$ the government budget constraint is the following:

$$M_t^g + B_t^g(1+i_t) + p_t g \leq M_{t+1}^g + B_{t+1}^g \tag{1}$$

where i_t is the nominal interest rate paid on nominal bonds lent by the government at time $t-1$, p_t is the price of one unit of the date t composite good, and M_0^g and initial debt liabilities $B_0^g(1+i_t)$ are given. A government policy is therefore a specification of $\{M_{t+1}^g, B_{t+1}^g, g\}$ for $t \geq 0$.

We assume that the economy is inhabited by a continuum of identical infinitely-lived households whose preferences over infinite sequences of consumption and labor can be represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{2}$$

where $c_t > 0$ denotes consumption at time t, n_t denotes labor at time t, and $0 < \beta < 1$ is the time discount factor. The utility function is assumed to satisfy standard assumptions, of being strictly increasing and strictly concave, and, for reasons that will become clear below, for most of the paper we assume it to be logarithmic, i.e., $u(c) = \log(c)$.

We assume that consumption in period t must be purchased using the currency carried over from period $t-1$ as in Svensson (1985). Notice that this timing of the cash-in-advance constraint implies that, when solving its maximization problem, the representative household takes both M_0 and $B_0(1 + i_0)$ as given.[1] Specifically, the cash-in-advance constraint faced by the representative household for every $t \geq 0$ is the following:

$$p_t c_t \leq M_t \tag{3}$$

To simplify the productive side of this economy we assume that, each period, labor can be transformed into either the private consumption good or the public consumption good on a one-to-one basis. Consequently, the competitive equilibrium real wage can be trivially shown to be $w_t = 1$ for all $t \geq 0$, and the economy's resource constraint is

$$c_t + g \leq n_t \tag{4}$$

for every $t \geq 0$.

Therefore, in each period $t \geq 0$ the representative household also faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - p_t c_t + B_t(1 + i_t) + p_t n_t \tag{5}$$

where M_{t+1} and B_{t+1} denote, respectively, the nominal money balances and the nominal government debt that the household carries over from period t to period $t+1$. Finally we assume that the representative household faces a no-Ponzi games condition

$$\lim_{T \longrightarrow \infty} \beta^T B_{T+1} = 0 \tag{6}$$

2.1 A competitive equilibrium

Definition 1 *A competitive equilibrium for this economy is a government policy,* $\{M^g_{t+1}, B^g_{t+1}, g,\}_{t=0}^{\infty}$*, an allocation* $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$*, and a price vector,* $\{p_t, i_{t+1}\}_{t=0}^{\infty}$*, such that:*

(i) given M^g_0 *and* $B^g_0(1 + i_0)$*, the government policy and the price vector satisfy the government budget constraint described in expression (1);*

[1] In the Lucas and Stokey (1983) timing both M_0 and B_0 can be chosen by the household.

(ii) when households take M_0, $B_0(1+i_0)$ and the price vector as given, the allocation maximizes the problem described in expression (2), subject to the cash-in-advance constraint described in expression (3), the household budget constraint described in expression (5), and the no-Ponzi games condition described in expression (6); and

(iii) the price vector is such that all markets clear, that is: $M_t^g = M_t$, $B_t^g = B_t$, and g and $\{c_t, n_t\}_{t=0}^{\infty}$ satisfy the economy's resource constraint described in expression (4), for every $t \geq 0$.

Given our assumptions on the utility function u, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the household budget constraint (5) and the economy's resource constraint (4) with equality, and that the first order conditions of the Lagrangian of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. Furthermore, it is also straightforward to show that, when $i_{t+1} > 0$, the cash-in-advance constraint (3) is binding, and that the competitive equilibrium allocation of this economy is completely characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, \tag{7}$$

$$1 + i_{t+1} = \beta^{-1}\frac{p_{t+1}}{p_t}, \tag{8}$$

$$c_t = \frac{M_t}{p_t}, \tag{9}$$

the government budget constraint (1), and the resource constraint (4). Moreover, these conditions, together with the households' no-Ponzi games condition (6), imply that the government present value budget constraint described in equation (10) is also satisfied in equilibrium.

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\beta}{\alpha} - n_t\right) = \frac{B_0(1+i_0)}{p_0}. \tag{10}$$

3 Optimal policy with indexed debt

First we study the case in which the outstanding stock of government debt is indexed. This case is the benchmark against which we compare the optimal policy that obtains when the outstanding stock of debt is nominal —that is, not indexed— which is the main focus of this article. When the debt is indexed, outstanding government liabilities are fixed in real terms. Let $b_{t+1} = B_{t+1}^g / p_t$ be the real value of the end-of-period stock of debt. Then the government budget constraint, (1) for $t \geq 0$, can be written as

$$\frac{M_{t+1}^g}{p_t} + b_{t+1} \geq \frac{M_t^g}{p_t} + b_t\beta^{-1} + g$$

where b_0 is the initial debt in real terms which has a real return of β^{-1}, since condition (8) also holds also in period zero when debt is indexed, that is,

$$1+i_t=\beta^{-1}\frac{p_t}{p_{t-1}},\ t\geq 0 \tag{11}$$

In other words, when debt is indexed and p_{t-1} is given, the government policy must be such that i_t adjusts to p_t in order to satisfy Fisher's equation even in period zero. This implies that the right hand side of the present value of the government budget constraint described in expression (10) becomes $b_0\beta^{-1}$ in equilibrium.

