On the Unstable Relationship between Exchange Rates and Macroeconomic Fundamentals[1]

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Abstract

It is well known from anecdotal, survey and econometric evidence that the relationship between the exchange rate and macro fundamentals is unstable. The aim of this paper is both to understand what drives this instability and to investigate some key implications. We develop a model where large time variation in the relationship between the exchange rate and macro fundamentals endogenously develops from a combination of incomplete information and very gradual changes in structural parameters of the economy. After calibrating the model to exchange rate and interest rate data for industrialized countries, we investigate the impact of parameter instability on the statistical properties of exchange rates and interest rates and on the ability to forecast exchange rates out of sample. We find the impact to be remarkably small, even when the relationship between exchange rates and fundamentals is highly unstable.

1 Introduction

"The dollar's resilience in the wake of recent dire US economic data has raised the prospect that the currency market may be experiencing one of its periodic changes in focus" (Financial Times, February 11, 2008)

"The dollar's latest stumble ... came despite optimistic economic data from the US. But analysts said the movement of the US currency was no longer driven by growth fundamentals. All the focus is on the deficit now..." (Financial Times, February 11, 2003)

As reflected in these quotes, it has been widely reported in the financial press that foreign exchange traders regularly change the weight they attach to different macro indicators. Yin-Wong Cheung and Menzie Chinn (2001) further confirm this through a survey of U.S. foreign exchange traders in 1998. It is also consistent with widespread evidence of parameter instability in the relationship between exchange rates and macro fundamentals, as documented for example by Wolff (1987), Schinasi and Swamy (1989), Rossi (2006) and Sarno and Valente (2008). The latter show that to achieve the best exchange rate forecast, one needs to continuously change the set of variables used. More generally there are numerous studies that document structural breaks and regime switching in nominal and real exchange rates.

The aim of this paper is both to propose an explanation for what drives this unstable relationship between exchange rates and fundamentals and to investigate some key implications. We first develop a model where large time-variation in the relationship between the exchange rate and macro fundamentals endogenously develops from a combination of incomplete information and very gradual changes in structural parameters of the economy. After calibrating the model to data on exchange rates and interest rates for industrialized countries, we investigate the impact of parameter instability on the statistical properties of exchange rates and interest rates and on the ability to forecast exchange rates out of sample.

We frame the analysis in a context of a setup common to many nominal exchange rate models. Equilibrium exchange rates are determined by two equations: an interest rate arbitrage equation and a relationship between the interest rate differential and a set of macro variables. The latter is usually derived from money market equilibrium or from monetary policy rules. We introduce two key elements to this setup. First, we allow for unobserved gradual changes in the structural parameters that characterize the relationship between interest rates and macro fundamentals. This can for example be the result of gradual changes in money demand, gradual policy changes or a gradual change in the structural relationship between variables targeted by policy makers and other macro variables.[1] Second, we also allow the interest differential to be affected by *unobserved* macroeconomic shocks which makes it harder for agents to learn about structural parameters associated with *observed* macro variables.[2]

We show that even when structural parameters change very slowly, the reduced form relationship between exchange rates and fundamentals can become highly unstable in the short to medium run. This is a result of estimation mistakes as agents continuously update their views of the structural parameters. Particularly the unobserved variables generate considerable confusion in the short to medium run. When the exchange rate fluctuates as a result of an unobserved macroeconomic shock, it can be optimal for agents to blame this on an observed macro fundamental by giving it more weight and therefore making it a “scapegoat”. For example, when the dollar depreciates it is natural to attribute it to a large current account deficit. This happens even when the depreciation is unrelated to the current account deficit.[3]

[1] That macroeconomic relationships are time varying has been widely documented. An early paper is Stock and Watson (1996). Recent contributions include Boivin (2006), Canova (2005), Clarida, Gali and Gertler (2000), Cogley and Sargent (2005), Del Negro and Otrok (2007), Inoue and Rossi (2007), Primiceri (2005), Sims and Zha (2006) and Fernandez-Villaverde and Rubio-Ramirez (2007).

[2] This could reflect unobserved shocks to money demand, to monetary policy or to any other part of the economy to the extent that it feeds into equilibrium interest rates (e.g. by affecting variables targeted by the central bank).

[3] In a previous short paper, Bacchetta and van Wincoop (2004), we developed the idea of such a scapegoat effect in the context of a simple static noisy rational expectations model in which some parameters are unknown. We showed that excessive weight could be given to a variable depending on the correlation between the noise shock and the fundamental shock. However, since that model is static it could not be used to address the unstable dynamic relationship between exchange rates and fundamentals and its implications. Apart from the dynamic setup, the model in this paper also differs in that there is no private information as in Bacchetta and van Wincoop

Having established that in an environment with incomplete information an unstable relationship between exchange rates and fundamentals naturally develops, we next investigate its implications. Our main focus is on the ability to forecast out of sample. It is exactly in this context that the issue of parameter instability has most frequently been discussed in the exchange rate literature.[4] Meese and Rogoff (1983) first showed that models do not outperform the random walk in forecasting future exchange rates, even when the actual future macro fundamentals are used to forecast. Their results have largely held up since then, even with a lot more data available.[5]

If the parameters that characterize the relationship between exchange rates and macro variables were known and constant, the model would by construction outperform the random walk.[6] Even if parameters were constant, the fact that they are not known may explain the Meese-Rogoff findings due to parameter estimation error in small samples. This has received considerable attention in the recent literature and tests have been developed to correct for small sample bias (e.g. Clark and West (2006)). It is remarkable though that even with much longer samples the Meese-Rogoff puzzle has continued to hold up. It is therefore natural to investigate the role of parameter instability as the alternative explanation for the Meese-Rogoff puzzle.[7]

We can use our model to investigate the extent to which parameter instability is responsible for the weak forecasting performance. Maybe surprisingly, we find that even when the relationship between exchange rates and fundamentals is highly unstable, this cannot explain the Meese-Rogoff findings.[8] The impact of parameter

(2004). Scapegoat effects naturally develop as long as there is incomplete information about parameters; the information does not need to be private.

[4]See Wolff (1987), Schinasi and Swamy (1989), Rossi (2005), Sarno and Valente (2008) and Meese and Rogoff (1988).

[5]See for example Cheung, Chinn and Pascual (2005) and Rogoff and Stavrakeva (2008).

[6]This is the case as long as the macro variables have any explanatory power at all.

[7]More broadly, the only other possible explanation involves changes in the model, which could involve either the structure of the model or its parameters. If the model remains the same, and so do its parameters, with a long enough sample we should be able to estimate these parameters and therefore outperform the random walk.

[8]Perhaps related to this, the literature on exchange rate parameter instability referred to in the opening paragraph finds that allowing for time-varying parameters does not systematically improve forecasting performance. But this may again be due to the difficulty in estimating

instability on forecasting performance is only large when the macro fundamentals have significant explanatory power to start with. But when that is the case, the model significantly outperforms the random walk in out of sample forecasts both with and without parameter instability, in contrast to the evidence. Our analysis implies that the Meese-Rogoff findings most likely result from a combination of the well-known weak explanatory power of observed macro fundamentals and small sample parameter estimation bias.

The next section presents the model. It also discusses the signal extraction method used to solve the model and the implications for the relationship between exchange rates and fundamentals. Section 3 calibrates the model based on data on interest rates and exchange rates for 13 industrialized countries and presents numerical results for the relationship between exchange rates and fundamentals based on simulations. Section 4 uses the model to analyze the impact of the unstable relationship between exchange rates and fundamentals on the statistical properties of exchange rates and interest rate and the ability to forecast out of sample. Section 5 concludes.

2 A Model with Unknown Parameters

We assume that agents know the structure of the model, but not the specific parameter values. The underlying parameters themselves gradually change over time, so that agents need to keep learning about them. While the underlying parameters change smoothly, their estimate may vary significantly. This affects the exchange rate and, more importantly, the relationship between exchange rates and fundamentals. In this section we first describe the model, then discuss how expectations about parameters are formed and finally consider the implications for the relationship between exchange rates and fundamentals.

2.1 Basic Framework

Many models of exchange rate determination can be reduced to two equations: an interest rate arbitrage equation and an equation for the interest rate differential. Together these imply a solution for the equilibrium exchange rate as the present

parameters in small samples, particularly when they are time-varying.

value of fundamentals. We write these two equations as

$$E_t s_{t+1} - s_t = i_t - i_t^* + \phi_t \quad (1)$$

$$i_t - i_t^* = -\mu(F_t + b_t) + \mu s_t \quad (2)$$

Equation (1) is the interest rate arbitrage equation. We denote E_t as the expectation of the representative investor, s_t as the log nominal exchange rate (domestic per foreign currency), i_t and i_t^* as the domestic and foreign nominal one-period interest rates and ϕ_t as the risk premium. This equation means that the expected excess return on the foreign bond is equal to a risk premium ϕ_t. Equation (2) gives an expression for the interest rate differential. Here F_t depends on a linear combination of observed macro fundamentals f_{it}, but is not observed directly, and b_t captures unobserved macro fundamentals.

Equation (2) is a reduced form equation that can be derived from interest policy rules, from a standard monetary model, or from a DSGE model (e.g., see Engel and West, 2005, or Nason and Rogers, 2008). It is usually assumed that all macro fundamentals are observed. We generalize this by allowing for both observed and unobserved fundamentals. There are many possible justifications for this. When derived from money market equilibrium, b_t could reflect the aggregate of idiosyncratic money demand components, which is not observed. When derived as a monetary policy rule, b_t could capture the idiosyncrasies of policy makers that are not easily connected to observed fundamentals. Yet another interpretation is that policy makers respond to a perfectly observable macro fundamental such as the unemployment rate, while the latter is driven by both observed and unobserved macro fundamentals. Under that interpretation $F_t + b_t$ would be the unemployment rate.

In the data both s_t and F_t are non-stationary and typically considered in first differences. We will therefore focus on the first difference of fundamentals and assume the following linear relationship[9]

$$\Delta F_t = \Delta \mathbf{f}_t' \boldsymbol{\beta}_t \quad (3)$$

[9] An alternative specification is $F_t = \mathbf{f}_t' \boldsymbol{\beta}_t$, so that $\Delta F_t = (\Delta \mathbf{f}_t)' \boldsymbol{\beta}_t + \mathbf{f}_t' \Delta \boldsymbol{\beta}_t$. In that case, however, the impact of small changes in parameters on F_t, and therefore on the exchange rate, eventually explodes to plus or minus infinity. The reason is that a parameter change is multiplied with the level of the fundamental, which is unbounded due to non-stationarity.

where $\mathbf{f}_t = (f_{1t}, f_{2t}, ..., f_{Nt})'$ is the vector of N observed macroeconomic fundamentals and $\boldsymbol{\beta}_t = (\beta_{1t}, \beta_{2t}, ..., \beta_{Nt})'$ is the vector of associated parameters. In contrast to most of the literature, we allow the coefficients associated with the observed fundamentals to vary over time. Moreover, we assume that investors do not know $\boldsymbol{\beta}_t$ and need to estimate it. However, the process of $\boldsymbol{\beta}_t$ is known (and is specified below).

Since F_t is not directly observable, there is uncertainty about the underlying model. Investors need to estimate current and future $\boldsymbol{\beta}_t$. They have two sources of information regarding $\boldsymbol{\beta}_t$. First, they know the process of $\boldsymbol{\beta}_t$. Second, by observing the exchange rate and the interest rate differential, they know $F_t + b_t$ from (2). We describe below how investors combine optimally these two sources of information to form expectations about $\boldsymbol{\beta}_t$. The use of $F_t + b_t$ gives more precise information on average. But it is also a source of estimation errors.

