Satisficing Contracts

by

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[PRELIMINARY AND INCOMPLETE DRAFT]

Abstract: We analyze a model of optimal contracting between two boundedly rational agents. Our model of bounded rationality is the one proposed in Bolton and Faure-Grimaud (2005) and relies on time-costs of deliberating current and future decisions. We show that optimal contracts may be incomplete and may allocate control rights to one or both parties to give them *the option to defer thinking* and to let them postpone deliberations on less important decisions to the time or event when they arise. We also show that it may be optimal for the parties to begin by writing a *preliminary agreement* before they continue negotiations on a more detailed final contract. Finally, the parties may also make *exploding contract offers* in equilibrium.

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1 Introduction

This paper considers a contracting problem between two boundedly rational agents. The basic situation we model is that of two agents who can team up to form a partnership or a new venture. The contract they write specifies in a more or less complete manner what action-plan they agree to undertake, how future decisions are made and by whom, and how the proceeds from the venture are to be shared.

As Oliver Hart and others have observed, ultimately to understand why contracts are incomplete and what determines the degree of incompleteness of contracts one needs to appeal to the contracting parties' bounded rationality:

"*In reality, a great deal of contractual incompleteness is undoubtedly linked to the inability of parties not only to contract very carefully about the future, but also to think very carefully about the utility consequences of their actions. It would therefore be highly desirable to relax the assumption that parties are unboundedly rational.*" [Hart, 1995, p. 81]

In our companion article (Bolton and Faure-Grimaud, 2005) we lay out a model of bounded rationality, which forms a basic building block of the contracting problem considered here. Our proposed model only allows for a minimal departure from rationality by introducing decision-making costs (or deliberation costs) in an otherwise fully rational framework.[1] Still, with this quasi-rational model of bounded rationality we are able to capture several important facets of incomplete contracts observed in practice.

In particular we are able to formalize the notion that boundedly rational agents write *satisficing contracts* rather than optimal contracts.[2] That is, when each party to a deal expects to receive a satisfactory payoff from the deal then the parties don't waste time writing a detailed contract and instead leave many decisions to be determined later. This is especially

[1] Our model builds on earlier work on decision-making with deliberation costs by Simon (1955) and Conlisk (1980, 1988, 1996) among others and also on the literature on multiarmed bandits by Gittins and Jones (1974), Rothschild (1974), Gittins (1979), Berry and Frystedt (1985) and Whittle (1980, 1982).

[2] We borrow Simon's terminology and notion of *satisficing* for decision problems of boundedly rational agents to describe a contracting problem between such agents (see Simon, 1955, Radner, 1975, and Radner and Rothschild, 1975). Interestingly, although the idea of satisficing has been explored extensively for decision problems it has not, as far as we know, been extended to a contracting problem. *A satisficing contract is an optimal contract when one takes into account the costs of optimizing the contractual relationship.*

the case when the parties have broadly aligned interests. More generally, boundedly rational agents tend to choose to leave out of the contract perfectly foreseeable and describable contingencies if they are sufficiently unlikely or distant, or if they don't affect overall expected payoffs very much. Also, we capture the observation that over time contracts will become more and more detailed.

Our paper provides rationale for the use of "satisficing" contracts, that is contracts that are incomplete in an environment where parties could have potentially written more sophisticated contracts. This allows us to shed light on recent empirical findings in the incomplete contract literature and also to help structure such studies by delivering some comparative static results. For instance, we will see that an important determinant of the degree of contractual incompleteness is how valuable the venture or deal is. Indeed, one of our basic predictions is that the more valuable the venture is overall, and the more aligned are the parties objectives, the more incomplete the contract is likely to be.

The major results from our analysis are, first, that control rights emerge as equilibrium contractual clauses even though a complete contract that specifies ex ante what action to take in every state of the world could be written by the contracting parties. The rationale for *control rights* in our model, defined as the allocation to some party of the right to choose between different actions, is that the holder of these rights benefits by having *the option to defer thinking* about decisions to when these decisions arise. Second, in our model the sharp distinction between contract negotiation and equilibrium contracts usually made in the contract theory literature is no longer warranted. Contracts are completed over time and negotiations about aspects that have been left out of the contract can be ongoing. In particular, an important feature of our model is that the contracting parties may choose to begin negotiations by writing a *preliminary agreement* specifying the broad outlines of a deal and committing the parties to the deal. The parties then continue with a phase of *due diligence* before finally agreeing to a detailed contract. Interestingly also, a party with all the bargaining power may choose to leave rents on the table to the other party, so as to meet its *aspiration level* and thus persuade it to sign on more quickly to a highly incomplete contract.

We also will identify conditions under which the parties instead sign a complete, but possibly *coarse contract.* A coarse contract is one where the parties do not rely on the finest information partition available to save on cognitive costs. In such situations, contracts can also be excessively complete.

These intuitive predictions are generally not consistent with the first generation of incomplete contracting theories following Grossman and Hart (1986) and Hart and Moore (1988), which assume that agents are fully rational, but that some states of nature or trades cannot be written into contracts due to enforcement, verifiability or describability constraints.[3] Indeed, these theories impose contractual incompleteness from outside and do not include considerations that the parties themselves may choose to leave the contract more or less complete.

The second generation of incomplete contracting theories, which includes the contributions of Anderlini and Felli (1994, 2002), Al Najjar, Anderlini and Felli (2002), MacLeod (2000), Battigalli and Maggi (2002), Bajari and Tadelis (2001), and Hart and Moore (2007), comes closer to explaining these characteristics of incomplete contracts.[4] Along with several of these studies, we take the view that a contract is incomplete when it leaves open the choice of an action in a future contingency, and specifies instead a governance procedure for the future choice of an action in this state of nature[5]. In closely related independent work, Tirole (2007) also considers a contracting problem between two boundedly rational agents. Contracts in his set up always specify a given action to be taken, but they are less likely to be renegotiated (more complete) when contracting parties have incurred larger cognitive costs. Similar themes and results emerge from his analysis, such as the endogenous incompleteness of contracts and the possible outcome of excessively complete contracts. However, the basic setup he considers is quite different. Unlike in our framework Tirole focuses on a holdup problem, where the value of thinking comes from the possibility to solve this problem and is always the most preferred course of action of one party and the least one for the other. In our framework, the value of thinking varies with the existence of conflicts, so that parties might disagree over how complete and detailed contracts should be. Like us, Tirole shows that sometimes contracts are incomplete *because* of the existence of some institutional arrangements, in his case relational contracting, a reverse causality argument. This

[3] Also, even in the presence of these enforceability constraints it may be possible for rational agents to write complete contracts and circumvent enforcement constraints by specifying sophisticated revelation schemes into the contract, as Maskin and Tirole (1999) have observed.

[4] See also the earlier theory of Dye (1985).

[5] A satisficing contract is contingent on all the information currently available to contracting parties, and in that sense is always a complete contract. However, a satisficing contract is not contingent on all the information potentially available to contracting parties, and to the extent that more information could be acquired it is also an incomplete contract. Importantly, from the perspective of an outside observer, who is unable to see all the information available to the contracting parties, a satisficing contract will have all the appearance of an incomplete contract when the contracting parties choose to base the contract on less information than is potentially available. Indeed, when they do so the contract will look like action choices in some states of nature have been left unspecified.

is reminiscent of our result that control rights enables incomplete contracts. Our dynamic framework also allows us to derive predictions about how parties will actually negotiate their deal over time, and shows how this might require the use of commonly encountered practices such as preliminary agreements, limited period for carrying out due diligence, and a sequence of contractual offers being made before a deal is struck.

The remainder of our paper is organized as follows. Section 2 presents our model of contracting between two boundedly rational agents. Section 3 characterizes satisficing contracts under the assumption of Non-transferable Utility. Section 4 considers satisficing contracts under the assumption of transferable utility. Section 5 concludes with a summary and directions for future research. Finally, an appendix contains the more involved proofs.

2 The Model

Two infinitely-lived agents can join forces to undertake a new venture at time $t = 0$. The venture requires initial funding of $I_k = I > 0$ from each agent $k = A, B$.[6] If investments are sunk at date $t \geq 0$ then at date $t + 1$ the venture may end up in one of two equally likely states: $\theta \in \{\theta_1, \theta_2\}$. When a state of nature θ_i is realized the two agents face a collective decision of which of two possible actions to take: a *safe* action with known payoff S_k, or a *risky* action with unknown payoff, $R_k \in \{\underline{R}_k, \bar{R}_k\}$. To keep things as simple as possible we shall allow for only two possible realizations of payoff on the risky action: either both agents get a payoff $\underline{R}_k$ or both get a payoff $\bar{R}_k$, and to make the problem non-trivial we assume that:

$$\bar{R} \equiv \bar{R}_A + \bar{R}_B > S \equiv S_A + S_B > \underline{R} \equiv \underline{R}_A + \underline{R}_B.$$

Each agent starts out with some prior belief $\nu_{ik} = \Pr(R_{ik} = \bar{R}_k)$ and can revise this belief by engaging in *thought-experimentation* over time, as in Bolton and Faure-Grimaud (2005). More precisely, each agent can get one draw of the payoff of the risky action per period: with probability λ_k $(k = A, B)$ the agent then finds out the true payoff associated with the risky action and with probability $(1 - \lambda_k)$ the agent learns nothing.

As long as neither agent has found out the payoff of the risky action, either agent can and may want to continue to engage in thought experimentation. As in the decision problem analyzed in Bolton and Faure-Grimaud (2005), in the contracting problem explored here the parties can choose to engage in thought experimentation before signing a contract, or after

[6] There is no loss of generality in assuming that $I_k = I$ given that we are free to choose any value for the investment returns of each party.

signing a contract, and before the state of nature θ is realized, or even after the state of nature is realized. Both parties discount future returns at the same discount factor $\delta \leq 1$. Without loss of generality, we assume that payoffs realized at the end of the period when thinking by either agent is successful are not discounted to the beginning of that period[7].

2.1 Timing of the Game

We shall make the following timing assumptions.

1. Technological Timing

At date 0, the parties can either invest I right away or they can engage in one round of thought experimentation. Investment can only be completed if both parties choose to invest. If only one party invests the project cannot be started. For simplicity we assume that as long as the project has not started investment is not sunk.

Subsequent periods are identical to date 0 until investment takes place. The only difference is that the parties may have been able to update their beliefs about the payoff of the risky action in state θ_i. Once investment has been completed, either state of nature θ_1 or θ_2 is realized one period later. At that point the parties either engage in more thinking, or choose an action. Once an action has been chosen the payoffs associated with that action are realized and the game ends.

2. Timing of the Negotiation Game

For expositional convenience we divide each period into two sub-periods: a first sub-period when a contract (or renegotiation) offer is made and possibly accepted, and a second sub-period as described in the technological timing above.

To keep the analysis of the negotiation game as simple as possible we shall make the extreme assumption that at the beginning of date 0 nature randomly gives one of the two parties (*the proposer*) all the bargaining power and the exclusive right to make all contract offers. In each period until the contract is signed the proposer can choose to wait or to make an offer of a contract to the other party (*the receiver*). If a contract offer is made the receiver can either accept or reject the contract. If the offer is rejected the game moves to the next period and starts over again. If the offer is accepted the contract is signed and the parties move on with the venture.

[7] Otherwise any thinking strategy suffers from the disadvantage of delaying returns by one period even when parties are perfectly rational ($\lambda_k = 1$). The same assumption is made in Bolton and Faure-Grimaud (2005) and we keep it here for consistency with our earlier work.

2.2 Information and Contracts

Given that parties can engage in thought experimentation they may acquire private information about payoffs over time. As is standard in most contracting problems, we shall assume that at date 0 there is no private information and that parties' beliefs ν_{ik} over payoffs of the risky action are common knowledge. However, if the parties engage in thought experimentation, what they learn about their payoffs is private information. Of course, each party can disclose what it has learned to the other party. We shall distinguish between the cases of *hard information*, which can be credibly disclosed, and *soft information*, which is pure cheap talk.

Similarly, we distinguish between two polar contracting environments: one where the parties' utility is perfectly transferable (*the TU case*) and the other where utility is non-transferable (*the NTU case*). In the TU case contracts can specify an action plan $a(\theta, \kappa_t^A, \kappa_t^B, t)$ and shares of profits from the venture $\alpha(\theta, \kappa_t^A, \kappa_t^B, t)$ for agent A (and $(1-\alpha)$ for agent B), where $\kappa_t^A \in \{\underline{R}_k, \overline{R}_k, \varnothing\}$ denotes the payoffs communicated by agent A (and κ_t^B the payoffs of agent B). In the NTU case contracts can only specify an action plan $a(\theta, \kappa_t^A, \kappa_t^B, t)$.

