Strategic Argumentation

Job Market Paper

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Abstract

I analyze a communication game in which an uninformed decision maker chooses an action based on the advice of an informed but possibly biased expert. The quality of each alternative is described by a set of arguments, and each argument favors one of two alternatives. Each argument is verifiable, but the number of arguments is not. The expert selects a subset of arguments to reveal to the decision maker.

In all equilibria the biased expert exaggerates his reports in favor of his preference, yet he does not suppress all of the unfavorable information. All equilibria with continuous belief function are outcome equivalent and most informative, and a unique equilibrium survives when a small cost of concealing information is added. In every continuous equilibrium, if the expert reports many arguments, the decision maker can infer the expert's bias, and she bases her choice solely on the number of arguments that favor the expert; otherwise the expert's report is ignored. If the decision maker expects the expert to be honest, the biased expert inflates his reports more.

If experts differ in the number of arguments they observe, a high-quality expert is better informed, but the decision maker may be able to infer more information from the low-quality expert's reports.

1 Introduction

Consider an investor who consults a financial adviser to help her choose between two investment options. The quality of each investment depends on many characteristics: average predicted returns, risk, correlation between the returns and the investor's income, liquidity, etc. The investor does not know the values of these characteristics, but more importantly, she does not even know what the characteristics are and how many are relevant. For example, she may not know whether a given investment is risky, but she also may not realize that some investments offer tax breaks or differ in liquidity. The financial adviser can credibly reveal any characteristics of the investments. However, since the investor does not know how many characteristics there are, the financial adviser can conceal some of them.[1] Additionally, the interests of the investor may not coincide with the interests of the financial adviser. The financial adviser may be honest, or he may have an agenda. For example, the financial adviser may always recommend an investment fund that pays him a commission for each persuaded investor.

Many situations exhibit similar features. A patient does not know what factors she should consider when choosing a treatment, and she relies on the information provided by her doctor, but the doctor may benefit financially if the patient chooses a particular option. An author of an academic article will not fabricate results, but he can present them selectively. A journalist may omit unfavorable information about his favorite candidate in an election. A lobbyist may hide an unfavorable analysis.

In this paper I formalize these situations in a communication game, focusing on two primary questions. First, I am interested in how much information is transmitted and whether there is room for persuasion when lying is not allowed and information manipulation takes place through selective disclosure. Second, I am interested in the details of communication. Anecdotal evidence suggests that even a biased expert reveals some information that is unfavorable to him. Even if the financial adviser wants the investor to choose a particular option, he is likely to mention some positive characteristics of the alternative option as well. Many commercials use two-sided messages; for example, an ad for dBase IV software attempted to persuade the consumers of the software's superiority by disclosing that it was more costly and poorer at handling errors than the competing products.[2] I am interested in whether these observations can be explained in a simple, game-theoretical framework.

[1] First, it may be illegal to misrepresent the facts. Second, the investor may be able to verify the aspects revealed by the adviser. On the Internet, almost any information is available to the decision maker, and the role of the expert is to identify the information that is relevant. For example, an investor may not think about looking at tax breaks, but once informed about their existence, she can easily verify whether a given investment offers a tax break.

[2] See Pechmann (1992). Other examples include Continental Airlines admitting a variety of problems such as canceled flights and lost luggage when trying to persuade the clients about its new commitment to quality (Crowley and Hoyer, 1994).

The existing literature is not well suited to analyze these questions. Information is usually modeled as a single variable, which can either be credibly disclosed[3] or is non-verifiable.[4] My motivating examples fit neither of these scenarios. Additionally, in order to understand whether a rational model can explain the use of unfavorable information by a biased expert, I need to model characteristics explicitly. And finally, in contrast to a large body of the literature, I want to analyze situations in which the preference of the expert is unknown to the decision maker.[5]

The communication game in this paper has the following structure. There is one expert and one decision maker. The decision maker chooses one of two alternatives–*Right* or *Left.* The quality of the alternatives is described by a set of arguments, each of which favors one alternative. The total number of arguments is a random variable and is known to the expert, but is unknown to the decision maker. The expert can credibly reveal any subset of the arguments; that is, he cannot misrepresent any argument, but he can hide some of them. The expert can be either an honest type, who reveals all of the arguments, or a persuader, who wants the decision maker to choose one particular alternative, independent of its quality. The type of the expert is private information. The decision maker cares about the quality of the alternatives, but she also has some prior preference bias. This bias is unknown to the expert, which implies that he does not know what quality suffices to make the decision maker choose a particular alternative.

I characterize all equilibria in a version of the game in which the expert can be either honest or biased in favor of *Right*, and show that there is a subset of equilibria for which the belief function is continuous. Moreover, across all equilibria in this set, the belief function is the same, which implies that all equilibria in this set are outcome equivalent. Apart from the obvious attractiveness of the continuity property, continuous equilibria exhibit other appealing properties. First, they are the most informative equilibria of the game. Second, in the general version of the game with the two-sided bias, the unique equilibrium that survives the introduction of a small cost of concealing information has continuous belief function. For most of the analysis I focus on the properties of the continuous equilibria.

The model delivers three main findings. First, in every equilibrium, the persuader biases his reports in favor of his preference, but he does not completely suppress all unfavorable information. The reason for that is the following. The decision maker is aware that the

[3] See Grossman and Hart (1979) and Grossman (1981).

[4] See Crawford and Sobel (1982), Krishna and Morgan (2001), Levy and Razin (2006), Battaglini (2002), and Chakraborty and Harbaugh (2005).

[5] The large body of the literature assumes that the preferences of the expert are common knowledge; see, for example, Crawford and Sobel (1982), Krishna and Morgan (2001), and Battaglini (2002). Sobel (1985), Benabou and Laroque (1992), Morris (2001), and Morgan and Stocken (2003) assume uncertainty about the expert's preference, but allow the bias to be only one-sided. Dimitrakas and Sarafidis (2003), and Li (2005) allow for two-sided bias in cheap-talk games. Only Wolinsky (2003) allows for two-sided bias in a game with partially verifiable signals.

persuader may conceal arguments; therefore, she tries to infer the preference of the expert. Since the honest expert reveals all of the arguments and he is very unlikely to have only arguments that favor one alternative, the persuader needs to reveal unfavorable information to convince the decision maker that he is honest.

Second, in a situation in which the expert can either be honest or be a persuader toward *Right*, the decision maker bases her decision solely on the number of arguments that favor *Right*, unless she observes a very balanced report. It is in the interest of both types of experts (honest and persuader) to reveal all arguments that favor *Right*, which is why the decision maker observes all arguments of this type. It is surprising, however, that she ignores the arguments that favor *Left.* Although these arguments carry some information about the quality of the alternatives, they do not carry any information about how many arguments of this sort have been concealed, and therefore they do not help the decision maker with her decision.

When the expert can be biased in favor of any alternative, the decision maker does not know whether the expert wants to conceal arguments that favor *Right* or those that favor *Left.* This may suggest that little information is revealed in the equilibrium. However, if the expert uses many arguments, in the equilibrium the decision maker is able to infer the bias of the persuader; only the honest expert and the persuader toward *Right* report many arguments that favor *Right.* Hence, the decision maker understands that there are no arguments that favor *Right* other than the ones that the expert discloses. On the other hand, when the expert reveals few arguments, the decision maker cannot infer the direction of the potential bias. She does not know whether she faces a persuader toward *Right,* in which case he is likely to have concealed many arguments that favor *Left,* or she faces the persuader toward *Left,* in which case he is likely to have concealed many arguments that favor *Right.* As a result, when she observes few arguments, the decision maker ignores them completely.

The ability of the decision maker to extract information depends crucially on her uncertainty about the choice problem. If the decision maker is familiar with the problem, she has a more precise estimate of how many arguments she should consider. For example, an experienced investor knows more about the complexity of investing than an inexperienced one. I show that the decision maker's utility increases with her familiarity with the problem.

The model easily extends to situations in which the expert receives only a fraction of the relevant arguments. Allowing the experts to differ in how many arguments they observe leads to surprising results. There is a trade-off between consulting a high-quality expert and a low-quality expert; in some cases, the decision maker may prefer to consult a low-quality expert. A high-quality expert receives a large fraction of the relevant arguments and a low-quality expert receives a small fraction of the relevant arguments. The information contained in the arguments of a high-quality expert is a precise measure of the state of nature. However, the decision maker knows more about how many arguments a low-quality expert is likely to observe; therefore, she can more easily extract the information from him. After observing a report, the decision maker can estimate more precisely how many arguments remain concealed. If the expert is likely to be honest, the first effect is more important. When the expert is likely to be a persuader, the decision maker may be willing to sacrifice some of the precision of the information for a better ability to extract it.

This paper makes three main contributions. First, it complements the existing literature. The papers that are closest to mine are Milgrom and Roberts (1986) and Shin (1994). Milgrom and Roberts (1986) propose a model in which each alternative is characterized by an unknown number of attributes, but they focus on the scope of information transmission when there are two experts with exactly opposing preferences. Competition between experts leads to full information revelation, and the lack of uncertainty about the experts' preferences makes it irrelevant how many unfavorable attributes they disclose. Shin (1994) analyzes a game in which experts manipulate information through selective disclosure, but he puts less structure on the signals that can be revealed. Also, in Shin (1994), the expert's bias is common knowledge; in my paper, it is not. Second, my paper provides insights into the structure of communication. It formalizes the casual observation that even a biased expert is likely to use arguments that oppose his interests. And third, the model presented here is a good starting point to analyze communication in more elaborate settings. The argument structure of information may be useful when analyzing two-sided communication in debates or bargaining situations.

The paper is organized as follows. In Section 2, I describe the game. In Section 3, I analyze a version of the model in which the expert is either honest or a persuader. In Section 4, I extend the analysis to the case in which the expert can be of three types. In Section 5, I provide some comparative statics results and extensions of the model. In Section 6, I highlight the similarities and differences between my model and the existing literature. Section 7 concludes.

2 The Model

The environment

There are two alternatives: *Right* and *Left.* The quality of each alternative is described by a set of N random variables, each of which can take value *Left* or *Right.* Each random variable is called an *argument.* An argument that takes value *Left* favors alternative *Left,* and an argument that takes value *Right* favors alternative *Right.* The interpretation of arguments follows the interpretation of alternatives. If alternative *Left* is "investing in option *Left*" and alternative *Right* is "investing in option *Right,*" then arguments are the relevant aspects of those investments. For example, an argument that "option *Left* had historically higher returns than option *Right*" favors option *Left.*

The number of arguments N is a random variable itself. Let R be the number of arguments that favor *Right* and L be the number of arguments that favor *Left;* hence $L + R = N$. For tractability, I assume that N, L and R are continuous variables, and $N \in [0, 1]$.[6] , That implies that N, L and R are fractions, but I will continue to say *number* when referring to them in order to remind the reader of their interpretation.

Arguments are distinct but equally important. This means that only the number of arguments that favor each alternative matters, not their identity. For example, the situation in which investment *Right* offers a tax break but is illiquid is equivalent to the situation in which investment *Right* is liquid but offers no tax breaks. As a result, we can represent the state space in a simple form: $S = \{(L, R) \in [0, 1] \times [0, 1] : R + L \leq 1\}$.[7]

The quality of an alternative is equal to the fraction of arguments that favor it, $q_R = \frac{R}{R+L}$ and $q_L = \frac{L}{R+L}$.[8] The state of nature is distributed according to the probability density function $f(L, R)$ which is continuous with full support. It satisfies the following regularity condition:

$$
\begin{aligned}
\frac{dE\left[\frac{R}{R+L}|R, L \leq \hat{L}\right]}{dR} &> 0 \text{ for all } R \text{ and } \hat{L}, \\
\frac{dE\left[\frac{L}{R+L}|L, R \leq \hat{R}\right]}{dL} &> 0 \text{ for all } L \text{ and } \hat{R}.
\end{aligned} \tag{1}
$$

Condition 1 says that observing an additional argument that favors one alternative increases the estimate of the quality of this alternative, all else equal.

The expert

The expert observes the state of nature (L, R), and sends a report (λ, ρ), where λ is the number of arguments that favor *Left* that the expert reveals, and ρ is the number of arguments that favor *Right* that the expert reveals. Let $\Sigma = \{(\lambda, \rho) : \lambda + \rho \leq 1,\ \lambda \geq 0,\ \rho \geq 0\}$ denote the set of all reports. A report $(\lambda, \rho) \in \Sigma$ is *feasible* in the state (L, R) if $\lambda \leq L$ and $\rho \leq R$. Let $V(L, R) = \{(\lambda, \rho) \in [0, 1] \times [0, 1] : \lambda \leq L, \rho \leq R\}$ be the set of all such reports. The structure of reporting implies the following. First, the expert must be truthful. Whenever he discloses an argument that favors *Right,* he cannot claim that it favors *Left.* Second, he cannot create arguments. The expert cannot credibly convey to the decision maker that he

[6] The maximum number of arguments is normalized to one, but this normalization is without loss of generality. All results would still hold for $N \in [0, \infty]$ if the same regularity conditions as imposed in the model hold.

[7] Glazer and Rubinstein (2001) show that if the decision maker commits to conditioning her actions on the identities of the arguments, then even if the arguments are ex ante identical, she can extract more information from the expert. In my model such a commitment is infeasible since the decision maker does not know ex ante the identities of the arguments. She cannot commit to choosing *Right* if she receives argument A and *Left* if she receives argument B, because she does not distinguish between arguments A and B, ex ante.

[8] This implies that quality is a relative measure, but it is straightforward to extend my model to a setting in which q_R and q_L are independent.

has disclosed all of the arguments.

There are three types of experts. The expert may be biased toward *Right,* P_r, biased toward *Left*, P_l, or an honest expert, H. An honest expert non-strategically reveals all of the arguments in each state of nature.[9] Biased experts are caller *persuaders.* A persuader toward *Right,* wants the decision maker to choose *Right* independent of the state of nature; that is, he maximizes $\Pr\{Right \text{ is chosen}\}$. The probability that the expert is of type $i \in \{P_l, P_r, H\}$ is π_i.

The reporting strategy of an expert of type i is a function $\sigma_i : (L, R) \to V(L, R)$. A report is *full* if $\sigma_i(L, R) = (L, R)$.

The decision maker

The decision maker does not know the realization of (L, R), but she holds a correct probabilistic belief. She chooses one of the alternatives, and her utility function is:

$$\begin{aligned} U(Right|L,R) &= \frac{R}{R+L} - \theta_i, \\ U(Left|L,R) &= \frac{L}{R+L} + \theta_i - 1, \end{aligned} \tag{2}$$

where $\theta_i \in [0, 1]$ is a preference parameter. This implies that decision maker is risk neutral, and she chooses alternative *Right* if and only if $E\left[\frac{R}{R+L}|\lambda, \rho\right] \geq \theta_i$. She cares only about the quality of each alternative; the total number of arguments does not enter into her utility function because it does not carry any relevant information. Keeping the quality constant, choosing alternative *Right* when the total number of arguments is high does not result in a different utility level than when the total number of arguments is low.

The parameter θ_i describes an ex ante preference of the decision maker. For example, the decision maker may have some intrinsic preference for Honda over Toyota when buying a car, an investor may prefer stocks of environmentally friendly companies, a voter may prefer a Republican candidate because of family tradition, other things equal. In other words, θ_i is the smallest quality of alternative *Right* that will make the decision maker choose *Right.* $\theta_i = \frac{1}{2}$ means that ex ante the decision maker is indifferent between the alternatives.

The decision maker knows her θ_i, but the expert does not. This assumption makes the model more interesting since it implies that the expert does not know exactly when the decision maker prefers *Right.* Let $h(\theta_i)$ be the probability density function of θ_i with the corresponding distribution function $H(\theta_i)$. $h(\theta_i)$ is continuous and has full support over $[0, 1]$.

The decision maker observes a report (λ, ρ), forms a belief about the state of nature, and then chooses the alternative that maximizes her utility. Let $\eta(\lambda, \rho)$ be the equilibrium belief

[9] Later, I show that many of the results are robust to variations in this assumption and would hold if the honest expert were strategic and maximized the utility of the decision maker.

of the decision maker about q_R if she observes a report (λ, ρ). The decision maker does not know the type of the expert.

The game

The triangle in Figure 1 represents the state space S and the report space Σ. If the state of nature is (L_0, R_0), then $V(L_0, \rho_0)$ is the set of all feasible reports. $Z(L_0, R_0)$ is the set of all states of nature that allow the expert to send a report (L_0, R_0).

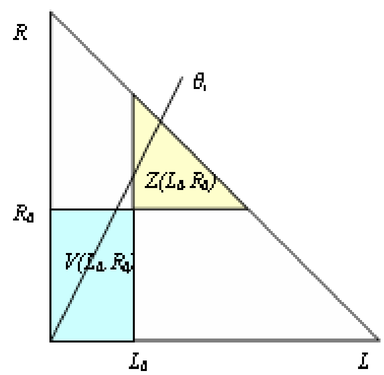


Figure 1: The state space. Each point in the triangle represents a state of nature. The ray from the origin represents the states in which $q_R = \theta_i$.

The line θ_i represents the states of nature that generate the same quality of alternative *Right,* $q_R = \theta_i$. The decision maker of type θ_i prefers to choose alternative *Right* if the state of nature lies above the θ_i line and alternative *Left* otherwise.

First, nature determines the type of the expert $i \in \{P_l, P_r, H\}$ and the set of arguments (L, R). Then, the expert observes his type and the state of nature (L, R), and sends a report (λ, ρ) to the decision maker. After observing the report, the decision maker chooses one of the alternatives.

I look for a perfect Bayesian equilibrium of this game. A perfect Bayesian equilibrium in this game is a reporting strategy for each type of the expert that maximizes his utility given the strategy of the decision maker, the belief function $\eta(\lambda, \rho) = E\left[\frac{R}{R+L}|\lambda, \rho\right]$, and the choice strategy of the decision maker $d(\lambda, \rho) \in \{\textit{Left}, \textit{Right}\}$ that maximizes her utility given her beliefs.

3 One-sided bias

In this section, I consider a situation in which the decision maker knows the direction of the potential bias of the expert. Let π be the probability that the expert is biased toward *Right,* i.e. of type P_r, and $1 - \pi$ be the probability that the expert is honest, i.e. of type H.

Often, the decision maker knows which alternative the expert favors. A sales representative may honestly advise the customer about the quality of his product, but certainly he is not interested in increasing the sales of the competing products. A lawmaker may propose a particular policy because it is beneficial to his constituency, but if it is not, clearly he prefers the policy to be adopted rather than rejected. The financial adviser may be honest, but if he is known to receive a higher commission from a specific investment fund for each persuaded investor, then the investor will be suspicious whenever the adviser recommends this fund. I analyze the case with three types of the expert in Section 4.

3.1 The properties of the equilibria

There is multiplicity of equilibria in this model, which is a typical feature of communication games. Based on the continuity of the equilibrium belief function, $\eta(\lambda, \rho)$, all equilibria can be divided into two groups. Proposition 1 describes common properties of all equilibria, and provides the details of the set of continuous equilibria.

