Asset Choice Regulation in Mutual Funds*

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Abstract

Should type of assets held by mutual funds be regulated? We investigate this issue in a costly-state verification model, where the regulator determines the class of assets in which mutual funds can invest, fund managers select the asset type under this constraint, and investors can, at a cost, control the performance of the manager. The optimal level of risk for the portfolio reflects an interesting trade-off: on the one hand risky assets magnify the manager's incentives to lie, on the other hand they enhance the incentives of the investor to monitor performance. We show that, if the mutual fund industry is not perfectly competitive, regulation helps protecting investors by restricting the discretion of the fund manager.

Keywords: mutual funds, asset choice, regulation

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1 Introduction

The existence of mutual funds has been usually justified by the presence of economies of scale in financial intermediation. Small investors[1] prefer to join a mutual fund than to manage directly their wealth, since fund managers are supposed to have a better knowledge of the market, due to their expertise and to "learning by doing" effects. Moreover, mutual funds can profit of large investment opportunities, for example a better diversification, the spreading of fixed operational costs over a larger asset base or the use of size as a tool to obtain better trading terms. However, an informational advantage generates problems of asymmetric information between the investor and the fund manager.[2] This implies that the cost of delegating the management of wealth could be particularly high for the investor. The regulator, as a partial solution to these problems, has often constrained the fund manager to some minimum requirements in the composition of the investment portfolio, for example by forcing the manager to have a compulsory minimum percentage of bonds in the portfolio.[3]

The goal of this paper is to present a theoretical investigation on the necessity and eventual effectiveness of regulatory constraints on portfolio composition in preventing fraudulent behaviors:[4] is it true that bonds offer a better protection to asymmetric information problems than stocks, or options? Can the inevitable freedom of the fund manager to choose the asset inside a given category allow him to gain information rents even when such constraints are imposed?

The importance of regulation in the mutual fund industry could be underestimated, since apparently agency costs can be easily controlled in mutual funds. Indeed, fund owners can redeem their investment with relatively little cost, trust and reputation play an important role in the industry

[1] Mutual funds are largely owned by small investors. The Investment Company Institute (2004) reports that nearly one out of every two U.S. households owns mutual funds. According to the report, a representative mutual fund owner is middle-aged (median age of 48 years), college-educated (56% has college or postgraduate degrees), employed (77%) and of moderate financial means (the median household income was $68,700).

[2] An interesting empirical test on the existence of a principal/agent relationship between the investor and the investment advisor is done in Golec (1992). The author links the parameters of the investment advisor's incentive contract to the portfolio characteristics and tests the relationship. He finds that the variables specified by the theoretical model enter the regressions with significant coefficients and often with the predicted sign.

[3] Even if in the last years the regulatory authorities in some countries have started to decrease constraints on manager's activities, investment constraints are still a common regulatory feature. For example, in the European Union mutual funds are regulated by the so-called "Undertakings for the Collective Investment of Transferable Securities" (UCITS) III directive, that specifies rules on the kinds of investments appropriated for mutual funds and on how these funds should be sold. Under UCITS, 90% of fund assets must be invested in publicly traded instruments and there are limits on a fund's borrowing rights. Similar rules apply in other countries, with restrictions on the use of derivatives and leverage being a common feature.

[4] We model manager's fraudulent behavior as misreporting of investment returns. This formulation includes other types of fraud that subtract rents to investors, like expense shifting ("soft dollar" arrangements and 12b-1 fees), stale price strategies (market timing and late trading) and cross-fund subsidization (preferential allocation and opposite trades).

and fund owners entrust fund providers with substantial amounts of their wealth, so they have large incentives to check manager's behavior. However, recent scandals (for example, Canary Capital or Putnam Investments) show that agency costs are a real problem in the mutual fund industry.[5] The trading practices of leading mutual fund families are under investigation, and lawsuits have been filed against certain of these companies. The SEC reckons that 10% of mutual funds group may have knowingly allowed the illegal practice of late trading and is considering different proposals aimed at reducing agency costs, especially for "soft-dollars" and late trading. Furthermore, also empirical evidence suggests that the agency problem in the mutual fund industry is hard to solve and that fund managers appropriate of large informational rents. For example, Livingston and Neal (1996) show that for "soft dollar" arrangements the expense ratio is positively correlated with brokerage commissions, while Siggelkow (2004) considers 12b-1 fees and obtains that they increase the expense ratio and do not benefit mutual fund owners. Other examples of agency costs are considered in Zitzewitz (2003) on stale prices and in Massa et al. (2006) on cross-fund subsidization.

There are different reasons for these difficulties in controlling the agency problem in mutual funds. First, as Zingales (2004) suggests, the investor's ability to withdraw money at the Net Asset Value eliminates the price signal from the market. Secondly, the deterrence effect of lawsuits is really limited since mutual fund companies manage a disproportionate amount of assets compared to their net wealth. Finally, load fees and search cost can be an obstacle to redemption as shown by Chordia (1996) or Sirri and Tufano (1998). These facts strengthen the idea that small investors have limited market power on the fund manager and suffer high switching costs when changing investment institutions, hence competition between fund managers is limited and managers can obtain positive rents due to their informational advantage.

When an investor decides to participate in a mutual fund, he delegates to the fund manager two different tasks: the management of the investment and the choice of the asset to invest in.[6] Investment delegation implies an asymmetry of information between the investor and the manager, since the fund manager is in contact with the market and has a superior knowledge of the market state, while the investor has only the manager's report to evaluate the investment performance. This asymmetry is reinforced when the asset choice task is delegated too, since the investor looses any control on his money and the manager chooses the type of investment that maximizes his own rents.

Our analysis proceeds in three different steps. First, we assume that the mutual fund manager can invest in only one type of financial instrument, e.g. only in bonds, or only in options. Then,

[5] A good survey of manager-investor conflicts in mutual funds is the paper of Mahoney (2004).

[6] Delegated asset management involves layering of agency relationship; in particular, a mutual fund implies two agency relationships: the *external relationship*, between the ultimate investor and the fund management firm, and the *internal relationship*, between the fund managers and the fund management firm. Our work is focused only on the external relationship, and assumes that the interests of the fund manager and the fund management firm coincide.

we analyze the decision of the fund manager, e.g. the nominal value of debt or the exercise price of the option, and compare the manager's selected values to the social optimum, e.g. the nominal value of debt or the exercise price of the option that maximizes gains from trade. Secondly, we consider the case where the fund manager can choose between different instruments. We consider the whole class of standard financial contracts: options, low risk debt, high risk bonds or stocks. We compare the manager's choice to the social planner's one, assuming again that social welfare is given by gains from trade. Finally, we introduce a regulatory authority in the model and we compare the fund manager's investment choice in the regulated and unregulated case.

Our model is a two-stage game. The timing is the following. First, the type of asset to be held is determined. Second, the contract between the investor and the fund manager is designed. This contract specifies the transfers to the manager and the investor contingent on the return of the asset as well as the monitoring policy.[7] Then, the manager invests in the asset class chosen at the first stage (there is no moral hazard on that dimension), and the return is realized. The manager finally has the choice between reporting the truth and lying, to try to capture the return on the investment.

The choice of the asset is governed by the following considerations: first, the greater the expected cash flow of the asset, the more gains from trade can be reaped. This captures the idea that financial markets exist to reshuffle wealth across agents, states and points in time. Second, an interesting trade-off arises: on the one hand, risky assets magnify the incentives of fund managers to lie (pretending that the state of the world was unfavorable, to pocket the large difference between high and low payoffs). This suggests that riskier assets magnify the asymmetric information problem, and thus should be avoided. On the other hand, the gap between high and low payoffs also enhances the incentives of the investor to incur costs to monitor his agent: the fund manager. This tends to discipline the manager.

In line with these considerations, when potential gains from trade are large and the monitoring cost is low, the socially optimal security is the stock. The latter enables large expected cash flows to be transferred, and thus large gains from trade to be reaped, and at the same time it induces intense monitoring, which is relatively cheap, and disciplines the manager. In contrast, when the

[7]In our model the investor chooses the contract for the fund manager, both in terms of transfers and monitoring policy. In the real world, the transfer scheme is normally chosen by the fund manager. However, if regulation on asset choice proves to be useful when the transfer scheme is the best for the investor, it should be even more useful if the fund manager can appropriate of more rents by using transfers. Since we want to focus on asset choice regulation, we do not consider problems related to transfer schemes and to their regulation.

Notice also that this modelization implies a high level of rationality for the small investor. However, the contract could be chosen also by the regulator without changing the structure of the model, provided that asset and contract choices are sequential in time. This assumption is reasonable, since laws on portfolio constraints are usually established ex-ante for all the investment funds, while contracts for a particular fund are specified case by case and can not be completely determined by law.

potential gains from trade are low and the cost of monitoring is high, the socially optimal security is low risk debt. This asset transfers less expected cash flow, but it also minimizes asymmetric information problems, which is important when monitoring costs are high.

Our paper also shows that if the security is chosen by the fund manager, in general it does not coincide with the socially optimal one: the manager often prefers riskier assets, for which he can obtain a larger rent. In particular, when monitoring costs are low the preferred asset is the call option, the asset with lowest gains from trade. This choice reflects the rentability/monitoring trade-off faced by the manager: higher gains from trade increase potential revenues from lying, but at the same time enhance the incentives of the investor to monitor the manager. As monitoring costs increase, this trade-off becomes less important and the preferred asset is in general the stock, the asset with highest gains from trade. The stock becomes optimal because the fund manager does not internalize the social cost of monitoring and prefers rentable assets that imply high information rents. Notice that, inside a given asset category, the asset value choice is also driven by the rentability/monitoring trade-off: the manager prefers a lower level of gains from trade (and so a lower level of asymmetric information) for low monitoring costs and a higher one for high monitoring costs.

Our model sheds light on the challenges faced by regulators. Regulators can help protecting investors by reducing the amount of risk that fund managers can take. Thus, they limit potential gains from fraud, even if they are unable to solve completely the delegation problem due to their incapacity to constrain the manager's choice within an asset class. This result implies that portfolio restrictions are an important regulatory instrument. They are particularly helpful when direct monitoring is not sufficient to control the fund manager's potential fraudulent behaviors. This is in line with the empirical findings of Almazan et al. (2004) on investment restrictions in mutual fund contracts: restrictions are more frequently present when it is relatively less beneficial to use direct methods to monitor the manager's behavior. Notice also that in our model investors are risk neutral. Hence, the optimality of risk regulation is not due to risk aversion. Rather it reflects the consequences of asymmetric information. Thus, the insights from our model can also apply to funds targeted at risk neutral investors, like hedge funds.

Our analysis offers also an insight on recent trends in the mutual fund industry, in particular on the "ongoing move away from traditional funds, in which a single asset manager controls a broad-base portfolio, towards specialist mandates focused on narrowly defined asset classes and clearly delineated investment styles" (Committee on the Global Financial System, 2003). An important downside of the increasing investment innovation and of the consequent broadening array of asset classes is that the potential for fraudulent behavior has increased as well. Our model suggests that financial innovation could increase the potential for fund manager's misbehavior and so could prevent investors from participating in mutual funds, especially if the manager can not commit to a certain asset choice within the portfolio. In the absence of strong regulatory constraints, the fund manager could prefer to self-constrain ex-ante his investment possibility set to induce investor's participation. This behavior could be enhanced by recent financial scandals on corporate malpractice (Enron, Parmalat) and financial analysts conflict of interests (Merrill Lynch) that could have strengthen investor's concerns on protection against fraudulent behavior by financial intermediaries.

We conclude by presenting some empirical predictions of our analysis. First, fraudulent activities in the mutual fund industry should be positively correlated with the degree of sophistication of the financial market and negatively correlated with the number of restrictions on asset choice imposed on the fund by the regulator. In a dynamic perspective, on the one side fraudulent activities should increase in time due to financial innovation; on the other side, they should decrease due to the growing practice of self-regulation on asset selection. It could be interesting to investigate if self-regulation is increasing fast enough to control for financial innovation. A second empirical implication derives from the fact that in our model regulation on asset choice is a substitute for competition: restrictions on asset choice should be negatively correlated with the competitiveness of the market. So, for example, the hedge fund industry, which is nearly not regulated in terms of asset choice, should be highly more competitive than the pension fund industry, where restrictions on investment choice are severe.

The remainder of our paper is organized as follows. Section 2 discusses how the paper relates to the previous literature. Section 3 introduces the model. Section 4 focuses on asset value choice for a given security class by analyzing both the social planner's and the fund manager's optimal choice. Three types of instruments are considered, call option, risky debt and stock, and the analysis is performed by differentiating low and high risk debt depending on the nominal debt value. Section 5 analyzes the asset class choice of both the fund manager and the social planner, while Section 6 focuses on the role of regulation. Finally, Section 7 presents some concluding remarks and discusses some empirical implications of the model. All the proofs are gathered in the Appendix.

2 Literature Framework

The model is quite different from the standard literature on contract theory and mutual funds, starting with the pioneering contribution of Bhattacharya and Pfleiderer (1985) and Starks (1987),[8] that have focused mainly on the optimal compensation scheme for the fund manager and on the impact of different compensation arrangements on the potential conflict of interests between investors and fund managers. Instead, we analyze in an adverse selection framework how different types of assets face the asymmetric information problem between the fund manager and the investor, and how

[8]Other contributions are Maug and Naik (1996) and Admati and Pfleiderer (1997) on benchmark-adjusted compensation contracts, Eichberger et al. (1999), Kapur and Timmermann (2005) and Heinkel and Stoughton (1994) on relative performance contract, Das and Sundaram (2001, 2002) on the signaling role of fees.

asymmetric information influences asset selection independently from differences in risk preferences between the investor and the manager. From a regulatory point of view, Das and Sundaram (2002) study the existence of a theoretical support to regulation on fund manager's compensation schemes, while we focus on the efficacy of regulatory policies that constrain the manager's portfolio choice.

Our analysis is related to contract theory literature on monitoring, and in particular to costly state verification models, like Townsend (1979) or Gale and Hellwig (1985), since the investor is allowed to learn at a cost the realized state of nature. A key difference is that in their works the monitoring policy is chosen by the manager who internalizes monitoring costs due to the assumption of perfectly competitive financial market. Instead, in our framework the monitoring policy is selected by the investor and the fund manager does not include monitoring costs in his objective function. So, in Gale and Hellwig (1985) the manager selects the minimum level of monitoring that implies incentive compatibility, and the debt contract is always optimal. On the contrary, in our model the debt contract is not always optimal, since a rentability/monitoring trade-off arises. This trade-off implies that, when the asymmetry of information can be easily controlled because of low monitoring costs, the socially optimal financial instrument is the stock, the asset that maximizes gains from trade. Moreover, the debt contract is not generally preferred when the investment choice is delegated to the fund manager, since the latter does not internalize monitoring costs and prefers riskier assets, like options or stocks, that allow higher information rents.

The paper presents also similarities with hierarchical models, in particular DeMarzo, Fishman and Hagerty (2000). In their framework, the regulatory policy is delegated to a self regulatory organization (SRO), that establishes the "rules of the game" (monitoring probabilities and eventual punishments) under which the investor chooses the optimal contract for the financial intermediary. In our model, on the contrary, investors select directly the monitoring policy and delegate to the fund manager the choice of the instrument to invest in. So, differently from DeMarzo, Fishman and Hagerty (2000), it is the choice of the financial instrument that establishes the "rules of the game" under which the fund manager's contract and the monitoring policy are chosen by the investor.

Our emphasis on portfolio strategy restrictions[9] is in line with Dybvig, Farnsworth and Carpenter (2002) and Gomez and Sharma (2006). However, their focus is different from ours. Dybvig, Farnsworth and Carpenter (2002) analyzes the introduction of restrictions on portfolio strategy in the fund manager's contract to induce manager's effort, while in our analysis portfolio strategy constraints are an additional ex-ante regulatory instrument that combined with monitoring can help preventing fraudulent behavior from the manager. Gomez and Sharma (2006) derives the optimal contract in the class of all linear contracts under short-selling constraints, while our model

[9]Mutual funds tend to be subject to regulatory or client-imposed investment constraints, like limits on investing in equities or in international assets, mandated investment in specific securities, diversification rules or limits on the use of derivatives. However, other types of constraints are generally imposed on mutual funds, like liquidity restrictions (use of restricted securities) or leverage-related restrictions (i.e. constraints on short sales, or on borrowing).

focuses on constraints on portfolio composition.

To summarize, our analysis, differently from the previous literature, focuses on portfolio constraints as an additional regulatory instruments, complementary to monitoring by regulatory authorities. In the following Section, we will introduce the model and show how it differentiates from both costly state verification and hierarchical models.

3 The Model

We assume for simplicity that in the market there is only one underlying asset that assumes the values R, $2R$, $3R$ with equal probability: its realized value is not publicly observable and is costlessly known only by the fund manager, while its probability distribution is public knowledge. Since the fund manager has an informative advantage, the investor has to prevent fraudulent behavior; at this scope he could simply use transfers, but, given the hypothesis of fund manager's limited liability, the latter would appropriate of the entire information rent.[10] Another solution to the asymmetric information problem is to use *costly monitoring*: truthtelling can be induced by checking the manager's report with a positive probability and by choosing the appropriate punishment in case of false report. So, we assume that the investor can discover the underlying asset realization by paying a cost $c(p) = cp^2/2$, with $c > 0$, where $p \in [0, 1]$ represents the probability to control the manager's report. The *Principle of Maximal Punishment*[11] applies, and, given the limited liability assumption, the manager caught lying will loose all the extra rents gained announcing the false report.[12] The type of security is selected by the mutual fund manager, while transfers and monitoring policy are chosen by the investor. We assume that the investor can commit ex-ante to the chosen monitoring policy by paying monitoring costs before the manager's announcement.[13]

[10] Our assumption of manager's limited liability implies that the fund manager can not be punished if the fund performs poorly, but he can just share losses. So, we do not consider explicitly the investor's possibility to shift assets between funds based on performance as an extra punishment in case of low returns. Our choice is motivated by the following considerations: first, we think that this possibility is limited for small investors due to high switching cost. Secondly, given that often mutual funds have a completely "flat" compensation scheme, we think that this type of punishment is already included in the performance based fee structure adopted in the model. Notice that this interpretation extends the applicability of our analysis to a wider range of investment funds than hedge funds.

[11] The *Principle of Maximal Punishment* has been coined by Baron and Besanko (1984).

[12] If unlimited liability was assumed, the full information equilibrium would have been easily achieved by imposing infinite punishment in case of false report. However, the hypothesis of no wealth for the manager prevents such a possibility.

[13] When the possibility for the principal to monitor the agent is introduced in the classical principal/agent framework, the implementation of the contract becomes very sensitive to strategic behavior from all the parties involved. First, the agent has an incentive to bribe the auditor if the latter is different from the principal. We prefer to avoid this complexity in the model by assuming an "honest" monitor, excluding the coalition case. For an analysis of collusion issues we refer to the works of Tirole (1986) or Kofman and Lawarrée (1993). Secondly, the principal faces a commitment problem in his audit policy, since his behavior presents a time inconsistency. After truthtelling has been

3.1 Agents

Three agents are active on the market: the issuer, the mutual fund manager and the investor. All agents are risk-neutral, to prevent any problem related to different risk preferences between the manager and the investor.

Issuer. The issuer proposes different derivatives based on the underlying asset, in particular risky debt, stocks and options are considered. Given the competitiveness of the financial market, every type of asset demanded by the investor or by the fund manager will be present on the market, if it has a positive social value. The issuer assigns to the asset a value equal to its expected cash flow multiplied by a discount factor $\delta \in [0, 1]$ that represents his time impatience, $\delta E(CF)$.[14]

Investor. The investor can invest his wealth directly in the financial market or indirectly in a mutual fund, but direct investment implies an high opportunity cost due to the loss of time needed by the direct management of the portfolio. His evaluation of the asset is given by the difference between the expected cash flow of the investment and the costs of delegation, $V = (1-\gamma)E(CF)$,where:

$$\gamma = \frac{delegation\ costs}{E(CF)} = \frac{E\left[c(p)\right] + transfers}{E(CF)}$$

The investor selects the compensation scheme of the fund manager and the eventual monitoring policy. So the costs of delegation will be equal to the expected transfers to the manager plus the costs of controlling the manager's work and will be endogenously determined.

Fund Manager. As already mentioned, the mutual fund manager owns no funds and can't buy directly assets. However, he owns an informational advantage since he knows costlessly the realization of the underlying, while the investor can costlessly know only the type of asset bought by the fund he has invested in. The fund manager selects the asset type to maximize his expected revenues: the manager's evaluation of the security is given by his information rent, therefore by the expected transfers received from the investor.

3.2 Classes of Assets

Depending on the return guaranteed to the investor, we differentiate three financial instruments based on the underlying asset:

induced, the principal will have ex-post a clear incentive not to do costly monitoring, the manager will anticipate this kind of behavior and the whole equilibrium will collapse. In the model we assume commitment, an interesting analysis on this issue is presented in Khalil (1997), and a joint analysis of the collusion and commitment problems can be found in Khalil and Lawarrée (2003).

[14]As DeMarzo and Duffie (1999), we assume that the issuer discounts future cash flows at a rate that is higher than the market rate. So, the discount factor δ represents the fractional value to the issuer of unissued assets.

