Messengers, Gatekeepers and Speakers: The Power of Intermediaries in Contracting.

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Abstract

In many settings, principals must use intermediaries to extract information from agents. For example, international organizations use NGOs or local leaders to acquire information about agents' preferences in developing countries, exporting firms employ local agent to obtain information about local market conditions, legislatures ask local representatives to report the preferences of the members of their constituency. In order to study the power of intermediaries in these different settings, this paper sets up a three-tier contracting model with a principal who chooses a policy, an intermediary and a agent who has a private valuation for the policy. We compare three levels of commitment for the intermediary: no commitment, partial commitment where she can condition her contract with the agent on her participation to the principal's contract, and full commitment where she can condition her contract with the agent on her message to the principal. In the case of a *pure intermediary*, who does not care about the principal's decision (or alternatively has a public valuation for this decision), we prove equivalence of full and partial commitment in a linear model, fully characterize the contracts in the two settings, and discuss conditions under which the principal prefers either form of commitment. In the case where the intermediary has private valuation over the decision, we show that the equivalence result for linear cases fails, and provide a partial characterization of the optimal contracts.

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1 Introduction

Decision-makers often rely on intermediaries to extract information from privately informed agents. Insurance companies use insurance brokers to obtain information about customers' risks, employers use recruitment specialists and psychologists to screen job candidates, investors use financial intermediaries to acquire information about risky projects, firms employ local subcontractors to learn about market conditions in foreign countries, international funding agencies rely on NGOs or local leaders to learn about the preferences of villagers over public projects. While the use of intermediaries in screening is thus widespread, little is known about the power of intermediaries, and the distortion that their presence may create in the design of optimal mechanisms. In this paper, our objective is to shed light on this issue by comparing different forms of intermediation, and providing analytical characterizations of optimal mechanisms with intermediaries.

We distinguish between three broad classes of intermediaries: messengers, whose only role is to transmit information; *gatekeepers* who choose whether or not to report the message to the decision-makers, and *speakers* who can commit on the content of the message sent to the decision-maker. These three forms of intermediation correspond to varying levels of commitment: a messenger is unable to commit, a speaker can commit on his participation to the contract with the decision-maker, and a speaker can commit on the message he sends to the decision-maker. In reality, all three forms of intermediaries exist.¹ In trying to learn about opportunities on foreign markets, firms often face messengers, local agents with little reputation, who are unable to commit because nobody expects them to stay around for long. Recruitment experts and financial intermediaries often act as gatekeepers – screening candidates and projects that they will or not report to the decision-maker. While they also pass along information about the agent, their inability to commit may be due to the fact that they lack information about the decision-maker's priorities, or that the decision is subject to external shocks that are outside their control. Finally, insurance brokers, local retailers, local representatives often have the ability to commit to the report sent to the decision-maker, and thus behave as "speakers". In this full commitment case, the decision-maker in fact delegates the decision to the intermediary, who can condition the transfer to the agent on the decision.

The three different levels of commitment (and the issues we study in this paper) are best illustrated in the context of Community Driven Development – the delegation of the choice among public projects by donors to local authorities.² In order to obtain information about preferences over public projects, funding agencies often enlist the services of intermediaries (local teams, NGOs) or ask a small sample of village leaders or village activists. If the intermediary is external to the community, and has little reputation, he might very well behave as a messenger, unable to commit about any aspect of the public project. If the intermediary does not control all elements of the decision-maker's choice (for

¹We take the existence of these three different institutions as given, but recognize that one may want to build models to explain the emergence of the different forms of intermediation.

 $^{^2}$ "community-based development" projects have become an important form of development assistance (the World Bank's portfolio alone approximating seven billion dollars).

example, the village may be competing against other villages for the provision of public projects, and external shocks may affect the decision-maker's choice among competing bids), he will behave as a gatekeeper. Finally, if the intermediary enjoys a stable reputation in the community (for example, if he is an established village leader coming from a reputable family), he will very likely be able to commit on the decision taken, and behave as a speaker. It seems important (both theoretically and for policy purposes) to understand how the presence of these different types of intermediaries affects the optimal contracts, the rents of the different agents, and the decisions of the international funding agency.

This paper considers a stylized three-tier model with a principal who has to choose a policy (such as the level of public good provision or the firm's production), an intermediary and an agent who has a private valuation for the policy. Contracts are nested: the principal first offers a contract, which is publicly observed by the other agents . In a second stage, the intermediary proposes a contract to the agent, conditional on the principal's contract. This contract may specify only a transfer (in the case of messengers, or no-commitment), a transfer and an acceptance policy (in the case of gatekeepers or partial commitment), or a transfer and a message (in the case of speakers or full commitment). In a third stage, The agents accepts or rejects the contract, sends his message to the intermediary and the contract between the intermediary and the agent is executed. In the final stage of the game, the intermediary accepts or rejects the principal's contract, sends his message to the principal, and the contract between the principal and the intermediary is executed.

We first consider the case of a *pure intermediary*, where the intermediary either does not care about the decision, or has a known valuation for the decision. We characterize the optimal contracts under no-commitment, partial and full commitment. With messengers, the contract is very restricted (a fixed transfer and constant decision), and the intermediary does not extract any rent. With gatekeepers, the optimal public decision schedule involves *bunching and a discontinuous jump at the bottom*. We show that there is a threshold value under which no policy is implemented. Above that, for low valuations a constant strictly positive policy level is chosen and there is *bunching at the bottom*, while for high valuations the policy level is increasing in the agent's valuation. Finally, with speakers, the optimal policy schedule is a close analog of the second-best direct contract, but involves double marginalization of rents, which result in a lower level of public good provision.

Comparing the different contractual arrangements, we show that the principal always prefers full commitment when the intermediary gets a positive value of the decision at the bottom. Otherwise, the principal may prefer partial commitment. The intermediary, if he were to choose between committing or not after observing the principal's contract, would always prefer to commit. However, this does not imply that he is always better off in the full commitment contract, as the contract proposed by the principal depends on the intermediary's commitment capacity. Finally, we prove an equivalence result for a restricted class of problems, and show that the optimal contracts under partial and full commitment are identical when the objective of the principal is linear in the decision. Finally, we illustrate our results using two examples: a benevolent social planner providing a public good and a monopolist choosing her production level. In the public good provision example, we find that, for intermediate types, the public good may be provided *more often* in the partial commitment case than in the direct contract. In the monopolist example, the production level is always smaller with an intermediary than in the direct contract.

Next, we consider the case when the intermediary's valuation of the policy is private information. In the partial commitment contract, we show that – in sharp contrast with the pure intermediary case – the policy choice and transfer from the principal cannot depend on the agent's type. Hence, information about the agent is no longer transmitted, and the principal's contract just attempts to extract some of the surplus from the intermediary. In the full commitment case, we show that the optimal contract depends on a one dimensional message (the sum of the intermediary's valuation and the agent's virtual valuation). When the principal tries to extract information about both agents' types, the full characterization of the optimal contract relies on the solution of a complex optimal control problem. In the particular case of a linear objective for the principal, we prove that the equivalence result obtained in the pure intermediary case no longer holds, and that the principal may in fact prefer the partial commitment contract.

There is a growing literature on mechanism design in the presence of intermediaries when the agent with privately known cost takes a productive action and the principal is the residual claimant (See Mookherjee (2005) for a great survey). Related work includes Melumad, Mookherjee and Reichelstein [MMR] (1995), Faure-Grimaud, Laffont and Martimort [FGLM] (2003), and Mookherjee and Tsumagari [MT] (2004) who explore the costs and benefits of delegated contracting in settings where agents can enter into collusive side contracts.

In particular, MT shows that the centralized contract even in the presence of collusion dominates contracting with an intermediary. They find a form of double-marginalization in their model in addition to a distortion of the allocation of production between the agent when their productive inputs are substitute. In a two type model, FGLM show the impact of collusion on the centralized contract and find that delegation to an intermediary is equivalent to direct contracting in the presence of collusion between the agent and the intermediary. Without collusion, MMR show that if the principal can monitor the contract between the intermediary and the agent, if the contract between the principal and the intermediary occurs *before* the subcontracting, and if the intermediary is not subject to limited liability then there is no cost of delegation.

In Faure-Grimaud and Martimort (2001), the principal delegates the design of the contract to the intermediary (regulator) to induce production by the productive agent (firm) that can be of three types (good, average and really bad). Since the design of the sub-contract offered by intermediary is not contractible, the intermediary can appropriate some of the information rent provided in the budget to permit production by one of the two most efficient firms, by playing a gamble and offering a contract that only the most efficient firm would accept. If the intermediary is lucky and the agent is of the most efficient type, she can pocket a surplus. The authors derives a form of agency costs directly from the limits in the contract design when an intermediary is needed to filter out an unwanted third type from producing.

On the topic of public good provision, Laffont and Martimort (2000) consider a public good provision problem with two agents who can collude. The agents' valuations for the public good takes one of two values and are private information. They show that there is no loss of generality in offering weakly collusion proof mechanisms, that the agents capture some rent even when their valuations are not independent and that the optimal policy moves continuously with the correlation.

Bardhan and Mookherjee (2000, 2005 and 2006) also study local and centralized provision of a public good and its allocation between different segments of population. The central government is actually assumed to use a bureaucrat as intermediary. In contrast the local government is perfectly informed but is more likely to be subject of elite capture in a Downsian model with some uninformed voters among the poor. They study the tradeoff between local information and elite capture.

Celik (2005) also studies a three-tier hierarchy and characterizes optimal contracts, albeit in a different model with collusive supervision.

2 Model Setup

2.1 Utilities and Actions

There is a single principal (denoted 0) and two agents (agents 1 and 2) where agent 1 is the the intermediary. The principal chooses an action x in the interval [0,1].³ Agents 1 and 2 have private valuations for the principal's action, denoted γ and θ , which are independently distributed according to probability distributions $F(\theta)$ and $G(\gamma)$. We assume throughout the paper that the hazard rates, $\frac{f(\theta)}{1-F(\theta)}$ and $\frac{g(\gamma)}{1-G(\gamma)}$ are monotonically increasing. We define the *virtual valuations* as: $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ and $K(\gamma) = \gamma - \frac{1-G(\gamma)}{g(\gamma)}$. All agents have quasi-linear utility functions, defined by

$$egin{array}{rcl} U_0 &=& U_0(x,\gamma, heta)+ au_0 \ U_1 &=& U_1(x,\gamma)+ au_1 \ U_2 &=& U_2(x, heta)+ au_2 \end{array}$$

where τ_0 , τ_1 and τ_2 denote the monetary transfers received by the agents.

Assumption 1 The utility of the agent and intermediary are multiplicatively separable: $U_1(x, \gamma) = \gamma x$ and $U_2(x, \theta) = \theta x$. The utility of the principal is given by $U_0(x, \gamma, \theta) = W(x, \gamma, \theta)$ where W is twice continuously differentiable, concave in x, the cross partial derivatives $\frac{\partial^2 W(x, \gamma, \theta)}{\partial x \partial \gamma}$ and $\frac{\partial^2 W(x, \gamma, \theta)}{\partial x \partial \theta}$ are positive and $W(x, \gamma, \theta)/x$ is non-increasing in x.

³We normalize the decision to lie in the interval [0, 1) with no loss of generality.

The specific form of the agent's and intermediary's utility (who only care about the *product* of the type and social decision) is a strong restriction, which will play a fundamental role in the later analysis. In particular, we will show that this formulation of the agent's utility function is a necessary condition for the single-crossing condition to hold in the contract of a pure intermediary with partial commitment.(We choose to make the same assumption on the intermediary's utility to maintain the symmetry of the model, even though it is not required in the analysis.) Notice that it also implies risk neutrality of the agent and intermediary

The conditions on the principal's utility function hold when the principal is a social planner who incurs a convex cost for the social decision, c(x) such that c'(x) > 0, $c''(x) \ge 0$ and c(0) = 0.

We now discuss two specific illustrative examples.