Let the *ambiguity area* be the set of all reports that are used by the persuader in an equilibrium, $\{(\lambda, \rho) : \sigma_{P_r}(L, R) = (\lambda, \rho) \text{ for some } (L, R) \in S\}$. Hence, the ambiguity area includes all reports that do not allow the decision maker to identify the type of the expert.

Proposition 1

a) *There are infinitely many equilibria in this game and all share the following features. In each equilibrium the ambiguity area is a strict subset of* Σ. *The persuader biases his reports toward* Right, *but he does not typically suppress all arguments that favor* Left. *Upon observing the report from the ambiguity area, the decision maker's belief is a function of the revealed number of arguments that favor* Right *only. The belief* $\eta(\lambda, \rho)$ *is weakly increasing in* ρ.

b) *The equilibria do not depend on the distribution of the decision maker's preference* θ_i.

c) *There is a unique equilibrium belief function* $\eta(\lambda, \rho)$ *that is continuous in reports* (λ, ρ). *All equilibria characterized by this function are outcome equivalent. In any such equilibrium* $\eta(\lambda, \rho)$ *is strictly increasing in* ρ, *and for each* R, *there exists* $\lambda_R > 0$, *defined by*

$$\eta^*_{R_0} \equiv \frac{R_0}{R_0 + \lambda_{R_0}} = \Pr(H|\lambda \leq \lambda_{R_0}, \rho = R_0)\, E[q_R|L \leq \lambda_{R_0}, R = R_0] + \Pr(P_r|\lambda \leq \lambda_{R_0}, \rho = R_0)\, E[q_R|R = R_0] \tag{3}$$

and

i. P_r *reveals all arguments that favor* Right*,* $\rho = R$*,*

ii. P_r *reveals a subset of arguments that favor* Left*; for* $L \leq \lambda_R$*, he sends a report* $\lambda = z_R(L) \leq \lambda_R$*, and for* $L > \lambda_R$*, he sends a report* $\lambda = s_R(L) \leq \lambda_R$*, where* s_R *and* z_R *solve*

$$\frac{R}{R+\lambda_R} = \frac{R}{R+\lambda} \Pr(L = \lambda | R) + \frac{R}{R+z_R^{-1}(\lambda)} \Pr\left(L = z_R^{-1}(\lambda) | R\right) + \frac{R}{R+s_R^{-1}(\lambda)} \Pr\left(L = s_R^{-1}(\lambda) | R\right), \tag{4}$$

with $z_R(0) = 0$ *and* $s_R(1-R) = 0$*.*

Proof The proof of this proposition and the detailed description of the discontinuous equilibria are in the Appendix. ■

Proposition 1 says that all equilibria have the following two properties. First, the strategy of the persuader includes revealing arguments unfavorable to him, a property predicted by casual observations. In terms of the motivating example, a financial adviser who is biased toward investment option *Right,* while saying that this investment option has high returns and is relatively less risky, will mention its low liquidity. Second, when the expert does not reveal himself to be honest, the decision maker forms her beliefs using only the number of arguments that favor *Right* that have been revealed to her. This is an interesting finding, given that the arguments that favor *Left* carry some information about each alternative. However, because the persuader does not necessarily reveal all arguments that favor *Left,* the decision maker does not know how many arguments of this type are concealed from her, and therefore λ does not help her in the decision making.

The equilibria do not depend on the distribution of the decision maker's preference type as long as this distribution is continuous with full support. The reason for that is simple: the persuader in favor of *Right* wants to generate as high belief about the quality of *Right* as possible, independent of the details of the distribution of θ_i.

Proposition 1 says that there is a unique equilibrium belief function that is continuous in reports. Figure 2 represents all continuous equilibria. The triangle represents all states of nature, S, and all reports, Σ, at the same time. The white area, which I call the *revealing area,* represents reports that are used in equilibrium only by H. The shaded region is the *ambiguity area* which includes all reports used by the persuader. The boundary of the ambiguity area is determined by λ_R defined by equation 3.

In equilibrium, the persuader reveals all of the arguments that favor *Right* and *some* arguments that favor *Left.* If the state of nature lies in the area of ambiguity, that is, if the quality of *Right* is high enough, the persuader reports according to $z_R(L)$. For example, if the state of nature is (L_1, R_0), the persuader sends $(z_{R_0}(L_1) \leq L_1, R_0)$. If the state of nature

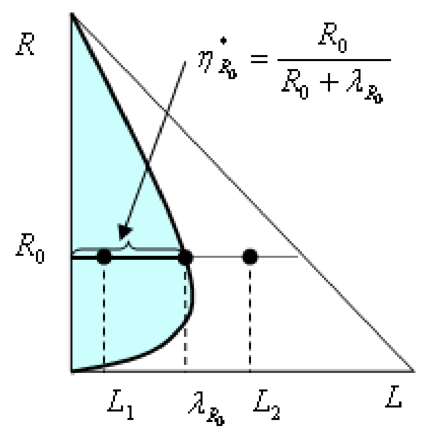


Figure 2: The details of the continuous equilibria. The shaded region represents the ambiguity area.

lies in the revealing area, the persuader reveals only a subset of the arguments that favor *Left* in such a way that his report lies in the ambiguity area. For example, if the state of nature is (L_2, R_0) the persuader reveals $(s_{R_0}(L_2) < \lambda_{R_0}, R_0)$.

The highest number of arguments that favor *Left* that the persuader reveals for any R is λ_R. After observing a report from the revealing area, the decision maker knows that the expert has revealed all arguments, and she believes $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda}$. After observing a report from the ambiguity area, she forms her belief based only on $\rho : \eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda_\rho}$.

The complete proof of Proposition 1 is in the Appendix, but I provide below the intuition for the shape of this equilibrium. First, in any equilibrium, for each R, there must exist some λ_R such that the decision maker holds the same belief for any report of a form $(\lambda \leq \lambda_R, R)$.[10] The reason for that is the following. Assume that for some $\rho = R_0$ the belief is strictly decreasing in λ, like in the first triangle in Figure 3.[11] In such a case P_r never sends any reports of a form $(\lambda \in (0, \varepsilon), R_0)$ (any reports that lie on the arrow); he prefers to send $(0, R_0)$ instead. Therefore, the belief that the decision maker forms when she sees $(0, R_0)$ must be smaller than 1. When the decision maker sees a report (ε, R_0), she knows that it was sent by the honest expert, and she believes $\eta(\varepsilon, R_0) = \frac{R_0}{R_0+\varepsilon}$. But as $\varepsilon \to 0$, we have $\eta(\varepsilon, R_0) \to 1$, and that contradicts the finding that $\eta(\varepsilon, R_0) < 1$. Alternatively, the belief may be strictly increasing in λ, as shown in the triangle on the right. Then the persuader, when sending reports of a form $(\lambda \in [0, \varepsilon), R_0)$ (from the arrow), sends the highest λ possible. In particular,

[10] The argument presented here holds for any equilibrium of the game.

[11] Although the argument presented here assumes a certain degree of continuity of beliefs, the formal proof presented in the Appendix does not.

he may send (ε, R_0) only in the states of nature that lie on the thick dashed line: when (L, R) are such that $\frac{R}{R+L} \geq \frac{R_0}{R_0+\varepsilon}$. That implies that $\eta(\varepsilon, R_0) \geq \frac{R_0}{R_0+\varepsilon}$. But then, for ε small enough, the belief must be arbitrarily close to 1, and that contradicts the assumption that the belief is strictly increasing in λ, since the belief can never be greater than 1.

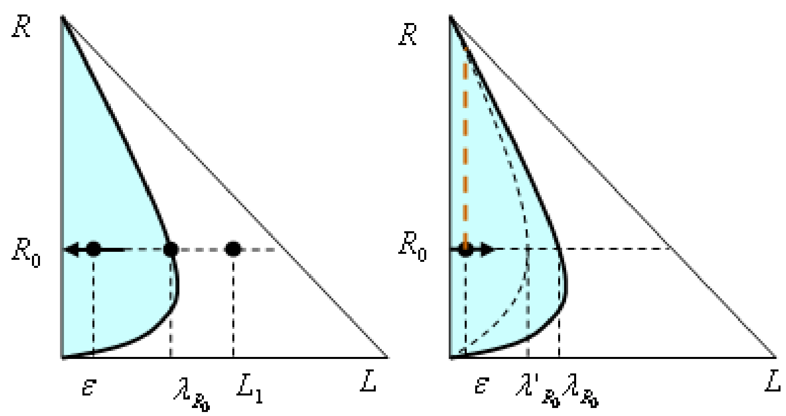


Figure 3: The shape of the equilibrium.

For each R, λ_R is unique, and the reason for that is the following. I have just established that the belief for any report of the form $(\lambda \leq \lambda_{R_0}, R_0)$ is the same, and that implies that it is equal to the belief the decision maker would hold if she knew that the report is of the form $(\lambda \leq \lambda_{R_0}, R_0)$, without knowing the exact λ. Such a belief is

$$\begin{aligned} E[q_R|\lambda \leq \lambda_{R_0}, \rho = R_0] &= \Pr(H|\lambda \leq \lambda_{R_0}, \rho = R_0)\, E[q_R|L \leq \lambda_{R_0}, R = R_0] \\ &\quad + \Pr(P_r|\lambda \leq \lambda_{R_0}, \rho = R_0)\, E[q_R|R = R_0]. \end{aligned}$$

In equilibrium, this belief must be equal to $\frac{R_0}{R_0+\lambda_{R_0}}$. To see that, consider $\lambda'_{R_0} < \lambda_{R_0}$, like in the right triangle in Figure 3. For such λ'_{R_0} we have $\frac{R_0}{R_0+\lambda'_{R_0}} > E\left[\frac{R}{R+L}|\lambda \leq \lambda'_{R_0}, R_0\right]$. But then in equilibrium only H sends a report $\left(\lambda'_{R_0} + \varepsilon, R_0\right)$; therefore, $\eta\left(\lambda'_{R_0} + \varepsilon, R_0\right) = \frac{R_0}{R_0+\lambda'_{R_0}+\varepsilon} > E\left[\frac{R}{R+L}|\lambda \leq \lambda'_{R_0}, R_0\right]$ for ε small enough. Hence, the persuader should always try to send $\left(\lambda'_{R_0} + \varepsilon, R_0\right)$ instead of any report from the ambiguity area, but this way it is impossible to generate a belief $E\left[\frac{R}{R+L}|\lambda \leq \lambda'_{R_0}, R_0\right] < \frac{R_0}{R_0+\lambda'_{R_0}}$ within the ambiguity area. Analogously, if $\lambda'_{R_0} > \lambda_{R_0}$ then $\frac{R_0}{R_0+\lambda'_{R_0}} < E\left[\frac{R}{R+L}|\lambda \leq \lambda'_{R_0}, R_0\right]$, but given that the persuader always reveals all arguments that favor *Right*, it is impossible to generate $\eta\left(\lambda'_{R_0}, R_0\right) > \frac{R_0}{R_0+\lambda'_{R_0}}$. That proves that λ_{R_0} must solve equation (3), and I show in the Appendix that the solution of this equation is unique.

To support a constant belief for all reports of the form $(\lambda \leq \lambda_\rho, \rho)$, the persuader must use a particular strategy when concealing information. For the belief to be constant for a given ρ and λs in the ambiguity area, it must be the case that the lower the quality of *Right* is, the more extreme the report of P_r is. Such a strategy solves equation (4). Since P_r is indifferent between all reports $(\lambda \leq \lambda_R, R)$, there are many strategies $s_R(L)$ and $z_R(L)$ that solve (4); sone such strategy is depicted in Figure 4.

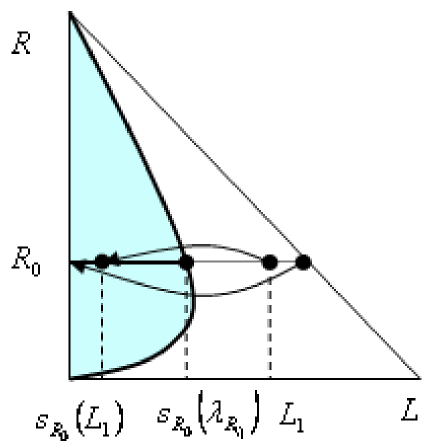


Figure 4: The strategy of the persuader. The arrows connect a state of nature with an equilibrium report.

The details of the strategy of the persuader shed some light on why the decision maker ignores the arguments that favor *Left* when the report lies in the ambiguity area. On the one hand, a smaller number of arguments that favor *Left* implies a higher quality of alternative *Right* if the expert happens to report fully. On the other hand, a smaller number of arguments that favor *Left* implies that more arguments of this type are concealed from the decision maker if the expert is not reporting fully. These two effects cancel each other out, and the decision maker holds the same belief independent of λ.

Figure 5 represents the equilibrium behavior of the decision maker in any continuous equilibrium. The line θ_i represents states in which $\frac{R}{R+L} = \theta_i$. The decision maker prefers alternative *Right* in the states that lie above this line and prefers alternative *Left* otherwise. R_i is the number of arguments that favor *Right* for which $\eta^*_{R_i} = \theta_i$. This means that if the decision maker observes R_i, she is indifferent between the alternatives. Seeing a report from the ambiguity area, the decision maker chooses *Right* if $R \geq R_i$, and she chooses *Left* if $R < R_i$. If she observes a report from the revealing area, she chooses *Right* if the report lies above the θ_i line.

In Figure 5 the shaded areas represent the states in which the decision maker chooses *Left*.

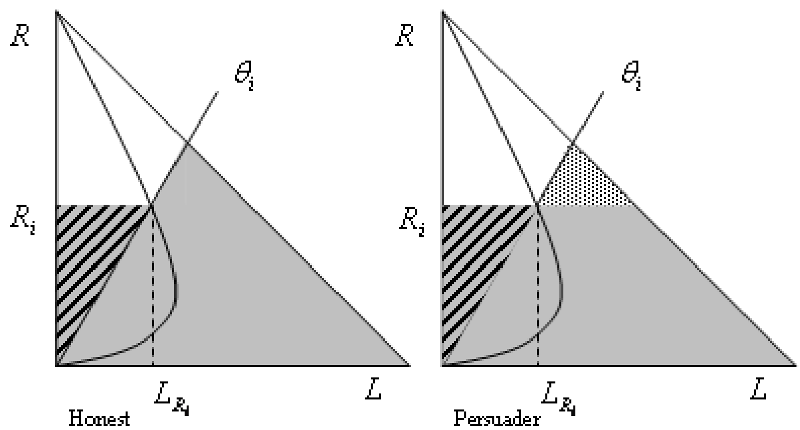


Figure 5: The behavior of the decision maker. The triangles represent the choice of the decision maker given the state of nature and given that the expert happens to be an honest type or a persuader, respectively.

When the number of arguments that favor *Right* is low enough, the decision maker chooses *Left* even if she receives an extreme report. The states in which *Right* is optimal but *Left* is chosen are represented by the striped area, which I call a *skeptic mistake.* The decision maker is skeptical about the quality of *Right* if the expert claims that it is high, but provides few arguments to support his claim. It is interesting that in the states of nature from the striped area every player of the game prefers the decision maker to choose alternative *Right,* but she nevertheless chooses *Left* in equilibrium. The dotted area represents *successful persuasion;* that is, the states in which *Left* is actually better but the decision maker nonetheless is persuaded to choose *Right.*[12]

Among all equilibria the set of equilibria with the continuous belief function stands out. First, all continuous equilibria are outcome equivalent, that is, they are characterized by the same belief function and by the same choice of the decision maker in each state of nature; they differ only in the details of the strategy of the expert. The same is not true about the set of discontinuous equilibria. Second, the proof of Proposition 1 reveals that all discontinuous equilibria require the persuader to play a rather elaborate strategy, even in the states of nature in which he is indifferent between doing that and revealing all of the arguments. This feature of the discontinuous equilibria is not very attractive; it seems natural to expect that an indifferent persuader would reveal all of the arguments. We can expect that revealing all of the arguments is easier or cheaper, for example, it may require a smaller mental effort than constructing an elaborate strategy, or there may be a fixed legal or reputation cost of

[12] Persuasion in my model happens despite full rationality of the decision maker. For examples of behavioral persuasion see Mullainathan and Shleifer (2005) and Murphy and Shleifer (2004).

concealing information, or the expert may experience guilt if he conceals information (see for example Gneezy 2005).

Proposition 2 says that if we perturb the game by adding a small fixed cost $\xi > 0$ of sending a report that is not *full,* measured in terms of utility, then there is a unique equilibrium in the perturbed game, which converges to a continuous equilibrium in the original game.[13,14] Let Γ be the original game, and $\Gamma(\xi)$ the perturbed game.[15]

Proposition 2 *The perturbed game $\Gamma(\xi)$ has a unique equilibrium which as $\xi \to 0$, converges to a continuous equilibrium with the properties described in Proposition 1, and with $z_R(L) = L$.*

Proof In the Appendix. ∎

In the rest of the paper I focus on the continuous equilibria. First, the only equilibrium that survives the introduction of a small cost of concealing information is continuous. Second, since in every discontinuous equilibrium we can find two reports arbitrarily close to each other that generate very different beliefs, it seems that such an equilibrium requires a lot of rationality on the part of the decision maker who has to observe and interpret the reports perfectly. Based on previous research one can expect that any noise in sending or interpreting the arguments should exclude the discontinuous equilibria.[16] Finally, focusing only on the continuous equilibria does not result in large loss of generality since all discontinuous equilibria display many qualitative features of the continuous equilibria, as seen in Proposition 1. In all equilibria the persuader biases his reports, but does not suppress all of the unfavorable arguments, and if the report favors *Right,* the belief is a function of the arguments that favor *Right* only.

Proposition 3 establishes another attractive feature of the continuous equilibria.

Proposition 3 *All continuous equilibria are equally informative, and they are the most informative equilibria of the unperturbed game (measured by the decision maker's ex-ante utility).*

Proof In the Appendix. ∎

[13] One may argue that the cost should depend on the number of arguments that are being concealed. The perturbed game with variable cost is more complicated to analyze, as any equilibrium in the pertubed game depends critically on the shape of the cost function. I conjecture, however, that if the variable cost is concave, there is a unique equilibrium in the perturbed game, and it converges to the same equilibrium. The same is not true, however, for convex cost.

[14] Kartik (2005) introduces an increasing cost of lying in a Crawford and Sobel (1982) type game and shows that only the most informative equilibrium can be a limit of monotone equilibria as the cost goes to zero. Kartik, Ottaviani and Squintani (2006) show that an increasing cost of lying when the state space is unbounded leads to a fully revealing equilibrium.

[15] Many economists have proposed refinements that restrict the set of equilibria in communication games (Farrell 1993, Sobel 1985, Rabin 1990, Matthews, Okuno-Fujiwara and Postlewaite 1991, Blume 1994).

[16] See, for example, Carlsson and van Damme (1993) and Battaglini (2002).