Consider for example the expectation of the parameter β_{nt} for fundamental n. While β_{nt} affects $F_t + b_t$, the latter is also affected by b_t, all current and past fundamentals and all current and past parameters. Therefore to the extent that $F_t + b_t$ is used as a source of information about β_{nt}, its expectation can change without any change in β_{nt} itself. We will see that it is this rational confusion that is the key driver behind the unstable relationship between exchange rates and observed fundamentals.

2.2 Exchange Rates and Fundamentals

After substituting (2) into (1) we obtain the following standard reduced form (e.g., see Engel and West, 2005, or Engel, Mark, and West, 2008):

$$s_t = (1-\lambda)F_t + \lambda z_t + \lambda E_t s_{t+1} \tag{4}$$

Here $\lambda = 1/(1+\mu)$ and $z_t = \mu b_t - \phi_t$. This can be used to solve for the equilibrium exchange rate as a familiar present value equation:

$$s_t = (1-\lambda)F_t + \lambda z_t + \sum_{i=1}^{\infty} \lambda^i E_t \left((1-\lambda)F_{t+i} + \lambda z_{t+i}\right) \tag{5}$$

For convenience, in the remainder of this section we consider the special case without a risk premium and where b_t and $\Delta\mathbf{f}_t$ are iid. More precisely, for the rest of this section we assume that: i) $\phi_t = 0$, $\forall t$; ii) $b_t = \varepsilon_t^b$ with $\varepsilon_t^b \sim N(0, \sigma_b^2)$; iii) $\Delta f_{nt} = \varepsilon_{nt}^f$ with $\varepsilon_{nt}^f \sim N(0, \sigma_f^2)$. A more general specification will be considered in the numerical analysis in the next section. We maintain the assumption throughout the paper that shocks to f_{nt}, b_t and parameters are uncorrelated with each other.

Under these assumptions, $\sum_{i=1}^{\infty} \lambda^i E_t(1-\lambda)F_{t+i} = \lambda E_t F_t$ because $E_t F_{t+i} = E_t F_t$. The first difference of the present value equation (5) then becomes:

$$\Delta s_t = (1-\lambda)\Delta F_t + (1-\lambda)\Delta b_t + \lambda(E_t F_t - E_{t-1}F_{t-1}) \tag{6}$$

When parameters $\boldsymbol{\beta}_t$ are known, $E_t F_t - E_{t-1}F_{t-1} = \Delta F_t$, so that (6) becomes:

$$\Delta s_t = \Delta \mathbf{f}_t' \boldsymbol{\beta}_t + (1-\lambda)\Delta b_t \tag{7}$$

In this case, the impact of a change in fundamental f_{nt} on the exchange rate is simply given by β_{nt}: $\Delta s_t = \beta_{nt}\Delta f_{nt}$.

When parameters are time varying, however, the expression $E_t F_t - E_{t-1}F_{t-1}$ is much more complex as it depends on expectations of parameters. In order to avoid having to compute expectations of parameter innovations going back to the infinite past, we assume that parameters are known after T periods. In practice we will set T very large. In that case, we can write (6) as:

$$\Delta s_t = \Delta \mathbf{f}_t' \left((1-\lambda)\boldsymbol{\beta}_t + \lambda E_t \boldsymbol{\beta}_t\right) + (1-\lambda)\Delta b_t + \lambda \sum_{i=1}^{T} \Delta \mathbf{f}_{t-i}' (E_t \boldsymbol{\beta}_{t-i} - E_{t-1}\boldsymbol{\beta}_{t-i}) \tag{8}$$

From (8) we see that the impact of fundamentals changes on exchange rate changes depends on the parameter expectation $E_t \boldsymbol{\beta}_t$ and on the change in expectations $E_t \boldsymbol{\beta}_{t-i} - E_{t-1}\boldsymbol{\beta}_{t-i}$ of past parameters.[10] More generally, if changes in fundamentals are not iid, Δs_t also depends on expectations about future values of the parameters. The general setup is discussed in the Appendix.

As can be seen from the first term in (8), the change $\Delta \mathbf{f}_t$ in the vector of current fundamentals is now multiplied by a weighted average of actual and expected parameter values. Since λ tends to be close to 1 (μ is small), most of the weight is on the expected value of parameters rather than the actual level of parameters. The

[10] The reason is that the impact of past changes in fundamentals on F_t depends on past parameter values, while agents continuously change expectations not only about current but also about past parameter values. This contrasts with the case of constant parameters where only current changes in fundamentals matter when fundamentals follow a random walk.

reason is that the exchange rate is forward looking and depends on expectations of future fundamentals. In this particular example, where fundamentals follow a random walk, expected future levels of F are equal to the expected level of F today, which depends on the expectation of the current set of parameters $\boldsymbol{\beta}_t$.

Moreover, we will show that changes in current fundamentals lead to changes in the expectation of both current and past parameters. Thus, changes in fundamentals also affect the last element of equation (8). The derivative of the exchange rate with respect to current fundamentals is then:

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}}{\partial \Delta f_{nt}} \tag{9}$$

2.3 Expectation of Parameters

In order to determine the impact of fundamentals on the exchange rate, we need to determine the expectation of current and past parameters. We do this by first assuming a process for the parameters and then solving a signal extraction problem.

We consider the case where a parameter β_{nt} depends on a finite number T of past innovations:

$$\beta_{nt} = \beta_n + \sum_{i=1}^{T} \theta_{ni} \varepsilon_{n,t-i+1} \tag{10}$$

where $\varepsilon_{nt} \sim N(0, \sigma_\beta^2)$. As discussed above, we assume that parameters at dates $t-T$ and earlier are known at date t. This assumption is for technical convenience only. It implies that only parameter innovations over the past T periods are unknown, which are the innovations that affect the current β_{nt}. By assuming that T is finite, the signal extraction problem becomes tractable as there is only a finite number of unknowns. However, we will set T very large in the numerical analysis in the next section, so that we allow for uncertainty in parameter innovations going far back in time.

In terms of vector notation (10) can be written as

$$\boldsymbol{\beta}_t = \boldsymbol{\beta} + \Theta \boldsymbol{\xi}_t \tag{11}$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, ..., \beta_N)'$ is a N-vector of constants; $\boldsymbol{\xi}_t$ is a NT vector that stacks all the vectors $\boldsymbol{\xi}_{nt} = (\epsilon_{nt}, ..., \epsilon_{n,t-T+1})'$; and Θ is a $N \times NT$ matrix with $\Theta[n, T(n-1)+1 : Tn] = \boldsymbol{\theta}'_n = (\theta_{n1}, \theta_{n2}, ..., \theta_{nT})$ and zeros otherwise.

In order to form expectations about current and past values of $\boldsymbol{\beta}_t$ we need to compute expectations about the vector $\boldsymbol{\xi}_t$ of current and past parameter innovations. Since the problem is linear and all the shocks are normal, we can use standard signal extraction techniques. Leaving some of the details to the Appendix, we sketch how this is done. We start from the knowledge that the unconditional distribution of $\boldsymbol{\xi}_t$ is normal with mean zero and variance $\sigma_\beta^2 \mathbf{I}_{NT}$, where $\mathbf{I}_{NT}$ is an identity matrix of size NT. We combine this with knowledge of $d_t = F_t + b_t$ over the past T periods. Defining $\mathbf{Y}_t = \left(d_t^*, .., d_{t-T+1}^*\right)$, where d_t^* subtracts the known components from d_t, we have

$$\mathbf{Y}_t = \mathbf{H}_t' \boldsymbol{\omega}_t \tag{12}$$

where $\boldsymbol{\omega}_t' = (\boldsymbol{\xi}_t', \varepsilon_t^b, \varepsilon_{t-1}^b, ..., \varepsilon_{t-T+1}^b)$ and $\mathbf{H}_t$ is a matrix that depends on current and lagged changes in observed fundamentals: $\Delta \mathbf{f}_{t-i}$ for $0 < i < T+1$. The precise form of $\mathbf{H}_t$ can be found in the Appendix.

The unconditional distribution of $\boldsymbol{\omega}_t$ is normal with mean zero and variance

$$\tilde{\mathbf{P}} = \begin{pmatrix} \sigma_\beta^2 \mathbf{I}_{NT} & \mathbf{0} \\ \mathbf{0} & \sigma_b^2 \mathbf{I}_T \end{pmatrix} \tag{13}$$

Combining this with (12), standard signal extraction[11] implies that the conditional distribution of $\boldsymbol{\omega}_t$ is normal with mean

$$\begin{aligned} E_t \boldsymbol{\omega}_t &= \mathbf{M}_t \mathbf{Y}_t \\ \mathbf{M}_t &= \tilde{\mathbf{P}} \mathbf{H}_t \left[\mathbf{H}_t' \tilde{\mathbf{P}} \mathbf{H}_t\right]^{-1} \end{aligned} \tag{14}$$

and variance

$$\mathbf{P} = \tilde{\mathbf{P}} - \mathbf{M}\mathbf{H}'\tilde{\mathbf{P}}$$

Therefore

$$E_t \boldsymbol{\omega}_t = \mathbf{C}_t \boldsymbol{\omega}_t \tag{15}$$

where $\mathbf{C}_t = \mathbf{M}_t \mathbf{H}_t'$. Given the definition of $\boldsymbol{\omega}_t$, we can determine $E_t \boldsymbol{\xi}_{t-i}$, $i = 1, 2, ..., T-1$, from (15) and use this to compute $E_t \boldsymbol{\beta}_{t-i}$ from (11).

We then have

$$E_t \boldsymbol{\beta}_{t-i} = \hat{\boldsymbol{\beta}}_{t-i} + \boldsymbol{\Omega}_{ti} \boldsymbol{\omega}_t \tag{16}$$

[11] See for example Townsend(1983, p.556).

Here $\hat{\boldsymbol{\beta}}_{t-i}$ is equal to $\boldsymbol{\beta}$ plus (for $i > 1$) a vector that depends on parameter innovations more than T periods ago that are known time t. The matrix $\boldsymbol{\Omega}_{ti}$ is equal to $\Theta' \widetilde{\mathbf{I}}_i \mathbf{C}_t$, where $\widetilde{\mathbf{I}}_i$ is a matrix of zeros and ones that maps $\boldsymbol{\omega}_t$ into the unknown elements of $\boldsymbol{\xi}_{t-i}$.

There are two important features to notice from (16). First, $E_t \boldsymbol{\beta}_{t-i}$ is determined by a combination of shocks contained in $\boldsymbol{\omega}_t$. Thus, the expectation of a specific coefficient β_{nt-i} depends on its own shocks, but also on current and past shocks to the noise vector $\mathbf{b}_t$ and to all other coefficients. Second, $\boldsymbol{\Omega}_{ti}$ depends on current and past $\Delta \mathbf{f}_t$ so that shocks to fundamentals affect parameter expectations.[12]

As we will see, the expectations of β_{nt} can change significantly over a relatively short period of time even when the true parameters change very slowly. What matters is not the monthly (or even annual) fluctuations in structural parameters but rather their potential to fluctuate over a very long period of time (decades or longer). The unconditional standard deviation of the parameters then becomes large even though changes from period to period are small. A large unconditional standard deviation of parameters, together with the difficulty in learning about their level, leaves a lot of room for large and frequent changes in expectations about these parameters. This allows expectations to become significantly disconnected from the true value of the parameters.