2.1.1 Public good provision

The principal is a social planner who provides a public good x. The intermediary is a representative or a village leader. In this case,

$$W(x,\gamma,\theta) = \frac{[\gamma+\theta]x - \lambda c(x)}{\lambda - 1}$$

where $\lambda > 1$ denotes the total cost of raising funds from taxpayers, and c(x) is an increasing and convex cost function with c(0) = 0. The difference $\lambda - 1$ is often called the *social cost of public funds.*⁴

2.1.2 Monopolist

The principal is a monopolist who wants to sell a consumption good in a foreign market and has to decide on the quantity x to produce. In this case,

$$W(x, \gamma, \theta) = -c(x)$$

where c(x) is an increasing and convex cost function with c(0) = 0.

2.2 Contracting and Commitment

We consider two types of contract (i) the direct contract where the principal deals directly with the agent, and (ii) the indirect contract where the principal contracts with the intermediary, and the intermediary with the agent. In the latter case, we analyze three different levels of commitment for the intermediary.

 $^{^4\}mathrm{See}$ for instance Laffont and Tirole (1986), Laffont and Martimort (2000) or Amstrong and Sappington (2005).

2.2.1 The Direct Contract

In the direct contract, the principal proposes contracts both to the intermediary and to the agent. A contract thus specifies the decision and the transfers as a function of the messages sent by the two agents,

 $C = \{M_1, M_2, x(m_1, m_2), t_1(m_1, m_2), t_2(m_1, m_2)\},\$

where M_1, M_2 denote the message spaces of the intermediary and the agent, t_1 is the transfer from the intermediary to the principal, and t_2 the transfer from the agent to the principal. In this standard setting, the time sequence is as follows:

- 1. The principal offers the contract C to agents 1 and 2
- 2. The two agents simultaneously and privately accept or reject the contract
- 3. If the contract is accepted, the two agents simultaneously and privately send the messages m_1 and m_2 to the principal.
- 4. The principal executes the contract and transfers are made.

2.2.2 The Indirect Contract

In the indirect contract setting, the principal offers a contract C_1 to the intermediary, and the intermediary a contract C_2 to the agent. The contracts are *nested*: the principal first offers her contract (which is publicly observed by the two other agents), and the intermediary proposes his contract next. Hence, as a Stackelberg follower, the intermediary designs his contract contingent on the contract received from the principal. As a Stackelberg leader, the principal designs her contract anticipating the response of the intermediary. Formally, the principal's contract specifies the decision and a transfer as a function of the message sent by the intermediary, $C_1 = \{M_1, x(m_1), t_1(m_1)\}$. The intermediary's contract specifies a decision (denoted y) and a transfer as a function of the message sent by the agent, $C_2 = \{M_2, y(m_2), t_2(m_2)\}$. We assume the following time sequence:

- 1. The principal offers contract C_1 to the intermediary. This contract is publicly observed by the intermediary and the agent.
- 2. The principal offers contract C_2 to the agent. This contract is not observed by the principal.
- 3. The agent accepts or rejects contract C_2 .
- 4. If the contract is accepted, the agent privately sends his message m_2 to the intermediary.
- 5. The intermediary executes the contract with the agent, transfer t_2 is made.
- 6. The intermediary accepts or rejects contract C_1
- 7. If the contract is accepted, the intermediary sends message m_1 to the principal

8. The principal executes the contract with the intermediary, transfer t_1 is made.

We consider three levels of commitment from the intermediary, corresponding to three possible actions y:

- No commitment Messengers $y = \emptyset$
- Partial commitment -Gatekeepers $y = q \in [0,1]$ is the probability of accepting contract C_1
- Full commitment Speakers $y = \mu$ where $\mu \in M_1$ is the (deterministic) message sent by the intermediary to the principal in contract C_1 .

Our typology of the three types of intermediaries can be reinterpreted as follows. Messengers are unable to commit, and can not condition the transfer t_2 on any decision. Gatekeepers can commit on the transmission of the message; but not on the content of the message. The transfer they require from the agent can only be conditioned on the probability of acceptance of contract C_1 , $t_2(q)$. Speakers can commit on the message sent to the principal, which will result in decision x. Hence, the transfer they require from the agent can be conditioned on x, $t_2(x)$.

3 Pure Intermediaries

In this Section, we consider the case of *pure information transmission*, when the type of the intermediary is known and the only piece of private information is the agent's type. Hence γ is publicly known by the agents and the principal, and the objective of the principal is to extract information about θ .

3.1 Direct Contracting: the second-best.

Using standard methods, we can easily compute the optimal contract offered by the principal when she can directly contract with the agent. As the intermediary has no private information, we clearly have $t_1 = \gamma x$. By the revelation principle, we can restrict attention to direct mechanisms. The principal chooses his decision x and transfer t_2 in order to maximize her expected utility

$$\int_{\underline{\theta}}^{\overline{\theta}} W(x,\gamma,\theta) dF(\theta),$$

subject to the agent's individual rationality and incentive compatibility constraints:

$$\begin{array}{rcl} \theta x(\theta) - t(\theta) & \geq & 0 \; \forall \; \theta \; \left(IR \right) \\ \theta x(\theta) - t(\theta) & \geq & \theta x(\widehat{\theta}) - t(\widehat{\theta}) \; \; \forall \; \theta, \widehat{\theta} \; \left(IC \right) \end{array}$$

The (standard) solution to this problem is given by the next Proposition.

Proposition 1 When the principal can directly contract with the agent, the optimal contract is given by:

$$x^{C}(\gamma,\theta) = \begin{cases} 0 & W_{x}(0,\gamma,\theta) + \gamma + J(\theta) < 0\\ 1 & if & W_{x}(1,\gamma,\theta) + \gamma + J(\theta) > 0\\ \{x|W_{x}(x,\gamma,\theta) + \gamma + J(\theta) = 0\} & otherwise \end{cases}$$
(1)

Transfers are given by

$$t_1^C(\gamma,\theta) = \gamma x^C(\gamma,\theta), t_2^C(\gamma,\theta) = \theta x^C(\gamma,\theta) - \int_{\underline{\theta}}^{\theta} x^C(\gamma,s) ds.$$
(2)

3.2 Contracting through the Intermediary

We now characterize the optimal indirect contract, considering in turn all possible levels of commitment of the intermediary.

3.2.1 Contracting through a Messenger

Suppose that the intermediary cannot commit to any action. In this very restrictive setting, the following proposition shows that the behavior of the principal and intermediary are severely constrained.

Proposition 2 When the intermediary cannot commit, the optimal contract is such that the principal offers a constant contract $x^N(\gamma)$ such that

$$\int_{\underline{\theta}}^{\overline{\theta}} W_x(x(x^N(\gamma)), \gamma, \theta) f(\theta) + \gamma = 0,$$

The transfers are given by $t_2^N(\gamma) = 0, t_1^N(\gamma) = \gamma x^N(\gamma, \theta).$

Proposition 2 is based on the observation that when the intermediary cannot commit, the agent's transfer has no impact on the intermediary's choice, and the agent thus has no incentive to pay a positive transfer. As the intermediary chooses his participation to the contract after he receives the agent's transfer, the principal and the intermediary contract as if the agent were not present, and the principal extracts all the rents from the intermediary.

3.2.2 Contracting through a Gatekeeper

We now suppose that the intermediary can commit to the probability q of accepting the principal's contract. We first define the *benevolent* choice of policy as the schedule

$$x^*(\gamma, \theta) = \begin{cases} 0 & W_x(0, \gamma, \theta) + \gamma < 0\\ 1 & if & W_x(1, \gamma, \theta) + \gamma > 0\\ \{x | W_x(x, \gamma, \theta) + \gamma = 0\} & \text{otherwise.} \end{cases}$$
(3)

This is the policy that the principal would choose even in the absence of any transfer coming (indirectly) from agent 2.

Proposition 3 Suppose that the intermediary can only commit on his acceptance of the principal's contract. Then, in the optimal contract, the principal chooses two thresholds θ^* and θ^{**} such that the optimal decision satisfies:

$$x^{P}(\gamma,\theta) = \begin{cases} 0 & \theta < \theta^{*} \\ x^{*}(\gamma,\theta^{**}) & if \quad \theta^{*} \leq \theta \leq \theta^{**} \\ x^{*}(\gamma,\theta) & \theta^{**} \leq \theta. \end{cases}$$
(4)

The thresholds are chosen in order to maximize

$$U_{0} = \int_{\theta^{*}}^{\theta^{**}} W(x^{*}(\gamma, \theta^{**}), \gamma, \theta) + \gamma x^{*}(\gamma, \theta^{**}) dF(\theta) + \int_{\theta^{**}}^{\overline{\theta}} W(x^{*}(\gamma, \theta), \gamma, \theta) + \gamma x^{*}(\gamma, \theta) dF(\theta) + [1 - F(\theta^{*})] x^{*}(\gamma, \theta^{**}) J(\theta^{*}).$$

Transfer t_1 is independent of θ and given by

$$t_1(\gamma) = \gamma x^P(\gamma, \theta) + x^{P*}(\gamma, \theta^{**})J(\theta^*).$$

Given this contract (C1), along the equilibrium path, the intermediary chooses a contract (C2) where $q(\theta) = 1$ for all $\theta \ge \theta^*$ and $q(\theta) = 0$ for all $\theta < \theta^*$. Finally, the transfer t_2 satisfies

$$t_2(\gamma, \theta) = \theta x^P(\gamma, \theta) - \int_{\theta^*}^{\theta} x^P(\gamma, s) ds,$$

for all $\theta \geq \theta^*$.

Proposition 3 completely characterizes the optimal contracts offered when the intermediary is a gatekeeper. Along the equilibrium path, the intermediary will never reject any contract when the public good is provided $(q(\theta) = 1)$ if and only if $x(\theta) > 0$. However, the principal anticipates the reaction of the intermediary, and this affects the shape of her optimal contract. For low types, the principal's contract may involve *bunching at the bottom* as the principal chooses a high level of policy which results in high transfers. For high types, the principal does not care about the transfers she receives from the agent through the intermediary, and chooses instead her "benevolent decision", which does not take into account the transfer received indirectly from the agent. Furthermore, notice that the principal does not necessarily receive a positive transfer from the intermediary, but may in fact give a transfer to the intermediary (as $J(\theta^*)$ may be negative). Finally notice that all three agents (including the intermediary) capture positive rents in the contract with partial commitment.

3.2.3 Contracting through a Speaker

We finally consider the case of an intermediary who is able to commit on the message he sends to the principal. To this end, we will further assume that the hazard rate is concave, and define the increasing function:

$$\mathcal{J}(\theta) = J(\theta) - J'(\theta) \frac{1 - F(\theta)}{f(\theta)}.$$

The following Proposition characterizes the optimal contract in this setting.

Proposition 4 Suppose that the hazard rate is increasing and concave. When the intermediary can commit on the message sent to the principal, there exists and optimal contract where $\mu(\gamma, \theta) = \theta$,

$$x^{F}(\gamma,\theta) = \begin{cases} 0 & W_{x}(0,\gamma,\theta) + \gamma + \mathcal{J}(\theta) < 0\\ 1 & if & W_{x}(1,\gamma,\theta) + \gamma + \mathcal{J}(\theta) > 0\\ \{x|W_{x}(x,\gamma,\theta) + \gamma + \mathcal{J}(\theta) = 0\} & otherwise. \end{cases}$$

The transfers are given by

$$t_1(\gamma,\theta) = (\gamma + J(\theta))x^F(\gamma,\theta) - \int_{\underline{\theta}}^{\theta} J'(s)x^F(\gamma,s)ds,$$

$$t_2(\gamma,\theta) = \theta x^F(\gamma,\theta) - \int_{\underline{\theta}}^{\theta} x^F(\gamma,s)ds.$$

Proposition 4 illustrates the well-known problem of *double marginalization* of rents. When the intermediary commits on the message, the situation is equivalent to a three-tier hierarchy where every agent (agents 1 and 2) extract a rent. The principal's optimal contract will result in a low level of public good provision, as the principal will try to limit the rents of the agents. In particular as $J(\theta) > \mathcal{J}(\theta)$, it is easy to check that the level of public good provision under full commitment will always be lower than under direct contracting,, $x^F(\gamma, \theta) < x^C(\gamma, \theta)$. The contract becomes equivalent to an incentive contract between the principal and an agent whose valuation is given by the virtual valuation $J(\theta)$. Notice that because $J(\theta)$ may be negative, the intermediary's utility may be *decreasing in* θ , forcing the principal to give a transfer to the intermediary for low levels of θ .

