Exclusive contracts and market dominance*

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Abstract

We develop a new theory of exclusive dealing. The theory rests on two realistic assumptions: that firms are imperfectly informed about demand, and that a dominant firm has a competitive advantage over its rivals. In this setting, exclusive contracts tend to be pro-competitive when the dominant firm's competitive advantage is small, but are anti-competitive when it is more pronounced. In this latter case, the dominant firm can profitably use exclusive dealing as a means to increase its market share at the expenses of its rivals, but without necessarily driving them out of the market, or impeding their entry. We discuss the implications of the results for competition policy.

Keywords: Exclusive dealing; Non-linear pricing; Antitrust; Dominant firm

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1 Introduction

Exclusive dealing has long been a controversial practice under the antitrust laws. Typically, the sort of situation that antitrust authorities are concerned with involves two main actors:

(i) a dominant firm that controls a substantial share of the market and has entered into some kind of exclusive arrangement with its customers, and

(ii) a smaller competitor (or group of competitors) that has been active in the industry for some time and in principle could have used exclusive contracts, too, but apparently has not.[1]

Until now, such situations have proven surprisingly resistant to formal analysis. In this paper, we propose a new theory of exclusive dealing that naturally explains those stylised facts. In the model, the dominant firm competes in non-linear prices with rivals supplying substitute goods. By requiring buyers not to purchase from its competitors, the firm can boost the demand for its product, raising its market share and profit at the expenses of rivals and customers alike.

Crucially, for the strategy to be profitable it is not necessary that the dominant firm's existing rivals are driven out of the market, or that the entry of new ones is impeded. This marks a key difference with alternative theories (discussed below) that view exclusive dealing as a means to deprive a rival of economies of scale. Ours is rather more reminiscent of analyses that pre-date the Chicago-school critique, and that in the last decades have fallen into disrepute as a result of heavy attacks by the Chicago school.[2] We show that those analyses can be vindicated under realistic assumptions.

Our two key assumptions are that firms are *incompletely informed about demand*, and that the dominant firm has a sizeable *competitive advantage* over its rivals in terms of lower costs, better products, or a combination of the two. Both assumptions are necessary for exclusive contracts to be profitable and anti-competitive in our model.

[1] A recent instance is the *Intel* case, which has spurred extensive litigation on both sides of the Atlantic and has led to the largest fine ever seen in the history of European competition policy, over a billion Euro. Intel holds a market share of not more than 80%, and its main competitor, AMD, has been operating for years in the microchip sector. A similar pattern recurs in classic antitrust cases such as *Standard Fashion* (where the incumbent had a market share of around 40%) or *Brown Shoe* (which controlled less than 10% of the relevant market).

[2] A typical Chicago-school critique would run as follows:

> The theory of exclusionary tactics underlying the law appears to be that firm X, which already has ten percent of the market, can sign up more than ten percent of the retailers, perhaps twenty percent, and, by thus foreclosing rivals from retail outlets, obtain a larger share of the market. But one must then ask why so many retailers are willing to limit themselves to selling X's product. Why do not ninety percent of them turn to X's rivals? Because X has greater market acceptance? But then X's share of the market would grow for that reason and the requirements contracts have nothing to do with it. Because X offers them some extra inducement? But that sounds like competition. It is equivalent to a price cut, and surely X's competitors can be relied upon to meet competition. (Bork and Bowman, 1965, p. 366-7)

See also Bork (1978) and Posner (1976).

Under complete information, a firm would extract, through non-linear pricing, all the surplus in excess of what the buyer can obtain by trading with the firm's competitors. Consequently, the firm would offer a contract that maximises such surplus – a property known as bilateral efficiency.[3] If buyers have a preference for variety so that efficiency requires that they purchase various products, exclusive dealing cannot be observed in equilibrium. However, this argument fails with incomplete information about demand.[4] Suppose, for example, that the buyer's demand can be either high or low, but the firm does not know which state is realised. The firm will then be unable to extract all the surplus when demand is high. Furthermore, in order to extract more surplus in the high-demand state, it must distort the contract that applies in the low-demand one. If the firm is restricted to non-linear pricing, all it can do is to reduce its own quantity below the efficient level. With exclusive contracts, however, the firm can set its rivals' volume to zero. By shifting the cost of the distortion onto the rivals, this often proves to be the most profitable strategy.

Against this backdrop, the effects of exclusive contracts crucially depend on the degree of asymmetry among the firms. When the dominant firm's competitive advantage is small, exclusive contracts actually intensify competition as firms will now compete for exclusives, i.e. in utility space, where the intensity of competition is not attenuated by product differentiation. Competitors of comparable size are therefore caught in a prisoner's dilemma: they have a unilateral incentive to offer exclusive contracts, but would actually benefit if such contracts were prohibited. These results generalise our findings in a related article (Calzolari and Denicolò, 2013), where the analysis is restricted to the case of identical firms.

However, antitrust authorities and the courts are rarely concerned with equally sized competitors that compete vigorously; more often, they are concerned with dominant firms facing smaller rivals. In this case, things are radically different. If the dominant firm's competitive advantage is sufficiently large, its less efficient rivals may not be able to compete for exclusives effectively. Furthermore, the information rent that the buyer obtains when the dominant firm acts as a monopolist may actually be larger than the rent that he could obtain by trading only with the dominant firm's competitors, even if the latters price as aggressively as possible. If this is so, then exclusive contracts are profitable for the dominant firm, which can achieve exclusion at no cost and be completely sheltered from competition. When the competitive advantage is smaller, but still sizeable, exclusive dealing entails a cost (as buyers must now be compensated for accepting it) but it may still be a profitable way of foreclosing rivals.

In any case, rivals will be foreclosed only from low-demand segments of the market. The reason for this is that when demand is highest the outcome must be

[3] See Bernheim and Whinston (1998) and O'Brien and Shaffer (1997), who also highlight that bilateral efficiency must hold even if buyers have some bargaining power, as long as bargaining is efficient.

[4] Full extraction of surplus may also be impeded by the firm being restricted to linear pricing – a case analysed by Mathewson and Winter (1987). For an excellent analysis of the role of contractual complexity in exclusive dealing arrangements, see Spector (2011).

efficient (a no-distortion-at-the-top property). Like under complete information, the most profitable strategy for the dominant firm is to maximise the surplus, and then extract it as best as it can.

Besides staying active and continuing to sell to high-demand buyers, in our model rivals actively resist being foreclosed from low-demand segments of the market. They do so by reducing their non-exclusive prices, and under certain circumstances also by offering exclusive contracts in their turn. However, the dominant firm can exploit its competitive advantage to offer exclusive deals so attractive that they cannot be matched by rivals. Thus, even when rivals offer exclusive contracts, only those offered by the dominant firm are accepted in equilibrium.

Buyers benefit from exclusive contracts when the dominant firm's competitive advantage is small, but are harmed (both in terms of higher prices and reduced variety) when it is large. A classic Chicago-school question is: Why do buyers sign exclusive contracts if they get harmed by doing so? The answer is, that these are the best contracts among those that are actually offered to them. Buyers are harmed with respect to the hypothetical equilibrium that would arise if exclusive contracts were prohibited. However, when exclusive contracts are permitted firms change their entire pricing strategies, and so non-exclusive contracts are no longer available at the same conditions.

The different effects of exclusive contracts with small and large asymmetry reflect a more fundamental difference in the way they are pinned down in equilibrium. When firms are symmetric, or nearly so, the structure of exclusive contracts is entirely determined by competition in utility space. Bertrand-like arguments then imply that exclusive prices must reflect the firms' costs. When the competitive advantage is large, by contrast, the dominant firm, being sheltered from the competitive pressure from rivals, can freely design its exclusive contracts so as to better screen the buyers. The resulting optimal screening problem is non standard in nature. It can be formulated as a multi-stage optimal control problem, and the solution technique that we delevop may be of independent technical interest.

We close the introduction with a brief discussion of the related literature. To the best of our knowledge, only two papers rely on asymmetric information to provide a rationale for exclusive contracts: Bernheim and Whinston (1998, sect. V), who propose a model of moral hazard, and Martimort (1996), who, like us, focuses on adverse selection. However, he posits symmetric firms and models the exclusive dealing case by assuming that each firm has access to a different retailer. Therefore, in his model firms do not really compete for exclusives.

Other post-Chicago theories that regard exclusive contracts as anti-competitive typically assume complete information and rely on the absence from the contracting game of some of the agents that are affected by exclusivity clauses. Their practical relevance depends on the realism of this crucial assumption. For example, in Aghion and Bolton (1987) and Rasmusen, Ramseyer and Wiley (1991) the missing agent is the dominant firm's competitor – in those models, this is a potential entrant that initially cannot contract with the buyers. Thus, these theories cannot apply to the many antitrust cases in which the dominant firm's competitors are already active. In Bernheim and Whinston (1998, sect. IV), the agents who are missing from the contracting stage are future buyers whose demand is essential for the dominant firm's rival to achieve economies of scale. Their explanation therefore applies to markets where a substantial increase in demand is expected in the not too distant future.

Furthermore, theories that rely on the absence of affected agents from the contracting stage explicitly or implicitly assume that players are committed to the signed contracts.[5] In fact, should the missing agents materialise, the parties would have an incentive to renegotiate – a point that has been forcefully made by Spier and Whinston (1995). In some cases, for exclusion to be prevented it may suffice that exclusivity clauses may be breached upon payment of reasonable damages (Simpson and Wickelgren, 2007).

Finally, theories that view exclusive dealing as a means to deprive a rival of economies of scale are faced with the difficulty that exclusivity clauses may not be necessary for that purpose. The same outcome can sometimes be achieved by simple non-linear pricing, e.g. *via* quantity forcing or quantity discounts. If this is so, then a prohibition of exclusive contracts might be easily overcome and could even be welfare reducing.

While these difficulties may not be insurmountable, they limit the applicability of existing anti-competitive theories in antitrust practice.[6] The theory developed in this paper does not suffer from any of these drawbacks. It is consistent with the stylised facts mentioned above, including the fact that all involved parties can often participate in the contracting game. It produces an equilibrium in which exclusive contracts are (trivially) renegotiation proof and need not be long-term to be effective, as they play no commitment role. It does not rely on the dominant firm's rivals being driven out of the market – so no proof of eviction or recoupment is needed. Finally, it uses as a benchmark the non-linear pricing equilibrium, which already allows for quantity discounts (including quantity forcing).

In sum, our theory seems more broadly applicable than alternative anti-competitive explanations. Having said this, it must be stressed that the anti-competitive effects of exclusive contracts that we uncover must be weighted not only against the possible pro-efficiency rationales well known in the literature,[7] but also against the pro-competitive effects that arise in our model when firms

[5] This is reflected in the emphasis that antitrust authorities and the courts sometimes place on the duration of exclusive contracts.

[6] See Whinston (2008) for a discussion. Sometimes one difficulty can only be addressed at the cost of exacerbating others. For example, Chen and Shaffer (2010) develop an interesting variant of the Rasmusen, Ramseyer and Wiley (1991) model, in which the incumbent uses as exclusionary devices market-share discounts rather than exclusivity clauses. They show that market-share discounts can be anti-competitive even if the entrant eventually enters, which makes their theory applicable to a broader set of cases. However, they assume that the incumbent can pre-commit to future prices – an assumption that is not made by Rasmusen, Ramseyer and Wiley (1991).

[7] See Whinston (2008) for an excellent discussion of these rationales. The pro-competitive explanations of exclusive contracts are especially plausible when such contracts are used also by the dominant firm's competitors. When they are not, however, efficiency-enhancing explanations may be viewed with some skepticism.

are not too asymmetric.

The rest of the paper is structured as follows. Section 2 sets out the model. In section 3, we discuss how the possibility of offering exclusive contracts changes the formulation of the firm's pricing problem. Section 4 analyses the case in which the dominant firm faces a competitive fringe, and section 5 the case of duopoly. Section 6 summarises the paper's results and discusses their implications for competition policy. Proofs are in an Appendix.

2 The model

We consider a one-period model of price competition. There are two substitute goods, A and B. Good A is supplied by firm A, whereas good B may be supplied either by firm B (the *duopoly model*) or by a competitive fringe (the *competitive fringe model*).

A buyer who buys q_A units of good A and q_B units of good B obtains a benefit, measured in monetary terms, of $u(q_A, q_B, \theta)$. The reservation payoff, $u(0, 0, \theta)$, is normalised to zero. We may think of buyers as downstream firms, and of u as their gross profits,[8] or as final consumers, with u as their utility function. The function u is symmetric and smooth. It is initially increasing in q_A and q_B, but in view of our normalisation of costs (see below) we assume that there exists a finite satiation point. The goods are imperfect substitutes, in the sense that $u_{q_i q_i}(q_A, q_B, \theta) < u_{q_i q_j}(q_A, q_B, \theta) < 0$, where subscripts denote partial derivatives. This implies that buyers have a preference for variety.

The one-dimensional parameter θ is the buyer's private information; it is distributed over an interval $[\theta_{\min}, \theta_{\max}]$ according to a distribution function $F(\theta)$ with density $f(\theta)$. We assume that higher values of θ correspond to higher demand and make the single-crossing assumption $u_{\theta q_i}(q_A, q_B, \theta) \geq 0$. Notice that since heterogeneity is one-dimensional, the demand for the two products is correlated.

We assume that firm A (the dominant firm) has a competitive advantage in terms of lower cost, better quality, or both. Firm A's marginal production cost is normalised to zero. With cost asymmetry, the unit production cost of product B is $c > 0$.[9] With quality asymmetry, the buyer's payoff becomes $u(q_A, q_B, \theta) - cq_B$ (with B's cost now set to zero). In this case, the parameter c can be interpreted as an index of vertical product differentiation, with product A being of better quality, and hence having a larger demand, than product B.

[8] In this interpretation, our analysis literally requires that downstream firms operate in separate markets and do not interact strategically with each other. Otherwise, contractual externalities might complicate the analysis: see, for instance, Fumagalli and Motta (2006), Simpson and Wickelgren (2007) and Wright (2009). However, our insights might apply also to situations in which downstream firms compete, as long as they have some market power.

[9] We abstract from fixed costs, and hence from economies of scale. As long as all firms remain active, this is with no loss of generality. Furthermore, in the competitive fringe model one can interpret c as the minimum average cost of a number of identical firms, thus allowing for economies of scale at the firm level.

The two formulations are equivalent, and in what follows we shall stick to the cost interpretation.

Firms compete by simultaneously and independently offering menu of contracts. We distinguish two different modes of competition according to the type of contract that the firms may offer. With simple *non-linear pricing*, the payment to each firm depends only on its own quantity. A strategy for firm i then is a function $P_i(q_i)$ in which q_i is the quantity firm i is willing to supply and $P_i(q_i)$ is the corresponding total payment it asks. With *exclusive contracts*, by contrast, a strategy for firm i comprises two price schedules, $P_i^E(q_i)$ and $P_i^{NE}(q_i)$. The former applies to exclusive contracts ($q_j = 0$), the latter to non exclusive ones ($q_j > 0$).[10]

Buyers have no bargaining power, but are large enough so that firms can monitor whether they purchase from their competitors. Buyer θ observes the firms' offers and then chooses the quantities $\{q_A(\theta), q_B(\theta)\}$ that maximise his net payoff. The buyer's equilibrium payoff, net of payments to the firms, is denoted by $U(\theta)$.

The full information, first-best quantities are

$$\left\{q_A^{fb}(\theta), q_B^{fb}(\theta)\right\} = \arg\max_{q_A, q_B} \left[u\left(q_A, q_B, \theta\right) - cq_B\right].$$

To make the analysis interesting, we assume that $q_B^{fb}(\theta_{\max}) > 0$; if this condition is violated, good B should not (and would not) be produced in equilibrium.

To simplify the exposition, we assume that the market is uncovered. This guarantees that in equilibrium the marginal buyer's demand is negligible, and so the price schedules which apply to the marginal buyer cannot involve any fixed fee or subsidy.[11] A sufficient condition for the market to be uncovered is that $q_A^{fb}(\theta_{\min}) = 0$ and $q_A^{fb}(\theta) > 0$ for all $\theta > \theta_{\min}$.[12] Additional regularity assumptions, which help simplify the analysis, will be introduced later.

In order to get explicit solutions, we shall at times focus on a uniform-quadratic specification of the model. In this specification, the parameter θ is assumed to be uniformly distributed over the interval $[0, 1]$, and the function u is taken to be:

$$u\left(q_A, q_B, \theta\right) = \theta(q_A + q_B) - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B. \tag{1}$$

The parameter γ captures the degree of substitutability among the products: it ranges from $\frac{1}{2}$ (perfect substitutes) to 0 (independent goods). The factor $\frac{1-\gamma}{2}$

[10] To guarantee that the buyer's maximisation problem has a solution, we assume that each price schedule P_i must be non decreasing in q_i (a free disposal assumption which also implies that price schedules must be differentiable almost everywhere), that it satisfies $P_i(0) = 0$, and that it is upper semi-continuous.

[11] This property was first noted by Wilson (1994) for the case of monopoly non-linear pricing, and Martimort and Stole (2009) for the case of duopoly. (The marginal buyer is the lowest type that purchases a positive quantity of at least one good.)

[12] In fact it suffices that there exists a $\theta \in [\theta_{\min}, \theta_{\max}]$ such that $q_A^{fb}(\theta) = 0$. If this holds, one can always choose $\theta_{\min}$ as the largest θ for which $q_A^{fb}(\theta) = 0$ and re-scale the distribution function accordingly.

in the middle term in (1) prevents changes in γ from affecting the size of the market.[13] In the uniform-quadratic specification, the condition $q_B^{fb}(\theta_{\max}) > 0$ becomes $c < \frac{1-2\gamma}{1-\gamma}$.

3 The pricing problem

Before comparing the equilibrium with and without exclusive contracts, it may be instructive to discuss how the possibility of using exclusive contracts changes the formulation of the firms' optimal pricing problem. Readers who are not interested in the technicalities of the analysis may jump directly to Section 4.2.

To fix ideas, in this section we shall focus on the dominant firm's best response. However, the same approach applies, in the duopoly model, to its rival.

