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The Role of Prices for Excludable Public Goods

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Abstract
When a public good is excludable it is possible to charge individuals for using the good. We study the role of prices for publicly provided excludable public goods within an extension of the Stern-Stiglitz version of the Mirrlees optimal income tax model.

We show that for a public consumer good there is a range of circumstances in which charging a price for the public good decreases welfare. We find that a necessary condition for a positive price to be desirable is that the marginal valuation of the public good is increasing in leisure. However, welfare is *initially* decreasing in the price, implying that charging a lower than optimal price may be less efficient than setting a zero price. Thus, even when there is a case for charging a price for the public good, an attempt to implement the optimum in practice may be risky, as even setting a modest price to avoid overshooting the optimum may be Pareto inferior to charging no price at all. The policy case for a price may thus appear rather weak. We also find that producers using an intermediate excludable public good as an input should *not* be charged a price for using the good.

Keywords: excludable public goods, public sector pricing, information constrained Pareto efficiency

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1. Introduction

In many countries there is growing concern about the size of public budgets and a search for alternatives to tax funding. Since excludable public goods are by definition such that it is possible to charge a price or fee for their use, even the well established case for free provision of public goods is being challenged in the case of excludable public goods. In this paper we shall address the provision of such goods.

There are many important examples of excludable public goods. Information is in general a public good and often it can be made excludable. Patent rights is one way to exclude. For information that is only of value in the fairly short term, like meteorological and hydrological forecasts, it would probably be easy to exclude people and only let those who subscribe to the service have access to the information. The services provided by the Central Bureau of Statistics is another example of information that can easily be made excludable. Radio and television broadcasts, many services provided on the internet, non-congested roads, public beaches, national parks and museums are some other examples of excludable public goods.

There are several questions that are of interest to study. First, since it is possible to have market provision of excludable public goods by private firms, there is the important issue of whether public or private provision is preferable. Second, if there is public provision, should it then be financed by an income tax or should it be partly, or completely, financed by a fee (price).[2] If the price instrument is indeed used, will this lead to more or less provision of the public good as compared to the situation without exclusion? Third, some excludable public goods can be regarded as final consumer goods. However, goods, like statistics or other information, have more the character of an intermediate good. It is conceivable that the rules for provision would depend on whether the excludable public good is a final consumer good or an intermediate good. We will not attempt to answer all these questions. We will focus on public provision of an excludable public good. However, we will consider both the case where the public good is a final consumer good and the case where it is an intermediate public good. In either case the central issue is the welfare effect of charging a price.

Empirically there are examples both of publicly provided excludable public goods being financed from general tax revenue and goods partly financed by prices. Weather forecasts are in many countries publicly provided and financed out of general tax revenue.

[2] One benefit of using prices to finance an excludable public good is that data will be generated that might be useful to derive the users' evaluation of the public good. In the analysis below we will not take this aspect into account. Hence, we will assume that individuals' preferences are known.

The same is true for many forms of statistics. However, there are examples, like Sweden, where recently substantial charges have been introduced for users to get access to Central Bureau of Statistics data. Indeed, we have the impression that in many countries there is a move away from financing excludable public goods via taxes towards financing via user charges.

There is a vast literature on public and/or private provision of nonexcludable public goods. However, public provision of excludable public goods has not been much studied. Of course, the first best rule does not depend on whether the public good is excludable or not. In a second best setting it may be different since there is one more instrument, a price on the public good, available when the public good is excludable. Fraser (1996) studied provision of excludable public goods. However, in his model the incomes are exogenous and there are no distortions from the income tax. There are several papers that have studied market provision of excludable public goods by private firms. Oakland (1974) studied a model where private firms operating under conditions of perfect competition provide an excludable public good. Brito and Oakland (1980) and Burns and Walsh (1981) consider a situation where a natural monopoly provide an excludable public good.

We believe it is important to acknowledge that taxes are not only used in order to finance public goods but also to achieve redistribution. We therefore use an extension of the Stiglitz-Stern version of the Mirrlees optimal income tax model.[3] We will assume there are two types of persons, high-skilled and low-skilled, and that it is desirable to redistribute from the high-skilled to the low-skilled. First considering the case where the public good is a final consumer good, we in section 2 describe consumer behavior and formulate the social optimization problem. In section 3 we derive and discuss the optimum price setting and public provision rules. The focus is on the conditions under which non-zero prices would be part of the optimal provision scheme. In section 4 we consider the case where the public good is an intermediate good. Section 5 concludes.

2. The Model

When the public good is excludable one can charge individuals for accessing it. Depending on the technology for excluding individuals the payment scheme might take different forms. Here we make two crucial assumptions. The first is that there is decentralization in the sense that the tax authorities gather information on labor income and collect taxes, while some other

[3] See Mirrlees (1971), Stern (1982) and Stiglitz (1982).

government authority, which does not have access to information on individual income, distribute the public good. This means that the provision scheme for the excludable public good cannot be income dependent. We believe this assumption to be more realistic than the alternative. We are not aware of any actual income dependent provision scheme for an excludable public good. The second assumption is that the price on the excludable public good must be non-negative. For many excludable public goods there is close to free disposal. If there were a negative price individuals would demand the maximum available amount. However, then the price would just function as a uniform lump-sum transfer. Hence, the negative price would be equivalent to a zero price and a lump-sum transfer.

2.1 Consumer behavior

The consumer enjoys one private good, one excludable public good and leisure. Let g^i , $i = 1,2$ denote the quantity of the public good consumed by a person of type *i* and let c^i be the consumption of the private good. Throughout the paper we assume that the goods and leisure are non-inferior. We normalize the price of the private good to one and let *q* denote the non-negative price on the public good.[4] Let G denote the quantity of the publicly provided public good, which can be interpreted in different ways. We may think of *G* as measuring the degree of detail of, say, information or forecasts made available. It may measure hours of broadcasting transmission or opening hours of a museum. More generally, it may reflect different quality levels of a public good. Exclusion implies that a consumer may have access only to a part of *G*, say only some pieces of available information, only part of the television network or access for only a limited period. The amount made available to consumer *i* is subject to the restriction that $g^i \le G$. We let L^i, H^i, w^i, Y^i and B^i denote leisure, hours of work, the wage rate, the before-tax and after-tax income of an individual of type *i*. $Z = L^i + H^i$ denotes the fixed time endowment. Since $Y^i = w^i H^i$, we can express H^i by Y^i / w^i.

