How Much Coordination is Possible via Correlation?*

Adam Brandenburger†

First Version: June 2006
Current Version: March 2007

Abstract

Players in a game are said to coordinate their behavior if they jointly choose strategies. (The idea goes back to von Neumann [18, 1928] and von Neumann-Morgenstern [20, 1944].) Correlation in behavior arises if players choose their strategies independently, but the choices are conditioned on observations of correlated signals (Aumann [1, 1974]). This note establishes an impossibility result: Some coordinated behavior in a game cannot be derived from correlated behavior. The result rests on an impossibility theorem from the hidden-variable program in quantum mechanics (Hardy [8, 1992]-[9, 1993], following Bell [2, 1964]). We explain the connection. We also discuss the implications for the relationship between cooperative and noncooperative game theory.

1 Introduction

Coordination and correlation are closely related concepts in game theory. The idea of coordinated behavior–which goes back to von Neumann [18, 1928] and von Neumann-Morgenstern [20, 1944]–is that two or more players in the game jointly choose a strategy profile. Aumann [1, 1974] introduced the idea of correlated behavior. Here, we think of the players as choosing their strategies independently, but the choices may be conditioned on correlated "signals" that they observe.

In this note we ask: Can all coordinated play in a game be understood as correlated play?

The question is relevant to the relationship between the noncooperative and cooperative branches of game theory. Von Neumann-Morgenstern [20, 1944, p.529] defined a (TU) cooperative game

*The first version of this note was called "A Connection Between Correlation in Game Theory and Quantum Mechanics." It owes a great deal to joint work with Amanda Friedenberg and Noson Yanofsky. I am indebted to Samson Abramsky for asking a question which stimulated this investigation, and to Eric Pacuit for detailed comments. John Asker, Jean de Valpine, Ariel Rubinstein, Gus Stuart, Johan van Benthem, and participants at the 7th Conference on Logic and the Foundations of Game and Decision Theory (University of Liverpool, July 2006) and the Colloquium on New Perspectives on Games and Interaction (Royal Netherlands Academy of Arts and Sciences, February 2007) provided important input. Financial support from the Stern School of Business is gratefully acknowledged. hmcp-03-16-07

†Address: Stern School of Business, New York University, 44 West Fourth Street, New York, NY 10012, adam.brandenburger@stern.nyu.edu, www.stern.nyu.edu/∼abranden

starting from a noncooperative game. (The characteristic function for a subset A of players is the maximin payoff to A in the associated zero-sum game between A and not-A.) They also showed [20, 1944, pp.530-532] that every superadditive TU characteristic function arises in this fashion.

Under this approach, then, cooperative games are noncooperative games with coordinated behavior.[1] But is coordinated behavior noncooperative behavior? We say it is, provided the coordination is indirect in that the players make their choices independently, with the coordination coming from signals they see. So, we come back to the question: Can all coordination be treated as correlation in this sense? If yes, then, it could be argued that cooperative theory reduces–not just formally, but conceptually–to noncooperative theory.

Of course, the question can also be considered on its own: What exactly is the nature of coordinated behavior in a game? Can it be thought of in terms of correlation? Or, is it–at least sometimes–a primitive notion of behavior, where players act jointly?

Our game-theory question is closely connected to hidden-variable analysis in quantum mechanics. Indeed, our main result is an adaptation of a "no-go" theorem on hidden variables (Hardy [8, 1992], [9, 1993]). After the fact at least, the connection is not surprising. The hidden-variable program in quantum mechanics concerns the question of whether 'behavior' of particles can be made (more) classical by adding variables to the analysis. Adding signals to a game is a similar maneuver. We explain the connection more precisely later.

2 The Question

Figure 1 depicts a situation where, prima facie, an observer (or a third player) thinks that Ann and Bob are coordinating their choices of strategy. The probability assessment shown assigns probability $\frac{1}{2}$ to the event that Ann and Bob coordinate on U and L, and probability $\frac{1}{2}$ to the event that they coordinate on D and R.

