Becoming the Neighbor Bidder:

Endogenous Winner's Curse in Dynamic Mechanisms

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Abstract

This paper addresses the problem of sequentially allocating time-sensitive goods, or one-period leases on a durable good, among agents who compete through time and learn about the common component of their valuation through experience. I show that efficiency is unattainable and I identify simple variations of sequential second-price or English auctions that implement the second best and the revenue-maximizing auctions. When the units are divisible, I identify the corresponding auctions that allow for double sourcing.

Keywords: Dynamic mechanism design, sequential auctions, interdependent values, multi-dimensional types, winner's curse, double sourcing

JEL Classification Numbers: D82, D86

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1 Introduction

Imagine that neighboring wildcat tracts are to be leased one by one, with a non-trivial amount of time elapsing between auctions. The winner of an auction wins the right to drill in the designated tract, which gives her access to further information about the geology of the neighboring tracts. Thus, the winner becomes the "neighbor" bidder (Wilson, 1969) in future auctions.

Or imagine two firms bidding to provide the Department of Defense (DoD) with a prototype weapon system, with the rights to produce the system also to be allocated by competitive bidding. The winner of the right to develop the prototype will acquire further information about the preferences and goals of the DoD, as well as about the cost of accommodating production regulations. This information is valuable in the subsequent competition for production.

These examples feature private garnering of information, information that is valuable to *all* agents, according to the interim outcomes of a sequential allocation process. Whoever gets the good first acquires an informational advantage over her opponents, the "non-neighbors." How should the successive auctions be designed in order to maximize social welfare or revenues?

This paper addresses this question in the context of sequential allocations of units of a time-sensitive good, or one-period leases on a durable good. Agents have multi-period demand, so they do not exit the market after trading. Their valuations in each period are the sum of a private-value component and a common-value component, the latter being unobserved ex-ante. Only the current winner gets to observe this common component, privately, before the next auction. Thus, there is an "information-access externality."

The analysis focuses on the case where there are only two periods. This case is directly relevant for the DoD example, and sheds light on related static problems. Similar insights and designs apply to the infinite-horizon case if the informational advantage of winning is always only one period ahead.[1]

I show that the first best is unattainable by means of standard mechanisms: Implementability implies inefficiency (Lemma 2). In particular, allocating the

[1]The details are available from the author upon request.

first-period unit leads to a lower expected future welfare. This trade-off between immediate surplus and expected future welfare can make it desirable to withhold the first-period unit, *even from the point of view of efficiency*.

While the sequential second-price or English auction is inefficient, a slight variation on the first-period auction can implement the second best (Theorem 2). Bidders are asked to pay a deposit before bidding. The deposit goes towards the winner's payment, and losers are reimbursed in full. However, it acts both as a bid floor and as an entry fee in the first-period auction. Thus, it excludes types of agents who would create too little surplus to justify the ensuing decrease in expected future welfare. Moreover, if only one bidder participates, she can get the good at a discount. This way, the auction also accommodates the *dynamic equilibrium externality*: Allocating the first-period unit to an agent reduces the continuation value of her opponents.

Another variation on the sequential second-price or English auction implements the revenue-maximizing mechanism (Theorem 3). The seller can raise the highest (feasible) expected future welfare, and captures it by charging personalized entry fees. If the seller cannot commit to excluding bidders who do not pay these fees, the second-period auction is a scoring-rule auction in which the reserve price for the non-neighbor is set by the neighbor's bid.

If the first-period unit is an input for the second-period unit, as in many procurement applications, first-period trade must take place to be able to hold the second-period auction. If the units are indivisible, a social planner cannot improve upon the sequential second-price or English auction. When the units are divisible, a more efficient option is *double sourcing*, namely splitting the allocation: Informational asymmetries are mitigated at a lower cost in terms of immediate surplus.[2] Double sourcing is socially desirable (profitable) when valuations (virtual utilities) are close to tied (Theorems 4 and 5).

The classical references on auctions with asymmetrically-informed bidders are Wilson (1969), Weverbergh (1979), Engelbrecht-Wiggans et al. (1983),

[2]The informational role of double sourcing in government procurements is pointed out in Gansler et al. (2009): "[...]Dual sourcing also increased the availability of supplier information between competitive suppliers that resulted in more aggressive bidding."

and Hendricks and Porter (1988). These papers study equilibria in one-shot common-value auctions where one bidder, the "neighbor," profits from having private information about the value of the object. Hernando-Veciana and Troge (2011) show that the neighbor is actually hurt by the informational asymmetry when the number of non-neighbors she faces is sufficiently high; they call this the "insider's curse." Goeree and Offerman (2003) consider auctions where all bidders receive partial signals about the common component. Other recent work includes the dynamic extensions of Wilson (1969) by Virag (2007) and Hörner and Jamison (2008).

In these papers, the informational asymmetry is exogenous, and can only decrease through time. In the present paper, the informational asymmetry is endogenous — bidders are ex-ante symmetric — and the information gap between them only widens with trade. Moreover, rather than looking at behavior in a given auction, I design welfare-maximizing and revenue-maximizing mechanisms.

This design problem concerns dynamic mechanisms under interdependent values and multi-dimensional types. Athey and Segal (2007) and Bergemann and Välimäki (2010) introduce efficient mechanisms for dynamic environments with independent, private values. Pavan et al. (2011) characterize implementability of dynamic mechanisms under interdependent valuations and unidimensional types. In a simple two-period model, I combine both interdependent values and multi-dimensional signals.

When there are both interdependent values and multi-dimensional types, social-choice functions that are Bayesian-Nash implementable are *generically* inefficient (Jehiel and Moldovanu, 2001), and only constant ones are *generically* ex-post implementable (Jehiel et al., 2006).

However, the notion of genericity in this literature is extremely demanding. As Bikhchandani (2006) shows, generic environments must feature both interdependent values and allocative externalities; agents must care *directly* about each other's information and portion of the outcome. Yet, many relevant economic problems are non-generic: In oil-tract auctions, the neighbor only cares about whether she wins the lease, and she knows her ex-post valuation for it. While Bikhchandani (2006) establishes the existence of non-trivial mechanisms that are ex-post implementable under private consumption, we lack a general characterization of implementability for non-generic problems. For the problem I study, I present a characterization that allows me to design second-best and revenue-maximizing auctions.

If the common-value signal represents ex-post verifiable information, the designer can resort to *royalty payments*. Such contingent-payment schemes are common in oil-tract auctions; the cost of a lease may be contingent on how much oil is found after drilling. Royalty payments can mitigate the winner's curse and contribute to efficiency. Tan (2012) shows that the neighbor may even willingly disclose the verifiable signal to the non-neighbor before bidding. However, my focus is on non-verifiable information, such as private readings about the probability of finding oil in adjacent tracts.

With non-verifiable information, implementability entails inefficiency in standard mechanisms — mechanisms where the same messages determine both allocations and payments. Mezzetti (2004) shows that efficiency is restored if additional reports can be sent to the mechanism ex-post, even in generic environments and even if signals are not verifiable.[3] First-round messages about types determine the allocation, while second-round messages about ex-post utilities determine the payments. To shed light on the limitations and strengths of more traditional auction formats, and to facilitate comparison with the literature, I do not consider ex-post reports in the paper.[4]

By restoring efficiency, ex-post reports can "lift" the winner's curse and thus eliminate the dynamic equilibrium externality inherent in standard mechanisms. A downside of the scheme in Mezzetti (2004) is that agents are indifferent between any of their ex-post messages. Incentives for truth-telling

[3]I am grateful to an anonymous referee for prompting me to clarify this last point.

[4]Mechanisms á la Mezzetti (2004) are considered in the online appendix. With two bidders, the following auction is efficient. Start with a second-price auction. The winner reports the value of the unit, and this report sets a second-period entry fee for the loser. In the second period, both bidders pay their opponent's bid, but the loser receives a bonus based on the value reported by the (new) winner.

The general consideration of ex-post payments requires a characterization of incentive compatibility; Mezzetti (2004) focuses on efficiency.

are weak, and weak incentives may lead to the existence of undesirable equilibria. Of course, this issue is a staple in mechanisms designed under *partial implementation*, and the mechanisms I propose are no exception — especially the auction in Theorem C1, if the neighbor fails to meet the reserve price (see discussion at the end of Section 5). However, in the second-best auction of Theorem 2, agents have stronger incentives than in Mezzetti (2004) to behave as the desired equilibrium dictates.

When allocating under exclusivity creates informational asymmetries, it might be desirable to allow for double sourcing. Anton et al. (2010) look at a static auction where the rationale for double sourcing is uncertainty about economies of scale. In both Klotz and Chatterjee (1995) and Hsieh and Kuo (2011), double sourcing can help finance entry costs. Klotz and Chatterjee (1995) also feature learning by doing. Valero (2013) rationalizes double sourcing as a way of reducing dependence on a supplier whose production is subject to shocks, by resorting to a more costly but more reliable competitor. I provide an informational rationale for double sourcing.

This paper is organized as follows. Section 2 describes the basic setup. Section 3 describes the class of mechanisms I consider, and discusses the characterization of implementability. Section 4 describes the second-best allocation rule and identifies a simple mechanism that implements it. Section 5 identifies a revenue-maximizing mechanism. Section 6 analyzes the case where the units can be double sourced. Section 7 concludes. Proofs are collected in Appendix A. Appendix B describes the sequential second-price auction benchmark. Appendix C features additional details on revenue maximization.

2 The Model

Two units of a time-sensitive good are to be allocated one at a time over two periods. There are $N \in \mathbb{N}$ agents, indexed by $i = i_1, \dots, i_N$. These agents have multi-period demand, and their valuations are the sum of a private-value and a common-value component. These components of the agents' valuations are renewed through time. There is a common discount factor $\delta > 0$. As the horizon is finite, we can allow for $\delta \geq 1$, giving future payoffs more weight than present payoffs. For instance, the first-period unit may be just a prototype, so the second-period unit carries more weight.

The common component in period t is represented by v_t, an *i.i.d.* draw from a random variable V with pdf f_V and compact support $[\underline{v}, \overline{v}] \subseteq \mathbb{R}_+$. The private components in period $t \in \{1, 2\}$ are $w_t := (w_{i_1 t}, \ldots, w_{i_N t})$. These are *i.i.d.* draws from a random variable W, independent of V, with pdf f_W and compact support $[\underline{w}, \overline{w}] \subseteq \mathbb{R}_+$. Both densities are assumed to be continuous, strictly positive on their support, and strictly log-concave. The (ex-post) valuation of agent i in period t is $u_{it} := v_t + w_{it}$; it is drawn from $U := V + W$, with density f_U given by the convolution of f_V and f_W.[5]

A high valuation for the neighbor may come from a high common-value or a high private-value component; only the first event is good news to non-neighbors. If the latter group could observe the draw $u = v + w$ of the neighbor's valuation, they would disentangle v and w by means of the conditional expectations $g(u) := E[V|U = u]$ and $h(u) := u - g(u) = E[W|U = u]$, respectively. Under strict log-concavity, these functions are continuous and strictly increasing.[6] Thus, a higher total valuation is indicative of higher values of both components. Conversely, a non-neighbor who faces a low-valuation neighbor would revise downwards her estimate of v; this is the typical winner's curse. Of course, non-neighbors do not observe u. However, they anticipate the value of the unit conditional on outbidding the neighbor, and g and h are relevant in characterizing their equilibrium behavior.

All signals are assumed to be non-verifiable. In the oil-tract example, the informational advantage represents insights on the geological properties of the field.[7] Similarly, in the DoD example, v captures the insights about preferences

[5]The assumption of additivity is made mainly for simplicity of exposition. The results in this paper generalize to valuations that are a linear combination of the signals, if the coefficients are commonly known and the same for both agents. Some results (Theorem B1 and the first part of Lemma 2) extend to strictly increasing aggregator functions.