Before addressing the social optimization problem we need to clarify the individual consumer behavior. To simplify notation we for the moment omit the superscript *i*. The utility maximization problem for an individual will take the form: Max $U(g, c, Y / w)$ *s.t.* the budget constraint $c + qg = B$ and the capacity constraint $g \le G$. It is useful to distinguish

[4] In some cases there may be a small cost of distributing the excludable public good to the consumers. In our analysis charging a price may in such cases be interpreted as the setting of a price over and above such a cost. For simplicity, we shall abstract from such costs and normalize the cost of distributing the good to zero.

between the demand functions that would result in absence of the constraint, the notional demands, and the demand functions that result when there is a capacity constraint, the effective demands. Conditioning on Y/w and neglecting the condition $g \le G$ we derive the notional conditional demand functions $\tilde{c}(q,B,Y/w)$ and $\tilde{g}(q,B,Y/w)$. We can, of course, also write these functions in terms of leisure. By definition, $\tilde{c}(q,B,Z-L) = \tilde{c}(q,B,H) = \tilde{c}(q,B,Y/w)$ and $\tilde{g}(q,B,Z-L) = \tilde{g}(q,B,H) = \tilde{g}(q,B,Y/w)$. For short we will in the following use $\partial\tilde{g}/\partial L$ for $\partial\tilde{g}(q,B,Z-L)$. The effective demand is given by: $g(q,B,Y/w,G) = \tilde{g}(q,B,Y/w)$ if $\tilde{g}(q,B,Y/w) \le G$ and $g(q,B,Y/w,G) = G$ if $\tilde{g}(q,B,Y/w) \ge G$. Sticking $g(q,B,Y/w,G)$ back into the direct utility function we obtain the conditional indirect utility function $V(B,Y/w,q,G)$. For ease of reference we summarize some properties of the indirect utility function and the notional and effective demand functions in the following lemma. Partial derivatives are indicated by subscripts.

Lemma 1: If the individual is unrationed, $\tilde{g} \le G$, the following properties are valid: $g_G = V_G = V_G / V_B = (U_g / U_c) - q = MRS_{gc} - q = 0$; $g_q = \tilde{g}_q$; $g_B = \tilde{g}_B$; $g_Y = \tilde{g}_Y$; $g_L = \tilde{g}_L$; $g_q^h = \tilde{g}_q^h$, where g^h denotes Hicksian demand. If the individual is rationed, $\tilde{g} > G$, the following properties are valid: $g_G = 1$; $V_G = U_g - qU_c > 0$; $V_G / V_B = (U_g / U_c) - q = MRS_{gc} - q > 0$; $g_q = g_B = g_Y = g_L = g_q^h = 0$. This also implies that the Slutsky decomposition is valid and can be used without any reservation about the case of rationing. We also note that Roy's identity holds irrespective of whether a person is rationed or not.

Proof: Most of the results in the lemma are obvious. We therefore only show that Roy's identity is true even if a person is rationed. If the person is rationed $g = G$ and $c = B - qG$. The indirect utility function is defined by $V(B,Y/w,q,G) \equiv U(G, B-qG, Y/w)$. Differentiating we obtain $\partial V/\partial q = -G(\partial U/\partial c)$ and $\partial V/\partial B = \partial U/\partial c$. It follows that $-(\partial V/\partial q)/(\partial V/\partial B) = G$.

It turns out that the characterization of different regimes that we do in section 3 depends crucially on how leisure affects the marginal valuation of g and the demand $\tilde{g}$. It is easy to show that $\partial(MRS_{gc})/\partial L > 0$ implies that $\tilde{g}_L > 0$ and that $\partial(MRS_{gc})/\partial L < 0$ implies that $\tilde{g}_L < 0$. If the direct utility function exhibits weak separability between leisure and market goods $\partial(MRS_{gc})/\partial L = 0$ and $\tilde{g}_L = 0$.

It is sometimes convenient to supress w in the indirect utility function. However, since individuals have different wage rates we then have to put a superscript *i* on the indirect utility function, i.e, $V^i(B^i, Y^i, q, G) = V(B^i, Y^i / w^i, q, G)$.

2.2 The social optimization problem

Turning to the social optimization problem, our concern will be with (information constrained) Pareto efficient policy. Even if our focus be on the optimal price of the public good, we have to analyze the price setting within a wider optimization problem that also involves income taxes. We assume a linear production technology and denote the producer price of the public good by p. Let N^1 and N^2 denote the number of type one and type two individuals, respectively. We assume $w^2 > w^1$, and that the social planner wants to redistribute from the high-skilled to the low-skilled type. In pursuing such a policy the planner will face an asymmetric information constraint. We assume that the identity of the two types is private information that is hidden to the planner. Thus the policy must be chosen subject to the restriction that incomes intended for type one do not accrue to type two individuals pretending to be of type one.[5] There is no reason to assume that there are systematic differences in preferences between high skill and low skill persons. Differences in demand for the public good depend only on differences in leisure (work effort) and income. We will explore the Pareto efficient allocations by maximizing the utility of persons of type one for a given utility level of type two persons. The optimization problem, which defines Pareto efficient taxation and provision of the public good, takes the form:

$$\underset{B^1, Y^1, B^2, Y^2, q, G}{Max} \quad V^1(B^1, Y^1, q, G) \tag{1}$$

$$s.t. \; V^2(B^2, Y^2, q, G) \geq \bar{V}^2 \tag{2}$$

$$V^2(B^2, Y^2, q, G) \geq V^m(B^1, Y^1, q, G) \tag{3}$$

[5] We could also have included a self-selection constraint that an individual of type one should not mimick a type two person. However, one can show that at most one of the self-selection constraints is binding. We make the usual assumption that it is the self-selection constraint that the high skill person should not mimic the low skill person that is binding.

$$N^2(Y^2 - B^2) + N^1(Y^1 - B^1) + q(N^1 g^1 + N^2 g^2) - pG \geq 0 \qquad (4)$$

The constraint (2) assigns a minimum utility level to individuals of type two. The constraint (3) is a self-selection constraint imposing that the vector of taxes, prices and level of G be such that type two persons do not gain by mimicking type one persons, i.e. by choosing the income intended for the low-skilled type of person. We use an index m to denote the "mimicker", i.e. $V^m(\,)$ is the utility of type two persons evaluated at the income point intended for type one persons. We assume the standard single crossing property that for any fixed gross and net income point in *Y, B* space the indifference curve of a low ability type of person is steeper than that of a high ability type[6]. In the analysis below we will also assume that q must be nonnegative. The government budget constraint is expressed by (4). It requires that revenue from income taxes and from charging a price on the public good is sufficient to finance the public good provision.

