

# Trading Volume with Career Concerns

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## Abstract

This paper shows that trade can occur in a market where all traders are rational and none of them is subject to exogenous shocks. We develop a model of delegated portfolio management that captures key features of the US mutual fund industry and we embed it into an asset pricing set-up. Fund managers differ in their ability to understand market fundamentals. In equilibrium, the presence of career concerns induces uninformed fund managers to *churn*, i.e. to engage in trading even when they face a negative expected return. As churning plays the role of noise trading, the asset market displays non-fully informative prices and positive (and high) trading volume.

## 1 Introduction

Any attempt to model financial trading faces the mighty obstacle of no-trade theorems.<sup>1</sup> Under general conditions, the arrival of new private information cannot generate trade among rational traders. The intuition is related to Akerlof's lemons problem. A trader who shows willingness to buy (sell) a given asset signals that he has private information indicating that the asset is worth more (less) than its market price. In equilibrium, this adverse selection problem results in zero trading. To get around the no-trade issue, the finance literature, beginning with Grossman and Stiglitz [10], has assumed the presence of noise trading. Noise traders are agents who must sell or buy because something has changed in their personal situation. For instance

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<sup>1</sup>See Brunnermeier [1] for an overview of the topic and for further references.

they are compelled by unforeseen circumstances to generate or utilize liquidity (hence, noise traders are sometimes referred to as liquidity traders) or they need to buy particular securities to hedge against new risks. The presence of noise traders reduces adverse selection and it allows for trade not only by noise traders but also by informed speculators.

However, noise trading theories have come under increasing attack for their perceived inability to explain the order of magnitude of financial trade. In such theories, total trading volume bears a connection with the amount of noise trade. While no conclusive evidence is available, many scholars are reluctant to accept that the trading volumes observed on modern stock markets (over \$10 trillion in 2002 on the New York Stock Exchange) can be explained by the kind of exogenous events that drive noise trading (Glaser and Weber [8]). De Bondt and Thaler [2, p. 392] go as far as to say that the high trade volume observed in financial markets “is perhaps the single most embarrassing fact to the standard finance paradigm. One solution to the trading volume puzzle is to abandon the rational paradigm, for instance by allowing for overconfidence (e.g. Kyle and Wang [15] and Glaser or Weber [8]).

Another approach, pioneered by Dow and Gorton [3], consists of looking at delegated portfolio management. The potential explanatory power of such an approach is high because most trading activity on modern security markets is carried out by institutional investors who manage other people’s money. Dow and Gorton embed an agency problem between investors and their fund managers into an asset pricing model and show that under the optimal contract fund managers have an incentive to trade even when they have no private information (*churning*). Churning can be viewed as noise trading because it is trade which provides no information on fundamentals. However, in Dow and Gorton’s model no churning occurs unless there is some exogenously driven trading activity (in this case deriving from the desire to hedge against newly arisen risks). Thus, they show that the presence of agency issues amplifies exogenously-driven trade.

Our paper goes a step further. It provides a solution to the no-trade puzzle based on agency problems *alone*. We develop a model in which all traders are rational and they are not subject to any form of exogenous shock, and we show that the fact that some of these traders are investing other people’s money is sufficient to generate positive (and considerable) trading volumes. The present contribution is two-fold. From a theoretical standpoint, this paper shows that the presence of career concerns is enough to side-step the

no-trade results. From a practical standpoint, as our assumptions on delegated portfolio management capture key features of the US mutual fund industry, our model identifies a potential explanation for the high trading volumes that we observe on the stock market.

The starting assumption is that some investors use active fund management. We are thus placing a two-fold restriction on the behavior of a class of investor, which is worth spelling out: (i) those investors use delegated portfolio management rather than trading directly; (ii) they use active portfolio management rather than passive management (like index tracker funds). It is worth stressing that this is a behavioral assumption: acting as in (i) and (ii) may not be in the best interest of investors. We should therefore see whether it is consistent with observed behavior. In the United States part (i) applies to the vast majority of beneficiaries of 401(k) pension plans, in the sense that they *must* use delegated portfolio management.<sup>2</sup> In 2002, 401(k) plans accounted for over \$1 trillion of investment in mutual funds (ICI [13]). This is also relevant for firm-sponsored defined benefit-pension plans, which are usually required by the pension covenant to outsource asset management. The second part of the assumption, (ii), is more problematic. Indeed, there is substantial empirical evidence (see, for example, Gruber [11] or Wermers [21]) that actively managed funds underperform index funds after accounting for expenses. Nevertheless, paradoxically, managed funds remain a popular choice amongst investors. In a recent survey, The Economist [4] concludes that in the U.S. only 12% of individual investors' delegated funds are in index funds, and the proportion is even smaller (2%) in Europe. We do not attempt to provide a solution to the "active management puzzle": we take it as a starting point to suggest a solution to the "trading volume puzzle". We show that the presence of active management is sufficient by itself to explain high trade volumes. In other words, the active management puzzle solves the trading volume puzzle.

In our model there are three classes of agents: investors who cannot

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<sup>2</sup>401(k) plans are the most common form of employer-sponsored defined contribution retirement schemes in the US. In theory, the employer can allow the beneficiary to use the plan money to buy stocks directly through brokers (self-directed investment). In practice, employers have a legal fiduciary duty with regards to 401(k) plans and they are concerned about the litigation potential of allowing employees access to powerful and complex financial instruments. For this reason, 86% of 401(k) schemes offer no self-direct option (Morgan Stanley [16]) and those who do tend to impose an upper limit to the share of funds that the beneficiaries can put in self-directed account.

trade directly (we refer to them as *investors*), traders who trade on behalf of investors (*fund managers*), and other investors who trade directly (we refer to them as *traders*). It is a dynamic model: in every period the investors select among available fund managers. Fund managers face career concerns, which are the driving force behind our results.

In the baseline model, the form of the payment from the investor to the fund manager is exogenously given and it does not depend directly on performance. First, this assumption applies by and large to US mutual funds because of legal restrictions on incentive fees.<sup>3</sup> Second, this allows us to make our main points in a simple, tractable model. Later in the paper we show that the results are still valid in an environment with endogenous contracting, as long as only short-term contracts are feasible.

There are two periods. In each period there is a market for a risky asset, which is liquidated at the end of the period. In the beginning of the first period, investors entrust a fund manager with a sum of money. The fund manager trades on their behalf, and at the end of the period the investors observe the trade and the liquidation value. At the beginning of the second period the investors can choose to retain the current fund manager or to replace him with a new one. Again, the fund manager trades on behalf of the investors.<sup>4</sup>

Fund managers are characterized by their ability to observe market fundamentals. A good fund manager is more likely to learn the liquidation value of the asset before the asset is liquidated. In equilibrium investors can attempt to infer the ability of their fund manager from the trades he has made and the outcome of the trades.

The rest of the market is made of a large number of uninformed traders. Each trader posts a bid price and an ask price. As traders are rational, there may be an endogenous bid-ask spread to account for adverse selection. In the baseline case, we make a simplifying assumption: each trader is short-lived and does not know what happened in the past (in particular, they do not

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<sup>3</sup>Performance payments are allowed only if they are symmetric (*fulcrum fees*), i.e. the positive bonus for high performance must be matched by a punishment of equal size in case of bad performance. For this reason, only 1.7% of mutual funds have some sort of the performance fee. All the others charge only a fixed fee (Elton et al. [5]).

