Markets with Cross Network Externalities as Vertically Differentiated Markets

Jean J. Gabszewicz*and Xavier Y. Wauthy‡

November 15, 2005

Abstract

In this note, we study duopoly competition between platforms in a market with cross network externalities. Network externalities are captured by a vertical differentiation setup. When agents patronize one platform only, we show that price competition allows for an equilibrium market outcome where two asymmetric platforms co-exist with positive profits. Product differentiation is therefore endogenous. When all agents are allowed to patronize the two platforms, we show that in equilibrium multi-homing takes place on one side of the market only. Moreover, the only equilibrium exhibiting positive profits for both platforms replicates the collusive outcome.

Keywords : cross network externalities, vertical differentiation, two-sided markets
JEL Classification :L13

*CORE, UCL

†CEREC, FUSL and CORE. X.Wauthy acknowledges the financial support of a SSTC grant from the belgian federal government under the IAP contract 5/26

‡We are grateful to Paul Belleflamme and Jean Tirole for stimulating comments on the first draft of this paper. We remain solely responsible for remaining errors.

1 Introduction

In markets where network externalities are present, the economic value of a particular good can be decomposed into two elements: the stand-alone value component and the network component. The first one refers to the benefits an individual obtains from the consumption of the good when the size of the network is nil, i.e. when network effects are not present. The second one refers to the specific added-value coming from the size of the network. For those goods which display network externalities, it is quite natural to view the size of the network as the key component of quality. In this respect, network industries might exhibit some of the key ingredients of vertical differentiation, i.e. *networks of different sizes attached to different goods actually induce a vertical ranking of these goods* in the eyes of would-be customers. Bental and Spiegel (1995) or Baake and Boom (1999) already developed models where the vertical differentiation framework is used to analyze competition in industries exhibiting network effects. In the present paper, we pursue along their line of research by considering the case of industries where cross network externalities are at work. More specifically, we have in mind the case of industries where the basic service provided by the firms consists of establishing various forms of connections between different types of agents. The value of the service for a particular type of agent exclusively depends on the number of other types to which they are possibly linked, i.e. the network effect is defined across types. Obviously, markets with these features belong to the class of the so-called "two-sided markets" (Rochet and Tirole, 2004).

A significant number of real-life markets operate under these features. Consider, as specific examples, shopping malls, exhibition centers, credit cards. The larger the number of shoppers attracted in a shopping mall, the higher the willingness of a retailer to locate in that shopping mall. Conversely, the larger the number of shops located in the shopping mall, the higher the willingness of shoppers to pay a visit to it. Similarly, in exhibition centers, the larger the number of visitors, the larger the number of exhibitors who want to participate. In the same manner, the larger the number of exhibitors in an exhibition center, the larger the number of visitors. Credit cards offer also a particularly well documented case in point. The plastic piece we hold in our wallet has no value as such. However one might be willing to pay for holding it, simply because it gives access to the network of retailers accepting the card as a means of payment. The larger the numbers of retailers who accept that card, the larger the value of holding this particular card. Notice that in all the above examples there is a priori no reason why a buyer or a seller should choose to interact through a single platform only. In the first example above, in which two shopping malls compete for attracting buyers or visitors, there is no reason why one should *a priori* prevent a particular shopper from visiting two shopping malls, and a particular retailer from settling a shop in each of them. Similarly, in the credit card example, nothing prevents a consumer from holding several credit cards or a retailer from accepting several of them. This behaviour of registration to several platforms has been coined as *multi-homing behaviour*.

While there already exists a vast literature analyzing markets with cross network externalities (see below), the connections between quality competition and two-sided competition has not been explicitely considered. In the present paper, we show that the standard vertical differentiation framework actually offers a very natural vehicle for capturing the basic mechanisms at work in two-sided industries. Building on the heterogeneity of agents on the two sides of the market, our model neatly captures the key features of two-sided markets: prices charged to the different types of agents determine equilibrium network sizes on both sides and therefore relative qualities of the services offered by the platforms. This in turn determines the market coverage, i.e. the size of the platform itself.