2.3 Assumptions on Payoffs

For simplicity we shall assume that the two parties' ex-ante beliefs are identical: $\nu_{ik} = \nu$ for $i = 1, 2$ and $k = A, B$. We denote by $\rho_k^* \equiv \nu \max\{\bar{R}_k, S_k\} + (1-\nu)\max\{\underline{R}_k, S_k\}$ each party's expected payoff under their preferred ex-post action choice and by $\rho_k \equiv \nu \bar{R}_k + (1-\nu)\underline{R}_k$ the expected payoff of the risky action.

When both parties engage in thought-experimentation in a given period, and share their thoughts, they can uncover the true payoff of the risky decision in state θ_i in a given period of time with probability:

$$1 - (1-\lambda_A)(1-\lambda_B) = \lambda_A + \lambda_B - \lambda_A\lambda_B \equiv \Lambda$$

Suppose that the parties find themselves in state θ_i without knowing the true payoff of the risky action. If the two parties then decide to delay any action choice and to engage in thought-experimentation until they have discovered the true payoff they can expect to get at most:

$$\Lambda\rho_k^* + \Lambda(1-\Lambda)\delta\rho_k^* + \Lambda(1-\Lambda)^2\delta^2\rho_k^* + ... = \widehat{\Lambda}\rho_k^*$$

where $\widehat{\Lambda} = \frac{\Lambda}{1-(1-\Lambda)\delta}$.

We then make the following assumptions on payoffs:

Assumption A1: $\rho_k > S_k$,

Assumption **A1** guarantees that both parties prefer the risky over the safe action given their prior beliefs (before they know the true payoff associated with the risky action). As will become clear below, this is not an essential assumption and our analysis can be extended straightforwardly to situations where prior beliefs are such that the parties prefer the safe over the risky action.

3 Satisficing Contracts under Non-transferable Utility

In this section we consider the polar case of contracting in a situation where utility is not transferable. In other words, in this setting the parties cannot make any monetary transfers. This is obviously an unrealistic assumption. The only purpose of this assumption is to simplify as much as possible the contracting problem and to reduce the characterization of the equilibrium contract negotiation and final contract to a determination of the equilibrium action-plan and allocation of *control rights* to the parties.

As the analysis even in this highly stylized setting is somewhat involved we simplify the setting further by starting from a situation where the two parties already know that they will get the same payoff in state θ_1 and we denote this payoff by $\pi \geq 0$. Thus the only uncertainty here is what the payoff of the risky action is in state θ_2. And the only potential conflict between the two parties is about which action to take in state θ_2 and when to take it.

Thus, to recapitulate, the contract between the two parties must ultimately specify an agreement that each is making an investment contribution of I at some point. Furthermore, the contract either specifies a control allocation, which determines who has the right to choose the action to be taken in state θ_2, or an explicit action choice in state θ_2.

We analyze the case where each party can *credibly disclose* what it has learned and will examine how our findings will change when information exchange is *cheap talk*.

3.1 Optimal Contracting with credible information disclosure

We shall consider in turn two different types of conflict that may arise between the two agents. The first is a disagreement on how much planning to do and on how promptly to act in response to new events. This type of disagreement naturally arises in our setup, but is not present in other models of incomplete contracts. The second conflict, on the other hand, is more standard and concerns disagreements among the two agents on the preferred action-plan. In general the two types of conflicts may be present simultaneously. However, for expositional reasons we shall only analyze situations where they arise in isolation.

3.1.1 Conflicts over cautiousness

We begin our analysis with the special case where the two parties know that they have the same ranking of payoffs in state θ_2 but they are uncertain about whether the risky action yields a higher payoff than the safe action:

Assumption A2:

$$\bar{R}_A > S_A > \underline{R}_A$$

and

$$\bar{R}_B > S_B > \underline{R}_B.$$

In this situation the only possible remaining conflict between the two parties is about *caution*, or in other words, about how much time to spend thinking on which decision to take in state θ_2. We shall show that the equilibrium outcome of the contracting game between the two parties under these circumstances may be for the more *impatient* party to relinquish control to the more *patient* party, as a way of accelerating the implementation of the project. The more patient party may agree to an earlier implementation of the project if she has control, as control gives her *the option to defer thinking* even after she has signed the contract and agreed to implement the project.

Two 'unbounded rationality' benchmarks We start by characterizing the optimal contract between unboundely rational agents. We define the parties to be *unboundedly rational* if: *i)* they begin the contracting game having already thought through all the consequences of their actions, so that either $\nu = 1$ or $\nu = 0$ at date 0; or, *ii)* the parties are uncertain as to the consequences of their actions, but they also are unable to reduce this uncertainty by thinking further about the contracting problem: $\lambda_k = 0$, $k = A, B$. Both polar cases can be interpreted as models of contracting between fully rational agents.

When either $\nu = 1$ or $\nu = 0$, the two parties play the contracting game by signing a contract requiring immediate investment and specifying the full action-plan at date 1. If $\nu = 1$ the contract specifies the risky action in state θ_2, and if $\nu = 0$ it specifies the safe action in state θ_2. The parties' respective payoffs are then given by:

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\bar{R}_k$$

when the risky action is optimal, and

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2} S_k$$

when the safe action is optimal.

To see that this is an equilibrium outcome of the contracting game note that when the proposer at date 0 offers a contract requiring immediate investment and specifying the full action-plan, the strict best response of the receiver is to accept this contract. And since such a contract maximizes the proposer's payoff this is an equilibrium offer.

Finally, when $\lambda_k = 0$, the parties sign a contract at date 0 agreeing to invest immediately and to take the risky action in state θ_2, since $\rho_k > S_k$ under assumption **A1**.

Importantly, under both polar cases there is no (strict) role for control rights and the entire action-plan is fully determined in the initial contract[8]. This is not surprising given that the two rational parties can write fully enforceable complete contracts[9].

In contrast, as we shall show below, when *boundedly rational agents* can write complete, state-contingent, fully enforceable contracts under symmetric information they may *choose to optimally specify an incomplete contract*, which leaves open the choice of the decision to implement in some states of nature and gives *de facto* control to one (or both) parties.

Contracting between boundedly rational agents and the role of control rights To establish that the contract is optimally incomplete we first need to specify the payoffs the parties can expect to get under the two possible thinking strategies that are open to them: *a)* invest right away and *think-on-the-spot* if state θ_2 arises, or *b) think ahead* about the optimal decision in state θ_2 before investing.

Thinking on the spot Under the first strategy the parties invest at date 0, and the state of nature is realized one period later. If state θ_1 is realized they each get a payoff of π, and if state θ_2 is realized they may start thinking about which of the *safe* or the *risky* action to take. If both parties think in parallel about the optimal action and share their thoughts they can expect to get each $\widehat{\Lambda}\rho_k^*$ in state θ_2 and, therefore, an expected ex-ante payoff of

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\widehat{\Lambda}\rho_k^* \tag{1}$$

[8] In our special setting, a contract giving full control to the proposer may also be an equilibrium contract. However, this contract can never be strictly preferred to the optimal complete contract.

[9] As is well known, when rational agents can write complete, state-contingent, fully enforceable contracts under symmetric information there is no role for control (see, e.g. Hart 1995, or Bolton and Dewatripont, 2005).

under the thinking on the spot strategy.

Thinking ahead Under this second strategy the parties only agree to invest once they have determined the optimal action in state θ_2. If both parties think in parallel about the optimal action, share their thoughts, and sign a contract agreeing to invest as soon as they have determined the optimal course of action they each can expect to get

$$\widehat{\Lambda}\left(-I+\frac{\delta\pi}{2}+\frac{\delta\rho_k^*}{2}\right) \tag{2}$$

under this strategy.

Control rights and the option to defer thinking Having determined the payoffs under the two thinking strategies we now turn to an analysis of the contracting game and to the characterization of equilibrium contracts. Recall that the formal contracting game starts with nature drawing one of the parties as the *proposer*. We shall take it that party A is selected to be the proposer of a contract at date 0. Agent B then responds by either accepting or rejecting the offer. If B accepts the offer, the continuation game is dictated by the terms of the contract. If B rejects the contract, then each party unilaterally decides whether to engage in one round of thought experimentation and communication before moving on to the next period. In the next period again A makes a new offer, and so on, until an offer is accepted by B.

The set of relevant contract offers for A in our simplified situation can be reduced to essentially six contracts, $\mathbb{C} = \{C_r, C_s, C_\sigma, C_\alpha, C_A, C_B\}$, and any probability distribution over $\mathbb{C}$, where:

1. C_r requires immediate investment and immediate choice of the risky action r in state θ_2;

2. C_s requires immediate investment and immediate choice of the safe action s in state θ_2;

3. C_σ requires immediate investment, followed by *thinking on the spot* in state θ_2 and unanimous agreement to select an action;

4. under C_α the parties agree to first determine an optimal action in state θ_2 by *thinking ahead* and to invest only once they have selected (possibly randomly) a new contract $C \in \{C_r, C_s\}$. We refer to this contract as a *preliminary agreement*;

5. C_A allocates all control rights to agent A. The controlling party can decide which action to take in state θ_2 at any time she wants;

6. C_B is identical to C_A except that it allocates all control rights to agent B.

Having laid out the formal description of the contracting game we now turn to the analysis of the subgame perfect equilibria of this game. Observe that the difference in payoffs (1) and (2) between the two thinking strategies only depends on the sign of the term $\left(-I + \frac{\delta\pi}{2}\right)$.

We begin by considering the case where assumption **A3** below holds:

Assumption A3: $I < \frac{\delta\pi}{2}$.

Under this assumption it is straightforward to see that the two contracting parties would not want to choose the *thinking ahead* strategy, as the opportunity cost in terms of delayed investment would then be too high for both agents:

Lemma 1: *Under assumption **A3** the contract C_α, which involves thinking ahead, is strictly dominated by one of the other contracts in $\mathbb{C}$ for both agents.*

Proof: Under contract C_α each agent's payoff is at most

$$\widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_k^*}{2}\right),$$

while under at least one of the contracts C_σ, C_A, C_B each agent's payoff is

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\widehat{\Lambda}\rho_k^*.$$

Now under assumption **A3** we have:

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\widehat{\Lambda}\rho_k^* > \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_k^*}{2}\right)$$

or, rearranging,

$$\left(-I + \frac{\delta\pi}{2}\right)(1 - \widehat{\Lambda}) > 0 \quad \blacksquare$$

Intuitively, when assumption **A3** holds the venture is so profitable that the parties will agree to a contract involving immediate investment so as to bring forward the time when they realize the returns from their investment.

Although the parties may agree to invest right away under assumption **A3,** and although they share the same ranking of underlying payoffs over the safe and risky action under assumption **A2**, they may still disagree about how much time to spend thinking about the optimal action. That is, one party may be more impatient than the other and may therefore prefer to act more quickly following the realization of the state of nature θ_2. This is the case, for example if the following assumption also holds:

Assumption A4: $\widehat{\Lambda}\rho_A^* < \rho_A$ and $\widehat{\Lambda}\rho_B^* \geq \rho_B$.

Indeed, under this assumption A is impatient to take the risky action in state θ_2 immediately, while B prefers a more cautious approach and to think before acting.

When the nature of the conflict between the two parties is about how cautiously to act – a form of conflict commonly encountered in reality – they may resolve this conflict optimally by allocating control rights to the more cautious party, as we now show.

Proposition 1 *Under assumptions **A1** to **A4**, the proposer (agent A) is the more impatient party. When condition*

$$-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B}{2} < \widehat{\Lambda}\left(-I + \frac{\delta}{2}(\pi + \rho_B^*)\right) \tag{3}$$

holds, the optimal contract is to invest immediately and to allocate control to agent B with probability y^* and to commit to choose the risky action in state θ_2 with probability $(1 - y^*)$, where y^* is given by:

$$-I + \frac{\delta\pi}{2} + y^*\widehat{\Lambda}\left(\frac{\delta\rho_B^*}{2}\right) + (1 - y^*)\frac{\delta\rho_B}{2} = \widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^*}{2}\right]$$

When the reverse condition holds, agent A optimally retains full control, invests immediately and always chooses the risky action in state θ_2.