<sup>4</sup>We interpret the firing of an existing fund managers and the hiring of a new one as a switch by the investor from one managed fund to another. The connection between career concerns and portfolio performance is well documented in the US mutual fund industry (Ellison and Chevalier [6]).

know if they find themselves in the first or in the second period). Later in the paper, we show that this assumption is not necessary if one considers a more complex model with overlapping generations of fund managers.

The main findings are:

1. *Without career concerns, there is no trading.* As a benchmark case, suppose that the fund manager has no career concerns (because the decision to replace him or retain him is exogenous). Then there is no equilibrium in which trade occurs. This benchmark case confirms the no-trade result in absence of career concerns.
2. *With career concerns, there is trading.* If the decision to replace or retain the fund manager is endogenous, there exists a *churning equilibrium* in which a young manager always trades. If he is informed, he trades correctly. If not, he randomizes among possible trades. If an uninformed young manager does not churn, he signals his lack of information and he gets replaced in the following period.  
From the viewpoint of the rest of the asset market, churners play the role of noise traders because their orders are not correlated with fundamentals. They lessen the adverse selection problem for informed traders, who now have opportunities for profitable trade. This closes the circle because ex post the investor has a strict incentive to retain a successful trader. The investor's gross expected return is zero because the profits of informed fund managers exactly offset the losses of churners.
3. *Trade volume is high.* In the churning equilibrium, trading volume is not only positive but also large: all young fund managers and all informed old managers engage in trading. The probability that an average manager trades goes from 50% if the average type of managers is lowest to 100% if the average type is highest.
4. *Trading equilibria exist as long as the contractual environment is not sufficiently rich to avoid career concerns.* If investors and fund managers can sign long-term (multi-period) contracts, churning disappears and trading volume is zero. This is because the investor can commit not to replace a bad manager, which kills career concerns. However, the churning equilibrium still exists when contracts are endogenous but only short-term contracting is available. Short-term contracts allow for

payment contingent on current performance but not payments contingent on future performance or on the choice to retain the manager. The fund manager is replaced if he underperforms, and this is sufficient to create career concerns and hence churning equilibria. In practice, the empirical findings of Ellison and Chevalier [6] suggest that the contractual arrangements that are in place in the US mutual fund industry generate substantial career concerns.

Besides the already discussed work by Dow and Gorton [3], the present paper is related to a path-breaking paper by Trueman [20]. He considers a delegated portfolio management model in which the fund manager's ability is unknown. Compensation depends on performance and on the posterior on the fund's manager ability. Trueman shows that there is a churning equilibrium in which uninformed fund managers trade. Our paper differs in two respects. First, Trueman assumes that the fund manager's future compensation depends on his posterior in an exogenously given way. Instead in our model, future compensation depends on the investor's retention decision, which is endogenous. Second, Trueman considers a partial equilibrium model (and therefore he cannot discuss trade volume) while we also take into account the feedback that the fund manager's trade has on the asset market.

This paper is inspired by the burgeoning literature on career concerns for experts (e.g. Scharfstein and Stein [19], Prendergast and Stole [18], and Ottaviani and Sørensen [17]). An expert is an agent whose type determines his ability to understand the state of the world. This differs from "classical career concerns" (Holmstrom [12]) in which the agent's type determines his ability to exert effort. Expert models are particularly suited to analyze agency relationships in financial setups, in which the key driver appears to be the ability to pick the right portfolio rather than pure effort exertion. Some expert models have been used to explain incentives to exaggerate private information (which can be linked to excessive trading). For instance, Prendergast and Stole [18] show that early in their career experts have an incentive to report a signal that is more extreme than the one which they have actually received. However, to the best of our knowledge no career concern models has been embedded in a financial market equilibrium setting.<sup>5</sup>

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<sup>5</sup>Scharfstein and Stein [19] develop a career concerns setup with multiple experts. Experts have an incentive to mimic the investment decision of other experts, irrespective of their private information. Scharfstein and Stein study a partial equilibrium setting. It would be interesting – but outside the scope of the present work – to extend their setup

The plan of the paper is as follows. The following section develops the simplest model which is sufficient to generate our main results. Section 3 extends the baseline model in various directions: existence of other equilibria besides the churning equilibrium; effects of endogenous contracting; infinite-horizon model; and positive net returns. Section 4 concludes.

## 2 The Baseline Case

To present the essence of our results, we begin by discussing a simple baseline model. The main assumptions are: (1) investors must use active management; (2) contracts are exogenously given; (3) there are only two periods and traders do not know in which period they are. Assumption (1) is essential to the whole paper, and it was discussed in the introduction. Assumptions (2) and (3) are made for analytical convenience and are relaxed in Section 3.

### 2.1 Model

Consider an economy with two periods,  $t = 1, 2$ . There is a single risk-neutral principal (investor) and a large pool of ex-ante identical risk-neutral agents (fund-managers). One of these is hired at random at  $t = 1$  to trade for the principal. At the end of the period, the principal may retain the agent or hire a new one of average quality from the pool. The agent can be of two types:  $\theta \in \{b, g\}$  with probabilities  $1 - \gamma$  and  $\gamma$  respectively. The type of the agent is unknown to both the principal and the agent.

At each time period  $t$ , there is exactly one risky asset with payoff  $v \in \{0, 1\}$  which occur with equal probability. Asset payoffs are independent across periods.  $v$  is realized at the end of each period and becomes publicly known. The agent's type  $\theta$  and the asset payoff  $v$  are independent.

At the end of each period, the principal can observe the action taken by the agent, as well as the (publicly observed) value of  $v$ . After such observation, the principal decides whether to retain or fire the fund manager.

There are a large number of risk-neutral short-lived uninformed rational traders who act as market-makers. Half of them operate in  $t = 1$ , the other half operate in  $t = 2$ . In each period  $t$ , the fund manager submits a market order  $a \in A = \{0, \emptyset, 1\}$ , where 0 stands for "sell one unit at highest available price", 1 stands for "buy one unit at lowest available price", and  $\emptyset$  represents

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to encompass the feedback from investment decisions into prices.

lack of activity. The traders observe the order and each of them announces a price. Thus, the action of a trader consists of setting two prices: an ask price  $p_A \in \left[\frac{1}{2}, 1\right]$  for  $a = 1$  and a bid price  $p_B \in \left[0, \frac{1}{2}\right]$  for  $a = 0$ . The bid-ask spread  $p_A - p_B$  may be positive. The fund manager is free to trade with any rational trader, and, when indifferent between them, chooses one at random. The traders are thus subject to Bertrand competition (as is now standard in the literature, following Glosten and Milgrom [9] and Kyle [14]) and each sets prices equal to the expected value of the asset conditional on the order.

We make one important simplifying assumption. Traders do not know whether they are in period 1 or 2. Therefore, they are unable to condition their action on the fund manager's seniority. In section 3 we show that this assumption is not necessary if we consider an infinite-horizon model of overlapping generations of fund managers.

