Model Uncertainty and the Gains from Coordinating Monetary Rules

Nicoletta Batini
International Monetary Fund

Alejandro Justiniano
International Monetary Fund

Paul Levine
University of Surrey

Joseph Pearlman
London Metropolitan University

May 10, 2005

Abstract

This paper empirically assesses the performance of forward-looking monetary rules for interdependent economies characterized by model uncertainty. We set out a two-bloc dynamic stochastic general equilibrium model with habit persistence (that generates output persistence), Calvo pricing and wage-setting with indexing of non-optimized prices and wages (generating inflation persistence), and the incomplete pass-through of exchange rate changes. We estimate a linearized form of the model by Bayesian methods using US and Euro-zone data. From the estimates of uncertainty we then examine monetary policy conducted both independently and cooperatively by the Fed and the ECB in the form Inflation-Forecast-Based interest rate rules. As in Batini *et al.* (2004b) which examined a closed economy only, the two central banks of this model use the estimated posterior probabilities to design rules to be robust with respect to the utility outcomes across all possible parameter combinations from a large sample of draws. The utility outcome in a closed-loop Nash equilibrium is then compared with the outcome from a coordinated design of policy rules. We find that current inflation rules perform better than forward-looking rules; there are modest gains from coordination, but only in a hypothetical US-Euro model where there is full goods market integration, and that for forward-looking rules robust policy design is essential to offer protection against the problem of indeterminacy.

JEL Classification: E52, E37, E58

Keywords: monetary policy coordination, robustness, inflation-forecast-based rules.

Contents

1 Introduction

The emergence of the new micro-founded open-economy macroeconomics has led naturally for the literature to revisit the economics of monetary policy interdependence. Following the seminal contribution of Obstfeld and Rogoff (1996), a number of papers have studied spillover effects and the resulting gains from policy coordination for interdependent economies (e.g., Betts and Devereux (2000b), Corsetti and Pesenti (2001), Obstfeld and Rogoff (2002), Clarida *et al.* (2002), Benigno and Benigno (2003)). We contribute to this literature by providing, in a rather more general setting than before, analytical results on spillover effects in a two-bloc set-up where there is incomplete exchange rate pass-through.

The main contribution of the paper however is empirical. The paper develops a two-bloc dynamic stochastic general equilibrium stochastic model to include habit persistence (that generates output persistence in the model), Calvo pricing with indexing of non-optimized prices (generating inflation persistence), and the incomplete pass-through of exchange rate changes. Wage stickiness is introduced using an analogous form of staggered wage setting. We estimate a linearized form of the model using Bayesian methods using US and Euro-zone data.

Throughout we focus on Taylor-type rules, and in particular on inflation-forecast-based (IFB) rules. These are 'simple' rules as in Taylor (1993), but where the policy instrument responds to deviations of expected, rather than current inflation from target. In most applications, the inflation forecasts underlying IFB rules are taken to be the endogenous rational-expectations forecasts conditional on an intertemporal equilibrium of the model. These rules are of interest because, as shown in Clarida *et al.* (2000) and Castelnuovo (2003), estimates of IFB-type rules appear to be a good fit to the actual monetary policy in the US and Europe of recent years. They are also of specific interest because similar reaction functions are used in the forecasting models of the Bank of Canada and the Reserve Bank of New Zealand, two prominent inflation-targeting central banks. In these countries and elsewhere, central bankers extol the virtues of IFB rules on the grounds that they "pre-empt inflation" and "enhance low-inflation credibility".

Using our estimated model probabilities and posterior densities of parameters, we then proceed to design IFB rules that are robust with respect to our *estimated* measures of model uncertainty in the sense that they guarantee stable and unique equilibria (thus avoiding indeterminacy[1] and, in addition, use the posterior parameter density functions to minimize an expected loss function of the central bank subject to this model uncertainty. Both cooperative and non-cooperative optimized IFB rules are computed. Comparisons between the outcomes under these two sets of rules provide an empirical assessment of the gains from coordination. Comparisons with an IFB rule with minimal feedback on expected inflation provides estimates of stabilization gains and finally a benchmark optimal rule that assesses both coordination and commitment enables us to evaluate the costs of restricting policy to IFB-type rules.

The rest of the paper is set out as follows: section 2 describes our model, the steady state and the linearization about the latter. Section 3 describes the estimation methodology and results. Section 4 provides a better understanding of the numerical results by analyzing a special case of our model involving the imposition of symmetry and other restrictions on parameters. Section 5 provides results for optimized IFB rules where model parameters are known with certainty. Section 6 tackles the case where there is parameter uncertainty and provides results for robust IFB rules. Section 7 summarizes our main results and suggests an agenda for further research.

2 The Model

There are two asymmetric unequally-sized blocs with the different household preferences and technologies. We assume complete asset markets. The exchange rate is perfectly flexible. The consumption index in each bloc is of Dixit-Stiglitz nested CES form with domestic and foreign components consisting of a basket of differentiated goods produced in each bloc. Goods producers and household suppliers of labour have monopolistic power. Wages and nominal domestic prices of both domestically produced and imported goods are sticky. Retail firms import foreign differentiated goods for which the law of one price holds at the docks. However in setting the domestic price of these goods, as with domestic producers, retail firms have monopolistic power which leads to a departure from the law of one price in both the long run and the short run.

[1] See Bernanke and Woodford (1997); Batini and Pearlman (2002); Giannoni and Woodford (2002); Carlstrom and Fuerst (1999), Benhabib *et al.* (2001), Woodford (2003), Batini *et al.* (2004a), BLP hereafter.

2.1 Households

There are ν_H households in the 'home' bloc and ν_F households in the 'foreign' bloc. A representative household r in the home bloc maximizes

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[\frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1-\sigma} + U_{M,t} \frac{\left(\frac{M_t(r)}{P_t}\right)^{1-\varphi}}{1-\varphi} - U_{N,t} \frac{(N_t(r) - H_{N,t})^{1+\phi}}{1+\phi} \right] \quad (1)$$

where $\mathcal{E}_t$ is the expectations operator indicating expectations formed at time t, β is the household's discount factor, $U_{C,t}$, $U_{M,t}$ and $U_{N,t}$ are preference shocks, $C_t(r)$ is a Dixit-Stiglitz index of consumption defined below in (4), $N_t(r)$ are hours worked, $H_{C,t}$ and $H_{N,t}$ represents the habit, or desire not to differ too much from other households, and we choose $H_{C,t} = h_C C_{t-1}$, where $C_t = \frac{1}{\nu_H} \sum_{r=1}^{\nu_H} C_t(r)$ is the average consumption index, $H_{N,t} = h_N \frac{N_{t-1}}{\nu_H}$, where N_t is aggregate labour supply defined after (3) below and $h_C, h_N \in [0, 1)$. When $h_C = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the intertemporal elasticity of substitution)[2]. $M_t(r)$ are end-of-period nominal money balances. An analogous symmetric intertemporal utility is defined for the 'foreign' representative household and the corresponding variables (such as consumption) are denoted by $C_t^*(r)$, etc.

The representative household r must obey a budget constraint:

$$P_t C_t(r) + \mathcal{E}_t(Q_{t+1} D_{t+1}(r)) + M_t(r) = (1 - T_t) W_t(r) N_t(r) + D_t(r) + M_{t-1}(r) + \Gamma_t(r) \quad (2)$$

where P_t is a Dixit-Stiglitz price index defined in (11) below, $D_{t+1}(r)$ is a random variable denoting the payoff of the portfolio $D_t(r)$, purchased at time t, and Q_{t+1} is the period-t price of an asset that pays one unit of domestic currency in a particular state of period $t+1$ divided by the probability of an occurrence of that state given information available in period t. $W_t(r)$ is the wage rate, T_t the income tax rate and $\Gamma_t(r)$ are dividends from ownership of firms. We first consider the case of flexible wages.

Assume the existence of nominal one-period riskless bonds denominated in domestic currency with nominal interest rate R_t over the interval $[t, t+1]$. Then arbitrage considerations imply that $\mathcal{E}_t Q_{t+1} = \frac{1}{1+R_t}$. In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand

[2] When $h_C \neq 0$, σ is merely an index of the curvature of the utility function.

for each consumer's labour supplied by ν_H identical households is given by

$$N_t(r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} \frac{N_t}{\nu_H} \tag{3}$$

where $W_t = \left[\frac{1}{\nu_H}\sum_{r=1}^{\nu_H} W_t(r)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ and $N_t = \left[\left(\frac{1}{\nu_H}\right)^{\frac{1}{\eta}}\sum_{r=1}^{\nu_H} N_t(r)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ are the average wage index and aggregate employment respectively.

Let the number of differentiated goods produced in the home and foreign blocs be n_H and n_F respectively. We assume that the the ratio of households to firms are the same in each bloc, so n_H and n_F (or ν_H and ν_F) are measures of size. Then the per capita Dixit-Stiglitz consumption index in the home bloc is given by

$$C_t(r) = \left[w_H^{\frac{1}{\mu}} C_{H,t}(r)^{\frac{\mu-1}{\mu}} + (1-w_H)^{\frac{1}{\mu}} C_{F,t}(r)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{4}$$

where

$$C_{H,t}(r) = \left[\left(\frac{1}{n_H}\right)^{\frac{1}{\zeta}} \sum_{f=1}^{n_H} C_{H,t}(f,r)^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \tag{5}$$

$$C_{F,t}(r) = \left[\left(\frac{1}{n_F}\right)^{\frac{1}{\zeta}} \sum_{f=1}^{n_F} C_{F,t}(f,r)^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \tag{6}$$

and

$$w_H = \frac{n_H \omega_H}{n_H \omega_H + n_F(1-\omega_H)} \tag{7}$$

In (7) $\omega_H \in [\frac{1}{2}, 1]$ is a parameter that captures the degree of 'bias' in the home bloc. If $\omega_H = 1$ we have autarky, while the lower extreme of $\omega_H = \frac{1}{2}$ gives us the case of perfect integration. If blocs are of equal size (as in BLP) then $n_H = n_F$, $w_H = \omega_H$ and consumption only favours home consumption if there is home bias.[3] In the absence of home bias $w_H = \frac{n_H}{n_H + n_F}$ and domestic/foreign consumption decisions depend only on relative size. As $\mu \to 1$ we approach a Cobb-Douglas utility function $C_t(r) = w_H^{-w_H}(1 - w_H)^{-(1-w_H)} C_{H,t}(r)^{w_H} C_{F,t}(r)^{1-w_H}$ as in BLP and Clarida *et al.* (2002).

If $P_{H,t}(f)$, $P_{F,t}(f)$ are the domestic prices of the two types of good produced by firm f in the relevant bloc, then the optimal intra-temporal decisions are given by standard results:

$$C_{H,t}(r,f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\zeta} C_{H,t}(r)\,; \;\; C_{F,t}(r,f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\zeta} C_{F,t}(r) \tag{8}$$

[3]The effect of home bias in open economies is also studied in Corsetti *et al.* (2002) and De Fiore and Liu (2002).

$$C_{H,t}(r) = w_H \left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t(r)\,; \;\; C_{F,t}(r) = (1-w_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t(r) \tag{9}$$

where aggregate price indices for domestic and foreign consumption bundles are given by

$$P_{H,t} = \left[\frac{1}{n_H}\sum_{f=1}^{n_H} P_{H,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \;;\;\; P_{F,t} = \left[\frac{1}{n_F}\sum_{f=1}^{n_F} P_{F,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{10}$$

and the domestic consumer price index P_t given by

$$P_t = \left[w_H(P_{H,t})^{1-\mu} + (1-w_H)(P_{F,t})^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{11}$$

The model considers departures from the law of one price i.e. prices in home and foreign blocs are linked by $\Psi_{H,t} = \frac{S_t P^*_{H,t}}{P_{H,t}} \neq 1$ and $\Psi_{F,t} = \frac{S_t P^*_{F,t}}{P_{F,t}} \neq 1$ necessarily, where $P^*_{H,t}$ and $P^*_{F,t}$ are the foreign currency prices of the home and foreign-produced goods and S_t is the nominal exchange rate. Let

$$P^*_t = \left[w_F(P^*_{F,t})^{1-\mu^*} + (1-w_F)(P^*_{H,t})^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} \tag{12}$$

be the foreign consumer price index corresponding to (11). Then it follows that the real exchange rate $E_t = \frac{S_t P^*_t}{P_t}$ and the terms of trade, defined as the domestic currency relative price of exports to imports, $\mathcal{T}_t = \frac{P_{H,t}}{P_{F,t}}$, are related by the relationship

$$E_t \equiv \frac{S_t P^*_t}{P_t} = \frac{\left[w_F \Psi_{F,t}^{1-\mu^*} + (1-w_F)(\Psi_{H,t}\mathcal{T}_t)^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}}}{\left[1 - w_H + w_H \mathcal{T}_t^{1-\mu}\right]^{\frac{1}{1-\mu}}} \tag{13}$$

Thus if the law of one price holds for differentiated goods; i.e., $\Psi_{H,t} = \Psi_{F,t} = 1$, and $\mu = \mu^*$, then $E_t = 1$ and the law of one price applies to the aggregate price indices iff $w_F = 1 - w_H$. The latter condition holds if there is no home bias. if there is home bias, the real exchange rate appreciates (E_t falls) as the terms of trade improves.

Maximizing (1) subject to (2) and (3), treating habit as exogenous, and imposing symmetry on households (so that $C_t(r) = C_t$, etc) yields standard results:

$$Q_{t+1} = \beta \frac{MU^C_{t+1}}{MU^C_t}\frac{P_t}{P_{t+1}} = \beta \frac{U_{C,t+1}(C_{t+1} - h_C C_t)^{-\sigma}}{U_{C,t}(C_t - h_C C_{t-1})^{-\sigma}}\frac{P_t}{P_{t+1}} \tag{14}$$

$$U_{M,t}\left(\frac{M_t}{P_t}\right)^{-\varphi} = \frac{(C_t - h_C C_{t-1})^{-\sigma}}{P_t}\left[\frac{R_t}{1+R_t}\right] \tag{15}$$

$$\frac{W_t(1-T_t)}{P_t} = -\frac{\eta}{(\eta-1)} \frac{MU^L_t}{MU^C_t} = \frac{\eta U_{N,t}}{(\eta-1)}\left(\frac{N_t}{\nu_H} - h_N \frac{N_{t-1}}{\nu_H}\right)^{\phi}(C_t - h_C C_{t-1})^{\sigma} \tag{16}$$

where MU_t^C and MU_t^L are the marginal utility of consumption and the marginal disutility of work respectively. Taking expectations of (14) we arrive at the following familiar Keynes-Ramsey rule adapted to take into account of the consumption habit:

$$1 = \beta(1+R_t)\mathcal{E}_t\left(\frac{U_{C,t+1}(C_{t+1}-h_C C_t)^{-\sigma}}{U_{C,t}(C_t-h_C C_{t-1})^{-\sigma}}\frac{P_t}{P_{t+1}}\right) \tag{17}$$

In (15), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. Given the central bank's setting of the latter and ignoring seignorage in the government budget constraint, (15) is completely recursive to the rest of the system describing our macro-model and will be ignored in the rest of the paper. (16) reflects the market power of households arising from their monopolistic supply of a differentiated factor input with elasticity η.

2.2 Domestic Producers

In the domestic goods sector each good differentiated good f is produced by a single firm f using only differentiated labour with another constant returns CES technology:

$$Y_t(f) = A_t\left[\left(\frac{1}{\nu_H}\right)^{\frac{1}{\eta}}\sum_{r=1}^{\nu_H} N_t(f,r)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \equiv A_t N_t(f) \tag{18}$$

where $N_t(f,r)$ is the labour input of type r by firm f and A_t is an exogenous shock capturing shifts to trend total factor productivity in this sector. Minimizing costs $\sum_{f=1}^{\nu_H} W_t(r)N_t(f,r)$ gives the demand for each household's labour by firm f as

$$N_t(f,r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta}\frac{N_t(f)}{\nu_H} \tag{19}$$

and aggregating over firms leads to the demand for labour as shown in (3)[4]. Total output in the home bloc is given by

$$Y_t = \sum_{f=1}^{n_H} Y_t(f) = A_t\left[\left(\frac{1}{\nu_H}\right)^{\frac{1}{\eta}}\sum_{r=1}^{\nu_H} N_t(r)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = A_t N_t \tag{20}$$

[4]Note that $N_t = \sum_{f=1}^{n_H} N_f(f) = \left[\left(\frac{1}{\nu_H}\right)^{\frac{1}{\eta}}\sum_{r=1}^{\nu_H} N_t(r)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ so in a symmetric equilibrium of identical firms and households $n_H N_t(f) = \nu_H N_t(r)$. Such a symmetric equilibrium applies to the flexi-price case of our model, but not to the sticky-price case where some firms and locked into contracts but others are revising their prices.

In a equilibrium of equal households, all wages adjust to the same level W_t. For later analysis it is useful to define the real marginal cost (MC) as the wage cost per unit of output relative to domestic producer price. Using (16) and (20) this can be written as

$$\begin{aligned} \mathrm{MC}_t &\equiv \frac{W_t}{A_t P_{H,t}} \\ &= \frac{U_{N,t}\eta}{(1-T_t)(\eta-1)A_t}\left(\frac{Y_t}{\nu_N A_t} - h_N \frac{Y_{t-1}}{\nu_H A_{t-1}}\right)^{\phi}(C_t - h_C C_{t-1})^{\sigma}\left(\frac{P_t}{P_{H,t}}\right) \end{aligned} \tag{21}$$

Turning to price-setting, we assume that there is a probability of $1-\xi_H$ at each period that the price of each good f is set optimally to $P^0_{H,t}(f)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation.[5] With indexation parameter $\gamma_H \geq 0$, this implies that successive prices with no re-optimization are given by $P^0_{H,t}(f),\ P^0_{H,t}(f)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H},\ P^0_{H,t}(f)\left(\frac{P_{H,t+1}}{P_{H,t-1}}\right)^{\gamma_H}, \ldots$. For each producer f the objective is at time t to choose $P^0_{H,t}(f)$ to maximize discounted profits

$$\mathcal{E}_t \sum_{k=0}^{\infty} \xi_H^k Q_{t+k} Y_{t+k}(f)\left[P^0_{Ht}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - P_{H,t+k}\mathrm{MC}_{t+k}\right] \tag{22}$$

where Q_{t+k} is the discount factor over the interval $[t, t+k]$, subject to a common[6] downward sloping demand from domestic consumers and foreign importers of elasticity ζ as in (8). The solution to this is

$$\mathcal{E}_t \sum_{k=0}^{\infty} \xi_H^k Q_{t+k} Y_{t+k}(f)\left[P^0_{Ht}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - \frac{\zeta}{(\zeta-1)}P_{H,t+k}\mathrm{MC}_{t+k}\right] = 0 \tag{23}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{H,t+1}^{1-\zeta} = \xi_H\left(P_{H,t}\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H}\right)^{1-\zeta} + (1-\xi_H)(P^0_{H,t+1}(f))^{1-\zeta} \tag{24}$$

2.3 Retail Firms

Following Monacelli (2003), retail firms import foreign differentiated goods for which the law of one price holds at the docks. The real marginal cost relative to the price $P_{F,t}$ set by retailers is therefore $\frac{S_t P^*_{F,t}}{P_{F,t}} = \Psi_{F,t}$. As for domestic producers, there is a probability of

[5]Thus we can interpret $\frac{1}{1-\xi_H}$ as the average duration for which prices are left unchanged.

[6]Note that we impose a symmetry condition $\zeta = \zeta^*$ at this point; i.e., the elasticity of substitution between differentiated goods produced in any one bloc is the same for consumers in both blocs.

