The Specification of Monetary Policy Inertia in Empirical Taylor Rules*

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Abstract

The inertia found in econometric estimates of interest rate rules is a continuing puzzle. Many reasons for it have been offered, though unsatisfactorily, and the issue remains open. In the empirical literature on interest rate rules, inertia in setting interest rates is typically modeled by specifying a Taylor rule with lagged policy rate on the right hand side. We argue that inertia in the policy rule may simply reflect the inertia in the economy itself. Since optimal rules typically inherit the inertia present in the model of the economy, empirical rules may simply reflect this. Our hypothesis receives some support from US data. . Hence, we agree with Rudebusch (2002) that monetary inertia is, at least partly, an illusion, but for different reasons.

JEL Classification: E52, E58

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1 Introduction

There is a conventional view that central banks adjust interest rates gradually in response to macroeconomic developments. The empirical evidence on the behaviour of central banks in the last two decades has been summarized as an inertial Taylor (1993) rule, where the nominal

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interest rate adjusts only partially to inflation and the output gap, as there is an interest rate smoothing component.[1]. A typical formulation has the policy rate responding to its own lagged value as well as a measure of the output gap and the inflation rate, such as the following:

$$i_t = \rho i_{t-1} + (1-\rho)\left(\mu_\pi \pi_t + \mu_y y_t\right)$$

Here i_t is some sort of nominal interest rate that is used as a policy instrument, π_t is a measure of the inflation rate, and y_t represents a measure of the output gap. The coefficient ρ ($\in [0,1]$) is taken to represent the degree of inertia or interest-rate-smoothing. (The coefficients μ_π and μ_y are the usual long-run responses of the policy rate to inflation and the output gap.)

Numerous explanations for smoothing have been offered, but they all seem in some sense unsatisfactory. The main reason for the unsatisfactoriness is that Central Banks say they do not do it.

A list of popular explanations for the apparent gradualism includes the following:

- Financial stability.[2] It is argued that by adjusting interest rates in small steps spread out over time, less pressure is put on the balance sheets of financial institutions which might otherwise be caught out by large unexpected changes.

- Financial markets may react adversely to frequent changes in the direction of movement of short-term interest rates (Goodfriend 1991). Frequent reversals may give the impression that the Central Bank is incompetent.

- Uncertainty about the structure of the macroeconomic model or about the values of its the parameters

- Measurement errors in relevant data.[3]

- The linkage between future monetary policy and aggregate demand can be exploited by central banks in order to stabilize the economy

[1] See for instance Clarida, Galì and Gertler (2000), who enphasize the empirical importance of including a lagged interest rate in a monetary policy rule for the United States. For a similar result for other industrial countries see Clarida, Galì and Gertler (1998).

[2] Reviews of this literature are provided by Cukierman (1992), Goodhart (1996), Walsh (2003), Sack and Wieland (2000).

[3] The importance of such uncertainties for gradualism is examined by Sack (1998,2000), Startz (2003), Orphanides (2003), Rudebusch (2001), Wieland (1998), among others.

optimally. When the current state of the economy is affected by expectations of future inflation (and other variables) it may be optimal to adjust the interest rate with some inertia.[4]

- It may be desirable to choose a central banker with an explicit interest rate smoothing objective, in a regime in which policy is delegated to a central banker who pursues policy in a discretionary (i.e., non-precommitted) manner.[5]

While many scholars accept that the apparent inertia is real, Rudebusch (2002) argues that it is an illusion. Since the coefficient of the lagged policy rate in empirical analyses frequently turns out to be large and highly significant, interest rates should be highly predictable.[6] However, on the basis of data on yield curves, he argues that they are not. He suggests that empirical Taylor rules may be misspecified and that what looks like inertia may actually be caused by serially correlated shocks.[7] English, Nelson and Sack (2003) show that it is possible to test directly the null hypothesis of serial correlated errors against the alternative of partial adjustment; but they are unable to reject the presence of either of them. Söderlind, Söderström and Vredin (2002) take up the question of predictability, and find further evidence against the inertial Taylor rule. They argue that a high coefficient of the partial adjustment component is a necessary but not sufficient condition for having a highly predictable interest rate. Predictability depends also on the other variables, namely the output gap and inflation. They find that, while it is relatively easy to predict these, it is very difficult to predict interest rates. They conjecture that this might result from the omission of an unpredictable variable from the Taylor rule.

In this paper we try to reconcile monetary policy inertia with the low predictability of short-term interest rates by proposing a different inertial Taylor rule than the one usually considered in the literature. We argue that the apparent inertia might arise from the the central bank's pursuing an optimal rule (or something of a similar character –

[4]See Woodford (1999).

[5]See Woodford (2003a). The previous two arguments for the optimality of monetary inertia considered in the text do not presume a central bank's loss function trading off objectives related to macroeconomic stability with an interest rate smoothing objective (usually interpreted as a financial stability motive).

[6]In the empirical literature the estimated coefficient for the lagged policy rate is ranging from .7 to .9. See Rudebusch (2002) for a review of the estimates found in the literature.

[7]See also Lansing (2002) for a theorical support of the 'illusion of monetary inertia' hypothesis, based on real-time estimation of trend output.

an 'optimal-ish' rule) for interest rates, which therefore inherits the inertia in the economy itself. If the evolution of the output gap and inflation depend on their own lagged values, the optimal rule for the interest rate will typically do so too. We will argue that, for given coefficient of partial adjustment, our alternative specification implies lower predictability of the interest rate than that implied by the standard specification of the inertial Taylor rule. In our empirical analysis we find support for the alternative specification against the standard specification. Moreover, in the alternative specification, the estimated coefficient of partial adjustment is below 0.5, which is lower than is usually found in the literature.

The structure of the paper is as follows. In section 2 we consider a simple empirical macro-economic model, along the lines of Svensson (1997), and derive the optimal interest rate rule for the central bank. We show that under certain conditions, this may be a simple rule that looks rather like an inertial Taylor rule. Section 3 discusses our empirical findings based on this alternative inertial Taylor rule. Section 4 makes some concluding observations and address future research.

2 A simple framework

2.1 The model

Here we use a simple framework for examining the optimal interest rate rule for a central bank, which is an extended version of the model used by Svensson (1997).[8] He argues that, even if there is no explicit role for private agents' expectations, the model has many similarities with more elaborate models used by central banks.[9]

Consider the following model[10]

$$\pi_{t+1} = \alpha_1 y_t + (1 - \alpha_2)\, \pi_t + \alpha_2 \pi_{t-1} + \epsilon_{t+1}, \tag{1}$$

and

[8] In the litereature, Svensson's (1997) model has been extended in several directions: for examining nominal income targeting (Ball 1999); for studying the implications of monetary policy for the yield curve (Ellingsen and Söderström 2001; Eijffinger, Schaling and Verhagen 2000); for examining model uncertainty, interest rate smoothing and interest rate stabilization - i.e. for studying the optimality of a more gradual adjustment of the monetary instrument (Svensson 1999). Moreover, Rudebusch and Svensson (1999) provide empirical estimates for a model similar to Svensson (1997) and use a calibrated version of the model in order to evaluate a large number of interest rate rules.

[9] See for instance the discussions in Rudebusch and Svensson (1999) and Rudebusch (2001).

[10] We have used the same notation as in Svensson (1997).

$$y_{t+1} = \beta_1' y_t - \beta_2 (i_t - E_t \pi_{t+1}) + \beta_3 y_{t-1} + \eta_{t+1}, \quad (2)$$

where π_t is the inflation rate, y_t is the output gap, i_t is the nominal repo rate, i.e. the monetary instrument of the central bank, and ϵ_t, η_t are i.i.d. shocks.[11] All the variables are considered as deviations from their long-run average levels, which are normalized to zero for simplicity.

After substituting $E_t\pi_{t+1}$ with the expectation of expression (1), expression (2) becomes:

$$y_{t+1} = \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1} + \eta_{t+1}, \quad (3)$$

with

$$\begin{aligned} \beta_1 &\equiv \beta_1' + \beta_2 \alpha_1; \\ \beta_4 &\equiv \beta_2 (1 - \alpha_2); \\ \beta_5 &\equiv \beta_2 \alpha_2. \end{aligned} \quad (4)$$

The coefficients in (1) and (3) are all assumed to be positive, with $0 < \alpha_2 < 1$. Equations (1) and (3) coincide with those considered in Svensson (1997) (equations 6.4 and 6.5 in his text) when $\alpha_2 = \beta_3 = 0$.[12] The restriction that the sum of the lag coefficients of inflation in (1) equals 1 is consistent with the empirical evidence.[13] An important feature of this model is the presence of lags in the transmission of monetary policy. In particular, the repo rate affects output with a one-period lag (where one period corresponds to one year), while affects inflation with a two-period lag. This feature is broadly consistent with the "stylized facts" of the impact of monetary policy on output and inflation.

Finally, suppose that monetary policy is conducted by a central bank with the following period loss function

$$L(\pi_t, y_t) = \frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right], \quad (5)$$

[11] See Svensson (1997) for the details on the model and in particular for the implications of substituting the long-term nominal rate with the repo rate.

[12] Contrary to Svensson we have assumed that the coefficient of one-period lagged inflation in (1) is less than 1, instead of equal to 1. McCallum (1997) has shown that when the coefficient is equal to 1 we may have problems of instability of nominal income rules that would not arise if expectations of current or future inflation were included in the model considered. See also Rudebusch (2002) and Jensen (2002) for further analyses of the performance of nominal income rules for monetary policy when a forward-looking price-setting behaviour is explicitly included in the analytical framework.

[13] See for instance Rudebusch and Svensson (1999) for a test of this restriction in a model similar to the one considered here.

where $\lambda > 0$ is the relative weight on output stabilization. The intertemporal loss function is

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, y_\tau). \tag{6}$$

The central bank minimizes the above intertemporal loss function by choosing a sequence of current and future repo rates $\{i_\tau\}_{\tau=t}^{\infty}$.