All continuous equilibria are equally informative because they are outcome equivalent. One can understand the intuition for the latter result by investigating the main difference between every discontinuous and every continuous equilibrium. Figure 6 shows an example of a discontinuous equilibrium. All other equilibria differ mainly in the number, size and location of the shaded trapezoids.[17] All reports lying in each shaded trapezoid induce the same belief. In all equilibria of this game, the belief given a report from the ambiguity area depends only on the number of revealed arguments that favor *Right*. Figure 6 shows that η^*_ρ is not strictly increasing in ρ in a discontinuous equilibrium. In the continuous equilibrium each ρ carries different information, while in any discontinuous equilibrium some ρs carry the same information; therefore, less information is revealed.

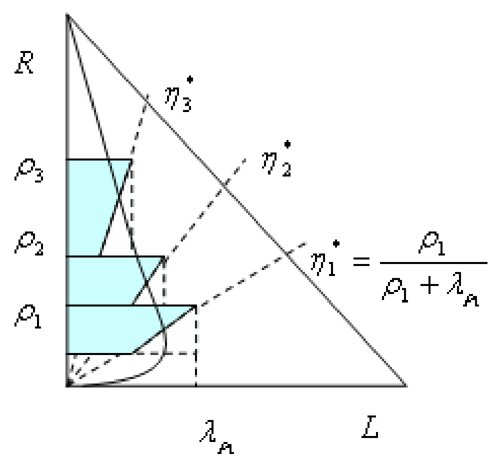


Figure 6: An example of a discontinuous equilibrium. Each shaded trapezoid consists of reports that induce the same belief. The curve represents the continuous equilibrium.

3.2 Benevolent expert

So far, I have assumed that the honest expert reveals all of the arguments. Alternatively, the honest expert may want to maximize the utility of the decision maker, i.e. he may be benevolent. One can see easily, however, that any continuous equilibrium of the original game is still an equilibrium of a game with a benevolent type of the expert, and the benevolent expert behaves like an honest expert. To see this, note that if the state of nature lies in the revealing area, the benevolent expert cannot do better than to report fully, because in

[17] Focus on one trapezoid from Figure 6. In an equilibrium it may be that also some reports from the rectangle that completes this trapezoid can generate the same belief as the one generated by the reports from the trapezoid.

this way he induces the correct belief. If the state of nature lies in the ambiguity area, the benevolent expert would like to induce a higher belief than the one induced in equilibrium, but there is no feasible report that can achieve that; therefore, again, full reporting is optimal.

The proposition below states an even stronger result; it says that the continuous equilibrium is also the unique equilibrium in the perturbed game with the strategic, benevolent expert.

Proposition 4 *Let the expert be either a persuader or a benevolent expert who maximizes the utility of the decision maker. Consider a perturbed game* $\Gamma'(\xi, \epsilon, B)$*, where* ξ *is a fixed cost of concealing information and* ϵ *is a fraction of honest experts. There exists a unique equilibrium in the perturbed game, and it converges to a continuous equilibrium of the original game with* $z_R(L) = L$*.*

Proof In the Appendix. ■

4 Two-sided bias

In this section, I consider a situation in which the expert can be biased toward either alternative. The expert can be P_r, P_l or H with probabilities π_r, π_l and π_H, respectively.

Sometimes the decision maker is not only uncertain whether the expert is honest, but she also does not know the potential bias of the persuader. A financial adviser may give honest advice, or he may have interest in promoting a particular investment fund, but the decision maker may not know which investment fund offers the adviser the highest commission. A scientist publishing a comparison of the performance of two drugs may be honest or biased, and the reader may not know which pharmaceutical company funded the research.

As we have seen in the previous section, when the expert is either honest or biased toward *Right*, the decision maker knows that all arguments that favor *Right* are revealed, and she uses those arguments to form her beliefs. Unless the expert reveals himself to be honest, she disregards the arguments that favor *Left* completely. When the expert can be biased in either direction, the decision maker cannot use the same logic; therefore, we can expect that much less information is revealed. This is only partially true, however. Proposition 5 describes the equilibrium that survives the introduction of a small cost of concealing arguments. In this equilibrium, the persuader toward *Right* and the persuader toward *Left* separate themselves if they happen to receive many arguments that favor their alternatives. In these states, the decision maker can use the same skeptical approach as in the one-sided case to infer information.

Proposition 5 *There exists a unique continuous equilibrium in the perturbed game $\Gamma(\xi)$. As $\xi \to 0$, it converges to the continuous equilibrium of the original game, characterized by the parameters $\bar{R}, \bar{L}$ and by the functions $\lambda_R, \rho_L, s_R(L), s_L(R)$, such that*

i. for all (L, R) such that $R \geq \bar{R}$, P_r reveals all arguments that favor Right*, and reveals a subset of arguments that favor* Left*: $\lambda = s_R(L)$ if $L \geq \lambda_R$ and $\lambda = L$ if $L < \lambda_R$, which solve*

$$\frac{R}{R+\lambda_R} = \frac{R}{R+\lambda} \Pr(L=\lambda|R) + \frac{R}{R+s_R^{-1}(\lambda)} \Pr\left(L = s_R^{-1}(\lambda)|R\right), \tag{5}$$

ii. for all (L, R) such that $L \geq \bar{L}$, P_l reveals all arguments that favor Left*, and reveals a subset of arguments that favor* Right*: $\rho = s_L(R)$ if $R \geq \rho_L$ and $\rho = R$ if $R < \rho_L$, which solve*

$$\frac{L}{\rho_L+L} = \frac{L}{\rho+L} \Pr(R=\rho|L) + \frac{L}{s_L^{-1}(\rho)+L} \Pr\left(R = s_L^{-1}(\rho)|L\right), \tag{6}$$

iii. there exists a double ambiguity area such that when $R < \bar{R}$, P_r sends reports from this area only, and when $L < \bar{L}$, P_l sends reports from this area only. The belief is constant for all reports in the double ambiguity area,

iv. λ_R and ρ_L solve the following equations

$$\begin{aligned}\eta^*_{R_0} \equiv \frac{R_0}{R_0+\lambda_{R_0}} &= \Pr(H|\rho = R_0, \lambda \leq \lambda_{R_0}) E[q_R|R=R_0, L \leq \lambda_{R_0}] \\ &+ \Pr(P_r|\rho = R_0, \lambda \leq \lambda_{R_0}) E[q_R|R=R_0],\end{aligned} \tag{7}$$

$$\begin{aligned}\eta^*_{L_0} \equiv \frac{\rho_{L_0}}{\rho_{L_0}+L_0} &= \Pr\left(H|\lambda = L_0, \rho \leq \rho_{L_0}\right) E\left[q_R|L=L_0, R \leq \rho_{L_0}\right] \\ &+ \Pr\left(P_r|\lambda = L_0, \rho \leq \rho_{L_0}\right) E[q_R|L=L_0].\end{aligned} \tag{8}$$

Proof in the Appendix. ■

Figure 7 represents the equilibrium for symmetric $f(L, R)$ and for $\pi_l = \pi_r$. In this equilibrium $\bar{R} = \bar{L}$ and $\eta\left(\bar{L}, \bar{R}\right) = \frac{1}{2}$. The light shaded areas represent the ambiguity area for P_r (along the vertical axis) and P_l (along the horizontal axis). The ambiguity area for P_r contains all reports used by P_r and H only, while the ambiguity area for P_l contains all reports used by P_l and H only. The dark square represents the set of reports that in equilibrium are used by all three types of the expert, the *double ambiguity area.*

As before, each type of persuader biases his reports; therefore, reports that consist of many relatively balanced arguments are sent only by the honest expert. Because each persuader biases the report toward his preferred alternative, in the equilibrium only H and P_r reveal

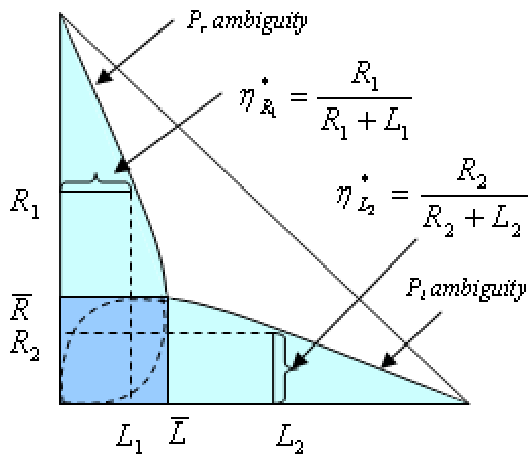


Figure 7: Two-sided bias.

many arguments that favor *Right,* and only H and P_l reveal many arguments that favor *Left.* Hence, reports that consist of many arguments reveal the potential bias of the expert. After observing many arguments that favor *Right* (reports from P_r ambiguity area), the decision maker knows that she does not face P_l, and hence, she knows that all arguments that favor *Right* have been revealed to her. Similarly, when she observes many arguments that favor *Left* (reports from P_l ambiguity area), she knows she does not face P_r, and therefore, she knows that all arguments that favor *Left* have been revealed to her.

Another interesting feature of this equilibrium is that the decision maker ignores reports that contain few arguments only. After receiving any report from the double ambiguity area, the decision maker cannot infer the potential direction of the expert's bias. Unable to use skepticism to form beliefs, she ignores such reports altogether. Therefore, for small R and L the equilibrium resembles a pure babbling equilibrium.

5 Comparative statics and extensions

In this section, I analyze how the parameters of the model, such as the probabilities of different types of the experts and the prior distribution of arguments, affect the equilibrium. How much information is transmitted should also depend on the quality of the expert's information, which so far has been assumed perfect. In the last part of this section, I extend my model to allow the expert to have imperfect knowledge about the true state of nature.

5.1 Varying the probability of facing the persuader

This section analyzes how changes in the probability of facing the persuader affect the agents' utilities and, more generally, the whole equilibrium. First, I look at what happens when the fraction of honest experts becomes negligible and what happens when the expert is honest with probability almost 1. Second, I analyze how the probability of facing a particular type of the persuader impacts the bias of his reports and the probability of persuading the decision maker.

Since there are three types of the expert, it is necessary to specify how the remaining probabilities change when the probability of facing the expert of type i changes. In the proposition below, I vary the probability of P_r and keep the conditional probability of facing the honest type given that the expert is not P_r constant. In such a case the shape of the ambiguity area for P_l remains the same.

Proposition 6 *In every continuous equilibrium, for P_r, $\lim_{\pi_H \to 1} \lambda_R = 0$, $\lim_{\pi_H \to 0} \lambda_R = \bar{\lambda}_R$, and for P_l, $\lim_{\pi_H \to 1} \rho_L = 0$ and $\lim_{\pi_H \to 0} \rho_L = \bar{\rho}_L$, where $\bar{\lambda}_R$ and $\bar{\rho}_L$ are such that $\frac{R}{R+\bar{\lambda}_R} = E[q_R|R]$ and $\frac{\bar{\rho}_L}{\bar{\rho}_L+L} = E[q_R|L]$. Keeping $\frac{\pi_H}{1-\pi_r}$ constant, as the probability of facing P_r decreases,*

i. the reports of the persuader P_r become more extreme,

ii. the utility of P_r increases,

iii. the utility of P_l decreases, and

iv. the expected utility of the decision maker increases.

Proof In the Appendix. ■

First, Proposition 6 states that as the probability of facing the honest expert increases, the ambiguity areas for both persuaders disappear, and the equilibrium converges to a fully informative equilibrium. On the other hand, as the probability of facing the honest expert goes down, λ_R converges to some $\bar{\lambda}_R < 1-R$ and ρ_L converges to $\bar{\rho}_L < 1-L$. That means that the ambiguity areas are always strict subset of Σ, and that the equilibrium never becomes a pure babbling equilibrium. The assumption that arguments are verifiable prevents equilibria from becoming completely uninformative in the L dimension. In the equilibrium, the decision maker learns how many arguments favor *Right* and nothing about how many arguments favor *Left.*

Figure 8 shows how the equilibrium changes as π_r decreases. π_1 and $\pi_0 > \pi_1$ are two different probabilities of facing P_r. The thick curves represent the initial equilibrium in which $\pi_r = \pi_0 = \pi_l$. Since the conditional probability of facing H is kept constant, the shape of the ambiguity area for P_l remains unchanged as π_r changes. As $\pi_r \to 0$, the ambiguity area for P_r becomes smaller, as represented by the thinner curve. The double ambiguity area for $\pi_r = \pi_0$ is the shaded region.

When π_r decreases, the reports of P_r become more extreme. The reason for this is the following. Sending a report that is not extreme works in two opposite directions. If the report comes from H, a less extreme report implies a lower q_R, hence it is less likely that the decision maker chooses *Right*. On the other hand, if the report comes from P_r, a less extreme report implies that P_r has concealed fewer arguments that favor *Left*, and therefore, the more likely it is that the decision maker chooses *Right.* As π_r decreases, the first effect dominates, and P_r has a higher incentive to bias his reports. In terms of my motivating example, this result says that the financial advisor biased toward an investment option that is not very popular among other advisers will not use very many arguments that oppose this option.

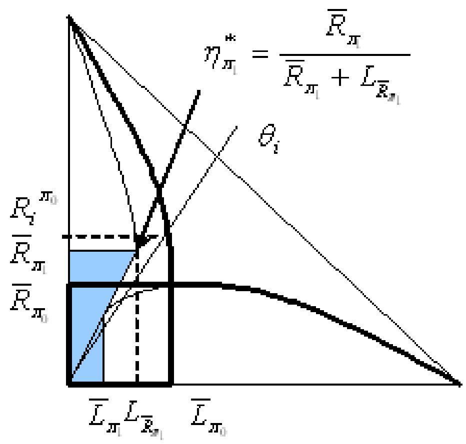


Figure 8: The effect of decreasing π_R, keeping $\frac{\pi_H}{1-\pi_R}$ constant.

From Figure 8 one can see that when the decision maker faces P_r, she chooses *Right* more often when π is low (always for π_1, and whenever $R \geq R_i^{\pi_0}$ for π_0). Therefore, the utility of the persuader toward *Right* increases. If the expert happens to be P_l, the decision maker chooses *Right* more often, which decreases the utility of the persuader toward *Left.* However, the utility of the decision maker increases, because she is more likely to face the honest expert.

Proposition 6 implies that a financial adviser biased toward a stock which is unpopular among other advisers is better at persuading the investor, while a financial adviser biased towards a popular stock is unlikely to be successful.

5.2 Varying the familiarity of the problem

Recall that $f(L,R)$ is the prior probability density function of the arguments, from which one can derive the prior density function of the total number of signals $g(N)$. The prior distribution of N reflects the decision maker's knowledge about the choice problem. It describes how the total number of arguments varies from situation to situation for the same decision problem. For example, in each election campaign different number of issues is important, which is the model can be represented by a relatively spread prior over N. Other choice problems are likely to be characterized by roughly the same number of arguments every time the decision maker faces them, for example, choosing an investment option or buying a car, and that is equivalent to a distribution of N concentrated around the mean. Alternatively, the prior distribution of N may describe the decision maker's knowledge about the problem. An investor with a very spread distribution of N knows little about the nature of the problem, while an experienced or educated investor is likely to have a very concentrated distribution of N.

In this section, I analyze how the prior distribution of arguments affects the utility of the decision maker, in cases in which the potential bias of the persuader is known, that is, when the expert can be either P_r or H. To isolate the effect of changing the distribution of N, keeping the distribution of the quality, q_R, unchanged, I reformulate the problem in terms of $(q_R, N) \equiv \left(\frac{R}{R+L}, L+R\right)$, and assume that q_R is uniformly distributed and independent of N, which implies that the joint density of quality and N is equal to the density of N :

$$g(q_R, N; z) = g(N; z).$$

Let $g(N;z)$ be symmetric around $\frac{1}{2}$, and z be a parameter that measures stochastic dominance: if $z_1 > z_2$ then $g(N;z_1)$ second-order stochastically dominates $g(N;z_2)$, and as $z \to \infty$, $g(\cdot)$ becomes degenerate at $N = \frac{1}{2}$.

Proposition 7 *For every preference type of the decision maker θ_i, and every $\pi > 0$, if $z_1 > z_2$ then the decision maker's utility is higher for $g(N;z_1)$ than for $g(N;z_2)$. As $z \to \infty$, there is full revelation of information.*

Proof in the Appendix. ■

Proposition 7 says that the lower the uncertainty about N is, the better-off the decision maker is. When the decision maker knows more about the arguments available to the expert, she can more easily extract his information: when she receives a report, she can estimate rather precisely how many arguments have been concealed from her. The decision maker is better off in situations that are standard or familiar to her. When faced with an unfamiliar choice, or a choice that is familiar but very different every time it presents itself, the decision maker chooses the suboptimal alternative more often, even when faced with an honest expert.

5.3 Extension: sampling of the arguments

So far I have assumed that experts know all relevant arguments; however, in reality experts may receive only a subset of them. How many arguments they know may depend on how well they have researched the topic, or how well they can remember the information they have been exposed to. In this section, I show that allowing the expert to have only a sample of the relevant arguments does not affect the nature of the equilibrium. Later, I analyze how the quality of the expert–his ability to sample arguments–affects the utility of the decision maker.

Given the state of nature (L, R), the expert samples $(l, r) : l \leq L$ and $r \leq R$, according to probability density function $s(l, r|L, R)$. Therefore, the prior density of (l, r) is $s(l, r) = s(l, r|L, R) f(L, R)$. Given that the expert knows only a subset of the relevant arguments, the information contained in his arguments is imprecise, which implies that $E\left[\frac{R}{R+L}|\frac{r}{r+l}\right] \neq \frac{r}{r+l}$. Let $\omega(l, r) = E\left[\frac{R}{R+L}|l, r\right] \in [0, 1]$. The only condition that is required on sampling is that $\frac{dE\left[\omega(l,r)|r,l\leq\hat{l}\right]}{dr} > 0$ for all r and $\hat{l}$ and $\frac{dE[\omega(l,r)|l,l\leq\hat{r}]}{d\hat{l}} < 0$ for all l and $\hat{r}$; that is, receiving an additional argument that favors some alternative increases the expected quality of this alternative. This is an equivalent of the regularity condition 1.

Proposition 8 states that the qualitative features of the continuous equilibria still hold.

Proposition 8 *There exists a unique continuous equilibrium in the perturbed game* $\Gamma(\xi)$*. As* $\xi \to 0$, *it converges to the continuous equilibrium which has the same qualitative features as the continuous equilibria in the original game. It is characterized by the parameters* $\bar{r}, \bar{l}$ *and by the functions* $\lambda_r, \rho_l, s_r(l), s_l(r)$, *with equation (5) replaced by (and analogously for equation 6)*

$$\omega(\lambda_r, r) = \omega(\lambda, r) \Pr(l = \lambda|r) + \omega\left(s_r^{-1}(\lambda), r\right) \Pr\left(l = s_r^{-1}(\lambda)|r\right),$$

and equation (7) replaced by (and analogously for equation 8)

$$\begin{aligned} \omega(\lambda_{r_0}, r_0) &= \Pr(B|\rho = r_0, \lambda \leq \lambda_{r_0}) E\left[\frac{R}{R+L}|r = r_0, l \leq \lambda_{r_0}\right] \\ &\quad + \Pr(P_r|\rho = r_0, \lambda \leq \lambda_{r_0}) E\left[\frac{R}{R+L}|r = r_0\right]. \end{aligned}$$

Proof Proof of this proposition is similar to the proof of Proposition 5. The crucial modifications are described in the Appendix. ■

5.3.1 Varying the quality of the expert

The experts may differ in how well they gather information. A high-quality expert may know more arguments, either because he is better at searching, or because he remembers more of them. In this section, I analyze how the quality of the expert affects the utility of the decision maker in the case when the expert can be either P_r or H.