2.4 The Scapegoat Effect

At times, the weight of a fundamental f_{nt} can increase even if its underlying parameter β_{nt} does not change. We refer to this phenomenon as a *scapegoat effect*. To see when this can occur, substitute (16) into (9). The derivative of Δs_t with respect to Δf_{nt} becomes

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial \boldsymbol{\Omega}_{ti}}{\partial \Delta f_{nt}} \boldsymbol{\omega}_t \tag{17}$$

This expression shows that the impact of a change in a fundamental on exchange rate depreciation depends only partly on the underlying coefficient β_{nt} (remember that λ is close to 1). The main elements determining this impact are the last

[12] Current and past $\Delta \mathbf{f}_t$ enter $\mathbf{H}_t$, which affects $\mathbf{M}_t$, which affects $\mathbf{C}_t$, which affects $\boldsymbol{\Omega}_{ti}$.

two terms, i.e., the expectation of β_{nt} and the change in expectations of $\boldsymbol{\beta}_{t-i}$ caused by the change in Δf_{nt}. Both of these factors contribute to instability in the relationship between exchange rates and fundamentals.

Consider for example a rise in b_t. This will increase $F_t + b_t$, which is observed. If at the same time the fundamental f_{nt} has increased (over the last T periods) relative to the other fundamentals, one can make this variable the scapegoat for the observed increase in $F_t + b_t$ by increasing the expectation of the parameter β_{nt}. This would happen even when β_{nt} has not changed at all.

The scapegoat effect can be seen more explicitly by linearizing equation (17). Let the last term in (17) be x_t. The second order term of its linear approximation gives (see Appendix):

$$x_t(2) = \kappa_{nt}\varepsilon_t^b \qquad\qquad \text{where } \kappa_{nt} = a\sum_{j=0}^{T-1} b_j \Delta f_{n,t-j}$$

i.e., κ_{nt} depends positively on current and past Δf_n. Thus, there is a scapegoat effect for fundamental n when there is a positive b shock combined with positive changes in the fundamental. This expression also shows that the scapegoat effect can be volatile since it depends on a combination of shocks.

The third order term in linearizing the last term in (17) gives

$$x_t(2) = \Gamma_t \boldsymbol{\xi}_t$$

where Γ_t depends positively on current and past fundamental shocks. Thus, shocks to other parameters combined with large values of fundamentals can create a scapegoat effect for variable n. The reason is that changes in parameters of other variables lead to a change in the observed $F_t + b_t$ that can be attributed to β_{nt} when the variable n is a convenient scapegoat.

In order to illustrate these points and show the magnitude of the scapegoat effect, we now turn to a calibration of the model that is grounded in monthly data on exchange rates and interest rates.

3 Numerical Analysis

3.1 Calibration

In the previous section, we considered a special case with no risk-premium shocks and where both b_t and $\Delta \mathbf{f}_{nt}$ are iid. For calibration purposes we now turn to a somewhat more general form of the model. We will consider one period to be a month.

First, we assume that b_t and Δf_{nt} follow AR(1) processes:

$$\begin{aligned}\Delta f_{nt} &= \rho_f \Delta f_{n,t-1} + \varepsilon_t^f \\ b_t &= \rho_b b_{t-1} + \varepsilon_t^b\end{aligned}$$

Second, in order to match observed exchange rate volatility we allow for a time-varying risk premium. Let v_t be the present discounted value of the risk premium:

$$v_t = \sum_{k=0}^{\infty} \lambda^k E_t \phi_{t+k}$$

To match the observed volatility and autocorrelation of Δs_t, we assume that v_t follows the process

$$v_{t+1} - v_t = \psi_1 (v_t - v_{t-1}) - \psi_2 v_t + \varepsilon_t^v \tag{18}$$

where $\varepsilon_t^v \sim N(0, \sigma_v^2)$.

We also need to be more precise about the process for the parameters β_{nt}, i.e., the values for θ_i in equation (10). For an interesting analysis, the underlying parameters should have two features. First, they should not be easily predictable. Otherwise there would be little model uncertainty. Second, they should not be too variable from period to period. Otherwise their intrinsic variability would fully explain the time variation in the relationship between exchange rates and fundamentals. It would appear highly unlikely that structural parameters change by large amounts from month to month, or from year to year, and on a continuous basis. In order to get these two features, we set $\theta_1 = \theta_T = 1$ and then choose the other parameters θ_i $(i = 2, .., T-1)$ such that we maximize the ratio of the unconditional standard deviation of β_{nt} relative to the standard deviation of changes in β_{nt}. This process implies that an innovation impacts the parameter β_{nt} slowly over time, building up to a maximum impact after $T/2$ periods.

Table 1 reports the parameters adopted for the benchmark parameterization. The first five parameters are associated with the process for b_t and v_t. These are set to closely match the standard deviation and first-order autocorrelation of the monthly exchange rate change and monthly interest differential in the data. We considered monthly data from 1973(1) to 2007(11) for exchange rates and interest differentials for 13 industrialized countries relative to the United States.[13] These moments can be seen from the first four rows of the first column of Table 2. In the second column of Table 2, we show the moments produced by the model with the benchmark parameters. As a by-product the model also generates a significant negative correlation between the change in the exchange rate and lagged interest differential. The Fama regression coefficient, reported in the fifth row of Table 2, is even slightly more negative than in the data.[14]

The next three parameters relate to the process for β_{nt}. We normalize by setting its mean value at $\beta = 1$. We set $T = 1000$, so that parameter innovations over the last 1000 months, or 83 years, are unknown. We set $\sigma_\beta = 0.000165$. As reported in the last row of Table 2, this implies a monthly standard deviation of the change in β_{nt} of 0.27% of the mean value of parameters, which is small. But there is considerable uncertainty about the level of parameters as their unconditional standard deviation is 1.2, or 120% of their steady state level. We will compare this both to the case where parameters are constant and where the standard deviation of parameters is twice that in the benchmark parameterization.

The next three parameters relate to the process of the fundamentals. We assume that $N = 5$, so there are five fundamentals. Under the benchmark parameterization we assume that fundamentals follow a random walk and the standard deviation of innovations is 0.2%. As can be seen from Table 2, this implies an R^2 of 0.04 of a regression of the monthly exchange rate change on the change in the five fundamentals (computed for a sample of 1300 months). This captures the well-known weak explanatory power of observed fundamentals for exchange rate fluctuations. At an annual level this corresponds to an R^2 of 0.11. These num-

[13]The countries are Australia, Austria, Belgium, Canada, Finland, Germany, Italy, Japan, Netherlands, Norway, Spain, Switzerland, United Kingdom.

[14]We emphasize that this is not intended as an explanation for the forward discount puzzle as it is due to entirely exogenous risk-premium shocks. It does imply though that the model is well grounded in the data as it conforms to the basic statistical properties of exchange rates and interest rates.

bers are not unrealistic, as documented for example in Bacchetta, Beutler and van Wincoop (2008). But we will conduct sensitivity analysis with respect to both the standard deviation and persistence of changes in fundamentals.

Finally, we have set μ relatively small at 0.03, implying a discount rate λ in the present value equation for the exchange rate of 0.97. This is consistent with evidence by Engel and West (2005) that the discount rate is close to 1.

3.2 Results

We simulate the model over 2300 months. All moments reported drop the first 1000 months in order to generate a prior history of shocks. Unless otherwise indicated, the moments reported in the Tables are based on the subsequent 1300 months.

Derivative Exchange Rate with Respect to Fundamentals

Figures 1 and 2 show $\partial \Delta s_t / \partial \Delta f_{nt}$ for each of the five fundamentals. From now on we simply refer to this as the derivative of the exchange rate with respect to fundamentals. Figure 1 does so for a 10-year period (observations 1601-1720 in the simulation), while Figure 2 does so for a 100-year period (observations 1001-2200 in the simulation). Both Figures also show β_{nt}, which would be the derivative of the exchange rate with respect to fundamentals if parameters were known.

It is evident from Figure 1 that the derivative of the exchange rate with respect to fundamentals is far more volatile than the underlying parameters. As reported in Table 2, the average standard deviation of monthly changes in the derivative is 25.66%, or 26% of the mean value of the derivative. By contrast, the standard deviation of monthly changes in the underlying parameters is only 0.27%, i.e., 100 times smaller. While Figure 1 would suggest that the derivative of exchange rates with respect to fundamentals is entirely disconnected from the true underlying parameters, Figure 2 shows that this is not the case when we take a much longer 100-year view. There are large changes in parameters over long cycles of several decades, while the derivative of the exchange rate with respect to the fundamentals broadly catches up with these long term swings. This implies that when there are persistent changes in parameters, agents do eventually learn about them.

But, as illustrated in both Figures 1 and 2, short-term fluctuations around such long-term cycles can be very large and even dominate the trend itself. It is precisely the possibility that parameters can change a lot in the long-run that creates significant uncertainty about their level and gives rise to scapegoat effects that lead to large changes in the derivatives over the short to medium run.

Expectation of Parameters

It is useful to recall equation (9), which is displayed here again for convenience:

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}}{\partial \Delta f_{nt}} \tag{19}$$

Since λ is close to 1, the derivative of the exchange rate with respect to fundamentals is primarily driven by the last two terms. The second term is proportional to the expectation $E_t\beta_{nt}$ of parameter n. Focusing on variable 3, Figure 3 compares the evolution of β_{3t} with $E_t\beta_{3t}$ over the samples of 10 and 100 years used in Figures 1 and 2. The top panels illustrate that $E_t\beta_{nt}$ can be significantly more volatile than the underlying parameter β_{nt}. But a comparison with Figures 1 and 2 also shows that the overall derivative $\partial\Delta s_t/\ \partial\Delta f_{nt}$ has even much larger fluctuations at high frequencies. This is illustrated in the bottom panels of Figure 3, which show β_{nt}, $E_t\beta_{nt}$ as well as $\partial\Delta s_t/\partial\Delta f_{nt}$.

It follows that the high frequency volatility in $\partial\Delta s_t/\partial\Delta f_{nt}$ is caused by the last term in (19). It is driven not so much by the expectation of parameters, but rather by the derivative of the expectation of parameters with respect to fundamentals. However, the weight of a given variable is best measured by $E_t\beta_{nt}$, since this is how agents would measure the importance of a variable. In this perspective, Figure 3 is consistent with anecdotal evidence that occasionally agents significantly change the weight they attach to certain macro variables in driving the exchange rate. The high volatility of the derivative of the exchange rate with respect to fundamentals at the monthly frequency should therefore not be misinterpreted as implying that every month agents completely change their view on the importance of parameters.

3.3 Sensitivity Analysis

We now examine the extent to which the results in the benchmark case are sensitive to changes in parameter values. We consider three types of parameters: the degree of parameter instability; the variability and persistence of fundamentals; and the horizon T after which parameters are known.

3.3.1 Sensitivity Moments to Parameter Instability

The instability of underlying parameters affects substantially the link between exchange rates and fundamentals. However, some basic moments involving exchange rates and interest rates are remarkably insensitive to the degree of parameter instability. This is illustrated in Table 2, which reports moments under two scenarios in addition to the benchmark. The third column shows the results with constant parameters, while the fourth column shows the case where the standard deviation of parameter innovations is twice that under the benchmark ($\sigma_\beta = 0.00033$). In the latter case the standard deviation of the derivative of the exchange rate with respect to fundamentals is 100%. For monthly changes in this derivative the standard deviation is 40.9%.