1. **Call Option:** a call option at time t is the right to buy at time $t+1$ at an exercise price E the underlying asset. So, a call option with exercise price E, with $E \in [2R, 3R]$, will give a return $(3R - E)$ or 0 with probabilities respectively 1/3 and 2/3.[15]

2. **Debt Contract:** investing in debt gives a return equal to a fixed repayment D, the nominal value of debt, if the issuer is solvent, while if this fixed payment cannot be met, the issuer has to declare bankruptcy and the investor will be the residual claimant of the issuer's assets. We differentiate:

   - **Low Risk Debt:** a low risk debt contract with nominal value $D \in [R, 2R]$ gives a return D or R with probabilities respectively 2/3 and 1/3.
   - **High Risk Debt:** a high risk debt contract with nominal value $D \in [2R, 3R]$ gives a return D, $2R$ or R with probability 1/3 each.

3. **Stock:** the stock is just a special case of the high risk debt contract, where $D = 3R$.

3.3 Asset Choice

From a social welfare point of view, investing in any type of asset should be valuable, since it always guarantees positive gains from trade, $W = (1 - \delta)E(CF) \geq 0$. However, due to the asymmetric information problem present on the market, the asset will be socially valuable only if it guarantees positive realized gains from trade, $W = (1 - \gamma - \delta)E(CF) \geq 0$. Since the asset choice is delegated to the manager who maximizes his own expected revenues, the positive social value of the security becomes only a necessary condition to induce both the investor and the issuer to participate in the market.

Our analysis is twofold: first, we assume that mutual funds can invest in only one type of instrument, e.g. only in options, and we analyze the choice of the asset value, e.g. the option exercise price, for a given asset type; secondly, we analyze the choice between different instruments (stock, debt, etc.).

We start by comparing for each security three different situations in term of asset value choice:

• **First Best:** there is neither intermediation nor information asymmetry and the investor buys directly the issuer's asset.

• **Second Best:** intermediation and asymmetry of information between the fund manager and the investor are assumed, but the social planner selects the asset value.

[15] We could also consider a call option with exercise price $E \in [R, 2R]$, that gives a return 0, $(2R - E)$ or $(3R - E)$ with equal probability. The introduction of this activity does not add anything to the qualitative results of the model, and is not included for simplicity. Moreover, the put option is omitted since it is totally symmetric to the call option and equivalent in social welfare/manager's rents terms.

• **Third Best:** asymmetry of information between the fund manager and the investor is maintained, and the choice of the asset value is delegated to the manager.

Then we consider the asset class choice in three different cases:

• **Social Planner:** the planner selects the asset that maximizes gains from trade, given that the optimal asset value is the second best one (so the planner chooses both the asset type and the asset value).

• **Fund Manager:** the fund manager selects the asset that maximizes his information rent, given that the asset value is the third best one (so the manager chooses both the asset type and the asset value).

• **Regulated Manager:** the planner selects the asset that maximizes gains from trade, but the asset value is the third best one (so the planner selects the asset type and the fund manager the asset value).

3.4 Timing

The game is modelled in the following way: at the initial stage the fund manager (or the social planner) proposes an asset type (debt, stock, option) and chooses a certain asset value for the proposed instrument. The selected asset must be socially valuable otherwise the issuer and the investor will not both participate in the market. At the second stage, the investor decides to enter or not in the mutual fund: if he decides not to enter, he will get an outside opportunity utility level normalized to zero due to the high opportunity cost of direct investment, while if he decides to enter, he will delegate the management of his wealth to the fund manager and he will choose a contract that specifies the compensation of the manager, depending on the return of the investment, and the eventual monitoring policy. The manager accepts or refuses the investor in the mutual fund. Then the manager discovers the realization of the underlying asset and makes a report to the investor. Depending on the reported value, the investor will monitor or not the manager's report. The contract is executed.

4 Asset Value Choice

The fund manager and the social planner, given a certain class of asset, choose the optimal asset value (and so the nominal value of debt or the exercise price of the option) by maximizing, respectively, information rents or realized gains from trade, taking into account the compensation schemes chosen by the investor at the second stage of the game.

4.1 First Best Choice (FB)

If the issuer and the investor could deal directly without intermediation, than the investor would choose the exercise price that maximizes gains from trade:

$$W = (1 - \delta)E(CF)$$

So, for each asset, the value that guarantees the higher return differential between the different states of nature would be selected:

- given $E \in [2R, 3R]$, the FB exercise price for the option is $E^{FB} = 2R$;
- given $D \in [R, 2R]$, the FB nominal value of low risk debt is $D^{FB} = 2R$;
- given $D \in [2R, 3R]$, the FB nominal value of high risk debt is $D^{FB} = 3R$ (the stock).

However, the necessity of an intermediary and the presence of an asymmetry of information can prevent such solutions.

4.2 Social Planner's Choice (Second Best)

Due to the asymmetric information problem, the Social Planner's choice will be generally different from the First Best one. The model is solved by backward induction starting from the investor's problem at the second stage to determine the optimal transfers and monitoring policy. Then, at the first stage, the social planner maximizes gains from trade given the optimal compensation and monitoring schemes chosen by the investor. Such schemes would clearly be influenced by the presence of an asymmetry of information between the manager and the investor: if the investor offers a constant fee to the manager, the latter would have an incentive to lie in good states of nature and so to report a bad realization, since he would receive the fee plus the return differential. So the compensation must be shaped in a way that induce truthtelling by the manager.

4.2.1 Model Structure in the SB Case

We present the model structure in the call option case as an example, the low and high risk debt cases are similar and are discussed in Appendix A.

Investor's Problem. At the second stage of the game, the investor, taking the exercise price of the option E as given, proposes to the fund manager a contract that maximizes his expected cash flow. So, he will choose the cash flow maximizing transfers to the manager, $\bar{t}$ and $\underline{t}$, respectively if a good or a bad realization of the option is announced, and the relative monitoring policy, $\overline{p}$ and $\underline{p}$, where $p \in [0, 1]$ represents the probability for the manager to be monitored. The investor's problem writes as:

$$
\begin{array}{llll}
\mathcal{P}_1: & \max\limits_{\bar{t},\underline{t},\bar{p},\underline{p}} & \multicolumn{2}{l}{\dfrac{1}{3}\left[(3R-E)-\bar{t}-c(\bar{p})\right]+\dfrac{2}{3}\left[-\underline{t}-c(\underline{p})\right]} \\
 & s.t. & (IR) & \dfrac{1}{3}\,\bar{t}+\dfrac{2}{3}\,\underline{t}\geq 0 \\
 & & (\underline{LL}) & \underline{t}\geq 0 \\
 & & (\overline{LL}) & \bar{t}\geq 0 \\
 & & (\overline{IC}) & \bar{t}\geq \underline{t}+(3R-E)(1-\underline{p}) \\
 & & (\underline{IC}) & \underline{t}\geq \bar{t}-(3R-E)(1-\bar{p})
\end{array}
$$

In maximizing his expected cash flow, the investor must satisfy some constraints. First, the investor has to be accepted in the fund, so the manager must get a non negative utility from the investor: the manager's participation constraint must be satisfied. Since the manager owns no wealth, transfers in any state of the world must be non negative: the two limited liability constraints[16] are imposed. Notice that the participation constraint of the fund manager is guaranteed by $(\underline{LL})$ and $(\overline{LL})$, that imply non negative ex-ante revenue for the manager. Finally, truthtelling has to be induced: the transfer obtained by the manager when the true realization is announced must be not inferior to the transfer received if a false realization is reported.

Social Planner's Problem. Taking the contract choice of the investor as given, the social planner maximizes realized gains from trade by solving the following problem:

$$
\begin{aligned}
\mathcal{P}_2: \quad & \max_{E} W^C = (1-\gamma-\delta)\frac{(3R-E)}{3} \\
& s.t.\ E\in[2R,3R] \\
& \quad\ \ \bar{t}^*,\underline{t}^*,\bar{p}^*,\underline{p}^*\ solve\ (\mathcal{P}_1) \\
& \quad\ \ W^C\geq 0
\end{aligned}
$$

The first best solution $E^{FB}=2R$ could not be feasible anymore, since it could provide a negative level of social welfare, and, even if still optimal, it provides a lower level of social welfare than in a first best world due to the delegation cost γ. By solving $(\mathcal{P}_2)$ we obtain the second best values of the exercise price, E^*_{SP}.

[16]The hypothesis of limited liability of the manager is crucial for the model. If unlimited liability had been assumed, the investor would have not needed monitoring to solve the asymmetric information problem and would have been able to reach the first best by simply imposing, for example, the following transfers:

$$\bar{t}=2R-\frac{2}{3}E\geq 0 \quad \text{and} \quad \underline{t}=\frac{E}{3}-R\leq 0$$

4.2.2 Second Best Choice

The model presented in the previous section for the call option case is solved for the three asset classes considered; the obtained results are summarized in the following Lemma and presented graphically:

Lemma 1 *When both monitoring costs and the time preference parameter δ are low, the social planner chooses the maximum value of the information asymmetry, while when they are high the social planner prefers to minimize the asymmetric information problem. In this last case, for particularly high values of δ, no trade occurs and a total market failure realizes for high risk debt.*

Call Option. The Social Planner still chooses the first best level of the exercise price, $2R$, if monitoring costs and the time preference parameter δ are low enough to ensure positive gains from trade (region I). Since realized gains from trade are a decreasing function of the monitoring cost parameter, the Social Planner selects a positive level of asymmetric information only for low values of the monitoring costs. If this is not the case, intermediation costs are higher than gains from trade and the Social Planner is forced to ensure at least a zero value of gains from trade by eliminating the asymmetric information problem and by choosing $E_P^* = 3R$ (region II). We have a *partial market failure*, since the Social Planner stays on the market but prefers to eliminate gains from trade to avoid the high intermediation costs.

– INSERT FIG.1 –

Low Risk Debt. The Social Planner chooses the first best nominal value of low risk debt if monitoring costs and the time preference parameter δ are sufficiently low to ensure higher gains from trade than $(1-\delta)R$ (region I). Otherwise, he prefers to eliminate the asymmetry of information by choosing $D_P^* = R$ and to obtain a social welfare level $W = (1-\delta)R$ (region II). The Social Planner's behavior is similar to the call option case: first best choice for low monitoring costs and no information asymmetry for high monitoring costs. Notice that low risk debt allows positive realized gains from trade even when the asymmetry of information is eliminated, since the investor is the residual claimant of the firm's asset and gets a minimum return of R. Hence, low risk debt allows the Social Planner to avoid the partial market failure of the call option case, since a strictly positive level of social welfare is always obtained.

– INSERT FIG.2 –

High Risk Debt. As for low risk debt, the Social Planner selects the first best level of the nominal debt value (the stock) if monitoring costs are sufficiently low to ensure a level of gains from trade greater than $W(2R)$ (region I). If this is not the case, his best option is to select the lower value of the nominal debt, $D = 2R$ (region II). However, by choosing $D = 2R$, the Social Planner is unable to eliminate completely the asymmetry of information and obtains only a low risk debt contract with nominal value equal to $2R$. This asset gives a negative welfare level for high values of δ and of the monitoring costs parameter, c. Hence, the Social Planner's problem will not have a solution (region III). We can have a *total market failure*: even if high risk debt is always potentially valuable, since it presents positive gains from trade, for high δ and c parameters the Social Planner prefers to stay out from the financial market due to the high intermediation costs. In the previous cases, this never happens because the Social Planner could always obtain a non negative welfare value by eliminating the asymmetry of information.

– INSERT FIG.3 –

4.3 Fund Manager's Choice (Third Best)

The fund manager is only interested in his own revenues, and does not care about social welfare. However, his choice is constrained by the necessity of investing in an asset that has non negative gains from trade, otherwise the issuer and the investor will refuse to participate in the mutual fund. As the social planner, the fund manager internalizes the compensation scheme and the monitoring policy chosen by the investor at the second stage.

4.3.1 Model Structure in the TB Case

Again, we present the solving methodology of the model in the call option case as an example, the other two cases are illustrated in Appendix A. Notice that the Investor's Problem in identical to the one in the Social Planner's case.

Fund Manager's Problem. The manager solves the following problem to choose the exercise price of the call option:

$$\begin{array}{lll} \mathcal{P}_3: & \max\limits_{E} & IR_M^C = \dfrac{1}{3}\,\bar{t} + \dfrac{2}{3}\,\underline{t} \\ & s.t. & E \in [2R, 3R] \\ & & 1 - \gamma - \delta \geq 0 \\ & & \bar{t}^*, \underline{t}^*, \bar{p}^*, \underline{p}^* \ solve\ (\mathcal{P}_1) \end{array}$$

The fund manager internalizes the investor's problem, and, given the optimal second-stage values of transfers and monitoring probabilities, maximizes expected transfers under a non negative social value constraint.

4.3.2 Third Best Choice

The solving methodology presented for the call option case is applied to the three asset classes considered and the analysis of each asset is presented graphically.

Call Option. The manager's and the social planner's choices coincide only for $\delta \leq 1/4$ and high monitoring costs (region $\mathrm{I_a}$). When both monitoring costs and the time preference parameter are low, the fund manager prefers an asymmetric information problem less severe than the Social Planner, and chooses a higher exercise price than the second best one (region II). This happens because monitoring costs are low: huge information rents, that depends positively on $(3R - E)$, induce the investor to monitor more and in this way lower manager's transfers. As δ increases, however, the necessity to induce the participation of both the investor and the issuer on the market (that implies a positive welfare level) starts to influence the manager's choice. The manager would have preferred a lower level of asymmetric information, and so a lower value of the exercise price, $E_M = 3R - c > E_M^* = 3R - 4c\delta$, that is not feasible anymore (region III). For $\delta \geq 1/2$ and low monitoring costs, the constraint binds even more: the manager is forced to choose an exercise price that implies a total control from the investor and so that cancels any information rent (region $\mathrm{I_b}$).

– INSERT FIG.4 –

For high values of both the monitoring cost and the time preference parameter, the probability of monitoring is always less than one and the fund manager would have preferred to maintain a positive level of information asymmetry. However, the social welfare constraint binds and the manager is forced to choose a value of the exercise price that eliminates any information asymmetry to ensure the participation on the market of the investor and the issuer: $E_M^* = E_P^* = 3R$ (region IV). Results are summarized in the following Lemma:

Lemma 2 *For low values of both the monitoring costs and the time preference parameter δ, the manager chooses $E_M^* > E_{SP}^*$, while in all the other cases E_M^* is equal to E_{SP}^*.*

Low Risk Debt. The manager's and the social planner's choices coincide only for $\delta \leq 1/4$ and high monitoring costs (region I). As in the call option case, the fund manager prefers a less severe asymmetric information problem than the Social Planner when monitoring costs are low. Hence, he chooses a lower nominal debt value than the second best one in order to avoid a high monitoring level (region II). As δ and c increase, the manager selects higher nominal debt value than the social planner (region III, IV), unless the social welfare constraint binds (region V).

An important difference between debt and call option is that the manager is never forced to cancel completely the information asymmetry to guarantee the investor's participation: since the investor is residual claimant of the issuer's assets, he obtains a strictly positive utility level even when $D = R$.[17] Given that gains from trade are decreasing for $D < R + c\delta/2$, the manager can obtain a strictly positive utility simply by choosing a debt level $R < D_M < R + c\delta/2$ and such that $W(D_M) \geq 0$. Notice also that, differently from the option case, the manager is never forced to choose a nominal debt value that implies a total control from the investor: again, it is always possible to choose a value of D close to R that satisfies the social welfare constraint and gives, at the same time, a strictly positive rent to the manager.

– INSERT FIG.5 –

High Risk Debt. The results obtained for the call option and the low risk debt hold also for high risk debt: the fund manager prefers a lower level of information asymmetry (a lower nominal debt value) than the planner if monitoring costs are low (region II), a greater one if monitoring costs are high (region III, IV), and is constrained in his choice by the necessity to induce investor's participation for high value of δ (region V, VI). Evidently, investment choice delegation worsens the asymmetric information problem and the fund manager can not improve on the social planner: a market failure occurs in the same region than for the planner (region VII). So, for high monitoring costs and a high level of the time preference parameter, the investor does not buy a high risk debt contract. The market failure is more severe than for the call option or the low risk debt: in those cases, the investor participates to the mutual fund, but the social welfare level is zero, while for high risk debt, the investor prefers to stay out of the fund, since participation implies a negative welfare value.

– INSERT FIG.6 –

The obtained results for low and high risk debt are summarized in the following Lemma:

Lemma 3 *When both monitoring costs and the time preference parameter δ are low, the fund manager chooses $D^*_M < D^*_{SP}$; when monitoring costs are high and the time preference parameter δ is low $D^*_M = D^*_{SP}$, while in the other cases $D^*_M > D^*_{SP}$. However, for particularly high values of δ, no trade occurs and a total market failure realizes for high risk debt.*

Stock. We have already remarked that high risk debt includes, as a special case, the stock: for $D = 3R$, high risk debt return replies exactly the underlying asset return. The market failure problem is clearly more severe for the stock: when monitoring costs are high, the asymmetric

[17] Obviously, this would not be the case if the underlying asset's low return was normalized to zero. However, we think that an intrinsic characteristic of debt is to guarantee a minimum positive return even in the worst nature's realization. On the opposite, it is typical of option to have a zero value if the bad state realizes.

information problem can not be reduced by choosing a lower value of the nominal debt. Hence, welfare becomes negative for lower values of the time preference parameter than in the high risk debt case. However, a stock performs better than high risk debt in welfare terms for low values of the monitoring costs. If this is the case, the manager prefers a lower value of nominal debt than the social planner to avoid a high probability of being monitored. The commitment to choose $D = 3R$ implied by the stock eliminates the manager's freedom to choose the nominal value of debt and increases realized gains from trade compared to high risk debt.

4.4 Conclusions

We have analyzed the asset value choice problem when the fund can invest in only one type of instrument for three different classes of securities: call option (C), low risk debt (LD) and high risk debt (HD). The social planner applies a bang-bang strategy: when potential gains from trade are high and the cost of controlling the asymmetric information problem is low, he prefers to maximize the amount of gains from trade since the manager's report can be easily checked. However, as monitoring costs increase, he prefers to minimize manager's incentives to misreport by reducing gains from trade to the minimal amount. In general, the fund manager's choice is different from the social planner's one and reflects a rentability/monitoring trade-off: the manager prefers a lower level of gains from trade (and so of information asymmetry) for low monitoring costs and a higher one for high monitoring costs. When monitoring costs are low, higher gains from trade increase potential revenues from lying, but at the same time enhance the incentives of the investor to incur monitoring costs to discipline the manager. As monitoring costs increase, investor's incentives to monitor decrease and the first effect dominates: the manager prefers a higher level of gains from trade than the social planner.

5 Asset Class Choice

If we consider the choice of the optimal asset class, it is evident that in the absence of intermediation the first best choice is the stock. In fact, if we denote by W_C, W_{LD} and W_{HD} the welfare levels of, respectively, a call option, a low risk debt or a high risk debt contract, calculated for the first best asset value, they have a precise ranking:[18]

$$W_C^{FB} = (1-\delta)\frac{R}{3} < W_{LD}^{FB} = (1-\delta)\frac{4}{3}R < W_{HD}^{FB} = (1-\delta)2R$$

Given that the optimal asset is the high risk debt contract and that the first best high risk debt nominal value is $D = 3R$, the optimal investment is the stock.

[18]This particular ranking is clearly due to the adopted formulation of the underlying asset: the difference between each possible asset realization is equal to R, the low state value. If a different specification was adopted, the stock would remain the preferred asset, but the first best ranking between call option and low risk debt contract could change.

However, when information asymmetry is introduced in the model, this is not necessarily the case. To analyze the effects of investment delegation, we compare the choice of the optimal asset class for the social planner and for the fund manager, to understand how the planner's choice differs from the first best one (the cost of delegating the management of the investment) and how the manager's choice differs from the planner's one (the cost of delegating the investment choice). The planner and the manager have completely different objectives: the social planner compares the realized gains from trade of the three assets class considered (call option, low risk debt, high risk debt), calculated for the second best asset value, while the manager compares the information rents that he obtains from the different securities, calculated for the third best asset value.

5.1 Social Planner's Optimal Asset Class

The social planner maximizes social welfare by comparing realized gains from trade of the three different securities, given that the asset value of each financial instrument maximizes social welfare (the second best asset value). If we denote by CF^P the cash flow of asset P, with $P \in \{C, LD, HD\}$, the planner's problem states as:[19]

$$\begin{aligned} \mathcal{P}_{10}: \quad \max_{P \in \{C,LD,HD\}} \quad & W_{SP}^P = (1-\gamma-\delta)E(CF^P) \\ s.t. \quad & E \ solves \ \mathcal{P}_2 \\ & D^{LD} \ solves \ \mathcal{P}_5 \\ & D^{HD} \ solves \ \mathcal{P}_8 \end{aligned}$$

The social planner chooses both the asset class and the asset value to maximize realized gains from trade. The following Proposition is obtained:

Proposition 1 *When both monitoring costs and the time preference parameter δ are low, the social planner selects $P_{SP}^* = HD$; while when they are high the planner chooses $P_{SP}^* = LD$. For intermediate values, low and high risk debt coincide and $P_{SP}^* = LD = HD$.*

Results are summarized in figure (7). The social planner faces the following trade-off: on the one hand risky assets magnify manager's incentives to lie and pocket the return differential, on the other hand a high gap between payoffs enhances the investor's incentives to incur monitoring costs to control the fund manager. In line with these consideration, when monitoring costs and the time preference parameter are low, the planner prefers the first best asset, high risk debt (HD), even when the delegation of the investment management, and so asymmetric information are introduced. Given the low monitoring costs, the planner can easily control the agency problem and chooses the stock as the optimal instrument (region I). As monitoring costs and δ increase, the social planner becomes indifferent between high and low risk debt (region II), since for the high risk debt contract

[19] Problems $\mathcal{P}_4 - \mathcal{P}_9$ are defined in Appendix A.

the optimal asset value is $D = 2R$ and so low and high risk debt coincide. For both monitoring cost and δ high, the planner chooses low risk debt (LD) and prefers to eliminate the information asymmetry by choosing $D = R$.