2.2 Optimal interest rate rule

In solving the optimization problem we use a convenient simplification. In the expression (3) of output the choice of i_τ affects y_{t+1}, but y_t, y_{t-1}, π_t and π_{t-1} are all predetermined. Thus we can write

$$y_{t+1} = \Delta_t + \eta_{t+1}, \tag{7}$$

with

$$\Delta_t \equiv \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1}. \tag{8}$$

As observed above, the repo rate affects affects inflation with a two-period lag. This can be seen by rewriting the expression (1) for inflation in the following way

$$\pi_{t+2} = \alpha_1 \Delta_t + (1 - \alpha_2)\pi_{t+1} + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + \epsilon_{t+2}, \tag{9}$$

where we have considered inflation at time $t+2$ and inserted expression (7). We can treat Δ_t as the control variable. Using dynamic programming, we can derive the optimal rule as the solution to the following problem

$$V(E_t\pi_{t+1}, \pi_t) = \min_{\Delta_t} E_t \left\{ \frac{1}{2}\left[\pi_{t+1}^2 + \lambda y_{t+1}^2\right] + \delta V(E_{t+1}\pi_{t+2}, \pi_{t+1}) \right\}, \tag{10}$$

subject to (7) and (9). The value function $V(E_t\pi_{t+1}, \pi_t)$ will be quadratic and in the present case, where constant terms are absent, it can be expressed without loss of generality as

$$V(E_t\pi_{t+1}, \pi_t) = \frac{1}{2}\gamma_1 \pi_{t+1}^2 + \gamma_2 \pi_{t+1}\pi_t + \frac{1}{2}\gamma_3 \pi_t^2 + k, \tag{11}$$

where the coeffcients γ_1, γ_2 and γ_3 need to be determined. The remaining constant k is a function of the variances of the shocks.

Here we have two state variables and one control variable. In general, the optimization problem cannot be solved analytically by means of dynamic programming if there is more than one state variable. In the simpler case with only one state variable, considered by Svensson, it is possible to get an analytical solution for the optimization problem.

Nevertheless, we can make a qualitative assessment of the form of the optimal rule. Svensson has shown that in the simpler case considered by him the optimal rule takes the form of the Taylor (1993) rule

$i_t = \phi_1 \pi_t + \phi_2 y_t$,

with $\phi_1 > 1$ and $\phi_2 > 0$. What emerges in the present case?

The first order condition with respect to Δ_t is given by

$$E_t y_{t+1} = -\frac{\alpha_1 \delta}{\lambda} \left(\gamma_1 E_t \pi_{t+2} + \gamma_2 E_t \pi_{t+1} \right), \tag{12}$$

where we have used (11).

The optimal interest rate can be derived by substituting (1) in (12) and using (3) to yield

$$\begin{aligned} i_t = {} & \alpha_2 \left[(1 + C) \pi_{t-1} + \frac{\beta_3}{\alpha_2 \beta_2} y_{t-1} \right] + \\ & (1 - \alpha_2) \left[(1 + A) \pi_t + \left(\frac{\beta_1}{(1 - \alpha_2) \beta_2} + B \right) y_t \right], \end{aligned} \tag{13}$$

with

$$\begin{aligned} A &\equiv \delta \alpha_1 \frac{(\gamma_1 + \gamma_2)(1 - \alpha_2) + \gamma_1 \alpha_2^2}{(1 - \alpha_2) \beta_2 (\lambda + \delta \alpha_1^2 \gamma_1)}; \\ B &\equiv \delta \alpha_1^2 \frac{\gamma_1 (1 - \alpha_2) + \gamma_2}{(1 - \alpha_2) \beta_2 (\lambda + \delta \alpha_1^2 \gamma_1)}; \\ C &\equiv \delta \alpha_1 \frac{\gamma_1 (1 - \alpha_2) + \gamma_2}{\beta_2 (\lambda + \delta \alpha_1^2 \gamma_1)}. \end{aligned} \tag{14}$$

In general, in a problem of this type, the optimal feedback rule can be represented as a linear function of the state variables, here $E_t \pi_{t+1}$, and π_t. So we could represent the rule for Δ_t as $\Delta_t = f_1 E_t \pi_{t+1} + f_2 \pi_t$. Since $E_t \pi_{t+1}$ can be represented as a function of current values and the first lag of the output gap and inflation, when we solve for the interest rate, the policy rule also emerges as a linear function of the same variables. It would be useful to be able to sign the parameters in the feedback rule (14). Since the value function is a positive definite quadratic form, it must be the case that $\gamma_1 > 0$, $\gamma_3 > 0$, and $\gamma_1 \gamma_3 - \gamma_2^2 > 0$, but it is not possible to sign γ_2. If the coefficients on the right hand side variables in (13) are all positive, and if the ratios of coefficients on the current variables (and) are the same as the ratios of coefficients on lagged variables (and), then the policy rule may have the form of a moving average of a simple Taylor rule. That is, (13) can be written as

$$i_t = \alpha_2 \left[\mu_3 \pi_{t-1} + \mu_4 y_{t-1}\right] + \\ \left(1 - \alpha_2\right) \left[\mu_1 \pi_t + \mu_2 y_t\right], \tag{15}$$

with $\mu_1 = (1 + A)$, $\mu_2 = \left(\frac{\beta_1}{(1-\alpha_2)\beta_2} + B\right)$, $\mu_3 = (1 + C)$, and $\mu_4 = \frac{\beta_3}{\alpha_2 \beta_2}$. If the pattern of coefficient were such that $\mu_1/\mu_2 = \mu_3/\mu_4$ then the actual rule could be thought of as a moving average of a simple rule $\bar{i}_t = \mu_1 \pi_t + \mu_2 y_t$.

2.3 Simple rules

During the past decade, the research on monetary policy design has focused on simple rules - among which Taylor's (1993) rule is a prominent example - as opposed to more complicated or fully optimal rules.[14] As argued by Woodford (2003b, p. 507), a rationale for this choice can be found in the greater transparency provided by simple rules, which may increase central bankers' accountability in terms of their commitment to the given policy rule.[15] Typically this literature has focused on simple rules based on two or three parameters (and variables) which are optimized for the given preferences and the given form of the rule assumed. For example Rudebusch and Svensson (1999) estimate a model similar to that presented here, with more lagged variables and an interest rate smoothing argument added in the loss function. They derive numerically the optimal policy rule, which looks more complicated than ours. Moreover they use the model to evaluate a large number of simple rules for setting the interest rate.

Two main findings of this literature are that simple rules perform nearly as well as fully optimal rules and that simple rules are more robust than more complicated rules to model misspecification.

In this vein, we can simplify the optimal rule in a way that approximates the behaviour of the optimal rule. In particular it is straightforward to see that the optimal rule (13) could be approximated by a simple rule of the following form

$$i_t = \rho \bar{i}_{t-1} + (1 - \rho) \bar{i}_t, \tag{16}$$

with

[14] For a review of this literature see for example Williams (2003).

[15] See Svensson (2003) for a discussion of the problems associated to using judgements in monetary policy based on simple instrument rules or targeting rules.

$$\bar{i}_t = \mu_\pi \pi_t + \mu_y y_t, \tag{17}$$

and $0 < \rho < 1$.

In the empirical literature the standard inertial Taylor rule takes instead the following form

$$i_t = \rho i_{t-1} + (1-\rho)\,\bar{i}_t, \tag{18}$$

with $\bar{i}_t$ equal to (17) or to a forward-looking version of (17) with future expected inflation. The term $\bar{i}_t$ is usually interpreted as an operating target for the policy rate.

The crucial difference of (16) with respect to (18) is that the inertial component is proportional to the lagged operating target, instead of the lagged interest rate. Hence, our alternative specification of the inertial policy rule implies that the central bank gradually adjusts the operating target for the policy rate.[16]

In our framework, substituting the lagged operating target with the lagged interest rate in the simple rule could improve the approximation of the optimal rule only if we had the lagged interest rate in the optimal rule. This only happens if we have an interest rate smoothing objective in the central bank loss function.

By using a model with forward-looking private sector, Woodford (2003a) has shown that it may be optimal to delegate monetary policy to a central bank that has an objective function with an interest rate smoothing motive. This is an interesting result. However, while there exist examples in the real world of institutional arrangements that penalize central banks for not achieving given inflation targets, there is less evidence of central banks being penalized for interest rate changes. The reference to a financial stability objective is very general and it is consistent also with an interest rate targeting objective without necessarily implying an interest rate smoothing objective.[17] Thus, to presume, as Woodford and others do, that central banks have preferences of this kind, which are unlike those specified in social loss functions, requires an explicit reference to an interest smoothing objective in the Law concerning central banks.

Sack (2000, pp. 230-231) provides a further argument against an explicit interest rate smoothing objective:

> "To describe this behaviour, which has been referred to as gradualism, many empirical studies of monetary policy incorpo-

[16] See Woodford (2003b, p. 96) for a discussion of interest rate rules with partial adjustment on lagged operating target.

[17] See for example Goodfriend (1987).

> rate an explicit interest-rate smoothing incentive in the objective function of the Fed. However, introducing this argument has little justification beyond matching the data. Furthermore, the above statistics provide evidence of gradualism only if the Fed would otherwise choose a random-walk policy in the absence of an interest-rate smoothing objective. Therefore, while establishing that the funds rate is not a random walk, these statistics do not necessarily provide evidence of gradualism in monetary policy".

Thus we can argue that it would be perfectly plausible to test empirically for alternative specifications of simple rules which do not necessarily include the lagged interest rate, but provide as well some degree of inertia reflecting the dynamic structure of the economy (and eventually the uncertainty surrounding that structure).

3 Empirical evidence

3.1 Estimation of inflation and output equations

In order to gain some insights into the parameters of the inflation and output equations used in the previous theoretical analysis we have first estimated the following empirical model based on Rudebusch and Svensson (1999):

$$\pi_t = \kappa_{\pi 1}\pi_{t-1} + \kappa_{\pi 2}y_{t-1} + \kappa_{\pi 3}\pi_{t-2} + \omega_t, \tag{19}$$

and

$$y_t = \kappa_{y1}y_{t-1} + \kappa_{y2}y_{t-2} + \kappa_{y3}\left(\widetilde{i}_{t-1} - \widetilde{\pi}_{t-1}\right) + \psi_t, \tag{20}$$

where the variables were de-meaned prior to estimation. The data used here are ex post revised quarterly data. Inflation is defined using the GPD-chain weighted price index (P_t), with $\pi_t = 400 \cdot (\ln P_t - \ln P_{t-1})$. The output gap is defined as the percentage difference betweeen actual real GDP (Q_t) and potential output (Q^*) estimated by the Congressional Budget Office. The interest rate i_t is the quarterly average of the Fed Funds rate.[18] The data are illustrated in Figures **??**, 2, and 3. In the text we do not discuss the stationarity or otherwise of the data. A note at the end of the appendix summarises some simple checks.

[18] While real GDP and the GPD-chain weighted price index were taken from FRED of the Federal Reserve of San Louis, the (effective) Fed Funds rate was taken from Datastream.

