Presidential Address: Discount Rates

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Abstract

Discount rate variation is the central organizing question of current asset pricing research. I survey facts, theories and applications.

We thought returns were uncorrelated over time, so variation in price-dividend ratios was due to variation in expected cashflows. Now it seems all price-dividend variation corresponds to discount-rate variation. We thought that the cross-section of expected returns came from the CAPM. Now we have a zoo of new factors.

I categorize discount-rate theories based on central ingredients and data sources.

Discount-rate variation continues to change finance applications, including portfolio theory, accounting, cost of capital, capital structure, compensation, and macroeconoimcs.

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1 Introduction

Asset prices should equal expected discounted cashflows. 40 years ago, Gene Fama (1970) argued that the expected part, "testing market efficiency," provided the framework for organizing asset-pricing research in that era. I argue that the "discounted" part better organizes our research today.

I start with facts: *How* discount rates vary over time and across assets. I turn to theory, *why* discount rates vary. I'll attempt a categorization based on central assumptions and links to data, analogously to Fama's "weak" "semi-strong" and "strong" forms of efficiency. Finally, I point to some applications, which I think will be strongly influenced by our new understanding of discount rates. In each case, I have more questions than answers. This paper is more of an agenda than a summary.

An apology: In the available space I cannot even cite let alone review all the deserving literature, or trace the development of all these ideas. My long reference list here only gives examples of relevant work.

2 Time-series facts

2.1 Simple DP regression

Discount rates vary over *time.* ("Discount rate" "risk premium" and "expected return" are all the same thing here.) Start with a very simple regression of returns on dividend yields,[1] shown in Table 1.

Horizon k	b	t(b)	R^2	$\sigma\left[E_t(R^e)\right]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

Table 1. Return forecasting regressions $R^e_{t\to t+k} = a + b \times D_t/P_t + \varepsilon_{t+k}$. Annual data, CRSP value weighted return less 3 month Treasury return 1947-2009. The 5 year regression t statistic uses the Hansen-Hodrick (1983) correction. $\sigma\left[E_t(R^e)\right]$ stands for $\sigma(\hat{b} \times D_t/P_t)$.

The one-year regression forecast doesn't seem that important. Yes, the t statistic is "significant," but there are lots of biases and fishing. The 9% R^2 isn't impressive.

In fact, this regression has huge economic significance. First, the coefficient estimate is large. One percentage point more dividend yield forecasts nearly *four* percentage points more return. Prices rise by an additional three percentage points.

Second, five and a half percentage point variation in expected returns is a lot. A six-percent equity premium was already a "puzzle."[2] The regression implies that expected returns *vary* by at least as much as their puzzling *level*.

[1] Fama and French (1988).

[2] Mehra and Prescott (1985).

By contrast, R^2 is a poor measure of economic significance in this context[3]. The economic question is "How much do expected returns vary over time?" There will always be lots of unforecastable return movement, so the variance of ex-post returns isn't a very informative comparison.

Third, the slope coefficients and R^2 rise with horizon. Figure 1 plots each year's dividend yield along with the subsequent seven years of returns. Read the dividend yield as prices upside down: prices were low in 1980 and high in 2000. The picture then captures the central fact: *High prices, relative to dividends, have reliably led to many years of poor returns. Low prices have led to high returns.*

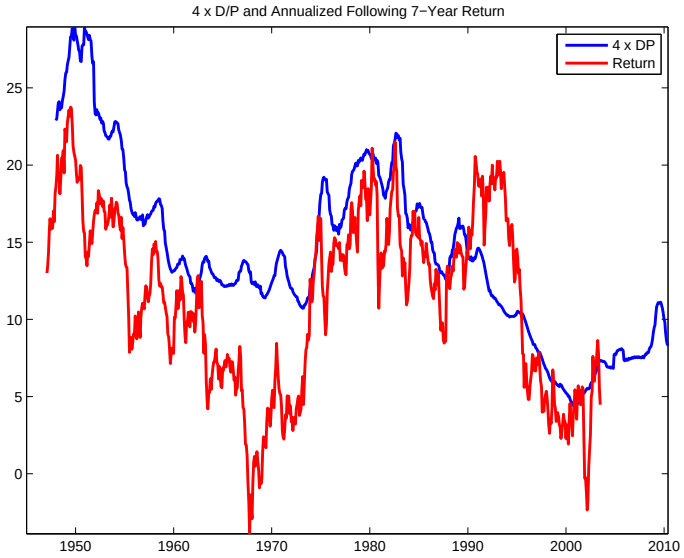

Figure 1: Dividend yield (multiplied by 4) and following annualized 7-year return. CRSP value-weighted market index.

2.2 Present values, volatility, bubbles, and long-run returns

Long horizons are interesting, really, because they tie predictability to volatility, "bubbles," and the nature of price movements. I make that connection via the Campbell-Shiller (1988) approximate present value identity,

$$dp_t \approx \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}, \tag{1}$$

where $dp_t \equiv d_t - p_t = \log(D_t/P_t)$, $r_{t+1} \equiv \log R$, and $\rho \approx 0.96$ is a constant of approximation. See the Appendix for details.

If we run regressions of weighted long-run returns and dividend growth on dividend yields, the present value identity (1) implies that the long-run regression coefficients must add up to one,

$$1 \approx b_r^{(k)} - b_{\Delta d}^{(k)} + \rho^k b_{dp}^{(k)}. \tag{2}$$

[3] Campbell (1991) makes this point, noting that a perpetuity would have very low short-run R^2.

Just run both sides of the identity (1) on dp_t. Here, $b_r^{(k)}$, $b_{\Delta d}^{(k)}$ and $b_{dp}^{(k)}$ denote long-run regression coefficients, i.e.

$$\begin{aligned} \sum_{j=1}^{k} \rho^{j-1} r_{t+j} &= a_r + b_r^{(k)} dp_t + \varepsilon_{t+k}^r \quad (3) \\ \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} &= a_d + b_d^{(k)} dp_t + \varepsilon_{t+k}^d \\ dp_{t+k} &= a_{dp} + b_{dp}^{(k)} dp_t + \varepsilon_{t+k}^{dp}. \quad (4) \end{aligned}$$

If we lived in an i.i.d. world, dividend yields would never vary in the first place as expected future returns and dividend growth would never change. If dividend yields vary at all, they must forecast long-run returns, long-run dividend growth, or a "rational bubble" of ever higher prices. The regression coefficients in (2) can be read as the fractions of dividend yield variation attributed to each source. To see this interpretation more clearly, multiply both sides of (2) by $var(dp_t)$, giving

$$var\left(dp_t\right) \approx cov\left[dp_t, \sum_{j=1}^{k} \rho^{j-1} r_{t+j}\right] - cov\left[dp_t, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}\right] + \rho^k cov\left(dp_t, dp_{t+k}\right) \quad (5)$$

The empirical question is, which is it? Table 2 presents long-run regression coefficients.

	$b_r^{(k)}$	$b_{\Delta d}^{(k)}$	$b_{dp}^{(k)}$
Direct regression , $k = 15$	1.01	-0.11	-0.11
Implied by VAR, $k = 15$	1.05	0.27	0.22
VAR, $k = \infty$	1.35	0.35	0.00

Table 2. Long-run regression coefficients, for example $\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a + b_r^{(k)} dp_t + \varepsilon_{t+k}^r$. Annual data 1947-2009. "Direct" estimates are based on 15-year ex-post returns. The "VAR" estimates infer long-run coefficients from one-year coefficients, using estimates in the right-hand panel of Table 3. (See the Appendix for details.)

The long-run return coefficients are all a bit larger than 1.0. The dividend-growth forecasts are small, insignificant, and positive point estimates go the "wrong" way – high prices relative to current dividends signal *low* future dividend growth. The 15-year dividend-yield forecast coefficient is also essentially zero, and has the "wrong" sign as well.

Thus the estimates say that *all price-dividend ratio volatility corresponds to variation in expected returns. None corresponds to variation in expected dividend growth, and none to "rational bubbles."*

In the 1970s, we would have guessed exactly the opposite pattern. On the idea that returns are not predictable, we would have supposed that high prices relative to current dividends reflect expectations that dividends will rise in the future, and so forecast higher dividend growth. That pattern is completely absent. Instead, high prices relative to current dividends entirely forecast low returns.

This is the true meaning of return forecastability.[4] This is the real measure of "how big" the point estimates are – return forecastability is "just enough" to account for price volatility. This is the natural set of units with which to evaluate return forecastability. What we expected to be 0 is 1; what we expected to be 1 is 0.

Table 2 also reminds us that the point of the project is to understand *prices*, the *right hand* variable of the regression. We put return on the left because the forecast error is uncorrelated with the forecasting variable. That choice does not reflect "cause" and "effect," nor does it imply that the point of the exercise is to understand (ex-post) return variation.

How you look at things matters. The long-run and short-run regressions are mathematically equivalent. Yet one transformation shows an unexpected economic significance. We will see this lesson repeated many times.

(Table 2 does not include standard errors, and sampling variation in long-run estimates is an important topic.[5] My point is the *economic* importance of estimates. One might still argue that we can't reject the alternative views. But when point estimates are 1 and 0, arguing we should believe 0 and 1 because that view can't be rejected is obviously a tough sell.

The variance of dividend yields or price-dividend ratios corresponds entirely to discount-rate variation, but as much as half of the variance of price changes $\Delta p_{t+1} = -dp_{t+1} + dp_t + \Delta d_{t+1}$ or returns $r_{t+1} \approx -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$ corresponds to current dividends Δd_{t+1}. This fact seems trivial but has caused a lot of confusion.)

2.3 A pervasive phenomenon

This pattern of predictability is pervasive across markets. For stocks, bonds, credit spreads, foreign exchange, sovereign debt and houses, a yield or valuation ratio translates one-for-one to expected excess returns, and does not forecast the cashflow or price change we may have expected. In each case our view of the facts have changed 100% since the 1970s.

- *Stocks.* Dividend yields forecast returns, not dividend growth.[6]
- *Treasuries.* A rising yield curve signals better one-year return for long-term bonds, not higher future interest rates. Fed fund futures signal returns, not changes in the funds rate.[7]
- *Bonds.* Much variation of credit spreads over time and across firms or categories signals returns not default probabilities.[8]
- *Foreign exchange.* International interest rate spreads signal returns, not exchange-rate depreciation.[9]
- *Sovereign debt.* High levels of sovereign or foreign debt signal low returns, not higher government or trade surpluses.[10]

[4] Shiller (1981), Campbell and Shiller (1988), Campbell and Ammer (1993), Cochrane (1991a), (1992), (1994) and review in (2005c).

[5] See Cochrane (2006) and its references.

[6] Fama and French (1988), (1989).

[7] Fama and Bliss (1987), Campbell and Shiller (1991), Piazzesi and Swanson (2008).

[8] Fama (1986), Duffie and Berndt (2011).

[9] Hansen and Hodrick (1980), Fama (1984).

[10] Gourinchas and Rey (2007).

- *Houses.* High price/rent ratios signal low returns, not rising rents or prices that rise forever.

Since houses are so much in the news, Figure 2 shows house prices and rents, and Table 3 presents a regression. High prices relative to rents mean low returns, not higher subsequent rents, or prices that rise forever. The housing regressions are almost the same as the stock market regressions. (Not everything about house and stock data is the same of course. Measured house price data are more serially correlated.)

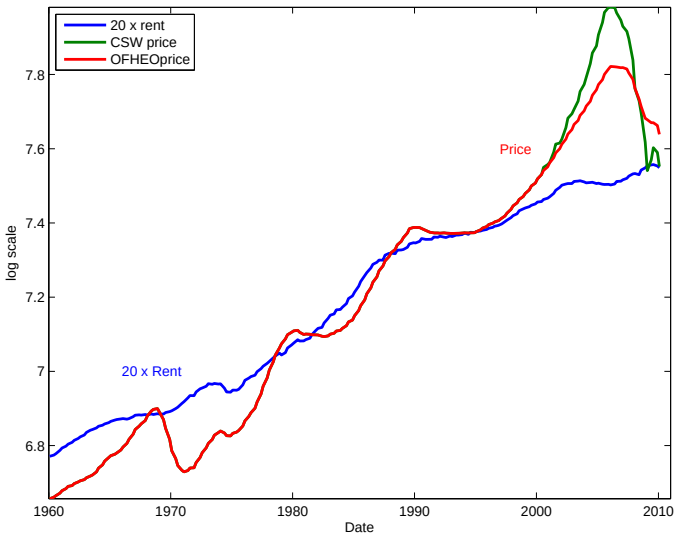

Figure 2: House prices and rents. OFHEO is the Office of Federal Housing Enterprise Oversight "purchase-only" price index. CSW are Case-Shiller-Weiss price data. Data from http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp

	Houses			Stocks		
	b	t	R^2	b	t	R^2
r_{t+1}	0.12	(2.52)	0.15	0.13	(2.61)	0.10
Δd_{t+1}	0.03	(2.22)	0.07	0.04	(0.92)	0.02
dp_{t+1}	0.90	(16.2)	0.90	0.94	(23.8)	0.91

Table 3. Left: Regressions of log annual housing returns r_{t+1}, log rent growth Δd_{t+1} and log rent/price ratio dp_{t+1} on the rent/price ratio dp_t, $x_{t+1} = a + b \times dp_t + \varepsilon_{t+1}$ 1960-2010. Right: Regressions of log stock returns r_{t+1}, dividend growth Δd_{t+1} and dividend yields dp_{t+1} on dividend yields dp_t, annual CRSP value weighted return data 1947-2010.

There is a strong common element and a strong business cycle association to all these forecasts.[11] Low prices and high expected returns hold in "bad times," when consumption, output and investment are low, unemployment is high, businesses are failing, and vice versa.

These *facts* bring a good deal of structure to the debate over "bubbles" and "excess volatility." High valuations correspond to low returns, and associated with good economic conditions. All a

[11] Fama and French (1989).

"price bubble" can possibly mean now is that the equivalent discount rate is "too low" relative to some theory. This fact channels us to a much more profitable discussion.

2.4 The multivariate challenge

This empirical project has only begun. We see that one variable at a time forecasts one return at a time. We need to understand their *multivariate* counterparts, on both the left and right hand sides of the regressions.

For example the stock and bond regressions on dividend yield and yield spread (ys) are

$$\begin{aligned} r^{\text{stock}}_{t+1} &= a_s + b_s \times dp_t + \varepsilon^s_{t+1} \\ r^{\text{bond}}_{t+1} &= a_b + c_b \times ys_t + \varepsilon^b_{t+1}. \end{aligned}$$

We have some additional predictor variables z_t, from similar univariate or at best bivariate (hence $[+b_s \times dp_t]$) explorations,

$$r^{\text{stock}}_{t+1} = a_s \ [+b_s \times dp_t] + d_s \times z_t + \varepsilon^s_{t+1}.$$

First, then, which of these variables are really important in a multiple regression sense? In particular, do the variables that forecast one return forecast another?

$$\begin{aligned} r^{\text{stock}}_{t+1} &= a_s + b_s \times dp_t + \boxed{c_s \times ys_t} + \boxed{d_s' z_t} + \varepsilon^s_{t+1}? \\ r^{\text{bond}}_{t+1} &= a_b + \boxed{b_b \times dp_t} + c_b \times ys_t + \boxed{d_b' z_t} + \varepsilon^b_{t+1}? \end{aligned} \tag{6}$$

(I put the variables we need to learn about in boxes.)

Second, how correlated are the right-hand terms of these regressions? What is the *factor structure of time-varying expected returns?* Expected returns $E_t(r^i_{t+1})$ vary across time t; how correlated is such variation across assets and asset classes j, and how can we best express that correlation as factor structure? As an example to clarify the question, suppose we find the stock return coefficients are all double those of the bonds,

$$\begin{aligned} r^{\text{stock}}_{t+1} &= a_s + 2 \times dp_t + 4 \times ys_t + \varepsilon^s_{t+1} \\ r^{\text{bond}}_{t+1} &= a_b + 1 \times dp_t + 2 \times ys_t + \varepsilon^b_{t+1} \end{aligned}$$

We would see a *one-factor model for expected returns,* with stock expected returns always changing by twice bond expected returns,

$$\begin{aligned} E_t\left(r^{\text{stock}}_{t+1}\right) &= 2 \times \text{factor}_t \\ E_t\left(r^{\text{bond}}_{t+1}\right) &= 1 \times \text{factor}_t. \end{aligned} \tag{7}$$

Third, we need to relate time-varying *expected* returns to covariances with pricing factors or portfolio returns.

$$E_t\left(r^i_{t+1}\right) = cov_t(r^i_{t+1}\mathbf{f}'_{t+1})\boldsymbol{\lambda}_t.$$

As a small step down this road, Cochrane and Piazzesi (2005) (2008) find that forward rates of *all* maturities help to forecast returns of *each* maturity – multiple regressions matter as in (6).

We found that the right-hand sides are almost perfectly correlated across left-hand maturities.[12] A single common factor describes 99.9% of the variance of *expected* returns as in (7). Finally, we find that the spread in time-varying expected bond returns across maturities corresponds to a spread in covariances with a single "level" factor, and the market prices of risk of slope, curvature, and expected-return factors are zero.

What similar patterns hold across broad asset classes? The challenge, of course, is that there are too many right hand variables, so we can't just go run huge multiple regressions. But these are the vital *questions*.

2.5 Multivariate prices

I advertised much of the point of running return regressions with prices on the right hand side was to understand those prices. How will a multivariate investigation change our picture of *prices and long-run* returns?

Again, the Campbell-Shiller present value identity

$$dp_t \approx \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \tag{8}$$

provides a useful way to think about these questions. Since this identity holds ex-post, it holds for any information set. Dividend yields are a great forecasting variable because they reveal market expectations of dividend growth and returns. However, dividend yields combine the two sources of information. A variable can help the dividend yield to forecast long-run returns if it *also* forecasts long-run dividend growth. A variable can also help to predict *one* year returns r_{t+1} without much changing long-run expected returns, if it has an offsetting effect on longer-run returns $\{r_{t+j}\}$. Such a variable signals a change in the *term structure of risk premia* $\{E_t r_{t+j}\}$.

I examine Lettau and Ludvigson's (2001a) (2005) consumption to wealth ratio cay as an example to explore these questions. Table 4 presents forecasting regressions

	Coefficients		t-statistics		Other statistics		
	dp_t	cay_t	dp_t	cay_t	R^2	$\sigma\left[E_t(y_{t+1})\right]\%$	$\frac{\sigma[E_t(y_{t+1})]}{E(y_{t+1})}$
r_{t+1}	0.12	0.071	(2.14)	(3.19)	0.26	8.99	0.91
Δd_{t+1}	0.024	0.025	(0.46)	(1.69)	0.05	2.80	0.12
dp_{t+1}	0.94	-0.047	(20.4)	(-3.05)	0.91		
cay_{t+1}	0.15	0.65	(0.63)	(5.95)	0.43		
$r_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$	1.29	0.033				0.51	
$\Delta d_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0.29	0.033				0.12	

Table 4. Forecasting regressions using dividend yield and consumption-wealth ratio, 1952-2009, annual data. Long-run coefficients are computed using a first-order VAR with dp_t and cay_t as state variables. Each regression includes a constant. Cay is rescaled so $\sigma(cay) = 1$. $\sigma(dp) = 0.42$.

Cay helps to forecast one-period returns. The t statistic is large, and it raises the variation of expected returns substantially. Cay only marginally helps to forecast dividend growth. (Lettau and Ludvigson report that it works better in quarterly data.)

