Fixed Price plus Rationing: An Experiment *

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Abstract

This paper reports a theoretical and experimental study on a fixed price mechanism by which, if aggregate demand exceeds supply, all bidders are proportionally rationed. If bidders face demand uncertainty, equilibrium requires them to overstate their true demand to prevent the effect of rationing; for prices sufficiently low to yield rationing under all demand scenarios, bidders should bid up to the upper limit. Our experimental study yields the following conclusion. Despite of a significant proportion of equilibrium play, subjects tend to (under)overbid the equilibrium strategy when rationing is (high) low, with only this latter effect being persistent over time. We explain the experimental evidence by a simple model in which the probability of a deviation is decreasing in the expected loss associated with it.

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1 Introduction

Prices are not always set such the market clears. Instead, we often observe non–price rationing of buyers, for different reasons. In initial public offerings, for example, the seller frequently sets a price at which he expects excess demand to be able to reward large investors, by way of some preferential treatment, for their information revelation.

In other situations, where demand is uncertain, the seller might not simply be able to set the market clearing price. In this case, two main classes of mechanisms have been proposed as optimal solution to this problem: auctions and fixed price mechanisms. As for the latter, since supply is fixed (and the price is chosen before actual demand reveals), the mechanism has to include a rationing device in case demand exceeds supply. While axiomatic properties of different rationing schemes have been explored extensively in the economic literature,[1] the strategic behavior of buyers who expect to be rationed has up to date received little attention.[2] The few papers that explicitly analyze the incentives in market games that may involve rationing of buyers find that those mechanisms are often desirable for the seller. In case a common value is sold, Bulow and Klemperer [6] show that prices which result by rationing can even be optimal. Gilbert and Klemperer [12] come to the same conclusion for situations where customers must make sunk investments to enter a market. In a private values setting, Bierbaum and Grimm [5] analyze a fixed price mechanism where (a continuum of) buyers are proportionally rationed in case of excess demand. They find that, if total demand is uncertain, bidders overstate their true demand in order to alleviate the effects of being rationed in high demand scenarios. This allows the seller to set the fixed price at a rather high level which yields the surprising result that the fixed price mechanism outperforms alternative selling mechanisms (such as a uniform price auction) with respect to a variety of criteria: revenue, variability of revenue in different demand scenarios, and minimum revenue that is raised if demand turns out to be low.

[1] See, for example, Herrero and Vilar [15], or Moulin [22]. Here rationing usually occurs because the allocating authority is not allowed to use prices in order to ration, e. g. in bankruptcy problems if claims are known but exceed the pie to be allocated.

[2] Noticeable exceptions are the papers of Chun [8], Dagan *et al.* [10], Moreno-Ternero [21] and Herrero [14]. Herrero *et al.* [16] provide an experimental study on the strategic behavior induced by rationing in the context of bankruptcy problems.

The above findings contribute to explain the frequent use of mechanisms that involve rationing of buyers.[3] This motivated us to experimentally study bidding behavior in a fixed price mechanism with proportional rationing (FPM) quite similar to the one analyzed in Bierbaum and Grimm [5]. Here we have to account for the fact that in our experiment we were restricted to the analysis of small markets (i.e. markets with a finite number of players). In small markets, however, the uniform price auction analyzed as a benchmark mechanism in Bierbaum and Grimm [5] is not an attractive choice for the seller, since bidders have an incentive to strategically reduce demand in order to lower the price they have to pay per unit.[4]

By analogy with Bierbaum and Grimm [5], our experimental design is based on a model where neither the buyers, nor the seller know total demand due to uncertainty about the number of (identical) buyers. The seller, who is endowed with a given quantity of a divisible good, sets a fixed price, and then, buyers are asked to submit a quantity bid at this price. They are proportionally rationed in case the total quantity bid for exceeds supply, otherwise they receive their bid.[5] In the experiment we were interested only in buyers' bidding behavior. Therefore, the seller's role was played by a computer, i. e. in each round a price was randomly chosen from the range where demand for the good was positive, which allowed us to extract complete bid functions.

We also study an "incentive compatible mechanism" (ICM), which only differs from FPM with respect to the fact that buyers are never rationed. Since truthful bidding strictly dominates any other strategy in ICM, we interpret ICM as a mechanism that reproduces the strategic properties of the uniform price auction in large markets. In addition, given that the two mechanisms only differ with respect to the presence of the rationing device, we can use ICM as a control treatment of the experimental results of FPM.

We shall now give a quick overview of our main results.

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[3]For example, Chemmanur and Liu [7] report that FPM is increasingly adopted as a selling mechanism in initial public offerings. They explain this evidence by constructing a two-period model: in period 1 the seller sets the price and bidders buy; in period 2 aftermarket takes place. Since the seller may have incentives to participate himself in the aftermarket, he sets in period 1 a higher price which is higher than the market-clearing price.

[4]See, for example, Ausubel and Cramton [4].

[5]This is basically the model analyzed in Bierbaum and Grimm [5]. The only differences are that Bierbaum and Grimm analyze large markets (whereas in our experiment the number of potential buyers is small) and moreover they allow for different types of buyers.

First, we show that Bierbaum and Grimm's [5] theoretical results on FPM are maintained in the context of small markets (i.e. a finite number of buyers). In particular, at high prices rationing never occurs and therefore bidding truthfully is optimal; at low prices bidders are always rationed and thus, in equilibrium, they demand the highest possible quantity (if any); at intermediate prices, where truthful bidding would yield to rationing only when demand is high, bidders overstate their true demand, but only moderately.

As for the experimental evidence, subjects play extremely well ICM, where truthful bidding emerges as unanimous behavior since the very beginning. In FPM, behavior converges to equilibrium for very high and very low prices, where the equilibrium strategy is relatively easy to figure out. For intermediate prices, where the equilibrium is strategically more complex, some noise remains. As time proceeds, bidders even move away a bit (but not far) from the risk neutral equilibrium prediction in the direction of overbidding. Given our experimental evidence, a profit maximizing seller would then opt for FPM (i. e. commit to a fixed price), not only for the theoretical reasons highlighted by Bierbaum and Grimm [5] (and confirmed by our theoretical analysis), but also because *overbidding with respect to equilibrium takes place exactly in the price range which maximizes seller's revenues*, yielding profits even higher what seller could extract if he could be able to act as a monopolist in all demand scenarios.

Overall, the explanatory power of the theory seems impressive, especially if compared with that of standard auction theory models.[6] These considerations notwithstanding, panel data estimations yield two significant deviations from the behavior predicted by the risk neutral Nash equilibrium (RNNE) of the game: at intermediate prices, bids are at a higher level but as price sensitive as predicted. At low prices we observe — contrary to the RNNE prediction — price sensitivity of bids and underbidding of RNNE.

We also find that these deviations cannot be explained by risk-attitude considerations, but are jointly consistent with the hypothesis of noisy directional learning (Anderson *et al.* [2]), where bidders adjust their actions in the direction of higher expected profits but do so subject to some exogenous noise (with the probability of an error being decreasing with the associated

[6]Experimental studies of multi unit auction formats find all kinds of out of equilibrium behavior that crucially affects the relative performance of different multi unit auction rules. See, e. g. Kagel and Levin [18], List and Lucking–Reiley [19], and Engelmann and Grimm [11].

expected loss). In the steady state equilibrium of this process, players' behavior is given by probability distributions over the strategy space that constitute a Quantal Response Equilibrium (QRE) of the game (McKelvey and Palfrey [20]). (Maximum likelihood) estimations of the corresponding QRE for each price level match the observed behavioral pattern: slight underbidding of RNNE together with some price sensitivity at low prices and simultaneously overbidding of RNNE at intermediate prices. They also confirm the intuition that behavior is less noisy at prices where the equilibrium is easier to figure out (i.e. high and low prices) than elsewhere. At the same time they explain why at extreme prices behavior converges to RNNE, while at intermediate prices it does not.

The remainder of the paper is arranged as follows. Theoretical properties of FPM is what we investigate first, in Section 2. Experimental conditions are described in Section 3. Section 4, devoted to experimental results, is divided in three parts. Descriptive statistics are presented first, followed by some panel data regressions in which we check the robustness of equilibrium predictions. We then conclude by checking whether risk aversion or bounded rationality may explain the discrepancy between theory and evidence. Conclusions and guidelines for future research are listed in Section 6, followed by an Appendix containing the proofs of the theoretical results of Section 2 and the experimental instructions.

2 Theoretical Background and Hypotheses

In section 2.1, we state a simple model and introduce the two mechanisms that will be compared in our experiment. Then, in sections 2.2 and 2.3 we characterize the equilibria of those mechanisms which leads to the theoretical predictions stated in section 2.4.

2.1 The Model

Consider a seller who has a fixed quantity (normalized to 1) of a perfectly divisible good and does not know the number of potential buyers interested in the good. By analogy with our experimental conditions let us assume that n, the number of buyers, is either 2 or 4, where the probability that n is 2 (4) is λ $(1-\lambda)$. Throughout the paper we shall refer to the case of $n = 2$ $(n = 4)$ as the "low" ("high") demand scenario. We assume that all potential buyers are identical. In particular, each buyer i has decreasing linear demand for the good,

$$x_i(p) = 1 - p. \tag{1}$$

In what follows we provide a theoretical analysis of two mechanisms: the Fixed Price Mechanism (FPM) and an Incentive Compatible Mechanism (ICM), which is identical to FPM apart from the fact that bidders are never rationed (i.e. they always get what they ask for).

2.2 FPM

We model FPM as a 3-stage, 4-player game with incomplete information. At Stage 0 Nature moves, deciding market size n. At this stage, Nature also chooses which players are informed about the existence of the good and thus, will participate in the market. Either all 4 players are informed (event with probability $1-\lambda$), or 2 players out of the 4, where each of the possible six pairs is informed with equal probability $\lambda/6$. Without loss of generality we (ex post) label the participants in a market with two players by "1" and "2". If $n = 4$ all players (then labeled "1" to "4") will participate in the market. By this thought exercise, the payoff functions for all 4 players are well defined in both scenarios and symmetric. Therefore, in what follows we look at the payoff of the representative player 1.

At the remaining two stages, the seller and the buyers move in sequence. At Stage 1, the seller announces a fixed price and an upper limit on individual bids $(p, \overline{d}) \in [0,1] \times [0, R]$. At Stage 2, each participating buyer i announces the quantity he demands at the posted price, $d_i \in [0, \overline{d}]$, which we will call buyer i's bid. If the aggregate quantity bid falls short of supply, each buyer obtains his bid, otherwise buyers are proportionally rationed. Each buyer has to pay the posted price for each unit he receives.

In order to formally describe proportional rationing, we need to introduce some notation. Let $d \equiv \{d_i\}$ denote the vector of bids and denote by $d_{-i} \equiv \{d_j\}_{j \neq i}$ the vector of bids by all players but i. Then, the aggregate quantity bid is given by $\sum_{i=1}^{n} d_i$, $n \in \{2, 4\}$. Under the proportional rule, buyer 1 demanding d_1 receives a final quantity of $d_1 Q^n(d)$, where

$$Q^n(d) = \min\{1, \frac{1}{\sum_{j=1}^{n} d_j}\}, \quad n \in \{2, 4\}. \tag{2}$$

We can now specify players' expected payoffs. Let "0" index the seller's player position and recall that we only consider the representative bidder "1". Now, for a given pair $(p, \overline{d})$, let $\pi_i : [0, \overline{d}]^4 \to R$ denote player i's expected payoff, given by

$$\pi_0(d) = \lambda Q^2(d) \sum_{j=1}^{2} d_j \cdot p + (1-\lambda) Q^4(d) \sum_{j=1}^{4} d_j \cdot p \tag{3}$$

and

$$\pi_1(d_1, d_{-1}) = \lambda \int_0^{d_1 Q^2(d_1, d_{-1})} (1-x-p)dx + (1-\lambda) \int_0^{d_1 Q^4(d_1, d_{-1})} (1-x-p)dx. \tag{4}$$

2.2.1 Stage 2: the bidding stage

We begin by characterizing bidders' optimal behavior for any given price p and upper limit on bids $\overline{d} \geq 1$. Proposition 1 shows that there is a unique symmetric equilibrium (in pure strategies) of the bidding stage for almost all prices.

Proposition 1 *Let* $p_e = \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$ *and* $p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.

- $p \in [\frac{3}{4}, 1]$: *unique equilibrium* $d_i(p) = 1 - p$ *for all* i.
- $p \in [0, p_e)$: *unique equilibrium* $d_i(p) = \overline{d}$ *for all* i.
- $p \in [p_e, p_m)$: *two equilibria,* $d_i(p) = \overline{d}$ *for all* i *and* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *for all* i.
- $p = p_m$: *one equilibrium where* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *and a continuum of equilibria where* $d_1 + d_2 \geq 1$*: all* d *with* $d_i = d_j$*, for all* i, j.
- $p \in (p_m, \frac{3}{4})$: *unique equilibrium* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *for all* i.

Proof. In the Appendix. ■

Figure 1 provides a graphical sketch of the structure of the game's equilibria, as characterized by Proposition 1.

Put Figure 1 about here

As Figure 1 shows, the interval of possible prices can be split up into three subintervals:

- *High prices:* $p \in [\frac{3}{4}, 1]$. Buyers' aggregate demand never exceeds supply. Therefore, rationing plays no role and buyers' optimal strategy is to simply to bid truthfully.
- *Low prices:* $p \in [0, p_e)$. Large excess demand in the high demand scenario (and, at prices below $\frac{1}{2}$, also excess demand in the low demand scenario) yields an incentive to overstate true demand high enough to lead to rationing in both scenarios. Thus, bids explode and the only equilibrium is that every buyer bids as much as possible.
- *Intermediate prices:* $p \in [p_e, \frac{3}{4})$. Excess demand in the high demand scenario is moderate, which still yields an incentive to overstate demand. The optimal bids solve a trade-off between getting too much in the low demand scenario (where no rationing takes place) and getting too little in the high demand scenario (where buyers are rationed).[7]

2.2.2 Stage 1: price and upper-bound fixing

In Stage 1 the seller chooses the profit maximizing price anticipating buyers' behavior at Stage 2, not knowing how many of them will participate in the market. Taking into account buyers' equilibrium bids, only prices in the interval $p \in [p_e, \frac{3}{4}]$ can be rational choices of the seller: at p_e he sells the whole quantity in both demand scenarios in any equilibrium of the continuation game and this would definitely lower his profit if he posted a lower price. Notice that $p = \frac{3}{4}$ is the linear monopoly price given high demand and thus, a higher price cannot be profit maximizing under demand uncertainty.