Proof: See the Appendix.■

Corollary 1: *If the more patient party (agent B) is the proposer then this party optimally retains full control.*

Proposition 1 is one of the main results of this paper. It establishes first that in a world where complete contracts are fully enforceable, boundedly rational agents optimally choose to write incomplete contracts which allocate control rights to the parties over future decisions left unspecified in the contract. As in other models of incomplete contracts, Proposition 1 also establishes that the way control is allocated is in part driven by the parties relative bargaining strengths. Thus, the proposer tends to appropriate more control other things equal. But, remarkably, Proposition 1 also establishes that an impatient proposer (who has full bargaining power) may optimally choose to give some control rights to the other (more patient) party, as a way of accelerating the closure of contract negotiations.

As is shown in the proof of Proposition 1, when condition (3) holds it is credible for agent B to reject all contract offers C_r, C_s and C_A and to keep thinking ahead while negotiations are ongoing, until the moment when agent B has been able to determine what the best action is in state θ_2. But agent A (the proposer) would prefer to get agent B to agree to invest right away. He can only convince agent B to sign on if agent B has the guarantee not to be forced into a hasty decision in state θ_2. Therefore he grants agent B some control rights by offering either contracts C_B or C_σ.

In other words, the *impatient* proposer prefers to give up some control, to prevent the *patient* agent from spending too much time fine tuning the details of the deal. By giving up control the proposer gives the more patient agent the option to fine tune details later if needed, and thus avoids paying too high an opportunity cost in delaying investment.[10]

Interestingly, this transfer of control may come at the cost of an efficiency loss in the exercise of control by the patient party. Indeed, consider what happens when the patient party ends up in control. Party B will only take an action once she knows the payoffs of the risky action. Of course, if she herself uncovers her true payoffs, she will stop thinking and choose the best option. However, when party A uncovers the true payoffs first, and they happen to favor the risky choice, there is potentially a credibility issue if A just sends a message to B telling her to choose r : if B follows A's recommendation at face value, A will have an incentive to claim that B should choose r even when A is in fact uninformed. Indeed, A is impatient and wants the risky action to be chosen even before its true payoffs have been found. Here, the assumption that the parties can communicate hard information is important to confer credibility to A's message. Party B is happy to follow A's recommendation when the safe action is recommended, otherwise she will ask for hard evidence.

One major potential complication in solving our negotiation game is that following a rejection of the initial contract offer, and after at least one round of thinking by both parties the continuation game is a game of asymmetric information, as neither party knows for sure what the other has learned. Still, under the assumption that the parties can communicate hard information this game is relatively straightforward to solve. Indeed, as we establish in the proof of proposition 1, a major simplification is that both parties have an incentive to disclose immediately anything they learn to the other party in any subgame. Therefore, the continuation game even after a deviation remains effectively a game of complete information.

While the greater impatience of the proposer (party A) can give rise to equilibrium

[10] In situations where the transfer of control rights may not be legally enforceable, party B can still impose her preferred action. The optimal contract would then specify a complete action-plan ex ante. In other words, equilibrium incomplete contracts emerge precisely when the legal environment permits the enforcement of control rights.

incomplete contracts (where party B gets some control rights) there may also be other outcomes where instead the equilibrium contract is *excessively complete*. Indeed, we describe a contracting situation below where party A is keen to close a deal immediately, party B prefers to think ahead, and where equilibrium play is such that party A completely caves in on party B's demands and ends up writing an *excessively complete contract*. This situation differs from the one we have considered in two ways:

1. assumption **A3** does not hold and is replaced by **Assumption AIII:** $I > \frac{\delta\pi}{2}$, and
2. assumption **A4** is strengthened to

   Assumption A4$^+$: $\widehat{\Lambda}\rho_A^* < \rho_A - \frac{2}{\delta}\left(I - \frac{\delta\pi}{2}\right)(1-\widehat{\Lambda})$ and $\widehat{\Lambda}\rho_B^* \geq \rho_B$.

Under assumption **AIII** there is a lower opportunity cost in thinking ahead and delaying investment. At the same time, under assumption **A4$^+$** party A is even keener to get the deal completed immediately. However, although party A has the exclusive right to make contract proposals, the bargaining power is effectively with party B, who can credibly reject all offers until after all information on action r's payoff has been uncovered. In this contracting situation the most preferred outcome for party A is to invest right away and choose action r without thinking, while the worst outcome is to invest immediately and think on the spot. As for party B, she prefers to think ahead to any other alternative. Remarkably, as the proposition below establishes, party B in this situation fully gets her way even though she has no power to propose contracts.

Proposition 2: *Under assumptions A1, A2, AIII and A4$^+$ the proposer has no choice but to follow party B's preferred path of action, which involves agreeing on a contract and investing only after all information regarding action r's payoff in state θ_2 has been uncovered. When this is the case the equilibrium contract is either C_r when $R_k = \overline{R}_k$ or C_s when $R_k = \underline{R}_k$.*

Proof: See the Appendix.■

Interestingly, the equilibrium outcome of the contracting game may be inefficient here. That is, the parties' joint welfare may be lower than under A's most preferred course of action. The reason is that B does not internalize A's opportunity cost of delayed investment and A is unable to compensate B to get her to accept A's preferred action.

3.1.2 Conflicts over action choice

We now turn to the analysis of the second, more familiar, type of conflict where the parties may disagree on the preferred choice of action in state θ_2. More precisely, we shall consider the situation characterized by assumption **A5** below:

Assumption A5:

a. $\overline{R}_A < S_A < \underline{R}_A$ and $\overline{R}_B > S_B > \underline{R}_B$

b. $\widehat{\Lambda}\rho_k^* \geq \rho_{_k}$ for $k \in \{A, B\}$

c. $I > \frac{\delta\pi}{2}$

Under assumption **A5.a** the two parties have opposite preferences on the best course of action in state θ_2. When $R_k = \overline{R}_k$, agent A prefers the safe action and agent B the risky action, and when $R_k = \underline{R}_k$, agent A wants to take the risky action, while B wants the safe action. This is, admittedly, an extreme situation of conflicting objectives, but it captures all the contracting issues that could arise when the parties anticipate that they may have conflicting objectives in the future.

Although they may disagree on the best action choice, the parties may still agree on the extent to which they want to engage in planning or thinking before acting.

Under assumption **A5.b** both parties prefer to think before acting in state θ_2 provided that they can implement their most preferred action once they have discovered which action is best for them. Also when both **A5.a** and **A5.b** hold, it is not always in the interest of a non-controlling party to always disclose what she has learned.

For example, if agent B has the right to choose the action in state θ_2 then if agent A learns that $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$ she may prefer not to disclose this information to B[11]. Therefore, when the parties have conflicting objectives, communication of information between the parties will be reduced and, consequently, the value of planning itself is reduced. This observation will lead to our second major result, which we establish below, that optimal contracts in situations where parties have conflicting objectives are more likely to take the form of complete but *coarse* contracts. In other words, optimal contracts leave no decisions

[11] *As we explain below, the incentive to suppress information can be present before investment is sunk, as a way of delaying a negative NPV outcome to which one of the parties is committed through a preliminary agreement or when a decision might be chosen in absence of hard information. Otherwise, each party has an incentive to bring forward the time when returns are realized, and therefore would not suppress any information.*

open for the future and determine a complete, but coarse, action-plan. We define a contract to be coarse when it prescribes the same action in state θ_2 irrespectively of whether $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$. Moreover, coarse action plans tend to prescribe *compromise actions*, which are *satisfactory* for both parties.

Under assumption **A5.c** there is no opportunity cost in thinking ahead, and the proposer may strictly prefer to think ahead. In contrast, when $I < \frac{\delta\pi}{2}$ both parties may prefer to invest as soon as possible without engageing in any detailed planning. In particular, as we have already observed in Lemma 1, when the parties have congruent objectives over action choice then any form of ex-ante planning may be dominated for both parties.

We note here that this remains true even when the two parties have conflicting ex-post objectives over actions. Indeed, when $I < \frac{\delta\pi}{2}$ and assumptions **A5.a** and **A5.b** hold it is easy to verify that the unique equilibrium contract is given by contract C_A, which requires immediate investment and gives full control to the proposer. For future reference we record this result in the lemma below:

Lemma 3: *When $I < \frac{\delta\pi}{2}$ then under assumptions **A5.a** and **A5.b** the unique equilibrium contract is contract C_A, which requires immediate investment and gives full control to the proposer.*

Proof: See the Appendix.■

Under assumption **A5.c** contract C_A may violate party B's participation constraint. In first generation models of incomplete contracting with limited or no transferable utility (e.g. Aghion and Bolton, 1992), when party B's participation constraint is violated under full control to party A, the optimal contract takes the form of shared control (or contingent allocation of control) to secure party B's participation. In contrast, as we show below, in our setting of complete contracting between boundedly rational agents, the optimal contract securing participation of agent B may not involve any transfer of control rights to B, but rather *explicitly restricts party A's discretion* by spelling out a more detailed action-plan in the initial contract. In some situations the optimal contract even eliminates any role for control and specifies a complete action plan.

Contracting between boundedly rational agents and the optimality of complete contracts We begin by characterizing a situation where one equilibrium outcome of the contracting game is for agents A and B to *think ahead* and to agree on a complete, state-contingent, resolution of their conflicts before investing in the venture. Although, the drafting of the complete contract may involve substantial time-costs it is still preferable for the proposer to incur these ex-ante contracting costs than to hand over substantial control benefits to the other party.

To see this, suppose that agent B's payoff satisfies the following condition:

Assumption A6:

$$-I + \frac{\delta\pi}{2} + \frac{\delta \underline{R}_B}{2} \geq 0$$

Under this assumption, B's eventual participation in the venture is always assured even if party A gets to choose her preferred action in state θ_2. Furthermore, assuming that the two parties credibly disclose what they have learned, agent A strictly prefers for both parties to think ahead under assumption **A5.c** than think on the spot in state θ_2, since then

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\widehat{\Lambda}\rho_A^* < \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_A^*}{2}\right).$$

The proposition below establishes that under these circumstances there exists an equilibrium of the contracting game where both agents think ahead and agent B credibly discloses what she has learned.

Proposition 3: *Under assumptions **A1**, **A5** and **A6**, the unique subgame-perfect equilibrium of the contracting game is such that:*

i) *the proposer delays his contract offer to the time when the parties have discovered whether* $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$;

ii) the receiver thinks ahead and credibly discloses what she has learned;

iii) once agent A has identified his preferred action in state θ_2 he offers a complete state-contingent contract to agent B, requiring immediate investment and implementation of A's preferred action-plan.

Proof: Suppose first that B follows a strategy of *thinking ahead* and of disclosing her findings to A, until the time when she gets a contract offer from A. Then clearly, agent A's best response is to also think ahead and to offer his preferred complete state-contingent contract to B as soon as A's preferred action has been identified. If A were to deviate from this strategy and to offer any other contract $C_r, C_s, C_\sigma, C_A, C_B$ it is straightforward to check that A would get a lower payoff than

$$V_A = \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_A^*}{2}\right)$$

under assumptions **A1** and **A5.**

Similarly, if A follows the equilibrium strategy then B's best response is to also think ahead and truthfully disclose what she has learned. The reason is simply that by pursuing this strategy B brings forward the time when investment takes place. Agent B can then hope to get a payoff

$$V_B = \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^-}{2}\right),$$

where

$$\rho_B^- = \nu S_B + (1-\nu)\underline{R}_B.$$

Any other strategy open to B, such as withholding information, or not engaging in any thinking, would only delay the implementation of the project and would lower B's payoff. Finally, under assumption **A6** agent B is always (weakly) better off accepting A's offer of A's preferred action plan. This establishes existence of the equilibrium.

The fact that no other equilibrium exists follows from the observation that no other strategy profile provides a higher payoff to agent A. Indeed, A might only make another offer if B could credibly threaten to conceal her information in some subgame. But, as we have shown, when responding to the proposer's delaying tactics, B cannot credibly threaten not to think ahead, nor to hide what she has learned. Moreover, in subgames where A chooses an action without knowing the risky action's true payoff, agent B still truthfully discloses her information. Indeed, if A is expected to choose the risky action, then B still prefers to disclose immediately that $R_k = \underline{R}_k$ since otherwise B would only get the same payoff one period later and $\underline{R}_k > 0$. And if A is expected to choose the safe action, B would of course prefer to immediately disclose that $R_k = \overline{R}_k$.