[12] Hansen and Hodrick (1983) and Stambaugh (1988) find similar structures.

Figure 3 graphs the one-year return forecast using dp alone, the one-year return forecast using dp and cay together, and the actual ex-post return. Adding cay lets us forecast business-cycle frequency "wiggles" while not much changing the "trend."

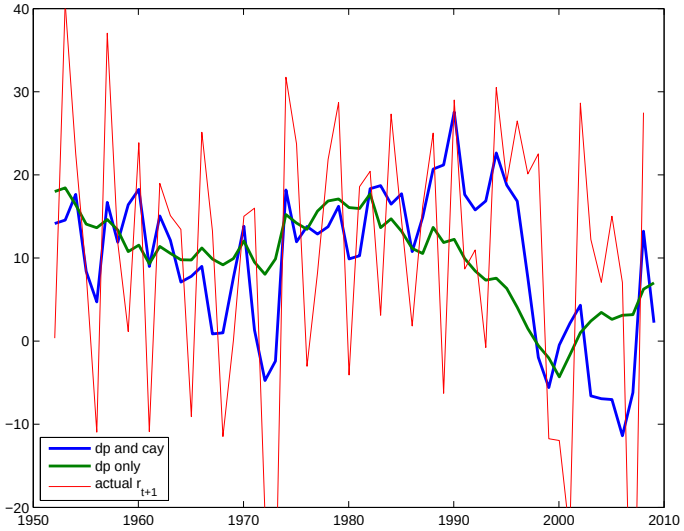

Figure 3: Forecast and actual 1 year returns. The forecasts are fitted values of regressions of returns on dividend yield and cay. Actual returns are plotted on the same date as their forecast, i.e. r_{t+1} is plotted at the same date as $a + b \times dp_t$.

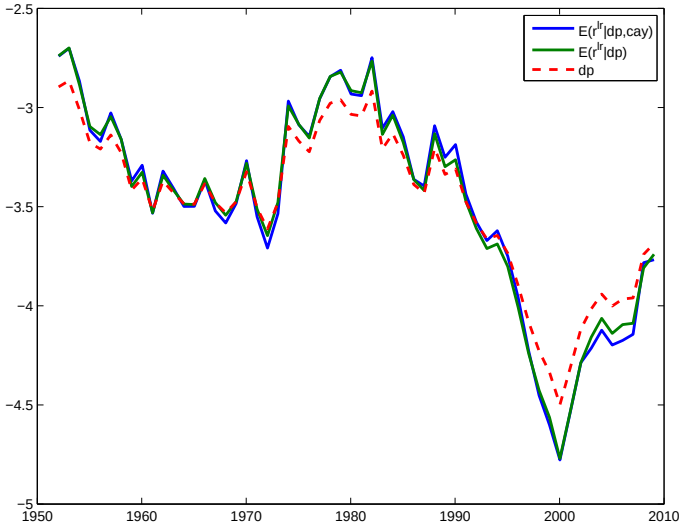

Figure 4: Dividend yield dp and forecasts of long-run returns $\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$.Return forecasts are computed from a VAR including dp, and a VAR including dp and cay.

Long-run return forecasts are quite different. Figure 4 contrasts long-run return forecast with and without cay. Though cay has a dramatic effect on *one-period* return r_{t+1} forecasts in Figure 3, cay has almost no effect at all on *long-run* return $\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$ forecasts in Figure 4.

Figure 4 includes the actual dividend yield, to show (by (8)) how dividend yields break into long-run return vs. dividend growth forecasts. The last two rows of Table 4 give the corresponding long-run regression coefficients. Essentially all price-dividend variation still corresponds to expected-return forecasts.

How can cay forecast one-year returns so strongly, but have such a small effect on prices? In the context of (8), cay alters the *term structure* of expected returns. We can display this behavior with impulse-response functions. Figure 5 plots responses to a dividend growth shock, a dividend yield shock, and a cay shock. In each case, I include a contemporaneous return response to satisfy the return identity $r_{t+1} = \Delta d_{t+1} - \rho dp_{t+1} + dp_t$.

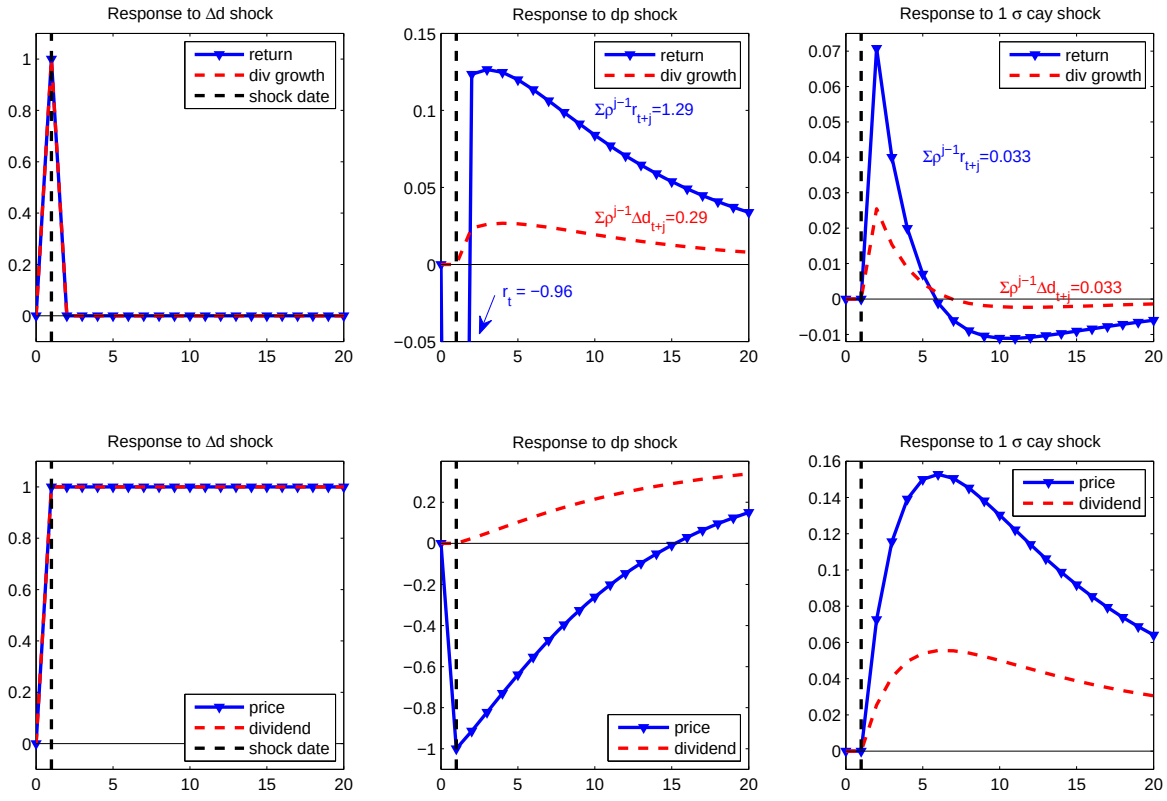

Figure 5: Response functions to dividend growth, dividend yield, and cay shocks. Calculations are based on the VAR of Table 4. Each shock changes the indicated variable without changing the others, and includes a contemporaneous return shock $r_{t+1} = \Delta d_{t+1} - \rho dp_{t+1} + dp_t$. The vertical dashed line indicates the period of the shock.

These plots answer the question, "what change in expectations corresponds to the given shock?" The dividend growth shock corresponds to permanently higher expected dividends with no change in expected returns. Prices jump to their new higher value and stay there. It is a pure "expected cashflow" shock. The dividend yield shock is essentially a pure discount rate shock. It shows a rise in expected returns with little change in expected dividend growth.

The cay shock in the rightmost panel of Figure 5 corresponds to a shift in expected returns from the distant future to the near future, with a small similar movement in the timing of a dividend growth forecast. It has almost no effect on long run returns or dividend growth. We could label it

a shock to the term structure of risk premia.[13]

So, cay strongly forecasts one-year returns, but has little effect on price-dividend ratio variance attribution. Does this pattern hold for other return forecasters? I don't know. In principle, consistently with the identity (8), other variables can help dividend yields to predict *both* long-run returns and long-run dividend growth. Consumption and dividends should be cointegrated, and since dividends are so much more volatile, the consumption-dividend ratio should forecast long-run dividend growth. Cyclical variables should work: at the bottom of a recession, both discount rates and expected growth rates are likely to be high, with offsetting effects on dividend yields. However, the lesser persistence of typical forecasters will work against much effect on price-dividend ratios. Cay's coefficient of only 0.65 on its own lag, and the fact that cay did not forecast dividend yields, are much of the story for cay's failure to affect long-run forecasts.

Even so, if additional variables help to forecast long-run dividend growth, they can only *raise* the contribution of long-run expected returns to price-dividend variation. It does not shift variance attribution from returns do dividends. A higher long-run dividend forecast must be matched by a higher long-run return forecast if it is not to affect the dividend yield.

This is a suggestive first-step, not an answer. We have a smorgasbord of return forecasters to investigate, singly and jointly, including additional predictability in additional lags of returns and dividend yields (see the Appendix). The point is this: Multivariate long-run forecasts and consequent price implications can be quite different from one-period return forecasts. As we pursue the multivariate forecasting question using the large number of additional forecasting variables, we should look at pricing implications, not just focus on short-run R^2 contests.

3 The Cross Section

In the beginning, there was chaos; practitioners thought one only needed to be clever to earn high returns. Then came the CAPM. Every clever strategy to deliver high average returns ended up delivering high market betas as well. Then anomalies erupted, and there was chaos again. The "value effect" was the most prominent anomaly.

Figure 6 presents Fama-French 10 book/market-sorted portfolios. Average excess returns rise from growth (low book/market, "high price") to value (high book/market, "low price"). This fact would not be a puzzle if the betas also rose. But the betas are about the same for all portfolios.

The absence of beta is really the heart of the value puzzle. It's perfectly natural that stocks which have fallen on hard times should have higher subsequent returns. If the market declines, these stocks should be particularly hard hit. They should have higher average returns – *and* higher betas. All puzzles are *joint* puzzles of expected returns and betas. Beta without expected return is just as much a puzzle – and profitable – as expected return without beta.[14] (The appendix shows how beta

Fama and French (1993), (1996) brought order once again with size and value factors. Figure 6 includes the results of multiple regressions on the market and Fama and French's hml factor,

$$R^{ei}_t = \alpha_i + b_i \times rmrf_t + h_i \times hml_t + \varepsilon_{it}.$$

The Figure shows the separate contributions of $b_i \times E\left(rmrf\right)$ and $h_i \times E\left(hml\right)$ in accounting for $E(R^{ei})$. Higher average returns *do* line up well with larger values of the $h_i$regression coefficient

[13] For impulse-responses, see Cochrane (1994). For the effect of cay, see Lettau and Ludvigson (2005).

[14] Frazzini and Pedersen (2010).

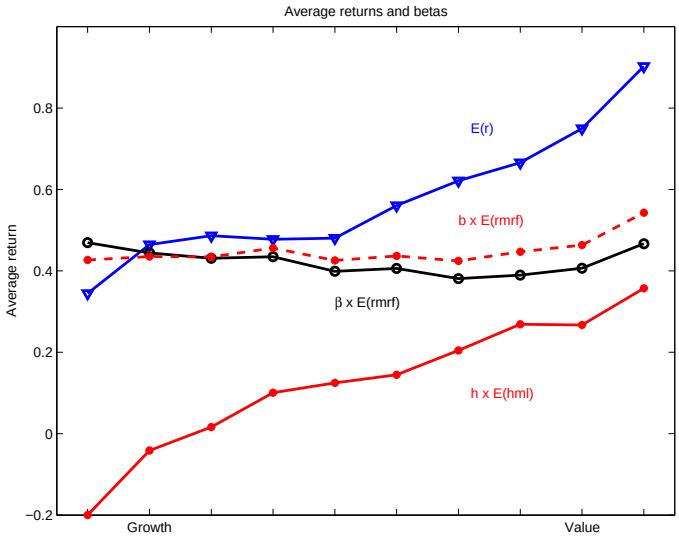

Figure 6: Average returns and betas for Fama - French 10 Book/Market sorted portfolios. Monthly data 1963-2010.

Fama and French's factor model accomplishes a very useful data reduction. Theories now only have to explain the hml portfolio premium, not the expected returns of individual assets.[15] This lesson has yet to sink in to a lot of empirical work, which still uses the 25 Fama French portfolios to test deeper models.

Covariance is in a sense Fama and French's central result: if the value firms decline, they all decline together. Where there is mean, there must be comovement, so that Sharpe ratios do not rise without limit in well-diversified value portfolios.[16] But theories now must also explain this *common movement* among value stocks. It is not enough to simply generate temporary price movements, a "fad" that produces high or low prices and then fades away rewarding contrarians. You need all the low-price securities to subsequently *rise and fall together* in the following month.

Finally, Fama and French found that *other* sorting variables, such as firm sales growth, did not each require a new factor. The three-factor model took the place of the CAPM for routine risk-adjustment in empirical work.

Order to chaos, yes, but once again, the world changed 100%. None of the cross-section of average stock returns corresponds to market betas. 100% corresponds to *hml* (and size) betas

Alas, the world is once again descending into chaos. Expected return strategies have emerged that *do not* correspond to market, value, and size betas. These include, among many others, momentum[17], accruals, equity issues and other accounting-related sorts,[18] beta arbitrage, credit risk, bond and equity market-timing strategies, foreign exchange carry trade, put option writing, and various forms of "liquidity provision."

[15] Daniel and Titman (2006), Lewellen, Nagel, and Shanken (2010).
[16] Ross (1976), (1978).
[17] Jegadeesh and Titman (1993).
[18] See Fama and French (2010).

3.1 The multidimensional challenge

We're going to have to repeat Fama and French's anomaly digestion, but with many more dimensions. We have a lot of questions to answer:

First, which characteristics really provide *independent* information about average returns? Which are subsumed by others?

Second, does each new anomaly variable also correspond to a new factor formed on those same anomalies? Momentum returns correspond to regression coefficients on a winner-loser momentum "factor." Carry-trade profits correspond to a carry-trade factor.[19] Do accruals return strategies correspond to an accruals factor? We should routinely look.

Third, how many of these new factors are really important? Can we again account for N independent dimensions of expected returns with $K < N$ factor exposures? Can we account for accruals return strategies by betas on some other factor, as with sales growth?

Now, factor structure is neither necessary nor sufficient for factor pricing. ICAPM and consumption-CAPM models do not predict or require that the multiple pricing factors will correspond to big common movements in asset returns. And big common movements, such as industry portfolios, need not correspond to any risk premium. There always is an equivalent single-factor pricing representation of any multifactor model, the mean-variance efficient portfolio. Still, the world would be much simpler if betas on only a few factors, important in the covariance matrix of returns, accounted for a larger number of mean characteristics.

Fourth, eventually, we have to connect all this back to the central question of finance, why do *prices* move?

3.2 Asset pricing as a function of characteristics/unification

To address these questions in the zoo of new variables, I suspect we will have to use different *methods.* Following Fama and French, a standard methodology has developed: sort assets into portfolios based on a characteristic, look at the portfolio means, especially the 1-10 portfolio alpha, information ratio, and t-statistic, and then see if the spread in means corresponds to a spread of portfolio betas against some factor. But we can't do this with 27 variables.

Portfolio sorts are really the same thing as nonparametric cross sectional regressions, using rather inefficient non-overlapping histogram weights. Figure 7 illustrates the point. For one variable, portfolio sorts and regressions both work. But we can't chop portfolios 27 ways, so I think we will end up running multivariate regressions.[20] (The Appendix gives a simple cross-sectional regression to illustrate.)

Having said that, you see that "time series" forecasting regressions, "cross-sectional" regressions and portfolio mean returns are really the same thing. All we are ever really doing is understanding a big panel-data forecasting regression

$$R^{ei}_{t+1} = a + \mathbf{b}'\mathbf{C}_{it} + \varepsilon^i_{t+1}.$$

We end up describe expected returns as a function of characteristics,

$$E(R^e_{t+1}|\mathbf{C}_t)$$

[19] Lustig, Roussanov, and Verdelhan (2010a).

[20] Fama and French (2010) already run such regressions, despite evident reservations over functional forms.

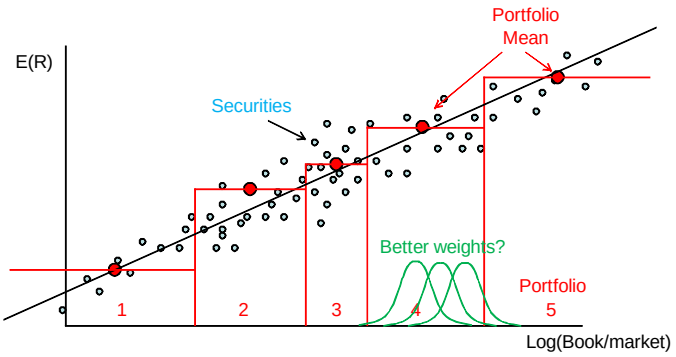

Figure 7: Portfolio means vs. cross -sectional regressions.

where $\mathbf{C}_t$ denotes some big vector of characteristics,

$$\mathbf{C}_t = [\text{size, b/m, momentum, accruals, d/p, credit spread....}].$$

Is value a "time-series" strategy that moves in and out of a stock as that stock's book/market changes, or a "cross-sectional" strategy that moves from one stock to another following the same signal? Well, both, obviously. They are the same thing. This is the managed-portfolio theorem:[21] an instrument z_t in a time series test $0 = E\left[\left(m_{t+1}R^e_{t+1}\right) z_t\right]$ is the same as an unconditional test of a managed portfolio $0 = E\left[m_{t+1}\left(R^e_{t+1}z_t\right)\right]$.

Once we understand expected returns, we have to see if expected returns line up with covariances of returns with factors. Sorted-portfolio betas are a nonparametric estimate of this covariance function

$$cov_t(R^{ei}_{t+1}, f_{t+1}) = g(C_{it}).$$

Parametric approaches are natural here as well, to address a multidimensional world. For example, we can run regressions

$$\left[R^{ei}_{t+1} - E(R^{ei}_{t+1}|C_{it})\right] f_{t+1} = c + d'C_{it} + \varepsilon^i_{t+1} \Rightarrow g(C) = c + d'C.$$

(The errors may not be normal, but they are mean-zero and uncorrelated with the right hand variable.) We want to see if the mean return *function* lines up with the covariance *function*.

$$E(R^e|C) = g(C) \times \lambda?$$

Underlying everything we're doing is an assumption that expected returns, variances and covariances are stable functions of characteristics, not (say) security name. That is an incredibly useful assumption–or, fact about the world. Without it, it's hard to tell if there is any spread in average returns at all. It means however, that asset pricing really is about the equality of two *functions*; the function relating means to characteristics and the function relating covariance to characteristics.