Proposition 2 (Equilibria of Stage 1) *(i) An equilibrium of FPM always exists.*

[7] For prices $p \in [p_e, p_m]$ there is also an equilibrium where demand explodes.

(ii) In every possible equilibrium of Stage 2 the entire quantity is sold at $p_e \leq \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$ in every demand scenario.

(iii) The upper bound on individual bids is $\bar{d} \geq 1$.

(iv) The seller's revenue is bounded below by p_e and may be higher.

(v) No equilibrium exists such that bidders have an incentive to trade after FPM was played.

Proof. In the Appendix. ■

2.3 ICM

As we already explained in the introduction, we also tested in the lab another fixed-price mechanism (we called it ICM) which only differs from FPM with respect to the fact that bidders also get what they ask for (i..e there is no rationing). In this case, bidder 1's payoff function (4) simplifies to

$$\pi_1(d_1, d_{-1}) = \int_0^{d_1} (1 - x - p)dx = \frac{1}{2}d_1(2 - 2p - d_1). \tag{5}$$

The absence of rationing basically breaks any strategic link among the players, who basically face a simple decision problem, whose solution is truthful bidding.

Proposition 3 *In ICM each bidder's optimal bid equals his true demand, i. e*

$$d_i^*(p) = 1 - p. \tag{6}$$

In our experiment, ICM mainly serves as a robustness check for our experimental design to evaluate whether subjects bid truthfully when it is a strictly dominant strategy to do so. The feature that in ICM bids have no impact on price replicates the situation of uniform prices auctions in large markets (e. g. IPOs), where bidders cannot lower the price by reducing demand and thus, truthful bidding is a dominant strategy. However, a crucial difference between those markets and ICM is that in large auctions bidders are well aware of the fact that they interact with other players, which may crucially influence their behavior.

2.4 Hypotheses from the theory

The theoretical analysis of Section 2 yields the following testable hypotheses.

1. *Individual behavior.* According to theory, subjects should bid truthfully in ICM, when they are not exposed to the risk of rationing. By contrast, in FPM, subjects should bid truthfully only for prices sufficiently high (i.e., for all $p \geq \frac{3}{4}$). Overbidding should be moderate for prices $\frac{3}{4} > p > p_m$, with demand exploding for all $p < p_e$.

2. *Symmetry.* As Proposition 1 shows, in FPM everybody should choose the same bid (i.e. all the equilibria of the game are symmetric).

3. *Seller's profit.* According to Proposition **??**, the profit maximizing price should be greater than p_e.

4. *FPM vs. ICM.* Since ICM replicates exactly the strategic framework of FPM except for the absence of rationing, we shall use ICM data as a robustness check of our experimental conditions. In the absence of rationing, truthful bidding becomes the "obvious way to play the game", insofar this behavior is strictly dominant (i. e. independent of others' behavior). Any significant difference between actual behavior and theoretical prediction in the ICM session should be properly "discounted" when analyzing the FPM data.

3 The experimental design

In what follows, we describe the features of the experiment in detail.

3.1 Subjects

The experiment was conducted in three subsequent sessions -two sessions devoted to FPM, one to ICM- in May, 2004. A total of 72 students (24 per session) were recruited among the undergraduate student population of the Universidad de Alicante -mainly, undergraduate students from the Economics Department with no (or very little) prior exposure to auction theory. The FPM sessions lasted approximately 120' each, while the ICM session was slightly shorter (100' approx.).

Subjects were given a written copy of the instructions in Spanish, together with a table indicating their monetary payoff associated with a grid of 21×21=441 representative price-quantities pairs.[8] Instructions were read aloud and we let subjects ask about any doubt they may have had. In addition, a self-paced, interactive computer program proposed three control questions, to make sure that subjects understood the main features of game.

3.2 Treatment

In each session, subjects played 84 rounds of the corresponding mechanism. As for the FPM sessions, subjects were divided into three *cohorts* of 8, with subjects from different cohorts never interacting with each other throughout the session. As for the ICM session, every subject can be considered as a "cohort of size one".

Compared with the scale used in Section 2, in the experiment, all prices and quantities were multiplied by 10.[9] We did this to mitigate "integer" frame problems. Within each round $t = 1, ..., 84$, group size, composition and prices were randomly determined. Let *period* $T_k = \{t : 21(k-1) < t \leq 21k\}, k = 1, ..., 4$, be the subsequence of the $k-$th 21 rounds. Within each period T_k, subjects experienced each and every possible price $p \in P = \{0, .5, 1, ..., 10\}$, the sequence of prices randomly selected within each period being different for each cohort. After being told the current price, subjects had to determine their bid, $d_i(p) \in [0, 10]$, for that round (subjects could not bid more than the entire supply). By this design, we are able to characterize 4 complete individual bid schedules, one for each period. Moreover, in each round t, a (uniform) random draw fixed the group size $n \in \{2, 4\}$ independently for each cohort (i.e. $\lambda = \frac{1}{2}$).

Given all these design features, we shall read the data under the assumption that the history of each individual cohort (6 for FPM, 24 for ICM) corresponds to an independent observation of the corresponding mechanism.

[8]The complete set of instructions, translated into English, can be found in the Appendix.

[9]Neverthelesss, in presenting the results, we shall not modify the scale to facilitate comparison with the content of Section 2.

3.3 Payoffs

Subjects participating in the FPM (ICM) sessions received 2000 (1500) ptas. (1 euro is approx. 166 ptas.) just to show up. These stakes were chosen to exclude the possibility of bankruptcy.

3.4 Ex-post information

After each round, subjects were informed on the payoff relevant information. As for FPM, this refers to group size, summary information on the aggregate behavior of their own group (both in terms of the total sum of individual bids, but also of the average bid(s) of the other component(s) of their group), the quantity of the good they actually received (FPM), together with the monetary payoff associated with it. As for ICM, subjects were simply told about the result of their individual bid. The same information was also given in the form of a *History Table*, so that subjects could easily review the results of all the rounds that they had played so far.

4 Results

In reporting our experimental results, we begin by looking at some descriptive statistics which summarize the evolution of subjects' aggregate behavior over time in the two experimental settings, ICM and FPM. We then estimate some dynamic panel data regressions to explore the behavioral properties of FPM and ICM. As for ICM, these regressions clearly show that equilibrium behavior explains almost perfectly subjects' behavior, at least in the last repetitions of the game. This is also true in the case of FPM, even if, in this case, our regressions unambiguously show consistent deviations from equilibrium behavior which do seem to persist over time. In short, people tend to overbid (underbid) the equilibrium strategy when rationing is less (more) severe.

4.1 Descriptive statistics

Figure 2 reports the descriptive statistics for our control treatment, ICM. Each row corresponds to a price, whereas columns list periods. Each cell of Figure 2 contains the average bid associated with the corresponding price-period pair (standard deviation reported within brackets).

Put Figure 2 about here

As Figure 2 shows, subjects played ICM extremely "well". Their behavior is close to equilibrium prediction since the very beginning, with some initial variance quickly vanishing over time. Out of 21 prices, in period 3 (4), *all* 24 subjects played *always* their dominant strategy in 19 (17) cases. Even when equilibrium play does not correspond to subjects' unanimous decision, deviations from the dominant strategy are negligible and only observed on behalf of few subjects.

Things are different when we move to FPM, whose aggregate statistics are reported in Figure 3. Consistently with the theoretical prediction, players now bid above their demand, even if they play the equilibrium less often than the ICM case.

Put Figure 3 about here

As we already know from Section 2, the structure of the equilibria of $FPM(p)$ crucially depends on the price level. Not surprisingly, also subjects' behavior is sensitive to prices, both with respect to aggregate behavior and its evolution over time.

By analogy with the discussion in Section 2, we shall present our experimental evidence for three broad price intervals, which turn out to be crucial not only in the theoretical analysis, but also to frame subjects' behavior in the experiment:

- $p \geq \frac{3}{4}$. Here we observe that, within the range of prices for which truthful bidding corresponds to the unique equilibrium, subjects start bidding slightly more than their demand, with overbidding gradually reducing in the last periods.
- $\frac{3}{4} > p \geq p_e \cong 0.568$. By contrast, for prices lower than $\frac{3}{4}$, subjects start bidding above their demand, with average bids increasing with time.
- $p < p_e$. Within the price-range for which demand explosion corresponds to the unique equilibrium, individual bids get very close to the maximum possible amount of 1. However, contrary to theoretical

prediction, average bids seem to be sensitive to prices: the lower the price the closer average bids get, in the last periods, to the upper limit.

Additional interesting information provided by Figure 3 comes from observing the evolution of standard deviations. By contrast with what happens in the case of ICM, they do not show a common time trend. We find prices for which standard deviation vanishes over time, but for others it decreases, fluctuates and in many cases it has no real trend. In other words, in the FPM sessions, subjects' play does not necessarily converge, as it happens in the case of ICM. Since standard deviations in the intermediate periods T_2 and T_3 are very similar to each other for most price levels, in the following discussion we shall focus our attention on the comparisons of standard deviations of the extreme periods T_1 and T_4.

Again, we shall discuss the issue separately for the three different price intervals:

- $p \geq \frac{3}{4}$. For high prices, initial variability is moderate and decreases with time. In this case, standard deviation evaluated in T_1 exceeds that evaluated in T_4 significantly, except for the case of $p = .9$, where the latter is slightly higher.

- $p < p_e$. For low prices, initial variability is higher than in the previous cases and decreases with time. This effect is more pronounced the lower the price. In particular, for $p \leq \frac{1}{4}$, standard deviation in T_1 is 50% higher than that evaluated in T_4.

- $\frac{3}{4} < p \leq p_e$. At intermediate prices, initial variability is still moderate and stays basically constant over time. Sometimes standard deviation evaluated in T_1 is higher than that evaluated in T_4, sometime is lower, but differences are small. This may suggest that, within this price range, subjects have not "learned" (at least, to the same extent as for other price ranges) how to play the equilibrium strategy.

This evidence suggests that equilibrium behavior is more likely to be achieved, at least in the long run, for very high and very low prices, with individual bids gradually converging to the corresponding equilibrium level, while for intermediate levels some initial variability persists.

Figure 4 provides a graphic sketch of the evolution of subjects' aggregate behavior, tracing the evolution of the inverse demand schedule in the 4 experimental periods.

Put Figure 4 about here

The y-axis of Figure 4 tracks prices, while the x-axis reports average bids. The dotted line corresponds to the equilibrium strategy as given by Proposition 1; the 4 grey lines correspond to aggregate average bid functions per period, with greyscale increasing with period. By analogy with the above discussion, subjects' aggregate behavior converges to equilibrium for prices at the extremes whereas, for intermediate prices, the reverse occurs.

In the following Figure 5 we trace the difference between actual behavior in FPM and theoretical prediction for the 4 time periods as a function of price (reported in the x-axis).

Put Figure 5 about here

As Figure 5 shows, p_e can be considered, in the context of our experimental evidence, as a crucial threshold value. For all prices higher than p_e, initial behavior consists in overbidding (with respect to the corresponding equilibrium strategy); for prices lower than p_e the reverse occurs. Also time trends are sensitive to prices. We can generally say that underbidding tends to reduce with time. Also overbidding is reduced, but only for prices sufficiently high. On the contrary, for intermediate prices (out of equilibrium) overbidding behavior gets stronger.

We now look at the experimental evidence from the seller's viewpoint. To this aim, Figure 6 traces the evolution of the seller's expected profits, given observed behavior.

Put Figure 6 about here

Figure 6 plots the evolution of expected profits (y-axis) as a function of the ruling price. By analogy with previous figures, the dotted line corresponds to the theoretical prediction, whereas the 4 grey lines report average profit per period.[10]

[10]Note that, in the range $[p_e, p_m]$, FPM has multiple equilibria and, therefore, also seller's profits are not uniquely determined.

As Figure 6 shows, for $p \leq p_m$, actual profits equals their equilibrium levels. This is basically due to the fact that, within this price range, out-of-equilibrium underbidding is not sufficient to prevent subjects to be rationed in both demand scenarios. As a consequence, the entire supply is always sold, independently on the demand scenario. In other words, out-of-equilibrium behavior has no effect for the seller within this price interval. Similar considerations hold for prices $p \geq \frac{3}{4}$, where expected profits start above the equilibrium level (due to overbidding), but converge to their equilibrium level very quickly.

Not surprisingly, out-of-equilibrium behavior has an impact on seller's profits, also in the long run, within the intermediate price range, that is, for $p_e < p < 7.5$. Here initial overbidding raises the seller's profits above their equilibrium levels. Moreover, as we know from Figures 1 and 5, overbidding within this price range increases with time. This, in turn, implies that also the seller's profits increase with time. Remember, from Section **??**, that the profit-maximizing price,p^*, lies precisely in the interval $[p_e, .75]$. Precisely, if $\lambda = \frac{1}{2}$ (as for our experimental conditions), $\hat{p} \cong .6606$ (.65, if we constrain prices to belong to the finite grid of our experiment). Therefore, *persistent overbidding takes place exactly within the price range that would be selected by a profit maximizing seller.* In consequence, actual profits always exceed the equilibrium level and, even, increase with time (up to 12% above the theoretical prediction, since actual and predicted behavior lead to profits of .65 and .583 respectively).

Suppose now that seller and buyers knew the market size, n. In such a case, the unique equilibrium would require the seller to set the monopoly price (i.e. either p=1/2 if $n = 2$, or $p = 3/4$ if $n = 4$) and buyers to bid truthfully.[11] Thus, the whole amount would be sold in both scenarios and the ex-ante expected revenue would be then the expected monopoly profit $MP = \frac{1}{2}\lambda + \frac{3}{4}(1 - \lambda)$. Since both scenarios are equally likely in our experiment (i.e. $\lambda = \frac{1}{2}$), $MP = .625$. Observe that the theoretical expected seller's revenues in FPM (.583) are lower than the expected linear monopoly profit. This may induce a rational seller to prefer a situation of full information. However, in our experiment, seller's profits are higher than the in the linear monopoly case.

[11] This assumption is somehow justified by our experiment on ICM, insofar when subjects face no demand uncertainty they bid truthfuly.