Now let $\widetilde{\Lambda}$ denote the rate at which agent A learns the risky action's payoff. Given that B (sometimes) truthfully discloses what she learns, we have that $\widehat{\lambda}_A < \widetilde{\Lambda} \leq \widehat{\Lambda}$. This inequality together with assumption **A5.b** guarantees that A never wants to choose an action in any subgame without knowing the risky action's true payoff, and knowing this agent B cannot credibly hide any information. ■

In this situation the equilibrium outcome is a complete state-contingent contract, since to begin with the venture does not appear to be that profitable ($I > \frac{\delta\pi}{2}$), and since the party with all the bargaining power is better off working out a complete plan before investing. Moreover, the other party, while benefiting less from working out a full plan cannot credibly threaten to walk away if the deal is not completed before a given deadline. It also cannot credibly threaten not to think about the project while negotiations are ongoing, or not to disclose what it has learned, since it also benefits from implementing the investment sooner rather than later.

The benefits from *thinking ahead* for the proposer, however, would be significantly reduced if the receiver prefers to walk away from the deal in a worst case scenario. This would be the case if the following condition holds for agent B's payoffs:

Assumption A7:

$$-I + \frac{\delta\pi}{2} + \frac{\delta \underline{R}_B}{2} < 0 < -I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}$$

As we show in the next section, in this situation the proposer cannot take full advantage of what he learns ahead of investing and therefore may strictly prefer to offer a complete but coarse contract, requiring immediate investment and binding the contracting parties to a *compromise* action in state θ_2.

The optimality of *preliminary agreements* and complete but *coarse* contracts When assumption **A7** holds agent A cannot get B to agree to choose action r in state θ_2 when both parties learn that the payoffs associated with that action are $\{\underline{R}_A; \underline{R}_B\}$. At that point the best contract for agent A that meets B's participation constraint, is to implement the $safe$ action in state θ_2 with at least probability x^*, so that:

$$-I + \frac{\delta\pi}{2} + \frac{\delta\left(x^* S_B + (1-x^*)\underline{R}_B\right)}{2} = 0,$$

or

$$x^* = \frac{2I - \delta(\pi + \underline{R}_B)}{\delta(S_B - \underline{R}_B)}.$$

The outcome of *thinking ahead* may then be an agreement on a *compromise action* for agent A. As a result the value of *thinking ahead* for A is reduced and she may then prefer to immediately agree to the complete but coarse contract C_r with agent B, which requires immediate investment and immediate choice of the risky action r in state θ_2. Under this contract the two agents' expected payoffs are $\left(-I + \frac{\delta\pi}{2} + \frac{\delta\rho_k}{2}\right)$, and as long as:

$$-I + \frac{\delta\pi}{2} + \frac{\delta\rho_A}{2} >$$

$$\widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\left(\nu S_A + (1-\nu)((1-x^*)S_A + x^*\underline{R}_A)\right)\right), \qquad (4)$$

and

$$-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B}{2} > \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\nu S_B\right) \qquad (5)$$

both agents prefer to immediately sign the coarse contract C_r than to think ahead and only agree to a contract once they have determined the payoffs of the risky action in state θ_2.

Although *thinking ahead* is then dominated, immediate agreement on the complete but coarse contract C_r is not necessarily an equilibrium outcome of the contracting game when conditions (4) and (5) hold. Indeed, another possible play of the contracting game, which may dominate, is for the two agents to begin the game by signing a *preliminary agreement*, continue by *thinking ahead*, and only commit to a complete action-plan once they have learned the payoffs of the risky action in state θ_2.

The purpose of the preliminary agreement is to secure agent B's participation *ex ante*, and thus to relax the ex-post participation constraint,

$$-I + \frac{\delta\pi}{2} + \frac{\delta\left(xS_B + (1-x)\underline{R}_B\right)}{2} \geq 0.$$

Provided the *preliminary agreement* is properly structured, it can then raise agent A's value from *thinking ahead* while still guaranteeing the participation of agent B. However, to be acceptable to agent B the *preliminary agreement* must guarantee that B gets a sufficiently high expected payoff in state θ_2 even though agent A gets to choose the risky action more often when it is her preferred action. The following type of *preliminary agreement* achieves this outcome.

Preliminary agreement:

1. *Commitment to invest:* Parties are committed to invest at some point in the future and agree on a final contract when they invest;

2. *Minimum guarantee:* the risky action in state θ_2 must be chosen with probability α greater than or equal to the threshold $\alpha_{t^*} > 0$;

3. *Limited negotiation phase*: party A makes the final contract offer no later than some pre-specified deadline t^*.

With this type of agreement, party A ends up offering a final contract committing to the risky action in state θ_2 when parties have learned that the payoffs are $\{\underline{R}_A, \underline{R}_B\}$. When instead, they learn that payoffs are $\{\overline{R}_A, \overline{R}_B\}$, party A would prefer to choose the safe decision, but the final contract must specify a minimum probability α_{t^*} of choosing the risky decision in state θ_2. Finally, the preliminary agreement can also specify that investment take place before some deadline t^*. Our discussion below explains the role of such a deadline.

This preliminary agreement, offered before parties have acquired any knowledge about what they prefer to do in state θ_2, allows the two agents to effectively trade payoffs across states of nature and thus achieve a higher ex-ante expected payoff, as with a standard insurance contract. Although they are both risk neutral there are gains from such an agreement by letting the parties trade commitments to choosing the risky action in situations when it is not their most preferred action. In this way the parties can make ex-post non-transferable utilities partially tradeable ex ante.

The role of a preliminary agreement is, thus, to overcome a form of *Hirshleifer effect*, where new information acquisition eliminates insurance or trading opportunities and thus results in a decline in ex-ante utility. Here, as the parties' information changes over time, so does the nature of the conflicts that oppose them. Absent a preliminary agreement, party B will be unwilling to invest when it expects to get $\underline{R}_B$ in state θ_2. Under the veil-of-ignorance concerning parties' true payoffs, they are able to find room for agreements, while once the information is revealed they are not.

There is, however, an important incentive constraint that limits the gains from trade the parties could obtain with such an agreement. Under the preliminary agreement, agent B ends up making a loss in the event that returns on the risky action are $\{\underline{R}_A; \underline{R}_B\}$, for then she gets a payoff $\left(-I + \frac{\delta\pi}{2} + \frac{\delta \underline{R}_B}{2}\right) < 0$ under assumption **A7**. Therefore, agent B has a strict incentive to try and postpone this outcome as much as possible.

Agent B can to some extent delay this negative outcome by withholding that she has learned $\{\underline{R}_A; \underline{R}_B\}$. In that case, a best response for agent B is to pretend that he has learned nothing. Because of agent B's incentive to suppress bad information in this situation, *thinking ahead* by the two agents collectively is slowed down and therefore less beneficial.

Interestingly, the two agents can mitigate this incentive problem by introducing a deadline for investment, t^*. Indeed, we show in the appendix that the negotiation phase cannot exceed some finite length of time. The reason is that as time passes and no evidence of the payoff of the risky decision is produced it becomes more and more likely to party A that B is withholding some information. Party A therefore becomes more convinced over time that the best course of action for her is to choose the risky decision in state θ_2. The value of thinking further for A is then reduced and at some stopping time t^* she will find it optimal to decide to invest. Thus to mitigate the incentive problem of withholding bad information the preliminary agreement could give party A the right to call an end to negotiations after some date t^*, or equivalently, it could specify that absent hard information on r's payoff, parties will have to invest by some date t^*.[12]

With these general observations in mind, we now turn to a more formal characterization of situations in which either of these two contracts will be equilibrium outcomes. In addition to assumption **A7,** we shall impose two other conditions on the two agents payoffs:

Assumption A8:

$$\frac{\underline{R}_A - S_A}{S_A - \overline{R}_A} < \frac{S_B - \underline{R}_B}{\overline{R}_B - S_B}$$

and:

Assumption A9:

a.

$$-I + \frac{\delta\pi}{2} + \frac{\delta\widehat{\Lambda}\rho_B^-}{2} < \widehat{\Lambda}\nu\left(-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}\right)$$

b.

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\max\{\widehat{\Lambda}\rho_A^*; \rho_A\} < \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\left(x^*\rho_A^* + (1 - x^*)S_A\right)\right),$$

As we have pointed out above a preliminary agreement on commitments to possible action choices in state θ_2 creates value added by effectively letting the parties *trade* ex-post non-transferable utilities. Under assumption **A8** the terms of trade favor agent A and this will affect the optimality of a preliminary agreement.

Assumption **A9.a** tells us that the receiver's expected payoff,

$$-I + \frac{\delta\pi}{2} + \frac{\delta\widehat{\Lambda}\rho_B^-}{2},$$

under the contract C_A[13] is lower than her expected payoff,

$$\widehat{\Lambda}\nu\left(-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}\right),$$

under the outcome of the contracting game where B rejects any contract offer until when the parties discover the true return on the risky action, and end up agreeing on the safe action choice when $R_k = \overline{R}_k$, and on the risky action with probability x^* and the safe action with probability $(1 - x^*)$ when $R_k = \underline{R}_k$.[14]

[12] An interesting issue with respect to the determination of the optimal deadline is whether the parties face a time-consistency problem. It is possible that the ex-ante optimal deadline is different from ex-post optimal stopping time t^*, since the specifycation of an ex-ante "binding" deadline that is tighter than t^* could assure B's participation at better terms (i.e. for a higher α). But note that a binding deadline is strictly less than t^*, only if B's expected utility is decreasing in t^*. The Appendix shows that this might be the case in some parameter region.

[13] Recall that $\rho_B^- = \nu S_B + (1 - \nu)\underline{R}_B$.

[14] Recall that under assumption **A5** the safe action is preferred by A and is acceptable to agent B when $R_k = \overline{R}_k$. And when $R_k = \underline{R}_k$ agent B is only willing to agree to the venture, in the absence of a preliminary agreement, if the safe action is chosen with a probability of at least $(1 - x^*)$.

Similarly for the proposer, under assumption **A9.b,** the outcome of the contracting game where A delays his contract offer to the time when the parties have discovered whether $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$ and end up agreeing on the above contract, is preferred to the outcome where the parties invest right away and he gets to pick his preferred action after thinking on the spot in state θ_2. The strategy of offering C_r is also dominated under **A9.b.** Thus, under assumption **A9** contract C_A or C_r cannot be an equilibrium outcome.

When the two agents' payoffs satisfy these assumptions the following result obtains.

Proposition 4: *Under assumptions **A1**, **A5**, **A7**, **A8** and **A9** the unique subgame-perfect equilibrium of the contracting game is such that:*

i) *the proposer delays his contract offer to the time when the parties have discovered whether* $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$;

ii) the receiver thinks ahead and credibly discloses what she has learned;

iii) once agent A has identified his preferred action in state θ_2 he offers the contract C_s when $R_k = \overline{R}_k$, and when $R_k = \underline{R}_k$ he offers the contract where action r is chosen in state θ_2 with x^* and action s with probability $(1 - x^*)$.

Proof: See the appendix.■

Corollary 2: *When all the assumptions in Proposition 4 hold, except that*

$$-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\widehat{\Lambda}\rho_A^* \leq \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta}{2}\left(x^*\rho_A^* + (1-x^*)S_A\right)\right) \leq -I + \frac{\delta\pi}{2} + \frac{\delta}{2}\rho_A$$

the unique subgame-perfect equilibrium of the contracting game is such that the proposer immediately offers contract C_r, and; *ii) the receiver immediately accepts. This contract is also renegotiation-proof.*

Proof: See the appendix.■

This latter contract is *coarse* in that it makes no use of the finest partition of the states of the world, nor does it involve any communication between the parties. As the parties have conflicting objectives, and since B can credibly threaten to walk away from the deal if A does not make concessions, the value of information for the contracting parties as a whole is essentially zero. There is then little point for the proposer to try to design a finely tuned contract. On the contrary, agreement is easier to obtain on a coarse proposal.

Under assumption **A8**, this coarse contract is also renegotiation-proof: Party A's value of information is reduced as he has to commit to choose the action that B prefers with sufficiently high probability to get B to accept to renegotiate. The value of information to A is then so low that he prefers to just carry on with the existing deal rather than thinking on the spot and renegotiate the contract. In a way, coarse contracts implement the opposite outcome to contracts granting control rights to one of the parties: while contracts with control rights implement deferred thinking, coarse contracts are a commitment to no thinking. The cost of the coarse contract for party A is that it will lead to some occasional mistakes, such as the choice of the risky action when he does not like it. This is, however, preferable to a more sophisticated contracting process that would involve some compromise with party B.

In contrast, when assumption **A8** is replaced by assumption **A10** below**,** the terms of trade favor agent A so that an incomplete *preliminary agreement* signed at date 0, and followed by a subsequent complete contract becomes the equilibrium outcome of the contracting game.