Looking at portfolio means rather than forecasting regressions was really the key to understanding *economic* importance of many effects, as was looking at long-horizon returns. For example, serial correlation with an R^2 of 0.01 doesn't seem that impressive. Yet is enough to account for

[21] Cochrane (2005c).

momentum: The last year's winners went up 100%, so an annual autocorrelation of 0.1, meaning 0.01 R^2, generates a 10% annual portfolio mean return. (An even smaller amount of time-series cross-correlation works as well.) Similarly, the information ratio for 1-10 (or 1-20, or 1-50) spread in portfolio mean returns is a persuasive metric for the *difference* in mean returns across a portfolio strategy. As another classic example, Lustig, Roussanov, and Verdelhan (2010a) translated carry-trade return-forecasting regressions to means of portfolios formed on the basis of currency interest differentials. This step led them to look for and find a factor structure of country returns that depends on interest differentials, a "high minus low" factor. This step followed Fama and French (1996) exactly, but no one thought to look for it in 30 years of running country-by-country time-series forecasting regressions

But the equivalence of portfolio sorts and regressions goes both ways. We can still calculate these measures of *economic* significance if we *estimate* panel-data regressions for means and covariances. From the spread of lagged returns, we can calculate the momentum portfolio implications directly. The 1-10 portfolio information ratio is the same thing as the Sharpe ratio of the underlying factor, or t-statistic of the cross-sectional regression coefficient. (See the Appendix.) We could study the covariance structure of panel-data regression residuals as a function of the same characteristics (interest rate spread, for example) rather than actually form portfolios.

Running multiple panel-data forecasting regressions is full of pitfalls of course. One can end up focusing on tiny firms, or outliers. One can get the functional form wrong. Uniting time-series and cross section will yield new insights as well. For example, variation in book/market over time for a given portfolio has a larger effect on returns than variation in book/market across the Fama-French portfolios, and a recent change in book/market also seems to forecast returns. (See the Appendix.) I didn't say it will be easy! But we must address the factor zoo, and it's hard to chop portfolios 27 ways.

3.3 Prices

Then, we have to answer the central question, what is the source of *price* variation?

When did our field stop being "asset pricing" and become "asset expected returning?" Why are betas exogenous?[22] A lot of price variation comes from discount factor news. What sense does it make to "explain" expected returns by covariation of expected return shocks with market expected return shocks? Market/book ratios should be our *left-hand* variable, the thing we're trying to *explain,* not a sorting characteristic for expected returns. Focusing on expected returns and betas rather than prices and discounted cashflows makes sense in a two-period or i.i.d. world, but much less so in a time-varying discount rate world.

A long-run, price-and-payoff perspective may also end up being simpler. As a hint of the possibility, solve the Campbell-Shiller identity for long-run returns,

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - dp_t.$$

So, *long-run* return uncertainty all comes from cashflow uncertainty. Long run betas are all cashflow

[22] Campbell and Mei (1993).

betas. The long run looks just like a simple one-period model with a liquidating dividend.

$$\begin{aligned} R_{t+1} &= \frac{D_{t+1}}{P_t} = \left(\frac{D_{t+1}}{D_t}\right) / \left(\frac{P_t}{D_t}\right) \\ r_{t+1} &= \Delta d_{t+1} - dp_t. \end{aligned}$$

A natural start is to forecast long-run returns and form price decompositions in the cross section, just as in the time-series; estimate forecasts such as

$$\sum_{j=1}^{\infty} \rho^{j-1} r^i_{t+j} = a + \mathbf{b}'\mathbf{C}_{it} + \varepsilon^i,$$

and then understand valuations with present value models as before.[23] (The Appendix includes two simple examples.) In the end, I would hope we end up studying prices and present values rather than expected returns and betas.

In a formal sense, of course, it doesn't matter whether you look at returns or prices. $1 = E_t(m_{t+1}R_{t+1})$ and $P_t = E_t \sum_{j=1}^{\infty} m_{t,t+j} D_{t+j}$ each imply the other. But, as I found with cay, our economic understanding may be a lot different in a price, long-run view than focusing on short-run returns. For example, since momentum amounts to a very small time-series correlation, I suspect it has little association with long-run returns and hence the level of prices. Long-lasting characteristics are likely to be more important.

What constitutes a "big" or "small" error is different as well. At a 2% dividend yield, $D/P = (r - g)$ implies that an "insignificant" 10bp/month expected return error is a "large" 12% price error, if it is permanent. Conversely, small transient price errors can have a huge impact on return measures. A tiny i.i.d. price error induces the appearance of mean reversion where there is none, and common procedures amount to taking many differences of prices, which amplify the error/signal ratio. A forward spread $f^{(n)}_t - y^{(1)}_t = p^{(n-1)}_t - p^{(n)}_t + p^{(1)}_t$ is already a triple-difference of price data.

4 Theories

Having viewed a bit of *how* discount rates vary, Let's think now about *why* discount rates vary so much.

4.1 A categorization, by ingredients and connection to data

It's useful to classify theories by their main ingredient, and by which data they use to measure discount rates. My goal is to produce for discount rates something like Fama's (1970) classification of informational possibilities.

1. Theories based on fundamental investors, with few frictions.
   (a) Macroeconomics – tie to macro data.
      i. Consumption, Aggregate risks.

[23] Vuolteenaho (2002) and Cohen Polk and Vuolteenaho (2003) are a start, with too-few followers.

ii. Risk sharing/background risks (Hedging outside income)

iii. Investment and production.

iv. General equilibrium, including macroeconomics

(b) Behavioral Irrational expectations. Tie to price data. Other data?

(c) Finance. Expected return-beta, return-based factors, affine term structure models. Tie to price data, returns explained by covariances.

2. Theories based on frictions.

(a) Segmented markets – different investors in different markets; limited risk bearing of active traders.

(b) Intermediated markets. Prices set by leveraged intermediaries; funding difficulties.

(c) Liquidity.

i. Idiosyncratic – easy to sell the asset.

ii. Systemic – times of market illiquidity.

iii. Information trading – value of securities in facilitating information trading.

"Macro" theories tie discount rates to macroeconomic data, such as consumption or investment, based on first-order conditions for the ultimate investors or producers.

The canonical consumption-based model with power utility relates discount rates to consumption growth,

$$\begin{aligned} m_{t+1} &= \beta \frac{u_c(t+1)}{u_c(t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}; \\ E_t(R^{ei}_{t+1}) &= R^f cov(R^{ei}_{t+1}, m_{t+1}) \approx \gamma cov(R^{ei}_{t+1} \Delta c_{t+1}). \end{aligned}$$

High expected returns (low prices) correspond to securities that pay off poorly when consumption is low. This model combines frictionless markets, rational expectations and utility maximization, risk-sharing so that only aggregate risks matter for pricing. It evidently ties discount rate variation to macroeconomic data.

A vast literature has generalized this framework, including (among others)[24] 1) Nonseparability across goods – durable and nondurable[25]; traded and nontraded; 2) Nonseparability over time, such as habit persistence,[26] 3) Recursive utility and long-run risks[27] 4) Rare disasters[28].

A related category of theories adds incomplete markets or frictions preventing some consumers from participating. Though close to the "frictions" category below, I categorize such models here because asset prices are still tied to some fundamental consumer or investor's economic outcomes. For example, if non-stockholders do not participate, we still can tie asset prices to the consumption decisions of stockholders who do participate.[29]

With incomplete markets, consumers still share risks as much as possible. The complete-market theorem that "all risks are shared," marginal utility is equated across people i and j, $m^i_{t+1} = m^j_{t+1}$,

[24] See Cochrane (2007a), Ludvigson (2011) for recent reviews.

[25] Recently, Yogo (2006),

[26] Campbell and Cochrane (1999) for example.

[27] Epstein and Zin (1989), Bansal and Yaron (2004), Hansen, Heaton and Li (2008).

[28] Rietz (1988), Barro (2006).

[29] For example, Mankiw and Zeldes (1991).

becomes "all risks are shared as much as possible." The projection of marginal utility on asset payoffs X is the same across people $proj(m^i_{t+1}|X) = proj(m^j_{t+1}|X) \equiv x^*$. We can still aggregate *marginal utility* rather than aggregate *consumption* and then take marginal utility. A discount factor $m_{t+1} = \int m^i_{t+1} di$ prices assets. For example with power utility we have

$$m_{t+1} = \beta E_{t+1}\left[\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\gamma}\right].$$

But this fact means that variation in the *distribution* of consumption matter to asset prices. Times in which there is more *cross-sectional risk* will be high-discount factor events.[30]

Outside or nontradeable risks are a related idea. If a mass of investors has jobs or businesses that will be hurt especially hard by a recession, they avoid stocks correlated with those risks.[31] Though in principle one could see such risks in consumption data, individual consumption data will always be so poorly measured that tying asset prices to more fundamental sources of risk may be more productive.

If we ask the "representative investor" in December 2008 why he or she is ignoring the buying opportunity of a lifetime in stocks and especially fixed income, the answer might well be "that's nice, but I'm about to lose my job, and my business might go under. I can't take any more risks right now, especially in securities that will lose value or become hard to sell if the recession gets worse." These extensions of the consumption-based model all formalize this sensible intuition – as opposed to the idea that these consumers have wrong expectations, or that they would have been happy to take risks but intermediaries were making all asset pricing decisions for them.

Investment-based models link asset prices to firms investment decisions, and general equilibrium models include production technologies and a specification of the source of shocks. This is clearly the ambitious goal towards which we are all aiming. It has to answer the vexing question, where do betas come from, and what makes a company a "growth" or "value" company in the first place.[32]

I think "behavioral" asset pricing's central idea is that people's expectations are wrong.[33] It takes lessons from psychology to find systematic patterns to the "wrong" expectations. There are some frictions in many behavioral models, but these are largely secondary and defensive, to keep risk-neutral "rational arbitrageurs" from coming in and undoing the behavioral biases. Often, simple risk aversion by the rational arbitrageurs would serve as well.

Behavioral models are also discount-rate theories. A distorted probability with riskfree discounting is mathematically equivalent to a different discount rate.

$$p = \sum_s \pi_s m_s x_s = \frac{1}{R^f}\sum_s \pi^*_s x_s$$

where

$$\pi^*_s \equiv \pi_s m_s R^f = \pi_s m_s / \sum_{s'} \pi_{s'} m_{s'},$$

s denote states of nature, π_s are true probabilities, m_s is a stochastic discount factor or marginal utility growth, and π^*_s are distorted probabilities.

[30] Constantinides and Duffie (1996) .
[31] Fama and French (1996), Heaton and Lucas (2000).
[32] A few good examples: Gomes, Kogan and Zhang (2003), Gala (2010), Gourio (2007)..
[33] See Barberis and Thaler (2003) and Fama (1998) for reviews.

It is pointless to argue "rational" vs. "behavioral" in the abstract. There is a discount rate and distorted probability that can rationalize any data. "The market went up, risk aversion must have declined" is as vacuous as "the market went up, sentiment must have increased." Any model only gets its bite by restricting discount rates or distorted expectations, ideally tying them to other data. The only thing worth arguing about is how persuasive those ties are in a given model and dataset, and whether it would have been easy to "predict" the opposite sign if the facts had come out that way.[34] And the line between recent "exotic preferences" and "behavioral finance" is so blurred[35] it describes academic politics better than anything substantive.

By and large, behavioral research so far largely ties prices to other prices; it looks for price patterns that are hard to understand with other models, such as "overreaction" or "underreaction" to news. Some behavioral research uses survey evidence, and survey reports of people's expectations are certainly unsettling. However, surveys are sensitive to language and interpretation. It doesn't take long in teaching MBAs to realize that the colloquial meanings of "expect" and "risk" are entirely different from conditional mean and variance. If people report the *risk-neutral* expectation, then expectations are in fact completely rational. An "optimistic" cash-flow growth forecast in an economic expansion is the same as a "rational" forecast, already discounted at a low rate, and leads to the correct decision, invest more. And the risk-neutral expectation, i.e. the expectation weighted by marginal utility, is the right sufficient statistic for many decisions. Treat painful things as if they were more probable than they are in fact.

The question, to which data does one connect discount factors, is an acid test of any theory. "Rational" theories struggle too. Changing expectations of consumption 10 years from now (long run risks) or changing probabilities of a big crash are hard to tell from changing "sentiment." At least, one can aim for more predictions than assumptions.

"Finance" theories tie discount rates to broad return-based factors. That's great for data reduction and practical applications. The more practical and "relative-pricing" the application the more "factors" we accept on the right hand side. For example, in evaluating a portfolio manager, hedging a portfolio, or finding the cost of capital for a given investment we routinely include momentum as a "factor" even though we don't have a deep theory of why the momentum factor is priced.

However, we still need the deeper theories for deeper "explanation." Even if the CAPM explained individual mean returns from their betas and the market premium, we would still have the equity premium puzzle – why is the market premium so large? (And why are betas what they are?) Conversely, even if we had the perfect utility function and a perfect consumption-based model, the fact that consumption data is poorly measured means we would still use finance models for most practical applications.[36]

The result is a nice division of labor. Empirical asset pricing in the Fama and French (1996) tradition boils down the alarming set of anomalies to a small set of large-scale systematic risks that generate rewards. "Macro" "behavioral" or other "deep" theories can then focus on why the factors are priced.

Models that emphasize frictions are becoming more and more popular, especially since the financial crisis. At heart, these models basically maintain the "rational" assumption. Admittedly, there are often "irrational" agents in such models. However, these agents are usually just convenient

[34] Fama (1998).

[35] For example, which of Epstein and Zin (1989), Barberis, Santos, and Huang (2001), Hansen and Sargent (2005), Laibson (1997), Hansen, Heaton and Li (2008), and Campbell and Cochrane (1999) is really "rational" and which is really "behavioral?"

[36] Campbell and Cochrane (2000) give a quantitative example.

shortcuts rather than central to the vision. A model may want some large volume of trade,[37] or to include some "noise traders," while focusing clearly on the delegated management problem, or the problem of leveraged intermediaries. For such a purpose, it's easy to simply allude to a slightly irrational class of trader rather than spell out their motives from first principles. However those assumptions are not motivated by deep reading of psychology or lab experiments. The focus is on the frictions rather than the risk-bearing ability of ultimate investors, or their psychological misperceptions.

I think it's useful to distinguish three categories of frictions: 1) Segmented markets and 2) Intermediated markets or "institutional finance"[38] and 3) Liquidity.

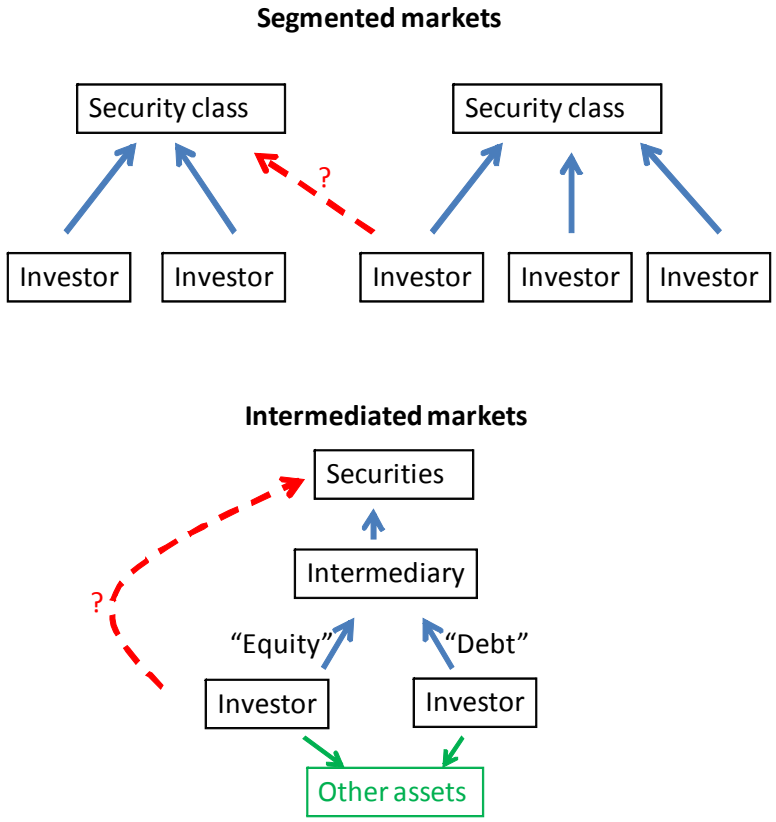

Figure 8: Segmented markets vs. intermediated markets.

I distinguish "segmented markets" from "intermediated markets," as shown in Figure 8. Segmented markets are really about limited risk sharing among the pool of investors active in a particular market.[39] They can generate "downward sloping demands," and average returns that depend on a "local" factor, little and poorly-linked CAPMs.[40] Given the factor zoo, that's an attractive idea.

[37] Scheinkman and Xiong (2003).

[38] Markus Brunnermeier coined this useful term.

[39] Some important examples: Burnnermeier and Pedersen (2009), Brunnermeier (2009), Gabaix, Krishnamurthy and Vigneron (2007), Duffie and Strulovici (2011), Garleanu and Pedersen (2009), He and Krishnamurthy (2010), Krishnamurthy (2008), Froot and O'Connell (2008), Vayanos and Vila (2011).

[40] For example, Gabaix, Krishnamurthy and Vigneron (2007).

"Intermediated markets" or "institutional finance" refers to a different, vertical rather than horizontal, separation of investor from payoff. Investors use delegated managers. Then, agency problems in delegated management spill over into asset prices. For example, suppose investors split their investments to the managers into "equity" and "debt" claims. When losses appear, the managers stave off bankruptcy by trying to sell risky assets. But since all the managers are doing the same thing, prices fall and discount rates rise. Colorful terms like "fire sale," "liquidity spirals" describe this process.[41]

Of course, we have to document and explain segmentation and intermediation. As suggested by the dashed arrows in Figure 8, there are strong incentives to undo any price anomaly induced by segmentation or intermediation. Models with these frictions often just rule out deep-pockets unintermediated investors – the sovereign wealth funds, pension funds, endowments, family offices, and Warren Buffets. Your "fire sale" is their "buying opportunity." Transactions costs, attention costs, or limited expertise suggest markets can be segmented until the "deep pockets" arrive, but they do arrive eventually. That observation suggests that segmentation is more important in the short run, after unusual events, or in more obscure markets. If I try to sell a truckload of tomatoes at 2 am in front of the Booth school, I am not likely to get full price. But if I do it every night, tomato buyers will start to show up. In the flash crash, it took about ten minutes for buyers to show up, which is either remarkably long or remarkably short, depending on your point of view.

A crucial question is, what *data* will this class of theories use to measure discount rates? Arguing over puzzling patterns of prices is weak (we've been at it for 40 years). Ideally, one should tie price or discount-rate variation to central items in the models, such as the balance sheets of leveraged intermediaries, data on who is actually active in segmented markets, and so forth. Yet such data is hard to find.[42]

We have long recognized that some assets have higher or lower discount rates in compensation for greater or lesser liquidity.[43] We have also long struggled to define and measure liquidity. There are (at least) three kinds of liquidity worth distinguishing. Liquidity can refer to the ease of buying and selling an individual security. Illiquidity can also be systemic: assets will face a higher discount rate if their prices fall when the market as a whole is illiquid, whether or not the asset becomes more or less illiquid. Finally, assets can have lower discount rates if they facilitate information trading, as money facilitates physical trading, an idea I explore a bit more below.

I think of "liquidity" as different from "segmentation" in that segmentation is about limited risk-bearing ability, while liquidity is about trading. Liquidity is a feature of individual assets, not the risks to which they are claims. Many theories of liquidity emphasize asymmetric information, not limited risk-bearing ability – assets become illiquid when traders suspect that anyone buying or selling knows something. Understanding liquidity requires us to unravel the puzzle of why people and institutions trade so vastly more than they do in our models.

All of these facts and theories are really about discount rates, expected returns, risk bearing, risk sharing and risk premiums. None are fundamentally about slow or imperfect diffusion of cash-flow information, i.e. informational "inefficiency." Informational efficiency isn't wrong or disproved. Efficiency basically won, and we moved on. When we see information, it is quickly incorporated in

[41] Brunnermeier (2009), Brunnermeier and Pedersen (2009) for example.