$1-\xi_F$ at each period that the price of each good f is set optimally to $P^0_{F,t}(f)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation with indexation parameter $\gamma_F \geq 0$. Following the same reasoning as before we arrive at the following counterparts to (23) and (24)

$$\mathcal{E}_t \sum_{k=0}^{\infty} \xi_F^k Q_{t+k} C_{F,t+k}(f) \left[P^0_{Ft}(f) \left(\frac{P_{F,t+k-1}}{P_{F,t-1}} \right)^{\gamma_F} - \frac{\zeta}{(\zeta-1)} P_{F,t+k} \Phi_{F,t+k} \right] = 0 \tag{25}$$

$$P^{1-\zeta}_{F,t+1} = \xi_F \left(P_{F,t} \left(\frac{P_{F,t}}{P_{F,t-1}} \right)^{\gamma_F} \right)^{1-\zeta} + (1-\xi_F)(P^0_{F,t+1}(f))^{1-\zeta} \tag{26}$$

2.4 Staggered Wage-Setting

We introduce wage stickiness in an analogous way. There is a probability $1-\xi_W$ that the wage rate of a household of type r is set optimally at $W^0_t(r)$. If the wage is not re-optimized then it is indexed to last period's CPI inflation. With a wage indexation parameter γ_W the wage rate trajectory with no re-optimization is given by $W^0_t(r)$, $W^0_t(r)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_W}$, $W^0_t(r)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_W}$, $\cdots$. The household of type r at time t then chooses $W^0_t(r)$ to maximize

$$\mathcal{E}_t \sum_{k=0}^{\infty} (\xi_W \beta)^k \left[W^0_t(r)(1-T_t) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_W} N_{t+k}(r)\Lambda_{t+k}(r) - \frac{\left(\frac{N_{t+k}(r)}{\nu_H} - \frac{N_{t+k-1}}{\nu_H} \right)^{1+\phi}}{1+\phi} \right] \tag{27}$$

where $\Lambda_t(r) = \frac{MU^C_t(r)}{P_t}$ is the marginal utility of nominal income and $N_t(r)$ is given by (3). The first-order condition for this problem is

$$\mathcal{E}_t \sum_{k=0}^{\infty} (\xi_W \beta)^k \; W^\eta_{t+k} N_{t+k} \Lambda_{t+k}(r) \Bigg[W^0_t(r)(1-T_t) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_W}$$
$$- \frac{\eta}{(\eta-1)} \frac{\left(\frac{N_{t+k}}{\nu_H} - h_N \frac{N_{t+k-1}}{\nu_H} \right)^{\phi}}{\Lambda_{t+k}(r)} \Bigg] = 0 \tag{28}$$

We can now use $\beta^k \Lambda_{t+k}(r) = Q_{t+k}$, obtained from (14), and $\Lambda_t(r) = \frac{MU^C_t(r)}{P_t}$ to write (28) as

$$\mathcal{E}_t \sum_{k=0}^{\infty} \xi_W^k Q_{t+k} \; W^\eta_{t+k} N_{t+k} \left[W^0_t(r)(1-T_t) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_W} - \frac{\eta}{(\eta-1)} P_{t+k} MRS_{t+k} \right] = 0 \tag{29}$$

where $MRS_t(r) = -\frac{MU^L_t(r)}{MU^C_t(r)}$ is the marginal rate of substitution between work and consumption for household r. Note that as $\xi_W \to 0$ and wages become perfectly flexible, only the first term in the summation in (29) counts and we then have the result (16) obtained previously. By analogy with (24) and (26), by the law of large numbers the evolution of the wage index is given by

$$W_{t+1}^{1-\eta} = \xi_W \left(W_t \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_W} \right)^{1-\eta} + (1-\xi_W)(W_{t+1}^0(r))^{1-\eta} \tag{30}$$

2.5 The Equilibrium

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the home consumer good and assuming that exogenous government, G_t, expenditure goes exclusively on home goods we obtain

$$Y_t = \nu_H C_{H,t} + \nu_F C_{H,t}^* + G_t = \nu(C_{H,t} + C_{F,t}) + \nu_F C_{H,t}^* - \nu_H C_{F,t} + G_t = \nu_H C_t + TB_t + G_t \tag{31}$$

where TB_t is the trade balance. A balanced government budget constraint[7]

$$P_{H,t} G_t = T_t N_t W_t = P_{H,t} T_t Y_t MC_t \tag{32}$$

that assumes all taxes are raised from wage income completes the model.

Given nominal interest rates R_t, R_t^* the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition. Then the equilibrium is defined at $t = 0$ as stochastic sequences C_t, C_{Ht}, C_{Ft}, P_{Ht}, P_{Ft}, P_t, M_t, W_t, Y_t, N_t, P_{Ht}^0, 11 foreign counterparts C_t^*, etc, E_t, and S_t, given past price indices and exogenous processes $U_{C,t}$, $U_{M,t}$, $U_{N,t}$, A_t, G_t and foreign counterparts.

From (14) and its foreign counterpart we have

$$Q_{t+1} = \beta \left(\frac{U_{C,t+1}(C_{t+1} - h_C C_t)^{-\sigma}}{U_{C,t}(C_t - h_C C_{t-1})^{-\sigma}} \right) \frac{P_t}{P_{t+1}} = \beta \left(\frac{U_{C,t+1}^*(C_{t+1}^* - h_C^* C_t^*)^{-\sigma^*}}{U_{C,t}^*(C_t^* - h_C^* C_{t-1}^*)^{-\sigma^*}} \right) \frac{P_t^* S_t}{P_{t+1}^* S_{t+1}} \tag{33}$$

Let $z_t = \frac{U_{C,t} S_t P_t^*}{U_{C,t}^* P_t} \frac{(C_t - h_C C_{t-1})^{-\sigma}}{(C_t^* - h_C^* C_{t-1}^*)^{-\sigma^*}}$. Then assuming identical holdings of initial wealth in the two blocs, (33) implies that $z_{t+1} = z_t = z_0 = \frac{E_0(C_0(1-h_C))^{\sigma}}{(C_0^*(1-h_C^*))^{\sigma^*}}$ where initial relative consumption in prices denominated in the home currency reflects different initial wealth

[7] We ignore seignorage and consistent with this we later ignore the utility from money balances in the household welfare function.

in the two blocs. Therefore[8]

$$\left(\frac{U_{C,t}(C_t - h_C C_{t-1})^{-\sigma}}{U^*_{C,t}(C^*_t - h_C C^*_{t-1})^{-\sigma^*}}\right) = \frac{z_0 P_t}{S_t P^*_t} = \frac{z_0}{E_t} \tag{34}$$

2.6 The Steady State

A deterministic zero-inflation steady state, denoted by variables without the time subscripts, $\mathcal{E}_{t-1}(U_{C,t}) = 1$ and $\mathcal{E}_{t-1}(U_{N,t}) = \kappa$ is given by

$$C_H = w_H \left(\frac{P_H}{P}\right)^{-\mu} C \tag{35}$$

$$C_F = (1 - w_H)\left(\frac{P_F}{P}\right)^{-\mu} C \tag{36}$$

$$P = \left[w_H P_H^{1-\mu} + (1 - w_H) P_F^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{37}$$

$$\left(\frac{M}{P}\right)^{-\varphi} = \frac{[(1 - h_C)C]^{-\sigma}}{P}\left(\frac{R}{1+R}\right) \tag{38}$$

$$\frac{W(1-T)}{P} = \frac{\kappa(1-h_N)^{\phi}(1-h_C)^{\sigma}}{1-\frac{1}{\eta}}\left(\frac{N}{\nu_H}\right)^{\phi} C^{\sigma} \tag{39}$$

$$1 = \beta(1+R) \tag{40}$$

$$Y = AN \tag{41}$$

$$P_H = P_H^0 = \frac{W}{A\left(1-\frac{1}{\zeta}\right)} \tag{42}$$

$$P_F = P_F^0 = \frac{SP_F^*}{\left(1-\frac{1}{\zeta}\right)} \quad \text{i.e., } \psi_F = \left(1 - \frac{1}{\zeta}\right) \tag{43}$$

$$Y = \nu_H C + G \tag{44}$$

plus the 10 foreign counterparts and

$$\mathcal{T} = \frac{P_H}{P_F} \tag{45}$$

$$E = \frac{SP^*}{P} \tag{46}$$

$$\frac{E(C(1-h_C))^{-\sigma}}{(C^*(1-h_C^*))^{-\sigma^*}} = z_0 \tag{47}$$

[8] (34) is the risk-sharing condition for consumption, because it equates marginal rate of substitution to relative price, as would be obtained if utility were being jointly maximized by a social planner (see Sutherland (2002)). Note that (17) and (34) together imply the stochastic UIP condition (see Benigno and Benigno (2001)).

This gives 23 equations to determine the steady state of 25 endogenous variables: C, C_H, C_F, P, M, W, N, R, Y, $P_H = P_H^0$, $P_F = P_F^0$, 11 foreign counterparts C^* etc, $\mathcal{T}$, S and E given G and z_0.

To pin down price levels we need to equate money demand in (38) and its foreign counterpart with exogenously set money supplies in the two blocs, which then gives us a determinate steady state of the model. It is convenient to assume that money supplies in our steady state are set so as to result in $S = 1$ and dispense with the money demand equations. Furthermore as is standard in general equilibrium models we choose units of output appropriately so that prices of the two goods in their own currencies are unity; i.e, $P_H = P_F^* = 1$. With these assumptions the law of one price gaps are $\Psi_H = \frac{SP_H^*}{P_H} = P_H^*$ and $\Psi_F = \frac{SP_F^*}{P_F} = \frac{1}{P_F}$. The foreign counterpart to (43) is a mark-up relationship

$$P_H^* = \frac{P_H}{S(1 - \frac{1}{\zeta})} \tag{48}$$

It follows that $\Psi_H = \frac{1}{\Psi_F}$ and the price of the imported good in a steady state equilibrium, P_I say, in either currency is given by

$$P_I = P_H^* = P_F = \frac{1}{\mathcal{T}} = \frac{1}{1 - \frac{1}{\zeta}} \tag{49}$$

Thus in our normalization and choice of exchange rate in the steady state the domestic good in each bloc is priced at unity and the imported good is marked above unity at $P_I = \frac{1}{1-\frac{1}{\zeta}}$. As exchange rate pass-through becomes complete, $\zeta \to \infty$ and both goods are priced at unity. The consumer price indices are

$$P = \left[w_H + (1 - w_H)P_I^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{50}$$

$$P^* = \left[w_F + (1 - w_F)P_I^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} \tag{51}$$

2.6.1 Conditions for a Symmetric Steady State

We now examine the conditions for which a symmetric steady state with $z_0 = E = 1$, $P = P^*$, $C = C^*$ etc exists. In such a steady state we assume that preferences are identical which includes the degree of bias in the two blocs and, as mentioned in describing household behaviour, we have assumed that the the ratio of households to firms are the same in each bloc. Normalizing the total world population and variety numbers at unity we can put $\nu_H = \nu$, $\nu_F = 1-\nu$, $n_H = n$, $n_F = 1-n$. Therefore we have

$$\frac{\nu}{n} = \frac{1-\nu}{1-n} \tag{52}$$

$$\omega_H = \omega_F = \omega \tag{53}$$

say. We further assume that $\frac{G}{G^*} = \frac{\nu}{1-\nu}$. With these symmetry assumptions the output equilibrium condition (44) and its foreign counterpart become

$$Y = \left(\nu w_H \left(\frac{1}{P}\right)^{-\mu} + (1-\nu)(1-w_F)\left(\frac{P_I}{P}\right)^{-\mu}\right) C \tag{54}$$

$$Y^* = \left((1-\nu) w_F \left(\frac{1}{P^*}\right)^{-\mu} + \nu(1-w_H)\left(\frac{P_I}{P^*}\right)^{-\mu}\right) C^* + \frac{1-\nu}{\nu} G \tag{55}$$

where

$$w_H = \frac{n\omega}{n\omega + (1-n)(1-\omega)}\,;\; w_F = \frac{(1-n)\omega}{(1-n)\omega + n(1-\omega)} \tag{56}$$

It follows that a symmetric equilibrium with $P = P^*$, consumption per household equal in the two blocs ($C = C^*$) and output per firm equal ($\frac{Y}{n} = \frac{Y^*}{1-n}$) requires that

$$w_H = w_F \tag{57}$$

$$(1-\nu)^2(1-w_F) = \nu^2(1-w_H) \tag{58}$$

It follows that with incomplete exchange rate pass-through so $P_I \neq 1$, a symmetric equilibrium for unequally sized blocs is only possible if there is autarky ($\omega = w_H = w_F = 1$), or the blocs are of equal size ($n = \nu = \frac{1}{2}$). To summarize:

Proposition 1

With symmetry conditions $\frac{\nu}{n} = \frac{1-\nu}{1-n}$, $\omega_H = \omega_F = \omega$ and $\frac{G}{G^*} = \frac{\nu}{1-\nu}$ a symmetric equilibrium of trading economies is only possible if autarky prevails, or the blocs are of equal size.

The intuition behind this result is straightforward. In an open economy the smaller a country is, the greater its CPI index as imported goods incur a second mark-up by retailers. It follows that in a symmetric equilibrium of open (but not autarkic) economies, equal CPI indices requires equally-sized blocs.

2.6.2 The Inefficiency of a Symmetric Steady State

Now consider a symmetric world equilibrium satisfying the requirements of proposition 1. After some manipulation, the steady-state level of output (the 'natural rate'), is given by

$$Y^{\phi}(Y-G)^{\sigma} = \frac{(1-T)\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right)A^{1+\phi}}{\kappa(1-h_C)^{\sigma}(1-h_N)^{\phi}} \tag{59}$$

Following the argument of Choudhary and Levine (2004), the social planner puts $C_t(r) = C_t$ and $N_t(r) = N_t = \frac{Y_t}{A_t}$ and maximizes household's utility in equilibrium. The efficient steady-state level of output Y^e, say, is then given by

$$(Y^e)^{\phi}(Y^e-G)^{\sigma} = \frac{(1-h_C\beta)A^{1+\phi}}{\kappa(1-h_N\beta)(1-h_C)^{\sigma}(1-h_N)^{\phi}} \tag{60}$$

Comparing (59) and (60), since $(Y)^{\phi}(Y-G)^{\sigma}$ is an increasing function of Y, we arrive at

Proposition 2

The natural level of output, Y, is below the efficient level, Y^e, if and only if

$$(1-T)\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right) < \frac{1-h_C\beta}{1-h_N\beta} \tag{61}$$

In the case where there is no habit persistence for both consumption and labour effort, $h_C = h_N = 0$, then (61) always holds. In this case tax distortions and market power in the output and labour markets captured by the elasticities $\eta \in (0,\infty)$ and $\zeta \in (0,\infty)$ respectively drive the natural rate of output below the efficient level. If $T = 0$ and $\eta = \zeta = \infty$, tax distortions and market power disappear and the natural rate is efficient. Another case where (61) always holds is where habit persistence for labour supply exceeds that for consumption; i.e., $h_N \geq h_C$. Some empirical estimates (though not in this paper) suggest that $h_C < h_N$ which leads to the possibility that the natural rate of output can actually be *above* the efficient level (see Choudhary and Levine (2004)).

2.7 Linearization

We now linearize around a baseline and, in general, asymmetric, steady state in which consumption, output, employment and prices in the two blocs are constant. Then inflation is zero. Output is then at its inefficient natural rate studied in the previous section and the nominal rate of interest is given by (40). Now define all lower case level variables, such as

C_t, Y_t, as proportional deviations from this baseline steady state. Rates of change, inflation and interest rates are expressed as absolute deviations.[9] Home producer and consumer inflation are defined as $\pi_{Ht} \equiv \frac{P_{Ht}-P_{H,t-1}}{P_{H,t-1}} \simeq p_{Ht} - p_{H,t-1}$ and $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}} \simeq p_t - p_{t-1}$ respectively. Similarly, define foreign producer inflation and consumer price inflation. Combining (23) and (24), we can eliminate P^0_{Ht} to obtain in linearized form

$$\pi_{Ht} = \frac{\beta}{1+\beta\gamma_H}\mathcal{E}_t\pi_{H,t+1} + \frac{\gamma_H}{1+\beta\gamma_H}\pi_{H,t-1} + \frac{(1-\beta\xi_H)(1-\xi_H)}{(1+\beta\gamma_H)\xi_H}mc_t \tag{62}$$

where

$$mc_t = w_t - p_{H,t} - a_t \tag{63}$$

$$\Delta mc_t = \Delta w_t - \pi_{H,t} - \Delta a_t \tag{64}$$

The linearized version of the marginal rate of substitution defined in (16) is given by

$$\begin{aligned} mrs_t &= -\left(\frac{\phi}{1-h_N}\right)a_t + \frac{h_N\phi}{1-h_N}a_{t-1} + \frac{\sigma}{1-h_C}(c_t - h_C c_{t-1}) \\ &+ \frac{\phi}{1-h_N}(y_t - h_N y_{t-1}) + u_{N,t} \end{aligned} \tag{65}$$

Then combining (29) and (30), we can eliminate W^0_t to obtain

$$\Delta w_t + \beta\gamma_W\pi_t = \beta\mathcal{E}_t\Delta w_{t+1} + \gamma_W\pi_{t-1} + \frac{(1-\beta\xi_W)(1-\xi_W)}{\xi_W}(mrs_t + p_t - w_t + t_t) \tag{66}$$

where $\Delta w_t = w_t - w_{t-1}$ is wage inflation, and from (25) and (26), we obtain

$$\pi_{F,t} = \frac{\beta}{1+\beta\gamma_F}\mathcal{E}_t\pi_{F,t+1} + \frac{\gamma_F}{1+\beta\gamma_F}\pi_{F,t-1} + \frac{(1-\beta\xi_F)(1-\xi_F)}{(1+\beta\gamma_F)\xi_F}\psi_{F,t} \tag{67}$$

Linearizing the remaining equations (17), (34) and (31) yields

$$\frac{1}{1+h_C}\mathcal{E}_t c_{t+1} - c_t + \frac{h_C}{1+h_C}c_{t-1} = \frac{1-h_C}{(1+h_C)\sigma}[r_t - \mathcal{E}_t\pi_{t+1} + \mathcal{E}_t u_{C,t+1} - u_{C,t}] \tag{68}$$

$$\frac{\sigma^*}{1-h^*_C}(c^*_t - h^*_C c^*_{t-1}) = \frac{\sigma}{1-h_C}(c_t - h_C c_{t-1}) - e_t + u_{C,t} - u^*_{C,t} \tag{69}$$

$$y_t = \alpha_H[c_t - \mu(p_{H,t} - p_t)] + \alpha_F[c^*_t - \mu(p^*_{H,t} - p^*_t)] + \alpha_G g_t \tag{70}$$

[9] That is, for a typical variable level X_t, $x_t = \frac{X_t - \bar{X}}{\bar{X}} \simeq \log\left(\frac{X_t}{\bar{X}}\right)$ where $\bar{X}$ is the baseline steady state. Rate variables such as the interest and tax rates however are expressed as an absolute deviation; i.e., $r_t = R_t - R$ and $t_t = T_t - T$.

where

$$\alpha_H = w_H \frac{\nu_H C}{Y}\left(\frac{P_H}{P}\right)^{-\mu} \tag{71}$$

$$\alpha_F = (1-w_F)\frac{\nu_F C^*}{Y^*}\left(\frac{P_H^*}{P^*}\right)^{-\mu^*} \tag{72}$$

$$\alpha_G = 1-\alpha_H-\alpha_F \tag{73}$$

$$g_t = t_t + y_t + mc_t \tag{74}$$

Let $\Psi_{H,t} \equiv \frac{S_t P_{H,t}^*}{P_{H,t}} = E_t \frac{P_{H,t}^*}{P_t^*}\frac{P_t}{P_{H,t}}$ be the relative price of the home good for foreign consumers relative to that for home consumers; i.e., the 'law of one price gap'. Similarly define $\Psi_{F,t} \equiv \frac{S_t P_{F,t}^*}{P_{F,t}}$. Then the foreign counterpart to $\Psi_{H,t}$ is $\Psi_{F,t}^* = \frac{P_{F,t}}{S_t P_{F,t}^*} = \frac{1}{\Psi_{F,t}}$. The linearized model is now completed with

$$\psi_{H,t} = e_t + p_{H,t}^* - p_t^* - (p_{H,t} - p_t) \tag{75}$$

$$\psi_{F,t} = e_t + p_{F,t}^* - p_t^* - (p_{F,t} - p_t) \tag{76}$$

$$\pi_t = w_H\left(\frac{P_H}{P}\right)^{1-\mu}\pi_{H,t} + (1-w_H)\left(\frac{P_F}{P}\right)^{1-\mu}\pi_{F,t} \tag{77}$$

$$p_t - p_{H,t} = -(1-w_H)\left(\frac{P_F}{P}\right)^{1-\mu}\tau_t \tag{78}$$

$$p_t - p_{F,t} = w_H\left(\frac{P_H}{P}\right)^{1-\mu}\tau_t \tag{79}$$

By analogy with (78) and (79) we have

$$p_t^* - p_{F,t}^* = -(1-w_F)\left(\frac{P_H^*}{P^*}\right)^{1-\mu^*}\tau_t^* \tag{80}$$

$$p_t^* - p_{H,t}^* = w_F\left(\frac{P_F^*}{P^*}\right)^{1-\mu^*}\tau_t^* \tag{81}$$

where $\tau_t^* = p_{F,t}^* - p_{H,t}^*$ are the terms of trade in linear-deviation form in the foreign country. Note first, from (75) and (76) that $\tau_t^* = \psi_{F,t} - \psi_{H,t} - \tau_t$. Second that the Keynes-Ramsey condition (68), its foreign counterpart and the risk-sharing condition (69) together imply

$$\mathcal{E}_t e_{t+1} - e_t = r_t - r_t^* - (\mathcal{E}_t\pi_{t+1} - \mathcal{E}_t\pi_{t+1}^*) \tag{82}$$

which is the UIP condition for the real exchange rate and real interest rates.