3.3 Comparing the contracts

In this subsection, we compare the optimal contracts under the different commitment régimes.

3.3.1 Utilities of the intermediary, principal and agent

We first observe that (as shown in the Proof of Proposition 7), whenever the policy schedule of the principal $x(\theta)$ is nondecreasing, the optimal choice of the intermediary is to adopt a *deterministic* acceptance strategy: $q(\theta) = 1$ for all $\theta \ge \theta^*$ where $\theta^* = \min \theta | x(\theta) > 0$. By committing to choosing $\mu(\theta) = \theta$) for $\theta \ge \theta^*$ and $\mu(\theta) = \theta^*$ for $\theta < \theta^*$, the intermediary could have generated the same outcome. Hence, by a revealed preference argument, faced with an increasing schedule $x(\theta)$, the intermediary weakly prefers to commit than not commit. This argument also shows that, in an extended game where the intermediary could endogenously choose whether or not to commit after observing the principal's contract C_1 , there will always be an equilibrium where he chooses to commit, and hence the full commitment contract would be the only outcome.

However, the preceding argument does not show that the equilibrium utility of the intermediary will always be higher in the full commitment contract, as the commitment capacity of the intermediary changes the contract proposed by the principal C_1 .

Concerning the principal, the revelation principle guarantees that she can do no worse by direct contracting than by the indirect contract. In the nocommitment case, the principal does not extract any rent from the agent, and hence does worse than in the partial and full commitment cases.

To compare the principal's utility in the game with gate keepers and speakers, notice that the policy schedule $x^P(\theta)$ can be implemented both in the partial and full commitment cases, with different transfers ,

$$\begin{split} t_1^P(\theta) &= \gamma x^P(\theta) + \theta^* x^P(\theta^*), \\ t_1^F(\theta) &= \gamma x^P + J(\theta) x^P(\theta) - \int_{\underline{\theta}}^{\theta} J'(s) x^P(s) ds \end{split}$$

A direct computation shows that

$$\begin{split} t_1^F(\theta) - t_1^P(\theta) &= x^P(\theta)J(\theta) - \int_{\theta^{*P}}^{\theta} x^P(s)J'(s)ds - x^P(\theta^{*P})J(\theta^{*P}) \\ &\geq (x^P(\theta) - x^P(\theta^{*P}))J(\theta^{*P}). \end{split}$$

Hence if at the threshold level θ^{*P} , $J(\theta^{*P}) \geq 0$, the principal can implement $x^{P}(\theta)$ under full commitment with larger transfers and hence prefers the full commitment contract to the partial commitment contract.⁵

Finally, the agent's utility is given by:

⁵However, notice that if $(J(\theta^{*P}) < 0)$, the converse statement is not necessarily true: the principal would always prefer the partial commitment contract to the full commitment contract only if the *expected value of* $x^{F}(\theta)J(\theta)$ were negative.

$$U_2(\theta) = \int_{\theta^*}^{\theta} x(s) ds$$

in all the contracts. As the policy schedules $x(\theta)$ cannot in general be ranked for all types θ , we do not expect any uniform ranking of the different types of contracts from the point of view of the agent.

3.3.2 An equivalence result for linear utility functions

We now specialize the model by supposing that the principal's objective is *linear* in x. (This would be the case if the principal was a monopolist or a social planner facing a linear cost, or if the social planner was choosing the probability to provide a discrete public good with fixed cost.)

When the objective function of the principal is linear, it is easy to check that the optimal contract always involves a threshold rule θ^* , such that $x(\theta) = 1$ for $\theta \ge \theta^*$ and $x(\theta) = 0$ otherwise.

In the direct and full commitment contracts, the thresholds are easily computed to satisfy

$$W_x(x,\gamma,\theta^{*C}) + \gamma + J(\theta^{*C}) = 0,$$

$$W_x(x,\gamma,\theta^{*F}) + \gamma + \mathcal{J}(\theta^{*F}) = 0.$$

In the partial commitment case, the principal chooses the threshold to maximize

$$U_0 = \int_{\theta^*}^{\overline{\theta}} W_x(x,\gamma,\theta) + \gamma dF(\theta) + [1 - F(\theta^*)]J(\theta^*).$$

Taking first order conditions with respect to the threshold θ^* , we obtain

$$W_x(x,\gamma,\theta^{*P}) + \gamma + \mathcal{J}(\theta^{*P}) = 0$$

Hence, the optimal contracts are identical in the partial commitment and full commitment cases.

It is important to point out that this equivalence results depend crucially on the linearity of the principal's objective function, which implies that her optimal decision rules is a threshold rule. The constraint that the transfer from the intermediary be invariant to the announcement in the partial commitment contract does not impose any cost to the principal. Furthermore, given that contract C_1 is a threshold contract, the behavior of the intermediary is the same in the partial and full commitment cases. He chooses optimally to partition the set of types into two sets: one where he participates in the contract (and fully reveals the agent's type), and one where he does not participate (or reveals a very low type). Hence the optimal contracts are identical in the full and partial commitment case.

3.3.3Examples

When the principal's objective is not linear in the public decision x, she would optimally choose a decision tailored to the type of the agent. In this case, the four types of contracts will result in different public policy schedules. The next two examples show that it may be impossible to rank the public decisions. Both examples assume a quadratic cost of providing the public decision, a parameter $\gamma = 0$ and a uniform distribution of the agent's type, $F(\theta) = \theta$ for $\theta \in [0, 1]$.

Example 1 Benevolent social planner

For a uniform distribution, $J(\theta) = 2\theta - 1$ and $\mathcal{J}(\theta) = 4\theta - 3$. Direct computations show that

$$\begin{split} x^{C}(\theta) &= \left\{ \begin{array}{cc} \frac{\theta(2\lambda-1)-(\lambda-1)}{\lambda} & if \quad \theta \geq \frac{\lambda-1}{2\lambda-1} \\ 0 & otherwise. \end{array} \right. \\ x^{F}(\theta) &= \left\{ \begin{array}{cc} \frac{\theta(4\lambda-3)-3(\lambda-1)}{\lambda} & if \quad \theta \geq \frac{3\lambda-3}{4\lambda-3} \\ 0 & otherwise. \end{array} \right. \end{split}$$

In the no-commitment case,

$$U_0 = \frac{1}{\lambda - 1} \int_0^1 (\theta x - \frac{\lambda x^2}{2}) d\theta$$

It is easy to check that $x^N = \frac{1}{2\lambda}$. Finally, in the partial commitment case, the benevolent policy schedule is given by $x^*(\theta) = \frac{\theta}{\lambda}$ and the principal chooses the two thresholds θ^* and θ^{**} to maximize

$$U_{0} = \int_{\theta^{*}}^{\theta^{**}} \left[\frac{\theta\theta^{**}}{\lambda(\lambda-1)} - \frac{\theta^{**2}}{2\lambda(\lambda-1)}\right] d\theta + \int_{\theta^{**}}^{1} \frac{\theta^{2}}{2\lambda(\lambda-1)} d\theta$$

+ $(1-\theta^{*})(2\theta^{*}-1)\frac{\theta^{**}}{\lambda}$
= $\frac{1}{\lambda} \left[\frac{\theta^{*}\theta^{**}(\theta^{**}-\theta^{*})}{2(\lambda-1)} + \frac{(1-\theta^{**3})}{6(\lambda-1)} + (1-\theta^{*})\theta^{**}(2\theta^{*}-1)\right].$

Tedious computations, given in the Appendix, allow us to compute the optimal thresholds as a function of λ .

1. If $1 \le \lambda \le \frac{5}{4}$, $\theta^* = \theta^{**} = \frac{6(\lambda - 1)}{8\lambda - 7}$ and $x(\theta) = \frac{\theta}{\lambda}$ for all $\theta \ge \theta^*$.

2. If
$$\frac{5}{4} \le \lambda \le \frac{3}{2}$$
, $\theta^* \frac{48\lambda^2 - 87\lambda + 39 - \sqrt{16\lambda^3 - 43\lambda^2 + 36\lambda - 9}}{45 - 108\lambda + 64\lambda^2}$ and $\theta^{**} = \frac{2(6(\lambda - 1) - \sqrt{16\lambda^3 - 43\lambda^2 + 36\lambda - 9})}{16\lambda - 15}$
 $x(\theta) = \frac{\theta^{**}}{\lambda}$ for all $\theta \in [\theta^*, \theta^{**}]$ and $x(\theta) = \frac{\theta}{\lambda}$ for all $\theta \in [\theta^{**}, 1]$.

3. If
$$\lambda \geq \frac{3}{2}$$
, $\theta^* = \frac{(6\lambda - 5)}{2(4\lambda - 3)}$, $\theta^{**} = 1$ and $x(\theta) = \frac{1}{\lambda}$ for all $\theta \in [\theta^*, 1)$.

The three different régimes can easily be interpreted.

[1] If $\lambda \leq \frac{5}{4}$, the cost of public good provision is low. The principal thus puts most weight on the provision of the public good, and very little weight on the transfer. She may even choose to give money to the intermediary $(t_1 < 0$ when $\theta^* < \frac{1}{2}$).

For intermediate values of the cost, $\lambda \in (\frac{5}{4}, \frac{3}{2})$, the principal chooses interior values of θ^* and th^{**} . In this case, the principal provides more than the value $x^*(\theta)$ for $\theta \in [\theta^*, t^*]$, and exactly $x^*(\theta)$ for $\theta \ge t^*$.

[3] Finally, when the social cost of public funds is high, $\lambda \geq \frac{3}{2}$, the weight of the transfer in the principal's objective becomes predominant. The principal then chooses to extract the highest possible transfer from the intermediary, setting a fixed level of public good, $\hat{x} = \frac{1}{\lambda}$ for all $\theta \geq \theta^*$.

Figure 1 illustrates the four contracts for $\lambda = \frac{4}{3}$. It shows that the optimal policy under no-commitment and partial commitment differ from the direct and full commitment policies, and may be *higher* than the second best policy for some values of the agent's type.

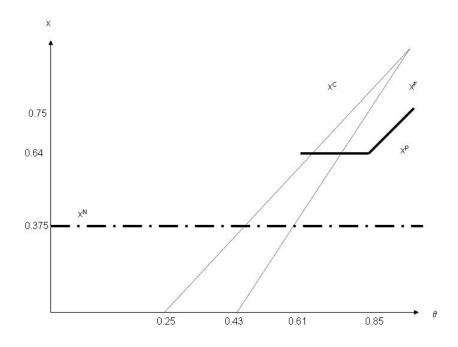


Figure 1: Optimal policy decision for a benevolent social planner

Example 2 Monopolist

Suppose that $W(x, \gamma, \theta) = -c(x)$ and let $\gamma = 0$. When the principal only incurs a cost of providing the good, the optimal policy in the direct contract is given by

$$x^{c}(\theta) = \{x | c'(x) = J(\theta),\$$

for all θ such that $c'(1) > J(\theta) > c'(0)$.

Similarly, the optimal policy in the contract with speakers is given by

$$x^{c}(\theta) = \{x | c'(x) = \mathcal{J}(\theta),\$$

for all θ such that $c'(1) > \mathcal{J}(\theta) > c'(0)$.

In the contract with messengers, it is easy to see that the optimal policy is to choose $x^N = 0$ for all θ .