3. Optimum price-setting and public good provision.

Our agenda is to discuss the optimum price-setting and the optimum provision of the excludable public good. We shall address these issues in two sub-sections below based on the first order conditions of the optimization problem above. Before deriving the first order conditions we can already establish that it will never be optimal to choose a combination of price and provision such that no individual consumes the total amount of the public good.

Proposition 1: An optimum is such that $G \leq \max\{\tilde{g}^1, \tilde{g}^2\}$.

Proof: We construct a proof by contradiction. Suppose there exists an optimum such that $G > \max\{\tilde{g}^1, \tilde{g}^2\}$. Decrease the level of G down to $\max\{\tilde{g}^1, \tilde{g}^2\}$. This will not harm any of persons one or two. It will not improve the situation of the mimicker, so the self-selection constraint will not be affected. Take the resources released and give to the actual person 2. In

[6] That is, for any admissible values of *B, Y, q, G* the utility function $V(B, Y/w^1, q, G)$ will have a steeper indifference curve than $V(B, Y/w^2, q, G)$ through a point *B,Y* in a diagram with *Y* measured along the horizontal and *B* along the vertical axis.

this way we have achieved a Pareto improvement. Hence, the initial situation with $G \geq \max\{\tilde{g}^1, \tilde{g}^2\}$ can not have been an optimum.

The Lagrangean of the optimization problem above can be written as:

$$\Lambda = V^1(B^1, Y^1, q, G) + \beta\left[V^2(B^2, Y^2, q, G) - \overline{V}^2\right] + \rho\left[V^2(B^2, Y^2, q, G) - V^m(B^1, Y^1, q, G)\right]$$

$$+ \mu\left[N^2(Y^2 - B^2) + N^1(Y^1 - B^1) - pG + q(N^1 g^1 + N^2 g^2)\right]$$

The first order conditions w.r.t. B^1, Y^1, B^2, Y^2, G and q are the following:

$$\frac{\partial \Lambda}{\partial B^1} = V_B^1 - \rho V_B^m - \mu N^1 + \mu q N^1 g_B^1 = 0 \tag{5}$$

$$\frac{\partial \Lambda}{\partial Y^1} = V_Y^1 - \rho V_Y^m + \mu N^1 + \mu q N^1 g_Y^1 = 0 \tag{6}$$

$$\frac{\partial \Lambda}{\partial B^2} = \beta V_B^2 + \rho V_B^2 - \mu N^2 + \mu q N^2 g_B^2 = 0 \tag{7}$$

$$\frac{\partial \Lambda}{\partial Y^2} = \beta V_Y^2 + \rho V_Y^2 + \mu N^2 + \mu q N^2 g_Y^2 = 0 \tag{8}$$

$$\frac{\partial \Lambda}{\partial G} = V_G^1 + \beta V_G^2 + \rho V_G^2 - \rho V_G^m - \mu p + \mu q(N^1 g_G^1 + N^2 g_G^2) = 0 \tag{9}$$

$$\frac{\partial \Lambda}{\partial q} = V_q^1 + \beta V_q^2 + \rho V_q^2 - \rho V_q^m + \mu(N^1 g^1 + N^2 g^2) + \mu q(N^1 g_q^1 + N^2 g_q^2) \leq 0\,, \tag{10}$$

and

$$q\frac{\partial \Lambda}{\partial q} = 0\,, \tag{11}$$

where Kuhn-Tucker conditions are used since $q \geq 0$ may well be a binding constraint.

We note that eqs. (5) – (8) and (10) as an equation look the same as corresponding equations in Edwards et al. (1994). However, note that there is only a *formal* similarity. In the present context individuals may be rationed and q must be non-negative.

Making use of Roy's identity and the Slutsky equation; we can rewrite (10) as

$$\frac{\partial \Lambda}{\partial q} = -V_B^1 g^1 - \beta V_B^2 g^2 - \rho V_B^2 g^2 + \rho V_B^m g^m + \rho V_B^m g^1 - \rho V_B^m g^1 + \mu N^1 g^1 + \mu N^2 g^2 + \mu q N^1 g_q^{h1} + \mu q N^2 g_q^{h2} + \mu q N^1(-g^1) g_B^1 + \mu q N^2(-g^2) g_B^2 \leq 0 \tag{12}$$

We then make use of (5) and (7) to eliminate terms:

$$\frac{\partial \Lambda}{\partial q} = \rho V_B^m (g^m - g^1) + \mu q (N^1 g_q^{h1} + N^2 g_q^{h2}) \leq 0 \tag{13}$$

3.1. Optimum price setting.

For discussing the case of a zero or negative impact of leisure on the demand for g the following lemma is helpful.

Lemma 2: A necessary condition for a positive price on g to be welfare improving is that the mimicker's consumption is larger than that of the type one person.

Proof: (By contradiction) Assume the mimicker consumes less than or the same amount as the type one person and that $q > 0$. From this situation, decrease the price to zero and decrease B^1 and B^2 by the amounts qg^1 and qg^2. This keeps the budget balanced and does not decrease utility for the two actual types of persons. The mimicker's utility either remains unchanged, or, is decreased if he consumes less than the person of type one. Accordingly the self-selection constraint is left unchanged, or is softened rendering possible a welfare impovement by removing the price. Hence, lemma 2 follows.

We can then draw the following conclusion.

Proposition 2: Nothing can be gained by a positive price on the excludable public good if $\tilde{g}_L^i \leq 0$.

Proof: The mimicker and the type one person have the same after tax income (B^1). The mimicker has more leisure as he can earn the same income in less time. It follows that a necessary condition for the mimicker to have larger consumption of the excludable public good is that $\tilde{g}_L^i > 0$. From lemma 2 we then deduce that proposition 2 is true.

It follows from Proposition 2 that efficiency can be improved by using the price instrument only if $\tilde{g}_L^i > 0$. However, as will be shown below this is a necessary, not a sufficient condition.

Proposition 3 :
Assume $\tilde{g}_L^i > 0$. Then there is no gain (or loss) from introducing a positive price as long as both types of persons remain rationed.

