Constitutional Rules*

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This version: September 2004
First Version: January 2004

Abstract

This paper proposes a normative theory of constitutional rules. The first-best cannot be achieved whenever constitutional rules cannot be made contingent on information about the costs and benefits of policy reforms. We characterize and welfare rank four classes of second best constitutions: constitutions that specify one rule for all types of decisions; constitutions that provide incentives for information about costs and benefits to be revealed; constitutions that allow for vetoes from interested parties and constitutions that specify different rules for different policy areas. In addition, we provide conditions for the existence of amendment rules that allow for changes to the original constitutional rules after the constitutional stage. Finally, we provide a new rationale for the existence of checks and balances.

Keywords: Constitutions, social contracts, majority rules, bill of righs, vetoes, referenda, amendments, checks and balances.

JEL Classification: H10, H11.

*We would like to thank Dhammika Dharmapala, Jordi Blanes i Vidal, Miltos Makris and participants at the June 2004 Political Economy Workshop in Birmingham, the 2004 North American Summer Meeting of the Econometrics Society; the 2004 Game Theory World Congress and the 15th Summer Festival on Game Theory at Stony Brook for helpful comments and suggestions. The usual qualifier applies.

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1. Introduction

A constitution is a social contract that sets out the rules that govern the way a society makes collective decisions. Despite differences in the details, the world's constitutions share a set of common features. All constitutions contain rules that govern how day-to-day decisions are made, with the most common decision rule being the simple majority rule. The vast majority of constitutions have a formal amendment process that allows the constitution itself to be changed.[1] Many constitutions single out specific policy areas that are governed by special rules and procedures. A leading example is bills of rights that make specific rights imperative. The requirement that a referendum is held in relation to certain types of decisions is another common feature of real world constitutions. Finally, most constitutions grant certain bodies or individuals the power to veto decisions or introduce other types of checks and balances.

This paper proposes a normative theory of constitutional design that can explain why constitutions embody a range of different procedures and rules. We are interested in understanding the multidimensionality or complexity of constitutional rules. In this regard we depart from most of the recent theoretical work on the subject which focus on a single rule or procedure at the time.[2] Our goal is to demonstrate that many of the rules that we observe in actual constitutions emerge endogenously in a simple theoretical framework where constitutional decisions are made from behind the veil of ignorance.

We consider a society with a continuum of individuals. These individuals must choose between two alternatives A and B. Alternative B – the status quo – yields the same level of welfare to all, while alternative A – a reform – creates winners and losers. The magnitude of the loss depends on the precise nature of the reform and in some cases – when the reform, for example, violates certain fundamental rights – the loss is particularly large. At the time when the constitutional rules are laid down, individuals do not yet know whether they will gain or lose from the reform, nor do they know how large the potential losses are going to be. Thus, the constitution is designed from the original position, behind a veil of ignorance.

The premise of our analysis is that societies have, repeatedly, to make decisions about policies that create winners and losers and that different policies are associated with different cost-benefit profiles. The first-best constitutional response is to introduce a separate decision rule for decisions with high and low costs, i.e., cost-dependent majority rules. This, however, requires that costs can be observed and can be verified before the decision is made. For some policies, the costs are known fairly well in advance, while for others it

[1] According to [24] (page 80), less than 4% of the world's constitutions lack such a provision. On the other hand, [14] shows that all the current constitutions in the 50 U.S. states and in a sample of countries around the world have been amended at some time or another.

[2] We discuss the literature in the next section.

is impossible to known the precise consequences until after implementation. In both cases, the welfare consequences of a reform cannot be verified objectively, and the constitutional rules cannot be contingent on the utility that individuals derive from the policy reforms adopted. We identify this as the fundamental problem that constitutional designers must address.

In the first part of the paper, we consider alternative constitutional responses to this fundamental problem. We argue that these correspond closely to solutions found in actual constitutions and identify the circumstances under which each of the alternatives is preferred to the others by the constitutional designer. The first and simplest alternative – a majority rule (MR) constitution – is to apply the same majority rule to all types of decisions. The problem with this alternative is that the constitutional designer wants to make it difficult to pass policy reforms with high costs, but in order to do so, he has to make it difficult to pass all policy reforms, including those with low costs. If some reforms, e.g., because they infringe with basic rights or property, impose very high costs on some individuals, the optimal single rule may be so strict that it effectively blocks all decision making – something which is clearly undesirable in itself. The constitutional designer may, therefore, explore other alternatives.

In those cases where the costs are known before a reform is actually implemented, the constitutional designer may choose to design rules that attempt to elicit this information truthfully. We consider two constitutional mechanisms that can achieve this: an incentive scheme (IS) constitution and a veto rule (VT) constitution. Both of these constitutions embody two majority rules: a default rule and an alternative that can only be applied under certain circumstances specified in the constitution.

The IS constitution allows individuals to request that the alternative rule is used, but only if they pay a cost, λ. This switching cost is chosen by the constitutional designer to insure that individuals would only make such a request if the reform involves truly high costs. This constitution, then, effectively elicits the actual costs associated with policy A and allows society to employ second-best cost-dependent majority rules. The down-side is that the switching is a deadweight loss and is incurred every time the need for a switch of rules arises. We show that the IS constitution is preferable to the MR constitution only in cases where reforms with high costs rarely come up for a vote, but when they occasionally do come up, they involve extremely high costs. We argue that the IS constitution is found in actual constitutions in the form of referenda or citizens' initiatives.

The VT constitution uses a different procedure to elicit cost information. Societies, typically, have to consider several policy reforms, say, one each period for a certain number of periods. The constitutional designer can take advantage of this. The basic idea is simple: each period, the constitution allows individuals to request that the alternative rule is used instead of the default rule at no cost, but they can only do so a specific number of times.

We interpret a request for a change of rule as a veto. By rationing the number of vetoes, the constitutional designer can provide incentives for truthful revelation of the actual costs. The point is that individuals, knowing that there are more decisions to be made than vetoes allowed by the constitution, are more likely to veto a policy reform with high costs than one with low costs.[3] In the limit when the number of decisions to be made is infinite, the VT constitution can implement the first-best. For a finite number of decisions, we show the VT constitution cannot be worse than the MR constitution and that it is at its best relative to the IS constitution when the cost difference between reforms with low and high costs is small and when both types of reforms come up fairly frequently. We argue that the VT constitution can be found in actual constitutions in the form of veto powers granted to certain institutions (such as a president or a minority in the legislature).

When the costs of a reform cannot be known until after it has been implemented, the constitutional designer can employ neither the IS nor the VT constitution, and the only alternative may appear to be the MR constitution. We argue, however, that it is often possible to define classes of reforms – policy areas – that can serve as proxies for the underlying cost of reforms that fall within those areas. Importantly, we assume that it is possible, at a cost, to establish objectively if a particular policy reform belongs to one areas or the other. This allows the constitutional designer to design policy area specific majority rules as an alternative to the single rule. We show that it is beneficial to do so when the cost of allocating decisions to the correct policy area is low and when policy areas are informative about the cost of reforms within those areas. We argue that policy area specific rules are often present in actual constitutions in the form of bills of rights or as special procedures such as those designed for dealing with ratification of international treaties or expropriation of property.

In the second part of the paper, we consider the important question of constitutional change. When all constitutional rules are designed behind the veil of ignorance, there is no role for constitutional change. A serious theory of constitutional change must, therefore, take on board the fact that amendment rules are meta-rules that specify how individuals can change the pre-determined rules of the original constitution after the veil of ignorance has been lifted. We consider situations where there exist non-verifiable, external threats to the welfare of certain individuals in society. When this threat – call it a crisis – varies with circumstances and when the original constitution cannot be made contingent on these circumstances, constitutional designers may want to introduce an amendment rule. This rule allows for constitutional changes to be implemented at a later stage when circumstances are more clear, but only if sufficiently many individuals support such amendments. We show that amendment rules provide the flexibility necessary to adapt the constitution to

[3]See [11] for a general analysis of mechanisms that link several decisions to generate revelation.

changing circumstances, but this flexibility comes at the cost that it must allocate decision making to a specific group of individuals rather than to someone who takes society's interest as a whole into account. Therefore, amendment rules are optimal only, from the original position's perspective, when it is sufficiently likely that the threat will be carried out. In other cases, when this threat is not so significant, we show that mechanisms that resemble the procedures for declaring a state of emergency are optimal.

The third part of the paper considers checks and balances, understood as different rules for different groups of individuals (as distinguished by wealth levels, geographical location etc.). The advantage of having checks and balances is that policy reforms cannot pass without sufficient support from all groups. This is in contrast to universal rules which allow support from one group to compensate for opposition from the other, thereby allowing policy reforms to pass if enough individual in society at large are in favor. We show that checks and balances are desirable in heterogeneous societies where some groups are more likely to suffer disproportionately from changes to the status quo than others.

We have chosen a normative approach to constitutional design for a variety of reasons. Firstly, this assumption maps nicely with the notion that constitutional designers are "founding fathers". If the individuals who design constitutions care sufficiently about the future, they will be careful not to design a constitution that makes it either too easy or too difficult to select reforms because while they might, for example, gain from today's reforms, they might lose from tomorrow's reforms. Secondly, any "positive" theory of constitutional design crucially relies on the details of the bargaining process between the parties. In contrast, with a normative approach, we are able to generate results which do not rely on anything else other than the veil of ignorance assumption.[4] Finally, a normative approach provides a benchmark against which actual outcomes might be measured and there exists ample anecdotal evidence that many of the constitutional rules that arise in our setup can be found in actual constitutions.

It is clear that our setup ignores many important features of the collective decision problem that societies faces in reality. In particular, in our model, we ignore all the agency problems that arise when decision making power is delegated to politicians. In addition, our policy space is very simple: in effect we have only two alternatives chosen by nature, which differ by the loss inflicted upon losers. This means that agenda setting is not an issue and that Condorcet cycles cannot arise (see [1]). On the other hand, all these simplifications allows use to isolate what we consider to be the fundamental constitutional problem: individuals that disagree about what should be done but need to make decisions. Understanding why a simple decision procedure such as the majority rule is, typically, not sufficient to enable societies to resolve this conflict adequately is of theoretical as well as

[4] See [25] for a survey of positive constitutional economics.

of practical importance.

The paper is organized as follows. In Section 2, we provide a short literature review. In Section 3, we introduce our model. In Section 4, we consider constitutional design under the assumption that the costs are observed before the decision is made. In Section 5, we consider constitutional design when the costs are not observed until after decisions are made and introduce the notion of a policy area. In Section 6, we present a new theory of constitutional amendment. In Section 7, we study the conditions under which checks and balances is optimal. Section 8 concludes. The appendix at the end contains many of the proofs.

2. Related Literature

In recent years, there has been a renewed interest in the fundamental questions related to constitutional design.[5] In this section, we offer a brief discuss of this literature and relate our analysis to what has gone on before.

- A number of recent papers view constitutions as incomplete social contracts.[6] Within this framework, [1] show that the optimal choice of a majority rule from behind the veil of ignorance is determined by a trade-off between two considerations. On the one hand, the desire to limit excessive ex-post redistribution whereby the majority expropriates the minority suggests that the majority rule should be strict. On the other hand, it is desirable to allow enough flexibility to circumvent ex-post vested interests that attempt to block socially desirable reforms. This suggests that the majority rules should be lax.[7] [15] considers a similar problem in the context of international organization but address the issue of self-enforcement and show that under certain condition unanimity is the optimal majority rule. [2] propose a related theory of endogenous political institutions but focus on rules that contain the power of political leaders. They show that the optimal degree of "insulation" measured as the share of votes needed to block legislation (or the size of the supermajority needed to pass legislation) is determined by a trade-off between allowing the political leader enough leeway to rule and restricting the scope for misuse of power. Our approach shares with these papers the assumption that constitutional choices are made from behind the veil of ignorance, yet our goal is different. We want to understand when and why particular constitutional rules emerge. Thus, rather than analyzing how

[5] The classical work in the area is [5]

[6] [7] presents an overview of this approach. See [13] for an exposition of the alternative view that considers constitutions as complete contracts and argues that the need for constitutions arises from information asymmetries. For a theoretical defence of the incomplete contract approach, see [10].

[7] This approach has been further developed by [8].

the strictness of one particular (decision) rule varies with changes in the economic environment, we are interested in the broader question of how the set of optimal rules itself varies with the environment.[8] Although our starting point is that constitutions are incomplete contracts in the sense that they do not necessarily provide a full state-contingent plan for all future events, we stress that appropriate responses to certain future events, in particular those that relate to fundamental rights, can be specified in the constitution. This is major departure from the previous work, but one, we argue, that provides valuable insights into the complexity of real world constitutions.

- Some recent papers have argued that constitutions are not written behind the veil of ignorance but by individuals who know their position in society. [16], for example, study a situation where the decision rule used to govern future decisions is itself decided by the majority rule. They find that supermajority rules emerge in an overlapping generations framework where the young can decide on the size of the supermajority that is going to be used to make decisions when they become old. Assuming that most public policies introduce immediate costs while benefits arrive later, older voters suffer more from reforms than young voters, and this provides an incentive for young voters to choose a strict rule that is going to apply when they are old. [23] analyze how economic factors (and in particular redistributive concerns) influence the choice of a majoritarian system versus a consensual system. [3] also study the endogenous choice of majority rule in a positive framework and derive conditions under which voting rules are self-sustaining, that is constitutions that are likely to endure. In particular, they provide a rationale for the existence of amendment rules since the flexibility they afford allows for more stable constitutions. We provide a rationale for amendment rules that emphasizes their flexibility as well, but argue that this comes at the cost of partiality and study how the trade-off between the two determines the nature of constitutional change.

- We show that checks and balances understood as different decision rules applied to different groups of individuals (e.g., as in bicameral systems) can be optimal if there is enough heterogeneity in the population. The function of checks and balances in our framework is very different from that of [20]. They focus on situations where checks and balances understood as separating decision making power between politicians can reduce agency problems. In our framework, checks and balances provide protection to groups of voters at risk of experiencing particularly large loses. This is so because checks and balances in our framework prevents support for alternative A to be transferred between groups.

[8] [1] take steps in this direction by analyzing when it would be desirable to introduction various minority protection rules such as equal tax rates and tax limits.

- Finally, [9] studies a formal model of constitutional change but assumes that changes are requested and determines the optimal amendment rules as a function of the pressures from other forms of change (interpretative interventions through legislation and the courts or, at the other extreme, the possibility of a complete constitutional crisis).[9] Our approach here is entirely different as we endogenize the possibility of change through the choice of the constitution itself.

3. The Basic Setup

We consider a society that must choose between two policies, $x \in \{A, B\}$. Policy B is the status quo and policy A is an alternative to the status quo. Policy A should be interpreted as a reform as in [2]. The society is populated by a continuum of individuals indexed by i. The population is partitioned into two disjoint groups, denoted W and W^C. The utility function of individual i depends on group affiliation and on the policy chosen by society, and can be written as:

$$u(i,x,c) = \begin{cases} w & if \quad x = A \text{ and } i \in W \\ -y & if \quad x = A,\ i \in W^C \text{ and } c = \underline{c} \\ -z & if \quad x = A,\ i \in W^C \text{ and } c = \overline{c} \\ 0 & if \quad x = B \end{cases}$$

where $w > 0$ and $z > y > 0$. The interpretation is as follows. If policy B is chosen, the status quo is preserved and all individuals obtain zero utility. If, on the other hand, policy A is chosen, those individuals who belongs to the set W (the "winners") gain utility w, while those who belongs to the set W^C (the "losers") experience a loss. How large this loss is depends on the nature of the policy reform. We assume that there exists two possible alternatives to the status quo. Which of these obtains is determined by the realization of the random variable c. For some policy reforms ($c = \overline{c}$), the loss is larger than for others ($c = \underline{c}$). The probability that the costs are low (y) is η and the probability that they are high (z) is $1-\eta$. What is important is that the precise nature of policy A cannot be known until after c has been realized. The timing of events can be summarized by the following time line:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.

2. Nature selects p. p is not observed by individuals.

3. Given p, individuals are partitioned in the two sets W and W^C by Nature. Individuals know to which set they belong, but this information is not verifiable.

[9] See also [19] and [4].

4. Individuals vote for or against policy A. The vote result is observed by everyone and is verifiable.

5. The policy outcome is determined according to the constitutional rules laid down in 1.

Individuals select a constitution (a mechanism) from behind a veil of ignorance without knowing neither if they will gain from policy A or not nor the precise nature of policy A (i.e., the realization of c). Once the constitution has been designed in stage 1, nature determines who the winners and losers are. This is done by first selecting a value p from a cumulative distribution function F with support on the unit interval and strictly positive density f (stage 2), and then, for each individual i determining whether $i \in W$ or not by a sequence of independent draws from a Bernoulli distribution with $p = \Pr(i \in W)$ in stage 3. With a continuum of individuals, p also represents the fraction of individuals who favor policy A over the status quo. In stage 4, voting takes place. Individuals vote in a state of aggregate uncertainty: they know whether they are winners or losers, but they do not know how many winners or losers there are until after the vote has taken place. We assume that an independent court can verify the number of votes in favor of policy A, that voting is sincere[10], and that an independent judiciary guarantees that the rules prescribed by the constitution will be enforced and the policy outcome is determined accordingly in stage 5.

The time line does not specify when the c associated with policy A becomes known. We consider two cases – both of which have considerable empirical relevance. For some policies, the welfare consequences are known ex ante, that is, before they are implemented, but after the is designed. In this case, c is observed by individuals at stage 3. Although the costs are observed, they cannot, in general, be verified by a court. For other policies, the welfare consequences are known only ex post, that is, after they are implemented. In this case, c is realized after voting takes place in stage 5. In both cases, the constitutional designer has to find ways to circumvent the fact that he cannot make the constitutional rules directly dependent on a key determinant of the desirability of reform (the realization of c). We begin our inquiry into this by considering the case in which the costs are observed ex ante and return to the case where they are observed only ex post in section 5.

[10]This assumption is needed only because our model assumes a continuum of voters. As a consequence, no individual voter is ever pivotal and any voting strategy, therefore, constitutes equilibrium behavior, including non-sincere ones. However, for any positive probability of being pivotal, sincere voting would obtain in our setting because there is only two alternatives. We assume throughout that voting is sincere.

4. Optimal Constitutions When c is Observed Ex Ante

In this section, we characterize optimal constitutions under the assumption that the costs become known to individuals (in stage 3) before the decision to adopt policy A or not is made. As a benchmark, we analyze the situation in which c is verifiable. We demonstrate that the first-best is attainable and can be implemented by cost-dependent majority rules. In the realistic case where c is not verifiable, the first-best cannot be attained. We consider three alternative solutions – a simple majority rule, an incentive scheme, and vetoes – that in different ways attempt to address this fact. We argue that these alternatives employ constitutional rules that correspond closely to solutions found in actual constitutions and identify the circumstances under which each of the alternatives is preferred to the others by the constitutional designer.

4.1. Constitutions with Verifiable c

Under the assumption that costs c can be verified by the courts, it is feasible to write constitutions that depend directly on c as well as p. We can define a cost-dependent constitution as follows:

Definition 1 *A cost-dependent (CD) constitution is a pair of majority rules* (m, n) *with* $m \in [0, 1]$ *and* $n \in [0, 1]$ *such that*[11]

$$x = \begin{cases} A \quad iff \quad ((p \geq m) \wedge (c = \underline{c})) \vee ((p \geq n) \wedge (c = \overline{c})) \\ B \quad \quad \quad \quad \quad \quad \quad \quad otherwise \end{cases}$$

The CD constitution employs two cost-dependent majority rules and policy reforms with low costs can pass with a m-majority, while policy reforms with high costs can pass with a n-majority. We are interested in the constitutional mechanism that achieves the first best outcome, that is, the constitutional mechanism that maximizes expected utility from the perspective of the original position. The following proposition shows that the first best constitution is, in fact, a CD constitution:

Proposition 1 *The CD constitution with majority rules*

$$m^*_{FB} = \frac{y}{y+w} \text{ and } n^*_{FB} = \frac{z}{w+z}$$

is the first-best constitution. Moreover, $m^*_{FB} < n^*_{FB}$.