On the ohter hand exchange rate volatility rises only slightly. The standard deviation of exchange rate changes rises from 2.94% to 3.20%, from the case of constant parameters to the extreme case where parameter volatility is twice that under the benchmark. The standard deviation of the interest rate differential, as well as the autocorrelation of monthly exchange rate change and the interest differential, are all virtually unaffected by parameter volatility. The same is the case for the monthly Fama regression coefficient of Δs_{t+1} on $i_t - i_t^*$.

The reason for these results is that most exchange rate volatility is unrelated to changes in fundamentals. For the benchmark parameterization the R^2 is 0.04 for monthly data and 0.11 for annual data (based on a long sample of 1300 months). We examine below what happens when we change the volatility of fundamentals.

3.3.2 Sensitivity to Process Fundamentals

We first examine the impact of the fundamentals process on the link between exchange rates and these fundamentals. We consider lower and higher standard deviations of the innovations of the fundamentals and positive persistence of changes in the fundamentals. We find that the volatility of $\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}$ decreases with σ_f. When we set the standard deviation of innovations five times as large as under the benchmark ($\sigma_f = 0.01$), the volatility of monthly changes in the derivative declines from 25.7% to 12.4%. Similarly, when the standard deviation of fundamental innovations is half of that under the benchmark ($\sigma_f = 0.001$), the derivative increases slightly to 26.4%.$

The explanation for these results is that when σ_f is larger, the signal $F_t + b_t$ becomes more informative as structural parameter innovations are multiplied by fundamentals that fluctuate more. As a result there is less confusion and the expectation of β_{nt} is closer to its actual value. The derivative $\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}$ is then more similar to β_{nt}, reducing volatility in the monthly and annual changes in the derivative.

Introducing persistence in Δf_{nt} has little effect on the overall volatility of $\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}$. If we set $\rho_f = 0.2$ (0.4) the volatility of monthly changes in the derivative increases to 28.9% (32.6%).

Changing the volatility of fundamentals also affects their explanatory power and the role of parameter instability. In particular the impact of parameter instability on exchange rate volatility is larger for a larger σ_f. For example, if we set the standard deviation of fundamental innovations at $\sigma_f = 0.01$, five times as large as under the benchmark, the standard deviation of the exchange rate increases from 3.6% for $\sigma_\beta = 0$ to 4.9% for the benchmark case. But in that case the R^2 is excessive: 0.59 for monthly data.[15]

3.3.3 Sensitivity to the horizon T

A smaller T implies that parameters β_{nt} become easier to predict. This reduces the scapegoat effect. This is illustrated in Figure 4, by comparing the case of $T = 1000$ to the case of $T = 300$. Figure 4 reports the correlation between the derivative $\partial \Delta s_t / \partial \Delta f_{nt}$ and β_{nt}.[16] This correlation would be 1 if parameters were known. A higher T further disconnects the derivative of the exchange rate with respect to fundamentals from the underlying structural parameters as reflected in a lower correlation. For example, over a 10-year sample the correlation between $\partial \Delta s_t / \partial \Delta f_{nt}$ and β_{nt} is 0.42 for $T = 300$ and 0.25 for $T = 1000$.

[15] Another way to increase the volatility of fundamentals is to make changes in f_{nt} persistent. This also increases the impact of parameter volatility, although the effect is rather modest. For example, for $\rho_f = 0.4$, the standard deviation of Δs_t rises from 3.0% to 3.2% when σ_β is raised from 0 to 0.000165.

[16] This correlation is computed as a function of the sample length based on the average of this correlation overall samples of that length. The first sample starts at observation $T + 1$.

4 Time-Varying Coefficients and Forecasting Performance

The previous section has shown that a significantly unstable relationship between exchange rates and fundamentals results from gradual changes in structural parameters coupled with the unobservability of the structural parameters. In this section we investigate the implications of this unstable relationship for out-of-sample forecasting and the corresponding Meese-Rogoff puzzle. If parameters were known and constant (and non zero), then by construction the model would outperform a random walk in predicting exchange rates. Since the empirical evidence shows that this is not the case, one would therefore have to assume either that parameters are not constant or that they are not known, or both.

4.1 Out-of-Sample Forecasting in the Data

4.1.1 The Meese-Rogoff experiment

In their seminal paper, Meese and Rogoff (1983a) conduct an out-of-sample forecasting exercise. It is not true forecasting as they forecast the future exchange rate using information about future macro fundamentals. The statistic they construct may be better called a measure of out-of-sample fit of the model. They first regress the exchange rate on a set of fundamental variables over a sample of L months, using the first L observations of their data. They use the estimate from this regression to compute a forecast at $L+1$, using the observed fundamentals at $L+1$.[17] Using rolling regressions, they repeat this P times, each time starting the sample one month later. They then compute the ratio of the resulting Mean Square Error (MSE) with the one obtained assuming that the exchange rate follows a random walk.[18] They assume $L = 45$ and $P = 55$, but subsequent studies have considered larger numbers for L and P as data samples became longer. For

[17] Meese and Rogoff (1983a) estimated the exchange rate equation in levels, using several lags of the exchange rate, but the subsequent literature has regressed the *change* in the exchange rate on fundamentals, sometimes including a cointegration term.

[18] More precisely, Meese and Rogoff (1983a) look at the RMSE which is the square root of MSE. They also look at the mean error and at the mean absolute error. They also consider the RMSE for forecasts further than 1 months ahead, in particular 6 and 12-month ahead forecasts.

example, in Molodstova and Papell (2008) $L = 108$ and $P = 292$.

The key result of Meese and Rogoff (1983a) is that the MSE ratio is generally above 1, so that the average forecast error is larger when using the fundamentals than adopting a random walk assumption. This result has largely held up to extensive scrutiny in the more than two decades of research that followed. For example, Cheung, Chinn and Pascual (2005) consider a longer sample of data, more currencies, and more fundamental variables. In only 2 out of 216 combinations that they consider does the model significantly outperform the random walk at a 10% significance level. Rogoff and Stavrakeva (2008) discuss recent models that have been somewhat more successful but continue to find that the MSE ratio is generally above 1 or just slightly below 1.

Figure 5 confirms this evidence. It shows the MSE ratio as we increase L from 40 to 220 and with $P = 200$. The fundamentals considered are money, industrial production and CPI inflation. Figure 5 shows the average over 5 exchange rates to the dollar (Canadian dollar, Japanese Yen, Swiss franc, British pound, and euro/DM). We see that the ratio is much higher than 1 when L is small, and that it decreases towards 1. However, it never goes below 1, meaning that the linear model does not beat the random walk.

4.1.2 Small Sample Bias

When parameters are constant, but their level is not known and has to be estimated, one can face small sample problems. This has been the focus of a lot of recent literature. Estimating an exchange rate equation over a short data sample can lead to spurious noise in the estimation of β even if it is constant. This can lead to a noisy forecast, raising the mean squared forecast error of the model compared to the random walk, which does not suffer from any estimation bias. This bias is also illustrated in Figure 5, where the MSE ratio is high for small L. This can indeed be a serious problem and statistics have been developed to correct for such small sample bias (e.g. Clark and West (2006)). However, even for relatively large values of L, involving more than two decades of data, it has been hard to outperform the random walk (e.g. Rogoff and Stavrakeva (2008)). This suggest that it is difficult to explain the Meese-Rogoff findings while maintaining the assumption of constant parameters.

4.2 Effect of Time-Varying Parameters: Simulation Results

In order to investigate the relationship between time-varying parameters and the Meese-Rogoff results we compute the MSE ratio in the benchmark model for different values of L and a large value of P equal to 1000. The first estimation sample uses observations $T+1$ to $T+L$ from the simulation to predict the exchange rate at $T+L+1$. We then use rolling regressions, as in Meese and Rogoff (1983a), with the last estimation sample using observations $T+P$ to $T+P+L-1$ from the simulation to predict the exchange rate at $T+P+L$.

Figure 6 reports the results for L ranging from 40 months to 300 months. Results are reported both for the benchmark parameterization with time-varying parameters and the case of constant parameters. It can be seen that the MSE ratio declines as the sample length L increases, as in the data in Figure 5. This illustrates the small sample bias.

A puzzling result emerges when we compare the MSE ratio of the time-varying coefficient model with the one of the constant coefficient model. We see that the forecasting performance is better with time-varying coefficients. There is, however, a straighforward explanation to this result. It turns out that in the specific set of simulation the average parameters β_{nt} are higher under time-varying coefficients. Under constant coefficients, the average of β_{nt} is 1 by assumption. But, with time varying coefficient the average of β_{nt} is higher than 1, as can be seen in Figure 2. As shown in Table 2, this implies that the R^2 is higher under time-varying coefficients (0.04 vs. 0.02).

To match the expectations of the underlying parameters, we consider the "adjusted" constant parameter case, where we set β_{nt} equal to the average of each β_n in the benchmark, time-varying, scenario.[19] With this adjustment, Figure 6 shows that the constant coefficient model has a slightly better forecasting performance, even though the difference is hard to distinguish.

4.3 Effect of Time-Varying Parameters: Explanation

[19] We use $\beta_{1t} = 1.795$, $\beta_{2t} = 1.531$, $\beta_{3t} = 1.621$, $\beta_{4t} = 1.659$, $\beta_{5t} = 2.005$

The limited explanatory power of the fundamentals is key in understanding the forecasting performance, both with time-varying and with constant coefficients. It is obvious that a larger explanatory power will give better predictions. This is illustrated in Figure 7, which considers two alternative values of σ_f : half that under the benchmark ($\sigma_f = 0.001$) and twice that under the benchmark ($\sigma_f = 0.004$). In these cases, the R^2 are 0.7% and 16.1%. The difference in forecasting performance is substantial.

On the other hand, the difference between time-varying coefficients and constant coefficients still remains very small (and is not shown in the graph). Thus, for time-varying coefficients to matter, the explanatory power of fundamentals should be much higher. But this would be inconsistent with the empirical fit of exchange rate equations. We examine these issues in more details in in Bacchetta, Beutler and van Wincoop (2008).[20]

5 Conclusion

Anecdotal, survey and econometric evidence all suggest that the relationship between the exchange rate and macro fundamentals is highly unstable. We have developed a model where this instability naturally results from a combination of incomplete information and very gradual changes in structural parameters of the economy. Nonetheless we find that even very large time-variation in the relationship between exchange rates and fundamentals has little impact on the statistical properties of exchange rates, the in-sample explanatory power of macro fundamentals and the ability to forecast out of sample.

[20] To facilitate our understanding, instead of the current model we consider a setup with exogenous time variation in the relationship between exchange rate and fundamentals.