For the exogenous shocks $u_{C,t} = U_{C,t} - 1$, $u_{N,t}$, a_t, g_t and foreign counterparts we assume AR(1) processes $u_{C,t} = \rho_C u_{C,t-1} + \epsilon_{C,t}$, $u_{N,t} = \rho_N u_{N,t-1} + \epsilon_{N,t}$, $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$ and $g_t = \rho_g g_{t-1} + \epsilon_{g,t}$ where $\epsilon_{i,t}$, $i = C, N, a, g$ is a white noise disturbance.

We finally close the model by assuming a standard Taylor-type rule for the conduct of monetary policy in each block, in which interest rates are set in an inertial manner to respond to inflation and the output gap, $\hat{y}$, (which is further described below)

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\kappa_\pi \pi_t + \kappa_y \hat{y}_t) + \epsilon_{r,t} \tag{83}$$

and similarly for the foreign block with parameters ρ_r^*, κ_π^* and κ_y^*.

2.8 State Space Representation

Taking first differences of $\tau_t = p_{H,t} - p_{F,t}$ and $\tau_t^* = p_{F,t}^* - p_{H,t}^*$ we have

$$\Delta\tau_t = \pi_{H,t} - \pi_{F,t} \tag{84}$$

$$\Delta\tau_t^* = \pi_{F,t}^* - \pi_{H,t}^* \tag{85}$$

and substituting (78) to (81) into (75) and (76) we have

$$\psi_{H,t} = e_t - w_F \left(\frac{P_F^*}{P^*}\right)^{1-\mu^*} \tau_t^* - (1-w_H)\left(\frac{P_F}{P}\right)^{1-\mu} \tau_t \tag{86}$$

$$\psi_{F,t} = e_t + (1-w_F) \left(\frac{P_H^*}{P^*}\right)^{1-\mu^*} \tau_t^* + w_H \left(\frac{P_H}{P}\right)^{1-\mu} \tau_t \tag{87}$$

We can write this system in state space form as

$$\begin{bmatrix} \mathsf{z}_{1,t+1} \\ \mathsf{z}_{2,t+1} \\ \mathcal{E}_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_{1,t} \\ \mathsf{z}_{2,t} \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{o}_t + C \begin{bmatrix} r_t \\ r_t^* \end{bmatrix} + D\epsilon_{t+1} \tag{88}$$

$$F\mathsf{o}_\mathsf{t} = H \begin{bmatrix} \mathsf{z}_{1,t} \\ \mathsf{z}_{2,t} \\ \mathsf{x}_t \end{bmatrix} \tag{89}$$

where $\mathsf{z}_{1,t} = [u_{C,t}, u_{C,t}^*, u_{N,t}, u_{N,t}^*, a_t, a_t^*, g_t, g_t^*]^T$ is a vector of predetermined exogenous variable, $z_{2,t} = [a_{t-1}, a_{t-1}^*, c_{t-1}, c_{t-1}^*, \pi_{H,t-1}, \pi_{H,t-1}^*, \pi_{F,t-1}^{'} \pi_{F,t-1}^*, \pi_t, \pi_t^*, y_{t-1}, y_{t-1}^*, \tau_{t-1}\tau_{t-1}^*,$ $mc_{t-1}, mc_{t-1}^*]^T$ is a vector of predetermined endogenous variables at time t, $\mathsf{x}_t = [a_t, a_t^*,$ $c_t, c_t^*, \pi_{H,t}, \pi_{F,t}, \pi_{H,t}^*, \pi_{F,t}^*, \Delta w_t, \Delta w_t^*]^T$ are non-predetermined variables, $\epsilon_t = \epsilon_{N,t}, \epsilon_{N,t}^*, \epsilon_{a,t}, \epsilon_{a,t}^*,$ $\epsilon_{g,t}, \epsilon_{g,t}^*]^T$ is a vector of white noise disturbances and $\mathsf{o}_t = [mrs_t, mrs_t^*, y_t, y_t^*, \psi_{H,t}, \psi_{F,t}, t_t, t_t^*, \pi_t, \pi_t^*,$ $w_t - p_t, w_t^* - p_t^*]^T$ is a vector of outputs. Matrices A, B, etc are functions of model parameters. Rational expectations are formed assuming an information set $\{z_{1,s}, z_{2,s}, x_s\}$, $s \leq t$, the model and the monetary rule.

For later use we require the *output gap* the difference between output for the sticky price-wage model obtained above and output when prices and wages are flexible and exchange rate pass-through is complete, $\hat{y}_t$ say. The latter, obtained by setting $\zeta = \infty$ and hence $P_H^* = P_F = 1$, $\xi_H = \xi_F = \xi_W = mc_t = \psi_{H,t} = \psi_{F,t} = mrs_t + p_t - w_t + t_t = 0$ and $\hat{\tau}_t = -\hat{\tau}_t^*$ in (65) to (81), is in deviation form given by[10]

$$
\begin{aligned}
-\left(1+\frac{\phi}{1-h_N}\right)a_t \quad &+ \quad \frac{h_N\phi}{1-h_N}a_{t-1} + \frac{\sigma}{1-h_C}(\hat{c}_t - h_C\hat{c}_{t-1}) \\
&+ \quad \frac{\phi}{1-h_N}(\hat{y}_t - h_N\hat{y}_{t-1}) + \hat{p}_t - \hat{p}_{H,t} + \hat{t}_t + u_{N,t} = 0 \qquad (90)
\end{aligned}
$$

$$
\begin{aligned}
\hat{e}_t &= -\frac{\sigma^*}{1-h_C^*}(\hat{c}_t^* - h_C^*\hat{c}_{t-1}^*) - \frac{\sigma}{1-h_C}(\hat{c}_t - h_C\hat{c}_{t-1})) + u_{C,t} - u_{C,t}^* && (91)\\
\hat{e}_t &= (1 - w_H - w_F)\hat{\tau}_t && (92)\\
\hat{y}_t &= \alpha_H[\hat{c}_t - \mu(\hat{p}_{H,t} - \hat{p}_t)] + \alpha_F[\hat{c}_t^* - \mu(\hat{p}_{H,t}^* - \hat{p}_t^*)] + \alpha_G g_t && (93)\\
g_t &= \hat{t}_t + \hat{p}_{H,t} - \hat{p}_t && (94)\\
\hat{p}_t - \hat{p}_{H,t} &= -(1 - w_H)\hat{\tau}_t && (95)\\
\hat{p}_t^* - \hat{p}_{H,t}^* &= -w_F\hat{\tau}_t && (96)
\end{aligned}
$$

with foreign counterparts. This can we written in state-space form

$$
\begin{aligned}
\hat{\mathsf{z}}_t &= J\hat{\mathsf{z}}_{t-1} + K\hat{\mathsf{o}}_t + L\epsilon_t && (97)\\
M\hat{\mathsf{o}}_t &= R\hat{\mathsf{z}}_t && (98)
\end{aligned}
$$

where $\hat{\mathsf{z}}_t = [\hat{c}_t, \hat{c}_t^*, \hat{y}_t, \hat{y}_t^*]^T$ and $\hat{\mathsf{o}}_t = [\hat{t}_t, \hat{t}_t^*, \hat{\tau}_t, \hat{e}_t]^T$.

The whole model can now be written in state space form as

$$
\begin{bmatrix} \mathsf{z}_{1,t+1} \\ \mathsf{z}_{2,t+1} \\ \hat{\mathsf{z}}_{2,t} \\ \mathcal{E}_t\mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_{1,t} \\ \mathsf{z}_{2,t} \\ \hat{\mathsf{z}}_{2,t-1} \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{o}_t + C\begin{bmatrix} r_t \\ r_t^* \end{bmatrix} + D\epsilon_{t+1} \qquad (99)
$$

$$
F\mathsf{o}_\mathsf{t} = H\begin{bmatrix} \mathsf{z}_{1,t} \\ \mathsf{z}_{2,t} \\ \hat{\mathsf{z}}_{2,t-1} \\ \mathsf{x}_t \end{bmatrix} \qquad (100)
$$

[10]Note that the zero-inflation steady states of the sticky and flexi-price steady states are the same.

3 Estimation

3.1 Data

We take the United States and the Euro Area to represent the theoretical domestic and foreign blocks of the model, respectively. The solution to the linearized version of the model is then fit to thirteen series: output, consumption, domestic and all-goods inflation, nominal wage inflation and interest rates observed both in the United States and Euro Area, as well as the real exchange rate. The model has fourteen structural shocks (technology, preferences, government, labour supply, the innovations to the monetary policy rule as above and, in addition, we add a mark-up shock to prices and wages with standard deviations $sd(m\pi)$ and $sd(mw)$ in the tables below). Rather than imposing a disturbance to the interest parity equation, which would contradict our complete markets assumption, we include in addition a white noise measurement error in the real exchange rate with standard deviation $sd(e)$ (as discussed below, this seems also natural given our construction of this series). Consequently, this last innovation does not enter the solution of the linear system but only the measurement equation in the state space representation.

Domestic and all-goods inflation for each block are given by the quarterly log-difference (annualized) in the consumption and GDP deflators respectively. Wage inflation also corresponds to the quarterly log-difference in nominal wages. Real private consumption and real output are expressed in log-deviations from a linear trend, while nominal interest rates are annualized. The log-difference in the real exchange rate is constructed in a model consistent manner by adding up quarterly Euro all-goods inflation and the (log) change in the bilateral nominal exchange rate and subtracting U.S .all-goods inflation. All series for the Euro Area are taken from the database for the Area-Wide Model (AWM) developed at the ECB (see Fagan *et al.* (2001)).[11]

[11] Corresponding to the mnemonics in the AWM database our foreign observables are given by YER (real GDP), PCR (real personal consumption), PCD (consumption deflator), YFD (GDP deflator), WRN (nominal wage-rate) and STI (short-term nominal interest rates). In contrast to Smets and Wouters (2003) who use the same dataset for Europe, we do not impose the same trend for consumption and output. For the domestic block, the series are obtained from Haver Analytics, with mnemonics GDPH (real GDP), CH (real personal consumption), JC (consumption deflator), JGDP (GDP deflator), LKPRIVA (average weekly earning of total private industry) and FFED (effective federal funds rate). As mentioned, the real exchange rate is constructed using the all-goods inflation measures and the bilateral exchange rate All

3.2 Normalizations

For purposes of the estimation, we make a number of convenient normalizations. First normalization total firms and households in the world economy at unity so that

$$\nu_H + \nu_F = n_H + n_F = 1 \tag{101}$$

and assume households per firm are the same in each bloc:

$$\frac{\nu_H}{n_H} = \frac{\nu_F}{n_F} \tag{102}$$

Then $\nu_H = n_H$ and $\nu_F = n_F$ are parameters indicating the *relative size* of the two blocs which we can calibrate using data on GDP. Second we use the *price normalizations* introduced in the steady-state section above, namely

$$\begin{aligned}
P_H &= P_F^* = 1 && (103)\\
P_I &= P_H^* = P_F = \frac{1}{\mathcal{T}} = \frac{1}{1 - \frac{1}{\zeta}} && (104)\\
P &= \left[w_H + (1 - w_H)P_I^{1-\mu}\right]^{\frac{1}{1-\mu}} && (105)\\
P^* &= \left[w_F + (1 - w_F)P_I^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} && (106)
\end{aligned}$$

Thus relative prices $\frac{P_H}{P}$, $\frac{P_F}{P}$,$\frac{P_F^*}{P^*}$, $\frac{P_H^*}{P^*}$ appearing the linearization can be expressed in terms of one further fundamental parameters to be estimated: ζ.

Included in the parameters to estimate are *the bias parameters* ω_H and ω_F. In principle these can be treated as any other parameter. However we adopt an alternative procedure using trade data so as to equate

$$\begin{aligned}
\frac{P_F C_F}{PC} = (1 - w_H)\left(\frac{P_F}{P}\right)^{1-\mu} &= \text{ imports share of consumption in H bloc} \equiv s_H && (107)\\
\frac{P_H^* C_H^*}{P^* C^*} = (1 - w_F)\left(\frac{P_H^*}{P^*}\right)^{1-\mu^*} &= \text{ imports share of consumption in F bloc} \equiv s_F && (108)
\end{aligned}$$

so shares are constant if the elasticities $\mu = \mu^* = 1$ but fall or rise with the price of the domestic good relative to the CPI index depending on whether domestic and imported goods are substitutes ($\mu, \mu^* > 1$) or complements ($\mu, \mu^* < 1$). Let s_H, s_F be these import

series are demeaned and the sample for the estimation runs from 1982q1 until 2003q4. For purposes of the estimation, the Kalman filter is initialized using data starting in 1975q2.

shares in the two blocs. Substituting for P and P^* we the arrive at the weights

$$w_H = \frac{(1-s_H)P_F^{1-\mu}}{s_H + (1-s_H)P_F^{1-\mu}} \tag{109}$$

$$w_F = \frac{(1-s_F)(P_H^*)^{1-\mu^*}}{s_F + (1-s_F)(P_H^*)^{1-\mu^*}} \tag{110}$$

Then the bias parameters, ω_H and ω_F, are obtained using (7) and its foreign counterpart.

3.3 Priors

We adopt a Bayesian approach to inference, as in Smets and Wouters (2003), Justiniano and Preston (2004), Batini *et al.* (2004b) and more recently Shorfheide and Lubik (2005). Therefore, the likelihood obtained from the solution to the linearized model is combined with the prior to obtain a posterior density. As usual, not all parameters can be estimated and hence the discount factor is calibrated to be the same in both blocks to the usual value of 0.99. We set $\nu_H = \nu_F = 0.5$ implying blocks of equal size, which is in line with calculations using GDP series at PPP (1995) values. Meanwhile the ratios of consumption and government to GDP are calibrated at the historical average over our estimation sample: 0.78 and 0.22 for the United States, and 0.73 and 0.27 for the Euro Area.[12] Similarly, the shares of foreign goods in GDP are fixed at the average over the sample given by $s_H = 0.14$ and $s_F = 0.37$ for the domestic and foreign blocks. Finally the elasticity of substitution amongst varieties of locally produced goods, ζ, is calibrated at 4, implying a markup of 0.33 which is higher than those reported by Rotemberg and Woodford (1997).

For the remaining parameters, our priors are given in Table A, where we report the mean and standard deviation of the densities together with 1% and 99% prior probability intervals. A-priori the domestic and foreign blocks are treated symmetrically, hence an identical prior is used for the same coefficient in each block. The prior for σ is intended to capture a fairly wide set of possible values for the parameter governing the curvature of the utility function. Ex-ante beliefs on ξ_H and ξ_F^* allow for prices in the currency of the producer country to be reoptimized roughly every 1.5 to 5 quarters, implying a broad degree of assumptions on price stickiness,while we further allow a-priori for some small to moderate degree of price indexation. Given the lively debate on the degree of pass-through

[12] The size of the Euro Area and the United States blocks obtained when aggregating the 1995 GDP series (at PPP) are 0.51 and 0.49.

and pricing assumptions in the international macro literature, we assume a more agnostic stance on the behavior of prices for imported goods as reflected in our priors for both the frequency of price re-optimization (ξ_F)and the extent to which prices are indexed (γ_F). Meanwhile, for wages our prior assigns small probability to the fully flexible or rigid wage scenarios, while permitting indexation to parallel that for domestically-produced goods. Regarding habit, our priors for consumption would well accord with estimates by Smets and Wouters (2003) and Christiano *et al.* (2001) and less with higher values reported in the literature. We assume a more agnostic stance for habit in labour, a parameter relatively unexplored by other papers in this field.

Both the inverse elasticity of labour supply (ϕ) and elasticity of domestic and foreign goods (μ) are notorious for the disagreements between the micro and macro evidence, which suggests considering a fairly broad set of possible parameter values. Priors for the coefficients of the Taylor rule are standard and entail a substantial degree of inertia in interest rates.

Regarding the AR(1) coefficients of the different shocks in the model, our priors reflect beliefs on very inertial stochastic processes mainly for technology and to a lesser degree government spending. Preference shocks, both affecting the intertemporal marginal rate of substitution and the disutility of labour, are assumed to be somewhat less persistent, although the prior for the latter type of disturbances encompasses a fairly wide range of possible values.[13]

Disturbances to technology and utility in consumption (ϵ_C and ϵ_{C^*}) are allowed to be correlated across countries, to allow for channels of transmission other than trade. For the correlation coefficient on technology shocks, $\upsilon_{\epsilon_a,\epsilon_{a^*}}$, an uninformative beta prior is specified. Meanwhile, our prior allows for a moderate to high degree of correlation across shocks to consumption utility, $\upsilon_{\epsilon_C,\epsilon_{C^*}}$.

3.4 Estimates

Having adopted a Bayesian perspective for inference we seek to characterize the posterior distribution of the parameters. An optimization algorithm (Chris Sims' csminwel) is used

[13]Recall that innovations to price and wage mark-ups, as well as the measurement error in the real exchange rate are assumed to be i.i.d.

to obtain an initial guess of the posterior mode. In the second step,a random walk metropolis algorithm, one particular class of Markov Chain Monte Carlo Methods (MCMC), generates chains of draws from the posterior densities. In this case, the Hessian obtained from the initial maximization is then scaled upwards and used as an over-dispersed distribution for randomly generating starting values for two chains of 40,000 simulations each, where we discard the initial 10,000 as a burn-in phase. These draws enable us to compute mean, medians and posterior probability intervals, which are reported in tables B1 and B2 for the domestic and foreign block respectively. Overall, the coefficient estimates are different from zero, fairly tightly pinned down and quite plausible. We provide a brief discussion of the main features of our coefficient estimates.

Posterior probability bands for σ and σ^* are indicate a very similar degree of curvature in the utility function across blocks. When combined with the posterior estimates for the degree of habit in consumption, h, one obtains elasticities of intertemporal substitution, given by $\frac{1-h}{\sigma}$, that well accord across the two blocks and that are in the neighborhood of 0.4, which is very similar with the estimates reported in the Euro area model with external habit of Smets and Wouters (2003).

In contrast, and in disagreement with our symmetric priors for domestic and foreign coefficients, posterior estimates indicate substantial differences in the degree of price stickiness for goods in the currency of producers. Notice that posterior intervals for ξ_H and ξ_F^* are tight (particularly for the latter) not overlapping and suggestive of a higher degree of price stickiness in the foreign block. According to the posterior medians, the average duration of price contracts in the United States is roughly two quarters, while close to four quarters in the Euro Area. Curiously, the degree of price indexation is inferred to be twice as large in the domestic than in the foreign block, with medians of 0.43 and 0.24. Together, these set of estimates point towards greater flexibility in the evolution of prices in the currency of the producer for the United States.

Turning to import prices, the behavior of the parameters governing Calvo pricing is closer in line with our symmetry assumption across blocks. However, posterior probability intervals are very concentrated far out in the right tails of our prior and point to a seemingly implausible degree of price stickiness (medians are 0.96 and 0.97 for the United States and the Euro Area). To compensate, in part, for the very low frequency of price adjustments, there is substantial indexation in the Euro Area (median is 0.53) while this is not the case in the domestic block. The results for the United States accord well with the view that there is substantial pricing to market by exporters into the U.S. market.

Regarding wages, our estimates suggest that the average duration of foreign wage contracts is approximately 5.8 quarters, which is higher than for prices of goods in the currency of producers. We note that this result is contrast to that obtained by Smets and Wouters (2003) who were somewhat puzzled by the robust finding that price contracts have longer duration than for wages. For the United States, the lower frequency of re-optimization in wages, relative to price, contracts is eve more startling, since our median parameters estimates suggest durations of 9 quarters, which seem to be in part compensated by a higher degree of wage indexation. Overall, our inferred parameters indicate a reasonable degree of nominal rigidities in wages and prices for the Euro Area. In the case of the domestic block, however, posterior estimates point to a greater role for wage contracts in generating nominal rigidities than for prices. In a future version of this paper we intend to analyze the implications of these discrepancies for the dynamic response to shocks, as well as to gauge the sensitivity of these results to our choice of priors.

