Long-Run Productivity Risk: A New Hope for Production-Based Asset Pricing*

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Abstract

This study examines the intertemporal distribution of productivity risk. Focusing on post-war US data, I show that the conditional mean of productivity growth is time-varying and extremely persistent. This generates uncertainty about the long-run perspectives of economic growth and affects asset prices. The data suggest that stock market prices are very sensitive to long-run news about productivity growth. After establishing this empirical link, I develop a production-based asset pricing model featuring long-run uncertainty about the productivity growth rate, convex adjustment costs, and recursive preferences à la Epstein-Zin. This model reproduces key features of both asset prices and macroeconomic quantities, including consumption, investment, and output. I also provide a detailed examination of the role of the intertemporal elasticity of substitution, relative risk aversion, and adjustment costs in this type of economy.

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"By 1995, the investment boom had gathered momentum, suggesting that earlier expectations of elevated profitability had not been disappointed ... Now, five years later, there can be little doubt that not only has productivity growth picked up from its rather tepid pace during the preceding quarter century but that the growth rate has continued to rise [...]", Alan Greenspan, 04/05/2000.

"PRODUCTIVITY. That word, more than any other, explains the phenomenal performance of United States stock markets over the last six years [...]", Alex Berenson, *The New York Times*, 02/10/2000.

"The identity of the macroeconomic risks that drive asset prices and expected returns is a central question of finance, and an important question for macroeconomics." J. Cochrane (1996).

1 Introduction

In every country, aggregate productivity is one of the most important macroeconomic indicators. Indeed, productivity is very important also from a financial point of view. Keeping everything else constant, higher productivity implies greater earnings and higher investment returns.

Despite the difficulty of the task, each quarter analysts try to forecast productivity with surgical precision. As soon as new information about productivity is acquired, financial traders update their expectations and rebalance their portfolios, and market prices move. Meanwhile, financial investors have to bear the price risk related to productivity news.

What is ultimately the link between stock market prices and productivity fluctuations? This is a question that has attracted the attention of several economists, who have been able to find interesting links between the movement in asset prices and real economic activity.

Fama (1981 and 1990), for example, is one of the first economists to study the empirical link between aggregate stock prices and macroeconomic variables. He finds that a substantial share of the variance of annual returns can be explained by aggregate output growth, the latter being a variable highly correlated with productivity that proxies for expected cash flows. Cochrane (1996) focuses on a specific component of output: investment. By employing a cross-section of assets, he shows that we cannot reject the assumption that investment returns—computed according to a convex adjustment costs function—are a significant risk factor for asset returns. More recently, Balvers and Huang (2007) successfully explain cross-sectional differences in asset returns focusing on innovations to US aggregate productivity measured at a business cycle frequency (HP-filtered productivity).

These three papers, among others that I discuss in the next section, represent important contributions describing the link between stock market prices and real variables at both an empirical and a theoretical level. All of these studies, however, share one common feature: they do not distinguish the specific impacts that different sources of productivity uncertainty can have on stock prices. In particular, they do not disentangle the role of predictable fluctuations that are active at low frequency and explain productivity growth rate swings over long horizons. I refer to this type of fluctuation as long-run productivity risk, according to a similar terminology that has been adopted by Barsky and DeLong (1993), and more recently by Bansal and Yaron(2004).

Barsky and DeLong (1993) show that a predictable and persistent component in dividends growth (namely, long-run dividend risk) can explain a large fraction of stock price fluctuations. This is true even if such a component is small and nearly undetectable over a short horizon. Bansal and Yaron (2004) explore the role of long-run risk in a fully specified endowment economy model in which the representative agent has recursive preferences à la Epstein-Zin, and a common long-run component simultaneously affects consumption and dividend growth rates. In this economy, they are able to reconcile several features of both macroeconomic quantities such as consumption and dividends, and asset prices. Above all, they are able to produce a high aggregate equity premium by assuming a low level of risk aversion, and by highlighting the role played by a moderate amount of long-run consumption risk.

Barsky and DeLong (1993) and Bansal and Yaron (2004) do not address the origins of the long-run component in consumption and in the assets' cash flow. In this paper, I consider the possibility that long-run consumption and dividend risks originate from a more primitive variable: long-run productivity risk.

To find evidence of the presence of long-run productivity uncertainty, I study the empirical intertemporal distribution of productivity risk for the US economy. I show that the conditional mean of annual productivity growth is time-varying and extremely persistent. This result is consistent with that found by Edge, Laubach and Williams (2007), who use real-time data to document time variation in the US long-run productivity growth forecast.[1] The slow fluctuations in the conditional mean of productivity growth produce

[1] They also study the macroeconomic implications of this uncertainty factor in a production economy in which the representative agent has standard CRRA preferences and limited information

uncertainty about the long-run perspectives of economic growth and affect asset prices. In particular, I show that stock market prices are very sensitive to long-run news for productivity growth: a 1% increase in the conditional mean of annual productivity growth is accompanied, on average, by an increase in the aggregate price-dividend ratio ranging between 11% and 22%. Overall, however, the predictable component in productivity growth is relatively small, and it explains at most 12% of the total variance of the innovations in the US price-dividend ratio.[2]

After demonstrating an empirical link between asset prices and productivity, I develop a production-based asset pricing model with complete markets featuring long-run uncertainty about the productivity growth rate. The main goal of this second step of my study is to explore in detail the theoretical implications of productivity growth predictability (long-run productivity risk) in the context of a fully specified model with production and recursive utility.

This theoretical approach is interesting because it bridges part of the gap between the current long-run risk literature and the macroeconomic literature. Standard production-based macroeconomic models try to explain the joint dynamics of quantities and productivity, focusing only on business cycle frequencies. In this literature, the implications for asset prices are usually neglected, and often they are simply counterfactual. In the finance literature, in contrast, long-run risk models are designed to explain key time-series properties of asset prices, keeping quantities as a given and focusing on the low-frequency fluctuations of cash flows and consumption. In this paper I propose a unifying framework in order to study the co-movements of asset-prices and quantities simultaneously over both the short and the long horizon.

For the sake of simplicity, the model is extremely parsimonious. It considers an infinitely lived representative agent with Epstein-Zin (1989) and Weil (1989) preferences. These preferences are important because they disentangle the intertemporal elasticity of substitution (IES) from the relative risk aversion coefficient (RRA) and are sensitive to the intertemporal distribution of risk.

The representative agent supplies labor and accumulates capital subject to adjustment costs as specified in Jermann (1998). Under the assumption of the existence of convex

about the long-run component.

[2]In a recent paper, Beaudry and Portier (2006) adopt a VAR approach to show *"how stock prices movements, in conjunction with movements in total factor productivity (TFP) can be fruitfully used to help shed new light on the forces driving business cycle fluctuations."* They use stock market prices to identify long-run shocks to productivity. Their results confirm the existence of a significant link between the predictable component in productivity growth and asset prices.

adjustment costs—see also Hayashi (1982) and Cochrane (1996)—firms needs to pay a cost in order to adjust their capital stock. This cost is an increasing function of the actual adjustment. In general, when convex adjustment costs are present, firms need to employ a more-than-proportional, increasing amount of resources in order to accumulate capital units. As a consequence, the supply curve of new capital is not perfectly flexible; the price of new capital is time-varying and increases when investment rises.

The importance of this kind of friction is twofold. On the one hand, it allows me to introduce time-variation in the price of capital, an important feature observed in the US data. On the other hand, it keeps my results comparable to those obtained in the existing asset pricing literature. Convex adjustment costs are a real investment friction that has already been studied in several other works (among others, see two recent papers by Campanale, Castro and Clementi (2006) and Kaltenbrunner and Lochstoer (2006)).

Productivity growth is exogenous and is affected by two different sources of uncertainty: a short-run shock that is *i.i.d.* (standard in the real business cycle literature), and a long-run component that is responsible for small but persistent fluctuations in the drift of productivity. The latter component is calibrated to introduce long-run productivity swings.

Under this set-up, the model successfully reproduces key features of both asset prices and macroeconomic quantities such as consumption, investment, and output. This is significant because the reconciliation of asset market factors with aggregate quantities behavior has proved a challenge for modern dynamic stochastic general equilibrium (DSGE) models. Rouwenhorst (1995), for example, shows that in a model with production and standard time-additive CRRA preferences as in Kydland and Prescott (1982), it is impossible to reproduce the US historical equity premium. Jermann (1998) shows that introducing adjustment costs in a model with CRRA preferences is not enough to produce relevant premia. At the same time, Jermann shows that when habit preferences and adjustment costs are combined, it is possible to obtain high equity premia and a low risk-free rate in an economy with production and fixed labor supply. Although this result represents a very important improvement in the production-based asset pricing literature, it is not completely satisfactory. As pointed out by Boldrin, Christiano and Fisher (2001) and Lettau and Uhlig (2000), Jerman's model produces a countercyclical response of labor to a persistent shock to productivity which is at odds with the data. Boldrin, Christiano and Fisher (2001) solve this problem by proposing a two-sector economy with limited labor and investment mobility. Contemporaneously, however, their model predicts a negative serial correlation for consumption growth and an excessively volatile risk-free rate. This is indeed a common problem shared by most models with habit preferences.

Given all these documented difficulties, it is remarkable that the model presented in this paper is able to reproduce key facts about both the first and the second moments of the equity premium and the risk-free rate. In particular, the model replicates the historical mean of both the US stock market excess returns and the risk-free rate, with a moderate amount of RRA and an IES slightly larger than one. Focusing on second moments, notice that the risk-free rate implied by the model is about 10 times less volatile than the excess returns, as observed in the data. Furthermore, the implied risk-free rate is as persistent as is observed in the data, while the excess returns are instead almost unpredictable, again consistent with the empirical evidence.

I show that in order to obtain these results the model needs a real friction that allows for time-variation in the marginal price of capital. Thanks to the long-run component, a moderate amount of capital adjustment cost is sufficient to produce sizable fluctuations in the stock price. The long-run shocks, indeed, are able to produce substantial shifts in the demand of new capital and generate relevant price movements even if the supply curve is not very steep or, in other words, even if the adjustment costs are not strong. As a matter of fact, in my benchmark calibration the elasticity of my adjustment costs function is about 4 times smaller than that in Jermann (1998) and is consistent with Abel (1980) empirical findings.

Although the long-run component plays a crucial role in producing high equity premia, it is important to stress the fact that the model predicts only a moderate contemporaneous correlation between price-dividend innovations and long-run news, similar to that observed in the data. Furthermore, this economy is able to produces high equity premia, keeping the correlation between consumption growth and excess returns close to that observed in the US. These two results are noteworthy because they highlight the fact that the model is able to produce the right amount of co-movements between asset prices and real variables.

At the same time, the model correctly reproduces the observed mean, volatility, and serial correlation of consumption growth. Furthermore, I devote section 3.4 of the paper to a demonstration of the way that long-run productivity risk can endogenously produce long-run consumption risk. In addition, investment co-movements with consumption are consistent with the data. The contemporaneous correlation between HP-filtered consumption and investment growth is about 85% (post-war sample). If we focus on raw data, the correlation of the growth rates drops to about 45%. While consumption and investment strongly co-move at business cycle frequencies, over the long horizon they do not.

Under the benchmark calibration, the model reproduces these co-movements given an IES other than 1, so that the income effect and substitution effect associated with future productivity do not cancel each other out. In particular, I show that when the IES is greater than 1, the substitution effect dominates: good news for the expected productivity growth rate provides a strong incentive to reduce consumption and invest more. Meanwhile, good news for the short-run implies an increase in both consumption and investment. The overall correlation between consumption and investment is positive and clearly in line with the data.

To my knowledge, this paper is one of the first to study the interaction between predictability in productivity growth and recursive preferences. For this reason I devote the final section of the paper to the influence of the RRA and of the IES on both quantities and prices.

I show that the IES has an extremely important impact on the dynamics of both quantities and prices. The RRA, in contrast, has only a marginal impact on quantities, while it has a strong impact on both the risk-free rate —because of precautionary savings motives— and the equity premium. I also study the case in which standard time-additive constant relative risk aversion (CRRA) preferences are employed, and I document their failure to produce a high market price of risk.

Finally, I explore the role of adjustment costs by comparing the model under the benchmark calibration to the case in which the supply curve of new capital is perfectly elastic. This experiment is particularly interesting because it relates asset-price implications to consumption-smoothing possibilities. While in the benchmark case investment is smoother than that observed in reality, when the adjustment costs are removed it becomes as volatile as in the data. Unfortunately, the results for both the asset prices dynamics and the co-movements between prices and quantities then become counterfactual. A real friction on investment is then required in order to generate reasonable predictions for the relationship between productivity and stock market returns.

1.1 Related literature

This paper is intended as a contribution to the long-run risk literature, the real business cycle (RBC) literature, and the literature regarding "production-based asset pricing"(Cochrane, 2005).[3] The most important papers related to this work are those of Tallarini (2000) and Bansal and Yaron (2004).

[3] For the sake of brevity, in this section I discuss only a subset of papers that are closely related to mine. This section is not – and does not aim to be – a complete review of the aforementioned literatures.

Tallarini (2000) is the first to employ risk-sensitive preferences in a production economy model in order to study the joint dynamics of prices and quantities. He works with a representative agent that has Epstein-Zin-Weil preferences, but he focuses only on the case in which the intertemporal elasticity of substitution is one.[4] Furthermore, productivity in his model follows a random walk with a constant drift, and the marginal price of new capital is fixed at one. He shows that his model has implications for macroeconomic quantities comparable to those obtained by Kydland and Prescott (1982) and that, for a given IES, the RRA has only second-order effects on this result. At the same time, Tallarini's model does not solve the equity premium puzzle, meaning that extremely high values of RRA (even 100) are required in order to justify the high Sharpe Ratio observed in the data. I show that this class of models can produce significantly improved results once we recognize the relevance of the following three additional economic factors: (1) the existence of predictability in productivity, (2) the relevance of time variation in the marginal value of capital, and (3) the interaction between the IES and income effect generated by future productivity shocks.

Bansal and Yaron (2004) demonstrate that in an exchange economy with long-run risk in consumption and dividends cash flow and Epstein-Zin-Weil preferences, it is possible to reconcile consumption and asset-price properties with moderate risk aversion and an IES slightly greater than one. I incorporate their insight in a simple production economy with capital adjustment costs in which the long-run risk is a primitive component of productivity. In my model, the long-run productivity risk optimally generates endogenous long-run risk in consumption and dividends and, as a consequence, high equity premia. More generally, analyzing this production economy allows me to bridge part of the gap between the current long-run risk literature, which takes consumption and dividends as given, and the business cycle literature, which often neglects the role of asset prices.

The interaction between long-run risk and production-based DSGE models has recently attracted significant attention. Campanale, Castro and Clementi (2006) characterize asset returns in a production economy with convex adjustment costs, trend-stationary productivity, and generalized recursive preferences à la Epstein-Zin (1989) and Gul (1991).[5]

Kaltenbrunner and Lochstoer (2006) also work with a production economy with convex

[4]This allows Tallarini to solve his model simply by applying the discounted linear exponential quadratic Gaussian control methodology developed by Hansen and Sargent (1995).

[5]When working with Epstein-Zin preferences, Campanale, Castro and Clementi (2006) calibrate their IES to a number very close to zero in order to match the relative volatility of investment and consumption. At the same time, however, their model produces a risk-free rate that is excessively volatile. Gul (1991) allows for disappointment aversion.

adjustment costs, and generalized recursive preferences à la Epstein-Zin. They show that in this environment, capital accumulation generates endogenous long-run consumption risk —a result confirmed in my investigation— and that the consumption-productivity ratio is an important factor of risk.