		Bob	
		L	R
Ann	U	1/2	0
	D	0	1/2

Figure 1

Following Aumann [1, 1974], such assessments are often explained as compatible with noncooperative play by adding "signals" to the given game (formally, payoff-irrelevant moves by Nature). For example, in Figure 2, a coin is tossed. Both players observe the coin toss, but each then chooses a strategy independently. The observer thinks that: (i) if the coin lands *Heads*, Ann chooses U

[1] To be complete, we should note that some of the time ([20, 1944, p.555 on]), von Neumann-Morgenstern treat the characteristic function as a primitive.

and Bob chooses L; and (ii) if the coin lands $Tails$, Ann chooses D and Bob chooses R. The observer's (degenerate) conditional probabilities are shown. (Note the identification of moves in the two subtrees in Figure 2. This is the basis for viewing the assessments in Figures 1 and 2 as equivalent.)

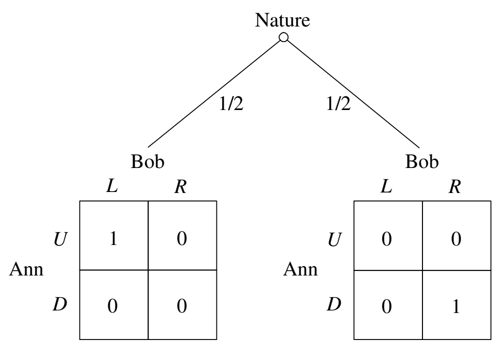

Figure 2

A natural question is then: Can all assessments in a game be explained in this fashion?

Not surprisingly, the answer depends on the answers to two other questions: What kinds of variables is it permitted to add to a game? And what conditions might be placed on such variables?

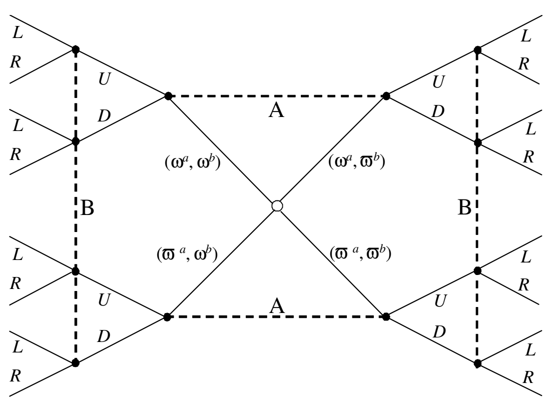

Figure 3

We suppose that, prior to the given game, Nature chooses a point (ω^a, ω^b) from some finite product space $\Omega^a \times \Omega^b$, and Ann (resp. Bob) observes the component ω^a (resp. ω^b).[2] See Figure 3 for an example.

Returning to Figure 1, in the format of Figure 3 we would set $\Omega^a = \{\omega^a, \overline{\omega}^a\}$ and $\Omega^b = \{\omega^b, \overline{\omega}^b\}$. The observer's assessment on the bigger game would then be given by $p(U, L, \omega^a, \omega^b) = p(D, R, \overline{\omega}^a, \overline{\omega}^b) = \frac{1}{2}$.

In general, let S^a (resp. S^b) be Ann's (resp. Bob's) strategy set. The condition we impose on

[2] Since we later establish an impossibility result, finiteness of Ω^a and Ω^b is, admittedly, a restriction.

our analysis is:

$$p(s^a, s^b|\omega^a, \omega^b) = p(s^a|\omega^a) \times p(s^b|\omega^b), \tag{1}$$

whenever $p(\omega^a, \omega^b) > 0$. This is a conditional independence-like requirement. Here is an easy fact (it is a special case of a result in Brandenburger-Friedenberg [4, 2004, Proposition 5.1]):

Proposition 2.1 *Under condition (1), if $p(\omega^a, \omega^b) = p(\omega^a) \times p(\omega^b)$ for all ω^a, ω^b, then $p(s^a, s^b) = p(s^a) \times p(s^b)$ for all s^a, s^b.*