Proof : For q such that $g^i \le G$ is strictly binding for both types of persons, the mimicker is also rationed since $\tilde{g}_L^i > 0$ (and $g^m = g^1 = G$), the compensated price derivatives are zero, and it follows immediately from (13) that $\partial \Lambda / \partial q = 0$.

Proposition 4 :
Assume $\tilde{g}_L^i > 0$ and that both persons are constrained up to price $\bar{q}$. Then, marginally increasing the price so that the quantity constraint of one type ($g^i \le G$) is just being relaxed, will make the allocation strictly less efficient.

Proof :
Suppose it is the quantity constraint for type i that is being relaxed first as the price increases. Just at the point of relaxation $g^i = \tilde{g}^i = G = g^m = g^j$ for j$\neq$i. Then, from eq. (13) we see that the change in welfare is given by $\partial \Lambda / \partial q = \mu q N^i g_q^{hi} < 0$.

It is of interest to compare this result with a corresponding result from the optimal commodity tax literature. Had the good in question been an ordinary private good x with $\partial x / \partial L > 0$ it would have been optimal to introduce a small tax on the good. (See e.g. Christiansen (1984)). This is because in the optimal commodity tax case, at a zero tax the size of the deadweight loss from a marginal tax would be of second order, whereas the benefits from the slackening of the self-selection constraint would be a first order effect. For a publicly provided excludable public good it is the other way around.

Since the marginal cost of providing one more unit of g to a person is zero as long as the person is not constrained, the price in our model resembles a tax. However, in our model, if $\tilde{g}_L > 0$, there would be a range of the price where there is neither a gain nor a loss from having a positive price. This holds as long as both persons are constrained. Then a further increase in q that results in the quantity constraint being relaxed for one type of person leads to a *decrease* in welfare due to the deadweight losses created by the price. This is quite different from the optimal commodity tax result and is due to the fact that we, as q is marginally increased above $\bar{q}$, come from a regime where the individuals are constrained. Looking at eq. (13) we see that at the price $\bar{q}$, where one type of person ceases to be constrained, $g^1 - g^m = 0$ and, initially, there is no gain in terms of slackening the self-selection constraint. However, as we increase the price above $\bar{q}$ the compensated price effects take on negative finite values, implying finite deadweight-losses from the price increase and a total decrease in welfare. As the price is increased further there will eventually be a difference between g^1 and g^m such that $g^m > g^1$ and an increase in q penalizes the mimicker more than it hurts the low-skilled person. It is then possible that the gain from slackening the self-selection constraint outweighs the deadweight losses.

We can then identify the possible optima.

Proposition 5 : If $\tilde{g}_L^i > 0$ the optimum can be of one of the following types : i. Persons of type one are not rationed in their demand for the excludable public good and a strictly positive price on the good is desirable. Persons of type two may or may not be rationed. ii. Both types of persons are rationed in their demand for the excludable public good and q assumes any value in the interval {0, $\bar{q}$ } where $\bar{q}$ is the price at which the rationing constraint ceases to bind. Thus we have an infinite number of equivalent optima.

Proof : It is not possible to have an optimum at which only type one is rationed. Ruling out satiation both types will be rationed if q=0. If q>0 and only type one is rationed, $\partial\Lambda / \partial q = \mu q N^2 g_q^{h2} < 0$ from (13), and (11) is violated. Thus, either type one is not rationed or both types are rationed. If type one is not rationed, $g^m > g^1 = \tilde{g}^i$, and it follows that (13) will only hold if q is strictly positive since the compensated price derivative is negative. If both types are rationed, the first order condition that $\partial\Lambda / \partial q = 0$ holds for the given price interval according to proposition 3.

We note that under the conditions of the model the price is within certain limits arbitrary. However, there is also an administrative cost of collecting a price which in practice will rule out indifference between a zero and a non-zero price.

To characterise further the positive price of regime *i* of Proposition 5, we note from (13) that for an interior optimum the f.o.c. $\partial\Lambda / \partial q = 0$, implies that

$$q = \frac{\mu^*(g^1 - g^m)}{N^1 g_q^{h1} + N^2 g_q^{h2}} \tag{14}$$

where $\mu^* = \rho V_B^m / \mu > 0$, and the denominator is strictly negative due to the substitution effects.

Equation (14) expresses a trade-off between two effects. Consider the case in which the demand for g is decreasing in labour and accordingly increasing in leisure ($\tilde{g}_L > 0$). If type one is not rationed $(\tilde{g}^1 < G)$, the mimicker, enjoying more leisure, will be the larger buyer of good g $(g^1 < g^m)$, and the price q can be used to relax the self-selection constraint. By increasing q and lowering the income tax to make person one equally well off, the mimicker, incurring a larger real income loss from the price increase, will be made worse off and the self-selection constraint (3) is being relaxed. This effect is reflected by the nominator of (14). It must be traded off against the effect captured by the denominator, which is the distortionary effect of pricing the public good in a way that drives the consumption below the available amount G.

Combining propositions 4 and 5 we note that a non-rationing must be characterised by a price which is set discretely above the level at which the rationing constraint ceases to bind. Let us see how this may happen. Consider the case described in proposition 4. Departing from the price $\bar{q}$ and then increasing the price will at first reduce efficiency as stated by proposition 4. But as q is increased and the consumption of the public good of person one declines, a discrepancy between $g^1$and g^m will occur. This discrepancy makes it possible to use the price q as a device for relaxing the self-selection constraint. On the other hand increasing q will discourage the consumption of g and distort the allocation. In order to obtain an efficiency gain from increasing q above $\bar{q}$, the benefits from softening the self-selection constraint must not only outweigh the cost of further distortions beyond some point, but it must also outweigh the latter by a margin which offsets the initial loss from increasing *q*. In this sense one may argue that there is a relatively weak case for regime *i* of proposition 5.

We may also note that the two effects discussed above are not entirely independent. One way that an increase in *q* may strongly discourage the consumption of good *g* by person one and thus create a discrepancy between g^1 and g^m is through a strong substitution effect, but this will also add to the consumption distortion and thus create a countervailing deadweight loss.