– INSERT FIG.7 –

5.2 Fund Manager's Optimal Asset Class

The fund manager maximizes his utility by comparing his information rents in the three different asset classes, given that the asset value of each security maximizes information rents (the third best asset value). If we denote by IR_M^P the manager's information rent when the asset type P is selected, with $P \in \{C, LD, HD\}$, the planner's problem states as:

$$\begin{aligned} \mathcal{P}_{11}: \quad \max_{P \in \{C,LD,HD\}} \quad & IR_M^P = E(t^P) \\ s.t. \quad & E \ solves \ \mathcal{P}_3 \\ & D^{LD} \ solves \ \mathcal{P}_6 \\ & D^{HD} \ solves \ \mathcal{P}_9 \end{aligned}$$

where $E(t^P)$, with $P \in \{C, LD, HD\}$, denotes the expected value of transfers received by the fund manager. The manager chooses both the asset class and the asset value to maximize his own utility, and does not consider gains from trade in his maximization process unless for ensuring the investor's and issuer's participation to the mutual fund.

Proposition 2 *When both monitoring costs and the time preference parameter δ are low, the fund manager selects $P_M^* = C$. For intermediate values of δ, the fund manager's choice depends on monitoring costs: for low costs, low and high risk debt coincide and $P_M^* = LD = HD$, while for high costs $P_M^* = HD$. When δ is particularly high, $P_M^* = LD$.*

Results are summarized in figure (8). The manager's asset class choice is nearly completely different from the investor's one. When both monitoring costs and δ are low, the manager prefers the call option (OC): even if such security is the worst from a social welfare point of view, it guarantees the highest information rent (region I). In general, high risk debt is preferred, since the asymmetric information problem is more severe than for the other assets (region III). The high risk debt contract can be equivalent for the fund manager to the low risk debt one (region II), since in that region the investor controls completely the high state realization D, and, from an information rent point of view, high debt reduces to a low debt contract with nominal value $2R$. Finally, as monitoring costs increase, gains from trade decrease, the positive welfare constraint starts to influence the manager's choice and low risk debt is preferred (region IV).[20]

[20] In Fig.8 there is a small region inside region IV, denoted by V, where high risk debt is preferred. In that region,

– INSERT FIG.8 –

Summarizing, the fund manager prefers a riskier asset class than the social planner, in order to gain higher information rents. However, for low monitoring costs, the manager selects a lower level of gains from trade in terms of asset values than the social planner, as we have shown in Lemma 2 and 3. The rentability/monitoring trade-off faced by the manager implies that not always higher gains from trade are beneficial. For example, the manager prefers the call option (the asset with the lowest gains from trade) when the monitoring policy can be particularly severe due to low monitoring costs. As monitoring costs increase, the trade-off becomes less important and the manager selects the stock (the asset with the highest gains from trade), since he does not internalize monitoring costs.

6 The Role of Regulation

We have seen how the planner's and the manager's asset choices differ, both in term of asset class and asset value. The regulator, to prevent such a behavior, has often constrained the manager's choice to a particular security class. To analyze if this policy is effective, we consider the asset choice problem of a regulated manager: the manager can choose freely the asset value, but he is constrained by a welfare maximizing planner to choose the security class that maximizes social welfare.

6.1 The Regulated Fund Manager

If we denote again by CF^P the cash flow of security P, with $P \in \{C, LD, HD\}$, the regulated manager's problem states as:

$$\begin{array}{llll} \mathcal{P}_{12}: & \max\limits_{P\in\{C,LD,HD\}} & W_{RM}^{P} = (1-\gamma-\delta)E(CF^{P}) \\ & s.t. & E \ solves \ \mathcal{P}_3 \\ & & D^{LD} \ solves \ \mathcal{P}_6 \\ & & D^{HD} \ solves \ \mathcal{P}_9 \end{array}$$

The regulated manager's problem is quite different from the social planner's one: even if he still maximizes realized gains from trade, the asset values maximize the manager's information rent.

for both (HD) and (LD) securities, the welfare constraint binds. Since both information rents have the same maximal value, $c/12$, and the constraint starts to bind before for (HD), we would expect (LD) to be always preferred. However, the information rent curve for low risk debt is steeper. So, we can have that $IR_M^{LD} < IR_M^{HD}$ for certain parameter values (region V).

Proposition 3 *When both monitoring costs and the time preference parameter δ are low, the regulated manager selects $P^*_{RM} = HD$; while when they are high he chooses $P^*_{RM} = LD$. When δ is particularly high, a partial market failure realizes since $W^P_{RM} \leq 0$ for $P \in \{C, LD, HD\}$.*

The results are summarized in figure (9). If we consider the regulated manager's asset choice, we notice that the regulator is unable to prevent partial market failure for high δ values (region V). For these values, high risk debt is not feasible since the implied welfare level would be negative, and both option and low risk debt give at most a zero welfare level. In the other regions, the regulated manager's choice is identical to the planner's one, even if social planner's region II is divided in two subregions in the regulated manager case, II and III, where the manager is not indifferent anymore between high and low risk debt but has a precise preference between the two.

– INSERT FIG.9 –

However, even if social planner's and regulated manager's asset class choices are apparently similar, regulation is not able to prevent the manager from choosing assets values significantly different from the second best ones, as it is evident from figure (10). So, regulation mitigates the agency conflict, but does not solve it, and in particular is unable to guarantee positive realized gains from trade when monitoring costs are particularly high.

– INSERT FIG.10 –

Notice that if we assume that the mutual fund industry is perfectly competitive, the unregulated mutual fund manager's problem becomes identical to the regulated fund manager's one. Due to competition, the manager is forced to select the asset class that is preferred by investors. So, he selects the asset class that maximizes realized gains from trade. However, the competitive fund manager can not be constrained in the choice of the asset type within an asset class.[21] This implies that competition, as regulation, is unable to solve completely the delegation problem due to the impossibility for the fund manager to commit ex-ante to buy a specific asset type within the selected asset class.

Competition is a perfect substitute for regulation only if perfect competition is assumed; if competition is less than perfect, a role for regulation arises. As we have already pointed out, small investors suffer switching costs when changing investment institutions. Moreover, empirical evidence suggests that agency costs are not easily controlled in the mutual funds industry. Hence competition between fund managers is limited and regulation is needed to protect investors.

[21]This is a crucial point of the paper. The hypothesis reflects reality: if the investor or the regulator could select on their own the asset type within a given asset class, there would be no reason for the existence of mutual funds.

7 Conclusions

We analyze in a principal/agent framework the investor/fund manager relationship and we focus on how the selection of a particular asset influences such relationship. Investment delegation creates an asymmetric information problem, that is more severe if the asset choice inside the portfolio is also delegated to the fund manager. Since the asset value chosen by a fund manager differs nearly always from the one selected by a social welfare maximizing planner, the welfare level would be lower and a market failure could occur if monitoring costs are high and the issuer is particularly impatient. The market failure can be partial or total: in the first case, the investor buys the asset but the welfare level is zero, while in the second one, the investor prefers not to buy the security and so not to participate in the financial market. So, asymmetric information and delegation costs create a difference between potential gains from trade and realized welfare levels, but they don't influence all the types of assets in the same way. High risk debt (for example, venture capital) seems the more exposed to delegation problems, while low risk debt (for example, bonds) seems less influenced by the Principal/Agent problem.

If we consider asset class selection, again the planner's and the manager's choices will differ nearly completely. Regulator's constraints on portfolio composition can be a way to reduce the welfare loss due to delegation even if they are unable to solve completely the asymmetric information problem, since the manager is still able to extract information rents by choosing the appropriate asset value within a given security class. In particular, regulation is unable to avoid partial market failure when gains from trade are low and monitoring costs are high. If we consider a perfectly competitive setting, the mutual fund will select the same asset classes than the regulator, so constraints on asset choice become useless. However, if competition in the mutual fund industry is not perfect, a role for regulation arises. Notice also that competition, as regulation, is unable to solve completely the delegation problem due to the impossibility for the fund manager to credibly commit ex-ante to buy a specific asset type within the selected asset class: the mutual fund manager obtains positive information rents even in a perfectly competitive environment.

A first empirical implication of our work is that fraudulent activity in the mutual fund industry should be positively correlated with the degree of sophistication of the financial market and negatively correlated with the number of restrictions on asset choice imposed on the fund by the regulator. The underlying idea is that financial sophistication increases the number of asset classes available to the fund manager. So, the fund manager can select in a bigger set of asset types the asset that allows greater possibilities to cheat on investment returns. However, the more the asset choice is restricted by the regulator, the less are the possibilities to cheat. In a dynamic perspective, on the one side fraudulent activity should increase due to financial innovation since the number of asset classes available to the fund manager augments. So, the potential for fraudulent behavior increases as well. On the other side, frauds should decrease due to the eventual increasing regulation on asset choice and to the growing practice of self-regulation on asset classes. An interesting line of investigation could be to analyze if regulation is increasing fast enough to control for financial innovation.

A second empirical implication of the model is that restrictions on asset choice should be negatively correlated with the competitiveness of the market. We have shown that if the mutual fund industry is perfectly competitive, there is no need of regulation. However, as we discussed in the introduction, competition seems not to be perfect in the mutual fund industry and regulation is needed to control the fund manager. So, regulation on asset choice is a substitute for competition in our framework: the less the market is competitive, the more it should be regulated. As an example, we can consider the hedge fund industry, which is not regulated in terms of asset choice, and the pension fund industry, where restrictions on investment choice are severe. We expect the hedge fund industry to be highly more competitive than the pension fund one.

An interesting but analytically complex extension of the model would be to consider portfolios that are combinations of different asset class, to understand how portfolio composition varies when the social planner or the manager selects the optimal portfolio. In this case, it would also be possible to analyze the effectiveness of a more realistic regulatory policy, where only a certain percentage of the fund must be invested in an asset class selected by the regulator.

Appendix A

Asset Choice: Low Risk Debt

Investor's Problem. At the second stage of the game the investor proposes a contract, taking the nominal debt value D as given. As for the call option, $(\bar{t}, \bar{p})$ and $(\underline{t}, \underline{p})$ denote transfers and monitoring policies if respectively a good or a bad realization is announced. The investor maximizes his expected cash flow by solving the following problem:

$$
\begin{aligned}
\mathcal{P}_4: \quad & \max_{\bar{t}, \underline{t}, \bar{p}, \underline{p}} && \frac{2}{3}\left[D - \bar{t} - c(\bar{p})\right] + \frac{1}{3}\left[R - \underline{t} - c(\underline{p})\right] \\
& s.t. && (IR) \quad \frac{2}{3}\bar{t} + \frac{1}{3}\underline{t} \geq 0 \\
& && (\underline{LL}) \quad \underline{t} \geq 0 \\
& && (\overline{LL}) \quad \bar{t} \geq 0 \\
& && (\overline{IC}) \quad \bar{t} \geq \underline{t} + (D - R)(1 - \underline{p}) \\
& && (\underline{IC}) \quad \underline{t} \geq \bar{t} - (D - R)(1 - \bar{p})
\end{aligned}
$$

The constraints are similar to the call option case previously presented: the investor has to be accepted in the mutual fund and so must select a transfer scheme that guarantees a non negative utility level to the manager. The manager's "no wealth" hypothesis implies the two limited liability constraints. Finally, truthtelling is induced by imposing the two incentive constraints.

Social Planner's Problem. At the first stage, the Social Planner maximizes gains from trade taking the investor's selected transfers and monitoring probabilities as given:

$$
\begin{aligned}
\mathcal{P}_5: \quad & \max_{D} W^{LD} = (1 - \gamma - \delta)\frac{(2D + R)}{3} \\
& s.t.\ D \in [R, 2R] \\
& \quad \bar{t}^*, \underline{t}^*, \bar{p}^*, \underline{p}^* \ solve\ (\mathcal{P}_4) \\
& \quad W^{LD} \geq 0
\end{aligned}
$$

Fund Manager's Problem. At the first stage, the fund manager maximizes expected transfers, anticipating the investor's choice at the second stage and internalizing the participation constraint of both the issuer and the investor:

$$
\begin{aligned}
\mathcal{P}_6: \quad & \max_{D} && IR_M^{LD} = \frac{2}{3}\bar{t} + \frac{1}{3}\underline{t} \\
& s.t. && D \in [R, 2R] \\
& && (1 - \gamma - \delta) \geq 0 \\
& && \bar{t}^*, \underline{t}^*, \bar{p}^*, \underline{p}^* \ solve\ (\mathcal{P}_4)
\end{aligned}
$$

Asset Choice: High Risk Debt

Investor's Problem. At the second stage of the game, the investor selects the contract terms taking, as in the low debt case, the nominal debt value D as given. Since the investment has now three possible returns, D, $2R$ and

R, the contract has to specify transfers and monitoring policies in each of the three possible realizations. Denote by $(\bar{t}, \bar{p})$, $(\widehat{t}, \widehat{p})$ and $(\underline{t}, \underline{p})$ transfers and monitoring policy if, respectively, D, $2R$ or R is reported by the manager. The investor's problem writes as:

$$
\begin{array}{llll}
\mathcal{P}_7: & \max\limits_{\bar{t},\widehat{t},\underline{t},\bar{p},\widehat{p},\underline{p}} & \multicolumn{2}{l}{\frac{1}{3}\left[D-\bar{t}-c(\bar{p})\right]+\frac{1}{3}\left[2R-\widehat{t}-c(\widehat{p})\right]+\frac{1}{3}\left[R-\underline{t}-c(\underline{p})\right]} \\
& s.t. & (IR) & \frac{1}{3}\bar{t}+\frac{1}{3}\widehat{t}+\frac{1}{3}\underline{t} \geq 0 \\
& & (\underline{LL}) & \underline{t} \geq 0 \\
& & (\widehat{LL}) & \widehat{t} \geq 0 \\
& & (\overline{LL}) & \bar{t} \geq 0 \\
& & (\overline{IC}_1) & \bar{t} \geq \widehat{t}+(D-2R)(1-\widehat{p}) \\
& & (\overline{IC}_2) & \bar{t} \geq \underline{t}+(D-R)(1-\underline{p}) \\
& & (\widehat{IC}_1) & \widehat{t} \geq \underline{t}+R(1-\underline{p}) \\
& & (\widehat{IC}_2) & \widehat{t} \geq \bar{t}-(D-2R)(1-\bar{p}) \\
& & (\underline{IC}_1) & \underline{t} \geq \widehat{t}-R(1-\widehat{p}) \\
& & (\underline{IC}_2) & \underline{t} \geq \bar{t}-(D-R)(1-\bar{p})
\end{array}
$$

The constraints are similar to the two cases already analyzed, but the presence of three possible realizations implies some differences. As before, the manager accepts the investor in the fund only if his utility is non negative: the participation constraint applies. Given the hypothesis of no wealth for the manager, the limited liability constraint in the three possible realizations has to be included. In order to induce truthtelling, however, two incentive constraints for each possible realization of the underlying asset are included in the problem: the manager can misreport in two different ways, and both have to be prevented to induce truth revelation.

Social Planner's Problem. In the first stage, the Social Planner maximizes gains from trade, anticipating the investor's choice in the second stage. The Planner's problem writes as:

$$
\begin{array}{ll}
\mathcal{P}_8: & \max\limits_{D} W^{HD} = (1-\gamma-\delta)\frac{(D+3R)}{3} \\
& s.t.\ D \in [2R, 3R] \\
& \quad \bar{t}^*, \widehat{t}^*, \underline{t}^*, \bar{p}^*, \widehat{p}^*, \underline{p}^* \ solve\ (\mathcal{P}_7) \\
& \quad W^{HD} \geq 0
\end{array}
$$

Fund Manager's Problem. At the first stage, the fund manager maximizes his expected rents anticipating the investor's contract and taking into account the social welfare constraint. The manager's problem writes as:

$$
\begin{array}{lll}
\mathcal{P}_9: & \max\limits_{D} & IR_M^{HD} = \frac{1}{3}\bar{t}+\frac{1}{3}\widehat{t}+\frac{1}{3}\underline{t} \\
& s.t. & D \in [2R, 3R] \\
& & (1-\gamma-\delta) \geq 0 \\
& & \bar{t}^*, \widehat{t}^*, \underline{t}^*, \bar{p}^*, \widehat{p}^*, \underline{p}^* \ solve\ (\mathcal{P}_7)
\end{array}
$$

Appendix B

Proof of Lemma 1

We analyze the social planner's choice for the three possible instruments considered in our analysis: call option, low and high risk debt. Problems ($\mathcal{P}_2$), ($\mathcal{P}_4$) and ($\mathcal{P}_6$) are solved by backward induction, starting from the investor's problem and substituting the obtained optimal values for transfers and monitoring policy in the social planner's problem.

<u>**Call Option**</u>

Investor's Problem. The binding constraints are ($\underline{LL}$) and ($\overline{IC}$), since ($\overline{LL}$) is implied by ($\overline{IC}$) and ($\underline{LL}$), ($\overline{LL}$) and ($\underline{LL}$) imply (IR) and, as standard in contract theory, ($\underline{IC}$) is omitted and checked ex post. Transfers are be equal to:

$$\underline{t}^* = 0 \quad \text{and} \quad \overline{t}^* = (3R - E)(1 - \underline{p})$$

After substituting the transfer values in the maximizing function and rearranging, we obtain:

$$\max_{\overline{p},\underline{p}} \ \frac{1}{3}\left[(3R - E)\underline{p} - c(\overline{p})\right] + \frac{2}{3}\left[-c(\underline{p})\right]$$

First order conditions are:

$$\frac{\partial V}{\partial \underline{p}} = \frac{1}{3}(3R - E) - \frac{2}{3}c\underline{p} \quad \text{and} \quad \frac{\partial V}{\partial \overline{p}} = -\frac{1}{3}c\overline{p}$$

Given that $\partial V/\partial \underline{p} \geq 0$ for $\underline{p} \leq (3R - E)/2c$, we obtain that $\underline{p}^* = (3R - E)/2c$ for $E > 3R - 2c$ and $\underline{p}^* = 1$ for $E \leq 3R - 2c$, while $\partial V/\partial \overline{p}$ is always negative and so $\overline{p}^* = 0$. ($\underline{IC}$) is satisfied.

Social Planner's Problem. We have to differentiate two cases depending on the monitoring costs parameter c:

<u>**Case 1: $\mathbf{c \geq R/2}$**</u>

In this case $2R \geq 3R - 2c$, so the probability of monitoring will be always less than one independently from the exercise price E chosen by the social planner. The latter maximizes the following function:

$$\max_{E} \quad (1 - \gamma - \delta)E(CF) = \left[1 - \frac{\frac{1}{3}(3R - E)\left(1 - \frac{3R - E}{2c}\right) + \frac{(3R - E)^2}{12c}}{\frac{1}{3}(3R - E)} - \delta\right]\frac{1}{3}(3R - E)$$

The first order condition is:

$$\partial W^C/\partial E = (2E - 6R + 4c\delta)/(12c)$$

The FOC is positive for $E \geq 3R - 2c\delta$, so we have two different subcases depending on the relative position of $2R$ and $3R - 2c\delta$: (1a) $3R - 2c \leq 2R \leq 3R - 2c\delta$ and (1b) $3R - 2c \leq 3R - 2c\delta < 2R$. Notice that $E = 3R$ provides a zero value of social welfare, $W^C(3R) = 0$.

Case 1a: $c \leq R/(2\delta)$

In this case $2R \leq 3R - 2c\delta$, so we have to compare $W^C(2R) = R(R-4c\delta)/(12c)$ and $W^C(3R)$, where $W^C(2R) =$ is positive for $c \leq R/(4\delta)$. We have two cases depending on the relative position of $R/(4\delta)$ and $R/2$:

1. If $\delta \leq 1/2$, then $R/(4\delta) \geq R/2$. The solution depends on the cost level:
   - $R/2 \leq c \leq R/(4\delta)$: $W^C(2R)$ is positive and $E^*_{SP} = 2R$.
   - $R/(4\delta) < c \leq R/(2\delta)$: $W^C(2R)$ is negative and $E^*_{SP} = 3R$.
2. If $\delta > 1/2$, then $R/(4\delta) < R/2$. In this case, $W^C(2R)$ is always negative and $E^*_{SP} = 3R$.

Case 1b: $c > R/(2\delta)$

In this case $2R > 3R - 2c\delta$ and $E^*_{SP} = 3R$, since the objective function is increasing for $E \geq 3R - 2c\delta$.