[11] Since we assume that voting is sincere, we can replace, without loss of generality, the measure of votes in favor of policy A with p, the measure of individuals in favor of policy A.

Proof. We can write expected utility for an individual behind the veil of ignorance as

$$\eta \int_0^1 \mu\left(\overline{c}, p\right) u_{\underline{c}}\left(p\right) dF\left(p\right) + (1-\eta) \int_0^1 \mu\left(\underline{c}, p\right) u_{\overline{c}}\left(p\right) dF\left(p\right)$$

where $\mu\left(c, p\right) = \Pr\left(x = A | c, p\right)$ is the probability that policy A will pass under the given mechanism, conditional on c and p and where

$$\begin{aligned} u_{\underline{c}}\left(p\right) &= pw - (1-p)y; \\ u_{\overline{c}}(p) &= pw - (1-p)z. \end{aligned}$$

Expected utility is maximized by any mechanism that satisfies the following conditions:

$$\begin{aligned} \mu\left(\underline{c}, p\right) &= \begin{cases} 1 & if \quad u_{\underline{c}}\left(p\right) \geq 0 \Leftrightarrow p \geq \frac{y}{y+w} \\ 0 & \qquad otherwise \end{cases} \\ \mu\left(\overline{c}, p\right) &= \begin{cases} 1 & if \quad u_{\overline{c}}(p) \geq 0 \Leftrightarrow p \geq \frac{z}{z+w} \\ 0 & \qquad otherwise \end{cases} . \end{aligned}$$

Given that individuals vote sincerely, the CD constitution with rules (m^*_{FB}, n^*_{FB}) implements the first-best. For $z > y$, $m^*_{FB} < n^*_{FB}$. ¤

The constitutional designer would ideally like to employ a lenient majority rule m^*_{FB} for reforms involving low costs and a strict majority rule n^*_{FB} for reforms involving high costs. We notice that these rules are independent of the distribution function F. Intuitively, this is because the constitution sets out the conditions under which policy A is chosen and these must be independent of the probability that such conditions obtain.

Cost-dependent constitutions are not feasible when c is unverifiable: whatever the true realization of c is, winners always have an incentive to claim that $c = \underline{c}$ and that the decision should be made with the lenient rule (m^*_{FB}), while the losers have the opposite incentive. Thus, when c is observable, but not verifiable, the constitutional design problem is to find ways to separate policies for which the costs to losers are high from those for which they are low. As we shall see, there exist mechanisms that can achieve this, but only at a cost. An implication, then, is that constitutions that depend on p only can in some cases be preferable.

4.2. Majority Rules

One immediate response to the fact that costs are not verifiable is to ignore the issue and focus on p-dependent constitutions that employ one single rule to all decisions and do not make any attempt to extract information about costs from individuals. Formally, we define such a constitution as follows:

Definition 2 *A Majority-Rule (MR) constitution is a majority rule $m \in [0,1]$ such that*

$$x = \begin{cases} A & iff \quad p \geq m \\ B & otherwise \end{cases}$$

The following proposition shows that an appropriately chosen MR constitution is the optimal constitution within the class of constitutions that solely depend on p.

Proposition 2 *The MR constitution with majority rule*

$$m^*_{MR} = \frac{z - \eta(z-y)}{z + w - \eta(z-y)}$$

is the optimal p-dependent constitution.

Proof. Write expected utility from behind the veil of ignorance as

$$\int_0^1 \mu(p) \left[\eta u_{\underline{c}}(p) + (1-\eta) u_{\overline{c}}(p)\right] dF(p)$$

where $\mu(p) = \Pr(x = A|p)$ is the probability that policy A will pass under the given mechanism, conditional on p alone. Expected utility is maximized by any mechanism that satisfies the following condition:

$$\mu(p) = \begin{cases} 1 & if \quad \eta u_{\underline{c}}(p) + (1-\eta) u_{\overline{c}}(p) \geq 0 \Leftrightarrow p \geq \frac{z-\eta(z-y)}{z+w-\eta(z-y)} \\ 0 & otherwise \end{cases}$$

Given that individuals vote sincerely, the MR constitution with m^*_{MR} implements this. ¤

The optimal MR constitution has one significant drawback: it applies the same rule to all decisions. To see the implications of this, suppose z is very large. In that case the MR constitution will effectively require unanimity to allow policy A to pass ($\lim_{z\to\infty} m^*_{MR} = 1$) – even when $c = \underline{c}$ and the loss is y rather than z. In effect, all policy making is blocked in this example because that is the only way to prevent some really "bad" outcomes from happening. Notice that this is true even if these really "bad" outcomes are very unlikely, but can happen (i.e., if $1 - \eta \approx 0$, but positive).

4.3. Incentive Schemes

The costs c are observed by individuals before they vote. Consequently, the constitutional designer might be able to design a constitutional mechanism through which c is revealed and in this way allow the constitutional rules to be made contingent on such information. To explore this possibility, we consider constitutions of the following type. The constitution prescribes two rules: a default majority rule that is used to make decisions unless some individuals, at a cost defined in the constitution, request that the decision is made using a different pre-specified rule. When the cost of switching from the default to the alternative rule is chosen appropriately, information about the realization of c can be elicited truthfully, and decisions can be made using cost dependent rules. The time line is modified to describe this situation as follows:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.
2. Nature selects p. p is not observed by individuals.
3. Given p, individuals are partitioned in the two sets W and W^C by Nature. Individuals know to which set they belong, but this information is not verifiable. Nature selects c which is observable but not verifiable.
4. A representative for the winners j and a representative for the losers k simultaneously decide how large a cost $\widehat{\sigma}_j \geq 0$ and $\widehat{\sigma}_k \geq 0$ they are willing to bear to change the default decision rule laid down in 1 to a pre-determined alternative rule.
5. Individuals vote for or against policy A. The vote result is observed by everyone and it verifiable.
6. The policy outcome is determined according to the constitutional rules laid down in 1.

The new feature is stage 4. It describes how much winners and losers are willing to pay to change the decision rules. For simplicity, we assume that winners and losers, respectively, are represented by one individual (denoted by j and k) and that these hold enough resources to cover the cost of changing the majority rule, if they so desire. This assumption allows us to abstract from free-rider problems, although one could argue that the costs of changing the majority rule include the costs associated with the free-rider problem itself. We consider the following class of constitutions that allow a change in the majority rule if and only if at least one of the representatives is willing to shoulder the cost of the change:

Definition 3 *An Incentive-Scheme (IS) constitution is a triple (m, λ, n) such that for $k \neq j \in \{1, 2\}$*

$$x = \begin{cases} A \quad iff & \begin{cases} (p \geq m) \wedge (\widehat{\sigma}_j \wedge \widehat{\sigma}_k < \lambda) \\ (p \geq n) \wedge (\widehat{\sigma}_j \vee \widehat{\sigma}_k \geq \lambda) \end{cases} \\ B & \qquad \text{otherwise} \end{cases}$$

where $m \in [0,1]$ and $n \in [0,1]$ are majority rules and $\lambda \geq 0$ is the cost of changing the decision rule from to m to n.[12]

Since the welfare gain (w) bestowed on the winners if policy A passes does not depend on c, only the representative for the losers has an incentive to incur the cost λ and only if by doing so, she can guarantee that a stricter majority rule is used to decide the faith of policy A.[13] The crucial task of the constitutional designer is to choose λ, m, and n to guarantee truthful revelation of the costs, allowing for the possibility that it may not be feasible to elicit the truth. The following proposition characterizes the optimal IS constitution with deadweight cost λ.

Proposition 3 *Assume that c is not verifiable and define*

$$\eta^* = \frac{(1-\psi)\, y\, (w+z)}{(z-y)\left((1-\psi)y + w\right)},$$

where $\psi = E_F(p)$.

1. For $\eta \geq \eta^*$, the IS constitution with majority rules
$$(m^*_{IS}, n^*_{IS}) = \left(\frac{y - \psi(1-\eta)y}{\eta\,(w+y)}, \frac{z - y(1-\psi)}{w+z}\right)$$
and switching cost
$$\lambda^*_{IS} = y\left[F\left(n^*_{IS}\right) - F(m^*_{IS})\right]$$
is the optimal revelation mechanism with deadweight costs.

2. For $\eta < \eta^*$, no revelation mechanism with deadweight costs is feasible and the IS constitution with $m = n = m^*_{MR}$ and switching cost $\lambda = 0$ is the optimal constitution.

3. η^* is a decreasing function of z with $\lim_{z\to\infty} \eta^* = \frac{(1-\psi)y}{w+y(1-\psi)}$ and $\eta^* = 1$ for $z = \frac{w(2-\psi)+y(1-\psi)}{w} y$.

Proof. See Appendix.

The proposition highlights an important distinction between situations in which reforms with high costs are likely ($\eta < \eta^*$) and situations in which such reforms are unlikely ($\eta >$

[12] Technically speaking, the IS constitution consists of the whole mechanism, not just the majority rules and the cost λ. However, for notational simplicity, we henceforth use the convention to refer to each mechanism by the majority rules, costs and announcements associated with it, rather than the whole game form. This is reasonable to do because these are the elements that have to be optimized by the constitutional designer.

[13] The logic can, however, be extended to the case in which w is also a function of c.

η^*). In the latter case, the constitutional designer selects rules that elicit the information about the cost truthfully. By defining the switching cost λ such that the losers want to incur it if and only if policy A is associated with the high cost, z, the constitution can use cost dependent rules to make decisions. This is what makes the IS constitution attractive. The downside, of course, is that the cost λ^*_{IS} is incurred whenever $c = \overline{c}$ – even if $p > n^*_{IS}$ (so that A would pass anyway) or $p < m^*_{IS}$ (so that A would not pass anyway). Moreover, the cost dependent rules are only second-best and, thus, not equal to the first-best rules defined in proposition 1. To see why, notice that $m^*_{IS} \geq m^*_{FB}$ and $n^*_{IS} \leq n^*_{FB}$ and that the constitutional designer could employ the first-best rules, but only by increasing the switching cost to $y\left[F\left(n^*_{FB}\right) - F(m^*_{FB})\right] > \lambda^*_{IS}$. Part 1 of the proposition shows that it is optimal to distort the two majority rules in order to reduce the switching cost. Thus, the constitutional designer trades off the utility cost of reducing the difference between the two majority rules against the benefit of reducing the switching cost.

Part 2 of the proposition shows that it is not always possible to elicit information about the costs. As noted above, the constitutional designer is willing to reduce the difference between the two cost-dependent majority rules to reduce the switching cost. When it is likely that reforms are associated with high costs ($\eta < \eta^*$) and the switching cost is incurred frequently, reducing the switching cost becomes the dominant concern. An implication then is that the constitutional designer reduces the switching cost to zero by employing the same majority rule to all decisions. This effectively corresponds to the optimal MR constitution (see proposition 2).[14] Part 3 of the proposition shows that the critical value η^* is lower the larger z is. This is because the benefits of using separate cost dependent rules is larger when the costs associated with the two types of reforms is large. However, there is a limit to what an increase in z can do: if η is too small ($\eta < \frac{(1-\psi)y}{w+y(1-\psi)}$), even very large values of z are not sufficient to make a constitution with two separate rules optimal.

It is important to emphasize that the switching cost λ^*_{IS} is a deadweight loss to society. If we were to allow for transfers between winners and losers, truthful revelation could be obtained irrespectively of the parameters of the model. However, there are several important reasons why we do not consider transfers. Firstly, in our model (and often in the real world), it is very difficult to verify the identify of winners and losers. Effectively the only possibility is to have an open ballot and write into the constitution that if majority rule n is to be used instead of m, then anyone who votes against policy A must make a payment to those who vote for policy A in exchange for the higher threshold. The problem is that an unavoidable consequence of open ballots is vote trading. In a direct democracy,

[14] Technically speaking, this corresponds to the corner solution with $\lambda = 0$.

vote trading might lead to inefficiencies[15] and bad redistributive outcomes[16] so that it is often forbidden. Secondly, even if ballots were open and vote trading were not a problem, in a direct democracy, any transfer system would be liable to significant deadweight losses of its own. Finally, monetary transfers are vulnerable to the fact that only rich individuals may be able to pay. This introduces a disparity between someone's ability to pay the necessary transfer and the willingness to do so.

Against this background, we argue that proposition 3 is mostly relevant in relation to direct democracy, and referenda and citizens' initiatives are two important examples of what we have in mind. These, typically, allow legislation to be passed (or to be cancelled) using a different (stricter) majority rule than the default (simple) majority rule, but to trigger a referendum or a citizens' initiative, a cost has to paid. In most cases, this cost is simply the requirement that the proposer collects a certain number of valid signatures to support the referendum, and the cost (including the cost of overcoming the free-rider problem) of doing so is clearly a deadweight cost.[17] In legislatures, on the other hand, open ballots are much more common because voters need information on the performance of their representatives, and the deadweight costs or fairness issues associated with transfers are less likely to be a problem. In these cases, we would expect informal bargaining arrangements (e.g. logrolling as in the US Congress) to provide an alternative the IS constitution described here. In conclusion, then, our analysis suggests that provisions for referenda and citizens' initiatives are included in real constitutions to deal with rare reforms which, if adopted, yield disproportionately large losses.

4.4. Vetoes

Societies need to make decisions about reforms repeatedly. This fact can be exploited by constitutional designers to overcome the problem that the costs of these reforms cannot be verified. To see the basic idea, which is described in detail in [11], suppose that society has to make t decisions, one in each of t periods.[18] As above, the constitution prescribes two

[15] [21], in the spirit of the Condorcet jury theorem, argues that voting can efficiently aggregate private signals about policy reforms. Thus, even though vote trading may increase efficiency by allowing individuals with little interest in a policy to sell their votes to individuals with more at stake, valuable information may be lost in the process.

[16] Open ballots are often associated with corruption and blackmail. [12] show how the post-war electoral system in Italy effectively allowed for open ballots. Since each voter could express a preference for up to four candidates within a particular party list, politicians could exchange favors with voters by requesting that they express their vote with a pre-determined pattern of preferences. When the ballots were counted, an accomplice of the politician, could then check that the voter had kept his or her end of the bargain. The system was perceived to be so corrupt that a referendum was passed in 1991 that changed the number of preferences that can be expressed from four to one.

[17] For an example, see chapter 2, title 4 of the Swiss Federal Constitution.

[18] [11] study a model which is much more general than the one considered here and show how veto mechanisms lead, asymptotically, to full revelation. In contrast, we apply the idea of a veto mechanism

rules: a default rule m and an alternative rule n. The new feature is that the constitution allows individuals (in equilibrium, losers) to request that rule n is used instead of rule m at no cost, but they can only do so a specified number of times $s \leq t$.[19] We interpret a switch of rule as a veto and s is then the number of vetoes allowed by the constitution. By rationing the number of vetoes, the constitutional designer can provide incentives for truthful revelation of the costs of reform. The point is that losers, knowing that there are more decisions to be made than vetoes, are more likely to veto a policy reform with high costs than one with low costs. To capture these ideas, we modify the time line is as follows:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.
2. Nature selects p. p is not observed by individuals.
3. Given p, individuals are partitioned in the two sets W and W^C by Nature. Individuals know to which set they belong, but this information is not verifiable.
4. Nature makes t draws of the random variable c – one for each of the t policy decisions to be made. All draws are observable, but not verifiable.
5. For each policy decision $\upsilon \in \{1, ..., t\}$ for which at least one veto is still available, the two representatives j and k simultaneously make announcements $\chi_j^\upsilon, \chi_k^\upsilon \in \{\varnothing, V\}$.
6. Individuals vote for or against policy A. The vote result is observed by everyone and it verifiable.
7. Policy outcome is determined according to the constitutional rules laid down in 1.

We interpret an announcement $\chi^\upsilon = V$ at time υ from one of the two representatives as a veto against the policy decision being made using the default majority rule m_{VT} and a (possibly implicit) request that the decision is made with the alternative majority rule n_{VT}. An announcement $\chi^\upsilon = \varnothing$ then corresponds to a decision not to veto. For simplicity, we assume that the winners and losers are the same each period, but that the policy reform

to our simple constitutional design problem and look for the optimal veto mechanism and compare it to the alternatives (the MR or the IS constitution) for a given t.

[19] Throughout, we assume that there is no discounting between periods. However, we can capture the effects of discounting by treating t as a parameter of the model and analyzing how the mechanism performs for different values of t with large t corresponding to a more patient society.

differs from period to period.[20] Let $s_{\widehat{v}}$ be the number of vetoes remaining at time $\widehat{v} \leq t$, i.e., the total number of vetoes (s) minus the number of vetoes already used. Formally:

$$s_{\widehat{v}} = s - \# \left\{ v < \widehat{v} | \chi_j^v \vee \chi_k^v = V \right\}.$$

We can then define a constitution with veto rules as a mechanism that allows representatives from winners and losers to veto the use of majority rule m_{VT}, but only if there are still vetoes left. Formally,

Definition 4 *A veto rule (VT) constitution is a triple (m_{VT}, n_{VT}, s_{VT}) such that in any period v for which $s_v = 0$*

$$x = \begin{cases} A & iff \quad (p \geq m_{VT}) \\ B & \quad otherwise \end{cases}$$

whereas in any period v for which $s_v \geq 1$

$$x = \begin{cases} A & iff \quad \begin{cases} (p \geq m_{VT}) \wedge \left(\chi_j^v \wedge \chi_k^v = \varnothing\right) \\ (p \geq n_{VT}) \wedge \left(\chi_j^v \vee \chi_k^v = V\right) \end{cases} \\ B & \quad otherwise \end{cases}$$

Before we proceed with the characterization of the optimal VT constitution, it is useful to define the following objects:

$$m(s) = \begin{cases} \frac{\left(\eta - \frac{1}{t}G(s,t)\right)y + z\frac{1}{t}H(s,t)}{\left(\eta - \frac{1}{t}G(s,t)\right)(w+y) + (w+z)\frac{1}{t}H(s,t)} & if \quad 0 \leq s \leq t-1 \\ m \in [0, m^*_{MR}] & if \quad s = t \end{cases} \tag{4.1}$$

and

$$n(s) = \begin{cases} \frac{\left((1-\eta) - \frac{1}{t}H(s,t)\right)z + y\frac{1}{t}G(s,t)}{\left((1-\eta) - \frac{1}{t}H(s,t)\right)(w+z) + (w+y)\frac{1}{t}G(s,t)} & if \quad 1 \leq s \leq t \\ n \in [m^*_{MR}, 1] & if \quad s = 0 \end{cases} \tag{4.2}$$

and

$$\begin{aligned} s^* \;=\; & \arg\max_{0 \leq s \leq t} \int_{m(s)}^{1} \left[\left(\eta - \frac{1}{t}G(s,t) \right) (p(w+y) - y) + \frac{1}{t}H(s,t)(p(w+z) - z) \right] dF(p) \\ & + \int_{n(s)}^{1} \left[\left(1 - \eta - \frac{1}{t}H(s,t) \right) (p(w+z) - z) + \frac{1}{t}G(s,t)(p(w+y) - y) \right] dF(p) \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} G(s,t) &= \sum_{x=0}^{s} \sum_{k=0}^{x-1} \binom{t}{k} (1-\eta)^k \eta^{t-k}, \\ H(s,t) &= (1-\eta)t + G(s,t) - s. \end{aligned}$$

With these definitions in place, we can characterize the optimal VT constitution as follows.

