New Eurocoin: Tracking Economic Growth in Real Time *

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This paper presents ideas and methods underlying the construction of a timely coincident index that tracks euro-area GDP growth, but, unlike GDP growth, (i) is updated monthly and almost in real time; (ii) is free from seasonal and shorter-run dynamics. We take as target the medium- long-run component of the GDP growth, defined in the frequency domain as including only waves of period larger than one year. We estimate the target by projecting it on generalized principal components extracted from a large panel of monthly macroeconomic series. The main contribution of the paper is that *current values* of our principal components, derived from a dynamic factor model, act as proxies for future values of GDP growth. In this way we improve with respect to the end-of-sample poor estimation which is typical with band-pass filters. Moreover, as it is defined as an estimate of a target which is observable (although with delay), the performance of our index at the end of the sample can be measured.

Keywords: coincident index, band-pass filter, large-dataset factor models, generalized principal components.

JEL classification: C51; E32; O30.

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1 Introduction

This paper presents a method for calculating real time estimates of the current state of the economy. The method is applied to the Euro area, the geographic focus being motivated by the creation of the European Monetary Union and the implementation of a common monetary policy. The resulting index, New Eurocoin (NE henceforth), is intended to replace the Eurocoin index proposed by Altissimo et al. (2001) and published monthly by the Centre for Economic Policy Research (see the website www.cepr.org).

The main goal of our index is an assessment of economic activity which is (a) comprehensive and non-subjective, (b) timely and (c) free from short-run fluctuations. None of the available macroeconomic series provides a measure of the state of the economy fulfilling all such criteria. GDP, the most comprehensive indicator of real activity, fails to meet (b) and (c). Regarding timeliness, GDP is only available quarterly and with a large delay. For instance, the preliminary estimate of the Euro area GDP for the first quarter of the year is usually available in May, so that in April we still do not have any idea about what happened since January. Moreover, GDP is affected by a sizable short-run component, so that, for example, the beginning of a positive medium-run wave cannot be distinguished from a transitory upward movement on a basically negative path.

Our proposed indicator is a real-time estimate of GDP growth, cleaned from short-run oscillations. Precisely:

(i) We focus on the growth rate of the GDP and define the *medium- long-run growth*, henceforth denoted by MLRG, as the component of the GDP growth rate which results by removing the oscillations of period shorter or equal to one year. This medium- long-run component of the GDP growth rate will be our target.
(ii) NE is an estimate of the MLRG for the Euro area, provided at monthly frequency and almost in real time. By the 20th of each month, we are able to produce a reliable estimate for the previous month and a good forecast for the current month.

Our definition of the MLRG is based on the spectral representation of a stationary process. Our target, being defined as including only the oscillations of period longer than one year, is a “smoothing” of GDP growth. As is well known, such a component can be obtained by applying to GDP growth a suitable band-pass filter, removing high-frequencies. However that the ideal band-pass filter is an infinite, centered, moving average. The effect of truncation is not uniform over the finite samples, the values at the end being badly estimated and subject to severe revisions as new data become available (see e.g. Baxter and King, 1999, Christiano and Fitzgerald, 2003).

A substantial mitigation of such conflict between timeliness and removal of the short-run fluctuations is the main contribution of the present paper. We obtain a good smoothing "cross-sectionally" by exploiting *current* information from a large macroeconomic dataset. The intuition is that the dataset contains variables that are leading with respect to current GDP. Therefore the information contained in the future of the GDP, which is unavailable, can be partially recovered by projecting the MLRG on a suitable set of linear combinations of current values of the variables.

Constructing such linear combinations is a crucial step of our procedure. We start with a large dataset, containing variables that are closely related to the MLRG. We might use a small number of them (as small as necessary, given the limited number of time observations) in a regression to estimate our unobserved component. But the macroeconomic series used as regressors would contain a good deal of idiosyncratic (i.e., specific to variable, country, sector, etc.) and short-run noise, which is harmful to estimate our index. The central idea is that instead of using a small number of macroeconomic variables we can employ a small number of linear combinations of the series in our large dataset, in such a way as to remove both variable-specific and short-run sources of fluctuation, while retaining cyclical and long-run movements. To do so, we use a particular kind of principal components, which are specifically designed to extract from the data set the common, medium- long-run information. More precisely, we take the linear combinations of the observable series whose fraction of common, medium- long-run variance is maximal.

Common medium- long-run variance is estimated using the Generalized Dynamic Factor Model (GDFM) proposed by Forni, Hallin, Lippi and Reichlin (2000) and Forni and Lippi (2001). The use of factor models is not new in the literature on coincident indicators, an important reference being Stock and Watson (1989), where the "cycle" is defined as the unique common factor loaded contemporaneously by a few coincident variables. By contrast, our model is designed to handle a large number of variables, affected by more than one common source of variation. Moreover, the factors are loaded with quite general impulse-response functions, so as to represent leading, coincident and lagging series (for models with these features, see also Stock and Watson 2002a, 2002b).

Let us point out that NE is not an estimate of a latent variable, as the coincident indexes constructed e.g. in Stock and Watson (1989) or those routinely produced by OECD and other international organizations. Rather, our index is defined as a real time estimate of the medium long-run component of GDP growth, and the latter is observable, although with delay. As a consequence, the performance of our index can be measured. More precisely, the value of the target, which is not available at the end-of-sample time T (the band-pass filter performing very poorly), becomes available with good accuracy at time $T + h$, for a suitable h. Therefore our index, produced at time T, can be compared with the target at T produced at $T + h$.[1] In this respect we improve upon the currently published Eurocoin.

Our dataset includes monthly series of production prices, wages, share prices, money, unemployment rates, job vacancies, interest rates, exchange rates, industrial production, orders, retail sales, imports, exports, consumer and business surveys for the Euro area countries and the Euro area as a whole (see Appendix B for details). The dataset has been organized taking into account the calendar of macroeconomic data releases which is typical in real situations, with the aim of reproducing the staggered flow of information available through time to policy makers and market forecasters. This lack of synchronism, though little considered in the literature, is crucial in assessing in a realistic way the performance of alternative real-time indexes.[2]

The paper is organized as follows. Section 2 collects some preliminary observations. Section 3 presents our target and discusses its interpretation. Sections 4 and 5 describe and motivate our estimation procedure. Section 6 shows the New Eurocoin index along with its real-time performance as compared with a few alternative estimates. Section 7 concludes. Technical details are presented in Appendix A. Appendix B describes the dataset and the data treatment.

2 Preliminary observations

To better appreciate the delay of GDP data and the potential usefulness of more timely information in assessing the current state of the economy, let us take a look at the typical calendar of some monthly series, together with GDP releases, for the Euro area (Table 1). This will be useful also to introduce some notation concerning time, illustrate how we treat the so called "end-of-sample unbalance" and how we deal with the problem of different time frequencies in our dataset.

As shown in the table, financial variables and surveys are the most timely data, while

[1] Recent papers providing direct estimates of current activity, as opposed to estimates of latent variables, are Mitchell et al. (2004) and Evans (2005).

[2] Important exception are Bernanke and Boivin (2003) and Giannone et. al. (2005).

Table 1: **The calendar of some macroeconomic series**

Time / Release time	DEC. 04	JAN. 05	FEB. 05	MAR. 05	APR. 05	MAY 05	JUN. 05	Delay
	Real time information sets →							
GDP	Q3 - 2004	Q3 - 2004	Q4 - 2004	Q4 - 2004	Q1 - 2005	Q1 - 2005	Q1 - 2005	*45-90 days*
Industrial Production	Oct. 04	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	*45-50 days*
Surveys (IFO, INSEE, ISAE)	Dec. 04 (20th day)	Jan. 05 (20th day)	Feb. 05 (20th day)	Mar. 05 (20th day)	Apr. 05 (20th day)	May. 05 (20th day)	Jun. 05 (20th day)	*0-25 days*
Retail sales	Dec. 04 (25th day)	Jan. 05 (25th day)	Feb. 05 (25th day)	Mar. 05 (25th day)	Apr. 05 (25th day)	May. 05 (25th day)	Jun. 05 (25th day)	*0-25 days*
Financial markets	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05	Jun. 05	*0 days*
CPI	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05	*15 days*
Car registrations	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05	*15 days*
Industrial orders	Oct. 04	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	*50 days*

industrial production and other real variables are usually available with longer delays. At the beginning of month $T+1$, when we produce the index for month T, surveys and financial variables are usually observed up to time T (thus with no delay), industrial production indexes up to $T-1$ and car registration and industrial orders up to $T-2$ or $T-3$. The GDP series is observed quarterly, so that its delay varies with time. In May, for instance, the delay is relatively short (just one month), since we already have the provisional estimate for the first quarter; in July, when we still have no data for the second quarter, the delay is three months.

Current produced indicators aiming at an early assessment of economic activity (such as Purchasing Managers Indexes, Consumer Surveys, Business Climate indexes, etc.) are based on the most recently delivered among the series above, but as a rule do not fulfill conditions (a) and (c) above. Surveys and standard series like industrial production and exports ignore large portions of economic activity. Moreover, all such series are affected by short-run fluctuations (see Fig. 1). As a result, none of them is fully satisfactory and "there is much diversity and uncertainty about which indicators are to be used" (Zarnowitz and Ozyildirim, 2002).