Case 2: $\mathbf{c < R/2}$

In this case $2R < 3R - 2c$, so for $E \in [3R - 2c, 3R]$, the objective function is the same than in case 1, and the Social Planner compares $W^C(2R)$ and $W^C(3R - 2c)$. However, for $E \in [2R, 3R - 2c]$ the welfare function assumes the following form:

$$\max_{E} \quad \left(1 - \frac{c/3}{(3R-E)/3} - \delta\right)\frac{1}{3}(3R - E)$$

The first order condition is:

$$\partial W^C / \partial E = -(1-\delta)/3$$

Given that the derivative is always negative and that $W^C(3R) \geq W^C(3R - 2c)$, in solving its maximization problem, the social planner will compare the welfare values of $3R$ and $2R$. As in the previous case $W^C(3R) = 0$, while $W^C(2R) = (1/3)\left[(1-\delta)R - c\right]$. Two cases are then considered:

Case 2a: $c \leq (1-\delta)R$

Since $W^C(2R) \geq 0$, the social planner will choose $E^*_{SP} = 2R$.

Case 2b: $c > (1-\delta)R$

Given that $c < R/2$, there are two possibilities:

1. If $\delta \leq 1/2$, then $(1-\delta)R \geq R/2$. This case is empty and $W^C(2R)$ is always positive for $c < R/2$.
2. If $\delta > 1/2$ then $(1-\delta)R < R/2$. In this case $W^C(2R)$ is always negative and $E^*_{SP} = 3R$.

To summarize the results:

$$(1)\ \textit{When}\ \delta \leq \frac{1}{2}: \begin{cases} if \quad c \leq R/(4\delta) \quad then \quad E^*_{SP} = 2R \\ if \quad c > R/(4\delta) \quad then \quad E^*_{SP} = 3R \end{cases}$$

$$(2)\ \textit{When}\ \delta > \frac{1}{2}: \begin{cases} if \quad c \leq (1-\delta)R \quad then \quad E^*_{SP} = 2R \\ if \quad c > (1-\delta)R \quad then \quad E^*_{SP} = 3R \end{cases}$$

Low Risk Debt

Investor's Problem. As in the call option case, $(\underline{LL})$ and $(\overline{IC})$ are the binding constraints, $(\underline{IC})$ is temporarily omitted and checked ex post. Transfers are equal to:

$$\underline{t}^* = 0 \quad \text{and} \quad \overline{t}^* = (D-R)(1-\underline{p})$$

After substituting for the optimal transfers, the investor solves:

$$\max_{\overline{p},\underline{p}} \ \frac{2}{3}\left[R + (D-R)\underline{p} - c(\overline{p})\right] + \frac{1}{3}\left[R - c(\underline{p})\right]$$

The first order conditions are:

$$\frac{\partial V}{\partial \underline{p}} = \frac{2}{3}(D-R) - \frac{1}{3}c\underline{p} \quad and \quad \frac{\partial V}{\partial \overline{p}} = -\frac{2}{3}c\overline{p}$$

So $\partial V/\partial \underline{p} \geq 0$ for $\underline{p} \leq 2(D-R)/c$, then $\underline{p}^* = 2(D-R)/c$ for $D < R + c/2$ and $\underline{p}^* = 1$ for $D \geq R + c/2$, while $\partial V/\partial \overline{p}$ is always negative and so $\overline{p}^* = 0$. $(\underline{IC})$ is clearly satisfied.

Social Planner's Problem. We have to consider two different cases:

Case 1: $\mathbf{c \geq 2R}$

In this case $2R \leq R + c/2$, so the probability of monitoring will be always less than one independently from the nominal value of debt selected by the social planner. The latter maximizes the following function:

$$\max_{D} \quad (1-\gamma-\delta)E(CF) = \left[1 - \frac{\frac{2}{3}(D-R)\left(1 - \frac{2(D-R)}{c}\right) + \frac{2}{3}\frac{(D-R)^2}{c}}{\frac{1}{3}(2D+R)} - \delta\right]\frac{1}{3}(2D+R)$$

The first order condition is:

$$\partial W^{LD}/\partial D = 4(D-R)/(3c) - 2\delta/3$$

The FOC is positive for $D \geq R + c\delta/2$, so we have two different subcases depending on the relative position of $2R$ and $R + c\delta/2$: (1a) $R + c\delta/2 \leq 2R \leq R + c/2$ and (1b) $2R < R + c\delta/2$. Notice that $D = R$ provides a strictly positive value of social welfare, $W^{LD}(R) = (1-\delta)R$.

Case 1a: $2R \le c \le 2R/\delta$

In this case $2R \ge R + (c/2)\delta$, so we have to compare $W^{LD}(2R)$ and $W^{LD}(R)$. It is easy to show that $W^{LD}(2R) = (1-\delta)R + (2/3)(R^2/c - \delta R)$, so $W^{LD}(2R) \ge W^{LD}(R)$ for $c \le R/\delta$. Two subcases are considered depending on the relative values of $2R$ and R/δ:

1. If $\delta \ge 1/2$ then $2R \ge R/\delta$. $W^{LD}(2R)$ is always less than $W^{LD}(R)$, so $D^*_{SP} = R$.

2. If $\delta < 1/2$ then $2R < R/\delta$. The planner's choice depends on the monitoring cost level:

   - $2R \le c \le R/\delta$: $W^{LD}(2R)$ is greater than $W^{LD}(R)$ and $D^*_{SP} = 2R$.
   - $R/\delta < c \le 2R/\delta$: in this case the opposite is true and $D^*_{SP} = R$.

Case 1b: $c > 2R/\delta$

This condition corresponds to $2R < R + c/2\delta$. Since the objective function is decreasing for $D < R + c/2\delta$, it is always true that $W^{LD}(2R) < W^{LD}(R)$ and $D^*_{SP} = R$.

Case 2: $\mathbf{c < 2R}$

In this case $2R > R + c/2$, so for $D \in [R, R + c/2)$, the objective function is the same than in case 1, while for $D \in [R + c/2, 2R)$ it assumes the following form:

$$\max_D \quad \left(1 - \frac{c/6}{(2D+R)/3} - \delta\right) \frac{(2D+R)}{3}$$

The first order condition is:

$$\partial W^{LD}/\partial D = 2(1-\delta)/3$$

Given that the derivative is always positive and that $W^{LD}(2R) \ge W^{LD}(R + c/2)$, to solve his maximization problem the social planner will compare the welfare values of R and $2R$. As in the previous case $W^{LD}(R) = (1-\delta)R$, while $W^{LD}(2R) = (1/3)\,[5(1-\delta)R - c/2]$, and $W^{LD}(2R) \ge W^{LD}(R)$ for $c \le 4(1-\delta)R$. Two cases are considered depending on the relative values of $2R$ and $4(1-\delta)R$:

1. If $\delta < 1/2$ then $2R < 4(1-\delta)R$. In this case it is always $W^{LD}(2R) > W^{LD}(R)$, and $D^*_{SP} = 2R$.

2. If $\delta \ge 1/2$ then $2R \ge 4(1-\delta)R$. Two sub-cases:

   - If $c \le 4(1-\delta)R$ then $D^*_{SP} = 2R$.
   - If $4(1-\delta)R < c < 2R$ then $D^*_{SP} = R$.

To summarize the results:

$$(1)\,When\ \delta < \frac{1}{2}: \begin{cases} if \quad c \leq R/\delta \quad then \quad D^*_{SP} = 2R \\ if \quad c > R/\delta \quad then \quad D^*_{SP} = R \end{cases}$$

$$(2)\,When\ \delta \geq \frac{1}{2}: \begin{cases} if \quad c \leq 4(1-\delta)R \quad then \quad D^*_{SP} = 2R \\ if \quad c > 4(1-\delta)R \quad then \quad D^*_{SP} = R \end{cases}$$

High Risk Debt

Investor's Problem. Notice that $(\underline{LL})$, $(\widehat{LL})$ and $(\overline{LL})$ imply (IR), that $(\overline{IC}_1)$ and $(\widehat{LL})$ imply $(\overline{LL})$ and that $(\widehat{IC}_1)$ and $(\underline{LL})$ imply $(\widehat{LL})$. Since, intuitively, the manager lies by reporting a worse result than the realized one, the following incentive constraints are temporarily ignored and checked ex post: $(\widehat{IC}_2)$, $(\underline{IC}_1)$ and $(\underline{IC}_2)$. The optimal transfers $\underline{t}^*$ and $\widehat{t}^*$ are then equal to:

$$\underline{t}^* = 0 \quad and \quad \widehat{t}^* = R(1-\underline{p})$$

The optimal transfer $\overline{t}^*$ is determined by the binding constraint between $(\overline{IC}_1)$ and $(\overline{IC}_2)$:

- $(\overline{IC}_1)$ $\quad \overline{t} \geq R(1-\underline{p}) + (D-2R)(1-\widehat{p})$
- $(\overline{IC}_2)$ $\quad \overline{t} \geq (D-R)(1-\underline{p})$

$(\overline{IC}_1)$ is the binding constraint if $\underline{p} \geq \widehat{p}$; assuming that this condition holds, the optimal transfer for a high report would be:

$$\overline{t}^* = R(1-\underline{p}) + (D-2R)(1-\widehat{p})$$

Given the optimal transfers and the assumption that $\underline{p} \geq \widehat{p}$, $(\underline{IC}_1)$ is satisfied, $(\widehat{IC}_2)$ is satisfied if $\widehat{p} \geq \overline{p}$ and $(\underline{IC}_2)$ is satisfied if $\widehat{p} \geq \overline{p}$ and $\underline{p} \geq \overline{p}$. Notice that incentive constraints imply a monotonicity constraint for monitoring probabilities: $\underline{p} \geq \widehat{p} \geq \overline{p}$. After substituting for the optimal transfers, the investor solves:

$$\max_{\overline{p},\widehat{p},\underline{p}} \ \frac{1}{3}\left[D - R(1-\underline{p}) - (D-2R)(1-\widehat{p}) - c(\overline{p})\right] + \frac{1}{3}\left[2R - R(1-\underline{p}) - c(\widehat{p})\right] + \frac{1}{3}\left[R - c(\underline{p})\right]$$

The first order conditions are:

$$\frac{\partial V}{\partial \underline{p}} = \frac{2}{3}R - \frac{1}{3}c\underline{p}; \quad \frac{\partial V}{\partial \widehat{p}} = \frac{1}{3}(D-2R) - \frac{1}{3}c\widehat{p} \quad and \quad \frac{\partial V}{\partial \overline{p}} = -\frac{1}{3}c\overline{p}$$

So: $\partial V/\partial \underline{p} \geq 0$ for $p \leq 2R/c$, then $\underline{p}^* = 2R/c$ for $c > 2R$ and $\underline{p}^* = 1$ for $c \leq 2R$; $\partial V/\partial \widehat{p} \geq 0$ for $p \leq (D-2R)/c$, then $\widehat{p}^* = (D-2R)/c$ for $D < 2R + c$ and $\widehat{p}^* = 1$ for $D \geq 2R + c$, while $\partial V/\partial \overline{p}$ is always negative and $\overline{p}^* = 0$. Notice that $\underline{p} \geq \widehat{p}$ if $D \leq 3R$, and this is always true since $D \in [2R, 3R]$. The monotonicity condition in probabilities, $\underline{p} \geq \widehat{p} \geq \overline{p}$, is satisfied, so the omitted constraints are all verified.

Social Planner's Problem. Notice that monitoring probabilities, if they are less than one, are $\widehat{p}^* = (D-2R)/c$ and $\underline{p}^* = 2R/c$. Evidently, $\underline{p} = 1$ if $c \leq 2R$ and $\widehat{p} = 1$ if $D \geq 2R + c$. It could be $\widehat{p} = 1$ only when $3R \geq 2R + c$, so if $c \leq R$. Three different cases are considered depending on the values assumed by $\widehat{p}$ and $\underline{p}$ (remember that $\widehat{p} \leq \underline{p}$):

- If $c \leq R$ then $\widehat{p} \leq 1$ and $\underline{p} = 1$.
- If $R < c \leq 2R$ then $\widehat{p} < 1$ and $\underline{p} = 1$.
- If $c > 2R$ then $\underline{p} < 1$.

<u>**Case 1: $\mathbf{c \leq R}$**</u>

In this case it can be $\widehat{p} = 1$. If the Planner chooses a value of $D < 2R+c$, he will maximize the following function:

$$\max_D \quad (1-\gamma-\delta)E(CF) = \left[1 - \frac{\frac{1}{3}(D-2R)\left(1-\frac{D-2R}{c}\right)+\frac{c}{6}+\frac{(D-2R)^2}{6c}}{(1/3)\,(D+3R)} - \delta\right]\frac{1}{3}(D+3R)$$

The first order condition is:

$$\partial W^{HD}/\partial D = (D-2R)/c - \delta/3$$

The FOC is positive for $D \geq 2R+c\delta$, and to obtain the solution we should compare $W^{HD}(2R)$ and $W^{HD}(2R+c)$. If the Social Planner's Problem selects a value $D > 2R + c$, his problem writes as:

$$\max_D \quad (1-\gamma-\delta)E(CF) = \left(1 - \frac{c/3}{(1/3)\,(D+3R)} - \delta\right)\frac{1}{3}(D+3R)$$

The first order condition is:

$$\partial W^{HD}/\partial D = (1-\delta)/3$$

The FOC is always positive, so the optimal value would be $D = 3R$. Given that $W^{HD}(3R) \geq W^{HD}(2R+c)$, to obtain the optimal solution we have to compare the welfare values of $D = 2R$ and $D = 3R$:

- $W^{HD}(2R) = 5(1-\delta)R/3 - c/6$
- $W^{HD}(3R) = 2(1-\delta)R - c/3$

Then, $W^{HD}(2R) \geq W^{HD}(3R)$ if $c \geq 2R\,(1-\delta)$. Notice that $W^{HD}(3R)$ is positive for $c \leq 6R(1-\delta)$ and so it will never be negative when $W^{HD}(3R) > W^{HD}(2R)$, while $W^{HD}(2R)$ is positive if $c \leq 10R(1-\delta)$. Given that $R < 2R\,(1-\delta)$ when $\delta < 1/2$ and that $R \leq 10R\,(1-\delta)$ when $\delta \leq 9/10$, we have three subcases:

(1) If $\delta < 1/2$ then $R < 2R(1-\delta)$.

Give that $W^{HD}(3R) > W^{HD}(2R)$ and that $W^{HD}(3R) \geq 0$, $D^*_{SP} = 3R$.

(2) If $1/2 \leq \delta \leq 9/10$ then $2R(1-\delta) \leq R \leq 10R(1-\delta)$.

The solution depends on the cost level:

- $c \leq 2R(1-\delta)$: since $W^{HD}(3R) > W^{HD}(2R) \geq 0$, $D^*_{SP} = 3R$
- $2R(1-\delta) < c \leq R$: since $W^{HD}(2R) > W^{HD}(3R)$ and $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.

(3) If $\delta > 9/10$ then $R > 10R(1-\delta)$.

The solution depends on the cost level:

- $c \leq 2R(1-\delta)$: since $W^{HD}(3R) > W^{HD}(2R) \geq 0$, $D^*_{SP} = 3R$.
- $2R(1-\delta) < c \leq 10R(1-\delta)$: since $W^{HD}(2R) > W^{HD}(3R)$ and $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.
- $10R(1-\delta) < c \leq R$: since $0 > W^{HD}(2R) > W^{HD}(3R)$, the problem has no solution.

Case 2: $\mathbf{R < c \leq 2R}$

In this case $\widehat{p}^*$ is always less than one, and the Social Planner's problem is identical to the problem in Case 1, when $D \leq 2R+c$. The FOC, as we show previously, is positive for $D \geq 2R+c\delta$, so there are two subcases depending on the relative values of $3R$ and $2R+c\delta$.

Case 2a: $c \leq R/\delta$

In this case $2R + c\delta \leq 3R$; we have to compare the welfare values of $D = 2R$ and $D = 3R$:

- $W^{HD}(2R) = 5(1-\delta)R/3 - c/6$
- $W^{HD}(3R) = (5/3 - 2\delta)R - c/6 + R^2/(6c)$

Then, $W^{HD}(2R) \geq W^{HD}(3R)$ if $c \geq R/(2\delta)$. $W^{HD}(2R)$ is positive if $c \leq 10R(1-\delta)$ and $W^{HD}(3R)$ is positive if $c^2 - 6c(5/3-2\delta)R - R^2 \leq 0$. So, $W^{HD}(3R)$ is non negative for $c \in [3R(5/3-2\delta) - f; 3R(5/3-2\delta)+f]$, where $f = \sqrt{9R^2(5/3-2\delta)^2 + R^2}$. Notice that $3R(5/3-2\delta) - f$ is always less than zero and that $3R(5/3-2\delta)+f > 2R$ for $\delta < 17/24$. Notice also that $2R \geq R/(2\delta)$ when $\delta \geq 1/4$ and that $R \leq R/(2\delta)$ for $\delta \leq 1/2$. We have several possible cases:

(1) If $\delta < 1/4$ then $2R < R/(2\delta) < R/\delta$.

Since $W^{HD}(3R) > W^{HD}(2R)$ and $W^{HD}(3R) \geq 0$, $D^*_{SP} = 3R$.

(2) If $1/4 \leq \delta \leq 1/2$ then $R \leq R/(2\delta) \leq 2R < R/\delta$.

The solution depends on the cost level:

- $R < c < R/(2\delta)$: since $W^{HD}(3R) > W^{HD}(2R)$ and $W^{HD}(3R) \geq 0$, $D^*_{SP} = 3R$.
- $R/(2\delta) \leq c \leq 2R$: since $W^{HD}(2R) \geq W^{HD}(3R)$, the solution will be $D = 2R$ if $W^{HD}(2R) \geq 0$. We have just to check that $2R \leq 10R(1-\delta)$: this is true for $\delta \leq 4/5$. So: $D^*_{SP} = 2R$.

(3) If $1/2 < \delta \leq 4/5$ then $R/(2\delta) < R < R/\delta < 2R$.

We are always in the case $W^{HD}(2R) \geq W^{HD}(3R)$, and, given that $W^{HD}(2R) \geq 0$, the solution is $D^*_{SP} = 2R$.

(4) If $4/5 < \delta \leq 1/2 + \sqrt{15}/10$ then $R/(2\delta) < R < R/\delta \leq 10R(1-\delta) < 2R$.

We are in the case $W^{HD}(2R) \geq W^{HD}(3R)$, and, given that $W^{HD}(2R) \geq 0$, the solution is $D^*_{SP} = 2R$.

(5) If $1/2 + \sqrt{15}/10 < \delta \leq 9/10$ then $\Rightarrow R/(2\delta) < R \leq 10R(1-\delta) < R/\delta < 2R$.

We are in the case $W^{HD}(2R) \geq W^{HD}(3R)$, the existence of a solution depends on the cost level:

- $R < c \leq 10R(1-\delta)$: since $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.
- $10R(1-\delta) < c \leq R/\delta$: since $W^{HD}(2R) < 0$, the problem has no solution.

(6) If $\delta > 9/10$ then $R/(2\delta) < 10R(1-\delta) < R \leq R/\delta < 2R$.

We are in the case $W^{HD}(2R) \geq W^{HD}(3R)$, but $W^{HD}(2R) < 0$ and so no solution exists.

Case 2b: $c > R/\delta$

In this case $2R + c\delta > 3R$; the derivative is always negative for $D \in [2R, 3R]$ and the Social Planner selects $D = 2R$ if $W^{HD}(2R) \geq 0$. However, we first have to check that, given $c \in [R, 2R]$, the case is not empty: $R/\delta \leq 2R$ for $\delta \geq 1/2$. If this is the case, we can differentiate the following subcases:

(1) If $1/2 \leq \delta \leq 4/5$ then $R/(2\delta) \leq R < R/\delta < 2R$.

We are always in the case $W^{HD}(2R) \geq W^{HD}(3R)$, and, given that $W^{HD}(2R) \geq 0$, the solution is $D^*_{SP} = 2R$.

(2) If $4/5 < \delta \leq 1/2 + \sqrt{15}/10$ then $R/(2\delta) < R < R/\delta \leq 10R(1-\delta) < 2R$.

We are in the case $W^{HD}(2R) \geq W^{HD}(3R)$, the solution depends on the cost level:

- $R/\delta < c \leq 10R(1-\delta)$: since $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.
- $10R(1-\delta) < c \leq 2R$: since $W^{HD}(2R) < 0$, the problem has no solution.

(3) If $\delta > 1/2 + \sqrt{15}/10$ then $10R(1-\delta) < R/\delta < 2R$.

We are in the case $W^{HD}(2R) \geq W^{HD}(3R)$, but $W^{HD}(2R) < 0$ and so no solution exists.

Case 3: c > 2R

In this case $\underline{p} < 1$, so the monitoring probabilities are both less than one, independently from the nominal debt value. The Social Planner's problem writes as:

$$\max_D \quad (1-\gamma-\delta)E(CF) = \left[1 - \frac{\frac{2}{3}R\left(1-\frac{2R}{c}\right) + \frac{1}{3}(D-2R)\left(1-\frac{D-2R}{c}\right) + \frac{2R^2}{3c} + \frac{(D-2R)^2}{6c}}{(1/3)\,(D+3R)} - \delta\right]\frac{1}{3}(D+3R)$$

The first order condition is:

$$\partial W^{HD}/\partial D = (D - 2R)/3c - \delta/3$$

The FOC is positive for $D \geq 2R + c\delta$, so there are two subcases depending on the relative values of $3R$ and $2R + c\delta$.