[20] This assumption can be relaxed. If p were drawn each period and a new set of winners and losers were determined, we get qualitatively similar results, but the veto constitution would be less efficient.

Proposition 4 *The optimal VT constitution is the triple*

$$(m^*_{VT}, n^*_{VT}, s^*_{VT}) = (m(s^*), n(s^*), s^*),$$

where $m(s^*)$ is given in equation (4.1), $n(s^*)$ is given in equation (4.2), and the optimal number of vetoes s^* is given in equation (4.3), with the following properties

1. $m^*_{VT} \in [m^*_{FB}, m^*_{MR}]$ and $n^*_{VT} \in [m^*_{MR}, n^*_{FB}]$.

2. s^* is unique and for $t > 1$, $0 < s^* < t$.

3. $\lim_{t\to\infty} EU(m^*_{VT}, n^*_{VT}, s^*_{VT}) = EU(m^*_{FB}, n^*_{FB})$.

Proof. See Appendix.

To develop intuition for these results, it is useful to begin by assuming that the number of decisions t is large enough to allow an accurate approximation of the binomial distribution with the normal. This, in turn, allows us to approximate the optimal number of vetoes by the solution to the following first order condition[21]

$$\Phi\left(\frac{s^*_{VT}-(1-\eta)t}{\sqrt{\eta(1-\eta)t}}\right)\vartheta_1(m^*_{VT}, n^*_{VT}) = \left(1-\Phi\left(\frac{s^*_{VT}-(1-\eta)t}{\sqrt{\eta(1-\eta)t}}\right)\right)\vartheta_2(m^*_{VT}, n^*_{VT}) \qquad (4.4)$$

where $\Phi(\bullet)$ is the cumulative distribution of the standard normal distribution and

$$\begin{aligned}\vartheta_1(m^*_{VT}, n^*_{VT}) &= \int_{m^*_{VT}}^{n^*_{VT}} (p(w+y)-y)\, dF(p)\\ \vartheta_2(m^*_{VT}, n^*_{VT}) &= -\int_{m^*_{VT}}^{n^*_{VT}} (p(w+z)-z)\, dF(p).\end{aligned}$$

Equation (4.4) reveals an important trade-off. The left hand side represents the expected costs of allowing too many vetoes. This cost arises because of the possibility that a decision with low costs is vetoed and thus determined by the stricter alternative rule when it should have been decided with the laxer default rule. The per-period cost of this is $\vartheta_1(m^*_{VT}, n^*_{VT})$ and the problem arises with probability $\Phi(.)$. The right hand side represents the expected cost of allowing too few vetoes. This cost arises because of the possibility that a decision with high costs is not vetoed and thus determined by the default rule when it should have been decided by the alternative rule. The per-period cost of this is given by $\vartheta_2(m^*_{VT}, n^*_{VT})$ and the problem arises with probability $1-\Phi(.)$. We, therefore, see that the optimal number of vetoes exactly balances these two concerns. Importantly, when more than one

[21] The solution to the first order condition will, in general, not be an integer. Thus, the approximate optimal number of vetoes is the nearest integer to the solution. The details are shown in the appendix where it is established that the second order condition is satisfied and that the solution is interior.

decision has to be made, it is never optimal to eliminate one of the two costs entirely by allowing either no vetoes or a veto for every decision. This is accomplished by adjusting the majority rules (and thus, the costs) accordingly.

An immediate consequence of the fact that $0 < s^* < t$ for $t > 1$ is that the MR constitution can never yield higher expected utility than the optimal VT constitution. To see this, notice that the MR constitution arises as a special case of the VT constitution when $s = 0$ or $s = t$. If $s = 0$, majority rule m is used for all decisions because no vetoes are allowed, while if $s = t$ all decisions will be vetoed by the representative of the losers and majority rule n is effectively used for all decisions. In both cases, the appropriate choice for the (single) majority rule is m^*_{MR}. The constitutional designer can, however, do better by allowing some, but not all, decisions to be vetoed. For example, suppose that $t = 2$. Then $s^* = 1$ and the optimal majority rules are

$$m^*_{FB} < m(1) < m^*_{MR} < n(1) < n^*_{FB}.$$

We see that $m(1)$ and $n(1)$ are closer to the first-best rules than m^*_{MR}. When the reforms in period 1 and 2 are associated with different costs, it is desirable to use majority rules that are closer to the first-best rules than the MR rule. However, when the two reforms are associated with the same cost, then one of the two decisions will be made with a rule that is worse than the MR rule. By appropriately choosing $m(1)$ and $n(1)$, the constitutional designer can always guarantee that the benefits are greater than the costs, and this is the reason why it is not, in general, optimal to use the first-best majority rules.

As the number of decisions (t) becomes larger, it is, however, easier for the constitutional designer to insure that there are neither too many nor too few vetoes available. In the limit (as $t \to \infty$) the optimal VT constitution achieves the first-best, as in [11].[22] An implication, then, is that the VT constitution yields higher expected utility than the IS constitution for a sufficiently large t, regardless of the other parameters of the model. The interesting question, therefore, is to ask: how many policy decisions t^* does it take for the VT constitution to yield higher expected utility than the IS constitution, and how does the critical value t^* depend on the cost z and the probability $(1-\eta)$ that reforms with such costs come up for a vote? To answer this question, we present some simulation results in Tables 4.1 and 4.2 showing t^* as a function of $(1-\eta)$ and z, respectively.[23]

[22] The main difference between the VT constitution and the mechanism considered by [11] is that we allow the rules to be determined optimally, while Jackson and Sonnenschein – using the notation of our model – set $s = (1-\eta)\,t$ and employ what would correspond to the first-best majority rules. Thus, for finite t the VT constitution welfare dominates this alternative, and it is, therefore, not surprising that the

$1-\eta$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$
$z = 200$	46	36	22	15	10	7	2	2	2
$z = 100$	41	30	18	12	8	4	2	2	2
$z = 10$	16	8	4	3	2	2	2	2	2

Table 4.1: The critical number of decisions as a function of the probability of high costs

z	10	20	30	40	50	60	70	80	90	100
$1-\eta = 1/4$	6	10	13	16	17	18	20	21	22	22
$1-\eta = 1/2$	2	3	4	5	6	7	7	8	8	8
$1-\eta = 16/27$	2	2	2	3	3	3	4	4	4	5

Table 4.2: The critical number of decisions as a function of z

The tables show that the critical value t^* is large when it is unlikely that reforms are associated with high costs and/or the cost z is large. This is as one would expect based on the analysis in section 4.3: the IS constitution performs well precisely in situation where reforms with high costs only come up for a vote at rare occasions, but when they do, the costs associated with them are very high. In contrast, when z is low and/or it is likely that reforms are associated with high costs, the VT constitution is better than the IS constitution even for 2 or 3 decisions.

Our analysis shows that the VT and the IS constitution cannot be worse than the MR constitution. At the same time, for $t < \infty$, all three constitutions fail to reach the first-best. One way then to measure the efficiency gains associated with the VT and the IS constitution relative to the MR constitution is to look at deviations from the first-best. To this end, we define the following efficiency scores

$$ES_i = 100 * \frac{(EU^*_{FB} - EU^*_{MR}) - (EU^*_{FB} - EU^*_i)}{(EU^*_{FB} - EU^*_{MR})} \quad i = IS,\ VT,$$

where EU^* indicated the optimized expected utility associated with the relevant constitution. This efficiency score has the advantage that it normalizes the efficiency loss associated with the IS and VT constitution relative to the benchmark loss associated with the MR constitution. Thus, EF_i is 100 if constitution i achieves the first-best and 0 if the constitution cannot improve upon the MR constitution. In table 4.3, we report efficiency scores for the IS and VT constitution for different values of z and $1-\eta$. Each cell has two entries:

VT constitution implements the first-best allocation when $t \to \infty$.

[23] The numerical simulations are performed in Mathematica version 5. The program is available upon request. All simulations set $w = y = 1$ and assume that F is uniform on $[0, 1]$. In the simulations, we do not make use of the normal approximation, and the exact optimal integer value of s is calculated from equation (4.3). Note that for certain parameter configurations, $\eta < \eta^*$ and the IS constitution becomes the MR constitution. For all these cases, clearly, $t^* = 2$.

$\frac{1-\eta}{z}$	$\frac{1}{100}$	$\frac{1}{25}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$
1.1	$\frac{22.62}{0}$	$\frac{44.30}{0}$	$\frac{58.88}{0}$	$\frac{67.67}{0}$	$\frac{71.22}{0}$	$\frac{72.79}{0}$	$\frac{73.18}{0}$	$\frac{72.58}{0}$	$\frac{70.79}{0}$	$\frac{66.96}{0}$	$\frac{57.80}{0}$
1.5	$\frac{25.60}{0}$	$\frac{47.80}{0}$	$\frac{61.03}{0}$	$\frac{68.82}{0}$	$\frac{71.77}{0}$	$\frac{72.93}{0}$	$\frac{72.95}{0}$	$\frac{71.97}{0}$	$\frac{69.77}{0}$	$\frac{65.47}{0}$	$\frac{55.68}{0}$
2	$\frac{28.94}{0}$	$\frac{51.36}{0}$	$\frac{63.78}{0}$	$\frac{70.53}{0}$	$\frac{72.78}{0}$	$\frac{73.31}{0}$	$\frac{72.74}{0}$	$\frac{71.11}{0}$	$\frac{68.39}{0}$	$\frac{63.58}{0}$	$\frac{53.17}{0}$
5	$\frac{42.93}{55.12}$	$\frac{62.82}{51.66}$	$\frac{71.30}{44.44}$	$\frac{74.32}{31.64}$	$\frac{74.53}{18.37}$	$\frac{73.33}{6.25}$	$\frac{71.14}{0}$	$\frac{67.82}{0}$	$\frac{63.30}{0}$	$\frac{56.34}{0}$	$\frac{41.60}{0}$
10	$\frac{54.94}{77.92}$	$\frac{69.54}{74.55}$	$\frac{76.08}{67.40}$	$\frac{76.44}{54.19}$	$\frac{74.56}{39.31}$	$\frac{72.05}{23.18}$	$\frac{68.58}{7.71}$	$\frac{64.08}{0}$	$\frac{58.33}{0}$	$\frac{49.04}{0}$	$\frac{33.79}{0}$
20	$\frac{65.13}{88.70}$	$\frac{77.27}{85.44}$	$\frac{78.88}{78.49}$	$\frac{77.22}{65.48}$	$\frac{73.80}{50.58}$	$\frac{69.78}{33.52}$	$\frac{65.18}{15.58}$	$\frac{59.78}{1.40}$	$\frac{52.56}{0}$	$\frac{41.67}{0}$	$\frac{24.68}{0}$
30	$\frac{69.49}{92.18}$	$\frac{80.12}{88.97}$	$\frac{80.12}{82.11}$	$\frac{76.68}{69.21}$	$\frac{73.05}{54.24}$	$\frac{68.60}{37.11}$	$\frac{63.42}{18.58}$	$\frac{57.04}{2.68}$	$\frac{48.75}{0}$	$\frac{37.33}{0}$	$\frac{19.42}{0}$
50	$\frac{75.66}{94.93}$	$\frac{82.01}{91.76}$	$\frac{80.92}{84.97}$	$\frac{76.42}{72.16}$	$\frac{71.24}{57.23}$	$\frac{66.15}{40.03}$	$\frac{60.21}{21.09}$	$\frac{53.32}{3.96}$	$\frac{44.74}{0}$	$\frac{32.65}{0}$	$\frac{14.83}{0}$
100	$\frac{82.98}{96.97}$	$\frac{84.34}{93.82}$	$\frac{80.72}{87.09}$	$\frac{74.80}{74.37}$	$\frac{69.13}{59.48}$	$\frac{62.87}{42.23}$	$\frac{56.30}{23.02}$	$\frac{48.87}{5.05}$	$\frac{38.31}{0}$	$\frac{25.37}{0}$	$\frac{9.77}{0}$

Table 4.3: The efficiency scores for the VT (upper entry) and IS (lower entry) constitutions for different values of z and 1-eta

the upper entry it ES_{VT} and the lower entry is ES_{IS}.[24]

The table shows a positive correlation between ES_{IS} and ES_{VT}. That is, the VT and IS constitution perform well in the same circumstances, namely, when reforms with high costs are unlikely, but when they come up, they are associated with very large costs. This is because this is precisely the circumstances under which the MR constitution performs poorly: it specifies a majority rule close to one to deal with the high value of z even though in most cases reforms only entail modest costs (y). It is interesting to consider the difference between the maximum efficiency and the minimum efficiency score for the two constitutions along the $1 - \eta$ dimension for different values of z. Consider, for example, $z = 100$. In this case, the maximum efficiency score for the VT (IS) constitution is 82.98 (96.97) for $1 - \eta = \frac{1}{100}$, but falls to 9.77 (0) for $1 - \eta = \frac{9}{10}$. Thus, both constitutions perform much worse when reforms with high costs (of 100) are likely than when they are unlikely. Contrast this with the case where $z = 1.1$. In this case, the difference between the maximum and the minimum efficiency score for the VT constitution is modest, while the IS constitution's performance is uniformly poor. Thus, the VT constitution performs well overall for low values of z while the IS constitution does not.

It is reasonable to assume that in actual constitutional design, when the difference in performance between a complex mechanism and a simple one is small, the latter is more likely to be chosen. In our model, IS and VT constitutions are never worse than MR constitutions, but they are much more complex. Table 4.3, then, suggests that the VT constitution will be particularly important for those cases in which z is relatively low since the cases in which z is relatively high are already well taken care of by the IS constitution

[24] The simulations are performed in Mathematica version 5. All simulations use $w = y = 1$ and $t = 30$ and assume that F is uniform. Clearly, the efficiency score of the VT constitution depends on t. We focus on the changes in efficiency given by changes in η and z for a given t and note that these comparative statics are qualitatively independent of precise value of t.

(when η is high) or the MR constitution (when η is low). Thus, we would expect real world constitutions to embody veto rules when the cost difference between different types of reforms is small.

5. Policy Areas

In some cases, the magnitude of the cost of a reform cannot be known until after the reform has actually been introduced. In such cases, the constitution can obviously not make use of information about costs or attempt to elicit them truthfully as above. The only possibility open to the constitutional designer may appear to be the MR constitution.

In reality, however, constitutions often include special provisions or rules that apply to particular policy decisions, along with a simple majority rule used for all other decisions. A careful look at these special provisions reveals that they are never contingent on the potential welfare consequences of a particular policy. Rather, special provisions apply to policy reforms that belong to defined classes of reforms – *policy areas*. A leading example of a policy area is a bill of rights, but many other examples could be given: the Danish constitution, for example, has special rules for decisions related to expropriation of property, while the German constitution specifies that some laws have to pass both the Bundesrat and in the Bundestag, while others can pass with a simple majority in the house alone.[25] Insofar as a policy area provide some, albeit, imperfect information about the potential costs associated with policy reforms in that area and reforms can reliably be attributed to the relevant policy area as they come up, there is scope for designing constitutions with policy area specific decision rules. This, of course, is very different from writing a cost dependent constitution.[26] The purpose of this section is to compare policy area (PA) constitutions with MR constitutions in a situation where the cost of reform is observed only *ex post*, i.e., after adaptation. We stress that it is costly, but not impossible, to determine accurately to which policy area a particular policy belongs.[27]

[25] The argument developed in this section is similar to the theory of constitutional rights developed by [17] and [18]. He makes the point that individuals, from behind the veil of ignorance, would want to apply stricter rules to policy decisions that entail particularly high costs than to other decisions.

[26] While a CD constitution says "if the cost to losers of policy A is low, then use majority rule m while if the cost is high, use majority rule n," a PA constitution may, for example, say "if policy A is about education, then use majority rule m whereas if it is about civil liberties, then use majority rule n".

[27] Policy areas can, in principle, be combined with the constitutional mechanisms considered in the previous sections. That is, policy areas can play a role also in situations where the costs can be observed, but not verified, before the policy decision is made. We do not consider this in detail but stress that real world constitutions typically contain a mixture of the various constitutions that we analyze. From an analytic point of view, it is, however, instructive to separate the different aspects out.

5.1. The set-up and timing of events

We assume that every policy reform A belongs to one of two policy areas, denoted $\phi \in \{\phi_1, \phi_2\}$. Each policy area is associated with a particular probability distribution of the costs of reform with

$$\rho_j = \Pr\left(c = \underline{c} | \phi = \phi_j\right).$$

The two policy areas are **cost informative** if $\rho_1 \neq \rho_2$ and are **cost equivalent** otherwise. We assume that $\rho_1 \geq \rho_2$, that is, policy area ϕ_1 is more likely to be associated with low cost reforms than policy area ϕ_2, but allow for cost equivalence. The unconditional probability that a policy reform belongs to area ϕ_1 is q. Accordingly, the probability that $c = \underline{c}$ obtains is

$$\Pr\left(c = \underline{c}\right) = q\rho_1 + (1-q)\,\rho_2 = \eta,$$

and $\Pr\left(c = \overline{c}\right) = 1 - \eta$. We can summarize the timing of events as follows:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.

2. Nature selects p which is not observed. Given p, individuals are partitioned in the two sets W and W^C by Nature. Individuals know to which set they belong, but this information is not verifiable.

3. Nature selects the policy area, ϕ. This can only be observed (and verified) by incurring the deadweight cost $\tau > 0$ (as described in stage 3c).

3a. Nature selects with equal probability a representative for the winners or the losers. This representative, indexed by j, makes an announcement about the policy area $\widehat{a}_j \in \{\phi_1, \phi_2\}$.

3b. Upon observing $\widehat{a}_j$, a representative for the other side, indexed by k, makes an announcement about the policy area $\widehat{a}_k \in \{\phi_1, \phi_2\}$.

3c. If $\widehat{a}_j \neq \widehat{a}_k$, the dispute is taken to the courts who insure that the policy decision is allocated to the correct policy area and all citizens incur the cost $\tau > 0$. If the two announcements coincide, the common recommendation determines which policy area the policy reform is allocated to and nobody pays the cost.