Definition 2 *For a given level of government expenditures, g, and initial values of currency, M_0, and real government debt, b_0, an optimal monetary equilibrium with indexed debt consists of a government policy, a price vector, and an implied allocation, such that: (i) the household utility is maximized, and (ii) the government policy, the allocation, and the price vector are a competitive equilibrium.*

We follow the standard implementability approach of having the government choose the allocation directly. That is, we replace $1+i_{t+1}$, p_{t+1}/p_t and M_t/p_t, for all $t\geq 0$, from equations (7), (9), and (11) into equation (1). Then, the household's utility is maximized when the government maximizes expression (22) subject to the implementability condition

$$u'\left(c_{t+1}\right)c_{t+1}\frac{\beta}{\alpha}+b_{t+1}=c_t+b_t\beta^{-1}+g,\ t\geq 0 \tag{12}$$

This problem is recursive only when $u(c)=\ln(c)$. In this case the price elasticity is unitary, and the nominal debt adjusts one-to-one to any change in the price level. As Nicolini (1998) has shown, in this case, the optimal monetary policy is time-consistent. More specifically, this problem can be written recursively as follows:

$$V(b)=\max_{c,b'}\left\{\log(c)-\alpha(c+g)+\beta V(b')\right\} \tag{13}$$

subject to

$$b'=c+\beta^{-1}b-\gamma, \tag{14}$$

where $\gamma\equiv\frac{\beta}{\alpha}-g$. The marginal condition for c is:

$$\frac{1}{c}-\alpha=-\beta V'(b') \tag{15}$$

The interpretation of this equation is that the marginal gain of consumption is equal to the marginal cost of one unit of future debt. Then, using the envelope theorem, we otatin taht

$$V'(b)=V'(b'), \tag{16}$$

and, substituting expression (15) into this expression we obtain that

$$\frac{1}{c}-\alpha=\frac{1}{c'}-\alpha \tag{17}$$

which implies that the optimal level of consumption, c^* is constant and equal to

$$c^* = \gamma - \left(\beta^{-1} - 1\right) b_0. \tag{18}$$

Notice that expression (15) implies that the initial value of the stock of indexed debt, b_0, is also stationary value of the real debt, $b^* = b_0$.

Finally, the stationary value of the nominal interest rate is

$$1 + i^{I*} = \left[\alpha\gamma - \alpha\left(\beta^{-1} - 1\right) b_0\right]^{-1}.$$

and the evolution of prices and money balances are given recursively by: $p_t = M_t/c^*$ and $M_{t+1} = \beta p_t/\alpha$.[2]

In this model economy the government has only one tax (the inflation tax) to service the stock debt and finance its expenditures. In Section 7, we discuss the optimal policy that obtains when the government can use additional taxes.

4 Optimal policy with nominal debt and full commitment

We now turn our attention to the case of nominal debt. In this section we start by studying the optimal monetary policy that obtains when the government can fully commit to its Ramsey (R) monetary policy implemented from a given period onwards. We denote this initial period as $t = 0$. Since the government can re-optimize at $t = 0$, it may choose a policy that differs from the policy implemented in the past. In fact, as we show below, when the value of the outstanding stock of nominal debt is positive, the government chooses to deviate from its previous policy. However, if this deviation had been anticipated by the households who took the decisions in the past, it would have had to be the case that the ex-ante real interest rate would have to be equal to the ex-post rate as in Chari and Kehoe (1999). To see this, consider the government budget constraint in period zero:

$$p_0 g + M_0^g + B_0^g(1 + i_0) \leq M_1^g + B_1^g \tag{19}$$

which, using the cash-in-advance constraint and the definition of b_1, can be written as

$$\gamma + b_1 - c_0 - \frac{B_0^g(1 + i_0)}{M_0^g} c_0 = 0 \tag{20}$$

In this case, the optimal policy implies choosing the value of c_0 that maximizes the representative household utility subject to (20). This results in $p_0 = M_0/c_0$ and real liabilities given by $B_0^g(1 + i_0)/p_0 = b_0(1 + r_0)$ where r_0 is the ex-post real interest rate.

Let $\overline{p}_0$ and $\overline{c}_0$ be such that they satisfy:

$$\frac{B_0^g(1 + i_0)}{\overline{p}_0} = \frac{B_0^g(1 + i_0)}{M_0^g} \overline{c}_0 = b_0\beta^{-1}. \tag{21}$$

That is, $\overline{p}_0$ is the price that satisfies Fisher's equation (8) in period zero, and $\overline{c}_0$ is the optimal consumption plan for $t = 0$ that is consistent with a real interest rate $r = \beta^{-1}$.

[2] Notice that this last equality has been obtained from expressions (7), (8) and (9).

Now we impose the additional consistency condition that $p_0 = \overline{p}_0$ in equilibrium. In other words, we impose the condition that the decisions taken in the past must satisfy a rational expectations consistency condition in period zero. This requires that there must be a fixed point between expectations and realizations —or, equivalently, between the ex-ante and the ex-post real interest rates— implicitely defined by $c_0 = \overline{c}_0$. Notice that imposing this rational expectations restriction in period zero, not only prevents the "free lunches" that result from surprise inflations, but it also allows us to compare the optimal policies that obtain in model economies with index debt and those that obtain in model economies with nominal debt, since we can compare equilibria that result under different regimes that have the same initial real government liabilities[3].