To capture the fact that even a low-quality expert is likely to have some information, I assume that $N \in [\underline{N}, 1]$.[18] $\underline{N}$ may represent the number of core arguments that are always present, or the number of arguments that are customary for an expert to know. For example, a financial adviser may not have detailed information about each particular company, but he is likely to know the general state of the economy and how it affects the prospects of a particular stock.

Let $n \equiv l + r$ be the total number of arguments received by the expert and $q_r \equiv \frac{r}{r+l}$ be the fraction of arguments that favor *Right* in his sample. Since I am interested in ordering experts according to how many arguments they are likely to sample, it is useful to reformulate the model in terms of (q_r, n) instead of (l, r). From the prior distribution of (l, r), $s(l, r)$, we can derive the distribution of $(q_r, n) : g(q_r, n; \alpha) = s((1 - q_r) n, q_r n; \alpha) n$. The parameter α measures the quality of the expert; $\alpha = 1$ means that the expert is of highest quality and he always receives $n = N$.

Recall that $\omega(l, r) = E\left[\frac{R}{R+L}|l, r\right]$ represents the expected quality of *Right* if the expert receives (l, r), and note that it can be rewritten in terms of q_R and $q_r : \omega(q_r; \alpha, \beta) = E[q_R|q_r, \alpha, \beta]$. If the expert always samples arguments in the correct proportion, then his arguments reflect the state of nature precisely, and $\omega(q_r; \alpha, \beta) = q_r$. If he samples arguments in a symmetric and unbiased way, then $\omega\left(\frac{1}{2}; \alpha, \beta\right) = \frac{1}{2}$ and $q_r - \omega(q_r; \alpha, \beta) = \omega(1 - q_r; \alpha, \beta) - (1 - q_r) > 0$ for $q_r < \frac{1}{2}$; that is, the expected quality given q_r is observed is less extreme than q_r, because extreme draws are likely to come from combination of arguments that imply a less extreme quality.

I make the following simplifying assumptions about the sampling procedure. I assume that sampling is such that $g(q_r, n; \alpha) = g(n; \alpha)$; that is, the sampled quality is independent of the number of arguments sampled, is distributed uniformly, and

$$\omega(q_r; \alpha, \beta) = \frac{1}{2} + \left(q_r - \frac{1}{2}\right)(1 - (1 - \alpha)\beta),$$

where $\beta \in [0, 1]$. The higher α means that the expert has less precise information, which implies that the expected quality of *Right* does not vary much with the information of the expert. β measures how the quality of the expert affects the precision of his information:

[18] Assuming that $\underline{N} > 0$ does not change anything qualitatively in the previous sections; $\underline{N}$ was set to 0 only for the purpose of the exposition.

high β means that decreasing the quality of the expert results in high loss of precision. In reality, the information precision loss of sampling depends on how the sampling is conducted. If the expert stops searching for information after he gathers a predetermined number of arguments, then his information is likely to be imprecise (high β). If he is exposed to all of the relevant arguments but remembers only a subset, then he may remember them in the proportion that resembles the true proportion (low β).

Additionally, let $g(n;\alpha) = \frac{g(n)}{G(\alpha)}$, where $g(\cdot)$ is the probability density function of N. I assume that $g(\cdot)$ is such that $\frac{G(n)}{ng(n)}$ is increasing.[19] This means that the expert of quality α receives only $n \in [\underline{N}, \alpha]$.

The proposition below states that consulting the expert with the highest quality is not always optimal.

Proposition 9 *For an unbiased decision maker, $\theta_i = \frac{1}{2}$, and every π and for every pair of experts with qualities α_1 and $\alpha_2 < \alpha_1$, there exists $\hat{\beta}(\pi)$ such that for all $\beta < \hat{\beta}(\pi)$ the decision maker prefers to consult the low-quality expert. $\hat{\beta}(\pi)$ is increasing in π.*

Proof in the Appendix. ■

The decision maker cares about the precision of the expert's information, but also about how well she can extract this information from him. If the expert is a persuader, then even if his information is perfect, the decision maker will form a noisy estimate of the true state of nature given the expert's report. But this estimate is more precise the lower the quality of the expert. The decision maker knows more about the number of arguments that such an expert is likely to receive; therefore, after observing his report, she knows more about how many arguments have been concealed from her.

If the expert is honest, he reveals all of his information, which implies that the decision maker is better off if the precision of the expert's information is high. If the expert is likely to be the persuader, in equilibrium much of his information is concealed from the decision maker. Therefore, the decision maker is willing to choose a low-quality expert in order to increase her ability to extract information.

A good example of such a trade-off is the market for news. Some newspapers have many reporters and gather an enormous amount of arguments, which allows them to manipulate the news by selective disclosure. Newspapers that have limited resources and specialize in providing the most important news only may have lower abilities to manipulate the news, as the readers know more about how many arguments they should expect.[20]

[19] $\frac{G(n)}{ng(n)}$ is increasing for a majority of standard $g(\cdot)$ functions, when $g' < 0$. This assumption allows me to prove the proposition below in a simple way, but is by no means a necessary condition for the proposition to hold. Essential for this exercise is that the distribution of n changes with α is such a way that given any r, there is less uncertainty about the total number of arguments that the expert has.

[20] Of course, some newspapers may gather all information, but reveal only a small subset due to limited space, in which case different results follow.

6 Comparison with the existing literature

The existing models on strategic information transmission differ mainly in two respects. First, information is usually described as a single variable, and the models differ in whether they assume that this variable can be credibly disclosed. Second, the preference of the expert can be private information or common knowledge.

Grossman and Hart (1979) and Grossman (1981) assume that information is verifiable and that the decision maker knows the preference of the expert. In these models there is full disclosure, since the decision maker can interpret the lack of information as information unfavorable to the expert, which in turn forces the expert to reveal all favorable information.

In their seminal paper, Crawford and Sobel (1982) assume that information is non-verifiable; the signals sent by the expert are cheap. In their model a purely uninformative equilibrium always exists, and all equilibria have partitional structure. Krishna and Morgan (2001) extend this model to a game with two experts who have opposing preferences.

In my model, the arguments are verifiable but their number is not, and this assumption has interesting implications. First, the pure babbling equilibrium cannot exist, as information becomes closer to verifiable in states characterized by many arguments. Second, if the decision maker knows the bias of the expert, the players behave as if the number of arguments that favor the preferred alternative were verifiable. The behavior along the other dimension has features of the babbling equilibrium: the decision maker makes the decision independent of how many arguments in this dimension the expert reveals.

The papers that, similar to my paper, model information as a collection of verifiable signals are Milgrom (1981), Milgrom and Roberts (1986), and Shin (1994). As in my model, experts manipulate information through selective disclosure. Milgrom (1981) analyzes a model in which a salesman can credibly disclose attributes of a product, but the number of attributes is common knowledge; this generates full information disclosure. Milgrom and Roberts (1986) propose a model in which each alternative is characterized by an unknown number of attributes, but they focus on the scope of information transmission when there are two experts with opposing preferences. Competition among experts leads to full information disclosure. Shin (1994) assumes that experts may have verifiable but imperfect signals about the state of nature, but the decision maker does not know how precise the experts' information is. Unlike me, he assumes that the preference of the expert is common knowledge and focuses on the equilibrium in which experts reveal all information favorable to them and suppress entirely the unfavorable information. Additionally, Shin (1994) allows for signals that convey any information about the state of nature, while the argument structure of my model restricts the signal space considerably.

Sobel (1985), Benabou and Laroque (1992), Morris (2001) and Morgan and Stocken (2003) extend cheap talk models to allow for uncertainty about the preference of the expert. These models assume that the expert can be either honest (strategic or not) or biased toward a particular alternative. Sobel (1985) and Benabou and Laroque (1992) analyze a dynamic game in which the truthfulness of the expert's recommendation is revealed after each period (with or without noise), and they establish that even the biased expert can reveal information truthfully in the initial rounds of the game. Morris (2001) assumes that the expert cares about his reputation, in which case even an honest expert has an incentive to distort information. In contrast, my model is static, and the expert does not have direct reputation concerns; nevertheless, I obtain a result that the biased expert chooses to mimic the honest expert. Morgan and Stocken (2003) extend Crawford and Sobel (1982), assuming that the expert may be biased or benevolent. Dimitrakas and Sarafidis (2003) allow for additional uncertainty in the strength of this bias, and Li (2005) assumes that the expert can be biased either upwards or downwards, and the direction of the bias is private knowledge.

The only paper that I am aware of that analyzes a two-directional bias in a communication game with partially verifiable information is Wolinsky (2003). He assumes that the expert may prefer to either maximize or minimize the magnitude of the decision maker's action. In his model, the state of nature is a unidimensional variable and the expert can certify that the state of nature lies above some threshold. In terms of my paper this assumption amounts to the expert being able to certify that the quality of one alternative, for example alternative *Right,* is higher than some threshold. In contrast, in the model of this paper a signal of this form is not available. It is an equilibrium result that in some states of nature each type of the persuader can credibly convey information that his preferred alternative has quality above a certain threshold. Wolinsky (2003) proposes an extension of his model to the case in which information is two-dimensional (like arguments in my model), but does not analyze such a game with uncertainty about the types of the expert.

Allowing for the uncertainty about the direction of the expert's bias in this paper leads to interesting findings. First, there are states in which experts with opposite biases separate themselves, and then the model becomes equivalent to the model with one-directional bias.[21] Second, in states in which the experts pool, no information is being revealed.

Despite the fact that information in this model is two-dimensional (dimension R and dimension L), the model differs significantly from multidimensional cheap talk literature (Battaglini 2002, Levy and Razin 2004, Chakraborty and Harbaugh 2004). First, these models assume that information is not verifiable in each dimension, while my model allows certain degree of verifiability. Second, the preference of the expert is common knowledge. Levy and Razin (2004) and Battaglini (2002) assume that the action space is also multidimensional; therefore, there is always a dimension in which the preferences of the expert and the decision maker agree. In my model, the action space is unidimensional. Chakraborty and Harbaugh

[21] In Wolinsky (2003) the persuaders do not always separate themselves.

(2004) allow for the decision to be unidimensional, but in a situation in which the expert and the decision maker agree on the ranking of the alternatives.

This paper shows that introducing a small cost on concealing information selects a maximally informative equilibrium. Kartik (2005) uses a similar tool in a cheap talk game. He introduces an increasing cost of misrepresenting information, and shows that a set of monotone equilibria converges to the most informative equilibrium of Crawford and Sobel 1982's model.

From a modelling perspective, the model in this paper is related to Glazer and Rubinstein (2001) and Glazer and Rubinstein (2004). They model information as a collection of arguments, each of which can be either credibly disclosed or can be verified by the decision maker. They focus on information transmission when there are exogenous constraints on how much information can be revealed.

7 Conclusion

The main finding of this paper is that even a biased expert uses two-sided reports that include arguments which oppose his interests. There is an extensive research on using two-sided arguments in marketing literature which usually finds that two-sided messages are more effective and increase the perceived truthfulness of the expert.[22] In my model, the persuader uses two-sided messages to gain credibility, but in equilibrium one-sided and two-sided messages induce the same belief. This is, however, only because the theoretical model assumes away some realistic interactions. For example, if we allow for a small fraction of naive decision makers who take the reports at their face value, in equilibrium the belief function would be such that a rational decision maker would be persuaded more often by two-sided reports and those reports would be deemed more credible.

At this point, I should mention some limitations of my model. First, the model assumes that the decision maker is uninformed, but clearly, it would be interesting to analyze the case where the decision maker has some prior information. Experiments on mass communication indicate that two-sided arguments are more effective when the audience is initially opposed to the expert's position, while one-sided arguments are more successful with listeners who are already disposed toward the expert's position.[23] Moreover, if the audience is later provided with arguments favoring the other position, those who were previously exposed to two-sided argumentation are less likely swayed away from this position than those initially exposed to one-sided argumentation.[24] These issues could be analyzed within my model if we endow the decision maker of the model presented in Section 5.3 with a small subset of arguments. Based

[22]See, for example, Smith and Hunt (1978) and Anderson and Golden (1984).
[23]See Hovland, Lumsdaine and Sheffield (1949).
[24]See Lumsdaine and Janis (1953).

on the analysis presented in this paper, I conjecture that such an extension would produce results that are consistent with these studies. Decision makers with information favoring one alternative should weight the arguments that reinforce their knowledge more than before, because they should expect the expert to have few opposing arguments. Those who have some opposing arguments should expect the honest expert to reveal such arguments, and if that is not the case, they should believe that the expert is likely to be a persuader. A counterpropaganda should be less effective if the expert first establishes himself to be honest by using two-sided messages.

Another limitation of my model is that it does not explain the trend toward polarization that seems to have been occurring recently. Many media outlets and politicians rely more on one-sided messages. This may be a result of competition or information overload–issues that are not addressed in this paper.

The model presented in this paper is a good starting point to analyze communication in more elaborate settings. Because of the argument structure, it is easy to define the differences in information that different players have, which makes this model especially well-suited to analyze two-sided communication. Moreover, since disclosing arguments requires time, the model has some natural timing structure build in, which means that it can be easily applied to debates and communication in bargaining.

A Appendix

Proof of Proposition 1

Every equilibrium takes the following form. For each ρ, there exist $\lambda_\rho > 0$, such that for all $(\lambda, \rho) : \lambda \leq \lambda_\rho$ we have $\eta(\lambda, \rho) \equiv \eta^*_\rho$. η^*_ρ is weakly increasing in ρ. If for all $\rho \in (\rho_1, \rho_2)$ η^*_ρ is constant, then λ_{ρ_2} is such that $\frac{\rho_2}{\rho_2 + \lambda_{\rho_2}} = \eta^*_\rho$, and for all other $\rho \in (\rho_1, \rho_2)$ we have $\frac{\rho}{\rho + \lambda_\rho} \leq \eta^*_\rho$, and $\eta(\lambda > \lambda_{\rho_2}, \rho) = \frac{\rho}{\rho + \lambda}$. The proof proceeds with the following steps.

Step 1 $\eta(0, \rho) < 1$ for all $\rho < 1$.

The existence of H experts makes $\eta(\lambda, \rho) < 1$ for all $\lambda > 0$. Assume there exists ρ such that $\eta(0, \rho) = 1$. Then for all $(L > 0, R) \in Z(0, \rho)$ P_r can induce belief 1 by sending $(0, \rho)$, on sending some other report $\eta(0, \rho')$ such that $\eta(0, \rho') = 1$. But then a rational decision maker would form a belief $\eta(0, \rho) < 1$ or a belief $\eta(0, \rho') < 1$, which is a contradiction.

Step 2 For all ρ there exist λ_ρ such that $\eta(\lambda, \rho) = \eta(0, \rho)$ for all $\lambda \leq \lambda_\rho$.

Assume that for all $\varepsilon > 0$ we can find $\lambda_0 < \varepsilon$ such that $\eta(\lambda_0, \rho) < \eta(0, \rho)$. Then by Step 1 we have $\eta(\lambda_0, \rho) < \eta(0, \rho) < 1$. Given that, P_r would never send (λ_0, ρ), and therefore, $\eta(\rho, \lambda_0) = \frac{\rho}{\lambda_0 + \rho}$, which goes to 1 as $\lambda_0 \to 0$, which is a contradiction.

Assume then that for all $\varepsilon > 0$ we can find λ_0 such that $\eta(\lambda_0, \rho) > \eta(0, \rho)$. Then P_r may send $(0, \rho)$ only if $(L, R) = (0, R \geq \rho)$ since he would rather send (λ_0, ρ) whenever he can. Therefore, $\eta(0, \rho) = 1$, but that contradicts Step 1.

Define $\eta^*_\rho \equiv \eta(0,\rho)$.

Step 3 η^*_ρ is weakly increasing in ρ.

Assume that there exist $\rho_2 > \rho_1$ such that $\eta^*_{\rho_2} < \eta^*_{\rho_1}$. Take $\varepsilon < \min\{\lambda_{\rho_1}, \lambda_{\rho_2}\}$. Then P_r never sends any messages of a form $(\lambda \leq \varepsilon, \rho_2)$, and this implies that for all those messages the belief is $\eta(\lambda, \rho_2) = \frac{\rho_2}{\rho_2+\lambda}$ which is not constant in λ. That contradicts Step 2.

Step 4 λ_ρ must be such that $\frac{\rho}{\rho+\lambda_\rho} \leq \eta^*_\rho$

Assume $\frac{\rho}{\rho+\lambda_\rho} > \eta^*_\rho$. Then for all $\varepsilon > 0$, we can find $\lambda_0 \in (\lambda_\rho, \lambda_\rho+\varepsilon)$ such that $\eta(\lambda_0,\rho) \neq \eta^*_\rho$. Assume first that $\eta(\lambda_0,\rho) < \eta^*_\rho$. Then only H sends (λ_0,ρ), and therefore, $\eta(\lambda_0,\rho) = \frac{\rho}{\rho+\lambda_0} > \eta^*_\rho$, a contradiction. Assume then that for all $\varepsilon > 0$, we can find $\lambda_0 \in (\lambda_\rho, \lambda_\rho+\varepsilon)$ such that $\eta(\lambda_0,\rho) > \eta^*_\rho$. But then (λ_0,ρ) is more attractive to P_r than any report of a form $(\lambda \leq \lambda_\rho, \rho)$. Therefore, P_r will send (λ_ρ,ρ) only if (L,R) is such that $\frac{R}{R+L} \geq \frac{\rho}{\rho+\lambda_\rho} > \eta^*_\rho$, which contradicts $\eta(\lambda_\rho,\rho) = \eta^*_\rho$.

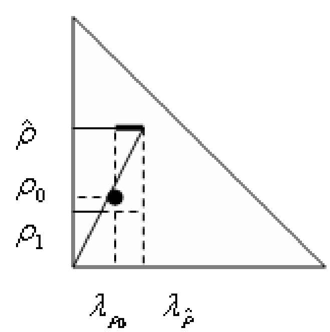
Step 5 Let $\hat{\rho} = \max\{\rho : \eta^*_\rho = \hat{\eta}\}$. Then $\lambda_{\hat{\rho}}$ is such that $\frac{\hat{\rho}}{\hat{\rho}+\lambda_{\hat{\rho}}} = \hat{\eta}$.

By Step 4 we have $\frac{\hat{\rho}}{\hat{\rho}+\lambda_{\hat{\rho}}} \leq \hat{\eta}$. Assume then $\frac{\hat{\rho}}{\hat{\rho}+\lambda_{\hat{\rho}}} < \hat{\eta}$. To generate a belief $\hat{\eta}$ for a report $(\lambda_{\hat{\rho}}, \hat{\rho})$ it must be that there exist $(L_1, R_1) \in Z(\lambda_{\hat{\rho}}, \hat{\rho})$ such that $\frac{R_1}{R_1+L_1} \geq \hat{\eta}$ and $\sigma_{P_r}(L_1, R_1) = (\lambda_{\hat{\rho}}, \hat{\rho})$. But that implies that $R_1 > \hat{\rho}$, and by definition of $\hat{\rho}$ we know $\eta^*_{R_1} > \hat{\eta}$; therefore, P_r would rather send $(0, R_1)$ than $(\lambda_{\hat{\rho}}, \hat{\rho})$. A contradiction.