[6]See Lemma 1 in Goeree and Offerman (2003), or Lemma 4 in Larson (2009). As a result, both g and h have continuous and strictly increasing inverses. Moreover, they satisfy the boundary conditions $g(\underline{v} + \underline{w}) = \underline{v}$, $g(\overline{v} + \overline{w}) = \overline{v}$, $h(\underline{v} + \underline{w}) = \underline{w}$, and $h(\overline{v} + \overline{w}) = \overline{w}$.

[7]Of course, these geological properties may be related to the amount of oil found on the

and goals of the DoD.

The timing is as follows. Each agent privately observes their period-t idiosyncratic component at the beginning of period t. The common-value component is not observed ex ante. Only the first-period winner gets to observe v_2, privately, before the second-period allocation is determined.

Without loss of generality, we can take $v_1 := E(V)$; to simplify notation, we can write $v := v_2$. I shall focus on the case $N = 2$, and discuss what changes when moving to the case $N \geq 3$ after the fact. The reason for this is twofold; it simplifies the exposition at little cost in terms of changes to the design of the auctions, and these changes are interesting in their own right. Figure 1 summarizes the timing and information structure for $N = 2$, where $a := i_1$ and $b := i_2$.

The assumption that v is perfectly observed from the allocation is without loss of generality. If the neighbor observes only a noisy signal of v, the analysis goes through replacing v with the conditional expectation of V given this signal. What is important is that the informational advantage is one-sided; namely, that the neighbor has nothing to learn from the non-neighbor.

The assumption of independence of V and W isolates experience with the allocation as the only source of information (beyond the priors). If these signals

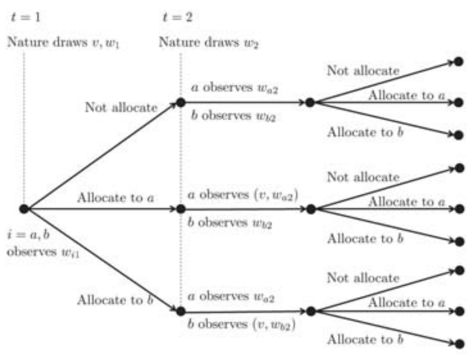


Figure 1: Timing and Information Structure.

tract, the latter being a verifiable signal. However, the informational advantage remains insofar as the amount of oil in a tract is not a perfect signal of oil deposits in nearby tracts.

were correlated, agents would update their guess of v from their first-period signals, as well as from their opponents' equilibrium behavior.

Independence across time is a restrictive assumption, and an extreme way to represent renewal of signals. A more flexible representation would allow for some amount of persistence or serial correlation. While there is no doubt that this would make the problem more general, realistic, and interesting, it would also introduce two complications.

If values are serially correlated, first-period behavior becomes informative about second-period signals — insofar as it conveys information about first-period signals. The mechanism designer could exploit this information, for instance, by making the second-period outcome contingent on first-period bids. Agents anticipate this impact of their current bids on the continuation of the mechanism. While there is no lack of commitment power, this situation formally resembles the "ratchet-effect" problem. This makes it hard to induce agents to reveal information; it is well known that, under ratchet-effect conditions, typically there is pooling in equilibrium.

But it is not only the mechanism designer that may want to "exploit" first-period information disclosed in equilibrium. The non-neighbor can learn about her competitor from the first-period outcome as well, and viceversa. Thus, there is potential for signalling: Bidders may want to bid aggressively if, by doing so, they can scare competitors away from the second-period auction.

These two difficulties obscure the effect of the informational impact of allocating the first unit, and can render the problem intractable. Serial independence isolates the effect of learning "from experience" from these other dynamic effects, and keeps the problem clean and tractable.

3 Implementability

The set of period-t allocations is $X := \{(0,0),(0,1),(1,0)\}$, where the i-th coordinate of $x_t \in X$ indicates whether agent i is allocated the period-t unit

($x_{it} = 1$) or not ($x_{it} = 0$).[8] In the oil-tract example, each unit is a lease; the government may grant a lease on neither or only on one of the tracts. In the DoD example, the first unit is typically an input for the second one, so the latter cannot be produced without the former. What can be done in this case, and what is common practice in procurements, is to award multiple contracts. This possibility is explored in Section 6.

The space of period-t transfer profiles is $T := \mathbb{R}^2$. Utilities are time-additive and linear in outcomes. The ex-post discounted payoff to agent i, when the signals are v, w_{i1}, and w_{i2}, is $(v_1 + w_{i1})x_{i1} - \tau_{i1} + \delta\left[(v + w_{i2})x_{i2} - \tau_{i2}\right]$.

A social-choice function is a pair of functions $f = (f_1, F_2)$. The function $f_1 : [\underline{w}, \overline{w}]^2 \to X \times T$ specifies the first-period outcome as a function of first-period private-component signals; we can write $f_1 = (q_1, \tau_1)$, where q_1 denotes the allocation rule, and τ_1 denotes the transfer function. Given w_1, $F_2(\cdot; w_1) : [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]^2 \to X \times T$ specifies the second-period outcome as a function of second-period signals. This second map satisfies the following *measurability* condition: For any $w_1, w_2 \in [\underline{w}, \overline{w}]^2$ and any $v, v' \in [\underline{v}, \overline{v}]$, $F_2(v, w_2; w_1) = F_2(v', w_2; w_1)$ if $q_1(w_1) = (0, 0)$. In words, the second-period outcome can only depend on information the agents indeed possess.

An important class of social-choice functions for the present problem is that of *simple social-choice functions*. These are social-choice functions where $F_2(\cdot, w_1) = F_2(\cdot, w_1')$ for any w_1 and w_1' such that $q_1(w_1) = q_1(w_1')$; second-period portions of simple social-choice functions depend on first-period signals only through the allocation the latter induce.

We may distinguish two kinds of histories at the beginning of period 2: history $h^{(0)}$, where the first-period unit is not allocated to any of the agents, and history $h^{(i)}$, where the first-period unit is allocated to agent i. Simple social-choice functions can be identified by the following pieces: a pair of functions q_1, τ_1, which specify the first-period outcome; functions $q_2^{(0)}, \tau_2^{(0)}$, which specify the second-period outcome at history $h^{(0)}$; and functions $q_2^{(i)}, \tau_2^{(i)}$, which specify the second-period outcome at $h^{(i)}$.

[8] Provided there is no need to do ironing, the restriction to deterministic mechanisms is without loss of generality with linear utilities.

In a dynamic mechanism, agents send messages each period, after observing their current private information. First-period messages determine the first-period outcome, which determines who, if any, observes v before sending second-period messages. For these mechanisms, I employ the notion of implementability in Perfect Bayesian Equilibrium (Brusco, 2006).

A dynamic mechanism implements a social-choice function f in *Perfect Bayesian Equilibrium* (PBE) if there is a Bayesian-Nash Equilibrium (BNE) of the induced game that, at any history, induces a BNE in the continuation game whose outcome coincides with f. For direct-revelation mechanisms, this notion of implementability is equivalent to interim incentive compatibility (Bergemann and Välimäki, 2010).[9]

This is a notion of *partial implementability*; existence of other equilibria that do not coincide with f is not ruled out. As noted in the introduction, existence of undesirable equilibria is a concern especially for the mechanism described in Theorem C1.

Corresponding to simple social-choice functions are *simple direct-revelation dynamic mechanisms*. These are direct-revelation dynamic mechanisms where the outcome function is a simple social-choice function; see Figure 2. The next lemma establishes that we can restrict our attention to simple direct-revelation dynamic mechanisms without loss of generality, for either welfare or revenue maximization. The key is the assumption of serial independence, which reduces the intertemporal link to the first-period allocation.

Lemma 1. *If a dynamic mechanism implements a social-choice function designed to maximize expected welfare or revenues, then so does a simple direct-revelation dynamic mechanism.*

There are two parts to this lemma. First, we can restrict attention to direct-revelation mechanisms. This is just the standard *Revelation Principle.*

[9] As signals are fully renewed in the second period, the posterior for second-period signals is the same as the prior. The bite in this notion lies in asking for sequential rationality at every history, so agents cannot commit to second-period messages beforehand.

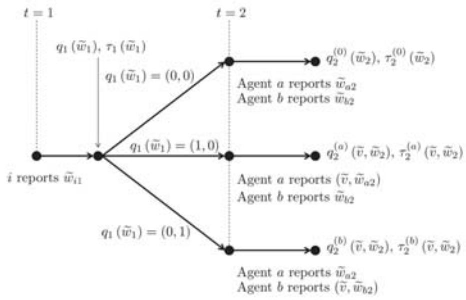


Figure 2: Simple direct-revelation dynamic mechanisms.

Second, for welfare or revenue maximization, we can simply ask agents to report their current private information, and conflate continuation games into histories $h^{(0)}$ or $h^{(i)}$. This follows from the serial-independence assumptions; restricting attention to simple direct-revelation mechanisms could involve loss of generality if private-value signals are serially correlated.

To address questions of mechanism design, we want to characterize the set of simple direct-revelation mechanisms that are implementable. At history $h^{(0)}$, neither agent has information that is directly relevant to the other, and their problem reduces to a standard independent-private-values problem.

At history $h^{(i)}$, agent i knows her ex-post valuation, so she faces a private-value problem. However, her type is two-dimensional: She observes both components of her valuation separately. Agent $-i$ has a single piece of information, but she faces a winner's-curse problem. The characterization of implementability follows from Jehiel and Moldovanu (2001).[10] In the present problem, it leads to the following consequence: Implementable mechanisms cannot discriminate among types with the same total valuation.

For each type $(v, w) \in [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]$, define the "iso-valuation" plane $H(v, w) := \{(v', w') \in [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}] : v' + w' = v + w\}$. Given a simple direct-revelation dynamic mechanism (q, τ), let $\overline{q}_{i2}^{(i)}$ denote the expected second-

[10] For more on multi-dimensional screening, see also Armstrong (1996), Rochet and Chone (1998), Jehiel et al. (1999), Krishna and Perry (2000), Carlier (2001), Basov (2005), Araujo et al. (2008), and Deneckere and Severinov (2009).

period allocation rule for agent i at history $h^{(i)}$.

Lemma 2. *If (q, τ) is an implementable simple direct-revelation dynamic mechanism, at history $h^{(i)}$, 1) All types of agent i in $H(v, w_{i2})$ enjoy the same second-period equilibrium surplus as (v, w_{i2}); 2) There is a function $\phi : [\underline{v} + \underline{w}, \overline{v} + \overline{w}] \to [0, 1]$ such that $\bar{q}_{i2}^{(i)}(v, w_{i2}) = \phi(v + w_{i2})$ for almost every $(v, w_{i2}) \in [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]$.*

We can think of Lemma 2 as an "equal treatment" result: Incentive compatibility in simple dynamic mechanisms entails treating different types with the same valuation equally. The result stems from the fact that different iso-valuation types must enjoy the same surplus, and that compensating any differential treatment in the allocation via differential treatment in transfers is infeasible from the point of view of incentives.[11]

From the point of view of the first period, the problem is an auction-design problem with externalities. The continuation payoff of an agent when her opponent becomes the neighbor is lower than when she is up against a non-neighbor; in the former scenario, her future payoffs are subject to the winner's curse. This *dynamic equilibrium externality* is an externality that arises in equilibrium, and takes the form of a consumption externality: Agents care about each other's first-period allocation due to its impact on their continuation values. As this externality depends on the reported type, not on the true type, standard characterizations of implementability continue to apply.

4 Inefficiency and Constrained Efficiency

An allocation rule is *first best* or *efficient* if it maximizes expected social discounted welfare. The first-best allocation rule, denoted by q^{FB}, dictates that each unit be allocated to the agent with the highest current idiosyncratic

[11]Lemma 2 is similar to Lemma 2 in Araujo et al. (2008), who assume that agents have multiplicative costs and focus on differentiable mechanisms with transfers that depend only on the allocation. Analogous results hold in Deneckere and Severinov (2009), who assume a single-crossing condition (Assumption 2) that rules out linear utility functions.

signal, who is also the one with the highest current valuation; the common component is disregarded.