Definition 3 *A Ramsey equilibrium with nominal debt is a competitive equilibrium and a value for expected consumption at time $t = 0$, $\overline{c}_0$, that satisfy the following conditions:*

(i) given $\overline{c}_0$, the real version of the equilibrium allocation that solves the following problem:

$$Max \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \alpha(c_t + g)\right] \tag{22}$$

subject to the implementability constraints

$$\gamma + b_1 - c_0 - c_0 b_0 \frac{\beta^{-1}}{\overline{c}_0} = 0 \tag{23}$$

$$\gamma + b_{t+1} - c_t - \beta^{-1} b_t = 0, \textit{ for } t \geq 1, \textit{ and,} \tag{24}$$

(ii) $c_0 = \overline{c}_0$.

A Ramsey equilibrium is characterized by the following conditions:

$$\frac{1}{c_0^*} - \alpha = \left[\frac{1}{c_1^*} - \alpha\right]\left[1 + b_0 \frac{\beta^{-1}}{c_0^*}\right] \tag{25}$$

$$c_{t+1} = c_1^*, \text{ for } t \geq 1.$$

Notice that, as long as $b_0 > 0$, c_0^* is smaller than c_1^*, and it is given by $c_0^* = c_1^* - \beta^{-1} b_0 (1 - \alpha c_1^*)$. Where, using (24) and (23), this —from period one on— constant value of consumption is given by:

$$c_1^* = \gamma \left[1 + \alpha\left(\beta^{-1} - 1\right) b_0\right]^{-1}$$

[3]Notice that, when the utility function is logarithmic in consumption, there is a one-to-one mapping between initial conditions (B_0, M_0) and b_0. Specifically, $b_0 \beta^{-1} = B_0/(\alpha M_0)$. This follows from the equalities $(1 + i_0)/p_0 = (1 + i_0)c_0/M_0 = 1/(\alpha M_0)$.

We can also describe the path of interest rates by rewriting (25) as

$$\frac{i_0^R}{1+\frac{(1+i_0^R)B_0}{M_0}} = i^R \tag{26}$$

It is useful to compare the Ramsey equilibrium allocation with that of an optimal policy with indexed debt, since indexed debt can be viewed as an extreme form of commitment.

The Ramsey equilibrium with nominal debt is characterized by an ex-post interest rate that in period $t = 0$ is higher than the one that obtains in the optimal policy with indexed debt, and that afterwards it is lower. The reason is that the government aims at taking advantage of the lump-sum character of monetizing the outstanding nominal debt since there is no time zero indexation that the government must internalize. In other words, while we maintain the assumption of a unitary price elasticity ($u(c) = \ln(c)$), nominal debt adjusts less than one-to-one to any price change and, therefore, it is part of an optimal tax policy to partially monetarize the debt in the initial period, when expectations are given and the price increase does not question the commitment to the new monetary policy. In the case of indexed debt (17) marginal values of consumption are equated, even in period zero. In contrast, with nominal debt —as (25) shows— the marginal value of consumption in period zero is *discounted* since, for example, a marginal reduction of consumption, through a higher price in period zero, results in a lower outstanding debt in period one. Notice, however, that —given b_0, and expectations $\bar{c}_0 = c^*$— the indexed debt solution, c^*, is a feasible solution for the Ramsey planner. But, with nominal debt, the Ramsey planner has an additional taxation instrument —to monetarize part of the debt— and c^* is not a best reply to the expectations $\bar{c}_0 = c^*$. As often happens with (Nash) equilibria, the fact that the government has an additional instrument does not imply that the Ramsey equilibrium with nominal debt (i.e., $\bar{c}_0 = c_0^*$) is more efficient than the equilibrium with indexed debt. In fact, as the following proposition states, the converse is true.

Proposition 4 *Assume $u(c) = \log(c)$. For any given $b_0 > 0$, an optimal policy with index debt is more efficient than an optimal policy with nominal debt in a full commitment Ramsey equilibrium.*

Proof. It is enough to show that the household's value achieved in the Ramsey equilibrium is lower than the value achieved in the indexed debt equilibrium. That is,

$$\begin{aligned} & \left\{\log(c_0^*) - \alpha(c_0^* + g) + \beta(1-\beta)^{-1}\left[\log(c_1^*) - \alpha(c_1^* + g)\right]\right\} \\ < \; & \left\{(1-\beta)^{-1}\left[\log(c^*) - \alpha(c^* + g)\right]\right\} \end{aligned}$$

however, given that preferences are linear in labor and strictly concave in consumption, the above inequality will follow follows from Jensen's inequality provided that $c^* = (1-\beta)c_0^* + \beta c_1^*$. But this equality follows immediately from the definitions of c_0^*, c_1^* and c^*, i.e.,

$$\begin{aligned} & (1-\beta)c_0^* + \beta c_1^* \\ = \; & (1-\beta)\left[c_1^* - b_0\beta^{-1}\left(1 - \alpha c_1^*\right)\right] + \beta c_1^* \\ = \; & \left(\gamma - \left(\beta^{-1} - 1\right) b_0\right) \\ = \; & c^* \end{aligned}$$

■

5 Optimal policy with nominal debt and no commitment

When the government can not commit to a monetary policy, the price level is decided in each period according to a policy function $p_t = p(b_t, M_t)$. Households have rational expectations and take as given the government policy function. Furthermore, their expected future prices, $\bar{p}_t$, are formed in period $t-1$, and are the same function of the state of the economy at the beginning of period t, i.e. $\bar{p}_t = p(b_t, M_t)$.