In the vertical differentiation set-up, the co-existence of asymmetric platforms emerges as a very natural equilibrium outcome: several platforms of different size might be active in equilibrium while enjoying strictly positive profits. Dominant platform equilibria where one platform only is active might also obtain. Moreover, the vertical differentiation model also allows to analyze the case where agents are allowed to patronize several platforms, i.e. agents "multi-home". When multi-homing is endogenous, we characterize the equilibrium in which both platforms are active and secure positive profits. This equilibrium displays multi-homing on one side of the market only, collusive prices in the multi-homing side of the market and marginal cost pricing on the other side. In equilibrium, platforms are therefore symmetric. Interestingly enough, this result suggests that the scope for asymmetric platforms as an equilibrium phenomenon might be tied to single-homing in an essential way.

In the literature which studies two-sided markets, Rochet and Tirole (2004) offer a precise discussion of the relevant issues pertaining to the analysis of such markets. We refer the reader to this paper for an extensive presentation of the state of the art and shall focus here of those papers which are most directly related to our analysis. Caillaud and Jullien (2003) study in detail the chicken and egg issue, i.e. the problem faced by a firm that needs to embark simultaneously both sides of the market for its platform to emerge. They consider a good whose value is entirely created by the intermediation service offered by the platforms and study the conditions under which these platforms manage to get the two sides of the market on board simultaneously. This problem is particularly crucial when products have zero stand-alone value. These authors consider a case where agents that make use of the platforms are homogeneous. By contrast, we assume they are heterogeneous. Moreover, they assume that all agents on both sides of the market "participate" (i.e. register to one firm at least) whereas we allow for an endogenous participation in each side of the market (i.e. not registrering to any firm is allowed, and is observed in equilibrium). Our results are clearly in line with those of Caillaud and Jullien (2003) while enlighting the impact of heterogeneity on platform competition. Armstrong (2005) and Armstrong and Wright (2004) allow for heterogeneous agents (possibly) on the two sides of the market. However, this heterogeneity is not at all related to the network externalities. Instead, products have positive stand-alone value and agents' preferences are heterogeneous with respect to this dimension only.[1] By contrast, in our model, heterogeneity of the agents' preferences is directly defined in terms of their valuation of network externalities and depend only on them.[2] Ambrus and Argenziano (2004) emphasize, like us, the connections existing between the analysis of two-sided markets and vertical differentiation. While they propose a very sophistiated analysis of the network formation issue, their model falls short of allowing for an explicit characterization of equilibria. By contrast, we have opted for a more simple setup. This allows us to obtain closed form solutions which neatly capture the main forces at work in markets with cross network externalities.

2 The Basic Setup

First, we present the formal model retained for the analysis. Then, we discuss the case of a monopoly platform in order to highlight the key intuitions underlying pricing strategies in

[1] Matutes and Vives (1996) propose a model of competition for deposits in banks where quality differentiation emerges in equilibrium as a result of bank competition. A key difference with our analysis is again that the heterogeneity of the agents is of the Hotelling type, i.e. horizontal differentiation.

[2] In Rochet and Tirole (2003) agents on both sides of the market are heterogeneous. However, their model focuses on usage externalities whereas the present one focuses on membership externalities.

two-sided markets.

3 The Model

The basic ingredients of the model are drawn from the standard literature on vertical differentiation. The specification of preferences we retain are those of Mussa and Rosen (1978). There are three types of agents:

- Platforms: they are denoted by i and sell product $i = 1, 2$. Product i is best viewed as a "device" designed to match different types of agents. For the sake of illustration, we shall refer here to the exhibition centers metaphor. Then one can think of product i as a commercial fair organized at an exhibition center i. Platforms are the organizers of the fairs in exhibition centers. They sell their product in two markets: the visitors' market and the exhibitors' market. The access permit paid by the visitors, as well as the rental fee paid to the platforms by exhibitors, allow visitors and exhibitors to trade if they succeed to match. We assume that platforms sell at zero marginal cost. Therefore they maximize sales revenue by setting access prices $p_i \geq 0$ in the visitors' market, and rental fees $\pi_i \geq 0$ in the exhibitors' one.[3]

- Visitors: they are denoted by their type θ. Types are uniformly distributed in the $[0, \overline{\theta}]$ interval. The total number of visitors is normalized to 1. They possibly buy product $i = 1, 2$ according to a utility function $U_i = \theta x_i^e - p_i$, with x_i^e denoting the expectation visitors hold about the number of exhibitors at platform i. When buying the access permits to *both* exhibition centers, a visitor enjoys a utility $U_{12} = \theta x_3^e - p_1 - p_2$. Parameter x_3^e depends on the expectation visitors hold about the number of exhibitors who exhibit in both centers. Holding no access permit yields a utility level normalized to 0.