Step 6 Step 4 and Step 5 imply that for all ρs such that $\eta^*_\rho = \hat{\eta}$ it must be that $\lambda_\rho \leq \lambda_{\hat{\rho}}$.

Step 7 Take all ρ such that $\eta^*_\rho = \hat{\eta}$. Then for all $\lambda \leq \lambda_{\hat{\rho}}$ it is $\eta(\lambda,\rho) \leq \eta^*_\rho$.

Assume that there exists a report (λ_0, ρ_0) depicted in figure below, such that $\eta(\lambda_0, \rho_0) > \hat{\eta}$.

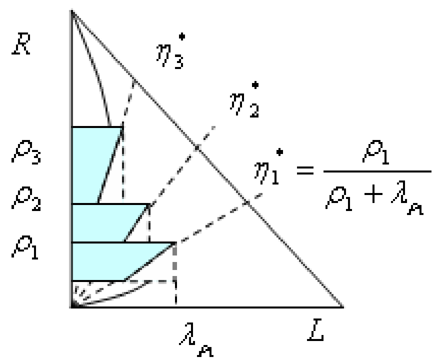


This means that P_r would never send reports of a form $(\lambda, \hat{\rho}) : \lambda \in (\lambda_0, \lambda_{\hat{\rho}})$ (thick line in the figure below), but then it is impossible that these reports generate belief $\hat{\eta}$, which contradicts Step 5.

Step 8 Let $\hat{\rho} = \max\{\rho : \eta^*_\rho = \hat{\eta}\}$. Then $\lambda_{\hat{\rho}}$ is such that $\frac{\hat{\rho}}{\hat{\rho}+\lambda_{\hat{\rho}}} = \hat{\eta}$. Then $\lambda_{\hat{\rho}} < 1 - \hat{\rho}$.

Assume $\lambda_{\hat{\rho}} = 1 - \hat{\rho}$. Then $\eta(\lambda_{\hat{\rho}}, \hat{\rho}) = \hat{\eta} = \frac{\hat{\rho}}{\hat{\rho}+\lambda_{\hat{\rho}}}$, but it is impossible to generate such a belief for all $(\lambda, \hat{\rho})$.

The steps above allow us to conclude that all equilibria look like in figure below, and differ only in the number, size and location of the shaded trapezoids. Each shaded trapezoid represents all reports that generate the same belief. Additionally, all reports lying in the rectangle that completes a trapezoid generate either the same belief as the trapezoid, or are sent only by H.



Take one trapezoid, for example the one associated with $\rho \in (\rho_1, \rho_2)$. Then for almost all reports lying outside of the rectangle that completes this trapezoid (dashed rectangle) the belief is $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda}$. To be precise, I have not excluded the possibility that there exists some (λ', ρ') from outside of this rectangle that generates a belief equal to is η_3^*. For that to be possible it must be that $\lambda_{\rho_3} > \lambda_{\rho_2}$ (unlike in the figure above) and $\lambda_{\rho_3} > \lambda'$. But then P_r would send this report in states with a very low quality of q_R, and therefore in generic situations it is impossible to induce belief η_3^*.

For a prior density function $f(L, R)$ that satisfies the regularity condition 1 the equilibrium belief function is discontinuous unless it is strictly increasing in ρ. To see that, note first that continuity requires that $\lambda_\rho = \frac{1-\eta_\rho^*}{\eta_\rho^*}\rho$; therefore λ_ρ is unique. That, together with continuity also implies that there is no $\lambda > \lambda_\rho$ such that $\eta(\lambda, \rho) \geq \eta_\rho^*$, which implies that only H sends reports of a form $(\lambda > \lambda_\rho, \rho)$.

Second, continuity implies that η_ρ^* is strictly increasing in ρ. To see that assume $\eta_\rho^* \equiv \hat{\eta}$ is constant over $\rho \in (\rho_1, \rho_2)$ and strictly increasing for $\rho \in (\rho_1 - \epsilon, \rho_1)$ and $\rho \in (\rho_2, \rho_2 + \epsilon)$ for some ϵ. Then for all $\varepsilon < \epsilon$ we have

$$\begin{aligned} \eta_{\rho_2+\varepsilon}^* &= \Pr(H|\rho = \rho_2 + \varepsilon, \lambda \leq \lambda_{\rho_2+\varepsilon}) E[q_R|R = \rho_2 + \varepsilon, L \leq \lambda_{\rho_2+\varepsilon}] \\ &\quad + \Pr(P_r|\rho = \rho_2 + \varepsilon, \lambda \leq \lambda_{\rho_2+\varepsilon}) E[q_R|R = \rho_2 + \varepsilon], \end{aligned}$$

$$\begin{aligned} \eta_{\rho_1-\varepsilon}^* &= \Pr(H|\rho = \rho_1 - \varepsilon, \lambda \leq \lambda_{\rho_1-\varepsilon}) E[q_R|R = \rho_1 - \varepsilon, L \leq \lambda_{\rho_1-\varepsilon}] + \\ &\quad \Pr(P_r|\rho = \rho_1 - \varepsilon, \lambda \leq \lambda_{\rho_1-\varepsilon}) E[q_R|R = \rho_1 - \varepsilon]. \end{aligned}$$

The regularity condition 1 implies that we can find some $\Delta > 0$ such that the right-hand side (RHS) of the first equation is bigger than the RHS of the second one by more than Δ for each $\varepsilon < \epsilon$. That implies $\eta_{\rho_2}^* > \eta_{\rho_1}^*$.

When η_ρ^* is strictly increasing, the strategy of P_r is to send $\rho = R$. He is indifferent between sending any number of arguments that support Left, as long as $\lambda \leq \lambda_\rho$. In equilibrium, however, his strategy must support the constant belief; therefore it must satisfy equation 4 for each R. There are many solutions to this equation, however, for each eligible $z_R(L)$ there exists a unique solution for $s_R(L)$.

It remains to show that for each R, η_R^* described by equation 3, exists and is unique. Equation 3 can be rewritten as follows:

$$\eta_R^* = \frac{\pi \int_0^{1-R} \frac{R}{R+L} f(L|R)\, dL + (1-\pi) \int_0^{\frac{1-\eta_R^*}{\eta_R^*} R} \frac{R}{R+L} f(L|R)\, dL}{\pi + (1-\pi) F\left(\frac{1-\eta_R^*}{\eta_R^*} R|R\right)} \tag{9}$$

The left hand side goes from R to 1. For the right hand side we have $RHS\,(\eta_R^* = R) = \bar{\eta}_R > R$ and $RHS\,(\eta_R^* = 1) = \bar{\eta}_R < 1$, where $\bar{\eta}_R = \int_0^{1-R} \frac{R}{R+L} f(L|R)\, dL$, therefore the solution exists.

The left-hand side (LHS) is strictly increasing. If we differentiate RHS with respect to η, we get

$$\begin{aligned}\frac{dRHS}{d\eta} &= \frac{(1-\pi) f\left(\frac{1-\eta}{\eta} R|R\right)}{\left(\pi + (1-\pi) F\left(\frac{1-\eta}{\eta} R|R\right)\right)^2} \frac{-1}{(\eta)^2} r \cdot \\ &\cdot \left(\pi(\eta - \bar{\eta}_R) + (1-\pi)\left(\eta F\left(\frac{1-\eta}{\eta} R|R\right) - \int_0^{\frac{1-\eta}{\eta} R} \frac{R}{R+L} f(L|R)\, dL\right)\right).\end{aligned}$$

For $\eta = R$ we have $\frac{dRHS}{d\eta} > 0$ and for $\eta = 1 - R$ we have $\frac{dRHS}{d\eta} < 0$, and when we evaluate $\frac{dRHS}{d\eta}$ at η_R^* satisfying equation 9 then

$$\frac{dRHS}{dx}(\eta = \eta^*) = \frac{(1-\pi) f(x|r)}{(\pi + (1-\pi) F(x|r))} (\eta^* - \eta^*) = 0.$$

That implies that the solution is unique. ■

Proof of Proposition 2

Proof of this proposition is a particular case of the proof of Proposition 5 when $\pi_l = 0$; therefore, I provide only the latter. Proposition 5 is the equivalent of 2 when there are three types of persuaders. ■

Proof of Proposition 5

Since H is nonstrategic, I will refer to P_l and P_r only throughout the entire proof; that is, when I say (λ, ρ) is sent only by P_i, I mean P_i and H. In the game with ξ cost the utility of P_r when the state is (L, R) and he sends a message (λ, ρ) is

$$u_R(\lambda, \rho|\,(L, R)) = \begin{cases} H(\eta(\lambda, \rho)) \text{ if } (\lambda, \rho) = (L, R) \\ H(\eta(\lambda, \rho)) - \xi \text{ if } (\lambda, \rho) \neq (L, R) \end{cases}.$$

First, I prove the following Lemmas.

Lemma 1 *There cannot exist an infinite sequence of reports* (λ_n, ρ_n) *such that* $(\lambda_n, \rho_n) \in Z(\lambda_{n-1}, \rho_{n-1})$ *and* $\eta(\lambda_n, \rho_n) \neq \frac{\rho_n}{\rho_n + \lambda_n}$.

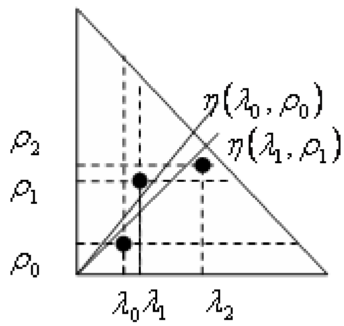
Proof We have $Z(\lambda_n, \rho_n) \subset Z(\lambda_{n-1}, \rho_{n-1})$; therefore, for some n, $Z(\lambda_n, \rho_n)$ is so small that the difference between $b_{\max} = \max_{(\lambda,\rho) \in Z(\lambda_n, \rho_n)} \frac{\rho}{\rho+\lambda}$ and $b_{\min} = \min_{(\lambda,\rho) \in Z(\lambda_n, \rho_n)} \frac{\rho}{\rho+\lambda}$ is smaller than the cost of concealing arguments ξ; therefore, all experts report fully, which implies that $\eta(\lambda_n, \rho_n) = \frac{\rho_n}{\rho_n + \lambda_n}$. ■

Lemma 2 *i) If* (λ, ρ) *is sent by* P_l *only when he reports fully, then* $\eta(\lambda, \rho) \leq \frac{\rho}{\rho+\lambda}$,

ii) If (λ, ρ) *is sent by* P_r *only when he reports fully, then* $\eta(\lambda, \rho) \geq \frac{\rho}{\rho+\lambda}$.

Proof I prove only part (i). Proof of part (ii) is analogous to the proof of part (i).

Assume (λ_0, ρ_0) is never sent by P_l when $(L, R) \neq (\lambda_0, \rho_0)$ and $\eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0 + \lambda_0}$, like shown in the Figure below.



Then there exists $(\lambda_1, \rho_1) \in Z(\lambda_0, \rho_0)$ such that $\frac{\rho_1}{\rho_1 + \lambda_1} \geq \eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0 + \lambda_0}$ and $\sigma_{P_r}(\lambda_1, \rho_1) = (\lambda_0, \rho_0)$, but for that to be optimal for P_r it must be that $\eta(\lambda_1, \rho_1) < \eta(\lambda_0, \rho_0) \leq \frac{\rho_1}{\rho_1 + \lambda_1}$ (P_r must prefer to send (λ_0, ρ_0) to (λ_1, ρ_1)). But then there exists $(\lambda_2, \rho_2) \in Z(\lambda_1, \rho_1)$ such that $\frac{\rho_2}{\rho_2 + \lambda_2} \leq \eta(\lambda_1, \rho_1)$ and $\sigma_i(\lambda_2, \rho_2) = (\lambda_1, \rho_1)$. Clearly $i = P_l$, since P_r would rather send (λ_0, ρ_0). For P_l to be optimal to send (λ_1, ρ_1) it must be that $\eta(\lambda_1, \rho_1) < \eta(\lambda_2, \rho_2)$. Then $\frac{\rho_2}{\rho_2 + \lambda_2} < \eta(\lambda_2, \rho_2)$ and only P_r can send (λ_2, ρ_2) while concealing arguments, but then set $(\lambda_0, \rho_0) \equiv (\lambda_2, \rho_2)$ and start the proof from this point. But if we continue like that, we end up with a sequence of (λ_n, ρ_n) such that $(\lambda_n, \rho_n) \in Z(\lambda_{n-1}, \rho_{n-1})$ and $\eta(\lambda_n, \rho_n) \neq \frac{\rho_n}{\rho_n + \lambda_n}$, which by Lemma 1 cannot be the case.

It is worthwhile to understand why the opposite argument does not work. If $\sigma_{P_l}(L, R) = (\lambda_0, \rho_0)$ only if $(L, R) = (\lambda_0, \rho_0)$ and $\eta(\lambda_0, \rho_0) < \frac{\rho_0}{\rho_0 + \lambda_0}$, then there must exist $(\lambda_1, \rho_1) \in Z(\lambda_0, \rho_0)$ such that $\frac{\rho_1}{\rho_1 + \lambda_1} \leq \eta(\lambda_0, \rho_0)$ and $\sigma_{P_r}(\lambda_1, \rho_1) = (\lambda_0, \rho_0)$. But it may still be the case that $\eta_1(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1 + \lambda_1}$ and P_r may prefer to send (λ_0, ρ_0) instead of (λ_1, ρ_1), $\sigma_{P_r}(\lambda_1, \rho_1) = (\lambda_0, \rho_0)$; therefore, we do not need an infinite sequence of arguments such that $\eta(\lambda_n, \rho_n) \neq \frac{\rho_n}{\rho_n + \lambda_n}$. ■

Corollary 1 *If $\sigma_{P_l}(L, R) = (\lambda_0, \rho_0)$ only if $(L, R) = (\lambda_0, \rho_0)$ and $\sigma_{P_r}(L, R) = (\lambda_0, \rho_0)$ only if $\frac{R}{R+L} \geq \frac{\rho_0}{\rho_0 + \lambda_0}$, then $\eta(\lambda_0, \rho_0) = \frac{\rho_0}{\rho_0 + \lambda_0}$.*

I use the above Lemmas to prove Proposition 5.

Step 1 i) If $\eta(\lambda_0, \rho) = \frac{\rho}{\rho + \lambda_0}$, then $\eta(\lambda, \rho) \geq \frac{\rho}{\rho + \lambda}$ for all $\lambda \geq \lambda_0$.

ii) If $\eta(\lambda, \rho_0) = \frac{\rho_0}{\rho_0 + \lambda}$, then $\eta(\lambda, \rho) \leq \frac{\rho}{\rho + \lambda}$ for all $\rho \geq \rho_0$.

Again, I prove only part (i); proof for part (ii) is analogous. I prove this by contradiction. Assume that there exists $\lambda_1 > \lambda_0$ and $\eta(\lambda_1, \rho) < \frac{\rho}{\rho + \lambda_1}$. Lemma 2 implies that only P_r may send (λ_1, ρ) when concealing information. But P_r would be better off sending (λ_0, ρ) instead of (λ_1, ρ) when concealing information, because by doing that he would induce the belief $\eta(\lambda_0, \rho) = \frac{\rho}{\rho + \lambda_0} > \frac{\rho}{\rho + \lambda_1} > \eta(\lambda_1, \rho)$. Therefore, $\sigma_{P_r}(L, R) = (\lambda_1, \rho)$ only if $(L, R) = (\lambda_1, \rho)$; hence by Lemma 2 we have $\eta(\lambda_1, \rho_0) \geq \frac{\rho_0}{\rho_0 + \lambda_1}$, a contradiction.

Step 2 i) For all $\rho < 1$ we have $\eta(0, \rho) < 1$.

ii) For all $\lambda < 1$ we have $\eta(0, \lambda) > 0$.

If $\xi < 1 - \eta(1 - \rho', \rho')$, then this proof is identical to the proof for Step 1 in the proof of Proposition 1 and an analogous proof holds for (ii).

Step 3 i) For all ρ, there exists $\varepsilon_0 > 0$ such that $\eta(\varepsilon, \rho) = \eta(0, \rho)$ for all $\varepsilon \leq \varepsilon_0$.

ii) For all λ, there exists $\varepsilon_0 > 0$ such that $\eta(\lambda, \varepsilon) = \eta(\lambda, 0)$ for all $\varepsilon \leq \varepsilon_0$

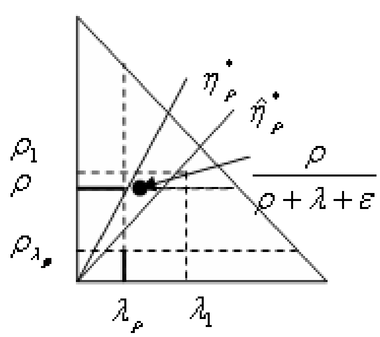
Assume that there exists ρ_0 such that for all ε_{ρ_0} we can always find $\varepsilon < \varepsilon_{\rho_0}$ such that $\eta(\varepsilon, \rho_0) < \eta(0, \rho_0) < 1$. Clearly, P_r prefers to send $\eta(0, \rho_0)$ instead of (ε, ρ_0) whenever he conceals information; therefore, $\sigma_{P_r}(L, R) = (\varepsilon, \rho_0)$ only if $(L, R) = (\varepsilon, \rho_0)$. But then for ε small enough, only P_l may send (ε, ρ_0) when concealing information, and then by Lemma 2 we have $\eta(\varepsilon, \rho_0) \geq \frac{\rho_0}{\rho_0 + \varepsilon}$, which converges to 1 as $\varepsilon \to 0$. But that contradicts $\eta(\varepsilon, \rho_0) < \eta(0, \rho_0) < 1$.

Assume then that for all ε_{ρ_0} we can always find $\varepsilon < \varepsilon_{\rho_0}$ such that $\eta(\varepsilon, \rho_0) > \eta(0, \rho_0)$. But as $\eta(0, \rho_0) < 1$ we know from Lemma 2 that $\sigma_{P_l}(L, R) = (0, \rho_0)$ only if $(L, R) = (0, \rho_0)$. Moreover, $\eta(\varepsilon, \rho_0) > \eta(0, \rho_0)$ implies that P_r sends $(0, \rho_0)$ only for $(0, \rho \geq \rho_0)$. But then by Corollary 1 we have $\eta(0, \rho_0) = 1$, which is a contradiction.

Define $\eta^*_\rho \equiv \eta(0, \rho)$ and $\eta^*_\lambda \equiv \eta(\lambda, 0)$. Let λ_ρ be the highest λ such that $\eta^*_\rho \equiv \eta(\lambda, \rho)$ for all $\lambda \leq \lambda_\rho$, and ρ_λ be the highest ρ such that $\eta^*_\lambda \equiv \eta(\lambda, \rho)$ for all $\rho \leq \rho_\lambda$. Clearly, $\lambda_\rho < 1 - \rho$ and $\rho_\lambda < 1 - \lambda$.