Consequently, in this case, the nominal interest rate will satisfy the following version of Fisher's equation:

$$1 + i_t = \frac{\overline{p}_t}{\beta p_{t-1}} = \frac{p(b_t, M_t)}{\beta p_{t-1}} \tag{27}$$

The implementability conditions can be written as:

$$\gamma + b_{t+1} = c_t + b_t(1 + i_t)\frac{p_{t-1}}{p_t} \tag{28}$$

From (27), we have

$$\gamma + b_{t+1} = c_t + b_t\beta^{-1}\frac{\overline{p}_t}{p_t} \tag{29}$$

Since $c_t = M_t/p_t$ and $\overline{c}_t = M_t/\overline{p}_t$, (29) takes the form

$$\gamma + b_{t+1} = c_t + b_t\beta^{-1}\frac{c_t}{\overline{c}_t} \tag{30}$$

Notice that the problem reduces to a problem with a single state variable b_t. The problem of the government is then to find a policy, $c = C(b)$, that solves

$$V(b) = \text{Max}\{\log(c) - \alpha(c+g) + \beta V(b')\} \tag{31}$$

s.t.

$$b' \leq c + b\beta^{-1}\frac{c}{\overline{C}(b)} - \gamma \tag{32}$$

with $C(b) = \overline{C}(b)$.

Definition 5 *A recursive monetary equilibrium for this economy is a value function $V(b)$, policy functions $\{C^*(b), b^*(b)\}$, and a function $\overline{\mathrm{C}}(b)$ such that*

(i) Given $\overline{\mathrm{C}}(b)$, the value function, $V(b)$, and the policy, $\{C^*(b), b^*(b)\}$, solve the problem described by expressions (31) and (32), and

(ii) $C^*(b) = \overline{\mathrm{C}}(b)$

To characterize the recursive monetary equilibrium notice that the first order conditions of (31)-(32) are, first:

$$\frac{1}{c} - \alpha = -\beta V'(b') \left[1 + b\beta^{-1} \frac{1}{\overline{C}\,(b)} \right] \tag{33}$$

This condition equates the marginal gain of one unit of consumption to its marginal cost associated with higher debt resulting from the additional debt, as if it was indexed, and the additional debt resulting from a lower price in the current period.

Second, using the envelope theorem,

$$V'(b) = V'(b') \left[\frac{c}{\overline{C}\,(b)} - \frac{c}{\overline{C}\,(b)} \frac{b\overline{C}'\,(b)}{\overline{C}\,(b)} \right] \tag{34}$$

or, given that in equilibrium $c = \overline{C}\,(b)$,

$$V'(b) = V'(b')\left[1 - \epsilon_c(b)\right] \tag{35}$$

That is, one marginal increase of b_t has value $V'(b_t)$, but the corresponding increase of b_{t+1} has two components, the direct effect of increasing the stock of debt —as in the indexed debt case— and the indirect effect due to the fact that higher values of debt are associated with higher interest rates, $\epsilon_c(b) \leq 0$, given that with a higher stock of nominal debt the incentive to monetize the debt is higher and, along a rational expectations equilibrium path, these distortions are anticipated.

Using (33) we can also express the last condition as

$$\frac{\frac{1}{c} - \alpha}{\left[1 + b\beta^{-1}\frac{1}{c}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + b'\beta^{-1}\frac{1}{c'}\right]} \left[1 - \epsilon_c(b')\right] \tag{36}$$

or

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}\right]} \left[1 - \epsilon_c(b')\right] \tag{37}$$

Notice that (37) shows that, in contrast with the case of indexed debt (17), where marginal values of consumption are simply equated, in a recursive monetary equilibrium with nominal debt, marginal values of consumption must be discounted, since a higher consumption means a lower price and therefore higher outstanding and future debt. Recall that, such discounting already appeared in the full commitment case in the evaluation of the marginal value of consumption in period zero (25). In the economy without full commitment, prices are re-optimized in every period and, therefore, marginal values of consumption must be discounted, as long as there is an outstanding debt. Condition (37) also shows that discounted marginal values of consumption are additionally distorted by the incentive to increase the current price when the outstanding debt, b', is positive: $\left[1 - \epsilon_c(b_{t+1})\right]$.

In the previous section we have shown that the optimal policy with indexed debt is more efficient than the optimal policy with full commitment in a Ramsey equilibrium;

that the later is more efficient than the optimal policy in a non-commitment recursive monetary equilibrium follows from the standard argument of comparing commitment and non-commitment policies with the same instruments and rational expectations consistency conditions: the Ramsey planner can choose the recursive equilibrium allocation —satisfying the required consistency condition— but such allocation is dominated by the Ramsey equilibrium allocation.

6 Numerical solutions

To carry out our numerical example, we use the following values for the model economy parameters: $\alpha = 0.45$, $\beta = 0.98$, $b_0 = 0.17865$ and $g = 0.00822$. Notice that our period corresponds to a year and that we choose a very high level of nominal debt in relation to government expenditures. As we will see, results for lower values of debt can be obtained from our computations. The results we obtain for the time paths of the stocks of debt, the nominal interest rates, and the levels of consumption in the three cases analyzed are reported in Figures 1, 2, and 3, respectively.