4.2 Panel-Data Regressions

In this section, our main concern is to check whether the discrepancy between the observed and predicted behavior is statistically significant. To this aim, we construct a panel containing all the decisions of all subjects at all times. Remember that each subject participated in 84 rounds of ICM (FPM), what creates a panel where subjects serve as the cross-sectional variable. As mentioned, the sample size is 24 (48) subjects for ICM (FPM) session(s).

As for the ICM data, we use a simple random-effect linear regression. The underlying model assumes subjects playing a linear bid function, one for each period $T_k, k = 1, ..., 4$. The model includes period as a regressor, individual (random) effects and idiosyncratic errors as follows:

$$d_{it} = \alpha + \beta p_t + \gamma T_k + \epsilon_i + \varepsilon_{it}, \tag{7}$$

where T_k denotes period as defined in Section 3.2; ϵ_i describes the unobserved time-invariant heterogeneity which characterizes subject i and ε_{it} is an idiosyncratic error term (we further assume that $\epsilon_i \perp \varepsilon_{it}$). Since, for ICM, the unique equilibrium corresponds to truthful bidding, null hypotheses for our tests are $\alpha = 1, \beta = -1$ and $\gamma = 0$.

Figure 7 reports the estimates of (7) (standard errors within brackets) for the whole ICM dataset, regression (I), and disaggregated for period, regressions (II-V).

Put Figure 7 about here

As it can be seen from the fits of regressions (I-V), bidders played very closely to the assumed linear function in all periods. In regression (I), our model explains more than 92% of subjects' behavior. The R^2 jumps from .735 in (II), to .9873 in (III) and stays above .99 in (IV-V). On the other hand, a very low fraction of variance is due to the individual effect of the experimental subjects (measured by ρ). In other words, it seems that all subjects learned very quickly to play the equilibrium, what leads to completely homogeneous play. Consequently, ρ reaches is basically 0 in the last two periods.

If we look at regression (I), we see that estimated parameters of α and β are .968 and $-.966$ against a theoretical prediction of 1 and -1, respectively. These differences are statistically significant, together with the estimated parameter of γ (positive γ meaning increasing bids across periods). However, if we look at regressions (II-V), we discover that only parameters of (II) are significantly different than their theoretical values that is, we cannot reject that the observed and the predicted behavior differ (neither independently, nor jointly) for regressions (III-V). This basically implies that learning mostly takes place in the first repetitions of the experiment and stabilizes from T_2 on. [12]

Finally, we restrict our attention to T_4 (regression(V)) and check the following hypotheses: $\alpha = 1$ and $\beta = -1$. Using Wald test, we cannot reject our null hypotheses, neither the independently, nor jointly (p-values .2089 and .3809, respectively, in the independent tests and .4219 in the joint test).

The FPM cross-sectional time-series analysis is more complex and results are less straightforward. By analogy with regressions (I-V), Figure 8 reports estimates of a model which assumes subjects playing a 3-piecewise linear bid function, as follows:

$$d_{it} = \alpha_0 + \alpha_1\eta_t + \alpha_2\theta_t + \beta_0 p_t + \beta_1 p_t\eta_t + \beta_2 p_t\theta_t + \gamma T_k + \epsilon_i + \varepsilon_{it} \qquad (8)$$

where η_t and θ_t are two index functions such that $\eta_t = 1$ if $p_t \leq .55$ and $\eta_t = 0$ otherwise, whereas $\theta_t = 1$ if $p_t \in (.55; .75)$ and $\theta_t = 0$ otherwise. Observe that these two dummies η_t and θ_t partition the price set into the same three subintervals object of our theoretical analysis. In consequence, we estimate three different -but interdependent, through the individual effect ϵ_i- linear bid functions, one for each subinterval. Precisely, β_0 and $(\beta_0 + \beta_1)$ measure the sensitivity of bids on price for high and low prices, respectively, whereas α_0 and $(\alpha_0 + \alpha_1)$ determine the constant terms. By analogy, the slope $(\beta_0 + \beta_2)$ and the constant term $(\alpha_0 + \alpha_2)$ describe the form of the estimated bid function at the intermediate subinterval.

It may be noticed at this stage that (8) can be interpreted as the natural extension of (7) to the case of FPM subject to some conditions, which we now discuss.

[12] We also run a regression analogous to (I) excluding observations coming from T_1. As expected, the null hypotheses on α, β and γ cannot be rejected, neither independently nor jointly.

First, recall from Section 2.2 that there is a multiplicity of equilibria for $p \in [p_e, p_m]$. Given the price grid and parameter values used in the experiment, multiplicity only occurs at $p = .6$ with equilibrium values being 1 and .461, respectively. In order to check which of these equilibria is somehow "more consistent" with our experimental evidence, we run a Wald test with following null hypotheses: $\overline{d}(.6) = 1$ ($\overline{d}(.6) = .461$). We can (not) reject the null hypothesis what suggests that in the experiment, subjects bid more consistently with the equilibrium where moderate bidding prevails. Consequently, we perform the regression of (8) including the $p = .6$ bids to the intermediate price interval.[13]

Second, as we know from Section 2.2, contrary to ICM, in FPM the equilibrium bid function is no longer linear. There are two price intervals, $p \leq p_e \cong 0.568$[14] and $p \geq .75$, where the equilibrium schedule is linear. However, for $p \in (p_e, .75)$ the equilibrium bid function is concave (see Proposition **??**). However, as Figure 1 shows, also for intermediate prices, the demand function may well be approximated by a simple line.

Third, notice that the presence of α_1 and α_2 in (8) allows for discontinuities at $p \in \{.55, .75\}$. We run two separate tests to check whether the continuity assumption is accepted by our data. In this case, our null hypotheses correspond to $\alpha_1 = \alpha_2$ and $\alpha_2 = 0$. We cannot reject the joint hypothesis in $T_1 - T_3$, regressions (VII-IX). However, the main interest is put on the overall estimation, (VI), and the regression of T_4, (X). In either case, we reject both independently and jointly our null hypotheses at any significance level. Thus, the continuity assumption implicit in the model is rejected by our experimental evidence.

Figure 8 reports the estimation results. By analogy with Figure 7, we also made estimations disaggregated for periods (regressions (VII-X)) to allow for inter-period comparisons.

Put Figure 8 about here

As Figure 8 shows, subjects' behavior is close to the assumed piecewise linear bid function, although not as close as for ICM. As time proceeds, we find only three parameters that change significantly between T_1 and

[13]P-values are 0 and .4787, respectively. In any case, we also run regressions excluding observation at $p = .6$. Results do not change (and are available on request).

[14]By analogy with the preceding paragraph, $p = .6$ belongs to the interemediate interval.

T_4: α_2 and β_2,which decrease, whereas the estimated value of β_1 increases. Remember that α_2 measures the difference between the constant terms of the bid function in the intermediate and high-price interval. Its growth supports the previous discussion on discontinuity. The estimated value increases gradually over time, so that the difference of the estimated α_2 in T_1 and T_4 is significant.

As for β_2, which measures the difference of slopes of the same price intervals, the high-price and the intermediate, its growth suggests that the slope of the estimated bid function in the intermediate interval decreases in comparison with the slope in the high-price interval.[15] Since at high prices, the bid-price relation stays almost constant over time (β_0), we conclude that the estimated slope of the intermediate interval increases.

On the contrary, β_1 grows with time. Therefore, the discrepancy between the slopes of low-price and high-price intervals gets stronger.

Standard errors decrease with time. In consequence, R^2 increases to a final .8976. Moreover, the fraction of variance due to the individual effects (ρ-statistics) fluctuates around 30%. This justifies the application of panel data techniques.[16]

We proceed by reporting the results of a statistical analysis of the difference between actual and equilibrium bids to check whether the estimated bid is close to equilibrium. Consistently with the previous discussions, we use the data of T_4 and test the relevant intervals independently:

- $p \geq \frac{3}{4}$. Here the null hypothesis is $\alpha_0 = 1$ (subjects bid 1 at $p = 0$) and $\beta_0 = -1$ (the bid function is $1 - p$ on this interval). We reject neither the joint (p-value .2530) nor the independent hypotheses (p-values .1219 and .1567). By analogy of our findings of Section 4.1, the behavior of experimental subjects converges to truthful bidding.

- $\frac{3}{4} > p \geq p_e \cong 0.568$. Within this interval, the estimated bid function coincides with equilibrium if $\alpha_0 + \alpha_2 = 1.255$ and $\beta_0 + \beta_2 = -1.324$, respectively. We reject the joint test at any significance level (p-value 0). In case of two independent tests, we reject the former hypothesis, but not the latter (p-values are .0296 and 0.1106, respectively). In

[15]Remember that in our case, all the slopes are negative.

[16]We run regressions analogous to (VI-X), excluding the individual effect in the model. In spite of significant values of ρ, ordinary least square estimations lead to the same qualitative results as panel data techniques do.

other words, the estimated bid function has a similar slope as the equilibrium one, while the estimated constant is bigger. In terms of our theoretical model, bidders overbid significantly wrt equilibrium, although the dependence on price remains constant.

- $p < p_e$. In this last interval, our null hypothesis corresponds to $\alpha_0 + \alpha_1 = 1$ and $\beta_0 + \beta_2 = 0$ (i.e. bids coincide with the upper bound and, therefore, are independent of prices). In this respect, Wald test leads to rejection of our joint hypothesis (p-value is 0). However, testing both hypotheses independently, we reject the independence of bids on prices, but not that at $p = 0$ bidders ask for the whole amount. As seen from the estimation, there is a negative relation between bids and prices.

A very good illustration of results of this Section can be Figure 9, which plots the estimated inverse bid functions disaggregated for periods (greyscale increasing over period), together with the equilibrium prediction (dashed line).

Observe that for high prices, the estimation starts above its equilibrium prediction, but over time, it shifts down until it gets very close to equilibrium. The figure shows that in T_4, the estimated line has a slightly lower slope (in absolute value). Nevertheless, this difference is not statistically significant.

For low prices, the estimated bid function shifts to the right. Bidders underbid and bids are equally sensitive to prices in all periods. Although the former phenomenon seems to disappear over time, the latter persists.

For intermediate prices, bidders overbid in all periods. In early periods, the estimated slope exceeds the equilibrium level. However, it decreases over time, until the estimated function becomes almost parallel to equilibrium in T_4. Consequently, it seems that the amount they overbid is independent of prices.

Put Figure 9 about here

To summarize, our panel-data analysis suggests that subjects played very well in the ICM session and at high prices in the FPM sessions (where the equilibria of both games actually coincide with truthful bidding). On the other hand, for the remaining prices the observed behavior in FPM is significantly different from equilibrium. For intermediate prices, bid are as sensitive to prices as equilibrium predicts. Bidders start overbidding insignificantly, but, as time proceed, the overbidding raises until it becomes significant. At low prices, bids negatively depend on price level, contrary to prediction. This dependence results in underbidding relative to equilibrium. Although underbidding tends to disappear (the estimated inverse bid function shifts to the right), dependence on prices remains.

5 Out-of-equilibrium behavior: risk aversion *vs.* bounded rationality

Our experimental results show that the equilibrium analysis developed in Section 2 is an (extraordinary) good predictor of subjects behavior (as far as ICM is concerned). This consideration notwithstanding, our regressions also show that subjects consistently deviate from equilibrium play, and that this deviation (with particular reference to overbidding at intermediate prices) does not seem to vanish over time. To understand this empirical regularity of our experimental evidence, we resort to two "usual suspects", often invoked to explain deviation from the risk-neutral equilibrium behavior in auction experiments, that is, risk aversion and bounded rationality.

5.1 Risk aversion

Risk-aversion has proved to be an important behavioral factor in explaining subjects' behavior in auction experiments (see, for example, [9] or [13]). In the context of FPM, player 1 has to trade-off the risk of getting too much in the low demand scenario against the risk of getting too little in the high demand scenario. These risks have to be pondered by the relative likelihood of each scenario (measured by λ). Clearly, risk aversion may play a role only when these risks affect player 1's payoffs in opposite direction (i.e. in the case of rationing only in the high demand scenario).[17] Even then, it is not obvious to predict how risk aversion should modify the equilibrium behavior of Section 2. By the above considerations, it is clear that, if λ is sufficiently high (low), we expect risk-averse players to over(under)bid with

[17]By the reason, risk aversion plays no role in ICM, where rationing never occurs.

respect of the equilibrium strategy, independently of what their degree of risk-aversion is. For intermediate values of λ, results will depend on how risk-aversion is formally defined.

By analogy with our experimental conditions, we shall explore equilibrium properties of FPM when (constant relative) risk aversion is taken into account and $\lambda \geq \frac{1}{2}$. To this aim, we modify the theoretical framework of Section 2 by considering preferences consistent with a (CRRA) function of expected payoffs, as follows:

$$u_i(d) = \frac{\pi_i^2(d)^{1-\rho}}{1-\rho}\lambda + (1-\lambda)\frac{\pi_i^4(d)^{1-\rho}}{1-\rho}, \tag{9}$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion. The case of $\rho = 0$ coincides with risk-neutrality (i.e. it covers the theoretical analysis we carried out in Section 2), with (CR) risk aversion increasing with ρ.

Proposition 4 *For all $\rho > 0$, the structure of equilibria of FPM when payoffs are defined by (9) is as follows:*

1. *$p \in [\frac{3}{4}, 1]$: unique equilibrium $d_i(p) = 1 - p$ for all i.*
2. *$p \in [0, p_e)$: unique equilibrium $d_i(p) = 1$ for all i.*
3. *$p \in (p_m, \frac{3}{4})$: unique equilibrium $\check{d}_i(p)$. If $\lambda \geq \frac{1}{2}$, then*
   (i) $\check{d}_i(p) > d_i^*(p)$, for all $\rho > 0$ and
   (ii) $\left|\check{d}_i(p) - d_i^*(p)\right|$ is bounded above by $\frac{1}{40}$.
4. *$p \in [p_e, p_m)$: two equilibria, $d_i(p) = 1$ and $\check{d}_i(p) < 1$.*
5. *$p = p_m$ continuum of equilibria: all d with $d_i = d_j$, for all i, j and $d_1 + d_2 \geq 1$*

Proof. In the Appendix. ■

As Proposition 4 shows, risk aversion has the effect of "discounting" the risk of getting too much against the risk of getting too little. It is important to notice that this does not necessarily imply bidding more than $d_i^*(p)$. Overbidding only occurs when λ is sufficiently high (e.g. when $\lambda \geq \frac{1}{2}$). In any case, deviations from equilibrium due to risk-aversion are negligible (less than half a decimal point in our parameter setting). This is the reason why, in what follows, we shall explore for an alternative justification for the discrepancy between theory and evidence in our experimental data.