Turning to the estimated inverse elasticities of labour supply, the likelihood prefers values quite smaller than our priors. The coefficients are very similar across blocks and point towards (close to) unitary elasticities. It is interesting notice that however, the degree of habit in labour is far larger for the United States, as evidenced by comparing the posterior probability bands.

As mentioned earlier, there is substantial disagreement on plausible values for the elasticity of substitution between domestic and foreign goods. In our case, median estimates are 1.39 and 1.11 for the domestic and foreign blocks, although posterior bands reveal this parameter is not well pinned down for the Euro-Area. These point estimates (keeping in mind the caveats on the dispersion of the latter) lie in the mid-range of vales in the literature: far smaller for instance than the chosen value of 6 in Obstfeld and Rogoff (2000) but higher than in Justiniano and Preston (2004) who estimate elasticities as low as 0.3 for small open economies.

Posterior medians for the Taylor Rule echo findings of a high degree of interest rate smoothing, which is estimated to be higher in the case of the Euro-area, judging by the posterior probability bands. The response coefficients on inflation adhere to the Taylor principle in that the monetary authority rises real interest rate rise in response to inflationary pressures. The greatest discrepancy across blocks emerges from the parameter governing the response to the output gap, which is very small for the foreign block (posterior probabilities cover the 0.01 to 0.12 interval). It is worth flagging that the countries comprising our foreign block were subject to common monetary rule only for a brief part of our sample. While the assumption of adopting a Taylor rule to describe the conduct of monetary policy in this region seems to hold rather well, this suggests however the need to be cautious in reading deeply into this particular result.[14]

Despite the inclusion of intrinsic mechanisms of persistence in the form of wage and price indexation, (external) habit in labour and consumption, and even deviations in the law of one price, the exogenous shocks are estimated to be very persistent. While close to unit root estimates in technology are not uncommon (as for the domestic block), ρ estimates above 0.9 for the disturbances to consumption and labour preferences are somewhat surprising and lie beyond (to the right) of the percentiles reported in our priors and result in tight posterior probability intervals. As mentioned, this highlights the model's inability to generate enough endogenous persistence to match the dynamics in the data and presents a challenge for introducing other sources of inertia (capital and adjustment costs, for instance) to this end.

Shocks to productivity and consumption preferences exhibit a similar degree of correlation across blocks. The inferred coefficients are seen to be different from zero and lie on the lower range of plausible values allowed by our prior, particularly for the case of disturbances to the consumption utility.

Regarding the volatilities, the large values of for the $sd(\epsilon_C)$ and $sd(\epsilon_N)$ (as well as their foreign counterparts) may at first seem quite striking, particularly considering the plausible range of values entertained by our prior.[15] However, it is important to remember that the stochastic processes of these shocks enter the structural equations multiplied by coefficients which are non-linear combinations of the degree of habit (consumption and leisure

[14]See the discussion in Smets and Wouters (2003) and references therein.

[15]$sd(\epsilon_C) = 4.13$ coupled with $\rho_C = 0.99$, which implies that $sd(u_C) = \frac{1}{\sqrt{(1-\rho_C^2)}} sd(\epsilon_C) = 7.09 \times 4.13$, is particularly implausible. In the simulations that follow we therefore imposed the same distributions on $sd(\epsilon_C)$ and $sd(\epsilon_C^*)$, and on ρ_C and ρ_C^*, using the estimated results for the foreign bloc.

accordingly) such as $\frac{1-h}{\sigma}$. This problem is generally resolved by imposing a normalization in the disturbances that will feature in future re-estimations of our models, although we emphasize that these should not affect the estimates other than for the standard deviations. We also note that posterior for the standard deviation of the measurement error in the real exchange resemble closely its prior density, suggesting that the role of this shocks is rather superfluous.

Parameters	Distribution	Mean	Std	1%	99%
σ, σ^*	N	2	0.8	0.14	3.86
ξ_H, ξ_F^*	B	0.65	0.1	0.4	0.87
γ_H,γ_F^*	B	0.5	0.1	0.27	0.73
ξ_F, ξ_H^*	B	0.5	0.2	0.09	0.91
$\gamma_F,\ \gamma_H^*$	B	0.5	0.2	0.09	0.91
ξ_W, ξ_W^*	B	0.5	0.1	0.27	0.73
γ_W, γ_W^*	B	0.5	0.1	0.27	0.73
h, h^*	B	0.5	0.1	0.27	0.73
h_N, h_N^*	B	0.5	0.15	0.17	0.83
ϕ, ϕ^*	N	2.5	0.6	1.1	3.9
μ, μ^*	N	2.5	0.6	1.1	3.9
$\kappa_\pi, \kappa_{\pi^*}$	N	1.8	0.25	1.22	2.38
κ_y, κ_y^*	N	0.4	0.15	0.05	0.75
ρ_r, ρ_r^*	B	0.75	0.1	0.49	0.93
ρ_a, ρ_a^*	B	0.85	0.1	0.55	0.99
ρ_g, ρ_g^*	B	0.75	0.1	0.49	0.93
ρ_N, ρ_N^*	B	0.6	0.1	0.36	0.81
ρ_C, ρ_C^*	B	0.6	0.15	0.25	0.9
$\upsilon_{\epsilon_a, \epsilon_{a^*}}$	B	0.5	0.2	0.08	0.91
$\upsilon_{\epsilon_C, \epsilon_{C^*}}$	B	0.4	0.2	0.41	0.86
$sd(\epsilon_r)$	I	0.3	0.1	0.12	0.58
$sd(\epsilon_a)$	I	0.6	0.4	0.06	1.9
$sd(\epsilon_g)$	I	0.6	0.4	0.06	1.9
$sd(\epsilon_N)$	I	0.6	0.2	0.23	1.16
$sd(\epsilon_C)$	I	0.5	0.2	0.15	1.08
$sd(m\pi)$	I	0.3	0.1	0.11	0.58
$sd(mw)$	I	0.3	0.1	0.11	0.58
$sd(e)$	I	0.2	0.05	0.1	0.33

Table A. Priors[16]

[16]Prior densities: N, normal, B, beta, I, Inverse-Gamma1. Third and fourth column report the mean and standard deviations, while the last two columns the 1st and 99th percentiles corresponding to each density. Coefficients are treated symmetrically across blocks.

Parameters	Median	Std	10%	90%
σ	1.43	0.17	1.23	1.66
ξ_H	0.45	0.04	0.41	0.53
γ_H	0.43	0.08	0.34	0.53
ξ_F	0.96	0.01	0.95	0.96
γ_F	0.13	0.05	0.07	0.19
ξ_W	0.89	0.02	0.86	0.91
γ_W	0.62	0.05	0.54	0.67
h	0.46	0.04	0.41	0.51
h_N	0.83	0.1	0.66	0.92
ϕ	1.34	0.28	0.91	1.64
μ	1.39	0.24	1.11	1.73
$\kappa_{\pi,}$	2.11	0.13	1.95	2.29
$\kappa_{y,}$	0.43	0.14	0.24	0.6
ρ_r	0.74	0.03	0.69	0.77
ρ_a	0.99	0.003	0.98	0.99
ρ_g	0.9	0.02	0.87	0.92
ρ_N	0.97	0.01	0.96	0.97
ρ_C	0.99	0.005	0.98	0.99
$\upsilon_{\epsilon_a,\epsilon_{a*}}$	0.16	0.06	0.08	0.24
$\upsilon_{\epsilon_C,\epsilon_{C*}}$	0.13	0.06	0.06	0.22
$sd(\epsilon_r)$	0.22	0.02	0.2	0.25
$sd(\epsilon_a)$	0.58	0.05	0.51	0.65
$sd(\epsilon_g)$	0.77	0.06	0.7	0.85
$sd(\epsilon_N)$	2.65	0.17	2.46	2.89
$sd(\epsilon_C)$	4.13	0.22	3.78	4.38
$sd(m\pi)$	1.15	0.07	1.07	1.24
$sd(mw)$	0.45	0.04	0.41	0.51
$sd(e)$	0.2	0.05	0.14	0.28

Table B1. Posterior: Domestic Bloc[17]

[17] Posterior medians, percentiles and standard deviations obtained with the Random Walk MCMC algorithm.

Parameters	Median	Std	10%	90%
σ^*	1.56	0.22	1.35	1.92
ξ_F^*	0.75	0.02	0.72	0.78
γ_F^*	0.24	0.06	0.17	0.32
ξ_H^*	0.97	0.01	0.96	0.98
γ_H^*	0.53	0.06	0.45	0.59
ξ_W^*	0.83	0.03	0.79	0.87
γ_W^*	0.34	0.07	0.26	0.43
h^*	0.4	0.07	0.31	0.48
h_N^*	0.37	0.11	0.22	0.51
ϕ^*	1.01	0.3	0.58	1.37
μ^*	1.12	0.36	0.66	1.61
κ_{π^*}	1.32	0.09	1.23	1.46
κ_y^*	0.05	0.04	0.01	0.12
ρ_r^*	0.82	0.02	0.8	0.84
ρ_a^*	0.91	0.03	0.86	0.94
ρ_g^*	0.91	0.02	0.88	0.93
ρ_N^*	0.94	0.02	0.92	0.96
ρ_C^*	0.93	0.01	0.92	0.95
$sd(\epsilon_r^*)$	0.14	0.01	0.13	0.16
$sd(\epsilon_a^*)$	0.58	0.05	0.51	0.65
$sd(\epsilon_g^*)$	0.66	0.05	0.6	0.73
$sd(\epsilon_N^*)$	1.23	0.17	1.03	1.47
$sd(\epsilon_C^*)$	1.50	0.15	1.33	1.7
$sd(m\pi^*)$	0.78	0.05	0.72	0.86
$sd(mw^*)$	0.46	0.04	0.5	0.51

Table B2. Posterior: Foreign Bloc [18]

4 Analysis of Special Case

In this special case we assume an entirely symmetrical model (equal-sized blocs, equal parameter values, preferences and identical stochastic processes though not realizations of shocks). With no habit persistence and indexation we put $h_C = h_N = 0 = \gamma_H = \gamma_F = \gamma_W = 0$. Wages are flexible so $\xi_W = 0$ and $w_t - p_t = mrs_t$. Another convenient simplifying assumption is to assume Cobb-Douglas household preferences with respect to domestic and imported goods and put $\mu = \mu^* = 1$. Then all terms involving $\left(\frac{1}{P}\right)^{1-\mu}$ and $\left(\frac{P_I}{P}\right)^{1-\mu}$ in the linearization are unity. We further put $G = 0$ and assume only technology

[18] Posterior medians, percentiles and standard deviations obtained with the Random Walk MCMC algorithm.

shocks a_t and a_t^*. With these simplifications the model reduces to

$$\mathcal{E}_t c_{t+1} = c_t + \frac{1}{\sigma}(r_t - \mathcal{E}_t \pi_{t+1}) \tag{111}$$

$$\mathcal{E}_t c_{t+1}^* = c_t^* + \frac{1}{\sigma}(r_t^* - \mathcal{E}_t \pi_{t+1}^*) \tag{112}$$

$$\beta \mathcal{E}_t \pi_{H,t+1} = \pi_{H,t} - \lambda_H mc_t \tag{113}$$

$$\beta \mathcal{E}_t \pi_{F,t+1}^* = \pi_{F,t}^* - \lambda_F^* mc_t^* \tag{114}$$

where $\lambda_H = \lambda_F^* = \frac{(1-\beta\xi_H)(1-\xi_H)}{\xi_H}$ and

$$mc_t = -(1+\phi)a_t + \sigma c_t + \phi y_t - (1-\omega)\tau_t \tag{115}$$

$$mc_t^* = -(1+\phi)a_t^* + \sigma c_t^* + \phi y_t^* - (1-\omega)\tau_t^* \tag{116}$$

$$\beta \mathcal{E}_t \pi_{F,t+1} = \pi_{F,t} + \lambda_F \psi_{F,t}^* \tag{117}$$

$$\beta \mathcal{E}_t \pi_{H,t+1}^* = \pi_{H,t}^* + \lambda_H^* \psi_{H,t} \tag{118}$$

where $\lambda_F = \lambda_H^* = \frac{(1-\beta\xi_F)(1-\xi_F)}{\xi_F}$ and

$$\psi_{H,t} = e_t - \omega\tau_t^* - (1-\omega)\tau_t \tag{119}$$

$$\psi_{F,t} = -\psi_{F,t}^* = e_t + (1-\omega)\tau_t^* + \omega\tau_t \tag{120}$$

$$e_t = -\sigma(c_t^* - c_t) \tag{121}$$

$$y_t = \omega(c_t - (1-\omega)\tau_t) + (1-\omega)(c_t^* + \omega\tau_t^*) \tag{122}$$

$$y_t^* = \omega(c_t^* - (1-\omega)\tau_t^*) + (1-\omega)(c_t + \omega\tau_t) \tag{123}$$

$$\pi_t = \omega\pi_{H,t} + (1-\omega)\pi_{F,t} \tag{124}$$

$$\pi_t^* = \omega\pi_{F,t}^* + (1-\omega)\pi_{H,t}^* \tag{125}$$

$$\Delta\tau_t = \pi_{H,t} - \pi_{F,t} \tag{126}$$

$$\Delta\tau_t^* = \pi_{F,t}^* - \pi_{H,t}^* \tag{127}$$

17 Equations (111) to (127) can be solved for 17 variables c_t, c_t^*, $\pi_{H,t}, \pi_{F,t}^*$, $\pi_{F,t}, \pi_{H,t}^*$, mc_t, mc_t^*, $\psi_{H,t}, \psi_{F,t}$, e_t, y_t, y_t^*, τ_t, τ_t^* and π_t, π_t^* given r_t, r_t^* and processes for a_t, a_t^*. However unlike the case of complete exchange rate pass-through, the need for relationships (126) and (127) means the state space formulation involves *structural dynamics*[19] This

[19]Monacelli (2003) eliminated these dynamics by in effect writing (126) as $\mathcal{E}_t\tau_{t+1} - \tau_t = \mathcal{E}_t(\pi_{H,t+1} - \pi_{F,t+1})$ and similarly for (127) but this is not correct as it treats τ_t as an extra independent jump variable.

makes the optimal policies difficult to calculate analytically even if we use the Aoki decomposition into sums and differences as in Batini *et al.* (2004a). However two useful exercises are analytically tractable and provide useful insights for understanding the numerical results on the full model. The first is the short-term and long-term effects of permanent unanticipated changes in domestic inflation in each of the two blocs. This exercise in particular identifies the nature of monetary spillovers and the role for monetary policy coordination. The second exercise is the study of the possible stability, instability and indeterminacy of inflation-based forecast rules. We consider these in turn.

4.1 Monetary Policy Spillovers

For symmetric economies it proves convenient to separately analyze the aggregate or sum system and the difference system. Define sums $c_t^s = c_t + c_t^*$ and similarly for variables y_t^s, r_t^s, mc_t^s and τ_t^*. Aggregate *domestic* and *imported* inflation rates are defined as $\pi_{D,t}^s = \pi_{H,t} + \pi_{F,t}^*$ and $\pi_{I,t}^s = \pi_{F,t} + \pi_{H,t}^*$ respectively and the aggregate deviations from the law of one price by $\psi_t = \psi_{H,t} + \psi_{F,t}^*$. Differences are defined by $c_t^d = c_t - c_t^*$ etc. Then (111) to (127) decomposes into the following *sum system*:

$$\mathcal{E}_t c_{t+1}^s = c_t^s + \frac{1}{\sigma}\left(r_t^s - \mathcal{E}_t \pi_{t+1}^s\right) \tag{128}$$

$$\beta \mathcal{E}_t \pi_{D,t+1}^s = \pi_{D,t}^s - \lambda_H \, mc_t^s \tag{129}$$

$$mc_t^s = -(1+\phi) a_t^s + \sigma c_t^s + \phi y_t^s - (1-\omega)\tau_t^s \tag{130}$$

$$\beta \mathcal{E}_t \pi_{I,t+1}^s = \pi_{I,t}^s + \lambda_F \, \psi_t^s \tag{131}$$

$$\psi_t^s = -\tau_t^s \tag{132}$$

$$y_t^s = c_t^s \tag{133}$$

$$\pi_t^s = \omega \pi_{D,t}^s + (1-\omega)\pi_{I,t}^s \tag{134}$$

$$\Delta \tau_t^s = \pi_{D,t}^s - \pi_{I,t}^s \tag{135}$$

and the following *difference system*:

$$\mathcal{E}_t c_{t+1}^d = c_t^d + \frac{1}{\sigma}\left(r_t^d - \mathcal{E}_t \pi_{t+1}^d\right) \tag{136}$$

$$\beta \mathcal{E}_t \pi_{D,t+1}^d = \pi_{H,t}^d - \lambda_H \, mc_t^d \tag{137}$$

$$mc_t^d = -(1+\phi) a_t^d + \sigma c_t^d + \phi y_t^d - (1-\omega)\tau_t^s \tag{138}$$

$$\beta \mathcal{E}_t \pi_{I,t+1}^d = \pi_{I,t}^d - \lambda_F \, \psi_t^d \tag{139}$$

$$\begin{aligned}
\psi_t^d &= 2e_t + (2\omega - 1)\tau_t^d && (140)\\
e_t &= \sigma c_t^d && (141)\\
y_t^d &= (2\omega - 1)c_t^d - 2\omega(1-\omega)\tau_t^d && (142)\\
\pi_t^d &= \omega\pi_{D,t}^d + (1-\omega)\pi_{I,t}^d && (143)\\
\Delta\tau_t^d &= \pi_{D,t}^d - \pi_{I,t}^d && (144)
\end{aligned}$$

4.1.1 The Sum System

We now examine the effect in the sum system of an unanticipated permanent increase in domestic inflation $\pi_{D,t}^s = \bar{\pi}_D^s,\ t \geq 1$. We start at the baseline steady state where all variables in deviation form are zero. There are no other shocks so we put $a_t = 0$. Consider first the first-period $t = 1$ response for which expected domestic and imported inflation and expected consumption are still zero so that $\mathcal{E}_1\pi_{D,2}^s = \mathcal{E}_1\pi_{I,2}^s = \mathcal{E}_1 c_2^s = 0$. From (129) to (133) we then have

$$\begin{aligned}
\pi_{D,1}^s = \lambda_H mc_1^s &= ((\sigma+\phi)y_1^s - (1-\omega)\tau_1^s)\\
\pi_{I,1}^s = \lambda_F\tau_1^s &= \lambda_F(\pi_{D,1}^s - \pi_{I,1}^s)
\end{aligned}$$

Putting $\pi_{D,1}^s = \bar{\pi}_D^s$ the *short-run* responses to aggregate domestic inflation, the terms of trade and output can be derived as

$$\begin{aligned}
\pi_{I,1}^s &= \frac{\lambda_F}{1+\lambda_F}\bar{\pi}_D^s && (145)\\
\tau_1^s &= \frac{1}{1+\lambda_F}\bar{\pi}_D^s && (146)\\
y_1^s &= \frac{(1+\lambda_F+\lambda_H(1-\omega))}{\lambda_H(\sigma+\phi)(1+\lambda_F)}\bar{\pi}_D^s \equiv \alpha_1^s\bar{\pi}_D^s && (147)
\end{aligned}$$

From (128) to engineer this rise in domestic inflation aggregate interest rates must be lowered and set at $r_1^s = -\sigma y_1^s$. For $\lambda_F < \infty$, equation (145) describes the incomplete pass-through of an aggregate domestic monetary expansion to imported inflation since $\pi_{I,1}^s < \bar{\pi}_D^s$. As $\lambda_F \to \infty$, the law of one price gap disappears and $\pi_{I,1}^s \to \bar{\pi}_D^s$. According to (146) this is associated with a rise in the aggregate terms of trade. In (147) there are two components of the stimulus to output, one associated with a domestic inflation surprise and one with an imported inflation surprise.