Proof. We have

$$\begin{aligned} p(s^a, s^b) = & \sum_{\{\omega^a,\omega^b : p(\omega^a,\omega^b)>0\}} p(s^a, s^b|\omega^a, \omega^b) p(\omega^a, \omega^b) = \\ & \sum_{\{\omega^a,\omega^b : p(\omega^a,\omega^b)>0\}} [p(s^a|\omega^a) \times p(s^b|\omega^b)] p(\omega^a, \omega^b) = \\ & \sum_{\{\omega^a : p(\omega^a)>0\}} p(s^a|\omega^a) p(\omega^a) \times \sum_{\{\omega^b : p(\omega^b)>0\}} p(s^b|\omega^b) p(\omega^b) = p(s^a) \times p(s^b), \end{aligned}$$

as required. ■

Taking the contrapositive, this says that if the observer assesses Ann's and Bob's strategy choices as dependent (not independent), then he must assess their signals as dependent. So, condition (1) captures the requirement that coordination in behavior comes from correlation in signals.

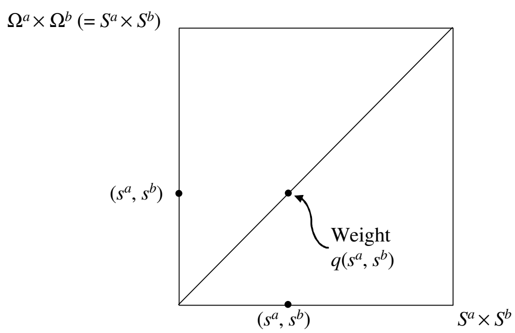

Figure 4

Starting with any matrix game[3] $S^a \times S^b$ and any assessment q on $S^a \times S^b$, it is always possible to find spaces Ω^a and Ω^b and a measure p on $S^a \times S^b \times \Omega^a \times \Omega^b$, such that p agrees with q on $S^a \times S^b$ and satisfies (1). Just set $\Omega^a = S^a$ and $\Omega^b = S^b$, and put p on the diagonal, as in Figure 4. (This is a standard game-theoretic construction, going back to the "revelation principle" of Myerson [16, 1986].)

[3] Strictly, game form, since we don't specify payoffs.

But does this construction work for more complicated games? In particular, we will consider a case where the underlying game itself already contains moves by Nature. Here is an example:

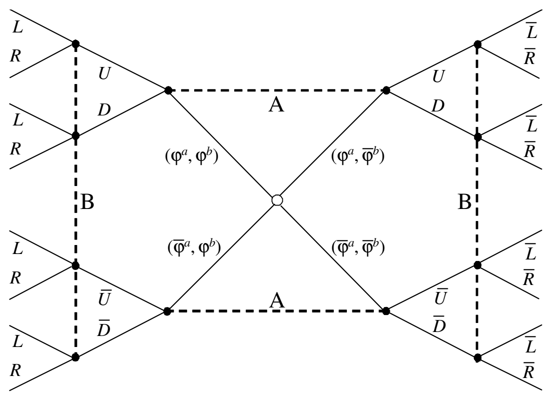

Figure 5

Note carefully that this is not the same as Figure 3. Here, we depict an underlying game, which happens to involve moves by Nature. These are depicted as (φ^a, φ^b), $(\overline{\varphi}^a, \varphi^b)$, $(\varphi^a, \overline{\varphi}^b)$, $(\overline{\varphi}^a, \overline{\varphi}^b)$, and we use φ rather than ω to distinguish these moves from moves by Nature in an augmented game (as in Figure 3).[4]

3 An Impossibility Result

Suppose in Figure 5 the observer has an assessment q satisfying:

$$q(\varphi^a, \varphi^b), q(\overline{\varphi}^a, \varphi^b), q(\varphi^a, \overline{\varphi}^b), q(\overline{\varphi}^a, \overline{\varphi}^b) > 0, \tag{2}$$

$$q(U, L, \varphi^a, \varphi^b) = q(\overline{U}, R, \overline{\varphi}^a, \varphi^b) = q(D, \overline{L}, \varphi^a, \overline{\varphi}^b) = 0, \tag{3}$$

$$q(\overline{U}, \overline{L}, \overline{\varphi}^a, \overline{\varphi}^b) > 0. \tag{4}$$

(We write "U" for the pair of strategies $(U, \overline{U})$ and $(U, \overline{D})$, and similarly for other terms in (3) and (4).)