The fund manager's information structure is common to both periods and it depends on the fund manager's type. A good fund manager receives a signal conveying the true liquidation value  $v$ , while a bad fund manager receives no signal. The signal  $s$  can take three values, 0, 1, and  $\emptyset$ , and it is determined as follows:

$$s(\theta, v) = \begin{cases} v & \text{if } \theta = g \\ \emptyset & \text{if } \theta = b \end{cases}$$

In the present setup,  $s$  reveals  $\theta$ . When the fund manager learns his signal he also learns his type. Instead, the investor does not observe either the signal or the type.

The fund manager incurs a cost of trading  $\epsilon > 0$  every time he buys or sells. The cost is introduced in order to break the indifference between trading and not trading in favor of the latter. Most of the results we present are obtained for the asymptotic case  $\epsilon \rightarrow 0$ .

In each period, the net return on investment is

$$\chi(a, p_A, p_B, v, \epsilon) = \begin{cases} 0 & \text{if } a = \emptyset \\ v - p_A - \epsilon & \text{if } a = 1 \\ p_B - v - \epsilon & \text{if } a = 0 \end{cases}$$

Write a time- $t$  mixed strategy for an agent as the mapping  $a_t : S \rightarrow \Delta A$ .

In this baseline version of the model, the contractual arrangement between the investor and the fund manager is exogenously determined. Given return  $\chi$ , the payment from the investor to the manager is

$$t = \alpha\chi(a, p, v) + \beta,$$



where  $\alpha \in (0, 1)$  and  $\beta \in (0, \infty)$ . In most of the results of the present section we focus on the case  $\alpha \rightarrow 0$ . However, the fact that  $\alpha > 0$  guarantees that when career concerns are absent, the interests of the fund manager are aligned with those of the investor.

For simplicity there is no discounting. The investor's payoff is given by  $\chi_1 - t_1 + \chi_2 - t_2$ . The fund manager's payoff is  $t_1 + t_2$ .

To summarize, timing is:

- $t = 1$ 
  - The fund manager learns  $s_1$  and chooses  $a_1$ ;
  - Traders observe  $a_1$  and set price;
  - The liquidation value  $v_1$  is observed by everyone; Payments to the fund manager are made.
- $t = 2$ 
  - The investor retains the incumbent or hires the challenger.
  - The fund manager learns  $s_2$  and chooses  $a_2$ ;
  - Traders observe  $a_2$  and set price;
  - The liquidation value  $v_2$  is observed by everyone; Payments to the fund manager are made.

## 2.2 No Trade without Career Concerns

We first establish the benchmark case without career concerns. Career concerns arise when the fund manager knows that his chance of being replaced depends on his behavior. Instead, to eliminate the career component, assume that the probability that the first period fund manager is kept is exogenously given by  $r \in [0, 1]$ . The event that the manager is kept is independent from any other variable in the model.

**Proposition 1** *For any exogenous  $r \in [0, 1]$ , there is no trade in equilibrium.*

**Proof.** For a fund manager with  $\theta = b$ , the expected value of the asset is  $E[v] = \frac{1}{2}$ . The bad fund manager never buys because

$$E[v] - \epsilon < \frac{1}{2} \leq p_A.$$

The good manager is willing to buy if

$$p_A \leq v - \epsilon.$$

If the good fund manager buys, it means that he knows  $v = 1$ . But then the ask price should be  $p_A = 1$ , which is a contradiction. An analogous contradiction arises when we consider selling instead of buying. ■

This result is a no-trade theorem. In the absence of career concerns, our model does not support positive trade volumes. When fund managers are not career-motivated, they trade optimally. Uninformed traders realize that because of adverse selection they can only lose from trading with fund managers. Trade cannot occur in equilibrium.

If the exogenous retention rate  $r$  is set to one, Proposition 1 can be interpreted as a situation in which the investor and the fund manager are the same person: an informed investor. We thus have an application of the no-trade theorem to our setting.

### 2.3 Positive Trading Volume with Career Concerns

Now we demonstrate that if instead the principal is forced to trade through an agent using contracts of the form specified above, there can be positive trading volume in equilibrium. Such trading activity occurs in the absence of any exogenous demand or supply shocks (noise trading) and occurs purely due to career concerns on the part of agents employed by the principal.

**Proposition 2** *For  $\alpha$  and  $\epsilon$  low enough, there exists an equilibrium in which.*<sup>6</sup>

1. *The investor retains only fund managers who have traded correctly.*
2. *A good fund manager always trades. A bad fund manager churns if  $t = 1$  and he does not trade if  $t = 2$ :*

$$\begin{aligned} a_t(s) &= s \text{ for } t = 1, 2 \\ a_1(\emptyset) &= \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases} \\ a_2(\emptyset) &= \emptyset \end{aligned}$$

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<sup>6</sup>Namely, the equilibrium exists if:

$$\alpha \leq \beta \text{ and } \epsilon < \frac{1}{2}(1 - \hat{\gamma}).$$

3. *Traders set prices:*

$$\hat{p}_A = \frac{1}{2}(1 + \hat{\gamma}) \quad \text{and} \quad \hat{p}_B = \frac{1}{2}(1 - \hat{\gamma})$$

where

$$\hat{\gamma} = \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2}.$$

**Proof.** *Fund manager's strategy at  $t = 2$ :* At  $t = 2$ , a bad manager never trades because  $\hat{p}_A > \frac{1}{2}$  and  $\hat{p}_B < \frac{1}{2}$ . A good fund manager with signal  $s = 1$  is strictly better off buying if  $1 - \hat{p}_A - \epsilon > 0$ , which is satisfied if

$$\epsilon < 1 - \hat{p}_A = \frac{1}{2}(1 - \hat{\gamma}) \equiv \hat{\epsilon} \tag{1}$$

A good fund manager with  $s = 0$  is better off selling if  $\hat{p}_B - \epsilon > 0$ , which is also satisfied under (1).

*Investor's belief:* After  $a$  and  $v$  are observed, the restrictions on the investor's posterior imposed by the requirement that beliefs are consistent are:

$$\Pr(\theta = g|a, v) \begin{cases} = 0 & \text{if } a = 1 - v \\ = \frac{\gamma}{\gamma + \frac{1}{2}(1 - \gamma)} = \frac{2\gamma}{\gamma + 1} & \text{if } a = v \\ \in [0, 1] & \text{if } a = \emptyset \end{cases}$$

The action  $a = \emptyset$  is off the equilibrium path at  $t = 1$ . Perfect Bayesian equilibrium imposes no restriction. We choose to set<sup>7</sup>

$$\Pr(\theta = g|a = \emptyset) = 0.$$

*Investor's retaining strategy:* Suppose (1) holds. A good fund manager generates a positive net return while a bad fund manager generates a zero net return. It is a best response for the investor to retain the fund manager if and only if

$$\Pr(\theta = g|a, v) \geq \gamma.$$

Combined with the posteriors above, this condition implies that the investor retains the fund manager if and only if the fund manager trades successfully.

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<sup>7</sup>The proof also works with any  $\Pr(\theta = g|a = \emptyset) \in (0, \gamma)$ .