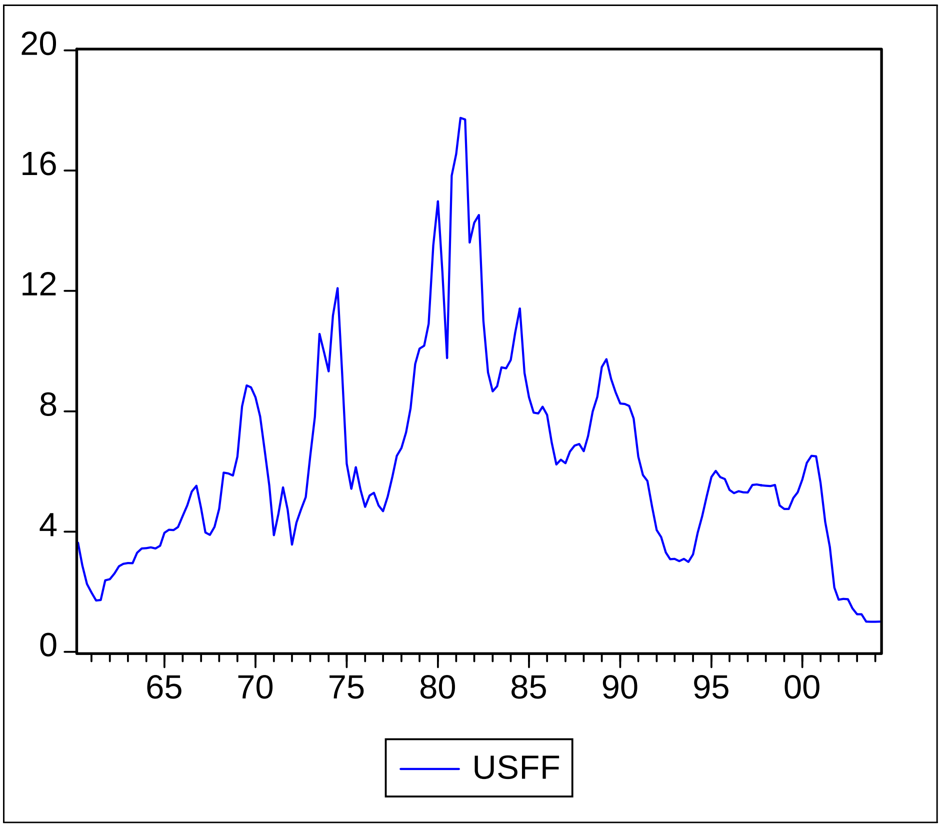

Figure 1: United States, Federal Funds Rate

In table 1 we report Ordinary Least Squares estimates of the above two equations over the period 1961 Q1 - 2004 Q2, with robust standard errors for the inflation equation. Following Rudebusch and Svensson the equations were estimated individually. In the output equation $\widetilde{i}_t = (1/4)\sum_{j=0}^{3} i_{t-j}$ and $\widetilde{\pi}_t = (1/4)\sum_{j=0}^{3} \pi_{t-j}$. The inflation equation is somewhat simpler compared to that estimated by Rudebusch and Svensson. According to the Wald test the null hypothesis that $\kappa_{\pi 3} = (1 - \kappa_{\pi 1})$ has a p-value of .15, therefore we have imposed this restriction in the estimation.

Table 1 Inflation and Output Equations with ex post revised data

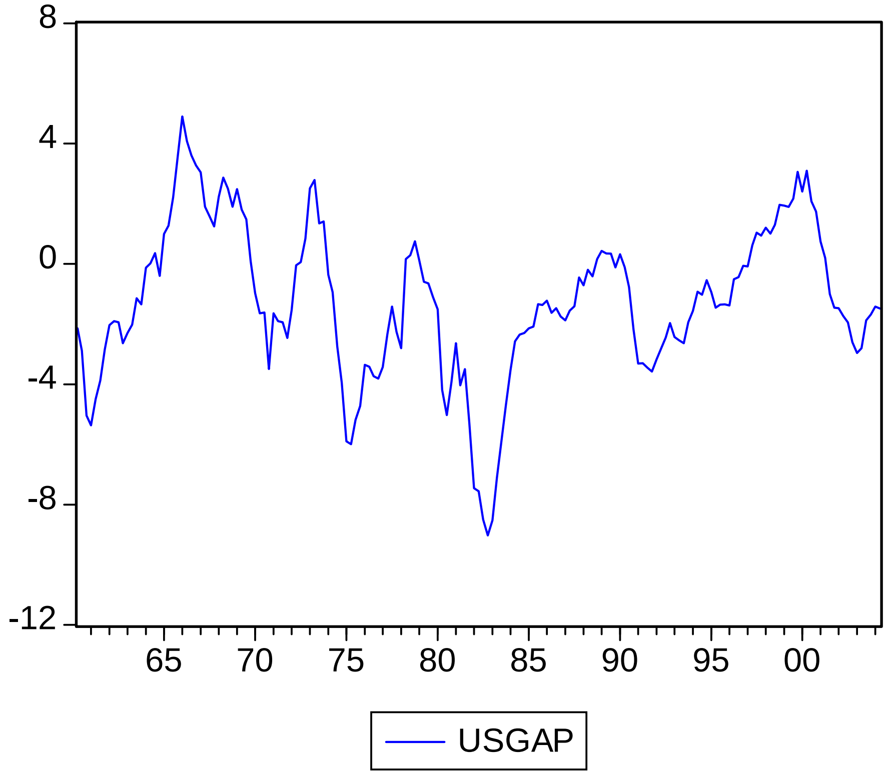

Figure 2: Output Gap

Inflation		Output	
$\kappa_{\pi 1}$	0.72 (7.80)	κ_{y1}	1.19 (16.52)
$\kappa_{\pi 2}$	0.09 (3.35)	κ_{y2}	-0.27 (-3.72)
		κ_{y3}	-0.06 (-2.12)
$\overline{R}^2$	0.81	$\overline{R}^2$	0.91
SE	1.08	SE	0.77

Notes: Ordinary Least Squares estimates. T statistics in paretheses. $\overline{R}^2$ and standard errors (SE) of residuals also reported. For the inflation equation T-statistics are based on heteroskedasticity- and serial correlation-corrected standard errors (Newey and West, 1987). Variables are de-meaned before estimation. Sample period 1961Q1 – 2004Q4.

The estimates in Table 1 have rather poor statistical properties. There is ample evidence of mis-specification and structrual instability

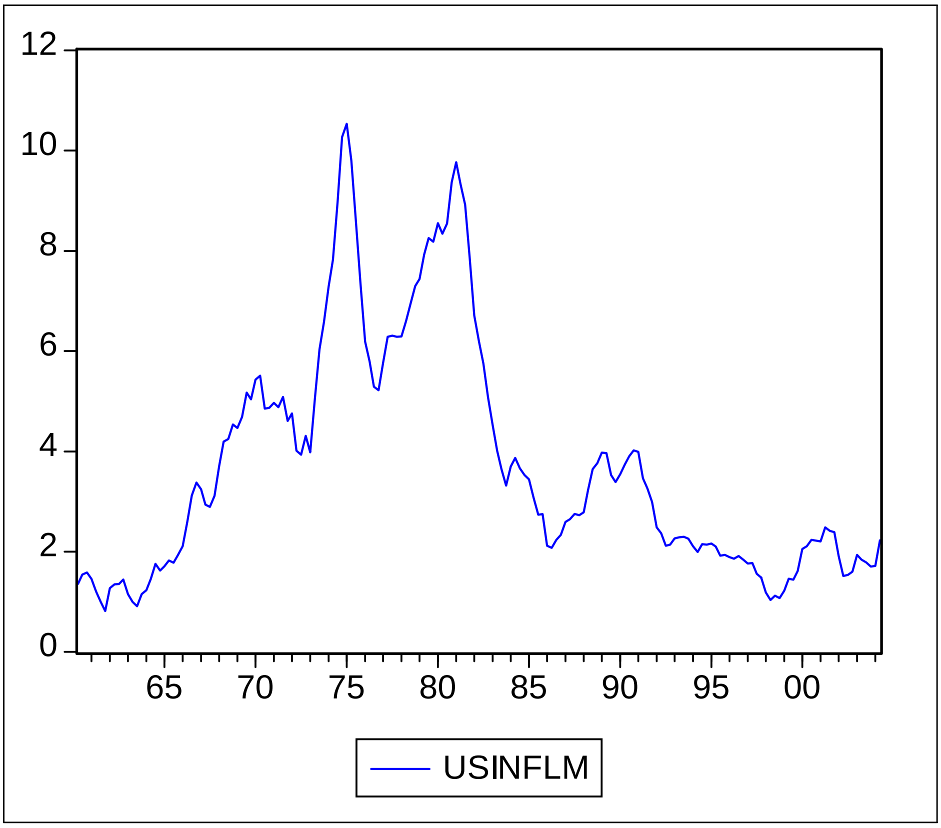

Figure 3: United States, Inflation Rate

over the sample period. The errors suffer from serial correlation. We offer these estimates as very crude indication of the orders of magnitude of the parameters in the Svensson model. They can be inserted into the optimzation exercise set out above to produce optimal policy rules for interest rates. The figures may give some indication as to whether the parameter restrictions above are satisfied and the optimal rule can be regarded as a first-order moving average of a simple Taylor rule.

3.2 Optimal Rules from the Estimated Macro Model

In the model above, equations (1) and (2), we insert parameter values as follows:

$$\pi_{t+1} = 0.09y_t + 0.72\pi_t + 0.28\pi_{t-1}$$

$$y_{t+1} = 1.19y_t - 0.06\,(i_t - E_t\pi_{t+1}) - 0.27y_{t-1}$$

When the welfare function weight $\lambda = 1$ is chosen (i.e., output and inflation deviations are equally weighted) in equation (5), we get the following results. The optimal rule for the effective control variable Δ_t can be expressed as a feedback on the state variables π_t and $E_t\pi_{t+1}$ (both of these are pre-determined variables):

$$\Delta_t = -0.698E_t(\pi_{t+1}) - 0.2011\pi_t$$

Thus policy can be expressed as an inflation forecast rule, as Svensson showed. This can be unscrambled to give a rule for the interest rate in terms of the current and past output gap and inflation:

$$i_t = (20.9)y_t + (12.4)\pi_t + (-4.5)y_{t-1} + (-3.0)\pi_{t-1}$$

The dynamics of expected inflation that result from applying this policy would be:

$$E_t(\pi_{t+2}) = 0.66E_t\pi_{t+1} + 0.26\pi_t$$

Thus the policy rule stabilises inflation. The rule itself involves a very strong response to current inflation and the output gap, which is then reversed next period. This suggests a volatile path of interest rates: the opposite of smoothing! This is something of a puzzle. However, the policy has the property that the coefficients on output and inflation approximately satisfy the condition $\mu_1/\mu_2 = \mu_3/\mu_4$ in (15) since $(20.9/12.4) \approx (4.5/3.0)$. Hence the policy rule could be a moving average of a simple Taylor rule.