3.1 Non-linear pricing

When exclusive contracts are prohibited, for any given price schedule offered by its rivals, $P_B(q_B)$, the dominant firm must solve a fairly standard problem of non-linear pricing. The only twist is that buyers can also purchase a substitute good, so they behave as if they had an "indirect utility function"

$$v(q_A, \theta) = \max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - P_B(q_B) \right], \tag{2}$$

which is the maximum payoff that buyer θ can obtain by purchasing q_A and then trading optimally with the dominant firm's rivals.[14]

Let us briefly review the solution technique with non-linear pricing. The firm maximises its profit $\int_{\theta_{\min}}^{\theta_{\max}} P_A(q_A(\theta)) f(\theta) d\theta$, where $q_A(\theta) = \arg\max_{q_A \geq 0} [v(q_A, \theta) - P_A(q_A)]$. By invoking the Revelation Principle,[15] we can reformulate the problem as if the firm could control $q_A(\theta)$ directly (i.e. a direct mechanism). Using the change of variables $U(\theta) = v(q_A(\theta), \theta) - P_A(q_A(\theta))$, the firm's objective function becomes $\max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - U(\theta) \right] f(\theta) d\theta$, where $\tilde{\theta} \geq \theta_{\min}$ is the lowest type served by the firm (chosen optimally). Provided that the indirect utility function satisfies the single-crossing condition $v_{\theta q_A}(q_A, \theta) \geq 0$, the incentive compatibility constraint $q_A(\theta) = \arg\max_{q_A \geq 0} [v(q_A, \theta) - P_A(q_A)]$ is equivalent to the requirements that $U'(\theta) = v_\theta(q_A, \theta)$ and that $q_A(\theta)$ is non-decreasing. The participation constraint is $U(\theta) \geq v(0, \theta)$, as the buyer can

[13] As argued by Shubik and Levitan (1980), this rules out spurious effects in the comparative statics analysis.

[14] The indirect utility function is similar to residual demand in models of linear pricing.

[15] The Revelation Principle applies even if principals (i.e. firms) compete, since we are focusing on a firm's best response to its rivals' given strategies.

choose to deal with the firm's competitors only.[16] The program then becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta),\theta) - U(\theta) \right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A(\theta),\theta) \\ U(\theta) &\geq v(0,\theta) \end{aligned} \tag{3}$$

and $q_A(\theta)$ non-decreasing. This is an optimal control program with $q_A(\theta)$ as the control variable and $U(\theta)$ as the state variable. Once the optimal quantity has been found, one can then recover the tariff that supports it.

3.2 Exclusive contracts

When exclusive contracts are permitted, the dominant firm can control not only $q_A(\theta)$, but also whether $q_B(\theta)$ may be positive or must be nil. This leads to a non standard screening problem, and one that calls for new methods of analysis.

In general, the firm can set $q_B(\theta)$ to nil (i.e. impose an exclusivity clause) for some types, and allow $q_B(\theta)$ to be positive for others. However, it is convenient to consider first the constrained problems in which $q_B(\theta)$ may be positive for all types, or must be nil for all types.

When $q_B(\theta)$ may be positive for all types, the firm's problem is similar to (3), except that the reservation utility is now $\max\{U_A^R(\theta), v(0,\theta)\}$, where[17]

$$U_A^R(\theta) = \max_{q_B \geq 0} \left[u\left(0, q_B, \theta\right) - P_B^E(q_B) \right].$$

The dominant firm's problem then becomes (following the same steps as above)

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta),\theta) - U(\theta) \right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A(\theta),\theta) \\ U(\theta) &\geq \max\{U_A^R(\theta), v(0,\theta)\} \end{aligned} \tag{4}$$

and $q_A(\theta)$ non-decreasing. We denote by $q_A^{NE}(\theta)$ the solution to this problem.

Next consider the constrained program in which the firm imposes an exclusivity clause on all buyers. Since $q_B(\theta)$ is thereby set to zero, the firm's problem is

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[u(q_A(\theta),0,\theta) - U(\theta) \right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_A(\theta),0,\theta) \\ U(\theta) &\geq \max\{U_A^R(\theta), v(0,\theta)\} \end{aligned} \tag{5}$$

[16] Thus, the buyer's reservation payoff when dealing with the firm can depend on his type. For a systematic treatment of type-dependent participation constraints in monopolistic screening problems see Jullien (2000).

[17] In equilibrium, it turns out that $P_B^E(q_B) \leq P_B^{NE}(q_B)$, so $U_A^R(\theta) \geq v(0,\theta)$.

and $q_A(\theta)$ non-decreasing. We denote by $q_A^E(\theta)$ the solution to problem (5).

Compared with problem (4), the indirect utility function $v(q_A(\theta),\theta)$ is now replaced by $u(q_A(\theta),0,\theta)$. Apart from the type-dependent participation constraint, the dominant firm can therefore behave as a monopolist. Notice that by construction we have $u(q_A(\theta),0,\theta) \leq v(q_A(\theta),\theta)$, reflecting the fact that exclusive contracts impose an unnecessary cost on buyers and thus reduce total surplus. However, they may allow a better screening: if high-type buyers value the opportunity to purchase both products more than low types, they have more to lose from accepting exclusive contracts.[18]

In fact, as noted above, the firm can impose an exclusivity clause only on subsets of buyers. Thus, one may view the firm as faced with a *multi-stage optimal control problem* involving two different control systems, (4) and (5), and the possibility of switching from one system to the other. To solve this problem, one needs to choose a sequence of control systems, the switching points, and the control function $q_A(\theta)$ for each system that maximise the firm's profit.[19] An additional incentive compatibility constraint that must be satisfied is that the control system chosen for a type must guarantee to him a (weakly) greater utility than the other. This reflects the fact that the firm just offers both exclusive and non-exclusive contracts, and buyers freely choose which type of contract to sign.[20] This additional constraint differentiates our problem from other multi-stage optimal control problems analysed in the literature so far.

Let us start from the choice of the optimal control function $q_A(\theta)$. We have:

Lemma 1 *Suppose that $q_A^{NE}(\theta)$ and $q_A^E(\theta)$ are strictly increasing and that the participation constraints bind only in the first stage of the problem (i.e., for the lowest types). Then, for any possible sequence of control systems and switching points, the optimal control function for the multi-stage problem coincides with $q_A^{NE}(\theta)$ whenever problem (4) applies, and with $q_A^E(\theta)$ whenever problem (5) applies.*

Lemma 1 means that the optimal control function for the multi-stage problem is formed by appropriately joining the control functions $q_A^{NE}(\theta)$ and $q_A^E(\theta)$ that are optimal for problems (4) and (5) taken separately. This *separation property* is remarkable: in general, the solution to a multi-stage problem depends on boundary conditions that may be affected by the sequence of control systems and the switching points.

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[18] From this viewpoint, exclusive dealing resembles the strategy of damaging one's goods analysed by Deneckere and McAfee (1996). One difference, however, is that with exclusive contracts most of the cost of the damage is borne by the firm's rivals, which makes the strategy more attractive.

[19] For an early economic analysis of a two-stage optimal control problem, in which there is only one possible switch between one control system and the other, see Tamiyama (1985). In our problem, the order in which the control problems apply is not pre-specified and, in principle, there could be multiple switches.

[20] The participation constraints guarantee that the buyer prefers to trade with the firm over trading with its competitors only. The standard incentive compatibility constraints guarantee that, within each menu of contracts (non-exclusive and exclusive), each buyer chooses the one intended for him. In addition, however, each buyer must choose the type of contract (non-exclusive or exclusive) intended for him, whence the new incentive compatibility constraint.

Mathematically, the separation property follows from the fact that the state variable enters linearly, and with the same coefficient, in both control problems (4) and (5). (Notice that this simple mathematical structure is shared by many screening problems, which suggests that the separation property applies well beyond the specific non-linear pricing problem at hand.) This implies that at the optimum, the costate variable is independent of both the exact sequence of the control systems, and the switching points.

Heuristically, if the only reason why the contract for a given type θ is distorted is to reduce the information rent for higher types, then all that matters is how many higher types there are relative to the current type, i.e. the hazard rate $\frac{1-F(\theta)}{f(\theta)}$. The exact contracts offered to higher (and lower) types do not matter.

The separation property greatly simplifies the analysis. Among other things, it may help determine the optimal sequence of control problems in specific examples. To see why, notice that the slope of the equilibrium rent function $U(\theta)$ (which by the standard incentive compatibility constraints is $v_\theta(q_A^{NE}(\theta),\theta)$ under non-exclusivity and $u_\theta(q_A^E(\theta),0,\theta)$ under exclusivity) can be calculated once the optimal functions $q_A^{NE}(\theta)$ and $q_A^E(\theta)$ have been found. Thus, one can verify whether the equilibrium rent function for one system is steeper or flatter than the other. For example, in the uniform-quadratic model the equilibrium rent function $U(\theta)$ turns out to be always steeper under non-exclusivity than under exclusivity. This implies that the optimal sequence is necessarily from exclusivity (for low-demand types) to non-exclusivity (for high-demand ones).

Let us now turn to the optimal switching points. We denote by $P_A^{NE}(q_A)$ and $P_A^E(q_A)$ the non-exclusive and exclusive tariffs, respectively.

Lemma 2 *At any optimal switching point $\hat{\theta}$, the following conditions must hold:*

$$u(q_A^E(\hat{\theta}),0,\hat{\theta}) - P_A^E(q_A^E(\hat{\theta})) = v(q_A^{NE}(\hat{\theta}),\hat{\theta}) - P_A^{NE}(q_A^{NE}(\hat{\theta})), \tag{6}$$

and

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{v_\theta(q_A^{NE}(\hat{\theta}),\hat{\theta}) - u_\theta(q_A^E(\hat{\theta}),0,\hat{\theta})} = \frac{1-F(\hat{\theta})}{f(\hat{\theta})}. \tag{7}$$

Since the tariffs $P_A^{NE}(q_A)$ and $P_A^E(q_A)$ must implement the optimal quantity schedules $q_A^{NE}(\theta)$ and $q_A^E(\theta)$, they are pinned down fully, except possibly for constant terms. Furthermore, the constant term of the tariff that applies to the marginal buyer $\tilde{\theta}$ must vanish, as we have noted above. Therefore, the two conditions (6) and (7) determine the first switching point $\hat{\theta}$ and the constant term of the tariff that applies for $\theta > \hat{\theta}$. If there are more switching points, conditions (6) and (7) apply recursively. In this way, they provide all the conditions that are needed to find a complete solution.[21]

[21] In the duopoly model, the equilibrium switching point is jointly determined by the pricing choices of the two firms. The corresponding equilibrium conditions will be derived below.

Condition (6) captures the new incentive compatibility constraint mentioned above. Condition (7) reflects the firm's optimisation.[22] It implies that marginal profitability always jumps up at a switching point. The economic intuition is simple. Consider an increase in the constant term of the tariff that applies for $\theta > \hat{\theta}$. (Notice that if the entire price schedule to the right of $\hat{\theta}$ is shifted up by a constant, local incentive compatibility is preserved, and hence the equilibrium quantities in that interval do not change.) Clearly, this move has a direct, positive effect on profits extracted from higher types, and an indirect effect due to the resulting increase in $\hat{\theta}$. The indirect effect would vanish if $P_A^{NE}(q_A^{NE}(\hat{\theta})) = P_A^E(q_A^E(\hat{\theta}))$. At the optimum, however, the indirect effect must be negative, as it must just offset the positive, direct effect. This implies that profitability must be greater to the right than to the left of the switching point. For example, when the denominator of the left-hand side of (7) is positive, the system optimally switches from exclusivity to non-exclusivity, and the dominant firm obtains more profits, at the margin, by serving type $\hat{\theta}$ under non-exclusivity than under exclusivity.

We now apply these ideas to the competitive fringe and the duopoly model.

4 Competitive fringe

In this section, we focus on the case in which the dominant firm A faces a competitive fringe. Besides often being realistic, this assumption serves two main purposes. First, it simplifies the analysis, as the competitive fringe will always price at cost (i.e., $P_B(q_B) = cq_B$), without imposing any exclusivity clause. Given the passive behaviour of the competitive fringe, finding the model's equilibrium is tantamount to finding the dominant firm's optimal pricing strategy.

Secondly, the dominant firm has no way to eliminate the competitive pressure from the fringe, which will always stand ready to supply product B at a unit price of c. This highlights the difference between our theory and other post-Chicago theories in which exclusive contracts serve to deter entry, or deprive a rival of economies of scale so as to drive it out of the market. In a competitive fringe model, the role of exclusive contracts must evidently be different.

The downside of the competitive fringe assumption is that firms that just break even cannot really be harmed. Thus, when exclusive contracts are anticompetitive, they will harm only the buyers. However, in the next section we shall see that the main insights carry over to the duopoly model, where exclusive contracts also impact the dominant firm's rival's profit.

[22] Mathematically, it is equivalent to the continuity of the Hamiltonian. This is a standard necessary condition in the multi-stage optimal control literature, and may be viewed as a generalisation of the Weierstrass-Erdmann corner conditions. Another standard necessary condition (i.e., continuity of the costate variable) holds trivially in our problem and is replaced by condition (6). Notice that the optimal quantity $q_A(\theta)$ may jump at a switching point.

4.1 Equilibrium

We first calculate the equilibrium with non-linear pricing, and then that with exclusive contracts.

4.1.1 Non-linear pricing

Consider first the optimal pricing strategy for the dominant firm when it is restricted to simple non-linear pricing. When $P_B(q_B) = cq_B$, the indirect utility function $v(q_A, \theta)$ in problem (3) is piecewise smooth, with two branches corresponding to the quantity

$$\tilde{q}_B(q_A, \theta) = \arg \max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - cq_B \right] \tag{8}$$

being nil or strictly positive, and a kink between the two branches. By a standard integration by parts, the firm's problem can be rewritten as

$$\max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q_A, \theta) \right] f(\theta) d\theta, \tag{9}$$

where the term inside square brackets is usually referred to as the "virtual surplus."

As we proceed, we shall impose several regularity conditions that serve to simplify the analysis. The first is:

A1. *The virtual surplus function*

$$v(q_A, \theta) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q_A, \theta) \tag{10}$$

is globally concave in q_A and has increasing differences in q_A and θ.

This assumption guarantees that the solution to the dominant firm's problem can be found by pointwise maximisation of the virtual surplus function. If it fails, an ironing procedure is needed, and the solution exhibits bunching. The condition can be reformulated in terms of the primitives of the model, but it involves third derivatives the economic interpretation of which is not immediate. However, it is easily met in the uniform-quadratic specification (this is true, in fact, for all the regularity conditions that we shall introduce).

Like the indirect utility function, the virtual surplus function has two branches with a kink in between. The maximum can occur on either branches, or at the kink. When the maximum occurs on the branch with $\tilde{q}_B(q_A, \theta) = 0$, it must coincide with the monopoly solution. We denote it by $q^m(\theta)$, and by $P^m(q)$ the corresponding price schedule.[23] When the maximum occurs on the branch corresponding to $\tilde{q}_B(q_A, \theta) > 0$, we obtain a "common representation" outcome.[24] The common representation quantities are denoted by $q_A^{cr}(\theta)$ and

[23] If not stated otherwise, the constant terms of this and the following price schedules must be understood to be nil.

[24] We call "common representation" the outcome in which the buyer buys a positive quantity of product B. This is a slight abuse of terminology, as the quantity of product A may actually be nil.

$q_B^{cr}(\theta) = \tilde{q}_B(q_A^{cr}(\theta), \theta)$, and $P_A^{cr}(q)$ is the price schedule that supports them. As for the kink, it is implicitly defined by the condition

$$u_{q_B}(q_A(\theta), 0, \theta) = c, \tag{11}$$

and therefore can be interpreted as a limit pricing solution. We denote the limit-pricing quantity (i.e. the solution to (11)) by $q^{\lim}(\theta)$, and by $P^{\lim}(q)$ the price schedule that implements it.

Clearly, we have $q^m(\theta) \geq q_A^{cr}(\theta)$ as the goods are substitutes. To reduce the number of cases that must be considered, we rule out multiple intersections between the curve $q^{\lim}(\theta)$ and the curves $q^m(\theta)$ and $q_A^{cr}(\theta)$. While this is not really necessary for our results, it simplifies the exposition considerably.[25]

A2. *The curves $q^m(\theta)$ and $q^{\lim}(\theta)$, and the curves $q^{\lim}(\theta)$ and $q_A^{cr}(\theta)$, intersect at most once.*

The dominant firm can actually engage in monopoly or limit pricing only if the competitive advantage parameter c exceeds critical thresholds, denoted by c^m and $c^{\lim}$, respectively (with $c^m > c^{\lim}$).[26]

Proposition 1 *In the competitive fringe model, there is a unique non-linear pricing equilibrium where the competitive fringe prices at cost ($P_B = cq_B$) and:*

- *when $0 \leq c \leq c^{\lim}$, firm A offers the price schedule*

$$P_A(q) = P_A^{cr}(q);$$

- *when $c^{\lim} \leq c \leq c^m$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^{\lim}(q) & \text{for } 0 \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

[25] In the uniform-quadratic model, the functions $q^m(\theta), q^{\lim}(\theta)$ and $q_A^{cr}(\theta)$ can be calculated explicitly and are

$$q^m(\theta) = \frac{2\theta - 1}{1 - \gamma},$$
$$q^{\lim}(\theta) = \frac{\theta - c}{\gamma},$$

and

$$q_A^{cr}(\theta) = 2\theta - 1 + c\frac{\gamma}{1 - 2\gamma}.$$

(When the above expressions are negative, quantities must be understood to be nil.) Since they are all linear, multiple intersections cannot occur. We also have

$$q_B^{cr}(\theta) = \theta\frac{1 - 2\gamma}{1 - \gamma} + \frac{\gamma}{1 - \gamma} - c\frac{1 - \gamma}{1 - 2\gamma}.$$

(To verify that $q^m(\theta) \geq q_A^{cr}(\theta)$, keep in mind that the expression for $q_A^{cr}(\theta)$ holds only when $q_B^{cr}(\theta) > 0$.)