Notice that the first best is a simple social-choice function. We can write it as $q_{i2}^{FB(i)}(w_2) = q_{i2}^{FB(-i)}(w_2) = q_{i2}^{FB(0)}(w_2) := I\left(w_{i2} > w_{-i2}\right)$, and $q_{i1}^{FB}(w_1) := I\left(w_{i1} > w_{-i1}\right)$, where I denotes the indicator function, taking the value 1 if the statement in the argument is true, and 0 otherwise. By disregarding the common-value component, the first best discriminates among different types of the neighbor with the same total valuation based on the idiosyncratic portion of their valuation. As a result of Lemma 2, we conclude that the first best is not implementable.

Corollary 1. *The first-best allocation rule is not incentive compatible.*

By disregarding v, the first-best allocation rule does not "punish" the neighbor for reporting a low value for v. As a result, this better-informed agent has an incentive to understate the common-value component and exaggerate her private-value component. Such misreporting leads to a higher probability of trade in her favor, and payments cannot fully undo this favor.[12]

We can visualize the conflict in terms of orders on the two-dimensional type space of the neighbor. For efficiency, any pair of types $(v, w), (v', w') \in [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]$ are to be treated equally if $w = w'$, while they are to be treated equally in implementable mechanisms if $v + w = v' + w'$, even if $w' > w$. See Figure 3.

In Jehiel and Moldovanu (2001), inefficiency obtains when at least two agents have private information about each other, information that is relevant to determine the efficient allocation: One-dimensional payments are insufficient to induce agents to disclose this relevant multi-dimensional information.

[12]The same intuition applies to a setting where information about v has *social* value, not just individual value. Imagine the designer must bear a cost $c > 0$ to carry out the sale of the units. It can be socially valuable to let one agent observe v and choose a reserve price such that the second sale is carried out only if it is worthwhile, at the cost of introducing an informational asymmetry. The neighbor has no incentive to report any value beyond the least value that yields the same outcome, and confound any excess common value with her own, idiosyncratic portion of the value.

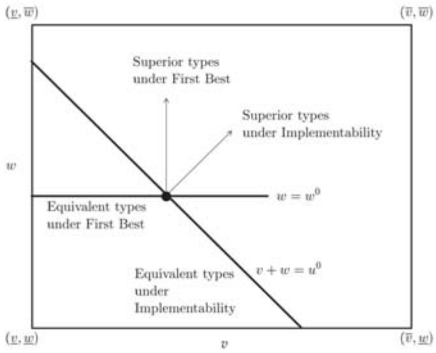


Figure 3: Tension between efficiency and incentive compatibility.

In the present problem, the non-neighbor has no private information about the neighbor's valuation. Moreover, the common-value component, the source of the asymmetry, is socially irrelevant.

An allocation rule is *constrained efficient* or *second best* if it maximizes expected welfare subject to the incentive-compatibility constraints. By Lemma 1, we focus on simple direct-revelation dynamic mechanisms. The following theorem characterizes the second-best allocation rule. Let $S_2^{\omega 0}$ and $S_2^{\omega 1}$ denote the expected continuation welfare levels stemming from history $h^{(0)}$ and histories $h^{(a)}, h^{(b)}$, respectively, and let $\Delta^\omega := S_2^{\omega 0} - S_2^{\omega 1}$ be the difference between them. Lemma A2 in Appendix A shows that $\Delta^\omega \geq 0$; the incentive constraint identified in Lemma 2 leads to a lower expected welfare at histories $h^{(a)}$, $h^{(b)}$ than at history $h^{(0)}$. For this reason, we can think of Δ^ω as the *welfare cost of implementability*, resulting from the endogenous winner's curse.

Theorem 1. *The second-best allocation rule, q^{SB}, is given as follows. In the second period, at history $h^{(0)}$, allocate the unit to the agent with the highest private-value signal: $q_2^{SB(0)} := q_2^{FB(0)}$; at history $h^{(i)}$, use u_{i2} to proxy for w_{i2} by $h(u_{i2})$, compare it to w_{-i2}, and allocate to the higher agent: $q_{i2}^{SB(i)}(u_{i2}, w_{-i2}) := I\left(h(u_{i2}) > w_{-i2}\right)$, and $q_{-i2}^{SB(i)}(w_{-i2}, u_{i2}) := I\left(h(u_{i2}) < w_{-i2}\right)$; finally, in the first period, allocate the unit to the agent with the highest signal, provided the expected surplus thus generated compensates for the welfare cost of imple-*

mentability: $q_{i1}^{SB}(w_1) := I\left(w_{i1} > \max\left\{w_{-i1}, \delta\Delta^{\omega} - E(V)\right\}\right)$.

The first-period portion of q_1^{SB} dictates allocating the unit to the agent with highest private component *if this results in a higher continuation welfare than withholding the unit*. In the second period, the allocation is efficient at history $h^{(0)}$. At history $h^{(i)}$, q^{SB} allocates the unit to agent i if her proxy private-value component is at least as high as agent $-i$'s idiosyncratic component. The welfare cost of implementability comes from the inefficiency present in allocating according to $h(u_{i2})$ rather than w_{i2}. When $w_{i2} > w_{-i2} > h(u_{i2})$ the second-period unit goes to agent $-i$, while i has a higher (ex-post) valuation for it. Similarly, when $w_{i2} < w_{-i2} < h(u_{i2})$, agent i receives the second unit, while agent $-i$ would produce a higher surplus.

The second-period portion of the second best coincides with the equilibrium outcome of a second-price or English auction (Theorem B1 in Appendix B). At history $h^{(i)}$, bidder i bids u_{i2}, her valuation, while $-i$ bids $h^{-1}\left(w_{-i2}\right)$, her guess of i's valuation at which both bidders are "tied" in terms of private-value components: $w_{-i2} = h\left(u_{i2}\right)$.[13] While "bidding to tie" is typical equilibrium behavior in common-value auctions, only the non-neighbor does the "guessing" here. The inefficiency arises in the auction through "mistakes" in these guesses: When $h^{-1}\left(w_{-i2}\right) > u_{i2} > u_{-i2}$, $-i$ overestimates v and outbids i, who should win; if $h^{-1}\left(w_{-i2}\right) < u_{i2} < v + w_{-i2}$, $-i$ has the higher valuation, but underestimates v and loses to i.

Just as the non-neighbor anticipates v from the equilibrium outcome in an auction, it is tempting to interpret $q_2^{SB(i)}$ as saying that i reports only her total valuation, and the planner guesses v from this report. However, this is not accurate: In the truthful equilibrium, the neighbor reports both signals to the planner. But, for the neighbor to be truthful, the planner cannot exploit the two signals separately. The welfare cost of implementability embodies this restriction, rather than the possibility of the planner making mistakes in guessing v.

[13] Alternatively, if i's valuation (hence, her bid) is equal to $h^{-1}\left(w_{-i2}\right)$, then $-i$ anticipates her valuation to be exactly tied with i's: $w_{-i2} + g\left(h^{-1}\left(w_{-i2}\right)\right) = h^{-1}\left(w_{-i2}\right)$.

While a second-price or English auction in the second period achieves the second best, the social planner can do better in the first period. A standard auction would always result in trade, at the expense of future expected welfare, even when the current surplus created is low. Therefore, the planner may want to exclude low types, taking into account that excluding a bidder from the auction shields her opponent from the future winner's curse.

The next theorem identifies a simple variation on the sequential second-price or English auctions that implements the second-best allocation rule. The variation consists in introducing deposit requirements to bid in the first period. Deposits are required to participate, for instance, in US General Services Administration (GSA) auctions. As in GSA auctions, the deposit is fully returned to the loser, and goes towards the winner's payment. If only one bidder participates, she is given the choice to purchase the good and get a partial reimbursement of her deposit, or to claim the full deposit but walk away empty-handed.

Denote the second-period equilibrium payoffs to a, b in $h^{(a)}$ by $S_{a2}(u_{a2})$, $S_{b2}\left(w_{b2}\right)$, respectively, and let $\overline{S}_2 := E\left[S_{a2}\left(U_{a2}\right)\right]$ and $\underline{S}_2 := E\left[S_{b2}\left(W_{b2}\right)\right]$ be the corresponding ex-ante payoffs. Let $S_2^0 := E\left[\max\{0, U_{a2} - U_{b2}\}\right]$ be the (counterfactual) expected payoff if both bidders observed v. Lemma B1 in Appendix B shows that $\overline{S}_2 \geq S_2^0 \geq \underline{S}_2$: Having unilaterally more information is always more profitable from the ex-ante perspective, even after taking into account how others may react to the informational asymmetry.[14] Therefore, $\Delta^S := \overline{S}_2 - \underline{S}_2 \geq 0$ represents the *informational value of winning*, the increase in continuation surplus due to the informational advantage that results from winning in the first period.

Theorem 2. *Define* $r^0 := \delta\left(\Delta^\omega + \Delta^S\right)$ *and* $r^1 := r^0 - \delta\left(S_2^0 - \underline{S}_2\right)$. *The second-best allocation rule is implemented by the following sequential auction. In the first period, bidders are asked to pay (simultaneously) a deposit of* r^0.

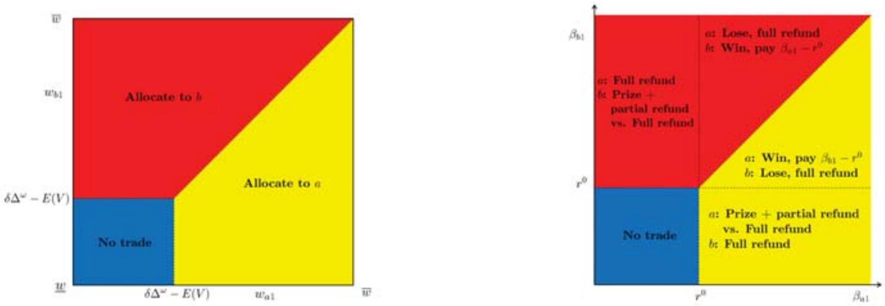
[14] We can think of $S_2^0 - \underline{S}_2$ as a decision-theoretic advantage: The behavior of the neighbor would not change if her opponent were also informed, so the additional information cannot hurt the non-neighbor; and of $\overline{S}_2 - S_2^0$ as a strategic advantage: The neighbor prefers facing a non-neighbor to another neighbor even after accounting for differences in behavior.

If both pay, they (simultaneously) submit bids; admissible bids are not lower than r^0. The highest bidder wins and pays the difference between the lowest bid and r^0; the loser gets her deposit back. If only one bidder pays, she is given the option to get the unit and a partial refund on her deposit of $r^0 - r^1 = \delta\left(S_2^0 - \underline{S}_2\right)$, or to get the full refund. No trade takes place if no bidder pays the deposit. The second-period auction is always a standard second-price or English auction.

See Figure 4. Allocating the first-period unit leads to a lower continuation welfare. By acting as entry fees and as reserve prices, the deposits discourage low-type agents, who would create too little surplus. Moreover, through contingent refunds, deposits accommodate the dynamic equilibrium externality.

Example. Consider the case where U, V are uniformly distributed on the interval $[0, 1]$.[15] Expected second-period payoffs in a second-price or English auction are $\overline{S}_2 = \frac{7}{24}$, $S_2^0 = \frac{1}{6}$, and $\underline{S}_2 = \frac{7}{48}$. Expected second-period welfare values are $S_2^{\omega 0} = \frac{7}{6}$ and $S_2^{\omega 1} = \frac{110}{96}$. Hence, we have $r^0 = \frac{\delta}{6}$ and $r^1 = \delta \frac{7}{48}$.