If an alternative welfare weight λ equal to 0.1 is chosen, then the results are

$$\Delta_t = -2.11E_t(\pi_{t+1}) - 0.6401\pi_t$$

$$i_t = (23.1)y_t + (36.7)\pi_t + (-4.5)y_{t-1} + (-9.5)\pi_{t-1}$$

$$E_t(\pi_{t+2}) = 0.53E_t\pi_{t+1} + 0.22\pi_t$$

Once again the rule approximates a moving average since $(23.1/36.7) \approx (4.5/9.5)$. However, in this case, with a lower welfare weight on output deviations, the response to the current state is even more aggressive, and there is a bigger reversal in the following period. Once again these policy rules are not romotely like estimated Taylor rules.

3.3 Estimates of Alternative Policy Rules

We now turn to estimating alternative forms of policy rule, the standard inertial Taylor rule

$$i_t = \rho i_{t-1} + (1-\rho)\,\bar{i}_t + \xi_t, \tag{21}$$

and the alternative inertial Taylor rule

$$i_t = \rho \bar{i}_{t-1} + (1-\rho)\,\bar{i}_t + \xi_t, \tag{22}$$

with

$$\bar{i}_t = \mu + \mu_\pi \widetilde{\pi}_t + \mu_y y_t, \tag{23}$$

and $0 < \rho < 1$. ξ_t is an i.i.d. error term. Following Taylor (1993) and Rudebusch (2002) the policy rate is assumed to react to the average inflation rate over four quarters, $\widetilde{\pi}_t$.

We allow for serial correlation in the errors in these two equations. As Rudebusch (2002) argues, a partial adjustment model and a model with serially correlated shocks can be nearly observationally equivalent. However English, Nelson and Sack (2003) find that both play an important role in describing the behaviour of the federal funds rate when they allow for both of these hypotheses in the estimation of the policy rule. The omission of a persistent, serially correlated variable that influences monetary policy could yield the spurious appearance of partial adjustment in the estimated rule. We assume that the shock ξ_t follows an AR(1) process:

$$\xi_t = \theta \xi_{t-1} + \varepsilon_t. \tag{24}$$

The combination of rule (21) with (24) yields the following expression for the first difference of the interest rate:

$$\Delta i_t = (1-\rho)\Delta \bar{i}_t - (1-\rho)(1-\theta)(i_{t-1} - \bar{i}_{t-1}) + \rho\theta \Delta i_{t-1} + \varepsilon_t. \tag{25}$$

This expression corresponds to that used by English, Nelson and Sack (2003). The combination of rule (22) with (24) yields the following expression for the first difference of the interest rate:

$$\Delta i_t = (1-\rho)\Delta \bar{i}_t - (1-\theta)(i_{t-1} - \bar{i}_{t-1}) + \rho\theta \Delta \bar{i}_{t-1} + \varepsilon_t. \tag{26}$$

Nonlinear Least Squares estimates of (25) and (26) are reported in tables 2 and 3, for the period 1987 Q4 - 2004 Q2, and for two subsamples of it. The point estimates of ρ and θ are both highly significant for all rules, suggesting that both partial adjustment and serially correlated errors are present. The coefficients on the output gap and inflation are largely consistent with other estimates from the literature, with a significant coefficient on the output gap and a coefficient on inflation greater than one. Moreover, both rules appear to fit the data relatively well.

Interestingly, the degree of inertia implied by the alternative inertial Taylor rule is systematically lower than that implied by the standard specification, with an estimated coefficient of partial adjustment ρ for the whole sample of .60 against one of .77. Meanwhile, the coefficient θ is systematically higher in the case of the alternative specification than in the standard specification. However, we have not tested whether these differences are significant statistically.

Thus, as in English, Nelson and Sack (2003), the empirical evidence suggests that specifications (25) and (26) of the policy rules perform no worse than the more usual specifications (21) and (22). The alternative specification suggests less monetary inertia but much greater importance of serially correlated errors than does the standard specification.

Table 2 Standard inertial Taylor Rule with ex post revised data

	1987Q4-1993Q4	1987Q4-2001Q2	1987Q4-2004Q2
μ_0	0.15 (0.12)	1.10 (0.94)	1.28 (0.89)
$\overline{\mu}_\pi$	2.31 (7.12)	1.85 (4.31)	1.66 (2.41)
$\overline{\mu}_y$	0.92 (5.61)	0.77 (3.94)	0.94 (3.49)
ρ	0.51 (7.58)	0.61 (7.34)	0.72 (6.49)
θ	0.34 (2.09)	0.80 (5.52)	0.77 (5.41)
$\overline{R}^2$	0.99	.097	0.98
SE	0.26	0.31	0.33

Note: Nonlinear least squares estimates. T-statistics in parentheses based on standard errors corrected for heteroskedasticity and serial correlation (Newey and West, 1987). $\overline{R}^2$ and standard errors (SE) of residuals are reported for the level of the funds rate.

Table 3 Alternative Inertial Taylor Rule with ex post revised data

	1987Q4-1993Q4	1987Q4-2001Q2	1987Q4-2004Q2
μ_0	0.41 (0.41)	1.70 (1.07)	−4.08 (−0.17)
$\overline{\mu}_\pi$	2.15 (9.69)	1.40 (5.04)	1.10 (3.50)
$\overline{\mu}_y$	0.78 (5.66)	0.65 (4.56)	0.67 (4.56)
ρ	0.48 (6.62)	0.59 (6.49)	0.60 (8.64)
θ	0.70 (6.18)	0.94 (17.29)	0.99 (26.68)
$\overline{R}^2$	0.99	.096	0.98
SE	0.28	0.35	0.36

Note: Nonlinear least squares estimates. T-statistics in parentheses based on standard errors corrected for heteroskedasticity and serial correlation (Newey and West, 1987). $\overline{R}^2$ and standard errors (SE) of residuals are reported for the level of the funds rate.

3.4 More general models

The two models – the standard rule and our revised rule – have been presented as two alternatives. However, they can both be represented as special cases of more general relations. The least restrictive is an unrestricted linear model involving lags of the change and level of the interest rate, and current and lagged values of the changes in the output gap and inflation and their lagged levels:

$$\begin{aligned}\Delta i_t = {} & c_0 + c_1\Delta i_{t-1} + c_2 i_{t-1} + c_3\Delta\widetilde{\pi}_t + c_4\Delta y_t + c_5\Delta\widetilde{\pi}_{t-1} + \\ & c_6\Delta y_{t-1} + c_7\widetilde{\pi}_{t-1} + c_8 y_{t-1} + u_t\end{aligned} \tag{27}$$

A set of restrictions that brings this closer to the Taylor rules above is to assume that the output gap and inflation rate enter through some sort of target interest rate $\overline{i_t}$, which is a a linear combination of the output gap and the inflation rate. The necessary restrictions are :

$$c_3/c_4 = c_5/c_6 = c_7/c_8.$$

When these hold, the change in the policy rate can be written as

$$\Delta i_t = c_1'\Delta i_{t-1} + c_2'(i_{t-1} - \overline{i}_{t-1}) + c_3'\Delta\overline{i}_t + c_4'\Delta\overline{i}_{t-1} + u_t \tag{28}$$

with

$$\overline{i_t} = \mu_0 + \mu_\pi\widetilde{\pi}_t + \mu_y y_t$$

This might be termed a semi-restricted model.

To get another step closer to the models estimated above, we can impose the restriction that there is a common factor in the lag polynomials for i_t and $\overline{i_t}$ so that the model can be represented as having a first-order autoregressive error term. This might be termed the hybrid model, as it takes the form of a linear combination of the two models set out above. The restriction that is imposed on the semi-restricted model above is that

$$\frac{c_3'}{1-c_3'} = \frac{c_2'}{1-c_3'-c_1'-c_4'} - \frac{c_4'}{c_1'+c_4'}$$

and when this restriction is valid we can reduce the four parameters c_1', c_2', c_3', and c_4' to three, ρ, ϕ, and θ, which satisfy

$$(1-\rho-\phi) = c_3', (1-\phi)(1-\theta) = c_2', \theta\phi = c_1', and \theta\rho \ = c_4'$$

In terms of the levels of the interest rate the hybrid model gives:

$$i_t = (1-\rho-\phi)\overline{i_t} + \rho\overline{i}_{t-1} + \phi i_{t-1} + \xi_t \qquad (29)$$

$$\overline{i_t} = \mu_0 + \mu_\pi \widetilde{\pi}_t + \mu_y y_t$$

$$\xi_t = \theta\xi_{t-1} + \varepsilon_t$$

As an expression for the change in the policy rate, the hybrid model gives:

$$\Delta i_t = (1-\rho-\phi)\Delta\overline{i}_t + (1-\phi)(1-\theta)(\overline{i}_{t-1} - i_{t-1}) + \theta\phi\Delta i_{t-1} + \theta\rho\Delta\overline{i}_{t-1} + \varepsilon_t$$

The models set out above are special cases of this hybrid model. If we assume $\rho = 0$, we get the "standard" type of inertial Taylor Rule. If instead we assume $\phi = 0$, we get the moving average form of Taylor rule, in which there is no real, only apparent inertia. If we assume $\theta = 0$, we are assuming that the error term is not serially correlated.

Estimated over the sample period 1987Q4 to 2004Q2, the unrestricted, semi-restricted, and hybrid models show that the hybrid model is an acceptable simplification of the unrestricted model. The relevant summary statistics are reported in Table 4

Table 4 Summary Statistics: Unrestricted, Semi-Restricted, and Hybrid Models

	Unrestricted	Semi-Restricted	Hybrid
R^2	0.62	0.61	0.61
$\overline{R}^2$	0.57	0.57	0.58
SE of Regression	0.31	0.31	0.310
Sum of squared residuals	5.65	5.86	5.86
Log Likelihood	-12.195	-13.42	-13.43
Akaike	0.63	0.61	0.58
Schwartz	0.93	0.84	0.78

Note: Sample period 1987Q4–2004Q2. R^2 measured for Δi_t.

However, when further restrictions are imposed on the hybrid model, they prove to be rejected by the data. Both the 'Normal Taylor' and the 'Alternative' models are rejected against the alternative hypothesis of the hybrid model. Consequently, for this sample period, neither model suffices. There appear to be elements of both in the data. The most that can be claimed is that, while structural inertia (represented by our alternative model) plays some role in explaining interest rate movements, there still appears to be an element of the 'inexplicable' inertia remaining.