[26] To be precise, c^m is the lowest c such that there exists at least one type θ for whom $q^m(\theta) > q^{\lim}(\theta)$, and $c^{\lim}$ is the lowest c such that there exists at least one type θ for whom $q_A^{cr}(\theta) > q^{\lim}(\theta)$. The existence of these thresholds is guaranteed as $q^{\lim}(\theta)$ decreases with c and vanishes if c is large enough, $q_A^{cr}(\theta)$ increases with c, and $q^m(\theta)$ is independent of c. Since $q^m(\theta) \geq q_A^{cr}(\theta)$, it also follows that $c^m > c^{\lim}$. In the uniform-quadratic model, the critical thresholds are $c^m = \frac{1}{2}$ and $c^{\lim} = \frac{1-2\gamma}{2-3\gamma}$.

where $\breve{\theta}_B$ *is highest type such that* $q_B^{cr}(\theta) = 0$ *and the constant guarantees the continuity of the price schedule;*

- *when* $c \geq c^m$, *firm* A *offers the price schedule*

$$P_A(q) = \begin{cases} P^m(q) & \textit{for } 0 \leq q \leq q^m(\theta^{\lim}) \\ P^{\lim}(q) + \textit{ constant} & \textit{for } q^m(\theta^{\lim}) \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \textit{ constant} & \textit{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

where $\theta^{\lim}$ *is the solution to* $q^m(\theta) = q^{\lim}(\theta)$ *and the constants guarantee the continuity of the price schedule.*

The equilibrium pattern is quite intuitive. When c is large, low-demand buyers are effectively captive, so the dominant firm can engage in monopoly pricing in the low-demand segment of the market. As demand increases, however, the buyer's temptation to purchase also product B increases. But if a buyer purchased a positive amount of product B, his demand for product A would decrease, as the products are substitutes. To prevent this, the dominant firm therefore engages in limit pricing, raising the sales of product A just up to the point where the buyer's marginal willingness to pay for product B equals the competitive fringe's cost c. Finally, when buyer's demand gets still higher, foreclosing the competitive fringe becomes unprofitable. The dominant firm therefore accommodates, and in equilibrium buyers purchase both goods.

As the competitive advantage c decreases, the dominant firm can no longer benefit from the presence of captive buyers. Thus, it engages in limit pricing in the low-demand segment of the market, and accommodates in the high-demand segment. When c is still lower, it is not even profitable to engage in limit pricing. In this case, low-demand buyers actually purchase product B only. The intuition for this is simple. Product A is less costly to produce, or of higher quality. However, product B is supplied competitively, whereas the dominant firm exercises its market power in the market for product A. When c is low, this latter effect must prevail.

Notice that the highest type, i.e. $\theta_{\max}$, always purchases both goods. In particular, it can be easily verified that he obtains the efficient quantities: $q_i^{cr}(\theta_{\max}) = q_i^{fb}(\theta_{\max}) > 0$. This no-distortion-at-the-top property must hold in all equilibria, with and without exclusive contracts. (It also hold under monopoly, and is preserved under competition.) It implies that the dominant firm's rivals are never driven out of the market in the absence of economies of scale.

4.1.2 Exclusive contracts

Let us now turn to the case in which exclusive contracts are permitted. Exploiting the separation property (Lemma 1), we first consider the case in which the firm imposes an exclusive arrangement on all buyer. The case where no such arrangement is imposed corresponds to the non-linear pricing solution that we have just characterised.[27]

By imposing an exclusive arrangement, the dominant firm can engage in monopoly pricing if the competitive pressure from the fringe is not too strong, that is, if the monopoly tariff lies below the cq line. In this case, the participation constraint $U(\theta) \geq U_A^R(\theta)$ in problem (5) is not binding. Otherwise, the dominant firm must undercut the competitive fringe, pricing just below c and selling an amount $q_c^e(\theta)$ of its product, where $q_c^e(\theta)$ is the solution to

$$u_{q_A}(q_A, 0, \theta) = c. \tag{12}$$

Again, to reduce the number of cases that must be considered, we rule out multiple intersections between the relevant curves.

A3. *The curves $q^m(\theta)$ and $q_c^e(\theta)$ intersect at most once.*[28]

Notice that $q^m(\theta_{\max}) > q_c^e(\theta_{\max})$ by the no-distortion-at-the-top property. Therefore, if the curves do intersect, the curve $q_c^e(\theta)$ must cut $q^m(\theta)$ from above. It follows that the solution to problem (5) may either coincide with $q^m(\theta)$, or it may be formed by two branches, i.e. $q_c^e(\theta)$ for low types and $q^m(\theta)$ for high types. Intuitively, the first pattern will emerge if the dominant firm's competitive advantage is large enough. To be precise, the condition is $c \geq c^m$.[29]

Having found the solution with and without exclusive contracts separately, it remains to determine which buyers are offered an exclusive contract, and which a non-exclusive one. From the no-distortion-at-the-top property, we know that the solution for high-demand types must be nearly efficient, which rules out exclusive dealing. However, exclusive dealing can be optimal for low-demand buyers, whose quantities are distorted more heavily. Our last simplifying assumption guarantees that the solution to the multi-stage control problem involves a unique switch, which must then necessarily be from exclusive to non-exclusive dealing.

A4. $v_\theta(q_A^{NE}(\theta), \theta) > u_\theta(q_A^E(\theta), 0, \theta)$.

In specific examples, such as the uniform-quadratic model, it is easy to verify that A4 holds. More generally, A4 will hold if, for example, $u_{\theta q_i}$ is constant provided that aggregate sales are greater under non exclusivity than under exclusivity.

Under our regularity assumptions, the equilibrium outcome with exclusive contracts is as follows.[30]

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[27] Since the competitive fringe always prices at cost, the participation constraint does not change.

[28] In the uniform-quadratic model, $q_c^e(\theta) = \frac{\theta - c}{1-\gamma}$, so condition A3 is satisfied.

[29] The reason why the critical threshold is again c^m is as follows. Since the goods are imperfect substitutes, it is clear from the definitions that

$$q_c^e(\theta) \leq q^{\lim}(\theta),$$

with equality only when both quantities vanish. The lowest c such $q^m(\theta)$ always exceeds $q_c^e(\theta)$ must therefore coincide with the lowest c such that $q^m(\theta)$ can exceed $q^{\lim}(\theta)$.

[30] Notice that while the equilibrium outcome is still unique, it can now be supported by different price schedules. The reason for this is that when the dominant firm offers both

Proposition 2 *With exclusive contracts, in the competitive fringe model there is a unique equilibrium outcome where $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$ for all $q_B \geq 0$. Furthermore, there exists a threshold $\bar{c} < c^m$ such that :*

- *when $c \leq \bar{c}$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = cq & \text{for } 0 \leq q \leq q_c^e(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q_c^e(\hat{\theta}) \end{array}$$

  where Φ_A is a constant term;

- *when $\bar{c} \leq c \leq c^m$, firm A offers the price schedules:*

$$P_A^E(q) = \begin{cases} cq & \text{for } 0 \leq q \leq q_A^m(\bar{\theta}) \\ P^m(q) + \text{ constant} & \text{for } q_A^m(\bar{\theta}) \leq q \leq q_A^m(\hat{\theta}) \end{cases}$$

  where $\bar{\theta}$ is the solution to $q_c^e(\bar{\theta}) = q_A^m(\bar{\theta})$, and the constant guarantees the continuity of the price schedule, and

$$P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A \qquad \text{for } q \geq q_A^m(\hat{\theta});$$

- *when $c \geq c^m$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = P^m(q) & \text{for } 0 \leq q \leq q_A^m(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q_A^m(\hat{\theta}). \end{array}$$

  In each case, $\hat{\theta}$ and Φ_A are determined by the equilibrium conditions (6) and (7) of Lemma 2.

Some general properties of the equilibrium are worth mentioning. First, the dominant firm does offer exclusive contracts, which are accepted by some buyers. By revealed preferences, then, exclusive contracts must be profitable. Secondly, when demand is highest, buyers are always served under common representation. Exclusive contracts are chosen only when demand is relatively low. This is a robust prediction of our model that simply reflects the no-distortion-at-the-top property. Thirdly, notice that when exclusive contracts are permitted, the limit pricing region, which exists under non-linear pricing, disappears.

Figure 1 depicts the critical thresholds for the uniform-quadratic model.[31] It shows, in the parameter space (γ, c), the various regions where different equilibrium patterns arise.

exclusive and non-exclusive contracts, some contracts are destined not to be accepted and may therefore be specified arbitrarily, at least to some extent. Accordingly, the following proposition specifies only the relevant parts of the equilibrium price schedules.

[31] The threshold $\bar{c}$ determines whether at the switching point, the solution under exclusive contracts is $q^m(\theta)$ or $q_c^e(\theta)$.

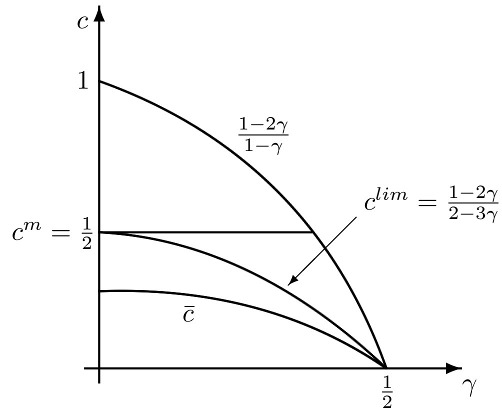

Figure 1: Critical thresholds for the competitive fringe model (uniform-quadratic specification).

4.2 Comparison

We can now compare the equilibrium with and without exclusive contracts. Since the role of exclusive contracts depends on the size of the dominant firm's competitive advantage, we discuss separately the case that this is large (which is our primary focus) or small.

4.2.1 Large competitive advantage ($c \geq c^{\text{lim}}$)

It is convenient to start from the sub-case $c \geq c^m$, which is depicted in Figure 2; we shall then briefly explain what changes when $c^{\text{lim}} \leq c < c^m$.

When $c \geq c^m$, the dominant firm's competitive advantage is sufficiently large so that in the non-linear pricing equilibrium the monopoly solution applies in the low-demand segment of the market. However, as demand increases the dominant firm must resort to limit pricing to foreclose its competitors (and it must accommodate when demand gets still higher). From the dominant firm's viewpoint, limit pricing is clearly a second best. It is less profitable than monopoly pricing, and must be adopted only because of the competitive pressure from the fringe.

The role of exclusive contracts in this case is simply to eliminate such competitive pressure. By imposing an exclusivity clause, the dominant firm can keep selling the profit-maximising monopoly quantity, without having to resort to limit pricing. In other words, *exclusive contracts allow the dominant firm to more efficiently foreclose its competitors from a segment of the market.*

Of course, buyers have the option of refusing the exclusivity clause and trading with the competitive fringe only. However, if the dominant firm's competitive advantage is large this option does not really constrain its pricing strategy. This is so because a buyer obtains an information rent even under exclusive dealing. When $c \geq c^m$, this is greater than the rent that he could obtain by trading with the competitive fringe only, so exclusive dealing effectively shelters the dominant firm from the fringe's competitive pressure at no cost.

In principle, the dominant firm could then impose exclusive dealing, and sell the monopoly quantity, to all buyers. However, the most profitable strategy is in fact to allow high-demand types to purchase quantities that are nearly efficient and then extract the surplus as best as it can. Thus, no exclusivity clause is imposed on high-demand buyers, who therefore buy both goods.

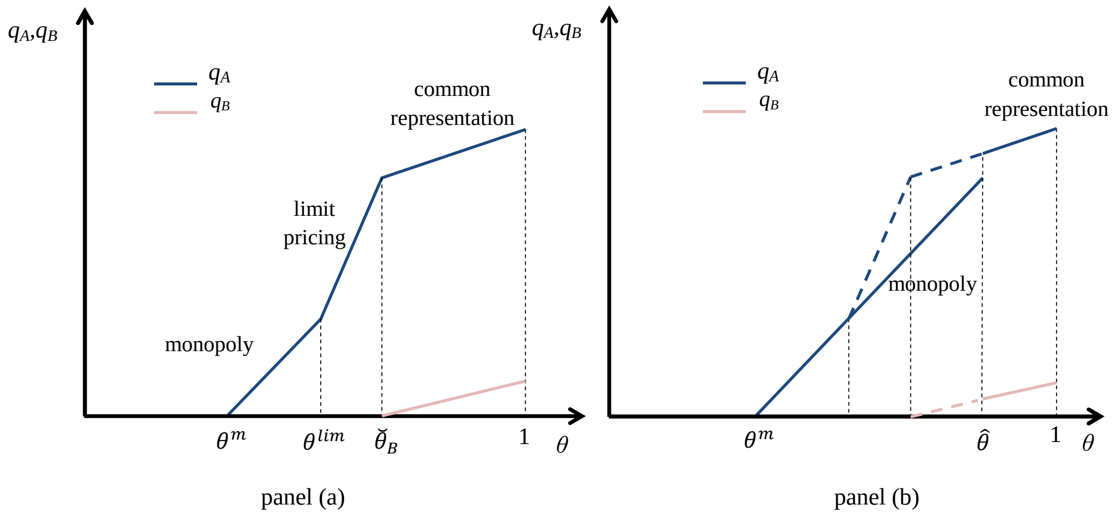

Figure 2: Equilibrium quantities in the competitive fringe model when the dominant firm's competitive advantage is large. Panel (a) non-linear pricing; panel (b) exclusive contracts.

In particular, the separation property (Lemma 1) implies that high-demand buyers will purchase exactly the same quantities as in the non-linear pricing equilibrium. However, the dominant firm extracts more surplus than under non-linear pricing by adding a fixed fee to its non-exclusive tariff. This fixed fee can be interpreted as a "tax" levied on product variety. Thanks to this tax, the dominant firm must actually extract, at the margin, more profits under common representation than under exclusivity (Lemma 2), even if in the latter case it charges monopoly prices.

Since exclusive dealing is more profitable than limit pricing as a foreclosure strategy, the dominant firm uses it more extensively. Fewer buyers purchase both goods when exclusive contracts are permitted than when they are prohibited (that is, $\breve{\theta}_B < \hat{\theta}$). Therefore, product variety is reduced.

Buyers are harmed by exclusive contracts, both in terms of higher prices and reduced variety. To be precise, low-demand buyers ($\theta \leq \theta^{\mathrm{lim}}$) are unaffected, as they purchase the monopoly quantity of good A only, both with and without exclusive contracts. However, some intermediate-demand buyers obtain the monopoly quantity of good A rather than the limit pricing quantity ($\theta^{\mathrm{lim}} < \theta \leq \breve{\theta}_B$), or the common representation quantities ($\breve{\theta}_B < \theta < \hat{\theta}$). In this latter case, buyers suffer both in terms of lower volumes (and hence higher prices) and reduced variety. Still higher types ($\theta \geq \hat{\theta}$) obtain the same quantities as in the non-linear pricing equilibrium, but they are left with lower rents because of the fixed fee that is added to the non-linear pricing equilibrium tariff.[32]

[32] When buyers are downstream firms, the extent to which their gains or losses are shifted onto final consumers may depend on how prices exactly change. Generally speaking, higher upstream prices tend to translate into higher downstream prices, so final consumers should also suffer from exclusive contracts when downstream firms do. However, if the only change is an increase in a fixed fee, there might be no effect on final consumers.

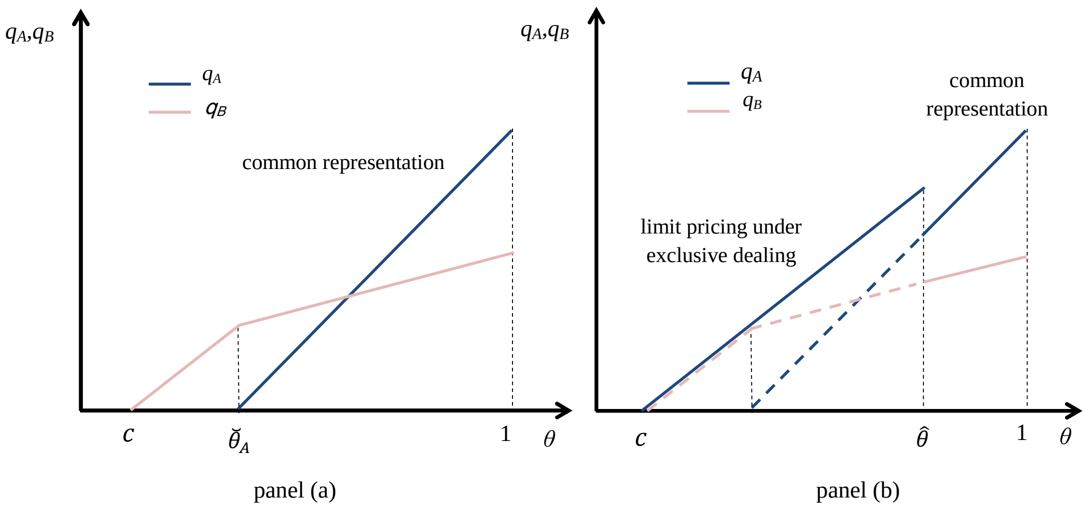

Figure 3: Equilibrium quantities in the competitive fringe model when the dominant firm's competitive advantage is small. Panel (a) non-linear pricing; panel (b) exclusive contracts (solid lines).

The impact of exclusive contracts on social welfare is also negative. This follows immediately from the fact that the equilibrium quantities are the same as under non-linear pricing for buyers of type $\theta \leq \theta^{\lim}$ and $\theta \geq \hat{\theta}$. However, under exclusive contracts intermediate types ($\theta^{\lim} < \theta \leq \hat{\theta}$) obtain the monopoly quantity, which entails a bigger distortion than either limit pricing or common representation.

When $c^{\lim} \leq c < c^m$, exclusive contracts are still used as a substitute for limit pricing. Now, however, the monopoly tariff lies above the cq line when q is small. Thus, the competitive pressure from the fringe forces the dominant firm to set the exclusive quantity at $q_c^e(\theta) > q^m(\theta)$ for some low-demand buyers. However, these buyers would have purchased the limit pricing quantity $q^{\lim}(\theta)$ under non-linear pricing. Since this is higher than $q_c^e(\theta)$, buyers are still harmed (and social welfare reduced) by exclusive contracts.