Backus, Routledge and Zin (2007) model a production economy with Epstein-Zin preferences and generalized predictability in productivity growth in order to reproduce the intertemporal co-movements of prices and quantities over the business cycle.[6]

Ai (2007) studies the asset-pricing and welfare implications of long-run productivity risk in a production economy in which the representative agent has limited information about the long-run component and has access to an AK technology function.[7] In such an economy, dividends equal total output. In my model, in contrast, total output and dividends differ from each other because of labor income and financial leverage.[8]

Finally, Uhlig (2007) explores the connection between leisure and the market price of risk in a DSGE model with recursive preferences.

2 Empirical evidence

I denote the level of productivity at time t as A_t. I use lowercase letters for log-units. In order to study the intertemporal distribution of risk related to productivity, I focus on the following model:

$$\begin{aligned} \Delta a_{t+1} &= \mu(1-\rho) + \rho\Delta a_t - b\epsilon_{a,t} + \sigma\epsilon_{a,t+1} \qquad (1) \\ \epsilon_{a,t+1} &\sim iidN(0,1). \end{aligned}$$

The productivity growth rate, $\Delta a_{t+1} \equiv \log(A_{t+1}/A_t)$, is an ARMA(1,1) that can be rewritten in the following way:

$$\Delta a_{t+1} = \underbrace{x_t}_{LRR} + \sigma \underbrace{\epsilon_{a,t+1}}_{SRR} \qquad (2)$$

$$x_t = E_t[\Delta a_{t+1}] = \mu(1-\rho) + \rho\Delta a_t - b\sigma\epsilon_{a,t}. \qquad (3)$$

[6] A previous version of their paper was circulated under the title "Leads, Lags, and Logs: Asset Prices in Business Cycle Analysis."

[7] An earlier version of Ai's paper was circulated under the title "Incomplete Information and Equity Premium in Production Economies" (2005).

[8] In this version of the model I keep the labor supply constant. The basic intuitions behind the model do not change when I allow labor to be endogenously time-varying. These results are available upon request.

The long-run risk component (LRR), x_t, captures persistent fluctuations in the conditional mean of the productivity growth rate. The short-run risk (SRR), $\epsilon_{a,t+1}$, instead is *i.i.d.*

I estimate equation (1) using post-war annual data.[9] Working with annual data is particularly important for two reasons. First, the data are not altered by any seasonal adjustment; second, they are more likely to contain a better signal and less noise related to the low-frequency component of productivity. This kind of approach is not new to the long-run risk literature. Harvey (1986) and, more recently, Bansal, Kiku and Yaron (2006) and Colacito and Croce (2007) use annual data to estimate long-run risk in consumption. I follow the same methodology, applying it to productivity.

In Table 1 I report my estimates for productivity growth. In the top panel I estimate the ARMA(1,1) model, assuming that the innovations are homoskedastic. This simple model explains about 25% of the total variance of productivity growth. Furthermore, the likelihood-ratio test rejects the null hypothesis that productivity growth is perfectly *i.i.d.* The forecast for productivity is extremely persistent: using the adjusted sample for the years 1951–2003, the implied annual persistence is about .84.

Several recent papers have documented a reduction in the volatility of aggregate variables.[10] To control for predictable time-variation in the volatility of productivity growth, I estimate the following model that includes a GARCH(1,1):

$$\begin{aligned} \Delta a_{t+1} &= \mu(1-\rho) + \rho \Delta a_t - b\sigma_t \epsilon_{a,t} + \sigma_{t+1}\epsilon_{a,t+1} \\ \sigma_{t+1}^2 &= \kappa + \rho_\sigma \sigma_t^2 + b_\sigma \sigma_t^2 \epsilon_{a,t}^2, \end{aligned} \tag{4}$$

In the lower portion of Table 1 I report the implied estimates. The predictable component continues to be extremely persistent, although less volatile, and it explains about 10% of the total variance of productivity growth.[11] The likelihood-ratio test confirms that we cannot reject the existence of predictability at a confidence level of 10%. In Fig. 1 I plot the productivity growth rates and the forecasts obtained from the ARMA(1,1) model with

[9]Data are from the Bureau of Labor Statistics; the sample spans 1948–2003. I use a multifactor productivity index that takes into account capital accumulation. In particular, the index measures the value-added output per combined unit of labor and capital input in private business and private nonfarm business.

[10]See, among others, Lettau, Ludvigson and Wachter (2006) and Fernandez-Villaverde and Rubio-Ramirez (2007).

[11]In the sample I consider, the estimate of the MA root, $\widehat{b}$, is greater than one. This would imply that the MA component of productivity growth is not invertible. In the Technical Appendix, I show that when both the predictable component in productivity growth and the sample size are small, this kind of result is quite frequent, even if the true data generating process has $b < \rho < 1$.

and without the GARCH(1,1) correction. The models give similar results: the conditional mean of productivity slowly declines until the 1970s, and then it begins to grow.

Both coefficients of the GARCH(1,1) are statistically different from zero. In Fig. 2 I depict the implied conditional volatility of productivity growth. The conditional volatility spikes up during the 1974 and 1983 crises, and then slowly declines. The flat, thin line represents the unconditional volatility of productivity growth measured on the whole sample. Since 1990, the conditional volatility of productivity has been moderate and has remained below its historical average. These findings are consistent with that observed for other macroeconomic quantities.

I now turn my attention to the impact of news about the conditional mean of the productivity growth rate on the aggregate stock market price. Both the aggregate stock market price and the long-run component in productivity are very persistent. They evolve as AR(1). In order to measure the influence of news to these two variables, I use the residuals of the following OLS regressions:

$$\begin{aligned} p_t - d_t &= \beta_{0,pd} + \rho_{pd}(p_{t-1} - d_{t-1}) + \epsilon_{pd,t} \\ x_t &= \beta_{0,x} + \rho_x x_{t-1} + \epsilon_{x,t}, \end{aligned} \tag{5}$$

where x_t is the ARMA(1,1) forecast of productivity growth. In a second step I estimate:

$$\epsilon_{pd,t} = \beta_0 + \beta_1 \epsilon_{x,t} + \epsilon_t. \tag{6}$$

The estimates are reported in Table 2. Fitted price-dividends ratio news, $\widehat{\epsilon}_{pd}$, and the fit of the regression in equation (6), $\widehat{\beta}_1 \widehat{\epsilon}_{x,t}$, are plotted in Fig. 3. I find a statistically significant positive relationship between price-dividend news and long-run productivity news. When I estimate the long-run component without including GARCH effects, I find that a 1% positive shock to long-run productivity can raise the price-dividend ratio by 11%. When I control for GARCH effects, the price-dividend ratio becomes even more sensitive, and $\widehat{\beta}_1 = 22$. This result is consistent with the idea of Barsky and DeLong (1993) and Bansal and Yaron (2004) that small but persistent innovations to the economy growth rate can produce a substantial adjustment in asset prices. This is a crucial empirical result that motivates and supports the model explored in the next sections.

3 The model

This section presents the model used to examine the link between productivity, asset prices, and other macroeconomic fundamentals. To keep the analysis as simple as possible, I focus only on the representative agent consumption-saving problem, and I keep constant the labor supply. The representative agent has preferences defined only by aggregate consumption:

$$\begin{aligned} U_t &= \left[(1-\delta) C_t^{1-\frac{1}{\Psi}} + \delta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\Psi}}}. \\ 0 &\leq C_t \end{aligned}$$

The consumption good is produced according to a constant returns-to-scale neoclassical production function:

$$Y_t = K_t^{\alpha} \left[A_t n_t \right]^{1-\alpha},$$

where K_t is the fixed stock of capital carried into date t, n_t is the labor input at t, and A_t is an aggregate productivity shock. The productivity growth rate, $\Delta a_{t+1} \equiv \log(A_{t+1}/A_t)$, has a long-run risk component and evolves as described below:

$$\begin{aligned} \Delta a_{t+1} &= \mu + x_t + \sigma \epsilon_{a,t+1} \\ x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\ \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right). \end{aligned}$$

Because this study examines the role of uncertainty about the conditional mean of productivity growth, I assume that the volatility of the shocks to productivity is constant.[12]

The specification above is consistent with that found in the empirical analysis. In particular, the conditional mean is allowed to be time-varying and persistent, and the Wold representation of total productivity growth is exactly an ARMA(1,1), as seen in the previous section. In order to perfectly disentangle the implications of the two sources of uncertainty, I assume that the short-run shocks and the long-run shocks are not correlated

[12] Fernandez-Villaverde and Rubio-Ramirez (2006) study the macroeconomic implications of time-varying volatility in productivity growth in the context of a production-based model with standard preferences.

with each other, and that the representative agent observes both the short-run shock and the long-run component. Hence, in my theoretical analysis I focus only on the case in which the representative agent has full information about the two components of productivity growth.[13]

The resource constraint of this economy is:

$$C_t + I_t \leq Y_t.$$

The capital stock evolves according to:

$$K_{t+1} = (1-\delta_k)K_t + G\left(\frac{I_t}{K_t}\right)K_t,$$

where

$$G\left(\frac{I_t}{K_t}\right) = \left[\frac{a_1}{1-\frac{1}{\tau}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\tau}} + a_2\right].$$

The rate of depreciation of capital is denoted by δ_k, and the function $G(.)$ transforms investment in new capital as in Jerman (1998). The agent is endowed with $\overline{n}$ units of time that she can devote to leisure (denoted by l_t) or labor according to the following constraint:

$$n_t + l_t \leq \overline{n}.$$

Since leisure does not appear in the utility function, the representative agent will always find it optimal to offer $n_t = \overline{n}$ units of labor.[14]

3.1 Equilibrium

In this economy, the allocation that solves the planner's problem can be decentralized by means of competitive markets.[15] It is then possible to find the competitive equilibrium allocation by solving the planner's problem. My computational methods are described in detail in the Technical Appendix. Once the planner's allocation is found, prices and

[13] There are several relevant papers that study the role of limited information about the long-run component. See, among others, Edge, Laubach and Williams (2007); Ai (2007); Croce, Lettau and Ludvigson (2005); Hansen and Sargent (2006); and Bansal and Shaliastovich (2008).

[14] I impose $\overline{n} = .18$. Like Tallarini (2000), I consider total employment multiplied by average weekly hours worked divided by the civilian non institutional population 16 years and older.

[15] See Sargent and Ljungqvist (2004).

returns can be derived as follows. The stochastic discount factor takes the following usual form (see Hansen, Heaton and Li (2005)):

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\Psi}} \left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma} \tag{7}$$

The second factor relates to news regarding the continuation value of the representative agent. Future utility is very sensitive to long-run news, and for this reason it can induce high volatility in the stochastic discount factor even for moderate amounts of risk aversion.[16] The risk-free rate is

$$R_t^f = E_t\left[M_{t+1}\right]^{-1}.$$

The marginal value of capital, q_t, is equal to the marginal rate of transformation between new capital and consumption:

$$q_t = \frac{1}{G'(\frac{I_t}{K_t})}.$$

The returns per unit of normalized capital are

$$R_{t+1} \equiv \frac{q_{t+1} + D_{t+1}}{q_t},$$

where

$$D_{t+1} \equiv \alpha\frac{Y_{t+1}}{K_{t+1}} - \delta_k q_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} G\left(\frac{I_{t+1}}{K_{t+1}}\right).$$

In accordance with Boldrin, Christiano and Fisher (1995), I introduce financial leverage. The levered excess return is:

$$R_{t+1}^d - R_t^f \equiv (R_{t+1} - R_t^f)\left(1 + \frac{\overline{B}}{S}\right),$$

where $\overline{B/S}$ is the average debt-share ratio of the firm. As in Boldrin, Christiano and Fisher (1995), I keep the leverage constant and assume $\overline{B/S} = 2/3$. Finally, wages equate to the marginal productivity of labor.

[16] For the interaction between Epstein-Zin preferences and long-run consumption risk, see, among others, Bansal and Yaron (2004); Hansen, Heaton and Li (2005); Croce(2006).

3.2 Benchmark calibration

As is typical in the long-run risk literature, I assume the representative agent has a monthly decision horizon. For this reason I calibrate the model to a monthly frequency, but I target quarterly statistics to obtain results that are comparable to those reported in the RBC literature.

Productivity growth in the US has an annual average of 2%. In order to match this moment at a monthly frequency, I set $\mu = .00165$. The annual volatility of the productivity growth rate is also about 2%, which implies a monthly volatility, σ, of approximately .006. The long-run component in productivity is calibrated so as to be relatively small but persistent, as seen in the previous section. I impose a monthly persistence of .98 (this is actually a conservative number, given that in annualized terms the persistence would be about .80). I allow the long-run component to explain only 7% of the total volatility of productivity growth at a monthly frequency. Given $\rho = .98$, I need to impose $\sigma_x = 5.5\sigma$.

The annualized capital depreciation rate is 6%. The parameter α is calibrated to match the capital income share. The elasticity of the supply curve of capital is equal to .98, a value in line with empirical evidence.[17]

In order to match the historical equity premium, the RRA is set to a value of 30. This is a plausible value, given the recent empirical findings in Bansal, Kiku, and Yaron (2006); Attanasio and Vissing-Jorgensen (2003); and Malloy, Moskowitz, and Vissing-Jorgensen (2006), but it is still reasonably low in light of the well-known difficulties of producing high equity premia in production economies.[18] Tallarini (2000), for example, using a risk aversion value of 100, obtains an annualized equity premium of just .04%, about 100 times smaller than that observed in the data. A similarly moderate level of relative risk aversion is adopted also by Lettau, Ludvigson and Wachter (2006).

In my benchmark calibration, I fix the IES at 2. This value is consistent with that estimated by Basal, Gallant, and Tauchen (2007); Colacito and Croce (2007); Bansal, Kiku, and Yaron (2006); and Attanasio and Vissing-Jorgensen (2003).

Finally, the annualized subjective discount factor is fixed at .98. This allows me to match the unconditional mean of the risk-free rate.

[17] Abel (1980) reports estimates that range between .5 and 1.14; Eberly (1997) has a 95% confidence interval for the US of $[1.08, 1.36]$.

[18] Malloy, Moskowitz and Vissing-Jorgensen (2006) exploit micro-level household consumption data and show that aggregation across stockholders affects the estimate of the RRA coefficient. When the authors focus only on wealthy stockholders, the RRA estimate is 10. When they consider all the stockholders, the RRA is about 20. When they use aggregate consumption, this estimate increases even more.

3.3 Predictions on quantities and prices

The main results produced by this calibration are reported in Table 3, panel A. In panel B I report first and second moments for both quantities and prices, while in panel C I focus on their co-movements.

The model reproduces quite well the observed unconditional mean of the growth rate of both investment and consumption. Furthermore, it implies an average income-output ratio of 25%, a number very close to that observed in the data. Models with standard time-additive CRRA preferences are not able to match the low level of the investment-output ratio and the low level of the risk-free rate simultaneously. Thanks to the Epstein-Zin preferences, however, this is not a problem. Under the benchmark calibration, the unconditional average of the risk-free rate is 1.2%, consistently with the data.

The implied equity premium, in log-units, is 4.8%. This value is slightly below the historical mean of the CRSP stock market excess returns.[19] However, it is in line with the estimates reported by Fama and French (2001). Furthermore, this is a remarkably high number compared to that obtained by Tallarini (2000) and Kaltenbrunner and Lochstoer (2006).[20]

To understand the reasons that the model is able to generate such a high equity premium, note that the Euler equation implies that $E_t[r_{t+1} - r_{f,t}] \approx -cov_t(m_{t+1}, r_{t+1})$. Capital must offer a higher equity premium when its returns move in a direction opposite to that of the discount factor. In Figs. 4 and 5 I show the impulse response function of both quantities and asset prices after short- and long-run shocks in order to better highlight their co-movements.

In Fig. 4 I examine the quantitative implications for the growth rates of consumption, C_t, investment, I_t, and output, Y_t. The plots show the percentage deviations from the steady state of the monthly growth rates realized after a single positive pulse shock to both the short-run component (left panels) and the long-run risk (right panels). The shocks materialize only at time 2, and they are normalized according to their standard deviations ($\epsilon_{a,2} = \sigma$ and $\epsilon_{x,2} = \sigma_x$).