From time $t \geq 2$ the permanent rise in inflation is anticipated and the dynamics of aggregate variables τ_t^s and $\pi_{I,t}^s$ are given by

$$\begin{bmatrix} 1 & 0 \\ \lambda_F & \beta \end{bmatrix} \begin{bmatrix} \tau_t^s \\ \mathcal{E}_t \pi_{I,t+1}^s \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_{t-1}^s \\ \pi_{I,t}^s \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{\pi}_D^s \tag{148}$$

with τ_1^s given by (146). The eigenvalues of (148) are those of the matrix

$$\begin{bmatrix} 1 & 0 \\ \lambda_F & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta & -\beta \\ -\lambda_F & 1+\lambda_F \end{bmatrix} \tag{149}$$

which can easily be shown to have one eigenvalue within the unit circle and one outside. The dynamic system (148) with one predetermined and one non-predetermined variable is therefore saddlepath stable.

Given the trajectories for τ_t^s, $\pi_{I,t}^s$ and $\pi_{D,t}^s = \bar{\pi}_D^s$, (128) to (134) now describes the trajectories for the rest of the sum system. This converges to a new steady state $\bar{\pi}_I^s$ etc with $\bar{\pi}_I^s = \bar{\pi}^s = \bar{r}^s = \bar{\pi}_D^s$, $\bar{\tau}^s = 0$ and consumption and output given by

$$\bar{c}^s \;=\; \bar{y}^s = \frac{(1-\beta)(\lambda_F + \lambda_H(1-\omega))}{\lambda_H \lambda_F(\sigma+\phi)} \bar{\pi}_D \equiv \bar{\alpha}^s \, \bar{\pi}_D^s \tag{150}$$

Thus for a discount factor $\beta < 1$ there is a long-run output-inflation trade-off, a familiar feature of New Keynesian DSGE models.

4.1.2 The Difference System

The short-run and long-run responses of the difference system following an unanticipated increase in $\pi_{D,t}^d$ can be found in the same way, though the algebra is not so straightforward. We first eliminate c_t^d and y_t^d from (140), (137) and (138) to obtain

$$\psi_t^d = \theta_1 mc_t^d + \theta_2 \tau_t^d \tag{151}$$

where

$$\theta_1 \;=\; \frac{2\sigma}{\sigma + (2\omega-1)\phi} \tag{152}$$

$$\theta_2 \;=\; \frac{\sigma + \phi((2\omega-1)^2 + 4(1-\omega)\omega\sigma)}{\sigma + (2\omega-1)\phi} \tag{153}$$

Proceeding as for the sum system the *short-run* responses are found from

$$\begin{aligned}
\pi^d_{D,1} = \lambda_H mc^d_1 &= \lambda_H((\sigma+\phi)y^d_1 - (1-\omega)\tau^d_1) \\
\pi^d_{I,1} = \lambda_F \psi^d_1 &= \lambda_F(\theta_1 mc^d_1 + \theta_2 \tau^d_1) \\
\tau^d_1 &= \pi^d_{D,1} - \pi^d_{I,1}
\end{aligned}$$

Putting $\pi^d_{D,1} = \bar{\pi}^d_D$ the short-run responses to domestic inflation, the terms of trade and output differences can be derived as

$$\pi^d_{I,1} = \frac{\lambda_F(\theta_1 + \lambda_H\theta_2)}{\lambda_H(1+\lambda_F\theta_2)}\bar{\pi}^d_D \tag{154}$$

$$\tau^d_1 = -\frac{\lambda_F\theta_1 - \lambda_H}{\lambda_H(1+\lambda_F\theta_2)}\bar{\pi}^d_D \tag{155}$$

$$\begin{aligned}
y^d_1 &= \frac{(1+\lambda_F\theta_2)(2\omega-1) + \lambda_H(1-\omega)(1-2\omega(1-\sigma))(\lambda_F\theta_1 - \lambda_H)}{\lambda_H(1+\lambda_F\theta_2)(\sigma+(2\omega-1)\phi)}\bar{\pi}^d_D \\
&\equiv \alpha^d_1 \bar{\pi}^d_D
\end{aligned} \tag{156}$$

To bring about these changes the difference in the interest rate must be set at $r^d_1 = -\sigma y^d_1$. The key feature of these short-term responses is the direction of change of the terms of trade. If $\lambda_F\theta_1 > \lambda_F$, that is, substituting for θ_1, if

$$\frac{2\sigma}{\sigma + (2\omega-1)\phi} > \frac{\lambda_H}{\lambda_F} \tag{157}$$

then exchange rate pass-through is sufficiently large in the first period to engineer a fall in the terms of trade difference. In the absence of consumption bias where $\omega = \frac{1}{2}$ this condition becomes simply $\lambda_F > \frac{\lambda_H}{2}$, so the degree of price stickiness in the retail market for imported goods needs to only half of that in the market for domestic goods for a fall in the terms of trade. For price stickiness in the retail market sufficiently large such that (157) no longer holds, then a unilateral monetary expansion on one bloc results in an *appreciation* of its terms of trade.[20] As we shall see this has important consequences for the direction of monetary spillovers on output.

From time $t \geq 2$ the permanent change in inflation differences is anticipated and the dynamics of aggregate variables τ^d_t and $\pi^d_{I,t}$ are given by

$$\begin{bmatrix} 1 & 0 \\ \lambda_F\theta_2 & \beta \end{bmatrix} \begin{bmatrix} \tau^d_t \\ \mathcal{E}_t\pi^d_{I,t+1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tau^d_{t-1} \\ \pi^d_{I,t} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -\lambda_F\theta_1 & 0 \end{bmatrix} \begin{bmatrix} \frac{(1-\beta)}{\lambda_F} \\ 1 \end{bmatrix} \bar{\pi}^d_D \tag{158}$$

[20] A similar result is obtained by Betts and Devereux (2000a) in a model of 'pricing-to-market'.

with τ_1^s given by (155). The dynamic matrix in (158) has the same form as for the sun system with one eigenvalue outside and one inside the unit circle. The difference system is therefore also saddlepath stable.

Given the trajectories for τ_t^d, $\pi_{I,t}^d$ and $\pi_{D,t}^d = \bar{\pi}_D^d$, (136) to (143) now describes the trajectories for the rest of the difference system. This converges to a new steady state $\bar{\pi}_I^d$ etc with $\bar{\pi}_I^d = \bar{\pi}^d = \bar{r}^d = \bar{\pi}_D^d$ and output and the terms of trade given by

$$\begin{aligned}\bar{y}^d &= \frac{(1-\beta)(\lambda_F\theta_2(2\omega-1)+\lambda_H(1-\omega)(1-2\omega(1-\sigma))(\lambda_F\theta_1-\lambda_H))}{\lambda_H\lambda_F\theta_2(\sigma+(2\omega-1)\phi)}\bar{\pi}_D^d \\ &\equiv \bar{\alpha}^d\,\bar{\pi}_D^d \qquad (159)\\ \bar{\tau}^d &= -\frac{(1-\beta)(\lambda_F\theta_1-\lambda_H)}{\lambda_H\lambda_F\theta_2}\bar{\pi}_D^d \qquad (160)\end{aligned}$$

4.1.3 Positive or Negative Spillovers?

We can now assess the sign of the monetary spillovers on output. Write the results obtained as

$$y_1^s \equiv y_1 + y_1^* = \alpha_1^s\,\pi_{D,1}^s \equiv \alpha_1^s(\pi_{H,1}+\pi_{F,1}^*) \qquad (161)$$

$$y_1^d \equiv y_1 - y_1^* = \alpha_1^d\,\pi_{D,1}^d \equiv \alpha_1^d(\pi_{H,1}-\pi_{F,1}^*) \qquad (162)$$

$$\bar{y}^s \equiv \bar{y} + \bar{y}^* = \bar{\alpha}^s\,\bar{\pi}_D^s \equiv \bar{\alpha}^s(\bar{\pi}_H+\bar{\pi}_F^*) \qquad (163)$$

$$\bar{y}^d \equiv \bar{y} - \bar{y}^* = \bar{\alpha}_1\,\bar{\pi}_D^d \equiv \bar{\alpha}^d(\bar{\pi}_H-\bar{\pi}_F^*) \qquad (164)$$

Hence the short-run and long-run responses on output in the home bloc to unanticipated (in the first period) permanent change in domestic inflation in both blocs is given by

$$y_1 = \frac{1}{2}\left[(\alpha_1^s+\alpha_1^d)\bar{\pi}_H + (\alpha_1^s-\alpha_1^d)\bar{\pi}_F^*\right] \qquad (165)$$

$$\bar{y} = \frac{1}{2}\left[(\bar{\alpha}^s+\bar{\alpha}^d)\bar{\pi}_H + (\bar{\alpha}^s-\bar{\alpha}^d)\bar{\pi}_F^*\right] \qquad (166)$$

Hence short-run spillovers are positive (negative) if $\alpha_1^s > (<)\alpha_1^d$ and similarly for long-run spillovers. Substituting for α_1^s, α_1^d, $\bar{\alpha}^s$, $\bar{\alpha}^d$ whether spillovers are positive or negative depends on the sign of

$$\alpha_1^s-\alpha_1^d = \frac{1}{\lambda_H}\left[\frac{(1+\lambda_F+\lambda_F(1-\omega))}{(\sigma+\phi)(1+\lambda_F)} - \frac{(1+\lambda_F\theta_2)(2\omega-1)+(1-\omega)(1-2\omega(1-\sigma))(\lambda_F\theta_1-\lambda_H)}{(1+\lambda_F\theta_2)(\sigma+(2\omega-1)\phi)}\right] \qquad (167)$$

for the short-run and on the sign of

$$\bar{\alpha}^s-\bar{\alpha}^d = \frac{(1-\beta)}{\lambda_H\lambda_F}\left[\frac{(\lambda_F+\lambda_H(1-\omega))}{(\sigma+\phi)} - \frac{(\lambda_F\theta_2(2\omega-1)+(1-\omega)(1-2\omega(1-\sigma))(\lambda_F\theta_1-\lambda_H))}{\theta_2(\sigma+(2\omega-1)\phi)}\right] \qquad (168)$$

for the long-run.

To assess these results first note that as $\omega \rightarrow 1$ and the two blocs cease to trade then, as logic demands, the spillovers disappear. Second, let us examine the case of complete exchange rate pass-through which is obtained from (167) and (168) by letting $\lambda_F \rightarrow \infty$. A little algebra then yields

$$\left[\alpha_1^s - \alpha_1^d\right]_{\lambda_F=\infty} = \left[\frac{\alpha^s - \alpha^d}{1-\beta}\right]_{\lambda_F=\infty} = \frac{4\omega\sigma(1-\omega)(1-\sigma)}{\lambda_H(\sigma+\phi)[\sigma+\phi((2\omega-1)^2+4\omega(1-\omega)\sigma)]} \tag{169}$$

Thus we have the following proposition generalizing the result of Clarida *et al.* (2002) which assumed no consumption bias $\omega = \frac{1}{2}$ to our case with $\omega \in [\frac{1}{2}, 1]$:

Proposition 3

For the case of complete exchange rate pass-through, the spillover effect of a monetary expansion on output is positive or negative depending on whether $\sigma < 1$ or $\sigma > 1$.

Now consider what happens as λ_F falls reducing the speed of exchange rate pass-through. Differentiating (167) we have

$$\frac{d}{d\lambda_F}\left[\alpha_1^s - \alpha_1^d\right] = \frac{(1-\omega)}{\lambda_H}\left[-\frac{\lambda_H}{(\sigma+\phi)(1+\lambda_F)^2} - \frac{(1-2\omega(1-\sigma))(\theta_1+\lambda_H\theta_2)}{(\sigma+(2\omega-1)\phi)(1+\lambda_F\theta_2)^2}\right] \tag{170}$$

If we now confine ourselves to the empirically realistic case of $\sigma > 1$, then $1-2\omega(1-\sigma) > 1$ and the derivative in (170) can be unambiguously signed as negative. Then together with the result of proposition 3 we can now assert that the effect of incomplete exchange rate pass-through (a fall in λ) is bring about a *reduction* in the negative spillover effect of monetary expansion on output. A similar result applies to the long-run spillover.

Finally we can determine an upper bound of the spillover by evaluating (168) at $\lambda_F = 0$. Some more algebra yields:

$$\left[\alpha_1^s - \alpha_1^d\right]_{\lambda_F=0} = \frac{2(1-\omega)\left[\sigma - \lambda_H\omega\left((\sigma-1)^2 - (1+\frac{(2\omega-1)}{\omega}\phi\right)\right]}{\lambda_H(\sigma+\phi)(\sigma+(2\omega-1)\phi)} \tag{171}$$

Thus for $\sigma = 1$ the case where spillovers disappeared for the complete exchange rate pass-through case, now the upper bound on the spillover effect is positive. More generally provided $\sigma - \lambda_H\omega(\sigma-1)^2 > 0$ the upper bound in (171) is positive. For instance if $\sigma = 3$, a large value, this condition becomes $\lambda_H\omega < 0.75$, a condition that is empirically plausible. Noting that the results also apply to the long-run, we can summarize these results as:

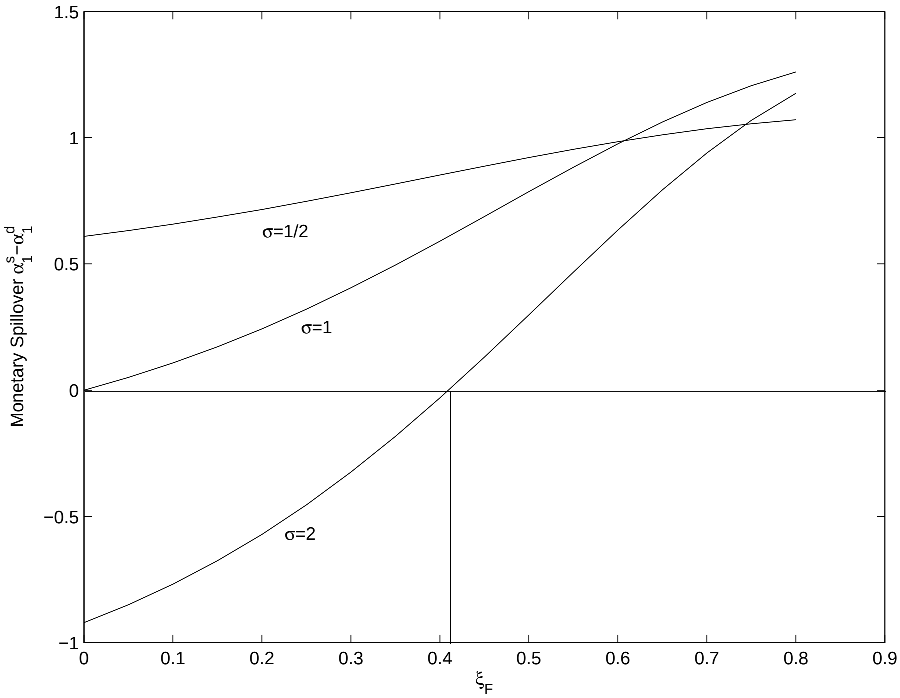

Figure 1: **Short-Run Monetary Spillovers on Output $\alpha_1^s - \alpha_1^d$.**

Proposition 4

For $\sigma > 1$, the effect of incomplete exchange rate pass-through is to reduce absolute magnitude of the negative short-run and long-run spillover effect of monetary expansion on output. If price-setting in the imported goods retail sector is sufficiently sticky, then the spillover can become positive provided $\lambda_H \omega < \frac{(\sigma-1)^2}{\sigma}$.

Figure 1 illustrates these results by plotting the short-run monetary spillover term $\alpha_1^s - \alpha_1^d$ against the probability ξ_F that the price in the imported retail sector is not optimized. ξ_F varies between zero (complete exchange rate pass-through) to $\xi = 0.8$ corresponding to an expected contract length of 5 quarters. For $\sigma < 1$ spillovers are positive. For $\sigma = 1$ spillovers disappear for the case of complete exchange rate pass-through. For $\sigma = 2$ spillovers are negative at this point, but as pass-through becomes incomplete the spillovers becomes less in magnitude, until at ξ_F a little over 0.4, corresponding to an expected contract length of just over 1.7 quarters, the spillovers become positive. All this is in accordance with propositions 3 and 4.

4.2 Stability and Determinacy of IFB Rules

This section studies an IFB-Taylor rule of the form

$$\begin{aligned} r_t &= \rho r_{t-1} + \theta(1-\rho)\mathcal{E}_t\pi_{t+j}\,;\ \rho \in [0,1),\, \theta > 0 \\ &= r_{t-1} + \Xi\mathcal{E}_t\pi_{t+j}\,;\ \rho = 1,\ \Xi > 0 \end{aligned} \tag{172}$$

and

$$\begin{aligned} r_t &= \rho r_{t-1} + \theta(1-\rho)\mathcal{E}_t\pi_{H,t+j}\,;\ \rho \in [0,1),\, \theta > 0 \\ &= r_{t-1} + \Xi\mathcal{E}_t\pi_{H,t+j}\,;\ \rho = 1,\ \Xi > 0 \end{aligned} \tag{173}$$

for the home bloc, where $j \geq 0$ is the forecast horizon, which is a feedback on single-period inflation over the period $[t+j-1, t+j]$. An analogous rule applies to the foreign bloc. With rule (172), policymakers set the nominal interest rate so as to respond to deviations of CPI inflation from target. With rule (173) the policymaker responds to domestic inflation. In addition, policymakers smooth rates, in line with the idea that central banks adjust the short-term nominal interest rate only partially towards the long-run inflation target, which is set to zero for simplicity in our set-up. The parameter $\rho \in [0,1]$ measures the degree of interest rate smoothing. If $\rho = 1$ we have an *integral rule* that guarantees that the long-run inflation target (zero in our set-up) is met, provided the rule stabilizes the economy. For $\rho < 1$, (172) can be written as $\Delta r_t = \frac{1-\rho}{\rho}[\theta\mathcal{E}_t\pi_{t+j} - i_t]$ which is a partial adjustment to a static IFB rule $r_t = \theta\mathcal{E}_t\pi_{t+j}$. j is the feedback horizon of the central bank. When $j = 0$, the central bank feeds back from current dated variables only. When $j > 0$, the central bank feeds back instead from deviations of forecasts of variables from target. Finally, $\theta, \Xi > 0$ are the feedback parameters for the non-integral and integral rules respectively: the larger is θ or Ξ, the faster is the pace at which the central bank acts to eliminate the gap between expected inflation and its target value.

We shall see in the next two sections that virtually all the optimal simple rules that we compute are of the integral form ($\rho = 1$). As a consequence we shall not address the more general rules ($\rho < 1$). However, we note that it is possible to derive the same general results as in BLP for the simplified sum and difference systems of this section: (i) for given $\rho < 1$ there exists a forward horizon J such that for any $j > J$ the system is suffers from indeterminacy; (ii) the critical J will be slightly greater than $1/(1-\rho)$; (iii) a similar result holds when *average* expected inflation is used as the feedback variable for the interest rate, but in this case the critical value J is slightly greater than $2/(1-\rho)$.

From these results, it is evident that for integral rules ($\rho = 1$), there will be no corresponding critical horizon that is ruled out by the requirement of determinacy. To see this more clearly we obtain the characteristic equations for the sum and difference systems under IFB rules that depend either on expected PPI or on expected CPI inflation.

As above, stability is addressed most easily by considering the sum and difference form separately. In each case the characteristic equation is formed from the matrices describing the z-transform of the systems; these matrices are displayed in Appendix A.