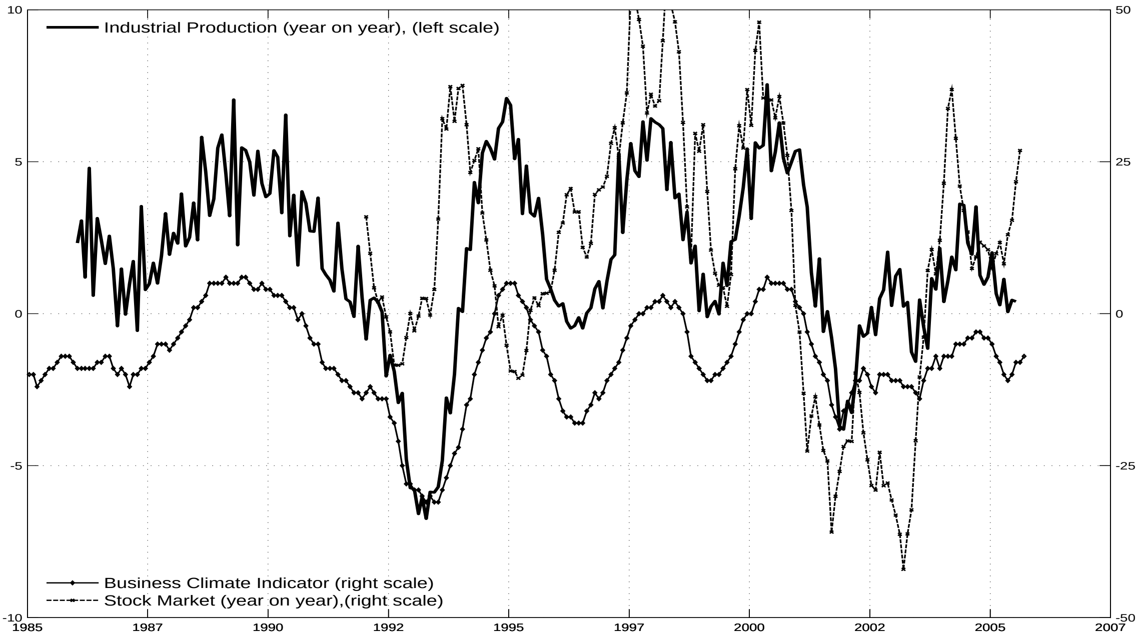

Figure 1: **Some economic indicators for the Euro area**

What we try to do here is joining the comprehensive and non-subjective information provided by GDP with the early information provided by surveys and other monthly series to obtain a reliable and timely picture of current economic activity.

Going back to Table 1, the fact that, at a given date, different series in a panel are available up to different times gives rise to an end-of-sample unbalance problem. We take it into account by shifting the monthly series (if necessary) in such a way that the last available figure is indexed with the last time period. Precisely, let again T be the month for which we are computing the index. Let x^*_{it}, $i = 1, \ldots, n$, be our monthly variables, after all transformations described in Appendix B, but before re-alignment. Suppose that the $i - th$ variable is released with k_i months of delay and therefore the last available observation is $x^*_{i,t-k_i}$. Then we set

$$x_{it} = x^*_{i,t-k_i}, \tag{1}$$

so that the last available observation of x_{it} is T for all i (of course for some variables we have to cut observations at the beginning of the sample). The beginning of our sample span is May 1987 for the transformed variables.

Since GDP is quarterly, we have to handle both monthly and quarterly data at the same time. To do this, it is convenient to think of GDP as a monthly series with missing observations. We assign to March the figure for the first quarter, that is the sum of the Gross Domestic Product of the months of January, February and March. Being available, the April figure would tell us the GDP of February, March and April. Hence there is a two-month overlapping between two subsequent elements of the series. Correspondingly, the GDP growth rate of April is the growth rate of the quarter February-April with respect to the quarter November-January.

Letting y_t be such monthly, quarter-on-quarter GDP growth rate for month t, the first observation of our sample is y_2 (June 1987) and y_t is observed only for $t = 3l - 1$, $l = 1, \dots, \lfloor T/3 \rfloor$ (where $\lfloor a \rfloor$ stands for the largest integer smaller than a). For instance, if $T = 8$ (Dec 1987), the last available observation is y_5 (Sept 1987), so that we have a three month delay, as described above, whereas if $T = 9$ (Jan 1988), the last observation is y_8 and the delay is just one month. In section 4 we show how this missing observations problem can be dealt with by using a suitable set of monthly regressors.

3 The MLRG and its interpretation

A natural way to define the medium- long-run fluctuation of a time series is to consider its spectral decomposition. Assuming stationarity, y_t can be represented as an average of infinitely many (an integral actually) mutually orthogonal sine and cosine waves of different frequency with stochastic weights. This is the well-known "spectral representation" (see e.g. Brockwell and Davis, 1991, ch. 4). We can distinguish long- and medium waves, say c_t, from short waves, say s_t, by splitting such integral into two parts, corresponding to complementary frequency intervals, separated by a threshold value. The choice of the threshold $\pi/6$ is quite natural in our context, since it corresponds to a period of one year: we are not interested in seasonality as well as higher frequency waves.

Here we do not formally introduce the spectral representation of y_t and go directly to the result (see e.g. Baxter and King, 1999, Christiano and Fitzgerald, 2003, for details). The medium- long-run component c_t is the following infinite, two-sided linear combination of the GDP growth series:

$$c_t = \beta(L) y_t = \sum_{k=-\infty}^{\infty} \beta_k y_{t-k}, \quad \beta_k = \begin{cases} \dfrac{\sin(k\pi/6)}{k\pi} & \text{for} \quad k \neq 0 \\ 1/6 \quad \text{for} & k = 0. \end{cases} \tag{2}$$

The filter $\beta(L)$ is called the low-pass filter selecting waves of frequency smaller than $\pi/6$. Our decomposition is then

$$y_t = c_t + s_t = \beta(L)y_t + [1 - \beta(L)]y_t. \tag{3}$$

Since $\beta(1) = 1$, the mean of the GDP growth series, say μ, is retained in c_t while the mean of s_t is zero. The variance of y_t is decomposed into the sum of a short-run variance and a medium- long-run variance, because c_t and s_t are orthogonal. The medium- long-run component c_t is our theoretical target MLRG.

Note that c_t is referred to as "medium- long-run growth" not as "growth-rate cycle" or "business cycle". Usually, in the definition of a "cycle" the oscillations of period longer than 8 years are also removed. This further refinement, though possible in principle, did not seem interesting to our purpose (for different definitions of the cycle, see Stock and Watson, 1999).

Formula (2) involves unobserved values of the GDP growth series, because of both missing values and finiteness of the sample span. Hence the target c_t can never be computed exactly. However, an approximation can be obtained by interpolating y_t and truncating the tails of the band-pass filter involving unavailable data (see Appendix A for details). We denote the approximation by $c_t^*(T)$, T being the last date of the sample, or c_t^* when no confusion can arise. We will see that the approximation provided by c_t^* is very poor at the end of the sample, almost perfect when t is, say, seven months away of T (see Section 4).

Figure 2 presents such approximation for the Euro zone GDP for $13 \leq t \leq T - 12$, along with y_t, $t = 3l - 1$, $l = 1, \dots, \lfloor T/3 \rfloor$, T corresponding to August 2005. We see that c_t^* tracks closely the GDP growth (MLRG captures about 70% of the variance of y_t). The main difference between MLRG and GDP growth is that the former, being free from short-run volatility, is by far smoother, so that it shows much more clearly the basic path along which economic growth is moving. An upturn (downturn) is always followed by several months of decreasing (increasing) growth. The slope changes slowly: a month characterized by a relatively large increase (decrease) in growth, say greater than 0.05%, is always followed by at least one further month (and often by several months) of increasing (decreasing) growth. As a consequence, observing MLRG in real time, besides being an assessment of the current state of the economy, would provide reliable information about what is going to happen in the near future.[3] This is why a measure of the signal behind the short-lived oscillations is useful for private and public decision makers.

[3]This is not surprising, given the occurrence of the future of GDP growth in formula (2).

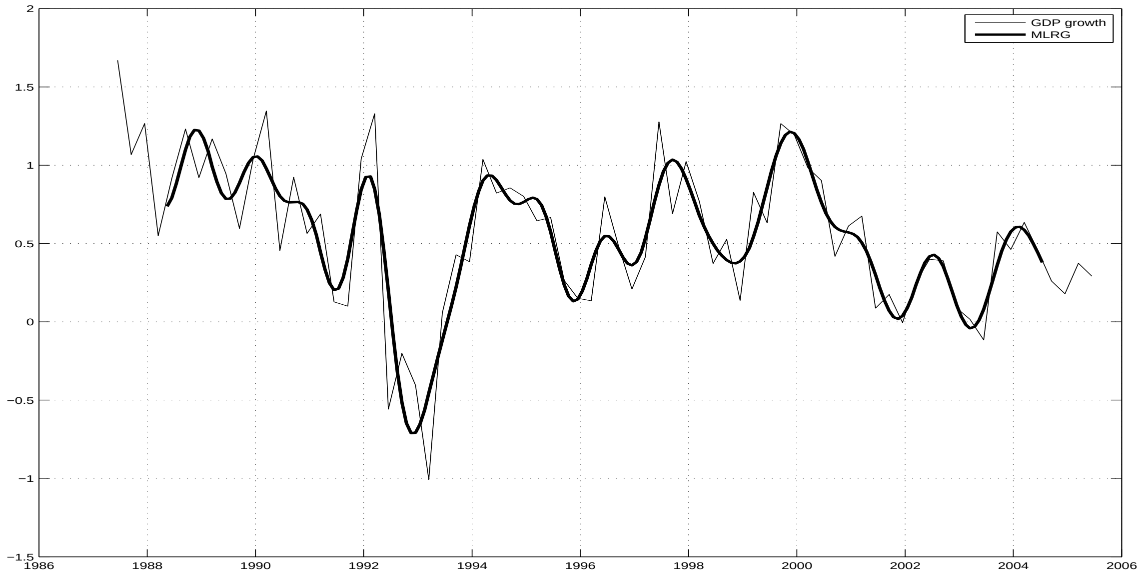

Figure 2: **The (approximate) MLRG and the GDP growth rate**

We conclude this section with a few observations about the relationship between MLRG and the year-on-year change of GDP, which is usually referred to as a good measure of medium- long-run growth. Indicating by z_t the year-on-year change of GDP, i.e. the difference between the quarter ending at t and the quarter ending at $t-12$ (divided by 4 to obtain quarterly rates) we have

$$z_t = \frac{y_t + y_{t-3} + y_{t-6} + y_{t-9}}{4}.$$

Hence z_t is a moving average of the y series which, unlike MLRG, is one-sided toward the past and hence not centered at t. As a result, z_t is lagging with respect to both y_t and MLRG by several months (precisely four and a half), as is apparent from Figure 3.

The phase shift is reduced if we compare MLRG with the future of z_t. In Section 6.4 we show that our index, which tracks MLRG, is a good predictor of future year-on-year growth.