Table 5 Estimates of Various Models

	Hybrid Model	Normal Taylor ($\rho = 0$)	Alternative ($\phi = 0$)
ρ	0.28 (0.10)	0	0.60 (0.10)
ϕ	0.48 (0.11)	0.71 (0.11)	0
μ_y	0.92 (0.21)	0.93 (0.38)	0.67 (0.11)
μ_π	1.15 (0.42)	1.65 (0.63)	1.10 (0.26)
θ	0.93 (0.06)	0.76 (0.14)	0.98 (0.03)
const	0.07 (0.10)	0.083 (0.09)	-0.04 (0.11)
R^2	0.61	0.56	0.47
$\overline{R}^2$	0.58	0.53	0.44
SE	0.310	0.325	0.36
Sum Squared Residuals	5.86	6.56	7.93
Log likelihood	-13.43	-17.23	-23.59
Akaike info criterion	0.58	0.66	0.85
Schwarz criterion	0.78	0.83	1.02

Notes: Sample Period 1987Q4–2004Q2

While neither model is acceptable for the period 1987Q4–2004Q2, it is possible to find shorter sample periods for which one or other of them is acceptable, as Table 6 shows. This table shows the p-values for the likelihood ratio test of the null hypothesis that the model is either the standard or the alternative inertial Taylor rule against the alternative hypothesis that the hybrid is the true model. Our alternative model is acceptable providing the sample starts in 1983Q4 and ends before 1999Q4. But if the sample begins in 1987Q4 the model is rejected. The standard inertial Taylor model by contrast is only accepted if the sample beings in 1987Q4 and ends by 1999Q4. All this points to considerable structural instability in these models, reflecting changing responses of interest rates to output gaps and inflation.

Table 6 Partial Adjustment and Correlated Shock Rules: p-values

Sample	Standard	Alternative Model
83Q4-93Q4	0.00	0.89
-96Q4	0.00	0.14
-99Q4	0.00	0.07
-04Q2	0.00	0.00
87Q4-93Q4	0.53	0.04
-96Q4	0.14	0.00
-99Q4	0.44	0.00
-04Q2	0.01	0.00

Note: The entries in the table are p-values of the Likelihood-Ratio Test. The null hypothesis is partial adjustment for the standard inertial Taylor Rule (columns headed 'Standard') or for the alternative inertial rule (columns headed 'Alt'), with and without serially correlated shocks.

The actual values of the interest rate and the fitted values for the hybrid model are displayed in Figure 4.

3.5 Forward-looking Policy Rules

In the literature there exists empirical evidence supporting the importance of forward-looking policy rules versus backward–looking ones - see for instance Orphanides (2001) and Clarida, Galì and Gertler (2000). Thus it might be useful to compare our estimated backward-looking policy rules with the estimates obtained from the standard specification of monetary inertia with expectations of future inflation in the implicit notional target. In Table 7 are reported Generalized Method of Moments (GMM) estimates of rule (21) with (24) and for the case when

$$\bar{i}_t = \mu + \mu_\pi E_{t-1}\widetilde{\pi}_{t+4} + E_{t-1}\mu_y y_t. \tag{30}$$

The instruments chosen were four lags each of inflation, the funds rate and the output gap. As it is possible to see from Table 7, the goodness-of-fit is not improved compared to the case of backward-looking policy rules. Moreover, unlike in the case of backward-looking policy rules, the estimates of the output coefficient are not always statistically significant.

Table 7 Forward-looking Inertial Rule with ex-post Revised Data

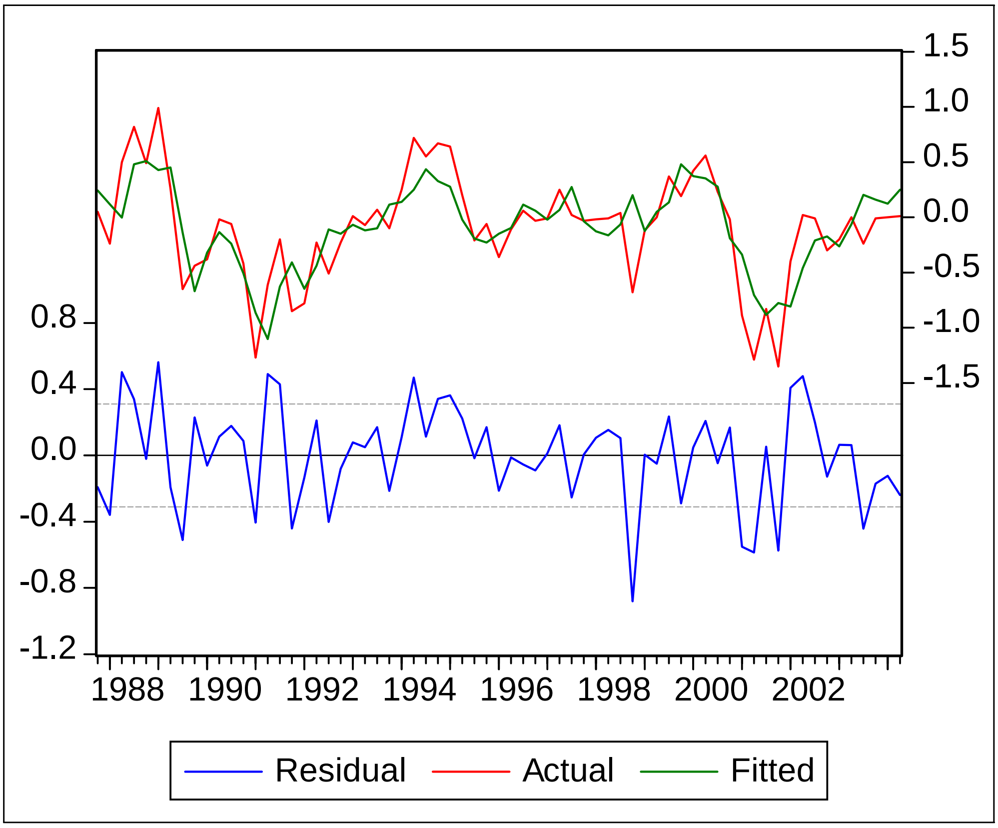

Figure 4: Hybrid Model, estimated on sample 1987Q4 2004Q2

	1987Q4-1993Q4	1987Q4-2001Q2	1987Q4-2003Q2
μ_0	-3.35 (-1.28)	0.64 (0.50)	-0.87 (-0.55)
$\overline{\mu}_\pi$	2.57 (3.51)	2.16 (4.16)	2.58 (3.83)
$\overline{\mu}_y$	-0.30 (-0.59)	0.62 (3.73)	0.74 (4.20)
ρ	0.79 (11.69)	0.66 (5.32)	0.68 (6.51)
θ	0.07 (0.37)	0.62 (3.71)	0.67 (5.08)
$\overline{R}^2$	0.94	.095	0.97
SE	0.60	0.37	0.38

Note: Generalised Method of Moments Estimates. Instruments are four lags of each of inflation, the funds rate, output gap. T-statistics shown in parentheses are based on standard errors corrected for heteroskedasticity and serial correlation (Newey and West, 1987). $\overline{R}^2$ and standard errors (SE) of residuals reported for the level of the funds rate. The earlier end date for the sample is required for the forward-looking specification.

3.6 Evidence from yield curves

Rudebusch argues that the partial adjustment of monetary policy by a central bank implies that the short-term interest rate should be highly predictable. However, term structure evidence based on futures contracts suggests that there is little if any information usually available in financial markets for predicting the Fed funds rate 3-6 months ahead and no information for predicting it 6-9 months ahead. On the contrary within a 3-month horizon the 3-month eurodollar forecasts relatively well the future change of the Fed funds rate (with an R^2 of 0.57 percent).

Söderlind, Södeström and Vredin (2003) show that the degree of predicatibility of the short-term interest rate depends crucially also on the degree of predictability of inflation and output and not only on the degree of monetary inertia. They show that, while it is relatively easy to predict the variables that enter the Taylor rule, it is very difficult to predict interest rates. They argue that this outcome might be related to an omitted variable problem in the Taylor rule, with the potentially omitted variable being not easily predictable.

In order to examine empirically the issue of predictability we consider our estimated equations for output, inflation and run recursive simulations for the Fed funds rate by using the different estimated policy rules for the 1987-2004 period. After having obtained one quarter, two quarters and three quarters ahead predictions of the Fed funds rate we estimate for the 1990 Q1 - 2004 Q2 period the following regressions:

$$\begin{aligned} i_{t+1} - i_t &= \psi_0 + \psi_1(E_t i_{t+1} - i_t) + \xi_{t+1,} \\ i_{t+2} - i_{t+1} &= \psi_0 + \psi_1(E_t i_{t+2} - E_t i_{t+1}) + \xi_{t+2,} \\ i_{t+3} - i_{t+2} &= \psi_0 + \psi_1(E_t i_{t+3} - E_t i_{t+2}) + \xi_{t+3.} \end{aligned} \tag{31}$$

The use of parameters estimated on the full sample should not matter if parameters are stable. There should not be any problem with this exercise, as recursive estimations starting from 1990 Q1 support parameters stability for the different policy rules considered. Equations (31) are the analogue of the equations considered by Rudebusch based on the forecasts implied by futures contracts.[19] In Table 8 are reported the estimated parameters and the corrected R^2 statistic for the two specifications. As it is possible to see our simple framework is capable of replicating quite closely the pattern found by Rudebusch.[20]

[19] Equations (15), (16) and (17) in Rudebusch (2002), pages 1172-1173.

[20] Also Favero (2002) has shown that the predictive regressions based on model projections and Fed Funds rate futures give very similar results. But he examines only the standard specification of the inertial forward-looking Taylor rule.

Table 8 Predictability of the Federal Funds Rate

	ψ_0	ψ_1	$\overline{R}^2$	SE
Standard Specification				
One quarter ahead	-0.02 -0.36	0.80 5.49	0.37	0.36
Two quarters ahead	-0.05 -0.55	0.56 3.03	0.14	0.43
Three quarters ahead	-0.07 -0.63	0.46 2.31	0.08	0.44
Alternative Specification				
One quarter ahead	-0.02 -0.37	0.83 4.44	0.30	0.38
Two quarters ahead	-0.04 -0.44	0.67 2.64	0.11	0.43
Three quarters ahead	-0.05 -0.50	0.63 2.36	0.09	0.44

Notes: OLS estimates. T-statistics shown below parameter estimates are based on Newey and West (1987) heteroskedasticity- and serial-correlation-corrected standard errors. $\overline{R}^2$ and standard errors of the residuals are reported for the first difference of the Federal Funds rate. Sample period for estimation 1990Q1–2004Q2.