Appendix

A Solving the General Model

In this Appendix we describe the model's solution in the more general case, where the processes for Δf_{nt}, b_t, and v_t are as specified in Section 3. A Technical Appendix provides further details towards the implementation of the simulations with Gauss. We start from the present value equation (5) of the exchange rate. We need to express it in way we can easily substitute the expectation terms. This equation can be rewritten as:

$$s_t = \frac{\mu}{1+\mu} F_t + \frac{\mu}{1+\mu} b_t - \frac{1}{1+\mu} \nu_t + \frac{\mu}{1+\mu} \sum_{k=1}^{\infty} \left(\frac{1}{1+\mu} \right)^k E_t \left(F_{t+k} + b_{t+k} \right) \quad (20)$$

First, consider the term involving the present discounted value of F. Use that

$$F_{t+k} = F_t + \sum_{n=1}^{N} \sum_{i=1}^{k} \beta_{n,t+i} \left(f_{n,t+i} - f_{n,t+i-1} \right) \quad (21)$$

Therefore

$$\sum_{k=1}^{\infty} \left(\frac{1}{1+\mu} \right)^k F_{t+k} =$$
$$\frac{1}{\mu} F_t + \frac{1+\mu}{\mu} \sum_{n=1}^{N} \sum_{i=1}^{\infty} \left(\frac{1}{1+\mu} \right)^i \beta_{n,t+i} \left(f_{n,t+i} - f_{n,t+i-1} \right) \quad (22)$$

The present value of b can be written as $\tilde{b} E_t b_t$, where

$$\tilde{b} = \frac{\mu}{1+\mu} \frac{\rho_b}{1+\mu-\rho_b} \quad (23)$$

Using this, (20) becomes

$$s_t = \frac{\mu}{1+\mu} F_t + \frac{1}{1+\mu} E_t F_t + \frac{\mu}{1+\mu} b_t - \frac{1}{1+\mu} \nu_t$$
$$\sum_{n=1}^{N} \sum_{i=1}^{\infty} \left(\frac{\rho}{1+\mu} \right)^i E_t \beta_{n,t+i} \left(f_{n,t} - f_{n,t-1} \right) + \tilde{b} E_t b_t \quad (24)$$

Therefore

$$\begin{aligned}
& s_t - s_{t-1} = \frac{\mu}{1+\mu}\sum_{n=1}^{N}\beta_{nt}\left(f_{nt} - f_{n,t-1}\right) + \frac{1}{1+\mu}\left[E_t F_t - E_{t-1}F_{t-1}\right] + \\
& \sum_{n=1}^{N} E_t\tilde{\beta}_{nt}\left(f_{n,t} - f_{n,t-1}\right) - \sum_{n=1}^{N} E_{t-1}\tilde{\beta}_{n,t-1}\left(f_{n,t-1} - f_{n,t-2}\right) + \\
& \frac{\mu}{1+\mu}(b_t - b_{t-1}) + \tilde{b}\left(E_t b_t - E_{t-1}b_{t-1}\right) - \frac{1}{1+\mu}\left(\nu_t - \nu_{t-1}\right)
\end{aligned} \tag{25}$$

where

$$\tilde{\beta}_{nt} = \sum_{i=1}^{\infty}\left(\frac{\rho}{1+\mu}\right)^{i}\beta_{n,t+i} \tag{26}$$

Finally, we can write

$$\begin{aligned}
& E_t F_t - E_{t-1}F_{t-1} = E_t(F_t - F_{t-1}) + \left[E_t F_{t-1} - E_{t-1}F_{t-1}\right] = \\
& \sum_{n=1}^{N} E_t\beta_{nt}\left(f_{nt} - f_{n,t-1}\right) + \sum_{n=1}^{N}\sum_{i=1}^{T}\left(f_{n,t-i} - f_{n,t-i-1}\right)\left[E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}\right]
\end{aligned} \tag{27}$$

Using (27) and collecting terms multiplying $f_{nt} - f_{n,t-1}$, (25) becomes

$$\begin{aligned}
& s_t - s_{t-1} = \sum_{n=1}^{N}\left(\frac{\mu}{1+\mu}\beta_{nt} + \frac{1}{1+\mu}E_t\beta_{nt} + E_t\tilde{\beta}_{nt}\right)\left(f_{nt} - f_{n,t-1}\right) + \\
& -\sum_{n=1}^{N} E_{t-1}\tilde{\beta}_{n,t-1}\left(f_{n,t-1} - f_{n,t-2}\right) + \\
& \frac{1}{1+\mu}\sum_{n=1}^{N}\sum_{i=1}^{T}\left(f_{n,t-i} - f_{n,t-i-1}\right)\left[E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}\right] + \\
& \frac{\mu}{1+\mu}(b_t - b_{t-1}) + \tilde{b}\left(E_t b_t - E_{t-1}b_{t-1}\right) - \frac{1}{1+\mu}\left(\nu_t - \nu_{t-1}\right)
\end{aligned} \tag{28}$$

Given the processes of $\boldsymbol{\beta}_t$ and b_t, the terms including expectations can be written as:

$$\begin{aligned}
& E_t\beta_{nt} - \beta = \hat{\omega}E_t\boldsymbol{\xi}_{nt} \\
& E_t\tilde{\beta}_{nt} - \frac{\rho}{1+\mu-\rho}\beta = \hat{\theta}E_t\boldsymbol{\xi}_{nt} \\
& E_t b_t = \hat{b}E_t\mathbf{b}_t + \rho_b^T b_{t-T} \\
& \sum_{i=1}^{T}\left(f_{n,t-i} - f_{n,t-i-1}\right)\left[E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}\right] = \\
& \qquad \sum_{i=1}^{T}\left(f_{n,t-i} - f_{n,t-i-1}\right)\theta_{T-i+1}\epsilon_{n,t-T} + \hat{h}_{nt}E_t\boldsymbol{\xi}_{nt} - \hat{f}_{n,t-1}E_{t-1}\boldsymbol{\xi}_{n,t-1}
\end{aligned}$$

where $\bar{\theta}$, $\hat{\theta}$, $\hat{b}$, $\hat{h}$ and $\bar{h}$ are 1 by T vectors with

$$\hat{\omega}(j) = \theta_j \tag{29}$$

$$\hat{\theta}(j) = \sum_{i=1}^{T-j} \theta_{j+i} \left(\frac{\rho}{1+\mu} \right)^i \tag{30}$$

$$\hat{b}(j) = \rho_b^{j-1} \tag{31}$$

$$\hat{h}_{nt}(j) = \sum_{i=1}^{j-1} (f_{n,t-i} - f_{n,t-i-1}) \theta_{j-i} \tag{32}$$

$$\hat{f}_{n,t-1}(j) = \sum_{i=1}^{j} (f_{n,t-i} - f_{n,t-i-1}) \theta_{j-i+1} \tag{33}$$

and $\hat{h}_{nt}(1) = 0$.

Substituting these results into (28) gives

$$\begin{aligned} s_t - s_{t-1} = & \sum_{n=1}^{N} \left(\frac{1+\mu}{1+\mu-\rho}\beta + \frac{\mu}{1+\mu}(\beta_{nt} - \beta) + \left[\frac{1}{1+\mu}\hat{\omega} + \hat{\theta} \right] E_t \boldsymbol{\xi}_{nt} \right) (f_{nt} - f_{n,t-1}) + \\ & - \sum_{n=1}^{N} \left(\frac{\rho}{1+\mu-\rho}\beta + \hat{\theta} E_{t-1} \boldsymbol{\xi}_{n,t-1} \right) (f_{n,t-1} - f_{n,t-2}) + \\ & \frac{1}{1+\mu} \sum_{n=1}^{N} \sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) \theta_{T-i+1} \epsilon_{n,t-T} + \frac{1}{1+\mu} \sum_{n=1}^{N} \left(\hat{h}_{nt} E_t \boldsymbol{\xi}_{nt} - \hat{f}_{n,t-1} E_{t-1} \boldsymbol{\xi}_{n,t-1} \right) + \\ & \frac{\mu}{1+\mu}(b_t - b_{t-1}) - \frac{1}{1+\mu}(\nu_t - \nu_{t-1}) + \tilde{b} \left(\hat{b}(E_t \mathbf{b}_t - E_{t-1} \mathbf{b}_{t-1}) + \rho_b^T (b_{t-T} - b_{t-T-1}) \right) \end{aligned} \tag{34}$$

The expectation terms can be derived from the signal extraction problem, where $E_t \boldsymbol{\omega}_t = \mathbf{C}_t \boldsymbol{\omega}_t$. This gives:

$$\begin{aligned} s_t - s_{t-1} = & \sum_{n=1}^{N} \left(\frac{1+\mu}{1+\mu-\rho}\beta + \frac{\mu}{1+\mu}(\beta_{nt} - \beta) + \left[\frac{1}{1+\mu}\bar{\omega}^n + \bar{\theta}^n \right] \mathbf{C}_t \boldsymbol{\omega}_t \right) (f_{nt} - f_{n,t-1}) + \\ & - \sum_{n=1}^{N} \left(\frac{\rho}{1+\mu-\rho}\beta + \bar{\theta}^n \mathbf{C}_{t-1} \boldsymbol{\omega}_{t-1} \right) (f_{n,t-1} - f_{n,t-2}) + \\ & \frac{1}{1+\mu} \sum_{n=1}^{N} \sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) \theta_{T-i+1} \epsilon_{n,t-T} + \frac{1}{1+\mu} \sum_{n=1}^{N} \left(\bar{h}_t^n \mathbf{C}_t \boldsymbol{\omega}_t - \bar{f}_{t-1}^n \mathbf{C}_{t-1} \boldsymbol{\omega}_{t-1} \right) + \\ & \frac{\mu}{1+\mu}(b_t - b_{t-1}) - \frac{1}{1+\mu}(\nu_t - \nu_{t-1}) + \tilde{b} \left(\bar{b}(\mathbf{C}_t \boldsymbol{\omega}_t - \mathbf{C}_{t-1} \boldsymbol{\omega}_{t-1}) + \rho_b^T (b_{t-T} - b_{t-T-1}) \right) \end{aligned} \tag{35}$$

Here $\bar{\theta}^n$ is a 1 by $(N+1)T$ vector with $\hat{\theta}$ in elements $T(n-1)+1$ through Tn and zeros otherwise. The vectors $\bar{\omega}^n$, $\bar{h}_t^n$ and $\bar{f}_{t-1}^n$ are defined analogously. $\bar{b}$ is a 1 by $(N+1)T$ vector with $\hat{b}$ in elements $NT+1$ through $NT+T$ and zeros otherwise.

Collecting terms in $\mathbf{C}_t\boldsymbol{\omega}_t$ and $\mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1}$, we can rewrite this as

$$
\begin{aligned}
s_t - s_{t-1} = & \sum_{n=1}^{N}\left(\frac{1+\mu}{1+\mu-\rho}\beta + \frac{\mu}{1+\mu}(\beta_{nt}-\beta)\right)(f_{nt}-f_{n,t-1}) + \\
& \left(\sum_{n=1}^{N}\left[\frac{1}{1+\mu}\bar{\omega}^n + \bar{\theta}^n\right](f_{nt}-f_{n,t-1}) + \frac{1}{1+\mu}\sum_{n=1}^{N}\bar{h}_t^n + \tilde{b}\bar{b}\right)\mathbf{C}_t\boldsymbol{\omega}_t - \\
& \left(\sum_{n=1}^{N}\bar{\theta}^n(f_{n,t-1}-f_{n,t-2}) + \frac{1}{1+\mu}\sum_{n=1}^{N}\bar{f}_{t-1}^n + \tilde{b}\bar{b}\right)\mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1} + \\
& -\sum_{n=1}^{N}\frac{\rho}{1+\mu-\rho}\beta(f_{n,t-1}-f_{n,t-2}) + \frac{1}{1+\mu}\sum_{n=1}^{N}\sum_{i=1}^{T}(f_{n,t-i}-f_{n,t-i-1})\theta_{T-i+1}\epsilon_{n,t-T} + \\
& \frac{\mu}{1+\mu}(b_t-b_{t-1}) + \tilde{b}\rho_b^T(b_{t-T}-b_{t-T-1}) - \frac{1}{1+\mu}(\nu_t-\nu_{t-1})
\end{aligned} \tag{36}
$$