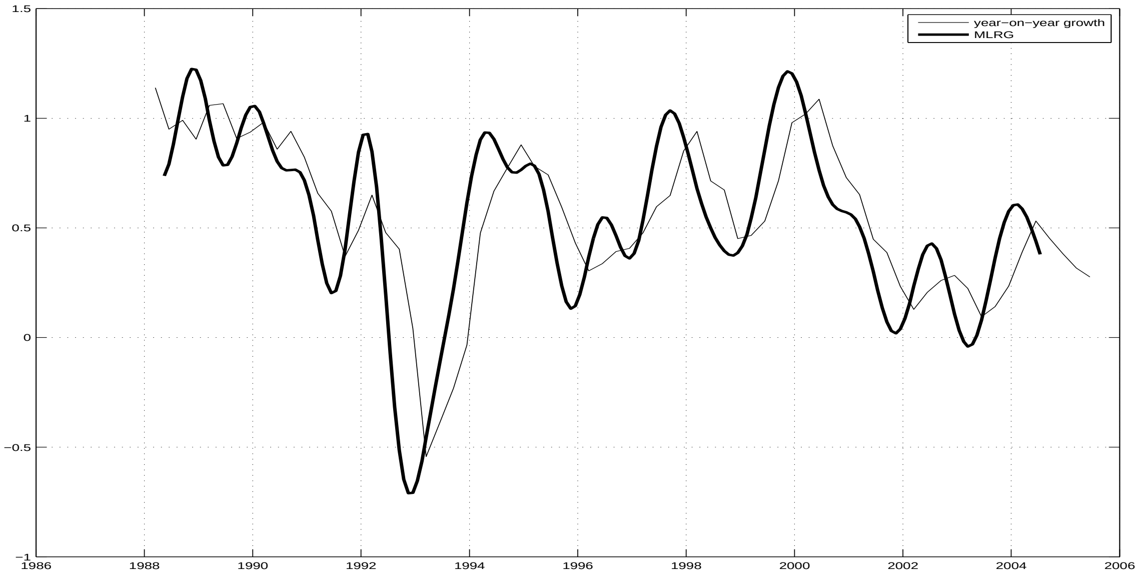

Figure 3: **The (approximate) MLRG and the year-on-year growth rate**

4 Estimation I: Projecting the MLRG on monthly series

Unfortunately, c_t^* is ill suited to produce reliable estimates of c_t in real time. Due to truncation, see Section 2, such estimates would be heavily biased toward the sample mean at the end and the beginning of the sample. This is why in Figures 2 and 3 we cut the first and the last year of c_t^*.

The solid line in Figure 4 shows the variance of $c_t(T) - c_t^*(T)$, normalized by the variance of c_t, for $T = t, t+1, \ldots, t+10$. We anticipate here that the variance of the difference between the target and NE is 16.7% (see dashed line in Figure 4). We see that the error, using $c_t^*(T)$, is huge at the end of the sample, almost 80% for $T = t$ and that NE has a substantial advantage for the first three months.[4]

The end-of-sample bias could in principle be reduced somewhat, as suggested by Chris-

[4]Figure 4 was suggested to us by Mark Watson. The solid line is a somewhat stylized representation. In real situations the shape of the solid line depends on the position of the month t within the quarter. For example, if t is January, c_t^* becomes more reliable than NE only in May.

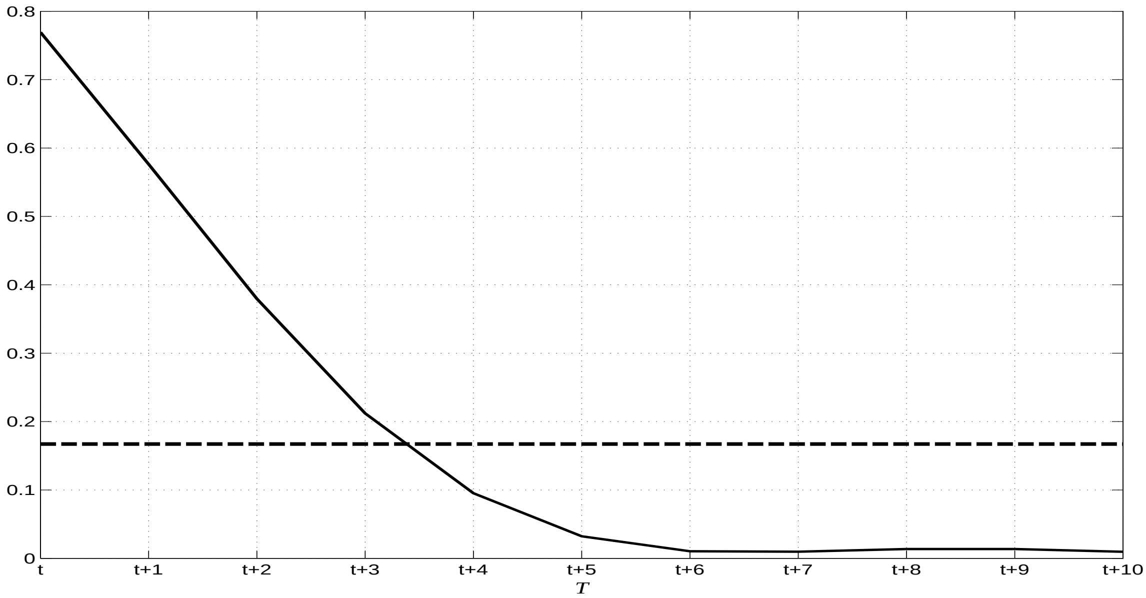

Figure 4: **Estimation of c_t: variance of the approximation error using c_t^* (solid line) and NE (dashed line).**

tiano and Fitzgerald (2003), by projecting the target on the available GDP growth data. This provides a filter (the "asymmetric band-pass filter") which, contrary to the truncated, symmetric band-pass filter, exploits the auto covariance structure of the original series. In practice however, such method does not improve upon the truncated filter in the present case (see below). Moreover, it exploits only GDP data, thus ignoring crucial information coming from more timely macroeconomic series.

Valle e Azevedo, Koopman and Rua (2006) propose a multivariate method with band-pass filter properties which exploits information from a relatively small number of variables. We are not far in spirit from their work, the difference being that our procedure is designed to extract information from a large panel of time series. Our basic idea is to construct a set of regressors which captures the relevant information, leading in particular, and project the MLRG on such regressors. In the following section we describe the method used to obtain the regressors, while in the remainder of the present section we explain how we compute the projection, given a set of regressors.

Let us start with a set of zero-mean time series w_{kt}^m, $k = 1, \ldots, r$, expressed in month-on-month changes or rates of change, just like most of the series in our data set. Since our target is expressed in quarter-on-quarter variations, we need a preliminary transformation in order to put the regressor on the same scale. This can easily be done by observing that if the flow variable w^m_{kt} is the month-on-month change $W^m_{kt} - W^m_{k,t-1}$, then the corresponding quarter-on-quarter change, $w_{kt} = W^m_{kt} + W^m_{k,t-1} + W^m_{k,t-2} - W^m_{k,t-3} - W^m_{k,t-4} - W^m_{k,t-5}$ is given by

$$w_{kt} = (1 + L + L^2)^2 w^m_{kt}. \tag{4}$$

A similar relation holds approximately for change rates, with the filter $(1 + L + L^2)^2/3$ replacing $(1 + L + L^2)^2$. We use the filter $(1 + L + L^2)^2$ for all regressors, since we are interested in the projection, which is invariant to the measurement unit.

The theoretical projection of c_t on the linear space spanned by the entries of $w_t = (w_{1t} \ \cdots \ w_{rt})'$ and the constant is

$$P(c_t|w_t) = \mu + \Sigma_{cw}\Sigma_w^{-1} w_t, \tag{5}$$

where Σ_{cw} is the row vector whose h-th entry is $\text{cov}(c_t, w_{ht})$ and Σ_w is the covariance matrix of w_t. NE is obtained by replacing the above population moments with appropriate estimators:

$$\hat{c}_t = \hat{\mu} + \hat{\Sigma}_{cw}\hat{\Sigma}_w^{-1} w_t. \tag{6}$$

The formulas for $\hat{\mu}$, $\hat{\Sigma}_w$ and $\hat{\Sigma}_{cw}$ are given in Appendix A. Here we only note that estimation of $\hat{\Sigma}_{cw}$ is not trivial, since c_t is not observed. Nevertheless, the cross-covariogram of a quarterly series with a monthly series (unlike the auto-covariogram) can be estimated consistently for any lag, despite the missing observations. Hence we can estimate the cross-spectrum of y_t and w_t. A consistent estimate of Σ_{cw} is then obtained by integrating such cross-spectrum over the relevant frequency band, without resorting to interpolation of y_t.

5 Estimation II: Constructing the regressors

Let us now come to the method adopted to construct the regressors w^m_{kt}, $k = 1, \ldots, r$. Our starting point is that extracting low-dimensional information from a large panel of time series can produce better results in forecasting (and nowcasting) than using standard forecasting methods with observable variables. This point of view is in line with the seminal work of Burns and Mitchell (1946), in which the comovements of macroeconomic variables at medium-run frequencies are strongly emphasized, and is the basis of a recent and growing literature on large factor models, including Bai (2003), Bai and Ng (2002), Boivin and Ng (2005), D'Agostino and Giannone (2005), Forni, Hallin, Lippi and Reichlin (2000, 2001, 2004, 2005 FHLR from now on), Forni and Lippi (2001), Stock and Watson (2002a, 2002b).

The importance of extracting low-dimensional information is fairly natural in our setting, namely predicting the MLRG using a large dataset. Indeed, regressing directly the MLRG on our dataset implies selecting a small number of series, given the short time span. On the other hand, whatever the choice of the series to be used as regressors, each of them would be partly affected by specific, idiosyncratic shocks, as well as short-run or seasonal variations, which have no relationship to the MLRG. Both issues, that is (i) reducing the dimension of the data set and (ii) extracting cyclical movements by removing idiosyncratic and short-run sources of fluctuation, will be solved here by taking as regressors a small number of linear combinations of all our dataset.