As we have already mentioned, the optimal monetary policy with indexed debt is stationary, while this is not the case when debt is nominal (see Figures 1 and 2).

The stock of debt, with indexed debt, is time-invariant, while when debt is nominal, it is optimal to reduce the initial stock. Under full commitment this debt reduction is only carried out during the first period, and under no commitment, the stock of debt is depleted progressively until it is completely cancelled (see Figure 1).

We also find that the long-run interest rate with indexed debt is higher than with nominal debt, and that, in this case, the long-run interest rate under full commitment is higher than the one when there is no commitment (see Figure 2).

Finally, when we compare the welfare levels in the three different regimes, we find that the value of the optimal consumption path is highest in the economy with indexed debt and no taxes. In particular, this value is 0.012% smaller when there is nominal debt and full commitment, and 0.133% smaller when there is nominal debt and no commitment.

7 Additional taxes

In most advanced economies seignorage is a very marginal tax, and government liabilities are financed mostly with consumption and income taxes. This leads us to generalize our model economy to include consumption taxes, τ^c[4] In this economy, a fiscal policy is a sequence $\{\tau_t^c\}_{t=0}^{\infty}$. Since fiscal policy is not as instantaneous as monetary policy, we assume that tax rates are defined at the beginning of the period, before monetary policy is decided. That is, given a current state (b_t, M_t, t), fiscal policy sets $\tau_t^c = \tau(b_t, M_t, t)$ and monetary policy $p_t = p(b_t, M_t, \tau_t^c, t)$. We consider first the case that fiscal policy is any arbitrary policy that allows the monetary policy to optimally adapt to it with nonnegative interest rates. We consider then the case in which fiscal authorities can

[4]As it will become clear from our analysis, the introduction of other additional taxes will not change the nature of our main results.

fully commit to an optimal policy, showing that is part of such policy to fully finance government liabilites with taxes, constraining the monetary authority to set nominal interst rates to zero.

In an economy with taxes the household problem becomes

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha n_t \right] \tag{38}$$

subject to

$$M_{t+1} + B_{t+1} \leq M_t + B_t(1+i_t) - p_t(1+\tau_t^c)c_t + p_t n_t \tag{39}$$

together with $\lim_{T \longrightarrow \infty} \beta^T B_{T+1} = 0$, and

$$p_t(1+\tau_t^c)c_t \leq M_t \tag{40}$$

With this new tax, the equations, (7)–(9), that characterize the consumer's choice, become

$$\frac{u'(c_{t+1})}{\alpha} = (1+i_{t+1})(1+\tau_{t+1}^c) \tag{41}$$

$$1 + i_{t+1} = \beta^{-1} \frac{p_{t+1}}{p_t} \tag{42}$$

and

$$c_t = \frac{M_t}{p_t(1+\tau_t^c)} \tag{43}$$

These conditions must hold for every $t \geq 0$. Notice that equation (41) reflects the fact that agents make their plans based on expectations on both interest rates and taxes. Fisher's equation (42) does not change[5] and the cash-in-advance constraint (43) now includes consumption taxes.

The intertemporal government budget constraint in this economy is now:

$$p_t g + M_t^g + B_t^g(1+i_t) \leq p_t \tau_t^c c_t + M_{t+1}^g + B_{t+1}^g \tag{44}$$

and the feasibility condition (4) remains the same.

We can consider the general case where fiscal and monetary policy rules take the form $\tau_t^c = \tau(b_t, M_t, t)$ and $p_t = p(b_t, M_t, \tau_t^c, t)$ and agents make their plans in period $t-1$ based on expectations $\overline{p}_t = p(b_t, M_t, \tau_t^c, t)$. In particular, for all $t \geq 0$,

$$1 + i_t = \frac{\overline{p}_t}{\beta p_{t-1}} = \frac{p(b_t, M_t, \tau_t^c, t)}{\beta p_{t-1}} \tag{45}$$

[5] It should be noticed that with labor taxes, $\tau_t^n = \tau^n(b_t, M_t, t)$, equation (42) will change to $1+i_{t+1} = \beta^{-1} \frac{p_{t+1}(1-\tau_{t+1}^n)}{p_t(1-\tau_t^n)}$ and therefore will be affected by fiscal policy. Regarding our results, this will only matter in the case that there is indexed debt and a non fully commited fiscal authority.

and planned consumption $\bar{c}_t$ satisfies (41) and for $i_t > 0$, the cash-in-advance constraint $\bar{c}_t = \frac{M_t}{\overline{p}_t(1+\tau_t^c)}$.In particular, households have rational expectations in the sense that, given (45), only their price expectations $\overline{p}_t$ matter, since $(1+i_t)(1+\tau_t^c) = \frac{1}{\alpha \bar{c}_t} = \frac{M_t}{\alpha \overline{p}_t(1+\tau_{t+1}^c)}$.

With this general formulation, indexed debt imposes the restriction that $p_t = \overline{p}_t$, for all $t \geq 0$; with nominal debt, full commitment to monetary policy imposes the restriction that $p_t = \overline{p}_t$, for all $t \geq 1$, while $p_0 = \overline{p}_0$ only has to be satisfied in equilibrium, and with nominal debt and no commitment to monetary policy $p_t = \overline{p}_t$, for all $t \geq 0$ only has to be satisfied in equilibrium.