5.2 Bounded rationality and Quantal Response Equilibrium

While risk-aversion explains deviation from predicting behavior as the result of a process of conscious (expected) utility maximization, in what follows, we shall assume that subjects' choices are also affected by other (unmodeled) external factors that make this process intrinsically *noisy.* This noise may be induced by the complexity of the game, limitation of subjects' computational ability, random preference shocks, etc... This kind of choice framework may be modeled by specifying the payoff associated with a choice as the sum of two terms. One term is the expected utility of a choice, given the choice probabilities of other players. The second term is a random variable that reflects idiosyncratic aspects of payoffs that are not modeled formally.

Clearly, properties of this alternative class of models crucially depend on the specific way in which the stochastic process that generates noise is formally defined. One approach that has received attention recently involves the concept of *quantal response equilibrium* (QRE), developed by McKelvey and Palfrey [20] in the context of finite games. A quantal response is, basically, a "smoothed-out best response", in the sense that agents are not assumed to select the strategy that maximizes their expected payoff with probability one. Instead, each pure strategy is selected with some positive probability, with this probability increasing in expected payoff.[18]

Some recent papers (such as [2], [13]) have modified the notion of QRE to deal with games with a continuum of pure strategies, as our ICM and FPM. A logit response function is often used to model the QRE. Formally, the standard derivation of the logit model is based on the assumption that payoffs are subject to unobserved preference shocks from a double-exponential distribution (e.g., Anderson *et al.* [1]). In this case, a (logit) QRE would be the fixed point

$$\delta_i(d_i) \equiv f_i(d_i|\delta_{-i}, \mu) = \frac{\exp\left[\pi_i(d_i, \delta_{-i})\mu\right]}{\int_0^1 (\exp\left[\pi_i(s, \delta_{-i})\mu\right])ds}, i = 1, ..., 4, \qquad (10)$$

where $\pi_i(d_i, \delta_{-i})$ is the expected payoff associated with the pure strategy d_i against $\delta_{-i} \in \Delta_{-i}$, and μ is the noise parameter. As $\mu \longrightarrow \infty$, the

[18] See also Rosenthal [23].

probability of choosing an action with the highest expected payoff goes to 1. Low values of μ correspond to more noise: if $\mu \longrightarrow 0$, the density function in (10) becomes flat over the entire support and behavior becomes random.

As we just noticed, a (logit) QRE is a then vector of densities that is a fixed point of (10). Continuity of the payoff function $\pi_i(.)$ ensures existence, both in the case of ICM and FPM. While Section 5.2.1 explicitly characterizes the (unique) logit equilibrium in the case of ICM, for FPM no explicit solution can be found. This is because FPM is a game with a continuum of pure strategies, for which logit equilibria can be calculated only for very special cases.[19] In this case, we are only able to evaluate a QRE numerically. This equilibrium has the property that, when $\mu \longrightarrow \infty$, it converges to the (unique) equilibrium we derived in Section 2.

5.2.1 ICM

Fix a price $p \in [0,1]$ and consider the associated game induced by $ICM(p)$. By (2), equilibrium distribution functions can be calculated as follows:

$$f(d_i|\mu) = \frac{\exp\left[\frac{\mu d_i(2-2p-d_i)}{2}\right]}{\int_0^1 \exp\left[\frac{\mu d_i(2-2p-y)}{2}\right] dy}. \tag{11}$$

In Figure 10 we use standard maximum-likelihood techniques to estimate the value of μ in each period. The second line of Figure 10 reports the estimation results using ICM data. The estimated noise parameter jumps dramatically between T_1 and T_2 and reaches its highest value at T_3. It then decreases at T_4, but is still significantly higher than at T_1.

Put Figure 10 about here

In Figure 11, we trace the equilibrium densities $f(d_i|\mu)$ for three price values: p=.2, p=.65 and p=.8. Every graph plots four curves, one for each period.

Put Figure 11 about here

[19]Such as potential games, as in Anderson *et al.* [3].

Not surprisingly, these distributions are unimodal at the value $(1-p)$ -the (equilibrium) pure strategy associated with the higher expected payoff- and become flatter as μ goes to zero. We are interested in the behavior of (equilibrium) expected bids $\hat{d}_i(p,\mu) = \int_0^1 d_i f(d_i|p,\mu) dd_i$. In the following Figure 12 we trace four $\hat{d}_i(p,\mu)$ with the same values of μ as estimated in Figure 10.

Put Figure 12 about here

The effect of the noise (whose magnitude is measured by μ) is to create underbidding (wrt to optimal behavior) when the price is low(er than .5), and overbidding when the price is high(er than .5). This threshold value is independent of μ. To see why, notice that equilibrium distributions (11) are *symmetric with respect to the mode (i.e. wrt* $x_i(p)$*)*. This is because payoff function is also symmetric wrt $x_i(p)$, given, by (??), $\frac{d\pi_i}{dd_i} = 1 - d_i - p$. This, in turn, implies that equilibrium average bids are biased toward the center: the cost (in terms of a payoff loss) of deviating by an ε is exactly the same whether deviation is upward or downward. The fact is that deviations toward the center are simply more likely (since, by (10), every pure strategy belongs to the support of the logit equilibrium).

The following Figure 13 is divided into four parts, one for each period. In every graphic, the dotted line trace the equilibrium prediction derived in Section 2, the smoothed curve plot the estimated QRE, while the broken line reproduces the observed behavior in the lab. T_1 graph shows that QRE predicts well the slight overbidding (underbidding) when prices are high (low). By analogy with QRE prediction, the observed threshold where the average bid switches from overbidding to underbidding as price decreases is situated around $p = .5$. From T_2 on, the three curves almost coincide, as we know already from Section 4.

Put Figure 13 about here

5.2.2 FPM

First, Figure 14 presents the maximum-likelihood estimations of the noise parameter μ in case of FPM. In every period, the estimations are divided into three parts: the first column lists one estimate per price, the second an estimate per price interval, and the third a unique estimate for the entire price spectrum.

Put Figure 14 about here

A first look at the table confirms the findings of Section 4. Let's start with the third column of each period. The estimated parameter raise gradually from 46 in T_1 to a final 150 in T_4. This suggests that as time proceeds, the observed behavior converges to the equilibrium prediction. In FPM, contrary to ICM, learning takes place in all four periods.

The per-interval estimations deserve a more detailed discussion. In the early periods the estimation of intermediate interval exceeds significantly both the low and high intervals. However, from T_2 on, it decreases slightly until it becomes the lowest in T_4. In general, the higher the price, the higher the estimate. The per-price estimations replicates the per-interval discussion. Only, the extremely high estimates of the highest prices are remarkable in this case.

As we previously mentioned, solutions for QRE in the case of FPM have beem evaluated numerically. The corresponding distributions are plotted in Figure 15.

Put Figure 15 about here.

Equilibrium distributions are analogous to the ICM case only for very high prices. By contrast, for very low prices, distributions are unimodal at 1. This is clearly due to rationing. More importantly, for prices $.75 > p \geq p_e$, the QRE distribution is not symmetric wrt to 1-p, but has a mode at a higher level and is skewed to the right. Given we cannot provide an explicit solution for the QRE in the case of FPM, we can only search for intuitions for this (numerical) result by getting back to FOCs in the "high-demand rationing case":

$$\frac{d\pi_1}{dd_1} = \lambda(1 - p - d_1) + (1 - \lambda)\frac{\left(\sum_{j>1} d_j\right)\left(\sum_{j>1} d_j - p\sum_j d_j\right)}{\left(\sum_j d_j\right)^3}. \qquad (12)$$

By (12), the derivative is decreasing and convex. This, in turn, implies that deviations in the direction of overbidding are relatively cheaper (and, therefore, by (10), overbidding wrt d_i^* is more likely to occur).Furthermore, the larger fraction of bidders overbids, the more attractive overbidding becomes for others. In other words, if overbidding strategies grow in probability, their payoff becomes relatively higher and this, by (10), reinforces the bias toward overbidding induced by the asymmetry in relative costs. These observations are well illustrated by Figure 16, which is the analogy with Figure 12 in case of FPM.

Put Figure 16 about here.

The qualitative features of Figure 16 reproduce our experimental evidence with remarkable accuracy, as Figure 17 shows.

Put Figure 17 about here

- $p \geq 7.5$. For very high prices, overbidding is basically due to the "drift effect" already discussed in Section ??.
- $p < p_e$. For very low prices, the drift effect yields underbidding (since mode correspond to the upper bound of the pure strategy space). Moreover, QRE predicts the observed sensitivity of bids on price level. This is due to the fact that the higher

the price the cheaper is to underbid the equilibrium prediction by the same amount. Therefore, it is more likely to observe such deviations the higher is the price.

- 7.5$< p \leq p_e$. For intermediate prices overbidding, due to the cost asymmetries highlighted in (12), persists and gets even stronger.

6 Conclusion

Two main conclusions can be drawn by our experiment: First, equilibrium analysis provides a very good description of subjects behavior, compared to the other experimental settings. Second, there are still deviations from equilibrium, for which QRE seems to produce a sufficiently consistent explanation.

We emphasize that these deviations make FPM even more attractive as a selling mechanism. Persistent overbidding of RNNE occurs exactly within the price range that would be selected by a profit maximizing seller. Revenues at this price turns out to be even higher than the expected monopoly profit.

A general and most important observation from our experimental data is that subjects were able to solve the problem well enough to achieve results closely resembling the theoretical predictions. This finding is important when it comes to the question when and where FPM should be used in practice. In this respect, two conclusions can be drawn. First, the theoretically appealing properties of FPM clearly survive (or even are improved on) in the laboratory, which suggests that FPM should be quite popular as a selling mechanism. Second, we have to keep in mind that those advantages of FPM can only be realized if the seller fixes the price correctly, anticipating buyers' bidding behavior. Thus, FPM should rather be observed in markets where sellers are experienced.

The latter observation points to a question for future research. While in our experiment we focused on the buyers' behavior, the seller's decision is certainly as relevant for evaluating the attractiveness of the mechanism. Two issues are of interest here. First, does the seller anticipate bidding behavior correctly and sets the price optimally given the behavior of the buyers? Second, does the fact that the seller is a real player (and not imitated by the computer) change buyers' behavior at the second stage of the game?

Another natural extension of the model studied in this paper could be the replacement of proportional rationing by a different rule. Two natural candidates are constrained equal losses and constrained equal awards. The former is a rule that makes losses as equal possible, under the condition that no participant ends up with negative transfers. In other words, this rule gives priority to higher bids. That is the reason why it is used for example in public good problems, or health care. In FPM with constrained equal losses, there are incentives to overbid in order to avoid excessive rationing. However, since this rule is advantageous for higher bids, we expect that the overbidding will be larger in case of bidders with low demand. Constrained equal awards is the dual rule to the constrained equal losses. In this case, the good is distributed such that each bidder receives the same fraction, subject to the condition that no buyer gets more than his bid.

It is not difficult to show that the equilibrium characterized in Section 2 remains as such under both rationing rule. But, in this case, multiple equilibria also occur. For example, under the constrained equal awards rule and sufficiently low prices, every strategy profile in which the minimum bid is higher than $\frac{1}{2}$ consitutes an equilibrium of Stage 2. How the presence of such strong strategic uncertainty may affect subjects' behavior in the lab is left to future research.

References

[1] Anderson, S., A. de Palma and J.-F. Thisse (1992), *Discrete Choice Theory of Product Differentiation*, Cambridge MA, MIT Press.

[2] Anderson, S., Goeree, J. K. and C. Holt (1999), *Stochastic Game Theory: Adjustment to Equilibrium under Noisy Directional Learning*, University of Virginia, mimeo.

[3] Anderson, S., Goeree, J. K. and C. Holt (2001), "Minimum-Effort Coordination Games: Stochastic Potential and Logit Equilibrium", *Games and Economic Behavior,* 34, 177-199.

[4] Ausubel, L. and C. Cramton (2002), *Demand Reduction and Inefficiency in Multi–Unit Auctions*, University of Maryland, mimeo.

[5] Bierbaum, J and V. Grimm (2004), *Selling Shares to Retail Investors: Auction vs. Fixed Price*, IVIE Working paper WP-AD 2004-08

[6] Bulow, J. and P. Klemperer (2002), "Prices and the Winner's Curse", *RAND Journal of Economics* 33, 1-21.

[7] Chemmanur, Thomas J. and Liu, Huanliang Mark (2003), *How Should a Firm Go Public? A Dynamic Model of the Choice Between Fixed-Price Offerings and Auctions in IPOs and Privatizations*, EFA 2003 Annual Conference Paper No. 366.

[8] Chun, Y. (1989), "A Noncooperative Justification for Egalitarian Surplus Sharing", *Mathematical Social Sciences,* 17, 245-261.

[9] Cox, J., Smith, V. L., and J. M. Walker (1988), "Theory and Individual Behavior of First-Price Auctions", *Journal of Risk and Uncertainty,* 1, 61-99.

[10] Dagan, N. Serrano, R. and Volij. O. (1997), A Noncooperative view of consistent bankruptcy rules, *Games and Economic Behavior*, 18: 55–72.

[11] Engelmann, D. and Grimm, V. (2004), *Bidding Behavior in Multi-Unit Auctions: An Experimental Investigation*, IVIE Working Paper WP-AD 2004-12.

[12] Gilbert, R. and P. Klemperer (2000), "An Equilibrium Theory of Rationing", *RAND Journal of Economics,* 31, 1-21.

[13] Goeree, J. K., Holt, C. A. and T. R: Palfrey (2000), *Quantal Response Equilibrium and Overbidding in Private-Value Auctions*, Caltech, mimeo.

[14] Herrero, C. (2003) Equal Awards Vs. Equal Losses: Duality in Bankruptcy, in M.R. Sertel and S. Koray (eds.), *Advances in Economic Design*, Springer-Verlag, Berlin, pp.413-426.

[15] Herrero, C. and A. Villar (2001), "The Three Musketeers. Four Classical Solutions to Bankruptcy Problems", *Mathematical Social Sciences,* 42, 307-328.

[16] Herrero, C., J. Moreno-Ternero and G. Ponti (2004), *On the Adjudication of Conflicting Claims: An Experimental Study*, Universidad de Alicante, mimeo.