*Good fund manager's strategy at  $t = 1$ :* A good fund manager who plays  $a = s$  produces net return

$$1 - \hat{p}_A - \epsilon = \hat{p}_B - \epsilon = \frac{1}{2}(1 - \hat{\gamma}) - \epsilon$$

in  $t = 1$ . He is retained and he produces again net return  $\frac{1}{2}(1 - \hat{\gamma}) - \epsilon$  in  $t = 2$ . His total payoff is

$$\pi_g(a = s) = 2 \left( \alpha \left( \frac{1}{2}(1 - \hat{\gamma}) - \epsilon \right) + \beta \right).$$

If  $\epsilon$  is small enough to satisfy (1), it is easy to see that  $\pi_g(a = s)$  is higher than the payoff that the fund manager would get if he plays  $a = \emptyset$ . It is also obvious that  $\pi_g(a = s)$  is higher than the payoff the manager would get if he plays  $a = 1 - s$ .

*Bad fund manager's strategy at  $t = 1$ :* A bad fund manager who does not trade generates a zero net return in  $t = 1$  and he is not retained. Therefore, his total payoff is

$$\pi_b(a = \emptyset) = \beta.$$

If instead the bad manager plays either  $a = 1$  or  $a = 0$  in  $t = 1$ , he is successful with probability  $\frac{1}{2}$ . His expected net return at  $t = 1$  is

$$\frac{1}{2} - \hat{p}_A - \epsilon = -\frac{1}{2} + \hat{p}_B - \epsilon = -\frac{\hat{\gamma}}{2} - \epsilon$$

If the churner is successful, he is retained and he does not trade at  $t = 2$ . His total expected payoff is

$$\pi_b(a \in \{0, 1\}) = \alpha \left( -\frac{\hat{\gamma}}{2} - \epsilon \right) + \beta + \frac{1}{2}\beta.$$

It is a best response to churn if

$$\alpha \leq \frac{\frac{1}{2}\beta}{\frac{\hat{\gamma}}{2} + \epsilon}.$$

As  $\epsilon$  is bounded above by (1), a sufficient condition for churning is

$$\alpha \leq \frac{\frac{1}{2}\beta}{\frac{\hat{\gamma}}{2} + \hat{\epsilon}} = \frac{\frac{1}{2}\beta}{\frac{\hat{\gamma}}{2} + \frac{1}{2}(1 - \hat{\gamma})} = \beta.$$

*Traders' pricing strategy:* The probability that the second-period fund manager is good is equal to the probability that the manager is good in  $t = 1$  (because he is retained for sure) plus the probability that the manager is bad and he is replaced with a good one:

$$\Pr(\theta = g|t = 2) = \gamma + \frac{1}{2}(1 - \gamma)\gamma.$$

Second-period managers trade only if they are good. First-period managers always trade, and churning randomize with equal probability between buying and selling. Thus, by symmetry,

$$\Pr(\theta = g|a = 1) = \Pr(\theta = g|a = 0) = \Pr(\theta = g|a \in \{0, 1\}).$$

A trader who receives a buy or sell order computes the following posterior

$$\begin{aligned} & \hat{\gamma} \\ \equiv & \Pr(\theta = g|a \in \{0, 1\}) \\ = & \frac{\Pr(a \in \{0, 1\}, \theta = g)}{\Pr(a \in \{0, 1\})} \\ = & \frac{\Pr(a \in \{0, 1\}, \theta = g, t = 1) + \Pr(a \in \{0, 1\}, \theta = g, t = 2)}{\Pr(a \in \{0, 1\}, t = 1) + \Pr(a \in \{0, 1\}, t = 2)} \\ = & \frac{\left[ \Pr(a \in \{0, 1\} | \theta = g, t = 1) \Pr(\theta = g|t = 1) + \Pr(a \in \{0, 1\} | \theta = g, t = 2) \Pr(\theta = g|t = 2) \right]}{\Pr(a \in \{0, 1\} | t = 1) + \Pr(a \in \{0, 1\} | t = 2)} \\ = & \frac{\Pr(\theta = g|t = 1) + \Pr(\theta = g|t = 2)}{1 + \Pr(\theta = g|t = 2)} \\ = & \frac{\gamma + \gamma + \frac{1}{2}(1 - \gamma)\gamma}{1 + \gamma + \frac{1}{2}(1 - \gamma)\gamma} \\ = & \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2}. \end{aligned}$$

The ask price is

$$\begin{aligned} \hat{p}_A &= \Pr(\theta = g|a \in \{0, 1\}) + \Pr(\theta = b|a \in \{0, 1\}) \frac{1}{2} \\ &= \frac{1}{2} + \frac{1}{2} \Pr(\theta = g|a \in \{0, 1\}) \\ &= \frac{1}{2}(1 + \hat{\gamma}), \end{aligned}$$

and the bid price is

$$\begin{aligned}
\hat{p}_B &= \Pr(\theta = g|a \in \{0, 1\})0 + \Pr(\theta = b|a \in \{0, 1\})\frac{1}{2} \\
&= \frac{1}{2}(1 - \Pr(\theta = b|a \in \{0, 1\})) \\
&= \frac{1}{2}(1 - \hat{\gamma}).
\end{aligned}$$

■

Proposition 2 identifies a churning equilibrium. All first-period fund managers trade. The good ones make correct trade by following their private information. The bad ones randomize between buying and selling.

The investor realizes that a successful trade may come from a lucky churner. Still, she knows that a good manager is more likely to be right and she revises her posterior upwards if she observes a successful trade. She also knows that a wrong trade can only come from a bad manager, and she believes that no-trade (an off-equilibrium event) is more likely to come from a bad manager. Given this set of beliefs, the investor retains the first-period manager if and only if he has traded successfully.

A good manager makes positive returns in both periods (provided the trading cost is low enough). He knows the liquidation value and he buys or sells at prices that are strictly between 0 and 1. He is also certain to be retained.

A bad manager faces a depressing choice between churning and non-trading. If he churns, he makes negative expected return  $-\frac{\hat{\gamma}}{2} - \epsilon$  but he has a 50% of being retained. If he does not trade, he makes a zero return and he is fired for sure. If the direct-stake parameter  $\alpha$  is low enough (in particular, lower than the fixed payment), the bad manager prefers to churn.

Traders know that a market order may come from a good manager who knows the liquidation value or a first-period bad manager who is churning. The price will be based on the probability that the order comes from a good manager conditional on the presence of an order, which is

$$\hat{\gamma} = \Pr(\theta = g|a \in \{0, 1\}) = \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2} \in (0, 1).$$

One can check that the posterior  $\hat{\gamma}$  is greater than the prior  $\gamma$ . This is due to two factors. First, a good manager is more likely than a bad manager to be retained. Second, a bad manager does not trade in the second period.

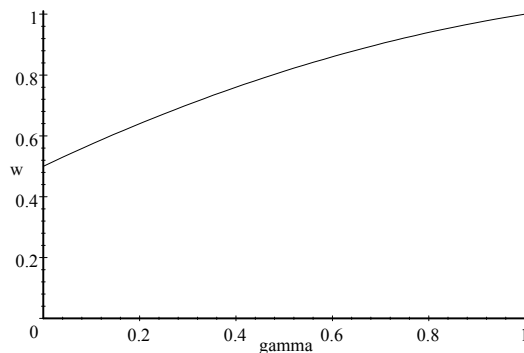
However,  $\hat{\gamma} \rightarrow \gamma$  when either  $\gamma \rightarrow 0$  or  $\gamma \rightarrow 1$ . Given the posterior  $\hat{\gamma}$ , traders compute the bid price and the ask price. The bid-ask spread is simply  $\hat{\gamma}$  and it is increasing in  $\gamma$ . It tends to 1 when  $\gamma \rightarrow 1$  and it tends to zero when  $\gamma \rightarrow 0$ .