- Exhibitors: they are denoted by their type γ. Types are uniformly distributed in the $[0, \overline{\gamma}]$ interval. Their total number is normalized to 1. They possibly exhibit in both exhibition centers. When they exhibit in center i, $i = 1, 2$, their utility is measured by $U_i^{'} = \gamma v_i^e - \pi_i$, with v_i^e denoting the expectation they hold for the number of visitors in center i. When deciding to exhibit in both centers, an exhibitor enjoys a utility $U_{12} = \gamma v_3^e - \pi_1 - \pi_2$. Again, v_3^e depends on the expectation about the total number of visitors in both centers. Refraining from exhibiting in any exhibition center yields a utility level normalized to 0.

Parameters γ and θ are best understood as an indirect measure of the value-added derived by agents in their respective markets from realizing a transaction. Exhibitors might be heterogeneous in this respect because of the unit value of the goods they exhibit for sale. Visitors on the other side are likely to be heterogeneous according to the number and the importance of the transactions they wish to perform. In any case, an exhibition without any exhibitor or without any visitor is completely useless and thereofre unattractive, i.e. the stand-alone value of the product is nil.

The intuition underlying our model is the following. From the viewpoint of an exhibitor, the willingness to rent a stand in the exhibition fair depends on his own type and on the number of additional sales this exhibitor may expect to realize by accepting to pay the rental

[3]Notice thus that our model corresponds to a "pure membership" model, following the terminology proposed by Rochet and Tirole (2004). Such an assumption is justified in cases where the platform does not benefit from an obvious way to monitor the transactions realized by the agents.

fee. This essentially depends on the number of visitors. On the other hand, the willingness to pay for holding some given access permit depends on the visitor's type and on the number of transactions he/she would realize by meeting exhibitors. This essentially depends on the number of exhibitors. The market is thus characterized by cross network externalities. From the viewpoint of one side of the market, say visitors, the two exhibition centers are viewed as selling vertically differentiated products whenever they differ in their number of exhibitors. Products hierarchy reflects the asymmetry in network sizes. The network externalities at work in the present framework can thus be viewed as defining two parallel vertically differentiated markets where quality in one market is endogeously determined by the size of the network in the other market. In the case where the expectations are that networks of the two platforms are of the same size, we shall assume that if prices are identical, agents are spread evenly across the two platforms.

As mentioned in the introduction, the recent literature on two sided markets has often focused on the formation of the networks themselves (see Caillaud and Jullien (2003)). Addressing this issue involves a very complex analysis of the coordination game played by the consumers themselves when they face the prices charged by the platforms. While we acknowledge the importance of this problem, we have opted for a simplified setup which allows us to better focus on the vertical dimensions of the problem. We only require that expectations consumers may hold are fullfilled in equilibrium. Obviously, the chief merit of this assumption is to keep the analysis reasonably tractable and intuitive. Moreover, as shown by Matutes and Vives (1996), this assumption is certainly acceptable when agents on one side of the market do not know how prices are set in the other one, for instance because they do not observe those prices or because they do not know the preferences of the agents on the other side. Notice indeed that contrary to the usual models displaying network externalities, in two-sided markets, the information which is relevant to an agent pertains to agents on the other side of the market (the prices these agents are charged but also their preferences). It is therefore far from obvious that this information is indeed available.

As should be clear by now, product quality in one market reflects the expectations of agents relative to the network size in the other market. We may therefore characterize optimal pricing strategies in each market, conditional on these expectations. To this end, we rely on the standard analysis of price competition under vertical differentiation (as developed for instance in Wauthy (1996)). We start by considering the monopoly case.