[42] Mitchell, Pedersen and Pulvino (2007) is a good example. They document who was active in convertible arbitrage markets through two episodes in which the specialized hedge funds left the market and it took months for the multi-strategy funds to move in.

[43] Acharya and Pedersen (2005), Amihud, Mendelson and Pedersen (2005), Cochrane (2005b), Pastor and Stambaugh (2003), Vayanos and Wang (2011).

asset prices. There is a lot of asset-price movement not related to visible information, but Hayek told us that would happen, and we learned that a lot of such price variation corresponds to expected returns. Little of the (large) gulf between the above models is really about information. Seeing the facts and the models as categories of discount-rate variation seems much more descriptive of most theory and empirical work.

Informational efficiency is much easier for markets and models to obtain than wide risk sharing or desegmentation. A market can become efficient with only one informed trader, who doesn't need to actually buy anything or take any risk. He should run in to a wall of indexers, and end up just bidding up the asset he knows is underpriced.[44] Risk sharing needs *everyone* to change their portfolios and bear a risk in order to eliminate segmentation. For example, if the small firm effect came from segmentation, the passively-managed small stock fund should have ended it – but it took the invention and marketing of such funds to end it. The actions of small numbers of arbitrageurs could not do so.

4.2 Recent performance

This is not the place for a deep review of theory and empirical work supporting or confronting theories. Instead, I think it will be more productive to think informally about how these classes of models might be able to handle big recent events.

4.3 Consumption

I still think the macro-finance approach is promising. Figure 9 presents the market price-dividend ratio, and aggregate consumption relative to a slow-moving "habit." The habit is basically just a long moving average of lagged consumption, so the surplus consumption ratio line is basically detrended consumption.[45]

As you can see, consumption and stock market prices *did* both collapse in 2008. Many high average-return-securities and strategies (stocks, mortgage-backed securities, low-grade bonds, momentum, currency carry) collapsed more than low-average-return counterparts. The basic consumption-model logic – that securities must pay higher returns, or fetch lower prices, if their values fall more when consumption falls – isn't drastically wrong.

The habit model captures the idea that people become more risk averse as consumption falls in recessions. As consumption nears habit, people are less willing to take risks that involve the same proportionate risk to consumption. Discount rates rise, and prices fall. Lots of models have similar mechanisms, especially models with leverage.[46]

In the habit model, the price-dividend ratio is a nearly log-linear function of the surplus consumption ratio. The fit isn't perfect, but the general pattern is remarkably good, given the hue and cry about how "the crisis invalidates all traditional finance."

[44] Milgrom and Stokey (1982).
[45] Campbell and Cochrane (1999)
[46] For example, Longstaff (2008).

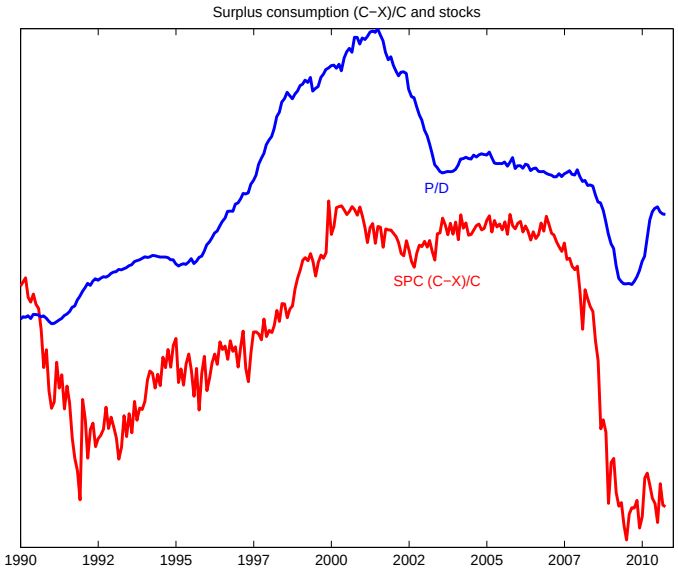

Figure 9: Surplus consumption ratio and price/dividend ratio. Surplus consumption is formed from real nondurable + services consumption using the Campbell and Cochrane (1999) specification and parameters. Price/dividend ratio is from the CRSP NYSE Value-Weighted portfolio.

4.4 Investment

The Q theory of investment is the off-the-shelf analogue to the simple power-utility model from the producer point of view. It predicts that *investment should be low when valuations (market to book) are low, and vice versa,*

$$1 + \alpha \frac{i_t}{k_t} = \frac{\text{market}_t}{\text{book}_t} = Q_t, \tag{9}$$

where i_t = investment and k_t = capital.

Figure 10 contrasts the investment/capital ratio, market/book ratio, and price/dividend ratio. The simple Q theory also links asset prices and investment better than you probably thought, both in the tech boom and the financial crisis.

Many finance puzzles are stated in terms of returns. To make that connection, one can transform (9) to a relation linking asset returns to investment growth. Many return puzzles are mirrored in investment growth as the q theory suggests.[47]

Q theory also reminds us that supply as well as demand matters in setting asset prices. If capital could adjust freely, stock values would never change, no matter how irrational investors are. Quantities would change instead.

I'm not arguing that consumption or investment *caused* the boom or the crash. Endowment-economy causal intuition does not hold in a production economy. These first-order conditions are happily consistent with a view, for example, that a the ultimate cause was rather small losses on subprime mortgages, amplified by a run on the shadow banking system and flight to quality[48]. The

[47] Cochrane (1991b), (1996) (2007a), Lamont (2000) Li, Livdan and Zhang (2008), Liu, Whithed and Zhang (2009), Belo (2010), Jermann (2010), Liu and Zhang (2011).

[48] Cochrane (2011)..

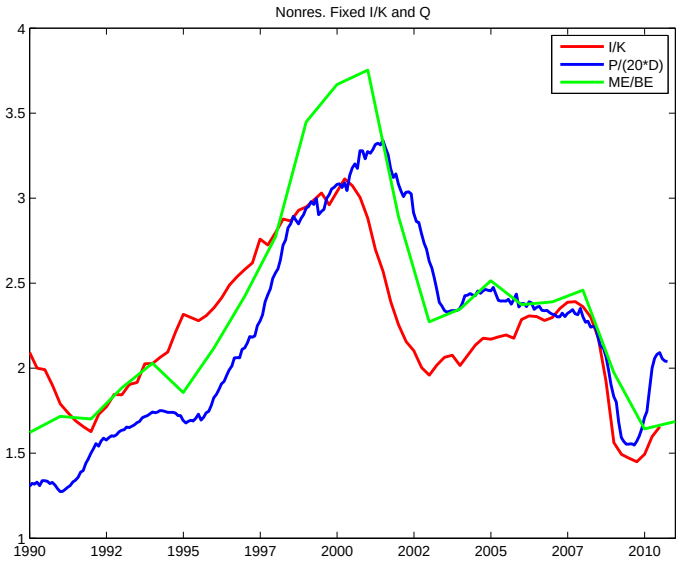

Figure 10: Investment/capital ratio, price/dividend ratio, and market/book ratio. Investment is real private nonresidential fixed investment. Capital is cumulated from investment with an assumed 10% annual depreciation rate. Price/dividend from CRSP, market/book from Ken French's website.

first-order conditions are consistent with many other views of the fundamental determinants of both prices and quantities. And I don't even pretend to have a full macro model that captures for these graphs, let alone to understand value or the rest of the factor zoo. But the graphs do argue that asset prices and discount rates are much better connected to big macroeconomic events than most people think. If people did not become more risk averse in recessions, and if firms could quickly transform empty houses into hamburgers, asset prices would not have declined as much. And they suggest that refining the very stylized models I used to make these graphs – a long literature already well under way – is not a hopeless endeavor.

4.5 Comparisons

Conversely, I think the other kinds of models, though good for describing particular anomalies, will have greater difficulty accounting for recent big-picture asset pricing events.

We see a *pervasive, coordinated* rise in the premium for systematic risk, common across all asset classes, and present in completely unintermediated and unsegmented assets. (The "systematic" adjective is important. People don't seem to drive a lot more carefully in recessions.) For example, Figure 11 plots government and corporate rates, and Figure 12 plots the baa-aaa spread with stock prices. You can see a huge credit spread open up and fade away along with the dip in stock prices.

Behavioral ideas – narrow framing, salience of recent experience, and so forth – are good at generating anomalous prices and mean returns in individual assets or small groups. They don't easily generate this kind of coordinated movement that looks just like a rise in risk premium. They don't naturally generate covariance either. For example, "extrapolation" generates the slight

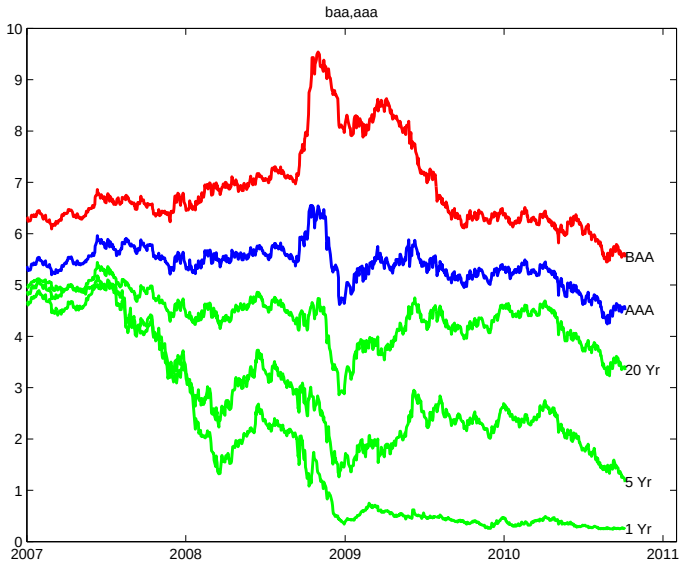

Figure 11: BAA, AAA, and Treasury yields.

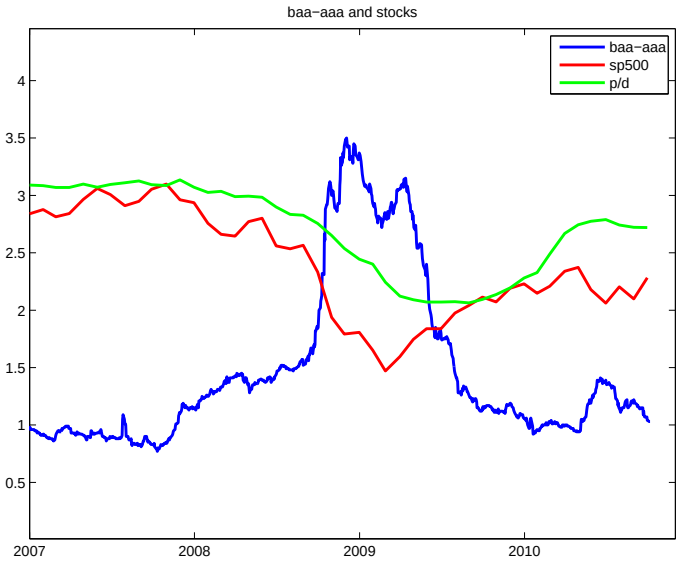

Figure 12: D/P, S&P500, BAA-AAA.

autocorrelation in returns that lies behind momentum. But why should all the momentum stocks then rise and fall together, just as if they are exposed to a pervasive, systematic risk?

Finance models don't help, of course, because we're looking at variation of the factors which they take as given.

Segmented or institutional models aren't obvious candidates to understand broad market movements. Each of us can easily access stocks and bonds through low-cost indices.

And none of these models naturally describe the strong correlation of discount rates with macro-

economic events. Is it a coincidence that people become irrationally pessimistic when the economy is in a tailspin, and they could lose their jobs, houses, or businesses if systematic events get worse?

Again, macro isn't everything – understanding the smaller puzzles is important. The point is only that *looking* for macro underpinnings for discount rate variation, through fairly simple models, isn't as hopelessly anachronistic as many seem to think.

4.6 Arbitrages?

One of the nicest pieces of evidence for segmented or institutional views is that arbitrage relationships were violated in the financial crisis.[49] Unwinding the arbitrage opportunities required one to borrow dollars, which intermediary arbitrageurs could not do.

Figure 13 gives one example. CDS plus Treasury should equal a corporate bond, and usually does. Not in the crisis.

Figure 13: Citigroup CDS and Bond spreads. Source: Fontana (2010).

Figure 14 gives another example, covered interest parity. Investing in the US vs. investing in Europe and returning the money with forward rates should yield the same thing. Not in the crisis. In both cases, profiting from the arbitrage requires one to borrow dollars, which was difficult in the crisis.

Similar patterns happened in many other markets, including even US treasuries.[50] Now, any arbitrage opportunity is a dramatic event. But in each case here the *difference* between the two ways of getting the same cashflow is dwarfed by the *overall* change in prices. And, though an "arbitrage," the price differences are not large enough to attract "long only" "deep pocket" money. If your precious cash is in a US money market fund, 20 basis points in the depth of a financial crisis is not enough to get you to investigate offshore investing with an exchange-rate hedging program.

[49] See also Fleckenstein, Longstaff, and Lustig (2010),
[50] Hu, Pan and Wang (2011).

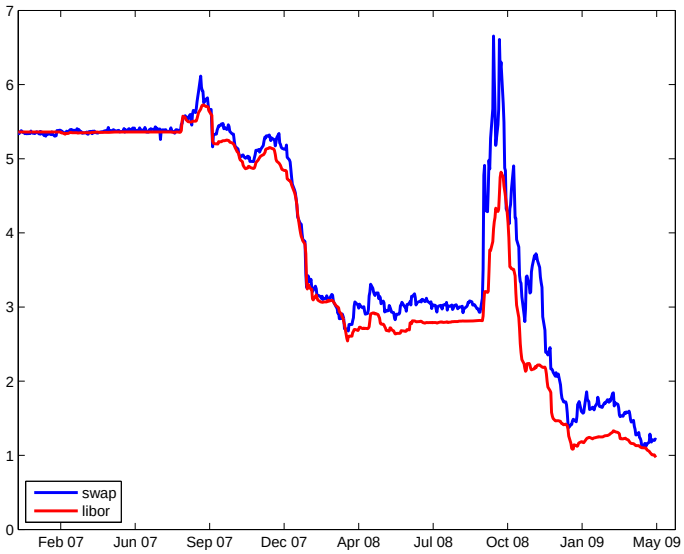

Figure 14: Three-month Libor and FX swap rate. Source: Baba and Packer (2009).

Analogously, the price of coffee displays arbitrage opportunities across locations at the ASSA meetings. The arbitrage reflects an interesting combination of transactions costs, short-sale constraints, consumer biases, funding limits, and other frictions. Yet we don't dream that this fact matters for big-picture variation in worldwide commodity prices.

So maybe it's possible that the "macro" view still builds the benchmark story of *overall* price change, with very interesting *spreads* opening up due to frictions.

4.7 Liquidity premia; trading value

Trading-related liquidity does strike me as potentially important for the big picture, and a potentially important source of the low discount rates in "bubble" events.[51]

I'm inspired by one of the most obvious "liquidity" premiums: Money is overpriced – lower discount rate – relative to government debt, though they are claims to the same payoff in a frictionless market. And this liquidity spread can be huge – hundreds of percent in hyperinflations.

Now, money is "special" for its use in transactions. But many securities are "special" in *trading*. Trading needs a certain supply of their physical shares. We cannot support large trading volumes by recycling one outstanding share at arbitrarily high speed. Even short sellers must hold a share for some short period of time.

When share supply is small, and trading demand is large, markets can support a lower discount rate or higher price for highly-traded securities, as they do for money. These effects have long been seen in government bonds, for example in the Japanese "benchmark" effect, the spreads between on-the-run and off-the-run Treasuries, or the spreads between Treasury and agency bonds.[52] Could

[51] Cochrane (2001), (2003), (2005b), Garber (2000), Krishnamurthy (2002), O'Hara (2008), Scheinkman and Xiong (2003).

[52] Boudoukh and Whitelaw (1991), Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2010).

these effects extend to other assets?

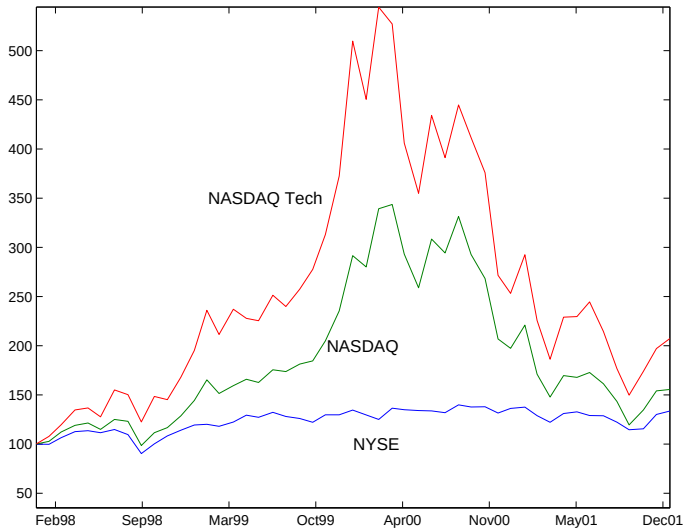

Figure 15: Nasdaq Tech, Nasdaq, and NYSE indeces Source: Cochrane (2003).

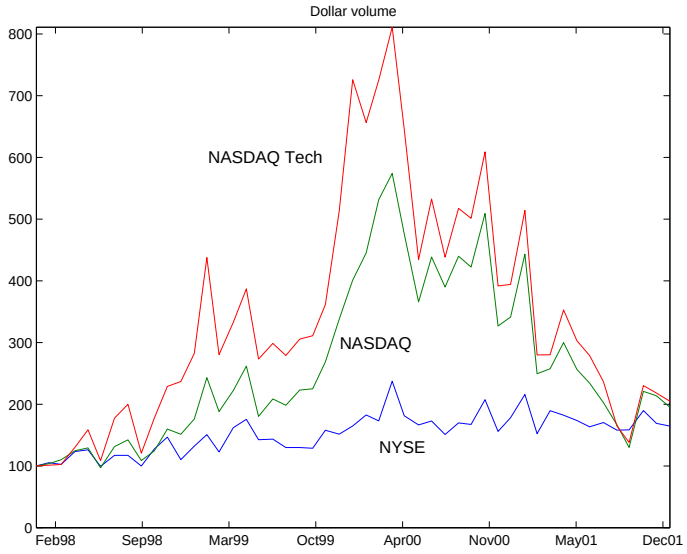

Figure 16: Dollar volume in Nasdaq tech, Nasdaq, and NYSE. Source: Cochrane (2003).

Figures 15 and 16 are suggestive. The stock *price* raise and fall of the late 1990s was concentrated in Nasdaq and Nasdaq Tech. The stock *volume* rise and fall was concentrated in the same place. *Every* asset price "bubble" – defined here by people's use of the label – has coincided with a similar trading frenzy, from Dutch tulips in 1620 to Miami condos in 2006.

Is this a coincidence? Do prices rise and fall for other reasons, and large trading volume follows, with no effect on price? Or is the high price – equivalently a low discount rate – explained at least in part by the huge volume; by the value of shares in facilitating a frenzy of information trading?

To make this a deep theory, we must answer why people trade so much. Verbally, we know the answer: The markets we study *exist* to support information-based trading. Yet, we really don't have good models of information-based trading.[53] Perhaps the question how information is incorporated in asset markets will come back to the center of inquiry!

