

Corporate Downsizing to Rebuild Team Spirit*

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Abstract

We propose a new theory for downsizing, based on strategic reasons, rather than technological ones. A crisis may lead to a decrease in the willingness to cooperate in an organization, and therefore to a bad equilibrium. A consensual downsizing episode may signal credibly that survivors are willing to cooperate, and thus, it may be optimal and efficiency-enhancing, as the empirical evidence suggests.

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1 Introduction

A usual response to crises in organizations is to reduce the number of its members, a phenomenon called *downsizing*. In this paper we propose a new theory to explain this phenomenon, based on strategic reasons, rather than technological ones.

Our first observation is that many activities in organizations involve strategic complementarities. Milgrom and Roberts (1995), for example argue that they are one of the reasons for the existence of firms.¹ These strategic complementarities naturally lead to multiple equilibria. Kreps (1990) has forcefully made the case that corporate culture is precisely a tool to create shared expectations, and thus, avoid strategic uncertainty, which then is useful to coordinate behavior in corporate games. Suppose now that, for some reason, members of an organization start to entertain doubts about the *ability to coordinate* of some of its fellow members on an action that is highly rewarding only if most other partners also take this action (which we will call the *cooperative* action). Then, it is rational for *everybody* in the organization to undertake a less cooperative action, if that one requires only individual effort to be profitable.

An obvious solution to this problem is to expel the non-cooperators (or *pathological individualists*, as we will later call them). This by itself would be a reason for downsizing. But notice that in the critical stage nobody in the organization is cooperating, so the *guilty* parties may not be easy to locate. They are simply doing what everybody else does. And even if pathological individualists were easy to locate, there is no guarantee that, after reorganization, the remaining members would in fact cooperate. This would seem especially doubtful if the reorganization were costly and traumatic, as this could perhaps create negative expectations about the behavior of survivors. Perhaps surprisingly, however, our explanation will crucially rely on the assumption that reorganization is costly.

Suppose then, that *holding everything else constant*, a downsizing reorganization of the firm is costly. The members of the organization are asked to vote on whether the reorganization will take place. The key point of our argument is that, since downsizing is costly, a yes-vote on reorganization is not a reasonable strategy (in game-theoretic terms, it is weakly dominated) if one intends not to cooperate after reorganization. It would be better to avoid re-organization and not cooperate. Thus, the approval of downsizing is a signal that yes-voters in the streamlined organization are ready to cooperate. If the benefit of enhanced cooperation in the re-organized firm is higher than the cost of reorganization, then the mem-

¹“strong complementarities make it more likely that [...] central strategic direction will be valuable,” Milgrom and Roberts (1995, p.190).

bers will approve the undertaking of this costly process, expecting that after reorganization, cooperation would be restored.

It may perhaps sound strange for an organization to put its reorganization plans to a vote. But in most European countries downsizing plans have to be consulted in a mandatory way with the workers. For a typical example, the following quote from the Financial Times² is enlightening: “Failing to consult a workforce properly before a big restructuring can prove costly. If an employer is shown to have flouted the rules, it is liable to pay three months’ pay to every affected worker. And that means not just those people that are dismissed, says Elaine Aarons at Eversheds, ‘but all those whose jobs change as a result’. In a large workforce, the potential for compensation is huge. So what must employers do? In non-unionised sectors, those with more than 20 employees must embark on a 30-day consultation with a body of elected workers’ representatives. Companies with more than 100 employees require a 90-day consultation period.”

At this point, it is worth to review briefly the stylized facts on downsizing. Oulton (1999) shows that the productivity is negatively correlated with size changes in a study of British manufacturing firms. Foster, Haltinwanger and Krizan (2000) has the same type of finding for American retail-trade firms. A natural explanation for this would be technological. If technology has decreasing returns to scale, downsizing would imply a gain in productivity. For this reason, it is interesting to turn to a study by Barnes and Haskel (2001, table 4) for a sample of British manufacturing firms since they decompose the effect of productivity between upsizing and downsizing firms. They show that, for firms that downsize the proportion whose productivity increases can be between 2 and 5 times higher than those whose productivity decreases depending on the period. Interestingly for us, the same qualitative finding is true for firms that upsize. This makes the purely technological story harder to take. Firms that downsize would have to be typically those in the decreasing returns part of the cost curve, and firms that upsize would have to be typically those in the increasing returns part of the curve. That sounds unlikely to be the case if reorganization is a response to a shift in demand, which is bound to be the case often. It is, however, consistent with a picture of firms mostly in the increasing returns part of the cost curve that downsize to recover coordination, as we posit in our model.