If $E(V)$ is high enough that first-period surplus always covers the discounted welfare cost of implementability, it is not desirable to restrict trade.



Second-best allocation rule | Auction in Theorem 2

Figure 4: Second-best allocation rule and auction design in Theorem 2.

[15] The density of the uniform distribution is not strictly log-concave. However, it yields conditional-expectation functions g, h that are continuous and strictly increasing.

In this case, the sequential second-price or English auction is second best, and the deposits in the auction of Theorem 2 are "non-binding" in equilibrium.

Corollary 2. *If $E(V) + \underline{w} \geq \delta\Delta^{\omega}$, in equilibrium, all types of bidders pay the deposit, and the mechanism in Theorem 2 leads to the same equilibrium outcome as the sequential second-price or English auction.*

A similar conclusion follows if the planner is unable to withhold the first unit. If both leases are to be granted, the sequence of second-price or English auctions is the best the government can do in terms of social surplus.

Remark 1. The equilibrium that implements the second best in Theorem 2 embodies a stronger notion than Perfect Bayesian Equilibrium. While not ex-post, the equilibrium is a posterior equilibrium with respect to the neighbor's bid (Green and Laffont, 1987): The non-neighbor has no regrets for the outcome after learning the neighbor's total valuation, which is the piece of second-period information equilibrium behavior reveals. In other words, the equilibrium outcome is the same that would prevail if the non-neighbor observed her opponent's total valuation before bidding.[16]

Remark 2. The informational asymmetry can be avoided by making the second-period allocation non-responsive to the second-period information. For instance, the government can adjoin the two tracts, and lease it as a single tract in the first period. However, the welfare impact of neglecting second-period information is at least as large as the welfare cost of implementability.[17] If

[16]Jehiel et al. (2007) show, by example, that posterior implementability can be possible even when ex-post implementability is generically impossible. In the present paper, the existence of non-trivial mechanisms that are posterior implementable might be due to this implementation notion being weaker than ex-post implementability, or due to the non-genericity of the framework. A question that remains open is under what conditions posterior implementability of non-trivial allocation rules is generically possible, or what is the maximal information feedback from non-trivial mechanisms that is generically consistent with posterior implementability. This is the subject of ongoing research.

[17]This is an immediate consequence of Theorem 1, as the allocation rule that is non-responsive to second-period signals is feasible. A direct argument is given in Lemma A3.

both units are available simultaneously, we could sell both units in the second period instead. However, this involves delay in trade, which may be costly.

An alternative way to implement the second best is to hold sequential English auctions where, instead of asking for deposits, the clock starts at a positive value and is adjusted downwards if only one of the bidders is willing to participate. In fact, when there are more than two agents, the second-period auction has to be of the open-bid format, where non-neighbors can observe if the neighbor is still active.

Theorem 2' *Let $N \geq 3$, $r^0 := \delta\left[\Delta^\omega + \Delta^S\right]$, and $r^1 := r^0 - \delta\left(S_2^0 - \underline{S}_2\right)$. The second-best allocation rule is implemented by the following sequential auction. In the first period, bidders are asked to pay (simultaneously) a deposit of r^0 to be entitled to submit a bid. If at least one pays, those who paid submit bids (simultaneously); the rest are excluded. Admissible bids are not lower than r^0, and the highest bidder wins and pays the difference between the second-highest bid and r^0; losers get their deposits back. If only one bidder pays, she is given the option to take the unit and a partial refund on her deposit of $r^0 - r^1 = \delta\left(S_2^0 - \underline{S}_2\right)$, or to get the full refund. No trade takes place if nobody pays the deposit. The second-period auction is always an English auction.*

The sealed-bid format fails at history $h^{(i)}$ because non-neighbors anticipate the possibility of winning being tied to other non-neighbors as well as to the neighbor. More than one non-neighbor outbidding the neighbor is "very" bad news about the common-value component, and leads them to bidding too low. This is avoided if non-neighbors can condition their behavior on whether the neighbor is active or not.

Now, when $N = 2$, the value of information is positive (Lemma B1). However, Hernando-Veciana and Troge (2011) show that the value of information is negative in an English auction if N is sufficiently large: High-type non-neighbors bid more aggressively than under symmetry, on average, and the distribution of non-neighbor bids becomes increasingly concentrated as they become more and more numerous. This means that, potentially, the seller might need to subsidize the first-period winner if this negative effect is large enough.

5 Maximizing Expected Revenue

Imagine now that the units are owned by a seller, who collects the transfers of the mechanism. As with the social planner, I assume that v is not payoff relevant to the seller, and that she does not know v. For revenue maximization, the issue of commitment power becomes central. I consider the case where the seller can make long-term commitments, and discuss what changes if she only enjoys short-term commitment power.[18]

The highest ex-ante social surplus that can be achieved in the second period is $S_2^{\omega 0}$. With intertemporal commitment power, the seller can create this surplus through a second-price or English auction, and capture it via entry fees charged at the end of the first period — before the second-period information arrives.[19] The next theorem identifies a revenue-maximizing mechanism.

Theorem 3. *The following mechanism maximizes expected revenues. The first-period auction is a second-price or English auction with reserve price $r^{\pi *} := (\phi^0)^{-1}(-E(V) + \delta\Delta^{\omega})$. After the bids are in and the winner is announced, but before allocating the unit, the winner is charged a second-period entry fee of $\mathrm{e}^1 := \delta\overline{S}_2$, and the loser, of $\mathrm{e}^2 := \delta\underline{S}_2$; if the auction results in no trade, both bidders are charged $\mathrm{e}^3 := \delta S_2^0$. If both bidders pay the corresponding entry fee, the first-period unit is allocated and a second-price or English auction follows. If only one bidder pays the fee, she gets the first-period unit and a rebate of $\mathrm{e} - \delta E(U)$, where e is the fee she paid; she also gets the second-period unit for free. If no bidder pays the fee, the first-period winner gets the first-period unit but the second-period unit is withheld.*

A benevolent social planner may restrict trade at most in the first period,

[18] The details for this second case are provided in Appendix C.

[19] Alternatively, instead of entry fees, the seller can resort to a deposit scheme. However, unlike in Theorem 2, all agents — even the loser — would get partial reimbursements.

to prevent the informational asymmetry from arising; a revenue-maximizing seller may restrict trade in the second period, to extract bidders' continuation values. For instance, the DoD might award a contract for a prototype, but then shut down production if none of the bidders are willing to pay the fees.

Example (Continued). In the case of long-term commitment, the first-period reserve price is $r^{\pi *} = \frac{1}{4} + \delta \frac{1}{96}$ and the entry fees are $e^1 = \delta \frac{7}{24}$, $e^2 = \delta \frac{7}{48}$, and $e^3 = \delta \frac{1}{6}$. The net payment when a single bidder enters is δ.

As before, when there are more than 2 agents, the second-period auction must be of the open format, allowing non-neighbors to condition their behaviour on whether the neighbor remains active. Nothing else changes; all of the bidders who pay the corresponding entry fees participate in an English auction for the second unit, while the rest are excluded.

The mechanism in Theorem 3 requires the seller to commit to excluding from the second-period auction bidders who did not pay the entry fee at the end of the first period. Otherwise, come the second period, the seller would not refuse to sell to bidders who failed to pay the corresponding fees. In this case, she faces a second-period auction-design problem.

If no bidder has observed v, the second-period problem is a standard auction-design problem, with valuations $w_{i2} + E(V)$ for $i = a, b$. At history $h^{(i)}$, the problem is an asymmetric auction-design problem with interdependent values. Following Myerson (1981), expected revenues are maximized by a second-price or English auction with reserve price at history $h^{(0)}$, and by the following *scoring-rule* auction at history $h^{(i)}$. Define the virtual-valuation functions:

$$\begin{aligned}
\phi^0(w) &:= w - \frac{1 - F_W(w)}{f_W(w)}, \\
\phi^1(u) &:= u - \frac{1 - F_U(u)}{f_U(u)}, \\
\phi^2(u) &:= \phi^1(u) - g(u).
\end{aligned}$$

The reserve price for bidder i is $r_i^{(i)} := (\phi^1)^{-1}(0)$. The reserve price for $-i$ is a function of i's second-period bid; this function is $r_{-i}^{(i)}(b) := (\phi^0)^{-1}(-g(b))$. If the profile of bids is (b_{i2}, b_{-i2}), b_{i2} is compared to $\psi(b_{-i2})$, where $\psi := (\phi^2)^{-1} \circ \phi^0$, and the highest of these determines the winner. If i wins, she pays $\psi(b_{-i2})$; if $-i$ wins, she pays $\psi^{-1}(b_{i2})$.

In this scoring-rule auction, bids determine scores based on virtual valuations; the agent with the highest scored bid is allocated the unit, provided this scored bid is sufficiently high. For the non-neighbor, how high her scored bid has to be to qualify depends on the neighbor's bid, insofar as the latter conveys information about v. In the first period, a similar deposit scheme as in Theorem 2 works (Theorem C1 in Appendix C).

Example (Continued). Virtual-utility functions are $\phi^0(w) = 2w-1$, $\phi^1(u) = \frac{3}{2}u - \max\left\{\frac{1}{u}, 1\right\}$, and $\phi^2(u) = u - \max\left\{\frac{1}{u}, 1\right\}$. Correspondingly, we have $\psi(w) = w - \frac{1}{2} + \sqrt{w^2 - w + \frac{5}{4}}$ for $w \in \left[0, \frac{1}{2}\right)$, and $\psi(w) = 2w$ for $w \in \left[\frac{1}{2}, 1\right]$. At history $h^{(0)}$, a second-price or English auction with reserve price of $\frac{1}{2}$ ensues. The auction at history $h^{(i)}$ features a reserve price for the neighbor of $r_i^{(i)} = \frac{\sqrt{6}}{3}$; the score for of the neighbor's bid that determines the qualification of the non-neighbor's bid, $r_{-i}^{(i)}$, is redundant in this example. If b_{i2}, b_{-i2} is the profile of bids, b_{i2} is compared to $\max\left\{\frac{\sqrt{6}}{3}, \psi\left(b_{-i2}\right)\right\}$; the latter is the amount bidder i pays if she wins. Bid b_{-i2} is compared against $\max\left\{0, \psi^{-1}\left(b_{i2}\right)\right\}$; if $-i$ wins, this is the amount she pays.

An undesirable feature of this mechanism is that the neighbor's bid is taken to be meaningful *even if it is below the reserve price* she faces. In other words, a "non-serious" neighbor, one whose valuation falls short of the reserve price for her, must bid "as if seriously." However, equilibrium behavior leaves the bids of non-serious bidders indeterminate. We must select the bids that correspond to the "serious-bidding" function; arbitrary non-serious bids convey little information about v.

6 Divisibility and Double Sourcing

The government cannot procure a new weapon system without first evaluating a prototype. Or it may find it prohibitively costly to fund production of an aircraft, or some other large asset, without first assessing a blueprint. With no choice but to allocate both units, the government cannot improve on the sequence of second-price or English auctions. However, the government may be able to award multiple contracts to develop prototypes, even if ultimately only one supplier is to be chosen. This is a more efficient way to prevent the monopoly on information of common interest between competitors.

I represent this possibility of *double sourcing* in the present framework by letting the units be divisible.[20] To keep the discussion simple, I will only consider the case $N = 2$; for government procurements, the assumption of few providers is not unrealistic. Moreover, I will allow only for a 50-50 split (the "split award" in Anton et al., 2010), and I will assume that both agents get to observe v in the event of double sourcing.[21]

With more than two agents, the government may choose a proper subset of agents among which to split the allocation; if all of the bidders selected observe the common-value signal, the government can ask each of them to report this signal, and threaten to exclude them from the second period if their reports do not match.