Trading volume is the expected number of assets traded. It can be defined as the average of the probability that a trade occurs at  $t = 1$  and the probability that a trade occurs at  $t = 2$ . From Proposition 2, we can easily compute trading volume in the churning equilibrium:

**Corollary 3** *The average trading volume is*

$$w = \frac{1 + \Pr(\theta = g|t = 2)}{2} = \frac{2 + 3\gamma - \gamma^2}{4}.$$

In the first period, there is always trade. In the second period, trade occurs only if the manager is good. Trading volume  $w$  is graphed below



Trading is at its lowest when almost all managers are bad. Still, the presence of churning guarantees that trading volume is always above  $\frac{1}{2}$ .

What are expected payoffs in this equilibrium? As the expected payoff of traders is zero, the gross expected return over the two periods must be zero as well. The expected net return, which is negative, is thus the expected number of trades  $2w$  times the trading cost:  $E(\chi_1 + \chi_2) = -2\epsilon w$ . The expected payment from the investor to the fund manager over the two periods is:

$$E(t_1 + t_2) = E(\alpha\chi_1 + \beta + \alpha\chi_2 + \beta) = 2(-\alpha\epsilon w + \beta).$$

The investor's expected payoff is:

$$E(\chi_1 - t_1 + \chi_2 - t_2) = -2((1 - \alpha)\epsilon w + \beta).$$

The investor's expected payoff is negative (but it becomes arbitrarily close to zero if  $\alpha$ ,  $\beta$ , and  $\epsilon$  tend to zero). The investor is hurt by using active portfolio management. He would be better off if he could commit to a no-trading strategy, which in our setting can be interpreted as using passive portfolio management. As discussed in the introduction, our paper builds on the active management puzzle. If, for some unmodeled reason, investors use active management even when it is against their interest (as they appear to do in reality), then trading can occur and it can be high even if there is no exogenously driven trade.

### 3 Discussion and Extensions

#### 3.1 The Agency Relationship

In this model we embed an agency relationship into an asset pricing framework. It is useful to isolate the two features of the model. To make this point transparent, we fix some arbitrary *interior* bid-ask prices (where the bid-ask spread is strictly less than 1) and consider only the strategies of the investor and the fund manager, and show that the presence of career concerns prevents the fund manager from trading optimally. We say that *the agency relationship has a sincere equilibrium* if – holding prices constant – there is a perfect Bayesian equilibrium in which the fund manager buys when  $v = 1$ , sells when  $v = 0$ , and does not trade when he is uninformed.

**Proposition 4 (non-existence of sincere equilibria)** *Fix any interior bid-ask prices  $p_A \in [\frac{1}{2}, 1)$  and  $p_B \in (0, \frac{1}{2}]$ . There exists an  $\hat{\alpha} > 0$  such that for all  $\alpha < \hat{\alpha}$  there exists an  $\hat{\epsilon}(\alpha)$  such that for all  $\epsilon < \hat{\epsilon}(\alpha)$  the agency relationship has no sincere equilibrium.*

**Proof.** Suppose a sincere equilibrium exists. For any interior bid-ask prices, there exists an  $\epsilon$  small enough that in the second period a good manager always trades and always generates a positive return. Instead, in the second period the bad fund manager never trades and generates zero expected return. If the investor knows the type of the first-period fund manager, she keeps him if and only if he is good. If the fund manager plays sincerely, his trade reveals his type. Thus, in a sincere equilibrium at  $t = 1$  the bad fund manager does not trade and he is replaced for sure. The payoff of a bad fund manager is thus  $\beta$ .



However, consider a possible deviation. At  $t = 1$  the bad fund manager churns: for instance he selects  $a = 1$ . The expected net return at  $t = 1$  is  $\frac{1}{2} - p_A - \epsilon$ , which is negative. However, with probability  $\frac{1}{2}$  the trade is correct, and the investor believes that  $\theta = g$  and she keeps the fund manager. This generates an expected benefit of at least  $\frac{1}{2}\beta$  (we have not specified what happens off the equilibrium path when the fund manager trades wrongly). The expected net benefit of the deviation is

$$\alpha \left( \frac{1}{2} - p_A - \epsilon \right) + \frac{1}{2}\beta,$$

which is strictly positive for  $\alpha$  sufficiently low. ■

In a sincere equilibrium the good fund manager trades and the bad fund manager does not. From the presence of trade, the investor infers the manager's type perfectly and she keeps only a good manager. But this creates a strong incentive for the bad fund manager to churn. If he trades and he is lucky, the bad fund manager appears good in the eyes of the investor and he is allowed to stay. If the direct stake in the return,  $\alpha$ , is low enough, churning is a profitable deviation and the sincere equilibrium does not exist.

Thus, as long as the bid-ask prices are interior (effectively, therefore, not strong form efficient) the presence of the agency relationship eliminates the possibility of sincere (optimal) trading by the fund manager. But such lack of sincerity simultaneously creates the possibility for prices to be interior: since managers sometimes churn, the market makers have an incentive to move prices (at least slightly) less in response to orders than in the case where actions fully reveal information. This is the underlying intuition behind the main result above.

However, it is clear that there may be other equilibria in the model, including ones in which there is no trade. In the appendix, we characterize some general properties of the equilibrium set of this game. We show that there are three types of equilibria: ones in which all agents trade, ones in which only good agents trade (but perversely, at a loss in period 1), and ones in which nobody trades. However, we also show that both the latter two classes of equilibria are “knife-edge” in the following sense: even if there is an arbitrarily small proportion of agents who are “trade lovers” (i.e., trade correctly when they have information, and churn when they do not), then all managers trade in the first period.

## 3.2 Endogenous Contracts

The baseline model postulates exogenously given linear contracts. We now remove this assumption and we consider endogenous contracting between the investor and the fund manager(s).

First, consider the following class of *long-term contracting*. A contract specifies the payment from the investor to the agent and a rule for retaining or replacing the fund manager. The payment can be contingent on all observables: trade  $a_t$  and liquidation value  $v_t$  at  $t = 1, 2$ . If the investor replaces the manager, she and the new manager agree on a new contract on the observables at  $t = 2$ .

The investor has the bargaining power: he makes a take-it-or-leave-it offer to the fund manager. To make things interesting, it is useful to assume that the fund manager must receive a minimum non-negative payment  $\bar{w}$  if he is employed (for every period he is employed). If this were not the case, the investor would just offer a zero payment in both periods and the fund manager would be entirely indifferent (and therefore he might as well behave optimally). As  $\bar{w} > 0$ , we can disregard the fund manager's participation constraint.<sup>8</sup>

Traders do not observe the contract signed by the investor and the fund manager. We can now prove the following:

**Proposition 5 (Long term contracts)** *With long-term contracting, trading volume is zero.*

**Proof.** Suppose there is an equilibrium with trading volume  $t > 0$ . In such equilibrium, the expected net return is  $-t\epsilon$ . The investor's expected payoff is bounded above by  $-2\bar{w} - t\epsilon$ . But the investor can always deviate to a different contract in which she offers a fix payment  $\bar{w}$  to the fund manager in each period, a positive amount  $\delta$  if the investor does not trade, and she commits not to replace him. Then, the trading volume is zero and the investor's expected payoff is  $-2\bar{w} - 2\delta$ . As  $\delta$  can be as small as we wish, this deviation is profitable. ■

The result relies on the investor's ability to commit to retain the current fund manager (or to replace him for sure). This kills off career concerns and therefore churning. Positive trading volumes cannot be supported in equilibrium.