When a short-run shock materializes, the representative agents finds it optimal to increase consumption and investment at the same time. Investing more allows the representative agent to temporarily increase the capital stock and smooth consumption over

[19]See Appendix A. for more details about the data.

[20]Kaltenbrunner and Lochstoer (2006) also consider pre–World War II data. While they have a lower relative risk aversion value of 5, they work with a more volatile, and thus riskier, consumption process.

time. This impulse response is fully consistent with the predictions of any standard RBC model.

The behavior of consumption and investment is very different with respect to the long-run component. First of all, notice that long-run shocks are very long-lasting, and for this reason they can have a very strong impact on savings decisions even if they are small. Furthermore, long-run news simultaneously generates both an income effect and a substitution effect, which work in opposite directions.

Higher expected long-run productivity generates a substitution effect that increases the opportunity cost of consumption, which tends to stimulate investment. Since output is predetermined, an increase in investment generates a contemporaneous drop in consumption.

At the same time, a positive long-run shock allows the agent to feel much richer and to desire an immediate increase in consumption. The long-run component, in fact, is highly persistent, and thus a single long-run shock is able to affect the flow of expected future utility over a very long time-horizon. Indeed, a positive long-run shock translates into a remarkable increase in the continuation value of the agent. For a given output, the income effect tends to produce an increase in consumption and a drop in investment.

When the IES is high, as in the benchmark calibration, the degree of substitutability between continuation value and current consumption is high also. In this case, the substitution effect dominates the income effect, and the agent finds it optimal to decrease consumption in order to accumulate more capital. This is the reason that after a positive long-run shock, consumption growth drops while investment growth increases (Fig. 4, right panels).

Under the benchmark calibration, investment rises after positive long-run and short-run shocks. As we can see in the lower panels of Fig. 5, this produces in both cases a pressure on the price of capital to appreciate (q_{t+1}/q_t measures the capital gain component of the return). As shown in the middle panels, both short-run and long-run shocks imply bigger stock market returns and a contemporaneous fall in the stochastic discount factor.[21] The stock market is indeed risky with respect to both sources of uncertainty, and for this reason it pays the high equity premium reported in Table 3.

The third column in panel B focuses on the volatility of both quantities and prices. The annualized volatility of quarterly consumption growth is 1.4%, exactly as observed

[21] The stochastic discount factor percentage deviation from its steady state is about −9% after short-run news and −24% when a long-run shock materializes. This is a key feature of the Epstein-Zin-Weil preferences, which are very sensitive to long-run shocks. See, among others, Bansal and Yaron (2004) and Croce, Lettau and Ludvigson (2005).

in the post-war data. Investment, in contrast, is less volatile than that observed in the data. In section 4.2 I show that this problem also affects economies with standard time-additive CRRA preferences, and in section 4.3 I show that the low volatility of investment is induced by the convex adjustment costs I adopt.

The excess returns implied by the model are less volatile than those observed in the data. This is due to the fact that both the price-dividend ratio and dividend growth are not volatile enough. Overall, however, thanks to the presence of adjustment costs, the volatility is much higher than that observed in standard frictionless production-based models.[22]

The risk-free rate has an annual standard deviation of .35%. Similar to that observed in the data, its volatility is about 10 times smaller than the volatility of market returns. This result is a success with respect to standard habit models, which are widely known to produce an excessively volatile risk-free rate.[23]

In the fifth column of Table 3, panel B we see one of the most interesting results of this theoretical analysis. The model is able to perfectly reproduce the persistence of both quantities growth rates and prices. The left panels of Fig. 4 indicate that short-run shocks tend to produce responses in the growth rates of consumption and investment that are not strongly persistent. The right panels, on the other hand, show that the long-run component is able to produce persistent adjustments. These latter movements, however, are relatively small and allow the autocorrelation function of both the quarterly consumption and investment growth to be maintained at about .35, consistent with the data. Meanwhile, the model reproduces the high persistence observed in the investment-output ratio, the price-dividend ratio, and the risk-free rate. The implied excess returns are instead almost *i.i.d.*, as in the data.

Finally, in panel C of Table 3 I focus on co-movements, an important dimension the model is able to explore. Working with unfiltered consumption and investment time-series, we obtain a moderate contemporaneous correlation between these two variables. Indeed, while at business cycle frequencies consumption and investment are highly correlated, at lower frequencies they are not.

The model is perfectly able to reproduce this moderate correlation thanks to the presence of the long-run component. As seen before, while short-run shocks induce a

[22] See, for example, Tallarini (2000), Kaltenbrunner and Lochstoer (2006), and the CRRA case in Jerman (1998).

[23] See, for example, Jerman (1998) and Boldrin, Christiano and Fisher (2001). Epstein and Zin (1989) preferences allow: (1) the conditional mean of the stochastic discount factor to have low volatility, and (2) the stochastic discount factor to have high conditional volatility.

perfect correlation between consumption and investment growth, long-run news forces consumption and investment to move in opposite directions. Overall, the correlation between these two variables implied by the model is still positive and perfectly in line with the empirical evidence.

In the data the correlation between stock market returns, investment, and output growth is small. This is mostly induced by the fact that dividend growth is quite volatile but not very correlated with the macroeconomics variables just mentioned. The model, not surprisingly, over predicts the correlation between stock market returns, investment growth, and output growth. In particular, what is at odds with the data is that capital income, dividend growth and returns all move in the same direction at exactly the same time after a short-run shock. Their contemporaneous correlation is higher than that observed in the data. In an exchange economy this problem is solved by introducing an idiosyncratic dividend-specific shock that reduces the correlation with macroeconomic fundamentals. In my production economy, however, this extra source of uncertainty for the dividends is not present.

The market returns are poorly correlated with consumption growth: in the data this correlation is about 20%, while in the model it is 29%. This result is extremely significative because it shows that the model produces high excess returns without altering the total correlation between consumption growth and stock market returns.

3.4 Properties of the consumption growth rate

The main goal of this paper is to study the implications of long-run productivity risk for asset prices in a general equilibrium model. It is also important, however, to consider its implications for the endogenous intertemporal distribution of consumption risk, a relevant component of the market price of risk.

This is of particular interest in the long-run risk literature, because in the data, consumption growth is almost *i.i.d.* at high frequencies. Bansal and Yaron (2004) document this empirical evidence and show that a long-run consumption risk model can be consistent with the data if the long-run component in consumption is small. In this section I show that the endogenous consumption process produced in this economy is consistent with that observed in the US.

If we allow $\widehat{c}_t \equiv C_t/A_t$ to denote the consumption-productivity ratio, the growth rate of consumption can be rewritten as follows:

$$\Delta c_{t+1} = \Delta a_{t+1} + \Delta \widehat{c}_{t+1}.$$

The first term shows movements in consumption growth induced by exogenous productivity growth fluctuations. The second term captures endogenous movements induced by the optimal response of the agent to the exogenous shocks. If the agent keeps constant the consumption-productivity ratio over time ($\Delta \widehat{c}_{t+1} = 0$ for any t), his consumption process has the same properties as the productivity process, and therefore it has the same exposure to both long- and short-run risk. However, this is not what the model predicts. The previous section, in fact, shows that consumption growth is smoother than productivity growth, implying that the agent moves $\widehat{c}_t$ counter-cyclically, allowing the consumption process to be less exposed to productivity shocks.

In Fig. 6 I plot the impulse response functions of the productivity growth and consumption-productivity ratios with respect to both short-run and long-run news under the benchmark calibration. When a positive short-run shock materializes (at time $t = 2$), the consumption-productivity ratio drops, because the agent responds by increasing her investment. After the first period, the growth rate of productivity goes back to precisely zero, while that of consumption remains positive, even if very close to zero. The agent, in fact, uses the extra capital accumulated in the first period to keep the growth rate of consumption above its steady-state level over a longer time-horizon. This effect is both very persistent —the decay rate of Δc_t after the initial shock is about .9953— and very small, almost invisible in the figure.

In order to capture the aforementioned result, in the lower-left panel of Fig. 6, I plot the spectral density of consumption growth generated by short-run risk. The same panel shows the theoretical spectral density of the productivity short-run risk.[24] The spectrum of short-run productivity shock is perfectly flat, while that of consumption growth has a spike at low frequencies. This shows that the model introduces a moderate amount of endogenous persistence in the growth rate of consumption even after a simple *i.i.d.* shock. This effect is also documented and studied in detail by Kaltenbrunner and Lochstoer (2006). My model, yields precise implications about the dynamics of the consumption-productivity ratio after long-run shocks. When this sort of shock arrives, the agent responds immediately by increasing investment and allowing the consumption-productivity ratio to decline. After the initial fall, consumption growth becomes positive but remains smaller than the growth rate of productivity for the remaining plotted periods. The growth rate of productivity, however, declines more quickly than the consumption growth rate and, after about 150 months, it becomes smaller than the consumption growth rate. In particular, after the initial shock, consumption growth decays at a rate of about .9838, while the productivity

[24]In the Technical Appendix, I show in detail how these spectral densities are computed.

growth rate's decay rate is $\rho = .98$. This is engendered by endogenous capital accumulation dynamics: the agent accumulates extra units of capital for many periods and, in a second moment (150 months in this case), begins to use them to sustain consumption growth over a longer time-horizon. The implied spectral density of consumption growth (bottom-right panel of Fig. 6) now has a pronounced spike at low frequencies, showing that long-run risk in productivity can generate long-run risk in consumption.

In Fig. 7 I show the contribution of both the short- and long-run components to the theoretical long-horizon variances in the productivity growth rate (right panel) and the consumption growth rate (left panel). I define the variance of the productivity growth rate at the horizon h as: $Var[\Delta a_{t+h|t}]/h$. Since I assume that long-run and short-run news are uncorrelated, the previous long-horizon variance is easily decomposed into two subcomponents:

$$Var[\Delta a_{t+h|t}]/h = \underbrace{Var\left[\sum_{k=1}^{h-1} x_{t+k-1}\right]/h}_{Var_h^{lrr}(\Delta a)} + \underbrace{Var\left[\sum_{k=1}^{h} \epsilon_{a,t+k}\right]/h}_{Var_h^{srr}(\Delta a)}.$$

In the right panel of Fig. 7, the solid line shows that the contribution of short-run risk to the long-horizon variance is constant across horizons, because the short-run risk is *i.i.d.* The dashed line, however, indicates that the variance of consumption attributable to the long-run component increases over longer time-horizons and actually becomes dominant after about 15 periods. This is due to the fact that the long-run component is persistent and its auto-covariances play an important role in amplifying the long-horizon productivity variance. I proceed in an analogous way to examine the long-horizon variance of the consumption growth rate.[25] The left panel of the figure confirms that, for any horizon h, the variances of both the long- and short-run components of consumption growth are smaller than those of productivity growth. Consumption is then smoother than productivity over both shorter and longer horizons. Note that the variance induced by the short-run shock is not flat across time-horizons. In accordance with my prior observations, variance actually increases slightly because of the small endogenous persistence in consumption growth introduced by capital accumulation. At the same time, the long-run productivity component is able to generate a strong increase in the variance of consumption over longer time-horizons similar to that which occurs in the productivity growth process.

[25] I describe the computations for the long-horizon variance of consumption in more detail in the Technical Appendix.

Finally, to better compare the model-generated consumption process to the data, in Fig. 8 I plot the spectrum of the *quarterly* growth rate of consumption. In particular, the dashed line shows the spectral density estimated from the data, while the other lines show the bottom 2.5% percentile, the median, and the top 97.5% percentile of the distribution of quarterly spectral densities generated by simulating the model with production.[26] If we focus on the business cycle frequencies, [.2–.8], we see that the median spectral density produced by the model is reasonably close to its empirical counterpart. More generally, the model is able to capture the decline of the empirical density over higher frequencies. Note also that the spectrum estimated from the data strays outside the confidence interval generated by the model only for frequencies higher than .75, equivalent to cycles with a duration shorter than 8 quarters. Overall, Fig. 8 shows that the model does a very good in job of replicating the observed consumption risk.

3.5 Price-dividend ratio and exposure to productivity risk

One of the main reasons that the model is able to reproduce the properties of the stock returns is that it is able to generate sizeable time-variation in the price of capital thanks to the adjustment costs. Furthermore, as in the empirical analysis above, the model produces fluctuations of the price-dividend ratio that are positively related to long-run productivity shocks.

In Fig. 9 I plot the impulse response function of the price-dividend ratio with respect to both a short-run and a long-run shock. After a short-run shock, the price-dividend ratio actually falls and then mean-reverts geometrically toward its steady state. In response to a long-run shock, in contrast, the price-dividend ratio jumps up and then slowly mean-reverts, following a more complicated trajectory.

In order to better understand these dynamics, let us use the Campbell-Shiller approximation for the price-dividend ratio:

$$p_t - d_t = \overline{pd} + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j E_t[r_{t+1+j}],$$

where k_d is a positive approximation constant, and $|k_d| < 1$. In the model described in

[26]In order to generate these distributions, I simulate the model with a production economy over a sample of 600 months, time-aggregate the quantities to quarterly frequencies, and compute the implied growth rates for consumption. I repeat this procedure 500 times to generate a sample of spectral densities.

this paper, the excess returns are almost constant over time. This allows me to write

$$p_t - d_t \approx \overline{pd}' + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j E_t[r^f_{t+1+j}].$$

Using equation (8) it is then possible to obtain:

$$p_t - d_t \approx \overline{pd}'' + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j \frac{1}{\Psi} E_t[\Delta c_{t+1+j}]$$

The price-dividend ratio is positively related to the conditional mean of future dividends growth and negatively related to the conditional mean of future consumption growth.

Under the benchmark calibration, the expected dividend growth induced by good long-run news is positive and is three times larger than that of consumption. Exactly as in Bansal and Yaron (2004), the positive long-run news about future dividends overcomes the movements in the discount factor related to the revisions of the expectations about future consumption growth. As a result, the price-dividend ratio increases.[27]

The opposite is true for the short run-shocks. After the realization of positive short-run news to productivity, $\epsilon_a > 0$, the expected future dividend growth falls. This effect is very small, almost invisible in the graph, but persistent. Moreover, the absolute value of this drop is five times larger than that of the consumption growth conditional mean. Overall, the endogenous negative leverage of future dividend growth with respect to the short-run productivity shock generates the drop in the price-dividend ratio shown in the graph.

Why does future dividend growth become negative after idiosyncratic good news to productivity? The answer is once again related to the capital accumulation dynamics. When a short-run shock materializes, the agent responds immediately by increasing investment. After the shock, the agent begins to sustain consumption growth above its unconditional mean by consuming the extra units of the capital just accumulated. The capital-productivity ratio, K_t/A_{t-1}, slowly declines back to its steady state, and the same happens to the total dividend-productivity ratio, D_t/A_{t-1}. The negative growth of the dividend-productivity ratio in turn pushes the growth rate of total dividends below its unconditional average.

At a monthly frequency, the long-run news at time t is simply identical to $\epsilon_{x,t}$. The

[27]The exposure of the expected dividends growth to long-run consumption risk is in reality time-varying. It is for this the reason that the price-dividend does not mean-revert geometrically.

relation of news to the price-dividends ratio, as defined in equation (5), is instead a nontrivial function of both $\{\epsilon_{a,k}\}_{k=0}^{t}$ and $\{\epsilon_{x,k}\}_{k=0}^{t}$. In order to easily recover $\{\epsilon_{pd,k}\}_{k=0}^{t}$, I simply simulate the model for six hundred months (i.e. figty years) and I estimate equation (5). At this point I am able to estimate equation (6) using the simulated data.