5 Applications

Finance is about practical application, not just deep explanation. Discount rate variation will change applications a lot.

5.1 Portfolio theory

A huge literature explores how investors should exploit the market-timing and intertemporal-hedging opportunities implicit in time-varying expected returns.[54]

But *the average investor must hold the market portfolio.* We can't all market-time, we can't all buy value, and we can't all be smarter than average. We can't even all rebalance. A useful and durable portfolio theory must be consistent with this theorem. Our discount-rate facts and theories suggest one, built on *differences* between people.

Consider Fama and French's (1996) story for value. The average investor is worried that value stocks will fall at the same time his or her human capital falls. But then some investors ("steel-workers") will be more worried than average, and should *short* value despite the premium; some others ("tech nerds") will have human capital correlated with growth stocks and buy lots of value, effectively selling insurance. A two-factor model implies a three-fund theorem, a three-dimensional multifactor efficient frontier as shown in Figure 17.[55] Investors have a difficult problem to figure out how much of three funds to hold.

And now we have *dozens* of such systematic risks for each investor to consider. Time-varying opportunities create more factors, as habits or leverage risk aversion shift some investor's risk aversion through time more or less than others.

Unpriced factors are even more important. Our steelworker should start by shorting a steel-industry portfolio, even if it has zero alpha. We academics should understand the variation across people in risks that are hedgeable by systematic factors, and find low-cost portfolios that span that variation.[56] Yet we've spent all our time looking for *priced* factors that are only interesting for the measure-zero mean-variance investor!

All of this sounds hard. That's good! We finally have a reason for a fee-based "tailored portfolio" industry to exist, rather than just to deplore it as folly. We finally have a reason for us to charge fat tuitions to our MBA students! We finally have an interesting portfolio theory that is not based on chasing zero-sum alpha!

[53] Milgrom and Stokey (1982).

[54] Merton (1971), Barberis (2000), Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (1999), (2002); Pastor (2000); see a revew in Cochrane (2007b).

[55] See Fama (1996), Cochrane (2007b).

[56] Heaton and Lucas (2000).

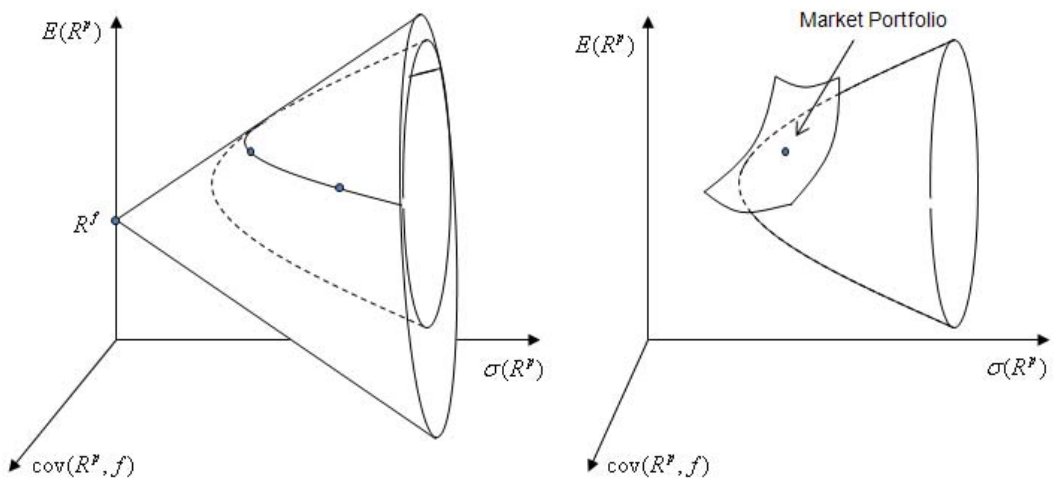

Figure 17: Multifactor efficient frontiers. Investors minimize variance given mean and covariance with the extra factor. A three-fund theorem emerges (left). The market portfolio is multifactor efficient, but not mean-variance efficient (right).

5.1.1 State Variables

Discount-rate variation means that state-variable hedging should matter. It is almost completely ignored in practice. Almost all hedge funds, active managers, and institutions still use mean-variance optimizers. This is particularly striking given that they follow active strategies, predicated on the idea that expected returns and variances vary a lot over time!

Perhaps state variable hedging seems nebulous, and therefore maybe small and easy to ignore. Here's a story to convince you otherwise. Suppose you are a highly risk averse investor, with a 10 year horizon. You are investing to cover a defined payment, say your 8 year old's tuition at the University of Chicago. The optimal investment is obviously a 10-year zero-coupon indexed Treasury (TIP).[57] Figure 18 tracks your investment through time.

Suppose now that bond prices plunge, and volatility surges, highlighted in the graph. Should you sell in a panic, to avoid the risk of further losses? No. You should tear up the statement. "Short term volatility" is irrelevant. Every decline in price comes with a corresponding rise in expected return. Evaluating bonds with a one-period mean-variance, alpha-beta framework is silly. (Though a surprising amount of the bond investing world does it!)

That's pretty obvious, but now imagine yourself a stock investor in December 2008 – say, your university's endowment. Stocks plummeted, shown in Figure 19, and stock volatility in Figure 20 rose dramatically, from 16% to 70%.

Should you sell? The standard formula says so! Picking a mean return and risk aversion to

[57] Campbell and Viceira (2001), Wachter (2003).

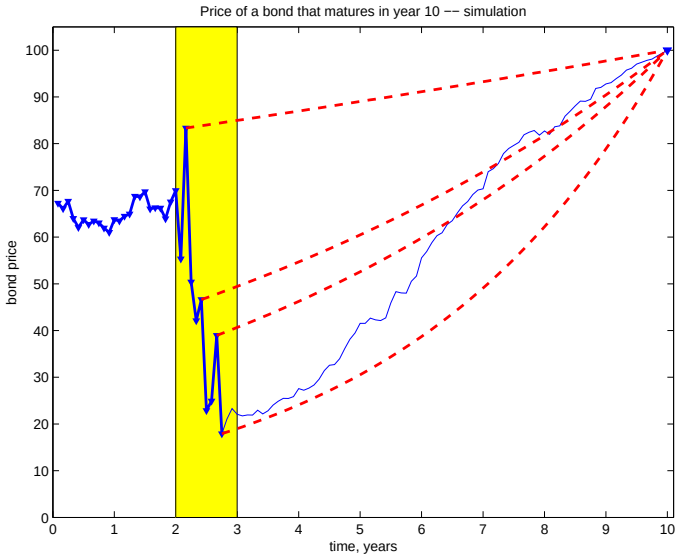

Figure 18: Bond price through time. A cautionary example.

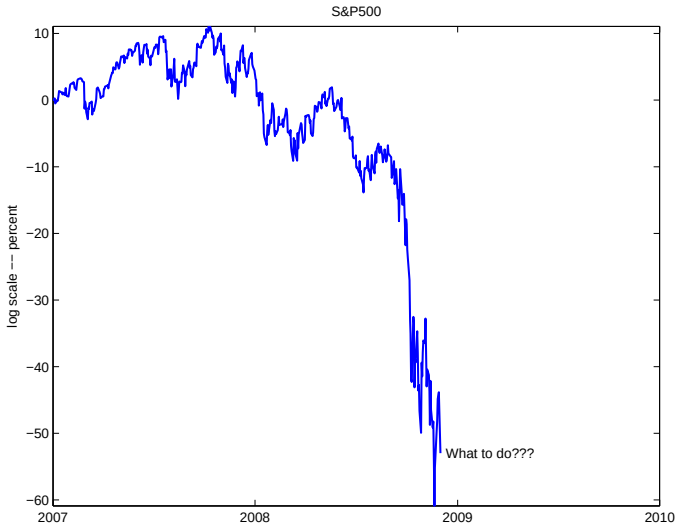

Figure 19: S&P500 price index in 2008

justify 60% stocks in normal times, you should reduce the equity share to 4%!

$$\text{share} = \frac{1}{\gamma}\frac{E(R^e)}{\sigma^2(R^e)}. \quad 0.6 = \frac{1}{2}\frac{0.04}{0.18^2} \Rightarrow \frac{1}{2}\frac{0.04}{\mathbf{0.70^2}} = \mathbf{0.04}?$$

(You might object that mean returns rose too. But they would have to have risen to $4 \times 0.70^2/0.18^2 = 60\%$ for this formula to tell you not to change allocation. Dividend yields did not rise that much! You also may object that many investors including endowments had leverage, tenured professor salaries to pay or other habit-like considerations for becoming more risk averse. Fair enough, but then mean-variance theory is particularly inappropriate in the first place.)

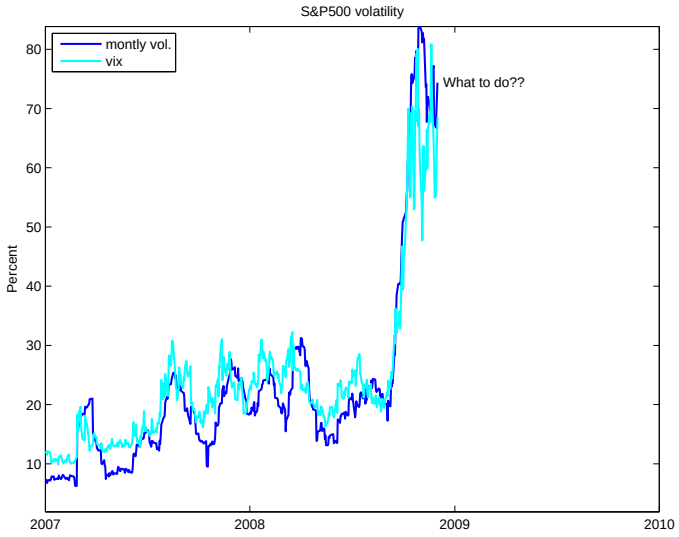

Figure 20: Volatility. VIX index, and 20 day realized daily volatility

But not everyone can do this – the market didn't fall 93%. If you're selling, who is buying? Is everyone else being stupid? Does it make sense to think that the market irrationally *over*valued in the midst of the financial crisis?

The answer, of course, is that one-period mean-variance analysis is completely inappropriate. If the world were i.i.d., volatility couldn't change in the first place. Stocks are a bit like bonds; price/dividend drops increase expected returns.[58] To some extent, "short run volatility" doesn't matter to a long-run investor. *State-variable hedging matters a lot, even for simple real-world applications.* And, by ICAPM logic, we should therefore expect multiple priced factors. Time-series predictability should be a strong source of additional pricing factors in the "cross section," and affect portfolios

5.1.2 Prices and payoffs

Or maybe not. Telling our bond investor to hold 10 year zeros because their price happens to covary properly with state variables for their investment opportunities just completely confuses the obvious. It's much clearer to look at the final payoff and tell him to *ignore* price fluctuations. Maybe dynamic portfolio theory overall might get a lot simpler if we look at payoff streams rather than looking at dynamic trading strategies that achieve those streams.

If you look at payoff streams, it's totally obvious that an indexed perpetuity (or annuity) is the risk-free asset for long-term investors, despite arbitrary time-varying return moments, just as the ten-year zero was obviously the riskfree asset for my bond investor. It's interesting that coupon-only TIPS are an exotic product, not the benchmark for every portfolio optimization in place of a money-market investment.

[58] Campbell and Vuolteenaho (2004).

How about risky investments? Here is a simple and suggestive step.[59] If utility is quadratic

$$\max_{\{c_t\}} E\sum_{t=0}^{\infty}\delta^t\left(-\frac{1}{2}\right)(c_t-c^*)^2$$

it turns out that we can still use two-period mean-variance theory to think about *streams of payoffs*, (loosely, streams of dividends) no matter how much expected returns vary over time.

Every optimal payoff stream combines an indexed perpetuity and a claim to the aggregate dividend stream. Less risk averse investors hold more of the claim to aggregate dividends, and vice versa.

Optimal payoffs lie on a long-run mean / long-run variance frontier, where I define "long run" means $\tilde{E}(x)$ that sum over time as well as states,

$$\tilde{E}(x)=\frac{1}{1-\beta}\sum_{j=0}^{\infty}\beta^j E(x_{t+j})$$

State variables disappear from portfolio theory, just as they did for our 10 year TIP investor, once he looked at the 10 year problem.

If our stock market investor thought this way, he would answer "I bought the aggregate dividend stream. Why should I buy or sell? I don't look at the statements." This is a lot simpler to explain and implement than deep time series modeling, value function calculation, and optimal hedge portfolios!

If investors have outside income, they first short a payoff stream most correlated with their outside income stream, and then hold the mean-variance efficient payoffs. Calculating correlations of income streams this way may be easier than trying to impute discount-rate induced changes in the present value of outside income streams, in order to calculate return-based hedge portfolios.

If investors have no outside income, long-run expected returns (payoffs divided by initial prices) line up with long-run market betas. A CAPM emerges, despite arbitrary time-variation in expected returns and variances. ICAPM pricing factors fade away as we look at longer horizons.

If investors do have outside income, an average-outside-income payoff emerges as a second priced factor, in the style of Fama and French's (1996) human capital story for the value effect.

Of course, quadratic utility is a troublesome approximation, especially for long-term problems. Still, this simple example captures the possibility that a price and payoff approach can give a much simpler view of pricing and portfolio theory than we get by focusing on the high-frequency dynamic trading strategy that achieves those payoffs in a given market structure.

5.2 Alphas, betas, performance evaluation

In the 1970 view, there is one source of systematic risk, the market index. Active management chases "alpha," which means uncovering inefficiently-priced assets.

Now we have dozens of dimensions of systematic risks. Many hedge fund strategies include an element of option writing. For example, Figure 21 shows the annual returns of "equity-market-neutral" hedge funds together with the market return. "Providing liquidity" looks a lot like writing out-of-the-money puts[60]!

[59] Results from Cochrane (2008).

[60] Mitchell and Pulvino (2001), Asness, Krail and Liew (2001), Agarwal,and Naik (2004).

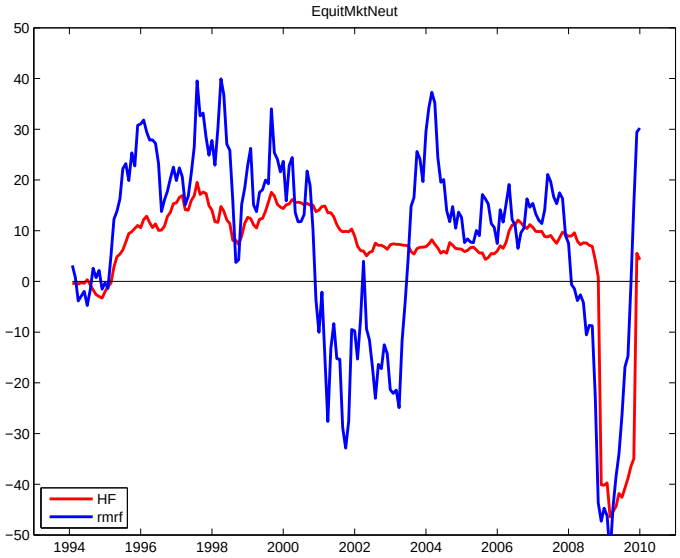

Figure 21: One year excess returns of the "equity market neutral" hedge fund index and the CRSP value-weighted portfolio. Data source: hedgeindex.com and CRSP.

I tried telling a hedge fund manager, "You don't have alpha. I can replicate your returns with a value-growth, momentum, currency and term carry, and short-vol strategy." He said, "'Exotic beta' *is* my alpha. I understand those systematic factors and know how to trade them. You don't." He has a point. How many investors have thought through their exposures to carry trade or short volatility "systematic risks," and are ready to consider those as "passive," mechanical investments? To an investor who hasn't heard of it and holds the market index, a new factor is alpha. And has nothing to do with informational inefficiency.

Most active management and performance evaluation just isn't well described by the alpha-beta, information-systematic, selection-style split anymore. There is no more alpha. There is just beta you understand and beta you don't understand, and beta you are positioned to buy vs. beta you are already exposed to and should sell.

5.3 Procedures, corporate, accounting, regulation

Time-varying discount rates and multiple factors deeply change many applications.

The first slide in a capital budgeting lecture looks something like this

$$\text{Value of investment} = \frac{\text{Expected payout}}{R^f + \beta\left[E(R^m) - R^f\right]},$$

with a 6% market premium. All of which, we now know, is completely wrong. The market premium isn't always 6%, but varies over time by as much as its mean. Expected returns don't line up with CAPM betas, but rather with multifactor betas. And since expected returns change over time, the discount rate is different for cashflows at different horizons.

It's interesting that investment decisions got so close to right anyway, with high investment following high stock prices. (Remember Figure 10.) Evidently, a generation of our MBAs figured out how to jigger the numbers and get the right answer despite a dramatically wrong model. Perhaps

what we often call "irrational" cash flow forecasts, optimistic in good times and pessimistic in bad times, are just a good way to offset artificially constant discount rates. Or perhaps they understood the Q theory lecture and just follow its advice.

I don't think the answer lies in multifactor betas[61] or discounting with dynamic present value models and time-varying risk premia, at least not yet. Capital budgeting is a "relative pricing" exercise – we want to use available information in asset markets to help us decide what the discount rate for a project should be. For this purpose, simply looking at average returns of "similar" securities is enough. Understanding discount rates as a function of characteristics – or, better, understanding valuations directly as a function of characteristics (the use of "comparables") – may end up being more fruitful. We don't have to *explain* discount rates – relate expected returns to betas, and understand their deep economics – in order to *use* them. Even when they are explained, the characterization (characteristic models) may be a better measure for practical relative-pricing than the explanation (beta models). And capital budgeting gives the same answer if discount rates are "wrong." When you shop for a salad, all you care about is the price of tomatoes. Whether tomatoes are expensive because the trucks got stuck in bad weather, or because of an irrational bubble in the tomato futures market makes no difference to your decision.

Many procedures in accounting, regulation and capital structure implicitly assume that returns are independent over time, and hence that prices only reflect cashflow information.

Suppose that a firm has a single cashflow in 10 years, and is funded by a zero-coupon bond and equity. In most accounting, capital structure, and regulation we would use the stock and bond prices to calculate the probability and distance to default. But if prices decline because discount rates rise, that fact has no implication for the probability or distance to default.

Perhaps banks' complaint that low asset prices represent "illiquidity" or "temporarily depressed valuations" rather than insolvency – a lesser chance of making future interest and principal repayments – make some sense. Perhaps capital requirements do not have to respond immediately to such events. Perhaps "hold to maturity" accounting is not as silly as it sounds. Perhaps the fact that firms change capital structures very slowly in response to changes in equity valuations also makes some sense.[62]

Of course, in such an event the *risk-neutral* probability of default has risen. Maybe regulators, bondholders, and capital structure *should* respond to a rise in the state-price of the default event exactly as they respond to a rise in the real probability of that event. Possibly, but at least it's a very different issue and worth asking the question.

I am not arguing that mark-to-market accounting is bad, or that fudging the numbers is a good idea. The point is only that what you *do* with a mark-to-market number might be quite different in a world driven by discount-rate variation than one driven by cashflow variation.[63] The mark-to-market value is no longer a sufficient statistic. Decisions need to incorporate expected future returns as well as current values.