We will show that, under certain conditions, there is no space $\Omega^a \times \Omega^b$ (where Ann observes ω^a and Bob observes ω^b) and measure p, such that p agrees with q on the underlying game. The interpretation will be that we can't explain the assessment in terms of correlated signals.

The conditions we impose are first:

$$p(s^a, s^b | \varphi^a, \varphi^b, \omega^a, \omega^b) = p(s^a | \varphi^a, \omega^a) \times p(s^b | \varphi^b, \omega^b), \tag{5}$$

whenever $p(\varphi^a, \varphi^b, \omega^a, \omega^b) > 0$. This is the conditional independence-like condition as before. The

[4] John Asker has suggested the nice terminology: Nature makes the chance moves (φ) in the underlying game, while Fate makes the extra chance moves (ω) in the augmented game.

difference is that now we take account of the fact that Ann observes both φ^a (from the underlying game) and ω^a (from the augmented game), and likewise for Bob.

Second, we require:

$$p(\varphi^a, \varphi^b, \omega^a, \omega^b) = p(\varphi^a, \varphi^b) \times p(\omega^a, \omega^b). \tag{6}$$

This says that (the observer assesses that) Nature's moves in the underlying game are independent of her moves in the augmented game. For example, in the underlying game, Ann and Bob might each privately toss a coin. (This would be denoted as φ^a vs. $\overline{\varphi}^a$ for Ann, and φ^b vs. $\overline{\varphi}^b$ for Bob.) Whatever process–of signals etc.–the observer has in mind that is captured in the space $\Omega^a \times \Omega^b$ should then, presumably, be viewed as independent of the coin tosses. Since we are about to prove an impossibility result, we don't have to argue that the independence condition must always hold (this would be unreasonable), but that there are natural scenarios where it holds. We suggest the above scenario is one.

Theorem 1 *There is no probability measure p satisfying (5) and (6) and agreeing with q.*

Proof. Suppose, contra hypothesis, there is a space $\Omega^a \times \Omega^b$ and a measure p satisfying (5) and (6), such that p agrees with q. Then

$$\begin{aligned}
0 &= q(U, L|\varphi^a, \varphi^b) = p(U, L|\varphi^a, \varphi^b) \\
&= \sum_{\{\omega^a,\omega^b : p(\varphi^a,\varphi^b,\omega^a,\omega^b)>0\}} p(U, L|\varphi^a, \varphi^b, \omega^a, \omega^b) p(\omega^a, \omega^b|\varphi^a, \varphi^b) \\
&= \sum_{\{\omega^a,\omega^b : p(\varphi^a,\varphi^b,\omega^a,\omega^b)>0\}} [p(U|\varphi^a, \omega^a) \times p(L|\varphi^b, \omega^b)] p(\omega^a, \omega^b|\varphi^a, \varphi^b) \\
&= \sum_{\{\omega^a,\omega^b : p(\omega^a,\omega^b)>0\}} [p(U|\varphi^a, \omega^a) \times p(L|\varphi^b, \omega^b)] p(\omega^a, \omega^b),
\end{aligned}$$

where the first line uses (3), the third line uses (5), and the fourth line uses (2) and (6) (twice). Letting $M = \{(\omega^a, \omega^b) : p(\omega^a, \omega^b) > 0\}$, we have shown that

$$p(U|\varphi^a, \omega^a) \times p(L|\varphi^b, \omega^b) = 0 \quad \text{on } M. \tag{7}$$

Similar arguments using $q(\overline{U}, R|\overline{\varphi}^a, \varphi^b) = 0$ and $q(D, \overline{L}|\varphi^a, \overline{\varphi}^b) = 0$ respectively, yield

$$p(\overline{U}|\overline{\varphi}^a, \omega^a) \times p(R|\varphi^b, \omega^b) = 0 \quad \text{on } M, \tag{8}$$

$$p(D|\varphi^a, \omega^a) \times p(\overline{L}|\overline{\varphi}^b, \omega^b) = 0 \quad \text{on } M. \tag{9}$$