In this economy with taxes the general implementability condition is

$$u'\left(c_{t+1}\right) c_{t+1} \frac{\beta}{\alpha} + b_{t+1} = b_t \beta^{-1} \frac{c_t}{\bar{c}_t} + c_t + g \tag{46}$$

which in the log case simplifies —as in (30)— to:

$$\gamma + b_{t+1} - c_t - b_t \beta^{-1} \frac{c_t}{\bar{c}_t} = 0 \tag{47}$$

With the additional restrictions that: *i)* if debt is indexed $\frac{c_t}{\bar{c}_t} = 1$,for all $t \geq 0$, and *ii)* if debt is nominal and there is full commitment to monetary policy $\frac{c_0}{\bar{c}_0} = 1$.

Consequently, for any level of monetary commitment the monetary authority faces the same problem with consumption taxes than the one faced when there was only seignorage. As a result, the allocations that obtain for the various monetary commitment technologies are exactly the same as those that we have discussed before. More specifically,

Consumption taxes and indexed debt. In this case policies are stationary and we obtain the stationary equilibrium allocation $c^* = \gamma - \left(\beta^{-1} - 1\right) b_0$. Notice, however that interest rates $i_t = i(b_t, \tau_t^c)$ are set as to satisfy

$$\frac{u'(c^*)}{\alpha} = (1 + i(b_t, \tau_t^c))(1 + \tau_t^c)$$

That is, nominal interest rates are given by:

$$i_t^{I_\tau} = i^I(b_0, \tau_t^c) = \left[\alpha\left(\gamma - \left(\beta^{-1} - 1\right) b_0\right)\left(1 + \tau_t^c\right)\right]^{-1} - 1$$

Where, as it has been said, we assume that $i^I(b_0, \tau_t^c) \geq 0$. The evolution of prices and money balances are recursively given by: $p_t = \frac{M_t}{c^*(1+\tau_t^c)}$ and $M_{t+1} = \frac{\beta}{\alpha} p_t$

Consumption taxes, nominal debt and full commitment to monetary policy. In this case we obtain the Ramsey equilibrium allocation $c_0^* = c_1^* - \beta^{-1} b_0 (1 - \alpha c_1^*)$ and $c_1^* = \gamma\left[1 + \alpha\left(\beta^{-1} - 1\right) b_0\right]^{-1}$, resulting in interest rates $i_t^{R_\tau} = i^R(b_t, \tau_t^c, t) = [\alpha c_1^*(1 + \tau_t^c)]^{-1} - 1$, for all $t \geq 1$, and $i_0^{R_\tau} == i^R(b_0, \tau_0^c, 0) = [\alpha c_0^*(1 + \tau_0^c)]^{-1} - 1$, with $i^R(b_t, \tau_t^c, t) \geq 0$. The intertemporal condition between interest rates in period zero —corresponding to (26)— is given by:

$$\frac{(1 + i_0^{R_\tau})(1 + \tau_0^c) - 1}{1 + (1 + i_0^{R_\tau})(1 + \tau_0^c)\frac{B_0}{M_0}} = (1 + i_1^{R_\tau})(1 + \tau_1^c) - 1 \tag{48}$$

Consumption taxes, nominal debt and no commitment to monetary policy. In this case policies are also stationary and we obtain the recursive equilibrium allocation described in Section 5. In particular, the intertemporal condition (37) now takes the form:

$$\frac{\frac{1}{c}-\alpha}{\left[1+(1+i)\,(1+\tau^{c})\frac{B}{M}\right]}=\frac{\frac{1}{c'}-\alpha}{\left[1+(1+i')\,(1+\tau^{c\prime})\frac{B'}{M'}\right]}\left[1-\epsilon_{c}(b')\right] \tag{49}$$

It follows that in the economy with nominal debt and no commitment to monetary policy, the path of depletion of the corresponding stock of real debt is the same as the one characterized in Section 5 (and computed in Section 6) even if, supposedly, tax revenues would allow for a faster depletion rate.

7.1 Optimal fiscal policy with commitment

As it can be seen, in all three regimes, just considered, there is an indeterminacy in how all equilibrium allocations are supported since only the 'effective' nominal return $(1+i)\,(1+\tau^{c})$ matters. In particular, it is always possible to set taxes in a way that the resulting monetary policy follows the Friedman rule of zero nominal interest rates, however in our economy there is no efficiency gain from following such a rule[6] More precisely, as long as, monetary responses to realized fiscal policies result in non negative interest rates, fiscal policy is not effective in this economy since the monetary authority can adapt to it as to attain the same outcome as in the economy without taxes, and such adaptation is its optimal policy reply. But this may not be the scenario in which monetary authorities operate.

