Do short-run efficiency and optimal capacity imply long-run efficiency?

Yves Balasko*

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Abstract

In a general equilibrium model with production, the long-run is defined by capacity being an endogenous variable. Associated with every long-run equilibrium is a short-run model where capacity is set at its long-run equilibrium value. The long-run equilibrium is then an equilibrium of the short-run model. But the converse is not always true. There exist short-run models that feature multiple (short-run) equilibria. Only one of these equilibria—the restriction of the long-run equilibrium—is long-run efficient. Rate-of-return regulation is a simple way, however, of preventing the market from selecting the long-run inefficient short-run equilibria.

1. Introduction

The past two decades have seen the privatization of a large number of state-owned monopolies in many countries all over the world. Often, these monopolies have flourished in the power and communication sectors, in water distribution, in railroad and road networks, all of them characterized by large infrastructures. It follows from Cournot's monopoly theory that a profit maximizing monopolist sets installed capacity at a level that is smaller than socially optimal and charges a price higher than the marginal cost. The goal of having a socially optimal installed capacity has therefore been used as a major theoretical justification for the state ownership of such monopolies. The search for (Pareto) efficiency in such a setup has led to a huge literature advocating the marginal cost pricing rule. But, at variance with profit maximization in a competitive environment, this rule gives no incentives for the firms to squeeze the largest production sets out of the available technologies. Many examples exist all over the world of industries for which a marked productivity increase has followed the passage from state ownership to private ownership and market competition. This explains the trend that has started more than two decades ago of privatizing state-owned monopolies with large infrastructures by: 1) setting installed capacity at a socially optimal level; 2) leaving the

*Department of Economics and Related Studies, University of York, York, YO10 5DD, UK. Email: yb501@york.ac.uk

day-to-day operations of the privatized industry to the forces of market competition. These two conditions are clearly necessary for long-run efficiency. But are they sufficient?

The answer is known to be positive for the partial equilibrium models with downward sloping demand curves used in industrial organization [4]. The main goal of this paper is to show that the answer is negative in more general setups. We consider a simple general equilibrium model where production is characterized by its installed capacity and a variable input. Long-run is defined by the property that installed capacity is endogenous and variable, short-run by installed capacity constant and exogenously given. A short-run model is associated with every level of installed capacity. A long-run equilibrium is an equilibrium of the long-run model and long-run installed capacity is a component of the long-run equilibrium. The restriction of the long-run equilibrium to the associated short-run model is obviously an equilibrium of the short-run model. The short-run equilibrium, restriction of the long-run equilibrium that defines the short-run model, is Pareto efficient in the long-run model if the first welfare theorem holds true. This short-run equilibrium is therefore long-run efficient. If this is the only equilibrium of the short-run model, the answer to the above question remains positive.

We show in this paper, however, that the uniqueness of short-run equilibria is not true in general. There exist economies such that the associated short-run models feature multiple (short-run) equilibria. All but one of the short-run equilibria of such economies are long-run inefficient. This implies that, from a practical standpoint, the above privatization scheme must be supplemented by some rule that prevents the market from selecting any one of the long-run inefficient short-run equilibria. The firm's profit being zero at the long-run efficient short-run equilibrium and strictly positive at all other short-run equilibria, a simple solution is to subject the privatized firm to rate-of-return regulation of the kind used for public utilities in the USA.

This paper is organized as follows. Section two is devoted to the main assumptions and definitions, and to setting the notation. This includes in particular the definition of the long-run and short-run (general equilibrium) models. The properties of the two models are addressed in Section three. The most important result is the example of a short-run model with multiple equilibria. Concluding comments end this paper with Section four.

2. The long and short-run models

The long-run model considered in this paper is a simplified version of the standard Arrow-Debreu model with production [2]. The simplification is justified by the desire to avoid lengthy mathematical developments without impairing the nature of the relationships between the long-run and short-run models and their equilibria.