4. Individuals vote for or against policy A. The vote result is observed by everyone.

5. The policy outcome is determined according to the constitutional rules laid down in 1.

6. Given ϕ, nature selects c and payoffs are realized.

This time line embodies two new features. First, the cost is not known until after the policy reform is adopted.[28] Second, the announcement stage, 3a-3c, is meant to capture the idea that it requires (costly) legal expertise or, as a minimum, time and effort from individuals to allocate reforms to the correct policy area. We denote this cost by $\tau > 0$. The cost is borne by all individuals. However, individuals can agree not to bother to find out about what the policy area is and, thereby, avoid the cost τ. Of course, by doing so, they cannot be sure that decisions are made with the intended rules which in itself is undesirable. To capture this in a simple way, we allow representatives of the winners and losers to make announcements about the policy areas.[29] Since they do not know what the correct policy area is, these announcements have no informational content as such, and serve the sole purpose of allowing individuals to decide not to incur the cost of finding out about the correct policy area.[30]

5.2. Optimal Constitutions When C is Observed Ex Post

We can now define formally what we mean by policy area specific majority rules:

Definition 5 *A Policy Areas (PA) constitution is a pair $\{m, n\}$ of majority rules with $m \in [0,1]$ and $n \in [0,1]$ such that*

$$x = \begin{cases} A \quad iff & \begin{cases} (p \geq m) \wedge \left[\left(\widehat{\phi}_j = \widehat{\phi}_k = \phi_1\right) \vee \left(\left(\widehat{\phi}_j \neq \widehat{\phi}_k\right) \wedge (\phi = \phi_1)\right)\right] \\ (p \geq n) \wedge \left[\left(\widehat{\phi}_j = \widehat{\phi}_k = \phi_2\right) \vee \left(\left(\widehat{\phi}_j \neq \widehat{\phi}_k\right) \wedge (\phi = \phi_2)\right)\right] \end{cases} \\ B & \text{otherwise} \end{cases}$$

A PA constitution specifies two majority rules m and n – one intended for each policy area. The m-majority rule is applied if the two representatives agree that this rule should

[28] Notice that neither the welfare costs nor the policy area are known until after individuals learn their status as winners or losers. This leaves room for renegotiation of the policy area specific majority rules specified in the original constitution after the veil of ignorance has been (partially) lifted in stage 2. The veil of ignorance construct, however, embodies the assumption that the original constitutional rules can be enforced by the courts after the veil has been lifted, and we, thus, rule out the possibility of renegotiation. It should, however, be noted that for certain specifications of the bargaining game describing the renegotiations (including generalized Nash Bargaining), our results would not be affected by allowing for renegotiation: the original constitutional designer would anticipate the outcome of the bargaining game and factor that into the original rules (which determines the fall back payoffs) in such a way as to implement the rules what would have been optimal without renegotiation.

[29] The presence of representatives that can make announcement on behalf of all winners or losers assumes away potential coordination problems.

[30] The sequential structure of the announcements is not important. Similar results can be obtained with simultaneous announcements. This is true because neither of the representatives knows the true policy area when they make their announcements.

be used or if, after a dispute, the court rules that the policy area is ϕ_1. Similarly for n. We notice that the MR constitution is a special case of this with $m = n$. The next proposition characterizes the optimal majority rules as a function of the cost of allocating decisions to the correct policy area (τ).

Proposition 5 *(PA constitutions) Let F be uniform on $[0,1]$, $\rho_1 > \rho_2$, $0 < q < 1$ and $\infty > z > y$. Then there exists a critical level of the cost $\Delta > 0$ such that the optimal PA constitution specifies policy area specific rules (m^*_{PA}, n^*_{PA}) for $\tau \in [0, \Delta]$ but specifies a single rule m^*_{MR} otherwise where*

$$m^*_{PA} \equiv \frac{\rho_1 y + (1-\rho_1) z}{w + \rho_1 y + (1-\rho_1) z};$$

$$n^*_{PA} \equiv \frac{\rho_2 y + (1-\rho_2) z}{w + \rho_2 y + (1-\rho_2) z};$$

$$m^*_{MR} = \frac{z - (q\rho_1 + (1-q)\rho_2)(z-y)}{w + z - (q\rho_1 + (1-q)\rho_2)(z-y)}.$$

Proof. See Appendix.

The proposition shows that policy area specific majority rules are optimal when the cost of allocating decisions to the correct policy area is low ($\tau \leq \Delta$). In this case, the two representatives are willing to incur the cost τ. This insures that the intended rule is always applied, and the constitutional designer can then tailor these to take into account policy area specific information about the distribution of the cost of reform. For $\tau > \Delta$, the situation is different. The representatives could, in principle, agree not to find out about the policy area (and avoid τ), but continue to apply policy area specific rules modified to take into account that decisions occasionally would be made using the "wrong" rule. The proposition shows that this is never optimal: for $\tau > \Delta$, it is better to use one rule for all decisions and that rule is effectively the one associated with the optimal MR constitution (compare to proposition 2).

The policy area specific majority rule for policy area 1 (m^*_{PA}) is more lenient than that for policy area 2 (n^*_{PA}). Intuitively, this is because it is more likely that losers incur the high cost z for policy reforms in this area than for reforms in the other area. However, if the two policy areas are cost equivalent, then the same majority rule applies to both areas, even if $\tau = 0 < \Delta$. Thus, a necessary condition for policy area specific majority rules is that policy areas are, in fact, cost informative and for $\tau \leq \Delta$, this is also sufficient. We can, however, say more than that by studying Δ for a given τ. The critical value Δ is defined by the difference in expected utility between a constitution that uses the optimal policy area specific rules (m^*_{PA}, n^*_{PA}) conditional on decisions being allocated correctly and a constitution that applies the best single rule m^*_{MR} to all decisions and is, thus, a function of z and ρ_1.

Proposition 6 *(Costs) Let F be uniform on $[0,1]$, $\rho_1 > \rho_2$, $0 < q < 1$ and $\tau \in (0, \tau_z)$.*[31] *Then there exist two critical values of z such that*

1. Policy area specific rules (m^*_{PA}, n^*_{PA}) are optimal for $z \in [z_1, z_2]$.

2. A single rule m^*_{MR} is optimal for $z \in (y, z_1)$ or $z \in (z_2, \infty)$.

Corollary 1 *(Bill of Rights). Let $\rho_1 = 1$ and $\tau > 0$. Then policy area specific rules (m^*_{PA}, n^*_{PA}) are optimal for all $z \geq z_3 > y$.*

Proof. See Appendix

The proposition shows how the constitutional choice of rules depends on the size of the potential cost z for a given τ. For low z, the optimal choice is to use a single rule. This is because the potential welfare consequences of different reforms are very similar (z close to y), and, as a consequence, it does not pay to incur the cost of separating the decisions along the lines of policy areas. More surprisingly, perhaps, the single rule is also optimal for extremely high costs. To see why notice from proposition 5 that it is almost impossible to pass any policy when z is large, independently of whether one or two rules are used. Accordingly, since most decisions are blocked in any case, it does not pay to incur the cost of separating decisions according to policy areas. The implication, then, is that policy area specific rules dominate a single rule for intermediate costs. In these cases, the optimal policy area specific rules are sufficiently different and the expected welfare gain of using a tougher rule for reforms in area 2 is sufficient to compensate for the cost τ.

More interestingly, perhaps, the corollary provides a rational for including a Bill of Rights in the constitution. Effectively a Bill of Rights separates out policy reforms that by infringing with fundamental rights often generate disproportionately large losses from those reforms that do not. Interpreting policy area 2 as the Bill of Rights and area 1 as all other decisions that never generates the high cost z ($\rho_1 = 1$), we see that it is optimal to include such a Bill if the potential cost associated with an infringement of these rights is sufficiently large.[32] In particular, if z is extremely large or if it is very likely that any change in the specified rights will generate the loss z, then the constitution would never allow reforms in this area, i.e., $n^*_{PA} \simeq 1$. This is, in fact, what many constitutions do: they single out a limited set of basic rights that cannot be violated by any policy reform.[33]

[31] τ_z is defined in the Appendix.

[32] Notice that with $\rho_1 = 1$, m^*_{PA} is independent of z and so the difference between n^*_{PA} and m^*_{PA} widens, rather than narrows, as z increases.

[33] Examples of this include Article 6 of the Dutch constitution of 1983 and Chapter 1 of the German constitution of 1995.

Proposition 7 *(Cost informativeness) Let F be uniform on $[0,1]$, $\infty > z > y$ and $\tau \in (0, \tau_{\rho_1})$.*[34] *Then there exists a critical value $\overline{\rho}_1 \in (\rho_2, 1)$ such that for $\rho_1 \geq \overline{\rho}_1$, policy area specific rules (m^*_{PA}, n^*_{PA}) are optimal, while for $\rho_1 \in (\rho_2, \overline{\rho}_1)$, the single rule m^*_{MR} is optimal.*

Proof. See Appendix.

Proposition 7 shows that policy area specific rules dominate the single rule when policy areas are sufficiently cost informative ($\rho_1 > \overline{\rho}_1$). The intuition is simple. As cost informativeness increases, the benefit of using policy area specific rules increases because the rules can be better tailored to the distribution of costs associated with each area. In turn, individuals are willing to incur larger costs to get decisions allocated to the correct policy area. Putting this together with the observation that policy area specific rules are likely to be better than a single rule for low τ, an interesting interaction effect may arise. If two policy areas are easily distinguishable, then it is reasonable to suppose that they are also cost informative. This makes it likely that τ is low **and** $(\rho_1 - \rho_2)$ is high at the same time.

For actual constitutions, then, our analysis suggests that special rules will mostly be used for policies that have to do with civil liberties, property rights, procedures for declaring war, procedures for signing international treaties, or the nomination/election of important institutions. These are easily distinguishable from all other policies. This is not only because it is easy to establish whether policies belong to these areas, but also because these policies are likely to be associated with substantially different cost distributions than other areas.

6. Constitutional Change

All the constitutions that we analyzed above share the common feature that no constitutional change is allowed. Some of the possibilities we have considered allow for different rules to be used depending on the circumstances, but these rules are all determined behind the veil of ignorance. In this section, we study **constitutional change**, by looking at constitutions that allow the rules to be modified depending on the circumstances after the veil of ignorance has been lifted, but where the new rules are **not** part of the original constitution. We show that amendment rules – meta-rules that allow individuals to change the original constitutional rules – emerge as an optimal response to the possibility of a constitutional crisis, but only if it is sufficiently likely that a crisis will occur. We introduce the possibility of a crisis to capture the notion that the relative political strength of different groups depends on other factors than group size (as captured by p). For example, it may

[34] τ_{ρ_1} is defined in the Appendix.

be the case that the winners have the numbers to enact a certain policy program, but that the losers have particularly strong interest groups or even a credible threat of recourse to violence that allow them to threaten a crisis if winners go ahead. Clearly, such threats are not always present and even when they are, loser may not be successful in carrying the threat through.

We assume that c can be verified. This is a very strong assumption, of course, but it allows us to focus on the question of constitutional amendments in isolation from the issues discussed in section 4. We model the constitutional crisis as follows. After the original constitution has been designed, but before policies are implemented, the conditions for a crises are determined, which are either favorable or unfavorable. If the state is $\underline{\omega}$, then conditions are unfavorable and a crisis is impossible. If, on the other hand, the state is $\overline{\omega}$, then conditions are favorable and a crisis may occur. The probability that conditions are favorable is $\gamma = \Pr(\omega = \overline{\omega})$. A crisis occurs if the constitution allows policies with $c = \overline{c}$ to pass with positive probability. The interpretation is that when $\omega = \overline{\omega}$, losers are in a position to generate a crisis, but will only do so if reforms with high costs can be implemented through the constitution. The fact that a crisis occurs does, however, not imply that it is successful: we assume that it is successful only with probability $1 - \alpha$. If it is successful, then B obtains regardless of the constitutional rules.[35] If a crisis is not successful, which happens with probability α, the policy outcome is decided by the constitutional rules in place at the time.[36] We can summarize this by the following (modified) time line:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.

2. Nature selects p. p is not observed by individuals.

3. Given p, individuals are partitioned in the two sets W and W^C by Nature. Individuals know to which set they belong, but this information is not verifiable. Nature selects ω. The realization of ω is observed by everyone.

4. If $\omega = \overline{\omega}$ and the constitution allows policy A to pass with some probability when $c = \overline{c}$, a constitutional crisis occurs and is successful with probability $1 - \alpha$. If it is successful, then all individuals get zero utility and the process ends. If it is not successful or if $\omega = \underline{\omega}$, then stage 5 applies.

5. Nature selects c which is observed by everyone and is verifiable.

[35] This insures that there is no scope for renegotiation after a successful crisis.

[36] In the model, we assume that p and ω are drawn independently by nature. In reality, p and ω may be negatively correlated because sheers numbers are important in democracies. Modelling this would complicate our model excessively but would not change our results in a significant way.

6. Individuals vote for or against A. The result of the vote is observed by everyone and is verifiable.

7. The policy outcome is determined according to the constitution in place at the time.

It is important that c, which controls the consequences of policy A, is observed after amendments to the constitution have been made and after any crisis has been resolved. One can think of policies A and B as bundles of decisions – policy programs – which differ in their cost to losers (depending on c), but generate the same set of winners and losers. The assumption that the value c is not revealed until after winners and losers (W and W^C) have been identified can then be interpreted as saying that the exact details of these programs are unknown at the time when individuals learn whether a change to the status quo will increase or decrease their welfare. In other words, what matters in determining whether a crisis occurs or not, is not whether losers suffer losses z, but whether the constitution allows such losses to occur.[37]

The key assumption, however, is that ω cannot be verified. While the factors (other than group size) that determine the actual strength of each group and their ability to generate a crisis may be easy to recognize ex post, they cannot easily be described ex ante and for that reason they cannot be part of the constitution. This is what leaves room for amendment rules that allow individuals to respond to circumstances as they arise. Before we demonstrate this point formally, it is useful to analyze the benchmark in which ω can, in fact, be verified. Throughout, we assume that F is uniform.[38]

6.1. Optimal Constitutions when ω is Verifiable

We begin by studying the case in which both ω and c can be verified. This allows us to characterize the first-best constitution. This obviously makes amendment procedures irrelevant, but it provides an useful benchmark. We refer to the first-best constitution with verifiable ω as the VFB constitution.

When a crisis is not possible ($\omega = \underline{\omega}$), the constitutional problem is identical to the one considered in section 4.1, i.e.,

$$\begin{aligned} &\max_{m,n} EU(m,n) \\ &= \eta \int_m^1 u_{\underline{c}}(p)\, dp + (1-\eta) \int_n^1 u_{\overline{c}}(p)\, dp. \end{aligned}$$

[37] As in section 5, the fact that c is unknown by the time winners and losers are determined leaves scope for renegotiation of the majority rules. We maintain the assumption that the courts will always be able to enforce the rules agreed upon behind the veil of ignorance. Note that this is a realistic assumption: in practice, renegotiating constitutional rules whenever a new policy decision has to be made would be impractical.

[38] The assumption that F is uniform can easily be relaxed.

If $\omega = \overline{\omega}$, however, the constitutional problem is more complex:

$$\max_{m,n} \begin{cases} EU\left(m,1\right) & if \quad n=1 \\ \alpha EU\left(m,n\right) & if \quad n<1 \end{cases}.$$

By blocking all reforms with high costs ($n = 1$), all crisis can be prevented and $EU(m, 1)$ is guaranteed. Alternatively, reforms with high costs may be allowed to pass with a n-majority leaving open the possibility that a crisis may occur and be successful. Thus, $EU(m, n)$ is obtained with probability α only. The next proposition follows immediately from these observations and is presented without proof.

Proposition 8 *Suppose ω is verifiable. The VFB constitution with majority rules*

$$\left(m^*_{VFB}, n^*_{VFB}\right)(\omega) = \begin{cases} \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & if \qquad \omega = \underline{\omega} \\ \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha \geq \alpha_0) \\ \left(\frac{y}{y+w}, 1\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha < \alpha_0) \end{cases}$$

where

$$\alpha_0 = \frac{\eta\left(w+z\right)}{\eta\left(w+z\right)+\left(w+y\right)\left(1-\eta\right)} < 1$$

is the first-best constitution.

To achieve first-best, the constitutional designer needs to tailor the rules to different realizations of ω (and c). The cut-off value, α_0, indicates the point at which, conditional on a crisis being possible ($\overline{\omega}$ obtaining), it is optimal for the constitutional designer to adopt a constitution that prevents all crises from happing ($n = 1$) rather than one that allows crises to happen ($n = \frac{z}{w+z}$). We note, therefore, that if it is sufficiently unlikely that a crisis will ever be successful (α is high), it is better to allow for the occasional crisis than to prevent all crises from happening at the cost of blocking policy reforms that potentially benefit a large number of people.

6.2. Optimal Constitutions when ω is Not Verifiable

In general, it is not possible to verify if the conditions for a crisis are favorable or not and it is, therefore, impossible to make the constitutions directly contingent on ω. However, since individuals do observe what the conditions are, the constitution can, in principle, make use of announcements made by members of society about ω and design the majority rules accordingly. To study this possibility, we allow representatives for the winners and losers to make costless announcements about what the conditions for a crisis are. These announcements are common knowledge. Formally, we add two new stages (3i and 3ii) to the time line after stage 3:

3i Nature selects with equal probability one of the two representatives j to make the announcement $\widehat{\omega}_j \in \{\underline{\omega}, \overline{\omega}\}$.

3ii Upon observing $\widehat{\omega}_j$, the other representative k makes the announcement $\widehat{\omega}_k \in \{\underline{\omega}, \overline{\omega}\}$.

Given these announcements, we can define a constitution with announcements (AFB) as a pair of announcement-specific majority rules $(m, n)(\widehat{\omega})$ where m is used for reforms with low costs $\underline{c}$ and n is used for reforms with high costs $\overline{c}$ and $\widehat{\omega} = (\widehat{\omega}_j, \widehat{\omega}_k)$. The difference between this and the VFB constitution, of course, is the dependency on the announced rather than the actual value of ω. The next proposition characterizes the optimal announcement constitution:

Proposition 9 *Suppose that ω is not verifiable. The optimal announcement constitution is:*

1. If $\alpha \geq \alpha_0$
$$(m^*_{AFB}, n^*_{AFB})(\widehat{\omega}) = \left(\frac{y}{y+w}, \frac{z}{z+w}\right)$$

2. If $\alpha \in (\alpha_2, \alpha_0)$
$$(m^*_{AFB}, n^*_{AFB})(\widehat{\omega}) = \begin{cases} \left(\frac{y}{y+w}, 1\right) & if \quad \widehat{\omega} = \{\overline{\omega}, \overline{\omega}\} \\ \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & otherwise \end{cases}$$

3. If $\alpha \in [0, \alpha_2]$
$$(m^*_{AFB}, n^*_{AFB})(\widehat{\omega}) = \begin{cases} \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & if \quad (\alpha \geq \alpha_1) \\ \left(\frac{y}{y+w}, 1\right) & if \quad (\alpha < \alpha_1) \end{cases}$$

where
$$\alpha_2 = \frac{(w+z)\,\eta y}{wz(1-\eta) + y(z+\eta w)} < \alpha_0$$

and
$$\alpha_1 = \frac{\alpha_0 - (1-\gamma)}{\gamma} < \alpha_0.$$

Proof. See Appendix ¤

The proposition shows that the majority rules used by the optimal announcement constitution depend critically on the probability of a successful crisis, $1-\alpha$. First, whenever $\alpha \geq \alpha_0$, a crisis is not very likely to be successful, and it never optimal to prevent it from happening and the constitution uses the rules $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ no matter what the announcements are, as in the first-best. Second, when $\alpha < \alpha_2$, a crisis is likely to be successful. The two representatives have opposite interests: the representative for the winners will always claim that conditions are favorable for a crisis (to get reforms blocked), while the representative for the losers will claim the opposite, no matter what the true conditions are. Thus, the announcements are not informative and the majority rules cannot be made contingent on them. The problem facing the constitutional designer then is:

$$\max_{m,n} \begin{cases} EU(m,1) & if \quad n = 1 \\ (\gamma\alpha + (1-\gamma)) EU(m,n) & if \quad n < 1 \end{cases}$$

This problem is similar to the one studied in section 6.1, except that the probability of avoiding a crisis when reforms with high costs are allowed to pass ($n < 1$) is $(\gamma\alpha + (1-\gamma))$ rather than α. This implies that it is optimal to set $n = 1$ for $\alpha < \alpha_1$ and to use $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ otherwise. We notice that the critical value α_1 is lower than α_0 – the critical value applicable when ω is verifiable. This is because the constitution cannot distinguish between favorable and unfavorable conditions for a crisis and so, if it blocks reforms with high costs, it does so, not only when a crisis is possible, as in the first-best, but also when a crisis is not possible. As a consequence, the constitutional designer is willing to run the risk of a crisis for lower values of α than in the first-best.