To see this, suppose debt is nominal and there is full commitment to monetary policy. Let fiscal authorities set $\tau(b_t, M_t, t) = \tau^*(b_0)$, where $\tau^*(b_0)$ corresponds to the tax rate that fully finances government liabilities in the indexed debt allocation. That is,

$$(1+\tau^*(b_0)) = \frac{u'(\gamma - \left(\beta^{-1}-1\right)b_0)}{\alpha} = \frac{u'(c^*)}{\alpha}$$

If the monetary authority tries to monetarize part of the existing nominal debt and use the resulting revenues to increase future consumption –say, maintaing a constant c_1– then, it must be that $c_0 < c^* < c_1$. But such allocation requires that $(1+i_0^{R_\tau})(1+\tau^*(b_0)) > (1+\tau^*(b_0)) > (1+i_1^{R_\tau})(1+\tau^*(b_0))$ which implies $i_0^{R_\tau} > 0 > i_1^{R_\tau}$. But negative interest rates can not be an equilibrium in this economy since then households would like to borrow unboundedly. But then, given that it is not possible to raise future consumption with negative taxes, there is no gain in partially monetarizing the nominal debt in period zero. It follows that, if fiscal authorities maximize (2), they will set $\tau(b_t, M_t, t) = \tau^*(b_0)$. The same argument applies when there is no commitment to monetary policy. In summary,

[6]This may not be true in a more general model. For example, it is not be true if in our model we introduce a distinction between cash and credit goods. In this case, the Friedman rule eliminates the distortion between cash and credit goods, nevertheless the distortions introduced by the presence of a positive stock of nominal debt remain, as in the economy with only cash goods.

Proposition 6 *Assume that fiscal authorities maximize the welfare of the representative household and can commit to their policies.The equilibrium allocation is the optimal equilibrium allocation with indexed debt even if debt is nominal and independently of the degree of commitment of monetary authorities.*

8 Concluding comments

This paper discusses the different ways in which nominal and real debt affect the sequential choice of optimal monetary policy in a general equilibrium monetary model where the costs of an unanticipated inflation arise from a cash-in-advance constraint. In our environment, as in Nicolini (1998), when the utility function is logaritmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a Ramsey government.

In contrast, when the initial stock of government debt is nominally denominated, a time inconsistency problem arises for the same specification of preferences. In this case, the government is tempted to inflate away its nominal liabilities. When the government cannot commit to its planned policies, the optimal sequential policy consists in depleting the outstanding stock of debt progressively, so that it converges asymptotically to zero. The optimal nominal interest rates in this case are also decreasing, and they converge to zero as long as there is no need to use seignorage to finance government expenditures different from debt servicing. Consequently, the optimal monetary policy in this economy coincides in the long term with the one that obtains in an economy which has no outstanding debt and from which these time-inconsistency distortions are obviously absent.

Such equilibrium path is not chosen when the initial stock of government debt is nominally denominated and the government can commit fully to its planned policies. In this case, it is optimal to increase the inflation tax in the first period and to keep a lower and constant inflation tax for the rest of the future.

We show that in the rational expectations equilibria of our economies there are no surprise inflations and that for a given initial level of outstanding debt, the most efficient equilibrium is the one that obtains when debt is indexed, the equilibrium with nominal debt and full commitment comes second, and the equilibrium with nominal debt and no commitment is the less efficient of the three. This result highlights the sense in which nominal debt is indeed a burden on optimal monetary policy.

It should be noted that the source of the inefficiencies and of the monetary policy distortions discussed in this paper is not the desire to run a soft budgetary policy that increases the debt liabilities of the government. Every policy discussed in this article is an optimal policy, subject to the appropriate institutional and commitment constraints, and it is implemented by a benevolent and far-sighted government who does not face either uncertainty or the need for public investments and who would, therefore, prefer to reduce debt liabilities. The source of the inefficiencies is the distortion created by the lack of commitment when there is an outstanding stock of nominal debt. Therefore, our results highlight the need to implement policy and institutional arrangements that either guarantee high commitment levels or that reduce the allowed levels of nominal debt. However, they also show that a constraint on deficits may be ineffective to reduce the distortions created by nominal debt since they are independent of the size of the deficits.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that monetary choices are made after the tax rates have been decided, we show that the equilibrium allocations that obtain when we introduce consumption taxes are the same as those that obtain when there is only seignorage, provided there is enough seignorage as to allow for an optimal monetary policy with non negative interest rates. However, if there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, choose to fully finance government liabilities and the resulting monetary policy is the Friedman rule of zero nominal interest rates and, as a result, the efficient equilibrium that obtains in the economy with index debt.

In summary, we show that fiscal discipline may be needed to achieve efficiency and price stability, even when monetary authorities pursue optimal policies. However, our analysis shows that fiscal discipline applies to the level of the debt and not to the level of the deficit or, alternatively, to the issuing of indexed debt. In contrast, for reasons beyond the scope of this paper, the use of nominal government debt and of 'constraints on fiscal deficits' (as in the EU Growth and Stability Pact) is widespread.

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9 Appendix: Computation

In order to compute the recursive monetary equilibrium defined in Section 5, we must solve the following dynamic program:

$$V\left(b\right)=\max_{c,b'}\{\log(c)-\alpha\left(c+g\right)+\beta V\left(b'\right)\} \tag{50}$$

s.t.

$$b'\leq c+b\beta^{-1}\frac{c}{\overline{C}\left(b\right)}+g-\frac{\beta}{\alpha} \tag{51}$$

for a given $\overline{C}\left(b\right)$.

However, computational cosiderations lead us to solve the following transformed problem:

$$V\left(x\right)=\max_{c,x'}\{\log(c)-\alpha\left(c+g\right)+\beta V\left(x'\right)\} \tag{52}$$

s.t.

$$\beta x'\hat{C}(x')\leq c(1+x)+g-\frac{\beta}{\alpha} \tag{53}$$

for a given $\hat{C}\left(b\right)$ and where $x=b/\beta\hat{C}(x)$.