It follows from our results that the welfare level can follow three different types of paths as *q* is being increased. These are depicted in the figures 1a-c below. We illustrate how the welfare of individual one changes with the level of *q*, keeping the utility of person 2 constant. As *q* is increased beyond a certain level efficiency declines and may never rise again (fig. 1a). For other properties of the economy efficiency may start picking up again at some level of *q*, but may never fully recover, and keeping *q* sufficiently low is optimal (fig. 1b). Only in the last case (fig 1c) does optimality require a strictly positive price. In this case the benefits of the slackening of the self-selection constraint outweigh the inefficiencies as captured by the substitution effects.

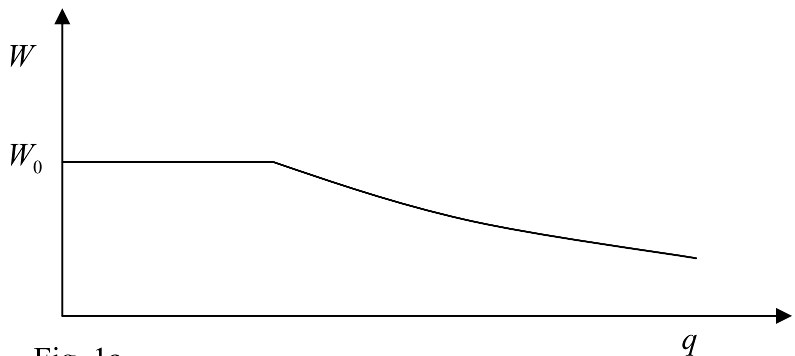

Fig. 1a

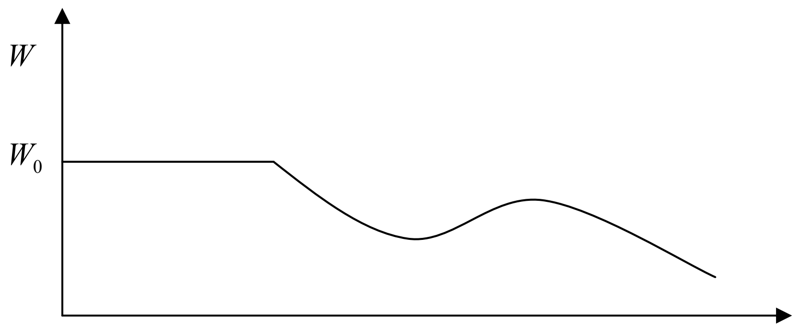

Fig. 1b

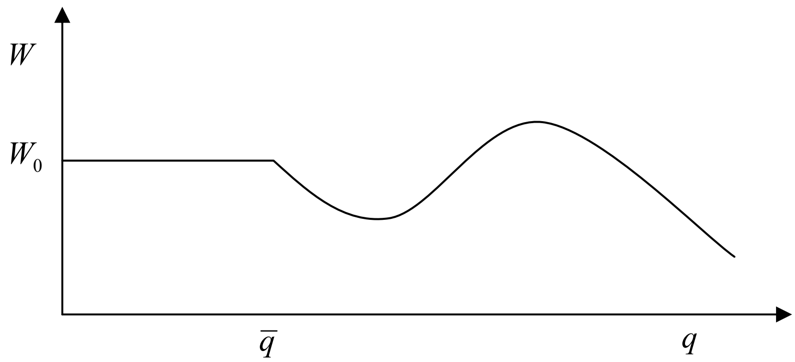

Fig. 1c

The basic message is that for a positive, but sufficiently small price the sole effect is to distort consumption of the public good and lower the utility level. Only when g^1 is sufficiently reduced will a slackening of the self-selection constraint occur to create a counter-veiling effect. We realise that for a strictly positive price to be optimal (Fig. 1 c) a price increase from zero must strongly discourage the consumption without inflicting a too heavy loss of utility on the consumers. Developing formally conditions that are conducive to a strictly positive price, turns out to be an extremely cumbersome exercise with which will we will burden the current presentation. In a companion paper (Blomquist and Christiansen (2001)) we have examined further the conditions that are favourable to a positive price We find that on the whole strong income and labour effects and weak substitution effects on the demand for g^1 as well as comparatively flat indifference curves for the mimicker in *Y,B*-space are conducive to the outcome in Fig. 1c. Such conditions will ensure that g^1 declines strongly in the former interval where utility is decreasing so that one gets to a position where the slackening of the self-selection constraint starts to dominate without sacrificing too much utility on the way. They will also imply that utility increases strongly beyond the local utility minimum. Some intuition may be invoked in support of these findings. We know from the basic theory of tax distortions that it is the substitution away from the taxed (highly priced) good that creates the deadweight loss, which suggests that a strong substitution effect is a poor case for a positive price. Moreover, the induced loss of utility (real income loss) has a negative income effect on consumption of g directly and via leisure effects. If this effect is strong it tends to discourage heavily the demand for g^1 for a given loss of utility, and possibly to an extent that may considerably soften the self-selection .

3.2. The optimum provision.

We next consider the rule for the optimal quantity of public good provision. For the reader's convenience we reproduce and rewrite eq. (9) as

$$V_G^1 + (\beta+\rho)V_G^2 - \rho V_G^m - \mu p + \mu q(N^1 g_G^1 + N^2 g_G^2) = 0 \tag{15}$$

Adding and subtracting $\rho V_B^m \left(\frac{V_G^1}{V_B^1}\right)$, we obtain:

$$(V_B^1 - \rho V_B^m)\frac{V_G^1}{V_B^1} + (\beta+\rho)V_B^2 \frac{V_G^2}{V_B^2} + \rho V_B^m \left(\frac{V_G^1}{V_B^1} - \frac{V_G^m}{V_B^m}\right) = \mu p - \mu q(N^1 g_G^1 + N^2 g_G^2) \tag{16}$$

Using eqs. (5) and (7) we rewrite this as:

$$\mu N^1 \frac{V_G^1}{V_B^1} + \mu N^2 \frac{V_G^2}{V_B^2} + \rho V_B^m \left(\frac{V_G^1}{V_B^1} - \frac{V_G^m}{V_B^m}\right) = \mu p - \mu q(N^1 g_G^1 + N^2 g_G^2) + \mu q\left(N^1 g_B^1 \frac{V_G^1}{V_B^1} + N^2 g_B^2 \frac{V_G^2}{V_B^2}\right) \tag{17}$$