After repeating this procedure five hundred times, I get an average $\widehat{\beta}_1$ that is positive, as in the data. Furthermore, the long-run news explains on average about 7% of the total variance of the price-dividend innovations. The model is then able to produce a limited amount of contemporaneous correlation between the long-run productivity news and price-dividend news, consistent with that observed in the data.[28]

4 Inspecting the mechanism

What is driving the results obtained in the previous section? Preferences or long-run risk? How does the long-run productivity risk interact with adjustment costs? In this section I perturb my benchmark calibration in order to study the role of the IES, the RRA, and the adjustment costs and answer the questions above. I show that the dynamics of the quantities are mainly affected by the IES coefficient and that the RRA is, conversely, very important for the mean of the risk-free rate and the equity premium. I also show that when the adjustment costs are not present and the price of capital is constant, the model is no longer able to replicate the stock market behavior.

4.1 The role of IES

The IES is a key parameter in this economy and in all the long-run risk models in general. Bansal and Yaron (2004) show that an IES greater than one is important to both maintain the unconditional mean of the interest rate at a low level and replicate the predictability of the excess returns in the presence of time-varying aggregate volatility. In this model I abstract from the presence of stochastic volatility, and for this reason my excess returns do not display any significant level of predictability. Still, the IES plays a crucial role in determining the response of endogenous quantities and, hence, the unconditional mean of

[28] These results are not directly comparable in absolute terms with those obtained in section 2. The reason for this is that here I work with monthly simulated data, while the empirical evidence is based on annual data. While it is possible to time-aggregate the simulated price-dividend ratio to an annual frequency and recover its own annual innovations, it is not possible to isolate annual long-run news. Time aggregation, in fact, generates a non linear and non-separable link between the annual growth of productivity and both the short-run and long-run productivity news.

the excess returns. In particular, I solve the planner's problem for the case in which IES $\in \{.8; 1; 2\}$. In panel A of Table 4, I detail the entire calibration and I show that all the other parameters are the same as those I used in the benchmark calibration.

Before analyzing the moments produced by the model, it is important to study the policy functions of the representative agent. In Fig. 10, I plot the optimal consumption-productivity ratio, C_t/A_{t-1}, and the optimal investment-productivity ratio, I_t/A_{t-1}, as a function of the exogenous states. The panels on the left describe consumption, while the panels on the right reflect investment. The upper panels refer to an economy in which the IES is 2; the middle panels reflects the results when the IES is 1; and the lower two panels show the results for an IES of .8. In each panel, there are two curves: the solid curve shows the slope of the given policy function with respect to variations in the long-run component, while the dashed curve shows the slope of the policy function with respect to the short-run shock. Capital is fixed at its steady-state value.

For every IES value considered in the figure, the agent finds it optimal to respond to positive short-run shocks by increasing consumption and investment simultaneously (all the dashed lines are upward sloping). The behavior of consumption and investment differs, however, with respect to the long-run component. As mentioned before, this is due to the interaction between the IES, the income effect, and the substitution effect, produced by long-run news.

Assume that a positive shock to the long-run productivity materializes. The continuation value then increases consistently. When the IES is low, current consumption and continuation value are complements: the income effect dominates, and the agent finds it optimal to increase consumption by reducing investment. This is the reason that the consumption policy function is upward-sloping with respect to the long-run risk, x.

When the IES is high, the degree of substitutability between continuation value and current consumption is high. As we have already seen, in this case the substitution effect dominates, and the agent finds it optimal to decrease consumption in order to increase investment and accumulate capital. The consumption policy function is downward-sloping with respect to the long-run risk, x.

Finally, when the IES is equal to one—the log-case—the demand for new capital tends not to move after long-run shocks. This is indeed a special case in which the long-run income effect and the long-run substitution effect perfectly offset each other. Under this calibration of the IES, which was the main focus of Tallarini (2000), the long-run productivity risk does not have any further impact on the investment and returns dynamics.

In order to better analyze the dynamics implied by the model when the IES is less than one, in Figs. 11 and 12 I plot the impulse response functions of the quantities and the market returns, respectively, when the IES is .8. The response of quantities and prices to the short-run shock are very similar to those obtained under the benchmark calibration with an IES greater than one. The responses with respect to the long-run component, in contrast, are very different because of the prevalence of the income effect. In particular, a positive long-run shock induces a drop in investment and negative market returns.

In Table 4 I report a list of relevant moments produced by simulating the model for the three values of the IES considered above. The annualized volatility of quarterly consumption growth is about 1.35%−1.4% when the IES differs from one, and it decreases to about 1.2% when the IES is equal to one. This is because when the IES is equal to one, the long-run shock does not have any immediate impact on consumption and investment. In panel A of Table 5, I report the volatility ratios produced by the model for consumption, output, and investment. The figures show that the IES has only a modest impact on volatility ratios.

In panel B of Table 5, I show contemporaneous correlations of quarterly growth rates of consumption, investment, output, and market returns. Let us focus now on the quantities. As already mentioned, the data show that consumption growth has a correlation of about 83% with output and 43% with investment. These correlations are typically significantly higher in standard RBC models that ignore the presence of long-run uncertainty in productivity. In standard models, in fact, all the variables move together in response to short-run news. This is exactly what happens in this model when the IES equals one. Even if there is long-run uncertainty, quantities do not react to long-run news because the income effect and the substitution effect offset each other. Hence, the correlations of the quantities are explained by their co-movements with short-run shocks. When the IES varies from one, the model instead produces lower contemporaneous correlations, since the long-run news moves consumption and investment in opposite directions, yielding a negative effect on their covariance.

On the other hand, a higher IES helps us to obtain higher excess returns and lower risk-free rates (Table 4, panel B). When the IES equals .8, the implied equity premium predicted by the model is zero, while the risk-free rate is about 4%. When the IES equals 2, the risk-free rate declines to 1.2% and the excess return is 4.5%. These results show that the market return is not particularly sensitive to the IES and, for this reason, a reduction in the risk-free rate produces almost a one-to-one increase in the excess return.

It is helpful for us to understand the negative link between the IES and the equity premium from an economic perspective (Fig. 12). When the IES is less than one, the stock market price actually falls after good news regarding long-run productivity. In this case, stocks provide insurance with respect to the long-run shock while producing risky returns in the short-run. Overall, the market premium is almost zero.

4.2 The role of RRA and the CRRA case

In the previous section we have seen that the responses of prices and quantities to productivity shocks depend crucially on the IES. What is the role of the RRA? We must first understand how the previous results change when the RRA decreases. In the third and the fourth columns in Table 6, panel B, I list the relevant moments produced by the model when the IES is equal to 2, but the RRA is decreased, respectively, to 10 and .5. The last case is particularly interesting because preferences collapse to the standard CRRA case.

The moments related to quantities are not significantly altered. Consumption has the same volatility and persistence and basically the same correlation with dividends, investment, and capital returns. The volatility ratios for consumption, output, and investment are nearly the same.

What is quite different is the level of the risk-free rate and the equity premium. When the RRA is lower, the precautionary savings incentives are less strong. The risk-free bond thus has to pay a higher interest rate at the equilibrium. At the same time, the stochastic discount factor is less sensitive to consumption growth shocks and is less volatile. Both the market price of risk and the total market return decline. The implied equity premium falls and becomes just .01% in the CRRA case. Furthermore, when IES = 1/RRA, the stochastic discount factor is no longer sensitive to shocks in the continuation value of the representative agent, and the only source of risk is the total change in consumption growth.

In Figs. A1 and A2, reported in the Technical Appendix, I show the impulse response function of quantities and prices when RRA = 1/IES = .5. Consumption growth and stock market returns respond to the short-run and long-run shocks, consistent with that seen for the benchmark calibration. Notice, however, that with CRRA preferences, after a positive shock to the long-run component both the market returns and the stochastic discount factor increase.[29] This time, stocks provide insurance against long-run shocks and for this reason pay a much lower equity premium.

[29]In the CRRA case, the stochastic discount factor is simply $M_{t+1} = (C_{t+1}/C_t)^{-\gamma}$. Since consumption growth falls after a positive long-run shock, the discount factor increases.

Finally, in the last column of Table 6 I report the moments produced by the model in the CRRA case, in which the IES is calibrated to .8. The reader can easily compare these results with those obtained in the second column of Table 4, panel B, where the IES is .8 and the RRA coefficient is 30. Once again, keeping the IES fixed implies that the moments concerning the volatility and the the quantities co-movements are not significantly altered. In Fig. A3 in the Technical Appendix, I show the quantities responses to the productivity shocks when RRA = 1/IES = 1.25. The impulse responses are very similar to that already seen in Fig. 11. Fig. A4 shows the response of asset prices in this last case. The stock is risky with respect to short-run shocks but provides insurance with respect to long-run productivity news. Under this specific calibration, with RRA = 1.25, the total equity premium is positive but very small. All these results confirm Tallarini's (2000) conclusion that the RRA has only second-order effects on the time-series properties of the macroeconomic quantities.

4.3 The role of adjustment costs

In Table 7 I compare the results obtained under the benchmark calibration with those produced by an economy in which there are no adjustment costs. All the preferences and productivity parameters are kept constant; the only parameter that is changed is τ, which measures the elasticity of the supply curve for new capital. The most important thing to notice is that the adjustment cost elasticity has an substantial impact both on quantities and on prices.

When the adjustment costs are not present, the representative agent can enhance consumption smoothing. The annualized volatility of consumption growth then drops to .9%. Investment and output, however, become much more volatile, as observed in the data. Removing the adjustment costs allows the agent to find it more convenient to intensively and persistently vary capital subsequent to both short-run and long-run shocks. As a consequence, the autocorrelation of consumption growth becomes too high.

The interaction between the IES and the elasticity of the adjustment costs function is crucial to an understanding of the co-movements of macroeconomic variables. In Fig. 13 I show that when the supply of new capital is flat and the IES is high enough, the substitution effect dominates the income effect also in the short run. When the adjustment of capital is costless, the opportunity cost of consumption increases significantly, even after short-run shocks. If the agent has a high elasticity of substitution, she will find it optimal to decrease consumption and also invest more after the realization of a transitory positive shock to productivity growth. Overall, this implies a counterfactual negative correlation between consumption and investment as well as between consumption and returns.

Without adjustment costs, the implications for prices deteriorate also. In this case, the price of capital is constant and equal to one (see the lower panels of Fig. 14). Capital appreciation is an important means by which to generate high equity premia in the long-run risk models, but in this general equilibrium set-up prices do not move. Returns are then totally driven by small adjustments in dividends and have much less exposure to productivity risk. The implied equity premium is about four times smaller.

All of these results suggest that economies with long-run risk and high IES cannot produce reasonable results with a perfectly flexible supply of new capital. On the one hand, the adjustment costs tend to make investment too smooth, but on the other, they are successfully able to reproduce both the positive link between long-run productivity and asset prices news we observe in the data, and the correct co-movements between the main components of output. To incorporate and motivate a real friction able to generate the correct amount of volatility for both real investment and stock prices simultaneously is an interesting economic challenge that I leave for future research.

5 Conclusion

This study has examined the intertemporal distribution of productivity risk. Focusing on post-war US data, I have shown that the conditional mean of productivity growth is time-varying and extremely persistent. This generates uncertainty about the long-run perspectives of economic growth and strongly affects asset prices. I developed a production-based asset pricing model featuring long-run uncertainty, convex adjustment costs, and recursive preferences à la Epstein-Zin. The model accounts for several key features of both prices and macroeconomic quantities, including consumption, investment, and output. I have also provided a detailed examination of the role of the intertemporal elasticity of substitution, relative risk aversion, and adjustment costs in this type of economy.

There are a number of directions in which this work could be fruitfully extended. First, in this paper I assume a constant supply of labor in order to keep the analysis as simple as possible. However, long-run labor fluctuations can have an important impact on human wealth returns (see, for example, Lustig and Van Nieuwerburgh (2006)) and should be taken into account in future research. In particular, it would be especially useful to study the implications of the model for the relationship between labor income, aggregate variables, and capital returns.

Second, as we have seen, the model predicts excessively smooth investment growth rates because of the presence of convex adjustment costs. Boldrin, Christiano, and Fisher (2001) show that a model with habits and real rigidities can partially solve this problem and simultaneously produce volatile returns and investment. It would be valuable to reexamine the relation between investment and returns in a model with long-run risk, Epstein-Zin preferences, and predetermined investment.

Finally, it would also be interesting to study the implications of time-varying volatility for productivity growth. This could help us to better understand the dynamics of aggregate excess returns.

Appendix A. Data.

To compute the spectral densities of consumption, I use data on real per capita consumption from Campbell (2003). Data comprise quarterly observations on private consumption of nondurables and services from 1970:Q1 to 1998:Q4. When I produce joint statistics about consumption and investment, I use the following quarterly data from Bureau of Economic Analysis (BEA):

(1) Real Private Fixed Investment, seasonally adjusted in billions of chained 2000 dollars, from 1947:Q1 to 2005:Q2, and

(2) Real Personal Consumption of Nondurables and Services, seasonally adjusted in billions of chained 2000 dollars, from 1947:Q1 to 2005:Q2.

I compute output as the sum of consumption and investment. I exclude government expenditures. All data are per-capita.

I measure inflation using the CPI index, from 1947:Q1 to 2005:Q2. In order to take into account seasonality, I smooth inflation by computing in every quarter the average of the prior four quarterly inflation rates.

The risk-free rate is measured by the 3-month T-Bill return minus the realized inflation, from 1947:Q1 to 2003:Q2.

Data on market returns (from 1947:Q2 to 2003:Q2), annualized price-dividend ratio (from 1947:Q1 to 2002:Q4), and dividends (from 1947:Q1 to 2003:Q3) are obtained from CRSP.

The time-series for annual productivity is from the Burau of Labor Statistics; the sample spans 1948–2003. I use a multifactor productivity index that takes into account capital accumulation. In particular, the index adopted measures the value-added output per combined unit of labor and capital input in private business and private nonfarm business (as Kaltenbrunner and Lochstoer (2006)).

References

[1] Abel, Andrew, 1980. Empirical investment Equations: An Integrative Framework, Carnegie-Rochester Conference Series on Public Policy, 12, 39–91.

[2] Ai, Hengjie, 2007. Information Quality and Long-run Risk: Asset Pricing and Welfare Implications. Working paper, the Fuqua School of Business, Duke University.

[3] Attanasio, O. P., and A. Vissing-Jorgensen, 2003. Stock Market Partecipation, Intertemporal Substitution and Risk Aversion. American Economic Review Papers and Proceedings, May 2003.

[4] Backus, David, Bryan Routledge, and Stan Zin, 2007. Asset prices in business cycle analysis. Working paper, The Tepper School of Business.

[5] Balvers, Ronald J., and Dayong Huang, 2007. Productivity-Based Asset Pricing: Theory and Evidence. Journal of Financial Economics, 86(2), 405–445.

[6] Bansal, Ravi, A. R. Gallant, and G. Tauchen, 2007. Rational Pessimism and Exuberance. Review of Economic Studies, October 2007, Vol. 74, Issue 4.

[7] -, -, Dana Kiku, and Amir Yaron, 2006. Risk For the long-run: Estimation and Inference. Working paper, the Wharton School.

[8] -, -, and Amir Yaron, 2004. Risk for the long-run: a potential resolution of asset pricing puzzles. The Journal of Finance, Vol. LIX, No. 4, August 2004, pages 1481–1509.

[9] -, - and Ivan Shaliastovich, 2008. Learning, Long Run Risks, and Asset Price Jumps. Working paper, The Fuqua School of Business.

[10] Robert B. Barsky, and J. Bradford DeLong, 1993. Why Does the Stock Market Fluctuate? Quarterly Journal of Economics 108: 2 (May), 291–312.

[11] Beaudry, Paul, and Frank Portier, 2006. Stock prices, news, and economic fluctuations. American Economic Review 96, 1293–1307.