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<sup>8</sup>The alternative is to assume that the bargaining power rests with the fund manager. But that would create a hard informed-principal problem.

We now move on to *short-term contracting*. The environment is the one above except that there are two short-lived investors. Investor 1 offers a contract  $b_1$  to the fund manager in the beginning of the first-period and she receives payoff  $\chi_1 - t_1$ . The payment between investor 1 and the manager may depend on all the observable at  $t = 1$  (but it cannot depend on what happens in  $t = 2$ ). Investor 2 is born in the beginning of the second period and she observes the outcome of the previous period: the trade  $a_1$  and the liquidation value  $v_1$  (she does not observe the contract used in the previous period). She then chooses between the incumbent manager and the challenger and he selects a contract  $b_2$ .

Given that the symmetry of the problem, we simplify analysis by assuming that contracts are symmetric too. The payment satisfy:

$$\begin{aligned} b_t(a_t = 1 = v_t) &= b_t(a_t = 0 = v_t) \equiv b_t(\text{success}) \\ b_t(a_t = 1 \neq v_t) &= b_t(a_t = 0 \neq v_t) \equiv b_t(\text{failure}) \\ b_t(a_t = \emptyset, v_t = 1) &= b_t(a_t = \emptyset, v_t = 0) \equiv b_t(\text{no trade}) \end{aligned}$$

We can then write a contract as a triple:

$$b_t = (b_t(\text{success}), b_t(\text{failure}), b_t(\text{no trade})),$$

under the constraint – discussed above – that all three values are not below  $\bar{w}$ .

- $t = 1$ 
  - Investor 1 specifies contract  $b_1$ ;
  - The fund manager learns  $s_1$  and chooses  $a_1$ ;
  - Traders observe  $a_1$  and set price;
  - The liquidation value  $v_1$  is observed by everyone; Payments to the fund manager are made.
- $t = 2$ 
  - Investor 2 observes  $a_1$  and  $v_1$ . She retains the incumbent or hires the challenger. She specifies contract  $b_2$ ;
  - The fund manager learns  $s_2$  and chooses  $a_2$ ;
  - Traders observe  $a_2$  and set price;
  - The liquidation value  $v_2$  is observed by everyone; Payments to the fund manager are made.

We show the following:

**Proposition 6 (Churning with endogenous short-term contracts)** *For any  $\bar{w} > 0$ , if the proportion of good traders  $\gamma$  and trading cost  $\epsilon$  are low enough, there exists a churning equilibrium in which:*

1. *Investor 1 selects a flat contract  $b_1 = (\bar{w}, \bar{w}, \bar{w})$ .*
2. *Investor 2 retains the fund manager if and only if he traded successfully. In either case, the investor selects a flat contract  $b_2 = (\bar{w}, \bar{w}, \bar{w})$ .*
3. *A good fund manager always trades. A bad fund manager churns if  $t = 1$  and he does not trade if  $t = 2$ .*
4. *Traders set prices:*

$$\hat{p}_A = \frac{1}{2}(1 + \hat{\gamma}) \quad \text{and} \quad \hat{p}_B = \frac{1}{2}(1 - \hat{\gamma}).$$

where

$$\hat{\gamma} = \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2}.$$

**Proof.** *Fund manager's trading strategy at  $t = 2$ .* The fund manager has no career concerns. If offered a flat payment, she is indifferent among trading or not trading. Hence, we can assume that a good manager trades successfully and a bad manager does not trade.

*Investor 2's contract choice.* As the fund manager is indifferent, the investor can obtain optimal behavior by offering a flat contract  $b_2 = (\bar{w}, \bar{w}, \bar{w})$ , which is clearly optimal.

*Investor 2's hiring choice.* The return at  $t = 2$  is  $\frac{1}{2}(1 - \hat{\gamma})$  if the manager is good and 0 if the manager is bad. Given a belief  $\Pr(\theta = g|a_1, v_1)$  on the incumbent's type, the expected net return from retaining the incumbent is

$$\Pr(\theta = g|a_1, v_1) \left( \frac{1}{2}(1 - \hat{\gamma}) - \epsilon \right),$$

while the expected net return from hiring the challenger is

$$\gamma \left( \frac{1}{2}(1 - \hat{\gamma}) - \epsilon \right).$$

It is a best response for the investor to retain the incumbent if and only if  $\Pr(\theta = g|a_1, v_1) \geq \gamma$ .

*Investor 2's belief.* The belief is the same of the churning equilibrium of Proposition 2, namely:

$$\Pr(\theta = g|a_1, v_1) = \begin{cases} \frac{2\gamma}{\gamma+1} & \text{if } a_1 = v_1 \\ 0 & \text{otherwise} \end{cases} .$$

*Fund manager's behavior at  $t = 1$ .* Given contract  $b_1$  and the continuation equilibrium at  $t = 2$ , the fund manager's expected payoff is:

$$\max_{a_1} E(b_1|a_1, s_1) + \Pr(\Pr(\theta = g|a_1, v_1) \geq \gamma|a_1, s_1) \bar{w}.$$

For a good manager ( $s_1 \in \{0, 1\}$ ), the expected payoffs are:

$$\begin{aligned} b_1(\text{success}) + \bar{w} & \quad \text{if } a_1 = s_1 \\ b_1(\text{failure}) & \quad \text{if } a_1 = 1 - s_1 \\ b_1(\text{no trade}) & \quad \text{if } a_1 = \emptyset \end{aligned}$$

For a bad manager, expected payoffs are

$$\begin{aligned} \frac{b_1(\text{success}) + b_1(\text{failure}) + \bar{w}}{2} & \quad \text{if } a_1 = \{0, 1\} \\ b_1(\text{no trade}) & \quad \text{if } a_1 = \emptyset \end{aligned}$$

The fund manager chooses  $a_1$  in order to maximize the payoffs above.

*Investor 1's contract choice.* If  $b_1 = (\bar{w}, \bar{w}, \bar{w})$ , the good manager chooses  $a_1 = s_1$  and the bad manager chooses  $a_1 = \{0, 1\}$ . Clearly, it is not in the interest of investor 1 to encourage the manager to choose  $a_1 = 1 - v_1$ . The only other possibility is to induce the good manager and/or the bad manager to play  $a_1 = \emptyset$ .