From (7), for every $(\omega^a, \omega^b) \in M$, either $p(U|\varphi^a, \omega^a) = 0$ or $p(L|\varphi^b, \omega^b) = 0$ (or both). Suppose $p(U|\varphi^a, \omega^a) = 0$. Then $p(D|\varphi^a, \omega^a) = 1$, so that $p(\overline{L}|\overline{\varphi}^b, \omega^b) = 0$, by (9). Similarly, if $p(L|\varphi^b, \omega^b) = 0$, then $p(R|\varphi^b, \omega^b) = 1$, so that $p(\overline{U}|\overline{\varphi}^a, \omega^a) = 0$, by (7). We see that for every $(\omega^a, \omega^b) \in M$, either

$p(\overline{L}|\overline{\varphi}^b, \omega^b) = 0$ or $p(\overline{U}|\overline{\varphi}^a, \omega^a) = 0$ (or both). But then

$$p(\overline{U}, \overline{L}|\overline{\varphi}^a, \overline{\varphi}^b) = \sum_{\{\omega^a, \omega^b : p(\omega^a, \omega^b) > 0\}} [p(\overline{U}|\overline{\varphi}^a, \omega^a) \times p(\overline{L}|\overline{\varphi}^b, \omega^b)] p(\omega^a, \omega^b) = 0,$$

whereas, by (4), $p(\overline{U}, \overline{L}|\overline{\varphi}^a, \overline{\varphi}^b) > 0$. Contradiction. ■

Some comments on the result:

i. The conditions are tight. If we ask only for condition (5), we can build an augmented structure with a measure agreeing with q on the underlying game. To do so, set $\Omega^a = S^a \times \Phi^a$ and $\Omega^b = S^b \times \Phi^b$, and define p by a diagonal construction like that in Section 2. For condition (6) only, just take Ω^a and Ω^b to be singletons. (See Brandenburger and Yanofsky [5, 2007, Theorems 5.1-5.2] for more general existence results of this type.)

ii. What matters for the impossibility result is that Nature's extra moves (ω) and Nature's moves in the underlying game (φ) are assessed as independent (condition (6)). It doesn't matter whether the ω-moves come before or after the φ-moves.[5] (In Figure 3, the extra moves were shown as coming before the underlying game.)

iii. We could imagine a more general augmented structure, viz.

$$S^a \times S^b \times \Phi^a \times \Phi^b \times \Omega,$$

for some (finite) space Ω, and partitions $\mathcal{H}^a$ (resp. $\mathcal{H}^b$) of Ω for Ann (resp. Bob). If, in the augmented game, Nature chooses a point ω, Ann (resp. Bob) observes the member of $\mathcal{H}^a$ (resp. $\mathcal{H}^b$) containing ω. We show in an appendix that, under analogs to conditions (5) and (6), the impossibility result still holds.

4 Discussion

Here we discuss some background to the impossibility result, and also its interpretation.

a. Theorem 1 is an adaptation to the game setting of a version–due to Hardy [8, 1992], [9, 1993]–of Bell's Theorem (Bell [2, 1964]) in quantum mechanics (QM). Here is a typical QM set-up (taken from an illustration in Laloë [11, 2001]):

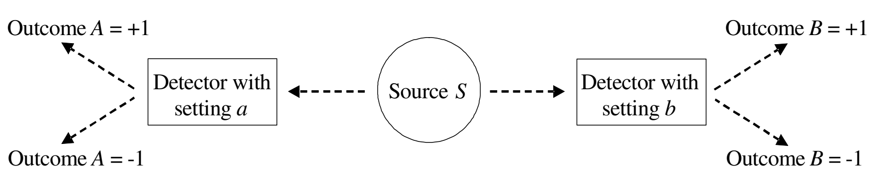

Figure 6

[5] Ariel Rubinstein kindly suggested this point.