4.2.1 The Sum System

Sum System: Interest Rate Responds to PPI Inflation

Taking z-transforms of the system (128) to (135) with the rule (173), the characteristic equation for this is given by

$$\begin{aligned}(z-1)[(z-1)^2(\beta z-1)^2-(z-1)(\beta z-1)z(\lambda_F+\lambda_H(1-\omega))\\ -\lambda_H\frac{\sigma+\phi}{\sigma}(\omega(z-1)(\beta z-1)-\lambda_F z)]\\ +\Xi z^{j+1}\lambda_H\frac{\sigma+\phi}{\sigma}((z-1)(\beta z-1)-\lambda_F z)=0\end{aligned} \tag{174}$$

Sum System: Interest Rate Responds to CPI Inflation

Similarly taking z-transforms of the system (136) to (144) and using the rule (172), the characteristic equation for this is given by

$$\begin{aligned}(z-1)[(z-1)^2(\beta z-1)^2-(z-1)(\beta z-1)z(\lambda_F+\lambda_H(1-\omega))\\ -\lambda_H\frac{\sigma+\phi}{\sigma}(\omega(z-1)(\beta z-1)-\lambda_F z)]\\ +\Xi z^{j+1}\lambda_H\frac{\sigma+\phi}{\sigma}(\omega(z-1)(\beta z-1)-\lambda_F z)=0\end{aligned} \tag{175}$$

4.2.2 The Difference System

Proceeding as for the sum system we have:

Difference System: Interest Rate Responds to PPI Inflation

The characteristic equation is given by

$$\begin{aligned}
&(z-1)[(z-1)^2(\beta z-1)^2-(z-1)(\beta z-1)z(\lambda_F+\lambda_H\omega) \\
&-\lambda_H(1-\omega)(1+2\omega\phi)z((z-1)(\beta z-1)-2\lambda_F z) \\
&-\frac{\lambda_H(2\omega-1)z}{\sigma}(\phi\omega(z-1)(\beta z-1)-(\sigma+\phi(2\omega-1))\lambda_F z)] \\
&+\Xi z^{j+1}\lambda_H[((z-1)(\beta z-1)-\lambda_F z(1+4\omega(1-\omega)\phi) \\
&+\frac{(2\omega-1)\phi}{\sigma}((z-1)(\beta z-1)-\lambda_F(2\omega-1)z))]=0 \qquad (176)
\end{aligned}$$

Difference System: Interest Rate Responds to CPI Inflation

The characteristic equation when there is no home bias, $\omega=\frac{1}{2}$, is given by

$$\begin{aligned}
&(z-1)[(z-1)^2(\beta z-1)^2-(z-1)(\beta z-1)z(\lambda_F+\lambda_H\omega) \\
&-\lambda_H(1-\omega)(1+2\omega\phi)z((z-1)(\beta z-1)-2\lambda_F z) \\
&-\frac{\lambda_H(2\omega-1)z}{\sigma}(\phi\omega(z-1)(\beta z-1)-(\sigma+\phi(2\omega-1))\lambda_F z)] \\
&+\frac{\Xi}{\sigma}z^{j+1}[(\lambda_H\omega(\sigma+\phi(2\omega-1)+2\lambda_F\sigma(1-\omega))(z-1)(\beta z-1) \\
&-\lambda_H\lambda_F z(\sigma+\phi(2\omega-1)^2+4\sigma(1-\omega)\omega\phi)]=0 \qquad (177)
\end{aligned}$$

With such a forward-looking system, stability is not an issue, but if there are too many stable eigenvalues (i.e., roots of the characteristic equation) of either the sum or difference system, then the system is indeterminate. A useful method for tracking the roots in the complex plane as Ξ increases, is the Root Locus method, invented by Evans (1954).[21] Unlike the work of Batini and Pearlman (2002) and BLP, the systems here are too complicated to analyze easily.[22] We note that all the characteristic equations are very similar to one another, so as a consequence we can draw some stylised root locus diagrams indicating the paths of the roots of the system as Ξ increases from 0 to ∞.

[21] See BLP for a users' guide to the Root Locus method.

[22] In fact it is possible to confirm that the numerical results that we obtain appear to be true in general. However it requires several intermediate diagrams to get to this point, so fuller discussion is omitted.

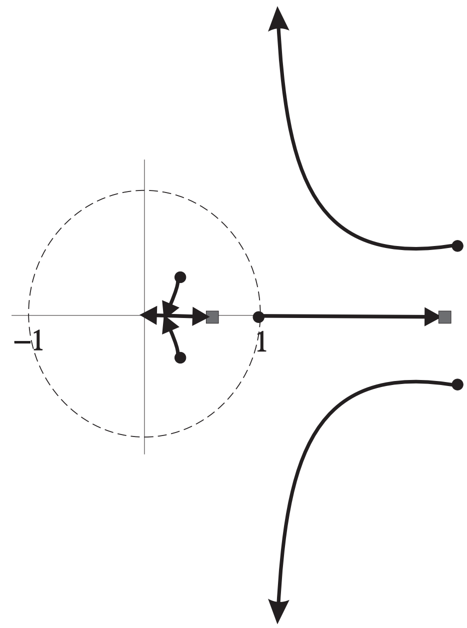

Figure 2: **The Position of Eigenvalues for Symmetrical IFB0 Rules.**

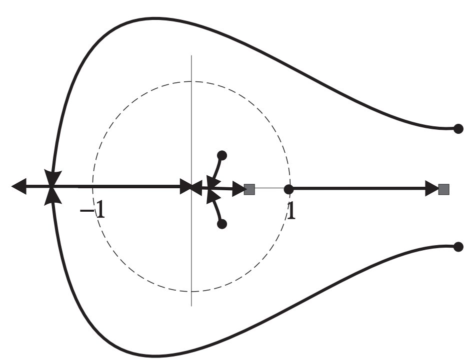

Figure 3: **The Position of Eigenvalues for Symmetrical IFB1 Rules.**

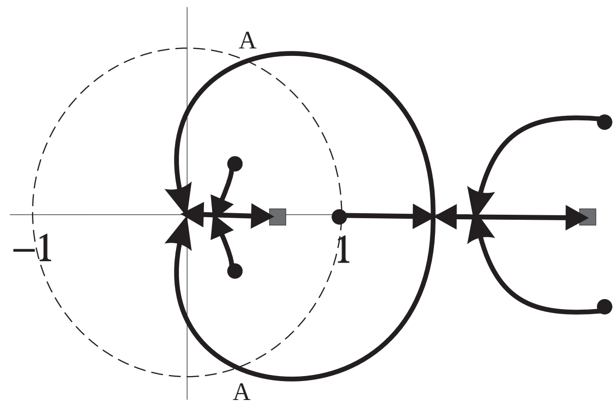

Figure 4: **The Position of Eigenvalues for Symmetrical IFB2 Rules.**

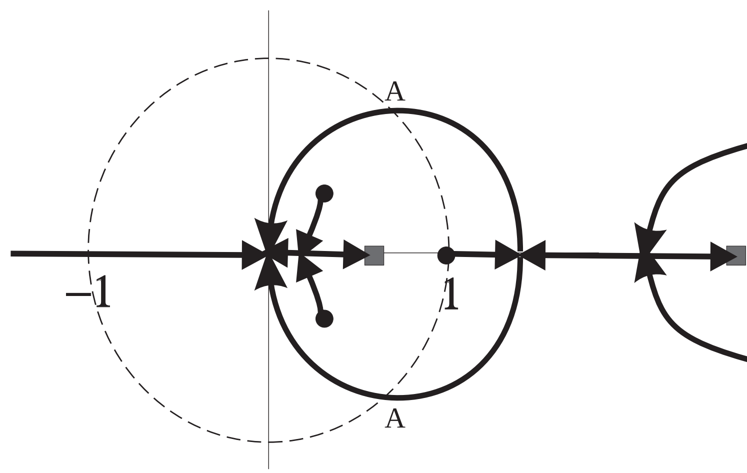

Figure 5: **The Position of Eigenvalues for Symmetrical IFB3 Rules.**

Figures 2-4 show the root locus diagrams for values $j = 0, 1, 2, 3$. These start at the roots of the system under no control (indicated by a black disc), and head off in the directions indicated by the arrows. The number of required stable roots corresponds to the number of predetermined variables in each of the two systems, of which there are two: τ and r. From 2 it is clear that there are always exactly two stable roots, so that for $j = 0$ there is never a problem of determinacy. For $j = 1$, there is indeterminacy only after the root locus crosses the unit circle at $z = -1$, while for $j = 2, 3$, the root locus first crosses the unit circle at the points labelled A. One can continue drawing these diagrams for all values of j, but they all have the same general appearance as Figs 4 and 5, apart from extra branches heading in from ∞. The conclusion that can be drawn is that there is always a conjugate pair of critical points on the unit circle corresponding to a particular value of Ξ beyond which there is indeterminacy. We summarise these observations as follows:

Proposition 5

The system under integral control is determinate (a) for all values of $\Xi > 0$ when $j = 0$ (b) over a bounded range of $\Xi > 0$ when $j > 0$.

Numerical simulations confirm an important property of the bound Ξ in proposition 5: the bound *decreases* as the rules become more forward-looking, i.e., as j increases.

An interesting knife-edge situation emerges when there is no home bias for home goods ($\omega = 1/2$), and when there is no inertia in setting the price of imported goods ($\xi_F = 0$). For the special case we are addressing this section, preferences across countries are Cobb-Douglas. It follows that, even though there is a mark-up ($\zeta/(\zeta-1)$) on imported goods, the consumer price index is essentially the same in each country, differing only by the nominal exchange rate i.e. PPP holds. Thus we have following result also obtained in BLP:

Proposition 6

For the case of no price inertia in imported goods, there is indeterminacy when CPI is used in the interest rate rule.

Proof

We can show this in two ways. Firstly consider the equations for the difference system. Combining (136) and (141) yields a UIP relationship. But for the difference system, the interest rate rule now merely feeds back on (expected) changes in the nominal interest rate. This implies a feedback rule that produces a path for the nominal exchange rate, but which is completely decoupled from other aspects of inflation. Thus π_D^d and π_I^d now evolve independently of the control rule, and display indeterminacy. An alternative method of showing this is to use (177), and setting λ_F to ∞. This yields a characteristic equation $(z - 1 - \Xi z^j)z((z-1)(\beta z - 1) - \lambda_H(1+\phi)z) = 0$ that is a product of two polynomials - one of them corresponding to the control rule on the nominal exchange rate, and the other corresponding to the dynamics of π_D^d and π_I^d, implying that the latter is unaffected by the control rule. $\square$

5 Optimized IFB Rules without Model Uncertainty

In this section we compute optimized IFB rules and optimal Taylor-type rules feeding back on either current producer price or consumer price inflation alone or on inflation and the output gap. The general form of the rule that covers integral and non-integral IFB as well as the Taylor-type rules is given for the home bloc and for CPI inflation by

$$i_t = \rho i_{t-1} + \Theta \mathcal{E}_t \pi_{t+j} \,; \;\; \rho \in [0,1],\, \Theta, \Theta_y > 0,\, j \geq 0 \tag{178}$$

and analogous rules apply for producer price inflation and for the foreign bloc.

In the absence of model uncertainty we assume that the policy problem of the home bloc central bank is to choose at time $t = 0$ in each period $t = 0, 1, 2, \cdots$ an interest rate r_t so as to minimize a standard expected loss function that depends on the variation of the output gap, CPI inflation and the level of the nominal interest rate:

$$\Omega_0^H = \mathcal{E}_0 \left[\frac{1}{2} \sum_{t=0}^{\infty} \beta_c^t \left[(\hat{y}_t - y_t)^2 + b\pi_t^2 + cr_t^2 \right] \right] \tag{179}$$

where β_c is the discount factor of the central bank. There is no ambitious output target that try to drive output closer to the efficient output level examined in section 3.6.2. Hence there is only a stochastic but no deterministic component of policy.[23] Given the estimated variance-covariance matrix of the white noise disturbances, an optimal combination (Θ, ρ) can be found for each rule defined by the time horizon $j \geq 0$.

With two central banks policy can either be set cooperatively or non-cooperatively. For cooperative rule of a particular type, the policymakers are assumed to jointly minimize an average loss function $(\Omega^H + \Omega^H)/2$. In the absence of cooperation, policymakers each independently choose an optimized feedback rule of a particular type given the choice of rule by the other. The resulting combination of rules will then be a *closed-loop Nash* equilibrium. The outcome under all rules are measured relative to an optimal baseline which is achieved if the two policymakers could both commit to the private sector and coordinate without being constrained to any particular simple form of rule. Details of all policy rules are provided in Appendix B.

[23] Since the IFB rule assumes a commitment mechanism, the policymaker in principle should be able to implement a policy $i_t = \bar{i}_t$ plus a feedback component such as (172) or (178) relative to $\bar{i}_t$, where the latter is the optimal deterministic trajectory.

5.1 Monetary Spillovers in the Estimated Model

Before we turn to the optimized rules it is instructive to examine the nature of the monetary spillovers in our estimated model. To carry out this exercise we run the model with a current CPI inflation rule of the form $r_t = -\bar{r} + 1.001\pi$, $r_t^* = 1.001\pi$ where the permanent decrease in the interest rate $\bar{r}$ is chosen so that the home bloc engineers an unanticipated increase of 1% in its domestic inflation rate in period 0. From our theory which applies to a simplified version of the model we then expect the spillover effect on foreign output to be positive for low rates of exchange rate pass-through and to be negative for high rates of exchange rate pass-through. Figure 6 shows that the former is the case. In figure 7 we simulate a hypothetical model with the same parameter values except that ξ_F, ξ_H^* are set at very low values and $\gamma_F = \gamma_H^* = 0$. This change then imposes *complete exchange rate pass-through* (i.e., PPP) on the model. Now the spillover effect on foreign output is positive, again as predicted by our theoretical analysis.

5.2 Optimized IFB Rules

The results are shown in table 1 for IFB rules feeding back on expected producer price inflation. In these results we put parameter values at their mean values in the posterior distribution of the estimated model and this is our baseline model. The weights in the loss function are welfare-based weights $b = 20.8$ and $c = 1.6$ taken from Woodford (2003).[24]

A number of interesting observations emerge from this table. First, from the output equivalent loss (relative to the optimal commitment outcome) of 'minimal feedback', the closest saddle-path stable integral rule using current domestic inflation to no feedback rule at all, we see that there are very significant gains from a stabilization policy amounting to around 7% output increase equivalent on average. These are most pronounced for the US. Second, simple current inflation rules are able to deliver almost all of these gains. If the policymaker can commit using a simple rule, the best one in this respect is a Taylor integral rule, and this realizes a large part of the potential stabilization gain. Third, for each

[24] Since these weights apply to a far simpler DSGE model of the US only, the results are only indicative as to actual welfare gains. Noting that our model is quarterly, the weights correspond to $b = 20.8/16$ and $c = 1.6/16$ in an annual model, values well within the range found in the literature. Future work will use the procedure for approximating a quadratic form of the consumers' utility based on Benigno and Woodford (2004).

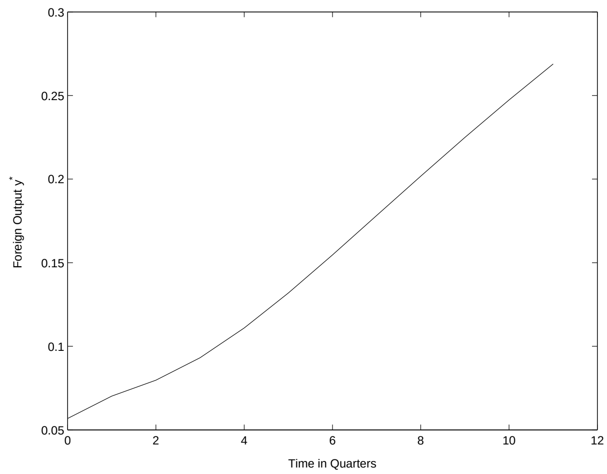

Figure 6: **Monetary Spillovers on Foreign Output from a Monetary Expansion Home Bloc H: Model B.**

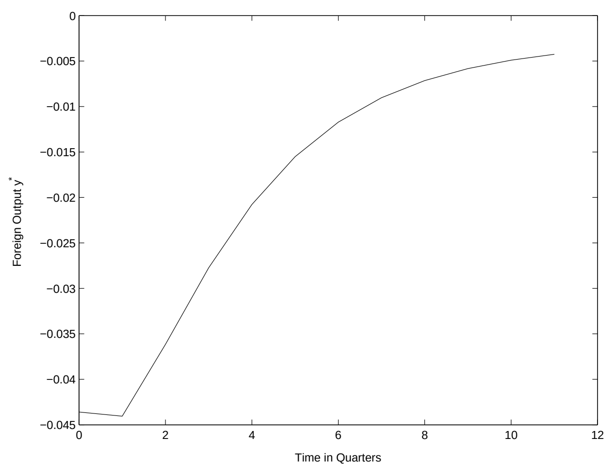

Figure 7: **Monetary Spillovers on Foreign Output from a Monetary Expansion Home Bloc H: Model B with PPP imposed.**

model we search for optimized rules within those that satisfy the determinacy conditions on ρ and θ for non-integral rules and on Θ for integral rules. We found that *integral rules consistently performed the best.* Fourth, our theory has shown that the requirement of determinacy severely constrains the range of possible stabilizing rules as the horizon j increases and as a result compared with the Taylor rule, IFBj rules *perform increasingly less well.* In our results the loss from IFB5 rules compared with a current inflation IFB0 rule is almost 2% on average. Finally for our estimated 2-bloc model the *gains from coordinating the design of IFBj rules is very small* amounting to a 0.02% output equivalent gain at most.

Rule	(ρ^H, ρ^F)	(Θ^H, Θ^F)	$(\Omega_0^H, \Omega_0^F); \bar{\Omega}_0$	% Output Equiv
Minimal Feedback	$(1, 1)$	$(10^{-3},\ 10^{-3})$	$(578, 436); 507$	$(9.06, 5.36); 7.2$
IFB0(C)	$(1, 1)$	$(0.28, 0.20)$	$(161, 187); 174$	$(0.72, 0.38); 0.54$
IFB0(NC)	$(1, 1)$	$(0.27, 0.29)$	$(167, 183); 175$	$(0.84, 0.30); 0.56$
IFB1(C)	$(1, 1)$	$(0.53, 0.28)$	$(156, 192); 174$	$(0.62, 0.48); 0.54$
IFB1(NC)	$(1, 1)$	$(0.51, 0.38)$	$(161, 189); 175$	$(0.72, 0.42); 0.56$
IFB2(C)	$(1, 1)$	$(1.24, 0.45)$	$(161, 202); 181$	$(0.72, 0.68)$;0.68
IFB2(NC)	$(1, 1)$	$(1.17, 0.58)$	$(165, 200); 182$	$(0.80, 0.64); 0.70$
IFB3(C)	$(1, 1)$	$(3.19, 0.74)$	$(169, 219); 194$	$(0.88, 1.04); 0.94$
IFB3(NC)	$(1, 1)$	$(3.06, 0.95)$	$(173, 217); 195$	$(0.96, 0.98); 0.96$
IFB4(C)	$(1, 1)$	$(2.68, 1.29)$	(193 ,242); 217	(1.36,1.48); 1.40
IFB4(NC)	$(1, 1)$	$(2.68, 1.65)$	(198,240); 219	$(1.46, 1.44); 1.40$
IFB5(C)	$(1, 1)$	$(1.31, 2.38)$	$(252, 268); 260$	$(2.54, 2.00); 2.26$
IFB5(NC)	$(1, 1)$	$(1.32, 3.0)$	$(256, 266); 261$	(2.62,1.96); 2.28
Optimal	n.a.	n.a.	(125,168)	(0,0)

Table 1. Baseline Model B[25]

Whilst the gains from coordinating IFB rules are small there are nonetheless interesting

[25] IFBj(C) and IFBj(NC) denote a j-period ahead IFB rule. Let under cooperation and non-cooperation rpectively. Let Ω^i be the loss for bloc i for any particular rule and Ω^i_Obe the loss from optimal cooperative rule with commitment. A 1% permanent fall in the output gap leads to a reduction in the loss function of $\frac{1}{2(1-\beta_c)} = 50$ in our calibration. The % output equivalent loss is then a measure of the degree of sub-optimality of each rule and is defined as $\frac{\Omega^i - \Omega^i_O}{41}$.

differences between the cooperative and non-cooperative rules. Under both cooperation and non-cooperation, the EU bloc uses monetary policy more aggressively in the face of high expected producer-price inflation. This is as one would expect from the more open of the two blocs. This aggressive use of monetary policy becomes more pronounced in the non-cooperative equilibrium resulting in a gain of as much as 0.06% for the EU at the expense of as much as 0.1% loss for the US. The net effect on the average of the non-cooperative compared with the cooperative loss is a small, as already noted.