The view that the stock price is driven by expected earnings lies behind stock-based executive compensation as well. It's already a bit of a puzzle that executives should be forced to hold the "systematic" risks due to market beta or commodity-price exposures, about which they can do nothing. Understanding that a large fraction of stock returns reflect changes in discount rates or new-factor beta exposures, makes the logic of such incentives even more curious. Perhaps stock-

[61] Fama and French (1997) try.
[62] Welch (2004).
[63] Heaton, Lucas and McDonald (2009).

based compensation has less to do with effort and operating performance, but more with incentives for risk management or tax treatment.

5.4 Macroeconomics

Large variation in risk premia implies exciting changes for macroeconomics.

Most of macroeconomics focuses on a single intertemporal price, "the" interest rate, which intermediates saving and investment, without worrying about risk premia. Yet in Figure 11, interest rates paid by borrowers (and received by any investors willing to lend) spiked up, while short term government rates went down. Recessions are all about changes in credit spreads, in the willingness to bear risk and the amount of risk to be borne, far more than they are about changes in the desire for current vs. future certain consumption. And most of the Federal Reserve's response to the Great Recession consisted of targeting risk premiums, not changing the level of interest rates or addressing a transactions demand for money.

Macroeconomics and finance have thought very differently about consumer (we call them investors) and firm behavior. For example, the consumers in the Cambpell and Cochrane (1999) habit model balance very strong precautionary saving motives with very strong intertemporal substitution motives, and have large and time-varying risk aversion. Their behavior is very far from the permanent-income intuition (or constrained alternative) in macroeconomic thinking.

As one simple story, macroeconomists often think about how consumers will respond to a change in "wealth," coming from a change in stock prices or house prices. Financial economists might suspect that consumers will respond quite differently to a decline in value coming from a discount rate rise – a temporary change in price with no change in capital stock or cashflow – than one that comes from a change in expected cashflows, or destruction of physical capital stock.

Financial models also emphasize adjustment costs or irreversibilities: If firms can freely transform consumption goods to capital, then stock prices (q) are constant. Yet, most "real business cycle" literature following King, Plosser and Rebelo (1988) left out adjustment costs, because they didn't need them to match basic quantity correlations. The first round of "new-Keynesian" literature abstracted from capital altogether, and much work in that tradition continues to do so.

But that omission can lead to basic differences in analysis. For example, a lot of macroeconomics worries right now about the "zero bound," that the real rate of interest, tied to marginal product of capital $f'(k)$, should be negative but nominal rates cannot be negative. With adjustment costs, however, the price of capital can always fall enough to give a positive real rate of interest.

As another simple story, Figure 10 linking investment to stock price/dividend ratios, together with the regression evidence of Table 1, strongly suggests that variations in the *risk premium* drive variations in the cost of capital and hence investment, not variation in the level of riskfree interest rates emphasized by macro policy-makers.

Formal macroeconomics has started to introduce some of the same ingredients that macro-finance researchers are using to understand discount rate variation, including "new" preferences, adjustment cost or other frictions in capital formation, and financial frictions in credit markets.[64] And macroeconomic models with financial frictions are all the rage since the financial crisis. Still,

[64] For example Christiano, Eichenbaum and Evans (2005). However, this is also a good example of remaining differences. They use a one-period habit, which does not generate slow-moving expected excess returns, and an adjustment cost tied to investment growth not the investment/capital ratio, which does not generate the q theory predictions of Figure 10 and related finance literature.

we are a long way from a single general-equilibrium model that matches basic facts from both quantity and asset prices.

Everyone is aware of the importance of the question. The job is just hard. Macro models are technically complicated. First-order approximations are hard enough to work out. To capture risk premia, you need second-order approximations, and to capture time-varying risk premia you need third order approximations. Putting "financial frictions" in such models is harder still. At a deeper level, successful "grand synthesis" models do not consist of just mixing all the popular ingredients together and stirring the pot; they must maintain the clear quantitative parable feature of good economics.

An asset-pricing perspective also informs monetary economics, and the interaction between monetary and fiscal policy. From a finance perspective, nominal government debt is "equity" in the government: it is the residual claim to primary fiscal surpluses. Hence, the price level must satisfy the standard asset pricing equation,

$$\frac{\text{Debt}_t}{\text{Price level}_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \text{ (real primary surplus}_{t+j}), \tag{10}$$

and inflation can absorb shocks to surpluses, just as as equity absorbs shocks to profit streams.

This fact is at least an important constraint on monetary policy, especially in a time of looming deficits.[65] This approach can determine the price level with no special or transactions status for money, no need for limited money supply, or even no money at all. The analogy to stocks suggests that variation in the *discount rate* $m_{t,t+j}$ for government debt will be important: A "flight to quality" such as in the recent recession lowers the discount rate for government debt. People want to hold more government debt, which means getting rid of goods and services, i.e. lower aggregate demand. We see a recession and pressure towards deflation on the left side of (10). This story links the "rising risk premium" which finance people see as the core of a recession with the "decline in aggregate demand" which macroeconomists see. The standard corporate finance perspective also then illuminates the choice of government debt maturity structure, denomination (foreign currency debt is debt, that must be repaid or defaulted rather than inflated) and the "control" or other rights that must accompany "government equity."

6 Conclusions

Discount rates vary a lot more than we thought. The puzzles and anomalies that we face amount to discount rate variation we don't understand. Our theoretical controversies are about how discount rates are formed. We need to recognize and incorporate discount rate variation in applied procedures.

We are really only beginning these tasks. The facts about discount rate variation need at least a dramatic consolidation. Theories are in their infancy. And most applications still implicitly assume i.i.d. returns and the CAPM, and therefore that price changes only reveal cashflow news.

[65] Sargent and Wallace (1981), Cochrane (2011).

7 References

Acharya, Viral and Lasse Heje Pedersen, 2005, Asset Pricing with Liquidity Risk, *Journal of Financial Economics* 77, 375-410.

Agarwal, Vikas and Narayan Naik, 2004, Risk and Portfolio Decisions Involving Hedge Funds, *Review of Financial Studies* 17, 63 - 98.

Amihud, Yakov, Haim Mendelson, and Lasse Heje Pedersen, 2005, Liquidity and Asset Prices, *Foundations and Trends in Finance* 1, 269-364.

Ang, Andrew, and Joe Chen, 2007, CAPM Over the Long Run: 1926-2001, *Journal of Empirical Finance* 14, 1-40.

Asness, Cifford, Robert Krail and John Liew, 2001, Do Hedge Funds Hedge? *Journal of Portfolio Management* 28, 6-19.

Baba, Naohiko and Frank Packer, 2009, Interpreting Deviations From Covered Interest Parity During the Financial Market Turmoil of 2007–08, *Journal of Banking and Finance* 33, 1953-1962.

Bansal, Ravi, and Amir Yaron, 2004, Risks For the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1481-1509.

Barro, Robert J., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics* 121, 823-866.

Barberis, Nicholas, 2000, Investing for the Long Run when Returns are Predictable," *Journal of Finance* 55, 225-264.

Barberis, Nicholas, Tano Santos, and Ming Huang, 2001, Prospect Theory and Asset Prices, *Quarterly Journal of Economics* 116, 1-53.

Barberis, Nicholas, and Richard Thaler, 2003, A Survey of Behavioral Finance, in George Constantinides, René. Stulz and Milton Harris eds., *Handbook of the Economics of Finance*, North Holland, Amsterdam, 2003.

Belo, Frederico, 2010, Production-Based Measures of Risk for Asset Pricing, *Journal of Monetary Economics* 57, 146-163.

Boudoukh, Jacob, and Robert Whitelaw, 1991, The Benchmark Effect in the Japanese Government Bond Market, *Journal of Fixed Income* 1, 52-59.

Brennan, Michael J., Eduardo S. Schwartz and Roland Lagnado, 1997, Strategic Asset Allocation, *Journal of Economic Dynamics and Control* 21, 1377-1403.

Brunnermeier, Markus, 2009, Deciphering the Liquidity and Credit Crunch 2007-2009, *Journal of Economic Perspectives* 23, 77-100.

Brunnermeier, Markus, and Lasse H. Pedersen, 2009, Market Liquidity and Funding Liquidity, *The Review of Financial Studies* 22, 2201-2238.

Campbell, John Y., 1991, A Variance Decomposition for Stock Returns, *Economic Journal* 101, 157–179.

Campbell, John Y., and John Ammer, 1993, What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns, *Journal of Finance* 48, 3–37.

Campbell, John Y., and John H. Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, *Journal of Political Economy* 107, 205-251.

Campbell, John Y., and John H. Cochrane, 2000, Explaining the Poor Performance of Consumption-Based Asset Pricing Models, *Journal of Finance* 55, 2863-78.

Campbell, John Y., and Jianping Mei, 1993, Where do Betas Come From? Asset Pricing Dynamics and the Sources of Systematic Risk, *Review of Financial Studies* 6, 567–592.

Campbell, John Y. and Robert J. Shiller, 1987, Cointegration and Tests of Present Value Models, *Journal of Political Economy* 95, 1062–1088.

Campbell, John Y., and Robert J. Shiller, 1988, The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors, *Review of Financial Studies* 1, 195–228.

Campbell, John Y., and Robert J. Shiller, 1991, Yield Spreads and Interest Rate Movements: A Bird's Eye View, *Review of Economic Studies* 58, 495–514.

Campbell, John Y., and Luis Viceira, 1999, Consumption and Portfolio Decisions When Expected Returns are Time Varying, *Quarterly Journal of Economics* 114, 433-495.

Campbell, John Y., and Luis M. Viceira, 2001, Who Should Buy Long-Term Bonds?,American Economic Review 91, 99-127.

Campbell, John Y., and Luis M. Viceira, 2002, *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors, Clarendon Lectures in Economics*, Oxford: Oxford University Press.

Campbell, John Y., and Tuomo Vuolteenaho, 2004, Bad Beta, Good Beta, *American Economic Review* 94, 1249-1275.

Chen, Long, Ralitsa Petkova, and Lu Zhang, 2008, The Expected Value Premium, Journal of Financial Economics 87, 269–280.

Cochrane, John H., 1991a, Volatility Tests and Efficient Markets: A Review Essay, *Journal of Monetary Economics* 27, 463-485.

Cochrane, John H., 1991b, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *Journal of Finance* 46, 207-234.

Cochrane, John H., 1992, Explaining the Variance of Price-Dividend Ratios, *Review of Financial Studies* 5, 243-280.

Cochrane, John H., 1994, Permanent and Transitory Components of GNP and Stock Prices, *Quarterly Journal of Economics* 109, 241-266

Cochrane, John H., 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy* 104, 572-621.

Cochrane, John H., 2001, Review of 'Famous First Bubbles: The Fundamentals of Early Manias', *Journal of Political Economy* 109, 150-1154.

Cochrane, John H., 2003, Stock as Money: Convenience Yield and the Tech-Stock Bubble,in William C. Hunter, George G. Kaufman and Michael Pomerleano, Eds., *Asset Price Bubbles.* Cambridge: MIT Press.

Cochrane, John H., 2005b, Liquidity, Trading and Asset Prices, NBER Asset Pricing Program Report, *NBER Reporter*. Cambridge: National Bureau of Econoimc Research.

Cochrane, John H., 2005c, *Asset Pricing*, Revised Edition. Princeton: Princeton University Press.

Cochrane, John H., 2006, The Dog That Did Not Bark: A Defense of Return Predictability, *Review of Financial Studies* 21, 1533-1575.

Cochrane, John H., 2007a, Financial Markets and the Real Economy,in Rajnish Mehra, Ed. *Handbook of the Equity Premium* Elsevier, 237-325.

Cochrane, John H., 2007b, Portfolio Theory, Manuscript, University of Chicago.

Cochrane, John H., 2008, A Mean-Variance Benchmark for Intertemporal Portfolio Theory, Manuscript, University of Chicago.

Cochrane, John H., 2011, Understanding Fiscal and Monetary Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic, *European Economic Review* 55, 2-30.

Cochrane, John H., and Monika Piazzesi, 2005, Bond Risk Premia, *American Economic Review* 95, 138-160.

Cochrane, John H., and Monika Piazzesi, 2008, Decomposing the Yield Curve, Manuscript, University of Chicago.

Cohen, Randolph B., Christopher K. Polk, and Tuomo Vuolteenaho, 2003, The Value Spread, Journal of Finance 58, 609-642.

Constantinides, George M. and Darrell Duffie, 1996, Asset Pricing with Heterogeneous Consumers, *Journal of Political Economy* 104, 219-240.

Davis, James L., Eugene F. Fama and Kenneth R. French, 2000, Characteristics, Covariances and Average Returns: 1929-1997, *Journal of Finance* 55, 389-406.

Daniel, Kent, and Sheridan Titman, 2006, Testing Factor-Model Explanations of Market Anomalies, Manuscript, Columbia University and University of Texas.

Duffie, Darrell, and Antje Berndt, 2011, Explaining Corporate Debt Returns, Manuscript, Stanford University.

Duffie, Darrell, and Bruno Strulovici, 2011, Capital Mobility and Asset Pricing, Manuscript, Stanford University and Northwestern University.

Epstein, Larry G. and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Fama, Eugene F., 1970, Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance* 25, 383-417.

Fama, Eugene F., 1984, Forward and Spot Exchange Rates, *Journal of Monetary Economics* 14, 319-38.

Fama, Eugene F., 1986, Term Premiums and Default Premiums in Money Markets, *Journal of Financial Economics* 17, 175-96.

Fama, Eugene F.,, 1996, Multifactor Portfolio Efficiency and Multifactor Asset Pricing, *Journal of Financial and Quantitative Analysis* 31, 441-465.

Fama, Eugene F.,1998, Market Efficiency, Long-Term Returns, and Behavioral Finance, *Journal of Financial Economics* 49, 283-306.

Fama, Eugene F. and Robert R. Bliss, 1987, The Information in Long-Maturity Forward Rates, *American Economic Review* 77, 680-92.

Fama, Eugene F. and Kenneth R. French, 1988, Dividend Yields and Expected Stock Returns, *Journal of Financial Economics* 22, 3-25.

Fama, Eugene F. and Kenneth R. French, 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics* 25, 23-49.

Fama, Eugene F. and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stock and Bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F. and Kenneth R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-84.

Fama, Eugene F. and Kenneth R. French, 1997, Industry Costs of Equity, *Journal of Financial Economics* 43, 153-93.

Fama, Eugene F., and Kenneth R. French, 2006, The Value Premium and the CAPM, *Journal of Finance* 61, 2163-85.

Fama, Eugene F. and Kenneth R. French, 2010, Dissecting Anomalies, *Journal of Finance* 63, 1653 - 1678.

Fleckenstein, Mattias, Francis A. Longstaff,, and Hanno Lustig, 2010, Why Does the Treasury Issue TIPS? The TIPS-Treasury Bond Puzzle, Manuscript, UCLA Anderson School of Management.

Fontana, Alessandro, 2010, The Persistent Negative CDS-Bond Basis During the 2007/08 Financial Crisis, Manuscript, Unversitá Ca Foscari, Venice.

Fontana, Alessandro, and M. Scheicher, 2010, An Analysis of Euro Area Sovereign CDS and their Relation with Government Bonds, European Central Bank, Working Paper No. 1271.

Frazzini, Andrea and Lasse Hege Pedersen, 2010, Betting Against Beta, Manuscript, New York University.

Froot, Kenneth. A., and P. O'Connell, 2008, On the Pricing of Intermediated Risks: Theory and Application to Catastrophe Reinsurance, *Journal of Banking and Finance* 32, 69-85.

Gabaix, Xavier, Arvind Krishnamurthy, and Olivier Vigneron, 2007, Limits of Arbitrage: Theory and Evidence from the Mortgage Backed Securities Market, *Journal of Finance,* 62, 557-596.

Garber, Peter M. 2000, *Famous First Bubbles: The Fundamentals of Early Manias*. Cambridge, Mass.: MIT Press

Gala, Vito, 2010, Irreversible Investment and the Cross-Section of Stock Returns in General Equilibrium, Manuscript, London Business School.

Garleanu, Nicolae, and Lasse Heje Pedersen, 2009, Margin-Based Asset Pricing and Deviations from the Law of One Price, Manuscript, New York University, Forthcoming *Review of Financial Studies*.

Gomes, Joao, Leonid Kogan, and Lu Zhang, 2003, Equilibrium Cross-Section of Returns, *Journal of Political Economy* 111, 693-732.

Gourio, Francois, 2007, Labor Leverage, Firms Heterogeneous Sensitivities to the Business Cycle, and the Cross-Section of Returns, Manuscript, Boston University.

Gourinchas, Pierre-Olivier and Hélène Rey, 2007, International Financial Adjustment, *Journal of Political Economy* 115, 665-703.

Gromb, Denis, and Dimitri Vayanos, 2010, Limits of Arbitrage: The State of the Theory, *Annual Review of Financial Economics* 2, 251-275.

Hansen, Lars Peter, and Robert J. Hodrick 1980, Forward Exchange-Rates As Optimal Predictors of Future Spot Rates - An Econometric-Analysis, *Journal of Political Economy* 88, 829-853.

Hansen, Lars Peter., John C. Heaton and Nan Li, 2008, Consumption Strikes Back?: Measuring Long Run Risk, *Journal of Political Economy* 116, 260-302.

Hansen, Lars Peter, and Thomas J. Sargent, 2005, Recursive Robust Estimation and Control under Commitment, *Journal of Economic Theory* 124, 258-301.

Heaton, John C., and Deborah J. Lucas, 2000, Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *Journal of Finance* 55, 1163–1198.

Heaton, John C., Deborah J. Lucas and Robert L. McDonald, 2009, Is Mark-to-Market Accounting Destabilizing? Analysis and Implications for Policy, Journal of Monetary Economics 57, 64-75.

He, Zhiguo and Arvind Krishnamurthy, 2010, Intermediary Asset Pricing, Manuscript, University of Chicago and Northwestern University.

Hu, Xing, Jun Pan and Jiang Wang, 2011, Noise as Information for Illiquidity, NBER Working Paper 16468.

Jegadeesh, Narasimhan and Sheridan Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency, *Journal of Finance* 48, 65 - 91.

Jermann, Urban, 2010, The Equity Premium Implied by Production,*Journal of Financial Economics* 98, 279-296.

King, Robert G., Charles I. Plosser and Sergio T. Rebelo, 1988, Production, growth and business cycles : I. The basic neoclassical model, Journal of Monetary Economics 21,195-232.

Koijen, Ralph and Jules van Binsbergen, 2010, Predictive regressions: A present-value approach, Journal of Finance 65 1439–1471.

Krishnamurthy, Arvind, 2002, The Bond/Old-Bond Spread, *Journal of Financial Economics* 66, 463-506.

Krishnamurthy, Arvind, 2008, Fundamental Value and Limits to Arbitrage, Manuscript, Kellogg Graduate School of Management.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2010, The Aggregate Demand for Treasury Debt, Manuscript, Kellogg Graduate School of Management.

Lacerda, Filipe, and Pedro Santa-Clara, 2010, Forecasting Dividend Growth to Better Predict Returns, Manuscript Universidade Nova de Lisboa.

Laibson, David, 1997, Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics* 62, 443-77.

Lamont, Owen, 2000, Investment Plans and Stock Returns, *Journal of Finance* 55, 2719-2745.

Lettau, Martin, and Sydney Ludvigson, 2001a, Consumption, Aggregate Wealth and Expected Stock Returns, *Journal of Finance* 56, 815-849.

Lettau, Martin and Sydney Ludvigson, 2001b, Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying, J*ournal of Political Economy* 109, 1238-1287.

Lettau, Martin, and Sydney Ludvigson, 2005, Expected Returns and Expected Dividend Growth, *Journal of Financial Economics* 76, 583-626.