We can therefore conclude that *when the dominant firm's competitive advantage is large ($c \geq c^{\lim}$), exclusive contracts are unambiguously anti-competitive.*

4.2.2 Small competitive advantage ($c < c^{\lim}$)

When the competitive advantage is small, the profitability of exclusive contracts rests on a different mechanism. Furthermore, even if exclusive contracts still harm buyers, their effects on social welfare are less clear cut.

The equilibrium quantities for this case are depicted in Figure 3 (the picture is actually drawn for the sub-case $c \leq \bar{c}$). When $c < c^{\lim}$, in the non-linear pricing equilibrium the marginal buyer purchases only product B. Only sufficiently high-demand types purchase also product A. This is disappointing from the point of view of the dominant firm (as well as being inefficient from the social viewpoint). If only the dominant firm could replace the competitive fringe in the low-demand segment of the market, it would save the production cost $cq_B(\theta)$ and increase its profits by the same amount.

However, to achieve that result the dominant firm should undercut the competitive fringe. With non-linear pricing, this would improve the high-demand buyers' contractual options, since buyers could now purchase a certain amount of product A at a unit price just below c, in addition to product B at a unit price of c. This possibility would reduce the rent that the firm can extract from high demand buyers, to such an extent that the resulting loss in the high-demand segment of the market would offset any gains in the low-demand one.

With exclusive contracts, however, the dominant firm can undercut the competitive fringe in the low-demand segment under the protection of an exclusivity clause. This leaves the high-demand buyers' contractual options substantially unchanged, as they can already purchase one product, namely B, at a price only nominally higher than that charged by firm A. It follows that by imposing an exclusivity clause the dominant firm can replace the competitive fringe in the low-demand segment of the market, without losing any profit on the high-demand segment.

However, the dominant firm cannot now restrict its supply to low-demand buyers to the monopoly quantity, as buyers would rather prefer to trade with the competitive fringe only. Therefore, even if it imposes an exclusivity clause, the dominant firm must offer a quantity of at least $q_c^e(\theta)$.[33]

Under the exclusivity clause, then, the dominant firm sells (at least) $q_c^e(\theta)$ units of product A. This guarantees a positive profit, which makes the dominant firm less eager to switch to common representation. Therefore, the dominant firm imposes exclusivity also on some buyers who would have purchased both products under non-linear pricing. As a result, these buyers suffer a loss in terms of both lower volumes and reduced variety.

Again, it is only for sufficiently high types that the dominant firm stops imposing the exclusivity clause. At that point, the equilibrium quantities must coincide with those of the non-linear pricing equilibrium. Not only do fewer buyers purchase both goods than in the non-linear pricing equilibrium, those who do also pay higher prices, as the dominant firm adds a positive fixed fee to the non-linear pricing equilibrium tariff, just as in the previous case.

Therefore, buyers are again harmed by exclusive contracts. The effect on social welfare is ambiguous, though. On the one hand, equilibrium quantities, which are already inefficiently low, are further reduced. This is bad for efficiency. A countervailing effect, however, is the replacement of the competitive fringe with the more efficient dominant firm in the low-demand segment of the market. This reduces total production costs. This cost-saving effect may make the total welfare effect of exclusive contracts ambiguous.

[33] When $c \leq \bar{c}$, this is greater than the monopoly quantity. When $\bar{c} \leq c \leq c^{\lim}$, however, the dominant firm can still sell the monopoly quantity to a subset of buyers.

5 Duopoly

In this section, we turn to the case in which there is only one supplier of product B (firm B), which will then have some market power. This opens up new possibilities. Firstly, since firm B may reap positive profits in equilibrium, it can be definitely harmed by the dominant firm's exclusionary strategy. Secondly, unlike the competitive fringe firm B can respond actively to the dominant firm's attempt at foreclosing it. In particular, it may offer exclusive contracts in its turn, lower its non-exclusive prices, or both. Therefore, the analysis now determines who offers exclusive contracts, and whose contracts are accepted in equilibrium, endogenously.

The duopoly model is more complex than the competitive fringe model. The main reason for this is that the solution to a firm's pricing problem (discussed in section 3) does not yield directly the equilibrium, but only its best response to its rival's strategy. Finding the equilibrium requires finding a fixed point of the best response correspondence. Given the extra complexity, to simplify the exposition we shall focus only on the uniform-quadratic specification of the model.

5.1 Non-linear pricing

To find the non-linear pricing equilibrium, we adapt to the asymmetric case the solution procedure proposed by Martimort and Stole (2009) for the symmetric case (i.e. $c = 0$). This is a "guess and check" procedure that starts from the conjecture that the equilibrium price schedules are (piecewise) quadratic and then verifies it by identifying the coefficients of the price schedules.[34]

The non-linear pricing equilibrium turns out to be similar to the competitive fringe model: depending on the size of its competitive advantage c and the intensity of demand θ, the dominant firm can engage in monopoly pricing, limit pricing, or it can accommodate its rival. The exact structure of the equilibrium is as follows:

Proposition 3 *In the duopoly model, the following is a non-linear pricing equilibrium. Firm B offers the price schedule*

$$P_B(q) = P_B^{cr}(q)$$

and:

[34] It is important to stress that this procedure makes a guess on the structure of the equilibrium, but does not restrict firms to quadratic price schedules. The drawback of the guess and check procedure is that it cannot find equilibria in which the price schedules do not conform to the guess, if there are any. However, this is not a serious problem for our purposes. If there were multiple non-linear pricing equilibria, for each there would exist a corresponding equilibrium with exclusive contracts, with the same comparative statics properties.

- *when* $c \leq \tilde{c}$,[35] *firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^{\lim}(q) & \text{for } 0 \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \ \text{constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B) \end{cases}$$

where $\breve{\theta}_B$ *is implicitly defined by the condition* $q_B^{cr}(\breve{\theta}_B) = 0$ *and the constant guarantees the continuity of the price schedule*;

- *when* $c \geq \tilde{c}$, *firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^m(q) & \text{for } 0 \leq q \leq q^m(\theta^{\lim}) \\ P^{\lim}(q) + \ \text{constant} & \text{for } q^m(\theta^{\lim}) \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P^{cr}(q) + \ \text{constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

where $\theta^{\lim}$ *is implicitly defined by the condition* $q^m(\theta^{\lim}) = q^{\lim}(\theta^{\lim})$ *and the constants guarantee the continuity of the price schedule.*

The monopoly schedule is exactly the same as in the competitive fringe model. The limit pricing schedule is similar, except that now the unit cost c is replaced by the marginal price that firm B charges for the first unit it offers, $P_B'^{cr}(0)$. As for the common representation quantities, now they are:[36]

$$q_A^{cr}(\theta) = \frac{\theta - \alpha}{1-\alpha} + c\frac{\gamma}{1-2\gamma}; \qquad q_B^{cr}(\theta) = \frac{\theta - \alpha}{1-\alpha} - c\frac{1-\gamma}{1-2\gamma}, \tag{13}$$

where $\alpha = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1-2\gamma+9\gamma^2}\right] \geq 0$ is a decreasing function of γ that vanishes when $\gamma = \frac{1}{2}$. The corresponding price schedules are

$$P_A^{cr}(q) = \alpha q + c\frac{\alpha\gamma}{1-2\gamma}q - \frac{\alpha}{2}q^2; \quad P_B^{cr}(q) = cq + \alpha\left[1 - \frac{c(1-\gamma)}{1-2\gamma}\right]q - \frac{\alpha}{2}q^2, \tag{14}$$

respectively, so $P_B'^{cr}(0) = c + \alpha\left[1 - \frac{c(1-\gamma)}{1-2\gamma}\right]$. Since firm B is now exercising its market power, $q_B^{cr}(\theta)$ is lower than in the competitive fringe model; accordingly, $q_A^{cr}(\theta)$ is higher.

The only qualitative difference with the competitive fringe model is that the case in which the marginal buyer purchases product B only can no longer arise in equilibrium. Low-demand buyers always purchase only product A. The reason for this is that now both firms exercise their market power.

[35] To be precise, the threshold $\tilde{c}$ is the lowest c such that there exists at least one type θ for whom $q^m(\theta) > q^{\lim}(\theta)$.

[36] Notice that the guess and check procedure is used only to find the common representation price schedules. The monopoly and (given $P_B'^{cr}(0)$) the limit pricing schedules are unique.

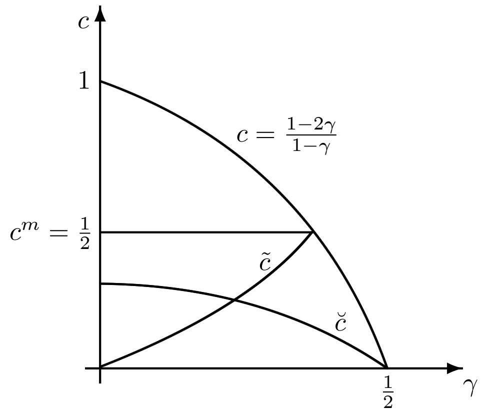

Figure 4: Critical thresholds for the duopoly model.

5.2 Exclusive contracts: large competitive advantage

With exclusive contracts, the nature of the duopoly equilibrium depends on the size of the dominant firm's competitive advantage even more profoundly than in previous cases. In particular, when the dominant firm's competitive advantage is large there is a unique equilibrium outcome, and exclusive contracts tend to be anti-competitive. When instead c is small the equilibrium outcome is no longer unique, and the effect of exclusive contracts is to reduce prices and profits. Because of these differences, it is convenient to deal with the two cases separately. Figure 4 shows the parameter values for which the different equilibrium patterns may arise.

We start from the case of a large competitive advantage: $c \geq \breve{c}$. Since firm B may charge different prices for exclusive and non exclusive contracts, we analyse separately the competition in exclusive and non-exclusive prices. In equilibrium, the former will apply to low-demand buyers, the latter to high-demand ones.

Let us consider the competition for exclusives first. Like in the competitive fringe model, when $c \geq c^m$ exclusive dealing completely shelters the dominant firm from any competitive pressure. That is, even if firm B priced as aggressively as it can, leaving to the buyer all the gain from trade, this would still be lower than the buyer's information rent when dealing with firm A only. The dominant firm can therefore safely engage in monopoly pricing: in this case, whether or not firm B offers exclusive contracts is, in fact, irrelevant.

When $c < c^m$, however, the monopoly tariff exceeds cq for a range of quantity levels. This means that firm B can now compete for exclusives. However, competition for exclusives is competition in utility space, where the firms' products effectively become homogeneous. In the ensuing Bertrand-like equilibrium, firm B always prices at cost, whereas firm A either undercuts firm B or engages in monopoly pricing – whichever leads to lower prices. Notice that even if firm B now offers exclusive contracts, only those offered by the dominant firm are accepted in equilibrium, and hence can be observed in practice. Therefore, the equilibrium outcome is still consistent with the stylised facts described in the introduction to this paper.

The condition that c exceeds the critical threshold $\breve{c}$ guarantees that the Betrand-like equilibrium is the only possible outcome of the competition for exclusives. Below the threshold, firms could manage to coordinate their pricing strategies to reduce the intensity of competition, as we shall see in the next subsection. This may lead to a multiplicity of equilibria. Above the threshold, however, there is no scope for coordination.[37]

Now consider the non-exclusive contracts. Except for constant terms, the non-exclusive tariffs coincide with the common representation tariffs (14) that arise in the non-linear pricing equilibrium. This result is reminiscent of the separation property, but in fact it rests on a subtler reasoning, as now firms interact strategically. The key idea is that the equilibrium quantities for high-demand types must be independent of whatever happens to low-demand buyers – a property that we call "type consistency." Intuitively, if there were a profitable deviation from the non-exclusive price schedules, this would also be a profitable deviation (*modulo* a constant term to account for the participation constraint) in the non-linear pricing game. But this contradicts the fact that $\{P_A^{cr}(q), P_B^{cr}(q)\}$ is, for high-demand types, an equilibrium of that game.

To complete the derivation of the equilibrium it remains to determine the constant terms of the non-exclusive tariffs, which we denote by Φ_A and Φ_B. These determine the critical buyer $\hat{\theta}$ who is just indifferent between exclusive and non-exclusive contracts. For this buyer, the following condition must hold

$$u(q_A^E(\hat{\theta}),0,\hat{\theta})-P_A^E(q_A^E(\hat{\theta})) = u(q_A^{cr}(\hat{\theta}),q_B^{cr}(\hat{\theta}),\hat{\theta})-P_A^{cr}(q_A^{cr}(\hat{\theta}))-P_B^{cr}(q_B^{cr}(\hat{\theta})-\Phi_A-\Phi_B. \tag{15}$$

Clearly, an increase in Φ_i will increase $\hat{\theta}$.

Intuitively, when choosing Φ_A and Φ_B, both firms are trading off market share and profitability. Consider, for instance, firm B. Since its exclusive contracts are not accepted (and in any case would not be profitable), it must try to induce more high-demand buyers, who value product variety more highly, to reject the exclusive contracts offered by firm A and buy both products. To get such buyers to purchase both products, firm B must lower its non-exclusive prices by adding a negative term (a fixed subsidy) to the tariff $P_B^{cr}(q)$. Firm A, by contrast, will add a fixed fee to the tariff $P_A^{cr}(q)$. The fixed fee must be sufficiently large that the dominant firm earns more, at the margin, from buyers who choose common representation than from those who choose exclusive dealing. This follows from arguments similar to those leading to Lemma 2.

More formally, consider the optimal choice of Φ_A and Φ_B. Firm A's profit is

$$\int_{\tilde{\theta}}^{\hat{\theta}} P_A^E(q_A^E(\theta))d\theta + \int_{\hat{\theta}}^{1} \left[P_A^{cr}(q_A^{cr}(\theta)) + \Phi_A\right] d\theta,$$

and firm B's is

$$\int_{\hat{\theta}}^{1} \left[P_B^{cr}(q_B^{cr}(\theta)) - cq_B^{cr}(\theta) + \Phi_B\right] d\theta.$$

[37] To be precise, the threshold $\check{c}$ is $\frac{2(1-2\gamma)}{5(1-\gamma)+\sqrt{1-2\gamma+9\gamma^2}}$.

Since $\hat{\theta}$ is determined by (15), the equilibrium conditions for Φ_A and Φ_B are:

$$\frac{P_A^{cr}(q_A^{cr}(\hat{\theta})) + \Phi_A - P_A^E(q_A^E(\hat{\theta}))}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_A^E(\hat{\theta})} = 1 - \hat{\theta}, \tag{16}$$

$$\frac{P_B^{cr}(q_B^{cr}(\hat{\theta})) + \Phi_B - c q_B^{cr}(\hat{\theta})}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_A^E(\hat{\theta})} = 1 - \hat{\theta}. \tag{17}$$

Condition (16) and (17) are the duopoly counterpart of condition (7) in Lemma 2. The economic intuition is similar. It can be confirmed that in equilibrium $\Phi_A > 0$, $\Phi_B < 0$ and $\Phi_A + \Phi_B > 0$.

We are now ready to provide the characterisation of the equilibrium when exclusive contracts are permitted and the dominant firm's competitive advantage is large.

Proposition 4 *The following is an equilibrium in the duopoly model when firms can use exclusive contracts and the dominant firm's competitive advantage is large, i.e.* $c > \breve{c}$.[38]

- *When* $\breve{c} < c < c^m$ *the two firms offer the following exclusive price schedules*

$$P_B^E(q) = cq$$
$$P_A^E(q) = \begin{cases} cq & \text{for } q \le q_c^e(\theta^m) \\ P_A^m(q) + \text{ constant} & \text{for } q > q_c^e(\theta^m) \end{cases}$$

  where θ^m *is such that* $q_c^e(\theta^m) = q_A^m(\theta^m)$ *and the constant guarantees the continuity of the price schedule, and the following non-exclusive price schedules*

$$P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A \quad \text{for } q \ge q_A^{cr}(\hat{\theta})$$
$$P_B^{NE}(q) = P_B^{cr}(q) + \Phi_B \quad \text{for } q \ge q_B^{cr}(\hat{\theta})$$

  where $\hat{\theta}$, Φ_A *and* Φ_B *are the solution to system (15)-(17).*

- *When* $c \ge c^m$ *the two firms offer the following price schedules*

$$P_A^E(q) = P^m(q)$$

  (firm B may not offer any exclusive contract at all), and

$$P_A^{NE}(q)[@P_A^{NL}(q)] = P_A^{cr}(q) + \Phi_A \quad \text{for } q \ge q_A^{cr}(\hat{\theta})$$
$$P_B^{NE}(q)[@P_B^{NL}(q)] = P_B^{cr}(q) + \Phi_B \quad \text{for } q \ge q_B^{cr}(\hat{\theta})$$

  where $\hat{\theta}$, Φ_A *and* Φ_B *are defined as in the previous case.*

[38] As we have already noted, if there were different equilibrium price schedules under common representation, $P_i^{cr}(q)$, for each of them there would be corresponding equilibria with exclusive contracts with the same structure as that described in Proposition 4. The same remark applies also to Proposition 5 below.

5.2.1 Comparison

When $c \geq c^m$, the role of exclusive contracts is exactly the same as in the competitive fringe model. Exclusive contracts, that is to say, are used by the dominant firm as a substitute for limit pricing – a more profitable means to foreclose the rival from a segment of the market.

The equilibrium quantities, with and without exclusive contracts, are qualitatively the same as in Figure 2. (The only difference is that under duopoly $q_A^{cr}(\theta)$ is greater, and $q_B^{cr}(\theta)$ smaller, than in the competitive fringe model.) By exactly the same arguments, we can therefore conclude that when the dominant firm's competitive advantage is large, exclusive contracts harm the buyers and are unambiguously anti-competitive. A novel implication of exclusive contracts is that they now also harm the dominant firm's rival. Firm B is harmed both because its market share falls, and because it must decrease its prices in order to resist being foreclosed from an even larger segment of the market. Notice that when $c \geq c^m$ firm B may well refrain from offering exclusive contracts, as they would not be accepted in any case.