2.1. Goods

There are only three goods. The first good is produced by the firm; no individual is endowed with that commodity. A second good can be thought of as capital. It is an intermediate commodity for which consumers have no direct utility. The third good is some composite commodity that aggregates all other goods in the economy like energy and labor. (Note, however, that in order to avoid sign problems, some readers may prefer to substitute leisure to labor in the definition of this third good.) This composite good is taken as the numeraire, which amounts to setting its price to 1.

2.2. Consumers

There is only a finite number $m + 1$ of consumers. Let (x_i, ξ_i) denote the consumption bundle of consumer i, with $0 \leq i \leq m$, where x_i and ξ_i are quantities of produced good and composite good (or numeraire) respectively.

Every consumption set is the strictly positive quadrant $\mathcal{X} = \mathbb{R}^2_{++}$. Preferences are represented by smooth utility functions $u_i : \mathcal{X} \to \mathbb{R}$ that satisfy the standard assumptions of smooth consumer theory: 1) u_i is smooth; 2) u_i is smoothly strictly increasing (i.e., $Du_i(x_i, \xi_i) \in \mathcal{X}$); 3) u_i is smoothly strictly quasi-concave (i.e., the inequalities $X^T D^2 u_i(x_i, \xi_i)\, X \geq 0$ and $X^T Du_i(x_i, \xi_i) \leq 0$ have only the solution $X = 0$, where $X^T = (x, \xi)$ is a row matrix); 4) the indifference curves are closed as subsets of $\mathbb{R}^2$ (which is equivalent to the coordinate axes being asymptotes of the indifference curves).

Consumer i is endowed with the quantity $\omega_i > 0$ of the composite good. There is no initial endowment of the consumption good produced by the firm.

2.3. The firm's long-run and short-run production sets with capital input

In the long-run, capacity is variable and endogenously determined. In the short-run, capacity is exogenously given. The firm produces one output out of the two inputs: capital and the composite good. The long-run production set Y consists of the technologically feasible triplets $(y, -\lambda, -K) \in \mathbb{R}^3$ where $y > 0$ and $-\lambda < 0$ and $-K < 0$ represent the output in the produced good and the composite good and capital inputs respectively.

The capital input $-K \leq 0$ defines the firm's installed capacity $K \geq 0$. Capacity is equal to the maximal quantity of output that can be produced for arbitrarily large inputs of the composite good. Given the installed capacity K, the firm needs γ units of the composite good to produce an additional unit of its output provided the total output is less than or equal to the installed capacity K. In other words, the short-run marginal cost is constant and equal to γ when output is less than or equal to the installed capacity K, and infinite beyond.
The long-run production set Y is therefore defined by the inequalities

$$\mu \geq \gamma y \quad , \quad y \leq K.$$

The short-run production set $Y(K)$ associated with the installed capacity K consists of the triplets $(y, -\mu, -K)$ satisfying the same inequalities as above for K given.

Note that the long-run production set Y is the union $\bigcup_{K \geq 0} Y(K)$ of the short-run production sets for all installed capacity levels.