When the task is summarizing information from a large dataset, ordinary principal components are the most popular linear combination. Ordinary principal components, however, accomplish only one half of the job: They cancel out the idiosyncratic components (Stock and Watson, 2002a, 2002b), but do not remove short-run fluctuations. Figure 5 illustrates the issue by showing the performance of ordinary principal component (filtered with $(1 + L + L^2)^2$ as explained above) in fitting c_t^*. We set the number of principal components to retain (12) by means of the Bai-Ng criterion PC2 ($r_{\max} = 25$) [5]. While the fit is acceptable (the correlation coefficient is as high as 0.89), the projection does not remove the very short-run fluctuations contained in the regressors, so that the resulting line is rather jagged.

To obtain smoothing, we solve a signal extraction problem: we take the linear combinations of the observable series whose fraction of common, medium-long-run variance is maximal.

Our definition of common, medium-long-run variance stems quite naturally from our reference model, the Generalized Dynamic Factor Model (FHLR 2000, Forni and Lippi, 2001). Let x_{it}, $t = 1, \ldots, T$ be the i-th series of our data set, after transformation and realignment. We assume that each series is the sum of two stationary, mutually orthogonal (at all leads and lags), unobservable components: the "common component", call it χ_{it},

[5] See Bai and Ng (2002).

Figure 5: **The projection of c_t on the largest 12 principal components of our data set**

and the "idiosyncratic component", ξ_{it}:

$$x_{it} = \chi_{it} + \xi_{it}. \tag{7}$$

The common component is driven by a small number, say q, of common shocks u_{ht}, $h = 1, \ldots, q$, which are the same for all the cross-sectional units, but are in general loaded with different coefficients and lag structures:

$$\chi_{it} = b_{1i}(L)u_{1t} + \cdots + b_{qi}(L)u_{qt}. \tag{8}$$

By contrast, the idiosyncratic component is driven by variable-specific shocks. The distinguishing feature of such shocks is that they are not "pervasive", so that the idiosyncratic components are "poorly" correlated at all leads and lags. For simplicity we further restrict the model by assuming that ξ_{it} and ξ_{jt} are mutually orthogonal at all leads and lags for $i \neq j$. [6]

[6]For details on the conditions imposed on the correlation structure of the common components, as well as on other assumptions of the model, we refer to FHLR (2000).

We can use the band-pass filter $\beta(L)$ introduced in Section 2 to further decompose the common components into the sum of a medium- long-run component $\phi_{it} = \beta(L)\chi_{it}$ and a short-run component $\psi_{it} = [1 - \beta(L)]\chi_{it}$ (see Altissimo et al., 2001), so that

$$x_{it} = \phi_{it} + \psi_{it} + \xi_{it}. \tag{9}$$

Correspondingly, the variance-covariance matrix of $x_t = (x_{1t} \ \cdots \ x_{nt})'$ is decomposed as follows:

$$\Sigma_x = \Sigma_\chi + \Sigma_\xi = \Sigma_\phi + \Sigma_\psi + \Sigma_\xi. \tag{10}$$

Consistent estimates $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ can be obtained following FHLR (2000); the formulas are given in Appendix A.3.

The matrices $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ are all we need to construct our smooth regressors. We start determining the linear combination of the x's such that the variance of the common component in the relevant spectral band is maximal. Then we determine another linear combination with the same property under the constraint of orthogonality to the first, and so on. Formally, setting $x_t = (x_{1t} \ \cdots \ x_{nt})'$, we look for the vectors v_k, $k = 1, \ldots, n$, and the corresponding linear combinations $w^m_{kt} = v'_k x_t$, solving the sequence of maximization problems

$$\max_{v \in R^n} v'\hat{\Sigma}_\phi v, \quad \text{s.t. } v'(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi)v = 1, \quad v'\left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_h = 0 \ \text{ for } h < k,$$

where $v_0 = 0$ and v_h solves problem h.

The solution of this sequence of problems is known and is given by the *generalized eigenvectors* $v_1, \ldots, v_n$ associated with the *generalized eigenvalues* λ_k, ordered from the largest to the smallest, of the couple of matrices $\left(\hat{\Sigma}_\phi, \ \hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right)$; i.e. the vectors satisfying

$$\hat{\Sigma}_\phi v_k = \lambda_k \left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_k, \tag{11}$$

with the normalization constraints $v'_k \left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_k = 1$ and $v'_k \left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_h = 0$ for $k \neq h$ (see Anderson, 1984, Theorem A.2.4, p. 590). The eigenvalue λ_k indicates the ratio of common, medium-long run to total variance explained by the k-th eigenvector. Of course, such a ratio is decreasing with k, so that, the greater is k, the less smooth and more idiosyncratic is w^m_{kt}.

Summing up, our indicator is obtained as follows. First, we compute $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ as explained in Appendix A.3. Second, we compute the generalized eigenvectors v_k, $k = 1, \ldots, r$ satisfying (11) and the associated linear combinations $w^m_{kt} = v'_k x_t$. Finally, we apply the filter (4) to get the quarter-on-quarter regressors w_{kt} and compute NE by using (6).

6 Results

6.1 The indicator

As shown by the formulas for $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ in Appendix A.3, the construction of the regressors w_{kt} requires the determination of three key parameters: the number of common shocks of the factor model (7), q, the lag window size, M, and the number of linear combinations to retain as regressors, r. Using the results of a pseudo real-time analysis (see below), we set $q = 2$, $M = 24$ and $r = 6$. We decided on the basis of a judgmental compromise among three performance measures: the mean-square error of the nowcast with respect to c_t^*, the percentage of correctly predicted slope signs (see Pesaran and Timmermann, 1992) and the mean-square revision error after one month.

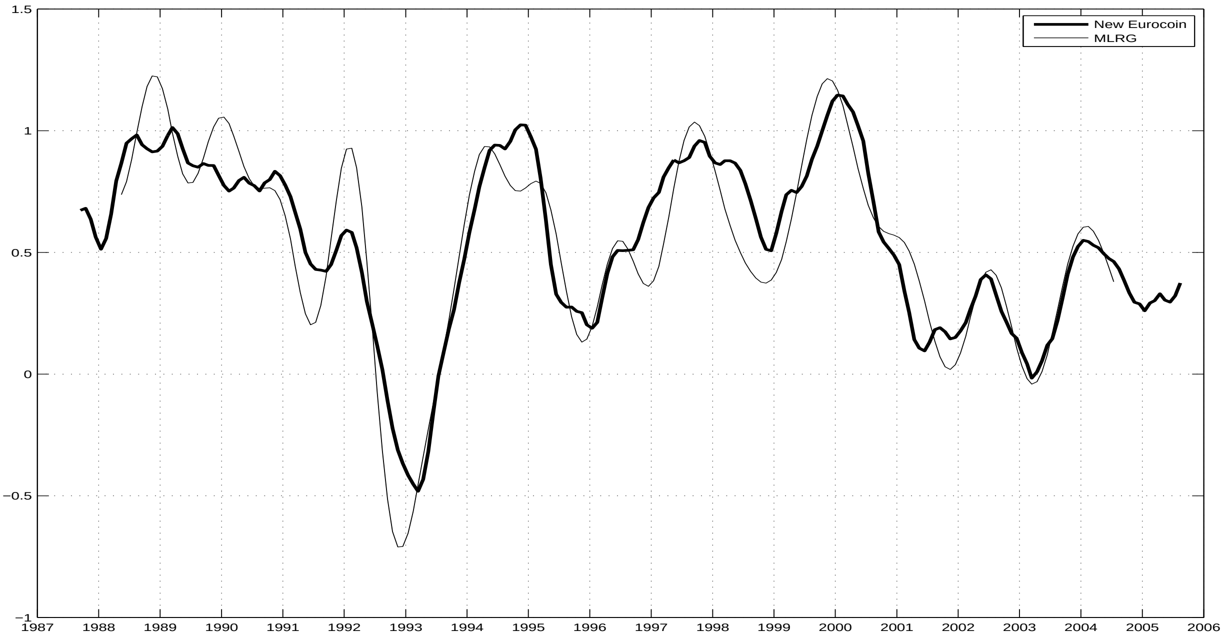

Figure 6: **New Eurocoin and MLRG**

Figure 6 shows the New Eurocoin indicator $\hat{c}_t$ (thick solid line) and c_t^*, $13 \leq t \leq T-12$, (thin solid line). Regarding the in-sample fit, NE is slightly better than the principal component index (PC) of Figure 5 (the correlation coefficient is 0.91, as against 0.89), though we use just 6 regressor (instead of 12). But the most noticeable improvement concerns smoothness: the total number of slope changes of NE is 40, as against 78 of the PC index.

It should be stressed that such a result is obtained by means of a contemporaneous, cross-sectional averaging, without resorting to the time dimension (except for the filter in (4), which was used also for the PC-based index). This may be surprising, given the definition of the target c_t as an average including both past and future values of GDP growth. To get an intuition of how such "cross-sectional" smoothing can work, we can refer to representation (8) of the common components, under the simplifying assumption of only one common shock u_t. Different variables load u_t with different lag distributions. In particular, the concentration of the weights at different displacement of the polynomial identify leading, lagging and coincident variables respectively. Under the reasonable assumption that the GDP growth is among the coincident variables, and that the dataset contains a good number of leading and lagging variables, the latter will act as proxies for future and past values of the GDP growth.