From the Lemma in section 2 we know that for an individual with consumption $\tilde{g}^i \le G$, g_G^i and V_G^i are zero. For an individual with $\tilde{g}^i > G$, g_B^i will be zero. Hence, the term $g_B^i \frac{V_G^i}{V_B^i}$ will always vanish. We can therefore rewrite the expression as:

$$\mu N^1 \frac{V_G^1}{V_B^1} + \mu N^2 \frac{V_G^2}{V_B^2} + \rho V_B^m \left(\frac{V_G^1}{V_B^1} - \frac{V_G^m}{V_B^m}\right) = \mu p - \mu q(N^1 g_G^1 + N^2 g_G^2) \tag{18}$$

From Lemma 1 in section 2 we also know that $\frac{V_G^i}{V_B^i} = MRS_{gc}^i - q$, which is zero if the person is not rationed. Inserting this result and dividing through by μ and rearranging we obtain:

$$N^1 MRS_{gc}^1 - N^1 q + N^2 MRS_{gc}^2 - N^2 q = MRT_{Gc} + \mu^*(MRS_{gc}^m - MRS_{gc}^1) - q(N^1 g_G^1 + N^2 g_G^2) \tag{19}$$

We now consider the interpretation of (19) under various assumptions.

Proposition 6. If $\tilde{g}_L^i = 0$ the Samuelson rule $\sum MRS = MRT$ applies.

Proof: The fact that $\tilde{g}_L^i = 0$ and non-inferiority of g imply that $\tilde{g}^1 = \tilde{g}^m < \tilde{g}^2$. It also implies that $MRS_{gc}^1 = MRS_{gc}^m$. Hence, the self-selection term in eq. (19) vanishes. From proposition 1 we know that $G \leq \tilde{g}^2$. Suppose first that $\tilde{g}^1 < G \leq \tilde{g}^2$, i.e., that the type one individual is not rationed. This implies in accordance with our assumption above that the Hicksian substitution effect is strictly negative. It follows that the first term on the right hand side of eq. (13) is zero, and the second term is strictly negative or zero according as q is strictly positive or zero. Hence, from (11) and (13) q is zero, and the q-term vanishes from (19). Suppose next that $G < \tilde{g}^1 < \tilde{g}^2$, that is, both types are rationed. Then $g_G^1 = g_G^2 = 1$ and the q-terms of (19) cancel out. This implies that we are left with the Samuelson rule.

Proposition 7: If $\tilde{g}_L^i < 0$ the rule for provision of the public good takes the form:

$$N^1 MRS_{gc}^1 + N^2 MRS_{gc}^2 = MRT_{Gc} + \mu^*(MRS_{gc}^m - MRS_{gc}^1)$$

Proof: If $\tilde{g}_L^i < 0$ it will be true that $\tilde{g}^m < \tilde{g}^1$, $\tilde{g}^m < \tilde{g}^2$ and $(MRS_{gc}^m - MRS_{gc}^1)$<0. If at least one of the actual persons is not rationed it follows from (11) and (13) that q is zero. If both actual persons are rationed $g_G^1 = g_G^2 = 1$ and the q-terms of (19) will cancel out. Hence, the q-terms will always disappear from the expression. However, the self-selection term remains.

Proposition 8: If $\tilde{g}_L^i > 0$ the following regimes are possible

i) Persons of type one and the mimicker are not rationed ($q > \overline{q}$ as defined in Proposition 5), and persons of type two are rationed. The public provision rule has the form $N^2 MRS_{gc}^2 = MRT_{Gc}$.

ii) Persons of type one are not rationed ($q \geq \overline{q}$), while the mimicker and persons of type two are rationed. The public provision rule has the form $N^2 MRS_{gc}^2 = MRT_{Gc} + \mu^*(MRS_{gc}^m - q)$.

iii) Persons of both types and consequently the mimicker are rationed ($q < \overline{q}$). The public provision rule has the form $N^1 MRS_{gc}^1 + N^2 MRS_{gc}^2 = MRT_{Gc} + \mu^*(MRS_{gc}^m - MRS_{gc}^1)$

iv) No type is actually rationed ($q > \overline{q}$). Only the mimicker is rationed. The public provision rule has the form $N^1 MRS_{gc}^1 + N^2 MRS_{gc}^2 = MRT_{Gc} + \mu^*(MRS_{gc}^m - MRS_{gc}^1)$.

Proof : The proof is straightforward by observing that the q-terms of a rationed person *i* cancel out in (19), and recalling that for a person who is not rationed MRS_{gc} =q.

The last case is a special one. Since $G \le \max\{\tilde{g}^1, \tilde{g}^2\}$, this case emerges only if we have the very special coincidence that $\tilde{g}^1 = \tilde{g}^2 = G$. Then there can only be a loss from increasing *G*. However, it is possible to reduce *G* by one unit, which will imply that both types of persons and the mimicker have to reduce consumption by one unit, and the production cost is reduced[7]. Optimality requires that there is no net gain from such a cutback as prescribed by the optimality condition above.

The intuition for the results above is pretty straightforward in most cases. There is a gain from deviating from the Samuelson rule if by doing so one succeeds in relaxing the self-selection constraint. This will happen if the low-ability person and the mimicker behave differently and can be discriminated between. If the mimicker, enjoying more leisure, has a lower marginal valuation of *g,* as in Proposition 7, the public good should be oversupplied compared to the Samuelson rule. Suppose that departing from the Samuelson optimum, an additional unit of *G* is supplied. The high- and low-ability persons are charged through their tax liabilities to be left equally well off. Then the mimicker is made worse off as his lower valuation of *G* does not compensate for the tax increase. Mimicking is being deterred. However, if the mimicker's valuation exceeds that of the low-ability person the mimicker will gain from a tax-paid increase in *G*, but will be made worse off if *G* is decreased. Hence *G* should be undersupplied relative to the Samuelson rule as prescribed by Proposition 8 ii-iv.

If the mimicker and the low-skill person have the same valuation of *g*, as in Proposition 6, a change in the supply of *G* and compensating tax changes will not discriminate between the two. Nothing can be gained by deviating from the Samuelson rule as stated by Proposition 6.

A special case arises when the low-ability type is not rationed. Then an increase in the provision of *G* will not affect the consumption of this type and the sum of marginal benefits of increasing *G* is equal to the sum taken solely across type two individuals. If the low ability type and the mimicker are both left unaffected by an increase in *G*, no discrimination of the mimicker can be achieved, we have a special case of the Samuelson rule as in case i of Proposition 8. If relaxing the mimicking constraint is possible case ii gives us a special case iii in Proposition 8.