A good deal of inspiration for our paper was obtained by reading Girard (1982). He argues that scapegoat episodes throughout both history and literature (where often there is an implicit historical basis in his view) share a few fundamental traits. One of them is that

²Financial Times (London), October 28, 2002, Monday London Edition 1, SECTION: INSIDE TRACK LAW & BUSINESS; Pg. 20.

prior to the episode there is a crisis and a breakdown of cooperation in the societies where they occur. The scapegoat episode serves as a way for society to regain cooperation. He also argues that scapegoats have a *mark*, which distinguishes them from the rest of society. This differentiating mark serves to coordinate on them as the victims.³ The coordinating aspect of scapegoats is connected with the discussion of our model so far. Less obvious is the connection with our model of the victimizing *mark*. Suppose, though, that the individuals affected by downsizing were not given exogenously. In this case, the collective choice problem would not simply be *whether* to downsize or not, but *who* would be expelled by the event. Under such a scenario a *mark* of distinction could be used as a focal point to coordinate expectations about who should be excluded from the organizations.

Section 2 describes the model while the results are presented in Section 3. Section 4 discusses the related literature and concludes.

2 The model

Productive technology An organization produces output in a way depends both on the size of the organization and its members' productive actions. We denote by $u_i(a_1, \dots, a_n; n)$ the individual output (net of effort costs) of a current member i of an organization composed of n members where member k takes action a_k . We suppose that u_i is multiplicatively separable in the vector (a_1, \dots, a_n) of productive actions and in the size n of the organization, that is,

$$u_i(a_1, \dots, a_n; n) = \phi_i(a_1, \dots, a_n) F(n)$$

The size effect function, $F(n)$, common to all organization members, is assumed to be increasing for some range $[0, \bar{n}]$. This reflects increasing returns at least below a critical

³More precisely, Girard's theory of scapegoating can be summarized as follows. First, it is argued that individual behavior within a given society is governed by *mimetic desire*, that is, the willingness for everyone to conform to others' behavior. In game-theoretic terms, payoffs are interdependent and display strategic complementarities among available actions. Second, mimetic desire (and its contagious counterpart) may bring about both social order (the societal consensus) or widespread violence (the mimetic crisis). In game-theoretic terms, the underlying supermodular game has at least two (Pareto-ranked) equilibria. Third, when a mimetic crisis arises, the collective choice of a common victim, the scapegoat, reunites the members of the society and restores the original consensus. At this stage, "stereotypes of persecution" (p. 12) are instrumental to select the arbitrary victim that serves the role of a scapegoat. Our paper precisely tries to make precise this last point on Girard's argument. For completeness, Girard further argues that ritual scapegoating, that is, a symbolic and systematic repetition of the original act of murder, helps the society to cope with the arbitrary nature of the original violence.

organization size.

For simplicity, there are only two productive actions for each member. The productive action space for member i is $A^i = \{0, 1\}$, where $a_i = 1$ represents the option to contribute to an activity which enhances productivity (*cooperative* action), and $a_i = 0$ represents the option not to contribute (*non-cooperative* action). For all productive action profile (a_1, \dots, a_n) , let $a = \sum_{i=1}^n a_i$ be the total number of contributors. The productive action effect varies across members in the following way:

$$\phi_i(a_i, a_{-i}) = \begin{cases} \tau(a), & \text{if } a_i = 1 \\ 1, & \text{if } a_i = 0 \end{cases}.$$

The individual return to choosing the non-cooperative action $a_i = 0$ is constant (and equal to one) irrespective of the choice made by other members. The return to the *cooperative* action $a_i = 1$ depends on the productive action profile as it varies with the total number of contributors. We assume that $\tau(a)$ is non-decreasing, with $\tau(a) = 0$ for all $a < n - \kappa$ and $\tau(n) > 1$. In words, the cooperative action yields non-zero returns whenever at most κ members in the organization choose *not* to contribute. The productive action effect thus resembles a public good game with a provision point as a minimal critical number of contributors (here, $n - \kappa$) is required to generate positive payoffs. Non-contributors can be excluded in our framework, thus limiting the distributional conflict. This is obviously a limitation, as that kind of conflict is probably an important part of the phenomenon we want to study. The limitation has the benefit that we want to focus on pure strategic uncertainty and other kinds of distributional conflict in our world arising purely as a consequence of downsizing.

The dilution of team spirit The team spirit of an organization is reflected in the profile a of productive actions taken by its members. Organizations with a sufficiently high proportion of members choosing $a_i = 1$ display a high team spirit, which translates into higher per capita payoffs for members building actively such team spirit.

Suppose now that, at some point in time, all members in the organization acquire the belief⁴ that, with some probability μ , each member i sticks to action $a_i = 0$, irrespectively of any other factor. We refer to the members with this kind of behavior as *pathological individualists*, and denote the set of members with this behavior by PI . Members in PI do not take part in team building activities under any circumstances.⁵ Prior to the revelation

⁴Perhaps because they observed a bad outcome last period.