When $\breve{c} \leq c < c^m$, competition for exclusives is fiercer, implying that for low volumes the dominant firm's equilibrium exclusive tariff is cq rather than $P^m(q)$. As a result, low-demand types buy $q_c^e(\theta)$ units of product A. This is still less than the limit pricing quantity that they would have obtained, under non-linear pricing, in the region $c < \tilde{c}$. However, in the region where $\breve{c} \leq c < c^m$ and $c \geq \tilde{c}$, some low-demand buyers would have obtained the monopoly quantity under non-linear pricing. These buyers may therefore now gain from exclusive contracts. Although high-demand buyers still lose, the welfare effect may therefore be ambiguous.

It may be interesting to elaborate on the reason why competition for low-demand buyers becomes tougher with exclusive contracts when $\tilde{c} \leq c < c^m$, as this help to understand what happens in the small competitive advantage case. Under non-linear pricing, in that region firm B does not actually compete for low-demand buyers as doing so would provide better outside options for high-demand buyers, reducing the rent that firm B can extract for them. With exclusive contracts, however, firm B can offer a discount conditioned on exclusivity. By doing so, it can compete for low-demand buyers, without losing any profit on the high-demand segment of the market. This forces firm A to match its rival's offer, to the benefit of low-demand buyers.

5.3 Exclusive contracts: small competitive advantage

We conclude the analysis by considering the case in which the dominant firm's competitive advantage is small ($c \leq \breve{c}$). This generalises the analysis of symmetric firms that we developed in our companion paper (Calzolari and Denicolò, 2013).

When firms are nearly symmetric, there is a multiplicity of equilibria that arises because the firms may or may not succeed in coordinating their strategies so as to extract the preference for variety and reduce the intensity of competition.

However, in all equilibria the effect of exclusive contracts is to reduce prices and profits.

For brevity, we shall not provide a complete characterisation of the set of equilibria, but we content ourselves with showing that exclusive contracts are pro-competitive. To this end, we shall focus exclusively on the "most cooperative" equilibrium, where prices and profits are largest, given that the firms actually play a non-cooperative game.[39]

To understand the coordination problems that the firms face, consider the outcome of the competition for exclusives in the large competitive advantage case: firm B prices at cost, whereas firm A undercuts it. Clearly, this is a possible equilibrium even when the competitive advantage is small. However, both firms can now obtain larger profits. This requires that the firms lower their non-exclusive prices in coordinated fashion, inducing some buyers to purchase both products. This move allows firms to extract the buyers' preference for variety.[40] If firms manage to coordinate their non-exclusive prices in this way, however, a new opportunity of coordination arises. Since certain exclusive contracts will no longer be accepted in equilibrium, firms have no longer an incentive to undercut one another's exclusive prices; therefore, they can also increase exclusive prices so as to reduce the intensity of competition.

These arguments imply that in the "most cooperative" equilibrium, where profits are largest, the exclusive and non-exclusive price schedules must be determined simultaneously. We now derive the conditions that must be satisfied in the equilibrium in which the firms extract the preference for variety, and reduce the intensity of competition, as best as they can given that they are playing a non-cooperative game.

Let $U^E(\theta)$ be the (type-dependent) reservation utility that buyer θ could obtain by choosing his most preferred exclusive contract. To extract the buyer's preference for variety, firms must introduce non-exclusive price schedules implicitly defined by the condition:

$$\max_{q_A, q_B} \left[u(q_A, q_B, \theta) - P_A^{NE}(q_A) - P_B^{NE}(q_B) \right] = U^E(\theta), \tag{18}$$

with a small tie-breaking discount if necessary. These price schedules apply to low-demand buyers; high-demand buyers will actually obtain more than $U^E(\theta)$ simply thanks to the competition in non-exclusive contracts. Notice that equation (18) does not pin down $P_A^{NE}(q_A)$ and $P_B^{NE}(q_B)$ uniquely. This reflects the fact that the preference for variety can be split between the two firms in different ways, provided that the firms do not ask, in the aggregate, for more than the buyer is willing to pay to purchase both goods. Since we look for the equilibrium in which firms' profits are largest, we shall focus on the case in which the firms maximise the rents that they extract from low-demand buyers.

[39] We refer the reader to our companion paper for a fuller discussion of other equilibria.

[40] Notice that while the function u always entails a preference for variety, the presence of the cost c means that when the intensity of demand is low, only good A must efficiently be produced. Thus, there is room for extracting the preference for variety only if the competitive advantage is not too large: to be precise, the condition is $c \leq \check{c}$.

This requires maximisation of the total surplus $u(q_A, q_B, \theta) - cq_B$, subject to the constraint that buyers must obtain $U^E(\theta)$. Using the envelope theorem, the constraint can be rewritten as

$$q_A(\theta) + q_B(\theta) = q^E(\theta), \tag{19}$$

where $q^E(\theta)$ is the optimal quantity under exclusivity. Notice that $q^E(\theta)$ depends on what exclusive prices are sustainable in the most cooperative equilibrium and hence must be determined jointly with all other variables.

Generally speaking, the more efficient firm must produce more than the less efficient one. In particular, the problem of total-surplus maximisation may have a corner solution in which some low-demand types must buy good A only. In this case, exclusive contracts must be accepted in equilibrium by those types, and so Bertrand competition in utility space implies that exclusive prices must fall to marginal costs. Therefore, for low-demand types $q_A(\theta)$ must coincide with $q_c^e(\theta)$, and $q_B(\theta)$ must vanish.

When instead the total-surplus maximisation problem has an interior solution, which is

$$q_A(\theta) = \frac{1}{2}q^E(\theta) + \frac{c}{2(1-2\gamma)}; \quad q_B(\theta) = \frac{1}{2}q^E(\theta) - \frac{c}{2(1-2\gamma)}, \tag{20}$$

buyers purchase both products. The corresponding exclusive contracts are not actually accepted in equilibrium, and so there may be room for coordinating also the exclusive prices. The reason for this is that exclusive contracts affect the equilibrium outcome even if they are not accepted: the less aggressively firms bid for exclusivity, the greater the payments firms can obtain for non-exclusive contracts. Thus, raising the exclusive prices is good for the firms' profits.

Let us denote by an upper bar the highest exclusive prices that can be sustained in a non-cooperative equilibrium. To find them, we can assume, with no loss of generality, that both firms offer the same exclusive price schedule $\bar{P}^E(q)$.[41] By construction, low-type buyers must be just indifferent between exclusive and non-exclusive contracts (equation (18)). Thus, any arbitrarily small discount would trigger a switch to an exclusive contract. In equilibrium, no such deviation can be profitable. This implies the following no undercutting conditions:

$$\begin{aligned} P^E(q^E(\theta)) &\leq P_A^{NE}(q_A^{cr}(\theta)); \\ P^E(q^E(\theta)) - cq^E(\theta) &\leq P_B^{NE}(q_B^{cr}(\theta)) - cq_B^{cr}(\theta), \end{aligned} \tag{21}$$

which in the most cooperative equilibrium must hold as equalities.

The most cooperative equilibrium is found by solving the system of equations (18)-(21). Specifically, denote by $\bar{q}^E(\theta)$ the optimal quantity associated with

[41] We can prove that this does not entail any loss of generality by contradiction. Suppose to the contrary that one firm offered more attractive exclusive contracts than its rival. Since these contracts are not accepted in equilibrium, the firm could increase its exclusive prices without losing any profits on its exclusive contracts. In fact, the buyers' reservation utility would decrease, allowing both firms to increase their profits from non-exclusive contracts.

the exclusive prices $\bar{P}^E(q)$, and by $\bar{q}_i^{cr}(\theta)$ the values of $q_i(\theta)$ given by (20) when $q^E(\theta) = \bar{q}^E(\theta)$. Rewrite (18) as

$$u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - \bar{P}_A^{NE}(\bar{q}_A^{cr}(\theta)) - \bar{P}_B^{NE}(\bar{q}_B^{cr}(\theta)) = u(0, \bar{q}^E(\theta), \theta) - \bar{P}^E\left(\bar{q}^E(\theta)\right)$$

and use the no-undercutting conditions (21) to get

$$\bar{P}^E\left(q^E(\theta)\right) = \left[u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - u(0, \bar{q}^E(\theta), \theta)\right] + c\left[\bar{q}^E(\theta) - \bar{q}_B^{cr}(\theta)\right].$$

The term inside the first square brackets on right-hand side can be interpreted as the preference for variety, while the term inside the second square bracket is the cost saving. Using (20), we finally get

$$\bar{P}^E(q) = \frac{c^2}{2(1-2\gamma)} + \frac{c}{2}q + \frac{1-2\gamma}{4}q^2, \tag{22}$$

and

$$\bar{P}_A^{cr}(q_A) = -cq + (1-2\gamma)q^2 + cq_c^e(\hat{\theta}); \quad \bar{P}_B^{cr}(q_B) = 2cq + (1-2\gamma)q^2, \tag{23}$$

where $\hat{\theta}$ is now the solution to $q_c^e(\hat{\theta}) = \bar{q}_A^{cr}(\hat{\theta})$ and the constant term in $\bar{P}_A^{cr}(q_A)$ guarantees a smooth-pasting condition from exclusive to non-exclusive contracts. The corresponding quantities are

$$\bar{q}^E(\theta) = \frac{2\theta - c}{3-4\gamma}, \tag{24}$$

and

$$\bar{q}_A^{cr}(\theta) = \frac{2\theta - c}{2(3-4\gamma)} + \frac{c}{2(1-2\gamma)}; \quad \bar{q}_B^{cr}(\theta) = \frac{2\theta - c}{2(3-4\gamma)} - \frac{c}{2(1-2\gamma)}. \tag{25}$$

Notice that in this case the switch from exclusive to non-exclusive contracts does not involve any jump, as at the switching point the non-exclusive contracts just replicate the payoff that the buyers could obtain through exclusive contracts.

We are now ready to provide the characterisation of the most cooperative equilibrium.

Proposition 5 *Suppose that the dominant firm's competitive advantage is small: $c \leq \check{c}$. Then, in the duopoly model the most cooperative equilibrium with exclusive contracts is as follows. Both firms offer the exclusive price schedules*

$$P_A^E(q) = P_B^E(q) = \begin{cases} cq & \textit{for } q \leq q_c^e(\hat{\theta}) \\ \bar{P}^E(q) & \textit{for } q > q_c^e(\hat{\theta}) \end{cases}$$

with firm A slightly undercutting firm B, though. Furthermore:

$$P_A^{NE}(q) = \begin{cases} \bar{P}_A^{cr}(q) & \textit{for } q \leq \bar{q}_A^{cr}(\bar{\theta}) \\ P_A^{cr}(q) + \textit{constant} & \textit{for } q \geq \bar{q}_A^{cr}(\bar{\theta}) \end{cases}$$
$$P_B^{NE}(q) = \begin{cases} \bar{P}_B^{cr}(q) & \textit{for } q \leq \bar{q}_B^{cr}(\bar{\theta}) \\ P_B^{cr}(q) + \textit{constant} & \textit{for } q \geq \bar{q}_B^{cr}(\bar{\theta}) \end{cases}$$

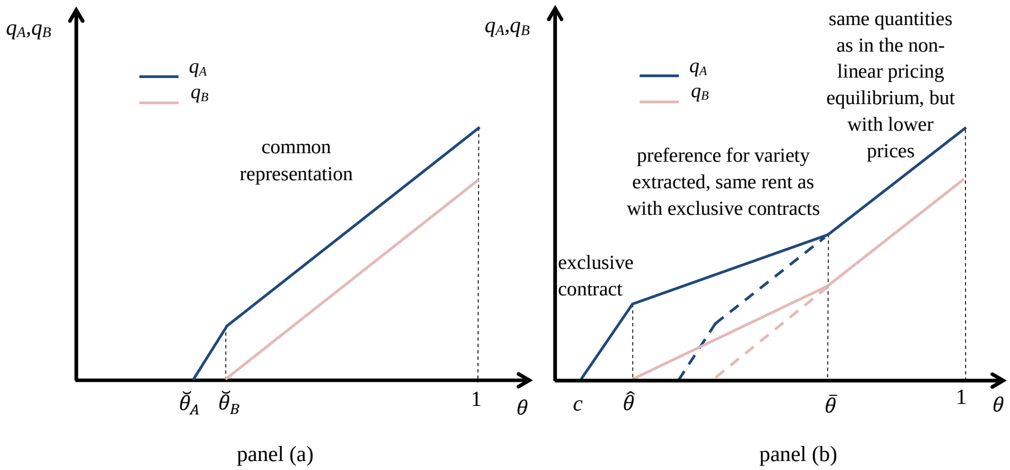

Figure 5: Equilibrium quantities in the duopoly model (small competitive advantage). Panel (a) non-linear pricing; panel (b) exclusive contracts (solid lines).

where $\hat{\theta}$ is the solution to $q_c^e(\hat{\theta}) = \bar{q}_A^{cr}(\hat{\theta})$ and $\bar{\theta}$ the solution to $\bar{q}_A^{cr}(\bar{\theta}) = q_A^{cr}(\bar{\theta})$ (and to $\bar{q}_B^{cr}(\bar{\theta}) = q_B^{cr}(\bar{\theta})$), and the constants guarantee the continuity of the price schedules.

The equilibrium quantities are depicted in Figure 5 (panel b) along with those of the non-linear pricing equilibrium (panel a). In the most cooperative equilibrium with exclusive contracts, buyers are divided into four groups. For $\theta \leq c$, buyers do not buy any product; for $c < \theta \leq \hat{\theta}$, buyers purchase $q_c^e(\theta)$ units of product A only at a price just below c; for $\hat{\theta} < \theta \leq \bar{\theta}$, buyers purchase $\bar{q}_A^{cr}(\theta)$ units of good A and $\bar{q}_B^{cr}(\theta)$ units of good B, and so obtain the same net surplus as if they had accepted an exclusive contract; finally, for $\bar{\theta} < \theta \leq 1$, buyers buy $q_A^{cr}(\theta)$ units of good A and $q_B^{cr}(\theta)$ units of good B, and strictly prefer their non-exclusive contracts to any exclusive one.

5.3.1 Comparison

Exclusive contracts are now unambiguously pro-competitive. To see why, notice that equilibrium quantities are larger than under non-linear pricing, and are everywhere closer to the first best. Since the social surplus (i.e., the sum of buyers' surplus and firms' profits) is concave in q_A and q_B, it is clear that exclusive contracts increase social welfare.

Buyers benefit from exclusive contracts. Low-demand buyers ($c < \theta < \bar{\theta}$) increase their purchases. High-demand buyers ($\theta \geq \bar{\theta}$) purchase the same quantities as in the non-linear pricing equilibrium, but they too are better off as they now have more attractive alternatives. The benefit is obtained *via* fixed subsidies that in the equilibrium with exclusive contracts are added to the non-linear pricing equilibrium price schedules.

However, both firms lose as compared to the non-linear pricing equilibrium. This follows from the fact that prices must be lower in order to support higher quantities. Since in the non-linear pricing equilibrium prices are already lower than under monopoly, the fact that they are further reduced means that exclusive contracts decrease firms' profits. Thus, firms are caught in a prisoner's dilemma: they have a unilateral incentive to offer exclusive contracts, but would actually benefit if such contracts were prohibited.

These conclusions are qualitatively similar to those obtained in Calzolari and Denicolò (2013) for the symmetric case.

6 Conclusion

In this paper, we have developed a new theory of exclusive dealing. We have argued that a dominant firm may find it profitable to use exclusive contracts just to increase its market share, without necessarily driving its existing rivals out of the market, or impeding the entry of new ones. This theory is valid under two assumptions. First, firms are imperfectly informed about demand. Second, the dominant firm has a sizeable competitive advantage over its rivals, in terms of lower cost, higher demand, or a combination of the two.

Not only are these assumptions realistic, but the model's predictions are also consistent with the stylised facts of many antitrust cases. In addition to a dominant firm that controls a substantial share of the market and has entered into some kind of exclusive arrangement with its customers, these often involve one or more smaller competitors, which have been active in the industry for some time and in principle could themselves use exclusive contracts, but apparently have not. Existing theories have found it difficult to explain this recurrent situation without making *ad hoc* assumptions. Ours, by contrast, can reproduce the stylised facts naturally.

Our theory offers new insights for competition policy. Since in our model exclusive contracts may be either pro or anti-competitive, the analysis does not call for a radical change in the current policy, which is based on the rule of reason. However, it may suggest that different factors should be considered for the purposes of antitrust evaluation.

In our model, the key factor is the size of the dominant firm's competitive advantage. This determines whether the dominant firm's rival can compete for exclusives effectively or not. If it can, exclusive contracts tend to be pro-competitive, reducing prices and profits and benefiting buyers. If it cannot, exclusive contracts are anti-competitive. The dominant firm gains, but both its rival and customers are harmed, and social welfare goes down.

While the size of the dominant firm's competitive advantage cannot be observed directly, it is correlated with variables that often can. One, for instance, is the dominant firm's market share. Another is the fraction of the market foreclosed. When they are large, an anticompetitive effect is more likely.

Factors other than the dominant firm's competitive advantage, which are often emphasised by antitrust authorities and the courts, turn out to be less important in our analysis. For example, the length of exclusive contracts is irrelevant, since contracts are not used for commitment purposes. Furthermore, if the negative impact of exclusive contracts on competition arises because rivals are driven, or kept, out of the market, then arguably claimants should be required to prove that eviction is likely, and that recoupment is possible. However, our analysis shows that exclusive contracts may have anti-competitive effects even if the dominant firm's rivals are, and stay, active. It also shows that the exclusionary strategy may not entail any short-term sacrifice on the part of the dominant firm. This implies that no proof of eviction and recoupment may be needed.

Finally, it is worth stressing that while our theory does not rely on the presence of economies of scale, if economies of scale are significant exclusive contracts can have anti-competitive effects under even broader circumstances. The reason for this is that in the duopoly model exclusive contracts always reduce the less efficient firm's profit – even if they intensify the competitive pressure on the dominant firm. With economies of scale, such more intense competition may drive the rival out of the market, in which case the welfare effects of exclusive contracts may well be negative.[42]

[42] The impact of exclusive contracts on the dominant firm's rival's profit can be significant even if the competitive advantage, and hence the fraction of the market that is foreclosed, are small. This is so because the effects of a small segment of the market being foreclosed reverberate throughout the entire market – for example, by inducing firms to reduce also their non-exclusive prices. This observation may suggest caution in providing "safe harbours." For example, it might seem that if exclusive dealing arrangements foreclose 30% of the market or less, the market that remains contestable should suffice for a rival to prosper. However, this argument overlooks the possibility that the dominant firm may retain the lion's share even of the market that is contestable, thereby forcing its competitor to exit.