Thus we can conclude that the issue of predictability of the short-term interest rate might be misleading. There seems to be nothing wrong with postulating a partial adjustment component in empirical Taylor rules, at least from the point of view of predictability. Clearly, so far in our analysis it remains still open the issue of which type of partial adjustment is the one actually followed by the Fed. In this perspective it is important to stress that, even if the illusion of monetary inertia hypothesis might not seem to be supported by the predictability argument, Rudebusch's hypothesis might still be supported by the emerging of evidence in favor of our alternative specification of monetary inertia. In fact, contrary to the standard specification, our specification of monetary inertia reflects more the dynamic structure of the economy, as it is inconsistent with an interest-rate smoothing objective in the central bank's loss function. Indeed, the partial adjustment component in the alternative specification could be termed "structural inertia", as opposed to "monetary inertia" for the partial adjustment component in the standard specification of the inertial Taylor rule. On the contrary the presence of serially correlated errors in the policy rule (for both specifications) should more reflect the presence of serially correlated variables (different from inflation and output gap) usually omitted in the literature on interest rate rules. As shown for instance by Driffill et al. (2006), likely candidates for these omitted variables are indicators of financial stress related to a financial stability motive.

In order to find more indirect evidence on the two specifications of the partial adjustment component we consider the Expectations hypothesis of the term structure of interest rates. This theory posits that the long rate $i_{t,T}$ is related to the current short rate and future expected short rates as follows:

$$i_{t,T} = \frac{1}{T-t} \sum_{j=1}^{T} E_t i_{t+j-1,t+j} + \Omega_t, \tag{32}$$

where Ω_t is a term premium often assumed constant or stationary. Let's compare the 3-month eurodollar rate taken at the beginnig of the quarter, i_t^{3M}, with the forecasted quarterly average Fed funds rate, $E_{t-1}i_t$, derived from the previous simulation excercise for the two specifications considered. This implies estimating the following equation

$$i_t^{3M} = \tau_0 + \tau_1 E_{t-1} i_t + \xi_t, \tag{33}$$

by assuming a constant term premium with $\Omega_t = \Omega = \tau_0$.

The Expectations hypothesis implies that the spread between current long and short rates should predict future changes in the short rate. Unfortunately, researchers have generally found absence of predictive information. This result has been widely interpreted as a rejection of the expectations hypothesis. An exception in the literature on the forecasting ability of yield spreads is represented by the overnight spread. As reported for instance in Rudebusch (1995), spreads between the overnight Fed's funds rate and one-month or three-month rates predict relatively well changes from the current daily overnight rate to the average daily rate over a one- or three-month horizon. Moreover, Mankiw and Miron (1986), focusing on the three-month and six-month yield spreads, have argued that the negligible predictive power of the term structure for future short rates is more an implication of the Fed's stabilization of short rates rather than a failure of the Expectations hypothesis. Rudebusch (1995) has provided a more rigorous empirical generalization of the link between Fed's behavior and the preditive content of the term structure, with the maintained hypothesis of rational expectations. All the above discussion implies that the 3-month eurodollar rate and the expected Fed funds rate should support the Expectations hypothesis.

Finally, Favero (2002) has argued that the rejection of the expectations model found in the literature is based on the estimation of single-equation models and on the assumption that realized returns are a valid proxy for expected returns. By taking into account short-term interest rates simulated forward from a small empirical macro model, similar to the one used in here, he provides evidence on the US term structure that does not lead to a rejection of the expectations model.

In Table 9 we have reported OLS estimates for this direct test of the Expectations hypothesis for the period 1990 Q1 -2004 Q2. If the theory formalized by (32) is correct we should have $\tau_1 = 1$. As it is possible to see from the Wald test reported in Table 9, the Expectations-hypothesis restriction is not rejected for both specifications of the inertial Taylor rule. In Table 9, the reported t-student have been corrected with Newey-West method as estimated errors exihibit serial correlation.

Unfortunately, the above testing procedure is based on a misspecified regression equation as there exists sound evidence that term premia are time-varying. For instance, studies of long- and short-term Treasury securities have shown that excess resturns in Treasury markets are significantly time-varying and predictable and correlated with the business cycle (see for instance Cocharane and Piazzesi 2002). While excess returns on federal funds futures have been found relatively well predicted by both macroeconomic indicators (like employment growth) and financial business-cycle indicators, and present a strong countercyclical pattern (see Piazzesi and Swanson 2004). Indeed the omission of a persistent, serially correlated variable that influences the term premium could affect the estimation of τ_1 in equation (33). In order to check for this problem we can use the same strategy used for the inertial Taylor rule in presence of serially correlated shocks faced by the central bank. Thus assuming that the shock ξ_t follows an AR(1) process

$$\xi_t = \theta \xi_{t-1} + \varepsilon_t, \tag{34}$$

now we estimate by means of NLS the following equation:[21]

$$\Delta i_t^{3M} = \tau_0 + \tau_1 \Delta E_{t-1} i_t - (1-\theta)\left(i_t^{3M} - \tau_1 E_{t-1} i_t\right) + \varepsilon_t. \tag{35}$$

In table 9 we have reported the NLS estimates for this new direct test of the Expectations hypothesis for the period 1990 Q1 -2004 Q2. As the Wald test in Table 9 shows, the expectations-hypothesis restriction is rejected only for the standard inertial specification of the Taylor rule. This finding suggests that rule (26) is more likely to be the rule actually implemented by the Fed.

Table 9 A Direct Test of the Expectations Hypothesis

[21]The starting values for NLS were derived by means of Two-Stage Least Squares (TSLS).

	Standard rule		Alternative rule	
	Baseline Equation	Serially Correlated Shocks	Baseline Equation	Serially Correlated Shocks
τ_0	0.09 0.56	0.05 0.55	0.05 0.33	0.08 0.76
τ_1	1.03 32.74	0.68 7.61	1.04 32.99	1.02 24.78
θ		0.94 18.95		0.51 4.01
$\tau_1 = 1$ (p-val)	0.28	0.00	0.19	0.69
$\overline{R}^2$	0.97	0.97	0.97	0.97
SE	0.33	0.32	0.36	0.32

Notes: OLS estimates for the baseline equaiton, non-linear least squares for the equations serially correlated errors. T-statistics beneath estimated parameters based on Newey-West (1987) heteroskedasticity- and serial-correlation-corrected standard errors. $\overline{R}^2$ and standard errors (SE) of residuals are reported for the level of the Federal Funds Rate; p-values of the F-statistic for the Wald test of the hypothesis $\tau_1 = 1$ are reported. Sample period for estimations is 1990Q1 – 2004Q2.

3.7 Real time data

We re-estimate specifications (25) and (26) using real-time data, instead of ex post revised data.[22] Real-time data on the output gap is shown in Figure 5. The two gap series differ in that the turning points in the real-time data occur later than they do in ex post revised data. There may be measurement errors in the estimates in Tables 2 and 3 if they are based on data that were not available to the Federal Reserve at the time of its policy decisions.[23] The real-time measures of the output gap and inflation used are based on the given quarter's releases of data for the

[22] For easing the comparison with the findings of English, Nelson and Sack (2003), we have used the same real-time data considered in their work. We thank Brian Sack for having kindly provided us the data. We have also used the real-time data used in Orphanides (2001), but we have got similar results. The estimations based on this last data set are not reported, but are available upon request. We thank Athanasios Orphanides for having kindly provided us the data.

[23] See Orphanides (2001) for an analysis of the informational problems related with the estimation of simple monetary policy rules. In particular he shows that estimates derived from ex post revised data differ remarkably from estimates derived from real-time data.

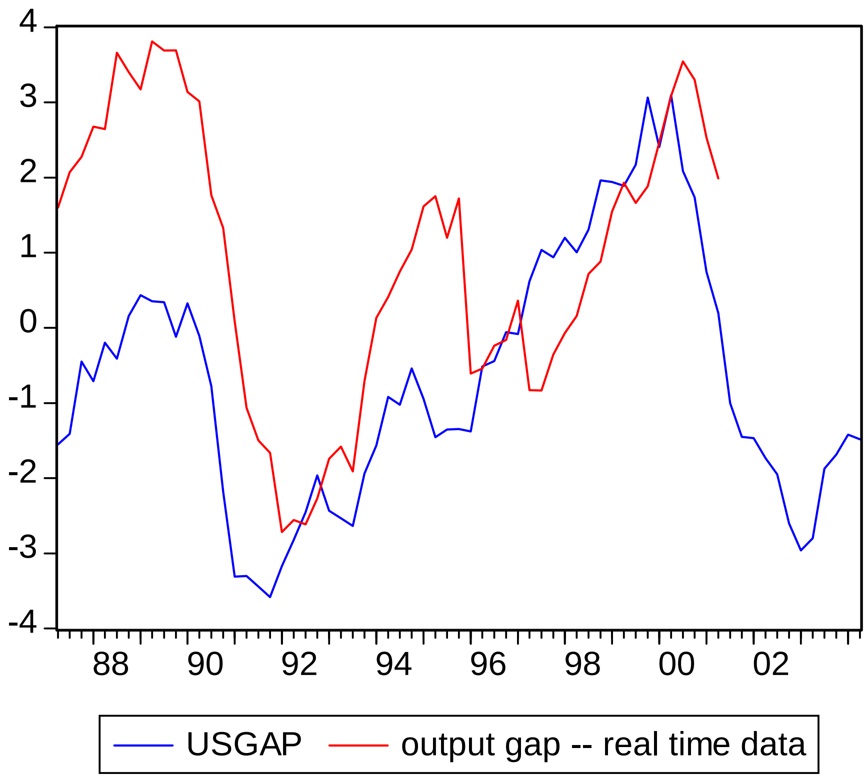

Figure 5: Measures of the output gap, on ex post revised and real time data

previous quarter.[24] In re-estimating specification (26) we have assumed that the lagged operating target $\bar{i}_{t-1}$ related to rule (22) is based on real-time data available at time $t-1$ and is not revised at time t.[25]

Table 10 reports Nonlinear Least Squares estimates of specifications (25) and (26) for the period from 1987 Q4 to 2001 Q2. The results confirm the presence of both partial adjustment and serially correlated

[24] The real-time data set is made available by the Federal Reserve Bank of Philadelphia.