In the contract with gate keepers, the benevolent policy must specify $x^*(\theta) = 0$. Hence, the principal will choose only one threshold θ^* and a fixed policy to maximize:

$$U_0 = [1 - F(\theta^*))(x^*J(\theta^*) - c(x^*)),$$

When the solution is interior, it satisfies the first order conditions:

$$c'(x^*) = J(\theta^*),$$

 $\frac{c(x^*)}{x^*} = \mathcal{J}(\theta^*),$

It is easy to check that (when the solutions are interior) $x^P(\theta) \leq x^C(\theta)$ for all θ . First observe that the threshold above which the good is produced is higher in the contract with gatekeepers than in the direct contract (the thresholds are determined by $c'(0) = \theta^*$ and $c'(x^*) = \theta^*$ respectively.) Furthermore, letting θ^{*P} denote the threshold in the partial commitment contract, $x^P(\theta) = x^P(\theta^{*P}) = x^C(\theta^{*P}) < x^c(\theta)$ for all $\theta > \theta^{*P}$.

The following graph (for a uniform distribution and a quadratic cost function) shows that the production schedules in other contracts may not be ranked uniformly for all θ .

4 Intermediary's type is Private Information.

We now consider the case where the intermediary cares for the public decision, and his valuation γ is not known by the other agents. We again discuss the optimal contracts for various contractual arrangements.

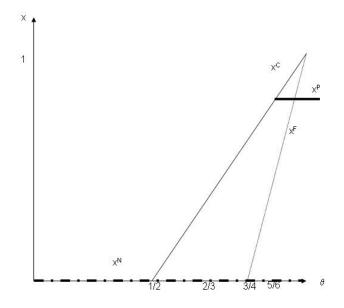


Figure 2: Optimal policy decision for a monopolist

4.1 Direct Contract

When the principal deals directly with both agents, the problem is a standard problem in Bayesian mechanism design. The principal asks agent $i \in \{1, 2\}$ to pay a transfer $t_i(\gamma, \theta)$ in exchange for providing a quantity $x(\gamma, \theta)$ of the public good. By appeal to the revelation principle, we restrict attention to direct mechanisms for which truth-telling is a Bayesian Nash equilibrium. The principal thus chooses a schedule $\{x(\gamma, \theta), t_1(\gamma, \theta), t_2(\gamma, \theta)\}$ in order to maximize her utility

$$U_0 = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\gamma}}^{\overline{\gamma}} [W(x(\gamma,\theta),\gamma,\theta) + t_1(\gamma,\theta) + t_2(\gamma,\theta)] dG(\gamma) dF(\theta).$$

subject to the agent's Bayesian individual rationality and incentive constraints:

$$\begin{split} E_{\theta}[x(\gamma,\theta)\gamma - t_{1}(\gamma,\theta) &\geq 0, \\ E_{\theta}[x(\gamma,\theta)\gamma - t_{1}(\gamma,\theta) &\geq E_{\theta}[x(\gamma',\theta)\gamma - t_{1}(\gamma',\theta)0, \forall \gamma' \neq \gamma, \\ E_{\gamma}[x(\gamma,\theta)\theta - t_{2}(\gamma,\theta) &\geq 0, \\ E_{\gamma}[x(\gamma,\theta)\theta - t_{2}(\gamma,\theta) &\geq E_{\gamma}[x(\gamma,\theta')\theta - t_{2}(\gamma,\theta')\forall \theta' \neq \theta. \end{split}$$

The standard solution to this problem is given by

Proposition 5 When the principal contracts directly with both agents, the optimal contract is characterized by the public decision $x^{C}(\gamma, \theta)$ which is chosen to maximize:

$$U_0 = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\gamma}}^{\overline{\gamma}} \left[W(x(\gamma,\theta),\gamma,\theta) + x(\gamma,\theta) \left(H(\gamma) + J(\theta) \right) \right] dG(\gamma) dF(\theta)$$

The transfers satisfy the following conditions:

$$E_{\theta}t_{1}(\gamma,\theta) = E_{\theta}x(\gamma,\theta)\gamma - E_{\theta}\int_{\underline{\gamma}}^{\gamma}x(s,\theta)ds,$$

$$E_{\gamma}t_{2}(\gamma,\theta) = E_{\gamma}x(\gamma,\theta)\theta - E_{\gamma}\int_{\theta}^{\theta}x(\gamma,s)ds.$$

4.2 Contracting through the Intermediary.

4.2.1 Contracting through a Messenger

Proposition 6 When the intermediary cannot commit, the optimal contract of the principal is independent of the report of the agent. The principal chooses the schedule $x(\gamma)$ such that

$$\int_{\underline{\theta}}^{\overline{\theta}} W_x(x(\gamma), \gamma, \theta) f(\theta) + K(\gamma) = 0.$$

Transfers are given by

$$t_1(\gamma) = \gamma x(\gamma) - \int_{\underline{\gamma}}^{\gamma} x(s) ds,$$

$$t_2(\gamma) = 0.$$

Proposition 6 generalizes Proposition 2 to the situation where the intermediary's type is unknown. The public decision is invariant with respect to the agent's type – but now depends on the type of the intermediary. We note that the rent collected by the intermediary is purely an informational rent, and not a rent due to the intermediary's position.

4.2.2 Contracting through a Gatekeeper

Proposition 7 When the intermediary can only commit on the probability of accepting the principal's contract, the principal's optimal contract $x(\gamma), t_1(\gamma)$ is independent of the agent's type. The principal optimally chooses an increasing public decision schedule $x(\gamma)$, the lowest value at which the public good is provided, γ^* and a constant t^* in order to maximize

$$\begin{split} U_0 &= \int_{\gamma^*}^{\overline{\gamma}} \int_{\theta^*(\gamma)}^{\overline{\theta}} W(x(\gamma), \gamma, \theta) dF(\theta) dG(\gamma) \\ &+ \int_{\gamma^*}^{\overline{\gamma}} (1 - F(\theta^*(\gamma))) (K(\gamma)x(\gamma) - x(\gamma^*)\gamma^* + t^*) dG(\gamma), \end{split}$$

where $\theta^*(\gamma)$ is implicitly defined by the equation:

$$x(\gamma)J(\theta)) + x(\gamma^*)\gamma^* - t^* + \int_{\gamma^*}^{\gamma} x(s)ds = 0.$$

The transfer $t_1(\gamma)$ is defined by:

$$t_1(\gamma) = x(\gamma)\gamma - x(\gamma^*)\gamma^* + t^* - \int_{\gamma^*}^{\gamma} x(s)ds.$$

Along the equilibrium path, the intermediary chooses a contract $q(\gamma, \theta), t_2(\gamma, \theta)$ where $q(\gamma, \theta) = 1$ if $\theta \ge \theta^* \gamma, q(\gamma, \theta) = 0$ otherwise and

$$t_2(\gamma, \theta) = \theta^*(\gamma) x(\gamma).$$

Proposition 7 characterizes the optimal contract in the partial commitment case. As in the case of pure intermediaries, the intermediary will *always* accept a contract for which the public good is provided, but the principal's choice of the region for which the public good is provided (namely values of (γ, θ) such that $\theta \ge \theta^*(\gamma)$), depends on the reaction of the intermediary. A remarkable feature of the optimal contract is that the principal's decision is *independent* of the agent's report. This result stands in sharp contrast to the case of pure intermediaries. Intuitively, in order to induce the intermediary to report truthfully her value γ , and knowing that the intermediary's participation decision is independent of the agent's type. Finally, we note that the optimization problem faced by the principal is very complex. It involves computing a schedule $x(\gamma)$ which is nondecreasing, a threshold value γ^* , and a constant for the transfer t^* . The general solution to this problem is not easy to compute using standard techniques of optimal control theory.

4.2.3 Contracting through a Speaker

Proposition 8 When the intermediary can commit on the message sent to the principal, there exists an optimal contract for the principal which only depends on $\delta = \gamma + J(\theta)$. Letting $[\underline{\delta}, \overline{\delta}]$ denote the support of δ , $H(\delta)$ the distribution,

and $\mathcal{L}(\delta) = \delta - \frac{1-H(\delta)}{h(\delta)}$, the principal chooses an increasing schedule $x(\delta)$ in order to maximize:

$$U_{0} = \int_{\underline{\delta}}^{\overline{\delta}} \int_{J^{-1}(\delta - \overline{\gamma})}^{J^{-1}(\delta - \underline{\gamma})} [W(x(\delta), \delta - J(\theta), \theta) + x(\delta)\mathcal{L}(\delta)] dF(\theta) dH(\delta).$$

The transfer is given by

$$t_1(\delta) = \delta x(\delta) - \int_{\underline{\delta}}^{\delta} x(s) ds.$$

The intermediary chooses to report the agent's message and his type truthfully, $\mu(\gamma, \theta) = (\gamma, \theta)$ and requires a transfer

$$t_2(\gamma, \theta) = \theta x(\gamma + J(\theta)) - \int_{\underline{\theta}}^{\theta} x(\gamma + J(s)) ds$$

Proposition 8 generalizes the optimal contract of Proposition 4 to the case where the intermediary's valuation of the public decision is unknown. As in the former contract, there is no loss of generality in assuming that the intermediary commits to truthfully reveal the two types to the principal. It turns out that the report of the intermediary can be summarized by $\delta = \gamma + J(\theta)$, so that the problem becomes a one-dimensional problem, and the optimal contract can be characterized using standard techniques. However, as in the case of the contract with gatekeepers, we notice that, in order to compute the optimal public policy, one has to solve a complex problem in optimal control theory.

4.3 Comparing the contracts

When the intermediary's type is private information, the comparison of the four different contracts raises additional problems. The direct contract relies on a different informational structure than the indirect contracts: in the direct contract, the agent and the intermediary ignore each other's type, whereas they know each other's type when the contract is mediated by the intermediary. Hence, the revelation principle (at least in its standard form) does not apply and the principal does not necessarily prefer the direct contract to an indirect contract.

It remains true that, as long as the contract x(.) is nondecreasing in θ , int he partial commitment contract, the intermediary will always choose q = 1whenever x > 0. Now, as $J'(\theta) > 0$, δ is increasing in θ . Hence, in the optimal contract under full commitment, the public decision is increasing in θ . This shows that, as in the case of pure intermediaries, the intermediary would always choose to commit, if she could endogenously choose whether or not to commit after observing the principal's contract.

More surprisingly, we will establish that when the principal has a linear objective function, the equivalence between partial and full commitment fails, and the principal prefers the partial commitment contract. To see this, notice that if $W(x, \gamma, \theta)$ is linear (and assuming that $\mathcal{L}(\delta)$ is increasing), the optimal policy of the principal in the full commitment contract is a threshold policy, with $x(\delta) = 1$ when $\delta \geq \delta^*$ and $x(\delta) = 0$ otherwise, where

$$\int_{J^{-1}(\delta-\overline{\gamma})}^{J^{-1}(\delta-\underline{\gamma})} [W_x(x,\delta-J(\theta),\theta) + \mathcal{L}(\delta)] dF(\theta) = 0$$

and $t_1(\delta) = \delta^*$ for all $\delta \ge \delta^*$.

Now, we claim that the principal could implement the same outcome in the partial commitment contract, by choosing $x(\gamma) = 1$ and $t_1(\gamma) = \delta^*$ for all γ . In fact, given this schedule, the principal will provide the public good if and only if:

$$J(\theta) + \gamma - \delta^* \ge 0.$$

so that the principal provides the public good (and extracts the same transfer) over exactly the same range of parameters as in the full commitment contract. This result shows that the principal can always do at least as well in the partial commitment than in the full commitment contract.

4.3.1 Examples

Example 3 Uniform distribution ; $W(x, \gamma, \theta) = 0$

This is the simplest example, and yet computations are hairy. Consider first the full commitment contract.