⁵Even when it would appear to an external observer that it is in their self interest to do so.

of their private type, the expected output for a member i that plans to choose action $a_i = 1$ ⁶ when every other player plans to choose $a_i = 1$ (if in the interim they are not revealed to be a PI -type), is $Eu_i = (1 - \mu) h(\mu, n) + \mu F(n)$, where:

$$h(\mu, n) = \sum_{j=0}^n \binom{n}{j} \mu^j (1 - \mu)^{n-j} \tau(n - j) F(n)$$

The corporate downsizing game Initially, an organization is composed of a set N of members.

The game consists of two stages. In the first stage, all organization members participate in the corporate downsizing procedure. Then, at an interim stage, a nature move determines the players' types. Each organization member has a probability μ of belonging to PI ; this is independent and identical across members. In the second stage, the remaining members in the organization choose their productive actions according to their types.

We have assumed at this point, for expositional simplicity, a common prior and revelation of information after downsizing is decided upon. This allows us to exclude the complexity of signalling-type phenomena at the downsizing stage. We later show that the basic thrust of the results still holds under a more natural informational assumption.

We now analyze the game for different scenarios about the first stage procedure.

3 The results

3.1 Approving or rejecting an elimination

One common procedure for corporate downsizing requires the membership to approve or reject the elimination of a certain subgroup. First let's look at the case where the candidate subgroup is exogenously given, then we will look at the case where the subgroup is endogenous.

In the first case, there is a candidate subgroup C for elimination. Each individual $i \in N$ casts a positive or negative vote on the elimination of C . The choice of C is exogenous to the approval procedure. Either the CEO or perhaps an outside consultant makes the proposal. The final decision is taken by k -majority voting, that is, for the set C to be eliminated, at

⁶Note that, if player i ends up being a pathological individualist once private types are revealed to players, she will not follow this intended plan of action and rather, given her type, select and stick to the non-cooperative alternative $a_i = 0$.

least k players have to vote for the elimination.⁷

Under these conditions we can characterize the undominated subgame perfect equilibria of the two-stage corporate downsizing game. Let c be the cardinality of C .

Proposition 1 *If $(1 - \mu)h(\mu, k) + \mu F(n - c) > F(n) > h(\mu, n)$, then all outcomes of subgame perfect equilibria in undominated strategies are such that C is eliminated and the remaining non-pathological players in the downsized organization choose to contribute, that is, $a_i = 1$, for all $i \in N \setminus \{C \cup PI\}$.*

Proof. First, observe that for all $i \notin C$ approving the elimination and then choosing $a_i = 0$ is weakly dominated by not approving the elimination and then choosing $a_i = 0$. Indeed, the expected payoff for player $i \notin C$ when nobody is eliminated and she chooses $a_i = 0$ is $F(n)$. The payoff for player $i \notin C$ when C is eliminated and she chooses $a_i = 0$ is $F(n - c)$, and $F(n - c) < F(n)$.⁸ Since casting a vote for the elimination of C by any player $i \notin C$ may be pivotal for this elimination, given the requirement of k -majority approval, the domination follows. Thus, any player who votes for the elimination of C will play $a_i = 1$ if in the interim stage $i \notin PI$. Therefore a lower bound for the expected payoff in case of elimination is given by $(1 - \mu)h(\mu, k) + \mu F(n - c)$.

Since $F(n) > h(\mu, n)$ is equivalent to $F(n) > (1 - \mu)h(\mu, n) + \mu F(n)$, it is dominant to choose $a_i = 0$ when nobody is eliminated. The condition $(1 - \mu)h(\mu, k) + \mu F(n - c) > F(n)$, guarantees that the player $i \notin C$ prefers (from an ex ante perspective) the situation where C is eliminated. Since casting a vote for the elimination of C by any player i may be pivotal for this elimination, it is dominant to vote for this elimination (and then choose $a_i = 1$). ■

Therefore, with exogenously given candidate subgroups for elimination of suitable size, downsizing is approved and members of the subsequent streamlined organization (excluding pathological individualists) choose the cooperative action. Notice that, net of contribution profiles and given our assumption on increasing returns in organization size, downsizing is produced at a cost borne by all members of the downsized organization, that is, $F(n - c) < F(n)$. Note also that expelled members need not necessarily be pathological individualists, as this trait is private information. The collective decision to downsize thus unfairly harms

⁷The “yes/no” voting procedure over two alternatives we adopt here rules out issues of strategic voting or cycling patterns.