Lettau, Martin and Jessica Wachter, 2011, The Term Structures of Equity and Interest Rates, *Journal of Financial Economics* Forthcoming.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2010, A Skeptical Appraisal of Asset Pricing Tests, *Journal of Financial Economics* 96, 175-194.

Li, Erica X. Dmitry Livdan, and Lu Zhang, 2008, Anomalies, *Review of Financial Studies* 22, 4301-4334

Liu, Laura Xiaolei, and Lu Zhang, 2011, Investment-Based Momentum Profits, Manuscript Hong Kong University of Science and Technology and Ohio State University

Liu, Laura Xiaolei, Toni M. Whited, and Lu Zhang, 2009, Investment-Based Expected Stock Returns, *Journal of Political Economy* 117, 1105-1139.

Longstaff, Francis A., 2004, The Flight to Liquidity Premium in U.S. Treasury Bond Prices, *Journal of Business* 77, 511-526.

Longstaff, Francis A., 2008, Flight-From-Leverage in Distressed Financial Markets, Manuscript, UCLA Anderson Scholl of Management

Ludvigson, Sydney, 2011, Advances in Consumption-Based Asset Pricing: Empirical Tests, in *Handbook of the Economics of Finance*, George Constantinides, Milton Harris and Rene Stulz Eds., vol. 2, Forthcoming. Elsevier Science B.V., North Holland, Amsterdam.

Lustig, Hanno, Nikolai L. Roussanov, and Adrien Verdelhan, 2010a, Common Risk Factors in Currency Markets, Manuscript, UCLA, MIT, and Wharton School

Lustig, Hanno, Nikolai L. Roussanov, and Adrien Verdelhan, 2010b, Countercyclical Currency Risk Premia, Manuscript, UCLA, MIT, and Wharton School.

Mankiw, N. Gregory and Stephen Zeldes, 1991, The Consumption of Stockholders and Non-Stockholders, *Journal of Financial Economics* 29, 97-112.

Mehra, Rajnish, and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145-161.

Merton, Robert C., 1971, Optimum Consumption and Portfolio Rules in a Continuous Time Model, *Journal of Economic Theory* 3, 373-413.

Milgrom, Paul, and Nancy Stokey, 1982, Information, Trade and Common Knowledge, *Journal of Economic Theory* 26, 17-27.

Mitchell, Mark, Lasse Heje Pedersen, and Todd Pulvino, 2007, Slow-Moving Capital, *American Economic Review* 97, 215-220.

Mitchell, Mark and Todd Pulvino, 2001, The Characteristics of Risk and Return in Risk Arbitrage, *Journal of Finance* 56, 2135-2176.

O'Hara, Maureen, 2008, Bubbles: Some Perspectives (and Loose Talk) from History, Review of Financial Studies 21, 11-17.

Pastor, Lubos, 2000, Portfolio Selection and Asset Pricing Models, *Journal of Finance* 55, 179-223.

Pastor, Lubos, and Robert F. Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, *Journal of Political Economy* 111, 642-685.

Piazzesi, Monika and Eric Swanson, 2008, Futures Prices as Risk-Adjusted Forecasts of Monetary Policy, *Journal of Monetary Economics* 55, 677-691.

Rietz, Thomas A. 1988, The Equity Risk Premium: A Solution, *Journal of Monetary Economics* 22, 117–31.

Ross, Stephen A., 1976, The Arbitrage Theory of Capital Asset Pricing, *Journal of Economic Theory* 13, 341-360.

Ross, Stephen A., 1978, A Simple Approach to the Valuation of Risky Streams, *Journal of Business* 51, 453-475.

Sargent, Thomas J., and Neil Wallace, 1981, Some Unpleasant Monetarist Arithmetic, Federal Reserve Bank of Minneapolis *Quarterly Review* 531.

Scheinkman, José A. and Wei Xiong, 2003, Overconfidence and Speculative Bubbles, *Journal of Political Economy* 111, 1183-1219.

Shiller, Robert. 1981, Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?, *American Economic Review* 71, 421-436.

Stambaugh, Robert, 1988, The Information in Forward Rates: Implications for Models of the Term Structure, *Journal of Financial Economics* 21, 41–70.

Vayanos, Dimitri, and Jiang Wang, 2011, Liquidity and Asset Prices: A Unified Framework—Part II, Manuscript, London School of Economics and Massachusetts Institute of Technology.

Vayanos, Dimitri, and Jean-Luc Vila, 2011, A Preferred-Habitat Model of the Term Structure of Interest Rates, Manuscript, London School of Economics

Vuolteenaho, Tuomo, 2002, What Drives Firm-Level Stock Returns?,*Journal of Finance* 57, 233–264.

Wachter, Jessica, 2003, Risk Aversion and Allocation to Long-Term Bonds, *Journal of Economic Theory* 112,325–333.

Welch, Ivo, 2004, Capital Structure and Stock Returns, Journal of Political Economy 112, 106-131.

Yogo, Motohiro, 2006, A Consumption-Based Explanation of Expected Stock Returns, Journal of Finance 61, 539 - 580

8 Appendix

8.1 Present values and identities

8.1.1 Return identity

To keep the presentation self-contained, I start with a derivation of the Campbell-Shiller (1988) linearization. The return is by definition,

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(1 + \frac{P_{t+1}}{D_{t+1}}\right)}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t}$$

Therefore the log return is

$$r_{t+1} = \log\left(1 + e^{-dp_{t+1}}\right) + dp_t + \Delta d_{t+1}$$

where $dp \equiv \log(D_t/P_t)$, $r = \log(R), d = \log(D)$. I Taylor expand the first term about a constant PD. This constant need not be the mean.

$$r_{t+1} = \log(1 + PD) - \frac{PD}{1 + PD}(dp_{t+1} - dp) + dp_t + \Delta d_{t+1}.$$

Denoting $\rho \equiv \frac{PD}{1+PD}$, we can write the approximation

$$r_{t+1} = \kappa - \rho dp_{t+1} + dp_t + \Delta d_{t+1} \tag{11}$$

where

$$\kappa = -(1-\rho)\log(1-\rho) - \rho\log(\rho).$$

In time-series applications where we will only consider second moments we interpret symbols as deviations from means and ignore κ leaving

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}. \tag{12}$$

8.1.2 Present value identity.

To derive the present value identity (1), rearrange the return identity as

$$dp_t = r_{t+1} - \Delta d_{t+1} + \rho dp_{t+1}.$$

Iterating forward

$$dp_t = \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}.$$

Assuming the latter term goes to zero – the "transversality condition" which rules out "rational bubbles"– we have

$$dp_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}. \tag{13}$$

This is an ex-post identity, so it also holds in conditional expectations using any information set.

8.1.3 Dividend construction.

I create dividends from the CRSP annual return series with and without dividends,

$$R_t \equiv \frac{P_{t+1} + D_{t+1}}{P_t};\ RX_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

Then,

$$\frac{D_{t+1}}{P_{t+1}} = \frac{R_{t+1}}{RX_{t+1}} - 1.$$

By using an annual horizon, I avoid the strong seasonal in dividend payments.

Annual dividend growth also includes some return information, because this definition one reinvests dividends to the end of the year at the market return. Sums of dividends are a good deal less volatile; and have the conceptual advantage of not mixing in any return information. However, if one uses sums of dividends over the year, then the identity $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$ does not hold. It's nice to use data definitions for which identities hold!

This definition of dividends, which cumulates returns, has a second practical advantage. Consider the sharp fall in stocks in the Fall of 2008. Now, using a simple sum of past dividends, we would see a large decline in price/dividend ratio. But much of that decline surely reflected news that dividends in 2009 were going to fall dramatically. In this way, the sum-of-dividend definition gives a measure that should forecast dividend growth as well as forecasting returns. By reinvesting dividends to the end of the year, the "dividend" series is lower than the sum; this price-dividend ratio already includes the information that dividends will decline next year and therefore produces a better return forecast.

I construct dividend growth by

$$\frac{D_{t+1}}{D_t} = \frac{(D_{t+1}/P_{t+1})}{(D_t/P_t)}.$$

For the VAR in Tables 2-4 I use instead dividend growth implied by the identity (11),

$$\Delta d_{t+1} = \kappa + r_{t+1} + \rho dp_{t+1} - dp_t.$$

Actual dividend growth gives very similar results. However, this construction means that Cambpell-Shiller approximate identities hold exactly, so it is easier to see their impact in the results. It's important to use "pure" returns rather than infer returns from dividend growth, otherwise approximation errors can show up as magic investment opportunities.

8.2 VARs

8.2.1 Shock definition

I identified the shocks in Figure 5 by setting changes to the other variables in turn equal to zero. The return identity (12) means that therefore some of the shocks must come with contemporaneous shocks to returns.

The dividend growth shock is a shock to dividend growth with no change in dividend yield or cay. Hence, it must come with a contemporaneous return shock,

$$\varepsilon^d_{t+1} = 1, \varepsilon^{dp}_{t+1} = 0, \varepsilon^{cay}_{t+1} = 0, \varepsilon^r_{t+1} = 1.$$

The dividend yield shock has no change in dividend growth or cay. If dividends don't change and dp rises, it means p and r fell a lot. Intuitively, a pure rise in discount rates lowers current return so it can raise subsequent returns.

$$\varepsilon^d_{t+1} = 0, \varepsilon^{dp}_{t+1} = 1, \varepsilon^{cay}_{t+1} = 0, \varepsilon^r_{t+1} = -\rho.$$

The cay shock is a change in cay_t with no change in dividend yield dp_t or dividend growth Δd_t, and hence no change in return r_t.

$$\varepsilon^d_{t+1} = 0, \varepsilon^{dp}_{t+1} = 0, \varepsilon^{cay}_{t+1} = 1, \varepsilon^r_{t+1} = 0.$$

I choose this definition of shocks. because it leads to nicely interpretable responses, e.g. "cash-flow" and "discount rate." Because dividends remain roughly unpredictable, this "short run" identification gives almost the same result as a "long run" identification. I.e., if rather than define the first shock as $\varepsilon^{dp}_{t+1} = 0, \varepsilon^{cay}_{t+1} = 0$, we had identified it by

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = 0,$$

we would have gotten nearly the same result. The resulting shocks are nearly uncorrelated, which is also convenient.

This VAR is very simple, since I left dividend growth out of the right hand side. My purpose is to distill the essential message of more complex VARs, and in such VARs, coefficients on dividend growth are small.

8.2.2 Identities in the cay VAR.

The present-value identity (13), conditioned down and reproduced here,

$$dp_t = E\left[\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} | I_t\right] - E\left[\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} | I_t\right],$$

suggests that an extra variable can only help dp to forecast long horizon returns if it forecasts long horizon dividend growth; it can help to forecast one year returns by changing the term structure of return forecasts as well. Here I show how that intuition applies algebraically to multiple regression coefficients and the impulse response function.

Regressing both sides of (13) on dp_t and z_t, we obtain the generalized restriction on long-run multiple regression coefficients,

$$1 = b^{lr}_r - b^{lr}_d \tag{14}$$
$$0 = c^{lr}_r - c^{lr}_d, \tag{15}$$

where the notation refers to regressions

$$\begin{aligned} r^{lr}_t &= b^{lr}_r dp_t + c^{lr}_r z_t + \varepsilon^r \\ \Delta d^{lr}_t &= b^{lr}_d dp_t + c^{lr}_d z_t + \varepsilon^d \end{aligned}$$

Equation (14) is the same as before, now applied to the multiple regression coefficient. Equation (15) expresses the idea that a new variable can only help to forecast long-run returns if it also helps to

forecast long-run dividend growth. But you see that the extra dividend growth and return forecasts will be perfectly negatively correlated. In this way, extra long-run dividend growth forecastability means *more* long-run return forecastability, not less.

In terms of individual long-horizon regressions

$$r_{t+j} = b_r^{(j)} dp_t + c_r^{(j)} z_t + \varepsilon_{t+j}^r$$

etc., (13) similarly implies

$$\begin{aligned} 1 &= \sum_{j=1}^{\infty} \rho^{j-1} b_r^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} b_d^{(j)} \\ 0 &= \sum_{j=1}^{\infty} \rho^{j-1} c_r^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} c_d^{(j)}. \end{aligned}$$

A variable can help to forecast *one-year* returns, $c_r^{(1)} \neq 0$ only if it correspondingly changes the forecast of longer-horizon returns, or dividend growth.

Since impluse-response functions are the same as regression coefficients of future variables such as r_{t+j} on shocks at time t, the impulse-response functions

$$(E_t - E_{t-1})\, r_{t+j} = e_{dp \to r}^{(j)} \varepsilon_t^{dp} + e_{z \to r}^{(j)} \varepsilon_t^z$$

where $E_t \equiv E(\cdot | dp_t, z_t)$ must obey the same relation,

$$\begin{aligned} 1 &= \sum_{j=1}^{\infty} \rho^{j-1} e_{dp \to r}^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} e_{dp \to \Delta d}^{(j)} \\ 0 &= \sum_{j=1}^{\infty} \rho^{j-1} e_{z \to r}^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} e_{z \to \Delta d}^{(j)}. \end{aligned}$$

This fact lets me easily interpret the change in forecastability by adding cay, in the context of the present value identity, by plotting the impulse responses. The numbers in Figure 5 are terms of this decomposition.

8.2.3 Results using Goyal-Welch predictors

To see if the pattern of the cay VAR holds more generally, I tried a number of the forecasting variables in Goyal and Welch (2008). The results are in Table A1. Each of these variables helps substantially to forecast one-period returns. Yet they mean-revert quickly and don't forecast dividends much, so the contribution to the variance of dividend yields is still almost all from the variance of long-run expected returns.

	dp	cay	eqis	svar	ik	dfy
c		2.21	-0.71	1.48	-5.30	5.25
$t(c)$		(1.73)	(-2.53)	(3.40)	(-0.85)	(1.86)
b	0.13	0.10	0.19	0.15	0.11	0.13
$t(b)$	(2.61)	(1.82)	(3.75)	(3.05)	(2.16)	(2.53)
R^2	0.10	0.16	0.19	0.15	0.11	0.13
$\sigma(E_t\Sigma_{j=1}^{\infty}\rho^{j-1}r_{t+j})$	0.52	0.46	0.49	0.42	0.53	0.49
$\sigma(E_t\Sigma_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j})$	0.17	0.13	0.16	0.11	0.17	0.14

Table A1. Multiple return-forecasting regressions and implied variance of long-horizon returns.

$$r_{t+1} = a + b \times dp_t + c \times z_t + \varepsilon_{t+1}$$

Data are from Goyal and Welch, 1947-2009. I calculate the variance of long-horizon expected returns and dividend growth from a bivariate VAR, and using actual (not identity) dividend growth forecasts. Equis, Percentage Equity Issuance is the ratio of equity issuing activity as a fraction of total issuing activity. Svar is stock variance, computed as sum of squared daily returns on the S&P 500. Ik is the investment to capital ratio, the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy. Dfy, the default yield spread, is the difference between BAA and AAA-rated corporate bond yields.

8.2.4 More lags of dividend growth and returns

An obvious first source of additional variables is less restrictive VARs than the simple first-order VAR that I presented in the text. Even in the information set of lagged $\{dp_t, r_t, \Delta d_t\}$, there may be more information.

The second lag of dividend yields is at least economically important. Table A2 presents the regressions

	dp_t	Δdp_t	$t(dp_t)$	$t\,(\Delta dp_t)$	R^2	$\sigma(E_t(y))\%$	$\frac{\sigma(E_t(y))}{E(y)}$
r_{t+1}	0.13	0.26	(2.45)	(1.83)	0.15	6.76	0.65
Δd_{t+1}	0.03	0.35	(0.62)	(3.27)	0.14	4.98	0.90
dp_{t+1}	0.93	0.10	(24.7)	(0.85)	0.91		
Δdp_{t+1}	-0.07	0.10	(-1.85)	(0.85)	0.06		

Table A2. Forecasts using dividend yield and change in dividend yield. CRSP value weighted return 1947-2009. $\Delta dp_t = dp_t - dp_{t-1}$

The change in dividend yield helps the return forecast, increasing R^2 from 0.09 to 0.15, and correspondingly increasing the more interesting measures of expected return variation.

The change in dividend yield really helps to forecast dividend growth, with a 3.27 t statistic, 5% standard deviation of forecast and forecast that varies by 90% of the mean. The 0.10 autocorrelation in Δdp_t however suggests that this will be a very short-lived signal, one with little impact on forecasts of long-run dividend growth or returns, and thus to our view of the sources of price-dividend ratio volatility.

Similarly, while *individual* r_{t-j} and Δd_{t-j} coefficients don't look big and don't have much pattern, they can nonetheless help as a *group*, or by sensibly restricting the pattern of lagged coefficients. In this vein, Lacerda and Santa-Clara (2010) and Koijen and van Binsbergen (2009) find that moving averages of past dividend growth help to forecast both returns and dividend growth (as they must, given the present value identity), almost doubling the return-forecast R^2.

8.2.5 VAR calculations

To find regression coefficients implied by a first-order VAR as in Table 2, I run

$$\begin{aligned} r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \qquad (16) \\ \Delta d_{t+1} &= b_d dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1} \qquad (17) \end{aligned}$$

Then I report

$$b_r^{(k)} = b_r \frac{1-(\rho\phi)^k}{1-\rho\phi}.$$

To calculate long run regression coefficients as in Table 4, with $z = cay$, I write the VAR as

$$\begin{aligned} \begin{bmatrix} dp_{t+1} \\ z_{t+1} \end{bmatrix} &= \begin{bmatrix} \phi_{dp,dp} & \phi_{dp,z} \\ \phi_{z,dp} & \phi_{z,z} \end{bmatrix} \begin{bmatrix} dp_t \\ z_t \end{bmatrix} + \begin{bmatrix} \varepsilon^{dp}_{t+1} \\ \varepsilon^{z}_{t+1} \end{bmatrix} \\ X_{t+1} &= \Phi X_t + \varepsilon^x_{t+1} \\ \begin{bmatrix} r_{t+1} \\ \Delta d_{t+1} \end{bmatrix} &= \begin{bmatrix} b_r & c_r \\ b_d & c_d \end{bmatrix} \begin{bmatrix} dp_t \\ z_t \end{bmatrix} + \begin{bmatrix} \varepsilon^{r}_{t+1} \\ \varepsilon^{d}_{t+1} \end{bmatrix} \end{aligned}$$

then

$$\begin{aligned} E_t \begin{bmatrix} r^{lr}_{t+1} \\ \Delta d^{lr}_{t+1} \end{bmatrix} &= \begin{bmatrix} b_r & c_r \\ b_d & c_d \end{bmatrix} \sum_{j=1}^{\infty} \rho^{j-1} \Phi^{j-1} \begin{bmatrix} dp_t \\ z_t \end{bmatrix} \\ E_t \begin{bmatrix} r^{lr}_{t+1} \\ \Delta d^{lr}_{t+1} \end{bmatrix} &= \begin{bmatrix} b_r & c_r \\ b_d & c_d \end{bmatrix} (I - \rho\Phi)^{-1} \begin{bmatrix} dp_t \\ z_t \end{bmatrix}. \end{aligned}$$

8.2.6 Univariate implications

While extending the VAR to additional variables adds a lot, the univariate implications of the VARs are much less than one might think. The middle panel of Figure 5 implies a completely temporary component of prices; if prices fall and dividends do not, that price movement is expected completely to melt away. However, the implied univariate representation of returns is almost

completely independent over time. Returns are predictable by dividend yields, but not by past returns. If we see a shock to returns without observing dividends, that shock is basically expected to last forever. As a result, stock return volatility does not decline substantially with investment horizon. One has to see the state variable dp to have any effect on forecastability or classic portfolio theory.