Case 3a: $c \leq R/\delta$

In this case $2R + c\delta \leq 3R$, and we have to check, given $c > 2R$, that the case is not empty: $2R < R/\delta$ only when $\delta < 1/2$. When this is the case, the relative welfare values of $D = 2R$ and $D = 3R$ have to be compared:

- $W^{HD}(2R) = (1 - 5\delta/3)\, R + 2R^2/(3c)$
- $W^{HD}(3R) = (1 - 2\delta)\, R + 5R^2/6c$

Then, $W^{HD}(2R) \geq W^{HD}(3R)$ if $c \geq R/(2\delta)$. Notice that $W^{HD}(3R)$ is always positive for $\delta \leq 1/2$, that $W^{HD}(2R)$ is always positive for $\delta \leq 3/5$ and that $R/(2\delta) > 2R$ only when $\delta < 1/4$. We have two possible cases:

(1) If $\delta < 1/4$ then $2R < R/(2\delta) < R/\delta$.

The solution depends on the cost level:

- $2R < c < R/(2\delta)$: since $W^{HD}(3R) > W^{HD}(2R)$ and $W^{HD}(3R) \geq 0$, $D^*_{SP} = 3R$.
- $R/(2\delta) \leq c \leq R/\delta$: since $W^{HD}(2R) \geq W^{HD}(3R)$ and $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.

(2) If $1/4 \leq \delta < 1/2$ then $R/(2\delta) \leq 2R < R/\delta$.

Since for $c \in (2R, R/\delta)$ we have that $W^{HD}(2R) \geq W^{HD}(3R)$ and $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.

Case 3b: $c > R/\delta$

In this case $2R + c\delta > 3R$, the derivative $\partial W^{HD}/\partial D$ is always negative for $D \in [2R, 3R]$ and the planner choice would be $D = 2R$ if $W^{HD}(2R)$ is positive. Notice that $W^{HD}(2R) \geq 0$ for $\delta \leq 3/5$ and for $\delta > 3/5$ if $c \leq \frac{2R}{5\delta - 3}$. Since $2R > \frac{2R}{5\delta - 3}$ for $\delta > 4/5$, we have three possible cases:

(1) $\delta \leq 3/5$

It is always true that $W^{HD}(2R) \geq 0$, so $D^*_{SP} = 2R$.

(2) $3/5 < \delta \leq 4/5$

Then $2R \leq \frac{2R}{5\delta - 3}$ and the solution depends on the cost level:

- $2R < c \leq \frac{2R}{5\delta - 3}$: since $W^{HD}(2R) \geq 0$, $D^*_{SP} = 2R$.
- $c > \frac{2R}{5\delta - 3}$: since $W^{HD}(2R) < 0$, the problem has no solution.

(3) $\delta > 4/5$

It is always $W^{HD}(2R) < 0$, so the problem has no solution.

<u>**To summarize the results**</u>:

$$(1)\ \textit{When}\ \delta \leq \frac{1}{2}\text{:} \begin{cases} if & c < R/(2\delta) \quad then \quad D^*_{SP} = 3R \\ if & c \geq R/(2\delta) \quad then \quad D^*_{SP} = 2R \end{cases}$$

$$(2)\ \textit{When}\ \frac{1}{2} < \delta \leq \frac{3}{5}\text{:} \begin{cases} if & c \leq 2\,(1-\delta)\,R \quad then \quad D^*_{SP} = 3R \\ if & c > 2\,(1-\delta)\,R \quad then \quad D^*_{SP} = 2R \end{cases}$$

$$(3)\ \textit{When}\ \frac{3}{5} < \delta \leq \frac{4}{5}\text{:} \begin{cases} if & c \leq 2(1-\delta)R \quad then \quad D^*_{SP} = 3R \\ if & 2(1-\delta)R < c \leq 2R/(5\delta-3) \quad then \quad D^*_{SP} = 2R \\ if & c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(4)\ \textit{When}\ \delta < \frac{4}{5}\text{:} \begin{cases} if & c \leq 2(1-\delta)R \quad then \quad D^*_{SP} = 3R \\ if & 2(1-\delta)R < c \leq 10R(1-\delta) \quad then \quad D^*_{SP} = 2R \\ if & c > 10R(1-\delta) \quad then\ no\ trade\ occurs \end{cases}$$

$\blacksquare$

Proof of Lemma 2

The investor's problem has been already solved in the proof of Lemma 1. Notice that the fund manager never chooses an exercise price such that $E < 3R - 2c$ (the monitoring probability would be equal to one and so he would receive no information rent), unless the participation constraint of the investor is satisfied only for this range of values. If this is the case, we assume that he will choose the value of E that is socially optimal. We have to consider two different cases:

<u>**Case 1: $\mathbf{c \geq R/2}$**</u>

In this case $2R \geq 3R - 2c$, so independently from the exercise price of the option chosen by the manager, the probability of monitoring will be always less than one and the manager will maximize the following function:

$$\max_E \quad \frac{1}{3}\bar{t} + \frac{2}{3}\underline{t} \quad \text{i.e.,} \quad \max_E \quad \frac{1}{3}(3R-E) - \frac{1}{6c}(3R-E)^2$$

Since:

$$1 - \gamma - \delta = 1 - \frac{(1/3)(3R-E)\left(1 - \dfrac{3R-E}{4c}\right)}{(1/3)(3R-E)} - \delta = \frac{3R-E}{4c} - \delta$$

the constraint $1 - \gamma - \delta \geq 0$ is equivalent to $E \leq 3R - 4c\delta$.

So the fund manager's problem can be rewritten in the following way:

$$\begin{aligned} \max_{E} \quad & \frac{1}{3}(3R - E) - \frac{1}{6c}(3R - E)^2 \\ s.t. \quad & E \le 3R - 4c\delta \\ & E \in [2R, 3R] \end{aligned}$$

For the problem to have a solution, we have to check that the set of the constrained possible values is not empty: $2R$ has to be lower than $3R - 4c\delta$ and higher than $3R - 2c$. It is easy to see that $3R - 2c \le 3R - 4c\delta$ only if $\delta \le 1/2$. If this is the case, we can compute the first order condition:

$$\partial IR_M^C / \partial E = -1/3 + (3R - E)/(3c)$$

The derivative will be positive for $E \le 3R - c$, so in the absence of any constraint the manager would choose $E = 3R - c$ if $3R - c \ge 2R$ ($c \le R$) or $E = 2R$ if $3R - c < 2R$ ($c > R$). The solution will depend on the relative position of $2R$, $3R$ and $3R - 4c\delta$: $2R \ge 3R - 4c\delta$ when $c \ge R/(4\delta)$, and given $\delta \le 1/2$ we know that $R/2 \le R/(4\delta)$, while $3R - c \le 3R - 4c\delta$ for $\delta \le 1/4$. We differentiate three cases:

1. If $\delta \le 1/4$ then $3R - c \le 3R - 4c\delta$. The solution depends on the cost level:

   - $R/2 \le c \le R$: since $2R \le 3R - c \le 3R - 4c\delta$, the optimal choice is $E_M^* = 3R - c$.
   - $R < c < R/(4\delta)$: since $3R - c < 2R < 3R - 4c\delta$, the optimal choice is $E_M^* = 2R$.
   - $c \ge R/(4\delta)$: since $2R \ge 3R - 4c\delta$, the manager will choose $E_M^* = 3R$ to satisfy the welfare participation constraint.

2. If $1/4 < \delta \le 1/2$ then $3R - 4c\delta < 3R - c$. The solution depends on the cost level:

   - $R/2 < c < R/(4\delta)$: since $2R < 3R - 4c\delta < 3R - c$, the optimal choice will be $E_M^* = 3R - 4c\delta$.
   - $c \ge R/(4\delta)$: since $2R \ge 3R - 4c\delta$, the manager will choose $E_M^* = 3R$.

3. If $\delta > 1/2$, the only possibility to satisfy the participation constraint is to choose $E_M^* = 3R$. The manager will give up any information rent.

Case 2: $\mathbf{c < R/2}$

In this case the manager will solve the same problem than in case 1 for $E \le 3R - 2c$, and will receive a zero information rent for $E > 3R - 2c$ since the monitoring probability will be equal to one. The participation constraint is still $E \le 3R - 4c\delta$ for $E \in [3R - 2c, 3R]$ and the manager optimal unconstrained choice is unchanged.

We have to differentiate three cases:

1. If $\delta \le 1/4$ then $3R - c \le 3R - 4c\delta$ and the optimal choice of the manager is $E_M^* = 3R - c$.

2. If $1/4 < \delta < 1/2$ then $3R - c > 3R - 4c\delta$; so I have to consider the relative position of $3R - 4c\delta$ and $2R$. We can easily show that $2R < 3R - 4c\delta$ for $c < R/(4\delta)$ and given that in this interval of values for δ it is always true that $R/2 < R/(4\delta)$ and that $3R - 2c < 3R - 4c\delta$, the optimal choice is $E_M^* = 3R - 4c\delta$.

3. If $\delta \geq 1/2$ then $3R - c > 3R - 4c\delta$; so the relative position of $3R - 4c\delta$ and $2R$ should be considered. We see that $2R < 3R - 4c\delta$ for $c < R/(4\delta)$, however it is important to notice that in this interval of values for δ, $R/2 \geq R/(4\delta)$ and $3R - 2c \geq 3R - 4c\delta$. The solution is the following:

   - $c < (1-\delta)R$: the only values that satisfy the participation constraint imply a monitoring probability of one; the fund manager is then indifferent and will choose the welfare maximizing value, $E_M^* = 2R$.
   - $(1-\delta)R \leq c < R/(4\delta)$: the welfare constraint is again satisfied only for exercise prices that imply a monitoring probability equal to one. The fund manager is indifferent and selects the welfare maximizing value, $E_M^* = 3R$.
   - $R/(4\delta) \leq c < R/2$: given that $2R \geq 3R - 4c\delta$, the only possible solution is $E_M^* = 3R$.

<u>**To summarize the results**</u>:

$$(1)\ \textit{When}\ \delta \leq \frac{1}{4}: \begin{cases} if \quad c \leq R \quad then \quad E_M^* = 3R - c \geq E_{SP}^* = 2R \\ if \quad R < c < R/(4\delta) \quad then \quad E_M^* = E_{SP}^* = 2R \\ if \quad c \geq R/(4\delta) \quad then \quad E_M^* = E_{SP}^* = 3R \end{cases}$$

$$(2)\ \textit{When}\ \frac{1}{4} < \delta < \frac{1}{2}: \begin{cases} if \quad c < R/(4\delta) \quad then \quad E_M^* = 3R - 4c\delta \geq E_{SP}^* = 2R \\ if \quad c \geq R/(4\delta) \quad then \quad E_M^* = E_{SP}^* = 3R \end{cases}$$

$$(3)\ \textit{When}\ \delta \geq \frac{1}{2}: \begin{cases} if \quad c \leq (1-\delta)R \quad then \quad E_M^* = E_{SP}^* = 2R \\ if \quad c > (1-\delta)R \quad then \quad E_M^* = E_{SP}^* = 3R \end{cases}$$

■

Proof of Lemma 3

The investor's problem is solved in the proof of Lemma 1. We differentiate low and high risk debt.

<u>**Low Risk Debt**</u>

The fund manager never chooses a debt nominal value $D \geq R + c/2$ (the monitoring probability would be equal to one and he would gain no rents), unless there is no other way to satisfy the positive social value constraint. If this is the case, the manager selects the socially optimal value. We differentiate two cases:

Case 1: $\mathbf{c > 2R}$

In this case $2R < R + c/2$ and the probability of monitoring is always less than one. The manager's problem is:

$$\max_{D} \quad \frac{2}{3}\bar{t} + \frac{1}{3}\underline{t} \quad \text{i.e.,} \quad \max_{D} \quad \frac{2}{3}(D-R) - \frac{4}{3c}(D-R)^2$$

The constraint $1 - \gamma - \delta \geq 0$ can be rewritten in the following way:

$$(1-\gamma-\delta) = 1 - \frac{\frac{2}{3}(D-R)\left(1 - \frac{2(D-R)}{c}\right) + \frac{2}{3}\frac{(D-R)^2}{c}}{(1/3)\,(2D+R)} - \delta = \frac{R + \frac{2}{3}\frac{(D-R)^2}{c} - \frac{2}{3}\delta D - \frac{\delta}{3}R}{(1/3)\,(2D+R)} \geq 0$$

The denominator is evidently always positive, while the numerator is positive when the following equation is satisfied:

$$2D^2 - 2(2R + c\delta)D + 3cR + 2R^2 - Rc\delta \geq 0$$

So we have to differentiate two cases:

a) If $\Delta < 0$ then $c < 6R(1-\delta)/\delta^2$ and W^{LD} is always non negative.

b) If $\Delta \geq 0$ then $c \geq 6R(1-\delta)/\delta^2$. W^{LD} is non negative for $D \leq R + (c/2)\,\delta - k$ and for $D \geq R + (c/2)\,\delta + k$, where $k = \sqrt{c^2\delta^2 + 6c(\delta-1)R}$. Notice that $R < R + (c/2)\,\delta - k$.

Case 1a: $\Delta < 0$ $\left(c < 6R(1-\delta)/\delta^2\right)$

We have to check that this case is not empty, and so to check if $2R < 6R(1-\delta)/\delta^2$ holds. This equation is true only for $\delta < (-3+\sqrt{21})/2$. In this case, the first order condition of the manager's unconstrained problem is:

$$\partial IR_M^{LD}/\partial D = 2/3 - 8(D-R)/(3c)$$

The derivative will be positive for $D \leq R + c/4$, so in the absence of any constraint the manager would choose $D = R + c/4$ if $R + c/4 \leq 2R$ ($c \leq 4R$) and $D = 2R$ if $R + c/4 > 2R$ ($c > 4R$). We differentiate two cases:

(1) If $\delta \in \left[0, \frac{-3+\sqrt{33}}{4}\right]$ then $6R(1-\delta)/\delta^2 \geq 4R$.

The manager's choice depends on monitoring cost:

- $2R < c \leq 4R$: the optimal choice is $D_M^* = R + c/4$.
- $4R < c < 6R(1-\delta)/\delta^2$: the manager's choice is $D_M^* = 2R$.

(2) If $\delta \in \left(\frac{-3+\sqrt{33}}{4}, \frac{-3+\sqrt{21}}{2}\right)$ then $6R(1-\delta)/\delta^2 < 4R$.

The manager selects $D_M^* = R + c/4$.

Case 1b: $\underline{\Delta \geq 0}$ $\left(c \geq 6R(1-\delta)/\delta^2\right)$

First, notice that $2R < 6R(1-\delta)/\delta^2$ if $\delta < (-3+\sqrt{21})/2$, as in the previous case. The manager's unconstrained problem solution is the same as before, so we have to control the relative values of $R + c/4$, $2R$, $R + (c/2)\,\delta - k$ and $R + (c/2)\,\delta + k$:

- $R + c/4 \leq R + (c/2)\,\delta - k$ if $\delta \geq 1/2$ and $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta - 1/4}$.
- $R + c/4 \geq R + (c/2)\,\delta + k$ if $\delta \leq 1/4$ or $1/4 < \delta < 1/2$ and $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta - 1/4}$.
- $2R \leq R + (c/2)\delta - k$ if $\delta \leq 3/5$ and $c \geq 2R/\delta$ or $3/5 < \delta \leq 3/4$ and $\frac{2R}{\delta} < c \leq \frac{2R}{5\delta - 3}$.
- $2R \geq R + (c/2)\delta + k$ if $\delta \leq 3/4$ and $c \leq 2R/\delta$ or $\delta > 3/4$ and $c \leq \frac{2R}{5\delta-3}$.

In the other cases, $R + (c/2)\delta - k < R + c/4 < R + (c/2)\delta + k$ and $R + (c/2)\delta - k < 2R < R + (c/2)\delta + k$. Note that: $IR_M^{LD}(R + (c/2)\delta - k) \geq IR_M^{LD}(R + (c/2)\delta + k)$ for $\delta \geq 1/2$.

(1) $\delta < 1/2$

In this case $R + c/4 > R + (c/2)\delta - k$ and $2R \leq R + (c/2)\delta - k$, since $\frac{2R}{\delta} < \frac{6R(1-\delta)}{\delta^2}$ for $\delta < 3/4$, and so $c \geq 2R/\delta$ and $\delta \leq 3/5$ is always satisfied. The manager's optimal choice is $D_M^* = 2R$.

(2) $1/2 \leq \delta \leq 3/5$

We have again $2R \leq R + (c/2)\delta - k$, since $\frac{2R}{\delta} < \frac{6R(1-\delta)}{\delta^2}$ for $\delta < 3/4$, and so $c \geq 2R/\delta$ and $\delta \leq 3/5$ is always satisfied. It can be that $R + c/4 \leq R + (c/2)\delta - k$, however it is still true that $\frac{6R(1-\delta)}{\delta^2} > 4R$ and so $R + c/4 > 2R$. The manager's optimal choice is still $D_M^* = 2R$.

(3) $3/5 < \delta < (-3+\sqrt{33})/4$

As in the previous case, $\frac{6R(1-\delta)}{\delta^2} > 4R$ and so $R + c/4 > 2R$. However, since this case is included in $3/5 < \delta \leq 3/4$, $2R \leq R + (c/2)\delta - k$ only for $\frac{2R}{\delta} < c \leq \frac{2R}{5\delta-3}$. Given that $\frac{2R}{\delta} < \frac{6R(1-\delta)}{\delta^2}$ for $\delta < 3/4$ and that $\frac{6R(1-\delta)}{\delta^2}$ is always lower than $\frac{2R}{5\delta-3}$, we have two cases:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{2R}{5\delta-3}$ then $D_M^* = 2R$.
- If $c > \frac{2R}{5\delta-3}$ then $D_M^* = R + (c/2)\delta - k$.

(4) $(-3+\sqrt{33})/4 \leq \delta \leq 7/10$

Now $\frac{6R(1-\delta)}{\delta^2} \leq 4R$ and in the interval $c \in \left[\frac{6R(1-\delta)}{\delta^2}, 4R\right]$ we have that $R + c/4 \leq 2R$. However, we have to check that is also $R + c/4 \leq R + (c/2)\delta - k$: this would be true for $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta-1/4}$. Note that $\frac{6R(1-\delta)}{\delta-1/4} \geq 4R$ for $\delta \leq 7/10$, so when $R + c/4$ is feasible, it satisfies the social welfare constraint. For $c > 4R$, we have to check that $2R \leq R + (c/2)\delta - k$: note that $4R \leq \frac{2R}{5\delta-3}$ for $\delta \leq 7/10$. So the inequality is satisfied for $4R < c \leq \frac{2R}{5\delta-3}$.

We have different cases:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq 4R$ then $D_M^* = R + c/4$.
- If $4R < c \leq \frac{2R}{5\delta-3}$ then $D_M^* = 2R$.
- If $c > \frac{2R}{5\delta-3}$ then $D_M^* = R + (c/2)\delta - k$.

(5) $7/10 < \delta \leq 3/4$

In this case, we have that: $\frac{6R(1-\delta)}{\delta^2} < \frac{2R}{5\delta-3} \leq \frac{6R(1-\delta)}{\delta-1/4} < 4R$, since $\frac{6R(1-\delta)}{\delta-1/4} \geq \frac{2R}{5\delta-3}$ for $\delta \in [7/10, 5/6]$. The following cases are possible:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + c/4$.
- If $c > \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + (c/2)\delta - k$.

(6) $3/4 < \delta \leq (-3+\sqrt{21})/2$

It is still true that $\frac{6R(1-\delta)}{\delta^2} < \frac{2R}{5\delta-3} \leq \frac{6R(1-\delta)}{\delta-1/4} < 4R$, however now $\frac{2R}{\delta} > \frac{6R(1-\delta)}{\delta^2}$ and $2R \geq R + (c/2)\,\delta + k$ for $c \leq \frac{2R}{5\delta-3}$. The solution is unchanged, since the manager prefers $R + c/4$ to any other value of D and such a value is feasible for $c \leq \frac{2R}{5\delta-3}$:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + c/4$.
- If $c > \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + (c/2)\delta - k$.

(7) $(-3+\sqrt{21}) < \delta \leq 5/6$

Now it still true that $\frac{6R(1-\delta)}{\delta^2} < \frac{2R}{5\delta-3} \leq \frac{6R(1-\delta)}{\delta-1/4} < 4R$, however $2R > \frac{6R(1-\delta)}{\delta^2}$, so we have to check the relative values of $2R$ and $\frac{6R(1-\delta)}{\delta-1/4}$. For $13/16 \leq \delta \leq 5/6$ we have that $2R \geq \frac{6R(1-\delta)}{\delta-1/4}$ and $D_M^* = R + (c/2)\delta - k$, while for $(-3+\sqrt{21}) < \delta < 13/16$ we have two cases:

- If $2R < c \leq \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + c/4$.
- If $c > \frac{6R(1-\delta)}{\delta-1/4}$ then $D_M^* = R + (c/2)\delta - k$.

(8) $\delta > 5/6$

We have: $\frac{6R(1-\delta)}{\delta^2} < \frac{6R(1-\delta)}{\delta-1/4} < \frac{2R}{5\delta-3} < 2R < 4R$, since $2R \geq \frac{2R}{5\delta-3}$ for $\delta \geq 4/5$. The manager selects $D_M^* = R + (c/2)\delta - k$.