Third, the most interesting case is when $\alpha \in (\alpha_2, \alpha_0)$. In this case, it is possible to elicit the information about the conditions for a crisis truthfully and, thus, to use the first-best majority rules. The point is that both representatives agree that blocking all reforms with high costs ($n = 1$) is optimal when conditions for a crisis are favorable ($\overline{\omega}$), but disagree when conditions are not favorable. Thus, by using the majority rule $n = 1$ if and only if the two representatives announce the same thing, the announcement constitution can mimic the VFB constitution. The reasons why the two representatives agree about the optimal rule when conditions for a crisis are favorable are, however, very different. The winners want $n = 1$ because, from their point of view, it is too likely that a crisis will be successful. By setting $n = 1$ and preventing a crisis for sure, reforms with low costs are allowed to pass with a m-majority and that is better than running the risk of not getting w at all. The losers, on the other hand, want $n = 1$ because, from their point of view, a crisis (which blocks all reforms) is not sufficiently likely to succeed, and it is better to insure that at least reforms with high costs are never adopted.

6.3. Optimal Constitutions with Amendment Rules

Another way to deal with the fact that the conditions for a crisis cannot be verified is to grant individuals the right to amend the original constitution provided that a sufficiently large majority – defined in the amendment rule of the original constitution – is in favor of an amendment. This is very different from the announcement constitution. In that constitution (and in all the other ones we have considered so far) the majority rules are determined from behind the veil of ignorance; that is, all rules are part of the original constitution. In contrast, amendment rules are meta-rule that define the circumstances under which a new set of majority rules (a new constitution) can be adopted **after** the veil of ignorance has been lifted. This allows individuals to respond to circumstances as they arise by change the rules that govern policy making.

To model the idea of constitutional amendment, we need to specify the details of the amendment procedure. Amendments take place after stage 3 in the time line as follows:

3I. Nature selects one of the representatives j to propose a new constitution, denoted by $\{\overline{m}_j(\underline{c}), \overline{n}_j(\overline{c}), r_j\}$ where $\overline{m}_j(\underline{c})$ and $\overline{n}_j(\overline{c})$ are the majority rules that apply to $\underline{c}$ and $\overline{c}$ respectively and r_j is an amendment rule. After that, k proposes another constitution $\{\overline{m}_k(\underline{c}), \overline{n}_k(\overline{c}), r_k\}$.

3II. Individuals vote sequentially for or against the proposed constitutional changes. Let $\{m, n, r\}$ be the original constitution and let v_j be the votes in favor of the new constitution proposed by representative j. If $v_j \geq r$, then $\{\overline{m}_j(\underline{c}), \overline{n}_j(\overline{c}), r_j\}$ replaces the current constitution.

3III. If $v_j \geq r$ and $v_k \geq r_j$ then $\{\overline{m}_k(\underline{c}), \overline{n}_k(\overline{c}), r_k\}$ replaces $\{\overline{m}_j(\underline{c}), \overline{n}_j(\overline{c}), r_j\}$ while if $v_j < r$ and $v_k \geq r$ then $\{\overline{m}_k(\underline{c}), \overline{n}_k(\overline{c}), r_k\}$ replaces $\{m, n, r\}$.[39]

We can now defined what an amendable (AM) constitution is.

Definition 6 *An Amendable (AM) constitution is a triplet $\{m, n, r\}$ with cost dependent majority rules $m \in [0,1]$ and $n \in [0,1]$, and an amendment rule $r \in [0,1]$ such that if for any $l \in \{j, k\}$, $v_l \geq r$, then the original constitution $\{m, n, r\}$ is replaced by $\{\overline{m}_l, \overline{n}_l, r_l\}$ and no replacement otherwise.*

This definition highlights two important points. First, an amendable constitution is not a ω-dependent mechanism as the original constitution $\{m, n, r\}$ does not depend on ω. This is crucial because an AM constitution is, in general, **not** outcome-equivalent to an announcement constitution. This provides a normative justification for amendment rules provided that the optimal AM constitution yields higher expected utility than the optimal constitution with announcements characterized in proposition 9. Second, the AM

[39] If no-one proposes an alternative to the original constitution, or if the proposals are all identical to the current constitution, then, trivially, the constitution is unchanged and steps 3II and 3III do not apply.

constitution takes the notion of constitutional amendment seriously in the sense that the amendment rules do not have direct policy implications but only through a change in the constitutional rules. In other words, r determines whether $\{m, n\}$ or $\{\overline{m}_l, \overline{n}_l\}$ apply, not whether $x = A$ or $x = B$. This is precisely why announcements and amendable constitutions can generate different outcomes.[40]

Proposition 10 *The optimal AM constitution is unique and given by*

$$\left(m^*_{AM}, n^*_{AM}, r^*_{AM}\right) = \left(\frac{y}{y+w}, 1, \frac{z}{z+w}\right)$$

and is it outcome-equivalent to a VFB constitution with

$$(m, n)(\omega) = \begin{cases} \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & if \quad \omega = \underline{\omega} \\ \left(\frac{y}{y+w}, \frac{z}{z+w}\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha \geq \eta) \\ \left(\frac{y}{y+w}, 1\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha < \eta) \end{cases}$$

Proof. See Appendix ¤

It is easy to give an intuition for this result. Given that the realization ω has nothing to do with $\underline{c}$, it is optimal to set $m = \frac{y}{y+w}$ just as we would have done had ω been verifiable. More interestingly, setting $r = \frac{z}{z+w}$ and $n = 1$ guarantees that if $\omega = \underline{\omega}$, a policy with $\overline{c}$ will obtain if and only if $p \geq \frac{z}{z+w}$ just as it would have done with a VFB constitution. This is because amendments are possible if and only if $p \geq r = \frac{z}{z+w}$ and given that the constitutional amendment passes, it is then optimal to set $m = \frac{y}{y+w}$ and to pick a $n \leq \frac{z}{z+w}$.

The difference between the VFB constitution and the AM constitution arises when $\omega = \overline{\omega}$. In this case, the change from $n = 1$ to some value $n \leq \frac{z}{z+w}$ can occur at lower values of α because individuals who know they are winners trade-off majority rules and the probability of a crisis differently from individuals behind the veil of ignorance. In particular, winners have more to lose from a majority rule that makes high costs to losers impossible than an individual behind the veil of ignorance who might actually turn out to be a loser.

The fact that optimal AM constitution is outcome-equivalent to a sub-optimal VFB constitution allows us to compare optimal AM and announcement constitutions against the optimal VFB constitution. Note that $\alpha_2 < \eta < \alpha_0$.

[40] We assume $w < z$. This guarantees that winners and losers cannot both satisfy the amendment rule by themselves.

1. For $\alpha < \eta$ the optimal AM constitution is equivalent to the optimal VFB constitution.

2. For $\alpha \in (\alpha_2, \alpha_0)$ the optimal announcement constitution is equivalent to the optimal VFB constitution. In particular, since $\alpha_2 < \eta$, the announcement constitution is the only constitution equivalent to the optimal VFB constitution in the interval $[\eta, \alpha_0)$.

3. For $\alpha \geq \alpha_0$ both the optimal AM and the announcement constitution are equivalent to the optimal VFB constitution.

Proof. Follows immediately from the analysis above ¤

The constitutional designer faces a choice between a flexible and a rigid constitution. The AM constitution is flexible because it allows the majority rules to be tailored to the realization of ω, but because this occurs only after the veil of ignorance is removed, it cannot be done with the impartiality that the veil of ignorance provides: the new constitution favors some individuals.[41] For small values of α, i.e. for any $\alpha < \eta$ the risk of a successful crisis is so high that winners can be trusted to make the right choice despite their impartiality. Thus, the optimal AM constitution fully replicates the VFB constitution and performs as if ω was verifiable.

The announcement constitution, on the other hand, is rigid in the sense that the majority rules are chosen from amongst a pre-determined set of rules. Nonetheless, it preforms better than the flexible AM constitution whenever the possibility of a successful crisis is neither too high nor too low ($\alpha \in [\eta, \alpha_0)$). The reason for this is that the announcement constitution can for probabilities of a successful crisis in this range elicit the true conditions of a crisis. Thus, a rigid constitution is the preferred mechanism for some cases in which the threat of a successful crisis is neither too high nor too low.

The above gives us interesting predictions regarding the degree of flexibility and rigidity in constitutions. The main message is that the greater flexibility associated with constitutional amendment should be associated with cases in which the stability of the political system is in more danger. With regard to AFB constitutions, note that the announcement phase in these constitutions is very similar to many of the procedures that actual constitutions use to declare states of emergency which often require different powers (e.g. executive and legislative) to agree.[42] Our results suggest that such mechanisms are particularly suited to situations when the risk to the stability of the political system are not too high.

[41] Notice that the majority rules of the VFB constitution depend on the realization of ω but how is determined already behind the veil of ignorance, Therefore, the VFB constitution provides flexibility and impartiality at the same time.

[42] [22], pages 143-144, argue that in most presidential systems, presidents have the power to declare that a state of emergency exists. However, it is interesting to note that with few exceptions (e.g. France) these power have to be exercised in agreement with the legislative branch.

7. Checks and Balances

A recent literature in political economics has emphasized on checks and balances as an instrument to reduce agency problems between politicians and citizens (see e.g., [20] for a fundamental contribution in this regard). While it is clear that checks and balances do play an important role in this regard, it is not the only role that they play. We argue that checks and balances can also serve to provide protection to identifiable groups of citizens by granting them a veto on decisions that are important to them. The leading example of this is a federation where some policy reforms must be approved by voters in each region separately to pass. In Belgium, for example, the constitution explicitly defines both linguistic communities and geographical regions which are granted specific rights. In particular, Articles 4 and 5 among other things state that federal executive power can only be imposed on a territory if it is approved by a separate majority vote in each of the four linguistic regions of the country. The European Union is another example of a polity where many key decisions require separate approval by each member state to pass.

The purpose of this section is to demonstrate that checks and balances, understood as separate decision rules for specific sub-groups of society, can be optimal. However, we also stress that there are costs as well as benefits associated with their introduction. Thus, we will show the mere fact that specific (linguistic or ethnic) groups can be identified and group membership can be verified is not sufficient to call for checks and balances.

7.1. The Setup

We assume that the population is partitioned into to disjoint groups, $h \in \{S, T\}$ according to some readily observable and verifiable characteristics. For example, group affiliation can be determined by geography or ethnicity. At the constitutional stage, individuals do not yet know their group affiliation, only that they face an equal chance of belonging to either group.[43] Group affiliation matters because of differences in the potential loss under policy A. We assume that $z_T > z_S$ and thus that individuals belonging to group T are more exposed to loses than individuals belonging to group S.[44] The fraction of individuals in group h that gains from policy A (the winners in group h) is denoted p_h, where p_S and p_T are drawn independently from the same cumulative distribution $F(p)$. To obtain closed form solutions, we assume that $F(p)$ follows a uniform distribution on $[0, 1]$. We focus on the case where the costs are observe ex post and allow for two policy areas, as in section 5. For simplicity, we assume that reforms can be allocated at zero cost to the

[43] The assumption that an individual from behind the veil of ignorance believes that he or she is as likely to belong to one group as to the other can be relaxed and would not change the results substantially.

[44] This is just one dimension in which groups might differ. An alternative is to assume that the likelihood of losing under policy A differs between groups. We conjecture that this would lead to results very similar to those presented in the text.

correct policy area and that $\rho_1 > \rho_2$. Thus, all the constitutions we consider below will be based on policy area specific rules of one sort or the other and we can without loss of generality ignore the announce stage (3a-3c). We can summarize the timing of events by the following time line:

1. From behind the veil of ignorance, a representative individual (the constitutional designer) designs a constitution.

2. Nature determines group affiliation. This is done by making a sequence of independent draws from a Bernoulli distribution with $\Pr(h = S) = \frac{1}{2}$. This is observable by everyone and is verifiable.

3. Nature selects p_h and ϕ. The realization of ϕ can be observed (and verified) at zero cost.

4. Given p_h, individuals in group $h \in \{S, T\}$ are partitioned into the two sets W and W^C by Nature.

5. Individuals vote for or against policy A. The vote result is observed by everyone.

6. The policy outcome is determined according to the constitutional rules laid down in 1.

7. Given ϕ, c is determined.

7.2. Optimal Constitutions

Can checks and balances be optimal? By checks and balances we mean a situation in which different decision rules apply to different groups and that policy A must therefore pass separate tests in the two groups to be adopted. In a sense, checks and balances give each group the right to veto a reform, but only if a sufficiently large number of individuals in the group supports the veto. In contrast, in the absence of checks and balances, policy A passes if it has sufficient support in the entire population, and consequently support from one group can compensate for opposition from the other: support is transferable.

Formally, we can define a Constitution with No Checks and Balances (NCB) is defined as follows:

Definition 7 *A NCB constitution is a pair $\{m, n\}$ with $m \in [0,1]$ and $n \in [0,1]$ such that*

$$x = \begin{cases} A \quad if \quad [p_S + p_T \geq 2m \vee \phi = \phi_1] \wedge [p_S + p_T \geq 2n \vee \phi = \phi_2] \\ B \qquad\qquad\qquad\qquad otherwise \end{cases}$$

This constitution requires that a m-majority be formed from either of the two groups to support a reform in policy area 1 for it to pass and similarly for reforms in policy area 2. In contrast, a Constitution With Checks and Balances (CB) requires that policy A obtains a majority separately in each of the two groups. Formally,

Definition 8 *A CB constitution is a set of group-specific majority rules* $\{m_S, m_T, n_S, n_T\} \in [0,1]^4$ *such that*

$$x = \begin{cases} A & if \quad (p_S \geq m_S \vee \; p_T \geq m_T \vee \phi = \phi_1) \wedge (p_S \geq n_S \vee \; p_T \geq n_T \vee \phi = \phi_2) \\ B & \qquad\qquad\qquad\qquad otherwise \end{cases}.$$

Thus, with checks and balances, a reform in policy area 1 passes only if a m_S-majority of group S and a m_T-majority of group T support it, and similarly for policies in area 2. Combinations of the two constitutions are, of course, possible and we can define a Constitution with Partial Checks and Balances (PCB) as follows:

Definition 9 *A PCB constitution is either a set of majority rules* $\{m, n_S, n_T\} \in [0,1]^3$ *such that*

$$x = \begin{cases} A & if \quad (p_S + p_T \geq 2m \vee \phi = \phi_1) \wedge (p_S \geq n_S \vee \; p_T \geq n_T \vee \phi = \phi_2) \\ B & \qquad\qquad\qquad\qquad otherwise \end{cases}$$

or a set of majority rules $\{m_S, m_T, n\} \in [0,1]^3$ *such that*

$$x = \begin{cases} A & if \quad (p_S \geq m_S \vee \; p_T \geq m_T \vee \phi = \phi_1) \wedge (p_S + p_T \geq 2n \vee \phi = \phi_2) \\ B & \qquad\qquad\qquad\qquad otherwise \end{cases}$$

For each of these constitutions, the constitutional designer selects the majority rules that maximizes expected utility from behind the veil of ignorance.[45] For the NCB constitution the following two rules are selected:

$$m^*_{NCB} = \frac{(1-\rho_1)z_E + \rho_1 w}{(1-\rho_1)z_E + (\rho_1 + 1)\, w} \tag{7.1}$$

$$n^*_{NCB} = \frac{(1-\rho_2)z_E + \rho_2 w}{(1-\rho_2)z_E + (\rho_2 + 1)\, w} \tag{7.2}$$

where $z_E \equiv \frac{z_S + z_T}{2}$. In contrast, the four that define the CB constitution are given by

$$m^*_{CB,h} = \frac{(1-\rho_1)z_h + \rho_1 w - \frac{w}{3}}{(1-\rho_1)z_h + (1+\rho_1)w} \tag{7.3}$$

[45] See the Appendix for more details. We have assumed to simply the algebra that $w = y$.

$$n^*_{CB,h} = \frac{(1-\rho_2)z_h + \rho_2 w - \frac{w}{3}}{(1-\rho_2)z_h + (1+\rho_2)w} \tag{7.4}$$

for $h \in \{S, T\}$. As we would expect, the rules associated with policy area 2 are stricter than those associated with area 1 in both cases, and under the CB constitution, group T, which has most to loss from policy A, are assigned stricter rules than group S with less to lose. The PCB constitution with checks and balance for one policy area only uses a combination of these rules: the NCB rules for the policy area without checks and balances and the CB rules for the area with checks and balances.

We can characterize the optimal constitutional choice:

Proposition 11 *Assume $\rho_1 > \rho_2$, $w = y$ and $z_T > z_S$. The optimal constitution is*

1. For $z_T \in [z_S, \overline{z}_T)$, the NCB constitution is optimal;
2. For $z_T \in [\overline{z}_T, \widehat{z}_T)$, the PCB constitution with checks and balances for policy area 2 is optimal;
3. for $z_T \geq \widehat{z}_T$, the CB constitution is optimal;

where

$$\overline{z}_T = \frac{2(1-\rho_2)z_S + (1+\rho_2)w}{(1-\rho_2)} > z_S;$$

$$\widehat{z}_T = \frac{2(1-\rho_1)z_S + (1+\rho_1)w}{(1-\rho_1)} > \overline{z}_T;$$

Proof. See Appendix.

The proposition shows that it is optimal to use checks and balances when the two groups are sufficiently different: for small cost differences, the NCB constitution continues to be optimal. For intermediate cost differences, it is optimal to use a constitution with a universal rule for decisions in policy area 1, but group-specific rules for decisions in policy area 2, i.e., to give the two groups a veto over policies in area 2. For large cost differences, it is optimal to use group-specific rules for all policy areas.

What is the intuition behind these results? The advantage of having checks and balances is that policy reforms cannot pass without sufficient support from all groups. In contrast, the advantage of universal rules is that they allow support from one group to compensate for opposition from the other, thereby allowing policy reforms to pass if most individuals in society at large, are in favor. Thus, the constitutional designer faces a trade off between giving veto rights to each group and making political support transferable. How this trade-off is resolved depends on the cost difference $z_T - z_S$. If the cost difference is large, the constitutional designer knows that if she keeps universal rules she takes on a large risk. If once the veil of ignorance is lifted, $c = \overline{c}$, and she is a loser in group T, policy A will pass too easily. On the other hand, if the cost difference is small, then it is desirable to avoid checks and balances and allow support to be transferable: the cost of allowing group T to block policy A is too large in the case where the constitutional designer belongs to the low cost group (group S). Since we assume that policies in policy area 2 are more likely to generate large losses, it is clear that it is, *ceteris paribus*, more beneficial to have checks and balances for this policy area than for policy area 1. This explains the intermediate regime with partial checks and balances: checks and balances are introduced only for policy areas for which it is likely that losses are large.

We have argued that policy reforms sometimes have very different consequences for different groups of citizens and shown that this line of reasoning provides an alternative rationale for having checks and balances in constitutions. Our analysis shows that societies might allow specific groups to hold a veto over policy reforms in some or in all policy areas. Put differently, optimal constitutions may require that policy reforms have sufficient support among all (relevant) groups in society. Checks and balances are optimal from the original position when group affiliation matters sufficiently. We focus on cost differences, but similar results can be derived when heterogeneity arises from the fact that individuals in one group are more likely to lose from reforms than individuals in other groups. An implication, then, is that checks and balance are of greater value in heterogeneous societies and may, in fact, be instrumental in preventing such societies from splitting up into smaller units.