[7] Mathematically the *g*-function is not continuously differentiable at *G* as a kink arises when the demand function hit the rationing constraint.

4. Excludable intermediate public good

We shall consider the same two-type, asymmetric information model as above with the following qualifications. An excludable public good is used as an intermediate good in the production sector, but we shall now include no public good in the consumption bundle.

There are two consumer goods. We assume that each good is provided by producers each producing a fixed quantity, normalised to unity. Let e_i be the labour cost in efficiency units per unit output of commodity *i*. The unit cost is assumed to be a function of the amount of the public good being used in the production of that commodity. The idea is that the use of the public good makes production more efficient and economizes on the use of labour.[8] We assume there is free entry and exit of firms. Hence, the quantities produced of the two consumption goods vary as the number of firms varies.

Assuming that each producer has a fixed output is a simplifying assumption. If a producer could make any acquired amount of *g* available at no further cost to all parts of the production, there would be economies of scale in production. Concentrating production would economise on the cost of acquiring the public good. However, there may be other disadvantages from having large production units, but with the cost of acquiring g still being one determinant of the optimum size. We do not want to model these various factors, which would take us far beyond the focus of our discussion. We observe that there are indeed sectors with many separate producers making use of public goods, and we want to consider such a setting without modeling too many details of the production structure.

We assume that we can tax the consumer goods using commodity taxes. The objective of the social planner is as before to achieve information-constrained Pareto efficiency. Given this setup one can derive the following proposition.

Proposition 9: Producers using the intermediate public good as input should face a zero price. Producers in both sectors are rationed ($\tilde{g}_i = G$) or there is rationing in one sector and satiation with respect to the public good in the other. The optimum supply of the intermediate public good is the amount that yields production efficiency in the first best sense.

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[8] An alternative perspective would be to assume that the effect of *g* is to raise the quality of the goods being produced rather than to lower their production cost. Essentially this is the same thing because producing higher quality at the same cost can be perceived as producing a larger amount at a given cost, which is equivalent to a lower cost per unit.

The intuition for this result is strong. We will therefore only give an informal argument here and present a rigorous proof in an Appendix.

The cost of charging a price for the intermediate excludable public good is that production gets distorted. We decrease the use of the public good although there is a zero social marginal cost of using it. The potential gain of charging a price is to improve efficiency by alleviating the asymmetric information. This is achieved by changing consumer prices so as to discourage mimicking. However, since we have access to taxes on the consumer goods we already have instruments available to set consumer prices. Hence the use of a price for an intermediate excludable public good only has costs and no gains beyond those already attainable.

If for some reason the desirable commodity taxes are not available there may be a case for achieving some of the same effect through a policy that violates production efficiency. Suppose it is desirable to increase the price of a commodity, and imposing a tax is not feasible. Then by making production in that sector less efficient, the price can be increased, but at the expense of production efficiency. However, we shall refrain from further analysis of such a regime.

5. Concluding remarks

We have used the Stiglitz-Stern model to study the respective conditions under which it is beneficial or harmful to set a positive price on a publicly provided excludable public good. As in the optimal commodity tax literature we find that the potential role of setting the price above marginal cost is to discourage high-skilled people from mimicking low ability individuals. Thus, whether the price instrument should be used or not hinges on how the evaluation of the public good depends on the amount of leisure, which is the only feature of consumption distinguishing a high-skilled mimicker from a low-skilled person. We first consider the case where the excludable public god is a final consumer good. We find that it can be gainful to use the price instrument only if the demand for an excludable public good increases with the amount of leisure.

The reason why charging a price may be beneficial in this case is that the mimicker has a larger consumption of the excludable public good and is hurt more than a low-skilled person by a price. As such this is not a surprising result. However, the positive leisure impact is not a sufficient condition for this outcome. Only if the demand for the excludable public good increases in leisure and the optimum is such that the low-skilled person is not rationed, is it desirable to set a positive price for the public good. A finding, which is somewhat surprising on intuitive grounds is that even when a strictly positive price is optimal, introducing a positive but too low price may be harmful compared to charging no price at all (even when we neglect the administrative cost of charging a price).

This is a result that ought to be of policy interest. It has been one of the achievements of second best theory to demonstrate that there may be a case for positive prices and taxes that are not valid in a first best regime. Implementing the optimum is a different story. There may be a long way to go to acquire the empirical knowledge required to hit the exact target in terms of an optimum. However, in many cases, e.g. in the conventional optimum commodity tax regime, we know that setting the price at a sufficiently modest level above marginal cost will at least dominate marginal cost pricing. In the present situation this is different. Not only is it uncertain whether there is a case for a positive price, but even if there is, undershooting the optimum level may prove more harmful than charging no price at all. Hence there is a heavy information requirement not only for hitting the optimum, but even for making sure that a reform is at all welfare improving. From a policy perspective it is tempting to conclude that in practice there is at best an uneasy case for a positive price for an excludable public good.

We also characterize how the Samuelson rule is modified depending on how the demand for the public good varies with the amount of leisure.

Finally, we have studied the case where the excludable public good is an intermediate good in a framework where optimal commodity taxes on the consumer goods are available. Given the optimal use of these instruments we find that nothing can be gained by using the quantity of the public good or the price on the public good for deterring mimicking. The rule for determining the quantity of the public good implies production efficiency.

When a public good is excludable private firms could provide the good. Such provision has been studied in Brito and Oakland (1980) and Burns and Walsh (1981). They study the case where there is a natural monopoly. Oakland (1974) studied the case where the provision is from firms operating under conditions of perfect competition. Both forms of provision involve inefficiencies. In the monopoly case we know that the quantity in general tends to be too low. In the case with perfect competition the fact that there must be a large number of firms means that the public good will not be produced under conditions that take full advantage of the fact that it is a public good. However, also the public provision scheme involves inefficiencies. It would therefore be of interest to compare public provision and provision from private firms. However, we leave that for future research.