With a uniform distribution $J(\theta) = 2\theta - 1$. Hence, the distribution of δ is the distribution of $\gamma + 2\theta - 1$ where γ and θ are both uniform distributions on [0, 1]. Painful but straightforward computations show that

$$H(\delta) = \begin{cases} \frac{1}{4}(\delta+1)^2 & -1 \le \delta \le 0\\ \frac{1}{4} + \frac{\delta}{2} & if \quad 0 \le \delta \le 1\\ 1 - \frac{1}{4}(2-\delta)^2 & 1 \le \delta \le 2. \end{cases}$$

So that

$$h(\delta) = \begin{cases} \frac{\delta+1}{2} & -1 \le \delta \le 0\\ \frac{1}{2} & if & 0 \le \delta \le 1 \\ \frac{(2-\delta)}{2} & 1 \le \delta \le 2. \end{cases}$$

and

$$\mathcal{L}(\delta) = \delta - \frac{1 - H(\delta)}{h(\delta)} = \begin{cases} \delta - \frac{2}{1+\delta} + \frac{1+\delta}{2} & -1 \le \delta \le 0\\ 5\frac{\delta}{4} - \frac{3}{8} & if \quad 0 \le \delta \le 1\\ \frac{3\delta}{2} - 1 & 1 \le \delta \le 2. \end{cases}$$

It is easy to check that \mathcal{L} is an increasing function: it is increasing in all the intervals, and exhibits upward jumps at $\delta = 0$ and $\delta = 1$. Hence, the conditions for the characterization of the optimal contract are satisfied, and one can see that $\mathcal{L}(\delta) = 0$ if and only if $\delta = \delta^* = \frac{3}{10}$. Hence, in the full commitment contract, the principal supplies the good when $\delta \geq \frac{3}{10}$ and does not supply the good otherwise.

Turning now to the partial commitment contract, the principal will choose the schedule $x(\gamma)$, the threshold γ^* and the transfer t^* in order to maximize:

$$U_0 = \int_{\gamma^*}^1 (1 - F(\theta^*(\gamma)))((2\gamma - 1)x(\gamma) - x(\gamma^*)\gamma^* + t^*)d\gamma,$$

where

$$\theta^*(\gamma) = \frac{1}{2} + \frac{t^* - \gamma^* x(\gamma^*) - \int_{\gamma^*}^{\gamma} x(s) ds}{x(\gamma)}$$

We thus see that the principal's utility is not linear in x because the decision $x(\gamma)$ also affects the set of parameters for which the intermediary accepts the contract, $\theta^*(\gamma)$. We conjecture, but haven't proven yet, that this non-linearity may result in the principal choosing an optimal schedule which is different from the simple threshold rule. This would then show that there exist conditions under which the principal strictly prefers the partial commitment contract to the full commitment contract.

5 Conclusion

This paper analyzes the power of intermediaries in screening models. It shows that the presence of the intermediary results in a double marginalization of rents, and hence reduces the utility of the principal. By comparing three different types of intermediaries (messengers who cannot commit, gatekeepers who commit on their participation, and speakers who commit on the report), the paper shows how different levels of commitment distort the contract offered by the principal. When the intermediary does not care about the public decision, the presence of a gatekeeper will result in an optimal contract with a discontinuous jump and bunching at the bottom, whereas speakers lead to a uniform decrease in the level of public good provision. When the intermediary cares about the public decision, in the presence of a gatekeeper, the contract is independent of the agent's type, whereas the contract with speakers depends on a one-dimensional parameter, the sum of the intermediary's valuation and the agent's virtual valuation. We show that, if the intermediary could endogenously choose whether to commit, he will always commit; that the principal may or may not prefer full commitment over partial commitment. When the principal's objective is linear in the decision, full commitment and partial commitment are equivalent in the case of a pure intermediary, but not when the intermediary's valuation of the public good is unknown.

While the analysis of this paper sheds some light on the role of intermediaries in mechanism design, we are aware of a number of open questions that could fruitfully be studied in the future. First, the model lends itself to the analysis of the trade-off between decentralization and delegation: should the principal choose to delegate the decision to one of the agents, or to decentralize it to an outsider who has no vested interest in the decision? Second, we should analyze by-pass in the model: supposing that the principal can (at a cost) directly contact the agent, and that he may do so after observing the intermediary's report, how can this by-pass possibility discipline the intermediary, and increase the principal's payoff? Finally, our model can be interpreted as a very simple model of strategic information transmission in networks. How would our results be affected if we increased the depth of the hierarchy? What would happen if the network displayed some cycles? All these questions seem to us to deserve further study.

6 Proofs of Section 3

In order to simplify notations, we omit the reference to the valuation of the intermediary, γ , in the contracts.

Proof of Proposition 2. Consider first the contract (C2) between the intermediary and the agent. As the intermediary's choices are in contract C1 are independent of the transfer t_2 received from the agent, the agent has no incentive to pay any transfer to the intermediary and $t_2 = 0$. Incentive compatibility then implies that the principal's decision, x is constant.

The intermediary's utility is given by

$$U_1\gamma x - t_1$$

Transfer t_1 can be chosen in order to eliminate all the rents of the intermediary:

$$t_1 = \gamma x.$$

Plugging back into the principal's utility function, we obtain the characterization given in the Proposition.

Proof of Proposition 3. The main difficulty of the proof is to characterize the optimal contract offered by the intermediary for all possible contracts C1 offered by the principal.

Contract between the intermediary and the agent: individual rationality and incentive compatibility

Suppose that $C1 = x(\theta), t_1(\theta)$ is given, and consider the contract offered by the intermediary to the agent. The individual rationality and incentive compatibility constraints of the agent are given by:

$$V_{2}(\theta) \equiv q(\theta)x(\theta)\theta - t_{2}(\theta) \ge 0 \quad \forall \ \theta \quad (IR)$$

$$V_{2}(\theta) \ge q(\widehat{\theta})x(\widehat{\theta})\theta - t_{2}(\widehat{\theta}) \ \forall \ \theta, \widehat{\theta} \quad (IC)$$

Incentive compatibility implies that $q(\theta)x(\theta)$ must be nondecreasing in θ . By the first order condition and the envelope theorem:

$$\frac{dV_2}{d\theta} = q(\theta)x(\theta).$$

Moreover, individual rationality implies that $V_2(\underline{\theta}) = 0$. Hence,

$$V_2(\theta) = \int_{\underline{\theta}}^{\theta} q(s)x(s)ds.$$

or

$$t_2(\theta) = q(\theta)x(\theta)\theta - \int_{\underline{\theta}}^{\theta} q(s)x(s)ds.$$

The intermediary enjoys the following expected utility:

$$U_{1} = \int_{\underline{\theta}}^{\overline{\theta}} [q(\theta) [\gamma x(\theta) - t_{1}(\theta)] + t_{2}(\theta)] dF(\theta);$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) [x(\theta)(\gamma + J(\theta) - t_{1}(\theta))] dF(\theta)$$

The problem of the intermediary is thus to maximize U_1 subject to $q(\theta)V_2(x(\theta))$ nondecreasing. Notice that if $x(\overline{\theta}) = 0$, this problem is degenerate and we can set $q(\theta) = 0$ for all θ . Hence, in what follows we will assume that $x(\overline{\theta}) > 0$ and denote

$$\hat{\theta} = \min\{\theta | x(\theta) > 0\}.$$
(5)

Without loss of generality, let $q(\theta) = 0$ for all $\theta < \hat{\theta}$.

Moreover, observe that, because $q(\theta)V_2(x(\theta))$ is increasing, either $q(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ or there exists a unique value $\theta^* \in [\widehat{\theta}, \overline{\theta}]$ such that $q(\theta) \leq 0$ for all $\theta < \theta^*$ and $q(\theta) > 0$ for all $\theta \geq \theta^*$. The optimal choice of the intermediary will be easier to solve after we characterize the contract between 0 and 1, so this is what we examine next.

Contract between the intermediary and the principal: incentive compatibility

In the contract between the intermediary and the principal, there is no individual rationality constraint, once the intermediary has committed to accepting the contract with the principal. We still need to consider the intermediary's incentive compatibility constraint. For the intermediary to reveal truthfully agent 2's type, it must be that for all $\theta, \theta' \in [\theta^*, \overline{\theta}]$,

$$\gamma x(\theta) - t_1(\theta) \ge \gamma x(\theta') - t_1(\theta') \forall \ \theta, \theta'.$$
(6)

Equation 6 implies that $\gamma x(\theta) - t_1(\theta)$ must be constant in θ so that

$$t_1(\theta) = \begin{cases} t_1 + \gamma x(\theta) & \forall \theta \end{cases} \begin{cases} \geq \theta^*. \end{cases}$$
(7)

Hence, the intermediary's expected utility (16) becomes

$$U_1 = \int_{\widehat{\theta}}^{\overline{\theta}} q(\theta) x(\theta) \phi(\theta) dF(\theta); \tag{8}$$

where

$$\phi(\theta) = \theta - \frac{1 - F(\theta)}{f(\theta)} - \frac{t_1}{x(\theta)} \ \forall \theta \ge \theta^*.$$
(9)

Contract between the intermediary and the agent: the optimal choice of the intermediary

We now compute the optimal choice $q(\theta)$ of the intermediary. The technical difficulty of this problem is that the intermediary's valuation, $\phi(\theta)$, is not necessarily increasing or positive. By the monotone likelihood assumption, we know

that it would be increasing if $x(\theta)$ were non-decreasing. However, in order to show that $x(\theta)$ is non-decreasing, we need to compute the optimal reaction of the intermediary, $q(\theta)$ for any choice of $x(\theta)$. This is the main difficulty of the following argument.

First, we define sequences of intervals where $\phi(\theta) \leq 0$. Let $\bar{b}_0 = \underline{\theta}$. Then, if $\phi(\theta) < 0$ for some $\theta \in [\underline{\theta}, \overline{\theta}]$, define recursively for k = 1, ...M

$$\underline{b}_{k} = \min\{\theta \in [\overline{b}_{k}, \overline{\theta}], \phi(\theta) < 0\}$$

$$\overline{b}_{k} = \min\{\theta \in [\underline{b}_{k}, \overline{\theta}], \phi(\theta') \ge 0 \forall \theta' > \theta\}.$$
(10)

Next, define

$$\Phi(a,b) \equiv \int_{a}^{b} \phi(\theta) dF(\theta)$$
(11)

for $a \ge b$ and

$$k^* = \begin{cases} \min\{k | \Phi(\bar{b}_k, \bar{b}_m) > 0 \ \forall m > k\} \text{ if such } k \text{ exists} \\ \emptyset \text{ otherwise} \end{cases}$$
(12)

As a first step we show that, without loss of generality, we can restrict attention to policies $q(\theta)$ such that $q(\theta) = 0 \forall \theta < \overline{b}_{k^*}$ and $q(\theta) > 0 \forall \theta \ge \overline{b}_{k^*}$.⁶

Lemma 1 If $k^* \neq \emptyset$, then there exists an optimal schedule for which $\theta^* = \overline{b}_{k^*}$. Otherwise, if $k^* = \emptyset$, $q(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ is optimal.

Proof. We start by showing that choosing $\theta^* \neq \bar{b}_k$ for some k is weakly dominated by choosing $\theta^* = \bar{b}_k$ for some k. Suppose first that $\phi(\theta^*) < 0$. Then, the intermediary can strictly increase his payoff by choosing $q(\theta) = 0$ for all $\theta \in [\theta^*, \min\{\theta > \theta^* | \phi(\theta) > 0\}]$. Suppose next that $\phi(\theta^*) = 0$. Then the intermediary's payoff will remain identical if he chose $q(\theta) = 0$ for all $\theta \in [\theta^*, \min\{\theta > \theta^* | \phi(\theta) > 0\}]$. Finally suppose that $\phi(\theta^*) > 0$. If $\theta^* \neq \bar{b}_k$, then there exists $\bar{b}_k < \theta^*$ such that $\phi(\theta) \ge 0$ for all $\theta \in [\bar{b}_k, \theta^*]$ and the intermediary's profit will either remain identical or increase if she chose $q(\theta) > 0$ for all $\theta \in [\bar{b}_k, \theta^*]$.

Next, we show that, if $k^* \neq \emptyset$, choosing a schedule where $\theta^* = \overline{b}_{k^*}$ gives at least as much payoff than $\theta^* = \overline{b}_k$ for $k \neq k^*$. While if $k^* = \emptyset$, setting $q(\theta) = 0$ for all θ dominates.