The derivative with respect to the current fundamental is:

$$
\begin{aligned}
\partial\Delta s_t/\partial\Delta f_{nt} = & \left(\frac{1+\mu}{1+\mu-\rho}\beta + \frac{\mu}{1+\mu}(\beta_{nt}-\beta)\right) + \\
& \frac{\partial\left(\sum_{n=1}^{N}\left[\frac{1}{1+\mu}\bar{\omega}^n + \bar{\theta}^n\right](f_{nt}-f_{n,t-1}) + \frac{1}{1+\mu}\sum_{n=1}^{N}\bar{h}_t^n + \tilde{b}\bar{b}\right)\mathbf{C}_t\boldsymbol{\omega}_t}{\partial\Delta f_{nt}}
\end{aligned} \tag{37}
$$

B Signal Extraction

The signal extraction problem is described in Section 2.3. The matrix $\mathbf{H}_t$ is defined as:

$$\mathbf{H}_t' = [\mathbf{A}_{1t}, ..., \mathbf{A}_{Nt}, \mathbf{B}] \tag{38}$$

with

$$
\mathbf{A}_{nt} = \begin{bmatrix} \hat{f}_{nt}(1) & \hat{f}_{nt}(2) & ... & \hat{f}_{nt}(T) \\ 0 & \hat{f}_{n,t-1}(1) & ... & \hat{f}_{n,t-1}(T-1) \\ & ... & ... & ... \\ 0 & 0 & ... & \hat{f}_{n,t-T+1}(1) \end{bmatrix}
$$

and

$$
\mathbf{B} = \begin{bmatrix} 1 & \rho_b & ... & \rho_b^{T-1} \\ 0 & 1 & ... & \rho_b^{T-2} \\ & ... & ... & ... \\ 0 & 0 & ... & 1 \end{bmatrix}
$$

C Linearization

We linearize the last element of (17). This basically requires the linearization of $\mathbf{C}_t = \mathbf{M}_t\mathbf{H}_t'$.

C.1 First order linearization

We have:

$$\mathbf{C}_t(1) = \mathbf{M}(\mathbf{0})\mathbf{H}_t'(1) + \mathbf{M}_t(1)\mathbf{H}'(0) \tag{39}$$

with:

$$\mathbf{H}'(0) = [\mathbf{0}, ..., \mathbf{0}, \mathbf{B}] \tag{40}$$

$$\mathbf{H}_t'(1) = [\mathbf{A}_{1t}, ..., \mathbf{A}_{Nt}, \mathbf{0}] \tag{41}$$

Using (14) we find that:

$$\mathbf{M}(\mathbf{0}) = \begin{pmatrix} \mathbf{0} \\ \mathbf{B}^{-1} \end{pmatrix} \tag{42}$$

We also have:

$$\mathbf{M}_t(1) = \tilde{\mathbf{P}}\mathbf{H}_t(1)\left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]^{-1}(0) + \tilde{\mathbf{P}}\mathbf{H}_t(0)\left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]^{-1}(1) \tag{43}$$

We first show that the second element of the RHS of (43) is equal to zero. Define $\mathbf{Q} = \left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]$. Since $\mathbf{Q}\mathbf{Q}^{-1}= \mathbf{I}$, we can write (from first order linearization):

$$\mathbf{Q}(0)\mathbf{Q}^{-1}(1) + \mathbf{Q}(1)\mathbf{Q}^{-1}(0)= \mathbf{0} \tag{44}$$

We note that $\mathbf{Q}(1) = \mathbf{H}_t'(1)\mathbf{H}_t(0) + \varphi\mathbf{H}_t'(0)\mathbf{H}_t(1) = 0$, where $\varphi = \sigma^2/\sigma_b^2$. Consequently, from (44) and the fact that $\mathbf{Q}(0)$ is full rank, $\mathbf{Q}^{-1}(1) = 0$. This implies that $\tilde{\mathbf{P}}\mathbf{H}_t(0)\left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]^{-1}(1) = 0$. It is then easy to see that:

$$\mathbf{M}_t(1) = \varphi\mathbf{H}_t(1)\left[\mathbf{B}'\mathbf{B}\right]^{-1} \tag{45}$$

Finally, using the above equations:

$$\mathbf{C}_t(1) = \begin{pmatrix} \mathbf{0} \\ \mathbf{B}^{-1}\mathbf{H}_t'(1) \end{pmatrix} + \varphi\begin{pmatrix} \mathbf{0} & \mathbf{H}_t(1)\mathbf{B}'^{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{NT} & \varphi\mathbf{D}_t \\ \mathbf{D}_t' & \mathbf{0}_T \end{pmatrix} \tag{46}$$

where

$$\mathbf{D}_t = \begin{pmatrix} \hat{f}_{1t}(1) & 0 & ..... & 0 \\ \hat{f}_{1t}(2) - \rho_b \hat{f}_{1t-1}(1) & \hat{f}_{nt}(1) & & 0 \\ ... & \hat{f}_{1t-1}(2) - \rho_b \hat{f}_{1t-2}(1) & & 0 \\ \hat{f}_{1t}(T) - \rho_b \hat{f}_{1t-1}(T-1) & & & 0 \\ ... & & & ... \\ \hat{f}_{Nt}(T) - \rho_b \hat{f}_{Nt-1}(T-1) & \hat{f}_{Nt-1}(T-1) - \rho_b \hat{f}_{Nt-2}(T-2) & ..... & \hat{f}_{Nt-T+1}(1) \end{pmatrix}$$

Using the above and the expectation of $\boldsymbol{\beta}_t$ from (16), we can write the derivative:

$$\frac{\partial E_t \beta_{nt}(1)}{\partial \Delta f_{nt}} = \left(\varphi \sum_{i=1}^{T} \theta_i^2 \right) \varepsilon_t^b \tag{47}$$

Similarly the derivative of the past parameter is:

$$\frac{\partial E_t \beta_{nt-1}(1)}{\partial \Delta f_{nt}} = \varphi \left(\sum_{i=1}^{T-1} \theta_i \theta_{i+1} \right) \varepsilon_t^b \tag{48}$$

On the other hand, $E_t \beta_{nt}$ is not affected by other fundamentals. Hence, we can compute the last term of equation (17) as:

$$\sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}(1)}{\partial \Delta f_{nt}} = \varphi \left(\sum_{j=0}^{T-1} \Delta f_{nt-j} \left(\sum_{i=1}^{T-j} \theta_i \theta_{i+j} \right) \right) \varepsilon_t^b \equiv \kappa_{nt} \varepsilon_t^b$$

C.2 Second order linearization

The second order term of $\mathbf{C}_t$ is:

$$\mathbf{C}_t(2) = \mathbf{M}(\mathbf{0})\mathbf{H}'_t(2) + \mathbf{M}_t(1)\mathbf{H}'(1) + \mathbf{M}_t(2)\mathbf{H}'(0) \tag{49}$$

It is to see that $\mathbf{M}(\mathbf{0})\mathbf{H}'_t(2)$, so that the first term is equal to zero. The second term can be derived from the results of the previous subsection. To derive the third term, notice that:

$$\mathbf{M}_t(2) = \tilde{\mathbf{P}}\mathbf{H}_t(0) \left[\mathbf{H}'_t \tilde{\mathbf{P}} \mathbf{H}_t \right]^{-1}(2) + \tilde{\mathbf{P}}\mathbf{H}_t(2) \left[\mathbf{H}'_t \tilde{\mathbf{P}} \mathbf{H}_t \right]^{-1}(0) + \tilde{\mathbf{P}}\mathbf{H}_t(1) \left[\mathbf{H}'_t \tilde{\mathbf{P}} \mathbf{H}_t \right]^{-1}(1) \tag{50}$$

The last two terms are equal to zero given the results in the above subsection. Again using $\mathbf{Q}\mathbf{Q}^{-1} = \mathbf{I}$, and taking a second order linearization, we find:

$$\mathbf{Q}^{-1}(2) = -\mathbf{Q}^{-1}(0)\mathbf{Q}(2)\mathbf{Q}^{-1}(0) \tag{51}$$

Thus:

$$\mathbf{M}_t(2) = -\tilde{\mathbf{P}}\mathbf{H}_t(0)\left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]^{-1}(0)\mathbf{H}_t'(1)\tilde{\mathbf{P}}\mathbf{H}_t(1)\left[\mathbf{H}_t'\tilde{\mathbf{P}}\mathbf{H}_t\right]^{-1}(0) \tag{52}$$

This implies:

$$\mathbf{C}_t(2) = -\varphi\begin{pmatrix}\mathbf{0}\\ \mathbf{B}^{-1}\end{pmatrix}\mathbf{A}_t\begin{pmatrix}\mathbf{0} & \mathbf{B}'^{-1}\end{pmatrix} + \varphi\begin{pmatrix}\mathbf{A}_{1t}'\\ ...\\ \mathbf{A}_{Nt}'\\ \mathbf{0}\end{pmatrix}\left[\mathbf{B}'\mathbf{B}\right]^{-1}\left[\mathbf{A}_{1t}, ..., \mathbf{A}_{Nt}, \mathbf{0}\right] \tag{53}$$

where

$$\mathbf{A}_t = \left[\mathbf{A}_{1t}, ..., \mathbf{A}_{Nt}, \mathbf{0}\right]\begin{pmatrix}\mathbf{A}_{1t}'\\ ...\\ \mathbf{A}_{Nt}'\\ \mathbf{0}\end{pmatrix}$$

Using this expression, we find that the second order term of the expectation of β_{nt} is given by:

$$E_t\beta_{nt}(2) = \beta + \sum_{i=1}^{T}\theta_i E_t\varepsilon_{n,t-i+1} = \beta + \varsigma_{nt}\sum_{i=1}^{T}\theta_i\hat{f}_{nt}(i) \tag{54}$$

where $\varsigma_{nt} = \sum_{k=1}^{N}\sum_{j=1}^{T}\sum_{i=1}^{j} b_{ni}\hat{f}_{k,t-i+1}(j-i+1)\varepsilon_{kt+1-j}$ and b_{ji} is the ij element of matrix $\left[\mathbf{B}'\mathbf{B}\right]^{-1}$. The derivative with respect to Δf_{nt} gives:

$$\frac{\partial E_t\beta_{nt}(2)}{\partial\Delta f_{nt}} = \varsigma_{nt}\sum_{i=1}^{T}\theta_i^2 + b_{nn}\left(\sum_{i=1}^{T}\theta_i\hat{f}_{nt}(i)\right)\sum_{k=1}^{N}\sum_{j=1}^{T}\theta_j\Delta f_{k,t-j+1}\varepsilon_{k,t-j+1} \tag{55}$$

We can see that the derivative depends on all current and past parameter shocks (up to $t-T+1$) for all variables, i.e. on the whole vector $\boldsymbol{\xi}_t$. We proceed similarly for β_{nt-i}, $1 < i < T-1$ and then multiply by Δf_{nt-i}. In the end, we get the expression:

$$\sum_{i=0}^{T}\Delta\mathbf{f}_{t-i}'\frac{\partial E_t\boldsymbol{\beta}_{t-i}(2)}{\partial\Delta f_{nt}} = \boldsymbol{\Gamma}_t\boldsymbol{\xi}_t \tag{56}$$

where $\boldsymbol{\Gamma}_t$ is a complex matrix depending on Δf_{nt-i}, θ_i, and b_{ji}.