Although we focus on the case where group affiliation is unknown at the constitutional stage (which allows us to use the veil of ignorance device to analyze the constitutional choice), our framework can be of guidance in thinking about the more common situation where group affiliation is known already at the constitutional stage but checks and balances might be necessary to ensure some group's participation. In the European Union, for example, a fundamental question is if member countries should be allowed to veto certain decisions or if decisions should be made by majority voting. Our results suggest that it is optimal to allow such vetoes for policy areas where national interests differ sufficiently (e.g. foreign policy and defense), but not for areas where costs and benefits are more evenly distributed across countries (e.g., immigration policy) just as is the case in the draft EU constitution.

It is clear that checks and balances of the kind considered above can only be introduced in situations where it is possible to identify and verify group affiliation at low cost. This raise the broader question of why checks and balances are not used more widely, e.g., why do modern constitutions not make a distinction between males and females and allow gender groups to have veto on specific decisions? Our model suggests that this may be due to the fact that for most policy areas men and women face a very similar distribution of costs and benefits, but it would be interesting to explore other reasons in future research.

8. Conclusions

In this paper we have developed a framework for analyzing optimal constitutional design from a normative perspective. Contrary to much of the recent literature on the issue, we do not focus on exogenously given constitutional mechanisms (e.g. majority rules) and studied how they change with changes in specific parameters in the environment. Instead, we study what constitutional mechanisms are optimal responses to the particular features of the environment.

The collective decision problem we have described in this paper is extremely simple. As mentioned above, the most obvious omissions is that we do not look at the agency issues involved in the relationship between citizens and politicians and we do not have an endogenous mechanism for developing policy proposal. A natural next step would then be to incorporate some of these features in a more sophisticated framework to see how the optimal constitutions would evolve in response to these features[46].

A second interesting area for future research would be an analysis of the issues from a positive perspective. As mentioned in the introduction this is not a simple task because there is always a danger that the results are not robust to the details of the constitutional bargaining game. Nevertheless, this is a necessary step if we wish to improve our understanding of the foundations of democracies.

[46] [2] take a first step in this direction by making the policy maker a politician who also cares about private benefits and can choose policies that provide him with private benefits. The optimal majority rules thus responds to the politician's incentives.

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Appendix A: Proofs

Proof of Proposition 3.

The constitutional designer selects rules $m \in [0,1]$, $n \in [0,1]$ and $\lambda \geq 0$ to maximize expected utility from behind the veil of ignorance, taking into account that cost dependent rules require that the cost realization is truthfully revealed in stage 4. Stage 4 describes a game between the two representatives. They simultaneously decides how large a cost they are willing to bear to change the default rule (m) to the alternative rule (n), that is, their strategies are $\sigma_j \in R^+$ and $\sigma_k \in R^+$.

Suppose, first, that the constitution specifies m, n and $\lambda \geq 0$ with $n \geq m$. It is a dominant strategy for the representative for the winners to play $\sigma_j = 0$. Consider the representative for the loser. We want to identify the values of λ that makes it a dominant strategy for the representative to play $\sigma_k = \lambda$ if and only if $c = \overline{c}$. Let

$$\mu(c,p) = \Pr(x = A|c,p)$$

be the probability that policy A is chosen as a function of c and p. Then, $\sigma_k = \lambda$ if and only if $c = \overline{c}$ requires the following incentive compatibility constraints to be satisfied:

$$\int_0^1 -z\mu(\overline{c},p)\,dF(p) - \lambda \geq \int_0^1 -z\mu(\underline{c},p)\,dF(p)$$

and

$$\int_0^1 -y\mu(\overline{c},p)\,dF(p) - \lambda \leq \int_0^1 -y\mu(\underline{c},p)\,dF(p).$$

The minimum switching cost consistent with these constraints is

$$\lambda = \int_0^1 y\left[\mu(\underline{c},p) - \mu(\overline{c},p)\right] dF(p)$$

Note that we must require that $\mu(\underline{c},p) > \mu(\overline{c},p)$ because the switching cost must be non-negative. The objective function of the constitutional designer can be written as

$$\eta \int_0^1 (p(w+y) - y)\,\mu(\underline{c},p)\,dF(p) + (1-\eta)\int_0^1 (p(w+z) - z)\,\mu(\overline{c},p)\,dF(p)$$
$$- \int_0^1 (1-p)dF(p)\left((1-\eta)\,y\int_0^1 \left[\mu(\underline{c},p) - \mu(\overline{c},p)\right] dF(p)\right),$$

where $\int_0^1 (1-p)dF(p) = 1-\psi$. This expression can be rearrange to obtain

$$\int_0^1 \mu(\underline{c},p)\left[\eta(p(w+y) - y) - (1-\psi)(1-\eta)\,y\right] dF(p)$$
$$+ \int_0^1 \mu(\overline{c},p)\left[(1-\eta)(p(w+z) - z + (1-\psi)y)\right] dF(p)$$

We see that this expression is maximized whenever

$$\mu\left(\underline{c},p\right)=\begin{cases}1 & if \quad \eta\left(p\left(w+y\right)-y\right)-\left(1-\eta\right)\left(1-\psi\right)y\geq 0\Leftrightarrow p\geq\frac{y-\psi(1-\eta)y}{\eta(w+y)}\\ 0 & otherwise\end{cases}$$

and

$$\mu\left(\overline{c},p\right)=\begin{cases}1 & if \quad \left(1-\eta\right)\left(p\left(w+z\right)-z+\left(1-\psi\right)y\right)\geq 0\Leftrightarrow p\geq\frac{z-y(1-\psi)}{w+z}\\ 0 & otherwise\end{cases},$$

respectively. Define

$$\eta^{*}=\frac{\left(1-\psi\right)y\left(w+z\right)}{\left(z-y\right)\left(\left(1-\psi\right)y+w\right)},$$

Then, for $\eta>\eta^{*}$, the constitution with $m=\frac{y-\psi(1-\eta)y}{\eta(w+y)}\geq n=\frac{z-y(1-\psi)}{w+z}$ and $\lambda=y\left[F(\frac{z-y(1-\psi)}{w+z})-F(\frac{y-\psi(1-\eta)y}{\eta(w+y)})\right]$ implements this. For $\eta<\eta^{*}$, this constitution is not feasible because λ would have to be negative. Given that, it is optimal to set $\lambda=0$ and use one common rule equal to m_{MR}^{*}.

Suppose, next, that the constitution specifies m, n and $\lambda\geq 0$ with $n<m$. It is a dominant strategy for the representative for the losers to play $\sigma_k=0$. Since the winners get w independent of c if policy A passes, there exists one and only one value of λ that satisfies both incentive compatibility constraints simultaneously:

$$\lambda=\int_{0}^{1}w\left[\mu\left(\underline{c},p\right)-\mu\left(\overline{c},p\right)\right]dF\left(p\right).$$

As a consequence, no matter how we break this indifference (always pay, never pay or randomize), truthful revaluation is not possible in all cases, and we can rule this situation out. Finally, the comparative statics and limits of η^{*} follow by inspection ¤

Proof of Proposition 4 We are looking at the class of mechanisms where there are two possible probabilities that policy A is chosen: $\mu\left(\varnothing,p\right)$ and $\mu\left(V,p\right)$ with $\mu\left(\varnothing,p\right)<\mu\left(V,p\right)$. Vetoes determine which of the two probabilities are chosen and we need to determine the optimal number of vetoes as well.

Given that there are s vetoes available, we can characterize the optimal behavior of the loser's representative in each period v. Note that for winners calling a veto at time v is always a (weakly) dominated strategy so that we focus on equilibria where winners never call vetoes. Consider losers: if $c=\overline{c}$ in period v and $s_v\geq 1$ then the veto will be used because in each subsequent period, expected costs $y\eta+z\left(1-\eta\right)$ are lower than the current costs z.

If $c=\underline{c}$, then by the argument above, there is a veto because the loss today y is smaller than the expected loss $y\eta+z\left(1-\eta\right)$ for any future period unless there are enough vetoes left to cover all remaining periods. That is, unless $s_\upsilon \geq t-\upsilon+1$. This defines the equilibrium strategies $\left(\chi_j^\upsilon\right)_{\upsilon=1}^t$ and $\left(\chi_k^\upsilon\right)_{\upsilon=1}^t$.

Now define the costs associated with this mechanism from the perspective of the original position. One is the cost of having a veto when there should not be one, ϑ_1 and the other is the cost of not having a veto when there should be one, ϑ_2. Formally

$$\begin{aligned}
\vartheta_1 &= \int_0^1 \left[\mu(\varnothing,p)-\mu(V,p)\right](p(w+y)-y)\,dF(p) \\
\vartheta_2 &= -\int_0^1 \left[\mu(\varnothing,p)-\mu(V,p)\right](p(w+z)-z)\,dF(p)
\end{aligned}$$

Given these costs, it is easy to see that the probability that exactly s realizations of $\overline{c}$ will occur in t periods is a Binomial distribution with mean $(1-\eta)t$ and variance $\eta(1-\eta)t$. Thus, the per-period expected utility as seen from the original position is

$$\begin{aligned}
&\eta\int_0^1 \mu(\varnothing,p)(p(w+y)-y)\,dF(p)+(1-\eta)\int_0^1 \mu(V,p)(p(w+z)-z)\,dF(p)- \\
&-\frac{1}{t}\vartheta_1(\mu(V,p),\mu(\varnothing,p))\left(\sum_{k=0}^{s-1}(s-k)\binom{t}{k}(1-\eta)^k\eta^{t-k}\right) \\
&-\frac{1}{t}\vartheta_2(\mu(V,p),\mu(\varnothing,p))\left(\sum_{k=s}^{t}(k-s)\binom{t}{k}(1-\eta)^k\eta^{t-k}\right)
\end{aligned}$$

or, equivalently,

$$\begin{aligned}
&\eta\int_0^1 \mu(\varnothing,p)(p(w+y)-y)\,dF(p)+(1-\eta)\int_0^1 \mu(V,p)(p(w+z)-z)\,dF(p) \\
&-\frac{1}{t}G(s,t)\int_0^1 \left[\mu(\varnothing,p)-\mu(V,p)\right](p(w+y)-y)\,dF(p) \\
&+\frac{1}{t}H(s,t)\int_0^1 \left[\mu(\varnothing,p)-\mu(V,p)\right](p(w+z)-z)\,dF(p)
\end{aligned}$$

where

$$\begin{aligned}
G(s,t) &= \sum_{k=0}^{s-1}(s-k)\binom{t}{k}(1-\eta)^k\eta^{t-k}=\sum_{x=0}^{s}\sum_{k=0}^{x-1}\binom{t}{k}(1-\eta)^k\eta^{t-k} \\
H(s,t) &= \sum_{k=s}^{t}(k-s)\binom{t}{k}(1-\eta)^k\eta^{t-k}=(1-\eta)t+\sum_{x=0}^{s}\sum_{k=0}^{x-1}\binom{t}{k}(1-\eta)^k\eta^{t-k}-s
\end{aligned}$$

Using these expressions and rearranging gives us

$$\begin{aligned}
&\int_0^1 \mu(\varnothing,p)\left[\left(\eta-\frac{1}{t}G(s,t)\right)(p(w+y)-y)+\frac{1}{t}H(s,t)(p(w+z)-z)\right]dF(p) \\
&+\int_0^1 \mu(V,p)\left[\left(1-\eta-\frac{1}{t}H(s,t)\right)(p(w+z)-z)+\frac{1}{t}G(s,t)(p(w+y)-y)\right]dF(p)
\end{aligned}$$

which is maximized if

$$\mu(\varnothing,p)=\begin{cases}1 \quad if & \begin{array}{c}\left(\eta-\frac{1}{t}G(s,t)\right)(p(w+y)-y)-\frac{1}{t}H(s,t)(p(w+z)-z)\geq 0\\ \Leftrightarrow p\geq \frac{\left(\eta-\frac{1}{t}G(s,t)\right)y+z\frac{1}{t}H(s,t)}{\left(\eta-\frac{1}{t}G(s,t)\right)(w+y)+(w+z)\frac{1}{t}H(s,t)}=m(s)\end{array}\\ 0 & otherwise\end{cases}$$

and

$$\mu(V,p)=\begin{cases}1 \quad if & \begin{array}{c}\left(1-\eta-\frac{1}{t}H(s,t)\right)(p(w+z)-z)-\frac{1}{t}G(s,t)(p(w+y)-y)\geq 0\\ \Leftrightarrow p\geq \frac{\left((1-\eta)-\frac{1}{t}H(s,t)\right)z+y\frac{1}{t}G(s,t)}{\left((1-\eta)-\frac{1}{t}H(s,t)\right)(w+z)+(w+y)\frac{1}{t}G(s,t)}=n(s)\end{array}\\ 0 & otherwise\end{cases}$$

Thus, given the veto strategies $\left(\chi_j^v\right)_{v=1}^t$ and $\left(\chi_k^v\right)_{v=1}^t$ our mechanism with $m(s^*)$ and $n(s^*)$, where

$$\begin{aligned}s^* &= \arg\max_{0\leq s\leq t}\int_{m(s)}^1\left[\left(\eta-\frac{1}{t}G(s,t)\right)(p(w+y)-y)+\frac{1}{t}H(s,t)(p(w+z)-z)\right]dF(p)\\ &\quad+\int_{n(s)}^1\left[\left(1-\eta-\frac{1}{t}H(s,t)\right)(p(w+z)-z)+\frac{1}{t}G(s,t)(p(w+y)-y)\right]dF(p)\end{aligned}$$

implements the optimal veto mechanism. In particular, it is easy to see that $m(s)$ and $n(s)$ are well-defined, strictly decreasing functions of s such that $m(s)>\frac{y}{y+w}$ and $n(s)<\frac{z}{w+z}$ for any $1\leq s\leq t-1$. Further, if $s=0$ then $m(0)=m_{MR}^*$ while $n(0)$ is not well defined. In this case, any $n>m_{MR}^*$ will do since vetoes are not available and n will never be used. Similarly, if $s=t$, then $n(t)=m_{MR}^*$ while $m(t)$ is not well-defined. In this case, any $m<m_{MR}^*$ will do since vetoes will always be used and m is redundant.

For large enough values of t, we can use the Normal approximation to the Binomial distribution to determine the optimal value of s. Let $\widetilde{\Phi}(s)$ be the cumulative of the Normal distribution with mean $(1-\eta)t$ and variance $\eta(1-\eta)t$. Note that

$$G(s,t)\approx\int_{-\infty}^s\widetilde{\Phi}(x)\,dx$$
$$H(s,t)\approx(1-\eta)t+\int_{-\infty}^s\widetilde{\Phi}(x)\,dx-s$$

So, using Leibnitz's rule, the FOC for s becomes

$$\begin{gathered}-\widetilde{\Phi}(s)\frac{1}{t}\int_{m(s)}^{n(s)}(p(w+y)-y)\,dF(p)-\left(1-\widetilde{\Phi}(s)\right)\frac{1}{t}\int_{m(s)}^{n(s)}(p(w+z)-z)\,dF(p)=0\\ \Leftrightarrow\widetilde{\Phi}(s)\,\vartheta_1(m(s),n(s))=\left(1-\widetilde{\Phi}(s)\right)\vartheta_2(m(s),n(s))\end{gathered}$$

while the second derivative of the objective function with respect to s is everywhere negative which guarantees that s^* is the unique global maximum.[47]

To see that $0 < s^* < t$, note that if we consider the objective function evaluated at any such s and subtract from it the case $s = t$, we have

$$\int_{m(s)}^{1}\left[\left(\eta - \frac{1}{t}G(s,t)\right)(p(w+y)-y) + \frac{1}{t}H(s,t)(p(w+z)-z)\right]dF(p)$$
$$+\int_{n(s)}^{1}\left[\left(1-\eta - \frac{1}{t}H(s,t)\right)(p(w+z)-z) + \frac{1}{t}G(s,t)(p(w+y)-y)\right]dF(p)$$
$$-\int_{n(t)}^{1}\left[(1-\eta)(p(w+z)-z) + \eta(p(w+y)-y)\right]dF(p)$$

since $m(s) < n(t) = m^*_{MR} < n(s)$ we can rewrite the above as

$$\int_{m(s)}^{n(t)}\left[\left(\eta - \frac{1}{t}G(s,t)\right)(p(w+y)-y) + \frac{1}{t}H(s,t)(p(w+z)-z)\right]dF(p)$$
$$+\int_{n(t)}^{n(s)}\left[\left(-\frac{1}{t}G(s,t)\right)(p(w+y)-y) + \left(\frac{1}{t}H(s,t) - (1-\eta)\right)(p(w+z)-z)\right]dF(p)$$

The first term is positive since it is positive for any $p > m(s)$. By the same argument, the second term is positive for any $p < n(s)$. An identical argument holds for the case in which $s = 0$.

Finally, we need to show that the expected utility with this mechanism converges to the first-best as t goes to infinity. It is clear that if this property applies to a veto mechanism for which $s = (1-\eta)t$, then it must also apply to the more efficient mechanism where $s = s^*$. Noting that $G((1-\eta)t,t) = H((1-\eta)t,t)$, we have that

$$EU(t) =$$
$$\int_{m((1-\eta)t)}^{1}\left[\left(\eta - \frac{1}{t}G((1-\eta)t,t)\right)(p(w+y)-y) + \frac{1}{t}G((1-\eta)t,t)(p(w+z)-z)\right]dF(p)$$
$$+\int_{n((1-\eta)t)}^{1}\left[\left(1-\eta - \frac{1}{t}G((1-\eta)t,t)\right)(p(w+z)-z) + \frac{1}{t}G((1-\eta)t,t)(p(w+y)-y)\right]dF(p)$$

is the per-period expected utility with $m(s)$, $n(s)$ and $s = (1-\eta)t$. We need to show that

$$\lim_{t\to\infty} EU(t) = \eta\int_{\frac{y}{y+w}}^{1}(p(w+y)-y)\,dF(p) + (1-\eta)\int_{\frac{z}{z+w}}^{1}(p(w+z)-z)\,dF(p)$$

[47]The proof of this is tedious and is omitted. Details are available upon request.

For our purposes, it suffices to show that $\frac{1}{t}G\left(\left(1-\eta\right)t,t\right)$ converges uniformly to zero because if it does, then both $m(\left(1-\eta\right)t)$ and $n(\left(1-\eta\right)t)$ converge uniformly to $\frac{y}{y+w}$ and $\frac{z}{z+w}$ respectively, while

$$\lim_{t\to\infty}\left[\left(\eta-\frac{1}{t}G((1-\eta)\,t,t)\right)(p\,(w+y)-y)+\frac{1}{t}G((1-\eta)\,t,t)\,(p\,(w+z)-z)\right]$$
$$+\lim_{t\to\infty}\int_{n}^{1}\left[\left(1-\eta-\frac{1}{t}G((1-\eta)\,t,t)\right)(p\,(w+z)-z)+\frac{1}{t}G((1-\eta)\,t,t)\,(p\,(w+y)-y)\right]=$$
$$\eta\,(p\,(w+y)-y)+(1-\eta)\,(p\,(w+z)-z)$$

Given these, the desired result follows immediately.