⁸Note that, if in the interim stage $i \in PI$, then $a_i = 0$ by our assumption on the behavior of pathological individualists.

some organization members, just as scapegoats are unfairly blamed for problems of any kind. Here, as in Girard's theory, arbitrary scapegoating reestablishes cooperation.

Let's now endogenize the choice of the set C . Prior to the approval or rejection of C , the members of the organization must choose a person who will propose the set to be subject to the approval/rejection process.

Proposition 2 *Consider any collective choice procedure that selects a delegate to make the proposal of C and suppose that $k + c \leq n$. Then, the delegate proposes a candidate C which maximizes $h(\mu, n - c)$ and the subgame perfect equilibrium outcomes in undominated strategies are such that C is eliminated and $a_i = 1$, for all $i \in N \setminus \{C \cup PI\}$.*

Proof. Once a delegate is chosen, Proposition 1 characterizes the behavior she expects for all proposals. Therefore, the delegate will propose a set C (not including herself) which maximizes $h(\mu, n - c)$, thus her expected payoff $(1 - \mu)h(\mu, n - c) + \mu F(n)$.⁹ ■

Downsizing does effectively rebuild team spirit even when the scapegoat is endogenously selected. The selection, here, consists of a two-stage procedure whereby members first choose a representative who then proposes a candidate subgroup for elimination to their approval. Note that the collective choice procedure for the selection of the representative is left unspecified implying that the result holds with a huge variety of institutional arrangements.

According to Proposition 1, sufficient conditions for downsizing to restore the cooperative action among members in the streamlined organization is to have $F(n) > h(\mu, n)$, and $(1 - \mu)h(\mu, k) + \mu F(n - c) > F(n)$ for some $n > c > 0$ and $k + c \leq n$. We now show with a simple example that both inequalities can hold simultaneously.

Example 1 *Majority approval.*

Suppose that the decision to downsize is taken by majority approval, that is, $k = n/2 + 1$ if n is even, while $k = (n + 1)/2$ if n is odd. Let $F(n) = n$ and suppose that $\tau(\cdot)$ is a step function, with $\tau(a) = 0$ if $a < n - \kappa$, and $\tau(a) = \tau$ otherwise.¹⁰ Table 1 computes the

⁹Note that the delegate is not in PI when choosing the candidate subgroup to elimination as types are defined at the interim stage between the voting stage and the contribution stage, that is, after the choice of C by the delegate.

¹⁰The actual value of τ is left unspecified and is fixed for each value of n, κ, μ so that the conditions $h(\mu, n - c) > F(n) > h(\mu, n)$ hold true for some c . See the proof of Proposition 3 for details.

optimal scapegoat size of Proposition 2 for different values of n, κ and μ .

$(n; \kappa)$	$\mu = 0.10$	$\mu = 0.15$	$\mu = 0.20$	$\mu = 0.25$	$\mu = 0.30$	$\mu = 0.35$
(50; 5)	7	22	24	24	24	24
(50; 10)	0	0	10	18	23	24
(100; 10)	20	47	49	49	49	49
(100; 15)	0	20	40	49	49	49
(100; 20)	0	0	20	36	46	49

Table 1. Optimal scapegoat size as a function of n, κ and μ .

A few things are interesting to note. First, the optimal scapegoat size is weakly increasing in the probability μ . That is, the higher the likely number of pathological individualist, the (weakly) bigger the size of the expelled group. Second, the values for the optimal scapegoat size range from 0 to $n/2 - 1$, which is the biggest size that can be expelled by majority approval.¹¹ The cells with zeros correspond to $\mu n < \kappa$, when the expected number of pathological individualists is lower than the provision point κ . When $\mu n \geq \kappa$, absent downsizing and given that pathological individualists always choose not to contribute, individual returns would always be zero irrespective of the choices of a_i . Given such bad prospects, downsizing is collectively approved and the optimal scapegoat size is $0 < c^* \leq n/2 - 1$. The upper bound $n/2 - 1$ is obtained with bad enough prospects, that is, with high enough values for μ .¹²

3.2 The case of unmediated downsizing

We now consider the general case where the scapegoat is endogenously and directly selected by organization members, without resorting to representatives. The general voting procedure we contemplate, where members do not simply cast a yes/no vote between two given alternatives but, rather, vote over the precise identity of each member to be expelled, potentially opens the door to strategic behavior during this voting stage.

The corporate downsizing procedure involves all initial members of the organization. Each member submits a list of individuals to be expelled from the organization. We denote

¹¹More precisely, the highest scapegoat size than can be expelled is equal to $n/2 - 1$ when the population size n is even, as in the numerical examples of Table 1. If, instead, n is odd, the highest possible scapegoat size is equal to $(n - 1)/2$.