A source S emits two particles. Ann performs a measurement a on one of the particles, while Bob performs a measurement b on the other particle. In a typical case, the possible outcomes A of Ann's measurement might be either $+1$ or -1, and likewise for Bob. (This is the spin of the particle in the direction measured.) The interesting case is when the two particles are "entangled." For example, it might be that if Ann measures spin $+1$ in a certain direction, Bob must measure spin -1 in the same direction, and vice versa.

We see that our φ^a, φ^b, s^a, s^b correspond to a, b, A, B, respectively. Of course, this is a formal correspondence only (more on this in Subsection d. below).

The question in QM is whether entanglement can be understood as reflecting the presence of extra variables–what in QM are called hidden variables. Einstein-Podolsky-Rosen [6, 1935] produced a QM set-up (our Figure 1 is similar as a game model) which they used to argue for the need to "complete" QM–e.g., via hidden variables. (Think of Figure 2 as a "completed" game model.) The general question in QM is usually posed under conditions exactly analogous to (5) and (6). Condition (5) is called locality, and says that the correlation between the particles is indeed due to hidden variables (like our ω^a and ω^b) and not 'direct.' (There is no "spooky action at a distance," in Einstein's famous phrase.) Condition (6) says that the process determining the hidden variables is independent of what measurements are conducted on the particles. Bell's Theorem ([2, 1964]) shows that there are correlations in QM that cannot be explained via hidden variables satisfying these conditions.

b. The situation in game theory is analogous. We gave a game (Figure 5) and an assessment on that game (any assessment satisfying (2)-(4)). For this game, there is no augmented game–with extra moves by Nature–and assessment on the augmented game, where this assessment satisfies (5) and (6) and agrees with the original assessment on the underlying game. In short, the dependence in the assessment on the original game cannot be explained as arising from the presence of extra variables.

c. Let us interpret this result, along the lines set out at the beginning of this note. The result says that coordinated behavior in a game cannot necessarily be understood as uncoordinated behavior in an augmented game with signals. Not all behavior can be reduced to individual behavior. Coordination must be viewed, at least sometimes, as a primitive. As for the relationship between noncooperative and cooperative game theory, the implication would seem to be that, conceptually at least, the latter is not reduceable to the former.

d. Just what kind of coordination is needed in our game of Figure 5? More precisely, what must an observer with an assessment satisfying (2)-(4) assume about how Ann and Bob play this game?

There is a growing literature on quantum games where players are assumed to have access to quantum devices. (Eisert-Wilkens-Lewenstein [7, 1999] and Meyer [15, 1999] are early papers. Landsburg [14, 2005] and La Mura [13, 2005] study in detail Nash and correlated equilibria respectively. There are many other papers.[6]) Certainly, if this was the case in the game of Figure 5, the

[6] More like the current note, Lambert-Mogiliansky, Zamir, and Zwirn [12, 2003] and Temzelides [17, 2005] apply

players could implement play corresponding exactly to the observer's assessment. (See again Hardy [8, 1992], [9, 1993] for the physical details.)

But we believe the observer's assessment is reasonable even if the game is completely classical. The key is that we allow coordination between Ann and Bob in the underlying game. (This is exactly the scenario we are studying.)

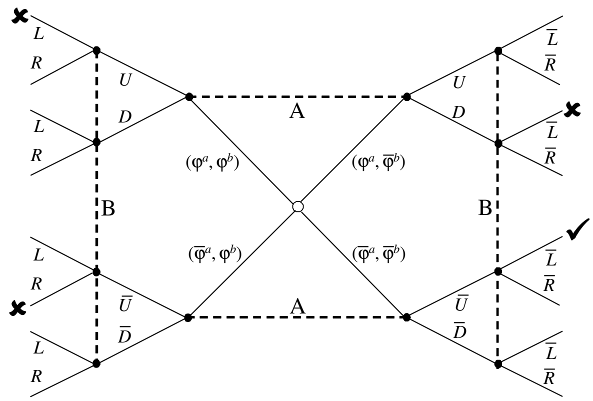

Figure 7

Figure 7 is Figure 5 again, with a check or a cross at a terminal node in accordance with (2)-(4). Suppose Ann observes φ^a and Bob observes φ^b. We imagine some process of coordination then takes place between them, which results in a coordinated choice which is not (U, L). Likewise with the other three cases. But let's look more closely.