Thus, we need to show that if $\Phi\left(x\right)$ is the cumulative of the standard normal distribution, then

$$\lim_{t\to\infty}\frac{\int_{-\infty}^{(1-\eta)t}\Phi\left(\frac{x-(1-\eta)t}{\sqrt{t\eta(1-\eta)}}\right)dx}{t}=0$$

This condition, using integration by parts, is equivalent to showing that

$$\lim_{t\to\infty}\frac{\int_{-\infty}^{(1-\eta)t}xd\Phi\left(\frac{x-(1-\eta)t}{\sqrt{t\eta(1-\eta)}}\right)}{t}=\Phi\left(0\right)\left(1-\eta\right)=\frac{1-\eta}{2}$$

But the left-hand side above can be rewritten as

$$\lim_{t\to\infty}\frac{1}{2t\sqrt{\eta\left(1-\eta\right)t}}\frac{\sqrt{2}}{\sqrt{\pi}}\times\frac{1}{2}t\left(1-\eta\right)\frac{1}{\sqrt{\frac{1}{\eta(1-\eta)t}}}$$
$$\times\lim_{x\to-\infty}\left(2\eta\exp\left(-\frac{1}{2}\frac{(x-t(1-\eta))^{2}}{\eta(1-\eta)t}\right)\sqrt{\frac{1}{\eta(1-\eta)t}}-2\eta\sqrt{\frac{1}{\eta(1-\eta)t}}+\sqrt{\pi}\sqrt{2}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\frac{x-t(1-\eta)}{\eta(1-\eta)t\sqrt{\frac{1}{\eta(1-\eta)t}}}\right)\right)$$

where erf represents the error function. This is equivalent to

$$\lim_{t\to\infty}\left(\frac{1}{2t\sqrt{\eta\left(1-\eta\right)t}}\frac{\sqrt{2}}{\sqrt{\pi}}\right)\times\left(\frac{1}{2}t\left(1-\eta\right)\frac{\sqrt{2}\sqrt{\pi}-2\eta\sqrt{\frac{1}{\eta(1-\eta)t}}}{\sqrt{\frac{1}{\eta(1-\eta)t}}}\right)$$
$$=\frac{1-\eta}{2}\frac{1}{\sqrt{\eta\left(1-\eta\right)}\sqrt{\frac{1}{\eta(1-\eta)}}}=\frac{1-\eta}{2}$$

as desired. ¤

Proof of proposition 5 First, consider the announcement stage 3a-3c. This represents a sequential game between the two representatives. The action spaces are announcements from the set $\{\phi_1,\phi_2\}$. We look for subgame perfect Nash equilibria. It is convenient to define the expected costs associated with the two policy areas as

$$u_i(p)=\rho_i y+(1-\rho_i)z,\quad i=1,2.$$

Assume that $n \geq m$ where n is the majority rule associated with area 2 and m is the rule associated with area 1. Note that the winners would like all decisions to be made with m while the losers would like all decisions to be made by n. Suppose Nature selects a winner to make the first announcement. Consider the strategies of the loser. If the winner announces ϕ_2, the the loser announces ϕ_2 as well. If the winner announces ϕ_1, the loser challenges this if and only if

$$[LC] \quad (1-q)\int_m^n u_2(p)dF(p) \geq \tau.$$

The condition says that the expected cost of using the rule m (wrongly) for reforms belonging to policy area 2 must be greater than the cost of a challenge. Given this, we can find the winner's equilibrium strategies. If condition $[LC]$ is satisfied, then it is optimal to announce ϕ_1 and induce a challenge if and only if

$$[WTC] \quad q\int_m^n wdF(p) \geq \tau.$$

This says that the expected cost of not obtaining policy A in area 1 because rule n is (wrongly) applied must outweigh the cost of the challenge. If condition $[WTC]$ fails (while condition $[LC]$ holds), it is optimal to announce ϕ_2. If condition $[LC]$ fails, it is optimal to announce ϕ_1. Suppose, next, that Nature selects a loser to make the first announcement. A similar analysis shows that the winners will challenge the loser's announcement of ϕ_2 if and only if

$$[WC] \quad q\int_m^n wdF(p) \geq \tau$$

and that the loser will triggered a challenge by announcing ϕ_2 if and only if

$$[LTC] \quad (1-q)\int_m^n u_2(p)dF(p) \geq \tau.$$

Notice that $[LC]$ and $[LTC]$ are identical etc. We can summarize the analysis as follows:

- The winner moves before the loser
  - [LC] and [WTC] holds: $\{m, n\}$ and a challenge results and the policy area is observed.
  - [LC] holds, but [WTC] fails: $\{n, n\}$ and no challenge
  - [LC] fails, but [WTC] holds: $\{m, m\}$ and no challenge
  - [LC] fails, but [WTC] fails: $\{m, m\}$ and no challenge

- The loser moves before the winner
  - [WC] and [LTC] holds: $\{n, m\}$ and a challenge results and the policy area is observed.
  - [WC] holds, but [LTC] fails: $\{m, m\}$ and no challenge
  - [WC] fails, but [LTC] holds: $\{n, n\}$ and no challenge
  - [WC] fails and [LTC] fails: $\{n, n\}$ and no challenge

It is easy to see that two separate cases can arise:

$$\text{Case } 1: q\int_m^n wdF(p) < (1-q)\int_m^n u_2(p)dF(p)$$

$$\text{Case } 2: q\int_m^n wdF(p) \geq (1-q)\int_m^n u_2(p)dF(p).$$

Consider case 1. The cost τ can fall in one of three regions. Region 1 is

$$\tau \leq q\int_m^n wdF(p)$$

in which a challenge will take place no matter who makes the first announcement and policy area is observed. Region 2 is

$$q\int_m^n wdF(p) < \tau < (1-q)\int_m^n u_2(p)dF(p).$$

Here, $[WC]$ and $[WTC]$ fail, but $[LC]$ and $[LTC]$ hold. Consequently, the equilibrium of the announcement game generates the announcements $\{\phi_2, \phi_2\}$ no matter who makes the first announcement. Thus, no challenge is being made and the same rule (n) is applied to all decisions. Region 3 is

$$q\int_m^n wdF(p) < (1-q)\int_m^n u_2(p)dF(p) < \tau$$

and all four conditions fail. The equilibrium of the announcement game is $\{m, m\}$ if the winner announces first and $\{n, n\}$ if the loser announces first. So, with probability $\frac{1}{2}$ the decision is made using the correct policy rule. A similar analysis applies to case 2 which completes the analysis of the announcement stage.

Second, to determine the optimal constitutional rules, we consider the decision problem of the constitutional designer in stage 1. In region 1 (of both case 1 and 2) the optimal PA constitution solves

$$\max_{m\in[0,1],n\in[0,1]} q\int_m^1 \overline{u}(p)dF(p) + (1-q)\int_n^1 \overline{v}(p)dF(p)$$

where

$$\begin{aligned}
\overline{u}(p) &= pw - (1-p)\left[\rho_1 y + (1-\rho_1) z\right] \\
\overline{v}(p) &= pw - (1-p)\left[\rho_2 y + (1-\rho_2) z\right]
\end{aligned}$$

and the rules

$$\begin{aligned}
m^*_{PA} &= \frac{\rho_1 y + (1-\rho_1) z}{w + \rho_1 y + (1-\rho_1) z} \\
n^*_{PA} &= \frac{\rho_2 y + (1-\rho_2) z}{w + \rho_2 y + (1-\rho_2) z}
\end{aligned}$$

The expected utility is $EU_{PA}(.) - \tau$. In region 2, the constitutional designer knows that only one rule is going to be applied at equilibrium and designs this rule to solve

$$\max_{\varsigma}\,(q\rho_1 + (1-q)\rho_2)\int_{\varsigma}^{1}\overline{u}(p)dF(p) + (1-(q\rho_1+(1-q)\rho_2))\int_{\varsigma}^{1}\overline{v}(p)dF(p)$$

which gives

$$\varsigma = m^*_{MR} = \frac{z - (q\rho_1 + (1-q)\rho_2)(z-y)}{w + z - (q\rho_1 + (1-q)\rho_2)(z-y)}.$$

In region 3, the constitutional designer knows that reforms are randomly allocated to the two policy areas and he solves

$$\begin{aligned}
&\max_{n,m}\eta\left(\frac{1}{2}\int_{m}^{1}\overline{u}(p)dF(p) + \frac{1}{2}\int_{n}^{1}\overline{v}(p)dF(p)\right) \\
&+(1-\eta)\left(\frac{1}{2}\int_{m}^{1}\overline{u}(p)dF(p) + \int_{n}^{1}\overline{v}(p)dF(p)\right)
\end{aligned}$$

with $\eta = (q\rho_1 + (1-q)\rho_2)$. Maximization yields

$$n = m = m^*_{MR} = \frac{z - (q\rho_1 + (1-q)\rho_2)(z-y)}{w + z - (q\rho_1 + (1-q)\rho_2)(z-y)}.$$

Thus, in the last two cases, the optimal constitution is effectively the MR constitution with expected utility $EU_{MR}(.)$. Notice that up to this point, the results are valid for any function F. However, to complete the proof, we need to assume that F is uniform.

Third, for the constitution with policy area specific rules to be optimal, two conditions must be satisfied. First, τ belongs in region 1 defined by the cut-off values

$$\begin{aligned}
A(.) &= q\int_{m^*_{PA}}^{n^*_{PA}} w dp = \frac{q(z-y)(\rho_1-\rho_2)w^2}{(w + z - \rho_2 z + \rho_2 y)(w + \rho_1 y + z - z\rho_1)}; \\
B(.) &= (1-q)\int_{m^*_{PA}}^{n^*_{PA}} u_2(p)dp = \frac{(1-q)(z-y)(\rho_2 y + z(1-\rho_2))(\rho_1-\rho_2)w}{(w + z - \rho_2 z + \rho_2 y)(w + \rho_1 y + z - z\rho_1)};
\end{aligned}$$

with $A(.) > B(.)$ if and only if $q > \frac{z-\rho_2(z-y)}{w+z-\rho_2(z-y)} \in (0,1)$. Second, the PA constitution must be better than the MR constitution, i.e.,

$$\begin{aligned}\Delta &= EU_{PA}(.) - EU_{MR}(.) \qquad\qquad (A.1)\\ &= \frac{w^2 q\,(-z+y)^2\,(\rho_1-\rho_2)^2\,(1-q)}{2\,(w+z-\rho_2(z-y))\,(w+z-\rho_1(z-y))\,(w+z-(q\rho_1+(1-q)\rho_2)(z-y))} \geq \tau.\end{aligned}$$

The following Lemma is very useful in establishing when these conditions are satisfied.

Lemma 1 $\Delta \leq \min[A(.), B(.)]$.

Suppose $q < \frac{z-\rho_2(z-y)}{w+z-\rho_2(z-y)}$ => $\min[A(.), B(.)] = A(.)$. A tedious calculation shows that

$$\text{sign}[\Delta - A(.)] = \text{sign}\left[q\,(z-y)\,(\rho_1-\rho_2)\,[g(.)]\,w^2\right]$$

where $g(.) \equiv (\rho_1(1+q) + (1-q)\rho_2)(z-y) - 2(w+z)$ which is clearly negative. Thus, $\Delta < A(.)$. Next, suppose that $q > \frac{z-\rho_2(z-y)}{w+z-\rho_2(z-y)}$=> $\min[A(.), B(.)] = B(.)$. Calculate A tedious calculation shows that

$$\text{sign}[\Delta - B(.)] = \text{sign}\,[w\,(1-q)\,(z-y)\,(\rho_1-\rho_2)\,f(z)]$$

where $f(z) = az^2 + b + c$ with

$$a = \left(-2\rho_2^2 - 2\rho_2\rho_1 q + 2\rho_1 q + 2\rho_2^2 q + 4\rho_2 - 2\rho_2 q - 2\right) < 0$$

$$b = \left(4\rho_2^2 + 4\rho_2\rho_1 q + 2\rho_2 q - 4\rho_2 - 2\rho_1 q - 4\rho_2^2 q\right) y - \rho_2 q w + q\rho_1 w - 2w + 2w\rho_2$$

$$c = \left(-2\rho_2\rho_1 q + 2\rho_2^2(q-2)\right) y^2 + (q(\rho_2-\rho_1) - 2\rho_2)\,wy < 0.$$

To see that $a < 0$, note that $a = 0$ for $\rho_1 q + (1-q)\rho_2 = 1$ and increasing in ρ_1. Thus, since $\rho_i \in [0,1]$ and $q \in [0,1]$, $\rho_1 q + (1-q)\rho_2 \leq 1$, it follows that $a < 0$. The sign of b is indeterminate. Note from Vièta's formulas that the two roots of f must be of the same sign and are positive if and only if $b > 0$. Note that $f(0), f(y), f(\pm\infty) < 0$ and that $\frac{\partial f}{\partial z}\big|_{z=0} = b$ and $\frac{\partial f}{\partial z}\big|_{z=y} = (w+2y)(\rho_1 q + (1-q)\rho_2 - (2-\rho_2)) < 0$. Thus, if the two roots are positive they must be smaller than y and so for all $z > y$, $f(z) < 0$. We conclude that $\Delta < B(.)$.

The proposition follows directly from the Lemma. For $\tau \leq \Delta < \min[A(.), B(.)]$ the optimal constitution specifies two policy area specific rules (m^*_{PA}, n^*_{PA}), while for $\tau > \Delta$, the optimal constitution specifies only one rule, m^*_{MR}.

Proof of Propositions 6 and 7 To establish these two Propositions, it is useful first to establish the following Lemma.

Lemma 2 *Let $\rho_1 > \rho_2$, $0 < q < 1$ and $\infty > z > y$. The critical cost Δ is a differentiable function of z and ρ_1 with the following properties*

1. For $\rho_1 < 1$, there exist a value $\infty > \overline{z} > y$ such that for $z \in (y, \overline{z}]$ $\frac{\partial \Delta}{\partial z} > 0$ and negative otherwise.

2. For $\rho_1 = 1, \frac{\partial \Delta}{\partial z} > 0$ for all $z > y$.

3. For $\rho_1 \leq 1$,
$$\frac{\partial \Delta}{\partial \rho_1} > 0$$

To establish part 1, calculate

$$\frac{\partial \Delta}{\partial z} = \frac{-w^2 q\,(z-y)\,(\rho_1-\rho_2)^2\,(1-q)\,(a_1 z^3 + a_2 z^2 + a_3 z + a_4)}{2\,(w+z-\rho_2(z-y))^2\,(w+z-\rho_1(z-y))^2\,(w+z-(q\rho_1+(1-q)\rho_2)(z-y))^2}$$

with

$$\begin{aligned}
a_1 &= (1-\rho_2)\,(1-\rho_1)\,(1-\rho_1 q-(1-q)\rho_2) > 0 \\
a_2 &= -3ya < 0 \\
a_3 &= \left(3q(1-\rho_2)\rho_1^2 + \left(-3\rho_2^2(1-q)+2(3\rho_2-q-1)\right)\rho_1 + 3\,(1-q)\,\rho_2^2 + 2\,(q-2)\,\rho_2\right) y^2 \\
&\quad + (\rho_2+\rho_1(1+q)+\rho_2(1-q)-3)\,(2wy+w^2) \\
a_4 &= -\left((1-\rho_2)q\rho_1^2 + (2-\rho_2+\rho_2 q)\,\rho_2\rho_1 + (1-q)\rho_2^2\right) y^3 + 2\,(-\rho_1(1+q)+\rho_2(q-2))wy^2 \\
&\quad + (\rho_2 q-\rho_1-2\rho_2-3-q\rho_1)\,w^2 y - 2w^3 \\
&< 0
\end{aligned}$$

From equation (A.1), we note that $\Delta(y) = \Delta(\pm\infty) = 0$ and that $\Delta(z) > 0$ for all $z > y$. It is clear that $\frac{\partial \Delta}{\partial z} = 0$ at $z = y$. For $z > y$, the sign of $\frac{\partial \Delta}{\partial z}$ is determined by the cubic equation $(a_1 z^3 + a_2 z^2 + a_3 z + a_4)$. Now, it will at least have one real root greater than y (because we know that $\Delta(y) = \Delta(\pm\infty) = 0$ and that $\Delta(z) > 0$ for all $z > y$), but it may have three roots. To rule this out it is sufficient to establish that a_3 is negative and apply Descartes' Rule of Signs. A sufficient condition is for a_3 to be negative is that

$$h(\rho_1) = \left(3q(1-\rho_2)\rho_1^2 + \left(-3\rho_2^2(1-q)+2(3\rho_2-q-1)\right)\rho_1 + 3\,(1-q)\,\rho_2^2 + 2\,(q-2)\,\rho_2\right) < 0$$

Notice that $h(\rho_2) = -3\rho_2\,(\rho_2-1)\,(-2+\rho_2) < 0$, $h(1) = -\rho_2 q + q + 2\rho_2 - 2 < 0$. Since $3q(1-\rho_2) > 0$, $h(\pm\infty) = +\infty$. Thus, $h(\rho_1) < 0$ for all $\rho_1 \in [\rho_2, 1]$. Hence, we conclude that there can only be one positive root to $a_1 z^3 + a_2 z^2 + a_3 z + a_4$, and, as a consequence, there is critical value of z where the derivative of $\Delta(z)$ changes sign as stated in the Lemma.

Part 2 of the Lemma follows by noting the $\rho_1 = 1$ implies that $a_1 = a_2 = 0$ and thus that $\frac{\partial \Delta}{\partial z} > 0$ for all $z > 0$. Part 3 of the Lemma follows immediately from

$$\frac{\partial \Delta}{\partial \rho_1} = \frac{(1-q)\left(2(w+z) + (\rho_1(1+q) + \rho_2(1-q))(y-z)\right)}{\left(w+z-\rho_1(z-y)\right)^2\left(w+z-(q\rho_1+(1-q)\rho_2)(z-y)\right)^2} > 0.$$

Proposition 6 follows from Part 1 of Lemma 2. Let $\tau_z = \max_z \Delta(z)$. Then the two critical values of z (z_1, z_2) are defined as the two solutions to the equation $\Delta(z) = \tau$ for $\tau \in (0, \tau_z)$. The corollary follows from part 2 of the Lemma 2 by noting that the equation $\Delta(z) = \tau$ allows only one solution z_3 for $\rho_1 = 1$. Proposition 7 follows from part 3 of Lemma 2. Let $\tau_{\rho_1} = \Delta(\rho_1)$ at $\rho_1 = 1$. Then the critical value $\overline{\rho}_1$ is the solution to $\Delta(\rho_1) = \tau$ for $\tau \in (0, \tau_{\rho_1})$ where $\overline{\rho}_1 = \rho_2$ at $\tau = 0$ and $\overline{\rho}_1 = 1$ at $\tau = \tau_{\rho_1}$.

Proof of Proposition 9 To determine the optimal AFB constitution the first thing to note is that, following the previous analysis, optimal constitutions, contingent on $\widehat{\omega}$, must always be of two types: either $(m,n)(\widehat{\omega}) = \left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ or $(m,n)(\widehat{\omega}) = \left(\frac{y}{y+w}, 1\right)$. The reason for this is that either the information conveyed by the announcements is such that it is optimal to run the risk of a crisis, in which case $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ is the best option or it isn't optimal to run the risk of a crisis in which case $\left(\frac{y}{y+w}, 1\right)$ is the best option.