Rule	(ρ^H, ρ^F)	(Θ^H, Θ^F)	$(\Omega_0^H, \Omega_0^F); \bar{\Omega}_0$	% Output Equiv
Minimal Feedback	$(1, 1)$	$(10^{-3}, 10^{-3})$	$(368, 563); 466$	$(3.84, 7.68); 5.76$
IFB0(C)	$(1, 1)$	$(0.10, 0.40)$	$(185, 231); 208$	$(0.18, 1.04); 0.60$
IFB0(NC)	$(1, 1)$	$(0.20, 0.61)$	$(192, 234); 215$	$(0.32, 1.10); 0.74$
IFB1(C)	$(1, 1)$	$(0.18, 0.52)$	$(184, 241); 213$	$(0.16, 1.24); 0.70$
IFB1(NC)	$(1, 1)$	$(0.41, 0.73)$	$(189, 249); 219$	$(0.26, 1.40); 0.82$
IFB2(C)	$(1, 1)$	$(0.38, 0.77)$	$(191, 259); 225$	$(0.30, 1.60); 0.94$
IFB2(NC)	$(1, 1)$	$(1.06, 1.09)$	$(196, 267); 232$	$(0.40, 176); 1.08$
IFB3(C)	$(1, 1)$	$(1.06, 1.23)$	$(201, 283); 242$	$(0.50, 2.08); 1.28$
IFB3(NC)	$(1, 1)$	$(3.40, 1.70)$	$(205, 288); 247$	$(0.58, 2.18); 1.38$
IFB4(C)	$(1, 1)$	$(3.40, 2.05)$	$(212, 306); 259$	$(0.72, 2.54); 1.62$
IFB4(NC)	$(1, 1)$	$(3.37, 2.80)$	(219,303); 262	$(0.86, 2.48); 1.68$
IFB5(C)	$(1, 1)$	$(1.76, 3.83)$	$(231, 324); 278$	$(1.10, 2.90); 2.00$
IFB5(NC)	$(1, 1)$	$(1.76, 4.88)$	$(236, 322); 279$	$(1.20, 2.86); 2.02$
Optimal	n.a.	n.a.	(176,179)	(0,0)

Table 2. Alternative Model with Full Trade Linkages

We have found that the gains from coordinating on IFB rules are extremely small for the two blocs. This perhaps is not surprising given the low trade linkages between the US and the Euro-zone.[26] In our next exercise we therefore ask the question: what *would* the gains be if the goods markets where completely integrated with import shares $s_H = s_F = 0.5$. Table 2 shows these results.

[26] Recall the calibrated import shares $s_H = 0.14$, $s_F = 0.37$.

In this hypothetical world of full trade linkages between the US and the Euro-zone table 2 shows that the gains from coordinating the design of current inflation and IFB rules are now significant ranging from a 0.12% output equivalent for the current inflation rules but falling as the rule becomes more forward-looking and j increases. The reason for this can be seen in proposition 5 and the numerical result alluded too just after the proposition: the upper bound on the feedback parameter necessary to avoid in determinacy falls as j increases, thus placing a increasingly tight constraint on the policymaker and forcing the cooperative and non-cooperative rules together.

Our estimates of gains from coordination are rather higher than those reported in Obstfeld and Rogoff (2002) in a far simpler model without many of the persistence mechanisms of that in this paper.[27] However their finding that *the coordination gains are far less than the stabilization gains* are borne out in our results.

6 Optimized IFB Rules with Model Uncertainty

6.1 Theory

In this section we consider model uncertainty in the form of uncertain estimates of the non-policy parameters of the model, Θ. Suppose the state of the world s is described by a model with $\Theta = \Theta^s$ expressed in state-space form as

$$\begin{bmatrix} \mathsf{z}^s_{t+1} \\ \mathcal{E}_t \mathsf{x}^s_{t+1} \end{bmatrix} = A^s \begin{bmatrix} \mathsf{z}^s_t \\ \mathsf{x}^s_t \end{bmatrix} + B^s \begin{bmatrix} r_t \\ r^*_t \end{bmatrix} + C^s \begin{bmatrix} \epsilon_{gt+1} \\ \epsilon_{at+1} \end{bmatrix} \tag{180}$$

$$o^s_i = E^s \begin{bmatrix} \mathsf{z}^s_t \\ \mathsf{x}^s_t \end{bmatrix} \tag{181}$$

where $\mathsf{z}^s_t =$ is a vector of predetermined variables at time t and x_t are non-predetermined variables in state s of the world. In (180) and (181) it is important to stress that variables are in deviation form about a zero-inflation steady state of the model in state s. For example output in deviation form is given by $y^s_t = \frac{Y^s_t - \bar{Y}^s}{\bar{Y}_s}$ where $\bar{Y}^s$ is the steady state of the model in state s defined by parameters Θ^s and $r^s_t = r_t - \bar{r}^s$ for the home bloc where the natural rate of interest in model s, $\bar{r}_s = \frac{1}{\beta^s} - 1$.

[27] Also, our results need to be treated with some caution as they depend on the estimates of the standard errors of the shocks which as we noted were implausible in one particular case.

Consider simple rules of the general form

$$\begin{bmatrix} r_t \\ r_t^* \end{bmatrix} = Dy_t = D \begin{bmatrix} z_t \\ x_t \end{bmatrix} \tag{182}$$

where D is constrained to be sparse in some specified way. Rule (B.20) can be quite general. By augmenting the state vector in an appropriate way it can represent a PID (proportional-integral-derivative) form of rule (though the paper is restricted to a simple proportional or integral form only).

For M-robustness, in general one sets up a *composite model* of outputs from each of the states $s = 1, 2, \cdots, n$ corresponding to the rival models and minimizes the expected loss across these states using estimated posterior probabilities. Because each model is linearized about a different steady state, we must now set up the model in state s in terms of the *actual* interest rate, not the deviation about the steady state. Then augmenting the state vector to become z_t^s we still have a state have a state-space form (180) and (181) and for the home bloc we minimize

$$\begin{aligned} \Omega_0 &= \frac{1}{2}\sum_{t=0}^{\infty}\beta_c^t \sum_{s=1}^{n} p_s \Big[(\bar{y}_{o,t}^s - k^s)^2 + b_s(\bar{\pi}_t^s)^2 + c_s(\bar{r}_t - \bar{r}_{t-1})^2 \\ &+ \mathcal{E}_0 \left[(\tilde{y}_{o,t}^s)^2 + b_s(\tilde{\pi}_t^s)^2 + c_s(\tilde{r}_t - \tilde{r}_{t-1})^2 \right] \Big] \end{aligned} \tag{183}$$

where $y_{o,t}^s = \hat{y}_t^s - \tilde{y}_t^s$ is the output gap in state s. Note that the inefficiency captured by k^s depends on the state. For P-robustness (183) is replaced with the average utility loss across a large number of draws from all models using both the posterior model probabilities and the posterior parameter distributions for each model.

In (183) the output target in state s of the world is given by $o_t^s = y_t^n + k^s$ where the ambitious output target k^s depends on s. In fact we will continue to assume that the central bank has no ambitious output targets and set $k^s = 0$ in its loss function. However with model uncertainty there is still a deterministic component of policy arising from differences in the natural rate of interest compatible with zero inflation in the steady state, $\bar{r}^s = \frac{1}{\beta^s} - 1$.[28] A non-integral rule specifying $r_t = \bar{r}^s$ in the long-run will only result in zero inflation in model s. From the consumers' Euler equation (14) in model s' with

[28]In fact in this paper we impose $\beta^s = 0.99$ for all states, so the point we make here is only potentially important if the β_s are estimated (as in Batini *et al.* (2004b)).

$\beta^s > \beta^{s'}$, implementing the rule designed for model s gives a steady state inflation rate $\bar{\pi}^{s'}$ given by

$$\frac{\beta^{s'}(1+\bar{r}^s)}{(1+\bar{\pi}^{s'})} = \frac{\beta^{s'}}{\beta^s(1+\bar{\pi}^{s'})} = 1 \quad \text{i.e., } \bar{\pi}^{s'} = \frac{\beta^{s'}}{\beta^s} - 1 > 0 \tag{184}$$

Our robust non-integral rule designed for any model specifies a natural zero inflation rate of interest $\bar{r}_R$, corresponding to a discount factor $\beta_R = \frac{1}{1+\bar{r}_R}$ to result in an expected long-run inflation rate across models of zero. This implies β_R is determined by

$$\sum_{s=1}^{n} p_s \left[\frac{\beta_s}{\beta_R} - 1\right] \Rightarrow \beta_R = \sum_{s=1}^{n} p_s \beta_s \tag{185}$$

That is, β_R is the expected value of β_s across the model variants. The need to specify a natural rate of interest, $\bar{r}_R$, only applies to non-integral rules. By contrast, a further benefit of integral rules is that the economy is automatically driven to a zero-inflation steady state whatever the state of the world without having to specify $\bar{r}_R$.

There is one final consideration first raised by Levine (1986) that is usually ignored in the literature. Up to now we have assumed that private sector expectations $\mathcal{E}_t \mathsf{x}^s_{t+1}$ are state s model-consistent expectations. In other worlds in each state of the world the private sector knows the state and faces no model uncertainty. In a more general formulation of the problem we can relax this assumption and assume that both the policymaker and the private sector faces model uncertainty. Suppose that in state s of the world the latter believes model s' is the correct one. Then $\mathcal{E}_t \mathsf{x}^s_{t+1}$ must be replaced by the composite expectation $\mathcal{E}_{t,s'} \mathsf{x}^s_{t+1}$ where the expectational operator at time t is now conditional on model s'. In state of the world s with the private sector believing state of the world s', the system under control (180), with the interest rate rules (believed by the private sector) given by (B.20), has a rational expectations solution with $x_t^{ss'} = -N^{s'} z_t^{ss'}$ where $N^{s'} = N^{s'}(D)$ is calculated on the basis of model s'. Hence

$$z_{t+1}^{ss'} = (G_{11}^s - G_{12}^s N^{s'}) z_t^{ss'} \tag{186}$$

where $G^s = \begin{bmatrix} G_{11}^s & G_{12}^s \\ G_{21}^s & G_{22}^s \end{bmatrix} = A^s + B^s D$ is partitioned conformably with $\begin{bmatrix} z_t^s \\ x_t^s \end{bmatrix}$. For M-robustness we now minimize we minimize

$$\begin{aligned} \Omega_0 &= \frac{1}{2} \sum_{t=0}^{\infty} \beta_c^t \sum_{s=1}^{n} \sum_{s'=1}^{n} p_{ss'} \Big[(\bar{y}_{o,t}^{ss'} - k^s)^2 + b_s (\bar{\pi}_t^{ss'})^2 + c_s (\bar{r}_t - \bar{r}_{t-1})^2 \\ &+ \mathcal{E}_0 \Big[(\tilde{y}_{o,t}^{ss'})^2 + b_s (\tilde{\pi}_t^{ss'})^2 + c_s (\tilde{r}_t - \tilde{r}_{t-1})^2 \Big] \Big] \end{aligned} \tag{187}$$

and the corresponding modification of P-robust rules is analogous.

6.2 P-Robust IFB Rules

In the results that follow we confine ourselves to model-consistent expectations and to P-robust rules with no ambitious output target. Table 3 sets out the P-robust rules for this case computed as described above. The notable features of these results are: first, as with optimized rules under certainty in table 1, integral rules or in the case of the current inflation rule IFB0, a near-integral rule, perform the best.

Rule	(ρ^H, ρ^F)	(Θ^H, Θ^F)
IFB0 (C)	(1,0.98)	(0.23,0.17)
IFB0 (NC)	(0.99,1)	(0.23,0.29)
IFB1 (C)	(1,1)	(0.45,0.29)
IFB1 (NC)	(1,1)	(0.42,0.45)
IFB2 (C)	(1,1)	(1.05,0.52)
IFB2 (NC)	(1,1)	(1.01,0.78)
IFB3 (C)	(1,1)	(2.08,0.94)
IFB3 (NC)	(1,1)	(2.08,1.04)
IFB4 (C)	(1,1)	(0.87,1.56)
IFB4 (NC)	(1,1)	(0.87,2.05)
IFB5 (C)	(1,1)	(0.52,2.0)
IFB5 (NC)	(1,1)	(0.53,2.01)

Table 3. P-Robust IFB Rules using Domestic Inflation.

Second, comparing the optimized rules with and without model uncertainty, the average degree of feedback under uncertainty is substantially lower. The need to exercise more caution in the conduct of stabilization policy where parameter values in the model are stochastic is a familiar result originating with Brainard (1967). It should be stressed however that this uncertainty induces caution results applies to the average response of the two blocs, but not necessarily to each of them. For example with the IFB4 rule, the

US is extremely cautious responding to a policy in the EU that is more aggressive under uncertainty.

Finally as in the absence of parameter uncertainty, the EU responds more aggressively in the non-cooperative equilibrium compared with cooperation. However, as the horizon j increases the upper bound constraint of IFBj rules highlighted in section 6.2 of our analysis kicks in with a consequence that the robust rules with and without cooperative draw closes so that for $j = 5$ they are almost identical.

In order to demonstrate the role of P-robustness in the design of optimized IFB rules we pick a number of interesting model variants from the draws of parameter combinations used to compute the P-robust rules. In the table that follows:

1. *Variant 1* has a combination of parameters with the minimum value of the important risk aversion, σ, in the H bloc at $\sigma = 0.89$.

2. *Variant 2* has a combination of parameters with the minimum value of the indexation parameter in the F-bloc's domestic sector, γ_F^*, at $\gamma_F^* = 0.077$.

3. *Variant 3* has a combination of parameters with the minimum value of the habit in labour supply, h_N^*, in the F-bloc at $h_N^* = 0.057$.

Table 4 sets out the outcomes under the rules. Non-robust rules $IFBj$, $j = 0, 1, \cdots 5$ are those from table 1 designed for parameter values from our baseline model with parameters set at the mean of the distribution. The first column then repeats the losses in table 1 for these rules. P-robust rules are those from table 3. Each row gives the value of the loss function for the H and F bloc followed by the average corresponding to each state of the world. Underneath are losses expressed as output equivalents.

Consider the outcome when a rule is designed for the baseline model, but an alternative model turns out to be the true state of the world. Then variant 1 that suppresses most of the inflation persistence in the EU bloc is the most determinacy-prone of our four alternative models. Variant 3 with low output persistence in the EU bloc generated by habit persistence in labour supply is the next most inflation-prone. In both cases IFBj rules which are designed to be optimal for the baseline model lead to indeterminacy if $j \geq 4$ and in the case of variant 1 for $j \geq 3$. Whether the blocs cooperative or not does not change this conclusion and the outcomes under cooperation and non-cooperation are

Rule	Model B	Variant 1	Variant 2	Variant 3
IFB0(C)	(161, 187); 174 (0.72, 0.38); 0.54	(396, 164); 280 (5.12, 0.26); 2.70	(342, 100); 221 (2.66, 0.50); 1.58	(102, 105); 104 (0.30, 0.40); 0.36
IFB0(NC)	(167, 183); 175 (0.84, 0.30); 0.56	(398, 161); 280 (5.16, 0.20); 2.70	(342, 100); 221 (2.66, 0.50); 1.58	(102, 105); 104 (0.30, 0.40); 0.36
IFB0(C,P-Robust)	(159, 192); 175 (0.68, 0.48); 0.56	(389, 166); 277 (4.98, 0.30); 2.64)	(325, 107); 216 (2.32, 0.64); 1.48	(104, 112); 108 (0.34, 0.54); 0.44
IFB0(NC,P-Robust)	(168, 183); 175 (0.86, 0.30); 0.56	(393, 161); 277 (5.06, 0.20); 2.64	(324, 86); 205 (2.30, 0.22); 1.26	(112, 97); 105 (0.34, 0.54); 0.38
IFB1(C)	(156, 192); 174 (0.62, 0.48); 0.54	(327, 166); 246 (3.74, 0.30); 2.02	(313, 106); 209 (2.08, 0.62); 1.34	(104, 111); 107 (0.30, 0.52); 0.42
IFB1(NC)	(161,189); 175 (0.72, 0.42); 0.56	((329, 163); 246 (3.78, 0.24); 2.02	(310, 94); 202 (2.02, 0.38); 1.20	(109, 103); 106 (0.44, 0.36); 0.40
IFB1(C,P-Robust)	(157, 191); 174 (0.64, 0.46); 0.54	(329, 166); 247 (3.78, 0.30); 2.04	(300, 104); 202 (1.82, 0.58); 1.20	(107, 110); 109 (0.40, 0.50); 0.46
IFB1(NC,P-Robust)	(165, 190); 177 (0.80, 0.44); 0.60	(335, 163); 248 (3.90, 0.24); 2.06	(295, 88); 192 (1.72, 0.26); 1.00	(115, 100); 107 (0.56, 0.30); 0.42
IFB2(C)	(161, 202); 181 (0.72, 0.68); 0.68	(273, 171); 222 (2.66, 0.40); 1.54	(302, 108); 205 (1.86, 0.66); 1.26	(113, 117); 115 (0.52, 0.64); 0.58
IFB2(NC)	(165, 200); 182 (0.80, 0.64); 0.70	(276, 167); 222 (2.72, 0.32); 1.54	(297, 98); 198 (1.76, 0.46); 1.12	(117, 110); 113 (0.60, 0.50); 0.54
IFB2(C,P-Robust)	(163, 201); 182 (0.76, 0.66); 0.70	(279, 169); 224 (2.78, 0.36); 1.58	(289, 102); 195 (1.60, 0.54); 1.06	)(117, 112); 115 (0.60, 0.54); 0.58
IFB2(NC,P-Robust)	(171, 204); 188 (0.92, 0.72); 0.82	(284, 165); 225 (2.88, 0.28); 1.60	(287, 90); 189 (1.56, 0.30); 0.94	(123, 105); 114 (0.72, 0.40); 0.56
IFB3(C)	(169, 219); 194 (0.88, 1.04); 0.94	indeterminacy	(303, 109); 206 (1.88, 0.68); 1.28	(123, 122); 123 (0.72, 0.74); 0.74
IFB3(NC)	(173, 217); 195 (0.96, 0.98); 0.96	indeterminacy	(299, 101); 200 (1.80, 0.52); 1.16	(126, 116); 121 (0.78, 0.62); 0.70
IFB3(C,P-Robust)	(175, 218); 197 (1.00, 1.00); 1.00	(254, 173); 213 (2.28, 0.44); 1.36	(276, 101); 189 (1.34, 0.52); 0.94	(134, 117); 125 (0.94, 0.64); 0.78
IFB3 (NC, P-Robust)	(177, 218); 198 (1.04, 1.00); 1.02	(254, 152); 213 (2.28, 0.42); 1.36	(277, 98); 187 (1.36, 0.46); 0.90	(135, 115); 125 (0.96, 0.60); 0.78
IFB4(C)	(193, 242); 217 (1.36, 1.48); 1.40	indeterminacy	(263,;108); 186 (1.08, 0.66); 0.88	indeterminacy
IFB4(NC)	(198, 240); 219 (1.46, 1.44); 1.40	indeterminacy	(263, 102); 183 (1.08, 0.54); 0.82	indeterminacy
IFB4(C,P-Robust)	(231, 243); 237 (2.12, 1.50); 1.80	(502, 183); 343 (7.24, 0.64); 3.96	(270, 104); 187 (1.22, 0.58); 0.90	(203, 127); 165 (2.32, 0.84); 1.58
IFB4(NC,P-Robust)	(236, 245); 240 (2.22, 1.54); 1.86	(509, 181); 345 (7.36, 0.60); 4.00	(271, 98); 184 (1.24, 0.46); 0.84	(207, 123); 165 (2.40, 0.76); 1.58
IFB5(C)	(252, 268); 260 (2.54, 2.00); 2.26	indeterminacy	(283, 107); 195 (1.48, 0.64); 1.06	indeterminacy
IFB5(NC)	(256, 266); 261 (2.62, 1.96); 2.28	indeterminacy	(283, 102); 192 (1.48, 0.54); 1.00	indeterminacy
IFB5(C, P-Robust)	(289, 272); 282 (3.28, 2.08); 2.72	(825, 197); 510 (13.7, 0.92); 7.30	(304, 113); 208 (1.90, 0.76); 1.32	(273, 144); 208 (3.72, 1.18); 2.44
IFB5 (NC,P-Robust)	(288, 276); 282 (3.26, 2.16); 2.72	(825, 197); 510 (13.7, 0.92); 7.30	(304, 113); 208 (1.90, 0.76); 1.32	(273, 144); 208 (3.72, 1.18); 2.44
Optimal Commitment	(125, 168); 147	(140, 151); 145	(209, 75); 142	(87, 85); 86

Table 4. Outcome with Model Uncertainty using Domestic Inflation

very similar. By contrast IFB0, IFB1 and IFB2 rules designed for the baseline model are remarkably robust across the model variants and there is little by way of increased robustness to be gained from using the P-robust rules.

For more forward-looking rule, IFBj with $j \geq 3$, P-robust rules by design offer protection against indeterminacy, but at a cost. If model B is the true model, P-robust rule with $j = 4$ results in a 0.5% equivalent output loss compared with the rule designed for model B. This rises to a 7.3% loss compared with an optimal rule designed for variant 1. The conclusion to be drawn from these results is that if a very forward-looking IFB as opposed to say a current inflation rule is employed, then P-robust rules become essential to avoid indeterminacy.