What if, for example, after observing $\overline{\varphi}^a$ and φ^b respectively, Ann and Bob decide on $(\overline{U}, L)$? Does the fact that we write $\overline{U}$ and not U tell Bob that Ann observed $\overline{\varphi}^a$ and not φ^a? One response is perhaps it does, but it isn't clear that this causes any difficulties for the scenario in question. Ann and Bob decide on $(\overline{U}, L)$, and that is that. It is also possible that the information isn't transmitted. This is because the analyst (who might be the observer) needs the bar notation to represent the game, but the players do not. They can just say: "Let's play *Up* and *Left*."

A second potential puzzle: After observing φ^a and φ^b, Ann and Bob do not play (U, L). (More precisely, this is what the observer thinks.) After observing $\overline{\varphi}^a$ and φ^b, Ann and Bob do not play $(\overline{U}, R)$. Can there be a difference in Bob's component of the (coordinated) choice, when his observation doesn't change? One answer is simply that, since we are imagining some process of coordination, all we can say is that the output of this process depends on both inputs, and there is no separability. But we can also give a more concrete answer. Perhaps, after observing φ^a, Ann says to Bob: "If you choose *Left*, I won't choose *Up*." After observing $\overline{\varphi}^a$, Ann says something different: "If you choose *Right*, I won't choose *Up*." Does this transmit information again (since Ann says different things in the two cases)? Perhaps, but, again, it isn't clear that this would cause any difficulties for our scenario.

QM formalism but not quantum devices to decisions and games.

All this said, we don't want to commit to details of the coordination process. We simply want to make clear that the situation in Figure 7 is conceptually coherent.

e. On a historical note, von Neumann [19, 1932, 1955] initiated the hidden-variable program in QM, and gave an impossibility argument. In fact, his argument is now known to use too-strong assumptions (Bell [3, 1966]). It is the modern impossibility results of Bell [2, 1964], Kochen-Specker [10, 1967], and others that are now considered decisive. Still, von Neumann's position was clear, as in the often-quoted: "[T]he present system of quantum mechanics would have to be objectively false, in order that another description of the elementary processes than the statistical one [i.e., in order that a hidden-variable description] be possible" ([19, 1955, p.325]).

The question in this note can be phrased as: Can extra variables in game theory explain all coordination? Our answer is no. To the best of our knowledge, von Neumann never (directly) asked this question in game theory.[7] But, just perhaps, the negative answer fits with what was clearly his sensibility in game theory. From his [18, 1928] paper on–including a large part of von Neumann-Morgenstern [20, 1944]–von Neumann's interest was in coordinated rather than purely noncooperative behavior in games.

Appendix

We consider an augmented structure

$$S^a \times S^b \times \Phi^a \times \Phi^b \times \Omega$$

for some finite space Ω, partitions $\mathcal{H}^a$ (resp. $\mathcal{H}^b$) of Ω for Ann (resp. Bob), and a measure μ. If Nature chooses a point ω, Ann (resp. Bob) observes the member $\mathcal{H}^a(\omega)$ of $\mathcal{H}^a$ (resp. the member $\mathcal{H}^b(\omega)$ of $\mathcal{H}^b$) containing ω.

The analogs to conditions (5) and (6) are now:

$$\mu(s^a, s^b|\varphi^a, \varphi^b, \mathcal{H}^a(\omega) \cap \mathcal{H}^b(\omega)) = \mu(s^a|\varphi^a, \mathcal{H}^a(\omega)) \times \mu(s^b|\varphi^b, \mathcal{H}^b(\omega)), \tag{A1}$$

whenever $\mu(\varphi^a, \varphi^b, \mathcal{H}^a(\omega) \cap \mathcal{H}^b(\omega)) > 0$, and

$$\mu(\varphi^a, \varphi^b, \mathcal{H}^a(\omega) \cap \mathcal{H}^b(\omega)) = \mu(\varphi^a, \varphi^b) \times \mu(\mathcal{H}^a(\omega) \cap \mathcal{H}^b(\omega)). \tag{A2}$$