Consider now the losers' representative. The loser would always prefer $n = 1$ whenever $\omega = \underline{\omega}$ while he would prefer $n = \frac{z}{z+w}$ if $\omega = \overline{\omega}$ whenever

$$\alpha\left(\eta(-y\left(1-\frac{y}{y+w}\right)) + (1-\eta)\,(-z\left(1-\frac{z}{z+w}\right))\right) \geq \eta\left(-y\left(\left(1-\frac{y}{y+w}\right)\right)\right)$$

$$\Leftrightarrow \alpha \leq \alpha_2 = \frac{\eta y\left(\frac{w}{y+w}\right)}{\eta y\left(\frac{w}{y+w}\right) + (1-\eta)z\left(\frac{w}{z+w}\right)}$$

while it will prefer $n = 1$ otherwise. Losers want to set $n = 1$ whenever $\omega = \underline{\omega}$ because it makes A less likely to obtain than setting $n = \frac{z}{w+z}$. If $\omega = \overline{\omega}$, however, setting $n = 1$ will guarantee that policies with $\overline{c}$ will not pass but at the same time allows policies with $\underline{c}$ to pass with probability $1 - \frac{y}{y+w}$). On the other hand, a $n < 1$ will trigger a crisis which, if the chance of its success is large enough, might be even better. Thus, if α is small enough, losers will want to have a constitution with $n < 1$ because they want the crisis to happen.

With winners, they will always want $n < 1$ with $\omega = \underline{\omega}$ because a crisis cannot happen and given that they want n to be as small as possible, they prefer $n = \frac{z}{z+w}$ to $n = 1$. If $\omega = \overline{\omega}$, they will want to set $n = \frac{z}{z+w}$ whenever

$$\alpha \geq \alpha_0 = \frac{\eta\left(\frac{w}{y+w}\right)}{\eta\left(\frac{w}{y+w}\right) + (1-\eta)\left(\frac{w}{z+w}\right)}$$

where $\alpha_0 > \alpha_2$ since $z > y$. These results allow us to characterize completely the preferences winners and losers have over n, as a function of α and the actual realization ω. These are summarized below:

$$\begin{array}{ccl} \text{if } \alpha \leq \alpha_2 & \textit{then} & \left\{ \begin{array}{l} \text{if } \omega = \underline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \succ_W 1 \text{ and } \frac{z}{w+z} \prec_{W^C} 1 \\ \text{if } \omega = \overline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \succ_{W^C} 1 \text{ and } \frac{z}{w+z} \prec_W 1 \end{array} \right. \\ \text{if } \alpha_2 < \alpha < \alpha_0 & \textit{then} & \left\{ \begin{array}{l} \text{if } \omega = \underline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \succ_W 1 \text{ and } \frac{z}{w+z} \prec_{W^C} 1 \\ \text{if } \omega = \overline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \prec_{W^C} 1 \text{ and } \frac{z}{w+z} \prec_W 1 \end{array} \right. \\ \text{if } \alpha \geq \alpha_0 & \textit{then} & \left\{ \begin{array}{l} \text{if } \omega = \underline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \succ_W 1 \text{ and } \frac{z}{w+z} \prec_{W^C} 1 \\ \text{if } \omega = \overline{\omega} \;\; \textit{then} \;\; \frac{z}{w+z} \prec_{W^C} 1 \text{ and } \frac{z}{w+z} \succ_W 1 \end{array} \right. \end{array}$$

Not being able to verify the identity of the announcers, and specifically whether they are winners or losers, the constitution can only select amongst the two possible values of n by making them conditional on whether $\widehat{\omega} = \{\underline{\omega}, \overline{\omega}\}, \{\underline{\omega}, \underline{\omega}\}$ or$\{\overline{\omega}, \overline{\omega}\}$. In other words, the constitution cannot distinguish between the case in which the winners' representative announces $\underline{\omega}$ and the losers' announces $\overline{\omega}$ from the case in which the losers' representative announces $\underline{\omega}$ and the winners', announces $\overline{\omega}$.

Consider the case $\alpha_2 < \alpha < \alpha_0$, and the following constitutional mechanism:

$$\begin{array}{ccc} \text{if } \widehat{\omega} = \{\underline{\omega}, \overline{\omega}\} \text{ or } \{\underline{\omega}, \underline{\omega}\} & \textit{then} & (m, n)(\widehat{\omega}) = \left(\frac{y}{y+w}, \frac{z}{z+w}\right) \\ \text{if } \widehat{\omega} = \{\overline{\omega}, \overline{\omega}\} & \textit{then} & (m, n)(\widehat{\omega}) = \left(\frac{y}{y+w}, 1\right) \end{array}$$

then the winner's representative has weakly dominant strategy in choosing announcement $\underline{\omega}$ whenever $\omega = \underline{\omega}$ and announcement $\overline{\omega}$ whenever $\omega = \overline{\omega}$ while the losers' representative has a weakly dominant strategy in selecting announcement $\overline{\omega}$ regardless of ω. This mechanism guarantees that if $\alpha_2 < \alpha < \alpha_0$, $(m, n) = \left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ whenever $\omega = \underline{\omega}$ and $(m, n) = \left(\frac{y}{y+w}, 1\right)$ whenever $\omega = \overline{\omega}$ and this is an optimal mechanism for the constitutional designers.

Consider now the case $\alpha \leq \alpha_2$. In this case, the constitutional designers will wish the mechanism to select $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ whenever $\omega = \underline{\omega}$ and $\left(\frac{y}{y+w}, 1\right)$ otherwise. However, no such mechanism exists. First of all, any such mechanism must be such that one of two possibilities (i.e. $n = 1$ or $n = \frac{z}{z+w}$) is associated with at least one announcement profile in $\{\{\underline{\omega}, \overline{\omega}\}, \{\underline{\omega}, \underline{\omega}\}, \{\overline{\omega}, \overline{\omega}\}\}$ and the other possibility is associated with the other two announcement profiles. Suppose w.l.o.g. that constitution $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ is associated with $\{\underline{\omega}, \overline{\omega}\}$ and $\left(\frac{y}{y+w}, 1\right)$ with the other two. This will not work, because if the losers' representative announces second, then whenever $\omega = \underline{\omega}$, it can always replicate the announcement of the winners' representative thus generating $\left(\frac{y}{y+w}, 1\right)$. On the other hand, whenever $\omega = \overline{\omega}$, it can always make an announcement different from that of the winners' representative, thus generating $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$. Since the mechanism cannot selects who announces first because identities are not verifiable, this mechanism provides the correct information with probability $\frac{1}{2}$ (the probability that the winner's representative announces second) only. Similar arguments apply for all other possible mechanisms and for the case $\alpha \geq \alpha_0$.

Thus, we've shown that for $\alpha \geq \alpha_0$ and $\alpha \leq \alpha_2$ any mechanism only reveals the true state of nature with probability $\frac{1}{2}$. We now show that this implies that for these values of α, it is optimal for the constitutional designer to either select constitution $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ or constitution $\left(\frac{y}{y+w}, 1\right)$ regardless of announcements. Let

$$\begin{aligned} u_1 &= EU\left(\frac{y}{y+w}, \frac{z}{z+w}\right) \\ u_2 &= EU\left(\frac{y}{y+w}, 1\right) \end{aligned}$$

where, by definition $u_1 > u_2$. Then a mechanism that reveals the truth with only $\frac{1}{2}$ probability gives us expected utility

$$\begin{aligned} &\gamma\left(\frac{1}{2}u_2 + \frac{1}{2}\alpha u_1\right) + (1-\gamma)\left(\frac{1}{2}u_2 + \frac{1}{2}u_1\right) \\ = &\ \frac{1}{2}u_2 + \frac{1}{2}u_1\left(\gamma\alpha + (1-\gamma)\right) \end{aligned}$$

while always having $\left(\frac{y}{y+w}, 1\right)$ gives expected utility u_2 and always having $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$ gives expected utility $u_1\left(\gamma\alpha + (1-\gamma)\right)$. It is easy to see that either

$$u_2 \geq \frac{1}{2}u_2 + \frac{1}{2}u_1\left(\gamma\alpha + (1-\gamma)\right)$$

or

$$u_1\left(\gamma\alpha + (1-\gamma)\right) > \frac{1}{2}u_2 + \frac{1}{2}u_1\left(\gamma\alpha + (1-\gamma)\right)$$

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Proof of Proposition 10 Consider representative k and suppose first that k represents the losers. Now, let $\{\overline{m}_l, \overline{n}_l, r_l\}$ represent $\{\overline{m}_j, \overline{n}_j, r_j\}$ if $v_j \geq r$ and $\{m, n, r\}$ otherwise. We know that any constitution $\{\overline{m}_k, \overline{n}_k, r_k\}$ proposed by k will pass only if $v_k \geq r_l$. The choice is between a constitution that will be unanimously agreed upon or one that will get the losers' vote[48]. The first possibility is always there as long as both $\overline{m}_l > 0$ and $\overline{n}_l < 1$ because, given that $z > y$ then losers can always offer winners a lower m in exchange for a higher n which will be a Pareto improvement over the previous constitutions. In particular, it is optimal for losers to propose $\overline{m}_k$ and $\overline{n}_k$ where the latter

[48] The case in which no proposal is made is a special case in which a constitution identical to the previous one is proposed.

is highest possible value of n such that

$$\begin{aligned} \eta\left(1-\overline{m}_l\right)+\left(1-\eta\right)\left(1-\overline{n}_l\right) &\leq \eta\left(1-\overline{m}_k\right)+\left(1-\eta\right)\left(1-\overline{n}_k\right) \\ \text{and } \overline{m}_k &\geq 0, \overline{n}_k \leq 1 \end{aligned}$$

The second possibility, for which we must have $p \leq 1 - r_l$ is to select any $\overline{m}_k > 1 - r_l$, $\overline{n}_k > 1 - r_l$ because this guarantees that A will not pass.[49] Which one of these two cases will be chosen depends on whether the constitution that makes A impossible (but requires that $p \leq 1 - r_l$) is preferable to the one that is agreed upon unanimously (but still allows for A).

Suppose now that k represents the winner. Again, renegotiation that will be unanimously agreed upon is always possible if along lines similar to the ones described above if both $\overline{m}_l > 0$ and $\overline{n}_l < 1$. Now, it is optimal for winners to propose $\overline{m}_k$ and $\overline{n}_k$ where the former is smallest possible value of m such that

$$\begin{aligned} \eta\left(1-\overline{m}_l\right)(-y)+\left(1-\eta\right)\left(1-\overline{n}_l\right)(-z) &\leq \eta\left(1-\overline{m}_k\right)(-y)+\left(1-\eta\right)\left(1-\overline{n}_k\right)(-z) \\ \text{and } \overline{m}_k &\geq 0, \overline{n}_k \leq 1 \end{aligned}$$

For a constitution to achieve just the winners' vote, we must have $p \geq r_l$ and given that, it is optimal to set $\overline{m}_k \leq r_l$ and $\overline{n}_k \leq r_l$ whenever $\omega = \underline{\omega}$ thus guaranteeing A. If $\omega = \overline{\omega}$, it is still optimal to set $\overline{m}_k \leq r_l$ but now we have $\overline{n}_k \leq r_l$ iff $\alpha \geq \eta$ and $\overline{n}_k = 1$ otherwise. To see this, note that by setting $\overline{n}_k \leq r_l$, the constitution allows the winner to get w for sure unless the crisis that will follow is successful. So the expected utility is αw. Alternatively, by selecting the best possible constitution that does not allow for a crisis - $\overline{m}_k \leq r_l$ and $\overline{n}_k = 1$ - the winners get w only with probability η, the probability that $c = \underline{c}$. Clearly, the first option is better than the second one iff $\alpha \geq \eta$.

Again, winners will have to choose before p is realized whether their expected utility from unanimous renegotiation is higher or lower than that from a change that only winners will vote for.

For the constitutional choice $\{\overline{m}_j, \overline{n}_j, r_j\}$, it is immediate to see that the same considerations apply.

We now focus on the problem faced by the constitutional designers behind the veil of ignorance. It is immediate to see that it is always optimal to set $m = \frac{y}{y+w}$. Consider now the choice $n = 1$, and $r = \frac{z}{w+z}$. This particular choice implies that if j represents the losers, there will be no change to the constitution. To see this, note first that n is j's preferred option, and that m and r are such that if $1 - p \geq r$, then $1 - p > m$ which means that there is no reason to change m because whenever that could be done, then A would not

[49] Since there is no opportunity for further amendment, r_k is irrelevant.

obtain anyway. No renegotiation can take place because $n = 1$. Finally, there is no reason to change r because whenever $(1-p) \geq r$, then we cannot have $p \geq r$ and this makes it impossible that the winners' representative k can make any changes that only winners would approve of. Thus, whenever j represents the losers, either no change is possible or no change is desired.

Now suppose j represents the winners. If $\omega = \underline{\omega}$ then $\overline{m}_j, \overline{n}_j \leq r$ and $r_j \geq r$ is optimal because it guarantees A by choosing the appropriate majority rules and by making sure that losers can never make any further changes. If $\omega = \overline{\omega}$ and $\alpha \geq \eta$ we have the same while no change will be necessary whenever $\alpha < \eta$. Again, in either case, no renegotiation is possible because $n = 1$.

The whole analysis implies that losers j or k will never change the constitution while winners j or k will change it whenever $\omega = \underline{\omega}$ or $\omega = \overline{\omega}$ and $\alpha \geq \eta$. To see that this is the optimal AM constitution note that this constitution is outcome-equivalent to a VFB constitution where instead of having the cut-off value α_0, we have the cut-off $\eta < \alpha_0$. Thus, this replicates the optimal VFB constitution except for those cases in which $\alpha \in [\eta, \alpha_0)$ where if $\omega = \overline{\omega}$ then the optimal VPA constitution is $\left(\frac{y}{y+w}, 1\right)$ while this constitution is $\left(\frac{y}{y+w}, \frac{z}{z+w}\right)$. It is immediate to see that there is no other AM constitution that improves upon this ¤

Proof of Proposition 11 The optimal rules are derived as follows. Define

$$\begin{aligned} u_h &= \rho_1 \left(p_h w - (1-p_h) w\right) + (1-\rho_1) \left(p_h w - (1-p_h) z_h\right), \\ v_h &= \rho_2 \left(p_h w - (1-p_h) w\right) + (1-\rho_2) \left(p_h w - (1-p_h) z_h\right), \end{aligned}$$

where we assume that $w = y$ to simplify the algebra. For a NCB constitution and for $\min\{m, n\} \geq \frac{1}{2}$, the constitutional designer maximizes

$$\begin{aligned} &\tfrac{q}{2}\left(\int_{2m-1}^{1}\int_{2m-p_T}^{1} u_S dp_S dp_T + \int_{2m-1}^{1}\int_{2m-p_S}^{1} u_T dp_T dp_S\right) \\ &+\tfrac{1-q}{2}\left(\int_{2n-1}^{1}\int_{2n-p_T}^{1} v_S dp_S dp_T + \int_{2n-1}^{1}\int_{2n-p_S}^{1} v_T dp_T dp_S\right) \end{aligned}$$

with solution given in equations (7.1) and (7.2) and expected utility $EU(m^*_{NCB}, n^*_{NCB})$. We note that $n^*_{NCB} > m^*_{NCB} \geq \frac{1}{2}$ confirming our assumption. For the CB constitution, the constitutional designer maximizes

$$\begin{aligned} &\tfrac{q}{2}\left(\int_{m_T}^{1}\int_{m_S}^{1} u_S dp_S dp_T + \int_{m_S}^{1}\int_{m_T}^{1} u_T dp_T dp_S\right) \\ &+\tfrac{1-q}{2}\left(\int_{n_T}^{1}\int_{n_S}^{1} v_S dp_S dp_T + \int_{n_S}^{1}\int_{n_T}^{1} v_T dp_T dp_S\right) \end{aligned}$$

with solutions given in equations (7.3) and (7.4) and expected utility $EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T})$. For the two PCB, the constitutional designer maximizes at combination of the two objective functions for the CB and the NCB constitution and expected utility of a PCB constitution with checks and balances for policy area 2 is $EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T})$ while expected utility for a PCB constitution with checks and balances for policy area 1 is $EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB})$.

We need to compare the differences in expected utility in the various cases. First, notice that

$$\begin{aligned}\Delta_1 &= EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) - EU(m^*_{NCB}, n^*_{NCB}) \\ &= EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) - EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB}).\end{aligned}$$

Some tedious, but straight forward calculations shows that

$$\Delta_1 = \frac{8\,(1-q)\,w^3\,(2(z_S^2+z_T^2)(1-\rho_2)^2-(z_S+z_T)w(1-\rho_2^2)-5z_Sz_T(1-\rho_2)^2-w^2(1+\rho_2)^2)}{27\,((1+\rho_2)2w+(1-\rho_2)(z_S+z_T))^2\,(z_1(1-\rho_2)+w(1+\rho_2)\,(z_T(1-\rho_2)+w(1+\rho_2)}.$$

We see that $\Delta_1 = 0$ if

$$\left\{z_T = \frac{1}{2}\frac{\rho_2 z_S - z_S + w + \rho_2 w}{\rho_2 - 1}\right\} \text{ or } \left\{z_T = \frac{2\rho_2 z_S - 2z_S - w - \rho_2 w}{\rho_2 - 1}\right\}.$$

We see that $\frac{1}{2}\frac{\rho_2 z_S - z_S + w + \rho_2 w}{\rho_2 - 1} < z_S$. Thus, $\Delta_1 > 0$ if $z_T > \frac{2(1-\rho_2)z_T+(1+\rho_2)w}{(1-\rho_2)} \equiv \overline{z}_T$. We see that $z_T > \overline{z}_T \Rightarrow EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{NCB}, n^*_{NCB})$ and $EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB})$ and the other way around for $z_T \leq \overline{z}_T$. Second, notice that

$$\begin{aligned}\Delta_2 &= EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB}) - EU(m^*_{NCB}, n^*_{NCB}) \\ &= EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) - EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T})\end{aligned}$$

with

$$\Delta_2 = \frac{8w^3q\,(2(z_S^2+z_T^2)(1-\rho_1)^2-(z_S+z_T)w(1-\rho_1^2)-(1+\rho_1)^2w^2-5z_Sz_T(1-\rho_1)^2)}{27\,(2(1+\rho_1)w+(1-\rho_1)z_S+(1-\rho_1)z_T)^2\,((1-\rho_1)\,z_T+(1+\rho_1)w)\,((1+\rho_1)w+(1-\rho_1)z_S)}$$

where $\Delta_2 = 0$ at $z_T^1 = \frac{w(1+\rho_1)+2(1-\rho_1)z_S}{1-\rho_1}$ and $z_T^2 = \frac{1}{2}\frac{-(1+\rho_1)w+(1-\rho_1)z_S}{1-\rho_1}$. Clearly, the relevant root is

$$\widehat{z}_T \equiv \frac{(1+\rho_1)w+2(1-\rho_1)z_S}{1-\rho_1} > \overline{z}_T$$

for $\rho_1 > \rho_2$. Thus, we have that $z_T > \widehat{z}_T \Rightarrow EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T})$ and $EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB}) > EU(m^*_{NCB}, n^*_{NCB})$ and the other way around for $z_T \leq \widehat{z}_T$. Combining these results, we conclude the following. For $z_T \in [z_S, \overline{z}_T)$, we get

$$EU(m^*_{NCB}, n^*_{NCB}) > EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T})$$

and

$$EU(m^*_{NCB}, n^*_{NCB}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB}).$$

This proves part 1 of the proposition. For $z_T \in [\overline{z}_T, \widehat{z}_T)$, we get

$$EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{NCB}, n^*_{NCB}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB})$$

$$EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}).$$

This proves part 2 of the proposition. For $z_T \geq \widehat{z}_T$, we get

$$EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) \geq EU(m^*_{NCB}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{NCB}, n^*_{NCB})$$

$$EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{CB,S}, n^*_{CB,T}) > EU(m^*_{CB,S}, m^*_{CB,T}, n^*_{NCB})$$

This proves part 3 of the proposition ¤