Step 4 $\eta^*_\rho \geq \frac{\rho}{\rho + \lambda_\rho}$ and $\eta^*_\lambda \leq \frac{\rho_\lambda}{\rho_\lambda + \lambda}$.

Assume $\eta^*_\rho < \frac{\rho}{\rho + \lambda_\rho}$, and this is represented by the straight line called $\hat{\eta}^*_\rho$ in the figure below. By Lemma 2 P_r must sometimes send $(\lambda \leq \lambda_\rho, \rho)$ while concealing information.



Take $(\lambda_\rho + \varepsilon, \rho)$. If $\eta(\lambda_\rho + \varepsilon, \rho) \geq \frac{\rho}{\rho + \lambda_\rho + \varepsilon}$ for all ε small enough, then $\eta(\lambda_\rho + \varepsilon, \rho) > \eta^*_\rho$ and therefore, P_r, when concealing information, prefers to send $(\lambda_\rho + \varepsilon, \rho)$ instead of $(\lambda \leq \lambda_\rho, \rho)$ whenever he can, which contradicts the finding that P_r must sometimes send $(\lambda \leq \lambda_\rho, \rho)$ while concealing information.

Step 5 For each ρ for all $\lambda > \lambda_\rho$ it is that $\eta(\lambda, \rho) < \eta^*_\rho$ and for each λ for all $\rho > \rho_\lambda$ it is that $\eta(\lambda, \rho) > \eta^*_\lambda$.

If there exists (λ_0, ρ) such that $\eta(\lambda_0, \rho) > \eta^*_\rho$, then P_l never sends (λ_0, ρ) when concealing information; he would rather send $(0, \rho)$ instead. Assume therefore that there exists (λ_0, ρ) such that $\eta(\lambda_0, \rho) = \eta^*_\rho$. Then

$\eta(\lambda_\rho+\varepsilon,\rho)<\eta^*_\rho$ for ε small enough, but that means that P_l would never send (λ_0,ρ) when lying. By Lemma 2 therefore $\eta(\lambda_0,\rho)\leq\frac{\rho}{\rho+\lambda_0}$, but that and Step 4 lead to a contradiction.

By now we have established that P_r always sends reports of the form $(\lambda\leq\lambda_\rho,\rho)$ if he conceals some arguments (see Step 1 and Lemma 2), and P_l sends reports of the form $(\lambda,\rho\leq\rho_\lambda)$ if he conceals some arguments. The remaining reports are sent only when reporting is full; therefore, Lemma 2 implies that $\eta(\lambda,\rho)=\frac{\rho}{\rho+\lambda}$ for those reports.

Therefore, we know that there are thresholds $\bar{\rho}$ and $\bar{\lambda}$ such that for all $\rho>\bar{\rho}$ only P_r sends reports of a form $(\lambda\leq\lambda_\rho,\rho)$ when concealing arguments and for $\lambda\geq\bar{\lambda}$ only P_l sends reports of a form $(\lambda,\rho\leq\rho_\lambda)$ when concealing information. Step 6 says that then $\eta^*_\rho=\frac{\rho}{\rho+\lambda_\rho}$, and $\eta^*_\lambda=\frac{\rho_\lambda}{\rho_\lambda+\lambda}$.

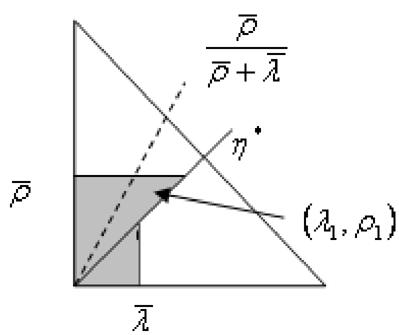
Step 6 If $\rho>\bar{\rho}$, then $\eta^*_\rho=\frac{\rho}{\rho+\lambda_\rho}$, and if $\lambda>\bar{\lambda}$, then $\eta^*_\lambda=\frac{\rho_\lambda}{\rho_\lambda+\lambda}$.

By step 5 $\eta^*_\rho\geq\frac{\rho}{\rho+\lambda_\rho}$ and $\eta^*_\lambda\leq\frac{\rho_\lambda}{\rho_\lambda+\lambda}$. But if only P_r sends (λ_ρ,ρ) when concealing information then by Lemma 2 it must be $\eta(\lambda_\rho,\rho)\leq\frac{\rho}{\rho+\lambda_\rho}$, which is a contradiction.

Step 7 By Step 5 we can conclude that there exist $\bar{\rho}>0$ and $\bar{\lambda}>0$ such that $\eta^*_\rho=\eta^*_\lambda\equiv\eta^*$ for all $\rho\leq\bar{\rho}$ and for all $\lambda\leq\bar{\lambda}$. We can have three situations, $\eta^*>\frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$, $\eta^*<\frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$ or $\eta^*=\frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$.

Let me consider $\eta^*<\frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$.

Step 8 If $\eta^*<\frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$ then for all (λ,ρ) such that they lie in the shaded area in the figure below, and only for those reports, we have $\eta(\lambda,\rho)=\eta^*$.



To see this notice that by step 5 $\eta(\lambda,\rho)=\eta^*$ for all $(\lambda,\rho):\lambda\leq\bar{\lambda}$ and $\rho\leq\bar{\rho}$. Take report (λ_1,ρ_1) : $\frac{\rho_1}{\rho_1+\lambda_1}>\eta^*$ in the figure above, and assume $\eta(\lambda_1,\rho_1)<\eta^*$; then only P_l may send (λ_1,ρ_1) when concealing information, but then by Lemma 2 we have $\eta(\lambda_1,\rho_1)\geq\frac{\rho_1}{\rho_1+\lambda_1}>\eta^*$, a contradiction.

Step 9 η^*_ρ is weakly increasing in ρ and η^*_λ is weakly decreasing in λ.

Assume there exist $\rho_2>\rho_1$ such that $\eta^*_{\rho_2}<\eta^*_{\rho_1}$. Then only P_l could send $(0,\rho_2)$ when concealing information, as P_r would prefer to send $(0,\rho_1)$. But then Lemma 2 implies that $\eta(0,\rho_2)\geq1$, which we have established cannot be. The proof that η^*_λ is weakly decreasing in λ is analogous.

Step 10 For $\rho>\bar{\rho}$, η^*_ρ is strictly increasing and for $\lambda>\bar{\lambda}$, η^*_λ is strictly decreasing.

By Step 9 we know that if $\rho_2>\rho_1>\bar{\rho}$ and $\eta^*_{\rho_1}=\eta^*_{\rho_2}$ then we have $\eta^*_\rho=\eta^*_{\rho_1}$ for all $\rho\in[\rho_1,\rho_2]$. Assume that ρ_1 and ρ_2 are the inf and the sup of ρ such that $\eta^*_\rho=\eta^*_{\rho_1}$. Because $\rho_2>\rho_1>\bar{\rho}$ only P_r send reports with $\rho\in(\rho_1,\rho_2)$ when concealing arguments and for all (λ,ρ) such that $\rho\in(\rho_1,\rho_2)$ and $\lambda\in[0,\lambda_\rho]$ the belief is constant. That means that the strategy of P_r must be such that for any such report the decision maker must attach the same belief. Moreover, if the state of nature is (L,R) such that $R \in (\rho_1, \rho_2)$ and $L \leq \lambda_R$, then P_r reports fully, as in this way he generates the highest possible belief and does not have to incur cost ξ. Therefore, we have

$$\eta^*_{\rho_2} \geq \Pr(H|\rho = \rho_2, \lambda \leq \lambda_{\rho_2}) E[q_R|R = \rho_2, L \leq \lambda_{\rho_2}] + \Pr(P_r|\rho = \rho_2, \lambda \leq \lambda_{\rho_2}) E[q_R|R = \rho_2],$$

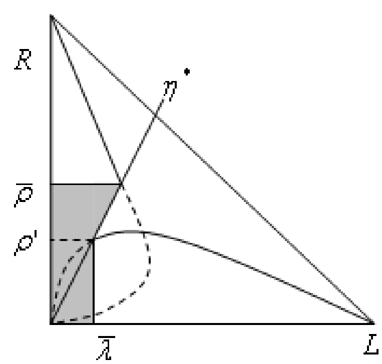
$$\eta^*_{\rho_1} \leq \Pr(H|\rho = \rho_1, \lambda \leq \lambda_{\rho_1}) E[q_R|R = \rho_1, L \leq \lambda_{\rho_1}] + \Pr(P_r|\rho = \rho_1, \lambda \leq \lambda_{\rho_1}) E[q_R|R = \rho_1],$$

and the regularity condition 1 requires that the RHS of the first inequality is bigger than the RHS of the second one, which implies $\eta^*_{\rho_2} > \eta^*_{\rho_1}$.

Step 11 $\eta^*_{\bar{R}+\varepsilon} \to \eta^*$

If not, then by the steps before we have $\eta^*_{\bar{R}+\varepsilon} > \eta^*$. We also have $\lambda_{\bar{R}+\varepsilon} \to \lambda_{\bar{R}}$. The only difference in the behavior of P_r when $R = \bar{R} + \varepsilon$ and when $R = \bar{R}$ is that in the latter case P_r does not have to send $\rho = \bar{R}$ when $L > \lambda_{\bar{R}}$, but that only increases the belief induced for $(\lambda \leq \lambda_{\bar{R}}, \bar{R})$. The only difference in terms of the behavior of P_l is that P_l never sends $\rho = \bar{R} + \varepsilon$ unless he reports fully, while he may send $\rho = \bar{R}$, but that can happen only for $(L,R) : \frac{R}{R+L} \geq \frac{\bar{R}}{\bar{R}+\lambda_{\bar{R}}}$, which additionally increases the belief induced for $(\lambda \leq \lambda_{\bar{R}}, \bar{R})$. Therefore, the only possibility is $\eta^*_{\bar{R}+\varepsilon} \to \eta^*$.

Now, we can summarize the shape of any equilibrium. If $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$ then the equilibrium looks like in the figure below.



In what follows below I use $\bar{R}$ instead of $\bar{\rho}$ and $\bar{L}$ instead of $\bar{\lambda}$ to reflect the fact that the belief is constant when the number of arguments received by the expert is below some threshold. The grey area represents all reports that generate the same belief η^*. The solid curves represent the areas in which only P_r and H or only P_l and H send reports. Therefore, in any equilibrium P_r sends $\sigma_{P_r}(L,R) = (L,R)$ if $L \leq \lambda_R$ as in this way he generates the highest possible belief and does not have to incur cost ξ. P_r reports fully also for (L,R) such that $L \leq \lambda_R + \psi(\xi)$, where $\psi(\xi)$ is defined as $\eta^*_R - \xi = \frac{R}{R+L+\psi}$, because of the cost of concealing information. For $R \geq \bar{R}$, P_r sends $\sigma_{P_r}(L,R) = (\lambda \leq \lambda_R, R)$ when $L > \lambda_R + \psi(\xi)$. The strategy for $R < \bar{R}$ is similar with a difference that when P_r conceals information he does not have to reveal all arguments that favor Right.

The behavior of P_l is analogous.

We need to establish the existence and the uniqueness of the equilibrium with the properties derived above. That is, we have to show that it is possible to construct the strategies of P_l and P_r so that the beliefs satisfy the properties derived above, and we need to show that these strategies are unique.

First, I show that it is possible to generate a flat belief η_R^* for each $R > \bar{R}$ (and analogously, η_L^* for $L > \bar{L}$). Take some $R > \bar{R}$. Experts H and P_l send reports of a form $(\lambda \leq \lambda_R, R)$ only when they report fully, and P_r sends such reports when he reports fully, but also when the true set of arguments is $(\lambda > \lambda_R + \psi(\xi), R)$. Therefore, λ_R must solve the following equation:

$$\eta_R^* = \frac{R}{R+\lambda_R} = \frac{p\left(\int_0^{\lambda_R} \frac{R}{R+L} f(L|R)\,dL + \int_{\lambda_R+\psi}^{1-R} \frac{R}{R+L} g(L|R)\,dL\right) + (1-p)\int_0^{\lambda_R} \frac{R}{R+L} f(L|R)\,dL}{p\left(1-F(\lambda_R+\psi|R) + F(\lambda_R|R)\right) + (1-p)F(\lambda_R|R)}, \qquad (10)$$

where p is the probability that the expert is P_r ($p = \frac{\pi_r}{1-\pi_l}$ if P_l never sends messages of a sort $(\lambda \leq \lambda_R, R)$ and $p = \pi_r$ otherwise) and $f(L|R)$ is the conditional distribution of L given R, derived from $f(L,R)$.

The RHS is defined for $\lambda_R \in [0, 1-R-\psi]$; therefore, the LHS of this equation is continuous, strictly decreasing and $LHS \in \left[\frac{R}{1-\psi}, 1\right]$. The RHS is continuous and

$$RHS(0) = \frac{\left(\int_\psi^{1-R} \frac{R}{R+L} f(L|R)\,dL\right)}{1-F(\psi|R)} = E\left[\frac{R}{R+L}|L \geq \psi, R\right] < 1$$

$$RHS(1-R-\psi) = \frac{\int_0^{1-R-\psi} \frac{R}{R+L} g(L|R)\,dL}{F(1-R-\psi|R)} = E\left[\frac{R}{R+L}|L \leq 1-R-\psi, R\right] > \frac{R}{1-\psi}$$

Therefore, the solution exists.

Taking the derivative of the RHS with respect to λ_R and evaluating it at λ_R that solves equation (10), we get

$$\frac{dRHS}{d\eta_R^*} =^{sign} pf(\lambda_R+\psi|R)\,\xi > 0.$$

Therefore, the solution to equation (10) is unique.

The easiest way to establish that $\frac{d\eta_R^*}{dR} > 0$ for $R > \bar{R}$ is to reformulate the problem in terms of (q_R, R) instead of (L,R), where $q_R = \frac{R}{L+R}$. Let $g(q_R, R)$ be the implied density function of (q_R, R). Then equation (10) takes the following form:

$$\eta_R^* = \frac{p\left(\int_R^{\eta_R^*-\xi} q_R g(q_R|R)\,dq_R + \int_{\eta_R^*}^1 q_R g(q_R|R)\,dq_R\right) + (1-p)\int_{\eta_R^*}^1 q_R g(q_R|R)\,dq_R}{p\left(1-G(\eta_R^*|R) + G(\eta_R^*-\xi|R)\right) + (1-p)\left(1-G(\eta_R^*|R)\right)},$$

and can be rewritten as

$$\begin{aligned} Y \equiv\ & \eta_R^*\left(p\left(1-G(\eta_R^*|R) + G(\eta_R^*-\xi|R)\right) + (1-p)\left(1-G(\eta_R^*|R)\right)\right) \\ & -p\left(\int_R^{\eta_R^*-\xi} q_R g(q_R|R)\,dq_R + \int_{\eta_R^*}^1 q_R g(q_R|R)\,dq_R\right) - (1-p)\int_{\eta_R^*}^1 q_R g(q_R|R)\,dq_R, \end{aligned}$$

By the implicit function theorem $\frac{d\eta_R^*}{dR} = -\frac{\frac{dY}{dR}}{\frac{dY}{d\eta_R^*}}$, and we have

$$\frac{dY}{d\eta_R^*} \equiv \left(1-G(\eta_R^*|R)\right) + p\left(\xi g(\eta_R^*-\xi|R) + G(\eta_R^*-\xi|R)\right) > 0,$$

$$\frac{dY}{dR} \equiv p\xi G_R(\eta_R^*-\xi|R) - p\int_{\eta_R^*-\xi}^{\eta_R^*} G_R(q_R|R)\,dq_R + p\int_R^1 G_R(q_R|R)\,dq_R + (1-p)\int_{\eta_R^*}^1 G_R(q_R|R)\,dq_R,$$

The regularity condition 1 implies $\frac{dE[q_R|R, q_R \geq \hat{q}_R]}{dR} \geq 0$ for all R and $\bar{q}_R$, but which in turn implies that

$\int_{\hat{q}_R}^{1} G_R(q_R|R)\, dq_R \leq 0$ for all $\hat{q}_R$. But this implies that for small ξ we have $\frac{dY}{dR} < 0$. Therefore, $\frac{dY}{dR} < 0$ which implies $\frac{d\eta_R^*}{dR} > 0$. As $\xi \to 0$ equation (10) converges to equation (7).

To generate a constant belief over $\lambda \in (0, \lambda_R)$ P_l must use a strategy, such that for $L \in (\lambda_R + \psi, 1 - R)$, the higher L he receives, the lower λ he sends. Let $s_R(L) = \lambda$ be this strategy for $\xi = 0$; then it must satisfy

$$\frac{R}{R+\lambda_R} = \frac{R}{R+\lambda} \Pr(L = \lambda|R) + \frac{R}{R + s_R^{-1}(\lambda)} \Pr\left(L = s_R^{-1}(\lambda)|R\right) \text{ for all } \lambda \in [0, \lambda_R],$$

which can be rewritten as

$$\frac{R}{R+\lambda_R} = \frac{f(\lambda|R)\frac{R}{R+\lambda} - \pi f\left(s_R^{-1}(\lambda)|R\right)\left(s_R^{-1}(\lambda)\right)' \frac{R}{R+s_R^{-1}(\lambda)}}{f(\lambda|R) - \pi f\left(s_R^{-1}(\lambda)|R\right)\left(s_R^{-1}(\lambda)\right)'}, \tag{11}$$

with the initial condition $s_R^{-1}(0) = 1 - R$. The above equation is an ordinary differential equation and satisfies the conditions for the existence and the uniqueness of the solution. Note also that $s_R^{-1}(\lambda_R) = \lambda_R$, as we can rewrite equation (11) in the following way

$$\left(\frac{R}{R+\lambda_R} - \frac{R}{R+\lambda}\right) f(\lambda|R) = \pi f\left(s_R^{-1}(\lambda)|R\right)\left(s_R^{-1}(\lambda)\right)' \left(\frac{R}{R+\lambda_R} - \frac{R}{R+s_R^{-1}(\lambda)}\right).$$

For any $\lambda < \lambda_R$ we have the LHS different from zero, therefore we need $\left(s_R^{-1}(\lambda)\right)' \neq 0$ and $s_R^{-1}(\lambda) \neq \lambda_R$. Therefore, for λ_R we must have $s_R^{-1}(\lambda_R) = \lambda_R$.

Now, I need to show that $\bar{R}$ and $\bar{L}$ are also unique. I show that for the limit case $\xi = 0$, the proof for small positive ξ is identical, only the formulas are more elaborate. Let all reports (λ, ρ) that generate η^* and either $\rho \leq \bar{R}$ or $\lambda \leq \bar{L}$ be called the double ambiguity area (DAA).