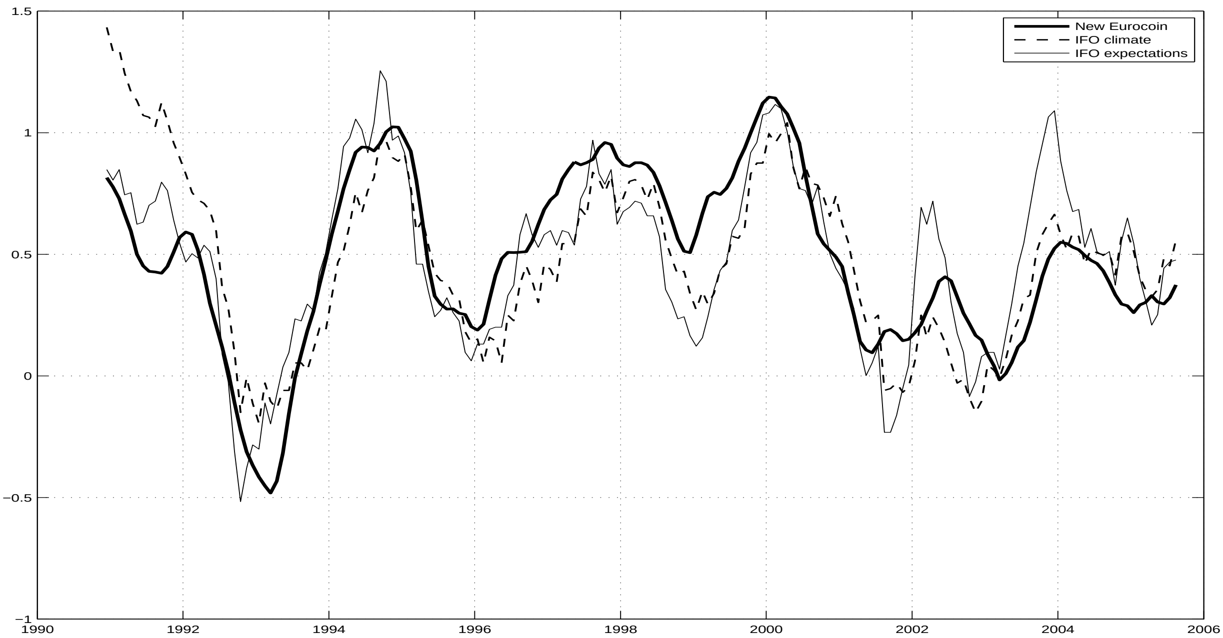

Figure 7: **New Eurocoin and two IFO indexes**

Figure 7 shows NE (bold solid line) along with two well-known German indexes of economic activity: the overall IFO index (dashed line) and the IFO index of business expectations (thin solid line). The IFO indexes are normalized in such a way as to have the same mean and variance as NE. The general message of NE and the two IFO indexes is essentially the same. However, there are two important differences. Firstly, our index is by far less jagged, so that in most cases it correctly signals whether growth is increasing or decreasing. Secondly, the IFO figures have no quantitative interpretation in terms of GDP growth.

6.2 The real-time performance

In this subsection we report a pseudo real-time performance evaluation of NE.[7] The exercise is run over the period November 1998-August 2005, where t denotes the running month, while T denotes August 2005. The estimate of NE for time t obtained using the data up to $t+h$ is denoted by $\hat{c}_t(t+h)$.

Figure 8, upper graph, illustrates the results. The long continuous line represents $c_t^*(T)$. The short line ending at t represents the three estimates: $\hat{c}_{t-2}(t)$, $\hat{c}_{t-1}(t)$ and $\hat{c}_t(t)$. Therefore the three points on the short lines over a given t are the first estimate and two revisions of NE at t, namely $\hat{c}_t(t)$, $\hat{c}_t(t+1)$ and $\hat{c}_t(t+2)$. The bullets indicate turning points and the diamonds indicate turning pont signals (see below). For comparison, the lower graph shows the end-of-sample estimates obtained by truncating the band-pass filter at the last available GDP observation. Therefore each short line represents $c_{t-2}^*(t)$, $c_{t-1}^*(t)$ and $c_t^*(t)$. Clearly the band-pass filter estimates (BP), though being perfectly smooth, exhibit a large bias towards the sample mean. NE estimates are much more accurate and the revision errors are by far smaller.

Let us now analyze results in detail. We are interested in

(a) the ability of $\hat{c}_t(t)$ to approximate (nowcast) $c_t^*(T)$, for the period $T-81 \leq t \leq T-12$, as measured by the mean-square error $\sum_{t=T-81}^{T-12}[\hat{c}_t(t) - c_t^*(T)]^2/70$;

(b) the ability of $\hat{c}_t(t) - \hat{c}_{t-1}(t) = \Delta\hat{c}_t(t)$ to signal the correct sign of the change, i.e. the sign of $\Delta c_t^*(T)$, as measured by the percentage of correct signs (see Pesaran and Timmermann, 1992);

[7] Here "pseudo" means that we do not use the true real-time preliminary estimates of the GDP, but the final estimates as reported in the GDP "vintage" available in September 2005. The same holds true for all other monthly variables

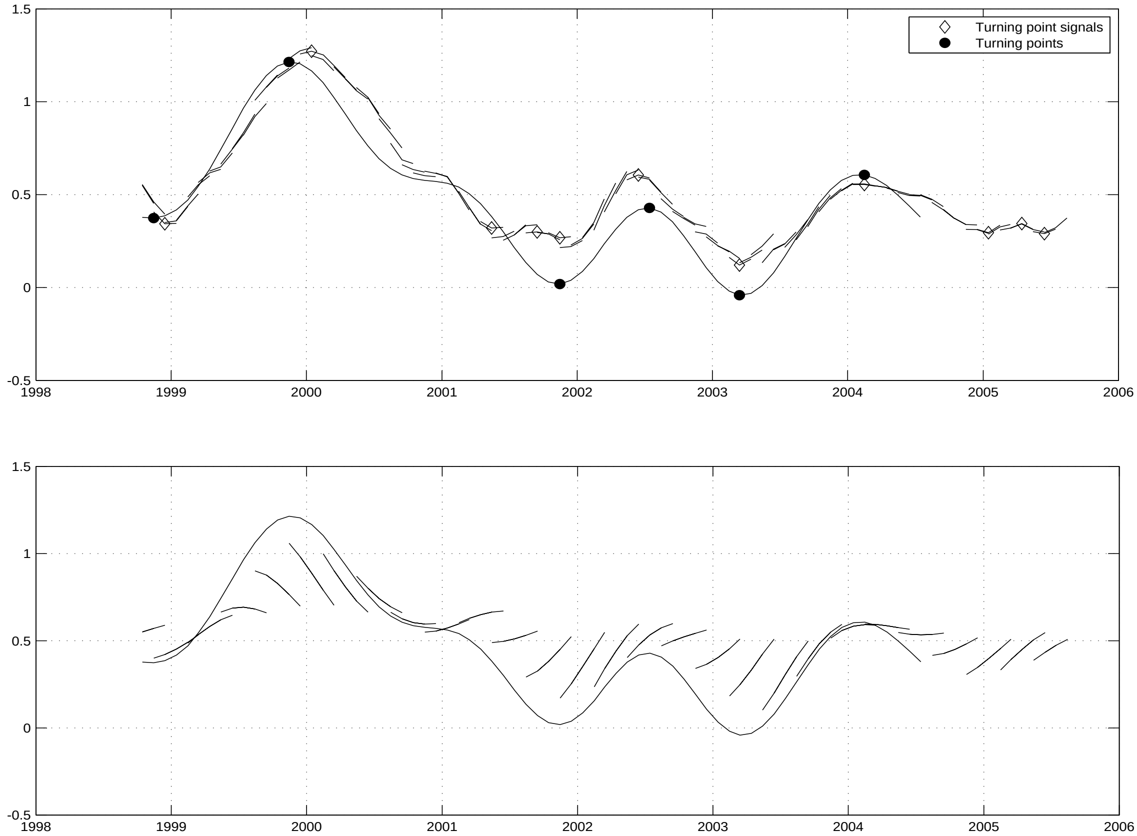

Figure 8: **Pseudo real-time estimates of MLRG, at the end of the sample, obtained with NE (upper panel) and the band-pass filter (BP)**

(c) the size of the revision errors after one month, as measured by the mean-square deviation $\sum_{t=T-81}^{T-1}[\hat{c}_t(t+1) - \hat{c}_t(t)]^2/81$;

For comparison with NE, we consider three alternative methods:

(BP) the truncated band-pass filter $c_t^*(t)$;

(ABP) the asymmetric filter proposed by Christiano and Fitzgerald (2003), applied to the GDP growth series after the same linear interpolation used for $c_t^*(t)$;

(PC) the estimates obtained by replacing the regressors w_{kt}^m with ordinary principal components (we use the first 12 principal components).

Table 2: **End of sample performance**

Indicator	Rmse with respect to c^*	% correct signs direction of c^*	Rmse Revision errors
NE	0.15	$0.86^{\dagger}$	0.03
BP	0.27	0.57	0.09
ABP	0.31	0.59	0.08
PC	0.20	0.64	0.04

Notes: Sample Nov.1998-Aug.2005. The first column reports the RMSE with respect to c^*. The second column reports the percentage of correct signs with respect to those of Δc^*. A (†) indicates that the null of no predictive performance is rejected at 5% significance level (Pesaran Timmerman, 1992). The third column reports the RMSE of the revision errors for the last estimate of NE after one month.

In Table 2 we see that NE scores remarkably better than BP and ABP regarding nowcasting of c^*, tracking the target direction of change, and in terms of size of revision error (points (a), (b) and (c) above). As expected, PC performs fairly well as far as (a) and (c) are concerned, but is by far outperformed by NE in terms of tracking the direction of change in the target (b). Hence, NE dominates all other indicators in terms of the criteria we selected.

6.3 The behavior around turning points

The figures in the second column of Table 2, concerning the percentage of correct signs, suggest that NE should perform well in signaling the turning points. In the remainder of the present section we analyze the quality of NE turning point signals. To this end, we need precise definitions of turning point, turning point signal and false signal.

To begin with, we define a *turning point* as a slope sign change of $c^*_t(T)$ within the interval $13 \leq t \leq T-12$. We have an *upturn (downturn)* at time t, $13 \leq t \leq T-12$, if $\Delta c^*_{t+1}(T) = c^*_{t+1}(T) - c^*_t(T)$ is positive (negative), whereas $\Delta c^*_t(T) = c^*_t(T) - c^*_{t-1}(T)$ is negative (positive). With this definition, we have 11 downturns and 10 upturns in the whole period $13 \leq t \leq T-12$, and 3 downturns and 3 upturns in the subsample involved in the pseudo real-time exercise.