<u>**Case 2: $\mathbf{c \leq 2R}$**</u>

The manager optimal unconstrained choice is the same than in case 1 and he receives now a zero information rent for $D \geq R + c/2$, since the monitoring probability will be equal to one. The social welfare constraint depends on the chosen value of D:

1) for $D \in [R, R+c/2)$ the same welfare constraint than in case 1 applies, so for $c < 6R(1-\delta)/\delta^2$, W^{LD} is always non negative, while for $c \geq 6R(1-\delta)/\delta^2$, W^{LD} is non negative for $D \leq R + (c/2)\delta - k$ and for $D \geq R + (c/2)\delta + k$, where $k = \sqrt{c^2\delta^2 + 6c(\delta-1)R}$.

2) for $D \in [R+c/2, 2R]$, W^{LD} is non negative if $D \geq \frac{c}{4(1-\delta)} - R/2$.

The manager always chooses a value of D less than $R + c/2$, if this is feasible given the social participation constraint. Since $R \leq R + (c/2)\delta - k \leq R + c/2$, there will always exist such a value and so the optimal solution will never be in the interval $[R + c/2, 2R]$.

Case 2a: $\Delta < 0$ $\left(c < 6R(1-\delta)/\delta^2\right)$

We know that $2R < 6R(1-\delta)/\delta^2$ for $\delta < (-3+\sqrt{21})/2$. There would be two cases:

(1) $\delta \leq (-3+\sqrt{21})/2$

W^{LD} is always non negative for $c \leq 2R$. Since $2R \geq R + c/4$ for $c \leq 4R$, the manager can always choose his unconstrained optimum: $D^*_M = R + c/4$.

(2) $\delta > (-3+\sqrt{21})/2$

Again W^{LD} is always non negative for $c < 6R(1-\delta)/\delta^2$ and the optimal manager's choice is $D^*_M = R + c/4$.

Case 2b: $\Delta \geq 0$ $\left(c \geq 6R(1-\delta)/\delta^2\right)$

For $(-3+\sqrt{21})/2 < \delta$ the case is empty, since $2R < 6R(1-\delta)/\delta^2$, while for $\delta > (-3+\sqrt{21})/2$ three cases can be differentiated:

(1) If $(-3+\sqrt{21})/2 \leq \delta < 13/16$ then $\frac{6R(1-\delta)}{\delta^2} < 2R < \frac{6R(1-\delta)}{\delta - 1/4}$.

The manager's optimal choice is $D^*_M = R + c/4$.

(2) If $13/16 \leq \delta \leq 5/6$ then $2R \geq \frac{6R(1-\delta)}{\delta-1/4}$.

There are two possibilities:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta-1/4}$ then $D^*_M = R + c/4$.
- If $\frac{6R(1-\delta)}{\delta-1/4} < c \leq 2R$ then $D^*_M = R + (c/2)\delta - k$.

(3) If $\delta > 5/6$ then $\frac{6R(1-\delta)}{\delta^2} < \frac{6R(1-\delta)}{\delta-1/4} < \frac{2R}{5\delta-3} < 2R < 4R$.

The solution depends on the cost level:

- If $\frac{6R(1-\delta)}{\delta^2} \leq c \leq \frac{6R(1-\delta)}{\delta-1/4}$ then $D^*_M = R + c/4$.

• If $\frac{6R(1-\delta)}{\delta-1/4} < c < \frac{2R}{5\delta-3}$ then $D_M^* = R + (c/2)\delta - k$ (note that in this case $2R \geq R + (c/2)\delta + k$, however $IR_M^{LD}(R + (c/2)\delta - k) \geq IR_M^{LD}(R + (c/2)\delta + k)$ for $\delta \geq 1/2$).

• If $\frac{2R}{5\delta-3} \leq c \leq 2R$ then $D_M^* = R + (c/2)\delta - k$.

<u>To summarize the results</u>:

(1) *When* $\delta \leq \frac{1}{4}$: $\begin{cases} if \quad c \leq 4R \quad then \quad D_M^* = R + c/4 \leq D_{SP}^* = 2R \\ if \quad 4R < c \leq R/\delta \quad then \quad D_M^* = D_{SP}^* = 2R \\ if \quad c > R/\delta \quad then \quad D_M^* = 2R > D_{SP}^* = R \end{cases}$

(2) *When* $\frac{1}{4} < \delta < \frac{1}{2}$: $\begin{cases} if \quad c \leq R/\delta \quad then \quad D_M^* = R + c/4 < D_{SP}^* = 2R \\ if \quad R/\delta < c \leq 4R \quad then \quad D_M^* = R + c/4 > D_{SP}^* = R \\ if \quad c > 4R \quad then \quad D_M^* = 2R > D_{SP}^* = R \end{cases}$

(3) *When* $\frac{1}{2} \leq \delta \leq \frac{3}{5}$: $\begin{cases} if \quad c \leq 4(1-\delta)R \quad then \quad D_M^* = R + c/4 < D_{SP}^* = 2R \\ if \quad 4(1-\delta)R < c < 4R \quad then \quad D_M^* = R + c/4 > D_{SP}^* = R \\ if \quad c \geq 4R \quad then \quad D_M^* = 2R > D_{SP}^* = R \end{cases}$

(4) *When* $\frac{3}{5} < \delta < \frac{7}{10}$: $\begin{cases} if \quad c \leq 4(1-\delta)R \quad then \quad D_M^* = R + c/4 < D_{SP}^* = 2R \\ if \quad 4(1-\delta)R < c \leq 4R \quad then \quad D_M^* = R + c/4 > D_{SP}^* = R \\ if \quad 4R < c < 2R/(5\delta-3) \quad then \quad D_M^* = 2R > D_{SP}^* = R \\ if \quad c \geq 2R/(5\delta-3) \quad then \quad D_M^* = R + (c/2)\delta - k > D_{SP}^* = R \end{cases}$

(5) *When* $\delta \geq \frac{7}{10}$: $\begin{cases} if \quad c \leq 4(1-\delta)R \quad then \quad D_M^* = R + c/4 < D_{SP}^* = 2R \\ if \quad 4(1-\delta)R < c < 6R(1-\delta)/(\delta-1/4) \quad then \quad D_M^* = R + c/4 > D_{SP}^* = R \\ if \quad c \geq 6R(1-\delta)/(\delta-1/4) \quad then \quad D_M^* = R + (c/2)\delta - k > D_{SP}^* = R \end{cases}$

<u>High Risk Debt</u>

Three different cases are considered depending on the values assumed by $\widehat{p}$ and $\underline{p}$ (remember that $\widehat{p} \leq \underline{p}$):

• If $c \leq R$ then $\widehat{p} \leq 1$ and $\underline{p} = 1$.

• If $R < c \leq 2R$ then $\widehat{p} < 1$ and $\underline{p} = 1$.

• If $c > 2R$ then $\underline{p} < 1$.

<u>Case 1: $\mathbf{c \leq R}$</u>

In this case it can be $\widehat{p} = 1$. The manager never chooses a nominal debt value $D \geq 2R + c$ if he can satisfy the social welfare constraint with a lower value of D, since his information rent is zero.

If the manager chooses a value of $D < 2R + c$, he will maximize the following function:

$$\max_D \quad \frac{1}{3}\bar{t} + \frac{1}{3}\widehat{t} + \frac{1}{3}\underline{t} \quad \text{i.e.,} \quad \max_D \quad \frac{1}{3}(D-2R)\left(1 - \frac{D-2R}{c}\right)$$

The constraint $(1 - \gamma - \delta) \geq 0$ can be rewritten in the following way:

$$(1-\gamma-\delta) = 1 - \frac{\frac{1}{3}(D-2R)\left(1-\frac{D-2R}{c}\right) + \frac{c}{6} + \frac{(D-2R)^2}{6c}}{(1/3)\,(D+3R)} - \delta$$

$$= \frac{\frac{1}{3}\,(1-\delta)\,(D+3R) - \frac{1}{3}\,(D-2R) + \frac{(D-2R)^2}{6c} - \frac{c}{6}}{(1/3)\,(D+3R)} \geq 0$$

The denominator is evidently always positive, while the numerator is positive when the following equation is satisfied:

$$D^2 - 2(2R + c\delta)D + (10 - 6\delta)cR + 4R^2 - c^2 \geq 0$$

So we have to differentiate two cases:

a) If $\Delta < 0$ then $c < \frac{10R(1-\delta)}{\delta^2+1}$: W^{HD} is always non negative.

b) If $\Delta \geq 0$ then $c \geq \frac{10R(1-\delta)}{\delta^2+1}$: W^{HD} is non negative for $D \leq 2R + c\delta - m$ and for $D \geq 2R + c\delta + m$, where $m = \sqrt{c^2\,(\delta^2+1) + 10c(\delta-1)R}$.

If the manager chooses a value of $D \geq 2R + c$, his revenues are always equal to zero since $\widehat{p} = \underline{p} = 1$, and he will be indifferent between any value of D. As before, we assume that he will choose the social welfare maximizing debt value, $D = 3R$ (see proof of *Lemma* 1, High Risk Debt, Case 1).

Case 1a: $\Delta < 0$ $\left(c < \frac{10R(1-\delta)}{\delta^2+1}\right)$

The first order condition of the manager's unconstrained problem is:

$$\partial IR_M^{HD}/\partial D = 1/3 - 2(D-2R)/(3c)$$

The derivative will be positive for $D \leq 2R + c/2$, so in the absence of any constraint the manager would choose $D = 2R + c/2$ if $R + c/2 \leq 3R$, so if $c \leq 2R$. Given $c \leq R$, this is always the case and $D_M^* = 2R + c/2$.

Case 1b: $\Delta \geq 0$ $\left(c \geq \frac{10R(1-\delta)}{\delta^2+1}\right)$

It is important to note that $R \geq \frac{10R(1-\delta)}{\delta^2+1}$ only when $\delta \geq \sqrt{34} - 5$, so for $\delta < \sqrt{34} - 5$ this case is empty. The manager's unconstrained problem solution is the same as before, so we have to compare the relative values of $2R + c/2$, $2R$, $2R + c\delta - m$ and $2R + c\delta + m$:

- $2R + c/2 \leq 2R + c\delta - m$ when $\delta \geq 1/2$ and $c \leq \frac{10R(1-\delta)}{\delta+3/4}$.
- $2R + c/2 \geq 2R + c\delta + m$ is never true for $\delta \geq 1/2$.
- $2R \leq 2R + c\delta - m$ when $c \leq 10R(1-\delta)$.
- $2R \geq 2R + c\delta + m$ is never true.

Note that $IR_M^{HD}(2R + c\delta - m) \geq IR_M^{HD}(2R + c\delta + m)$ for $\delta \geq 1/2$, so always in this case. We also know that $\frac{10R(1-\delta)}{\delta+3/4}$ is always greater than $\frac{10R(1-\delta)}{\delta^2+1}$, that $\frac{10R(1-\delta)}{\delta+3/4} \leq R$ for $\delta \geq 37/44$, that $R < 10R(1-\delta)$ for $\delta < 9/10$ and that $10R(1-\delta) > \frac{10R(1-\delta)}{\delta+3/4}$ for $\delta > 1/4$. We have three subcases:

(1) If $\sqrt{34} - 5 \leq \delta < 37/44$ then $\frac{10R(1-\delta)}{\delta^2+1} \leq R < \frac{10R(1-\delta)}{\delta+3/4}$.

In this case $2R + c/2 \leq 2R + c\delta - m$, so the manager's optimal choice is $D_M^* = 2R + c/2$.

(2) If $37/44 \leq \delta < 9/10$ then $\frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} \leq R < 10R(1-\delta)$.

The solution depends on the cost level:

- $\frac{10R(1-\delta)}{\delta^2+1} \leq c \leq \frac{10R(1-\delta)}{\delta+3/4}$: since $2R + c/2 \leq 2R + c\delta - m$, $D_M^* = 2R + c/2$.
- $\frac{10R(1-\delta)}{\delta+3/4} < c \leq R$: since $2R + c/2 > 2R + c\delta - m$, $D_M^* = 2R + c\delta - m$.

(3) If $\delta \geq 9/10$ then $\frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} < 10R(1-\delta) \leq R$.

The solution depends on the cost level:

- $\frac{10R(1-\delta)}{\delta^2+1} \leq c \leq \frac{10R(1-\delta)}{\delta+3/4}$: since $2R + c/2 \leq 2R + c\delta - m$, $D_M^* = 2R + c/2$.
- $\frac{10R(1-\delta)}{\delta+3/4} < c \leq 10R(1-\delta)$: since $2R + c/2 > 2R + c\delta - m$, $D_M^* = 2R + c\delta - m$.
- $c > 10R(1-\delta)$: since $2R > 2R + c\delta - m$, the only possible solution is $D = 3R$. However, $W^{HD}(3R) = 2(1-\delta)R - c/3$ and is positive only for $c \leq 10R(1-\delta)$. The problem has no solution.

Case 2: $\mathbf{R < c \leq 2R}$

In this case $\widehat{p} < 1$ and $\underline{p} = 1$. The manager's unconstrained problem is the same that in Case 1 for $D < 2R + c$. The constraint $(1 - \gamma - \delta) \geq 0$ is satisfied, as before, when the following inequality holds:

$$D^2 - 2(2R + c\delta)D + (10 - 6\delta)cR + 4R^2 - c^2 \geq 0$$

So we have to differentiate two cases:

a) If $\Delta < 0$ then $c < \frac{10R(1-\delta)}{\delta^2+1}$ and W^{HD} is always non negative.

b) If $\Delta \geq 0$ then $c \geq \frac{10R(1-\delta)}{\delta^2+1}$ and W^{HD} is non negative for $D \leq 2R + c\delta - m$ and for $D \geq 2R + c\delta + m$, where $m = \sqrt{c^2(\delta^2+1) + 10c(\delta-1)R}$.

Case 2a: $\Delta < 0$ $\left(c < \frac{10R(1-\delta)}{\delta^2+1}\right)$

It is important to note that $R < \frac{10R(1-\delta)}{\delta^2+1}$ for $\delta < \sqrt{34}-5$, so for $\delta \geq \sqrt{34}-5$ this case is empty.

The social welfare constraint is always satisfied. The first order condition of the manager's unconstrained problem is:

$$\partial IR_M^{HD}/\partial D = 1/3 - 2(D-2R)/(3c)$$

The derivative will be positive for $D \leq 2R + c/2$, so in the absence of any constraint , and given that $c \in (R, 2R]$, the manager chooses $D_M^* = 2R + c/2$.

Case 2b: $\Delta \geq 0$ $\left(c \geq \frac{10R(1-\delta)}{\delta^2+1}\right)$

Notice that $2R \geq \frac{10R(1-\delta)}{\delta^2+1}$ only when $\delta \geq (\sqrt{41}-5)/2$, so for $\delta < (\sqrt{41}-5)/2$ this case is empty. Remember also that $R \geq \frac{10R(1-\delta)}{\delta^2+1}$ only when $\delta \geq \sqrt{34}-5$. The manager's unconstrained problem solution is the same as before, so we have to consider the relative values of $2R+c/2$, $2R$, $2R+c\delta-m$ and $2R+c\delta+m$. Given the same relationships between the values as in Case 1, we have six subcases. Note that, as before, $IR_M^{HD}(2R+c\delta-m) \geq IR_M^{HD}(2R+c\delta+m)$ for $\delta \geq 1/2$, so always in this case. We also know that $\frac{10R(1-\delta)}{\delta+3/4}$ is always greater than $\frac{10R(1-\delta)}{\delta^2+1}$, that $10R(1-\delta) > \frac{10R(1-\delta)}{\delta+3/4}$ for $\delta > 1/4$, that $\frac{10R(1-\delta)}{\delta+3/4} \leq R$ for $\delta \geq 37/44$ and $\frac{10R(1-\delta)}{\delta+3/4} \leq 2R$ for $\delta \geq 17/24$, that $R < 10R(1-\delta)$ for $\delta < 9/10$ and $2R < 10R(1-\delta)$ for $\delta < 4/5$. The subcases are:

(1) If $(\sqrt{41}-5)/2 \leq \delta \leq 17/24$ then $R < \frac{10R(1-\delta)}{\delta^2+1} \leq 2R \leq \frac{10R(1-\delta)}{\delta+3/4}$.

In this case $2R + c/2 \leq 2R + c\delta - m$, so the manager's optimal choice is $D_M^* = 2R + c/2$.

(2) If $17/24 < \delta < 4/5$ then $R < \frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} < 2R < 10R(1-\delta)$.

The solution depends on the cost level:

- $\frac{10R(1-\delta)}{\delta^2+1} \leq c \leq \frac{10R(1-\delta)}{\delta+3/4}$: since $2R + c/2 \leq 2R + c\delta - m$, $D_M^* = 2R + c/2$.
- $\frac{10R(1-\delta)}{\delta+3/4} < c \leq 2R$: since $2R + c/2 > 2R + c\delta - m$, $D_M^* = 2R + c\delta - m$.

(3) If $4/5 \leq \delta < \sqrt{34}-5$ then $R < \frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} < 10R(1-\delta) \leq 2R$.

The solution depends on the cost level:

- $\frac{10R(1-\delta)}{\delta^2+1} \leq c \leq \frac{10R(1-\delta)}{\delta+3/4}$: since $2R + c/2 \leq 2R + c\delta - m$, $D_M^* = 2R + c/2$.
- $\frac{10R(1-\delta)}{\delta+3/4} < c \leq 10R(1-\delta)$: since $2R + c/2 > 2R + c\delta - m$, $D_M^* = 2R + c\delta - m$.
- $10R(1-\delta) < c \leq 2R$: since $2R > 2R + c\delta - m$, $W^{HD}(2R) < 0$. The only other possible solution is $D = 3R$. From the proof of *Lemma* 1, High Risk Debt, Case 2, we know that $W^{HD}(2R) \geq W^{HD}(3R)$ if $c \geq R/(2\delta)$ and $R \geq R/(2\delta)$ for $\delta \geq 1/2$. So it will always be $W^{HD}(3R) < W^{HD}(2R) < 0$ and the problem has no solution.

(4) If $\sqrt{34}-5 \leq \delta < 37/44$ then $\frac{10R(1-\delta)}{\delta^2+1} \leq R < \frac{10R(1-\delta)}{\delta+3/4} < 10R(1-\delta) \leq 2R$.

The solution is the same as in case (3), but the first cost interval starts from R instead of $\frac{10R(1-\delta)}{\delta^2+1}$.

(5) If $37/44 \leq \delta < 9/10$ then $\frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} \leq R < 10R(1-\delta) \leq 2R$.

The solution depends on the cost level:

- $R < c \leq 10R(1-\delta)$: since $2R + c/2 > 2R + c\delta - m$, $D_M^* = 2R + c\delta - m$.
- $10R(1-\delta) < c \leq 2R$: $W^{HD}(3R) < W^{HD}(2R) < 0$ and the problem has no solution.

(6) If $\delta \geq 9/10$ then $\frac{10R(1-\delta)}{\delta^2+1} < \frac{10R(1-\delta)}{\delta+3/4} < 10R(1-\delta) \leq R < 2R$.

In this case $W^{HD}(3R) < W^{HD}(2R) < 0$ and the problem has no solution.

Case 3: c > 2R

In this case both monitoring probabilities are strictly less than one. The manager's unconstrained problem writes as:

$$\max_D \quad \frac{1}{3}\bar{t} + \frac{1}{3}\hat{t} + \frac{1}{3}\underline{t} \quad \text{i.e.,} \quad \max_D \quad \frac{2}{3}R(1 - \frac{2R}{c}) + \frac{1}{3}(D - 2R)\left(1 - \frac{D-2R}{c}\right)$$

The constraint $(1-\gamma-\delta) \geq 0$ can be rewritten in the following way:

$$(1-\gamma-\delta) = 1 - \frac{\frac{2}{3}R\left(1 - \frac{2R}{c}\right) + \frac{1}{3}(D-2R)\left(1 - \frac{D-2R}{c}\right) + \frac{2R^2}{3c} + \frac{(D-2R)^2}{6c}}{(1/3)\,(D+3R)} - \delta$$

$$= \frac{\frac{1}{3}(1-\delta)(D+3R) - \frac{1}{3}D + \frac{2R^2}{3c} + \frac{(D-2R)^2}{6c}}{(1/3)\,(D+3R)} \geq 0$$

The denominator is evidently always positive, while the numerator is positive when the following equation is satisfied:

$$D^2 - 2(2R + c\delta)D + 6c(1-\delta)R + 8R^2 \geq 0$$

So we have to differentiate two cases:

a) If $\Delta < 0$ then $c < \frac{(3-5\delta)R+g}{\delta^2}$: W^{HD} is always non negative, where $g = R\sqrt{(5\delta-3)^2 + 4\delta^2}$.

b) If $\Delta \geq 0$ then $c \geq \frac{(3-5\delta)R+g}{\delta^2}$: W^{HD} is non negative for $D \leq 2R + c\delta - h$ and for $D \geq 2R + c\delta + h$, where $h = \sqrt{c^2\delta^2 + 12c(5\delta-3)R - 4R^2}$.

Case 3a: $\Delta < 0$ $\left(c < \frac{(3-5\delta)R+g}{\delta^2}\right)$

We have to check that this case is not empty, and so to check if $2R \leq \frac{(3-5\delta)R+g}{\delta^2}$ holds: the inequality is satisfied for $\delta \leq (\sqrt{41}-5)/2$. The first order condition of the manager's unconstrained problem is :

$$\partial IR_M^{HD}/\partial D = 1/3 - 2(D-2R)/(3c)$$

The derivative will be positive for $D \leq 2R + c/2$, so in the absence of any constraint the manager would choose $D = 2R + c/2$ if $R + c/2 \leq 3R$, so if $c \leq 2R$, or $D = 3R$ if $c \leq 2R$. In our case, the optimal unconstrained choice is clearly $D^*_M = 3R$.