The minimal amount that the investor must pay in order to make the good manager play  $a_1 = \emptyset$  is  $\bar{w}$ . The minimum amount she must pay to make the bad manager play  $a_1 = \emptyset$  is  $\frac{\bar{w}}{2}$ . Thus, the lower bound to the additional expected payment needed to induce *any* fund manager to play  $a_1 = \emptyset$  is

$$C_{\min} = (1 - \gamma) \frac{\bar{w}}{2}.$$

The highest net return investor 1 can hope for is when the good manager trades correctly and the bad manager does not trade. This is:

$$\gamma \left( \frac{1}{2} (1 - \hat{\gamma}) - \epsilon \right)$$

The equilibrium expected net return is instead:

$$\gamma\left(\frac{1}{2}(1-\hat{\gamma})-\epsilon\right)-(1-\gamma)\frac{1}{2}(\hat{\gamma}+2\epsilon).$$

Thus, the upper bound to the additional net return that the investor can get from using *any* other contract is

$$R_{\max}=(1-\gamma)\frac{1}{2}(\hat{\gamma}+2\epsilon)$$

An upper bound to the net benefit of inducing the manager to change his action is:

$$\begin{aligned} R_{\max}-C_{\min} &= (1-\gamma)\frac{1}{2}(\hat{\gamma}+2\epsilon)-(1-\gamma)\frac{\bar{w}}{2} \\ &= (1-\gamma)\frac{1}{2}(\hat{\gamma}+2\epsilon-\bar{w}), \end{aligned}$$

which is negative if

$$\bar{w} > \hat{\gamma} + 2\epsilon,$$

which is satisfied for  $\gamma$  low enough and  $\epsilon$  low enough. Then, investor 1 prefers to offer  $b_1 = (\bar{w}, \bar{w}, \bar{w})$ . ■

Even with endogeneous contracting, the churning equilibrium of Proposition 2 is still an equilibrium if: (i) only short-term contracts are possible; and (ii) the proportion of good managers is low. Such equilibrium has the same high levels of trading volume identified in Corollary 3.

Churning hurts the first-period investor, who faces a negative expected return (plus trading cost). If churning stops, the investor makes an expected gain

$$R_{\max}=(1-\gamma)\frac{1}{2}(\hat{\gamma}+\epsilon).$$

The investor can eradicate churning by offering an appropriate contract. The benefit of churning to a bad manager is given by a 50% chance of being hired again in the next period:  $\frac{1}{2}\bar{w}$ . To persuade him to stop trading, the investor needs to set

$$b_1(\text{no trade}) > \frac{1}{2}\bar{w}.$$

The expected cost of eliminating churning is thus

$$C_{\min}=(1-\gamma)\frac{1}{2}\bar{w}.$$

If  $\frac{\hat{\gamma}}{2} + \epsilon$  is small enough, the difference  $R_{\max} - C_{\min}$  is negative. The investor cannot benefit from eradicating churning.

The damage of churning *per churner* on investor 1 is  $\frac{1}{2}(\hat{\gamma} + 2\epsilon)$ . It is lowest when the proportion of good types  $\gamma$  is low because the bid-ask spread is narrow. Churning is least costly when there are many churners. Therefore, if  $\gamma$  is low enough, the benefit of stopping churning is lower than the cost of reimbursing the bad manager for the lost career opportunity.

The result may also be understood in terms of inefficiencies generated by bilateral contracting in an environment with more than two agents. Churning increases the probability that the fund manager is retained in the second period. This creates an additional rent to the incumbent which in part is paid for by investor 2 (who cannot tell for certain between a good and a bad incumbent) and by the challenger (who is hired with a lower probability). As investor 1 and the incumbent do not internalize the cost that churning imposes on the other two parties, full contracting among them can lead to socially inefficient outcomes.

### 3.3 Infinite Horizon

We now consider an infinite horizon model. At each period  $t$ , there are: one incumbent fund manager and one challenger; one short-lived investor; and a large number of short-lived rational traders.

As before, the type of a fund manager is  $\theta \in \{b, g\}$  and the prior is  $\gamma$ . In each period a potentially immortal fund manager is born. If the fund manager is not hired or he is replaced, he dies. In every period, there is a probability  $\delta \in (0, \frac{1-\gamma}{2})$  that a good fund manager becomes bad. A bad fund manager stays bad.<sup>9</sup> The fund manager maximizes the expected sum of future payments (because of  $\delta$  there is no need for further discounting).

In every period  $t$  there is a short-lived investor who observes all the past trades and liquidation values. The investor chooses whether to retain the incumbent from  $t - 1$  or hire the challenger who is born at  $t$ . As in the baseline model, the contract between the investor and the fund manager is exogenously given. At the end of  $t$  the investor pays the fund manager:

$$x_t = \alpha\chi(a_t, p_t, v_t) + \beta,$$

where the net return  $\chi(a_t, p_t, v_t)$  is as in the baseline model.

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<sup>9</sup>This is a minor simplifying assumption that is not necessary for the results.

In every period  $t$ , there are a large number of short-lived rational traders who observe all the past trades and liquidation values. As before, the fund manager submits a market order and each trader offers an ask price  $p_A$  and a bid price  $p_B$ .

To summarize the stage game at  $t$  is:

- a. Investor  $t$  observes  $a_{t-1}$  and  $v_{t-1}$ . She retains the incumbent or she hires a challenger with prior  $\gamma$ .
- b. The incumbent observes  $s_t$  and he selects  $a_t$ .
- c. The valuation  $v_t$  is realized. With probability  $\delta$  a good incumbent becomes bad. A bad incumbent stays bad.

We prove the existence of a churning equilibrium:

**Proposition 7 (Infinite horizon)** *For  $\alpha$  and  $\epsilon$  low enough, there is a perfect Bayesian equilibrium in which:*

1. At  $t$ , the investor and the traders have belief about the type of the current incumbent:

$$\Pr(\theta_t = g | a_{t-1}, v_{t-1}) = \begin{cases} (1 - \delta) \frac{2\tilde{\gamma}_{t-1}}{\tilde{\gamma}_{t-1} + 1} & \text{if } a_{t-1} = v_{t-1} \\ 0 & \text{otherwise} \end{cases} ;$$

2. The investor at  $t$  retains the incumbent if and only if  $\Pr(\theta_t = g | a_{t-1}, v_{t-1}) \geq \gamma$ ;
3. At  $t$  a good fund manager trades correctly ( $a_t = v_t$ ) and a bad fund manager churns ( $a_t \in \{0, 1\}$  with equal probability);
4. The traders offer prices

$$\hat{p}_{A,t} = \frac{1}{2}(1 + \tilde{\gamma}_t) \quad \text{and} \quad \hat{p}_{B,t} = \frac{1}{2}(1 - \tilde{\gamma}_t).$$

The variable  $\tilde{\gamma}_t$  is defined recursively as follows:

$$\tilde{\gamma}_t = \begin{cases} (1 - \delta) \frac{2\tilde{\gamma}_{t-1}}{\tilde{\gamma}_{t-1} + 1} & \text{if at } t \text{ the incumbent is retained} \\ \gamma & \text{if at } t \text{ the challenger is hired} \end{cases} ;$$

and  $\gamma_1 = \gamma$ .



**Proof.** The upper bound of  $\tilde{\gamma}_t$  occurs after observing an infinite sequence of successful trades. Letting

$$\tau(\gamma) = (1 - \delta) \frac{2\gamma}{\gamma + 1},$$

the upper bound is the limit of the following sequence

$$\gamma, \tau(\gamma), \tau(\tau(\gamma)), \dots$$

which can be computed by setting

$$\gamma = (1 - \delta) \frac{2\gamma}{\gamma + 1}$$

The solution is

$$\tilde{\gamma}_{\max} = 1 - 2\delta.$$

It is immediate to check that the belief is consistent given equilibrium play. Given the belief, the investor's strategy is optimal.