3.1 The Monopoly Case

Define an agent to be *active* if he/she visits, or exhibits in at least one commercial fair. Obviously the number of active agents in each market depends negatively on the product price and positively on the expected size of the relevant network in the other market. The set of active agents in one market is defined by those types who enjoy a positive surplus when buying the product. For instance, in the visitors' market, given some price p and expectation x^e, the set of active visitors is $[\hat{\theta}, 1]$ where $\hat{\theta}$ solves $\theta x^e - p = 0$. Using our specification of agents' preferences, we may derive demand addressed to the platform by the visitors as a function of the expected number of exhibitors. Denoting this expected number by x^e, we get

$$D^v(p, x^e) = 1 - \frac{p}{\overline{\theta} x^e}.$$

Regarding demand addressed to the platform by the exhibitors, given an expected number of visitors v^e, we have:

$$D^x(\pi, v^e) = 1 - \frac{\pi}{\overline{\gamma} v^e}.$$

Given expectations x^e and v^e, the objective of the monopolist is to maximize the function

$$\max_{p,\pi} pD^v(p, x^e) + \pi D^x(\pi, v^e).$$

Using this objective function, we derive optimal prices, conditional on expectations. The optimal strategies are $p^*\left(x^e, v^e\right) = \overline{\theta}\frac{x^e}{2}$ and $\pi^* = \overline{\gamma}\frac{v^e}{2}$. At these prices, the size of the network on each market is equal to $\frac{1}{2}$. Requiring that expectations are fullfilled at equilibrium, we obtain a candidate optimal strategy for the monopolist as $p^* = \frac{\overline{\theta}}{4}$ and $\pi^* = \frac{\overline{\gamma}}{4}$. The monopolist's payoff in this case is equal to $\frac{\overline{\theta}}{8} + \frac{\overline{\gamma}}{8}$. Regarding the participation of visitors and exhibitors to the market, we notice that half of the visitors and half of the exhibitors (those with high θ and γ) are active.

In order to characterize the optimal strategy of the monopolist, it remains to consider the case where the monopolist sets different prices from on side to the other. A candidate optimal stragegy in this case consists in charging a low price on one side in order to increase market coverage on this side, which in turn allows to charge a higher price to the other side of the market. Given our specification of agents' preferences, a monopolist should decide to charge a high price on that side of the market which displays the highest willingness to pay for quality. Suppose for instance that $\overline{\gamma} > \overline{\theta}$, then a candidate optimal strategy is $p^{**} = 0$, $\pi^{**} = \frac{\overline{\gamma}}{2}$. It is then immediate to see that this candidate optimum dominates the above one whenever $\overline{\gamma} > \overline{\theta}$. We may therefore summarize our findings in the following proposition:

Proposition 1 *Suppose $\overline{\gamma} > \overline{\theta}$, the optimal strategy for the monopolist is to price at zero on the visitors' side and set $\pi^{**} = \frac{\overline{\gamma}}{2}$ on the exhibitors' side. Suppose $\overline{\gamma} < \overline{\theta}$, the optimal strategy for the monopolist is to price at zero on the exhibitors' side and set $p^{***} = \frac{\overline{\theta}}{2}$ on the visitors' side. Suppose $\overline{\gamma} = \overline{\theta} = z$, the monopolist is indifferent between the two above strategies and setting prices $p^* = \pi^* = \frac{z}{4}$.*

Proposition 1 makes transparent the mechanism at work in this model. Pricing "low" in one side of the market increases the number of active agents in this market. This makes the platform more attractive to the other side of the market, which allows to charge higher prices there. A firm may therefore expect to recoup margins lost in one market by the extra margin it allows for in the other market. Whenever the two sides differ in the absolute willingness to pay for quality, the monopolist extracts all of its surplus from the "rich" side of the market. This example nicely illustrates the fact that in markets with cross network externalities, it is the price structure, rather that the level of prices, which really matters (see Rochet and Tirole (2004) on this point). An interesting property of the model lies in the fact that symmmetry of the two sides is a possible source of market failure. The monopolist is indeed indifferent between a symmetric and an asymmetric price structure in this case. However in the first case only half of the population in both markets is active. This is clearly less efficient than asymmetric pricing strategies because of a lower market coverage. Fortunately enough, a limited asymmetry among the two sides of the market is sufficient to get rid of this form of inefficiency.

4 Duopoly Competition with Single-Homing

We assume now that there exist two platforms which compete in prices. However, in this subsection, we suppose that active agents are not allowed to patronize two platforms simultaneously, i.e. they have to "single-home".