[25] It could be argued that it would be more plausible to assume that the central bank revises also the lagged operating target by considering the real time values of past data available at time t. Nevertheless, for inflation our assumption can be viewed as a good approximation, as inflation is revised relatively less heavily than output. Thus the assumption adopted is satisfactory given our purpose to show that, opposite to the standard specification, with our alternative specification of the Fed's policy rule the inflation coefficient becomes significant.

errors in the estimated interest rate rules. However, like the findings of Orphanides (2001), in the case of rule (25) the coefficient of inflation falls below one and is not statistically different from zero.[26] This is unfortunate! As Henderson and McKibbin (1993) and Clarida, Galì and Gertler (2000) show, a coefficient on inflation greater than one is required for stability in macroeconomic models with policy rules of this type.[27] However, in rule (26) the coefficient of inflation is statistically different from zero. This suggests that, on the basis of real-time data, rule (25) is misspecified, while the correct specification is more likely to be rule (26). There is still the problem that the coefficient of inflation in rule (26) is greater than one only for the subsample 1987 Q4 - 1993 Q4, where it equals 1.04. The fact that it is not greater than one for the period 1987 Q4 - 2001 Q2 could be due to the Federal Reserve reacting to more timely information than the lagged GDP deflator. Forecasts from surveys or alternative indicators of inflation might be included in the information set available for the policy maker. The null hypothesis that the coefficient of inflation in rule (26) is equal to 1.04 also for period 1987 Q4 - 2001 Q2 is not rejected using a Wald test.

Table 10 Inertial Taylor Rules with Real Time Data

	Standard specification		Alternative specification	
	87Q4–93Q4	87Q4–01Q2	87Q4–93Q4	87Q4–01Q2
μ_0	3.61 (1.98)	3.66 (4.57)	2.05 (2.21)	2.81 (1.65)
$\overline{\mu}_\pi$	0.47 (0.73)	0.47 (1.36)	1.04 (4.17)	0.71 (3.92)
$\overline{\mu}_y$	0.95 (3.50)	0.64 (1.90)	0.69 (6.39)	0.54 (5.43)
ρ	0.67 (3.95)	0.65 (2.71)	0.43 (2.99)	0.37 (3.62)
θ	0.26 (1.63)	0.73 (2.17)	0.65 (3.80)	0.94 (10.15)
$\overline{R}^2$	0.98	0.96	0.97	0.95
SE	0.37	0.34	0.41	0.38

[26] In the working paper version of their analysis, of 2002, also English, Nelson and Sack report a not significant coefficient for inflation in the standard inertial Taylor rule in the estimates based on real-time data (see table 3 in their text).

[27] The principle that interest rate rules should respond more than one for one to changes in inflation is called "Taylor principle": see for instance Walsh (2003). However, Bullard and Mitra (2002) and Woodford (2003b) have shown that in general the necessary and sufficient condition required for stability may have a more complex form than that expressed by the Taylor principle. In particular it is possible to show that $\overline{\mu}_\pi > 1$ is only a necessary condition for the determinacy of the rational expectations equilibrium, and even values of $0 < \overline{\mu}_\pi < 1$ can be consistent with stability. However, as argued by Woodford (2003b, p. 254) the Taylor principle continues to be a crucial condition for determinacy if it is reformulated as: "[...] *At least in the long run, nominal interest rates should rise by more than the increase in the inflation rate*".

Notes: Non-linear least squares estimates. T-statistics beneath estimated parameters based on Newey-West (1987) heteroskedasticity- and serial-correlation-corrected standard errors. $\overline{R}^2$ and standard errors (SE) of residuals are reported for the level of the Federal Funds Rate. The data set used here is the same as that used by English *et al* (2003).

4 Conclusions

In this paper we have attempted to add to the many already-existing explanations for inertia in empirical Taylor rules. Our proposal is that the optimal interest rate rule for stabilising inflation and the output gap will typically inherit the inertia in the economic system itself. If the evolution of the output gap and inflation depends on their own lagged values, then the rule for the control variable, the interest rate, will typically do the same. When estimated empirically, a rule in which the interest rate depends on current and lagged values of the state variables – the output gap, inflation, and so on – may look rather like one in which the interest rate depends on its own lagged values. The picture is likely to be further confused by omitted autocorrelated variables which engender a serially correlated error term in the estimated equation. We have derived a rule from a simple macroeconomic model. The optimal interest rate rule implied by crude estimates of this model looks something like a modified form of Taylor rule with inertia. When we estimate alternative forms of interest rules directly, our alternative formulation is not wholly inconsistent with the data. While it does not completely supplant the standard Taylor rule, neither does the standard rule explain the data satisfactorily. A hybrid model containing elements of both appears to perform rather better than either alone.

Rudebusch and others have pointed to the inconsistency between the apparent forecastability of interest rates implied by the inertia in estimated Taylor rules, and the lack of forecastability implied by yield curves. The future interest rates implicit in yield curves for Trasury Bills are not good forecats of future interest rates. However, it turns out that, with the modified form of inertial Taylor rule, allowing for the need to forecast the output gap and inflation that enter the rule, there does not appear to be significant inconsistency between the implications of the yield curve data and the direct estimates of the Taylor rule.

The results obtained here are suggestive rather than conclusive. This line of enquiry needs to be developed in a number of ways. The macroeconomic model we used contains no forward looking behaviour or other nods in the directions of microeconomic foundations, and the empirical estimates of it a crude in the extreme. We need to use a more conceptually coherent model and to obtain better quality estimates of it.

We need to examine the implied forecastability of interest rates from alternative pieces of data more carefully.

APPENDIX

The equation for inflation can be written as

$$\pi_{t+2} = \alpha_1 \Delta_t + (1 - \alpha_2)\, \pi_{t+1} + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + \epsilon_{t+2}, \tag{36}$$

and this can be converted to a system of first order difference equations so that it can be written as a standard dynamic programming problem. The choice of Δ_t is made at time t knowing π_t, $E_t(\pi_{t+1})$, y_t and so on. So we write the equation as

$$E_{t+1}(\pi_{t+2}) = \alpha_1 \Delta_t + (1 - \alpha_2)\, E_t(\pi_{t+1}) + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + (1 - \alpha_2)\epsilon_{t+1}$$

and we supplement the system with

$$\pi_{t+1} = E_t(\pi_{t+1}) + \epsilon_{t+1}$$

then we have a first order system in the two variables $E_t(\pi_{t+1})$ and π_t. It can be written as

$$z_{t+1} = Az_t + Bu_t + \nu_{t+1}$$

where we have defined $z_t \equiv \begin{bmatrix} E_t(\pi_{t+1}) \\ \pi_t \end{bmatrix}$, $u_t \equiv [\Delta_t]$ and $\nu_{t+1} \equiv \begin{bmatrix} \alpha_1 \eta_{t+1} + (1 - \alpha_2)\epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}$ and the parameter vectors and matrices are $A = \begin{bmatrix} 1 - \alpha_2 & \alpha_2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \alpha_1 \\ 0 \end{bmatrix}$.

The period loss function is

$$L_t = (1/2)(\pi_t^2 + \lambda y_t^2)$$

and we try to minimize $E_t \left(\sum_{s=t}^{\infty} \delta^{s-t} L_s \right)$ by choosing a sequence of $\Delta_t, \Delta_{t+1}, \Delta_{t+2}, \ldots.$ So we can write the period loss function (for period t+1) as

$$L_{t+1} = (1/2) \left((E_t(\pi_{t+1}) + \epsilon_{t+1})^2 + \lambda(\Delta_t + \eta_{t+1})^2 \right)$$

In terms of expected values as of date t, we have

$$E_t(L_{t+1}) = (1/2) \left\{ z_t' R z_t + u_t' \lambda u_t + (\sigma_\epsilon^2 + \lambda \sigma_\eta^2) \right\}$$

Now the problem can be written in a standard form. We choose Δ_t so that

$$V_t(E_t(\pi_{t+1}, \pi_t) = \min_{\Delta_t} E_t \left\{ z_t' R z_t + u_t' \lambda u_t + (\sigma_\epsilon^2 + \lambda \sigma_\eta^2) + \delta V_{t+1}(E_{t+1}(\pi_{t+2}), \pi_{t+1}) \right\},$$

subject to the equation of motion of the system given above. The cost-to-go function $V_t(E_t(\pi_{t+1}, \pi_t)$ has the form

$$V_t(E_t(\pi_{t+1}, \pi_t) = z_t' v_t z_t + k_t$$

where k_t is a constant (whose value depends on the variance terms). So we can write the problem as

$$z_t' v_t z_t + k_t = \min_{u_t} E_t \left\{ z_t' R z_t + u_t' \lambda u_t + (\sigma_\epsilon^2 + \lambda \sigma_\eta^2) + \delta(z_{t+1}' v_{t+1} z_{t+1} + k_{t+1}) \right\},$$

This is the standard textbook formulation of the dynamic programming problem. The first order condition gives

$$E_t \left[\lambda u_t + \delta B v_{t+1} z_{t+1} \right] = 0$$

or

$$\lambda u_t + \delta B' v_{t+1}(A z_t + B u_t) = 0$$

hence the feedback rule

$$u_t = -(\lambda + \delta B' v_{t+1} B)^{-1} \delta B' v_{t+1} A z_t$$

which is conventionally written as

$$u_t = F_t z_t$$

with

$$F_t \equiv -(\lambda + \delta B' v_{t+1} B)^{-1} \delta B' v_{t+1} A$$

Putting the feedback rule back into the expression for the cost-to-go function above gives

$$v_t = R + F_t' \lambda F_t + \delta(A + BF_t)' v_{t+1}(A + BF_t)$$

In the infinite horizon case, assuming the system can be controlled and we have convergence, $v_t = v_{t+1} = v$, and

$$v = R + A'[\delta v - \delta v B(\lambda + B' \delta v B)^{-1} B' \delta v] A$$

and

$$F \equiv -(\lambda + \delta B' v B)^{-1} \delta B' v A$$

What does all this imply for the interest rate rule? We have from the above that

$$\Delta_t = f_1 E_t(\pi_{t+1}) + f_2 \pi_t$$

where $F = [f_1 \, f_2]$. Since the control variable Δ_t is defined as

$$\Delta_t \equiv \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1}$$

and since

$$E_t(\pi_{t+1}) = \alpha_1 y_t + (1-\alpha_2)\,\pi_t + \alpha_2\pi_{t-1}$$

the rule for the interest rate becomes

$$i_t = \frac{\beta_1 - f_1\alpha_1}{\beta_2}y_t + \frac{\beta_4 - f_1(1-\alpha_2) - f_2}{\beta_2}\pi_t + \frac{\beta_3}{\beta_2}y_{t-1} + \frac{f_1\alpha_2 + \beta_5}{\beta_2}\pi_{t-1}$$

Are Interest Rates, the Output Gap, and Inflation Stationary?