For a good manager it is always a best response to trade. A bad manager who churns receives at least additional expected payoff (this is the expected payoff for a bad manager who churns at  $t$  and does not trade at  $t + 1$  if retained):

$$-\left(\frac{1}{2}\tilde{\gamma}_t + \epsilon\right)\alpha + \frac{1}{2}\beta$$

which is nonnegative if

$$\left(\frac{1}{2}\tilde{\gamma}_t + \epsilon\right)\alpha \leq \frac{1}{2}\beta$$

This in turn is true if

$$\left(\frac{1}{2}(1 - 2\delta) + \epsilon\right)\alpha \leq \frac{1}{2}\beta$$

If  $\epsilon$  and  $\alpha$  are low enough the inequality is satisfied.

Given  $\tilde{\gamma}_t$ , it is easy to see that the bid-ask prices are optimal. ■

### 3.4 Positive Net Returns for Investors

In all versions of our model, delegation to active fund managers leads to negative ex ante expected return for investors.<sup>10</sup> This is in keeping with the

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<sup>10</sup>It is worth noting, however, that when  $(\epsilon, \alpha, \beta) \rightarrow 0$ , the investor is essentially indifferent between delegation and direct investment.

empirical findings (Gruber [11] or Wermers [21] for example) that investors in actively managed funds obtain negative risk-adjusted returns, and is a direct consequence of the fact that there are no exogenous traders in our model who are willing to lose money. Thus, our analysis in current form is consistent with the observed participation in actively managed mutual funds (an assumption in the model) on the one hand, and the high volume of trade observed in financial markets (a result from the model) on the other. We are agnostic about the reasons for which investors put their money in actively managed funds. We treat this as a behavioural feature of the class of investors we consider.

However, it is not difficult to relax this assumption by introducing exogenous noise trading. It is easy to see that even the smallest amount (any small but positive probability) of exogenous noise trade would lead to strictly positive returns for investors (since  $\beta$  can be as small as we like). This would allow us to eliminate the initial behavioural assumption for investors, but at the cost of making our analysis inconsistent with the empirical findings on the underperformance of actively managed funds. Nevertheless, it is worth noting that since trading volume can be maximal even in the complete absence of exogenous noise trade, our results with noise traders would be immune to the order of magnitude critique: even the smallest amount of noise trade would be capable of generating maximal amounts of trade, which simultaneously leads to strictly positive returns for the investor.

## 4 Conclusions

It is a well-established empirical anomaly that individual investors invest heavily in actively managed funds despite the fact that such funds are known to be inferior to their passively managed index counterparts. Simultaneously, the volume of trade observed in financial markets is also an established puzzle: existing models of rational behaviour cannot explain it. Taking the first anomaly as a starting point, we resolve the second puzzle: we show that fully rational behaviour will result in a large volume of trade, even in the absence of any exogenous “noise” trading. The trade arises from an underlying agency relationship between the investors and the fund manager. The fund manager has career concerns: he wants to retain his job. Not trading makes him look “dumb” and can lead to him being fired. Thus, he tends to trade sometimes even without information. But the fact that he does so makes

rational traders, who cannot identify the type of manager they are dealing with, willing to be counterparty to such trades at more favourable terms, which in turn makes trading possible.

## 5 Appendix

We characterize here some more general properties of the equilibrium set of the basic model outlined in Section 2 above. The baseline model has equilibria with zero trading volume. The no-trading equilibria are supported by beliefs that penalize trading like the following:

$$\Pr(\theta = g|a, v) = \begin{cases} \gamma & \text{if } a = \emptyset \\ 0 & \text{otherwise} \end{cases} .$$

The investor retains a fund manager who did not trade. If  $\alpha$  is low enough, then the fund manager should never trade. We can also show:

**Proposition 8** *If  $\alpha$  and  $\epsilon$  are low enough, either everyone trades in the first period or bad traders do not trade.*

**Proof.** Let  $\beta_\theta$  denote the probability that type  $\theta$  trades at  $t = 1$ .

If  $\beta_b > 0$ , the bid-ask spread is less than one and informed trade generates a positive net return. It is a best response for the investor to retain (replace) the fund manager if the posterior is strictly higher (lower) than  $\gamma$ . If  $\beta_g < \beta_b$ , then

$$\hat{\gamma}(\text{no trade}) = \frac{\gamma(1 - \beta_g)}{\gamma(1 - \beta_g) + (1 - \gamma)(1 - \beta_b)} > \gamma$$

and thus the investor retains non-traders and it is strictly better for bad agents not to trade, which implies that  $\beta_b = 0$ , a contradiction. Therefore, it must be that  $\beta_g \geq \beta_b$ . If  $\beta_g > \beta_b$ , the posterior for nontraders is below  $\gamma$ , and the investor replaces all non-traders. But then for small  $\alpha, \epsilon$ , it is a best response for all bad agents to churn, and thus,  $\beta_b = 1$ , which is a contradiction.

So if  $\beta_b > 0$  it must be either that  $\beta_b = \beta_g = 1$  (the churning equilibrium) or  $\beta_g = \beta_b \in (0, 1)$ . Suppose that the latter is true. This means that both good and bad managers are strictly indifferent between trading and not trading, but this is impossible because good managers benefit strictly more than bad managers from trade. ■

There are equilibria in which bad traders never trade (and some perverse equilibria in which only good traders trade, and they do it at a loss). One can argue that these equilibria are knife-edge. Suppose that a small but positive proportion of fund managers, both good and bad, like trading *per se*. Formally assume that, with probability  $\eta > 0$  the fund manager is a *trade lover*. A lover chooses  $a = s$  if he is good and he randomizes between  $a = 0$  and  $a = 1$  otherwise.

**Proposition 9** *For any  $\eta > 0$ , if  $\alpha$  and  $\epsilon$  are low enough, all fund managers trade in the first period.*

**Proof.** Suppose that no fund manager trades at  $t = 1$  except trade lovers. The investor's posterior is

$$\Pr(\theta = g|a, v) = \begin{cases} 0 & \text{if } a = 1 - v \\ \frac{2\gamma}{\gamma+1} & \text{if } a = v \\ \gamma & \text{if } a = \emptyset \end{cases} .$$

As some bad types are trading the bid-ask spread is smaller than one:

$$\hat{p}_A < 1 \text{ and } \hat{p}_B > 1.$$

If  $\epsilon$  is small enough, it is a best response for the investor to retain a successful trader.

But then, a good fund manager at  $t = 1$  gains by deviating from  $a = \emptyset$ . As his trade will be successful, he will be retained, plus he will benefit from the trade. Thus, there do not exist equilibrium which no fund manager trades at  $t = 1$  except trade lovers.

Suppose that no bad manager trades but  $\beta_g > 0$ . Still, because  $\eta > 0$  the bid-ask price is smaller than one:

$$\hat{p}_A < 1 \text{ and } \hat{p}_B > 1.$$

If  $\epsilon$  is small enough, for any  $\beta_g > 0$  it is a best response for the investor to replace a non-trader. It must also be that for  $a = v$  and/or  $a = 1 - v$  the manager is retained. But then for  $\alpha$  low enough it is a best response for the bad fund manager to churn. ■

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