[17] Kagel, J. (1995), "Auctions: a Survey of Experimental Research", in Kagel, J. and Roth, A. (eds.), *The Handbook of Experimental Economics*, Princeton NJ, Princeton University Press.

[18] Kagel, J. H. and Levin, D. (2001), "Behavior in Multi–Unit Demand Auctions: Experiments with Uniform Price and Dynamic Auctions", *Econometrica,* 69, 413 – 454.

[19] List, J. A. and Lucking-Reiley, D. (2000), "Demand Reduction in Multiunit Auctions: Evidence from a Sportscard Field Experiment", *American Economic Review*, 90, 961–972

[20] McKelvey, R. D., and T. R: Palfrey (1995), "Quantal Response Equilibria for Normal Form Games", *Games and Economic Behavior,* 10, 6-38.

[21] Moreno-Ternero, J.D. (2002) *Noncooperative Support for the Proportional Rule in Bankruptcy Problems*, mimeo, Universidad de Alicante.

[22] Moulin, H. (2000), "Priority Rules and Other Asymmetric Rationing Methods", *Econometrica,* 68, 643-648.

[23] Rosenthal, R. (1989)., "A Bounded-Rationality Approach to the Study of Noncooperative Games", *International Journal of Game Theory,* 18, 273-292.

7 Appendix

7.1 Proofs

The proof of Proposition 1 is obtained by way of four lemmas, characterizing bidders optimal behavior at all prices. Fix a pair $(p, \overline{d})$ chosen by the seller at Stage 1 and consider the corresponding vector of bids $d(p)$ at price p. Since, at a given price, the buyer's demand is just a quantity, in what follows we omit the dependency on the price in order to simplify notation.

Differentiating player 1's payoff function (4) with respect to d_1 yields

$$\begin{aligned} \frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} &= \lambda \left[Q^2(d_1, d_{-1}) + d_1 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} \right] (1 - d_1 Q^2(d_1, d_{-1}) - p) \quad (13) \\ & +(1-\lambda) \left[Q^4(d_1, d_{-1}) + d_1 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} \right] (1 - d_1 Q^4(d_1, d_{-1}) - p) \end{aligned}$$

.Note that

$$\left[Q^n(d_1, d_{-1}) - d_1 \frac{\partial Q^n(d_1, d_{-1})}{1} \right] = \begin{cases} 1 & \text{if } \sum_{j=1}^n d_j < 1, \\ \frac{\sum_{j=2}^n d_j}{(\sum_{j=1}^n d_j)^2} & \text{if } \sum_{j=1}^n d_j \geq 1. \end{cases} \quad (14)$$

Thus, (14) is always positive whenever at least one bidder ask for a positive quantity. In the following Lemma 5, we establish that no bidder has an incentive to ask for less than her demand (independently on others' behavior):

Lemma 5 *$d_1 < 1 - p$ is strictly dominated by $d_1 = 1 - p$ for all $p \in [0, 1]$ and all $\overline{d} \in [0, R]$.*

Proof. If $d_1 < 1 - p$, then $(1 - d_1 Q^n(d) - p) > 0$, since, by (2), $Q^n(d) \leq 1$. Thus, for $d_1 \in [0, 1 - p)$, all terms in (13) are strictly positive, and thus, $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} > 0$. ■

Given Lemma 5, in what follows we shall restrict our attention to strategy profiles $d = \{d_i \geq 1 - p\}$. The remaining three lemmas establish uniqueness of the equilibrium of the bidding stage at almost all prices. We proceed by partitioning the price set [0,1] into three subintervals: *i*) prices $p \in (\frac{3}{4}, 1]$ above the market clearing price in case of 4 buyers; *ii*) prices $p \in [0, \frac{1}{2}]$ below the market clearing price in case of 2 buyers, and finally, *iii*) prices $p \in (\frac{1}{2}, \frac{3}{4}]$ in between the two market clearing prices.

Lemma 6 *Let* $p \in (\frac{3}{4}, 1]$. $\pi_1(d_1, d_{-1})$ *is strictly decreasing in* d_1 *for all* d_{-1} *such that* $d_j > x_j$ *for at least one* $j \neq 1$ *and* $d_1 \geq \max\{d_{-1}\}$.

Proof. If $\sum_{j=1}^{4} d_j > 1$, then, by (2) it must be that $1 \geq Q^2(d) > Q^4(d)$. Thus, if $1 - d_1 Q^4 - p \leq 0$ then $1 - d_1 Q^2 - p < 0$. Since we assume $\max\{d_{-1}\} > 1 - p = x_1$, any bid $d_1 \geq \max\{d_{-1}\}$ yields a supply to bidder 1 of $d_1 Q^4 \geq \frac{1}{4} \geq 1 - p$ for all $p \in [\frac{3}{4}, 1]$ in the high demand scenario. Thus, $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 0$ and, therefore, $1 - d_1 Q^2 - p < 0$. This, in turn, implies $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} < 0$.

Assume instead $\sum_{j=1}^{4} d_j \leq 1$. Then, it must be $1 = Q^2(d) = Q^4(d)$. Again, since $\max\{d_{-1}\} > 1 - p$ by assumption, a bid $d_1 \geq \max\{d_{-1}\}$ yields a supply to bidder 1 of $d_1 Q^n > 1 - p$ in any demand scenario. Thus, $1 - d_1 Q^n - p < 0$ which implies $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} < 0$. ■

Lemma 6 implies that, at prices $p \in (\frac{3}{4}, 1]$, any buyer has a strict incentive to underbid the highest bid of his opponents if the latter exceeds true demand. Together with Lemma 5 this implies that the only equilibrium at high prices is truthful bidding.

Lemma 7 *Let* $p \in [0, \frac{1}{2}]$. $\pi_1(d_1, d_{-1})$ *is strictly increasing in* d_1 *for all* d_{-1} *such that* $d_j \geq 1 - p$ *and* $d_1 \leq \min\{d_{-1}\}$.

Proof. At prices $p \in [0, \frac{1}{2}]$, since the bidders bid at least their true demand, it holds that $Q^4 < Q^2 \leq 1$ (<1 if $p < \frac{1}{2}$). Thus, if $1 - d_1 Q^2(d_1, d_{-1}) - p \geq 0$, then $1 - d_1 Q^4(d_1, d_{-1}) - p > 0$. If $d_1 \leq \min\{d_{-1}\}$, then $d_1 Q^2 \leq \frac{1}{2} \leq 1 - p$. Therefore, $1 - d_1 Q^2(d_1, d_{-1}) - p \geq 0$, which, in turn, implies $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} > 0$. ■

It follows from Lemma 7 that, at low prices, every bidder strictly wants to outbid the lowest bidder given any vector of reasonable bids of the opponents (i. e. bids above true demand). Thus, the only equilibrium at low prices is that everyone's bid equals the upper limit.

Lemma 8 *Let* $p \in (\frac{1}{2}, \frac{3}{4}]$.

1. *If there is no rationing in case* $n = 2$, *i. e.* $d_1 + d_2 \leq 1$, *then*

   (i) $\pi_1(d_1, d_{-1})$ *is decreasing for all* $d_1 \geq \frac{x_1}{Q^4}$.

   (ii) $\pi_1(d_1, d_{-1})$ *is strictly concave in* d_1 *for all* $d_1 \in [0, \frac{x_1}{Q^4}]$, d_{-1} *such that* $d_j \geq x_j$, *for all* $j \neq 1$.

2. *If there is rationing in case $n = 2$, i. e. $d_1 + d_2 \geq 1$, then*

   (i) *$\pi_1(d_1, d_{-1}, p)$ is strictly increasing for all d_{-1} such that $d_j \geq x_j$ and $d_1 \leq \min\{d_{-1}\}$ if $p < \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.*

   (ii) *$\pi_1(d_1, d_{-1}, p)$ is strictly decreasing for all d_{-1} such that $d_j \geq x_j$ and $d_1 \geq \max\{d_{-1}\}$ if $p > \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.*

   (iii) *At $p = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$, $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = 0$ for any d such that $d_i = d_j$ for all i, j.*

3. *Every pure strategy equilibrium of $FPM(p)$ is symmetric.*

Proof. Part 1(i). Since $Q^4 < Q^2 = 1$, it holds that, if $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 0$, then $1 - d_1 Q^2(d_1, d_{-1}) - p < 0$, and thus, by (13), $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} < 0$. $d_1 \geq \frac{x_1}{Q^4}$ yields $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 1 - \frac{x_1}{Q^4} Q^4 - p = 0$, which proves the first part of the lemma.

Part 1(ii). The second derivative of π_1 with respect to d_1 is given by

$$\frac{\partial^2 \pi_1(d_1, d_{-1})}{\partial d_1^2} = \tag{15}$$

$$- \lambda \left[Q^2(d_1, d_{-1}) + d_1 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} \right]^2 - (1 - \lambda) \left[Q^4(d_1, d_{-1}) + d_1 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} \right]^2$$
$$+ \lambda \left[2 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^2(d_1, d_{-1})}{{}^{2}_{1}} \right] (1 - d_1 Q^2(d_1, d_{-1}) - p)$$
$$+ (1 - \lambda) \left[2 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^4(d_1, d_{-1})}{{}^{2}_{1}} \right] (1 - d_1 Q^4(d_1, d_{-1}) - p).$$

By (14), the first two terms of the LHS of (15) must be negative. It remains to show that the sum of the last two terms is also negative. Note that

$$\left[2 \frac{\partial Q^n(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^n(d_1, d_{-1})}{\partial d_1^2} \right] = \begin{cases} 0 & \text{if } \sum_{j=1}^n d_j < 1, \\ -2 \frac{\sum_{j=2}^n d_j}{(\sum_{j=1}^n d_j)^3} & \text{if } \sum_{j=1}^n d_j \geq 1. \end{cases} \tag{16}$$

Thus, if no rationing occurs in the low demand scenario ($\sum_{j=1}^2 d_j < 1$), the third term is equal to zero. The fourth term is negative for $d_1 \in [0, \frac{x_1}{Q^4}]$, since for those bids it holds that $1 - d_1 Q^4(d_1, d_{-1}) - p \geq 0$.

Part 2(i). We substitute (14) into (13) to get

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = \lambda \frac{d_2}{(d_1+d_2)^2}(1 - \frac{d_1}{d_1+d_2} - p) + (1-\lambda)\frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2}(1 - \frac{d_1}{\sum_{j=1}^4 d_j} - p). \tag{17}$$

Note that, if $d_1 \leq \min\{d_j\}$, then $\frac{d_2}{(d_1+d_2)^2} \geq \frac{1}{4}$, $\frac{d_1}{d_1+d_2} \leq \frac{1}{2}$, $\frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2} \geq \frac{3}{16}$, and $\frac{d_1}{\sum_{j=1}^4 d_j} \leq \frac{1}{4}$. Substituting in yields

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} \geq \lambda \frac{1}{4}(1 - \frac{1}{2} - p) + (1-\lambda)\frac{3}{16}(1 - \frac{1}{4} - p) > 0 \quad \Leftrightarrow p < \frac{1}{4}\frac{9-\lambda}{3+\lambda}.$$

Part 2(ii). By (17), if $d_1 \geq \max\{d_j\}$, then $\frac{d_2}{(d_1+d_2)^2} \leq \frac{1}{4}$, $\frac{d_1}{d_1+d_2} \geq \frac{1}{2}$, $\frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2} \leq \frac{3}{16}$, and $\frac{d_1}{\sum_{j=1}^4 d_j} \geq \frac{1}{4}$. Substituting in yields

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} \leq \lambda \frac{1}{4}(1 - \frac{1}{2} - p) + (1-\lambda)\frac{3}{16}(1 - \frac{1}{4} - p) < 0 \quad \Leftrightarrow p > \frac{9-\lambda}{4(3+\lambda)},$$

which proves part 2(ii) of the lemma.

Part 2(iii). For any vector of equal bids such that $d_1 + d_2 \geq 1$, (13) simplifies to

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = \frac{1}{d_1}\left[\lambda \frac{1}{4}(1 - \frac{1}{2} - p) + (1-\lambda)\frac{3}{16}(1 - \frac{1}{4} - p)\right],$$

wit $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = 0$ iff $p = \frac{9-\lambda}{4(3+\lambda)}$.

Part 3. In any equilibrium, either all bidders are rationed in the low demand scenario, or none of them is rationed, because their joined quantity determines the same rationing factor for everyone. In both cases (rationing if $n = 2$, or no rationing), we have shown that there is a unique best-reply to any given strategy profile of bidder 1's opponent, d_{-1}. Since bidders' payoff functions of all bidders are symmetric, also the equilibria of the game must be symmetric. ■

From Lemma 8, it follows that at any price but one in the interval $(\frac{1}{2}, \frac{3}{4}]$, $FPM(p)$ can have at most two equilibria in pure strategies, and all of them it must be symmetric (by part 3 of the lemma). An equilibrium where all buyers bid $d_i(p) = 1$ exists for all prices $p \leq p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$, but not for higher prices (part 2 of the lemma). At $p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$ also any quadruple of equal bids that leads to rationing in both scenarios is an equilibrium of the game. A symmetric equilibrium without rationing in the low demand scenario exists whenever the solution to $\max_{d_1} \pi_1(d_1, d_{-1})$ s. t.. $d_1 = d_j \; \forall \; d_j \in d_{-1}$ ensures that $d_1 + d_2 \leq 1$, which is the case for prices $p \geq p_e = \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$.

We are now in the position to prove Proposition 1.

Proof. [Proof of Proposition 1.] Existence and uniqueness of equilibrium at prices $p \in [0, p_e)$ and $p \in [\frac{3}{4}, 1]$ has already been shown in lemmas5 − 8 . Also, all remaining equilibria where $d_i = 1$, $\forall i$ and the continuum of equilibria at p_m have been derived in Lemma 8, part 2. It remains to solve for those equilibria where no rationing takes place in the low demand scenario at prices $p \in [\frac{1}{2}, \frac{3}{4}]$.