Suppose first that either $k < k^*$ or $k^* = \emptyset$. If $k^* = \emptyset$, set $k^* = M$ for this argument. Observe that $\Phi(\bar{b}_{k^*-i}, \bar{b}_{k^*}) < 0$ for $i = 1, ..., k^* - k$. To see this, note that we must have that $\Phi(\bar{b}_{k^*-1}, \bar{b}_{k^*}) < 0$ by definition of k^* in (12). Now, consider $n \in [2, k^* - k]$ and suppose that $\Phi(\bar{b}_{k^*-i}, \bar{b}_{k^*}) < 0$ for all $i \in \{1, n\}$. If $\Phi(\bar{b}_{k^*-n}, \bar{b}_{k^*}) \ge 0$, then $\Phi(\bar{b}_{k^*-n}, \bar{b}_{k^*-i}) = \Phi(\bar{b}_{k^*-n}, \bar{b}_{k^*}) - \Phi(\bar{b}_{k^*-i}, \bar{b}_{k^*}) > 0$ for all 1 < n < r, so that $\Phi(\bar{b}_{k^*-n}, \bar{b}_m) > 0 \ \forall m > k^* - n$, contradicting the definition of k^* .

⁶If $\phi(t) = 0$ over some interval, or $\Phi(\theta^*, b_m) = 0$ there might be other optimal schedules, resulting in the same payoff for the intermediary, which start at values above \bar{b}_{k^*} and result in different payoffs for the principal.

Now, for any interval $[\overline{b}_{k-1}, \overline{b}_k]$, $\phi(\theta) \ge 0$ for $\theta \in [\overline{b}_{k-1}, \underline{b}_k)$ and $\phi(\theta) < 0$ for $\theta \in [\underline{b}_k, \overline{b}_k)$. As $q(\theta)x(\theta)$ is increasing, this implies that $\int_{\overline{b}_{k-1}}^{\underline{b}_k} q(\theta)x(\theta)\phi(\theta)f(\theta)d\theta \le q(\underline{b}_k)x(\underline{b}_k)\Phi(b_{k-1}, \underline{b}_k)$ and that $\int_{\underline{b}_k}^{\overline{b}_k} q(\theta)x(\theta)\phi(\theta)f(\theta)d\theta \le q(\underline{b}_k)x(\underline{b}_k)\Phi(\underline{b}_k, \overline{b}_k)$. Thus,

$$\int_{\overline{b}_{k}}^{\overline{b}_{k^{*}}} q(\theta)x(\theta)\phi(\theta)f(\theta)d\theta \leq \sum_{i=k+1}^{k^{*}} q(\underline{b}_{i})x(\underline{b}_{i})\Phi(\overline{b}_{i-1},\overline{b}_{i})$$

$$\leq q(\underline{b}_{k+1})x(\underline{b}_{k+1})\Phi(\overline{b}_{k},\overline{b}_{k^{*}}) + \sum_{i=k+2}^{k^{*}-1} [q(\underline{b}_{i+1})x(\underline{b}_{i+1}) - q(\underline{b}_{i})x(\underline{b}_{i})]\Phi(\overline{b}_{i},\overline{b}_{k^{*}}).$$

Again, as $q(\theta)x(\theta)$ is nondecreasing, $q(\underline{b}_{i+1})x(\underline{b}_{i+1}) - q(\underline{b}_i)x(\underline{b}_i \ge 0$. Since furthermore $\Phi(\overline{b}_i, \overline{b}_{k^*}) < 0$ for $i = k, ...k^* - 1$,

$$\int_{\overline{b}_k}^{\overline{b}_k*} q(\theta) x(\theta) \phi(\theta) f(\theta) d\theta < 0$$

Hence, this schedule is dominated by a schedule where $q(\theta) = 0 \forall \theta \in [\overline{b}_k, \overline{b}_{k^*}]$.

Next suppose that $k > k^*$. By definition of k^* , $\int_{\overline{b}_{k^*}}^{\overline{b}_k} \phi(\theta) f(\theta) d\theta \ge 0$. Hence by setting $q(\theta)x(\theta) = q(\overline{b}_k)x(\overline{b}_k)$ for all $\theta \in [\overline{b}_{k^*}, \overline{b}_k]$, the payoff of the intermediary weakly increases. Next suppose that $k > k^*$. By definition of k^* , $\int_{\overline{b}_{k^*}}^{\overline{b}_k} \phi(\theta) f(\theta) d\theta \ge 0$. Hence by setting $q(\theta)x(\theta) = q(\overline{b}_k)x(\overline{b}_k)$ for all $\theta \in [\overline{b}_{k^*}, \overline{b}_k]$, the payoff of the intermediary weakly increases.

We now assume that $k^* \neq \emptyset$. In the next step, we construct sequences of intervals over which the intermediary optimally chooses a constant schedule. Let $\overline{a}_1 = \overline{\theta}$. Then define recursively for j = 1, ...N

$$\underline{a}_{j} = \max\{\theta \leq \overline{a}_{j} | \Phi(\theta, \overline{a}_{j}) \geq 0 \text{ and } \Phi(\theta', \overline{a}_{j}) < 0 \text{ for all } \theta < \theta' < \overline{a}_{j}\};$$
$$\overline{a}_{j+1} = \max_{k > k^{*}} \{\overline{b}_{k} | \overline{b}_{k} \leq \underline{a}_{j}\}.$$

Lemma 2 For any $\theta \in [\underline{a}_j, \overline{a}_j]$, in any optimal schedule $q(\theta)x(\theta) = q(\underline{a}_j)x(\underline{a}_j)$ for all j = 1, ..N.

Proof. Assume not. Let $\theta' = \min\{\arg \max_{\theta \in [\underline{a}_j, \overline{a}_j]} q(\theta) x(\theta)\}$ and assume that $q(\theta')x(\theta') > q(\underline{a}_j)x(\underline{a}_j)$. Hence,

$$\begin{split} &\int_{\underline{a}_{j}}^{\overline{a}_{j}}q(\theta)x(\theta)\phi(\theta)dF(\theta) = \int_{\underline{a}_{j}}^{\theta'}q(\theta)x(\theta)\phi(\theta)dF(\theta) + q(\theta')x(\theta')\Phi(\theta',\overline{a}_{j})\\ < &\int_{\underline{a}_{j}}^{\theta'}q(\theta)x(\theta)\phi(\theta)dF(\theta) + (q(\theta') - \epsilon)x(\theta')\Phi(\theta',\overline{a}_{j}). \end{split}$$

for some $\epsilon > 0$. This last inequality follows from the fact that $\Phi(\theta', \overline{a}_j) < 0$ since $\theta' > \underline{a}_k$. Hence, the intermediary could increase its payoff by reducing $q(\theta)$ for $\theta \in [\theta', \overline{a}_k]$, a contradiction.

In order to conclude the characterization of the intermediary's optimal decision, we now recall that incentive compatibility of contract C2 implies that the 'effective provision of good $x' q(\theta)x(\theta)$ is non decreasing. We thus define for $\theta \geq \hat{\theta}$ the following non decreasing function

$$l(\theta) = \frac{\min\{x(t)|t \in [\theta, \overline{\theta}]\}}{x(\theta)}.$$
(13)

Note that $\min\{x(t)|t \in [\theta, \overline{\theta}]\} = x(\theta)$ if x is non decreasing for valuation larger than θ . Hence, $l(\theta)$ represent the largest 'effective provision of the good x' $q(\theta)x(\theta)$ that the intermediary can choose for any valuation θ without violating the non-decreasing constraint.

Lemma 3 If $k^* \neq \emptyset$, there is an optimal schedule where $q(\theta) = l(\theta)$ for any $\theta \geq \overline{b}_{k^*}$ and $\notin [\underline{a}_k, \overline{a}_k]$.

Proof. The intermediary cannot choose $q(\theta) > l(\theta)$ for some $\theta \in [\overline{b}_{k^*}, \overline{\theta}]$. Otherwise there exists $\theta' > \theta$ such that $q(\theta)x(\theta) > q(\theta')x(\theta')$ (since $q(\theta') \le 1$) which is not incentive compatible for the agent.

Now, assume that $q(\theta) < l(\theta)$ for some $\theta \in [\overline{b}_{k^*}, \overline{\theta}]$ and $\notin [\underline{a}_k, \overline{a}_k]$. As $\theta \in [\overline{a}_{j+1}, \underline{a}_j]$ for some j, by definition of \overline{a}_{j+1} and because $\phi(\underline{a}_j) > 0$, $\phi(\theta) > 0$. Let $\theta' = \min\{t|q(t) > q(\theta)\}$ if it exists or $\overline{\theta}$ otherwise. From Lemma 2, we know that $\theta' \in [\overline{a}_{j+1}, \underline{a}_j)$. Hence, $\Phi(\theta, \theta') > 0$. It follows that

$$\int_{\theta}^{\theta'} q(\theta) x(t) \phi(t) dF(t) = q(\theta) x(\theta) \Phi(\theta, \theta') < q(\theta') x(\theta') \Phi(\theta, \theta')$$

Hence, the intermediary could earn strictly more profit by increasing q(t) to $q(\theta')\frac{x(\theta')}{x(t)}$ for all $t \in [\theta, \theta')$

Putting together Lemmas (1), (2) and (3), we can characterize the optimal response of the intermediary for any contract $C1 = (x(\theta), t_1(\theta))$:

There exists an optimal schedule for the intermediary such that [1] $q(\theta) = 0$ for all $\theta < \theta^*$ if $k^* \neq \emptyset$ and for all θ otherwise; [2] $q(\theta) = l(\theta)$ for any $\theta \ge \overline{b}_{k^*}$ and $\notin [\underline{a}_k, \overline{a}_k]$; [3] $q(\theta) = q(\underline{a}_k) \frac{x(\underline{a}_k)}{x(\theta)}$ for any $\theta \ge \overline{b}_{k^*}$ and $\in [\underline{a}_k, \overline{a}_k]$.

Contract between the principal and the intermediary: the optimal choice of the principal

We now compute the optimal choice of the principal, $x(\theta)$. First, using the characterization of the intermediary's response, we show that the optimal choice is non-decreasing.

Lemma 4 There exists an optimal contract between 0 and 1, $(x(.), t_1)$ where x(.) is non-decreasing.

Proof. Consider a contract $(x(.), t_1)$ where x(.) is decreasing and define a new contract $(x'(.), t'_1)$ with $t'_1 = t_1$ and

$$x'(\theta) = \begin{cases} 0 & \text{for all } \theta < \overline{b}_{k^*} \\ l(\theta)x(\theta) & \text{for any } \theta \ge \overline{b}_{k^*} and \notin [\underline{a}_k, \overline{a}_k] \\ \text{for any } \theta \ge \overline{b}_{k^*} and \in [\underline{a}_k, \overline{a}_k]. \end{cases}$$

Let ϕ' and q' be the valuation and acceptance probability of the intermediary associated with the new scheme x'.

Notice that the new contract $x'(\theta)$ is nondecreasing and therefore that $\phi'(\theta) = J(\theta) - \frac{t_1}{x'(\theta)}$ is nondecreasing. As $\bar{b}_{k^*} \notin [\underline{a}_k, \overline{a}_k], x(\bar{b}_{k^*}) \geq x'(\bar{b}_{k^*}) = l(\bar{b}_{k^*})$. Hence,

$$\phi'(\overline{b}_{k^*}) \ge \phi(\overline{b}_{k^*}) > 0.$$

We thus conclude that $\phi'(\theta) > 0$ for all $\theta > \overline{b}_{k^*}$. Since $x'(\theta)$ is nondecreasing, $q'(\theta) = 1$ for all $\theta > \overline{b}_{k^*}$. Hence, for any $\theta \notin [\underline{a}_k, \overline{a}_k]$,

$$q'(\theta)x'(\theta) = l(\theta) = q(\theta)x(\theta).$$

Furthermore, for any $\theta \in [\underline{a}_k, \overline{a}_k]$,

$$q'(\theta)x'(\theta) = q(\underline{a}_k)x(\underline{a}_k) = q(\theta)x(\theta).$$

However, as $q'(\theta) = 1$, we necessarily have: $q'(\theta) \ge q(\theta)$ for all $\theta > \overline{b}_{k^*}$ and hence $x'(\theta) \le x(\theta)$.