Next we need a rule to decide when a slope sign change of our index $\hat{c}$ has to be interpreted as a reliable signal of a turning point in c^*. We examine the sign of the last two changes of the current estimate, and the sign of the last two changes of the previous

Table 3: **Classification of signals**

	$\Delta\hat{c}_{t-2}(t-1)$	$\Delta\hat{c}_{t-1}(t-1)$	$\Delta\hat{c}_{t-1}(t)$	$\Delta\hat{c}_{t}(t)$	consistency	signal type
1	−	−	−	+	yes	**upturn** at $t-1$
2	+	−	−	+	yes	uncertainty
3	−	−	−	−	yes	deceleration
4	+	−	−	−	yes	slowdown
5	+	+	+	−	yes	**downturn** at $t-1$
6	−	+	+	−	yes	uncertainty
7	+	+	+	+	yes	acceleration
8	−	+	+	+	yes	recovery
9	−	−	+	−	no	trembling deceleration
10	+	−	+	−	no	downturn at $t-2$ shifted
11	−	−	+	+	no	missed upturn
12	+	−	+	+	no	downturn at $t-2$ not confirmed
13	+	+	−	+	no	trembling deceleration
14	−	+	−	+	no	upturn at $t-2$ shifted
15	+	+	−	−	no	missed downturn
16	−	+	−	−	no	upturn at $t-2$ not confirmed

estimate, that is

$$\Delta\hat{c}_{t-1}(t) \qquad \Delta\hat{c}_{t}(t) \tag{12}$$

$$\Delta\hat{c}_{t-2}(t-1) \qquad \Delta\hat{c}_{t-1}(t-1) \tag{13}$$

A sign change makes (12) a candidate as a *signal at t locating a turning point at $t-1$*. However, we accept the sign change in (12) as a turning point signal only if
(a) there is consistency between (12) and (13) at $t-1$, i.e. the signs of $\Delta\hat{c}_{t-1}(t-1)$ and $\Delta\hat{c}_{t-1}(t)$ coincide,
(b) there is no sign change in (13) between $t-2$ and $t-1$, i.e. the signs of $\Delta\hat{c}_{t-2}(t-1)$ and $\Delta\hat{c}_{t-1}(t-1)$ coincide.

The reason for conditions (a) and (b) is that we want to be strict enough to rule out sign changes that may be caused by unstable estimates rather than by true turning points. Condition (a) is obvious. Condition (b) rules out a sign change between $t-1$ and t that follows the opposite change between $t-2$ and $t-1$ in the previous estimate.

Table 5 provides a classification for the 8 consistent and the 8 inconsistent signals. Only two out of the eight sign changes in (12) are confirmed as turning point signals, namely the first and the fifth, an upturn and a downturn respectively.

Finally, we say that an upturn (downturn) signal at t locating a turning point at $t-1$ is *false* if c^* has no upturns (downturns) in the interval $[t-3, t+1]$, *correct* otherwise.

With this definition, an upturn signal leading or lagging the true upturn by a quarter or more is false, whereas a two-month error is tolerated.

Table 4: **Real time detection of turning points (TP)**

Target	consistent signals	uncertainty signals	TP signals	TP signals excl. endpoints	Correct TP	% correct TP	% missed TP
NE	81	0	11	8	6	75	0
BP	81	0	4	4	1	25	83
ABP	77	0	3	3	2	67	67
PC	76	16	17	14	5	36	17

Notes: the first column reports the number of consistent signals (out of 81). The fourth column reports the number of turning point signals when excluding the last 12 signals. The fifth column counts the number of correct turning point signals, i.e. those matching the ones in the target. The last shows the percentage of turning points in the target which are missed by each indicator.

Table 4 shows results for the competing indexes in our real time exercise, which spans over 82 months from November 1998 to August 2005. The first signal is missing as the previous estimate is lacking, hence, overall, we have 81 signals. Interestingly, across methods most signals in real-time are consistent, all of them for NE and BP, 76 with the index based on simple principal components (PC). The latter also provides 16 uncertain signals. Coming to the turning points identified by each indicator (third column), NE indicates 11 turning points, 8 of which identified before the last twelve months, the period over which we cannot compute the c^* index. Among these, 6 correspond to all the turning points in the target. The PC index continues to perform poorly, not only in signaling too often a turning point which does not correspond to actual movements in the target, but also missing one of them.

6.4 The forecasting properties of the indicator

In Section 3 we argued that we should expect a close match between NE and the GDP growth rate, in particular once the latter is smoothed with a moving average such as the one induced by the year-on-year transformation, and adjusted for the phase shift.[8] The last two columns of table 5 show that this feature is indeed confirmed in the data. While the RMSE of NE with respect to quarter on quarter GDP growth (first column) is 0.20, the same statistics with respect to year-on-year growth is 0.18 (second column)

[8] A similar idea is exploited in Cristadoro et al. (2005) to motivate their result that a core inflation index obtained as a smoothed projection of CPI inflation on factors is a good forecaster of the CPI headline inflation.

and reduces to 0.13 and 0.17 when we adjust for the phase shift by considering future year-on-year growth (third and fourth column)[9]. The competing indicators do not have similar forecasting properties.

Table 5: **How to relate the monthly indicator to actual GDP growth**

	RMSE with respect to different growth rates (%)			
Indicator	quarter-on-quarter current quarter	year-on-year current quarter	year-on-year 1 quarter ahead	year-on-year 2 quarters ahead
NE	0.20	0.18	0.13	0.17
BP	0.34	0.26	0.27	0.28
ABP	0.32	0.27	0.26	0.29
PC	0.21	0.20	0.17	0.21

Notes: Sample Nov.1998-Aug.2005.

To better gauge the forecasting ability of NE we compare it with the one of univariate ARMA models of quarterly GDP growth, selected by standard in-sample criteria. Such models are often used as benchmark in forecasting studies (Stock and Watson, 2002b).

Table 6: **Pseudo real-time forecast performance**

	Target growth rates (%)		
Model	quarter-on-quarter current quarter	year-on-year current quarter	year-on-year 1 quarter ahead
NE	0.20	0.18	0.13
AR (AIC)	0.35	0.19	0.25
AR (BIC)	0.36	0.18	0.24
ARMA (AIC)	0.35	0.21	0.25
ARMA (BIC)	0.35	0.15	0.23
Random walk	0.31	0.18	0.19

Notes: Sample Nov.1998-Aug.2005. The first column reports the root mean square forecast error with respect to *current* quarter-on-quarter GDP growth rate, the second with respect to *current* year-on-year GDP growth rate, the third with respect to *next* quarter year-on-year GDP growth rate. NE is the New Eurocoin forecast obtained using the monthly dataset with information updated at most up to last month of the current quarter. The AR and ARMA models are selected at each step according to their in sample performance (in parenthesis the selection criterion used), and are estimated on the quarterly GDP series.

As shown in Table 6, for quarter-on-quarter GDP growth (first column) and for the year-on-year growth rate one quarter ahead (third column) the forecast error of the indicator is very much lower than the one of ARMA models and the random walk model.

[9]Clearly, we can compare our monthly indicator with actual GDP growth rates only at the end of each quarter.

7 Summary and conclusion

Our coincident index NE is a timely estimate of the medium- long-run component of the Euro area GDP growth. The latter, our target, has been defined as a centered, symmetric moving average of the GDP growth, whose weights are designed to remove all fluctuations of period shorter than one year (the band-pass filter). As we have observed in Section 3, our target, which has a rigorous spectral definition, leads the "popular" measure of medium- long-run change, namely the year-on-year GDP growth, by several months.

We avoid the large end-of-sample bias typical of two-sided filters by projecting the target onto suitable linear combinations of a large set of monthly macroeconomic variables. Such linear combinations are designed as to discard useless information, namely idiosyncratic and short-run noise, and retain relevant information, i.e. common, cyclical and long-run waves. Both the definition and the estimation of the common, medium-long-run waves are based on recent factor model techniques. Embedding the smoothing into the construction of the regressors is in our opinion an important contribution of the present paper.

The performance of NE as a real-time estimator of the target has been presented in detail in Section 6. The indicator is smooth and easy to interpret. In terms of turning points detection, it scores much better than the competitors that naturally arise as estimators of the medium- long-run component of GDP growth in real-time. The reliability of the signal is reinforced by the fact that the revision error of our indicator is extremely small compared to the competitors. We have also shown that NE is a very good forecaster of year on year GDP growth one and two quarters ahead; it also scores well in forecasting quarter on quarter GDP growth, with a RMSE of 0.20, which ranks well even in comparison with best practice results.

Appendix A: Technical details

A.1 The formula for c_t^* (Section 3)

The approximation c_t^* is computed by using the formula

$$c_t^* = \hat{\mu} + \beta(L)Y_t, \tag{14}$$

where

$$\hat{\mu} = \sum_{l=1}^{\lfloor T/3 \rfloor} y_{3l-1} / \lfloor T/3 \rfloor \tag{15}$$

and

$$Y_t = \begin{cases} y_t - \hat{\mu} \quad \text{for } t = 3l-1, \quad 1 \le l \le \lfloor T/3 \rfloor \\ \frac{2}{3} y_{3l-1} + \frac{1}{3} y_{3l+2)} - \hat{\mu} \quad \text{for } t = 3l, \quad 1 \le l \le \lfloor T/3 \rfloor - 1 \\ \frac{1}{3} y_{3l-1} + \frac{2}{3} y_{3l+2} - \hat{\mu} \quad \text{for } t = 3l+1, \quad 1 \le l \le \lfloor T/3 \rfloor - 1 \\ 0 \quad \text{for } t < 1 \text{ and } t > 3\lfloor T/3 \rfloor - 1. \end{cases}$$

In words, Y_t is obtained by centering (de-meaning) y_t, filling the in-sample missing values by linear interpolation and adding zeros outside the sample period. T denotes the last observation available for our monthly series after the end-of-sample unbalance treatment described in Section 2. Note that formula (14) takes into account the publication delay of the GDP series described in Section 2.

The approximation c_t^* is affected by two sources of errors. First, the above formula implies a truncation of the infinite filter $\beta(L)$ at the end of the sample. This error is negligible for values of t located in an appropriate interval $1 < t_1 \le t \le t_2 < T$, since for such values of t we truncate tails with very small coefficients. Cutting one year at the beginning and the end of the sample is sufficient to obtain a very good approximation in the present case, so that we set $t_1 = 13$ and $t_2 = T - 12$.