¹²Formally, denote by $\lceil \frac{n-1}{2} \rceil$ the highest integer smaller or equal than $\frac{n-1}{2}$. The requirement of majority approval implies that the scapegoat size c takes on values in $\{0, 1, \dots, \lceil \frac{n-1}{2} \rceil\}$. The optimal size c^* that maximizes $h(\mu, n - c)$ on $\{0, 1, \dots, \lceil \frac{n-1}{2} \rceil\}$ can either be an ‘interior’ point, that is, $1 \leq c^* < \lceil \frac{n-1}{2} \rceil$, or a ‘corner’ solution, that is, $c^* \in \{0, \lceil \frac{n-1}{2} \rceil\}$. Consistent with Proposition 3, $c^* = 0$ when $\mu n < \kappa$. When $\mu n \geq \kappa$, c^* is an ‘interior’ point for low values of μ , while $c^* = \lceil \frac{n-1}{2} \rceil$ when μ is high enough.

by N the set of initial members of the organization and by $S_i \subseteq N$ the list submitted by member i . The case $S_i = \emptyset$ corresponds to i submitting a list with no names.

Given a collection of submitted lists $\mathcal{S} = (S_1, \dots, S_n)$, a collective choice procedure determines the group $E(\mathcal{S})$ that is actually expelled. $E(\mathcal{S})$ is selected the following way. If there is a unique group S' that receives the (absolute) majority of votes, then $E(\mathcal{S}) = S'$. In all other cases, $E(\mathcal{S}) = \emptyset$.

Lemma 3 *Suppose $F(n) > h(\mu, n)$. For all $i \notin PI$, submitting a list $S_i \neq \emptyset$ and not contributing whenever S_i is eliminated is a weakly dominated strategy.*

Proof. Consider the following strategy for player i : submit a list $S_i \neq \emptyset$, then after some history where S_i is eliminated, $a_i = 0$ (we do not need to specify the full strategy). Consider an alternative strategy that behaves as the previous one after all histories, except that it submits $S_i = \emptyset$. We show that the latter dominates the former.

First note that the expected payoff of a player when nobody is eliminated and she chooses $a_i = 0$ is $F(n)$. The payoff for player $i \in N \setminus \{E(\mathcal{S}) \cup PI\}$ when $E(\mathcal{S}) \neq \emptyset$ is eliminated and she chooses $a_i = 0$ is $F(n - \#E(\mathcal{S}))$, and $F(n - \#E(\mathcal{S})) < F(n)$. Note also that $F(n) > h(\mu, n)$ implies that it is dominant to choose $a_i = 0$ when nobody is eliminated.

Consider now some list of submissions S_{-i} . We distinguish two cases. First, player i 's choice is pivotal. If she votes $S_i \neq \emptyset$, then $E(S_i, S_{-i}) \in \{S_i, \emptyset\}$, whereas if she votes $S_i = \emptyset$, then $E(\emptyset, S_{-i}) = \emptyset$. Thus, the payoff accruing to i is never lower under the second strategy, and strictly higher when $E(S_i, S_{-i}) = S_i$. The alternative case is trivial. The choice of i is not pivotal, and behavior (thus payoffs) after all histories is the same for both strategies. ■

We denote by $e(\mathcal{S})$ the cardinality of $E(\mathcal{S})$.

Proposition 4 *In all undominated subgame perfect equilibria where $E(\mathcal{S}) \neq \emptyset$, all players $i \notin PI$ such that $S_i = E(\mathcal{S})$ choose $a_i = 1$, whenever $h(\mu, n - e(\mathcal{S})) > F(n)$.*

This shows that obtaining the efficient outcome is a possibility, but, unlike in the centralized case, not the only reasonable possibility. But since the pressure to coordinate is large, a focal point could be used to choose a group to be eliminated. As we already discussed, Girard (1982) argues that scapegoats have a *mark*, which distinguishes them for the rest of society. This could serve as the focal point, but notice that what is focal need not be what is efficient, as the focal group for elimination may be larger or smaller than the one maximizing $h(\mu, n - c)$. In reality, firms have typically a policy for layoffs, most usually the last hired are the first to be laid off. Another intriguing possibility, which we explore in the following subsection more formally is that *history* can serve to *efficiently* focus expectations.

Learning dynamics or an algorithm for efficiency The preceding discussion leaves open the possibility that a fully decentralized downsizing procedure may lead to suboptimal equilibrium allocations. One could think that this scarcely matters, since we have shown in Proposition 2 that a choice through a delegate already reaches the maximum organizational output. Yet, there may be circumstances where the direct route would be better. We have not considered, for example, the case where delegates are corruptible which could get in the way of the delicate belief-coordinating process.