Let us then derive demands addressed to platforms by the exhibitors. They depend on exhibitors' expectations (v_1^e, v_2^e). Assume $v_2^e > v_1^e$; then we get

$$D_1^x(\pi_1, \pi_2) = \frac{\pi_2 v_1^e - \pi_1 v_2^e}{v_1^e(v_2^e - v_1^e)} \frac{1}{\overline{\gamma}},$$

$$D_2^x(\pi_1, \pi_2) = 1 - \frac{\pi_2 - \pi_1}{v_2^e - v_1^e} \frac{1}{\overline{\gamma}}.$$

These are the demand functions of a vertical differentiation model where the quality of the products are exogenously defined by $v_2^e > v_1^e$. A similar demand specification $D_i^v(p_1, p_2)$ can be obtained in the visitors' market given expectations $x_2^e > x_1^e$.

Conditional on expectations v_1^e, v_2^e, $v_2^e > v_1^e$, and $x_1^e, x_{2,}^e x_2^e > x_1^e$, the payoff functions are then derived as

$$p_i D_i^v(p_1, p_2) + \pi_i D_i^x(\pi_1, \pi_2), \ i = 1, 2.$$

Formally, we define a Nash equilibrium in our model as follows:[4]

Definition 1 *A Nash Equilibrium is defined by two quadruples* (p_i^*, π_i^*) *and* (v_i^*, x_i^*) *with* $i = 1, 2$, *such that*

1. *given expectations* $(v_1^*, v_2^*, x_1^*, x_2^*)$, $\left(p_{i,}^* \pi_i^*\right)$ *is a best reply against* $\left(p_{j,}^* \pi_j^*\right)$, $i \neq j$, *and vice-versa ;*

2. $D_i^v(p_1^*, p_2^*) = x_i^*$*;* $D_i^x(\pi_1^*, \pi_2^*) = v_i^*$, $, i = 1, 2$.

This definition allows firms to deviate simultaneously in the two components of the strategies at their disposal. Obviously, it implies that, at a Nash equilibrium, each pair of prices $(p_1^*, p_2^*), (\pi_1^*, \pi_2^*)$ also defines a price equilibrium in its respective market. Notice that when the two pairs (p_1^*, p_2^*) and (π_1^*, π_2^*) define each a price equilibrium in the visitors' and exhibitors' market respectively, the pair of strategies $(p_1^*, \pi_1^*), (p_2^*, \pi_2^*)$ must satisfy the first condition in our definition. The second part of the definition requires that expectations are fullfilled in equilibrium.

We now derive the price equilibrium on the exhibitors' market, conditional on expectations $v_1^e < v_2^e$:

$$\pi_2(v_1^e, v_2^e) = \overline{\gamma} \frac{2v_2^e(v_2^e - v_1^e)}{4v_2^e - v_1^e},$$

$$\pi_1(v_1^e, v_2^e) = \overline{\gamma} \frac{v_1^e(v_2^e - v_1^e)}{4v_2^e - v_1^e},$$

with corresponding demands:

$$D_2^x(v_1^e, v_2^e) = \frac{2v_2^e}{4v_2^e - v_1^e},$$

[4] This definition essentially extends the definition of Katz and Shapiro (1984) to the context of a market with cross network externalities.

$$D_1^x(v_1^e, v_2^e) = \frac{v_2^e}{4v_2^e - v_1^e}.$$

Obviously, the symmetry of our model allows us to directly infer the price equilibrium, conditional on expectations, $x_2^e > x_1^e$, on the visitors' market, as well as conditional equilibrium demands. We obtain

$$D_2^v(x_1^e, x_2^e) = \frac{2x_2^e}{4x_2^e - x_1^e},$$

$$D_1^v(x_1^e, x_2^e) = \frac{x_2^e}{4x_2^e - x_1^e}.$$

Then it remains to solve the model for fullfilled expectations. This is done by solving the system

$$x_2 = \frac{2D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)},$$

$$x_1 = \frac{D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}.$$

Straighforward computations yield $x_1^* = v_1^* = \frac{2}{7}$ and $x_2^* = v_2^* = \frac{4}{7}$, and corresponding prices $\pi_1^* = \overline{\gamma}\frac{2}{49}$, $\pi_2^* = \overline{\gamma}\frac{8}{49}$ on the exhibitors' side, $p_1^* = \overline{\theta}\frac{2}{49}$, $p_2^* = \overline{\theta}\frac{8}{49}$ on the visitors' one.