[12] Boldrin, Michele, Lawrence J. Christiano, and Jonas D. I. Fisher, 1995. Asset pricing Lessons for modeling Business Cycles. NBER Working Paper 5262.

[13] Boldrin, Michele, Lawrence J. Christiano, and Jonas D. I. Fisher, 2001. Habit Persistence, Asset Returns, and the Business Cycle. American Economic Review, 91(1), 149–166.

[14] Campanale, Claudio, Rui Castro, and Gian Luca Clementi, 2006. Asset pricing in a general equilibrium production economy with Chew-Dekel Preferences.

[15] Campbell, J., 2003. Consumption-based asset pricing. Handbook of the Economics of Finance. George Constantinides, Milton Harris, and Rene Stulz, eds. North-Holland, Amsterdam.

[16] Cochrane, John, 1996. A Cross-Sectional Test of an Investment Based Asset Pricing Model. Journal of Political Economy, 104, 572–621.

[17] -, -, 2005. Financial markets and the real economy. Foundations and Trends in Finance, 2005, 1(1), pp. 1–101.

[18] Colacito, Riccardo, and Mariano M. Croce, 2007. Risk for the long-run and the real exchange rate. Working paper, Kenan-Flagler School of Business, University of North Carolina at Chapel Hill.

[19] Croce, Mariano M., 2006. Welfare Costs and Long-Run Consumption Risk in a production Economy. Working paper, New York University.

[20] Croce, Mariano M., Martin Lettau, and Sydney C. Ludvigson, 2005. Investor Information, Long-Run Risk, and the Duration of Risky Flows. Working paper, New York University.

[21] Eberly, Janice, 1997. International Evidence on investment and Fundamentals. European Economic Review, 41(6), 1055–78.

[22] Edge, Rochelle, Thomas Laubach, and John C. Williams, 2007. Learning and Shifts in Long-Run Productivity Growth. Journal of Monetary Economics, 54, 2421-2438.

[23] Epstein, Larry, and Stan Zin, 1989. Substitution Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. Econometrica, 57, 937–965.

[24] Fama, Eugene F., 1981. Stock Returns, Real Activity, Inflation, and Money. American Economic Review, 71(4), 545–65.

[25] Fama, Eugene F., 1990. Stock Returns, Expected Returns, and the real activity. The Journal of Finance, 45(4), 1089–1108.

[26] -,-, and K. R. French, 2001. The Equity Premium. Working Paper.

[27] Fernandez-Villaverde, J., and J. Rubio-Ramirez, 2006. Estimating Macroeconomic Models: A Likelihood Approach. Review of Economic Studies, 74(4), 1059–1087.

[28] Gul, F., 1991: A Theory of Disappointment Aversion Econometrica, 59, 667-686.

[29] Hansen, Lars P., John C. Heaton and Nan Li, 2005. Consumption strikes back? Measuring Long-Run Risk. University of Chicago.

[30] -, -, Thomas J. Sargent, 1995. Discounted linear exponential quadratic gaussian control. University of Chicago.

[31] -, -, Thomas J. Sargent, 2006. Fragile beliefs and the price of model uncertainty. Working paper, University of Chicago.

[32] Harvey, C. R., 1986. Recovering expectations of consumption growth from an equilibrium model of the term structure of interest rates. Ph.D. thesis, The University of Chicago.

[33] Jermann, Urban J., 1998. Asset pricing in production economies. Journal of Monetary Economics, 41(2), 257–275.

[34] Kaltenbrunner, G., and L. Lochstoer, 2006. LongRun Risk through Consumption Smoothing. Working Paper, London Business School.

[35] Kydland, Finn E., and Edward C. Prescott, 1982. Time to build and aggregate fluctuations. Econometrica, 50(6), pp.1345–1370.

[36] Lettau, Martin, Sydney Ludvigson, and Jessica Wachter, 2006. The Declining Equity Premium: What Role Does Macroeconomic Risk Play? Review of Financial Studies, forthcoming.

[37] Lettau, Martin, and Harald Uhlig, 2000. Can Habit Formation be Reconciled with Business Cycle Facts? Review of Economic Dynamics 3(l) , 79–99.

[38] Lustig, Hanno, and Stijn Van Nieuwerburgh, 2006. The Returns on Human Wealth: Good News on Wall Street is Bad News on Main Street. Review of Financial Studies, forthcoming.

[39] Malloy Christopher, Tobias J. Moskowitz, Annette Vissing-Jrgensen, 2006. Long-Run Stockholder Consumption Risk and Asset Returns. Working paper.

[40] Rouwenhorst, K. G., 1995. Asset pricing implications of equilibrium business cycle models. Frontiers of Business Cycle Research. Princeton University Press, princeton, NJ, 294–330.

[41] Sargent, Thomas J., and Lars Ljungqvist, 2004. Recursive Macroeconomics Theory. The MIT Press.

[42] Tallarini, Thomas D. Jr., 2000. Risk-sensitive real business cycles. Journal of Monetary Economics, 45, 507–532.

[43] Uhlig, Harald, 2007. Leisure, Growth and Long Run Risk. Working paper.

[44] Weil, Philippe, 1989. The Equity Premium Puzzle and the Risk-Free Rate Puzzle. Journal of Monetary Economics, 24(3), 401–421.

Table 1

Predictability in Productivity Growth

$$\Delta a_{t+1} = \mu(1-\rho) + \rho\Delta a_t - b\epsilon_{a,t} + \sigma\epsilon_{a,t+1}$$

ρ	b	$R^2_{\Delta a}$	LR	ρ_x	R^2_x	ρ_σ	b_σ
			No GARCH(1,1)				
.965	1.191	.250	10.801	.835	.720	-	-
(.061)	(.149)	-	-	(.073)	-	-	-
[.000]	[.000]	-	[.004]	[.000]	-	-	-
			With GARCH(1,1)				
.892	.965	.106	7.74	.894	.954	.520	.420
(.043)	(.014)	-	-	(.073)	-	(.156)	(.142)
[.000]	[.000]	-	[.10]	[.000]	-	[.003]	[.004]

In the first three columns I report, respectively, the estimates of the AR root, the MA root, and the R^2 of the regression for Δa_{t+1}. Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p-values. In the fourth column I report the likelihood ratio of the estimated model versus the following model:

$$\Delta a_{t+1} = \mu + \sigma\epsilon_{a,t+1}.$$

In columns 5 and 6 I report the estimated AR coefficient and the R^2 of the following regression:

$$x_t = \mu(1-\rho_x) + \rho_x x_{t-1} + \epsilon_{x,t}$$

where x_t is the ARMA(1,1) forecast of Δa_{t+1}. In the last two columns I report the estimates of the AR and the MA root of the conditional volatility that is assumed to follow a GARCH(1,1). In the lower portion of the table I report the *joint* estimates of both the ARMA(1,1) and the GARCH(1,1) coefficients. Data are annual (1948–2003).

Table 2

Long-Run Productivity Risk and P/D Ratio News

$\epsilon_{pd,t} = \beta_0 + \beta_1 \epsilon_{x,t} + \epsilon_t$

β_1	F-stat	$R^2_{\Delta a}$
	No GARCH(1,1)	
11.107	6.426	.112
(5.222)	-	-
[.038]	[.014]	-
	With GARCH(1,1)	
22.967	3.581	.065
(12.135)	-	-
[.064]	[.064]	-

The variables ϵ_x and ϵ_{pd} are the residuals of the following regressions:

$$\begin{aligned} p_t - d_t &= \beta_{0,pd} - \rho_{pd}(p_{t-1} - d_{t-1}) + \epsilon_{pd,t} \\ x_t &= \beta_{0,x} - \rho_x x_{t-1} + \epsilon_{x,t} \end{aligned}$$

where x_t is the ARMA(1,1) forecast of the productivity growth rate, Δa_{t+1}. In the upper portion of the table I report the OLS estimates obtained using as the measure of productivity long-run risk the forecast provided by the ARMA(1,1) model not corrected for GARCH effects. In the lower portion of the table I report the forecast obtained by controlling for GARCH effects. Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p-values. Data are annual; the adjusted sample is for the years 1951–2003.

Table 3

Production Economy: Benchmark Calibration

Panel A: Calibration

μ	σ	σ_x	ρ	δ_k	α	τ	δ^{12}	γ	Ψ
.165%	.60%	5.5%σ	.98	.5%	.33	.98	.98	30	2

Panel B: General Statistics

	Mean		St. Dev.		ACF$_1$	
	Model	Data	Model	Data	Model	Data
C_{t+1}/C_t	2.0	2.0	1.4	1.4	.35	.32
I_{t+1}/I_t	2.0	2.4	1.8	7.0	.30	.5
I_t/Y_t	25	20	0.5	2.0	.98	.97
$r^d_{t+1} - r^f_t$	4.8	6.0	4.0	16.0	.05	.01
r^f_t	1.2	1.1	.34	1.5	.97	.90
$p_t - d_t$	3.25	3.2	.11	.35	.98	.96

Panel C: Cross-Correlations

DATA

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.85		
ΔC_t	.83	.43	
r^d_t	.12	.02	.18

MODEL

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.75		
ΔC_t	.94	.51	
r^d_t	.50	.76	.29

Panel A shows the benchmark monthly calibration. In panels B and C, all the statistics are annualized and multiplied by 100. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. Data are quarterly (1947:Q1–2003:Q2); more details can be found in Appendix A. In this table $\bar{l} = .82$.

Table 4
The Role of IES (Ψ)

Panel A: Calibration

μ	σ	σ_x	ρ	δ_k	α	τ	δ^{12}	γ
.165%	.60%	5.5%σ	.98	.5%	.33	.98	.98	30

Panel B: Statistics

	DATA	$\Psi = .8$	$\Psi = 1$	$\Psi = 2$
$\mathrm{E}[C_{t+1}/C_t]$	02.00	01.91	01.92	01.93
$\mathrm{std}[C_{t+1}/C_t]$	01.30	01.34	01.22	01.40
$\mathrm{ACF}_1[C_{t+1}/C_t]$	32.00	35.02	36.01	35.00
$\rho_{\Delta c,\Delta d}$	10.00	97.00	98.31	87.00
$\mathrm{std}[D_{t+1}/D_t]$	12.00	04.06	04.04	04.06
$\mathrm{E}[r^d_{t+1} - r^f_t]$	06.00	-0.12	01.40	04.80
$\mathrm{E}[r^f_t]$	01.10	04.04	03.12	01.20
$\mathrm{std}[r^d_{t+1} - r^f_t]$	16.40	02.50	02.40	04.00
$\mathrm{std}[r^f_t]$	01.35	00.74	00.62	00.34
$\mathrm{std}[p_t - d_t]$	40.00	14.70	13.10	11.00
$\mathrm{ACF}_1[p_t - d_t]$	98.00	98.00	98.00	98.00

Panel A shows the benchmark monthly calibration. In panel B, all the statistics are annualized and multiplied by 100. Data are quarterly (1947:Q1–2003:Q2); more details can be found in Appendix A. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table, $\bar{l} = .82$.

Table 5

The Role of IES (Ψ) in Quarterly Second Moments

Panel A: Volatility Ratios

	$\sigma_{\Delta Y}/\sigma_{\Delta C}$	$\sigma_{\Delta I}/\sigma_{\Delta C}$
DATA	1.5	5
$\Psi = 2$	1.01	1.33
$\Psi = 1$	1.00	0.99
$\Psi = .8$	0.98	1.01

Panel B: Cross-Correlations

DATA

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.85		
ΔC_t	.83	.43	
r_t^d	.12	.02	.18

$\Psi = 2$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.75		
ΔC_t	.94	.51	
r_t^d	.50	.76	.29

$\Psi = 1$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.99		
ΔC_t	.99	.99	
r_t^d	.70	.70	.70

$\Psi = .8$

	ΔY_t	ΔI_t	ΔC_t
ΔI_t	.93		
ΔC_t	.99	.88	
r_t^d	.65	.69	.61

Panel A reports contemporaneous cross-correlations between quantities for different calibrations of the IES (Ψ). All the other parameters of the model are calibrated as in Table 4. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table, $\bar{l} = .82$. Panel B reports volatility ratios.

Table 6

The Role of the RRA (γ)

Panel A: Calibration

μ	σ	σ_x	ρ	δ_k	α	τ	δ^{12}
.165%	.60%	5.5σ	.98	.5%	.33	.98	.98

Panel B: Statistics

	DATA	IES=2 RRA=10	IES=2 RRA=.5	IES=.8 RRA=1/.8
$\mathrm{E}[C_{t+1}/C_t]$	02.00	01.93	01.92	01.92
$\mathrm{std}[C_{t+1}/C_t]$	01.30	01.40	01.41	01.34
$\mathrm{ACF}_1[C_{t+1}/C_t]$	32.00	35.00	35.00	35.90
$\rho_{\Delta c,\Delta d}$	10.00	87.01	82.03	97.03
$\mathrm{std}[D_{t+1}/D_t]$	12.00	04.68	05.20	03.74
$\mathrm{E}[r_{t+1} - r_t^f]$	06.00	01.74	00.01	00.13
$\mathrm{E}[r_t^f]$	01.10	01.29	02.93	04.36
$\mathrm{std}[r_{t+1} - r_t^f]$	16.40	03.71	03.74	02.50
$\mathrm{std}[r_t^f]$	01.35	00.36	00.36	00.72
$\mathrm{std}[p_t - d_t]$	40.00	14.07	16.04	12.10
$\mathrm{ACF}_1[p_t - d_t]$	98.00	98.50	98.50	98.90
$\rho_{\Delta c,\Delta i}$	43.00	47.00	44.00	88.00
$\rho_{\Delta c,\Delta r^d}$	12.00	26.20	24.00	62.00
$\sigma_{\Delta y}/\sigma_{\Delta c}$	1.50	1.00	.95	.98
$\sigma_{\Delta i}/\sigma_{\Delta c}$	5	1.33	1.34	1.01

Panel A shows the calibration of the unchanged parameters. In Panel B, all the statistics are annualized and multiplied by 100. Data are quarterly (1947:Q1–2003:Q2); more details can be found in Appendix A. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table, $\bar{l} = .82$.

TABLE 7

THE ROLE OF THE ADJUSTMENT COSTS (τ)

Panel A: Calibration

	μ	σ	σ_x	ρ	δ_k	α	δ^{12}	Ψ	γ
	.165%	.60%	5.5σ	.98	.5%	.33	.98	2	30

Panel B: Statistics

	DATA	$\tau = .98$ (Benchmark)	$\tau = +\infty$ (No Adj. Costs)
$\mathrm{E}[C_{t+1}/C_t]$	02.00	01.93	01.92
$\mathrm{std}[C_{t+1}/C_t]$	01.30	01.40	00.91
$\mathrm{ACF}_1[C_{t+1}/C_t]$	32.00	35.00	60.00
$\rho_{\Delta c,\Delta d}$	10.00	87.01	-28.61
$\mathrm{std}[D_{t+1}/D_t]$	12.00	04.06	04.24
$\mathrm{E}[r_{t+1} - r_t^f]$	06.00	04.80	01.23
$\mathrm{E}[r_t^f]$	01.10	01.20	01.85
$\mathrm{std}[r_{t+1} - r_t^f]$	16.40	04.00	00.28
$\mathrm{std}[r_t^f]$	01.35	00.34	00.32
$\mathrm{std}[p_t - d_t]$	40.00	11.00	11.81
$\mathrm{ACF}_1[p_t - d_t]$	98.00	98.00	97.98
$\rho_{\Delta c,\Delta i}$	43.00	51.00	-44.95
$\rho_{\Delta c,\Delta r^d}$	12.00	29.00	-76.00
$\sigma_{\Delta y}/\sigma_{\Delta c}$	1.50	1.01	1.53
$\sigma_{\Delta i}/\sigma_{\Delta c}$	5	1.33	5.82

Panel A shows the calibration of the unchanged parameters. In panel B, all the statistics are annualized and multiplied by 100. Data are quarterly (1947:Q1–2003:Q2); more details can be found in Appendix A. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to a quarterly frequency. In this table, $\bar{l} = .82$.