Given these possibilities, is there any way to justify a fully decentralized downsizing? Suppose all agents are sufficiently strategically savvy to behave as in Proposition 4, but they have more trouble guessing at the particular way others will vote, and have to learn their way through equilibrium by a trial-and-error process. More specifically, we assume that agents play the game repeatedly. Each individual starts by playing some arbitrary (pure) voting-strategy (as we said before they still behave as in Proposition 4 after the voting stage) and before each repetition of the game they have an opportunity to change their vote with some probability. The dynamics of voting will be fully described when one identifies the transition probabilities between strategies. Instead of fully describing the process we enumerate a set of assumptions that are sufficient for the results of the paper.

D1 The transition probabilities depend exclusively on the present voting profile.

D2 One individual chosen at random is given the chance to update her vote every period.

D3 If the individual is given the chance to update her strategy, any vote that best-responds to the present voting profile is adopted with positive probability.

D4 A vote which does not improve upon the strategy currently in use is adopted with zero probability.

D5 An individual changes a vote which leads to the maximal payoff with zero probability.

These assumptions permit us to obtain clear-cut results in a relatively simple fashion. Assumption D1 simplifies the analysis by making the strategy profile of a certain period the state variable of the system, but it is not essential for the results. It would suffice if the system had a finite memory, for example. Assumption D2 is necessary because the voting game is such that except when voters are pivotal, any vote is a best response. The fact that every vote is a best response, coupled with assumption D3 would make it impossible to find any stable outcome, as all voters could simultaneously switch to anything else and destroy the stability of any outcome.

Assumptions D3, D4 and D5 are designed to exploit a special characteristic of the voting game. While an agent is not pivotal, she can change her vote while choosing a best response. This easily leads to a situation in which someone is pivotal. At that point only votes that are “efficient” are taken.

Properties D1 to D4 make our dynamics similar to the ones in Kim and Sobel (1995). Assumption (D2) corresponds to their assumption (I), Assumption D3 corresponds to their assumption (BR), assumption D4 to their assumption (NL). Our dynamics are also closely related to the ones in Hurkens (1995) and Gilboa and Matsui (1991).

Proposition 5 *Let \mathcal{S}^C be any set of voting profiles which leads to a set of eliminated individuals C that maximizes $h(\mu, n - c)$, let \mathcal{C} be the union of all such sets, and let $\mathcal{S}^C = \cup_{C \in \mathcal{C}} \mathcal{S}^C$. Given dynamics that satisfy properties D1, D2, D3, D4, D5;*

(a) *If $\mathcal{S}(0)$ is such that $\mathcal{S}(0) \notin \mathcal{S}^C$, then for all $C \in \mathcal{C}$, $\Pr(\text{for some } t', \mathcal{S}(t) \in \mathcal{S}^C, \forall t \geq t') > 0$.*

(b) *$\Pr(\text{for some } t', \mathcal{S}(t) \in \mathcal{S}^C, \forall t \geq t') = 1$.*

Proof. To prove (a) we have to look at a number of cases:

Case (1) Suppose that $\mathcal{S}(0)$ is such that nobody is eliminated and nobody can change the majority. Note that votes which do not change the outcome are always a best response. Then, by assumption D2 and D3, there is positive probability of a sequence of one period moves where all members of the population sequentially change their vote to $S_i = \emptyset$ so that we end up in a state of the population where $S_i(t) = \emptyset, \forall i$. Then, from that state and for any C^* , there is, by assumptions D2 and D3 a positive probability that all members of $N \setminus C^*$ get sequentially a chance to vote and they choose $S_i = C^*$, which is a best-response even when this means a change in state from eliminating not eliminating anybody to eliminating C^* . Once $\mathcal{S}(t)$ is such that C^* is eliminated, no member of $N \setminus C^*$ changes her vote.

Case (2) Suppose that $\mathcal{S}(0)$ is such that set $C \notin \mathcal{C}$ is eliminated. Then by assumption D2 and D3, there is a positive probability of a sequence of one period moves where all members of the population sequentially change their vote to $S_i = C$ so that we end up in a state of the population where $S_i(t) = C, \forall i$. From that point, there is a positive probability of a sequence of one period moves where all members of the population sequentially change their vote to $S_i = \emptyset$ so that we end up in a state of the population where $S_i(t) = \emptyset, \forall i$. The only thing needed for this to work is that the person in the sequence who changes from a C majority to a \emptyset majority is an agent $i \in C$. Once in state $S_i(t) = \emptyset, \forall i$ we can apply the same reasoning as in Case (1).

Case (3) Suppose that $\mathcal{S}(0)$ is such that nobody is eliminated and some agents can change the majority to eliminating set C . Then by assumption D2, one of the agent i

already voting for C gets to move and changes her vote to $S_i = \emptyset$. From that point on we are in a situation like Case (1).