The results in Section 3 apply to the allocation space $X' := X \cup \{(1/2, 1/2)\}$. Sharing the unit leads to a new continuation game, however; let h^D be the history after the unit is split in the first period, and denote the second-period portion of simple social-choice functions at history h^D by $f_2^D = \left(q_2^D, \tau_2^D\right)$.

With both agents having observed v, a second-price or English auction is efficient; it is irrelevant whether agents are equally informed or equally uninformed.[22] Hence, in the second period, the second-best allocation rule is

[20]The insights are the same if we represent double sourcing as awarding multiple units that are costly for the government to award and only one of them is socially valuable.

[21]Similar insights arise when we allow for a continuum of shares, the amount of information about the common-value components depending on the size of these shares. The details are available in the online appendix to this paper.

[22]If V and W are not independent, being equally uninformed or equally informed leads to

still q_2^{SB}, and the continuation welfare levels are still $S_2^{\omega 0}$ and $S_2^{\omega 1}$.

Now, the second best never wastes a unit: The informational asymmetry can be prevented by means of double sourcing. Nonetheless, there remains a trade-off. If first-period signals are w_{i1}, w_{-i1}, double sourcing creates an immediate surplus of $\frac{w_{i1}+w_{-i1}}{2}$, which is never higher than the immediate surplus created under exclusivity, $\max\{w_{i1}, w_{-i1}\}$; but double sourcing leads to an expected continuation welfare of $S_2^{\omega 0}$ instead of $S_2^{\omega 1}$. The net welfare from allocating to i exclusively compared to double sourcing is:

$$w_{i1} + \delta S_2^{\omega 1} - \frac{w_{i1} + w_{-i1}}{2} - \delta S_2^{\omega 0} = \frac{w_{i1} - w_{-i1}}{2} - \delta \Delta^{\omega}.$$

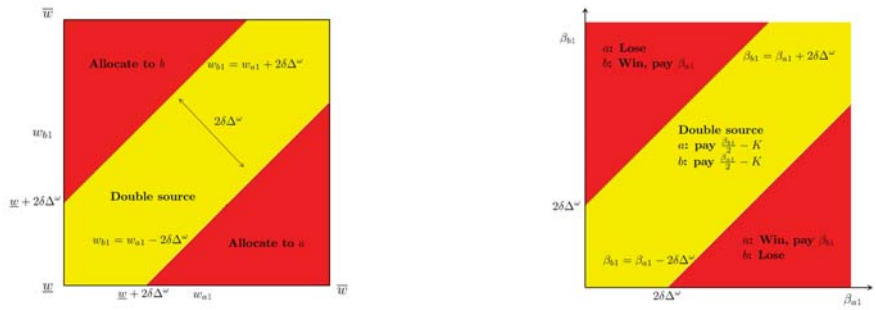
The next theorem is the counterpart of Theorem 2; it presents a simple variation of a sequential second-price or English auction that implements the second-best allocation rule with double sourcing. Define $K := \delta\Delta^{\omega} + \frac{\delta}{2}\left[\left(\overline{S}_2 - S_2^0\right) - \left(S_2^0 - \underline{S}_2\right)\right]$; Lemma A4 in Appendix A shows that $K \geq 0$.

Theorem 4. *The second-best allocation rule under divisibility is implemented by the following sequential auction. In the first-period, both bidders (simultaneously) submit bids. If the highest bid is higher than the lowest bid by at least $2\delta\Delta^{\omega}$, the highest bidder wins exclusively and pays the lowest bid. If the bids are within $2\delta\Delta^{\omega}$ of each other, the unit is split and each bidder pays half her opponent's bid minus a discount of K. The second-period auction is always a second-price or English auction.*

See Figure 5. Bidders are asked to submit a single bid, and whether there is double sourcing or exclusive dealing is contingent on how the two bids compare. In the auction analyzed in Anton et al. (2010), agents submit separate bids for exclusive dealing and for the split award.

Example (Continued). For double-sourcing the first unit, the bid spread is

different scenarios. If no agent observes v, they update their beliefs from their idiosyncratic-component signals and anticipate the value of v conditional on winning. If all agents observe v, their valuations are correlated but private, and there is no "guessing."



Second best with double sourcing — Auction in Theorem 4

Figure 5: Second best with double sourcing and auction in Theorem 4.

$2\delta\Delta^\omega = \frac{\delta}{24}$. In the event of double sourcing, bidders pay half their opponent's bid minus a discount of $K = \delta\frac{7}{96}$.

For a revenue-maximizing seller, double sourcing is desirable when agents are close to tied; but the relevant comparison is in terms of virtual valuations, not actual valuations. Unlike the social planner, a seller might still find it profitable to exclude some types, to save on information rents.

Theorem 5. *Under long-term commitment, the following mechanism maximizes revenues. The first-period auction is a scoring-rule auction: If the profile of bids is $b_1 = (b_{a1}, b_{b1})$, with $b_{i1} > b_{-i1}$, bidder i wins exclusively if $\phi^0(b_{i1}) > \phi^0(b_{-i1}) + 2\delta\Delta^\omega$, provided that $b_{i1} \geq r^{\pi*}$, where $r^{\pi*}$ is defined in Theorem 3; the unit is split if $|\phi^0(b_{i1}) - \phi^0(b_{-i1})| < 2\delta\Delta^\omega$ and $\frac{\phi^0(b_{i1})+\phi^0(b_{-i1})}{2} \geq -E(V)$. Otherwise, there is no trade. Payments for exclusivity and double sourcing are, respectively,*

$$p^{E*}(b_{-i1}) := E(V) + \frac{B_+(b_{-i1}) + B_-(b_{-i1})}{2},$$
$$p^{DS*}(b_{-i1}) := \frac{E(V) + B_-(b_{-i1})}{2},$$

where:

$$B_{-}(b) := \max\left\{\left(\phi^{0}\right)^{-1}\left(\phi^{0}(b)-2\delta\Delta^{\omega}\right), \left(\phi^{0}\right)^{-1}\left(-\phi^{0}(b)-2E(V)\right)\right\},$$
$$B_{+}(b) := \max\left\{\left(\phi^{0}\right)^{-1}\left(\phi^{0}(b)+2\delta\Delta^{\omega}\right), \left(\phi^{0}\right)^{-1}\left(-\phi^{0}(b)-2E(V)\right)\right\}.$$

After the bids are in and the winner is announced, but before allocating the unit, the winner is charged a second-period entry fee of $e^{1} := \delta\overline{S}_{2}$*, and the loser, of* $e^{2} := \delta\underline{S}_{2}$*; if the auction results in either double sourcing or in no trade, both bidders are charged* $e^{3} := \delta S_{2}^{0}$*. The rest of the second-period auction is as in Theorem 3.*

Example (Continued). Under long-term commitment, the first-period unit is allocated to bidder i if $b_{i1} > b_{-i1} + \delta\frac{1}{24}$, if her bid mets the reserve price. Otherwise, provided that $b_{i1} + b_{-i1} \geq \frac{1}{2}$, the unit is double sourced. Payments are based on $B_{-}(b) = \max\left\{b - \delta\frac{1}{48}, \frac{1-2b}{2}\right\}$ and $B_{+}(b) = \max\left\{b + \delta\frac{1}{48}, \frac{1-2b}{2}\right\}$.

Without intertemporal commitment power, at history h^{D}, second-period expected revenues are maximized by a second-price or English auction with a reserve price of $r^{1*}(v) := \left(\phi^{0}\right)^{-1}(-v)$ (See Theorem C2 in Appendix C).

7 Conclusion

This paper looks at the problem of sequentially allocating time-sensitive units of a good. Agents' valuations are the sum of both a private-value and a common-value component. This common component is unobserved ex-ante; it is only revealed through experience, and privately.

While the sequential second-price or English auction is inefficient, the second best can be implemented by a slight variation on this mechanism in which bidders are asked to pay deposits in the first period. If the first-period surplus always covers the welfare cost of implementability, or if both units must be allocated, then no variation is necessary: The sequential second-price or English auction implements the second best. If the units can be double sourced, bidders submit single bids directly, and the bid gap determines whether the contract is split or awarded exclusively.

Another variation on the sequential second-price or English auction, with personalized entry fees, implements the revenue-maximizing mechanism. When the seller cannot commit to excluding bidders who fail to pay the entry fees, then a scoring-rule auction with endogenous reserve prices is held after allocating the first unit exclusively.

One important question remains open for future research: What kind of social-choice functions can be implemented when private-value signals are persistent? The planner may wish to exploit this persistence, and condition the second-period allocation or the payments on the first-period outcome; but then, agents may have incentives to pool and hide information in the first place.

A Proofs

Proof of Lemma 1. By the Principle of Optimality, we can maximize welfare or revenues recursively. Due to the independence of the agents' first- and second-period signals, the second-period state variables are the contemporaneous signals and the first-period allocation outcome, not the first-period signals. The rest follows from the same argument behind the Revelation Principle. $\square$

For Lemma A1 and for the proof of Lemma 2 below, let $S_{i2}^{(i)}$ denote the truthful-equilibrium payoff function of agent i at history $h^{(i)}$.

Lemma A1. *Assume that there is a function $\Phi : [\underline{v}+\underline{w}, \overline{v}+\overline{w}] \to \mathbb{R}$ such that $S_{i2}^{(i)}(v, w_{i2}) = \Phi(v+w_{i2})$ for every (v, w). If $S_{i2}^{(i)}$ is differentiable at (v, w), Φ is differentiable at $v+w$.*

Proof. Assume that (v, w) is an interior point; for boundary points, the same argument works focusing on limits in the right directions. Take $h > 0$ small enough that $v+w+h \in [\underline{v}+\underline{w}, \overline{v}+\overline{w}]$, $v+\frac{h}{2} \in [\underline{v}, \overline{v}]$ and $w+\frac{h}{2} \in [\underline{w}, \overline{w}]$ and notice that $h = \left\| \begin{bmatrix} \frac{h}{2} & \frac{h}{2} \end{bmatrix}' \right\|$. Define the difference-quotient map, $\delta_{(v,w)}$, as:

$$\delta_{(v,w)}(h) := \frac{\Phi(v+w+h) - \Phi(v+w)}{h} = \frac{S_{i2}^{(i)}\left(v + \frac{h}{2}, w + \frac{h}{2}\right) - S_{i2}^{(i)}(v,w)}{\left\| \begin{bmatrix} \frac{h}{2} & \frac{h}{2} \end{bmatrix}' \right\|}$$
$$= \frac{S_{i2}^{(i)}\left(v + \frac{h}{2}, w + \frac{h}{2}\right) - S_{i2}^{(i)}(v,w) - \nabla S_{i2}^{(i)}(v,w)' \begin{bmatrix} \frac{h}{2} & \frac{h}{2} \end{bmatrix}'}{\left\| \begin{bmatrix} \frac{h}{2} & \frac{h}{2} \end{bmatrix}' \right\|} + \overline{q}_{i2}^{(i)}(v,w).$$

Since $S_{i2}^{(i)}$ is differentiable at (v,w), the limit of the first term as h approaches 0 is 0. Thus, $\delta_{(v,w)}(h)$ has limit $\overline{q}_{i2}^{(i)}(v,w)$ as h approaches 0. □

Proof of Lemma 2. (*Idea for argument suggested by Alejandro Manelli.*) The first part is immediate, and means that we can write $S_{i2}^{(i)}$ as a function $\widetilde{S}_{i2}^{(i)}$ of the sum of the arguments. By incentive compatibility, $S_{i2}^{(i)}$ is convex and hence differentiable almost everywhere on $[\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]$. By Lemma A1, if $S_{i2}^{(i)}$ is differentiable at $(v, w_{i2}) \in [\underline{v}, \overline{v}] \times [\underline{w}, \overline{w}]$, then $\widetilde{S}_{i2}^{(i)}$ is differentiable at $v + w_{i2}$. At such a point, we have:

$$\nabla S_{i2}^{(i)}(v, w_{i2}) = x_i(v, w_{i2}) = \overline{q}_{i2}^{(i)}(v, w_{i2}) \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \widetilde{S}_{i2}^{(i)\prime}(v + w_{i2}) \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The result follows by taking $\phi := \widetilde{S}_{i2}^{(i)\prime}$. □