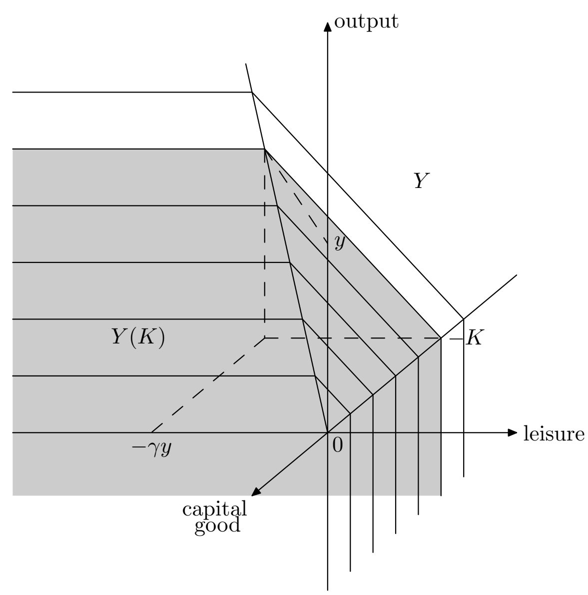

Figure 1: Production set with capital good input

2.4. The firm's *reduced* long-run and short-run production sets

Capital is an intermediate good and, as such, is not an argument of consumers' utility functions. We assume that in order to produce one unit of installed capacity, the firm needs ρ units of the composite good. The *reduced* long-run and short-run production sets then consist of the technologically feasible activities $(y, -\lambda)$ where the only input $-\lambda$ is the composite good while y represents the quantity of produced good. These sets result from the elimination of the intermediary capital good from the model. The capital good being not an argument of the consumers' utility functions, its elimination simplifies the study of the properties of both long-run and short-run models.
The reduced long-run production set Z is therefore defined by the inequalities

$$\lambda \geq (\rho + \gamma)y \quad , \quad 0 \leq y.$$

The reduced short-run production set $Z(K)$ associated with installed capacity K satisfies the inequalities

$$\lambda \geq \rho K + \gamma y \quad , \quad 0 \leq y \leq K.$$

The input $-\lambda$, with

$$\lambda = \begin{cases} \rho K + \gamma y & \text{for } y \leq K \\ +\infty & \text{for } y > K \end{cases}$$

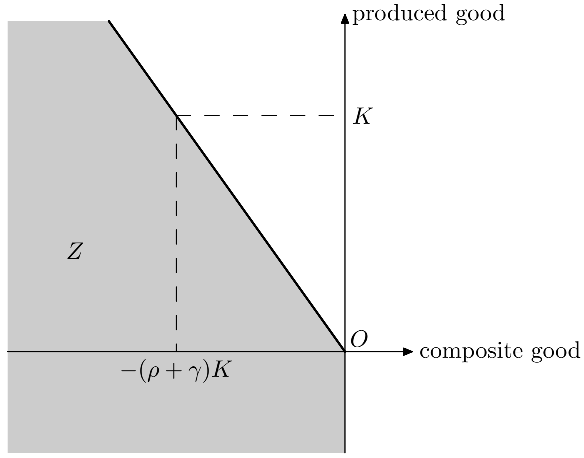

Figure 2: Long-run *reduced* production set with composite good input

represents the firm's *cost function* associated with capacity K for producing the quantity y of the produced good. The graph of this function in the coordinates $(y, -\lambda)$ coincides with the efficient boundary of the *reduced* short-run production set $Z(K)$. The set $Z(K)$ is not convex because of the fixed setup cost ρK. We have $Z = \bigcup_{K \geq 0} Z(K)$.

2.5. The firm's long-run and short-run equilibria

Let $(p, 1)$ be the price system. Then, the firm's profit associated with the *reduced* activity vector $(y, -\lambda)$ is equal to $py - \lambda$. The activity vector $(y, -\lambda)$ is a long-run (resp. short-run) equilibrium activity of the firm if it maximizes profit subject to the constraint $(y, -\lambda) \in Z$ (resp. $(y, -\lambda) \in Z(K)$).

The long-run profit associated with the output y being equal to $(p - \gamma - \rho)y$, the firm has a non-zero long-run equilibrium activity (i.e., $y > 0$) if and only if $p = p^* = \rho + \gamma$. Profit is then nil. If $p > p^*$, there is no long-run equilibrium for the firm. (Profit would then be arbitrarily large.) If $p < p^*$, the only long-run equilibrium is the zero activity. Long-run profit is always 0 at equilibrium.

The short-run profit associated with the output y given the installed capacity K and the price $p > 0$ of the produced good is equal to $(p - \gamma)y - \rho K$. Note that, at variance with the long-run equilibrium of the firm, there always exists a short-run equilibrium for the firm for any given price $p > 0$: the activity vectors $(0, 0)$ for $p \leq p^*$ and $(K, -(\rho + \gamma)K)$ for $p \geq p^*$ respectively. The short-run profit is always ≥ 0. It is strictly > 0 for $p > p^*$.