If $Q^2(d) = 1$, (13) simplifies to

$$\lambda \left[1 - d_i - p\right] + (1 - \lambda) \left[Q^4(d_i, d_{-i}) - d_i \frac{\partial Q^4(d_i, d_{-i})}{i}\right] (1 - d_i Q^4(d_i, d_{-i}) - p) = 0. \tag{18}$$

Substituting $Q^4(d_i, d_{-i}) = \frac{1}{\sum_{j=1}^{4} d_j}$ in (18), and imposing symmetry yields

$$d_i = \frac{1}{2}(1 - p) + \sqrt{\frac{1 - \lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1 - p)^2}. \tag{19}$$

A symmetric profile (19) can be an equilibrium only if $2d_i \leq 1$, which is the case for all $p \geq p_e$,with $p_e = \frac{9+7\lambda}{4(3+5\lambda)}$. ■

Proof. [Proof of Proposition 2.] If the upper limit $\overline{d}$ is high enough not to affect revealed demand at prices in $\left[p_e, \frac{3}{4}\right]$, equilibrium demand of buyer 1 at price $p \in \left[p_e, \frac{3}{4}\right]$ is at least d_i, as given by equation (19). At price p_e the whole supply is sold in both scenarios, which implies that seller's expected revenue is safe and equal to p_e (which proves part (ii) of the proposition). Since there is only 1 unit for sale, setting a price below p_e is strictly dominated for the seller. Thus, the seller's revenue is bounded below by p_e and may be even higher (part (iii)). Since $\overline{d}$ can only reduce the demanded quantity, the seller strictly prefers a limit that does not affect revealed demand by any bidder at the posted price (part (iv)).

We have already shown in proposition 1 that for any upper bound on bids an equilibrium of the bidding stage exists at all prices $p \in [0, 1]$, and that any such equilibrium is symmetric. Under proportional rationing any bidder who has bid the same quantity receives the same. Recall that all bidders have the same demand function. Therefore, their willingness to pay for the next unit is the same and no aftermarket trade among the bidders will occur (part (v)).

Finally, for any equilibrium played in the continuation game, there is a (not necessarily unique) profit maximizing price. Thus, an equilibrium of FPM always exists (part (i)), where the seller chooses the profit maximizing price $p^\star$ given the play at the second stage, and chooses $\overline{d}$ higher than the bidders' (unrestricted) bids at $p^\star$. ■

Proof. [Proof of Proposition 3.] Assume that player 2, 3 and 4 play the equilibrium strategy, i.e. $x_i = d_i^*(p)$). If $p \geq \frac{3}{4}$, if player 1 selects a quantity sufficiently close to $d_1^*(p)$, rationing does not occur in either scenario. This simplifies the derivative of $u_i(x)$ wrt d_1 to the following:

$$\frac{du_i(x)}{dd_1} = 2^\rho(1-p-d_1)(2x_1(1-p-\frac{1}{2}d_1))^{-\rho}, \tag{20}$$

that is, FOCs equivalent to the case of risk neutrality.

Assume $p < p_e$ and $d_j(p) = 1, j \neq 1$. Then, for all $d_1 > 0$ (since player 1 is rationed in both scenarios), differentiating $u_i(x)$ wrt x_1 yields the following:

$$\frac{du_i(d)}{dd_1} = 2^\rho \left(\begin{array}{c} \frac{\lambda(1-p(1+d_1))}{(1+d_1)^3\left(d_1 Q_2^2(2+d_1-2p(1+d_1)\right)^\rho} + \\ \frac{3(1-\lambda)(3-p(3+d_1))}{(1+d_1)^3\left(d_1 Q_4^2(6+d_1-2p(3+d_1)\right)^\rho} \end{array} \right) > 0. \tag{21}$$

We know, from Proposition 1, that, when rationing takes place in the low demand scenario only, $\pi_i(d)$ is a strictly concave function. This, in turn, implies that, $u_i(d, \rho)$ is also a strictly concave function (i.e. the equilibrium is unique). In this case, first-order conditions (21) correspond to

$$\frac{du_i(d)}{dd_1} = 2^\rho \left(k(d,\rho)\frac{\lambda}{(1-\lambda)}(1-p-d1) + \frac{(\sum_{j>1} d_j)(\sum_{j>1} d_j - p\sum_j d_j)}{\left(\sum_j d_j\right)^3} \right), \tag{22}$$

with $k(d,\rho) \equiv \frac{\left(d1\left(2pd_1-d_1+2(1-p)\left(\sum_{j>1} d_j\right)\right)\right)^{-\rho}}{(d_1(2-2p-d1))^\rho\left(\sum_{j>1} d_j\right)^3}$. Condition (22) is equivalent to the FOC under risk-neutrality (13) when $k(d) = 1$. Notice that $0 < k(d) \leq 1$, and increases (decreases) with p (ρ). If $\lambda \geq \frac{1}{2}$, then (22) implies that, if a symmetric equilibrium $\frac{1}{4} \leq \check{d}_i(p,\rho) \leq \frac{1}{2}$ exists, it must be $\check{d}_i(p) \geq d_i^*(p,\rho)$. This already implies (given $p \geq p_m$), $k(d,\rho) \geq \frac{12p-11}{16d_i^*(p,\rho)(2p-2+d_i^*(p))} \geq \frac{3}{4}$.

This, in turn, implies

$$\left|\check{d}_1(p,\rho) - d_1^*(p)\right| \leq \frac{1}{8}\left(2\sqrt{7-12p+4p^2} - \sqrt{25-44p+16p^2}\right) \leq \frac{1}{8}\left(2\sqrt{2}-\sqrt{7}\right) \cong .023.$$

■

7.2 The experimental instructions

Welcome to the experiment!

This is an experiment to study how people solve decision problems.

Our unique goal is to see how people act on average; not what you, particularly, do. Do not think, then, that we expect you to take any specific behavior.

On the other hand, you should take into account that your behavior will affect the amount of money you will earn throughout the experiment. It is, therefore, your own interest to do your best.

This sheet contains the instructions explaining the way the experiment works and the way you should use your computer.

Please Do not disturb the other participants during the course experiment. If you need any help, please, raise your hand and wait in silence. You will be attended as soon as possible.

How can you earn money?

You will have to play 84 rounds of a simple game described as follows. In each round, you will be part of a group of 2 or 4 people (including you) of this room. Whether the group will be of 2 or 4 people will be decided randomly and it will change within each round.[20]

During the experiment, 50% of times you will be in a group of 4 and 50% of times in a group of 4. It is crucial to keep in mind that **the composition as well as the size of your group will change at each round!**

[20]As there were no groups in ICM session, the last two phrases of this paragraph and all the following one are omitted in the ICM insructions. All the rest of the insructions is slightly modified in parts where we talk about "the other members of your group" in order not to confuse the subjects participating to the ICM session.

In each round, you and each of the other members of your group will have to make a choice. Your decision (together with the decisions of the others in your group) will determine the amount of money you will earn at the end of that round.

We will also give you a show-up fee of 2000 ptas[21]. At the end of the experiment, you will be paid the exact amount you have earned throughout its course plus the show-up fee.

How to play the game?

In each round, you will participate to a market together with the other members of your group (who can be one or three). In this market, 10 units of a product are put in sale.

In each round, a price between 0 and 10 will appear in the screen of your (as well as your group members') computer. This price does not necessarily have to be an integer, and has been determined randomly. You and the other group members have to decide the amount of the product you want to bid at this price[22].

How can you get the product?[23]

You will not always get the amount of product you have bid!!!

The amount of good you will get depends on your bid and the bids of the other group members. Keep in mind that you will take part of a group that will be formed of 2 or 4 members (including you). At the moment you will have to decide your bid you will not know the size of your group!!!

In each round, we will sum the bid amounts by all your group members. Do not forget that the **maximum** amount we can distribute is 10 units.

In case that the sum of the bids of all the members of your group (including yourself) does not exceed 10, each member receives what he demanded.

Otherwise, that is, if the sum exceeds 10 units, each member receives a lower amount than what he demanded, although each

[21]The show-up fee of the ICM session was 1500 pesets, since the control treament was strategically simpler than FPM.

[22]As a next paragraph the following text in Bold appears in the ICM instructions: You will always receive the amount you have bidded!!!

[23]This chapter - together with the following Summary and the Control questions 1 and 2 - does not appear in the ICM instructions, since there is no rationing.

member get the same percentage of his bid. This percentage is determined from the relation between the available amount and the aggregate demand of your group.

Example: Suppose that:

- the price of this round is 5.5,
- your bid was 2 units,
- each of the other members of your group demanded 6 units.

If the size of your group was 2, your group's aggregate bid would be 2+6=8. Since this amount is lower than 10 (the available amount), you will receive 2 units and the other one gets 6 units that is, what you both bid..

If the size of your group was 4, the aggregate demand would be 2+6+6+6=20. Since this amount is higher than 10 (the available amount), each member of your group receives 50% of what he has bid. This is because the available amount, 10, is 50% of the amount demanded by the whole group, 20. That is: you will get 1 unit and the others receive 3 units each.

Summary

If the aggregate bid of the group is less or equal to 10, each member gets what he has bid..

If the aggregate bid is higher than 10, each member receives the same percentage as he bid.. This percentage is determined from the relation between the available amount (10 units) and the aggregate bid (e.g. 20 units in our example).

This implies that always when a person bid more than an other one this person gets more units than the other one.

Control question # 1: If you bid 6 and each of the other members of your group bids 6,

How many units do you get if the size of your group is 4?
How many units do you get if the size of your group is 2?

Control question # 2: If you bid 8 and each of the other members of your group bids 4,

How many units do you get if the size of your group is 4?
Do you get what you bid if the size of your group is 2?

How much money you can earn?

Look at the table we give you together with these instructions. In this table, you can check how much money you earn for each quantity you get at each price. The first column of the table shows the different prices that can appear during the experiment. In the first row, you have different quantities between 0 and 10 units. In each cell , you find your profit if you get the corresponding quantity at the corresponding price.

For instance, if you like to know how much money you earn if you receive 4.5 units at price 4, have a look at the cell that corresponds to the row of price 4 and to the column corresponding to the quantity of 4.5 units. By doing so, you will see that you earn 16.88 ptas..

Control question # 3: How much money do you earn if you get 8 units at price 2.5?[24]

In each round, you can bid any amount between 0 and 10, but it has to be a number with at most 2 decimals. You are not forced to only bid the amounts listed in the table.

It can also happen that the quantity you get corresponds to a number between two of the quantities listed in the table. In such a case, your profit will also be between the two corresponding profits.

Summary[25]

In each round, you and other members of your group will participate in a market where 10 units of product are being sold.

[24]Obviosly, this is the unique control question that appears in the ICM instructions.

[25]Due to the simplicity of the control treatment, we have not found it necessary to place a Summary part to the instructions of the ICM session.

The size and the composition of your group will change in each round and they will, always, be determined randomly. The size of your group can be 2 or 4 (including yourself). In each round, both possibilities have the same probability (i.e.50% of times your form part of a group of 2 and 50% of a group of 4).

In each round, you and the other members of your group will face a different price.

At this price, you have to bid a quantity and you will get:

- What you have bid if the sum of bids of whole group is lower or equal to 10.
- If the aggregate bid is higher than 10, each member receives the same percentage of the total amount (10) as his bid (compared to the total sum of bids).

This implies that who bids more always receives more.

You can check your profits in the table enclosed with these instructions.

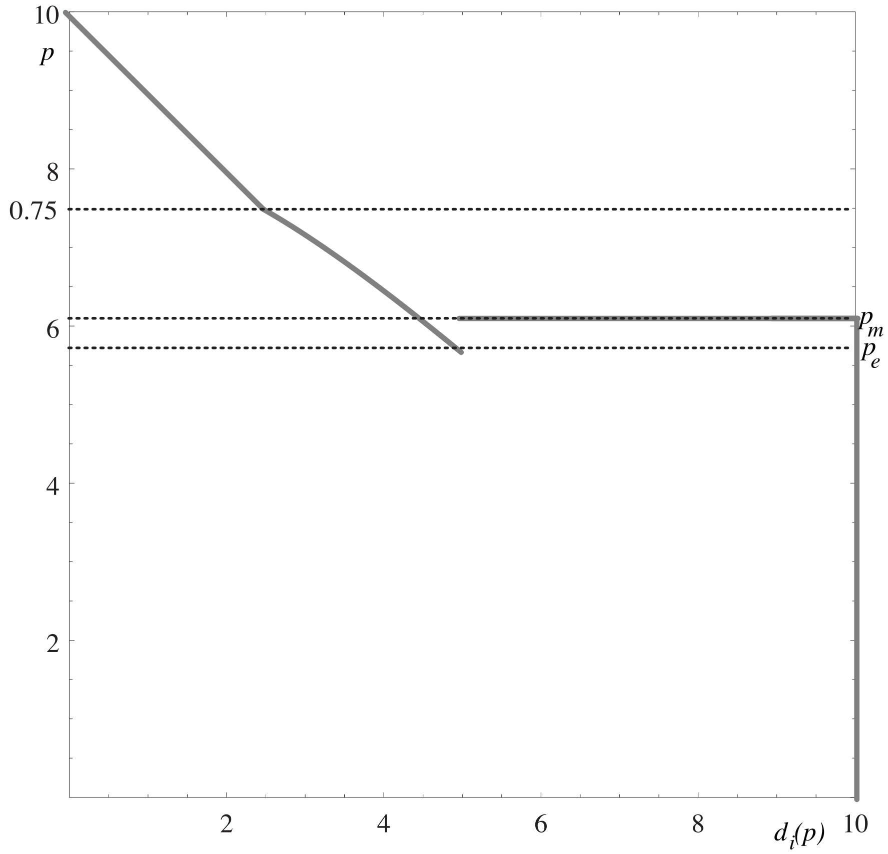

Figure 1. FPM: Equlibrium bid function(s)

p \ T_i	1	2	3	4
0	9.583 (2.041)	10 (0)	10 (0)	10 (0)
0.5	8.125 (3.073)	9.521 (0.102)	9.5 (0)	9.5 (0)
1	8.19 (2.212)	9.021 (0.102)	9 (0)	9 (0)
1.5	7.811 (2.167)	8.502 (0.01)	8.542 (0.204)	8.5 (0)
2	7.76 (1.228)	7.771 (1.123)	8 (0)	7.75 (1.225)
2.5	7.044 (1.707)	7.292 (1.021)	7.5 (0)	7.5 (0)
3	6.675 (1.114)	7 (0)	7 (0)	7 (0)
3.5	5.842 (1.933)	6.5 (0)	6.5 (0)	6.5 (0)
4	5.752 (1.022)	6 (0)	6 (0)	6 (0)
4.5	5.437 (0.224)	5.417 (0.319)	5.5 (0)	5.5 (0)
5	5.101 (1.286)	5 (0)	5 (0)	5 (0)
5.5	4.502 (1.076)	4.498 (0.01)	4.5 (0)	4.5 (0)
6	4.498 (1.623)	4 (0)	4 (0)	4 (0)
6.5	3.542 (0.751)	3.492 (0.052)	3.5 (0)	3.542 (0.204)
7	3.603 (1.68)	3 (0.002)	3 (0)	3 (0)
7.5	2.498 (1.191)	2.501 (0.004)	2.521 (0.102)	2.479 (0.102)
8	1.958 (0.204)	2 (0)	2 (0)	2 (0)
8.5	1.482 (0.571)	1.5 (0)	1.5 (0)	1.5 (0)
9	0.967 (0.163)	0.962 (.184)	1 (0)	0.979 (0.102)
9.5	1.148333 (2.114274)	0.485 (0.076)	0.5 (0)	0.5 (0)
10	0.537 (2.075)	0 (0)	0 (0)	0 (0)