Proof of Theorem 1. Let the expected second-period welfare in a simple direct-revelation dynamic mechanism be $s_2^{\omega 0}(q_2), s_2^{\omega 1}(q_2)$, and let $s_1^{\omega}(q)$ denote the corresponding expected overall welfare:

$$\begin{aligned} s_2^{\omega 0}(q_2) &:= E\left[(E(V) + W_{i2})q_{i2}^{(0)}(W_2) + (E(V) + W_{-i2})q_{-i2}^{(0)}(W_2)\right], \\ s_2^{\omega 1}(q_2) &:= E\left[U_{i2}q_{i2}^{(i)}(U_{i2}, W_{-i2}) + (V + W_{-i2})q_{-i2}^{(i)}(W_{-i2}, U_{i2})\right], \\ s_1^{\omega}(q) &:= E\left[q_{a1}(W_1)\left(E(V) + W_{a1} + \delta\left(s_2^{\omega 1}(q_2) - s_2^{\omega 0}(q_2)\right)\right)\right. \\ &\quad \left. + q_{b1}(W_1)\left(E(V) + W_{b1} + \delta\left(s_2^{\omega 1}(q_2) - s_2^{\omega 0}(q_2)\right)\right)\right] + \delta s_2^{\omega 0}(q_2). \end{aligned}$$

Start by analyzing the second-period allocation. At history $h^{(0)}$, there is no asymmetry or interdependencies in the information held by the agents, and the first-best allocation can be achieved: $s_2^{\omega 0}(q_2) \leq E(V) + E\left[\max\left\{W_{i2}, W_{-i2}\right\}\right] = s_2^{\omega 0}\left(q_2^{FB}\right) = S_2^{\omega 0}$. At history $h^{(i)}$, by Lemma 2, the planner only condition the second-period allocation rule on the sum of agent i's reports. We have:

$$
\begin{aligned}
s_2^{\omega 1}(q_2) &= E\left[q_{i2}\left(U_{i2}, W_{-i2}\right) U_{i2} + q_{-i2}\left(U_{i2}, W_{-i2}\right)\left(V + W_{-i2}\right)\right] \\
&\leq E\left[q_{i2}\left(U_{i2}, W_{-i2}\right)\left(U_{i2} - V - W_{-i2}\right)\right] + E(U) \\
&= E\left\{E\left[q_{i2}\left(U_{i2}, W_{-i2}\right)\left(U_{i2} - V - W_{-i2}\right)\right| U_{i2}, W_{-i2}\right]\right\} + E(U) \\
&= E\left[q_{i2}\left(U_{i2}, W_{-i2}\right)\left(h\left(U_{i2}\right) - W_{-i2}\right)\right] + E(U) \\
&\leq E\left[\max\left\{0, h\left(U_{i2}\right) - W_{-i2}\right\}\right] + E(U) = s_2^{\omega 1}\left(q_2^{SB}\right) = S_2^{\omega 1}.
\end{aligned}
$$

Moving back to the first period, fixing q_2^{SB} as continuation, $s_1^{\omega}\left(q_1, q_2^{SB}\right) \leq \delta S_2^{\omega 0} + E\left[\max\left\{\max\left\{W_{a1}, W_{b1}\right\} + E(V) - \delta\Delta^{\omega}, 0\right\}\right] = s_1^{\omega}\left(q^{SB}\right)$. □

Proof of Corollary 1. This is an immediate consequence of Lemma 2. □

Lemma A2. $\Delta^{\omega} \geq 0$.

Proof. We can write expected second-period social welfare as follows:

$$
\begin{aligned}
S_2^{\omega 0} &= E\left[\max\left\{U_{i2}, U_{-i2}\right\}\right], \\
S_2^{\omega 1} &= E\left[\max\left\{U_{i2}, g(U_{i2}) + W_{-i2}\right\}\right].
\end{aligned}
$$

For each $u_{i2} \in [\underline{v} + \underline{w}, \overline{v} + \overline{w}]$ and $w_{-i2} \in [\underline{w}, \overline{w}]$, we have:

$$
\begin{aligned}
\max\left\{u_{i2}, g\left(u_{i2}\right) + w_{-i2}\right\} &= \max\left\{u_{i2}, E\left[V + w_{-i2}\left| U_{i2} = u_{i2}, W_{-i2} = w_{-i2}\right.\right]\right\} \\
&\leq E\left[\max\left\{U_{i2}, U_{-i2}\right\}\left| U_{i2} = u_{i2}, W_{-i2} = w_{-i2}\right.\right].
\end{aligned}
$$

The result follows by taking expectations. □

Lemma A3. *Let $S_1^{\omega} := s_1^{\omega}\left(q^{SB}\right)$ and $\widetilde{S}_1^{\omega} := E\left[\max\left\{W_{a1}, W_{b1}\right\}\right] + E(V) + \delta E(U)$; $\widetilde{S}_1^{\omega}$ is the highest possible expected welfare when the second-period allocation is fixed beforehand. Then, $S_1^{\omega} \geq \widetilde{S}_1^{\omega}$.*

Proof. Take a pair $w_{a1}, w_{b1} \in [\underline{w}, \overline{w}]$ such that trade takes place under q_1^{SB}, we have $\max\{w_{a1}, w_{b1}\} + E(V) + \delta E(U) \leq \max\{w_{a1}, w_{b1}\} + E(V) + \delta S_2^{\omega 1}$. If the signals induce no trade under q_1^{SB}, namely, if $\max\{w_{a1}, w_{b1}\} + E(V) < \delta\Delta^\omega$, then $\max\{w_{a1}, w_{b1}\} + E(V) + \delta E(U) < \delta\Delta^\omega + \delta E(U) = \delta S_2^{\omega 0} - \delta S_2^{\omega 1} + \delta E(U) \leq \delta S_2^{\omega 0}$. The result follows by taking expectations. □

Proof of Theorem 2. Consider bidder i's problem of type w_{i1}, w_{i2}, if bidder $-i$ of type w_{-i1}, w_{-i2} adopts the following strategy:

- At history $h^{(0)}$, bid $w_{-i2} + E(V)$.
- At history $h^{(-i)}$, given v, bid $u_{-i2} = v + w_{i2}$.
- At history $h^{(i)}$, bid $h^{-1}(w_{-i2})$.
- If both bidders have paid the deposit, bid $w_{-i1} + E(V) + \delta\Delta^S$.
- If only $-i$ has paid the deposit, accept if $w_{-i1} + E(V) + \delta\overline{S}_2 - r^1 \geq \delta S_2^0$.
- Pay the deposit if $w_{-i1} \geq \underline{w}^* := \delta\Delta^\omega - E(V)$.

As first-period signals are uninformative of v, beliefs for agent i at histories $h^{(0)}$ and $h^{(-i)}$ are given by the priors. At history $h^{(0)}$, the symmetric equilibrium in the second-price auction has both bidders bidding their idiosyncratic signals augmented by $E(V)$. Straightforward bidding gives a bid of $u_{i2} = w_{i2} + v$ at history $h^{(i)}$; Theorem B1 establishes that i bids $h^{-1}(w_{i2})$ at history $h^{(-i)}$.

Turn to the first period. If bidder i is the only one who paid the deposit, she will be offered the first-period unit and the rebate. This offer is accepted by all types w_{i1} such that $w_{i1} + E(V) + \delta\overline{S}_2 - r^1 \geq \delta S_2^0$, or, equivalently, $w_{i1} \geq \delta\Delta^\omega - E(V) = \underline{w}^*$. If both bidders are active, they bid according to Theorem B1.

Finally, consider the participation problem. Start with the case $w_{i1} < \underline{w}^*$. If $w_{-i1} < \underline{w}^*$ and bidder i pays the deposit, she will be the only one to do so and can get the first-period unit for a net payment of r^1, her outside option being δS_2^0. However, this is not profitable: $w_{i1} + E(V) + \delta\overline{S}_2 - r^1 < \delta S_2^0$. An opponent of type $w_{-i1} \geq \underline{w}^*$ would pay the deposit and take the offer if unopposed. Hence, i is better off not paying the deposit: $w_{i1} + E(V) + \delta \overline{S}_2 - \beta_1(w_{-i1}) \leq w_{i1} + E(V) + \delta \overline{S}_2 - r^0 < \delta \underline{S}_2$. In either case, type $w_{i1} < \underline{w}^*$ cannot do better than not paying the deposit.

If $w_{i1} \geq \underline{w}^*$ and her opponent is of type $w_{-i1} < \underline{w}^*$, she will be the only bidder in the auction, should she choose to participate. She can get the first-period unit for a net payment of r^1, which yields a payoff of at least δS_2^0. This payoff is exactly her outside option, so she cannot profit by withholding the deposit. When her opponent's type is also above $\underline{w}^*$, her outside option is $\delta \underline{S}_2$. If she participates in the auction, both bidders will be present and the (interim) payoff to i will be $\max\{w_{i1} - w_{-i1}, 0\} + \delta \underline{S}_2 \geq \delta \underline{S}_2$.

It follows that the suggested strategy is a PBE. In fact, the strategies remain best responses if the information disclosed in equilibrium were made public beforehand. However, there would be ex-post regret if both components of agent i's valuation were made public at history $h^{(i)}$.

It is immediate that the equilibrium outcome at history $h^{(0)}$ coincides with the second-best allocation rule; the same is true at history $h^{(i)}$. When both bidders are active in the first period, the unit goes to the highest-value bidder. The deposits and reimbursements are chosen to exclude the right types. $\square$

Proof of Corollary 2. Let $\underline{w}^* = \delta\Delta^\omega - E(V)$ be the lowest type that pays the deposit in equilibrium. If $E(V) + \underline{w} \geq \delta\Delta^\omega$, then $\underline{w} \geq \underline{w}^*$. $\square$

Proof of Theorem 2'. The portion of the argument that correspond to the continuation game at histories $h^{(a)}, h^{(b)}$ is a special case of Proposition 1 in Hernando-Veciana and Troge (2011), so the details are omitted. As for the first period, the same argument as in the proof of Theorem 2 applies, with $\overline{w}_{-i1} := \max_{j\neq i} w_{j1}$ replacing w_{-i1} in the analysis of agent i's behavior. $\square$

Proof of Theorem 3. As the seller can exclude bidders who do not pay the entry fee for the second period, bidders' outside option at the moment of paying the fees is 0. As the fees do not extract more than the corresponding continuation values, it is individually rational (for all types) to pay the fee.

Hence, the seller captures the full continuation welfare. Seller's profits are:

$$\Pi(q_1, \tau_1) = E\left\{q_{a1}(W_1)\left[E(V) + \phi^0(W_{a1}) + \delta S_2^{\omega 1}\right]\right.$$
$$\left. + q_{b1}(W_1)\left[E(V) + \phi^0(W_{b1}) + \delta S_2^{\omega 1}\right] + \left[1 - q_{a1}(W_1) - q_{a2}(W_1)\right]\delta S_2^{\omega 0}\right\}.$$

Following Myerson (1981), the first-period portion of the mechanism is a second-price auction with reserve price $r^{\pi *}$. $\square$

Lemma A4. $K \geq 0$.