Second, there is an error induced by linear interpolation. The size of such error depends on the autocorrelation structure of the original series, which cannot be estimated at all lags for y_t because of the missing values. However this error is probably negligible, due to the high autocorrelation induced by the two-month overlapping of the series with its first lag. To get an indirect confirmation of this, we can take a monthly series, compute its quarter-on-quarter growth rate z_t, compute the interpolated series Z_t, and compare $\beta(L)z_t$

with $\beta(L)Z_t$. For the industrial production index of the Euro zone we got a correlation coefficient of 0.9987. Similar results are obtained for other series.

We conclude that, for the interval $13 \leq t \leq T-12$, we can retain c_t^* as an almost perfect approximation of our theoretical target c_t.

A.2 Formulas for $\hat{\mu}$, $\hat{\Sigma}_{cw}$ and $\hat{\Sigma}_w$ (Section 4)

As observed in the main text, μ and Σ_w can be trivially estimated by their sample counterparts (15) and

$$\hat{\Sigma}_w = \sum_{t=1}^{T} w_t w_t' / (T-1). \tag{16}$$

Estimation of Σ_{cw} is less obvious, since c_t is not observed. We proceed as follows. First, we estimate the covariance of y_t and w_t at lags $k = -M, \ldots, M$ as

$$\hat{\Sigma}_{yw}(k) = \sum_{l} y_{3l-1} w'_{3l-1-k} / (\lfloor (T-k)/3 \rfloor - 1), \tag{17}$$

where l varies from $\max[1, 1 + \lfloor (k+1)/3 \rfloor]$ to $\min[\lfloor T/3 \rfloor, \lfloor (T-k)/3 \rfloor]$. Note that the cross-covariances $\Sigma_{yw}(k)$ can be consistently estimated for any lag k, despite the fact that y_t is only observed quarterly.

Then, we estimate the cross-spectrum over the relevant frequency interval, at the $2J+1$ equally spaced points θ_j, by using the Bartlett lag-window estimator

$$\hat{S}_{yw}(\theta_j) = \frac{1}{2\pi} \sum_{k=-M}^{M} W_k \, \hat{\Sigma}_{yw}(k) \, e^{-i\theta_j k}, \tag{18}$$

where $W_k = 1 - \frac{|k|}{M+1}$ and $\theta_j = \frac{\pi j}{3(2J+1)}$, $j = -J, \ldots, J$. Note that the larger frequency estimated is not $\pi/6$, but the middle point of the $(2J+1)$-th interval, ending at $\pi/6$. Finally, we estimate Σ_{cw} by averaging the cross-spectrum over such points, i.e.

$$\hat{\Sigma}_{cw} = \frac{2\pi}{2J+1} \sum_{j=-J}^{J} \hat{S}_{yu}(\theta_j). \tag{19}$$

For New Eurocoin we set $J = 60$ and $M = 24$.

A.3 Formulas for $\hat{\Sigma}_\phi$, $\hat{\Sigma}_\chi$ and $\hat{\Sigma}_\xi$ (Section 5)

To get an estimate of Σ_χ we have first to estimate the spectral density matrix of the vector of monthly variables $x_t = (x_{1t} \ \cdots \ x_{nt})'$. We estimate the covariance matrices of x_t at lags $l = -M, \ldots, M$, as

$$\hat{\Sigma}_x(l) = \sum_t x_t x'_{t-l}/(T-l),$$

where t varies from $\max[1, 1+l]$ to $\min[T, T-l]$. Then we estimate the spectrum of x_t at the $2J+1$ equally spaced frequencies θ_j by using the Bartlett lag-window estimator

$$\hat{S}_x(\theta_j) = \frac{1}{2\pi} \sum_{l=-M}^{M} W_l \ \hat{\Sigma}_x(l) \ e^{-i\theta_j l}, \tag{20}$$

where $W_l = 1 - \frac{|l|}{M+1}$ and $\theta_j = \frac{2\pi j}{2J+1}$, $j = -J, \ldots, J$. Again we set $J = 60$ and $M = 24$.

As a second step, we compute the eigenvalues and eigenvectors of $\hat{S}_x(\theta)$ at each frequency. Let $\lambda_j(\theta)$ be the j-th largest eigenvalue of $\hat{S}_x(\theta)$ and $U_j(\theta)$ be the corresponding eigenvector. Moreover, let $\Lambda(\theta)$ be the $q \times q$ diagonal matrix having on the diagonal the first q eigenvalues in descending order and $U(\theta)$ be the matrix having on the columns the first q eigenvectors, i.e. $U(\theta) = [U_1(\theta)U_2(\theta)\cdots U_q(\theta)]$. Our estimate of Σ_χ is

$$\hat{S}_\chi(\theta) = U(\theta)\Lambda(\theta)\tilde{U}(\theta) \tag{21}$$

where tilde denotes conjugation and transposition. Given the correct choice of q, consistency results for the entries of this matrix as both n and T go to infinity can easily be obtained from Forni, Hallin, Lippi and Reichlin (2000).

Third, we average $\hat{S}_\chi(\theta)$ over all points θ_j to get our estimate of Σ_χ and average $\hat{S}_\chi(\theta)$ over the relevant frequency band $[-\frac{2\pi}{12}, \frac{2\pi}{12}]$ to get our estimate of Σ_ϕ:

$$\hat{\Sigma}_\chi = \frac{2\pi}{2J+1} \sum_{j=-J}^{J} \hat{S}_\chi(\theta_j); \tag{22}$$

$$\hat{\Sigma}_\phi = \frac{2\pi}{2J+1} \sum_{j=-10}^{10} \hat{S}_\chi(\theta_j). \tag{23}$$

Finally, our estimate of the idiosyncratic variance-covariance matrix Σ_ξ is simply obtained as

$$\hat{\Sigma}_\xi = \operatorname{diag}\left(\hat{\Sigma}_x - \hat{\Sigma}_\chi\right), \tag{24}$$

diag(A) being the diagonal matrix having on the diagonal the diagonal elements of A. We set to zero the off-diagonal terms because we are assuming mutual orthogonality of the idiosyncratic components.

Appendix B: Data set and treatment

The data set is made up by 145 series from Thomson Financial Datastream, referring to the Euro area as well as its major economies. For the Euro area GDP we used data from Fagan et al. (2001) until the first quarter 1991, from then on we used the official Eurostat series (the first of the two time series have been riporportioned so as to avoid a sudden change in level in 1991). The database is organized into homogeneous blocks, i.e. industrial production indexes (41 series), prices (24), money aggregates (8), interest rates (11), financial variables (6), demand indicators (14), surveys (25), trade variables (9) and labour market series (7).

All series were transformed to remove outliers, seasonal factors and non-stationarity. Regarding outliers, we eliminated from each series those points that were more than 5 standard deviation away from the mean and replaced them with the sample average of the remaining observations. Seasonal adjustment was obtained by regressing variables on a set of seasonal dummies. We did not resort to other more sophisticated procedures (e.g. Seats or X12) to avoid the use of two-sided filters, which would imply large revisions in the seasonally adjusted series and therefore in the indicator. Non stationarity was removed following an automatic procedure: all the series in a given economic class (e.g. industrial production, prices and so on) were treated in the same way.

Finally, the series were normalized subtracting the mean and dividing for the standard deviation as usually done in the large factor model literature. The detailed list of the variables and the related transformation is reported in the table below.