Case 3b: $\underline{\Delta \geq 0}$ $\left(c \geq \frac{(3-5\delta)R+g}{\delta^2}\right)$

The manager unconstrained choice, given $c < 2R$, is still $D = 3R$. We have to compare the relative values of $2R$, $3R$, $2R + c\delta - h$ and $2R + c\delta + h$:

- $2R \leq 2R + c\delta - h$ for $\delta \leq 3/5$ or for $\delta > 3/5$ and $c \leq \frac{2R}{5\delta-3}$.
- $2R \geq 2R + c\delta + h$ is never true.
- $3R \leq 2R + c\delta - h$ for $\delta \leq 1/2$ and $c > R/\delta$ or for $1/2 < \delta \leq 6/7$ and $R/\delta < c \leq \frac{5R}{6(2\delta-1)}$.
- $3R \geq 2R + c\delta + h$ for $\delta \leq 6/7$ and $c < R/\delta$ or for $\delta > 6/7$ and $c \leq \frac{5R}{6(2\delta-1)}$.

Note that $IR^{HD}_M(2R + c\delta - h) \geq IR^{HD}_M(2R + c\delta + h)$ for $\delta \geq 1/2$.

We also know that $2R > R/\delta$ for $\delta > 1/2$, that $2R \geq \frac{5R}{6(2\delta-1)}$ for $\delta \leq 1/2$ and $\delta \geq 17/24$, that $2R \geq \frac{2R}{5\delta-3}$ for $\delta \geq 4/5$, that $\frac{5R}{6(2\delta-1)} > \frac{2R}{5\delta-3}$ for $\delta \in (1/2, 3/5)$ and that $\frac{5R}{6(2\delta-1)} > \frac{(3-5\delta)R+g}{\delta^2}$ for $\delta > 1/2$.

We have many subcases depending on the δ parameter:

(1) If $\delta \leq 1/2$ then $\frac{5R}{6(2\delta-1)} \leq 2R \leq R/\delta \leq \frac{(3-5\delta)R+g}{\delta^2}$.

Notice that $\frac{R}{\delta} \leq \frac{(3-5\delta)R+g}{\delta^2}$ is always satisfied for $\delta \leq 1/2$. In this case $3R \leq 2R + c\delta - h$, so the manager's optimal choice is $D^*_M = 3R$.

(2) If $1/2 < \delta \leq 3/5$ then $R/\delta < 2R < \frac{(3-5\delta)R+g}{\delta^2} < \frac{5R}{6(2\delta-1)}$.

The solution depends on the cost level:

- $\frac{(3-5\delta)R+g}{\delta^2} \leq c \leq \frac{5R}{6(2\delta-1)}$: since $3R \leq 2R + c\delta - h$, $D^*_M = 3R$
- $c > \frac{5R}{6(2\delta-1)}$: since $2R + c\delta - h < 3R < 2R + c\delta - h$, $D^*_M = 2R + c\delta - h$

(3) If $3/5 < \delta \leq \frac{\sqrt{41}-5}{2}$ then $R/\delta < 2R < \frac{(3-5\delta)R+g}{\delta^2} < \frac{5R}{6(2\delta-1)} < \frac{2R}{5\delta-3}$.

The solution depends on the cost level:

- $\frac{(3-5\delta)R+g}{\delta^2} \leq c \leq \frac{5R}{6(2\delta-1)}$: since $3R \leq 2R + c\delta - h$, $D^*_M = 3R$.
- $\frac{5R}{6(2\delta-1)} < c \leq \frac{2R}{5\delta-3}$: since $2R \leq 2R + c\delta - h < 3R < 2R + c\delta - h$, $D^*_M = 2R + c\delta - h$.
- $c > \frac{2R}{5\delta-3}$: since $2R + c\delta - h < 2R < 3R < 2R + c\delta - h$, the problem has no solution.

(4) If $(\sqrt{41} - 5)/2 < c \leq 17/24$ then $\frac{(3-5\delta)R+g}{\delta^2} < 2R \leq \frac{5R}{6(2\delta-1)} < \frac{2R}{5\delta-3}$.

The solution depends on the cost level:

- $2R \leq c \leq \frac{5R}{6(2\delta-1)}$: since $3R \leq 2R + c\delta - h$, $D^*_M = 3R$.

- $\frac{5R}{6(2\delta-1)} < c \leq \frac{2R}{5\delta-3}$: since $2R \leq 2R + c\delta - h < 3R < 2R + c\delta - h$, $D^*_M = 2R + c\delta - h$.

- $c > \frac{2R}{5\delta-3}$: since $2R + c\delta - h < 2R < 3R < 2R + c\delta - h$, the problem has no solution.

(5) If $17/24 < c \leq 4/5$ then $\frac{(3-5\delta)R+g}{\delta^2} < \frac{5R}{6(2\delta-1)} < 2R \leq \frac{2R}{5\delta-3}$.

The solution depends on the cost level:

- $2R < c \leq \frac{2R}{5\delta-3}$: since $2R \leq 2R + c\delta - h < 3R < 2R + c\delta - h$, $D^*_M = 2R + c\delta - h$.

- $c > \frac{2R}{5\delta-3}$: since $2R + c\delta - h < 2R < 3R < 2R + c\delta - h$, the problem has no solution.

(6) If $c > 4/5$ then $\frac{(3-5\delta)R+g}{\delta^2} < \frac{5R}{6(2\delta-1)} < \frac{2R}{5\delta-3} < 2R$.

Given that $2R + c\delta - h < 2R < 3R < 2R + c\delta - h$, the problem has no solution.

<u>To summarize the results</u>:

$$(1)\ \ When\ \delta \leq \frac{1}{4}: \begin{cases} if \quad c \leq 2R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad 2R < c \leq R/(2\delta) \quad then \quad D^*_M = D^*_{SP} = 3R \\ if \quad c > R/(2\delta) \quad then \quad D^*_M = 3R > D^*_{SP} = 2R \end{cases}$$

$$(2)\ \ When\ \frac{1}{4} < \delta < \frac{1}{2}: \begin{cases} if \quad c \leq R/(2\delta) \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad R/(2\delta) < c \leq 2R \quad then \quad D^*_M = 2R + c/2 > D^*_{SP} = 2R \\ if \quad c > 2R \quad then \quad D^*_M = 3R > D^*_{SP} = 2R \end{cases}$$

$$(3)\ \ When\ \frac{1}{2} \leq \delta \leq \frac{3}{5}: \begin{cases} if \quad c \leq 2(1-\delta)R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad 2(1-\delta)R < c \leq 2R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 2R \\ if \quad 2R < c \leq 5R/\left[6(2\delta-1)\right] \quad then \quad D^*_M = 3R > D^*_{SP} = 2R \\ if \quad c > 5R/\left[6(2\delta-1)\right] \quad then \quad D^*_M = 2R + c\delta - h > D^*_{SP} = 2R \end{cases}$$

$$(4)\ \ When\ \frac{3}{5} < \delta < \frac{17}{24}: \begin{cases} if \quad c \leq 2(1-\delta)R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad 2(1-\delta)R < c \leq 2R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 2R \\ if \quad 2R < c \leq 5R/\left[6(2\delta-1)\right] \quad then \quad D^*_M = 3R > D^*_{SP} = 2R \\ if \quad 5R/\left[6(2\delta-1)\right] < c \leq 2R/(5\delta-3) \quad then \quad D^*_M = 2R + c\delta - h > D^*_{SP} = 2R \\ if \quad c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(5)\ \ When\ \frac{17}{24} \leq \delta \leq \frac{4}{5}: \begin{cases} if \quad c \leq 2(1-\delta)R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad 2(1-\delta)R < c \leq 10R(1-\delta)/(\delta+3/4) \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 2R \\ if \quad 10R(1-\delta)/(\delta+3/4) < c \leq 2R \quad then \quad D^*_M = 2R + c\delta - m > D^*_{SP} = 2R \\ if \quad 2R < c \leq 2R/(5\delta-3) \quad then \quad D^*_M = 2R + c\delta - h > D^*_{SP} = 2R \\ if \quad c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(6)\ \textit{When}\ \delta > \frac{4}{5}: \begin{cases} if \quad c \leq 2(1-\delta)R \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 3R \\ if \quad 2(1-\delta)R < c \leq 10R(1-\delta)/(\delta+3/4) \quad then \quad D^*_M = 2R + c/2 < D^*_{SP} = 2R \\ if \quad 10R(1-\delta)/(\delta+3/4) < c \leq 10R(1-\delta) \quad then \quad D^*_M = 2R + c\delta - m > D^*_{SP} = 2R \\ if \quad c > 10R(1-\delta) \quad then\ no\ trade\ occurs \end{cases}$$

■

Proof of Proposition 1

We have first to compute the social welfare levels for the three possible asset types, given the best choice of the asset value for the social planner determined in the proof of Lemma 1. We obtain the following values:

Call Option

$$(1)\ \textit{When}\ \delta \leq \frac{1}{2}: \begin{cases} if \quad c \leq R/2 \quad then \quad W^C_{SP} = (R/3)(1-\delta) - c/3 \\ if \quad R/2 < c \leq R/(4\delta) \quad then \quad W^C_{SP} = (R/12c)(R - 4c\delta) \\ if \quad c > R/(4\delta) \quad then \quad W^C_{SP} = 0 \end{cases}$$

$$(2)\ \textit{When}\ \delta > \frac{1}{2}: \begin{cases} if \quad c \leq (1-\delta)R \quad then \quad W^C_{SP} = (R/3)(1-\delta) - c/3 \\ if \quad c > (1-\delta)R \quad then \quad W^C_{SP} = 0 \end{cases}$$

Low Risk Debt

$$(1)\ \textit{When}\ \delta < \frac{1}{2}: \begin{cases} if \quad c \leq 2R \quad then \quad W^{LD}_{SP} = (1/3)\left[5(1-\delta)R - c/2\right] \\ if \quad 2R < c \leq R/\delta \quad then \quad W^{LD}_{SP} = (1-\delta)R + 2(R^2/c - \delta R)/3 \\ if \quad c > R/\delta \quad then \quad W^{LD}_{SP} = (1-\delta)R \end{cases}$$

$$(2)\ \textit{When}\ \delta \geq \frac{1}{2}: \begin{cases} if \quad c \leq 4(1-\delta)R \quad then \quad W^{LD}_{SP} = (1/3)\left[5(1-\delta)R - c/2\right] \\ if \quad c > 4(1-\delta)R \quad then \quad W^{LD}_{SP} = (1-\delta)R \end{cases}$$

High Risk Debt

$$(1)\ \textit{When}\ \delta \leq \frac{1}{4}: \begin{cases} if \quad c \leq R \quad then \quad W^{HD}_{SP} = 2(1-\delta)R - c/3 \\ if \quad R < c \leq 2R \quad then \quad W^{HD}_{SP} = (5/3 - 2\delta)R - c/6 + R^2/6c \\ if \quad 2R < c \leq R/(2\delta) \quad then \quad W^{HD}_{SP} = (1-2\delta)R + 5R^2/(6c) \\ if \quad c > R/(2\delta) \quad then \quad W^{HD}_{SP} = (1 - 5\delta/3)R + 2R^2/3c \end{cases}$$

$$(2)\ \textit{When}\ \frac{1}{4} < \delta < \frac{1}{2}: \begin{cases} if \quad c \leq R \quad then \quad W^{HD}_{SP} = 2(1-\delta)R - c/3 \\ if \quad R < c \leq R/(2\delta) \quad then \quad W^{HD}_{SP} = (5/3 - 2\delta)R - c/6 + R^2/6c \\ if \quad R/(2\delta) < c \leq 2R \quad then \quad W^{HD}_{SP} = 5(1-\delta)R/3 - c/6 \\ if \quad c > 2R \quad then \quad W^{HD}_{SP} = (1 - 5\delta/3)R + 2R^2/3c \end{cases}$$

$$(3)\ \textit{When}\ \frac{1}{2} \leq \delta \leq \frac{3}{5}: \begin{cases} if \quad c \leq 2R(1-\delta) \quad then \quad W^{HD}_{SP} = 2(1-\delta)R - c/3 \\ if \quad 2R(1-\delta) < c \leq 2R \quad then \quad W^{HD}_{SP} = 5(1-\delta)R/3 - c/6 \\ c > 2R \quad then \quad W^{HD}_{SP} = (1 - 5\delta/3)R + 2R^2/3c \end{cases}$$

$$(4)\ \textit{When}\ \frac{3}{5} < \delta < \frac{4}{5}\text{:}\ \begin{cases} if \quad c \le 2R(1-\delta) \quad then \quad W_{SP}^{HD} = 2(1-\delta)R - c/3 \\ if \quad 2R(1-\delta) < c \le 2R \quad then \quad W_{SP}^{HD} = 5(1-\delta)R/3 - c/6 \\ if \quad 2R < c \le 2R/(5\delta-3) \quad then \quad W_{SP}^{HD} = (1-5\delta/3)\,R + 2R^2/3c \\ if \quad c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(5)\ \textit{When}\ \delta \ge \frac{4}{5}\text{:}\ \begin{cases} if \quad c \le 2R(1-\delta) \quad then \quad W_{SP}^{HD} = 2(1-\delta)R - c/3 \\ if \quad 2R(1-\delta) < c \le 10R(1-\delta) \quad then \quad W_{SP}^{HD} = 5(1-\delta)R/3 - c/6 \\ if \quad c > 10R(1-\delta) \quad then\ no\ trade\ occurs \end{cases}$$

To obtain the Proposition and the graph, we just compare the welfare values of the three possible asset types and choose the highest one, for each possible value of δ and c.

■

Proof of Proposition 2

We have first to compute the manager's information rents for the three possible security types, given the manager's best choice of the asset value determined in the proofs of Lemma 2 and 3. We obtain the following results:

Call Option

$$(1)\ \textit{When}\ \delta \le \frac{1}{4}\text{:}\ \begin{cases} if \quad c \le R \quad then \quad IR_M^C = c/6 \\ if \quad R < c \le R/(4\delta) \quad then \quad IR_M^C = (R/3)(1 - R/(2c)) \\ if \quad c > R/(4\delta) \quad then \quad IR_M^C = 0 \end{cases}$$

$$(2)\ \textit{When}\ \frac{1}{4} < \delta < \frac{1}{2}\text{:}\ \begin{cases} if \quad c \le R/(4\delta) \quad then \quad IR_M^C = (4/3)c\delta(1-2\delta) \\ if \quad c > R/(4\delta) \quad then \quad IR_M^C = 0 \end{cases}$$

$$(3)\ \textit{When}\ \delta \ge \frac{1}{2} \quad then \quad IR_M^C = 0$$

Low Risk Debt

$$(1)\ \textit{When}\ \delta \le \frac{3}{5}\text{:}\ \begin{cases} if \quad c \le 4R \quad then \quad IR_M^{LD} = c/12 \\ if \quad c > 4R \quad then \quad IR_M^{LD} = (2/3)R(1-2R/c) \end{cases}$$

$$(2)\ \textit{When}\ \frac{3}{5} < \delta < \frac{7}{10}\text{:}\ \begin{cases} if \quad c \le 4R \quad then \quad IR_M^{LD} = c/12 \\ if \quad 4R < c \le 2R/(5\delta-3) \quad then \quad IR_M^{LD} = 2R(1-R/(2c))/3 \\ if \quad c > 2R/(5\delta-3) \quad then \quad IR_M^{LD} = (c\delta - 2k)(1-\delta+2k/c)/3 \end{cases}$$

$$(3)\ \textit{When}\ \delta \ge \frac{7}{10}\text{:}\ \begin{cases} if \quad c \le 6R/(\delta - 1/4) \quad then \quad IR_M^{LD} = c/12 \\ if \quad c > 6R/(\delta - 1/4) \quad then \quad IR_M^{LD} = (c\delta - 2k)(1-\delta+2k/c)/3 \end{cases}$$

where $k = \sqrt{c^2\delta^2 + 6c(\delta-1)R}$.

High Risk Debt

$$(1)\ When\ \delta \leq \frac{1}{2}: \begin{cases} if \quad c \leq 2R \quad then \quad IR_M^{HD} = c/12 \\ if \quad c > 2R \quad then \quad IR_M^{HD} = R - 5R^2/(3c) \end{cases}$$

$$(2)\ When\ \frac{1}{2} < \delta < \frac{3}{5}: \begin{cases} if \quad c \leq 2R \quad then \quad IR_M^{HD} = c/12 \\ if \quad 2R < c \leq 5R/\left[6(2\delta - 1)\right] \quad then \quad IR_M^{HD} = R - 5R^2/(3c) \\ if \quad c > 5R/\left[6(2\delta - 1)\right] \quad then \quad IR_M^{HD} = \overline{h} + 2(1 - R/(2c))/3 \end{cases}$$

$$(3)\ When\ \frac{3}{5} \leq \delta \leq \frac{17}{24}: \begin{cases} if \quad c \leq 2R \quad then \quad IR_M^{HD} = c/12 \\ if \quad 2R < c \leq 5R/\left[6(2\delta - 1)\right] \quad then \quad IR_M^{HD} = R - 5R^2/(3c) \\ if \quad 5R/\left[6(2\delta - 1)\right] < c \leq 2R/(5\delta - 3) \quad then \quad IR_M^{HD} = \overline{h} + 2\left[1 - R/(2c)\right]/3 \\ if \quad c > 2R/(5\delta - 3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(4)\ When\ \frac{17}{24} < \delta \leq \frac{4}{5}: \begin{cases} if \quad c \leq 10R(1 - \delta)/(\delta + 3/4) \quad then \quad IR_M^{HD} = c/12 \\ if \quad 10R(1 - \delta)/(\delta + 3/4) < c \leq 2R \quad then \quad IR_M^{HD} = \overline{m} \\ if \quad 2R < c \leq 2R/(5\delta - 3) \quad then \quad IR_M^{HD} = \overline{h} + 2\left[1 - R/(2c)\right]/3 \\ if \quad c > 2R/(5\delta - 3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(5)\ When\ \delta > \frac{4}{5}: \begin{cases} if \quad c \leq 10R(1 - \delta)/(\delta + 3/4) \quad then \quad IR_M^{HD} = c/12 \\ if \quad 10R(1 - \delta)/(\delta + 3/4) < c \leq 10R(1 - \delta) \quad then \quad IR_M^{HD} = \overline{m} \\ if \quad c > 10R(1 - \delta) \quad then\ no\ trade\ occurs \end{cases}$$

where $\overline{h} = (1/3)(c\delta - h)(1 - \delta + h/c)$, with $h = \sqrt{c^2\delta^2 + 12c(5\delta - 3)R - 4R^2}$, and $\overline{m} = (1/3)(c\delta - m)(1 - \delta + m/c)$, with $m = \sqrt{c^2\left(\delta^2 + 1\right) + 10c(\delta - 1)R}$.

To obtain the Proposition, we just compare the information rents of the three securities and choose the highest one, for any value of δ and c.