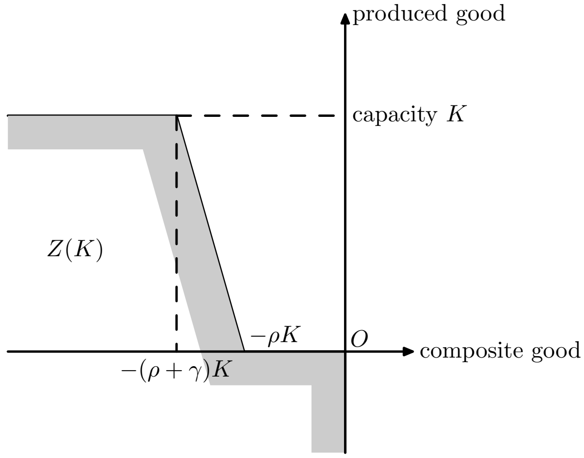

Figure 3: Short-run *reduced* production set with composite good input

2.6. The long-run and short-run models

The unique firm is owned by consumer 0. Therefore, consumer 0 is entitled to the full profit $py - \lambda$ given the activity vector $(y, -\lambda)$.

Consumer 0 maximizes the utility $u_0(x_0, \xi_0)$ subject to the budget constraint

$$px_0 + \xi_0 = \omega_0 + py - \lambda.$$

Consumer i, with $i \neq 0$, maximizes $u_i(x_i, \xi_i)$ subject to the budget constraint

$$px_i + \xi_i = \omega_i.$$

Given any price vector $(p, 1)$, the solution of the consumer i's maximization problem is denoted by (x_i, ξ_i), with $0 \leq i \leq m$.

The long-run and short-run (general equilibrium) models $\mathcal{P}$ and $\mathcal{P}(K)$ are defined by having the firm equipped with the long-run and short-run production sets Z and $Z(K)$ respectively.

3. Properties of the long-run and short-run models

3.1. Long-run and short-run equilibria

In both models $\mathcal{P}$ and $\mathcal{P}(K)$, the price vector $(p, 1)$ is an equilibrium price vector if the equalities

$$\sum_{i=0}^{m} x_i = y \tag{1}$$

$$\sum_{i=0}^{m} \xi_i = \sum_{i=0}^{m} \omega_i + \lambda. \tag{2}$$

are satisfied, with (x_i, ξ_i) representing consumer i's demand for $0 \leq i \leq m$ and $(y, -\lambda)$ being an equilibrium of the firm in the long-run and short-run respectively.

The equilibria of the long-run (resp. short-run) model are called long-run (resp. short-run) equilibria for obvious reasons. In both models, the two equations (1) and (2) have a unique unknown, namely the price p of the produced good. Walras law implies, however, that these two equations are not independent.

3.2. Long-run equilibria

The long-run model $\mathcal{P}$ is the model of a private ownership convex production economy in the sense of [2]. Existence and Pareto efficiency of long-run equilibria is then a straightforward consequence of general theorems. See, for example, Theorems 16.2 and 16.3 of [3].

The model $\mathcal{P}$ is so simple, however, that it is best studied through a direct elementary approach. First of all, at equilibrium, each consumer's consumption of the produced good is > 0, which implies that no-activity is incompatible with equilibrium. This implies that the only possible equilibrium price vector $(p, 1)$ is such that $p = p^* = \gamma + \rho$, the long-run marginal cost. It is also obvious that $p = p^*$ does define an equilibrium price vector $(p^*, 1)$ of the long-run production economy $\mathcal{P}$ and this equilibrium is unique. We denote by K^* the installed capacity defined by this long-run equilibrium.