Figure 2. ICM: Average aggregate bids (with standard deviations)

T_i / p	1	2	3	4
0	9.41 (1.433)	9.479 (1.407)	9.437 (2.007)	9.771 (1.139)
0.5	8.585 (2.241)	9.469 (1.537)	9.604 (1.276)	9.719 (1.051)
1	8.713 (2.039)	9.432 (1.43)	9.448 (1.555)	9.760 (.799)
1.5	8.217 (2.022)	9.506 (1.085)	9.437 (1.633)	9.594 (1.17)
2	8.855 (1.692)	9.065 (1.687)	9.184 (1.807)	9.604 (1.317)
2.5	7.396 (2.405)	8.9375 (1.844)	8.886 (1.853)	9.44 (1.229)
3	8.039 (1.964)	8.552 (2.052)	8.633 (1.977)	9.115 (1.874)
3.5	7.474 (1.95)	8.017 (2.014)	8.648 (1.864)	9.046 (1.66)
4	6.904 (1.907)	7.767 (2.)	8.322 (2.082)	8.402 (1.879)
4.5	6.807 (2.031)	6.694 (1.897)	7.516 (2.109)	8.21 (2.093)
5	6.323 (2.063)	6.52 (2.116)	6.924 (2.105)	7.414 (2.094)
5.5	5.15 (1.771)	6.38 (1.93)	6.523 (2.11)	6.339 (2.086)
6	4.764 (1.315)	4.957 (1.43)	4.934 (1.409)	5.611 (1.756)
6.5	4.514 (1.757)	4.577 (1.34)	4.823 (1.678)	5.136 (1.846)
7	3.809 (1.281)	3.528 (1.054)	3.824 (1.346)	3.897 (1.259)
7.5	3.16 (1.115)	2.83 (1.215)	2.672 (0.702)	2.859 (0.858)
8	2.2 (0.859)	2.104 (0.385)	2.063 (0.286)	2.083 (0.315)
8.5	2.226 (1.948)	1.56 (0.303)	1.729 (1.333)	1.63 (0.399)
9	1.078 (0.456)	0.984 (0.182)	0.969 (0.16)	1.094 (0.741)
9.5	1.073 (1.581))	0.595 (0.262)	0.522 (0.101)	0.51 (0.072)
10	0.51 (1.278)	0.658 (0.361)	0.005 (0.036)	0. (0.001)

Figure 3. FPM: Average aggregate bids (with standard deviations)

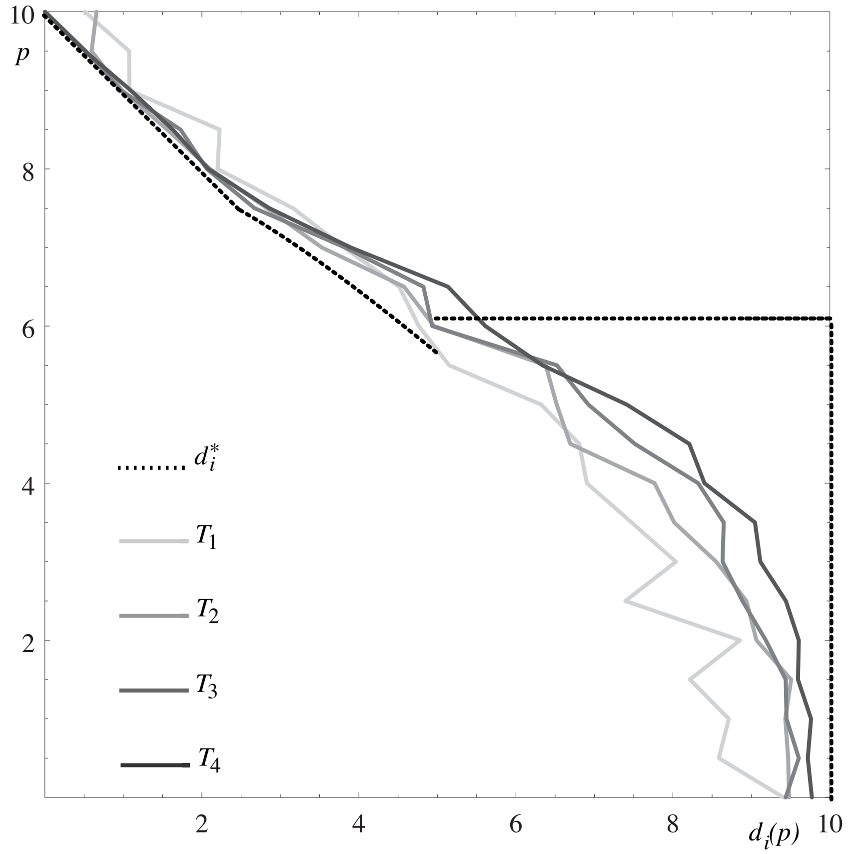

Figure 4. FPM: Evolution of aggregate bids

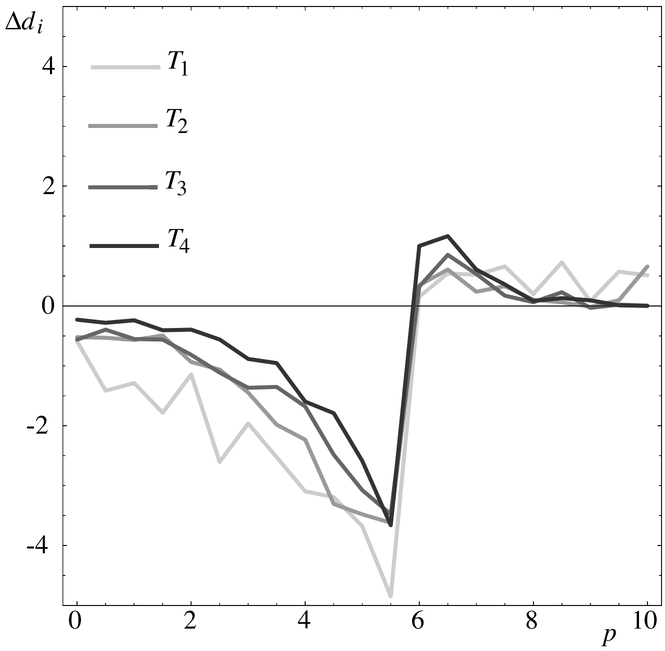

Figure 5. FPM: Evolution of deviations from equilibrium

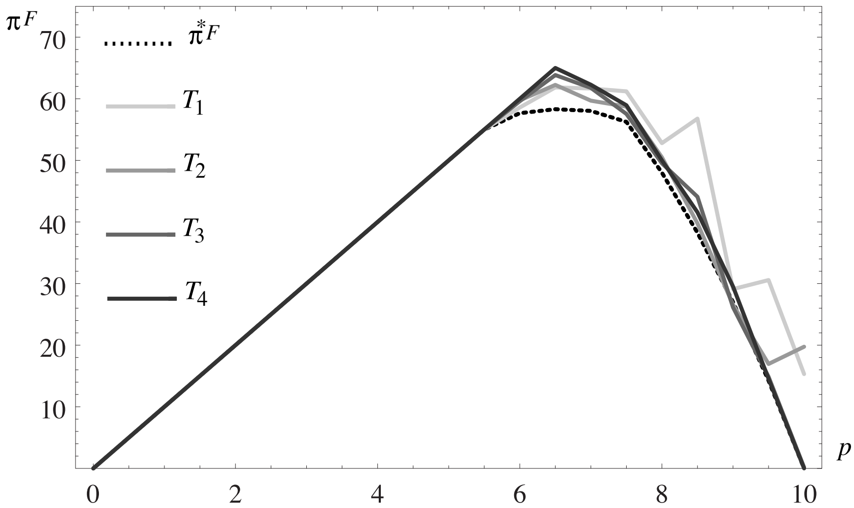

Figure 6. FPM: Evolution of the seller's profits

	T_i	α	β	γ	p-value	ρ	R^2
I	$T_1 - T_4$	.968 (.006)	$-.966$ (.006)	.004 (.001)	0	.037	0.9240
II	T_1	.921 (.016)	$-.871$ (.022)	-	0	.108	.735
III	T_2	.995 (.003)	$-.995$ (.004)	-	0	.044	.9873
IV	T_3	-1 (.0004)	1 (.0007)	-	0	0	.9997
V	T_4	.997 (.002)	$-.996$ (.004)	-	0	0	.9919

Figure 7. ICM: Panel Data Estimation

	T_i	α_0	α_1	α_2	β_0	β_1	β_2	γ	p-value	ρ	R^2
VI	$T_1 - T_4$	.993 (.044)	$-.049$ (.042)	.252 (.101)	-1.028 (.048)	.439 (.050)	$-.273$ (.150)	.021 (.001)	0	.248	.798
VII	T_1	1.059 (.094)	$-.119$ (.094)	$-.003$ (.225)	-1.015 (.106)	.382 (.112)	.061 (.332)	-	0	.232	.7194
VIII	T_2	.943 (.082)	.065 (.082)	.420 (.195)	$-.912$ (.092)	.268 (.098)	$-.517$ (.288)	-	0	.267	.8081
IX	T_3	1.068 (.079)	$-.065$ (.079)	.105 (.188)	-1.069 (.089)	.530 (.094)	$-.039$ (.278)	-	0	.333	.8162
X	T_4	1.113 (.073)	$-.078$ (.073)	.488 (.175)	-1.117 (.082)	.576 (.087)	$-.597$ (.258)	-	0	.288	.8553

Figure 8. FPM: Panel Data Estimations.