Assumption A10:

$$\frac{\underline{R}_A - S_A}{S_A - \overline{R}_A} > \frac{S_B - \underline{R}_B}{\overline{R}_B - S_B}$$

Proposition 5: *Under assumptions* ***A1**, **A5**, **A7**, **A9*** *and* ***A10**,* *and provided that* $\left(\widehat{\Lambda} - \widehat{\lambda}_A\right)$ *is small enough, the unique subgame-perfect equilibrium of the contracting game is such that:*

i) agent A offers a ***preliminary agreement*** *to agent B with the following terms: a) the parties commit to invest at the latest by some finite deadline t^*, and at the earliest once they have discovered whether $R_k = \overline{R}_k$ or $R_k = \underline{R}_k$; b) if they discover that $R_k = \underline{R}_k$ then action r is chosen in state θ_2; c) if they discover that $R_k = \overline{R}_k$ then action s is chosen with probability α_{t^*} and action r with probability $(1 - \alpha_{t^*})$ in state θ_2, where α_{t^*} solves agent B's participation constraint at date 0;*

ii) agent B responds by accepting the preliminary agreement, then thinks ahead until either of the two parties has discovered the value of R_k;

iii) if agent B is the first to discover R_k she discloses truthfully to agent A when $R_k = \overline{R}_k$, and when $R_k = \underline{R}_k$ she best-responds by withholding her findings;

iv) agent A also best-responds by thinking ahead and if the true payoffs are not discovered by date t^* then investment takes place and action r is implemented in state θ_2.

Proof: See the appendix.■

We refer to the equilibrium contract in Proposition 5 as a *preliminary agreement*, since the contract is signed before any of the parties has done any thinking (or *due diligence*) and since the contract spells out a commitment to a contingent action-plan which depends on the outcome of the due diligence. In this respect the contract has the appearance of preliminary agreements in merger and acquisition transactions. However, in many other respects this contract is very different from the simple two or three-page letter of intent that is commonly signed at the early stages of an acquisition transaction. In particular, the preliminary agreement defined above is a complete state-contingent contract.

To see intuitively why Proposition 5 obtains, note that under assumption **A9** agent A would be worse off proposing an incomplete contract at date 0 where he has full control, which would give him an expected payoff of only

$$-I + \frac{\delta\pi}{2} + \frac{\delta\widehat{\Lambda}\rho_A^*}{2}.$$

Similarly, proposing a complete but coarse contract that implements the risky decision r in state θ_2 would only give him an expected payoff of

$$-I + \frac{\delta\pi}{2} + \frac{\delta\rho_A}{2},$$

which is even lower, as $\widehat{\Lambda}\rho_A^* > \rho_A$ by assumption **A5**. Under the preliminary agreement, however, he would get an expected payoff:

$$\begin{aligned} V_A^{pr} &= \nu\widehat{\Lambda}\left[\alpha_p^*\left(-I + \frac{\delta\pi}{2} + \frac{\delta S_A}{2}\right) + (1-\alpha_p^*)\left(-I + \frac{\delta\pi}{2} + \frac{\delta\overline{R}_A}{2}\right)\right] \\ &\quad +(1-\nu)\widehat{\lambda}_A\left[-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_A}{2}\right] + W_A(t^*) \end{aligned}$$

where,

$$\begin{aligned} &W_A(t^*) = \delta^{t^*} \times \\ &\left[\nu(1-\Lambda)^{t^*}\left[\left(-I + \frac{\delta\pi}{2} + \frac{\delta\overline{R}_A}{2}\right) - \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\alpha_p^* S_A + (1-\alpha_p^*)\overline{R}_A}{2}\right)\right]\right. \\ &\left. + (1-\nu)(1-\lambda_A)^{t^*}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_A}{2} - \widehat{\lambda}_A\left(-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_A}{2}\right)\right]\right]. \end{aligned}$$

The value of α_p^* is determined by party B's participation constraint (given in the Appendix). The term $W_A(t^*)$ represents the added value of the deadline t^* for the proposer. Without the deadline, the proposer's payoff would be given by only the first two terms in V_A^{pr}. In that case agent B only reveals her thinking with probability ν. The last term in $W_A(t^*)$ may be positive and increasing in t^* so that party A may be better off with a deadline.

The preliminary agreement is the proposer's preferred choice, when V_A^{pr} exceeds what party A would get by postponing all negotiation to the time where the payoffs of the risky action r have been uncovered. In that case he would get the payoff:

$$\widehat{\Lambda}\left(-I+\frac{\delta\pi}{2}+\frac{\delta}{2}\left(x^*\rho_A^*+(1-x^*)S_A\right)\right).$$

This payoff is smaller than V_A^{pr} when λ_A is close to Λ. As $\Lambda=\lambda_A+\lambda_B-\lambda_A\lambda_B$, this is the case when λ_B is small. Intuitively, the drawback of the preliminary contract is that party B sometimes hides what she learns, while she does not in the absence of a preliminiray agreement. This is all the more costly when party B's thinking is more essential for uncovering the true payoffs. Similarly, a preliminary agreement is less attractive when ν is small; that is, when it is more likely that party B will want to hide what she has learned.

3.2 Optimal Contracting under cheap talk

When the parties' communication is *cheap talk* the aggregate value of thinking before acting is generally reduced, as the parties can no longer be trusted to reveal the results of their thinking truthfully. As a result, the equilibrium play of the contracting game will change towards earlier investment and coarser contracts than under credible disclosure.

One important implication of this partial breakdown in communication is that the true payoff of the risky action is only verifiable when the risky decision is taken. As a result, contracts granting control rights to one (or both) parties can now *strictly dominate* complete contingent contracts. When information can be credibly disclosed, on the other hand, a contract granting control rights is at best weakly optimal, as such a contract can always be replicated by a complete contract contingent on the information disclosed[15].

3.2.1 Conflict over cautiousness

More specifically, when the two parties have a *conflict over cautiousness* then the more patient party can no longer rely on what the impatient party is saying, as the impatient party always strictly prefers to claim to have found that the risky action is optimal rather

[15] In particular, preliminary agreements take the form of an incomplete contract when information is not fully verifiable.

than concede that he has not learned anything. Only communication that the payoff of the risky action is low ($R_k = \underline{R}_k$) is then credible.

The partial breakdown in communication under *cheap talk* only affects some of the results derived in the previous section. In particular, Lemma 1 continues to hold unchanged. As for Proposition 1 and the Corollary, analogous results obtain under cheap talk with albeit slightly stronger assumptions. If we change assumption **A4** to assumption **A4b** below then we obtain an analogous Proposition **1b**.

Under assumption **A4b** the cautious agent (agent B) is less likely to obtain control. A stronger assumption is required due to the partial breakdown in communication. There are two opposing effects of reduced communication. On the one hand, the cautious agent's threat to reject all offers until she has determined the payoff of the risky action is less credible. The reason is that collective learning about the payoff is slowed down under cheap talk, as agent A (the more impatient agent) communications are not always credible. With slower learning the value of thinking ahead is reduced and therefore agent B is keener to accept a reasonable offer from agent A. On the other hand, the value of control for agent B is also reduced under cheap talk, for the same reason. Thus, although agent B is more likely to accept a contract without any control rights, when she requires control rights to sign on she requires more control rights than under credible information disclosure.

When agent B has the right to choose the action in state θ_2 she learns the true payoff of the risky action in any given period with only probability

$$\xi_B = \lambda_B + (1-\nu)(1-\lambda_B)\lambda_A.$$

Agent A, in contrast, learns the true payoff with the higher probability $\Lambda > \xi_B$. This difference in probabilities reflects the agents' different incentives to truthfully reveal what they have learned.

Although at any one time communication that $R_k = \overline{R}_k$ by agent A is not credible, agent B puts more weight on this information following multiple repetitions of this information by agent A. In other words, agent A is able to eventually *persuade* agent B, following enough repetitions of the message that $R_k = \overline{R}_k$. The reason is that after a while it is less and less likely that agent A has not learned anything and is simply pretending to have found the payoff $R_k = \overline{R}_k$. And when agent A has been able to identify the true payoff he prefers to truthfully reveal it. Thus, after t^* periods agent B's posterior belief that the risky action is best is sufficiently close to one, for agent B to prefer to immediately invest and settle on the risky action.

More formally, when agent B is in control, she prefers to think on the spot before acting if and only if:

$$\nu\overline{R}_B\left[\lambda_B\left[1+(1-\lambda_B)\delta+(1-\lambda_B)^2\delta^2+...+(1-\lambda_B)^{t^*-1}\delta^{t^*-1}\right]+\delta^{t^*}(1-\lambda_B)^{t^*}\right]+$$

$$(1-\nu)\left(S_B\Lambda\left[1+(1-\Lambda)\delta+(1-\Lambda)^2\delta^2+...+(1-\Lambda)^{t^*-1}\delta^{t^*-1}\right]+\delta^{t^*}(1-\Lambda)^{t^*}\underline{R}_B\right)\geq\rho_B.$$

Let

$$\Lambda_k\equiv[\widehat{\lambda}_k\left(1-(1-\lambda_k)^{t^*}\delta^{t^*}\right)-\delta^{t^*}(1-\lambda_k)^{t^*}]$$

and

$$\Gamma_B\equiv\nu\overline{R}_B\left(\frac{\Lambda_T-\Lambda_B}{1-(1-\Lambda)^{t^*}\delta^{t^*}}\right),$$

then, this condition can be written more compactly as follows[16]:

$$\widehat{\Lambda}\rho_B^*-\Gamma_B\geq\rho_B$$

As for agent A, he prefers to act immediately rather than think on the spot if and only if

$$\widehat{\Lambda}\rho_A^*-\Gamma_A<\rho_A,$$

where Γ_A is defined in the same way as Γ_B.

Under the assumption **A4b** below, then we obtain an analog to Proposition 1 (and to its Corollary):

Assumption A4b: $\widehat{\Lambda}\rho_A^*-\Gamma_A<\rho_A$ and $\widehat{\Lambda}\rho_B^*-\Gamma_B\geq\rho_B$.

Proposition 1b *Under assumptions **A1, A2, A3** and **A4b** Agent A strictly prefers to allocate some control to agent B when the following condition holds:*

$$-I+\frac{\delta\pi}{2}+\frac{\delta\rho_B}{2}<\widehat{\Lambda}\left(-I+\frac{\delta}{2}(\pi+\rho_B^*)\right)-\nu\left(-I+\frac{\delta}{2}(\pi+\overline{R}_B)\right)\left(\frac{\Lambda_T-\Lambda_B}{1-(1-\Lambda)^{t^*}\delta^{t^*}}\right)\quad(6)$$

When the reverse condition holds, agent A optimally retains full control.

3.2.2 Conflict over actions

When the two parties have a conflict over action choice (assumption **A5** holds) there can be no credible communication at all via cheap talk, as each party always strictly prefers to claim that its preferred action is the best choice. As a result the two agents may duplicate their cognitive efforts. In this situation again analogous results to Propositions 3, 4 and 5 can be obtained under again stronger assumptions to account for the slower speed of effective

[16]It is easy to verify that $\Lambda_T>\Lambda_B$.

learning. But, equilibrium contracts are generally more likely to be *coarse*. In particular, it is straightforward to establish the following lemma:

Lemma 4: When the equilibrium contract under verifiable information is a *coarse* contract, then it is also the equilibrium contract when information is not verifiable.

There is no learning involved before taking an action under a coarse contract and therefore the parties' payoffs are unaffected by whether the information disclosed is verifiable or not. At the same time, their payoffs in any play of the contracting game that involves some thinking are weakly higher when information is verifiable than when it is not. Therefore, if the coarse contract provides the highest payoffs when information is verifiable, then a fortiori it provides the highest payoff when it is not.

Finally, another difference with the situation where information is verifiable is that negotiations may last forever under non-verifiable information even when the receiver (party A) has been able to uncover the true payoff of the risky action. Indeed, the repeated disclosure of the same information by party A now cannot *persuade* the receiver in any way that the sender's information is reliable. The reason is that the receiver now cannot infer anything from the repeated disclosure of the same information, about the payoff the sender is likely to have observed. Given that the receiver's beliefs do not change following repeated communication rounds, the receiver will not want to stop after some finite deadline. This, in turn, reduces the value of a preliminary agreement.