Some readers may be curious as to whether the variables we have used a stationary or have unit roots. In some sense, if the US Federal Reserve is pursuing an effective policy to keep inflation low and output close to capacity, all three variables are highly likely to be stationary. In most of the empirical analysis in the paper it is assumed that the variables are stationary. However, in some of the estimated equations the dependent variables have been expressed in first differences, such as the change in the interest rate; and the independent variables have been expressed in changes and in linear combinations of lagged levels, which are stationary even if some of the individual component variables are not, providing the US Federal Reserve is following something like a Taylor Rule in the long run.

For the output gap, for the sample 1960Q4 – 2004Q2, we obtain an augmented Dickey-Fuller (ADF) test statistic of -3.55, with a p-value of 0.0076 for the null hypothesis of a unit root. On this test, a unit root is rejected. For the Federal Funds rate, over a sample 1961Q1 – 2004Q2, the ADF test statistic is -2.41, with a p-value of 0.14. Here a unit root cannot be ruled out. For inflation, over the sample 1961Q3 to 2004Q2, the ADF test statistic is -2.23, with a p-value of .20. Again, a unit root cannot be rejected. The non-rejection of a unit root in inflation and nominal interest rates is not unexpected. Both have been persistent, and there was a marked rise in both until the late seventies and early eighties, since when both have drifted back down to low single figures (at an annual percentage rate). The non-rejection may just reflect the meeting of stationary but persistent series with a test of known low power.

If a unit root in these were accepted, then it would be legitimate to estimate a long run Taylor rule from a regression of the interest rate on inflation and the output gap. Doing that for the sample 1987Q4–2004Q2 yields

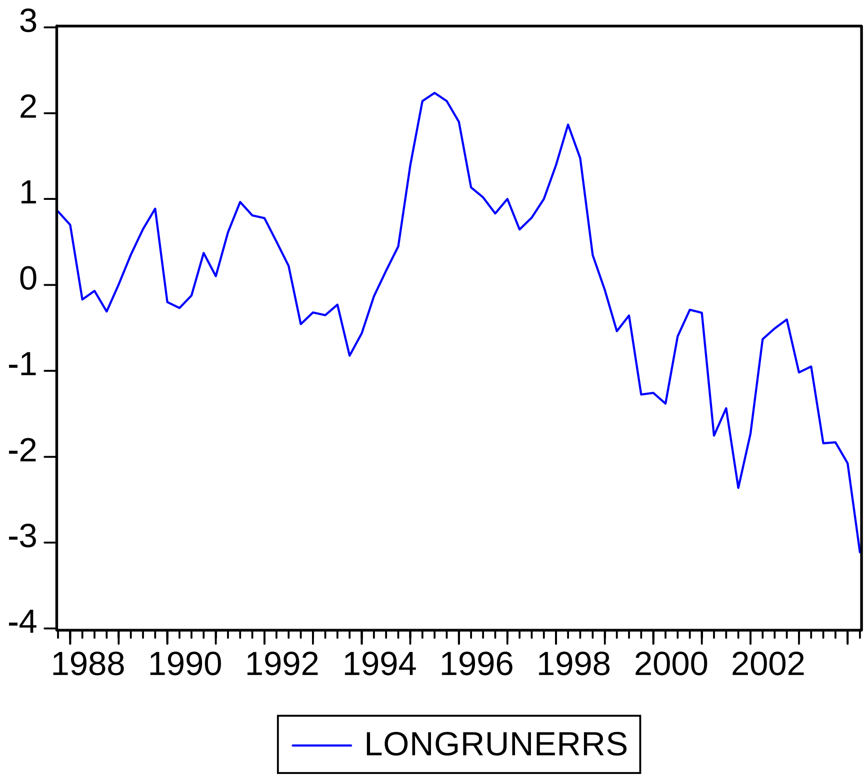

Figure 6: Residuals in estimated long run Taylor rule

$$i_t = \underset{1.36}{0.58} + \underset{9.68}{0.83} y_t + \underset{11.93}{2.14} \pi_t$$
$$R^2 = .75, DW = 0.21,$$

(T-statistics beneath estimated parameters.) The parameters are not massively dissimilar from the 'Taylor' values of 0.5 and 1.5. Instead we have 0.83 amd 2.14, implying a stronger long-run response. The errors from this equation are shown in figure 6. They do not look particularly stationary. The persistent fall from around 1995 to 2004 may reflect the under-measurement of the output gap as the economy grew more strongly than expected without inflation taking off, and the Fed's allowing interest rates to remain low.

REFERENCES

Ball, L., 1999. Efficient rules for monetary policy. *International Finance.* 2:63-83.

Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. *Journal of Monetary Economics.* 49:1105-1129.

Clarida, R., Galì, J., Gertler, M., 1998. Monetary rules in practice: some international evidence. *European Economic Review.* 42:1033-1067.

Clarida, R., Galì, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. *Quarterly Journal of Economics.* 115:147-180.

Cochrane, J.Y., Piazzesi, M., 2002. Bond risk premia. NBER working paper, no. 9178.

Cukierman, A., 1992. *Central bank strategy, credibility, and independence - theory and evidence.* MIT Press, Cambridge.

Driffill, J., Rotondi, Z., Savona, P., Zazzara, C., 2006. Monetary policy and financial stability: What role for the futures market? *Journal of Financial Stability.* 2:95-112.

English, W., Nelson, W., Sack, B., 2002. Interpreting the significance of the lagged interest rate in estimated monetary policy rules. Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series.

English, W., Nelson, W., Sack, B., 2003. Interpreting the significance of the lagged interest rate in estimated monetary policy rules. *Contributions to Macroeconomics.* 3:1-16.

Eijffinger, S., Schaling, E., Schaling, V., 2000. The term structure of interest rates and inflation forecast targeting. CEPR discussion paper, no. 2375.

Ellingsen, T., Söderström, U., 2001. Monetary policy and market interest rates. *American Economic Review.* 91:1594-1607.

Favero, C.A., 2002. Taylor rules and the term structure. IGIER working paper series, no. 195. Forthcoming *Journal of Monetary Economics.*

Goodfriend, M., 1987. Interest rate smoothing and price level trend-stationarity. *Journal of Monetary Economics.* 19: 335-348.

Goodfriend, M., 1991. Interest rates and the conduct of monetary policy. *Carnegie-Rochester Conference Series on Public Policy.* 34:7-30.

Goodhart, C., 1997. Why do monetary authorities smooth interest rates? In: Collignon, S. (Ed.), *European monetary policy.* 119-178. Pinter, Washington, DC.

Goodhart, C., 2005. The Monetary Policy Committee's reaction function: an exercise in estimation. *Topics in Macroeconomics.* 5:1-40.

Jensen, H., 2002. Targeting nominal income growth or inflation? *American Economic review.* 92:928-956.

Lansing, K., 2002. Real-time estimation of trend output and the illusion of interest rate smoothing. *Federal Reserve Bank of San Francisco Economic Review.* 17-34.

Mankiw, N.G., J.A. Miron, 1986. The changing behavior of the term strucutre of interest rates. *Quarterly Journal of Economics.* 101:211-228.

McCallum, B.T., 1997. The alleged instability of nominal income targeting. NBER working paper, no. 6291.

Newey, W., West, K., 1987. A simple positive-definite heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica.* 55: 703-708.

Orphanides, A., 2001. Monetary policy rules based on real-time data. *American Economic Review.* 91:964-985.

Oprhanides, A., 2003. Monetary policy evaluation with noisy information. *Journal of Monetary Economics.* 50:605-31.

Piazzesi, M., Swanson, E., 2004. Futures prices as risk-adjusted forecasts of monetary policy. NBER working paper, no. 10547.

Rudebusch, G., 1995. Federal Reserve interest rate targeting, rational expectations, and the term structure. *Journal of Monetary Economics.* 35:245-274.

Rudebusch, G., 2001. Is the Fed too timid? Monetary policy in an uncertain world. *Review of Economics and Statistics.* 83:203-217.

Rudebusch, G., 2002. Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Policy.* 49:1161-1187.

Rudebusch, G., 2005. Monetary policy inertia: fact of fiction? Mimeo Federal Reserve Bank of San Francisco.

Rudebusch, G., Svensson, L.E.O., 1999. Policy rules for inflation targeting. In: Taylor, J., (Ed.), *Monetary policy rules.* 203-46. University of Chicago Press, Chicago.

Sack, B., 1998. Uncertainty, learning and gradual monetary policy. FEDS working paper, no. 34, Federal Reserve Board.

Sack, B., 2000. Does the Fed act gradually? A VAR analysis. *Journal of Monetary Economics.* 46:229-256.

Sack, B., Wieland, V., 2000. Interest rate smoothing and optimal monetary policy: a review of recent empirical evidence. *Journal of Economics and Business.* 52:205-228.

Söderlind, P., Söderström, U., Vredin, A., 2003. Taylor rules and the predictability of interest rates. CEPR discussion paper, no. 3934.

Startz, R., 2003. Partial adjustment as optimal response in a dynamic Brainard model. Mimeo University of Washington. Available also at: http://www.ssrn.com.

Svensson, L.E.O., 1997. Inflation forecast targeting: Implementing and monitoring inflation targets. *European Economic Review.* 41:1111-1146.

Svensson, L.E.O., 1999. Inflation targeting: some extensions. *Scandinavian Journal of Economics.* 101:337-361

Svensson, L.E.O., 2003. What is wrong with Taylor rules? Using judgement in monetary policy through targeting rules. *Journal of Economic Literature.* 41:426-477.

Taylor, J., 1993. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy.* 39:195-214.

Walsh, C.E., 2003. *Monetary theory and policy.* MIT Press, Cambridge, MA.

Wieland, V., 1998. Monetary policy and uncertainty about the natural unemployment rate. FEDS discussion paper, no. 22, Federal Reserve Board.

Williams, J.C., 2003. Simple rules for monetary policy. *Federal Reserve Bank of San Francisco Economic Review.* :1-11.

Woodford, M., 1999. Optimal monetary policy inertia. *The Manchester School Supplement.* 67:1-35.

Woodford, M., 2003a. Optimal interest rate smoothing. *Review of Economic Studies.* 70:861-86.

Woodford, M., 2003b. *Interest and prices: Foundations of a theory of monetary policy.* Princeton University Press, Princeton and Oxford.