In order to solve this problem we use the following algorithm:

- Step 1: Define a discrete grid on x
- Step 2: Define a decreasing discrete function $\hat{C}_0(x)$
- Step 3: Iterate on the Bellman operator described in equations (32) and (53) until we find the converged $V^*(x)$, $x'^*(x)$,$c^*(x)$
- Step 4: If $c^*(x)=\hat{C}_0(x)$, we are done. Else, let $\hat{C}_0(b)=c^*(b)$ and goto Step 3.

Finally, to recover the policy functions of the original problem we undo the transofrmation as follows: From $x=b/\beta\hat{C}(x)$, we obtain that $\hat{b}(x)=\beta x\hat{C}(x)$, which can be computed directly from the solution to the transformed problem described above. Next we invert $\hat{b}(x)$ and we obtain $x=\hat{b}^{-1}(b)$. Finally, we use this expression to obtain $C(b)=\hat{C}[\hat{b}^{-1}(b)]$ and $b'(b)=\hat{b}\{x'[\hat{b}^{-1}(b)]\}$.

Figure 1: The optimal stocks of indexed debt and of nominal debt with full commitment and with no commitment

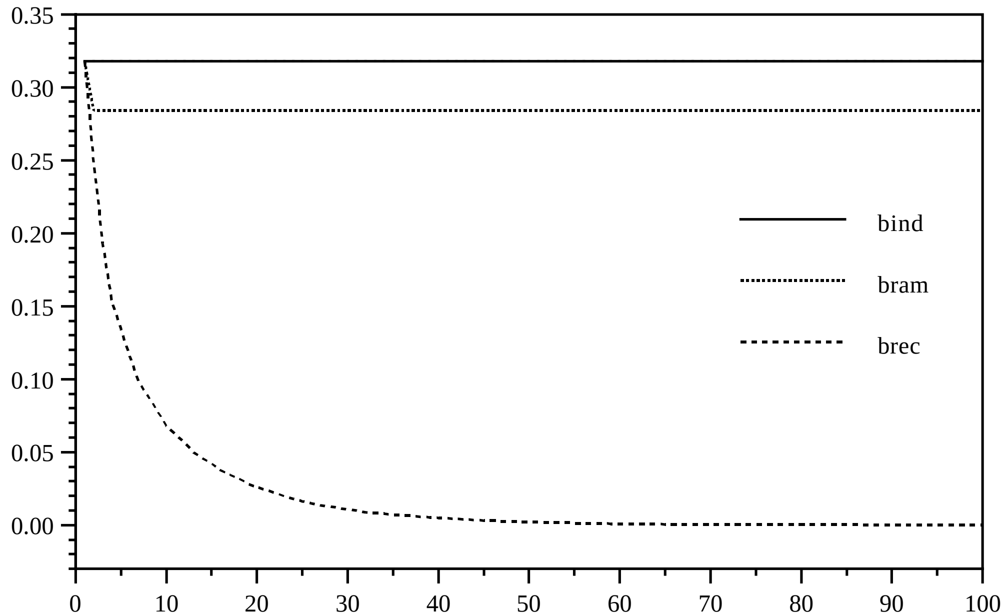

Figure 2: The optimal paths of nominal interest rates with indexed debt and with nominal debt with full commitment and with no commitment

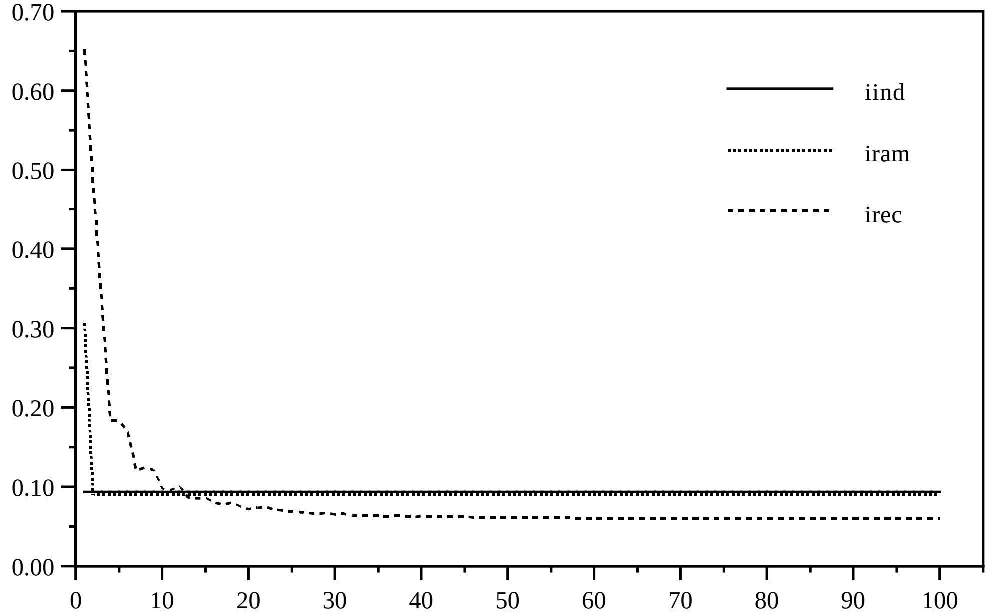

Figure 3: The optimal paths of consumption with indexed debt and with nominal debt with full commitment and with no commitment