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Appendix

This Appendix contains the proofs omitted in the text.

Proof of Lemma 1. For the purposes of this Lemma, we assume that the switching points are exogenous, and are not controlled either by the firm or by the buyers. Hence, the switching points are not necessarily optimal or even just incentive compatible. The result holds for arbitrary switching points (and so also when they are optimal and incentive compatible).

Assuming a finite number of switches, denoted by N, the multistage optimal control problem can be compactly written as follows: the firm maximises

$$\sum_{i=0}^{N} \int_{\hat{\theta}_i}^{\hat{\theta}_{i+1}} \left[v^{r_i}(q_A(\theta), \theta) - U(\theta) \right] f(\theta) d\theta$$

where $\hat{\theta}_0 = \tilde{\theta}$, $r_i \in \{E, NE\}$ is the regime that applies to types $\theta \in [\theta_i, \theta_{i+1}]$, $v^E(q_A(\theta), \theta) = u(q_A(\theta), 0, \theta)$ and $v^{NE}(q_A(\theta), \theta) = v(q_A(\theta), \theta)$, subject to the following constraints:

$$\begin{aligned} \frac{dU}{d\theta} &= v_\theta^{r_i}(q_A(\theta), \theta) \text{ for any } \theta \in [\theta_i, \theta_{i+1}] \text{ and all } i = 0, 1, ..., N \\ U(\theta) &\geq \bar{U}^{r_i}(\theta), \end{aligned}$$

and $q_A(\theta)$ non decreasing, where $\bar{U}^E(\theta) = \max\{U_A^R(\theta), v(0, \theta)\}$ and $\bar{U}^{NE}(\theta) = v(0, \theta)$.

Consider first the relaxed problem without the monotonicity constraint and in which the participation constraint is just $U(\theta) \geq v(0, \theta)$. The objective function can be rewritten as follows

$$\sum_{i=0}^{N} \int_{\hat{\theta}_i}^{\hat{\theta}_{i+1}} v^{r_i}(q_A(\theta), \theta) f(\theta) d\theta - \int_{\tilde{\theta}}^{\theta_{\max}} U(\theta) f(\theta) d\theta$$

Let $\mu(\theta)$ denote the co-state variable associated with the state variable $U(\theta)$. By Pontryagin's maximum principle, we have

$$\dot{\mu}(\theta) = -\frac{\partial H}{\partial U} = f(\theta)$$

Since $U(\theta_{\max})$ is unconstrained, the transversality condition $\mu(\theta_{\max}) = 0$ must hold. Integrating we get

$$\mu(\theta) = F(\theta) - 1$$

Therefore, the Hamiltonian is

$$v^{r_i}(q_A(\theta), \theta) f(\theta) - (1 - F(\theta)) v_\theta^{r_i}(q_A(\theta), \theta) + \text{constants}.$$

This must be maximised pointwise, which is equivalent to pointwise maximisation of the virtual surplus

$$v^{r_i}(q_A(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta^{r_i}(q_A(\theta),\theta).$$

The Lemma then simply follows by noting that this is also the optimality condition for problems (4) and (5) separately, provided that the monotonicity constraint and the participation constraints do not bind. The assumption that $q_A^E(\theta)$ and $q_A^E(\theta)$ are strictly increasing guarantees that the monotonicity constraint does not bind (otherwise, there would be bunching and the solution would be constant over a non degenerate interval).

To conclude the proof, notice that if the participation constraints binds only in the first stage of the problem, i.e. for $\theta \in [\theta_0, \theta_1]$, then it will affect the solution to the multi-stage problem exactly in the same way as it affects the solution in regime r_0. ■

Proof of Lemma 2. Now we assume that the switches are determined optimally, subject to appropriate incentive compatibility constraints. In particular, the "across-regimes" incentive compatibility constraints must guarantee that no type has an incentive to choose a regime different from that intended for him. This means that for any $\theta \in [\theta_i, \theta_{i+1}]$ and any $\theta' \in [\theta_j, \theta_{j+1}]$

$$v^{r_i}(q_A(\theta),\theta) - P^{r_i}(q_A(\theta)) \geq v^{r_j}(q_A(\theta'),\theta) - P^{r_j}(q_A(\theta')).$$

At a switching point $\hat{\theta}$, the above weak inequality must hold as an equality. (To see this, let $j = i+1$ and $\theta = \theta' = \hat{\theta}$, and note that the weak inequality must hold in both directions.) Therefore:

$$u(q_A^E(\hat{\theta}),0,\hat{\theta}) - P_A^E(q_A^E(\hat{\theta})) = v(q_A^{NE}(\hat{\theta}),\hat{\theta}) - P_A^{NE}(q_A^{NE}(\hat{\theta})).$$

This proves that condition (6) must hold. Consider next condition (7). This can be derived as follows. Let Φ be a parallel shift in all the price schedules that apply for $\theta \geq \hat{\theta}$. Notice that a change in Φ will not affect the quantities nor the switching points to the right of $\hat{\theta}$. Therefore, a small increase $d\Phi$ in Φ will increase profits by $\left[1 - F(\hat{\theta})\right] d\Phi$. On the other hand, a change Φ will change $\hat{\theta}$. By implicit differentiation, the associated change in profits is

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{v_\theta(q_A^{NE}(\hat{\theta}),\hat{\theta}) - u_\theta(q_A^E(\hat{\theta}),0,\hat{\theta})} f(\hat{\theta}) d\Phi,$$

irrespective of whether the switch is from exclusivity to non exclusivity, or viceversa. At an optimum, profit must be locally constant and so

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{v_\theta(q_A^{NE}(\hat{\theta}),\hat{\theta}) - u_\theta(q_A^E(\hat{\theta}),0,\hat{\theta})} = \frac{1-F(\hat{\theta})}{f(\hat{\theta})}. \quad ■$$

Proof of Proposition 1. Because the proof is based on the use of direct mechanisms, it is convenient to report the equilibrium quantities first. They are:

- when $c \le c^{\lim}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le \breve{\theta}_A \\ q_A^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_A \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \le c \\ q_c^e(\theta) & \text{for } c \le \theta \le \breve{\theta}_A \\ q_B^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_A, \end{cases}$$

- when $c^{\lim} \le c \le c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le \breve{\theta} \\ q^{\lim}(\theta) & \text{for } \breve{\theta} \le \theta \le \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \le \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_B, \end{cases}$$

- when $c \ge c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le \theta^m \\ q^m(\theta) & \text{for } \theta^m \le \theta \le \theta^{\lim} \\ q^{\lim}(\theta) & \text{for } \theta^{\lim} \le \theta \le \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \le \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \ge \breve{\theta}_B, \end{cases}$$

where the thresholds $\breve{\theta}_A$ and $\breve{\theta}_B$ are implicitly defined as the largest θ such that $q_A^{cr}(\theta) = 0$ and $q_B^{cr}(\theta) = 0$, respectively, θ^m is the marginal buyer under monopoly, $\breve{\theta}$ is defined by the condition $u_{q_A}(0, 0, \theta) = c$ (or, equivalently, $q_c^e(\theta) = 0$) and hence is the marginal buyer under limit pricing, and $\theta^{\lim}$ is implicitly defined by the condition $q^m(\theta) = q^{\lim}(\theta)$. The marginal buyer is $\breve{\theta}$ when $c \le c^m$, and θ^m when $c \ge c^m$.

Obviously, the competitive fringe will always price at cost: $P_B(q_B) = cq_B$. To prove the proposition, it then suffices to show that the dominant firm's equilibrium pricing strategy is indeed optimal. To do so, we shall focus on direct mechanisms and hence find the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported above. It is then straightforward to conclude that the price schedules that support these quantities, which are the equilibrium price schedules, are indeed optimal.

To begin with, consider the indirect utility function $v(q_A, \theta)$ when $P_B(q_B) = cq_B$. This is piecewise smooth, with two branches corresponding to the cases in which the quantity

$$\tilde{q}_B(q_A, \theta) = \arg\max_{q_B \ge 0} \left[u\left(q_A, q_B, \theta\right) - cq_B\right]$$

is 0 or is strictly positive, and a kink between the two branches. It can be easily checked that the function v is globally concave in q_A. It also satisfies the single-crossing condition $v_{\theta q_A}(q_A, \theta) \ge 0$, since we have

$$v_\theta(q_A, \theta) = u_\theta\left(q_A, \tilde{q}_B(q_A, \theta), \theta\right)$$

and hence:

$$\begin{aligned} v_{\theta q_A} &= u_{\theta q_A} + \frac{d\tilde{q}_B(q_A,\theta)}{dq_A} u_{\theta q_B} \\ &= u_{\theta q_A} - \frac{u_{q_B q_A}}{u_{q_B q_B}} u_{\theta q_B} \geq 0, \end{aligned}$$

where the inequality follows by the fact that the goods are imperfect substitutes.

The single-crossing condition guarantees that the participation constraint binds only for the marginal buyer, whom we indicate here as $\tilde{\theta}$, and that firm A's optimisation program (3) can be written as

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta),\theta) - U(\theta) \right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A,\theta) \\ U(\tilde{\theta}) &= v(0,\tilde{\theta}) \end{aligned}$$

By a standard integration by parts, the problem reduces to finding the function $q_A(\theta)$ that pointwise maximises the indirect virtual surplus:

$$s(q_A,\theta) = v(q_A,\theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q_A,\theta).$$

Like the indirect utility function, the indirect virtual surplus has two branches and a kink at $q_A = q^{\lim}(\theta)$.

Generally speaking, for any θ the maximum can occur in either one of the two quadratic branches, or at the kink. Let $q^m(\theta) = \arg\max_{q_A} s(q_A,\theta)$ when the maximum lies on the first branch, and $q_A^{cr}(\theta) = \arg\max_{q_A} s(q_A,\theta)$ when it lies on the second. Notice that the kink $q^{\lim}(\theta)$ is implicitly defined by the condition $u_{q_B}\left(q^{\lim}(\theta),0,\theta\right) = c$.

Since $q_A(\theta)$ must pointwise maximise the virtual surplus, we can conclude that $q_A(\theta) = q_A^m(\theta)$ if the maximum is achieved on the upper branch, $q_A(\theta) = q_A^{cr}(\theta)$ if the maximum is achieved on the lower branch, and $q_A(\theta) = q^{\lim}(\theta)$ if the maximum is achieved at the kink. By assumption A1, $s(q_A,\theta)$ is globally concave in q_A. This implies that if $q^m(\theta) > q^{\lim}(\theta)$, then $s(q_A,\theta)$ is increasing at the kink and the maximum is achieved at $q^m(\theta)$. If instead $q^m(\theta) < q^{\lim}(\theta)$, then $s(q_A,\theta)$ is decreasing to the right of the kink, and one must further distinguish between two cases. If $q_A^{cr}(\theta) > q^{\lim}(\theta)$, then $s(q_A,\theta)$ is increasing to the left of the kink and so the maximum is achieved at the kink $q^{\lim}(\theta)$. If instead $q_A^{cr}(\theta) < q^{\lim}(\theta)$, the maximum is achieved to the left of the kink and is $q_A^{cr}(\theta)$.

It remains to find out when each type of solution applies. By A2, the condition $q^m(\theta) > q^{\lim}(\theta)$ is equivalent to $\theta < \theta^{\lim}$. Since $q^m(\theta)$ is positive only for $\theta > \theta^m$, the monopoly solution is obtained if and only if the interval $\theta^m \leq \theta \leq \theta^{\lim}$ is not empty. This is true if only if $c > c^m$ (recall that c^m is defined as the lowest c such that $q^m(\theta) > q^{\lim}(\theta)$ for some θ). In this case, then, we have $q_A(\theta) = q^m(\theta)$ for $\theta^m \leq \theta \leq \theta^{\lim}$. Of course, the corresponding equilibrium quantity of good B must be nil.

Now suppose that $\theta > \theta^{\lim}$, so that $q^m(\theta) < q^{\lim}(\theta)$. In this case, the solution depends on whether $q_A^{cr}(\theta)$ is larger or smaller than $q_A^{\lim}(\theta)$. The limit pricing solution can emerge only if $q_A^{cr}(\theta) > q_A^{\lim}(\theta)$. By A2, the condition $q_A^{cr}(\theta) > q^{\lim}(\theta)$ reduces to $\theta < \breve{\theta}_B$. Since $q^{\lim}(\theta)$ is positive only for $\theta > \breve{\theta}$, the limit pricing solution is obtained if and only $\breve{\theta} < \breve{\theta}_B$. This condition is equivalent to $c \geq c^{\lim}$ (recall that $c^{\lim}$ is the lowest c such that $q_A^{cr}(\theta) > q^{\lim}(\theta)$ for some θ). When this condition holds, there exists an interval of types to whom the limit pricing solution applies. Again, the corresponding equilibrium quantity of good B must be nil.

Finally, consider the case in which $\theta \geq \breve{\theta}_B$, so that $q_A^{cr}(\theta) \leq q^{\lim}(\theta)$ and the maximum is achieved on the lower branch of the virtual surplus function. Here, we must distinguish between two sub-cases, depending on whether the solution is interior, or is a corner solution at $q_A(\theta) = 0$. Clearly, the solution is interior, and is $q_A^{cr}(\theta)$, when $\theta \geq \breve{\theta}_A$. In this case, the corresponding equilibrium quantity of good B is $q_B^{cr}(\theta) = \tilde{q}_B(q_A^{cr}(\theta), \theta)$. Now, notice that when $c < c^{\lim}$ we have $\breve{\theta}_B < \breve{\theta}_A$, whereas the inequality is reversed when $c \geq c^{\lim}$. This means that if $c \geq c^{\lim}$ and the maximum is achieved in the lower branch, it must necessarily be an interior solution. However, when $c < c^{\lim}$ we have $\breve{\theta}_B < \breve{\theta}_A$. In this case, for $\breve{\theta} \leq \theta \leq \breve{\theta}_A$, we have a corner solution for q_A, and the corresponding equilibrium quantity of good B is $q_c^e(\theta)$; for $\theta \geq \breve{\theta}_A$, the solution is again interior.

This completes the derivation of the optimal quantities in all possible cases. It is then easy to check that they coincide with the equilibrium quantities reported above, and that they are implemented by the equilibrium price schedules. This completes the proof of the Proposition.[43] ■

Proof of Proposition 2. Since the proof will use direct mechanisms, we start again by reporting the equilibrium quantities. They are:

- when $c \leq \bar{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta} \\ q_c^e(\theta) & \text{for } \breve{\theta} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

- when $\bar{c} \leq c \leq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta} \\ q_c^e(\theta) & \text{for } \breve{\theta} \leq \theta \leq \theta^+ \\ q^m(\theta) & \text{for } \theta^+ \leq \theta \leq \hat{\theta} \\ q^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

[43] Notice that since equilibrium quantities are everywhere continuos, the equilibrium price schedules must be continuous. The constant terms that guarantee continuity are all negative, i.e. fixed subsidies. In fact, it can be verified that the equilibrium price schedules are also everywhere smooth.

when $c \geq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \theta^m \\ q^m(\theta) & \text{for } \theta^m \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

where $\hat{\theta}$ is defined in the text of the Proposition and θ^+ is the solution to $q_c^e(\theta) = q^m(\theta)$ (this is unique by A3).

The strategy of proof is the same as for Proposition 1. Obviously, the competitive fringe will always price at cost, i.e. $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$. As for firm A, we shall focus on direct mechanisms and hence look for the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported above.

Lemma 2 implies that the solution to the dominant firm's problem is formed by appropriately joining the solution to the maximisation program (5) and that to the maximisation program (4). By assumption A4, the former applies to low-demand buyers ($\theta < \hat{\theta}$), the latter to high-demand buyer ($\theta > \hat{\theta}$). The solution to problem (4), which has been characterised in the proof of Proposition 1 (notice that since $P_B^E(q_B) = P_B^{NE}(q_B)$, the constraint $U(\theta) \geq U^R(\theta)$ is already subsumed into the indirect utility function), implies that the participation constraints binds only for the lowest type. Together with A1, this implies that the assumptions of Lemma 2 indeed hold true.

We therefore start by focusing on problem (5). This is a standard monopolistic non-linear pricing problem with a utility function $u(q_A, 0, \theta)$, except that buyers now have a type-dependent reservation utility

$$U_A^R(\theta) = \max\left[u(0, q, \theta) - cq\right].$$

Thus, the problem becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_0^1 \left[u(q_A(\theta), 0, \theta) - U(\theta)\right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_A(\theta), 0, \theta) \\ U(\theta) &\geq U_A^R(\theta). \end{aligned} \tag{A.1}$$

Its solution is given in the following.

Lemma 3 *When $c \geq c^m$, the solution to problem (A.1) is*

$$q_A(\theta) = \begin{cases} 0 & \textit{for } 0 \leq \theta \leq \theta^m \\ q_A^m(\theta) & \textit{for } \theta \geq \theta^m. \end{cases}$$

When instead $c \leq c^m$, the solution is

$$q_A(\theta) = \begin{cases} 0 & \textit{for } \theta \leq \check{\theta} \\ q_c^e(\theta) & \textit{for } \check{\theta} \leq \theta \leq \theta^+ \\ q_A^m(\theta) & \textit{for } \theta \geq \theta^+. \end{cases}$$

Proof. Consider first the unconstrained problem. Clearly, its solution is $q_A^m(\theta)$.

When $c \geq c^m$, we have $U^m(\theta) \geq U_A^R(\theta)$ for all θ, so the unconstrained solution applies. To show this, notice first of all that it follows from our definitions that

$$q_c^e(\theta) \leq q^{\lim}(\theta),$$

with equality only when both quantities vanish. Thus, $\check{\theta}$ is the largest θ such that $q_c^e(\theta) = q^{\lim}(\theta) = 0$. The condition $c \geq c^m$ guarantees that $\theta^m \geq \check{\theta}$. By A3, this implies that $q_A^m(\theta) > q_c^e(\theta)$ for $\theta > \theta^m$. Since

$$U^m(\theta) = \int_{\theta^m}^{\theta} u_\theta(q_A^m(s), 0, s)ds$$

whereas

$$U_A^R(\theta) = \int_{\theta^m}^{\theta} u_\theta(q_c^e(s), 0, s)ds$$

it follows by the sorting condition $u_{\theta q_A} \geq 0$ that the participation constraint is always satisfied.