Cases (1), (2) and (3) exhaust all the possible cases to show part (a), so the result follows. To establish part (b) notice that part (a) establishes that from any $\mathcal{S}(t)$ there is a lower bound $\epsilon > 0$ on the probability of reaching \mathcal{S}^C , and staying there forever in a number of steps smaller than some fixed and finite k . So the probability of not reaching \mathcal{S}^C in kn steps is bounded above by $(1 - \epsilon)^{kn}$. Since $\lim_{n \rightarrow \infty} (1 - \epsilon)^{kn} = 0$, part (b) follows. ■

3.3 A different timing for type revelation

The timing of type revelation may be seen as somewhat awkward in the game we just presented. One could perhaps expect that individuals knew whether they were PI at the time of voting, and not just when choosing whether to cooperate. It is not completely clear, however, that this is the clearly the best assumption, as there is substantial evidence (Johansson-Stenman and Svedsäter 2003) and theorizing (Santos-Pinto and Sobel 2002) about the importance of self-image for economic agents, and voting as a PI could negatively affect the individual's self-image in a way that actually acting as a PI would not. Nevertheless, we will now see that the results are very similar when one uses the alternative assumption that individuals know their type at the time of participating in the collective decision scheme.

We will assume that collective choice procedures are anonymous, in the sense that voting or related activities will be *secret*. So agents will only know *how many* others are in favor or against different options, not *who* is it that favors them. Besides being realistic, this trait would help true revelation of information. If voting were not secret, a PI -type would be concerned that signalling his type through voting would expose her to exclusion from the organization.

In addition, we assume, that the participants will only be informed about whether the proposal was passed or not. This is not quite as realistic, but the alternative provides the opportunity for much more conditioning of actions by the workers on the results of voting, and hence the possibility for strange equilibria to arise. We feel that those equilibria are probably unreasonable, but cannot give a more formal rebuttal, whereas under the assumption we use, the results are sharper, and it is still a feasible mechanism, which gives it, at a minimum, normative relevance.

The belief that other individuals are PI is now dependent on the history of play. Let \mathcal{N} be the outcome of voting under which C is *not* eliminated. Denote by π the cardinality of PI , by $p(\cdot | \mathcal{N})$ the posterior distribution over the size of PI and by $p_0(\cdot)$ the prior distribution over π , which depends on μ .

With this we can now proceed to review our results.

Proposition 6 *Assume that $\tau(k)F(n - c) > F(n) > h(\mu, n)$. Then at all equilibria where no player uses weakly dominated strategies, it is weakly dominant for all $i \in N \setminus \{C \cup PI\}$ to vote for elimination of C and choose $a_i = 1$, after the elimination.*

Proof. Notice that for PI types it is always dominant to reject an elimination. Indeed, their payoff after elimination is $F(n - c)$, which is smaller than $F(n)$, the payoff without elimination, and their vote may be pivotal.

We will now show that non- PI types choose to eliminate and then cooperate when. First, voting to eliminate and then choosing $a_i = 0$ is dominated for the reasons we have mentioned in other cases. Therefore, we are left to show that voting not to eliminate cannot be optimal when $\tau(k)F(n - c) > F(n) > h(\mu, n)$. We proceed in a number of steps.

Step 1. After observing an elimination, the distribution of the number of PI -types is a first-order stochastically dominating shift over the prior distribution.

Proof. Denote by ρ_S the cardinality of the number of non- PI players who decide not to cooperate after observing the outcome \mathcal{N} under strategy profile S . Given such strategy profile S , let P be the corresponding probability that the outcome is \mathcal{N} . Let $m = n - k$ and $p_0^i = p_0(\pi = i)$. For $x \leq m$, we have that

$$\begin{aligned}
p(\pi \leq x | \mathcal{N}) &= \frac{\sum_{i=0}^x p_0^i P(\rho_S + i \geq m + 1)}{\sum_{i=0}^x p_0^i P(\rho_S + i \geq m + 1) + \sum_{i=x+1}^{n-k} p_0^i P(\rho_S + i \geq m + 1) + \sum_{i=n-k+1}^n p_0^i} \\
&\leq \frac{\sum_{i=0}^x p_0^i P(\rho_S + x \geq m + 1)}{\sum_{i=0}^x p_0^i P(\rho_S + x \geq m + 1) + \sum_{i=x+1}^{n-k} p_0^i P(\rho_S + x \geq m + 1) + \sum_{i=n-k+1}^n p_0^i} \\
&\leq \frac{\sum_{i=0}^x p_0^i}{\sum_{i=0}^x p_0^i + \sum_{i=x+1}^{n-k} p_0^i + \frac{\sum_{i=n-k+1}^n p_0^i}{P(\rho_S + i \geq m + 1)}} \\
&\leq \frac{\sum_{i=0}^x p_0^i}{\sum_{i=0}^x p_0^i + \sum_{i=x+1}^{n-k} p_0^i + \sum_{i=n-k+1}^n p_0^i} \\
&= \sum_{i=0}^x p_0^i = p_0(\pi \leq x)
\end{aligned}$$

where the first inequality follows because the function $\frac{x_1}{x_1 + x_2 + x_3}$ is increasing in x_1 , and decreasing in x_2 , and we have substituted $P(\rho_S + i \geq m + 1)$ in the positions corresponding to x_1 and x_2 by something respectively bigger and smaller. **Q.E.D.**