Proof. By Lemma A2, the first term in K is non-negative; we are done if we show that the second term is non-negative as well. Define the ex-post surplus functions:

$$\overline{S}_2(w_2, v) := \max\left\{0, w_{a2} - w_{b2} + v - g\left(h^{-1}\left(w_{b2}\right)\right)\right\},$$
$$S_2^0(w_2) := \max\{0, w_{a2} - w_{b2}\},$$
$$\underline{S}_2(w_2, v) := \max\left\{0, w_{a2} - w_{b2} + g\left(v + w_{b2}\right) - v\right\},$$

so $\underline{S}_2 = E\left[\underline{S}_2(W_2, V)\right]$. For each w_2, v, define $p(\cdot; w_2, v) : [0,1]^2 \to \mathbb{R}$ as:

$$p(x; w_2, v) := \max\left\{0, w_{a2} - w_{b2} + \left[v - g\left(h^{-1}\left(w_{b2}\right)\right)\right]x_1 + \left[g\left(u_{b2}\right) - v\right]x_2\right\}.$$

Notice that we can write $\overline{S}_2(w_2, v) = p((1,0); w_2, v)$, $S_2^0(w_2, v) = p((0,0); w_2, v)$, and $\underline{S}_2(w_2, v) = p((0,1); w_2, v)$. As the upper envelope of affine functions, it follows that $p(\cdot; w_2, v)$ is convex. Thus, $\frac{1}{2}p((1,0); w_2, v) + \frac{1}{2}p((0,1); w_2, v) \geq p\left(\left(\frac{1}{2}, \frac{1}{2}\right); w_2, v\right)$, or:

$$\overline{S}_2(w_2, v) + \underline{S}_2(w_2, v) \geq \max\left\{0, 2(w_{a2} - w_{b2}) + g\left(v + w_{b2}\right) - g\left(h^{-1}\left(w_{b2}\right)\right)\right\}.$$

By taking expectations over V conditional on w_2 and on $V > h^{-1}\left(w_{b2}\right) - w_{b2}$, Jensen's Inequality and affiliation imply that $\max\left\{0, 2(w_{a2} - w_{b2})\right\}$ is at most:

$$E\left[\overline{S}_2(w_2, v) | w_2, V > h^{-1}\left(w_{b2}\right) - w_{b2}\right] + E\left[\underline{S}_2(w_2, v) | w_2, V > h^{-1}\left(w_{b2}\right) - w_{b2}\right]$$

Finally, taking expectations, we get that $\frac{\left(\overline{S}_2 - S_2^0\right) - \left(S_2^0 - \underline{S}_2\right)}{2} = \frac{\overline{S}_2 + \underline{S}_2}{2} - S_2^0 \geq 0$. $\square$

Proof of Theorem 4. The payoff to w_{i1} when she bids b and $-i$ bids b' is:

$$s_{i1}(b, b'; w_{i1}) = \delta \underline{S}_2 + \begin{cases} w_{i1} + E(V) + \delta\Delta^S - b' & b > b' + 2\delta\Delta^\omega, \\ \frac{w_{i1}+E(V)+\delta\Delta^S}{2} - \frac{b'}{2} + \delta\Delta^\omega & |b - b'| \leq 2\delta\Delta^\omega, \\ 0 & b < b' - 2\delta\Delta^\omega. \end{cases}$$

Define $\mathbf{w}_{i1} := w_{i1} + E(V) + \delta\Delta^S$. We have $\mathbf{w}_{i1} - b' > \frac{\mathbf{w}_{i1}-b'}{2} + \delta\Delta^\omega$ if and only if $\mathbf{w}_{i1} > b' + 2\delta\Delta^\omega$; and $\frac{\mathbf{w}_{i1}-b'}{2} + \delta\Delta^\omega > 0$ if and only if $\mathbf{w}_{i1} > b' - 2\delta\Delta^\omega$. $\square$

Proof of Theorem 5. As in the proof of Theorem 3, the seller always captures the continuation welfare. (Recall that the continuation welfare is the same at histories h^D and $h^{(0)}$.) If first-period signals are w_1, interim profits are now $\phi^0(w_{a1}) + E(V) + \delta S_2^{\omega 1}$ if $q_1(w_1) = (1, 0)$; $\phi^0(w_{b1}) + E(V) + \delta S_2^{\omega 1}$ if $q_1(w_1) = (0, 1)$; $\frac{\phi^0(w_{a1})+\phi^0(w_{b1})}{2} + E(V) + \delta S_2^{\omega 0}$ if $q_1(w_1) = \left(\frac{1}{2}, \frac{1}{2}\right)$; and $\delta S_2^{\omega 0}$ if $q_1(w_1) = (0, 0)$. The following allocation rule maximizes expected revenues:

$$q_1^\pi(w_1) = \begin{cases} (1,0) & w_{a1} > w_{b1},\ w_{a1} \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(w_{b1}) + 2\delta\Delta^\omega\right), r^{\pi*}\right\}, \\ (0,1) & w_{b1} > w_{a1},\ w_{b1} \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(w_{a1}) + 2\delta\Delta^\omega\right), r^{\pi*}\right\}, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & |\phi^0(w_{a1}) - \phi^0(w_{b1})| < 2\delta\Delta^\omega,\ \frac{\phi^0(w_{a1})+\phi^0(w_{b1})}{2} \geq -E(V), \\ (0,0) & \text{otherwise}, \end{cases}$$

where $r^{\pi*}$ is as in Theorem 3. This allocation function satisfies the monotonicity condition for implementability; if the profile of first-period types is such that the unit is double sourced, then an increase in the signal of one of the agents will never cause her to be excluded. In other words, in moving from double sourcing to exclusive allocation, the favored agent always meets the reserve price $r^{\pi*}$: $\phi^0(w_{i1}) \geq \phi^0(w_{-i1}) + 2\delta\Delta^\omega$ and $\phi^0(w_{i1}) + \phi^0(w_{-i1}) \geq -2E(V)$ imply $\phi^0(w_{i1}) \geq -E(V) + \delta\Delta^\omega$, or $w_{i1} \geq r^{\pi*}$. The transfers that yield an implementable mechanism follow from the envelope formula for payoffs. For agent i, for the double-sourcing allocation, we have:

$$\tau_{i1}^\pi(w_1) = \frac{E(V) + B_-(w_{-i1})}{2};$$

for exclusivity,

$$\tau_{i1}^{\pi}(w_1) = E(V) + \frac{B_+(w_{-i1}) + B_-(w_{-i1})}{2};$$

otherwise, $\tau_{i1}^{\pi}(w_1) = 0$. These correspond to the proposed mechanism. $\square$

B Sequential Second-price or English Auctions

Theorem B1. *The following strategy profile is the unique strategy profile that survives iterated deletion of weakly-dominated strategies. In the second period, the neighbor bids her total valuation, while the non-neighbor bids according to $\beta_2 = h^{-1}$. In the first period, both bid according to $\beta_1(w) = w + E(V) + \delta\Delta^S$.*

Proof. Assume, without loss of generality, that a is the neighbor in the second period. Straightforward bidding is a weakly dominant strategy for her. If b observed u_{a2} before bidding, she would update her valuation to $w_{b2} + g(u_{a2})$. Her payoff from bid b would be $I(b > u_{a2})[w_{b2} - h(u_{b2})] \leq \max\{0, w_{b2} - h(u_{b2})\}$; this upper bound is uniquely attained at $b = h^{-1}(w_{b2})$.

In the first period, the bidders' problem is equivalent to a problem of independent and private values; bidding their valuation, $w_{i1} + E(V) + \delta\Delta^S$, is a weakly-dominant strategy. $\square$

Lemma B1. $\overline{S}_2 \geq S_2^0 \geq \underline{S}_2$.

Proof. Giving more information to the non-neighbor allows her to refine her bids without changing the neighbor's behavior; hence, $S_2^0 \geq \underline{S}_2$. Next, consider $\overline{S}_2(w_2, v) = \max\{0, w_{a2} - w_{b2} + v - g(h^{-1}(w_{b2}))\}$ and $S_2^0(w_2) := \max\{0, w_{a2} - w_{b2}\}$, so that $\overline{S}_2 = E\left[\overline{S}_2(W_2, V)\right]$, $S_2^0 = E\left[S_2^0(W_2)\right]$. By Jensen's Inequality and affiliation, $E\left[\overline{S}_2(w_2, V)\,|w_2, U_{a2} \geq h^{-1}(w_{b2})\right]$ is at least

$$\max\left\{0, w_{a2} - w_{b2} + E\left[V\,\middle|U_{a2} \geq h^{-1}(w_{b2})\right] - E\left[V\,\middle|U_{a2} = h^{-1}(w_{b2})\right]\right\},$$

which is at least $S_2^0(w_2)$. The result follows taking expectations. $\square$

C Short-Term Commitment

Lemma C1. *The second-period revenue-maximizing mechanism at $h^{(i)}$ is implemented by the following auction. The reserve price for i is $r_i^{(i)} := (\phi^1)^{-1}(0)$. The reserve price for $-i$ is a function of bidder i's second-period bid; this function is $r_{-i}^{(i)} := (\phi^0)^{-1} \circ (-g)$. If the profile of bids is (b_{i2}, b_{-i2}), b_{i2} is compared to $\psi(b_{-i2})$, where $\psi := (\phi^2)^{-1} \circ \phi^0$, and the highest of these determines the winner. If i wins, she pays $\psi(b_{-i2})$; if $-i$ wins, she pays $\psi^{-1}(b_{i2})$.*

Proof. Under implementable mechanisms, revenues are bounded above by the following expression: $E\left[\max\left\{\phi^0\left(W_{-i2}\right) + g\left(U_{i2}\right), \phi^1\left(U_{i2}\right), 0\right\}\right]$. This bound is attained by:

$$q_{i2}^{\pi,(i)}\left(u_{i2}, w_{-i2}\right) = I\left(\phi^1\left(u_{i2}\right) > \max\left\{\phi^0\left(w_{-i2}\right) + g\left(u_{i2}\right), 0\right\}\right),$$
$$q_{-i2}^{\pi,(i)}\left(u_{i2}, w_{-i2}\right) = I\left(\phi^0\left(w_{-i2}\right) + g\left(u_{i2}\right) > \max\left\{\phi^1\left(u_{i2}\right), 0\right\}\right).$$

Agents i, $-i$ pay, respectively,

$$\underline{u}^i\left(w_{-i2}\right) := \inf\left(\left\{u \in [\underline{v} + \underline{w}, \overline{v} + \overline{w}] : q_{i2}^{\pi,(i)}\left(u, w_{-i2}\right) = 1\right\}\right),$$
$$\underline{w}^{-i}\left(u_{i2}\right) := \inf\left(\left\{w \in [\underline{w}, \overline{w}] : q_{-i2}^{\pi,(i)}\left(u_{i2}, w\right) = 1\right\}\right)$$

per unit; these prices represent the lowest types that trade. With these transfer functions, both agents have incentives to bid truthfully. □

For the next lemma, let π_2^0 be the seller's second-period expected profits when no agent has observed v, and π_2^1, when a single agent has:

$$\pi_2^0 := E\left[\max\left\{\max\left\{\phi^0(W_{a2}) + E(V), 0\right\}, \max\left\{\phi^0(W_{b2}) + E(V), 0\right\}\right\}\right],$$
$$\pi_2^1 := E\left[\max\left\{\max\left\{\phi^1(U_{a2}), 0\right\}, \max\left\{\phi^0(W_{b2}) + g(U_{a2}), 0\right\}\right\}\right].$$

Let $\overline{S}_2^\pi$, $S_2^{\pi 0}$ and $\underline{S}_2^\pi$ be the ex-ante second-period surplus of the neighbor, of either agent if both are equally (un)informed, and of the non-neighbor, respectively:

$$\begin{aligned}
\overline{S}_2^{\pi} &:= E\left[\max\left\{U_{a2} - \max\left\{\left(\phi^1\right)^{-1}(0), \psi\left(W_{b2}\right)\right\}, 0\right\}\right], \\
S_2^{\pi 0} &:= E\left[\max\left\{W_{a2} - \max\left\{W_{b2}, \left(\phi^0\right)^{-1}(-E(V))\right\}, 0\right\}\right], \\
\underline{S}_2^{\pi} &:= E\left[\max\left\{W_{b2} - \max\left\{\left(\phi^0\right)^{-1}\left(-g\left(U_{a2}\right)\right), \psi^{-1}\left(U_{a2}\right)\right\}, 0\right\}\right].
\end{aligned}$$