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DATA DESCRIPTION

Group	Description	Type of treatment
Industrial Production	DE INDUSTRIAL PRODUCTION-INTERMEDIATE GOODS	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODUCTION	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODN - MANUFACTURE OF CHEMICAL & CHEMICAL PRDS.	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODN. -MANUFACTURE OF RUBBER & PLASTIC PRDS.	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODUCTION-MANUFACTURE OF BASIC METALS	*(1-L)log*
Industrial Production	DE INDUSTRIAL PRODN -MANUFACTURE OF ELEC. MACH. & APPARATUS	*(1-L)log*
Industrial Production	BG INDUSTRIAL PRODUCTION INCL. CONSTRUCTION	*(1-L)log*
Industrial Production	BG INDUSTRIAL PRODUCTION EXCL CONSTRUCTION	*(1-L)log*
Industrial Production	BG INDUSTRIAL PRODUCTION-INTERMEDIATE PRODUCTS	*(1-L)log*
Industrial Production	BG INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODN - MANUFACTURE OF PULP, PAPER & PAPER PRD.	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODN - MANUFACTURE OF CHEMICAL & CHEMICAL PRDS.	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODUCTION-MANUFACTURE OF BASIC METALS	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODN -MANUFACTURE OF MACHINERY AND EQUIPMENT	*(1-L)log*
Industrial Production	EA INDUSTRIAL PRODUCTION	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION-MANUFACTURE OF BASIC METALS	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODN -MANUFACTURE OF MACHINERY AND EQUIPMENT	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION-INTERMEDIATE GOODS	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION-CAPITAL GOODS	*(1-L)log*
Industrial Production	ES INDUSTRIAL PRODUCTION-OTHER NON	*(1-L)log*
Industrial Production	FN INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION	*(1-L)log*
Industrial Production	FR INDUDSTRIAL PRODUCTION (UUSTED)	*(1-L)log*
Industrial Production	FR INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	FR INDUSTRIAL PRODUCTION-CONSUMER GOODS	*(1-L)log*
Industrial Production	FR INDUSTRIAL PRODUCTION-ENERGY PRODUCTS	*(1-L)log*
Industrial Production	FR INDUSTRIAL PRODUCTION-INVESTMENT GOODS	*(1-L)log*
Industrial Production	FR INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	IR INDUSTRIAL PRODUCTION-MANUFACTURING	*(1-L)log*
Industrial Production	IR INDUSTRIAL PRODUCTION-INDUSTRIES	*(1-L)log*
Industrial Production	IT INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION	*(1-L)log*
Industrial Production	IT INDUSTRIAL PRODUCTION: CONSUMER GOODS	*(1-L)log*
Industrial Production	IT INDUSTRIAL PRODUCTION: INVESTMENT GOODS	*(1-L)log*
Industrial Production	IT INDUSTRIAL PRODUCTION	*(1-L)log*
Industrial Production	NL INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION	*(1-L)log*
Industrial Production	NL INDUSTRIAL PRODUCTION	*(1-L)log*
Industrial Production	PT INDUSTRIAL PRODUCTION (ADJUSTED FOR WORKING DAYS)	*(1-L)log*
Prices	DE PPI: ENERGY	*(1-L)log*
Prices	DE PPI: INDUSTRY (EXCLUDINGCONSTRUCTION)	*(1-L)log*
Prices	DE PPI: MANUFACTURING	*(1-L)log*
Prices	DE PPI: NON - DURABLE CONSUMER GOODS	*(1-L)log*
Prices	DE PPI: INDUSTRIAL PRODUCTS	*(1-L)log*
Prices	BG PPI: DURABLE CONSUMER GOODS	*(1-L)log*
Prices	BG PPI: ENERGY	*(1-L)log*
Prices	BG PPI: INDUSTRY (EXCLUDINGCONSTRUCTION)	*(1-L)log*
Prices	BG PPI: MANUFACTURING	*(1-L)log*
Prices	EA PPI: TOTAL MANUFACTURING -DOMESTIC MARKET	*(1-L)log*
Prices	EA CPI (DS CALCULATED BEFORE 1990, HARMONISED)	*(1-L)log*
Prices	EA INDUSTRIAL PPI-EXCLUDING CONSTRUCTION	*(1-L)log*
Prices	ES PPI: DURABLE CONSUMER GOODS	*(1-L)log*
Prices	ES PPI: ENERGY	*(1-L)log*
Prices	ES PPI: INDUSTRY (EXCLUDINGCONSTRUCTION)	*(1-L)log*
Prices	ES PPI: MANUFACTURING	*(1-L)log*
Prices	ES PPI: NON - DURABLE CONSUMER GOODS	*(1-L)log*
Prices	FN PPI: INDUSTRY (EXCLUDING CONSTRUCTION)	*(1-L)log*
Prices	GR PPI: MANUFACTURING	*(1-L)log*
Prices	IR PPI: INDUSTRY (EXCLUDING CONSTRUCTION)	*(1-L)log*
Prices	IT PPI: ENERGY	*(1-L)log*
Prices	IT PPI: NON - DURABLE CONSUMER GOODS	*(1-L)log*
Prices	NL PPI-MANUFACTURED GOODS	*(1-L)log*
Prices	NL PPI-INTERMEDIATE GOODS OUTPUT	*(1-L)log*
Trade	DE EXPORTS FOB	*(1-L)log*
Trade	DE IMPORTS CIF	*(1-L)log*
Trade	BG EXPORTS (FOB)	*(1-L)log*
Trade	BG IMPORTS (CIF)	*(1-L)log*
Trade	ES EXPORTS FOB	*(1-L)log*
Trade	ES IMPORTS CIF	*(1-L)log*
Trade	FR EXPORTS FOB	*(1-L)log*
Trade	IT EXPORTS FOB	*(1-L)log*
Trade	NL IMPORTS (CIF)	*(1-L)log*

DATA DESCRIPTION (*continued*)

Group	Description	Type of treatment
Surveys	DE CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS - GERMANY	--
Surveys	DE INDUSTRY SURVEY: ORDER BOOK POSITION-GERMANY	--
Surveys	BG INDUSTRIAL CONFIDENCE INDICATOR-BELGIUM	--
Surveys	BG INDUSTRY SURVEY: ORDER BOOK POSITION-BELGIUM	--
Surveys	ES CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS-SPAIN	--
Surveys	ES ECONOMIC SENTIMENT INDICATOR-SPAIN	--
Surveys	ES INDUSTRY SURVEY: ORDER BOOK POSITION-SPAIN	--
Surveys	ES INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH. AHEAD-SPAIN	--
Surveys	FR CONSTRUCTION CONFIDENCE INDICATOR-FRANCE	--
Surveys	FR INDUSTRIAL CONFIDENCE INDICATOR-FRANCE	--
Surveys	FR SURVEY: INDUSTRY-ORDERBOOK & DEMAND	--
Surveys	FR SURVEY: MANUFACTURING OUTPUT LEVEL-GENERAL OUTLOOK	--
Surveys	FR SURVEY: INDUSTRY-RECENT OUTPUT TREND	--
Surveys	FR SURVEY: INDUSTRY-PROBABLE OUTPUT TREND	--
Surveys	IT ISAE HOUSEHOLD CONFIDENCE INDEX: NET OF IRREGULAR COMPONENTS	--
Surveys	IT CONSUMER CONFIDENCE INDICATOR-ITALY	--
Surveys	IT INDUSTRY SURVEY: STOCKS OF FINISHED GOODS-ITALY	--
Surveys	IT INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH. AHEAD-ITALY	--
Surveys	NL CONSUMER SURVEY: ECONOMIC SITUATION LAST 12 MTH - NETHERLANDS	--
Surveys	DE ASSESSMENT OF BUSINESS SITUATION: CONSTRUCTION	--
Surveys	DE ASSESSMENT OF BUSINESS SITUATION	--
Surveys	DE BUSINESS EXPECTATIONS	--
Surveys	DE ASSESSMENT OF BUSINESS SITUATION: MANUFACTURING	--
Surveys	DE BUSINESS CLIMATE INDEX: MANUFACTURING	--
Surveys	DE ASSESSMENT OF BUSINESS SITUATION: TRADE	--
Interest Rates	DE MONEY MARKET RATE (FEDERAL FUNDS)	*(1-L)*
Interest Rates	DE GOVT BOND YIELD-LONGTERM	*(1-L)*
Interest Rates	DE MORTGAGE BANK LENDING TODOMESTIC NON - BANKS	*(1-L)log*
Interest Rates	BG CENTRAL GOVERNMENT BOND-5 YEAR YIELD	--
Interest Rates	BG TREASURY BILL RATE	*(1-L)*
Interest Rates	FN MONEY MARKET RATE (FEDERAL FUNDS)	*(1-L)*
Interest Rates	FR MONEY MARKET INT.RATES -AVERAGE YEARLY MONEY MARKET RATE	*(1-L)*
Interest Rates	GR TREASURY BILL RATE	*(1-L)*
Interest Rates	IT TREASURY BOND NET YIELD- SECONDARY MKT. (EP)	*(1-L)*
Interest Rates	NL LENDING RATE (PRIME RATE)	*(1-L)*
Interest Rates	NL GOVT BOND YIELD-LONGTERM	*(1-L)*
Wages	DE UNIT LABOUR COSTS, RELATIVE NORMALIZED	*(1-L)log*
Wages	DE WAGE & SALARY RATES: MONTHLY - PRODUCING SECTOR(PAN DE M0191)	*(1-L)log*
Wages	IT HOURLY RATES IN INDUSTRY	*(1-L)log*
Wages	NL HOURLY WAGE RATE MANUFACTURING	*(1-L)log*
Money Supply	DE MONEY SUPPLY-M2 (CONTINUOUS SERIES)	*(1-L)log*
Money Supply	DE MONEY SUPPLY-M3 (CONTINUOUS SERIES)	*(1-L)log*
Money Supply	EA MONEY SUPPLY: M1	*(1-L)log*
Money Supply	EA MONEY SUPPLY: M3	*(1-L)log*
Money Supply	FR MONEY SUPPLY-M1 (NATIONAL CONTRIBUTION TO M1)	*(1-L)log*
Money Supply	FR MONEY SUPPLY-M3 (NATIONAL CONTRIBUTION TO M3)	*(1-L)log*
Money Supply	IT MONEY SUPPLY: M1-ITALIAN CONTRIBUTION TO THE EURO AREA	*(1-L)log*
Money Supply	IT MONEY SUPPLY: M3-ITALIAN CONTRIBUTION TO THE EURO AREA	*(1-L)log*
Exchange Rates	DE REAL EFFECTIVE EXCHANGE RATES	*(1-L)*
Exchange Rates	EA U.S. $ TO 1 EURO (ECU PRIOR TO 1999)	*(1-L)*
Exchange Rates	IT REAL EFFECTIVE EXCHANGE RATES	*(1-L)*
Finance	FR SHARE PRICES-SBF 250	*(1-L)log*
Finance	US STANDARD AND POORS COMPOSITE INDEX (EP)	*(1-L)log*
Finance	US DOW JONES INDUSTRIALS SHARE PRICE INDEX (EP)	*(1-L)log*
Demand Indicators	DE RETAIL SALES EXCLUDING CARS (DERETT0TF FOR 2000=100)	*(1-L)log*
Demand Indicators	DE VACANCIES (PAN DE FROM JAN 1994)	*(1-L)*
Demand Indicators	DE WHOLE SALES TURN OVER, NOMINAL VALUE	*(1-L)log*
Demand Indicators	BG NEW CAR REGISTRATIONS	*(1-L)log*
Demand Indicators	BG RETAIL SALES	*(1-L)log*
Demand Indicators	BG RETAIL SALES-FOOD	*(1-L)log*
Demand Indicators	ES EMPLOYMENT PROMOTION CONTRACTS: IN PRACTICE	*(1-L)*
Demand Indicators	ES NEW CAR REGISTRATIONS	*(1-L)log*
Demand Indicators	FR NEW CAR REGISTRATIONS	*(1-L)log*
Demand Indicators	FR HOUSEHOLD CONSUMPTION-DURABLE GOODS	*(1-L)log*
Demand Indicators	FR HOUSEHOLD CONSUMPTION-MANUFACTURED GOODS, RETAIL GOODS	*(1-L)log*
Demand Indicators	FR HOUSEHOLD CONSUMPTION-MANUFACTURED GOODS	*(1-L)log*
Demand Indicators	IT NEW CAR REGISTRATIONS	*(1-L)log*
Demand Indicators	IT RETAIL SALES	*(1-L)log*
Unemployment	DE STANDARDIZED UNEMPLOYMENT RATE	*(1-L)*
Unemployment	FR UNEMPLOYMENT RATE-UNDER 25 YEARS	*(1-L)*
Unemployment	IR STANDARDIZED UNEMPLOYMENT RATE	*(1-L)*