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Table 1 Benchmark Parameter Assumptions*

σ_b	2.6
ρ_b	0.95
σ_υ	2.7
ϕ_1	0.1
ϕ_2	0.1
σ_β	0.0165
β	1
T	1000
σ_f	0.2
ρ_f	0
N	5
μ	0.03

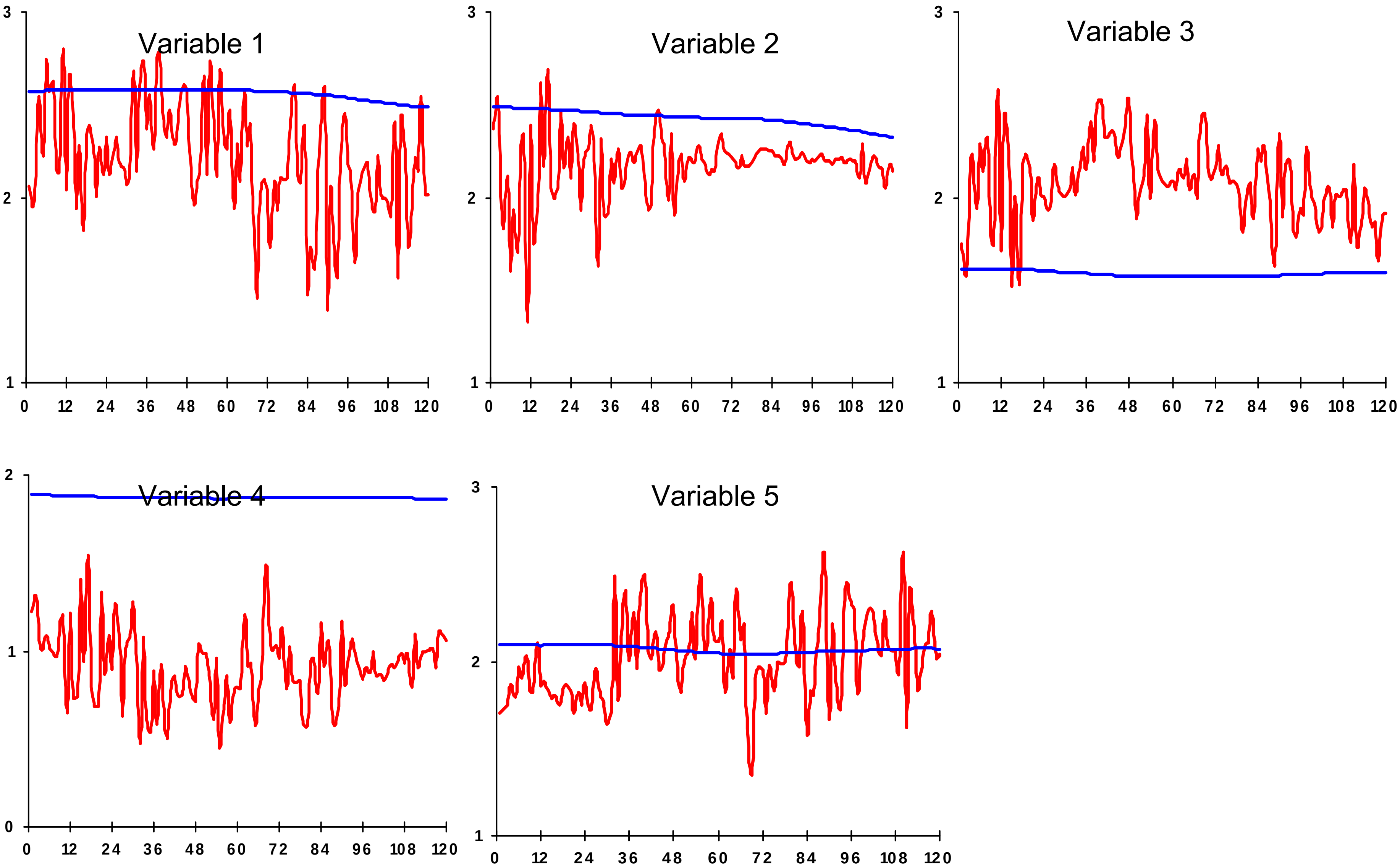
*** Standard deviations are given in %.**

Table 2 Moments: Data and Model*

	Data	Benchmark	σ_β=0	σ_β=0.033
Standard Deviation Δs_t	3.08	3.08	2.94	3.20
Corr(Δs_t, Δs_{t-1})	0.06	0.06	0.08	0.05
Standard Deviation $i_t - i_t^*$	0.25	0.24	0.25	0.24
Corr($i_t - i_t^*, i_{t-1} - i_{t-1}^*$)	0.94	0.93	0.93	0.92
cov($\Delta s_t, i_{t-1} - i_{t-1}^*$) / var($i_{t-1} - i_{t-1}^*$)	-0.91	-1.56	-1.47	-1.59
R^2 monthly	-	0.04	0.02	0.08
R^2 annual	-	0.11	0.07	0.16
s.d. $\partial \Delta s_t / \partial \Delta f_{nt}$	-	50.31	0	100.02
s.d. Monthly Change $\partial \Delta s_t / \partial \Delta f_{nt}$	-	25.66	0	40.93
s.d. Monthly Change $\Delta \beta_{nt}$	-	0.27	0	0.53

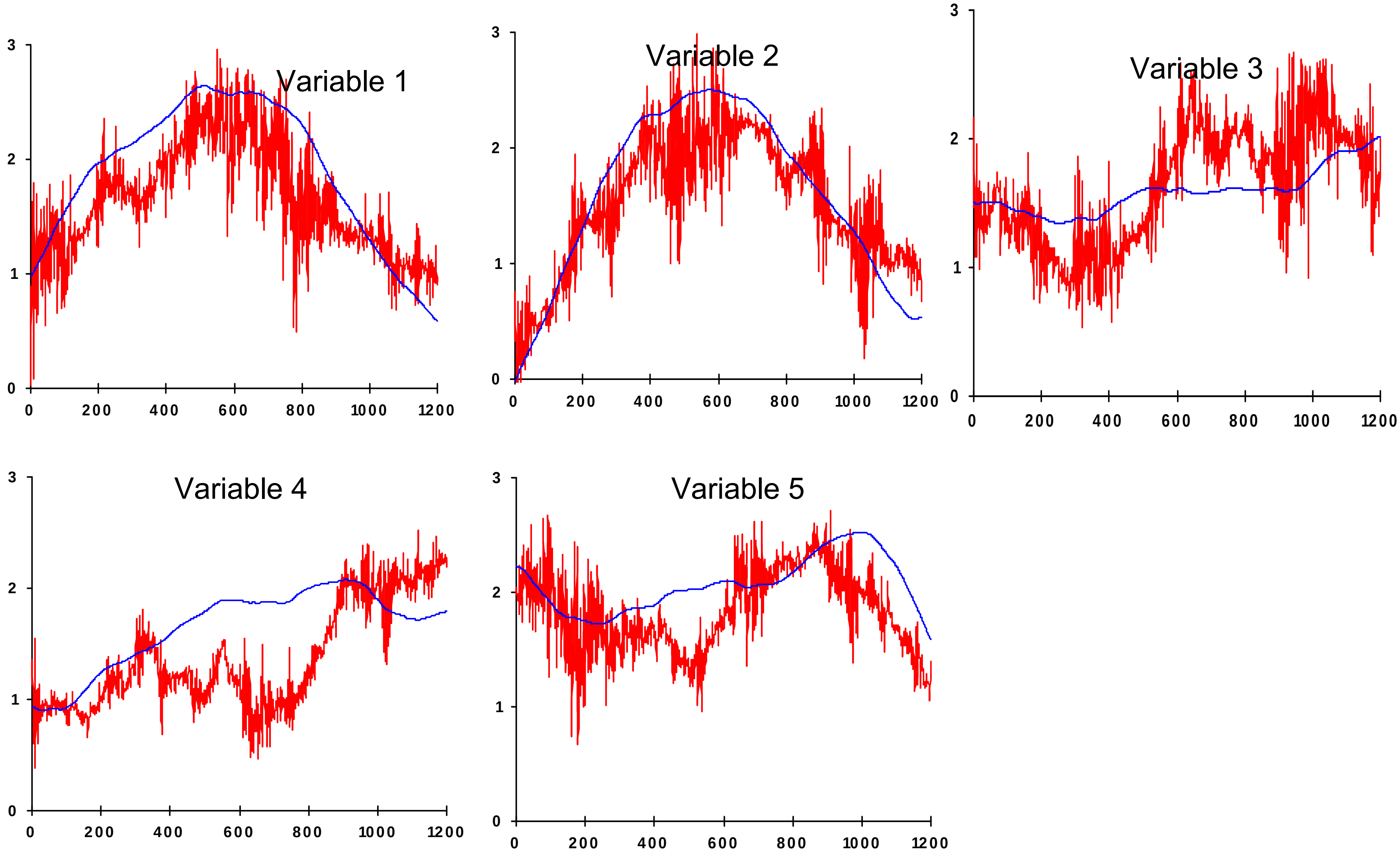
*** Standard deviations are given in %.**

Figure 1 Derivative Δs_t with respect to Δf_{nt} (10 years)*



* The smooth line is β_{nt}, while the volatile line represents the derivative of Δs_t with respect to Δf_{nt}.

Figure 2 Derivative Δs_t with respect to Δf_{nt} (100 years)*



* The smooth line is β_{nt}, while the volatile line represents the derivative of Δs_t with respect to Δf_{nt}.

Figure 3 Expectations β_{nt} (variable 3)