Lemma C2. *The first-period allocation and transfer functions that maximize expected revenues are given by* $q_{i1}^{\pi}(w_1) = I\left(\phi^0(w_{i1}) + \Delta^{\pi} > 0\right) I\left(w_{i1} > w_{-i1}\right)$ *and by* $\tau_{i1}^{\pi}(w_1) = q_{i1}^{\pi}(w_1)\left[E(V) - \delta\left(\underline{S}_2^{\pi} - \overline{S}_2^{\pi}\right) + \max\left\{w_{-i1}, \left(\phi^0\right)^{-1}\left(-\Delta^{\pi}\right)\right\}\right] +$ $(1 - q_{-i1}^{\pi}(w_1))\delta\left(S_2^{\pi 0} - \underline{S}_2^{\pi}\right)$*, where* $\Delta^{\pi} := E(V) - \delta\left(S_2^{\pi 0} - \overline{S}_2^{\pi} + \pi_2^0 - \pi_2^1\right)$*.*

Proof. The seller's revenues, $\Pi(q_1, \tau_1)$, can be bound above by:

$$E\left[\max\left\{\phi^0(W_{a1}), \phi^0(W_{b1})\right\} + \Delta^{\pi}\right] - S_{a1}(\underline{w}) - S_{b1}(\underline{w}) + \delta\left(2S_2^{\pi 0} + \pi_2^0\right).$$

The bound is attained by $q_{i1}^{\pi}(w_1) = I\left(\phi^0(w_{i1}) + \Delta^{\pi} > 0\right) I\left(w_{i1} > w_{-i1}\right)$; the proposed transfers guarantee implementability. □

Theorem C1. *Let:*

$$\begin{aligned}
r^{\pi 0} &:= E(V) - \delta\left(\underline{S}_2^{\pi} - \overline{S}_2^{\pi}\right) + \left(\phi^0\right)^{-1}\left(-\Delta^{\pi}\right), \\
r^{\pi 1} &:= r^{\pi 0} - \delta\left(S_2^{\pi 0} - \underline{S}_2^{\pi}\right), \\
r^{\pi 2} &:= E(V) + \left(\phi^0\right)^{-1}(-E(V)).
\end{aligned}$$

The following mechanism maximizes expected revenues under short-term commitment. Both bidders are asked to pay (simultaneously) a deposit of $r^{\pi 0}$*. If both pay, they (simultaneously) submit bids; admissible bids are not lower than* $r^{\pi 0}$*. The highest bidder wins and pays the difference between the lowest bid and* $r^{\pi 0}$*; the loser gets her deposit back. If only one bidder pays, she is given the option to take the unit for a final price of* $r^{\pi 1}$*, or a full refund. In either case, if the unit is sold, the scoring-rule auction of Lemma C1 follows. The unit is* *withheld if nobody pays the deposit. If the unit is not sold, a second-price or English auction with reserve $r^{\pi 2}$ follows.*

Proof. Consider the following strategy for bidder i's of type w_{i1}, facing bidder $-i$ of type w_{-i1}:

- In the second period, bid as in the proof of Theorem 2 and Lemma C1.
- If both bidders are active in the first period, bid $w_{-i1}+E(V)-\delta\left(\underline{S}_2^\pi-\overline{S}_2^\pi\right)$.
- If only $-i$ has paid the deposit, accept if $w_{-i1}+E(V)+\delta\overline{S}_2^\pi-r^{\pi 1}\geq\delta S_2^{\pi 0}$.
- Pay the deposit if $w_{-i1}\geq\underline{w}^{**}:=\left(\phi^0\right)^{-1}\left(-\Delta^\pi\right)$.

That this strategy identifies a symmetric equilibrium follows by a very similar argument as in the proof of Theorem 2, so the details are omitted. It can be checked that the equilibrium outcome coincides with the social-choice function identified in Lemmas C1 and C2. □

Allowing for double sourcing, at history h^D, second-period expected revenues are maximized by a second-price or English auction with reserve price $r^{1*}(v):=\left(\phi^0\right)^{-1}(-v)$; the reserve price is $r^{1*}(E(V))$ instead if the first unit has been withheld. Denote by π_2^D, S_2^D the ex-ante second-period profits and surplus at h^D:

$$\begin{aligned}
\pi_2^D &:= E\left[\max\left\{\max\left\{\phi^0(W_{a2})+V,0\right\},\max\left\{\phi^0(W_{b2})+V,0\right\}\right\}\right],\\
S_2^D &:= E\left[\max\left\{W_{a2}-\max\left\{W_{b2},\left(\phi^0\right)^{-1}(-V)\right\},0\right\}\right].
\end{aligned}$$

Let $\Delta^{D1}:=\delta\left(2S_2^D+\pi_2^D-\overline{S}_2^\pi-\underline{S}_2^\pi-\pi_2^1\right)$ be the difference in discounted continuation welfare at h^D versus $h^{(i)}$; $\Delta^{D2}:=\delta\left(2S_2^{\pi 0}+\pi_2^0-\overline{S}_2^\pi-\underline{S}_2^\pi-\pi_2^1\right)$, that at $h^{(0)}$ versus $h^{(i)}$; and $\Delta^{D3}:=\delta\left(2S_2^{\pi 0}+\pi_2^0-2S_2^D-\pi_2^D\right)=\Delta^{D2}-\Delta^{D1}$, at $h^{(0)}$ versus h^D. Finally, define the reserve price $r^D:=\left(\phi^0\right)^{-1}\left(\Delta^{D2}-E(V)\right)$.

Theorem C2. *The following mechanism maximizes expected revenues under short-term commitment. In the first period, both bidders submit bids. If the* *profile of bids is $b_1 = (b_{a1}, b_{b1})$, with $b_{i1} > b_{-i1}$, bidder i wins exclusively if $\phi^0(b_{i1}) > \phi^0(b_{-i1}) + 2\Delta^{D1}$ and $b_{i1} \geq r^D$; the unit is split if $|\phi^0(b_{i1}) - \phi^0(b_{-i1})| \leq 2\Delta^{D1}$ and $\frac{\phi^0(b_{i1})+\phi^0(b_{-i1})}{2} \geq \Delta^{D3} - E(V)$. If bidder i wins exclusively and $b_{-i1} \geq r^D$, her payment is:*

$$p^{E*}(b_{-i1}) := E(V) + \frac{C_+(b_{-i1}) + C_-(b_{-i1})}{2} - \delta\left(\underline{S}_2^\pi - \overline{S}_2^\pi\right),$$

where:

$$\begin{aligned} C_-(b) &:= \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(b) - 2\Delta^{D1}\right), \left(\phi^0\right)^{-1}\left(-\phi^0(b) + 2\Delta^{D3} - 2E(V)\right)\right\}, \\ C_+(b) &:= \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(b) + 2\Delta^{D1}\right), \left(\phi^0\right)^{-1}\left(-\phi^0(b) + 2\Delta^{D3} - 2E(V)\right)\right\}. \end{aligned}$$

Otherwise, with bidder $-i$ disqualified, she is offered the unit for a price of $p^\pi := \delta\left(\overline{S}_2^{\pi 0} - S_2^{\pi 0}\right)$. In either case, bidder $-i$ pays nothing. If the unit is double sourced, bidder i pays:

$$p^{DS}(b_{-i1}) := \frac{E(V) + C_-(b_{-i1})}{2} + \delta\left(S_2^D - \underline{S}_2^\pi\right),$$

and similarly for bidder $-i$. The unit is withheld if nobody meets the reserve price. The second-period auction is as in Theorem C1, with the corresponding reserve price $r^{1*}(\cdot)$.

Proof. Given w_1, revenues are $\delta\left(\overline{S}_2^\pi + \underline{S}_2^\pi + \pi_2^1\right) + E(V) + \phi^0(w_{a1})$ if $q_1(w_1) = (1,0)$; $\delta\left(\overline{S}_2^\pi + \underline{S}_2^\pi + \pi_2^1\right) + E(V) + \phi^0(w_{b1})$ if $q_1(w_1) = (0,1)$; $\delta\left(2S_2^D + \pi_2^D\right) + E(V) + \frac{\phi^0(w_{a1})+\phi^0(w_{b1})}{2}$ if $q_1(w_1) = \left(\frac{1}{2}, \frac{1}{2}\right)$; and $\delta\left(2S_2^{\pi 0} + \pi_2^0\right)$ if $q_1(w_1) = (0,0)$. The allocation rule that maximizes revenues is given by:

$$\begin{aligned} &q_1^\pi(w_1) \\ &:= \begin{cases} (1,0) & w_{a1} > w_{b1}, \quad w_{a1} \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(w_{b1}) + 2\Delta^{D1}\right), r^D\right\}, \\ (0,1) & w_{b1} > w_{a1}, \quad w_{b1} \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(w_{a1}) + 2\Delta^{D1}\right), r^D\right\}, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & |\phi^0(w_{i1}) - \phi^0(w_{-i1})| < 2\Delta^{D1}, \ \frac{\phi^0(w_{i1})+\phi^0(w_{-i1})}{2} \geq \Delta^{D3} - E(V), \\ (0,0) & \text{otherwise.} \end{cases} \end{aligned}$$

As in Theorem 5, this allocation function satisfies the monotonicity condition for implementability. This is the proposed allocation rule. The payoff to w_{i1} from the proposed auction when she bids b, and $-i$ bids b', is:

$$
\begin{aligned}
& s_{i1}(b,b';w_{i1}) \\
&= \delta \underline{S}_2^{\pi} + \begin{cases}
w_{i1} - \frac{C_+(b')+C_-(b')}{2} & \begin{array}{c} b \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(b') + 2\Delta^{D1}\right), r^D\right\}, \\ b' \geq r^D; \end{array} \\
w_{i1} + \delta\left(\overline{S}_2^{\pi 0} - \underline{S}_2^{\pi}\right) - p^{\pi} & \begin{array}{c} b \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(b') + 2\Delta^{D1}\right), r^D\right\}, \\ b' < r^D, \text{ accept}; \end{array} \\
\delta(S_2^{\pi 0} - \underline{S}_2^{\pi}) & \begin{array}{c} b \geq \max\left\{\left(\phi^0\right)^{-1}\left(\phi^0(b') + 2\Delta^{D1}\right), r^D\right\}, \\ b' < r^D, \text{ reject}; \end{array} \\
\frac{w_{i1} - C_-(b')}{2} & \begin{array}{c} |\phi^0(b) - \phi^0(b')| < 2\Delta^{D1}, \\ \frac{\phi^0(b)+\phi^0(b')}{2} \geq \Delta^{D3} - E(V); \end{array} \\
0 & \text{otherwise;}
\end{cases} \\
&= \delta \underline{S}_2^{\pi} + \begin{cases}
w_{i1} - \frac{C_+(b')+C_-(b')}{2} & b \geq C_+(b'), b' \geq r^D, \\
w_{i1} + \delta(S_2^{\pi 0} - \underline{S}_2^{\pi}) & b \geq C_+(b'), b' < r^D, \text{ accept}, \\
\delta(S_2^{\pi 0} - \underline{S}_2^{\pi}) & b \geq C_+(b'), b' < r^D, \text{ reject}, \\
\frac{w_{i1} - C_-(b')}{2} & b \in [C_-(b'), C_+(b')), \\
0 & \text{otherwise.}
\end{cases}
\end{aligned}
$$

Bidding straightforwardly is a weakly dominant strategy. In the event of being the only serious bidder, the offer to buy for price p^{π} is never rejected. □

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