7 Conclusions

We summarize the main results of the paper as follows:

1. Analysis using a simplified symmetrical model without persistence mechanisms and wage stickiness showed that, if we assume (as supported by our estimation) that the risk-parameter $\sigma > 1$, then the spillover effect of a monetary expansion in one bloc on output in the other is negative. The effect of incomplete exchange-rate pass-through is to reduce the absolute magnitude of these spillovers. For a sufficient departure from the law of one price, spillovers become positive.

2. Numerical Results from the full estimated model are

    (a) Forward-looking IFB rules designed for the baseline model, whether cooperative or non-cooperative are outperformed by current inflation rules both in terms of their performance when the model is known, and their robustness when there is parameter uncertainty.

    (b) There are only very small overall gains from cooperation in terms of the average loss, but under non-cooperation we find a significant increase in the aggressiveness of monetary policy in the EU at the expense of the US.

    (c) In a hypothetical world of full trade linkages between the US and the Euro-zone the gains from coordinating the design of current inflation and IFB rules are now significant ranging from a 0.12% output equivalent for the current inflation rules but falling as the rule becomes more forward-looking and the horizon j increases. The reason for this is a determinacy constraint on the policymaker that becomes increasingly tight, forcing the cooperative and non-cooperative rules together.

(d) The coordination gains are far less than the stabilization gains, a result in agreement with Obstfeld and Rogoff (2002).

(e) Under cooperation or non-cooperation a P-robust rule is essential for very forward-looking rules offering protection against indeterminacy in all states of the world. This protection however comes at a significant output equivalent cost.

There are a number of limitations of our research which future research will seek to redress. First, the model has a number of deficiencies such as the absence of a rest of the world bloc, the absence of capital and the allowance for incomplete asset markets. Second, in common with much of the literature in computing optimized rules, we optimize using a plausible policymakers' loss function that penalizes deviations from zero of the output gap and inflation, and changes in the interest rate. Rules that optimize the welfare of households would provide an interesting comparison.[29] Third, as we have pointed out, in considering model uncertainty we still imposed model-consistent expectations. Finally in the closed-economy model of Batini *et al.* (2004a) which assumed flexible prices but no habit persistence in labour supply, we were able to find a number of rival models that were accepted by the data. Current work is attempting the same for our two-bloc model.

References

Batini, N. and Pearlman, J. (2002). Too Much Too Soon: Instability and Indeterminacy With Forward-Looking Rules. Bank of England External MPC Discussion Paper No. 8.

Batini, N., Levine, P., and Pearlman, J. (2004a). Indeterminancy with Inflation-Forecast-

[29]In our linear-quadratic framework this would draw upon Benigno and Woodford (2004). Applying their techniques to a complex two-bloc country such as that in this paper is not straightforward and would require computational methods; see Levine *et al.* (2005).)

Based Rules in a Two-Bloc Model. ECB Discussion Paper no 340 and FRB Discussion Paper no 797, presented at the International Research Forum on Monetary Policy in Washington, DC, November 14-15, 2003.

Batini, N., Justiniano, A., Levine, P., and Pearlman, J. (2004b). Robust Inflation-Forecast-Based Rules to Shield against Indeterminacy. IMF Discussion Paper, forthcoming, presented at the 10th International Conference on Computing in Economics and Finance, Amsterdam, July 8-10, 2004, available at www.econ.surrey.ac.uk/people/index.htm.

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. (2001). Monetary Policy and Multiple Equilibria. *American Economic Review*, **91**(1), 167–86.

Benigno, G. and Benigno, P. (2001). Implementing Monetary Cooperation through Inflation Targeting. New York University, Mimeo.

Benigno, G. and Benigno, P. (2003). Designing Targetting Rules for International Monetary Policy Cooperation. ECB Working Paper no. 279, Mimeo.

Benigno, P. and Woodford, M. (2004). Optimal Monetray and Fiscal Policy: A Linear-Quadratic Approach. ECB Discussion Paper no 341, presented at the International Research Forum on Monetary Policy in Washington, DC, November 14-15, 2003.

Bernanke, B. and Woodford, M. (1997). Inflation Forecasts and Monetary Policy. *Journal of Money Credit and Banking*, **24**, 653–684.

Betts, C. and Devereux, M. (2000a). Exchange Rate Dynamics in a Model of Pricing-to-Market. *Journal of International Economics*, **50**, 215–244.

Betts, C. and Devereux, M. (2000b). International Monetary Policy Coordination and Competitive Depreciaition: A Re-evaluation. *Journal of Money, Credit and Banking*, **32**, 722–746.

Brainard, W. (1967). Uncertainty and the Effectiveness of Policy. *American Economic Journal*, **47**(2), 411–425.

Carlstrom, C. T. and Fuerst, T. S. (1999). Real Indeterminacy in Monetary Models with Nominal Interest Rate Distortions. Federal Reserve Bank of Cleveland working paper.

Castelnuovo, E. (2003). Taylor Rules and Interest Rate Smoothing in the US and EMU. Mimeo, Bocconi University.

Choudhary, A. and Levine, P. (2004). Idle Worship. *Economic Letters.* Forthcoming.

Christiano, L. J., Eichenbaum, M., and Evans, C. (2001). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. NBER Working Paper, no. 8403.

Clarida, R., Galí, J., and Gertler, M. (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics*, **115**(1), 147–80.

Clarida, R., Galí, J., and Gertler, M. (2002). A Simple Framework for International Monetary Policy Analysis. *Journal of Monetary Economics* , **49**, 679–904.

Corsetti, G. and Pesenti, P. (2001). Welfare and Macroeconomic Interdependence. *Quarterly Journal of Economics*, **CXVI**, 421–446.

Corsetti, G., Dedola, L., and Leduc, S. (2002). Consumption and Real Exchange Rates with Goods and Asset Markets Frictions. University of Rome III, Mimeo.

Currie, D. and Levine, P. (1993). *Rules, Reputation and Macroeconomic Policy Coordination.* CUP.

De Fiore, F. and Liu, Z. (2002). Openness and Equilibrium Determinancy under Interest Rate Rules. European Central Bank, WP. no. 173.

Fagan, G., Henry, J., and Mestre, R. (2001). An area-wide model (AWM) for the Euro Area. ECB WP No. 42.

Giannoni, M. and Woodford, M. (2002). Optimal Interest-Rate Rules: I. General Theory. NBER Working Paper no. 9419.

Justiniano, A. and Preston, B. (2004). Small Open Economy DSGE Models: Specification, Estimation and Model Fit. Mimeo, Columbia University.

Levine, P. (1986). The Formulation of Robust Policies for Rival Rational Expectations Models of the Economy. *Journal of Economic Dynamics and Control*, **10**, 93–97.

Levine, P., Pearlman, J., and Pierse, R. (2005). A Users' Guide to Linear Approximation in Macroeconomics. Mimeo, forthcoming.

Monacelli, T. (2003). Monetary Policy in a Low Pass-Through Environment. European Central Bank, Working Paper No. 227.

Obstfeld, M. and Rogoff, K. (1996). *Foundations of International Macroeconomics.* MIT Press, Cambridge, Massachusetts. London, England.

Obstfeld, M. and Rogoff, K. (2000). The Six Major Puzzles in International Finance: Is There a Common Cause? NBER Macroeconomics Annual, 15.

Obstfeld, M. and Rogoff, K. (2002). Global implications of self-oriented national monetary rules. *Quarterly Journal of Economics*, pages 503–535.

Rotemberg, J. and Woodford, M. (1997). An Optimization-Based Econometric Framework for the Evaluation of Monetary Rules. *NBER Macroeconomics Annual*, **12**, 297–346.

Shorfheide, F. and Lubik, T. (2005). A Bayesian Look at New Open Economy Macroeconomics. Presented at the NBER Macro Annual Conference.

Smets, F. and Wouters, R. (2003). An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, **1**(5), 1123–1175.

Sutherland (2002). International Monetary Policy Coordination and Financial Market Integration. Mimeo, Board of Governors of the Federal Reserve System.

Taylor, J. B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, **39**, 195–214.

Woodford, M. (2003). *Foundations of a Theory of Monetary Policy.* Princeton University Press.

A Derivation of Characteristic Equations

We obtain the characteristic equations of the sum and difference systems from the determinant of the z-transform matrix (where z is the forward operator) on the endogenous variables describing each of the systems (128)-(135), (172) and (136)-(144), (173). For the ordering c, π_D, π_I, τ, r these matrices are given by:

Sum System:

$$\begin{bmatrix} z-1 & \frac{\omega z}{\sigma} & \frac{(1-\omega)z}{\sigma} & 0 & -\frac{1}{\sigma} \\ \lambda_H(\phi+\sigma) & \beta z - 1 & 0 & -\lambda_H(1-\omega) & 0 \\ 0 & 0 & \beta z - 1 & \lambda_F & 0 \\ 0 & -z & z & z-1 & 0 \\ 0 & -\Xi z^{j+1}\Upsilon_1^s & -\Xi z^{j+1}\Upsilon_2^s & 0 & z-1 \end{bmatrix} \tag{A.1}$$

where $\Upsilon_1^s = \omega$, $\Upsilon_2^s = 1-\omega$ for CPI inflation rules and $\Upsilon_1^s = 1$, $\Upsilon_2^s = 0$ for PPI inflation rules.

Difference System:

$$\begin{bmatrix} z-1 & \frac{\omega z}{\sigma} & \frac{(1-\omega)z}{\sigma} & 0 & -\frac{1}{\sigma} \\ \lambda_H(\sigma+\phi(2\omega-1)) & \beta z - 1 & 0 & -\lambda_H(1-\omega)(1+2\omega\phi) & 0 \\ 2\lambda_F\sigma & 0 & \beta z - 1 & \lambda_F(2\omega-1) & 0 \\ 0 & -z & z & z-1 & 0 \\ 0 & -\Xi z^{j+1}\Upsilon_1^d & -\Xi z^{j+1}\Upsilon_2^d & 0 & z-1 \end{bmatrix} \tag{A.2}$$

where $\Upsilon_1^d = \omega$, $\Upsilon_2^d = 1-\omega$ for CPI inflation rules and $\Upsilon_1^d = 1$, $\Upsilon_2^d = 0$ for PPI inflation rules.

B The Policy Rules

Substituting out for outputs (89), the state-space representation (88) in deterministic form is:

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t \tag{B.1}$$

where z_t is an $(n-m)\times 1$ vector of predetermined variables including non-stationary processes, z_0 is given, $w_t = [r_t, r_t^*]^T$ is a vector of policy variables, x_t is an $m \times 1$ vector of non-predetermined variables and $x^e_{t+1,t}$ denotes rational (model consistent) expectations of x_{t+1} formed at time t. Then $x^e_{t+1,t} = x_{t+1}$ and letting $y_t^T = [z_t, x_t]^T$, (B.1) becomes

$$y_{t+1} = Ay_t + Bw_t \tag{B.2}$$

Define target variables s_t by

$$s_t = My_t + Hw_t \tag{B.3}$$

and the policymakers' loss function under cooperation at time t by

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty} \lambda^t [s_{t+i}^T Q_1 s_{t+i} + w_{t+i}^T Q_2 w_{t+i}] \tag{B.4}$$

which we rewrite as

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty} \lambda^t [y_{t+i}^T Q y_{t+i} Q y_{t+i} + 2y_{t+i}^T U w_{t+i} + w_{t+i}^T R w_{t+i}] \tag{B.5}$$

where $Q = M^T Q_1 M$, $U = M^T Q_1 H$, $R = Q_2 + H^T Q_1 H$, Q_1 and Q_2 are symmetric and non-negative definite R is required to be positive definite and $\lambda \in (0,1)$ is discount factor. The procedures for evaluating the three policy rules are outlined in the rest of this appendix (or Currie and Levine (1993) for a more detailed treatment).

B.1 The Optimal Policy: Cooperation with Commitment

Consider the policy-maker's *ex-ante* optimal policy at $t = 0$. This is found by minimizing Ω_0 given by (B.5) subject to (B.2) and (B.3) and given z_0. We proceed by defining the Hamiltonian

$$H_t(y_t, y_{t+1}, \mu_{t+1}) = \frac{1}{2}\lambda^t (y_t^T Q y_t + 2y_t^T U w_t + w_t^T R w_t) + \mu_{t+1}(Ay_t + Bw_t - y_{t+1}) \tag{B.6}$$

where μ_t is a row vector of costate variables. By standard Lagrange multiplier theory we minimize

$$L_0(y_0, y_1, \ldots, w_0, w_1, \ldots, \mu_1, \mu_2, \ldots) = \sum_{t=0}^{\infty} H_t \tag{B.7}$$

with respect to the arguments of L_0 (except z_0 which is given). Then at the optimum, $L_0 = \Omega_0$.

Redefining a new costate vector $p_t = \lambda^{-1}\mu_t^T$, the first-order conditions lead to

$$w_t = -R^{-1}(\lambda B^T p_{t+1} + U^T y_t) \tag{B.8}$$

$$\lambda A^T p_{t+1} - p_t = -(Qy_t + Uw_t) \tag{B.9}$$

Substituting (B.8) into (B.2)) we arrive at the following system under control

$$\begin{bmatrix} I & \lambda B R^{-1} B^T \\ 0 & \lambda(A^T - UR^{-1}B^T) \end{bmatrix} \begin{bmatrix} y_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} A - BR^{-1}U^T & 0 \\ -(Q - UR^{-1}U^T & I \end{bmatrix} \begin{bmatrix} y_t \\ p_t \end{bmatrix} \tag{B.10}$$

To complete the solution we require $2n$ boundary conditions for (B.10). Specifying z_0 gives us $n-m$ of these conditions. The remaining condition is the 'transversality condition'

$$\lim_{t\to\infty} \mu_t^T = \lim_{t\to\infty} \lambda^t p_t = 0 \tag{B.11}$$

and the initial condition

$$p_{20} = 0 \tag{B.12}$$

where $p_t^T = \left[\begin{array}{cc} p_{1t}^T & p_{2t}^T \end{array} \right]$ is partitioned so that p_{1t} is of dimension $(n-m) \times 1$. Equation (B.3), (B.8), (B.10) together with the $2n$ boundary conditions constitute the system under optimal control.

Solving the system under control leads to the following rule

$$w_t = -F \left[\begin{array}{cc} I & 0 \\ -N_{21} & -N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{B.13}$$

$$\left[\begin{array}{c} z_{t+1} \\ p_{2t+1} \end{array} \right] = \left[\begin{array}{cc} I & 0 \\ S_{21} & S_{22} \end{array} \right] G \left[\begin{array}{cc} I & 0 \\ -N_{21} & -N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{B.14}$$

$$N = \left[\begin{array}{cc} S_{11} - S_{12}S_{22}^{-1}S_{21} & S_{12}S_{22}^{-1} \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{array} \right] = \left[\begin{array}{cc} N_{11} & N_{12} \\ N_{21} & N_{22} \end{array} \right] \tag{B.15}$$

$$x_t = -\left[\begin{array}{cc} N_{21} & N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{B.16}$$

where $F = -(R + B^T SB)^{-1}(B^T SA + U^T), G = A - BF$ and

$$S = \left[\begin{array}{cc} S_{11} & S_{12} \\ S_{21} & S_{22} \end{array} \right] \tag{B.17}$$

partitioned so that S_{11} is $(n-m) \times (n-m)$ and S_{22} is $m \times m$ is the solution to the steady-state Ricatti equation

$$S = Q - UF - F^T U^T + F^T RF + \lambda(A - BF)^T S(A - BF) \tag{B.18}$$

The cost-to-go for the optimal policy (OP) at time t is

$$\Omega_t^{OP} = -\frac{1}{2}(\mathrm{tr}(N_{11}Z_t) + \mathrm{tr}(N_{22}p_{2t}p_{2t}^T)) \tag{B.19}$$

where $Z_t = z_t z_t^T$. To achieve optimality the policy-maker sets $p_{20} = 0$ at time $t = 0$. At time $t > 0$ there exists a gain from reneging by resetting $p_{2t} = 0$. It can be shown that $N_{22} < 0$, so the incentive to renege exists at all points along the trajectory of the optimal policy. This is the time-inconsistency problem.

B.2 Optimized Simple Rules

We now consider simple sub-optimal rules of the form

$$w_t = Dy_t = D \left[\begin{array}{c} z_t \\ x_t \end{array} \right] \tag{B.20}$$

where D is constrained to be sparse in some specified way. Rule (B.20) can be quite general. By augmenting the state vector in an appropriate way it can represent a PID (proportional-integral-derivative)controller (though the paper is restricted to a simple proportional controller only).

First consider the design of cooperative simple rules. Substituting (B.20) into (B.5) gives

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty} \lambda_t y_{t+i}^T P_{t+i} y_{t+i} \tag{B.21}$$

where $P = Q + UD + D^T U^T + D^T RD$. The system under control (B.1), with w_t given by (B.20), has a rational expectations solution with $x_t = -Nz_t$ where $N = N(D)$. Hence

$$y_t^T P y_t = z_t^T T z_t \tag{B.22}$$

where $T = P_{11} - N^T P_{21} - P_{12}N + N^T P_{22}N$, P is partitioned as for S in (B.17) onwards and

$$z_{t+1} = (G_{11} - G_{12}N)z_t \tag{B.23}$$

where $G = A + BD$ is partitioned as for P. Solving (B.23) we have

$$z_t = (G_{11} - G_{12}N)^t z_0 \tag{B.24}$$

Hence from (B.25), (B.22) and (B.24) we may write at time t

$$\Omega_t^{SIM} = \frac{1}{2} z_t^T V z_t = \frac{1}{2}\mathrm{tr}(VZ_t) \tag{B.25}$$

where $Z_t = z_t z_t^T$ and V satisfies the *Lyapunov* equation

$$V = T + H^T V H \tag{B.26}$$

where $H = G_{11} - G_{12}N$. At time $t = 0$ the optimized simple rule is then found by minimizing Ω_0 given by (B.25) with respect to the non-zero elements of D given z_0 using a standard numerical technique. An important feature of the result is that unlike the previous solution the optimal value of D is not independent of z_0. That is to say

$$D = D(z_0)$$

For the non-cooperative case, in a closed-loop Nash equilibrium we assume each policymaker chooses rules $w_t = Dy_t$ and $w_t^* = D^* y_t$ independently taking the rule of the other bloc as given. The equilibrium is then computed by iterating between the two countries until the solutions converge.

B.3 The Stochastic Case

Consider the stochastic generalization of (B.1)

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t + \begin{bmatrix} u_t \\ 0 \end{bmatrix} \tag{B.27}$$

where u_t is an $n \times 1$ vector of white noise disturbances independently distributed with $\mathrm{cov}(u_t) = \Sigma$. Then, it can be shown that certainty equivalence applies to all the policy rules apart from the simple rules (see Currie and Levine (1993)). The expected loss at time t is as before with quadratic terms of the form $z_t^T X z_t = \mathrm{tr}(X z_t, Z_t^T)$ replaced with

$$\mathcal{E}_t \left(\mathrm{tr} \left[X \left(z_t z_t^T + \sum_{i=1}^{\infty} \lambda^t u_{t+i} u_{t+i}^T \right) \right] \right) = \mathrm{tr} \left[X \left(z_t^T z_t + \frac{\lambda}{1-\lambda} \Sigma \right) \right] \tag{B.28}$$

where $\mathcal{E}_t$ is the expectations operator with expectations formed at time t.

Thus for the optimal policy with commitment (B.19) becomes in the stochastic case

$$\Omega_t^{OP} = -\frac{1}{2} \mathrm{tr} \left(N_{11} \left(Z_t + \frac{\lambda}{1-\lambda} \Sigma \right) + N_{22} p_{2t} p_{2t}^T \right) \tag{B.29}$$

For the simple rule, generalizing (B.25)

$$\Omega_t^{SIM} = -\frac{1}{2} \mathrm{tr} \left(V \left(Z_t + \frac{\lambda}{1-\lambda} \Sigma \right) \right) \tag{B.30}$$

The optimized cooperative simple rule is found at time $t = 0$ by minimizing Ω_0^{SIM} given by (B.30). Now we find that

$$D^* = D^* \left(z_0 + \frac{\lambda}{1-\lambda} \Sigma \right) \tag{B.31}$$

or, in other words, the optimized rule depends both on the initial displacement z_0 and on the covariance matrix of disturbances Σ. The non-cooperative rule for the stochastic case follows as before.