4 Satisficing Contracts under transferable utility

Consider now the contracting problem where utility is fully transferable through (state-contingent) monetary payments. When the contracting parties can transfer utility by making side-payments, a preliminary agreement is likely to have even greater benefits and to correspond much more closely to the letters of intent typically observed in practice. Indeed, by first specifying the broad terms of the deal, the preliminary agreement now has the important purpose of aligning the contracting parties' objectives in the venture. Once the parties have aligned objectives they will agree on the extent of due diligence they wish to undertake and will more readily accept to invest without working out a complete future action-plan.

Lemma 5 (The Congruence Principle): it is weakly optimal for the contracting parties to begin by signing a preliminary agreement which establishes how the parties will share the profits from the venture.

Once the parties have agreed on how they will share the surplus from the venture they have aligned objectives in determining how far ahead they want to plan, what they would like to specify in the contract ahead of investment, and what decisions they would like to leave open until when they arise. In other words, after the preliminary agreement the two parties act as a team and the optimal contracting problem reduces to a problem of determining the team's optimal plan of action. Communication is therefore no longer a problem and there is no longer any difference between the cases of soft and hard information. Any actions that the contracting parties determine by thinking ahead will be specified in the contract and any decisions to be determined when they arise will be taken jointly by the two parties.

Obviously, since the parties have perfectly aligned objectives there is no need to actually specify anything in a contract beyond the preliminary agreement. Note, however, that under even a very small risk of an unforeseen change in one of the parties' preferences that might result in a conflict ex-post, the parties would strictly prefer to explicitly spell out what future action choices they have agreed to in a contract. For the same reason the parties would strictly prefer to specify a governance structure that defines the process by which future decisions are taken.

With only one state of nature (θ_2) where some thinking is needed, the contracting problem with transferable utility collapses to a simple decision of the type studied in Bolton and Faure-Grimaud (2005). With more than one problem to solve (e.g. in state θ_1 too parties must choose between two actions), and with more than one thinking party, the team problem is of independent interest as it raises a number of organisational design questions. For instance one might wonder who should think about what problem? Or, whether the team is more efficient having both parties thinking through the same problem at the same time, or rather opting for some form of parallel thinking? We refer the reader for answers to some of those questions to our companion paper, Bolton and Faure-Grimaud (2007).

To summarize, when utility is perfectly transferrable the parties begin the contracting process by signing a preliminary agreement which perfectly aligns their objectives. They then proceed to contract negotiations which end with a more or less complete contract depending on the perceived value of the venture. If the venture is perceived to be very valuable the parties don't waste much time in specifying a complete contingent contract and instead leave many decisions to be determined later. On the other hand, if the venture is not perceived to be very profitable they will spend more time thinking through the details of the venture and will write a more complete contract.

5 Conclusion

The model we have considered is extremely simple, involving only a sequence of two decisions and only two states of nature. The main advantage of this set-up is that it is relatively tractable to characterize equilibrium satisficing contracts. But the drawback is that it oversimplifies most contracting situations in reality and it oversimplifies the analysis of boundedly rational agents that are confronted with these contracting problems. [TO BE COMPLETED]

References

Aghion, P. and P. Bolton (1992) "An Incomplete Contracts Approach to Financial Contracting." *Review of Economic Studies*, 59, 473–94.

Al Najjar, N., L. Anderlini, and L. Felli (2002): "Unforeseen Contingencies," STICERD Discussion Paper TE/02/431, London School of Economics.

Anderlini, Luca and L. Felli (1994) "Incomplete Written Contracts: Undescribable States of Nature", *Quarterly Journal of Economics*, 109 (4), pp. 1085-124.

Anderlini, Luca and L. Felli (1999). "Incomplete Contracts and Complexity Costs", *Theory and Decision*, 46 (1), pp. 23-50.

Bajari, Patrick and S. Tadelis, (2001) "Incentives versus Transaction Costs: A Theory of Procurement Contracts", *RAND Journal of Economics*, Vol. 32, No. 3, 387-407

Battigalli, P. and G. Maggi (2002) "Rigidity, Discretion and the Costs of Writing Contracts", *American Economic Review*, 92 (4), pp. 798-817.

Berry, D. A. and D. Fristedt (1985), *Bandit Problems: Sequential Allocation of Experiments*, London, Chapman Hall.

Bolton, Patrick and Antoine Faure-Grimaud (2005), "Thinking Ahead: The Decision Problem", NBER Working Paper No. W11867, http://ssrn.com/abstract=872723

Conlisk, John. "Costly Optimizers Versus Cheap Imitators" *Journal of Economic Behavior and Organization*, 1980, 275-293.

Conlisk, John. "Optimization Cost" *Journal of Economic Behavior and Organization*, 1988, 213-228.

Conlisk, John. "Why bounded rationality?" *Journal of Economic Literature*, June 1996, 34 (2), pp.669-700.

Dye, Ronald (1985) "Costly Contract Contingencies", *International Economic Review*, 26 (1), pp. 233-50.

Gittins, J.C. and D. M. Jones (1974), "A Dynamic Allocation Index for the Sequential Design of Experiments.", in *Progress in Statistics*, Gani, Sarkadi and Vince (eds.), New York, North Holland

Gittins, J.C. (1979) "Bandit processes and dynamic allocation indices", *Journal of the Royal Statistical Society* B, 41, 2, p.148-77.

Grossman, S.J. and O.D. Hart (1986). "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, 94, 691–719.

Hart, O. D. (1995). *Firms, Contracts, and Financial Structure.* Clarendon Lectures in Economics. Oxford University Press.

Hart, O., and J. Moore. (1988) "Incomplete Contracts and Renegotiation." *Econometrica*, 56, 755–85.

Hart, O., and J. Moore. (2007), "Contracts as Reference Points", mimeo Harvard University

MacLeod, W. B. (2000): "Complexity and Contract," *Revue d'Economie Industrielle*, 92, 149-178.

Maskin, E. and J. Tirole (1999): "Unforeseen Contingencies and Incomplete Contracts," *Review of Economic Studies*, 66, 83-114.

Radner, R. (1975) "Satisficing", *Journal of Mathematical Economics*, 2 (2), 253-62.

Radner, R. and M. Rothschild (1975), "On the Allocation of Effort", *Journal of Economic Theory*, 10 (3), 358-76.

Rothschild, M. (1974) "A Two-Armed Bandit Theory of Market Pricing," *Journal of Economic Theory*, 9, 185-202.

Simon, Herbert. "A behavioral model of rational choice," *Quarterly Journal of Economics*, 1955, 69 (1).

Tirole, Jean (1999) "Incomplete Contracts: Where Do We Stand?" *Econometrica*, 67 (4), pp. 741-81.

Tirole, Jean (2007) "Bounded Rationality and Incomplete Contracts" *mimeo.*

Whittle, P. (1980) "Multi-armed Bandits and the Gittins Index", *Journal of the Royal Statistical Society*, 42, 143-149

Whittle, P. (1982), *Optimization over time: Dynamic Programming and Stochastic Control*, John Wiley, New York

APPENDIX

Proof of Proposition 1: We first establish a series of preliminary results that simplify the argument.

Lemma 2: *Under assumptions **A2** to **A4**, it is always a weakly dominant strategy for a party to reveal what she has learned to the other party.*

Proof: This observation follows immediately from the facts that: i) once the information is shared parties have fully congruent objectives under the stated assumptions; and ii) not revealing what a party has learned can only delay the time at which payoffs are received and cannot result in higher payoffs. As each party gets strictly positive payoffs (under assumption ***A3***) it follows that immediate truthful disclosure of what a party learns is a weakly dominant strategy.■

Claim 1: *Let $U_{\min}^{RFI}$ be the lowest payoff that the receiver can guarantee herself in any subgame under complete information. Then, either the proposer implements her most preferred contract or the receiver gets exactly $U_{\min}^{RFI}$.*

Proof: Observe first that $U_{\min}^{RFI} = 0$ in the absence of a pre-existing contractual agreement. Suppose now that the claim is not true and that there exists an SPNE where under full information, the receiver gets some payoff $\widehat{U} > U_{\min}^{RFI}$, and the proposer is not offering his most preferred plan of action. For this to be true, it must be that the receiver rejects any offer that gives her less than $\widehat{U}$. But, given that the proposer is not making his most preferred offer, it must then be the case that the receiver is just indifferent between accepting and rejecting the offer giving her $\widehat{U}$. Otherwise, the receiver could offer a lottery that would put some weight ε on his most preferred contract and $(1-\varepsilon)$ on the offer currently providing $\widehat{U}$ to the receiver. Therefore it must be that along the equilibrium path in such an equilibrium, at any date t, $\widehat{U}_t = \delta \widehat{U}_{t+1}$. Iteration of this argument requires $\widehat{U}_{t+\tau}$ to go to infinity as τ goes to infinity, which is impossible.■

Claim 2: *Denote by U^{RFI} the unique subgame perfect equilibrium payoff that the receiver obtains in any subgame under complete information. Then, in any subgame where the payoffs of the risky action are unknown, either the proposer offers her most preferred contract, or the receiver gets $U_{\min}^{R} = \widehat{\Lambda} U^{RFI}$.*

Proof: Suppose again this is not true. As in the proof of Claim 1, it then follows that the receiver must be indifferent at any date t between accepting or rejecting the offer that gives the receiver some utility level $\widetilde{U}_t^R > U_{\min}^R$. In particular, it then must be the case that

$$\widetilde{U}_t^R = \Lambda U^{RFI} + (1-\Lambda)\delta \widetilde{U}_{t+1}^R.$$

And, if $\widetilde{U}_t^R = \widetilde{U}_{t+1}^R$, then $\widetilde{U}_t^R = \frac{\Lambda}{1-(1-\Lambda)\delta} U^{RFI} = \widehat{\Lambda} U^{RFI}$, a contradiction. Alternatively, iterating the same the argument, we would find that

$$\widetilde{U}_{t+\tau}^R = \frac{\widetilde{U}_t^R}{(1-\Lambda)^\tau \delta^\tau} - \widehat{\Lambda} U^{RFI} \frac{1-(1-\Lambda)^\tau \delta^\tau}{(1-\Lambda)^\tau \delta^\tau}$$

which, when $\widetilde{U}_t^R > U_{\min}^R$ requires $\widetilde{U}_{t+\tau}^R$ to go to infinity when τ goes to infinity. Again, this leads to a contradiction.■

We now make use of these observations to establish Proposition 1.

Note first that under assumptions **A2** and **A4** party B's minimum guaranteed payoff is

$$U_{\min}^B = \widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^*}{2}\right].$$

If condition (3) in proposition 1 does not hold, so that

$$I + \frac{\delta\pi}{2} + \frac{\delta\rho_B}{2} \geq \widehat{\Lambda}\left(-I + \frac{\delta}{2}\left(\pi + \rho_B^*\right)\right),$$

then the proposer's most preferred contract– C_r –gives a higher expected payoff to B than $U_{\min}^B$. Therefore B's best response is to accept this offer.

Now suppose that condition (3) holds. Then, from claim 1, the receiver gets exactly $U_{\min}^B$ in equilibrium.

To complete the proof of proposition 1 it remains to show that the stochastic contract offer that gives A the highest possible payoff while guaranteeing $U_{\min}^B$ to B, takes the form described in the proposition, namely that both parties agree to invest immediately, party B gets control with probability y^* and the risky action is chosen in state θ_2 with probability $(1-y^*)$.

There are several types of stochastic contracts that can implement $U_{\min}^B$. A first contract is to give full control to party B (draw contract C_B) with probability y and to take the risky action in state θ_2 with probability $(1-y)$.[17] An alternative offer is to give B control in every period with some probability z and to take the risky action in state θ_2 with probability $(1-z)$. As we show below these two contracts are in fact equivalent. To see this, note that under the latter contract party k expects to receive:

$$(1-z)\rho_k + z\Lambda\rho_k^* +$$

$$z(1-\Lambda)\delta\left[z(1-\Lambda)\delta\left[(1-z)\rho_k + z\Lambda\rho_k^* + z(1-\Lambda)\delta[...]\right]\right] =$$

[17] An equivalent contract is to draw contract C_α with probability y and to take the risky action in state θ_2 with probability $(1-y)$.

$$\frac{z\Lambda}{1-z(1-\Lambda)\delta}\rho_k^* + \frac{1-z}{1-z(1-\Lambda)\delta}\rho_k.$$

Now setting $y = \frac{z\Lambda}{\widehat{\Lambda}(1-z(1-\Lambda)\delta)}$ this reduces to $y\widehat{\Lambda}\rho_k^* + (1-y)\,\rho_k$. (Note also that there is no loss of generality in considering only stationary strategies $z_t = z$ for all t).