Now consider the expected utility of the principal under the two contracts:

$$U_{0}(x,t_{1}) = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) [W(x(\theta),\gamma,\theta) + t_{1}(\theta)] dF(\theta)$$

$$= \int_{\theta^{*}}^{\overline{\theta}} [q(\theta)x(\theta) \frac{W(x(\theta),\gamma,\theta)}{x(\theta)} + \gamma + q(\theta)t_{1}] dF(\theta)$$

$$U_{0}(x',t_{1}) = \int_{\theta^{*}}^{\overline{\theta}} [q'(\theta)x'(\theta) \frac{W(x'(\theta),\gamma,\theta)}{x'(\theta)} + \gamma + q'(\theta)t_{1}] dF(\theta)$$

Now, as $t_1 \ge 0$ and $q'(\theta) \ge q(\theta)$ for all $\theta > \overline{b}_{k^*}$,

$$q(\theta)t_1 \le q'(\theta)t_1.$$

Furthermore, as $\frac{W(x(\theta), \theta)}{x(\theta)}$ is non-increasing in x and $x'(\theta) \le x(\theta)$,

$$\frac{W(x(\theta), \boldsymbol{\theta})}{x(\theta)} \leq \frac{W(x'(\theta), \gamma, \theta)}{x'(\theta)}.$$

Hence, $U_0(x', t_1) \ge U_0(x, t_1)$ proving the Lemma.

Knowing that the principal offers a non-decreasing provision of the public good $x(\theta), \phi(\theta)$ is increasing in θ . Assuming that $\phi(\overline{\theta}) \ge 0, \theta^* = \min\{\theta \ge \widehat{\theta} | \phi(\theta) \ge 0\}$. Moreover, $l(\theta) = 1$ for all θ . Hence, the intermediary's response simplifies to

$$q(\theta) = \begin{cases} 1 for \theta \ge \theta^* \\ 0 for \theta < \theta^* \end{cases}$$
(14)

Using the intermediary's response, we can rewrite the principal's payoff as

$$U_{0} = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) [W(x(\theta), \gamma, \theta) + t_{1}(\theta)] dF(\theta)$$

=
$$\int_{\theta^{*}}^{\overline{\theta}} [W(x(\theta), \gamma, \theta) + t_{1} + \theta_{1}x(\theta)] dF(\theta)$$

Mimicking the classical argument on individual rationality, it is easy to check that $\phi(\theta^*) = 0$. (Otherwise, $\phi(\theta) > 0$ for all $\theta \ge \theta^*$ and the principal could increase her profit by increasing slightly t_1 without affecting the intermediary's response). Hence,

$$t_1 = x(\theta^*)J(\theta^*).$$

Hence, the principal's problem consists in choosing a non-decreasing schedule $\{x(\theta)\}$ and a threshold θ^* to maximize

$$U_{0} = \int_{\theta^{*}}^{\overline{\theta}} [W(x,\gamma,\theta) + t_{1}(\theta)] dF(\theta);$$

$$= \int_{\theta^{*}}^{\overline{\theta}} [W(x(\theta),\gamma,\theta) + \gamma x(\theta)] dF(\theta) + [1 - F(\theta^{*})] x(\theta^{*}) J(\theta^{*}). \quad (15)$$

Recall that we define the benevolent policy choice $x^*(\gamma, \theta)$ as

$$x^*(\gamma, \theta) = \{x | W_x(x, \gamma, \theta) + \gamma = 0\},\$$

when this interior solution exists. We claim that the optimal policy of the principal must involve bunching for the low types and follow the benevolent policy choice for the high types.

Lemma 5 If $x^*(\theta) \le x(\theta^*)$, then at the optimum, $x(\theta) = x(\theta^*)$ [bunching for the low types]. If $x^*(\theta) > x(\theta^*)$, then at the optimum, $x(\theta) = x^*(\theta)$ [benevolent provision for the high types].

Proof. Notice that, in the absence of the non-decreasing constraint on $x(\theta)$, pointwise maximization leads to choosing $x^*(\theta)$. Moreover $x^*(\theta)$ is non-decreasing. It follows that if $x^*(\theta) \ge x(\theta^*)$, the constraint would indeed not bind such that $x(\theta) = x^*(\theta)$. Now, if $x^*(\theta) < x(\theta^*)$ then the non-decreasing constraint for $x(\theta)$ binds. In this case, $x(\theta) = x(\theta^*)$.

The preceding argument implies that there are three policy regimes depending on the agent's valuation θ . Below the threshold θ^* either the principal would choose a zero policy or the intermediary rejects the contract so that no policy is implemented. For $\theta^* \leq \theta^{**}$ (where θ^{**} is defined by $x^*(\theta^{**}) = x(\theta^*)$, the principal chooses a constant policy in order to extract higher transfers from the intermediary. For $\theta > \theta^{**}$, the principal cares mostly about policy, and will set her decision at the benevolent level $x^*(\theta)$. Hence, the principal will end up choosing the two threshold levels θ^* and θ^{**} in order to maximize

$$U_{0} = \int_{\theta^{*}}^{\theta^{**}} W(x^{*}(\gamma, \theta^{**})) + \gamma x^{*}(\gamma, \theta^{**}) dF(\theta) + \int_{\theta^{**}}^{\overline{\theta}} W(x^{*}(\gamma, \theta)) + \gamma x^{*}(\gamma, \theta) dF(\theta) + [1 - F(\theta^{*})]x^{*}(\gamma, \theta^{**})J(\theta^{*}).$$

as stated in the Proposition.

Proof of Proposition 4.

We characterize the optimal contracts starting with contract C_2 . The Individual Rationality and Incentive Compatibility conditions are given by

$$\begin{aligned} x(\mu(\theta))\theta - t_2\theta &\geq 0, \\ x(\mu(\theta))\theta - t_2\theta &\geq x'(\theta')\theta - t'_2(\theta') \end{aligned}$$

By a standard argument, $x(\mu(\theta))$ must be nondecreasing and the transfer of the agent must be equal to:

$$t_2(\theta) = x(\mu(\theta))\theta - \int_{\underline{\theta}}^{\theta} x(\mu(s))ds$$

Plugging back into the expected utility of the intermediary, we obtain:

$$U_{1} = \int_{\underline{\theta}}^{\overline{\theta}} [\gamma x(\mu(\theta)) - t_{1}(\mu(\theta))] + t_{2}(\theta) dF(\theta);$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} [x(\mu(\theta))(\gamma + J(\theta)) - t_{1}(\mu(\theta))] dF(\theta).$$
(16)

We now argue that the revelation principle holds, and that we can assume without loss of generality, that the intermediary reveals truthfully the type of the agent. Suppose that there exists a sequence of optimal contracts $(x(\theta), t_1(\theta), \mu(\theta), t_2(\theta))$ where $\mu(\theta) \neq \theta$. Consider the new sequence of contracts $x' = (x \circ \mu), t'_1 =$ $(t_1 \circ \mu), \mu' = identity, t'_2 = t_2$. As $x'(\theta')\theta - t'_2(\theta') = x(\mu(\theta'))\theta - t_2\theta'$ for all θ, θ' , the new contract C'_2 between the intermediary and the agent satisfies the Individual Rationality and Incentive Compatibility conditions. As $\gamma x'(\theta') + t'_2(\theta) - t'_1(\theta') = x(\mu(\theta')) + t_2(\theta) - t_1(\mu(\theta'))$ for all θ, θ' , the new contract C'_2 results in the same expected utility for the intermediary as contract C_2 (and is hence optimal). Furthermore, the Individual rationality and Incentive Compatibility constraints of the intermediary in contract C'_1 are satisfied. Finally, as $x'(\theta) = x(\mu(\theta))$ and $t'_1(\theta) = t_1(\mu(\theta))$, the principal obtains the same expected utility in contracts C'_1 and C_1 , proving the claim.

We can thus restrict attention to optimal contracts where the intermediary truthfully reveals the agents' type, i.e. the optimal contract of the intermediary involves $\mu(\theta) = \theta$. Pointwise maximization of the principal's expected utility then implies that

$$\begin{aligned} x(\theta)(\gamma + J(\theta)) - t_1(\theta) &\geq 0, (IR') \\ x(\theta)(\gamma + J(\theta)) - t_1(\theta) &\geq x(\theta')(\gamma + J(\theta)) - t_1(\theta')(IC'), \end{aligned}$$

Hence, the principal's problem is to choose a contract maximizing her expected payoff under the modified individual rationality and incentive compatibility constraints (IR') and (IC'). Standard arguments can be used to show that $x(\theta)$ must be nondecreasing, and that the transfer t_1 satisfies:

$$t_1(\theta) = x(\theta)(\gamma + J(\theta)) - \int_{\underline{\theta}}^{\theta} J'(s)x(s)ds.$$

Plugging back into the principal's objective:

$$U_{0} = \int_{\underline{\theta}}^{\overline{\theta}} \left[W(x(\theta), \gamma, \theta) + x(\theta)[\gamma + J(\theta)] - \int_{\underline{\theta}}^{\theta} J'(s)x(s)ds \right] dF(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[W(x(\theta), \gamma, \theta) + x(\theta)[\gamma + \mathcal{J}(\theta)] \right] dF(\theta).$$

we obtain the characterization of the Proposition. Finally, we note that when the hazard rate, $\eta(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is increasing and concave,

$$\mathcal{J}(\theta) = \theta - \frac{2}{h(\theta)} - \frac{h'(\theta)}{h(\theta)^3},$$

is increasing so that $x(\theta)$ is nondecreasing.

7 Proofs of Section 4

Proof of Proposition 6.

The proof follows the same steps as the proof of Proposition 2. The contract between the intermediary and the agent must specify a zero transfer and an invariant decision $x(\gamma)$.

In the contract between the principal and the intermediary, the individual rationality and incentive constraints are given by

$$\begin{array}{ll} \gamma x(\gamma) - t_1(\gamma) & \geq \\ \gamma x(\gamma) - t_1(\gamma) & \geq \gamma x(\gamma') - t_1(\gamma') \forall \gamma, \gamma'. \end{array}$$

Following standard arguments, the transfer $t_1(\gamma)$ is given by

$$t_1(\gamma) = \gamma x(\gamma) - \int_{\underline{\gamma}}^{\gamma} x(s) ds$$

Replacing in the principal's utility function,

$$U_0 = \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\underline{\theta}}^{\overline{\theta}} W(x,\gamma,\theta) dF(\theta) dG(\gamma) + \int_{\underline{\gamma}}^{\overline{\gamma}} K(\gamma) x(\gamma) dG(\gamma),$$

and pointwise maximization gives the desired result.

Proof of Proposition 7.

We first claim that the intermediary cannot gain by hiding his type to the agent, so that there is no informed principal problem.

Claim 1 With no loss of generality, we may assume that the agent knows the intermediary's type.

Proof. In the contract between the agent and the intermediary, the intermediary chooses a decision q and a transfer t_2 given the contract x, t_1 chosen by the principal. The utility of both the intermediary and the agent are quasi-linear,

$$U_1 = q(\gamma x - t_1) + t_2,$$

$$U_2 = q\theta x - t_2.$$

Following Maskin and Tirole (1990), we analyze a game where the intermediary initially proposes a mechanism assigning a random outcome $\nu(\theta, \gamma)$ (a probability distribution over probabilities q and transfers t_2) as a function of the intermediary's type and agent announcement. Next consider, as in Maskin and Tirole (1990), the full information program where the intermediary selects the random outcome in order to maximize her expected utility:

$$U_1(\gamma) = \int_{\theta^*}^{\overline{\theta}} (q(\theta, \gamma)(x(\theta, \gamma)) - t_1(\gamma, \theta)) + t_2(\gamma, \theta),$$

subject to the individual rationality and incentive constraints

$$U_{2}(\gamma, \theta^{*}) = \theta^{*}q(\gamma, \theta^{*})x(\gamma, \theta^{*}) - t_{2}(\gamma, \theta^{*}) = 0,$$

$$\theta q(\gamma, \theta)x(\gamma, \theta) - \int_{\theta^{*}}^{\theta} x(\gamma, s)ds - t_{2}(\gamma, \theta) = 0.$$

where $\theta^*(\gamma) = \min \theta | x(\gamma, \theta) > 0$ is the minimal value of the agent's type for which the principal provides the public good, given the intermediary's type γ . Let $\rho(\gamma, \theta^*)$ and $\tau(\gamma, \theta)$ denote the shadow prices of the individual rationality and incentive compatibility constraints. Writing the Lagrangian and maximizing with respect to the (deterministic) transfers, we obtain $\tau(\gamma, \theta) = 1$ for all $\theta \neq \theta^*$ and $\rho(\gamma, \theta^*) + \tau(\gamma, \theta^*) = 1$, so that these shadow prices are *independent* of the intermediary's type. By the same argument as the argument in proposition 11 page 401 in Maskin and Tirole (401), this suffices to show that the unique equilibrium payoff is the full information payoff, so that we may as well assume that the intermediary's type is known to the agent.