■

Proof of Proposition 3

We have first to compute the social welfare levels for the three possible security types, given that the asset value is chosen by the manager in a way that maximizes his information rent, so as determined in the proofs of Lemma 2 and 3. We obtain the following values:

Call Option

$$(1)\ When\ \delta \leq \frac{1}{4}: \begin{cases} if \quad c < R \quad then \quad W_{RM}^C = (c/12)(1 - 4\delta) \\ if \quad R \leq c \leq R/(4\delta) \quad then \quad W_{RM}^C = (R/12c)(R - 4c\delta) \\ if \quad c > R/(4\delta) \quad then \quad W_{RM}^C = 0 \end{cases}$$

$$(2)\ When\ \frac{1}{4} < \delta < \frac{1}{2} \quad then \quad W_{RM}^C = 0$$

$$(3)\ \textit{When}\ \delta > \frac{1}{2}\text{:}\ \begin{cases} if & c \le (1-\delta)R \quad then \quad W^{C}_{RM} = (R/3)(1-\delta) - c/3 \\ if & c > (1-\delta)R \quad then \quad W^{C}_{RM} = 0 \end{cases}$$

Low Risk Debt

$$(1)\ \textit{When}\ \delta < \frac{3}{5}\text{:}\ \begin{cases} if & c \le 4R \quad then \quad W^{LD}_{RM} = (1-\delta)R + c(1/4-\delta)/6 \\ if & c > 4R \quad then \quad W^{LD}_{RM} = (1-\delta)R + 2R(R/c-\delta)/3 \end{cases}$$

$$(2)\ \textit{When}\ \frac{3}{5} \le \delta \le \frac{7}{10}\text{:}\ \begin{cases} if \quad c < 4R \quad then \quad W^{LD}_{RM} = (1-\delta)R + c(1/4-\delta)/6 \\ if \quad 4R \le c \le 2R/(5\delta-3) \quad then \quad W^{LD}_{RM} = (1-\delta)R + 2R(R/c-\delta)/3 \\ c > 2R/(5\delta-3) \quad then \quad W^{LD}_{RM} = 0 \end{cases}$$

$$(3)\ \textit{When}\ \delta > \frac{7}{10}\text{:}\ \begin{cases} if & c \le 6R(1-\delta)/(\delta-1/4) \quad then \quad W^{LD}_{RM} = (1-\delta)R + c(1/4-\delta)/6 \\ if & c > 6R(1-\delta)/(\delta-1/4) \quad then \quad W^{LD}_{RM} = 0 \end{cases}$$

High Risk Debt

$$(1)\ \textit{When}\ \delta \le \frac{1}{2}\text{:}\ \begin{cases} if & c \le 2R \quad then \quad W^{HD}_{RM} = 5(1-\delta)R/3 - c(3/4-\delta)/6 \\ if & c > 2R \quad then \quad W^{HD}_{RM} = (1-2\delta)R + 5R^2/(6c) \end{cases}$$

$$(2)\ \textit{When}\ \frac{1}{2} < \delta < \frac{3}{5}\text{:}\ \begin{cases} if & c \le 2R \quad then \quad W^{HD}_{RM} = 5(1-\delta)R/3 - c(3/4-\delta)/6 \\ if & 2R < c \le 5R/\left[6(2\delta-1)\right] \quad then \quad W^{HD}_{RM} = (1-2\delta)R + 5R^2/(6c) \\ if & c > 5R/\left[6(2\delta-1)\right] \quad then \quad W^{HD}_{RM} = 0 \end{cases}$$

$$(3)\ \textit{When}\ \frac{3}{5} \le \delta \le \frac{17}{24}\text{:}\ \begin{cases} if & c \le 2R \quad then \quad W^{HD}_{RM} = 5(1-\delta)R/3 - c(3/4-\delta)/6 \\ if & 2R < c \le 5R/\left[6(2\delta-1)\right] \quad then \quad W^{HD}_{RM} = (1-2\delta)R + 5R^2/(6c) \\ if & 5R/\left[6(2\delta-1)\right] < c \le 2R/(5\delta-3) \quad then \quad W^{HD}_{RM} = 0 \\ if & c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(4)\ \textit{When}\ \frac{17}{24} < \delta \le \frac{4}{5}\text{:}\ \begin{cases} if & c \le 10R(1-\delta)/(\delta+3/4) \quad then \quad W^{HD}_{RM} = 5(1-\delta)R/3 - c(3/4-\delta)/6 \\ if & 10R(1-\delta)/(\delta+3/4) < c \le 2R/(5\delta-3) \quad then \quad W^{HD}_{RM} = 0 \\ if & c > 2R/(5\delta-3) \quad then\ no\ trade\ occurs \end{cases}$$

$$(5)\ \textit{When}\ \delta > \frac{4}{5}\text{:}\ \begin{cases} if & c \le 10R(1-\delta)/(\delta+3/4) \quad then \quad W^{HD}_{RM} = 5(1-\delta)R/3 - c(3/4-\delta)/6 \\ if & 10R(1-\delta)/(\delta+3/4) < c \le 10R(1-\delta) \quad then \quad W^{HD}_{RM} = 0 \\ if & c > 10R(1-\delta) \quad then\ no\ trade\ occurs \end{cases}$$

To obtain the Proposition and the graph, we just compare the welfare values of the three possible securities and choose the highest one, for each possible value of δ and c.

■

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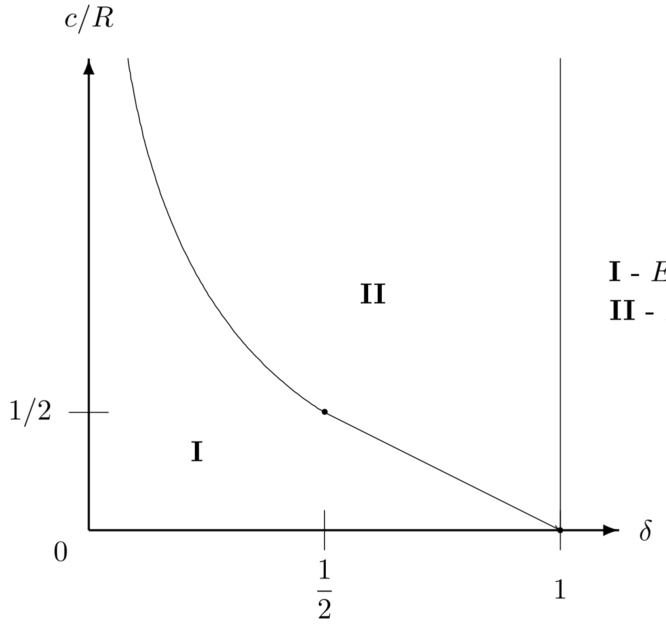
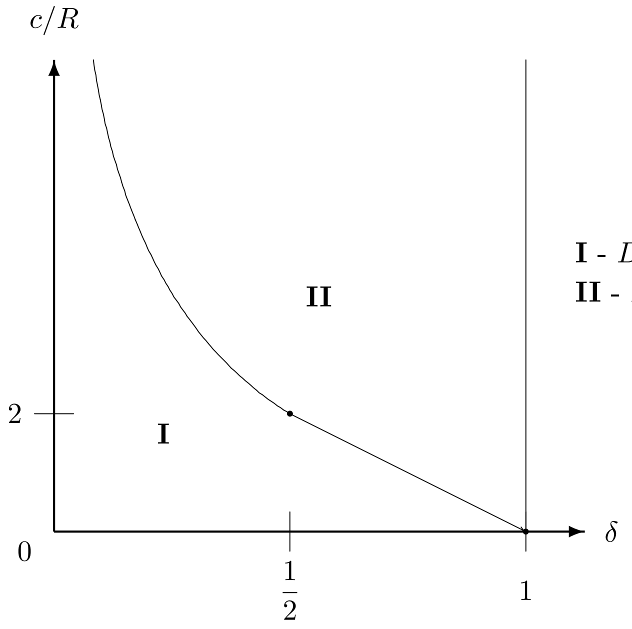
- Figures -

Fig 1 - Call Option: Social Planner



I - $E^{\star}_{SP} = 2R$
II - $E^{\star}_{SP} = 3R$

Fig 2 - Low Risk Debt: Social Planner



I - $D^{\star}_{SP} = 2R$
II - $D^{\star}_{SP} = R$

Fig 3 - High Risk Debt: Social Planner

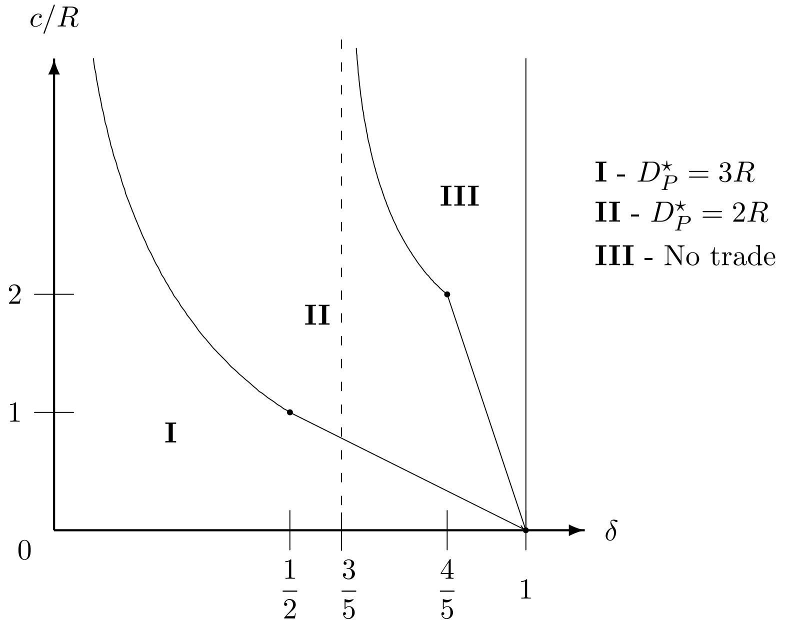


Fig 4 - Call Option: Manager

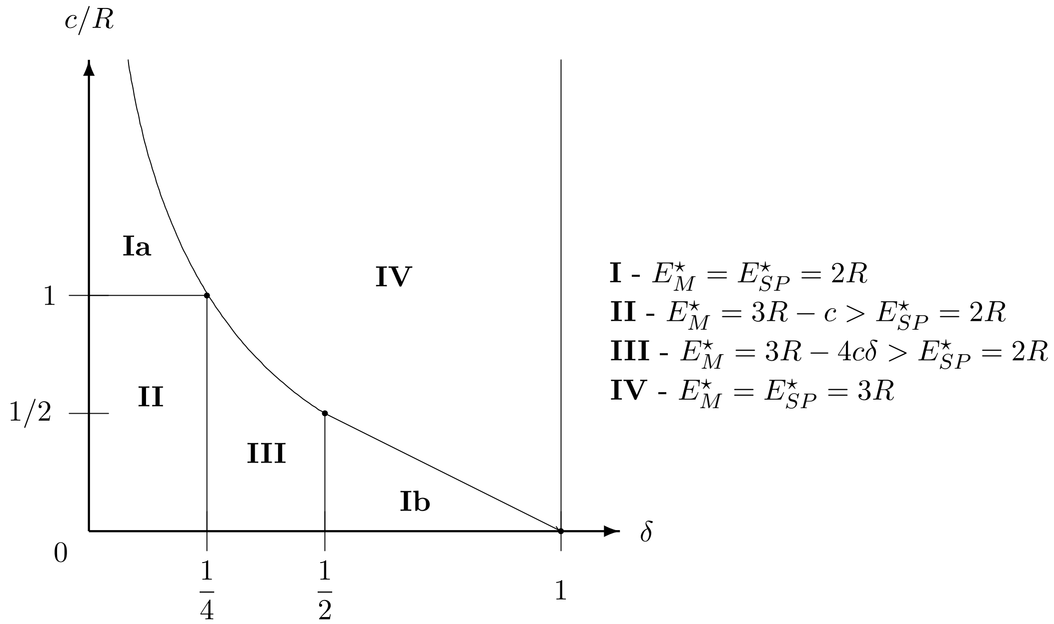
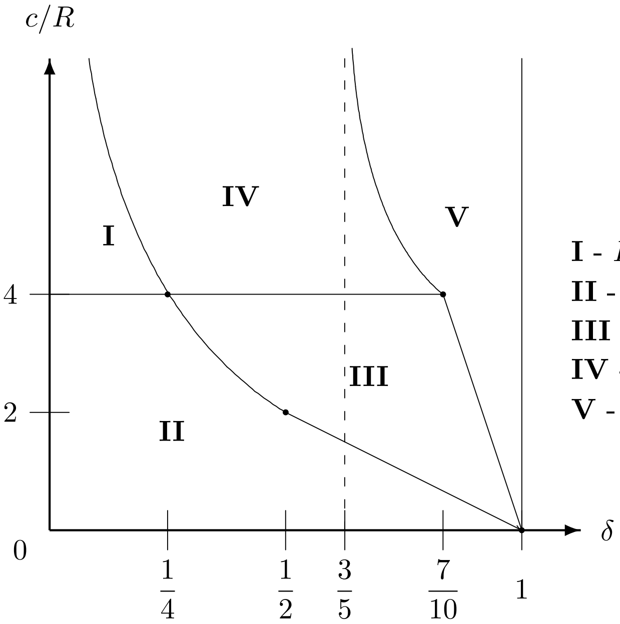
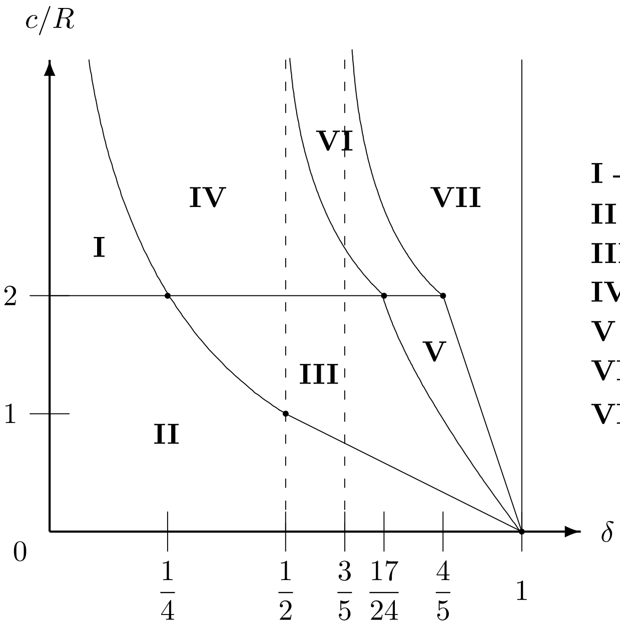


Fig 5 - Low Risk Debt: Manager



I - $D^{\star}_{M} = D^{\star}_{SP} = 2R$
II - $D^{\star}_{M} = R + c/4 < D^{\star}_{SP} = 2R$
III - $D^{\star}_{M} = R + c/4 > D^{\star}_{SP} = R$
IV - $D^{\star}_{M} = 2R > D^{\star}_{SP} = R$
V - $D^{\star}_{M} = R + c\delta/2 - k > D^{\star}_{SP} = R$

Fig 6 - High Risk Debt: Manager



I - $D^{\star}_{M} = D^{\star}_{SP} = 3R$
II - $D^{\star}_{M} = 2R + c/2 < D^{\star}_{SP} = 3R$
III - $D^{\star}_{M} = 2R + c/2 > D^{\star}_{SP} = 2R$
IV - $D^{\star}_{M} = 3R > D^{\star}_{SP} = 2R$
V - $D^{\star}_{M} = 2R + c\delta - m > D^{\star}_{SP} = 2R$
VI - $D^{\star}_{M} = 2R + c\delta - h > D^{\star}_{SP} = 2R$
VII - No trade

Fig 7 - Social planner (SP)

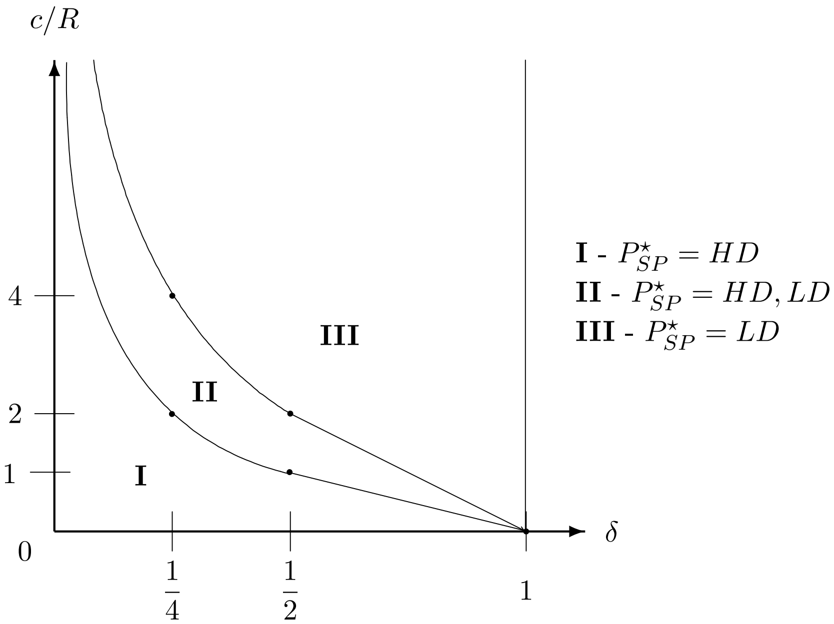


Fig 8 - Manager (M)

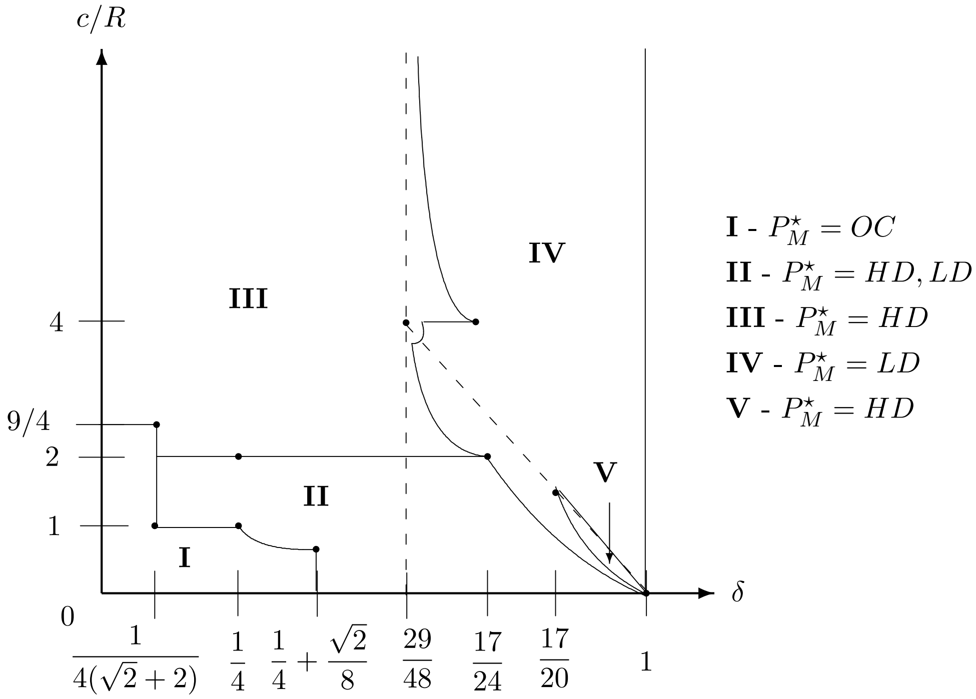
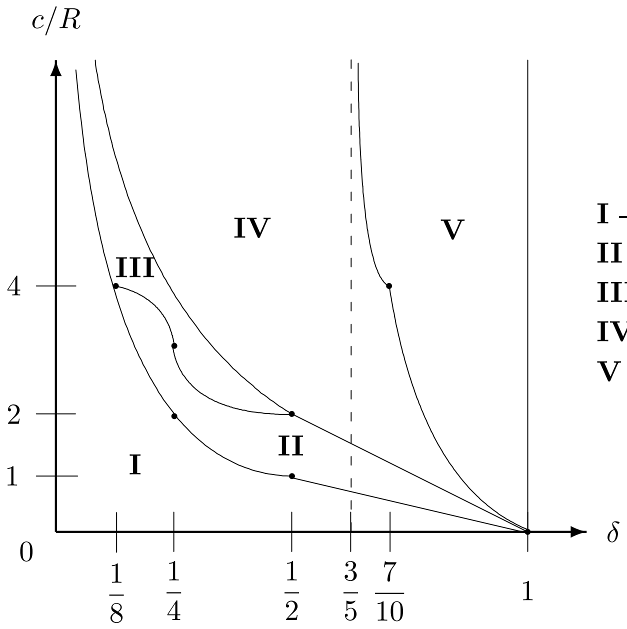
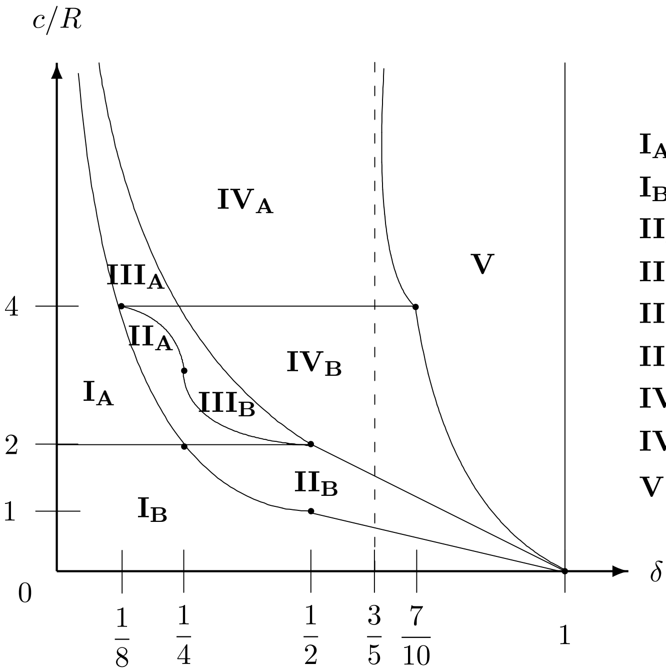


Fig 9 - Regulated Manager (RM)



I - $P^{\star}_{RM} = P^{\star}_{SP} = HD$
II - $P^{\star}_{RM} = HD$; $P^{\star}_{SP} = HD, LD$
III - $P^{\star}_{RM} = LD$; $P^{\star}_{SP} = HD, LD$
IV - $P^{\star}_{RM} = P^{\star}_{SP} = LD$
V - Market Failure ($P^{\star}_{SP} = LD$)

Fig 10 - Asset choice in the RM portfolio



$\mathbf{I_A}$ - $D^{\star}_{RM} = D^{\star}_{SP} = 3R$
$\mathbf{I_B}$ - $D^{\star}_{RM} = 2R + c/2 < D^{\star}_{SP} = 3R$
$\mathbf{II_A}$ - $D^{\star}_{RM} = 3R > D^{\star}_{SP} = 2R$
$\mathbf{II_B}$ - $D^{\star}_{RM} = 2R + c/2 > D^{\star}_{SP} = 2R$
$\mathbf{III_A}$ - $D^{\star}_{RM} = D^{\star}_{SP} = 2R$
$\mathbf{III_B}$ - $D^{\star}_{RM} = R + c/4 < D^{\star}_{SP} = 2R$
$\mathbf{IV_A}$ - $D^{\star}_{RM} = 2R > D^{\star}_{SP} = R$
$\mathbf{IV_B}$ - $D^{\star}_{RM} = R + c/4 > D^{\star}_{SP} = R$
V - Market Failure