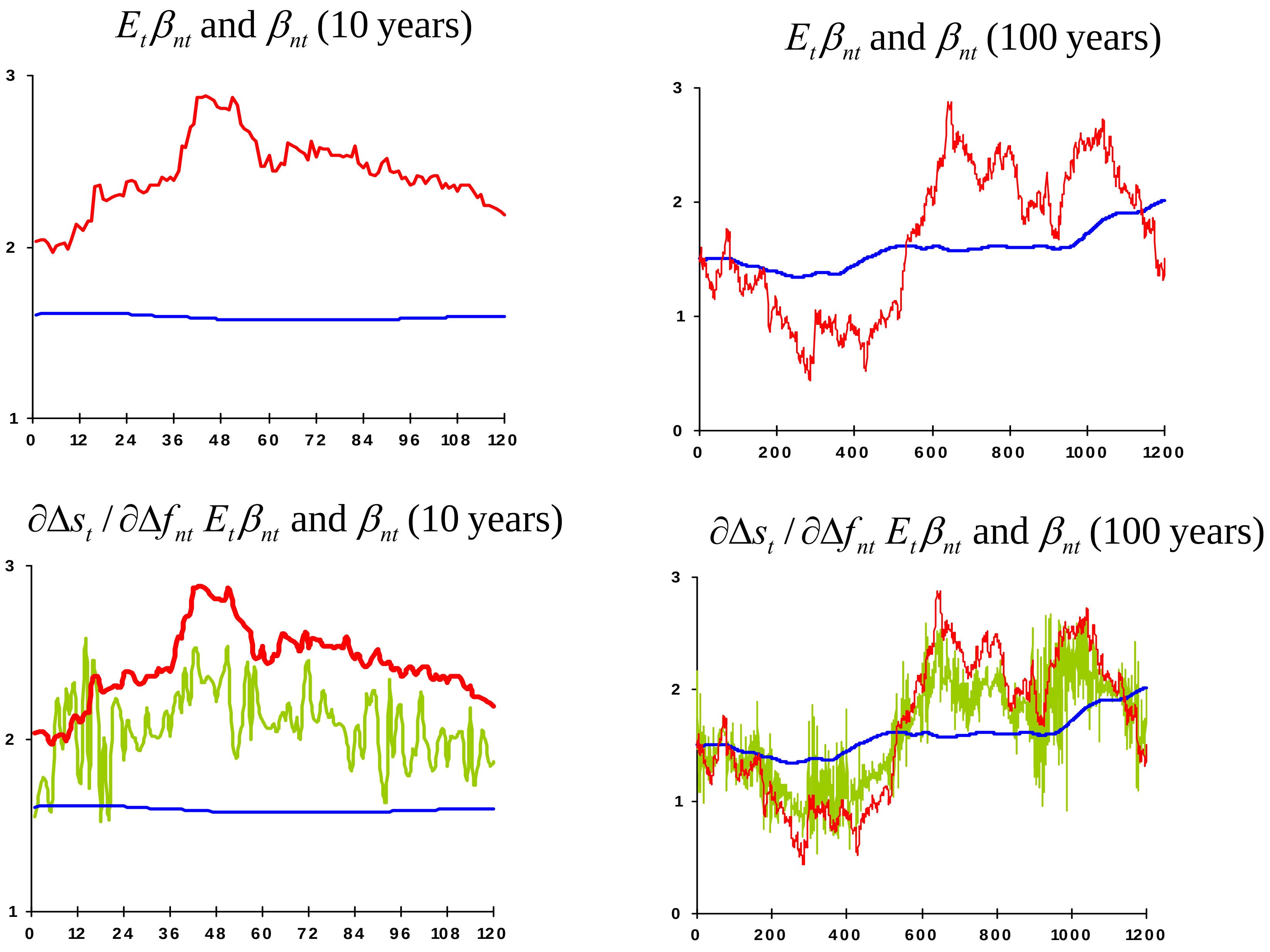


Figure 4 Correlation between $\partial \Delta s_t / \partial \Delta f_{nt}$ and β_{nt}

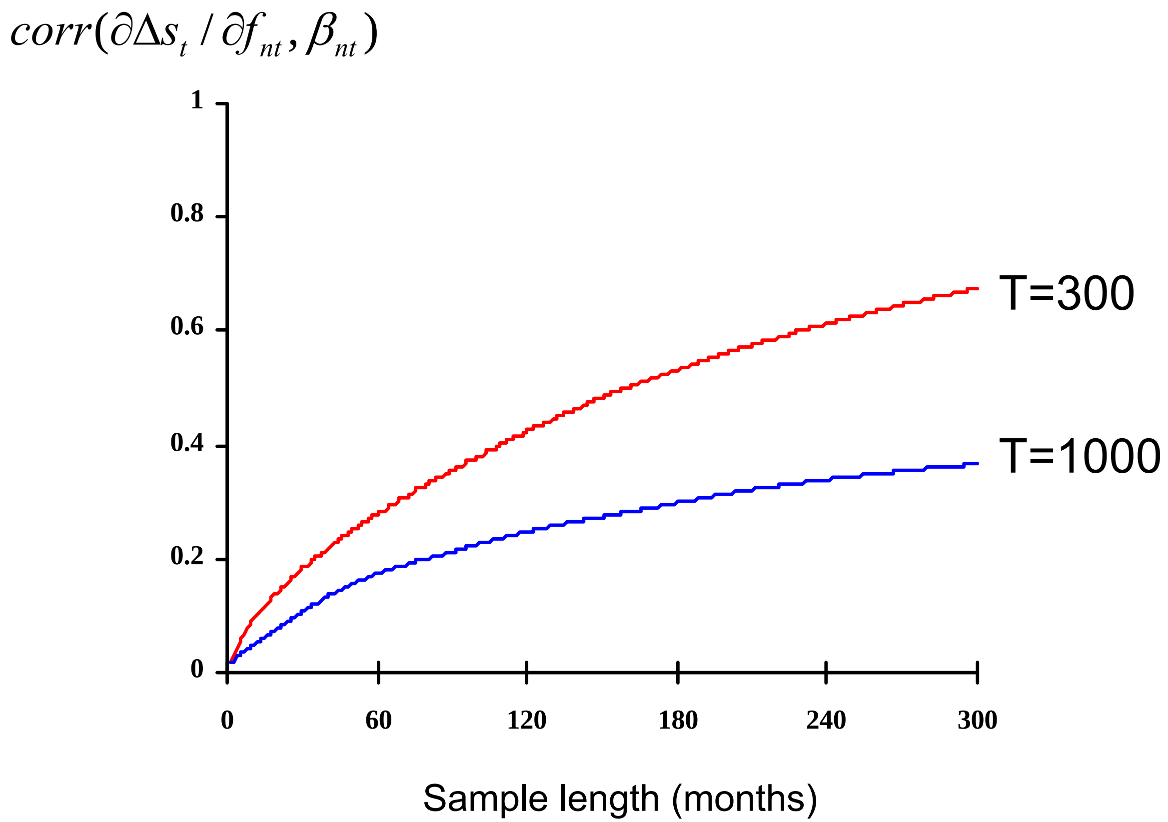
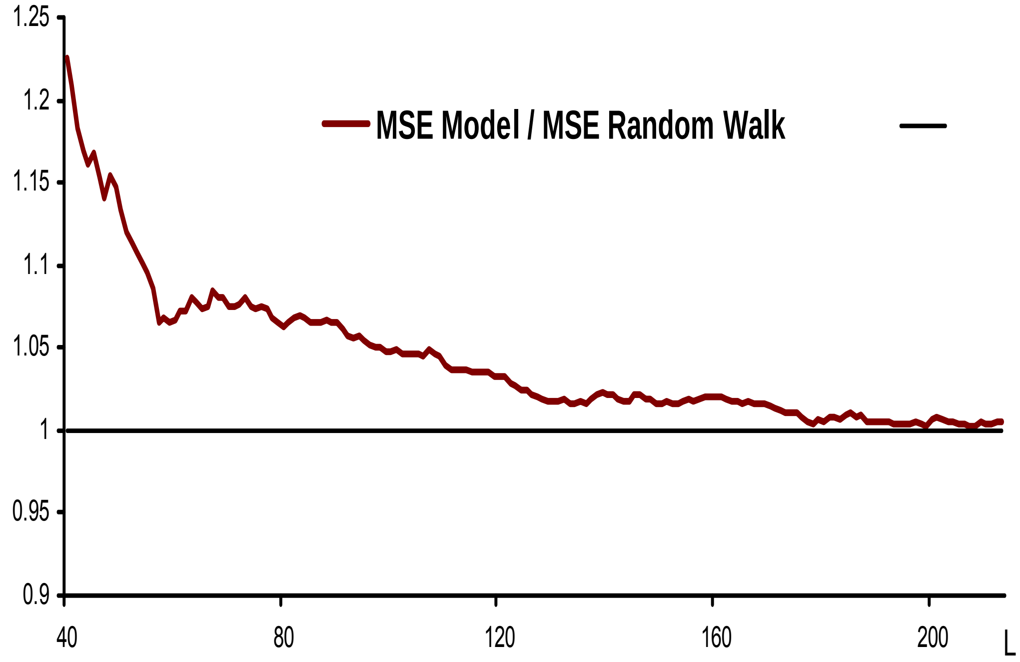


Figure 5 Out-of-Sample Fit and Sample Size
Empirical Evidence



Note: Mean-Square Error (MSE) of one month ahead exchange rate forecasts from model including money, output and inflation estimated by rolling regressions relative to MSE of random walk forecast. The reported line is an average for bilateral US Dollar exchange rate with Canadian Dollar, Japanese Yen, Swiss Franc and British Pound. Forecasting sample is 200 periods. Sample : 1973M3 - 2007M10. Data sources: IFS (exchange rates, industrial production index, CPI and money supply for Japan), OECD and Bank of England (money supply).

Figure 6 Out of Sample Forecasting:
MSE Model/MSE Random Walk

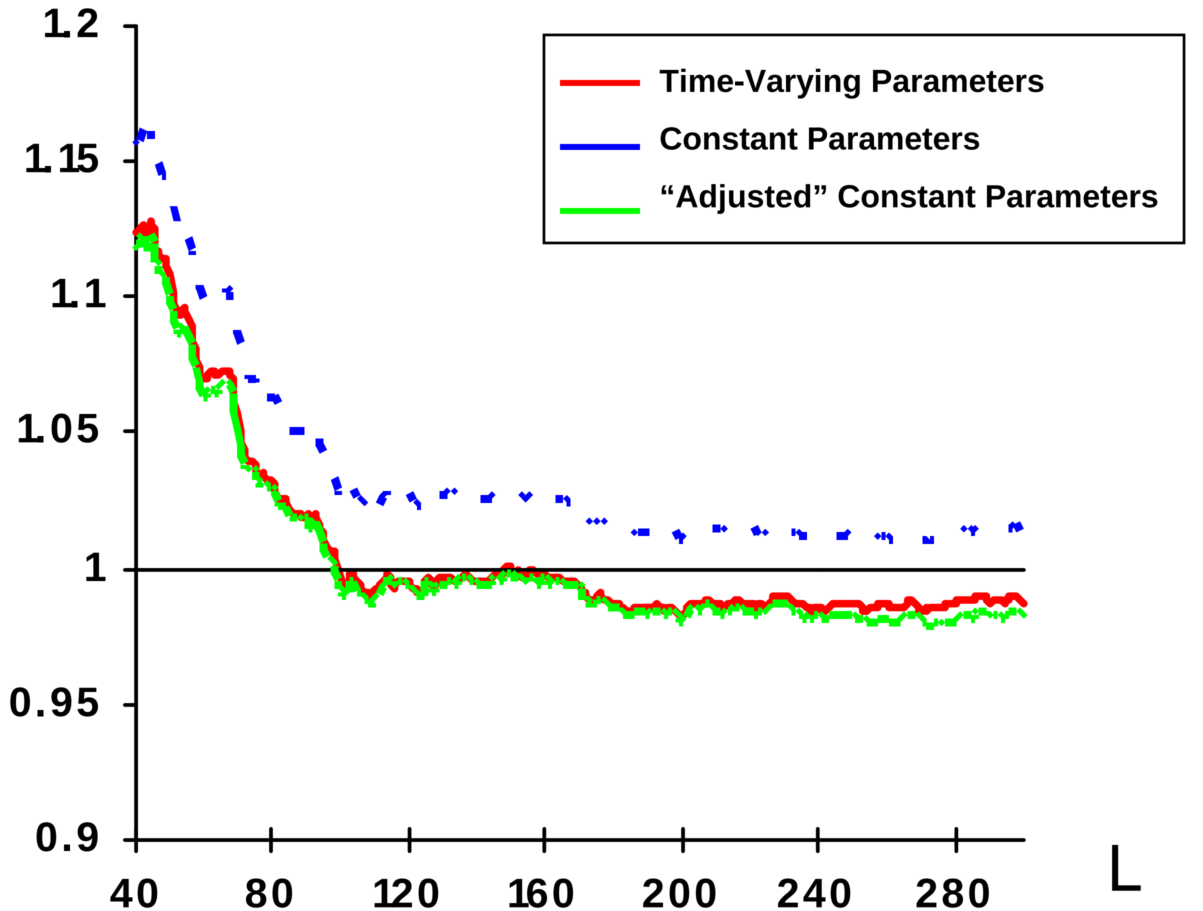


Figure 7 MSE Model/MSE Random Walk
Different volatility in fundamentals