We now characterise the highest payoff available to A under the constraint that B gets $U_{\min}^B$. Agent A's control variables are the probability x of engaging in thinking ahead before investing and the probability y of engaging in thinking on the spot in state θ_2 before chosing an action. Therefore agent A is looking for the solution to the constrained maximization program:

$$\begin{aligned} MP_A \;\; \equiv \;\; & \max_{x,y} x\widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_A^*}{2}\right] \\ & + (1-x)\left[-I + \frac{\delta\pi}{2} + y\widehat{\Lambda}\left(\frac{\delta\rho_A^*}{2}\right) + (1-y)\frac{\delta\rho_A}{2}\right] \end{aligned}$$

subject to:

$$\widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^*}{2}\right] \leq$$

$$x\widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^*}{2}\right] + (1-x)\left[-I + \frac{\delta\pi}{2} + y\widehat{\Lambda}\left(\frac{\delta\rho_B^*}{2}\right) + (1-y)\frac{\delta\rho_B}{2}\right].$$

Other contracts that involve for instance choosing the safe action before learning whether it is optimal or, choosing the sub-optimal action once parties have learned which action is best are dominated for both parties and cannot therefore maximize A's payoff under the constraint that B obtains at least $U_{\min}^B$.

Forming the Lagrangian, and taking its partial derivatives with respect to x and y we obtain that:

$$\frac{\partial L}{\partial x}(1-x) = (1-y)\frac{\partial L}{\partial y} - (1-x)(1+\vartheta)(1-\widehat{\Lambda})(-I + \frac{\delta\pi}{2})$$

where $\frac{\partial L}{\partial x}$ (resp. $\frac{\partial L}{\partial y}$) is the partial derivative of the Lagrange function with respect to x (resp. y) and ϑ is the Lagrange multiplier of the constraint.

From the last inequality it is apparent that the solution to this program is $x^* = 0$ and $y^* \in (0,1)$ if and only if:

$$-I + \frac{\delta\pi}{2} + \widehat{\Lambda}\left(\frac{\delta\rho_B^*}{2}\right) > \widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B^*}{2}\right],$$

which is true under assumptions **A2** to **A4**. This establishes that the most efficient way for A to deviate from his preferred course of action is to invest right away, to choose the risky action in state θ_2 with probability $(1-y^*)$ and to think on the spot with probability y^*. This action-plan is implemented by offering party B control with probability y^*, as party B would then want to think on the spot in state θ_2. Finally, the exact value of y^* is given by:

$$-I+\frac{\delta\pi}{2}+y^*\widehat{\Lambda}\left(\frac{\delta\rho_B^*}{2}\right)+(1-y^*)\frac{\delta\rho_B}{2}=$$
$$\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\frac{\delta\rho_B^*}{2}\right]$$

To summarize, the following strategies support this subgame-perfect equilibrium:

- **Equilibrium strategy for** A : at date 0, offer a stochastic contract committing to immediate investment and that implements C_r with probability $1-y^*$ and C_B with probability y^*. If the contract is accepted, invest at date 0 and if state θ_2 is realized and A has control, implement decision r. If B has control, think and credibly reveal any new information to B.

If the offer is rejected, think and again credibly reveal any new information to B. If A uncovers the optimal decision in state θ_2 reveal it to B and offer the first-best optimal complete contract to B (either C_r or C_s depending on whether A uncovers that r or s is optimal). Similarly, if B reveals the optimal decision in state θ_2 offer the first-best complete contract to B.

If A learns nothing during that second sub-period of period 0 (from his own thinking or from B) repeat at date 1 the same strategy as at date 0 and continue doing so until investment takes place.[18]

- **Equilibrium strategy for** B : at date 0, accept all contract offers with immediate investment that take support in $\mathbb{C}\backslash\{C_s\}$, provided that those offers put a weight of at least y^* on the choice of C_B. In state θ_2, when B has control think on the spot and implement the optimal decision. Following a rejection at date 0, think in the second sub-period of date 0 and reveal any information to A. Then accept all first-best complete contract offers. Similarly, if A reveals that decision s (resp. r) is optimal in state θ_2, accept all first-best complete contract offers. If neither party learns anything, repeat at date 1 the same strategy as at date 0 and continue doing so until a contract is accepted.■

Proof of Corollary 1: immediate from previous results and noticing that now necessarily under ***A2*** to ***A4***,

$$-I+\frac{\delta\pi}{2}+\frac{\delta\widehat{\Lambda}\rho_A^*}{2}>\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\frac{\delta\rho_A^*}{2}\right]=U_{\min}^A.$$

[18] Note that nothing is changed if party A offers initially C_A instead of C_r, or C_α instead of C_B.

Therefore, the proposer B must obtain her most preferred path of action, i.e. C_B.

Remark: if the optimal path of actions followed in the SPNE is uniquely pinned down, this is not so for the contractual offers. From A's perspective, C_r and C_A are equivalent. From B's perspective C_σ and C_B are also equivalent. This last equivalence, however, will cease to be true in the case of soft information.

Proof of Proposition 2: As above party B can guarantee herself $\underline{U}_B$ by following the simple strategy of not accepting a contract before the payoffs of action r are uncovered. Based on the previous arguments, and the fact that Assumption $\mathbf{A}_4^+$ implies that no other path of action can provide B with a higher payoff than $\underline{U}_B$, the only possibility for A is to make a contractual offer only after the payoffs have been uncovered. More precisely, the following strategies form a SPNE:

- Equilibrium strategy for B : at date 0, think. If B's thinking uncovers the optimal decision to take in state θ_2, credibly reveal it to A, and offer in the newly granted sub-period C_r (resp. C_s) if B's thinking uncovers that decision r is optimal (resp. s). If B's thinking is unsuccessful during that second sub-period of period 0, but A's is and A reveals what decision is optimal, again offer either C_r or C_s according to what A reveals.

If thinking by either party is unsuccessful, repeat at date 1 the same strategy as at date 0 and continue doing so until investment takes place.

- Equilibrium strategy for A : at date 0, accept do not accept any contractual offer. Think in the second sub-period of date 0. Reveal any information uncovered by A's thinking and accept C_r (resp. C_s) if A finds out that decision r (resp. s) is optimal in state θ_2. If B credibly reveals that decision r (resp. s) is optimal in state θ_2, accept C_r (resp. C_s). If thinking by either partner is unsuccessful, repeat at date 1 the same strategy as at date 0 and continue doing so until investment takes place.

Proof of Lemma 3: Consider an equilibrium where A never takes an action before some hard evidence has been uncovered. To be subgame perfect, the best response of B must specify that B reveals what she has learned at any date: concealing information can only delay the time at which A makes a decision but cannot change that decision. When $I < \frac{\delta\pi}{2}$, all payoffs to B are strictly positive and delaying those is dominated. If B reveals what she learns, A's most preferred course of action is to keep control in state θ_2 and indeed to think before acting as $\widehat{\Lambda}\rho_A^* > \rho_A$. Finally B is better off accepting a contract C_A than rejecting it as again this can only result in her getting:

$$\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\frac{\delta}{2}\rho_B^-\right]$$

less than $-I+\frac{\delta\pi}{2}+\frac{\delta}{2}\widehat{\Lambda}\rho$. It is straightforward to identify the equilibrium strategies as in the previous proof that support this SPNE.

Notice that there could be multiple equilibria in the situation examined in Lemma 3. To see this, suppose that A believes that B will never reveal that $R_k=\overline{R}_k$. This will reduce the speed of learning from B's point of view to $\nu\widehat{\lambda}_A+(1-\nu)\widehat{\Lambda}$. It could be now that, even under *A5.b,*

$$\rho_A>\left(\nu\widehat{\lambda}_A+(1-\nu)\widehat{\Lambda}\right)\rho_A^*$$

and that therefore the best response of A is indeed to offer C_r right away. As thinking is not called for, B is playing a best response on the equilibrium path. Moreover in any sugame out of the equilibrium path where B expects A to choose the risky decision at the next date, B's strategy of hiding $R_k=\overline{R}_k$ is also subgame perfect.

It is possible however to specify sufficient conditions to guarantee unicity. For instance under:

$$\mathbf{A11}:\begin{cases}\delta\overline{R}_B<S_B\\ \delta S_B<\underline{R}_B\end{cases}$$

whatever B thinks about A's next course of action (as induced by A's beliefs about B's information out of the equilibrium path), she is better off revealing the truth immediately. Indeed, B is never hurt by revealing $R_k=\overline{R}_k$ as even if A chooses then her least preferred option, the safe decision, the first inequality implies that she is better off revealing her information. The second inequality rules out the possibility for B to hide her knowledge under the belief that this will be conducive to A choosing the safe decision instead of the risky one.

Proof of Proposition 4:

As a preliminary step, we characterize the policy that gives the highest payoff to A under a strategy of thinking ahead, while guaranteeing some minimum level of utility to B, say V_B^*. Formally, we are looking for α (respectively β), the probability to choose decision r when finding out that r returns $\{\overline{R}_A,\overline{R}_B\}$ (respectively $\{\underline{R}_A,\underline{R}_B\}$) as solutions to:

$$\begin{aligned}MPA &= \max_{\alpha,\beta}\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\left(\nu\left(\alpha\overline{R}_A+(1-\alpha)S_A\right)+(1-\nu)\left(\beta\underline{R}_A+(1-\beta)S_A\right)\right)\right]\\ \text{subject to} &:\end{aligned}$$

$$\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\nu\left(\alpha\overline{R}_B+(1-\alpha)S_B\right)+(1-\nu)\left(\beta\underline{R}_B+(1-\beta)S_B\right)\right]\geq V_B^*$$

Agent A would prefer $\alpha=0$ and $\beta=1$: indeed choosing $\alpha>0$ involves a merginal cost of $(S_A-\overline{R}_A)$ and a marginal return of $(\overline{R}_A-S_A)$. Similarly, choosing $\beta<1$ involves a marginal cost $(\underline{R}_A-S_A)$ and a marginal return of $(S_A-\underline{R}_A)$. It is therefore best to have $\beta=1$ and $\alpha=0$ when

$$\frac{S_B-\underline{R}_B}{\underline{R}_A-S_A}>\frac{\overline{R}_B-S_B}{S_A-\overline{R}_A},$$

which is Assumption ***A8***. Also notice that the solution to the above program when $V_B^*=\widehat{\Lambda}\nu\left(-I+\frac{\delta\pi}{2}+\frac{\delta S_B}{2}\right)$ is $\alpha=0$ and $1-\beta=x^*$.

Under assumption ***A9,*** a pair of strategies where the parties do not sign a contract prior to finding the true payoff of the risky action, if part of an equilibrium, gives more to A than what could be obtained under a preliminary agreement which stipulates some $\alpha>0$. Moreover, such a pair of strategies gives both A and B a higher expected payoff than an offer of a coarse contract that stipulates C_r. Finally A's payoff also exceeds what he could get when thinking on the spot.

In light of these observations equilibrium strategies that support a SPNE as described in Proposition 4 are:

- For A : Do not make any offer until the true payoff of the risky action is uncovered. Always disclose what A learns. Once the true payoffs are known, either offer C_s upon learning that r returns $\{\overline{R}_A,\overline{R}_B\}$, or a stochastic contract that puts a weight x^* on C_s and $(1-x^*)$ on C_r upon learning that r returns $\{\underline{R}_A,\underline{R}_B\}$.

- For B : Always accept C_B. Always disclose what she learns. At $t=0$, accept any offer that provides at least $\widehat{\Lambda}\nu\left(-I+\frac{\delta\pi}{2}+\frac{\delta S_B}{2}\right)$, reject otherwise. At $t>0$, in any subgame where the true payoff of the risky action has not been uncovered, reject any offer that puts a weight less than x^* on C_s. At $t>0$, accept any offer upon learning that r returns $\{\overline{R}_A,\overline{R}_B\}$. Upon learning that r returns $\{\underline{R}_A,\underline{R}_B\}$ accept any offer that puts a weight of at least x^* on C_s, reject otherwise.

Remark: an offer at t when parties do not know the true payoff of the risky action is an out-of-equilibrium move. We assume that B's beliefs are then that the risky action returns $\{\underline{R}_A,\underline{R}_B\}$.■

Proof of Corollary 2: under the conditions spelled out in the Corollary, A's best reply to B's equilibrium strategy is to offer at $t=0$ to invest immediately and to choose C_r. This is immediately accepted by B as

$$-I + \frac{\delta\pi}{2} + \frac{\delta\rho_B}{2} > \widehat{\Lambda}\nu \left(-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2} \right)$$

At any other date, B will reject such an offer so A is better off offering it at $t = 0$. The equilibrium strategy for A spelled out in the proof of Proposition 4, amended to include a first offer of C_r at $t = 0$, together with B's equilibrium strategy constitute a SPNE.