Now suppose that $c < c^m$, so that the type-dependent participation constraint must bind for a non-empty set of types. To deal with this constraint, we use the results of Jullien (2000), and in particular his Proposition 3. To apply that proposition, we must show that our problem satisfies the conditions of Weak Convexity, Potential Separation, Homogeneity, and Full Participation. Weak Convexity requires that $U^m(\theta)$ is more strongly convex than $U_A^R(\theta)$. This is implied by assumption A3. Following Jullien (2000), define the virtual surplus function

$$s^E(g, q_A, \theta) = u(q_A, 0, \theta) - \frac{g - F(\theta)}{f(\theta)} u_\theta(q_A, 0, \theta)$$

where the "weight" $g \in [0, 1]$ accounts for the possibility that the participation constraint may bind over any subset of the support of the distribution of types. Pointwise maximisation of the virtual surplus function yields

$$\ell^E(g, \theta) = \arg\max_{q_A} s^E(g, q_A, \theta).$$

Potential Separation requires that $\ell^E(g, \theta)$ is non-decreasing in θ, which is obviously true. Homogeneity is obvious, as it requires that $U_A^R(\theta)$ can be implemented by a continuous and non decreasing quantity; in our case, this is by construction $q_c^e(\theta)$. Finally, the condition of Full Participation requires that in equilibrium all types $\theta > \check{\theta}$ obtain positive quantities, which is obvious given that their reservation utility is strictly positive.

Proposition 3 in Jullien (2000) then implies that the solution to problem (5) is

$$q_A(\theta) = \begin{cases} q_c^e(\theta) & \text{for } \check{\theta} \leq \theta \leq \theta^+ \\ q_A^m(\theta) & \text{for } \theta \geq \theta^+, \end{cases}$$

and obviously $q_A(\theta) = 0$ for $\theta \leq \check{\theta}$. ■

Next, we proceed to the characterisation of the optimal switching point, $\hat{\theta}$. To begin with, observe that condition A4 guarantees that the equilibrium rent function $U(\theta)$ is steeper under non-exclusivity than under exclusivity. This implies that the solution to the hybrid optimal control problem involves a unique switch from problem (A.1) (which applies to low-demand types) to problem (4) (which applies to high-demand types).

The next lemma says that the switch must be from exclusive dealing to a common representation equilibrium. In other words, at the switching point the solution to problem (4) is given by the common representation quantities $q_A^{cr}(\theta), q_B^{cr}(\theta) > 0$. This rules out the possibility that the switch occurs for types who obtain the monopoly or limit pricing quantity of product A.

Lemma 4 *When $\theta > \hat{\theta}$, both $q_A(\theta)$ and $q_B(\theta)$ are strictly positive.*

Proof. From condition (7), it is clear that when $v_\theta(q_A^{NE}(\theta), \theta) > u_\theta(q_A^E(\theta), 0, \theta)$ (which is guaranteed by A4) it must be $P_A^{NE}(q_A^{cr}(\hat{\theta})) > P_A^E(q_A^E(\hat{\theta}))$, so the dominant firm extracts more rents, at the margin, from buyers who accept non-exclusive contracts than from those who accept exclusive ones. From this, it follows immediately that that $q_A^{NE}(\hat{\theta}) > 0$ (otherwise, $P_A^{NE}(q_A^{NE}(\hat{\theta}))$ must be nil). The proof that also $q_B^{NE}(\hat{\theta}) > 0$ is equally simple. If the solution to problem (4) entails $q_B(\theta) = 0$, it must be either $\max[q_A^m(\theta), q_c^e(\theta)]$ or $q_A^{\lim}(\theta)$. In the former case, the dominant firm would obtain the same rent from buyers who accept non-exclusive contracts as from those who accept the exclusive one; in the latter, it would actually obtain less. Since we have just shown that it must obtain more, these two cases are not possible. ■

While for $\theta > \hat{\theta}$ we always have the common representation quantities, for $\theta < \hat{\theta}$ we can have either the monopoly quantity $q_A^m(\theta)$ or the quantity $q_c^e(\theta)$. The former case arises when $c > \bar{c}$, the latter when $c \leq \bar{c}$, where the threshold $\bar{c}$ is implicitly defined as the solution to $\hat{\theta}(c) = \theta^+(c)$ and hence satisfies $\bar{c} < c^{\lim}$.

This completes the derivation of the equilibrium quantities in all possible cases. It is then easy to check that these equilibrium quantities are implemented by the price schedules reported in the statement of the Proposition. ■

The uniform-quadratic model. Before proceeding, we provide the explicit derivation of the equilibrium for the competitive fringe model under the uniform-quadratic specification and check that all of our assumptions are satisfied in this case.

Consider the non-linear pricing equilibrium first. The indirect utility function is piecewise quadratic, with two branches corresponding to the cases in which the quantity

$$\tilde{q}_B(q_A, \theta) = \max\left[0, \frac{\theta - c - \gamma q_A}{1 - \gamma}\right]$$

is 0 or is strictly positive, and a kink between the two branches. That is:

$$v(q_A,\theta)=\begin{cases}\theta q_A-\dfrac{1-\gamma}{2}q_A^2 & \text{if } \tilde{q}_B(q_A,\theta)=0 \text{ or, equivalently, } q_A\geq q_A^{\lim}(\theta)\\ A_0+A_1q_A+A_2q_A^2 & \text{if } \tilde{q}_B(q_A,\theta)>0 \text{ or, equivalently, } q_A\leq q_A^{\lim}(\theta),\end{cases}$$

where

$$A_0=\frac{(\theta-c)^2}{2(1-\gamma)},\quad A_1=\frac{c\gamma+\theta(1-2\gamma)}{1-\gamma},\quad \text{and } A_2=-\frac{1-2\gamma}{2(1-\gamma)}.$$

On both branches, the coefficients of the quadratic terms are negative. Furthermore,

$$\begin{aligned}\left.\frac{\partial v(q_A,\theta)}{\partial q_A}\right|_{q_A<q_A^{\lim}(\theta)} &= \frac{c\gamma+\theta(1-2\gamma)}{1-\gamma}-\frac{1-2\gamma}{(1-\gamma)}q_A^{\lim}(\theta)\\ &\geq \left.\frac{\partial v(q_A,\theta)}{\partial q_A}\right|_{q_A>q_A^{\lim}(\theta)}=\theta-(1-\gamma)\,q_A^{\lim}(\theta),\end{aligned}$$

so the function v is globally concave in q_A.

It can also be easily checked that the single-crossing condition $v_{\theta q_A}(q_A,\theta)\geq 0$ is satisfied since:

$$v_{\theta q_A}(q_A,\theta)=\begin{cases}1 & \text{if } q_A\geq q^{\lim}(\theta)\\ \frac{1-2\gamma}{1-\gamma} & \text{if } q_A<q^{\lim}(\theta).\end{cases}$$

Like the indirect utility function, the indirect virtual surplus $s(q_A,\theta)=v(q_A,\theta)-(1-\theta)v_\theta(q_A,\theta)$ is a piecewise quadratic function, with two branches and a kink at $q_A=q^{\lim}(\theta)$. Since the additional term $(1-\theta)v_\theta(q_A,\theta)$ is linear in q_A and $v(q_A,\theta)$ is globally concave, $s(q_A,\theta)$ is also globally concave in q_A.

Generally speaking, for any θ the maximum can occur on either one of the two quadratic branches, or at the kink. It is easy to verify that $q^m(\theta)$, $q^{\lim}(\theta)$ and $q_A^{cr}(\theta)$ are given precisely by the expressions reported in footnote 27:

$$q^m(\theta)=\frac{2\theta-1}{1-\gamma},$$

$$q^{\lim}(\theta)=\frac{\theta-c}{\gamma},$$

$$q_A^{cr}(\theta)=2\theta-1+c\frac{\gamma}{1-2\gamma}.$$

We also have

$$q_B^{cr}(\theta)=\theta\frac{1-2\gamma}{1-\gamma}+\frac{\gamma}{1-\gamma}-c\frac{1-\gamma}{1-2\gamma}.$$

The critical thresholds are $c^m=\frac{1}{2}$, $c^{\lim}=\frac{1-2\gamma}{2-3\gamma}$, $\theta^m=\frac{1}{2}$, $\theta^{\lim}=\frac{c(1-\gamma)-\gamma}{1-3\gamma}$, $\breve{\theta}=c$, $\breve{\theta}_A=\frac{1}{2}+c\frac{\gamma}{2(1-2\gamma)}$ and $\breve{\theta}_B=c\frac{(1-\gamma)^2}{(1-2\gamma)^2}-\frac{\gamma}{1-2\gamma}$.

With exclusive contracts, the utility function in problem (5) is

$$u(q_A, 0, \theta) = \theta q_A - \frac{1-\gamma}{2} q_A^2,$$

and the type-dependent reservation utility is

$$U_A^R(\theta) = \frac{(\theta - c)^2}{2(1-\gamma)}.$$

The virtual surplus function for problem (A.1) is

$$s^E(g, q_A, \theta) = (2\theta - g) q_A - \frac{1-\gamma}{2} q_A^2.$$

Pointwise maximisation yields

$$\ell^E(g, \theta) = \frac{2\theta - g}{1-\gamma}.$$

Straightforward calculations show that Weak Convexity always holds.

To show that condition A4 holds, i.e. that the equilibrium rent function $U(\theta)$ is steeper under non-exclusivity than under exclusivity, it suffices to notice that the slope of $U(\theta)$ is the sum of the equilibrium quantities. It is easy to verify that $q_A^{NE}(\theta) + q_B^{NE}(\theta) \geq q_A^E(\theta)$, with a strict inequality whenever $q_B^{NE}(\theta) > 0$.

The explicit expression for $\bar{c}$ and $\Phi_A > 0$ are complicated and are reported in a Mathematica file which is available from the authors upon request.

Proof of Proposition 3. As usual, we start by reporting the equilibrium quantities, which are

- when $c \leq \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq P_B'^{cr}(0) \\ q^{\lim}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

- when $c > \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q^m(\theta) & \text{for } \frac{1}{2} < \theta \leq \theta^{\lim} \\ q^{\lim}(\theta) & \text{for } \theta^{\lim} < \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta > \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

where

$$P_B'^{cr}(0) = \alpha + c\left[1 - \frac{\alpha(1-\gamma)}{1-2\gamma}\right].$$

Like in Section 4, $\breve{\theta}_B$ is implicitly defined by the condition $q_B^{cr}(\breve{\theta}_B) = 0$ and θ^m by the condition $q^m(\theta^m) = q^{\lim}(\theta^m)$; now, however, the explicit expressions are different as $q_B^{cr}(\breve{\theta}_B)$ and $q^{\lim}(\theta^m)$ in the duopoly model differ from the competitive fringe model. The explicit solutions are

$$\breve{\theta}_B = \alpha + c\frac{(1-\gamma)(1-\alpha)}{1-2\gamma}$$

and

$$\theta^m = \frac{(1-\gamma)}{1-3\gamma} P_B'^{cr}(0) - \frac{\gamma}{1-3\gamma}.$$

To prove the proposition, we must show that the equilibrium price schedules satisfy the best response property. Given its rival's price schedule, a firm is faced with an optimal non-linear pricing problem that can be solved by invoking the Revelation Principle and thus focusing on direct mechanisms. The strategy of the proof is to show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$, given $P_{-i}(q_{-i})$, coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules that support these quantities must be equilibrium price schedules.

Given $P_{-i}(q_{-i})$, firm i faces a monopolistic screening problem where type θ has an indirect utility function

$$v^i(q_i, \theta) = \max_{q_{-i} \geq 0} \left[u(q_i, q_{-i}, \theta) - P_{-i}(q_{-i}) \right],$$

which accounts for any benefit he can obtain by optimally trading with its rival. Since u is quadratic and $P_{-i}(q_{-i})$ is piecewise quadratic, v_i is also piecewise quadratic. It may have kinks, but we shall show that any such kink preserve concavity, so the indirect utility function is globally concave.

Provided that the single-crossing condition holds, firm i's problem reduces to finding a function that pointwise maximises the "indirect virtual surplus"

$$s^i(q_i, \theta) = v^i(q_i, \theta) - c_i q_i - (1-\theta) v_\theta^i,$$

where c_i is zero for $i = A$ and c for $i = B$. It is then easy to verify ex post that the maximiser $q_i(\theta)$ satisfies the monotonicity condition.

Consider, then, firm A's best response to the equilibrium price schedule of firm B, $P_B(q_B)$. The indirect utility function is piecewise quadratic, with two branches corresponding to the case in which $\arg\max_{q_B \geq 0} \left[u(q_A, q_B, \theta) - P_B(q_B) \right]$ is 0 or is strictly positive, and a kink between the two branches:

$$v^A(q_A, \theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2} q_A^2 & \text{if } q_B = 0 \text{ or, equivalently, } q_A \geq q^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_B > 0 \text{ or, equivalently, } q_A < q^{\lim}(\theta). \end{cases}$$

The coefficients A_0, A_1 and A_2 can be calculated as

$$\begin{aligned}
A_0 &= \frac{\left[(\theta - c)(1-2\gamma) - \alpha(1 - c(1-\gamma) - 2\gamma)\right]^2}{2(1-\gamma-\alpha)(1-2\gamma)^2}, \\
A_1 &= \gamma\frac{c(1-2\gamma) + \alpha(1 - c(1-\gamma) - 2\gamma)}{(1-\gamma-\alpha)(1-2\gamma)} + \theta\frac{1-2\gamma-\alpha}{1-\gamma-\alpha} \\
A_2 &= -\frac{1-2\gamma+\alpha(1-\gamma)}{2(1-\gamma-\alpha)} < 0.
\end{aligned}$$

On both branches of the indirect utility function, the coefficients of the quadratic terms are negative. In addition, it can be easily checked that

$$\left.\frac{\partial v^A(q_A,\theta)}{\partial q_A}\right|_{q_A \le q_A^{\lim}(\theta)} \ge \left.\frac{\partial v^A(q_A,\theta)}{\partial q_A}\right|_{q_A > q_A^{\lim}(\theta)},$$

so the function $v^A(q_A,\theta)$ is globally concave in q_A. It can also be checked that the sorting condition $\frac{\partial^2 v^A}{\partial\theta\partial q_A} > 0$ is satisfied as

$$\frac{\partial^2 v^A}{\partial\theta\partial q_A} = \begin{cases} 1 & \text{if } q_A \ge q_A^{\lim}(\theta) \\ \dfrac{1-2\gamma-\alpha}{1-\gamma-\alpha} > 0 & \text{if } q_A < q_A^{\lim}(\theta). \end{cases}$$

We can therefore obtain A's best response by pointwise maximising the virtual surplus function $s^A(q_A,\theta)$. Like the indirect utility function, the virtual surplus function is piecewise quadratic with a kink. The maximum can occur in either one of the two quadratic branches, or at the kink. To be precise:

$$\arg\max_{q_A(\theta)}[\sigma^A(q_A,\theta)] = \begin{cases} \dfrac{2\theta-1}{1-\gamma} & \text{if } \gamma < \frac{1}{3} \text{ and } \frac{1}{2} \le \theta \le \theta^{\lim} \\ \dfrac{\theta - P_B'^{cr}(0)\alpha}{\gamma} & \begin{array}{l}\text{if } \gamma < \frac{1}{3} \text{ and } \theta^{\lim} \le \theta \le \breve{\theta}_B \\ \text{or if } \gamma \ge \frac{1}{3} \text{ and } P_B'^{cr}(0) \le \theta \le \breve{\theta}_B\end{array} \\ \dfrac{\theta-\alpha}{1-\alpha} + \dfrac{c\gamma}{1-2\gamma} & \text{if } \theta \ge \breve{\theta}_B. \end{cases}$$

But these are precisely the monopoly, limit-pricing and common representation quantities defined in the main text. Note also that the case in which $\gamma < \frac{1}{3}$ and $\frac{1}{2} \le \theta \le \theta^m$ cannot arise if $c < \tilde{c}$. In this case, the optimum is never achieved on the upper branch of the indirect utility function; in other words, firm A's best response never involves setting the quantity at the monopoly level. It is therefore apparent that firm A's best response is to offer precisely the equilibrium quantities. This can be achieved by offering the equilibrium price schedules. This verifies that firm A's equilibrium price schedule satisfies the best response property.

Consider now firm B. The procedure is the same as for firm A, but now we must distinguish between two cases, depending on whether A's price schedule comprises the lowest, monopoly branch or not.

Consider first the case in which there is no monopoly branch in A's price schedule. The indirect utility function of a buyer when trading with firm B then is

$$v^B(q_B,\theta) = \begin{cases} \theta q_B - \frac{1-\gamma}{2} q_B^2 & \text{if } q_B \geq q_B^{\lim}(\theta) \\ \hat{B}_0 + \hat{B}_1 q_B + \hat{B}_2 q_B^2 & \text{if } \check{q}_B(\theta) \leq q_B < q_B^{\lim}(\theta) \\ B_0 + B_1 q_B + B_2 q_B^2 & \text{if } 0 < q_B \leq \check{q}_B(\theta) \end{cases}$$

where

$$\begin{aligned} q_B^{\lim}(\theta) &= \frac{\theta-\alpha}{\gamma} - \frac{\alpha c}{1-2\gamma} \\ \check{q}_B(\theta) &= \frac{\theta-\alpha-c(1-\alpha)}{\gamma} + \frac{\alpha c}{1-2\gamma}. \end{aligned}$$

The first branch corresponds to firm B acting as a monopolist. Along the second branch, firm B competes with firm A's limit-pricing price schedule. Clearly, neither case can occur in equilibrium. Finally, the third branch corresponds to the case in which firm A accommodates.