First, by Step 11 η^*, $\bar{R}$ and $\bar{L}$ must be such that $\eta^*_{\bar{R}+\varepsilon} \to \eta^*$ and $\eta^*_{\bar{L}+\varepsilon} \to \eta^*$, which means that they must satisfy equation (10) for $\xi = 0$:

$$\eta^* = \frac{\frac{\pi_r}{1-\pi_l}\int_0^{1-\bar{R}} \frac{\bar{R}}{\bar{R}+L} f\left(L|\bar{R}\right) dL + \frac{(1-\pi_r-\pi_l)}{1-\pi_l}\int_0^{\lambda_{\bar{R}}} \frac{\bar{R}}{\bar{R}+L} f\left(L|\bar{R}\right) dL}{\frac{\pi_r}{1-\pi_l} + \frac{(1-\pi_r-\pi_l)}{1-\pi_l} F\left(\lambda_{\bar{R}}|\bar{R}\right)}, \tag{12}$$

$$\eta^* = \frac{\frac{\pi_l}{1-\pi_r}\int_0^{1-\bar{L}} \frac{R}{R+\bar{L}} f\left(R|\bar{L}\right) dR + \frac{(1-\pi_l-\pi_r)}{1-\pi_r}\int_0^{\rho_{\bar{L}}} \frac{R}{R+\bar{L}} f\left(R|\bar{L}\right) dR}{\frac{\pi_l}{1-\pi_r} + \frac{(1-\pi_l-\pi_r)}{1-\pi_r} F\left(\rho_{\bar{L}}|\bar{L}\right)}. \tag{13}$$

Note that equation (12) and equation (13) imply that $\eta^* \in [0, 1]$.

For all reports in DAA to generate the same belief η^*, this belief must satisfy

$$\begin{aligned}\eta^* &= P(P_r|DAA)\, E\left[\frac{R}{R+L}|DAA, P_r\right] + \\ &+ P(P_l|DAA)\, E\left[\frac{R}{R+L}|DAA, P_l\right] + P(H|DAA)\, E\left[\frac{R}{R+L}|DAA, H\right].\end{aligned}$$

Recall Figure A; the equation above can be rewritten as follows (where $\pi_H = (1 - \pi_l - \pi_r)$ and f is used instead of $f(L,K)$ to shorten the formula):

$$\eta^* = \frac{\pi_r \int_0^{\bar{R}} \int_0^{1-R} \frac{R}{R+L} f dL dR}{\pi_r \int_0^{\bar{R}} \int_0^{1-R} f dL dR + \pi_l \int_0^{\bar{L}} \int_0^{1-L} f dL dR + \pi_H \left(\int_{\frac{\eta^*}{1-\eta^*}\bar{L}}^{\bar{R}} \int_0^{\frac{1-\eta^*}{\eta^*}R} f dL dR + \int_0^{\frac{\eta^*}{1-\eta^*}\bar{L}} \int_0^{\bar{L}} f dL dR \right)} \tag{14}$$

$$+ \frac{\pi_l \int_0^{\bar{L}} \int_0^{1-L} \frac{R}{R+L} f dL dR}{\pi_r \int_0^{\bar{R}} \int_0^{1-R} f dL dR + \pi_l \int_0^{\bar{L}} \int_0^{1-L} f dL dR + \pi_H \left(\int_{\frac{\eta^*}{1-\eta^*}\bar{L}}^{\bar{R}} \int_0^{\frac{1-\eta^*}{\eta^*}R} f dL dR + \int_0^{\frac{\eta^*}{1-\eta^*}\bar{L}} \int_0^{\bar{L}} f dL dR \right)}$$

$$+ \frac{\pi_H \left(\int_{\frac{\eta^*}{1-\eta^*}\bar{L}}^{\bar{R}} \int_0^{\frac{1-\eta^*}{\eta^*}R} \frac{R}{R+L} f dL dR + \int_0^{\frac{\eta^*}{1-\eta^*}\bar{L}} \int_0^{\bar{L}} \frac{R}{R+L} f dL dR \right)}{\pi_r \int_0^{\bar{R}} \int_0^{1-R} f dL dR + \pi_l \int_0^{\bar{L}} \int_0^{1-L} f dL dR + \pi_H \left(\int_{\frac{\eta^*}{1-\eta^*}\bar{L}}^{\bar{R}} \int_0^{\frac{1-\eta^*}{\eta^*}R} f dL dR + \int_0^{\frac{\eta^*}{1-\eta^*}\bar{L}} \int_0^{\bar{L}} f dL dR \right)}$$

The LHS is continuous, strictly increasing, and $LHS \in [0,1]$. The RHS is continuous and for $\eta^* \to 0$ equation (12) implies that $\bar{R} \to 0$ and $\bar{L} \to 1$; therefore the $RHS \to \frac{\int_0^1 \int_0^{1-L} \frac{R}{R+L} f dL dR}{\int_0^1 \int_0^{1-L} f dL dR} > 0$. Similarly, as $\eta^* \to 1$ then $\bar{R} \to 1$ and $\bar{L} \to 0$ therefore $RHS \to \frac{\int_0^1 \int_0^{1-R} \frac{R}{R+L} f dL dR}{\int_0^1 \int_0^{1-R} f dL dR} < 1$. Therefore, the solution exists. To see that the solution is unique, we can take the derivative of the RHS with respect to η^* and evaluate it at the point at which $\eta^* = RHS(\eta^*)$. We have

$$\frac{dRHS}{d\eta} = \frac{\partial RHS}{\partial \eta} + \frac{\partial RHS}{\partial \bar{R}} \frac{d\bar{R}}{d\eta} + \frac{\partial RHS}{\partial \bar{L}} \frac{d\bar{L}}{d\eta}$$

and using equation (12) and equation (13) we can show

$$\frac{\partial RHS}{\partial \eta} (\eta = \eta^*) = 0$$

$$\frac{\partial RHS}{\partial \bar{R}} (\eta = \eta^*) = 0$$

$$\frac{\partial RHS}{\partial \bar{L}} (\eta = \eta^*) = 0$$

Therefore, every time $\eta^* = RHS(\eta^*)$, the derivative $\frac{dRHS}{d\eta} = 0$; therefore, there is at most one solution. ■

Proof of Proposition 3

The proof compares any discontinuous equilibrium with the continuous one. All variables that represent the continuous equilibrium have superscript c, $\lambda_\rho^c, \eta_\rho^{*c}$. First, note that by the construction of the continuous equilibrium we have

$$\frac{\pi E[q_R|R] + (1-\pi) P(q_R > \theta_i) E[q_R|q_R > \theta_i, R]}{\pi + (1-\pi) P(q_R > \theta_i)} \begin{cases} > \theta_i \text{ if } \theta_i < \eta_R^{*c} \\ < \theta_i \text{ if } \theta_i > \eta_R^{*c} \end{cases} \tag{15}$$

Take ρ such that $\eta_\rho^* < \eta_\rho^{*c}$. Therefore, for this ρ only the decision maker with $\theta_i \in (\eta_\rho^*, \eta_\rho^{*c})$ is affected[25], and she chooses *Left* under the discrete equilibrium, and she chooses *Right* in the continuous equilibrium when she faces P_r, and she chooses optimally otherwise.

[25]In fact if other θ_i's are affected that works to a clear disadvantage of the discrete equilibrium; therefore I omit those cases.

The change in the utility given that r happens is

$$\begin{aligned}
U_{discrete} - U_{cont} &= EU\left(Left|R\right) - \pi EU\left(Right|R\right) - (1-\pi) E\left[\max\left\{U\left(Left|R\right), U\left(Right|R\right)\right\}\right] < \\
&= E\left[\theta_i - q_R|R\right] - \pi E\left[q_R - \theta_i|R\right] \\
&\quad - (1-\pi) P\left(q_R < \theta_i|R\right) E\left[\theta_i - q_R|q_R < \theta_i, R\right] \\
&\quad - (1-\pi) P\left(q_R > \theta_i|R\right) E\left[q_R - \theta_i|q_R > \theta_i, R\right] \\
&= \underbrace{2\pi\left(\theta_i - E\left[q_R|R\right]\right) + 2(1-\pi) P\left(q_R > \theta_i\right)\left(\theta_i - E\left[q_R|q_R > \theta_i, R\right]\right)}_{<0 \text{ using } (15)} < 0
\end{aligned}$$

Take ρ such that $\eta_\rho^* > \eta_\rho^{*c}$. Therefore, for $R = \rho$ the decision maker with $\theta_i \in \left(\eta_\rho^{*c}, \eta_\rho^*\right)$ is affected. In the continuous equilibrium she chooses *Left*. In the discrete equilibrium she chooses *Right* if she faces the persuader (as the persuader can generate at least belief η_ρ^*), and she chooses at best optimally if she faces H[26].The change in the utility given that $R = \rho$ is

$$\begin{aligned}
U_{discrete} - U_{cont} &= (1-\pi) E\left[\max\left\{U\left(Left|R\right), U\left(Right|R\right)\right\}\right] + \pi U\left(Right|R\right) - U\left(Left|R\right) = \\
&= (1-\pi) P\left(q_R < \theta_i|R\right)\left(\theta_i - E\left[q_R|q_R < \theta_i, R\right]\right) + \\
&\quad + (1-\pi) P\left(q_R > \theta_i|R\right)\left(E\left[q_R|q_R > \theta_i, R\right] - \theta_i\right) + \\
&\quad + \pi E\left[q_R - \theta_i|R\right] - E\left[\theta_i - q_R|R\right] \\
&= 2\pi\left(E\left[q_R|R\right] - \theta_i\right) + 2(1-\pi) P\left(q_R > \theta_i|R\right)\left(E\left[q_R|q_R > \theta_i, R\right] - \theta_i\right) < 0
\end{aligned}$$

In a highly irregular equilibrium also some $\theta_i > \eta_\rho^*$ can be affected, but in such a way that the formula above is still negative. ■

Proof of Proposition 4

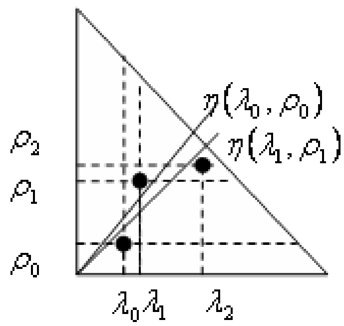
This proof requires only slight modifications to the proof of Proposition 5. Lemma 1 does not rely on H being nonstrategic. Below, I prove Lemma 2 for H being a strategic, benevolent expert. All of the steps of the proof of Proposition 5 then follow.

Lemma 3 *i) If (λ, ρ) is sent by P_l only when he reports fully, then $\eta(\lambda, \rho) \leq \frac{\rho}{\rho+\lambda}$,*
ii) If (λ, ρ) is sent by P_r only when he reports fully, then $\eta(\lambda, \rho) \geq \frac{\rho}{\rho+\lambda}$.

I prove only part (i). Proof of part (ii) is analogous to the proof of part (i).

Assume (λ_0, ρ_0) is never sent by P_l when $(L, R) \neq (\lambda_0, \rho_0)$ and $\eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0+\lambda_0}$, as shown in the Figure below.

[26] If the equilibrium has a more complicated form then the decision maker may not be able to choose optimally when she faces H, but that would only decrease the informativeness of a discrete equilibrium.



Then there exists $(\lambda_1, \rho_1) \in Z(\lambda_0, \rho_0)$ such that $\frac{\rho_1}{\rho_1+\lambda_1} \geq \eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0+\lambda_0}$ and $\sigma_i(\lambda_1, \rho_1) = (\lambda_0, \rho_0)$ for $i \in (P_r, H)$. There are three cases: 1. $\eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, 2. $\eta(\lambda_1, \rho_1) > \frac{\rho_1}{\rho_1+\lambda_1}$, 3. $\eta(\lambda_1, \rho_1) < \frac{\rho_1}{\rho_1+\lambda_1}$. If Case 1, $\eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, then only P_r could use a strategy $\sigma_{P_r}(\lambda_1, \rho_1) = (\lambda_0, \rho_0)$, but only if $\eta(\lambda_0, \rho_0) > \eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, which is not the case. If Case 2, $\eta(\lambda_1, \rho_1) > \frac{\rho_1}{\rho_1+\lambda_1}$, then P_l never sends (λ_1, ρ_1) while concealing information, and then call $(\lambda_0, \rho_0) = (\lambda_1, \rho_1)$ and start the analysis from the beginning, which would require an infinite sequence of reports violating Lemma 1. Therefore, it must be that Case 3, $\eta(\lambda_1, \rho_1) < \frac{\rho_1}{\rho_1+\lambda_1}$, and moreover $\eta(\lambda_1, \rho_1) < \eta(\lambda_0, \rho_0)$, otherwise neither H nor P_l would send (λ_1, ρ_1). But then there exists $(\lambda_2, \rho_2) \in Z(\lambda_1, \rho_1)$ such that $\frac{\rho_2}{\rho_2+\lambda_2} \leq \eta(\lambda_1, \rho_1)$ and $\sigma_i(\lambda_2, \rho_2) = (\lambda_1, \rho_1)$. But then $i \in (H, P_l)$, as P_r would prefer to send (λ_0, ρ_0). Therefore, again, we have three cases. Case 1, $\frac{\rho_2}{\rho_2+\lambda_2} = \eta(\lambda_2, \rho_2)$, is impossible. If Case 2, $\eta(\lambda_2, \rho_2) > \frac{\rho_2}{\rho_2+\lambda_2}$, then it must be that $\eta(\lambda_2, \rho_2) > \eta(\lambda_1, \rho_1)$, otherwise neither H nor P_l would send $\sigma_i(\lambda_2, \rho_2) = (\lambda_1, \rho_1)$. But then set $(\lambda_0, \rho_0) = (\lambda_2, \rho_2)$ and start the analysis from the beginning, which would again violate Lemma 1. Therefore, Case 3, $\eta(\lambda_2, \rho_2) < \frac{\rho_2}{\rho_2+\lambda_2}$, must be the case. If we continue this reasoning we will end up with an infinite sequence of reports such that $(\lambda_n, \rho_n) \in Z(\lambda_{n-1}, \rho_{n-1})$ and $\eta(\lambda_n, \rho_n) < \frac{\rho_n}{\rho_n+\lambda_n}$, but that violates Lemma 1. ■

Proof of Proposition 6

Equation (7) takes the following form

$$\eta_R^* \equiv \frac{\frac{\pi_r}{1-\pi_l}\int_0^{1-R} \frac{R}{R+L} f(L|R)\, dL + \frac{\pi_H}{1-\pi_l}\int_0^{\lambda_R} \frac{R}{R+L} f(L|R)\, dL}{\frac{\pi_r}{1-\pi_l} + \frac{\pi_H}{1-\pi_l} F(\lambda_R|R)} \tag{16}$$

Equation (16) does not depend on π_l, therefore we can take the limit of equation (16) keeping π_r constant, and we get $\lim_{\pi_H \to 1} \eta_R^* = E\left[\frac{R}{R+L}|R\right]$, and $\lim_{\pi_H \to 0} \eta_R^* \to 1$. The definition of $\eta_R^* \equiv \frac{R}{R+\lambda_R}$ implies that $\lim_{\pi_H \to 0} \lambda_R = 0$; that is, the reports of the persuader become more extreme, and that $\lim_{\pi_H \to 1} \lambda_R = \bar{\lambda}_R = \frac{1-E[q_R|R]}{E[q_R|R]} R < 1 - R$. The analogous holds for P_l.

The utility of each persuader depends on what belief he can induce given the number of arguments that favor his alternative. That depends on the shape of the ambiguity area for this persuader and on the belief induced in the double ambiguity area. First, since $\frac{\pi_H}{1-\pi_r}$ is kept constant, the shape of the ambiguity area for

P_l remains unchanged, which can be seen if we expand equation (8):

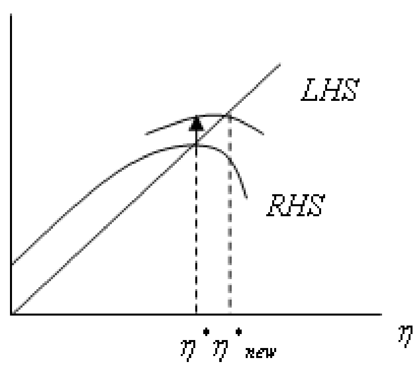
$$\eta_L^* \equiv \frac{\frac{\pi_l}{1-\pi_r}\int_0^{1-L}\frac{R}{R+L}f\left(R|L\right)dR+\frac{\pi_H}{1-\pi_r}\int_0^{\rho_L}\frac{R}{R+L}f\left(R|L\right)dR}{\frac{\pi_l}{1-\pi_r}+\frac{\pi_H}{1-\pi_r}F\left(\rho_L|L\right)}.$$

Keeping $\frac{\pi_H}{1-\pi_r}$ constant implies that π_H increases as π_r decreases, therefore equation (16) implies that the ambiguity area of P_r shrinks. That means that $\lim_{\pi_r\to 0}\lambda_R \to 0$, which means that the reports of the persuader toward *Right* become more extreme. It remains to show what happens to the double ambiguity area. The proof of proposition 2 shows that the shape of the ambiguity area is determined by η^*, therefore we have to show that η^* increases as π_r goes down.

η^* is determined by equation 14. If we take the derivative of the RHS of this equation with respect to π_r and evaluate at η^* we get

$$\frac{dRHS}{d\pi_r} = \frac{\frac{1}{(1-\pi_r)^2}\int_0^{\bar{R}}\int_0^{1-R}\left(\frac{R}{R+L}-\eta^*\right)fdLdR+\frac{\pi_H}{1-\pi_r}\left(\int_0^{\frac{1-\eta^*}{\eta^*}\bar{R}}\left(\frac{R}{R+L}-\eta^*\right)fdL\right)\left(\frac{d\bar{R}}{d\pi_r}\right)}{\frac{\pi_r}{1-\pi_r}\int_0^{\bar{R}}\int_0^{1-R}fdLdR+\frac{\pi_l}{1-\pi_r}\int_0^{\bar{L}}\int_0^{1-L}fdLdR+\frac{\pi_H}{1-\pi_r}\left(\int_{\frac{\eta^*}{1-\eta^*}\bar{L}}^{\bar{R}}\int_0^{\frac{1-\eta^*}{\eta^*}R}fdLdR+\int_0^{\frac{\eta^*}{1-\eta^*}\bar{L}}\int_0^{\bar{L}}fdLdR\right)} < 0.$$

Recall that η^* is the point of intersection of the $LHS\left(\eta\right)$ and the $RHS\left(\eta\right)$ of equation 14 for the initial π_r, like in the picture below.



The fact that $\frac{dRHS}{d\pi_r} < 0$ implies that as π_r decreases the function $RHS\left(\eta\right)$ shifts up, which implies that the new $\eta_{new}^* > \eta^*$.

Now we can conclude that P_r is better off, and P_l is worse off when π_r goes down. As π_r decreases, η^* and η_R^* for each R increases, and for each R the persuader toward *Right* can induce a higher belief, or in other words, he can persuade more decision makers. Since the ambiguity area for P_l has the same shape, for big L, P_l can induce the same belief. However, the belief attached to the double ambiguity area is higher, and also it is achieved for lower $\bar{L}_{new} < \bar{L}$ which means that for $L < \bar{L}$, P_l induces a higher belief, which means less decision makers choose *Left*.