3.3. Short-run equilibria

The price vector $(p^*, 1)$ is also an equilibrium price vector of the short-run model $\mathcal{P}(K^*)$. The corresponding equilibrium allocation is a Pareto optimum of the long-run model or, in other words, is long-run efficient. Parenthetically, equilibrium for the short-run model $\mathcal{P}(K^*)$ always exists despite the fact that the short-run production set $Z(K^*)$ is not convex.

3.4. Existence of multiple short-run equilibria

Let us show that the short-run model $\mathcal{P}(K^*)$ may have multiple equilibria despite the fact that the long-run equilibrium is unique.

The proof goes as follows. Let $(p, 1)$ be an equilibrium price vector of the short-run model $\mathcal{P}(K^*)$. It follows from the assumptions that the output of the produced good y is necessarily > 0 at equilibrium. Profit maximization for the firm implies that we have $y = K^*$.

Consumer 0 maximizes $u_0(x_0, \xi_0)$ subject to the budget constraint

$$px_0 + \xi_0 = \omega_0 + (p - \gamma - \rho)K^*$$

which can be rewritten as

$$px_0 + \xi_0 = pK^* + \big(\omega_0 - (\gamma + \rho)K^*\big).$$

This budget constraint is equivalent to consumer 0 being endowed with the resource $(K^*, \omega_0 - (\gamma + \rho)K^*)$ instead of owning the firm.

The equilibrium equation of the short-run model with production $\mathcal{P}(K^*)$

$$\sum_{i=0}^{m}(x_i, \xi_i) = \big(K^*, -(\gamma + \rho)K^*\big) + \sum_{i=0}^{m}(0, \omega_i) \tag{3}$$

can be rewritten as

$$\sum_{i=0}^{m}(x_i, \xi_i) = \big(K^*, \omega_0 - (\gamma + \rho)K^*\big) + \sum_{i=1}^{m}(0, \omega_i). \tag{4}$$

This proves that the short-run model $\mathcal{P}(K^*)$ with production is equivalent to the pure exchange economy $\mathcal{E}(K^*)$ made of the same $m + 1$ consumers, consumer 0 being endowed with the resource $\big(K^*, \omega_0 - (\gamma + \rho)K^*\big)$. Only the price vectors $(p, 1)$ of the pure exchange economy $\mathcal{E}(K^*)$ that satisfy the inequality $p \geq p^* = \gamma + \rho$ are equilibria of the short-run production economy $\mathcal{P}(K^*)$.

Because of this equivalence between the short-run production economy $\mathcal{P}(K^*)$ and the pure exchange economy $\mathcal{E}(K^*)$, the multiplicity issue is reduced to the same problem for a pure exchange economy with the additional restriction that only the price of the produced good that satisfy the inequality $p \geq p^*$ are equilibria of the production economy $\mathcal{P}(K^*)$.

It is known that pure exchange economies can have an arbitrary number n of equilibria. Even the most elementary textbooks offer pictures of intersecting offer curves in the Edgeworth box featuring three or more equilibria and this is sufficient for our purpose. Figure 4 is an Edgeworth box representation of a pure exchange economy $\mathcal{E}(K^*)$ associated with the short-run production economy $\mathcal{P}(K^*)$ in the case $m = 1$ (i.e., there are two consumers) that features three equilibria. Recall also that the number n of equilibria is generically odd.

Consider therefore an economy like the one represented in Figure 4. Sort the n equilibria of the pure exchange economy $\mathcal{E}(K^*)$ by the equilibrium price of the produced good: $p^{(1)} < p^{(2)} < \cdots < p^{(n)}$. It then suffices to pick the parameters γ and ρ such that the sum $p^* = \gamma + \rho$ is equal to $p^{(j)}$ for some $j < n$ to get a short-run production economy $\mathcal{P}(K^*)$ whose short-run equilibrium allocations associated with price $p^{(k)}$, with $j < k \leq n$, are not long-run efficient. There are $n - j \geq 1$ such long-run inefficient equilibria.