The intermediary will thus offer a contract $q(\gamma, \theta), t_2(\gamma, \theta)$ where the transfer satisfies:

$$t_2(\gamma, \theta) = x(\gamma, \theta)\theta - \int_{\theta^*(\gamma)}^{\theta} x(\gamma, s)ds,$$

where $\theta^*(\gamma) = \min \theta | x(\gamma, \theta) > 0$ is the minimal value of the agent's type for which the principal provides the public good, given the intermediary's type γ . Furthermore, the probability $q(\gamma, \theta)$ is chosen to maximize

$$\int_{\theta^*(\gamma)}^{\overline{\theta}} q(\gamma,\theta) \phi(\theta) dF(\theta),$$

where as before,

$$\phi(\gamma, \theta) = x(\gamma, \theta)(\gamma + J(\theta)) - t_1(\gamma, \theta).$$

We now show that, in the two-dimensional case, the contract between the principal and the intermediary *does not depend on the type of the agent*

Lemma 6 When $x(\gamma, \theta)$ and $t_1(\gamma, \theta)$ are piecewise differentiable, $x(\gamma, \theta) = x(\gamma, \theta')$ and $t_1(\gamma, \theta) = t_1(\gamma, \theta')$ for almost all θ, θ' .

Proof. The incentive compatibility constraint of the intermediary is given by

$$\gamma(x(\gamma,\theta)) - t_1(\gamma,\theta) \ge \gamma x(\gamma',\theta') - t_1(\gamma',\theta') \forall \gamma,\gamma',\theta,\theta',$$

Suppose by contradiction that there exists an open neighborhood for which $x(\gamma, \theta) \neq x(\gamma, \theta')$ for all $\theta \theta'$ in the neighborhood. By incentive compatibility, we must have

$$\gamma(x(\gamma,\theta)) - t_1(\gamma,\theta) = \gamma x(\gamma,\theta') - t_1(\gamma,\theta'),$$

Differentiating with respect to θ ,

$$\frac{\partial x}{\partial \theta}\gamma - \frac{\partial t_1}{\partial \theta} = 0,$$

and differentiating once again with respect to γ ,

$$\frac{\partial^2 x}{\partial \theta \partial \gamma} \gamma + \frac{\partial x}{\partial \theta} - \frac{\partial^2 t_1}{\partial \theta \partial \gamma} = 0,$$

On the other hand, by incentive compatibility,

$$\frac{\partial x}{\partial \gamma}\gamma - \frac{\partial t_1}{\partial \gamma} = 0,$$

implying that

$$\frac{\partial^2 x}{\partial \theta \partial \gamma} \gamma - \frac{\partial^2 t_1}{\partial \theta \partial \gamma} = 0,$$

so that $\frac{\partial x}{\partial \theta} = 0$, contradicting our original assumption.

Lemma 6 stands in sharp contrast to the case where the intermediary's value γ , is known. In the latter situation, the schedule $x(\theta)$ is not constant, and incentive compatibility is satisfied as long as $t_1(\theta) = t_1 + \gamma x(\theta)$. When the type of the intermediary is privately known, the requirement that $x(\gamma, \theta)$ be piecewise continuously differentiable in both variables and satisfy incentive compatibility in γ implies that the public decision is independent of the agent's type. We can thus compute

$$\phi(x,\gamma,\theta) = x(\gamma)(\gamma + J(\theta)) - \tau_1(\gamma))$$

which is non-decreasing in θ .

This fact enables us to side-step a number of the difficulties arising in the case where γ is known, and to immediately conclude that an intermediary of type γ chooses $q(\gamma, \theta) = 1 (= 0)$ for all $\theta \ge \theta^*(\gamma)$ where $th^*(\gamma)$ is defined by:

$$\phi(x,\gamma,\theta^*(\gamma)) = \gamma + J(\theta^*(\gamma)) - \frac{\tau_1(\gamma)}{x(\gamma)} = 0.$$

Once the intermediary has accepted the contract with the principal, the design of the optimal contract $x(\gamma), t_1(\gamma)$ is based on standard arguments. Given the incentive constraint of the intermediary,

$$x(\gamma)\gamma - t_1(\gamma) \ge x(\gamma')\gamma - t_1(\gamma'),$$

We can deduce that $x(\gamma)$ must be nondecreasing and $t_1(\gamma)$ satisfies:

$$\tau_1(\gamma) = x(\gamma)\gamma - \underline{V_1} - \int_{\underline{\gamma}}^{\gamma} x(s)ds.$$
(17)

where $\underline{V_1} = x(\underline{\gamma})\underline{\gamma} - \tau_1(\underline{\gamma}).$

Notice that, because the intermediary is not facing a standard individual rationality constraint, the utility at the lowest type, V_1 is not necessarily equal to zero, and will in fact be chosen optimally by the principal. Replacing in the principal's utility, we obtain

$$U_{0} = \int_{\gamma^{*}}^{\overline{\gamma}} \int_{\theta^{*}(\gamma)}^{\overline{\theta}} W(x(\gamma), \gamma, \theta) dF(\theta) dG(\gamma) + \int_{\gamma^{*}}^{\overline{\gamma}} (1 - F(\theta^{*}(\gamma))) (K(\gamma)x(\gamma) - x(\gamma^{*})\gamma^{*} + t^{*}) dG(\gamma),$$

leading to the characterization in the Proposition.

Proof of Proposition 8.

We can easily replicate the argument of Claim 1 to show that, given that the intermediary and the agent have quasi-linear utilities, we may as well assume that the agent knows the intermediary's type. We characterize the contract by repeating the steps of the proof of Proposition 4. The contract between the intermediary and the agent can be computed by standard arguments, yielding transfers

$$t_2(\gamma, \theta) = x(\mu(\gamma, \theta))\theta - \int_{\underline{\theta}} \theta x(\mu(\gamma, s))ds,$$

where $x \circ \mu$ is nondecreasing in θ . The expected utility of the intermediary can be computed as:

$$U_1(\gamma) = \int_{\underline{\theta}}^{\overline{\theta}} x(\mu(\gamma, \theta))(\gamma + J(\theta)) - t_1(\mu(\gamma, \theta)).$$
(18)

By the same argument as in the proof of Proposition 4, without loss of generality, we can assume that the intermediary reveals truthfully the information. Furthermore, let

$$\delta = \gamma + J(\theta)$$

denote the valuation of the intermediary for the public decision. We claim that without loss of generality, we can restrict attention to optimal contracts which only depend on δ (and not on the two parameters (γ, θ)).

Claim 2 Without loss of generality there is an optimal contract $x(\gamma, \theta), t_1(\gamma, \theta)$ such that $x(\gamma, \theta) = x(\gamma', \theta')$ and $t_1(\gamma, \theta) = t_1(\gamma', \theta')$ whenever $\gamma + J(\theta) = \gamma' + J(\theta')$.

Proof. The incentive compatibility constraints of the intermediary are given by

$$(\gamma + J(\theta)x(\gamma, \theta) - t_1(\gamma, \theta) \ge (\gamma + J(\theta)x(\gamma', \theta') - t_1(\gamma', \theta') \forall \gamma, \gamma', \theta, \theta'.$$

Assuming that $x(\gamma, \theta)$ and $t_1(\gamma, \theta)$ are piecewise differentiable, we compute

$$\begin{array}{lll} \displaystyle \frac{\partial U_1}{\partial \gamma} & = & x(\gamma,\theta) \\ \displaystyle \frac{\partial U_1}{\partial \theta} & = & J'(\theta)x(\gamma,\theta) \\ \displaystyle \frac{\partial^2 U_1}{\partial \gamma \partial \theta} & = & \displaystyle \frac{\partial x}{\partial \theta} = J'(\theta)\frac{\partial x}{\partial \gamma}. \end{array}$$

Now consider the set of (γ, θ) such that $\gamma + J(\theta) = K$ and a variation along this curve, i.e. a variation such that $d\gamma + J'(\theta)d\theta = 0$.

$$dx = \frac{\partial x}{\partial \theta} d\theta + \frac{\partial x}{\partial \gamma}$$
$$= \frac{\partial x}{\partial \gamma} (J'(\theta) d\theta + d\gamma),$$
$$= 0.$$

so that $x(\gamma, \theta)$, and hence $t_1(\gamma, \theta)$ must be constant for all (γ, θ) such that $\gamma + J(\theta) = K$.

We have thus reduced the set of parameters to a one-dimensional set. The contract between the principal and the intermediary becomes a standard contract $x(\delta), t_1(\delta)$ which must respect the intermediary's modified individual rationality and incentive constraints:

$$\begin{split} \delta x(\delta) - t_1(\delta) &\geq 0, \\ \delta x(\delta) - t_1(\delta) &\geq \delta x(\delta') - t_1(\delta') \forall \delta, \delta'. \end{split}$$

Hence,

$$t_1(\delta) = \delta x(\delta) - \int_{\underline{\delta}}^{\delta} x(s) ds$$

and the principal's problem is to choose an increasing $x(\delta)$ in order to maximize

$$\int_{\underline{\delta}}^{\overline{\delta}} \int_{J^{-1}(\delta-\overline{\gamma})}^{J^{-1}(\delta-\underline{\gamma})} W(x(\delta), \delta - J(\theta), \theta) + x(\delta)\mathcal{L}(\delta)dF(\theta)dH(\delta).$$

8 Computations for the Examples

Example 1.b.

From the utility of the principal we get

$$\begin{split} \frac{\partial U_0}{\partial \theta^{**}} &= -\frac{(\theta^{**} - \theta^*)^2}{2\lambda(\lambda - 1)} + \frac{(1 - \theta^*)(2\theta^* - 1)}{\lambda}, \\ \frac{\partial U_0}{\partial \theta^*} &= \theta^{**}[\frac{\theta^{**} - 2\theta^*}{2\lambda(\lambda - 1)} + \frac{3 - 4\theta^*}{\lambda}]. \end{split}$$

Hence, if $\theta^* < 1/2$, $\frac{\partial U_0}{\partial \theta^{**}} < 0$ and $\theta^{**} = \theta^*$. If $\theta^* > \frac{2\lambda - 1}{4\lambda - 3}$, then $\frac{\partial U_0}{\partial \theta^{**}} > 0$ and $= \theta^{**} = 1$. Otherwise, the optimal value θ^{**} is an interior point in $[\theta^*, 1]$. On the other hand,

$$\theta^* = \frac{6\lambda - 6 + \theta^{**}}{8\lambda - 6} \in [0, 1]$$

Using the first order conditions, we derive

$$\begin{array}{lll} \theta * * & = & \frac{2(6(\lambda - 1) - \sqrt{16\lambda^3 - 43\lambda^2 + 36\lambda - 9})}{16\lambda - 15} \\ \theta^* & = & \frac{48\lambda^2 - 87\lambda + 39 - \sqrt{16\lambda^3 - 43\lambda^2 + 36\lambda - 9})}{45 - 108\lambda + 64\lambda^2} \end{array}$$

We can easily check that $\theta^* = \frac{1}{2}$ if and only if $\lambda = \frac{5}{4}$ and $\theta^* = \frac{2\lambda-1}{4\lambda-3}$ if and only if $\lambda = \frac{3}{2}$. This enables us to distinguish between the three régimes mentioned in the example.

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