Define an augmented structure of the form in the text by setting

$$\begin{aligned} \Omega^a &= \{h^a : h^a \in \mathcal{H}^a\}, \\ \Omega^b &= \{h^b : h^b \in \mathcal{H}^b\}, \end{aligned}$$

[7] Jean de Valpine brought to my attention that there are some references–of a philosophical kind–to quantum mechanics in von Neumann-Morgenstern [20, 1944, pp.3, 33, 148, 401].

and building a measure p on $S^a \times S^b \times \Phi^a \times \Phi^b \times \Omega^a \times \Omega^b$ by

$$p(s^a, s^b, \varphi^a, \varphi^b, h^a, h^b) = \mu(s^a, s^b, \varphi^a, \varphi^b, h^a \cap h^b).$$

It is immediate that p and μ agree on $S^a \times S^b \times \Phi^a \times \Phi^b$, and that (5) holds if (A1) holds and (6) holds if (A2) holds.

References

[1] Aumann, R., "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics,* 1, 1974, 76-96.

[2] Bell, J., "On the Einstein-Podolsky-Rosen Paradox," *Physics*, 1, 1964, 195-200.

[3] Bell, J., "On the Problem of Hidden Variables in Quantum Theory," *Reviews of Modern Physics*, 38, 1966, 447-452.

[4] Brandenburger, A., and A. Friedenberg, "Intrinsic Correlation in Games," 2004. Available at www.stern.nyu.edu/∼abranden.

[5] Brandenburger, A., and N. Yanofsky, "A Classification of Hidden-Variable Properties," 2007. Available at www.stern.nyu.edu/∼abranden.

[6] Einstein, A., B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review*, 47, 1935, 770-780.

[7] Eisert, J., M. Wilkens, and M. Lewenstein, "Quantum Games and Quantum Strategies," *Physical Review Letters*, 83, 1999, 3077-3088.

[8] Hardy, L., "Quantum Mechanics, Local Realistic Theories, and Lorentz-Invariant Realistic Theories," *Physical Review Letters*, 68, 1992, 2981-2984.

[9] Hardy, L., "Nonlocality for Two Particles without Inequalities for Almost All Entangled States," *Foundations of Physics*, 13, 1993, 1665-1668.

[10] Kochen, S., and E. Specker, "The Problem of Hidden Variables in Quantum Mechanics," *Journal of Mathematics and Mechanics*, 17, 1967, 59-87.

[11] Laloë, F., "Do We Really Understand Quantum Mechanics? Strange Correlations, Paradoxes, and Theorems," *American Journal of Physics*, 69, 2001, 655-701.

[12] Lambert-Mogiliansky, A., S. Zamir, and H. Zwirn, "Type Indeterminacy: A Model for the KT (Kahneman-Tversky)-Man," 2003. Available at www.ma.huji.ac.il/∼zamir.

[13] La Mura, P., "Correlated Equilibria of Classical Strategic Games with Quantum Signals," *International Journal of Quantum Information*, 3, 2005, 183-188.

[14] Landsburg, S., "Nash Equilibria in Quantum Games," 2005. Available at www.landsburg.com.

[15] Meyer, D., "Quantum Strategies," *Physical Review Letters*, 82, 1999, 1052-1055.

[16] Myerson, R., "Bayesian Equilibrium and Incentive-Compatibility: An Introduction," in Hurwicz, L., D. Schmeidler, and H. Sonnenschein, eds., *Social Goals and Social Organization*, Cambridge University Press, 1985, 229-259.

[17] Temzelides, T., "An Uncertainty Principle for Social Science Experiments," 2005. Available at www.pitt.edu/~tedt.

[18] von Neumann, J., "Zur Theorie der Gessellshaftspiele," *Mathematische Annalen*, 100, 1928, 295-320.

[19] von Neumann, J., *Mathematische Grundlagen der Quantenmechanik*, Springer-Verlag, 1932. (Translated as *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, 1955.)

[20] von Neumann, J., and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 1944 (60th Anniversary Edition, 2004).