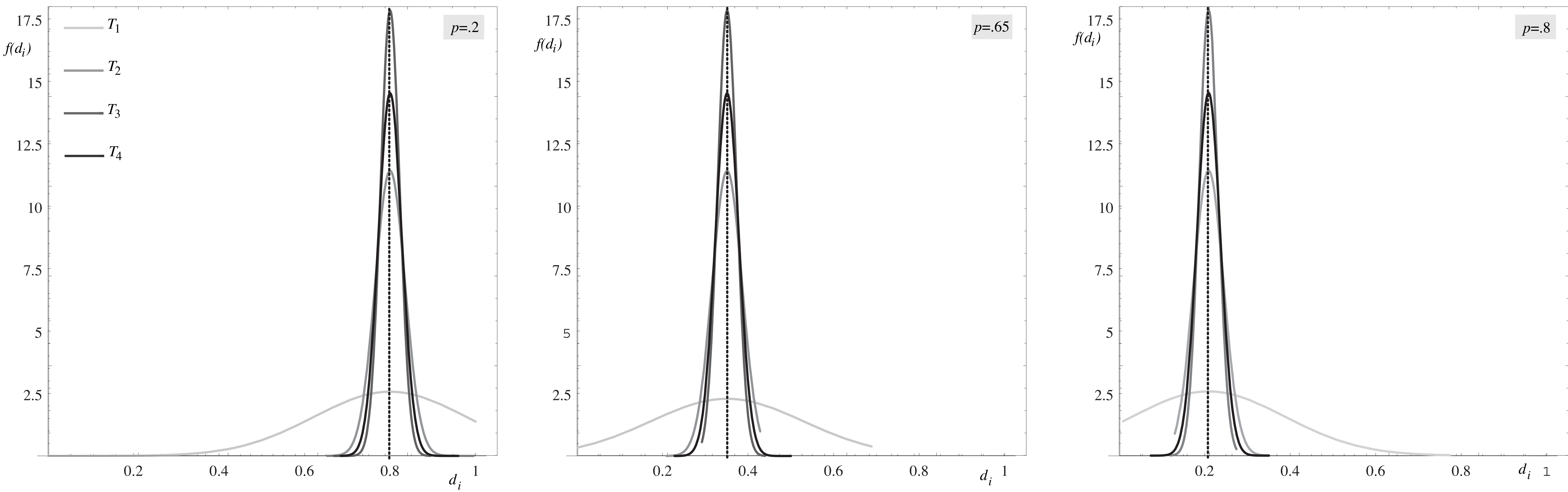

Figure 11. ICM: estimated QRE distributions

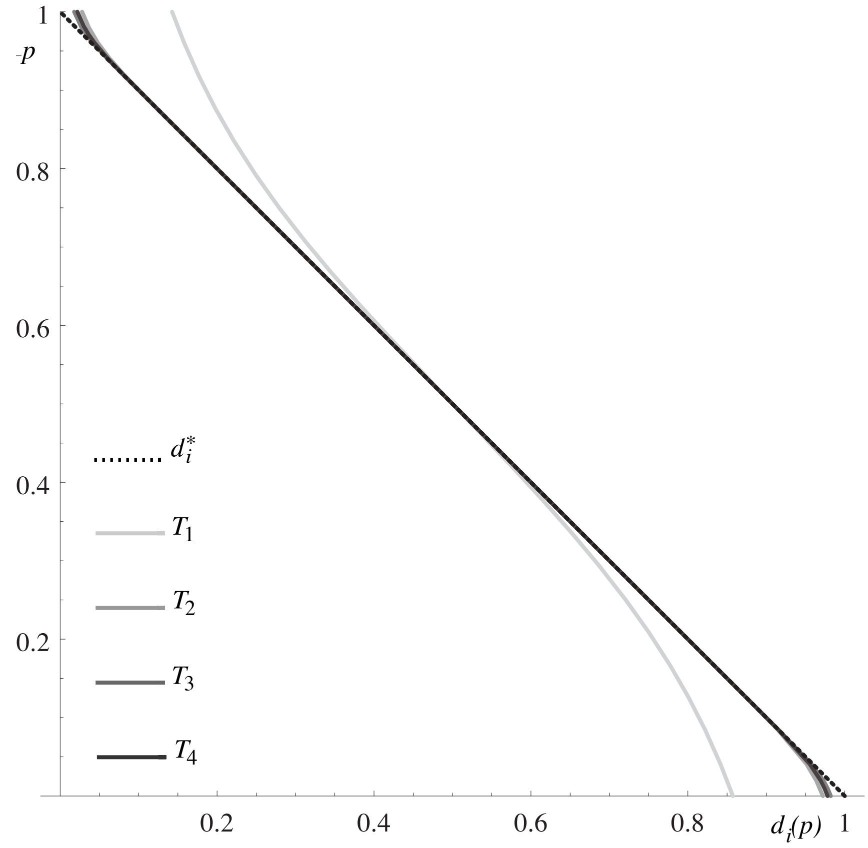

Figure 12. ICM: estimated QRE average bids

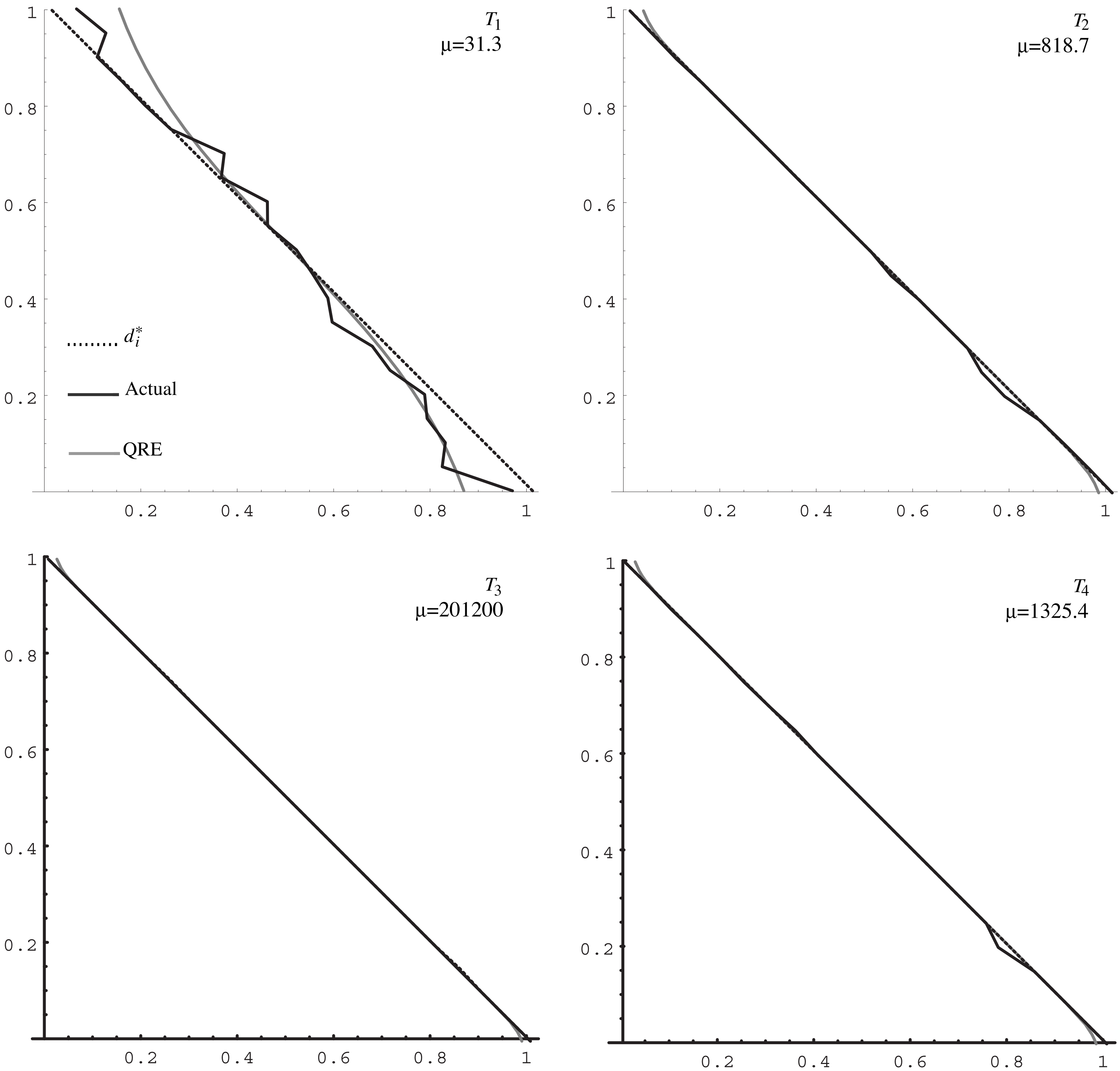

Figure 13. FPM: evolution of estimated QRE average bid functions

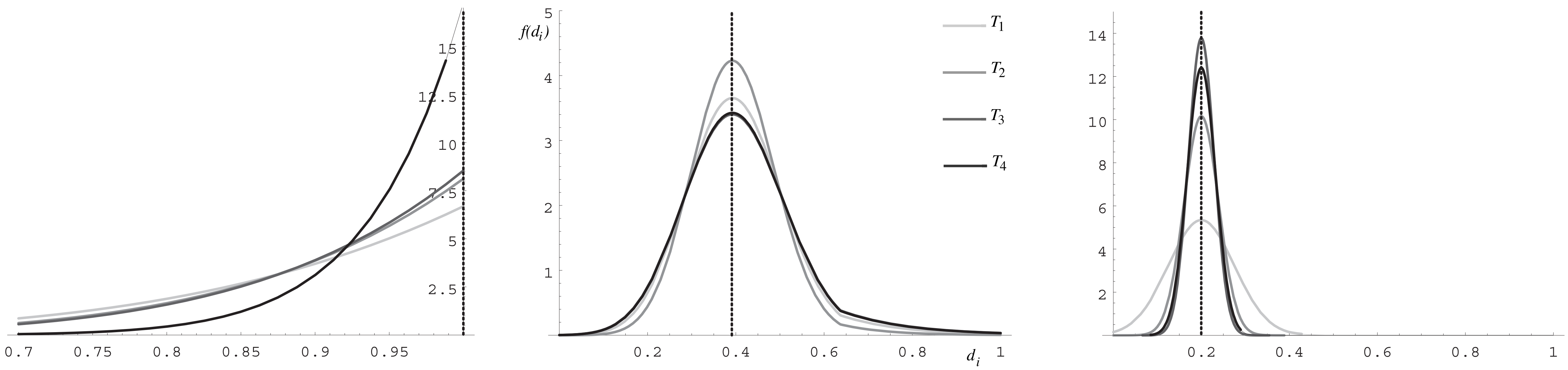

Figure 15. FPM: QRE distributions

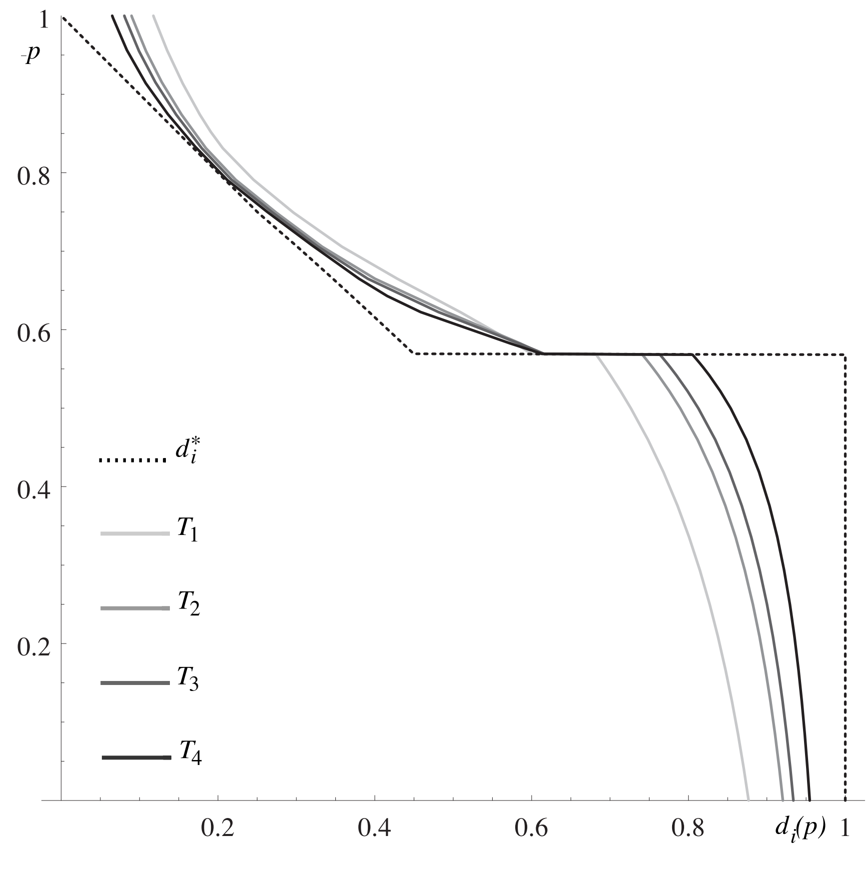

Figure 16. FPM: estimated QRE average bids

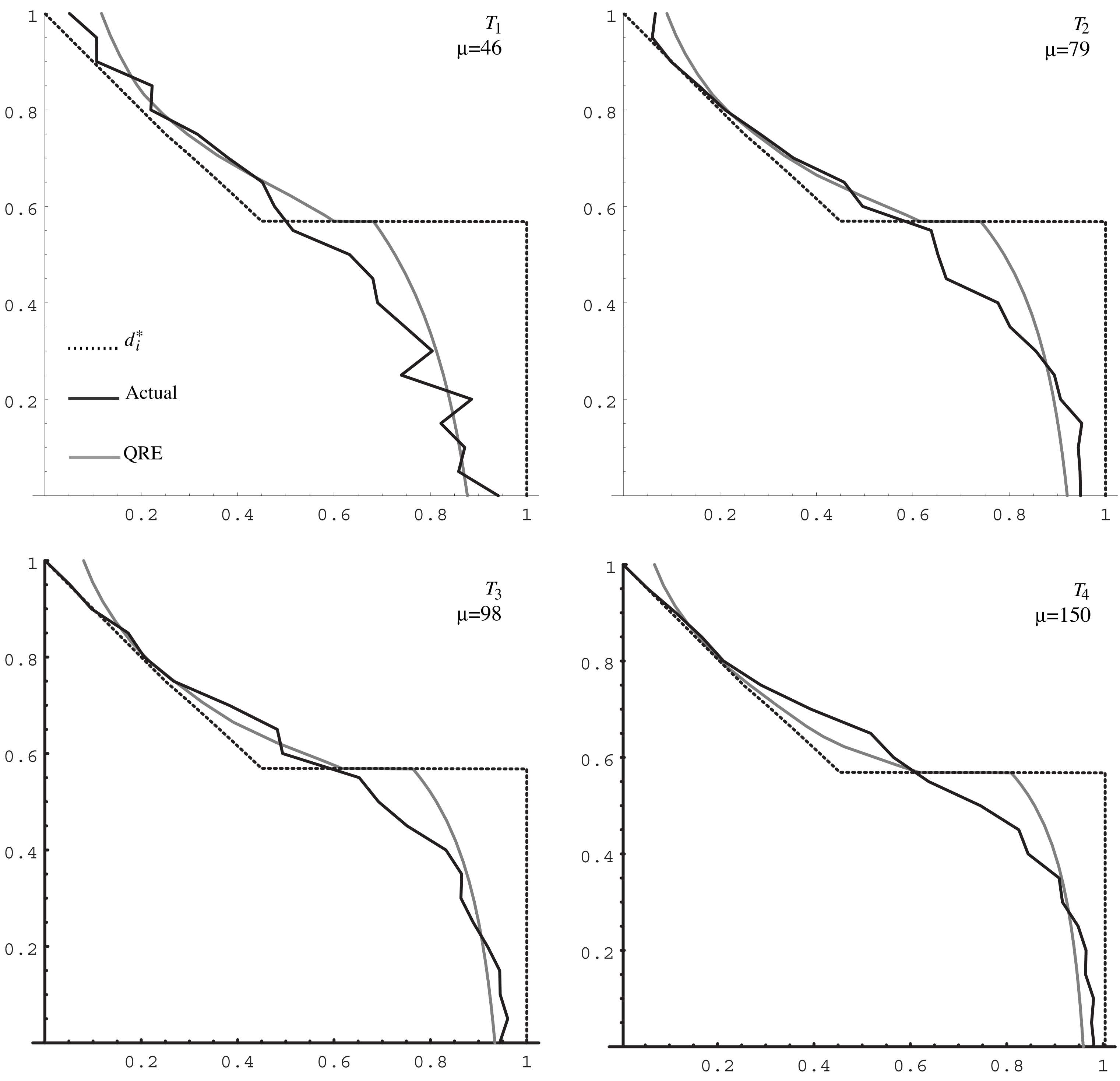

Figure 17. FPM: evolution of estimated QRE average bid functions

T_k	1	2	3	4
$\widehat{\mu}$	31.3	818.7	201200	1325.4

Figure 10. ICM: Estimated values of μ

p \ T_k	1			2			3			4		
0	91.2			102			64.8			206.5		
.05	31.5			99			142.9			212.4		
.1	41.4			109			103.1			303.8		
.15	30.8			155.5			105.2			180.5		
2	62.3			78.2			82.9			180.3		
.25	19.8			70.7			68.9			165.5		
.3	40.5	37.5		55.1	70.6		60.9	76		92.8	129	
.35	33.4			46.4			75			111.6		
.4	29.6			47			64.8			74.1		
.45	34.8		46	33.8		79	49.5		98	73.4		150
.5	37.3			40			49.7			62.4		
.55	45.9			70.4			58.9			60.1		
.6	205.2			254.4			309.4			197.2		
.65	134.4	148.2		175.4	182		118.6	146.5		120	126	
.7	135.6			153.2			125.6			114.6		
.75	105.5			155.7			294.2			184.1		
.8	178.3			648			1192.7			966.7		
.85	31.3	61		1072.7	81		69	358		577.2	401	
.9	430.7			3072			3840			137.1		
.95	36.9			1078.8			9569.3			191200		
1	54.2			19			76800			48×10^6		

Figure 14. FPM: Estimated values of μ (the 1st column of each period corresponds to estimations per prices, the 2nd per intervals and the last is restricted to unique μ per whole bid function)