Appendix

The producer will minimize the unit cost of production including the cost of acquiring g_i, $e_i(g_i) + g_i q$, implying that in the case of an interior solution

$$-e_i' = q . \tag{A1}$$

The cost saving at the margin is equated to the price, or the producer may be rationed such that

$$-e_i'(g_i) > q \text{ and } g_i = G, \ i = 1,2 \tag{A2}$$

Neglecting the public input we would have the standard two-type, non-linear income tax model with two consumption commodities. (See for instance Edwards et al. (1994). We know that in general efficiency a Pareto improvement can be achieved by supplementing the income tax with commodity taxes (Christiansen (1984), Edwards et al. (1994)). Let us assume that such commodity taxes are in place, let t_i denote the unit tax imposed on commodity *i*, and let p_i be the consumer price. Under competitive, free-entry conditions the equilibrium market price equals the producers' marginal and unit cost which consist of the production cost e_i (in terms of efficiency labour units), the cost of buying the required input of the public good $(g_i q)$ and the tax.

$$p_i = e_i(g_i) + g_i q + t_i \tag{A3}$$

The Pareto efficiency problem can then be formulated as

$$\max V^1(B^1, Y^1, p_1, p_2)$$

w.r.t.

$$B^1, Y^1, B^2, Y^2, G, t_1, t_2, q$$

s.t

$$V^2(B^2, Y^2, p_1, p_2) \geq \overline{V}^2 \tag{A4}$$

$$V^2(B^2, Y^2, p_1, p_2) \geq V^m(B^1, Y^1, p_1, p_2) \tag{A5}$$

$$N^1(Y^1 - B^1) + N^2(Y^2 - B^2) + t_1 x_1 + t_2 x_2 + g_1 q x_1 + g_2 q x_2 - G \geq 0 \tag{A6}$$

$$p_i = e_i(g_i) + g_i q + t_i \quad i = 1,2 \tag{A7}$$

$$-e_i'(g_i) = q \text{ and } g_i \le G\text{, } i = 1{,}2 \tag{A8}$$

or $e_i'(g_i) > q$ and $g_i = G,\ i = 1{,}2$

A necessary condition for a Pareto efficient allocation is that it is impossible to use the instruments of the government in such a way that both types of persons are kept equally well off while the government revenue is increased. If that were possible the additional revenue could be used to increase the disposable income of the high-skilled person. The self-selection constraint would not be violated and a Pareto improvement would be obtained. We will now focus on such necessary conditions, which will allow us to draw some important conclusions about price setting without going through a detailed optimization with respect to all the instruments of the model above.

We do the following exercise. We consider changes in *q* and *G*. At the same time we keep the gross and net incomes of both types of persons unchanged and adjust the commodity taxes in such a way that the consumer prices of both goods remain the same. Then obviously both types of persons stay equally well off, the mimicking constraint remains satisfied and demand for the consumer goods is left unchanged. We can then concentrate on the effect on government revenue.

Pegging the consumer prices at fixed values $\overline{p}_1$ and $\overline{p}_2$ the commodity taxes are restricted by

$$e_i(g_i) + g_i q + t_i = \overline{p}_i \quad i = 1{,}2 \tag{A9}$$

The necessary adjustments in response to changes in *q* and *G* are given by

$$\frac{\partial t_i}{\partial q} = -\left(e_i' + q\right)\frac{dg_i}{dq} - g_i \tag{A10}$$

$$\frac{\partial t_i}{\partial g_i} = -e_i' - q \tag{A11}$$

Let us then explore how the government revenue is affected by changing *q* and *G* subject to the constraint imposed above. Taking net and gross incomes and hence the income tax revenue as fixed we focus on the net revenue from *G* and commodity taxes

$$R = t_1 x_1 + t_2 x_2 + q g_1 x_1 + q g_2 x_2 - G \tag{A12}$$

We differentiate with respect to *q* and then insert the expressions for commodity tax changes derived above to obtain

$$\frac{\partial R}{\partial q} = x_1 \frac{\partial t_1}{\partial q} + x_2 \frac{\partial t_2}{\partial q} + g_1 x_1 + g_2 x_2 + q x_1 \frac{dg_1}{dq} + q x_2 \frac{dg_2}{dq}$$

$$= x_1\left(-e_1^{'} - q\right)\frac{dg_1}{dq} - g_1 x_1 + x_2\left(-e_2^{'} - q\right)\frac{dg_2}{dq}$$
$$- g_2 x_2 + g_1 x_1 + g_2 x_2 + q x_1 \frac{dg_1}{dq} + q x_2 \frac{dg_2}{dq}$$

$$= -e_1^{'} \frac{dg_1}{dq} - e_2^{'} \frac{dg_2}{dq} = 0$$

implying that

$$-\frac{dg_i}{dq} = 0 \text{ or } e_i^{'} = 0. \qquad \text{(A13)}$$

Either the producer is not rationed, and the price is zero (such that $e_i^{'} = 0$), or the producer is rationed (such that $-dg_i / dq = 0$).

The optimum amount of G is characterized by the first order condition

$$\frac{\partial R}{\partial G} = x_1 \frac{\partial t_1}{\partial g_1}\frac{dg_1}{dG} + x_2 \frac{\partial t_2}{\partial g_2}\frac{dg_2}{dG} + q x_1 \frac{dg_1}{dG} + q x_2 \frac{dg_2}{dG} - 1$$

$$= x_1\left(-e_1^{'} - q\right)\frac{dg_1}{dG} + x_2\left(-e_2^{'} - q\right)\frac{dg_1}{dG} + q x_1 \frac{dg_1}{dG} + q x_2 \frac{dg_2}{dG} - 1$$

$$= -e_1^{'} x_1 \frac{dg_1}{dG} - e_2^{'} x_2 \frac{dg_2}{dG} - 1 = 0,$$

which is equivalent to

$$-e_1^{'} x_1 \frac{dg_1}{dG} - e_2^{'} x_2 \frac{dg_2}{dG} = 1 \qquad \text{(A14)}$$

We know from above that that q=0 or producers are rationed. If q=0 and there is no rationing also $e_i^{'} = 0$ and (A14) does not hold. Hence we realize that producers in both sectors are rationed, or one producer is rationed and one producers is at a satiation point where $e_i^{'} = 0$. For a rationed producer $dg_i / dG = 1$.

We can conclude that

$$-e_1^{'} x_1 - e_2^{'} x_2 = 1 \qquad \text{(A15)}$$

which holds with at least one rationed producer. We note that the gain from using an extra unit of g in the production of good i is a cost saving per unit equal to $-e_i^{'}$. The total cost saving at an output level x_i is then $-e_i^{'}x_i$. The sum of marginal benefits is then equal to the left hand side of (A15). The right hand side is the marginal cost of providing *G*, so (A15) is the Samuelson rule for an intermediate public good. We can conclude that the Samuelson rule is valid. We can summarize the findings as in Proposition 9 in section 4.

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