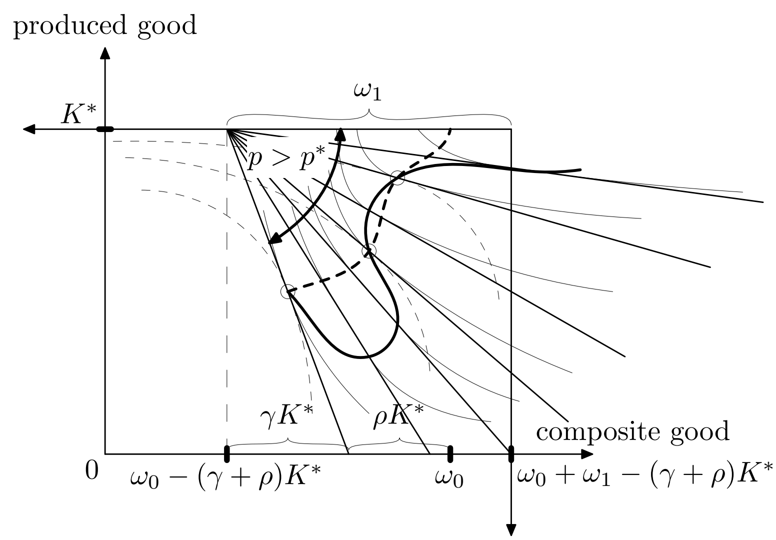

Figure 4: Edgeworth box representation of the pure exchange economy $\mathcal{E}(K^*)$

Incidentally, it is worth noting that, at variance with convex production economies like $\mathcal{P}$ where the number of equilibria is generically odd, the number of equilibria of the short-run production economy $\mathcal{P}(K^*)$ can be even for an open set of endowments and odd for another set. In particular, the parity of the number of short run equilibria is not an invariant of the economy.

The long-run inefficient short-run equilibria are easy to characterize in terms of the short-run production model $\mathcal{P}(K^*)$. The inequality $p > p^{(j)} = p^*$ is equivalent to the firm's profit being strictly positive for the price p. Therefore, the short-run equilibria that are long-run inefficient are precisely those that yield a strictly positive profit for the firm.

4. Concluding comments

The long-run production economy $\mathcal{P}$ considered in this paper has a unique (long-run) equilibrium that corresponds to $p = p^* = \gamma + \rho$. This uniqueness property is not general and hinges on the fact that the number of goods of the reduced model is equal to 2. With more than two goods, there may exist multiple long-run equilibria. The corresponding equilibrium allocations are all Pareto efficient. But they entail different wealth distribution among the consumers. The choice of one of these equilibria involves distributional issues and is beyond the scope of this paper. But, once such an equilibrium has been selected, it is straightforward to extend the line of reasoning of the current paper to that case. Therefore, the main conclusion that unregulated short-run market competition may fail to lead to long-run efficiency even when capacity is set at some long-run optimal value holds true for an arbitrary number of goods, consumers and producers.

Existence of multiple short-run equilibria is worth more than a footnote. It follows from properties of the pure exchange model that the number of equilibria is larger than one whenever the vector of net trades at equilibrium is large enough. (See [1], Th. 4.5.4 and, in the case of two consumers, Th. 6.3.1.) In the current setup, this net trade vector cannot be made arbitrarily small—a way to ensure uniqueness—since there are no individual endowments of the produced good. In fact, examples featuring multiple equilibria, even with very ordinary preferences (excluding those defined by log-linear utility functions), can easily be devised by having the net trade vector sufficiently large .

Since the existence of long-run inefficient short-run equilibria cannot be excluded, short-run market competition must be supplemented by some form of regulation. The long-run inefficient short-run equilibria are precisely those that bring a strictly positive profit to the firm. An unregulated profit-maximizing firm will employ its market power at selecting one of these long-run inefficient equilibria. A simple countermeasure is to prevent the firm from making a strictly positive profit. Rate-of-return regulation of the kind used in the USA for public utilities suffices to prevent the market mechanism from selecting the long-run inefficient short-run equilibria.

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