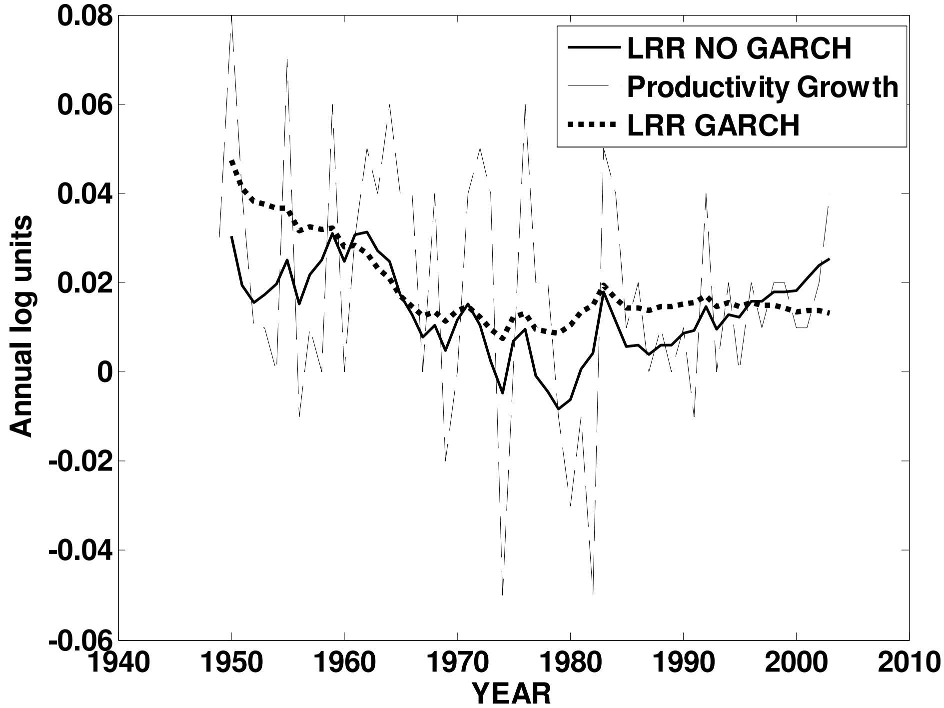

FIG. 1 – THE LONG-RUN PRODUCTIVITY RISK

The dashed line shows the annual growth rates of aggregate US productivity (source: BLS; sample: 1948–2003). The solid thick line shows the fit from an ARMA(1,1) model with homoskedastic innovations. The dotted line shows the estimated conditional mean of the productivity growth rate obtained by estimating the following ARMA(1,1)-GARCH(1,1) model:

$$\begin{aligned} \Delta a_{t+1} &= \mu(1-\rho) + \rho \Delta a_t - b\sigma_t \epsilon_{a,t} + \sigma_{t+1}\epsilon_{a,t+1} \\ \sigma^2_{t+1} &= \kappa + \rho_\sigma \sigma^2_t + b_\sigma \sigma^2_t \epsilon^2_{a,t} \\ \epsilon_{a,t+1} &\sim iidN(0,1), \end{aligned}$$

where Δa_{t+1} denotes the log growth rate of productivity between time t and $t+1$. All the parameter estimates are reported in Table 1.

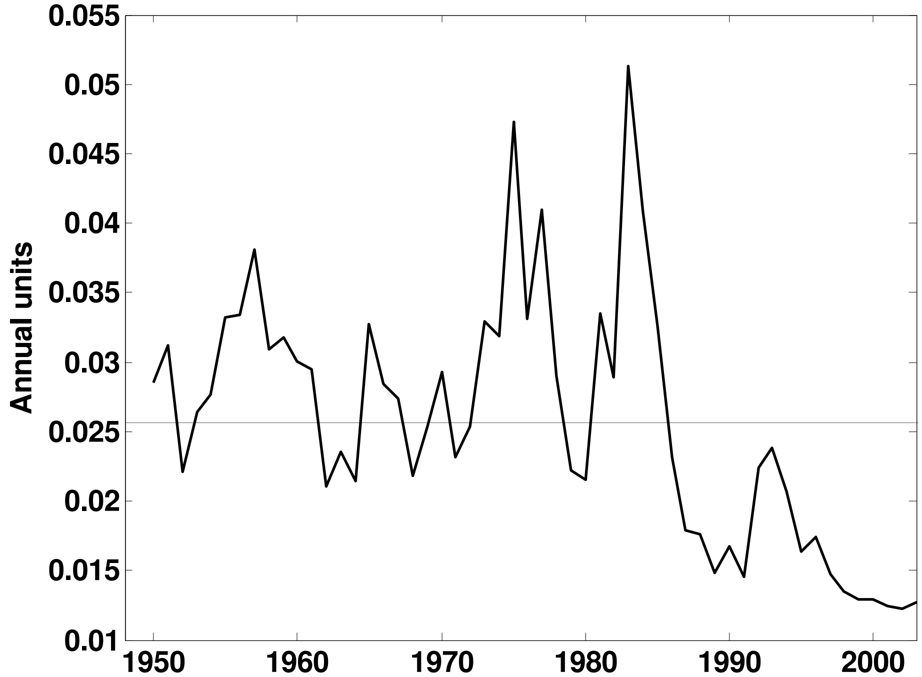

Fig. 2 – Volatility of Productivity Growth Rates

The thick line shows the fitted conditional volatility, $\sqrt{\widehat{\sigma_t^2}}$, of the annual growth rate of aggregate US productivity (source: BLS; sample: 1948–2003). The model employed is the following:

$$\begin{aligned}
\Delta a_{t+1} &= \mu(1-\rho) + \rho \Delta a_t - b\sigma_t \epsilon_{a,t} + \sigma_{t+1}\epsilon_{a,t+1} \\
\sigma_{t+1}^2 &= \kappa + \rho_\sigma \sigma_t^2 + b_\sigma \sigma_t^2 \epsilon_{a,t}^2 \\
\epsilon_{a,t+1} &\sim iidN(0,1),
\end{aligned}$$

where Δa_{t+1} denotes the log growth rate of productivity between time t and $t+1$. The thin flat line shows the unconditional volatility of the productivity growth rate in the same sample. All the parameter estimates are reported in Table 1.

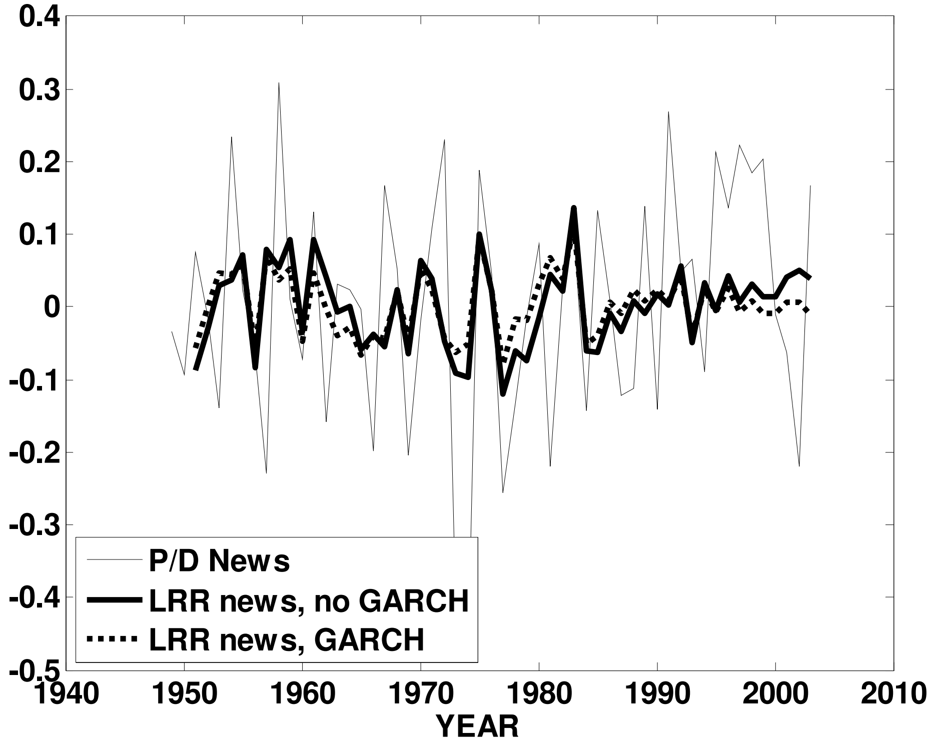

Fig. 3 – Long-Run Productivity News and P/D News

The thin solid line shows fitted annual P/D news, $\epsilon_{pd,t}$, computed according to the following model:

$$p_t - d_t \quad = \quad \beta_{0,pd} + \rho_{pd}(p_{t-1} - d_{t-1}) + \epsilon_{pd,t}.$$

Let x_t denote the annual conditional mean of productivity growth, and let $\epsilon_{x,t}$ denote the innovations to the following AR(1) representation of x_t:

$$x_t \quad = \quad \beta_{0,x} + \rho_x x_{t-1} + \epsilon_{x,t},$$

I estimate:

$$\epsilon_{pd,t} = \beta_0 + \beta_1 \epsilon_{x,t} + \epsilon_t.$$

The solid thick line shows $\widehat{\beta}_1 \widehat{\epsilon}_{x,t}$ when the productivity growth is assumed to follow an ARMA(1,1) with homoskedastic innovations. The dotted line shows $\widehat{\beta}_1 \widehat{\epsilon}_{x,t}$ when productivity growth follows an ARMA(1,1) and the conditional volatility of the productivity innovations follows a GARCH(1,1). The estimates for β_1 are reported in Table 2.

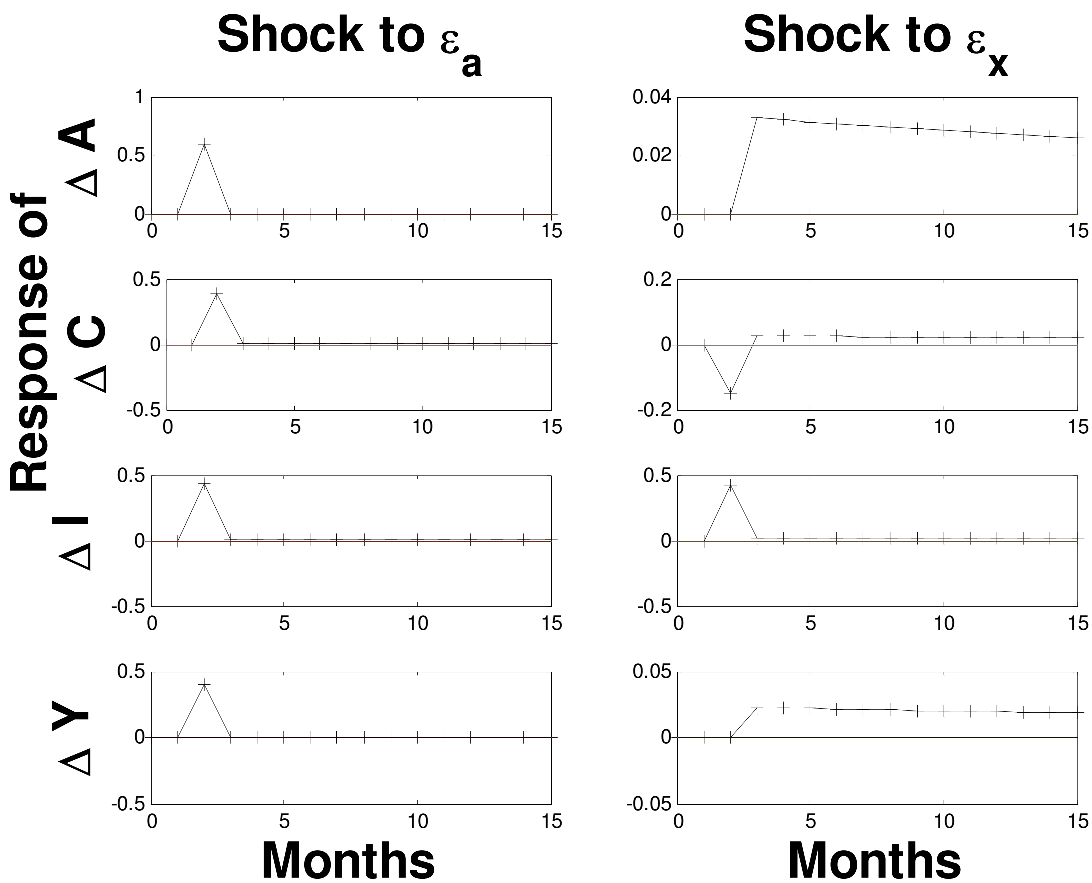

Fig. 4 – Quantities Impulse Response Functions (IES = 2)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 3. The policy functions are computed numerically.

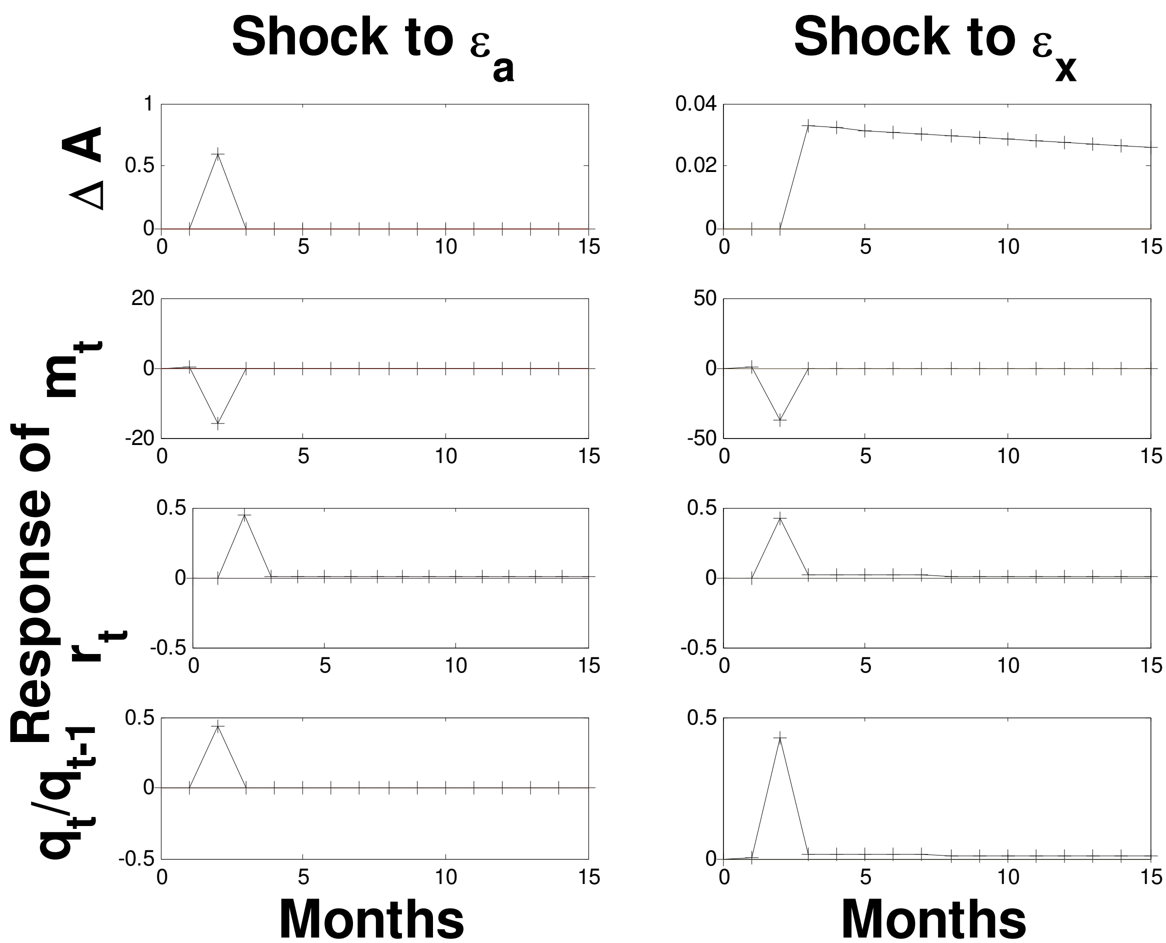

Fig. 5 – Prices Impulse Response Functions (IES = 2)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 3. The policy functions are computed numerically. Returns are not levered.