The coefficients of the lower branches of the indirect utility functions are

$$\hat{B}_0 = \frac{(\theta-c)^2}{2\gamma}; \quad \hat{B}_1 = c; \quad \hat{B}_2 = -\frac{1-2\gamma}{2}$$

and

$$B_0 = \frac{2\theta-1}{2(1-\gamma)}; \quad B_1 = \theta - \frac{\gamma}{1-\gamma}; \quad B_2 = -\frac{1-\gamma}{2}.$$

All branches are concave. Global concavity can be checked by comparing the left and right derivatives of $v^B(q_B,\theta)$ at the kinks, as we did for firm A. The sorting condition can also be checked as for firm A. We can therefore find B's best response by pointwise maximisation of the virtual surplus function.

It is easy to verify that there is never an interior maximum on the upper or intermediate branch of the virtual surplus function. This is equivalent to saying that firm B is active only when firm A supplies the common representation quantity $q_A^{cr}(\theta)$. Pointwise maximisation of the relevant branch of virtual surplus function then leads to

$$\arg\max[\sigma^B(q_B,\theta)] = \frac{\theta-\alpha}{1-\alpha} - c\frac{1-\gamma}{1-2\gamma}.$$

This coincides with $q_B^{cr}(\theta)$, thereby confirming that the equilibrium price schedule $P_B(q_B)$ is indeed its best response to firm A's strategy.

The case where firm A's price schedule comprises also the monopoly branch is similar. The indirect utility function $v^B(q_B,\theta)$, and hence the virtual surplus $s^B(q_B,\theta)$, now comprise four branches (all quadratic). The equation of the fourth branch, which corresponds to $0 < q_A < q^m(\theta)$, is

$$v^B(q_B,\theta) = \tilde{B}_0 + \tilde{B}_1 q_B + \tilde{B}_2 q_B^2$$

where

$$\tilde{B}_0 = \frac{(2\theta-1)^2}{4(1-\gamma)}; \quad \tilde{B}_1 = \frac{\theta+\gamma(1-3\gamma)}{1-\gamma}; \quad \tilde{B}_2 = -\frac{1-\gamma(2+\gamma)}{2(1-\gamma)}.$$

However, it turns out that the optimum still lies on the same branch as before and that it therefore entails a quantity equal to $q_B^{cr}(\theta)$. This observation completes the proof of the Proposition. ■

To avoid repetitions, it is now convenient to take up Proposition 5 before Proposition 4.

Proof of Proposition 5. The equilibrium quantities are:

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq \hat{\theta} \\ \bar{q}_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq \bar{\theta} \\ q_A^{cr}(\theta) & \text{for } \bar{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ \bar{q}_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq \bar{\theta} \\ q_B^{cr}(\theta) & \text{for } \bar{\theta} \leq \theta \leq 1, \end{cases}$$

where $\hat{\theta}$ and $\bar{\theta}$, which are defined in the text of the Proposition, are given by

$$\begin{aligned} \hat{\theta} &= \frac{c(2-3\gamma)}{1-2\gamma}, \\ \bar{\theta} &= \frac{c(1-2\gamma)+\alpha[3-c-2(2-c)\gamma]}{\alpha+2(1-2\gamma)}. \end{aligned}$$

The claim that this is the most cooperative equilibrium is justified in the main text. Here, we just verify that this is indeed an equilibrium of the game. The logic of the proof is the same as for Proposition 3. We must show that for each firm the equilibrium price schedules satisfy the best response property. When calculating the best response, we take $\{P_{-i}^E(q), P_{-i}^{NE}(q)\}$ as given and hence can invoke the Revelation Principle and focus on direct mechanisms. We must therefore show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$ coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules $P_i^E(q), P_i^{NE}(q)$ that support these quantities must be equilibrium price schedules.

Given its rival's exclusive and non exclusive price schedules, a firm must solve a monopolistic screening problem in which the buyer has an indirect utility function

$$v^i(q_i,\theta) = \max_{q_{-i}\geq 0} \left[u\left(q_i, q_{-i}, \theta\right) - P_{-i}^{NE}(q_{-i})\right],$$

and a reservation utility

$$U_i^R(\theta) = \max_{q_{-i}} \left[u(0, q_{-i}, \theta) - P_{-i}^E(q_{-i})\right].$$

Since firm i can impose exclusivity clauses, it must solve a hybrid optimal control problem in which the two control systems are

$$\begin{aligned} & \max_{q_i} \int \left[v^i(q_i, \theta) - U(\theta) - c_i q_i \right] d\theta \\ \text{s.t.} \quad \frac{dU}{d\theta} &= v^i_\theta(q_i, \theta) \\ U(\theta) &\geq U_i^R(\theta) \end{aligned} \tag{A.2}$$

if $q_{-i}(\theta) > 0$, and

$$\begin{aligned} & \max_{q_i} \int \left[u(q_i, 0, \theta) - U(\theta) - c_i q_i \right] d\theta \\ \text{s.t.} \quad \frac{dU}{d\theta} &= u_\theta(q_i, 0, \theta) \\ U(\theta) &\geq U_i^R(\theta) \end{aligned} \tag{A.3}$$

if $q_{-i}(\theta) = 0$. In both cases, $q_i(\theta)$ must be non-decreasing.

Problem (A.3) is relevant only for the dominant firm. When it sets $q_B(\theta) = 0$, noting that problem (A.3) coincides with problem (A.1) in the proof of Proposition 2, we can apply Lemma 3 and conclude that

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq 1 - c \\ q_A^m(\theta) & \text{for } \theta \geq 1 - c. \end{cases}$$

It is then easy to verify that $\hat{\theta}$ is now lower than $1 - c$, so the only relevant part of the solution is $q_c^e(\theta)$.

Consider now problem (A.2). Several properties of the solution to this problem must hold for both firms. By construction, the indirect utility functions $v^i(q_i, \theta)$ are almost everywhere differentiable. At any point where the derivatives exist, by the envelope theorem we have

$$v^i_\theta(q_i, \theta) = q_i + \tilde{q}_{-i}(q_i, \theta),$$

where

$$\tilde{q}_{-i}(q_i, \theta) = \arg\max_{q_{-i} > 0} \left[u\left(q_i, q_{-i}, \theta\right) - P_{-i}^{NE}(q_{-i}) \right]$$

Generally speaking, the indirect utility functions $v^i(q_i, \theta)$ have two branches, according to whether $\tilde{q}_{-i}(q_i, \theta) \leq \bar{q}_{-i}(\bar{\theta})$ or $\tilde{q}_{-i}(q_i, \theta) \geq \bar{q}_{-i}(\bar{\theta})$ respectively. When $\tilde{q}_{-i}(q_i, \theta) \leq \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = \bar{P}_{-i}^{cr}(q_{-i})$. When $\tilde{q}_{-i}(q_i, \theta) \geq \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = P_{-i}^{cr}(q_{-i})$ (plus a constant).

The indirect utility functions $v^i(q_i, \theta)$ are continuous, almost everywhere differentiable, and satisfy $v^i_{\theta q_i}(q_i, \theta) > 0$. Continuity and a.e. differentiability follows directly from the definition of $v^i(q_i, \theta)$. To prove the sorting condition, observe that

$$v^i_{\theta q_i}(q_i, \theta) = 1 - \gamma \frac{\partial \tilde{q}_{-i}(q_i, \theta)}{\partial q_i} \geq 0.$$

Consider the two branches of the indirect utility function in turn. When $\tilde{q}_{-i}(q_i,\theta) \leq \bar{q}_{-i}(\bar{\theta})$,

$$v^i_{\theta q_i}(q_i,\theta) = 1 + \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial \theta}(-\gamma)^2 = \frac{3-6\gamma}{3-5\gamma} > 0.$$

When instead $\tilde{q}_{-i}(q_i,\theta) \geq \bar{q}_{-i}(\bar{\theta})$ the sorting condition is immediately verified since

$$v_{\theta q_i}(q_i,\theta) = \frac{1-\alpha-2\gamma}{1-\alpha-\gamma} \geq 0.$$

Now consider problem (A.2). Because of the type-dependent participation constraint, following Jullien (2000) we define the virtual surplus function:

$$\sigma^i(g,q_i,\theta) = v^i(q_i,\theta) - (g-\theta)\, v^i_\theta(q_i,\theta)$$

where the "weight" $g \in [0,1]$ accounts for the possibility that the participation constraint may bind for a whole set of types. Let

$$\ell_i(g,\theta) = \arg\max_{q_i \geq 0} \sigma^i(g,q_i,\theta)$$

be the maximiser of the virtual surplus function. This solution is still in implicit form, as it depends on the value of g, which is still to be determined. This can be done by exploiting Proposition 5.5 of Jullien (2000).

To apply that Proposition, we first prove the following lemma.

Lemma 5 *Problem (A.2) satisfies the conditions of Potential Separation, Homogeneity and Weak Convexity.*

Proof. Potential Separation requires that $\ell_i(g,\theta)$ is non-decreasing in θ. This follows from the fact that the virtual surplus function has increasing differences. To show this, consider each branch of the indirect utility function separately. First, when $\tilde{q}_{-i}(q_i,\theta) \leq \bar{q}_{-i}(\bar{\theta})$ we have

$$\sigma^i_{q_i\theta}(q_i,\theta) = v^i_{q_i\theta}(q_i,\theta) - \left[1 + \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial q_i}\right] \frac{d}{d\theta}(g-\theta).$$

The first term is positive, as we have just shown. The second term is positive because $\frac{d}{d\theta}(g-\theta) < 0$ and

$$1 + \frac{\partial \tilde{q}_{-i}}{\partial q_i} = \frac{1-2\gamma}{3-5\gamma} > 0.$$

Second, when $\tilde{q}_{-i}(q_i,\theta) \geq \bar{q}_{-i}(\bar{\theta})$ the indirect utility function coincides, modulo a constant, with the one arising in the equilibrium with non-linear pricing. In this case, it is immediate to show that $\sigma^i_{q_i\theta}(q_i,\theta) > 0$. This completes the proof that problem (A.2) satisfies the condition of Potential Separation.

Homogeneity requires that $U^R_i(\theta)$ can be implemented by a continuous and non decreasing quantity. This is obvious, since $U^R_i(\theta)$ is implemented by $q^E(\theta)$, where $q^E(\theta)$ is the optimal quantity given the exclusive price schedule $P^E_{-i}(q)$:

$$q^E(\theta) = \begin{cases} q^e_c(\theta) & \text{if } \theta \leq \hat{\theta} \\ \bar{q}^E(\theta) & \text{if } \theta > \hat{\theta}. \end{cases}$$

To prove Weak Convexity, we first show that $\ell_i(0,\theta)+\tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$ for all $\theta \in [0,1]$. By definition,

$$\ell_i(0,\theta) = \arg\max_{q_i} \left[v^i(q_i,\theta) + \theta v^i_\theta(q_i,\theta)\right].$$

Thus, $\ell_i(0,\theta)$ is implicitly defined by the first order condition

$$v^i_{q_i}(q_i,\theta) + \theta v^i_{\theta q_i}(q_i,\theta) = 0.$$

Since $v^i_{\theta q_i}(q_i,\theta) > 0$, this implies that $v^i_{q_i}(q_i,\theta) < 0$, or $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) < 0$. In other words, $\ell_i(0,\theta)$ exceeds the satiation consumption $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) = 0$. The quantity $q^E(\theta)$, on the contrary, is lower than the satiation consumption. It follows that $\ell_i(0,\theta) + \tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$.

In addition, Weak Convexity requires that the curve $q^E(\theta)$ cuts the curve $\ell_i(1,\theta) + \tilde{q}_{-i}(\ell_i(1,\theta),\theta) = q^{cr}_A(\theta) + q^{cr}_B(\theta)$ from above. Noting that $\ell_i(1,\theta) = q^{cr}_i(\theta)$, the fact that $q^E(\theta)$ can only cut the curve $q^{cr}_A(\theta) + q^{cr}_B(\theta)$ from above as

$$\frac{d[q^{cr}_A(\theta) + q^{cr}_B(\theta)]}{d\theta} \geq \frac{dq^E(\theta)}{d\theta},$$

irrespective of whether $q^E(\theta)$ is $q^e_c(\theta)$ or $\bar{q}^E(\theta)$. This finally proves Weak Convexity and hence the lemma. ■

With these preliminary results at hand, let us now consider the dominant firm's problem. The solution when $q_B(\theta) = 0$ has been already characterised. If $q_B(\theta) > 0$, Proposition 5.5 in Jullien (2000) guarantees that generally speaking the solution partitions the set of types into three sets: buyers who are excluded, buyers who obtain their reservation utility $U^R_A(\theta)$, and buyers whose payoff is strictly greater than $U^R_A(\theta)$. Clearly, the first set is always empty: if $q_B(\theta) > 0$, we always have $q_A(\theta) > 0$.

Next consider the second group of buyers. When the participation constraint binds, firm A can guarantee to each low type consumer his reservation utility $U^R_A(\theta)$ in two ways. First, it can offer an exclusive price schedule that just undercuts that of firm B. Alternatively, it can implement *via* non-exclusive prices the quantities that satisfy the condition

$$\bar{q}^{cr}_A(\theta) + \bar{q}^{cr}_B(\theta) = \bar{q}^E(\theta),$$

which by the envelope theorem guarantees that the participation constraint is met as an equality. The maximum payment that firm A can requested for $\bar{q}^{cr}_A(\theta)$ is

$$\bar{P}^{cr}_A\left(\bar{q}^{cr}_A(\theta)\right) = -c\bar{q}^{cr}_A(\theta) + (1-2\gamma)\left[\bar{q}^{cr}_A(\theta)\right]^2 + cq^e_c(\hat{\theta}).$$

The second strategy is at least as profitable as the first one if

$$\bar{P}^{cr}_A\left(\bar{q}^{cr}_A(\theta)\right) \geq \bar{P}^E(\bar{q}^E(\theta)),$$

which is precisely the no-undercutting condition (21) which holds by construction. This shows that offering $\bar{P}^{cr}_A(q_A)$ is indeed a best response for firm A when the participation constraint is binding.

Finally, when the participation constraint does not bind, the solution to firm A's program is obtained simply by setting $g = 1$. Assume that $\ell_A(1, \theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$ (this will be proven shortly). Since the virtual surplus function $\sigma_A(1, q_A, \theta)$ is exactly the same as in the non-linear pricing equilibrium, modulo a constant, the maximisers of the virtual surplus functions must coincide and the optimal quantity is

$$\ell_A(1, \theta) = q_A^{cr}(\theta).$$

Finally, the cutoff $\bar{\theta}$ is implicitly given by the condition

$$\bar{q}^E(\bar{\theta}) = \ell_A(1, \bar{\theta}) + \tilde{q}_B(\ell_A(1, \bar{\theta}), \bar{\theta}).$$

This also establishes that $\ell_A(1, \theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$.

To complete the verification of the best response property for firm A, it remains to consider the switch from exclusive to non-exclusive contracts. By the no-deviation condition (21), which in the most cooperative equilibrium holds as an equality, firm A is just indifferent between imposing an exclusivity clause or not for all $\theta \leq \bar{\theta}$. Exclusive dealing arises just when $\bar{q}_B^{cr}(\theta) \leq 0$, which is equivalent to $\theta \leq \hat{\theta}$. Because firm A is indifferent between the exclusive and non-exclusive regimes, at the switching point a smooth-pasting condition must now hold, which implies that aggregate quantities must be continuous, and hence that $P_A^{NE}(\bar{q}_A^{cr}(\hat{\theta})) = P_A^E(\bar{q}_A^E(\hat{\theta}))$.

The problem faced by firm B is similar, except that firm B can never make a profit by selling under an exclusivity clause. Thus, we can focus on problem (A.2). Proceeding as for firm A, one can show that the optimal quantity is $\bar{q}_B^{cr}(\theta)$ when the participation constraint $U(\theta) \geq U_A^R(\theta)$ is binding, and $q_B^{cr}(\theta)$ when it is not.

These arguments complete the proof that the solution to the problem of firm i coincides with $q_i(\theta)$ as shown in the text of the proposition. By construction, this solution can be implemented by firm i using the equilibrium price schedules $\left(P_i^E(q_i), P_i^{NE}(q_i)\right)$.

This solution is well defined when the three intervals $[c, \hat{\theta})$, $[\hat{\theta}, \bar{\theta}]$ and $(\bar{\theta}, 1]$ are non-empty. This requires $c \leq \hat{\theta}$, $\hat{\theta} \leq \bar{\theta}$ and $\bar{\theta} \leq 1$. It is immediate to show that the first and the last of these inequality always hold. Thus, the solution is well defined if and only is $\hat{\theta} \leq \bar{\theta}$, which is equivalent to

$$c \leq \check{c} \equiv \frac{2(1 - 2\gamma)}{5(1 - \gamma) + \sqrt{1 - 2\gamma + 9\gamma^2}}. \blacksquare$$

We can now finally proceed to the proof of Proposition 4.

Proof of Proposition 4. As usual, we start by reporting the equilibrium quantities, which are:

- when $\check{c} \leq c \leq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq \theta^m \\ q_A^m(\theta) & \text{for } \theta^m \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1, \end{cases}$$

- when $c > c^m$

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^m(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1. \end{cases}$$

The strategy of the proof is the same as for Proposition 2. Many of the arguments are indeed the same as in previous proofs and so need not be repeated here. In particular, notice that:

- first, when $c > \check{c}$, there is no longer any scope for coordinating exclusive prices (this was shown in the proof of Proposition 2). Hence, firm B always sets exclusive prices at the competitive level $P_B^E(q_B) = cq_B$. This implies that when firm A imposes an exclusivity clause, the buyers' reservation utility is exactly the same as in the competitive fringe model. It follows that the solution to problem (5) is still given by Lemma 3;

- second, without exclusivity the problems that are faced by the firms are exactly the same as in the proof of Proposition 2 when the participation constraint does not bind.

These remarks imply that Proposition 4 can be proved simply by combining arguments already presented in the proofs of Proposition 1 and Proposition 2. The only difference is that now the switch from the exclusive to the non-exclusive regime is the result of the interaction between the pricing choices of firm A and firm B. This point, however, has already been discussed in the main text, which shows that the equilibrium switching point must satisfy conditions (15)-(17). The explicit expressions for Φ_A and Φ_B are complicated and are reported in a Mathematica file that is available upon request from the authors.■