Step 2. The payoff for a non- PI -type for cooperating after observing the rejection of an elimination is bounded above by $h(\mu, n)$.

Proof. Given that PI -types do not cooperate, the best possible case for cooperation is when all non- PI types cooperate. Thus the payoff for cooperation is bounded by $\sum_{i=0}^n p(\pi = i|\mathcal{N})\tau(n-i)F(n)$. We know by Step 1 that $p(\pi = i|\mathcal{N})$ is a first order stochastically dominating shift over $p_0(\pi = i)$. Since $\tau(n-i)$ is a monotonic function, this implies (Mas-Colell, Green and Whinston 1995, definition and proposition 6.D.1), that the payoff for cooperation is bounded by $\sum_{i=0}^n p_0(\pi = i)\tau(n-i)F(n) = h(\mu, n)$. **Q.E.D.**

Step 3. When $F(n) > h(\mu, n)$, cooperating for a non- PI after observing \mathcal{N} is not optimal.

Proof. $F(n)$ is the payoff for cooperating after \mathcal{N} , and $h(\mu, n)$ is an upper bound to the payoff under cooperation, by Step 2. **Q.E.D.**

Step 4. When $\tau(k)F(n-c) > F(n)$, voting for the elimination of C (and then choosing $a_i = 1$ after elimination) is weakly dominant if all players avoid the use of weakly dominated strategies.

Proof. Indeed, when C is eliminated, at least k players cast the yes vote. Then, $\tau(k)F(n-c)$ is a lower bound for players in $C \cup PI$ that contribute after C is eliminated. **Q.E.D. ■**

With this we can now proceed to the following result, analogous to Proposition 2.

Corollary 7 *Consider any collective choice procedure that selects a delegate to make the proposal of C and suppose that $k + c \leq n$. Then, if the delegate is not of a PI -type, she proposes a candidate C which maximizes the ex-ante payoff of all $i \in N \setminus \{C \cup PI\}$.*

4 Related literature

Our paper is closely related to the literature that relates the existence of firms and corporate cultures to the solution of coordination problems. Seminal papers in this literature are those of Kreps (1990) and Milgrom and Roberts (1995). Downsizing in our model has the property that it induces people to believe that others will play the high payoff (but risky) action. In this sense it acts in the way that Kreps (1990) envisioned (good) corporate culture. Our mechanism, though, is different from the one in that paper, which relied mostly on experiences shared by organization members to coordinate beliefs on the right action. Here, by contrast, a shared experience of cheating induces people to believe that a large enough number of pathological individualists are present in the organization. Only the downsizing event will change those beliefs for the good.

The mechanism that induces cooperation in our paper is connected to forward induction, as in the papers of Van Damme (1989) and Ben-Porath and Dekel (1992). Notice, though, that we only need one round of deletion of weakly dominated strategies (Dekel and Fudenberg 1990, Börgers 1994). Furthermore, unlike in Ben-Porath and Dekel (1992), the potential for self-sacrifice is not sufficient to obtain the desired outcome. The sacrifice has to be effective.

The relationship between scapegoats and organizations has also been studied by Winter (2001). He studies the incentive effects in a team-production problem carried out by a hierarchy of selecting certain individuals for punishment in case of an organizational failure. The mechanism design problem consists of finding the best possible structure of punishments for a given organization. López-Pintado, Ponti, and Winter (2003) study Winter (2001) in an experimental context.

5 Conclusion

We have shown in this paper that downsizing can have efficiency enhancing effects in organizations, by focusing expectations on the cooperating outcome. A number of extensions are possible.

But most importantly, the conclusions are based on the assumptions that agents are intelligent enough to avoid dominated strategies, and realize that others will do so. Since there is conflicting experimental evidence on whether this is actually true in games where the iterated deletion of weakly dominated strategies leads to a unique solution (see e.g. Balkenborg 1998, Brandts and Holt 1995 and Brandts, Cabrales and Charness 2002) it would be a good idea to test experimentally the predictions of this paper. Beyond this narrow test, further work, needs to be done to see whether the ideas in this paper have empirical support.

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