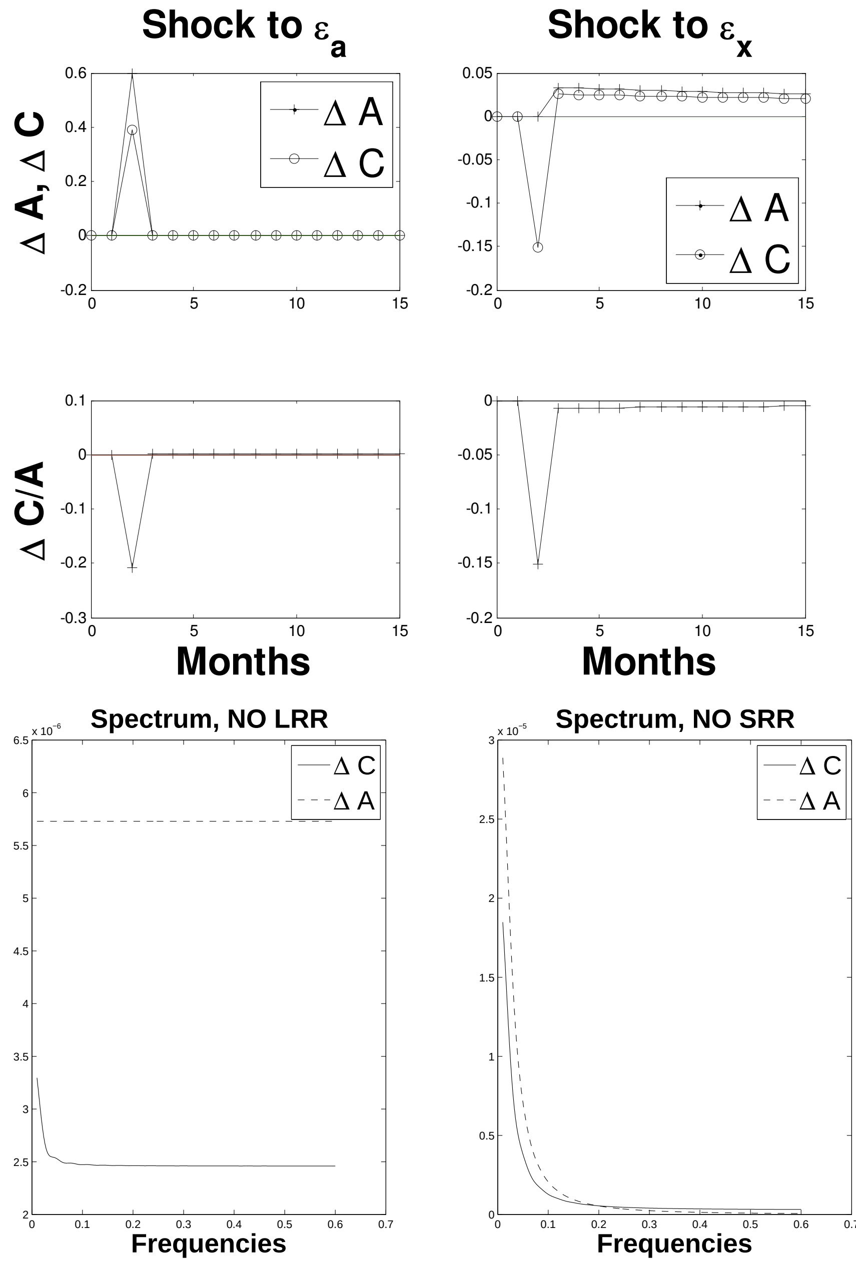

FIG. 6 – QUANTITIES IRFS AND SPECTRAL DENSITIES

This figure shows monthly log-deviations from the steady state of both consumption and productivity growth after the arrival of short-run news (top left panels) and long-run news (top right panels). Units are multiplied by 100. The lower two panels show the implied spectral densities. All the parameters are calibrated to the values reported in Table 3. The policy functions are computed numerically.

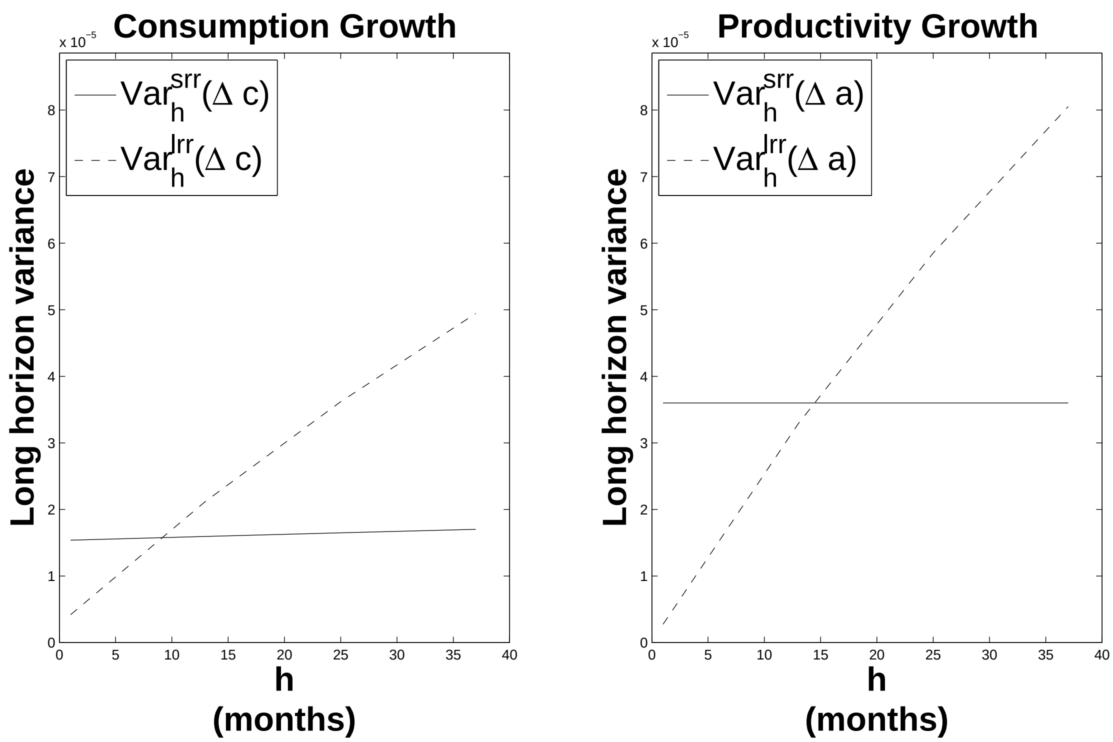

Fig. 7 – Long Horizon Variances (IES = 2)

This figure shows long-horizon variances for the monthly growth rates of consumption and productivity. In the right panel, $Var_h^{lrr}(\Delta a) \equiv Var\left[\sum_{k=1}^{h-1} x_{t+k-1}\right]/h$ and $Var_h^{srr}(\Delta a) \equiv Var\left[\sum_{k=1}^{h} \epsilon_{a,t+k}\right]/h$. I adopt analogous definitions for $Var_h^{lrr}(\Delta c)$ and $Var_h^{srr}(\Delta c)$. The productivity growth rate Δa is calibrated as in Table 3. The consumption growth rate Δc is obtained by solving the model with production according to the calibration reported in Table 3.

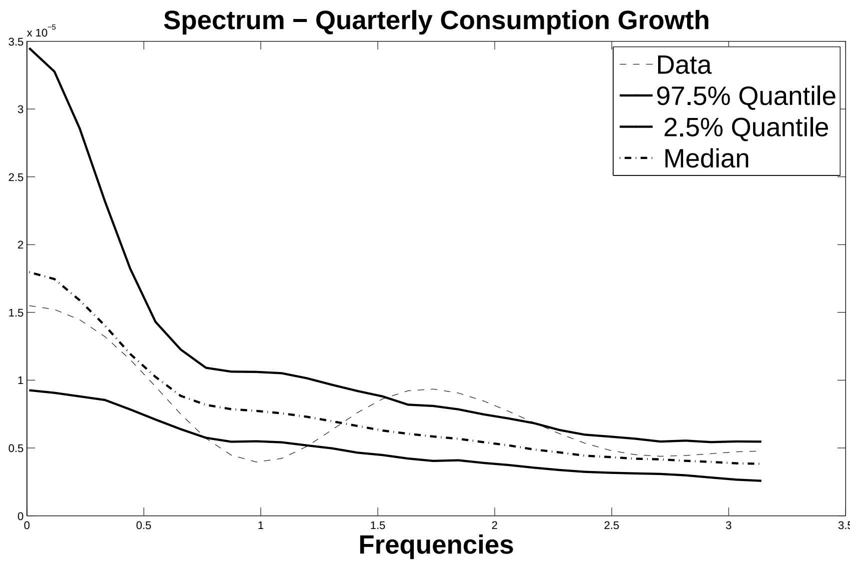

Fig. 8 – Spectrum of Quarterly Consumption

The dashed line shows the spectral density of US quarterly consumption of non-durables and services from 1970:Q1 to 1998:Q2. The solid lines show the percentiles 2.5 and 97.5 of the distribution of the spectral density of time-aggregated quarterly consumption growth generated by simulating samples of 600 months 500 times. The remaining line shows the median of the distribution of the spectrum generated by the model with production. The parameters adopted are reported in Table 3.

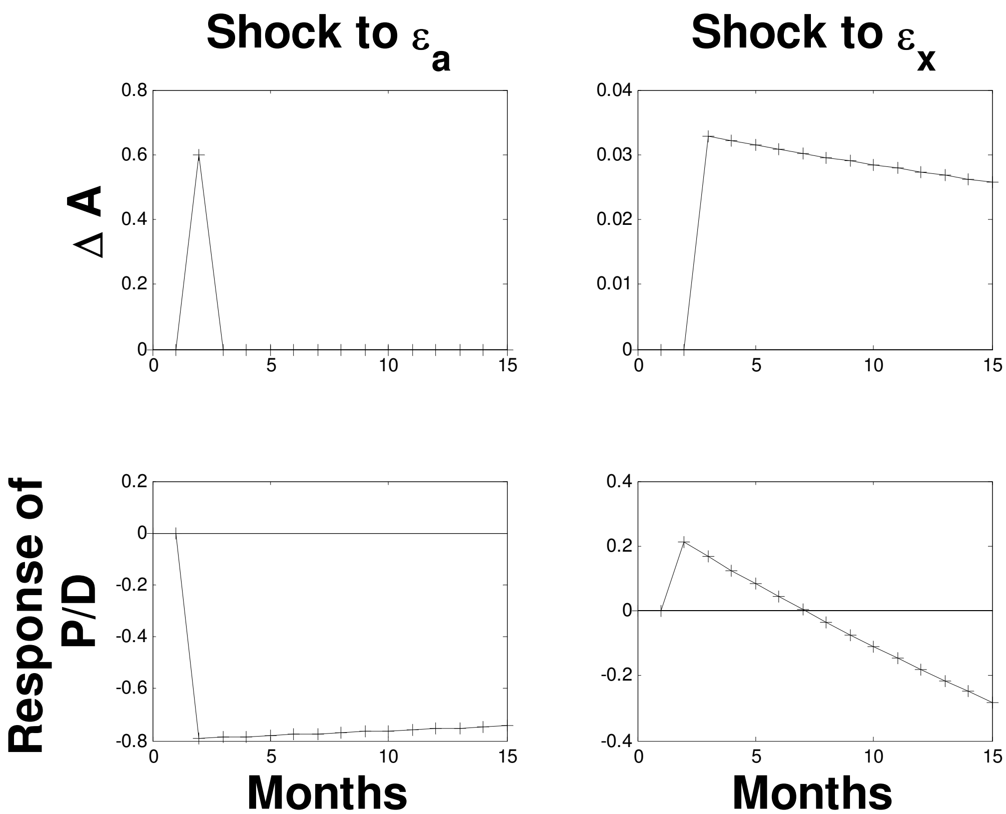

FIG. 9 – PRICE-DIVIDEND IMPULSE RESPONSE FUNCTIONS

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 3. The policy functions are computed numerically.

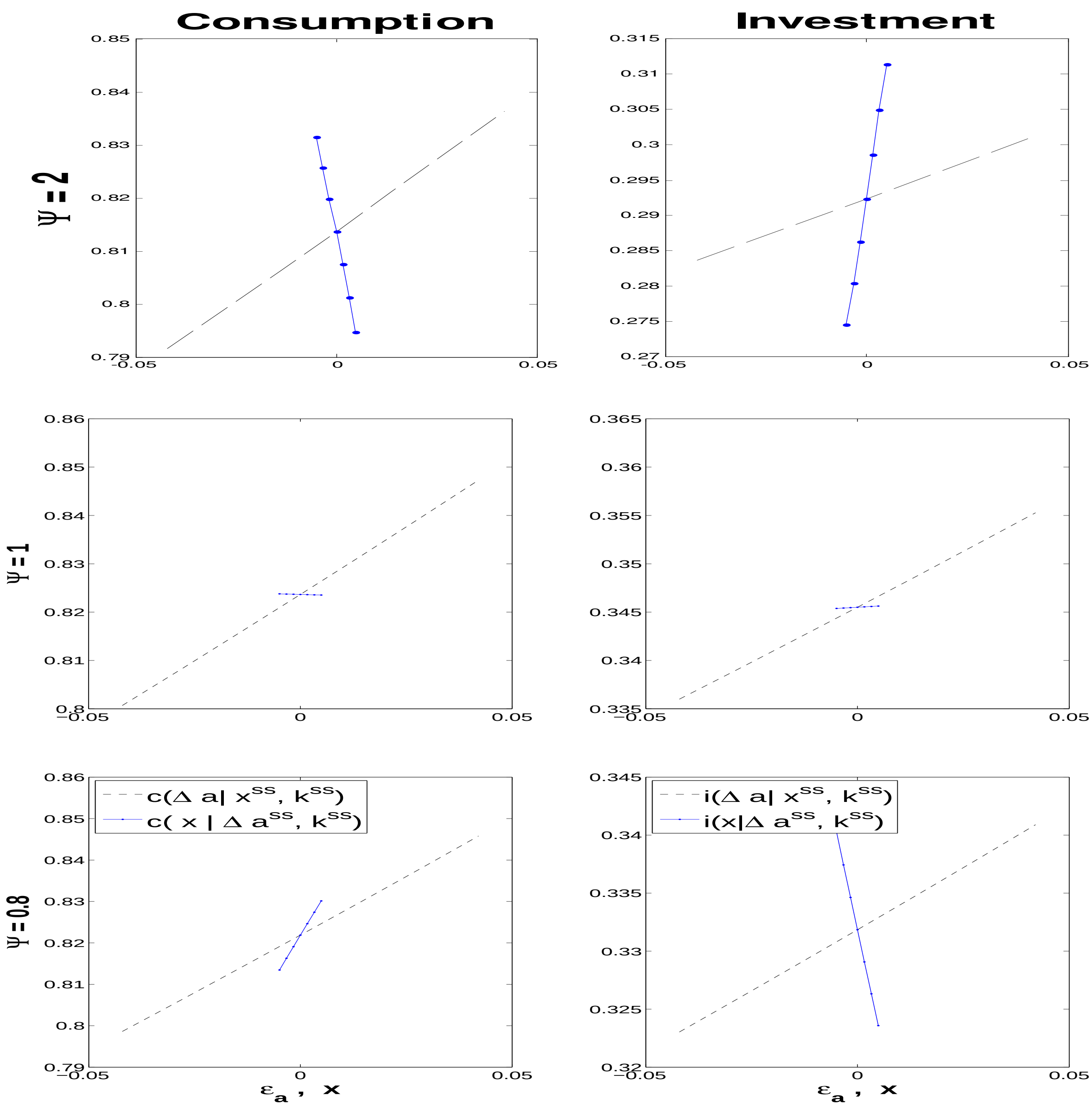

FIG. 10 – POLICY FUNCTIONS AND IES (Ψ)

All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically. In every panel, the solid line shows the policy function as a function of x. The dashed line shows the policy function as a function of ϵ_a. Note that in this figure, $i_t \equiv I_t/A_{t-1}$ and $c_t \equiv C_t/A_{t-1}$.