Along the equilibrium path, there is scope for renegotiation: after an initial offer of C_r and investment at $t = 0$, when parties find themselves in state θ_2, they know that none of them is informed. Party A could then make a new offer, guaranteeing ρ_B to party B but opening the possibility for A to think on the spot. We now establish that such a move would not be profitable for A. Indeed, to be acceptable to B, A needs to offer to choose r when finding out that r returns $\{\overline{R}_A, \overline{R}_B\}$ (resp. $\{\underline{R}_A, \underline{R}_B\}$) with probability α (resp. β) so that:

$$\widehat{\Lambda} \left(\nu \left(\alpha \overline{R}_B + (1-\alpha) S_B \right) + (1-\nu) \left(\beta \underline{R}_B + (1-\beta) S_B \right) \right) \geq \rho_B$$

and under Assumption ***A8,*** the best way to do so is first to reduce β and keep $\alpha = 0$. And if the condition is not satisfied when $\beta = \alpha = 0$, also increase α. If $\alpha = 0$ and $\beta \in (0, 1)$ the left hand side of the above condition is less than $\widehat{\Lambda} S_B$ which is strictly less than ρ_B. When $\alpha > 0$ and $\beta = 0$, then A gets less than $\widehat{\Lambda} S_A$, which again is strictly less than ρ_A. In other words, there is no renegotiation offer that makes both parties better off. We have assumed that if renegotiation takes place parties will always truthfully disclose, a best case scenario. This completes the proof.

Proof of Proposition 5:

When Assumption ***A8*** is not satisfied, the solution to MPA as defined in the proof of Proposition 4 involves setting $\alpha > 0$ and maintain $\beta = 1$. This is only possible in the presence of a binding preliminary agreement, since when the true payoff of the risky action is known to be $\{\overline{R}_A, \overline{R}_B\}$, party A would rather offer C_s.

Consider a strategy for B such that:

- At $t = 0$: accept any offer to invest immediately, provided that it gives B at least

  $$\nu\widehat{\Lambda} \left[-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2} \right]$$

  Once in state θ_2, always truthfully disclose. If an offer is made at $t = 0$ that does not involve immediate investment, accept only the following *preliminary* contract: investment will take place when either A discloses the payoff of the risky action, or some deadline t^* is reached. If the true payoff is $\{\overline{R}_A, \overline{R}_B\}$ then implement s in state θ_2 with probability α_p^* and r with probability $(1-\alpha_p^*)$. If the true payoff is $\{\underline{R}_A, \underline{R}_B\}$, then implement r in state θ_2. Agent B discloses $\{\overline{R}_A, \overline{R}_B\}$ when she finds this to be the true payoff, and witholds what she learns otherwise. The probability α_p^* is given by:

$$V_B^{pr} = \nu\widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}\right]$$

where,

$$\begin{aligned} V_B^{pr} &= \nu\widehat{\Lambda}\left[\alpha_p^*\left(-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}\right) + (1-\alpha_p^*)\left(-I + \frac{\delta\pi}{2} + \frac{\delta\overline{R}_B}{2}\right)\right] \\ &\quad +(1-\nu)\widehat{\lambda}_A\left[-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_B}{2}\right] + W_B(t^*) \end{aligned}$$

where

$$\begin{aligned} & W_B(t^*) = \delta^{t^*} \times \\ & \left[\nu(1-\Lambda)^{t^*}\left[\left(-I + \frac{\delta\pi}{2} + \frac{\delta\overline{R}_B}{2}\right) - \widehat{\Lambda}\left(-I + \frac{\delta\pi}{2} + \frac{\delta\alpha_p^* S_B + (1-\alpha_p^*)\overline{R}_B}{2}\right)\right]\right. \\ & \left. + (1-\nu)(1-\lambda_A)^{t^*}\left[-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_B}{2} - \widehat{\lambda}_A\left(-I + \frac{\delta\pi}{2} + \frac{\delta\underline{R}_B}{2}\right)\right]\right]. \end{aligned}$$

- At $t > 0$: if no contract has been accepted at date $t = 0$, and if the true payoff of the risky action is not known, always accept CR^B, and stochastic contracts that implement C_r with probability $x \leq x^*$ and C_s with probability $1-x$. If an offer is made that is accompanied with disclosure that the true payoff is $\{\underline{R}_A, \underline{R}_B\}$, then B accepts CR^B, and stochastic contracts that implement C_r with probability $x \leq x^*$ and C_s with probability $1-x$. If an offer is made that is accompanied with disclosure that the true payoff is $\{\overline{R}_A, \overline{R}_B\}$, accept all offers. In addition, B always truthfully reveals what she learns. The value of x^* is given by:

$$-I + \frac{\delta\pi}{2} + \frac{\delta x^*\underline{R}_B + (1-x^*)S_B}{2} = 0.$$

Consider now the following strategy for agent A:

- At any date $t \geq 0$, always truthfully reveal what he learns,

- At $t = 0$ offer a preliminary agreement that stipulates that investment will take place when either the true payoff of the risky action is known, or some deadline represented by a date t^* is reached. The preliminary contract stipulates that if evidence is shown by either agent that r's payoff is $\{\overline{R}_A, \overline{R}_B\}$, then implement s with probability α_p^* and r with probability $(1 - \alpha_p^*)$. If evidence is produced that r's payoff is $\{\underline{R}_A, \underline{R}_B\}$, then implement r.

- If no contract has been accepted at date $t = 0$, then at any date $t > 0$, do not make any offer until the true payoff of the risky action is uncovered. Once the true payoffs are known, either offer C_s upon learning that r returns $\{\overline{R}_A, \overline{R}_B\}$, or a stochastic contract that puts a weight of x^* on C_s and $(1-x^*)$ on C_r upon learning that r returns $\{\underline{R}_A, \underline{R}_B\}$.

In addition, assume that if no contract has been accepted at $t = 0$ (an out-of-equilibrium outcome), and if A does not learn the true payoff A's beliefs are that B knows that r returns $\{\underline{R}_A, \underline{R}_B\}$.

These two strategies form a SPNE as:

- Party B cannot credibly reject the preliminary offer since

$$V_B^{pr} = \nu\widehat{\Lambda}\left[-I + \frac{\delta\pi}{2} + \frac{\delta S_B}{2}\right]$$

is the best payoff that B can expect if she rejects the preliminary offer, given A's strategy. The preliminary offer maximizes A's expected payoff. Out of the equilibrium path, the two strategies are best responses, from previous arguments and our assumption about out-of-equilibrium beliefs.

We now show that the best preliminary offer for A necessarily includes a possibility to end negotiations at some date t^* when no evidence of r's payoff has been produced. Denote by $\widehat{\nu}(t)$ the updated probability at date t that A puts on the possibility that action r returns $\{\overline{R}_A, \overline{R}_B\}$. Given that B conceals her knowledge only when she finds that r returns $\{\underline{R}_A, \underline{R}_B\}$, we have

$$1 - \widehat{\nu}(t) = \frac{(1-\nu)}{\nu(1-\lambda_B)^t + 1 - \nu}$$

and this tends to one when t goes to infinity. Suppose that contrary to what we claim, there is no such finite date t^* at which even in absence of hard evidence of r's payoff, A continues to think ahead. This would mean that his payoff is:

$$\widehat{\nu}(t)\widehat{\Lambda}\left(-I+\frac{\delta\pi}{2}+\frac{\delta\alpha_p^*S_A+(1-\alpha_p^*)\overline{R}_A}{2}\right)+$$

$$(1-\widehat{\nu}(t))\,\widehat{\lambda}_A\left(-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}\right)$$

while under immediate investment he gets:

$$-I+\frac{\delta\pi}{2}+\frac{\delta}{2}\left[\widehat{\nu}(t)\overline{R}_A+(1-\widehat{\nu}(t))\underline{R}_A\right]$$

and so for t large enough the first payoff tends to $\widehat{\lambda}_A\left(-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}\right)$ strictly less than the limit of the second, $-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}$.

As a result, party A's expected utility is:

$$\begin{aligned} V_A^{pr} &= \nu\widehat{\Lambda}\left[\alpha_p^*\left(-I+\frac{\delta\pi}{2}+\frac{\delta S_A}{2}\right)+(1-\alpha_p^*)\left(-I+\frac{\delta\pi}{2}+\frac{\delta\overline{R}_A}{2}\right)\right] \\ &\quad +(1-\nu)\widehat{\lambda}_A\left[-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}\right]+W_A(t^*) \end{aligned}$$

For simplicity, rewrite

$$\begin{aligned} &W_A(t^*)=\delta^{t^*}\times \\ &\left[\nu(1-\Lambda)^{t^*}\overline{w}_A+(1-\nu)\left(1-\lambda_A\right)^{t^*}\underline{w}_A\right] \end{aligned}$$

where

$$\overline{w}_A = \left(-I+\frac{\delta\pi}{2}+\frac{\delta\overline{R}_A}{2}\right)-\widehat{\Lambda}\left(-I+\frac{\delta\pi}{2}+\frac{\delta\alpha_p^*S_A+(1-\alpha_p^*)\overline{R}_A}{2}\right)$$

and

$$\underline{w}_A = \left[-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}\right](1-\widehat{\lambda}_A)>0$$

If $\overline{w}_A>0$ then $W_A(t^*)>0$. We establish that $W_A(t^*)$ must be positive even when $\overline{w}_A<0$. The fact that $\overline{w}_A<0$, and that $(1-\Lambda)<(1-\lambda_A)$ imply that $W_A(t)$ is decreasing in t^* for t^* large enough. Indeed:

$$\frac{W_A(t+1)}{W_A(t)} = \delta\frac{\nu(1-\Lambda)^{t+1}\overline{w}_A+(1-\nu)\left(1-\lambda_A\right)^{t+1}\underline{w}_A}{\nu(1-\Lambda)^{t}\overline{w}_A+(1-\nu)\left(1-\lambda_A\right)^{t}\underline{w}_A}$$

$$= \delta\frac{\left(1-\lambda_A\right)^{t+1}\left[\nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t+1}\overline{w}_A+(1-\nu)\underline{w}_A\right]}{\left(1-\lambda_A\right)^{t}\left[\nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t}\overline{w}_A+(1-\nu)\underline{w}_A\right]}$$

$$= \delta\left(1-\lambda_A\right)\frac{\nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t+1}\overline{w}_A+(1-\nu)\underline{w}_A}{\nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t}\overline{w}_A+(1-\nu)\underline{w}_A}$$

We have $\frac{W_A(t+1)}{W_A(t)} \leq 1$ if and only if:

$$\delta\left(1-\lambda_A\right)\left[\nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t+1}\overline{w}_A+(1-\nu)\underline{w}_A\right] \leq \nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t}\overline{w}_A+(1-\nu)\underline{w}_A$$

or

$$0 \leq \nu\left(\frac{1-\Lambda}{1-\lambda_A}\right)^{t}\overline{w}_A\left[1-\delta\left(1-\lambda_A\right)\left(\frac{1-\Lambda}{1-\lambda_A}\right)\right] +(1-\nu)\underline{w}_A\left[1-\delta\left(1-\lambda_A\right)\right]$$

which is true for t large enough. Moreover, either the last inequality is true everywhere or $W_A(t^*)$ is first increasing and then decreasing in t^*. Notice that when t^* goes to infinity, $W_A(t^*)$ goes to zero. Therefore $W_A(t^*)$ is always positive. Moreover when $\widehat{\lambda}_A$ is close to $\widehat{\Lambda}$, V_A^{pr} tends to:

$$V_A^{pr} = \nu\widehat{\Lambda}\left[\alpha_p^*\left(-I+\frac{\delta\pi}{2}+\frac{\delta S_A}{2}\right)+\left(1-\alpha_p^*\right)\left(-I+\frac{\delta\pi}{2}+\frac{\delta\overline{R}_A}{2}\right)\right] +(1-\nu)\widehat{\Lambda}\left[-I+\frac{\delta\pi}{2}+\frac{\delta\underline{R}_A}{2}\right]+W_A(t^*)$$

and exceeds $\widehat{\Lambda}\left(-I+\frac{\delta\pi}{2}+\frac{\delta}{2}\left(x^*\rho_A^*+(1-x^*)S_A\right)\right)$ under assumption ***A8.***