The reason is easy to see in a simplified example. Write the VAR as

$$\begin{aligned} r_{t+1} &= b_r \times dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= 0 \times dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= \phi \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

and note that the return identity (12) means

$$\begin{aligned} \varepsilon^r_{t+1} &= \varepsilon^d_{t+1} - \rho\varepsilon^{dp}_{t+1} \\ b_r &= 1 - \rho\phi + 0. \end{aligned}$$

Now, examine $(1-\phi L)r_{t+1}$

$$\begin{aligned} (1-\phi L)r_{t+1} &= b_r(1-\phi L)dp_t + \varepsilon^r_{t+1} - \phi\varepsilon^r_t \\ (1-\phi L)r_{t+1} &= (1-\rho\phi)\,\varepsilon^{dp}_t + \left(\varepsilon^d_{t+1} - \rho\varepsilon^{dp}_{t+1}\right) - \phi\left(\varepsilon^d_t - \rho\varepsilon^{dp}_t\right) \\ (1-\phi L)r_{t+1} &= -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1} + \varepsilon^{dp}_t - \phi\varepsilon^d_t \end{aligned}$$

Thus, r_{t+1} is an ARMA(1,1) in its univariate representation.

$$(1-\phi L)r_{t+1} = (1-\theta L)v_{t+1}.$$

Now, $\rho \approx 0.96$ and $\hat{\phi} \approx 0.94$, and is biased down. Hence $\rho = \phi$ is a good approximation. In that case, and with $cov(\varepsilon^d, \varepsilon^{dp}) = 0$ which is also very close to the data,

$$\begin{aligned} var\left[(1-\theta L)v_{t+1}\right] &= \left(1+\theta^2\right)\sigma^2_v = \left(1+\rho^2\right)\left(\sigma^2_{dp} + \sigma^2_d\right) \\ cov\left[(1-\theta L)v_{t+1}, (1-\theta L)v_t\right] &= -\theta\sigma^2_v = -\rho\left(\sigma^2_{dp} + \sigma^2_d\right) \end{aligned}$$

Hence, $\theta = \rho$ and returns are uncorrelated over time,

$$r_{t+1} = v_{t+1}.$$

In general, with $\phi \neq \rho$, we still have θ very near ϕ (between ϕ and ρ in fact), so returns follow an ARMA with very slight mean-reversion and a large permanent component.

8.3 Asset Pricing as a function of characteristics

8.3.1 Portfolio spreads.

In the text, I related 1-10 portfolio means to Sharpe ratios of underlying factors. Here is the result. Consider the ideal world for such an investigation: Expected returns rise with a characteristic C_i (for example b/m)

$$E(R^i) = a + b \times C_i,$$

and this variation corresponds exactly to a factor (for example, hml)

$$R_t^i = \beta^i \times f_t + \varepsilon_t,$$

with betas that also rise with the characteristic

$$\beta^i = \frac{a}{E(f)} + \frac{b}{E(f)} \times C_i,$$

and uncorrelated residuals

$$cov(\varepsilon^i, \varepsilon^j) = 0.$$

Now, consider the usual 1-10 or 1-20 portfolio difference

$$\begin{aligned} E(R^i - R^j) &= b(C_i - C_j) \\ \sigma^2(R^i - R^j) &= \left(\beta^i - \beta^j\right)^2 \sigma_f^2 + 2\frac{\sigma_\varepsilon^2}{N} = \frac{b^2}{E(f)^2}\left(C^i - C^j\right)^2 \sigma_f^2 + 2\frac{\sigma_\varepsilon^2}{N} \end{aligned}$$

where N is the number of securities in each portfolio. Therefore, the Sharpe ratio, which is proportional to the t statistic $\sqrt{T}E/\sigma$for the mean spread-portfolio return, is

$$\frac{E(R^i - R^j)}{\sigma(R^i - R^j)} = \frac{E(f)}{\sigma_f} \frac{b(C_i - C_j)}{\sqrt{b^2\left(C^i - C^j\right)^2 + 2\frac{\sigma_\varepsilon^2}{N}\frac{E(f)^2}{\sigma_f^2}}}$$

This Sharpe ratio rises as we look at further separated portfolios. As $C_i - C_j$ increases, it approaches the pure Sharpe ratio of the factor $E(f)/\sigma(f)$. It does not increase forever. Splitting in to finer portfolios can get the magic 1% per month portfolio mean or alpha, but cannot arbitrarily raise Sharpe ratios or t statistics. Splitting the portfolio more finely reduces N, so splits that are too fine end up reducing the Sharpe ratio and t statistic by including too much idiosyncratic risk.

Having seen this analysis, of course, we see that it's more efficient simply to examine the statistical significance of the cross-sectional regression coefficient $\hat{b}$, which uses information in all the securities, not just the end portfolios. Since $\hat{b} = cov(E(R^i), C^i)/var(C^i) = E(R^i \times \left[C^i - E(C^i)\right])/var(C^i)$ this regression coefficient is the same thing as testing the mean $E(f)$ of a factor which is also formed as a linear function of the characteristic $f_t = R_t^i \times \left[C^i - E(C^i)\right]$.

8.3.2 Value, betas, and samples.

In the text, I emphasized that all puzzles are joint puzzles of expected returns and betas, and cautioned that the value puzzle does not hold in pre- 1963 US data. Figure 22 presents the CAPM in the Fama-French 10 book/market portfolios before and after 1963. In the left hand panel, you see the familiar failure of the CAPM – average returns are higher in the value portfolio, but there is no association between the wide spread in average returns and market betas. The right-and panel shows average returns and betas before 1963. Here the CAPM is working remarkably well. The big change is *not* in the pattern of average returns. Value still earns more than growth, 45 bp compared to roughly 60 bp in the post 63 period. The big change is betas – in the pre-63 period value firms have higher betas, just as they "ought" to do.[66]

[66] Davis, Fama, and French (2000), Cambpell and Vuolteenaho (2004), Ang and Chen (2007), Fama and French (2006).

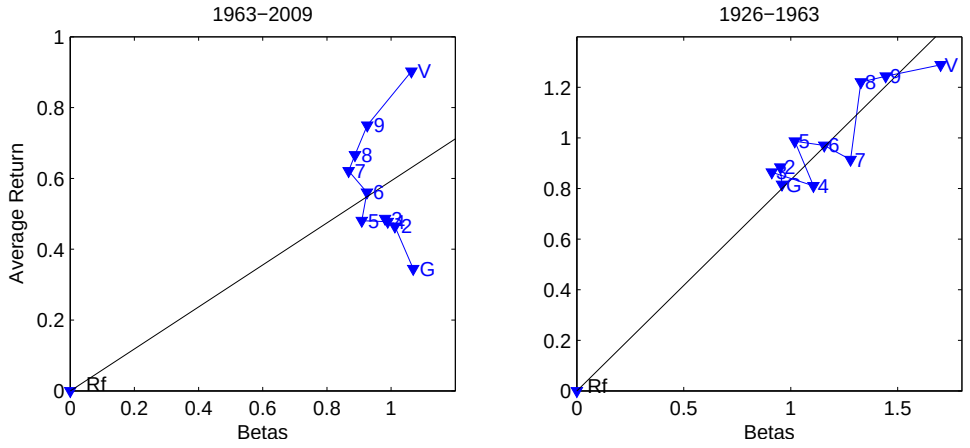

Figure 22: Value effect before and after 1963. Average returns on Fama - French 10 portfolios sorted by book/market equity vs. CAPM betas. Monthly data. Source: Ken French's website.

8.3.3 Time series and cross section

As a first step towards understanding mean returns as a function of characteristics, and to help make the ideas concrete, Table A3 presents regressions using the Fama-French 25 size and book-market portfolios. I use log book/market and log size relative to the market portfolio.

The first row of Table A3 gives a pure cross-sectional regression. The fitted values of this regression fit the portfolio average returns quite well, with a 77% R^2 (One does better still with a $size \times bm$ cross-term, allowing the growth portfolios to have a different slope on size than the value portfolios.)

	size_t	bm_t	Δsize_t	Δbm_t
1. Cross section	-0.030	0.27		
2. Pooled	-0.022	0.55		
3. Time dummies	-0.031	0.29		
4. Portfolio dummies	-0.087	1.48		
5. Pooled	-0.030	0.46	-0.38	1.11

Table A3 Regressions of Fama-French 25 size and B/M portfolio returns on size and book/market characteristics. The regression specification is in the first row is

$$E\left(R^{ei}_{t+1}\right) = a + b \times E\left(size_{it}\right) + c \times E(bm_{it}) + \varepsilon_i;\ i = 1, 2, ..25$$

The remaining rows are

$$R^{ei}_{t+1} = a + (a_t) + (a_i) + b \times size_{i,t} + c \times bm_{i,t} + d \times (size_{i,t} - size_{i,t-12}) + e \times (bm_{i,t} - bm_{i,t-12}).$$

Terms in parentheses only appear in some regressions. *size* is log(market equity/total market equity). *bm* is log(book/market). Monthly data 1947-2009. Data from Ken French's website.

The second row of Table A3 gives a pooled forecasting regression, which is the most natural way to integrate time series and cross section. The size coefficient is a little smaller, and the bm coefficient is much larger.

To diagnose the difference between the cross-section and pooled regressions, rows 3 and 4 present a regression with time dummies and a regression with portfolio dummies. Variation *over time* in a *given* portfolio's bm is a much stronger signal of return variation than the same size variation *across portfolios* in average bm.

When we run such regressions for individual firms, we can't use dummies, since the average return of a specific company over the whole sample is meaningless. The goal of this regression is to mirror portfolio formation and remove firm-name completely from the list of characteristics. The last line of Table A3 gives a way to capture the difference between time-series and cross section without dummies – it allows an independent effect of recent changes in the characteristics. This specification accounts quite well for the otherwise unpalatable time and portfolio dummies. The portfolio dummy regression coefficient that captures time-series variation is quite similar to the sum of the level and recent-change coefficients. It is also gratifyingly similar to the "recent-change" effect in aggregate dividend-yield regressions of Table A2. One could of course capture the same phenomenon with portfolios, by sorting based on level and recent change of characteristics. But my goal is to explore the other direction of this equivalence.

Next, we want to run regressions like this on individual data, and find similar characterization of the covariance matrix as a function of characteristics. Then, we can expand to multiple right-hand variables.

8.3.4 Prices in the cross section

Section 3.3 suggested merging time-series and cross-sectional approaches, to understand the variation in prices (price-dividend ratios) across time and portfolios by exploring long-run return predictability in the cross-section. How much of the difference between one asset's price-dividend, price-earnings, book-market, etc. ratio and another's is due to variation in expected returns, and how much to expected dividend growth or other cashflow expectations?[67]

To explore this question and clarify the idea, I examine the 10 Fama-French book/market portfolios. Eventually, we're looking for an estimates that are functions of size and other characteristics in individual data. Figure 23 presents the average return, dividend growth, and dividend yield of the portfolios.

Over long horizons, dividend yields are stationary so long-term average returns come from dividend yields and dividend growth. Taking unconditional means of the return identity (12),

$$E\left(r^i\right) = (1-\rho)\, E\left(dp^i\right) + E(\Delta d^i).$$

Figure 23 shows that value portfolio returns come roughly half from greater dividend growth and half from a larger average dividend yield. From a valuation perspective, this is a surprising result. High prices – low dividend yields – should correspond to higher subsequent dividend growth, not lower.[68]

Our objective is to produce variance decompositions across time and securities as with the

[67] Vuolteenaho (2002), Cohen, Polk and Vuolteenaho (2003).

[68] Chen, Petkova and Zhang (2008) section 2.2.2 discuss this puzzle.

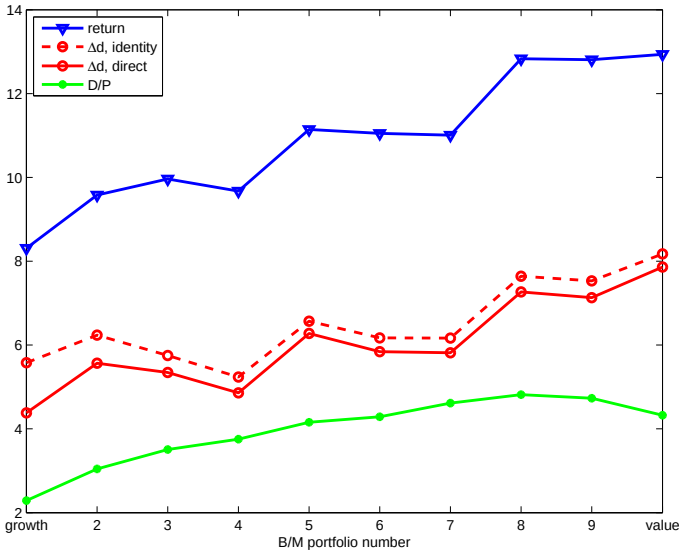

Figure 23: Average return r_{t+1}, dividend growth Δd_{t+1}, and dividend yield dp_t for the Fama - French 10 book/market portfolios, 1947-2008. The dashed Δd line gives mean dividend growth implied by the approximate identity $\Delta d_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$.

market return. Flipping this around, we have

$$E\left(dp^i\right) = \frac{1}{1-\rho}\left[E\left(r^i\right) - E(\Delta d^i)\right]. \tag{18}$$

Now, we can see that a purely cross-sectional regression of average returns, dividend growth on dividend yields must obey

$$1 = \frac{b_r^{cs}}{1-\rho} - \frac{b_d^{cs}}{1-\rho},$$

where the b are the cross-sectional regression coefficients of the terms in (18). We can interpret these coefficients as the fraction of *cross-sectional* dividend yield variation

$$var_{cs}(dp) = \frac{1}{N}\sum_{i=1}^{N}\left[E(dp^i) - E(dp)\right]^2$$

driven by discount rates and driven by dividend growth. (Vuolteenaho (2002) uses a different present value identity to understand variation in the book / market ratio directly, rather than use dividend yields as I have. This is a better procedure for individual stocks, which often do not pay dividends. I use dividend yields here for simplicity.)

The first column of Table A4 presents this cross-sectional regression. The results are quite similar to the time-series regressions for the market portfolio from Tables 2 - 4: *More than all of the cross-sectional variation in average dividend yields of these portfolios comes from cross-sectional variation in expected returns* (1.33). Expected dividend growth goes "the wrong way" – low prices correspond to high dividend growth, as seen in Figure 23. (Sample means obey the identity

$$E\left(dp_t^i\right) = \frac{1}{1-\rho}\left[E\left(r_{t+1}^i\right) - E\left(\Delta d_{t+1}^i\right) + \rho\frac{1}{T}\left(dp_T^i - dp_1^i\right)\right].$$

The last term is fairly large, which is why the $b/(1-\rho)$ column does not add up to one.)

	Cross section		Portfolio dummies		Time dummies		Pooled		Pooled	
	b	$\frac{b}{1-\rho}$	b	$\frac{b}{1-\rho\phi}$	b	$\frac{b}{1-\rho\phi}$	b	$\frac{b}{1-\rho\phi}$	dp_{t-1}	Δdp_{t-1}
r	0.053	1.33	0.107	0.903	0.044	0.325	0.095	0.967	0.090	0.074
Δd	0.036	0.89	-0.005	-0.041	-0.083	-0.611	0.004	0.042	-0.004	0.073
Δd^*	0.026	0.64	-0.011	-0.097	-0.092	-0.675	-0.003	-0.033	-0.012	0.076
dp			0.92		0.90		0.94		0.94	0.002

Table A4. Cross-sectional regression coefficients of average returns $E(r^i_{t+1})$, average dividend growth $E(\Delta d_{t+1})$, and dividend yield change on dividend yields $E(dp^i)$, 10 Fama-French B/M portfolios. Implied dividend growth Δd^* is calculated from the approximate identity $\Delta d^*_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$. I use $\rho = 0.96$

We can, of course, ask how much of the *time-variation* in these dividend yields around their *portfolio average* corresponds to return vs. dividend growth forecasts. A regression that includes portfolio dummies, shown next in Table A4, addresses this question. The 0.11 return-forecasting coefficient for *portfolios* is almost the same as the return forecasting coefficient for the market as a whole seen in Tables 2-4. The dividend growth forecast is also nearly zero. So *all variation in book/market sorted portfolio dividend yields over time, about portfolio means, corresponds to variation in expected returns,* much like that of market returns.

The regression with time dummies, next in Table A4, paints a different picture. The return coefficient is smaller at 0.044, and ϕ is smaller as well, so expected returns only account for 33% of the variation in dividend yields. Finally we see an important dividend growth forecast, with the right sign, -0.08-0.09, accounting for 61-68% of dividend yield volatility. The strong contrast of this result with the pure cross sectional regression means that *a time of unusually large cross-sectional dispersion in dividend yields corresponds to an unusually high dispersion in dividend growth forecasts.* Dividend yields of these portfolios move overall with a very slow trend. About this trend, there are times in which the dispersion widens and tightens. This widening and tightening does not add up to a large variation in individual dividend yields about their portfolio means, so we don't see it in the regression with portfolio dummies.

This is an important regression, in that it gives us a sense that there is a component of variation in valuations that does correspond to dividend growth forecasts. The unusual dispersion in dividend growth forecasts adds up to zero, so this kind of variation cannot be seen in the aggregate dividend yield and its forecasting relations. We get a sense here that there is individual-security variation in forecastable dividend growth, which drives some individual variation in prices, but which averages out across all securities, so that the aggregate dividend yield is driven primarily by expected returns.

A pooled regression with no dummies looks much like the time-series regression with portfolio dummies. There is more time-variation in dividend yields than cross-sectional variation, so adding them up evenly the time-variation dominates the pooled regression.

The last column of Table A4 takes a hint from Table A3, to try to unite time-series and cross-sectional variation without using dummies. It shows a very similar result, with the Δdp_{t-1} variable accounting for much of the dividend growth forecastability. The next step is to calculate the price implications of this multivariate regression, as I did with cay, but that takes us too far afield of this simple example.

The Fama-French size portfolios, shown in Table A5, present a quite different picture. The pure cross-sectional regression (first column) finally shows cashflow effects: Higher pieces (low dividend yields) are associated with higher subsequent dividend growth, which by one measure fully accounts for the dividend yield variation! However, with portfolio dummies we again see that practically all dividend yield variation *over time for a given portfolio* comes from expected return variation, just as for the market as a whole. With time dummies, variation *across portfolios in a given time period* is split between return and dividend growth forecasts.

	Cross section		Portfolio dummies		Time dummies		Pooled	
	b	$\frac{b}{1-\rho}$	b	$\frac{b}{1-\rho\phi}$	b	$\frac{b}{1-\rho\phi}$	b	$\frac{b}{1-\rho\phi}$
r	-0.014	-0.355	0.077	1.022	0.023	0.267	0.067	0.947
Δd	-0.030	-0.757	0.016	0.207	-0.045	-0.525	0.011	0.150
Δd^*	-0.048	-1.197	0.002	0.022	-0.063	-0.733	-0.004	-0.053
dp			0.963		0.952		0.968	

Table A5. Cross-sectional regression coefficients of average returns $E(r^i_{t+1})$, average dividend growth $E(\Delta d^i_{t+1})$, and dividend yield change on dividend yields $E(dp^i)$, 10 Fama-French ME (size) portfolios. Implied dividend growth is calculated from the approximate identity $\Delta d_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$.