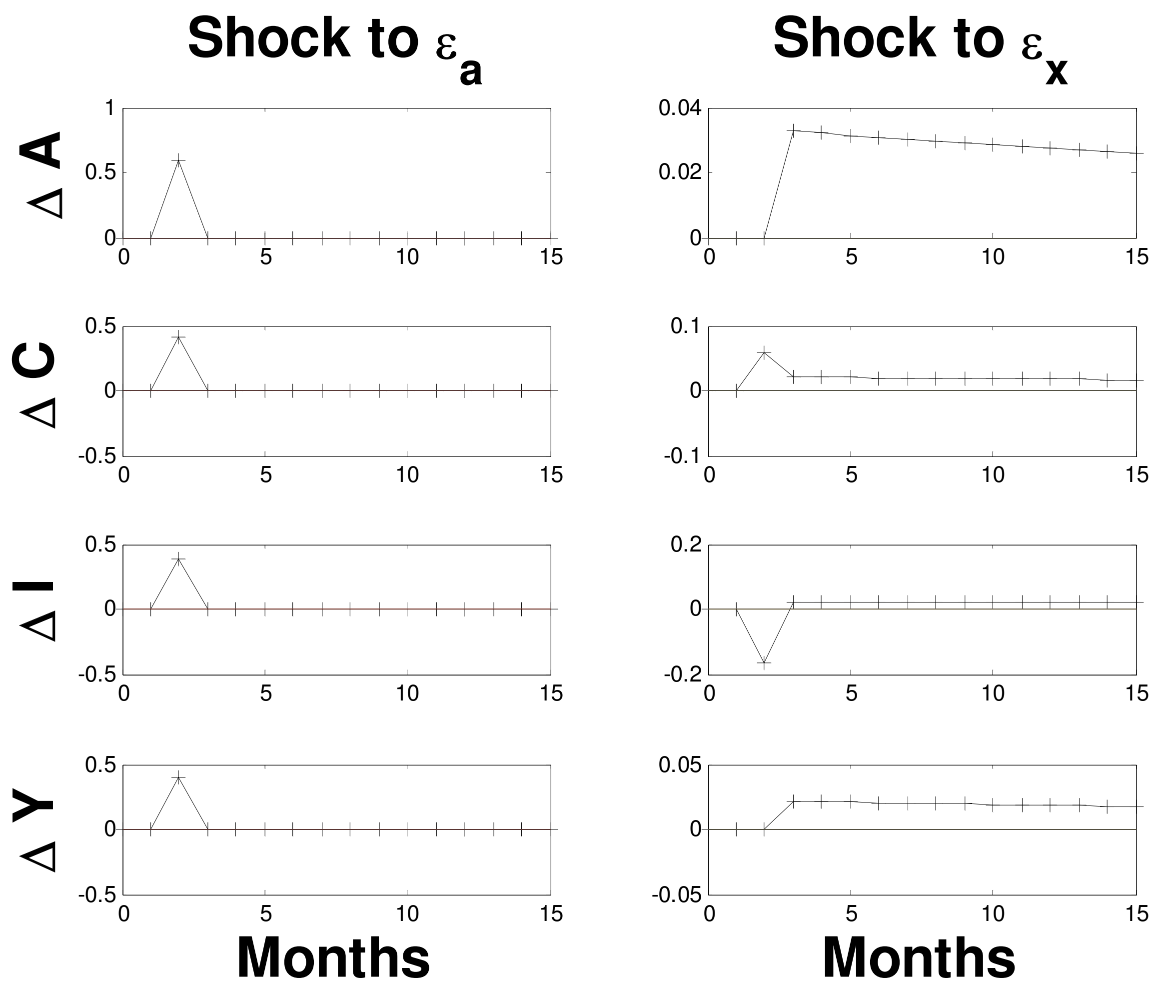

Fig. 11 – Quantities Impulse Response Functions (IES = .8)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically.

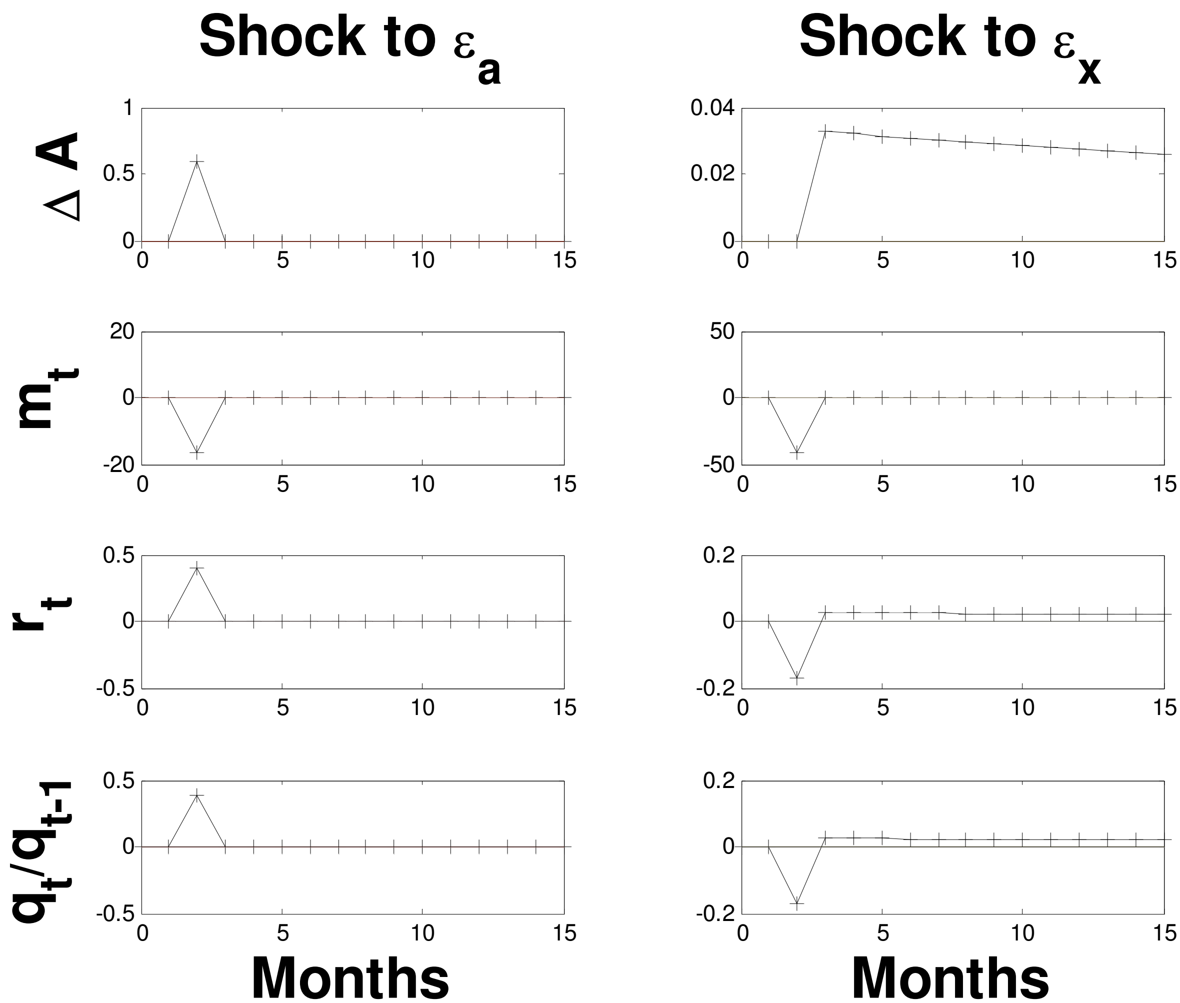

Fig. 12 – Prices Impulse Response Functions (IES = .8)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 4. The policy functions are computed numerically. Returns are not levered.

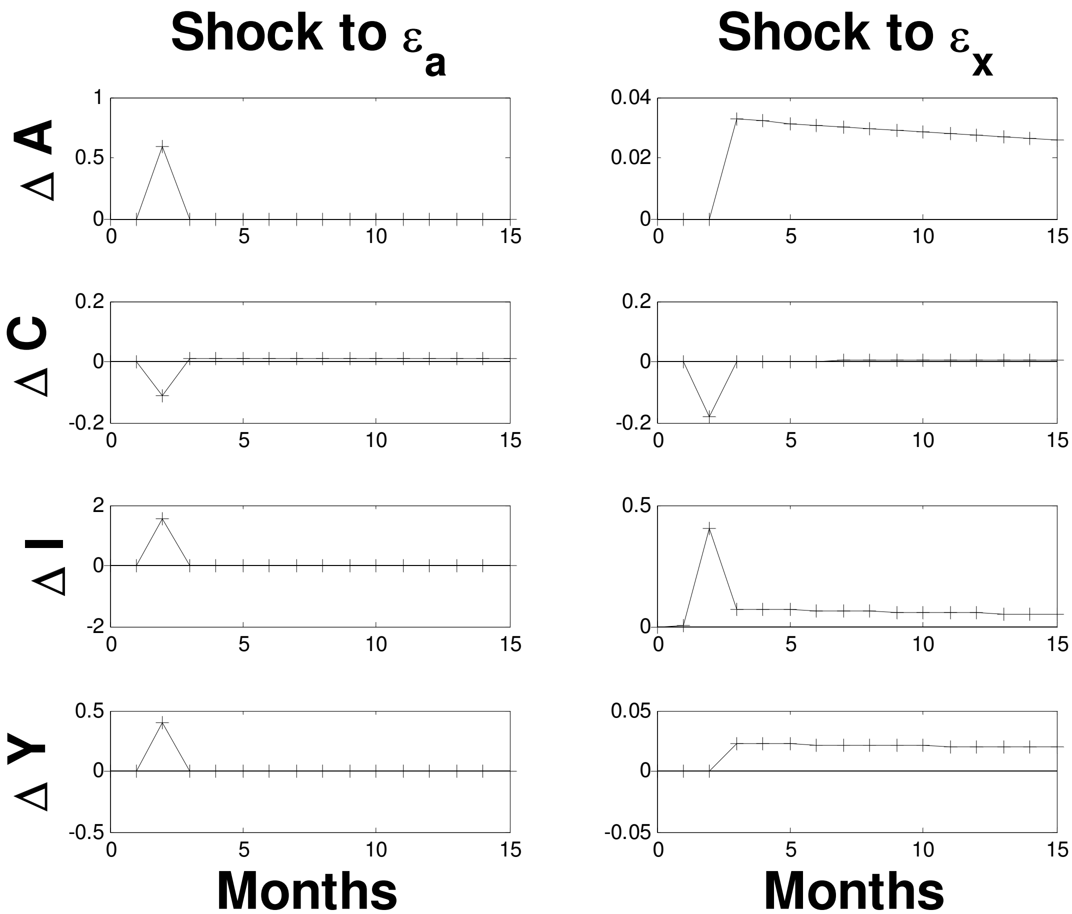

Fig. 13 – Quantities Impulse Response Functions (no adjustment costs)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 7 and $\tau = +\infty$. The policy functions are computed numerically.

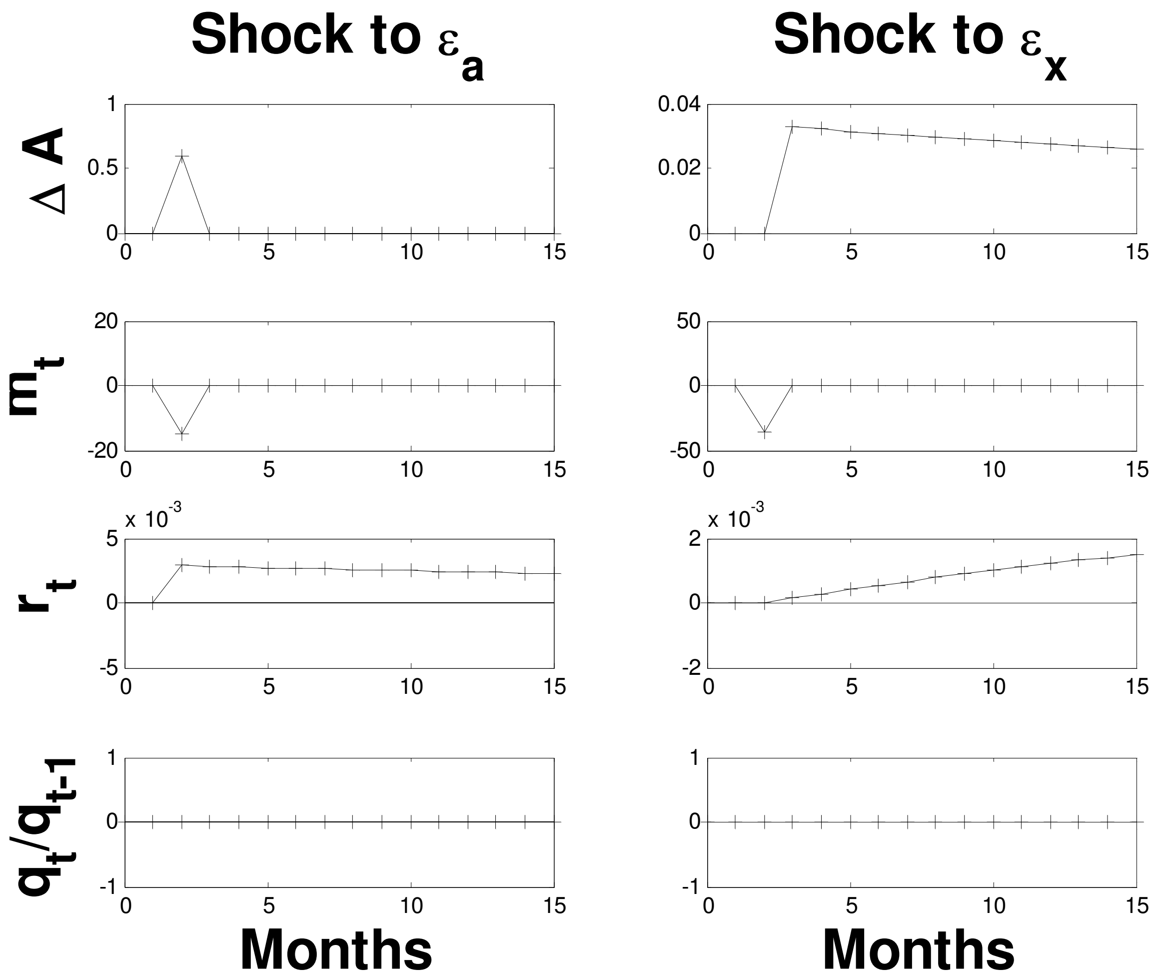

FIG. 14 – PRICES IMPULSE RESPONSE FUNCTIONS (NO ADJUSTMENT COSTS)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 7 and $\tau = +\infty$. The policy functions are computed numerically. Returns are not levered.