Showing that the utility of the decision maker increases requires some tedious algebra, which I omit here, but the result is straightforward, since it is more likely that the decision maker faces the honest expert. ■

Proof of Proposition 7

Given the assumptions on $g(q_R, N; z)$, we can derive the conditional distribution of N given $R: g(N|R;z) = \frac{g(N;z)}{N}$. Therefore, R_i, the threshold R for which the decision maker with parameter θ_i is indifferent between alternatives, is defined by

$$\theta_i = \frac{\pi \int_{R_i}^1 \frac{R_i}{N}\frac{1}{N} g(N;z)\, dN + (1-\pi)\int_{\frac{R_i}{\theta_i}}^1 \frac{R_i}{N}\frac{1}{N} g(N;z)\, dN}{\pi \int_{R_i}^1 \frac{1}{N} g(N;z)\, dN + (1-\pi)\int_{\frac{R_i}{\theta_i}}^1 \frac{1}{N} g(N;z)\, dN} > \frac{\int_{R_i}^1 \frac{R_i}{N}\frac{1}{N} g(N;z)\, dN}{\int_{R_i}^1 \frac{1}{N} g(N;z)\, dN}.$$

But the last expression can be rewritten as

$$\frac{\int_{R_i}^1 \frac{R_i}{N}\frac{1}{N} g(N;z)\, dN}{\int_{R_i}^1 \frac{1}{N} g(N;z)\, dN} = \frac{\hat{R}E\left[\frac{1}{N^2}|N > R_i; z\right]}{E\left[\frac{1}{N}|N > R_i; z\right]} > \frac{R_i}{\bar{N}} \Rightarrow 2R_i < \theta_i.$$

The utility of the decision maker with the preference parameter θ_i is

$$\begin{aligned} U &= \int_0^{R_i}\int_0^1 (1+\theta_i - q_R)\, g(N;z)\, dN dq_R \\ &+\pi \int_{R_i}^1 \int_0^{\frac{R_i}{q_R}} (1+\theta_i - q_R)\, g(N;z)\, dN dq_R + \pi \int_{R_i}^1 \int_{\frac{R_i}{q_R}}^1 (1-\theta_i + q_R)\, g(N;z)\, dN dq_R \\ &+ (1-\pi)\int_{\hat{R}}^{\theta_i}\int_0^1 (1+\theta_i - q_R)\, g(N;z)\, dN dq_R + (1-\pi)\int_{\theta_i}^1\int_0^{\frac{R_i}{q_R}} (1+\theta_i - q_R)\, g(N;z)\, dN dq_R \\ &+ (1-\pi)\int_{\theta_i}^1\int_{\frac{R_i}{q_R}}^1 (1-\theta_i + q_R)\, g(N;z)\, dN dq_R, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} U &= \int_0^{R_i} (1+\theta_i - q_R)\, dN d\eta + \pi \int_{R_i}^1 (1-\theta_i + q_R)\, dq_R \\ &+ (1-\pi)\int_{\hat{R}}^{\theta_i} (1+\theta_i - q_R)\, dN dq_R + (1-\pi)\int_{\theta_i}^1 (1-\theta_i + q_R)\, dN dq_R \\ &+\pi \int_{R_i}^1 2(\theta_i - q_R)\, G\left(\frac{R_i}{q_R}; z\right) dN dq_R + (1-\pi)\int_{\theta_i}^1 2(\theta_i - q_R)\, G\left(\frac{R_i}{q_R}; z\right) dq_R \end{aligned}$$

R_i is chosen to maximize $\bar{U}$; therefore,

$$\frac{dU}{dz} = \frac{\partial U}{\partial z} + \frac{\partial U}{\partial R_i}\frac{dR_i}{dz} = \frac{\partial U}{\partial z}.$$

We get

$$\frac{dU}{dz} = \pi \int_{R_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dN dq_R + (1-\pi)\int_{\theta_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dq_R.$$

The second expression is positive, as $2(\theta_i - q_R) < 0$ and $G_z\left(\frac{R_i}{q_R}; z\right) < 0$ for $q_R \in (\theta_i, 1)$. The first expression can be rewritten as

$$\begin{aligned} \int_{R_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dN dq_R &= \int_{R_i}^{2R_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dq_R \\ &+ \int_{2R_i}^{\theta_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dq_R \\ &+ \int_{\theta_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R}; z\right) dq_R \end{aligned}$$

We have $G_z\left(\frac{R_i}{q_R};z\right)<0$ if and only if $q_R>2R_i$. Hence, the first and the last expressions are positive. Note that $R_i<\frac{R_i}{1-\frac{R_i}{\theta_i}}<2R_i$; therefore, we can rewrite

$$\begin{aligned}\frac{dU}{dz} &= \int_{R_i}^{\frac{R_i}{1-\frac{R_i}{\theta_i}}} 2\left(\theta_i-q_R\right)G_z\left(\frac{R_i}{q_R};z\right)dq_R+\int_{\frac{R_i}{1-\frac{R_i}{\theta_i}}}^{2R_i} 2\left(\theta_i-q_R\right)G_z\left(\frac{R_i}{q_R};z\right)dq_R \\ &\quad +\int_{2R_i}^{\theta_i} 2\left(\theta_i-q_R\right)G_z\left(\frac{R_i}{q_R};z\right)dq_R+\int_{\theta_i}^{1} 2\left(\theta_i-q_R\right)G_z\left(\frac{R_i}{q_R};z\right)dq_R.\end{aligned}$$

By symmetry of F we have

$$\int_{\frac{R_i}{1-\frac{R_i}{\theta_i}}}^{2R_i} G_z\left(\frac{R_i}{q_R};z\right)dq_R+\int_{2R_i}^{\theta_i} G_z\left(\frac{R_i}{q_R};z\right)dq_R=0,$$

where $G_z\left(\frac{R_i}{q_R};z\right)>0$ in the first integral. Also (θ_i-q_R) is greater for any q_R from the first integral; therefore, we get $\int_{\frac{R_i}{1-\frac{R_i}{\theta_i}}}^{2R_i} G_z\left(\frac{R_i}{q_R};z\right)dq_R+\int_{2R_i}^{\frac{1}{2}} G_z\left(\frac{R_i}{q_R};z\right)dq_R>0$. That completes the argument that $\frac{dU}{dz}>0$.

Full revelation of information in the case of $\lim z=\infty$ is a standard result. ■

Proof of Proposition 8

As mentioned in the text, the proof of this proposition is very similar to the proof of Proposition 5. The crucial difference is that $\frac{\rho}{\rho+\lambda}$ should be replaced by $\omega(\lambda,\rho)$. Here I will provide the proof of Step 2 which is the only one that differs significantly. The entire proof available upon request.

Step 2 i) There exists $\bar{\rho}\in[0,1)$ such that for all $\rho<\bar{\rho}$ we have $\eta(0,\bar{\rho})\equiv$ *const* and for all $\rho\in(\bar{\rho},1)$ we have $\eta(0,\rho)<\omega(0,\rho)$.

ii) There exists $\bar{\lambda}\in[0,1)$ such that for all $\rho<\hat{\lambda}$ we have $\eta(0,\bar{\rho})\equiv$ *const* and for all $\lambda\in\left(\bar{\lambda},1\right)$ we have $\eta(0,\lambda)<\omega(0,\lambda)$

Assume that there exists $\rho_0>\bar{\rho}$ such that $\eta(0,\rho_0)\geq\omega(0,\rho_0)$. This means that for some $\rho\in(\bar{\rho},\rho_0)$ either $\eta(0,\rho)<\eta(0,\rho_0)$ or $\eta(0,\rho)>\eta(0,\rho_0)$. If $\eta(0,\rho)>\eta(0,\rho_0)=\omega(0,\rho_0)>\omega(0,\rho)$ then P_l never sends $(0,\rho)$ when lying and when the state of nature is high enough to generate such a high belief (he would rather send $(0,\rho_0)$ and this is feasible). Therefore by Lemma 2 that cannot be the case. If $\eta(0,\rho)<\eta(0,\rho_0)$ and $\eta(0,\rho_0)>\eta(0,\rho_0)$, then there must exist $(\lambda_1,\rho_1)\in z(0,\rho_0)$ and $\omega(\lambda_1,\rho_1)>\eta(0,\rho_0)$ such that $\sigma_{P_r}(\lambda_1,\rho_1)=(0,\rho_0)$. But then it must be that $\eta(\lambda_1,\rho_1)<\omega(\lambda_1,\rho_1)$, and since P_r does not send (λ_1,ρ_1) when lying, that contradicts Lemma 2.

If $\eta(0,\rho)<\eta(0,\rho_0)$ and $\eta(0,\rho_0)=\eta(0,\rho_0)$, then by Step 1 and Lemma 2 we have that $\eta(0,\rho_0)>\eta(\lambda,\rho_0)$ for all $\lambda\in(0,1-\rho_0)$; therefore P_r sends $(0,\rho_0)$ when $\eta(\lambda,\rho_0)$ for λ large enough. But then again, to generate $\eta(0,\rho_0)=\eta(0,\rho_0)$ there must exist $(\lambda_1,\rho_1)\in z(0,\rho_0)$ and $\omega(\lambda_1,\rho_1)>\eta(0,\rho_0)$ such that $\sigma_{P_r}(\lambda_1,\rho_1)=(0,\rho_0)$. But then it must be that $\eta(\lambda_1,\rho_1)<\omega(\lambda_1,\rho_1)$ and since P_r does not send (λ_1,ρ_1) when lying, that contradicts Lemma 2.

An analogous proof holds for (ii). ■

Proof of Proposition 9

I present only the proof for the case in which the threshold R for which the decision maker is indifferent between the alternatives, $\hat{R}$, is smaller than $\underline{N}$. The proof is analogous for the other case. $\hat{R}$ is defined by the following equation.

$$0=\pi\int_{\underline{N}}^{\alpha}\left(1-2\frac{\hat{R}}{n}\right)\frac{1}{n}\frac{g(n)}{G(\alpha)}dn+(1-\pi)\int_{\underline{N}}^{2\hat{R}}\left(1-2\frac{\hat{R}}{n}\right)(1-(1-\alpha)\beta)\frac{1}{n}\frac{g(n)}{G(\alpha)}dn \tag{17}$$

The expected utility of the decision maker is

$$\begin{aligned}U \;=\; & \pi(1-(1-\alpha)\beta)\int_{\underline{N}}^{\alpha}\left(\int_{\frac{\hat{R}}{n}}^{1}\left(q_r-\frac{1}{2}\right)dq_r-\int_{0}^{\frac{\hat{R}}{n}}\left(q_r-\frac{1}{2}\right)dq_r\right)\frac{g(n)}{G(\alpha)}dn\\ & +(1-\pi)(1-(1-\alpha)\beta)\int_{\underline{N}}^{2\hat{R}}\left(\int_{\frac{\hat{R}}{n}}^{1}\left(q_r-\frac{1}{2}\right)dq_r-\int_{0}^{\frac{\hat{R}}{n}}\left(q_r-\frac{1}{2}\right)dq_r\right)\frac{g(n)}{G(\alpha)}dn\\ & +(1-\pi)(1-(1-\alpha)\beta)\int_{2\hat{R}}^{\alpha}\int_{\frac{1}{2}}^{1}\left(q_r-\frac{1}{2}\right)dq_r\frac{g(n)}{G(\alpha)}dn\\ & -(1-\pi)(1-(1-\alpha)\beta)\int_{2\hat{R}}^{\alpha}\int_{0}^{\frac{1}{2}}\left(q_r-\frac{1}{2}\right)dq_r\frac{g(n)}{G(\alpha)}dn,\end{aligned}$$

which can be rewritten as

$$\begin{aligned}U \;=\; & \pi(1-(1-\alpha)\beta)\int_{\underline{N}}^{\alpha}\frac{\hat{R}}{n}\left(1-\frac{\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}dn\\ & +(1-\pi)(1-(1-\alpha)\beta)\int_{\underline{N}}^{2\hat{R}}\frac{\hat{R}}{n}\left(1-\frac{\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}dn\\ & +(1-\pi)(1-(1-\alpha)\beta)\frac{1}{4}\int_{2\hat{R}}^{\alpha}\frac{g(n)}{G(\alpha)}dn,\end{aligned}$$

therefore,

$$\begin{aligned}\frac{dU}{d\alpha}\frac{1}{(1-(1-\alpha)\beta)} \;=\; & \beta\frac{U}{(1-(1-\alpha)\beta)^2}\\ & -\frac{g(\alpha)\hat{R}}{(G(\alpha))^2}\left(\pi\int_{\underline{N}}^{\alpha}\left(1-\frac{2\hat{R}}{n}\right)\frac{g(n)}{n}\frac{G(n)}{ng(n)}dn+(1-\pi)\int_{\underline{N}}^{2\hat{R}}\left(1-2\frac{\hat{R}}{n}\right)\frac{g(n)}{n}\frac{G(n)}{ng(n)}dn\right),\end{aligned}$$

where I used integration by parts to simplify the last part. Denote the first expression I and the second T, as the first shows the utility gain due to higher information precision and the second shows the loss due to lower transparency. $I>0$ and $T>0$, because T is similar to equation 17, but the expression inside the integral is multiplied by $\frac{G(n)}{ng(n)}$. As $\frac{G(n)}{ng(n)}$ is increasing the parts that are positive have higher weight than in equation 17. We have

$$\frac{dT}{d\beta}=-(1-\alpha)\frac{T}{(1-(1-\alpha)\beta)}<0$$

as $\frac{d\hat{R}}{d\beta}=0$, and we have

$$\frac{dI}{d\beta}=\frac{U}{(1-(1-\alpha)\beta)}>0$$

Additionally, if $\beta=0$ then $I=0$ and $\frac{dU}{d\alpha}<0$. Therefore, there exists $\hat{\beta}(\pi)$ such that for all $\beta<\hat{\beta}(\pi)$, the decision maker chooses a low-quality expert.

We want to show now that

$$\frac{d\beta(\pi)}{d\pi} = -\frac{\frac{d}{d\pi}\left(\frac{dU}{d\alpha}\right)}{\frac{d}{d\beta}\left(\frac{dU}{d\alpha}\right)} > 0,$$

therefore, we need to show $\frac{d}{d\pi}\left(\frac{dU}{d\alpha}\right) > 0$. We have

$$\frac{dI}{d\pi} = \frac{\beta}{(1-(1-\alpha)\beta)}\frac{dU}{d\pi} < 0$$

$$\begin{aligned}
\frac{dT}{d\pi}\left(\frac{G(\alpha)}{(1-(1-\alpha)\beta)g(\alpha)}\right) &= \hat{R}\int_{2\hat{R}}^{\alpha}\frac{1}{n}\left(1-\frac{2\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn \\
&+\frac{d\hat{R}}{d\pi}\pi\int_{\underline{N}}^{\alpha}\frac{1}{n}\left(1-\frac{4\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn+ \\
&+\frac{d\hat{R}}{d\pi}(1-\pi)\int_{\underline{N}}^{2\hat{R}}\frac{1}{n}\left(1-4\frac{\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn
\end{aligned}$$

$$\frac{d\hat{R}}{d\pi} = \frac{\int_{2\hat{R}}^{\alpha}\left(1-2\frac{\hat{R}}{n}\right)\frac{1}{n}\frac{g(n)}{G(\alpha)}dn}{\pi\int_{\underline{N}}^{\alpha}2\frac{1}{n^2}\frac{g(n)}{G(\alpha)}dn+(1-\pi)\int_{\underline{N}}^{2\hat{R}}2\frac{1}{n^2}\frac{g(n)}{G(\alpha)}dn}$$

$$\begin{aligned}
\frac{dT}{d\pi}\left(\frac{G(\alpha)}{(1-(1-\alpha)\beta)g(\alpha)}\right) &> \hat{R}\int_{2\hat{R}}^{\alpha}\frac{1}{n}\left(1-\frac{2\hat{R}}{n}\right)\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn \\
&-\frac{d\hat{R}}{d\pi}\pi\int_{\underline{N}}^{\alpha}\frac{2\hat{R}}{n^2}\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn \\
&-\frac{d\hat{R}}{d\pi}(1-\pi)\int_{\underline{N}}^{2\hat{R}}\frac{2\hat{R}}{n^2}\frac{g(n)}{G(\alpha)}\frac{G(n)}{ng(n)}dn
\end{aligned}$$

$$\begin{aligned}
\frac{dT}{d\pi}\left(\frac{G(\alpha)}{(1-(1-\alpha)\beta)g(\alpha)}\right)\frac{G^2(\alpha)}{2\hat{R}} &> \left(\int_{2\hat{R}}^{\alpha}\left(1-\frac{2\hat{R}}{n}\right)\frac{g(n)}{n}\frac{G(n)}{ng(n)}dn\right)\left(\int_{\underline{N}}^{\alpha}\frac{g(n)}{n^2}dn\right) \\
&-\left(\int_{2\hat{R}}^{\alpha}\left(1-2\frac{\hat{R}}{n}\right)\frac{g(n)}{n}dn\right)\left(\int_{\underline{N}}^{\alpha}\frac{g(n)}{n^2}\frac{G(n)}{ng(n)}dn\right) \\
&+\left(\int_{2\hat{R}}^{\alpha}\left(1-\frac{2\hat{R}}{n}\right)\frac{g(n)}{n}\frac{G(n)}{ng(n)}dn\right)\int_{\underline{N}}^{2\hat{R}}\frac{g(n)}{n^2}dn \\
&-\left(\int_{2\hat{R}}^{\alpha}\left(1-2\frac{\hat{R}}{n}\right)\frac{g(n)}{n}dn\right)\int_{\underline{N}}^{2\hat{R}}\frac{g(n)}{n^2}\frac{G(n)}{ng(n)}dn,
\end{aligned}$$

and the RHS is positive. To see that the RHS is positive note that if we set $\alpha = 2\hat{R}$ in the limits of integration then the RHS is zero. Set $b \equiv \alpha$ as the upper limit of integration and take the derivative with respect to that. For the last two terms we get

$$\frac{d\Delta_1}{db} = \left(1-\frac{2\hat{R}}{b}\right)\frac{g(b)}{b}\int_{\underline{N}}^{2\hat{R}}\frac{g(n)}{n^2}\left(\frac{G(b)}{bg(b)}-\frac{G(n)}{ng(n)}\right)dn > 0,$$

and for the first two,

$$\begin{aligned}
\frac{d\Delta_1}{db} &= \frac{g(b)}{b}\left(1-\frac{2\hat{R}}{b}\right)\left(\int_{\underline{N}}^{b}\frac{g(n)}{n^2}\left(\frac{G(b)}{bg(b)}-\frac{G(n)}{ng(n)}\right)dn\right) \\
&\quad -\frac{g(b)}{b^2}\int_{2\hat{R}}^{b}\frac{g(n)}{n}\left(1-\frac{2\hat{R}}{n}\right)\left(\frac{G(b)}{bg(b)}-\frac{G(n)}{ng(n)}\right)dn \\
&> \frac{g(b)}{b}\left(\int_{2\hat{R}}^{b}\left(\frac{1}{n}-\frac{1}{b}\right)\hat{R}g(n)\left(\frac{G(b)}{bg(b)}-\frac{G(n)}{ng(n)}\right)dn\right) > 0.
\end{aligned}$$

That completes the proof. ■

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