The presence of heterogeneity in both markets allows for an interior equilibrium where both platforms enjoy strictly positive networks and profits. Notice however that in addition to this equilibrium we also identify the "dominant firm" equilibria where one exhibition center monopolizes the markets by pricing at marginal cost on one side and charging the corresponding monopoly price on the other side. Obviously, this equilibrium replicates the monopoly equilibrium of Proposition 1. Last, we cannot rule out the pure Bertrand equilibrium where both platforms sell their products at marginal costs in the two markets. In this equilibrium all visitors and exhibitors are active and the market is shared evenly. Platforms make no profit at this equilibrium.

Proposition 2 *With single-homing only, the set of Nash Equilibria obtains as*

- *the quadruples* $(x_1^* = v_1^* = \frac{2}{7},\ x_2^* = v_2^* = \frac{4}{7})$ *and* $(\pi_1^* = \overline{\gamma}\frac{2}{49}, p_1^* = \overline{\theta}\frac{2}{49}\ ,\ \pi_2^* = \overline{\gamma}\frac{8}{49}, p_2^* = \overline{\theta}\frac{8}{49})$ *which define the unique (up to permutation) interior equilibrium in which both platforms enjoy positive profits ;*
- *The "dominant firm" equilibria which replicate the outcomes described in Proposition 1 ;*
- *and the Bertrand equilibrium* $(v_i^* = x_i^* = \frac{1}{2})$, $(p_i^* = \pi_i^* = 0)$, *with* $i = 1, 2$.

Proposition 2 clearly illustrates the links that relate markets with cross network externalities and vertically differentiated industries. When setting different prices, platforms actually attract different types of agents on both sides of the market and thereby fix the size of the networks. In equilibrium, the size of the network endogenously determines the willingness of the consumers to participate in one of the two platforms. When the population of agents on both sides of the market is heterogeneous in its willingness to pay for network sizes, asymmetric equilibria naturally emerge. In these equilibria the two platforms are clearly ranked by size but nevertheless enjoy positive market shares and profits. On each side of the market, equilibrium outcomes resemble those obtained in standard models of vertical differentiation: one firm is perceived by all agents as better than the other but not all agents register to that firm because of the price differential. The relative sizes of the platform can thus be viewed as the qualities attached to these platforms, so that by playing on agents' heterogeneity, a dominated platform can survive by charging lower prices, without inducing the dominant platform to price aggressively and preempt the market. A key difference with standard models of vertical differentiation is obviously that realized qualities are not directly controlled by the firms but depend on their prices and the network externalities that cross from one side of the market to the other.

4.1 Duopoly Competition with Multi-Homing

Suppose now that exhibitors may opt for exhibiting in both centers, and/or visitors may decide to visit both centers, i.e. agents are allowed to multi-home. To what extent does this possibility alter our previous analysis? Intuitively, the willingness to pay for a second purchase on one side of the market depends on the multi-homing behaviour of the other side. Suppose for instance that most exhibitors attend the two fairs. Then, a visitor's willingness to visit exhibition 1 *in addition to* exhibition 2 must be almost equal to zero. Indeed, the number of additional transactions that a visitor may realize because he holds two visiting permits instead of one is almost zero. On the other hand, if no exhibitor rents a stand simultaneously in the two fairs, then the added-value of visiting a second one is the largest. To put it differently, the added-value of multi-homing in one market depends negatively on the extent of multi-homing which is expected to take place in the other market. Let us consider the viewpoint of visitors. Given expectations $x_1 < x_2$, we may define the specific value of multi-homing x_3 by the difference between the number of exhibitors renting a stand in one fair at least minus the number of those who exhibit at fair 2. Suppose that no exhibitor rents simultaneously in the two fairs ; then visiting the two allows for a number of possible transactions equal to $x_2 + x_1$. By contrast, if all exhibitors who rent in fair 1 also rent in fair 2, the added-value of a joint visit as compared to visiting 2 only is nil. In other words, we have $x_3 = x_2$. Obviously, in this last case, we do not expect visitors to multi-home. Thus, the willingness to pay for an additional visit in the visitor's market is negatively related to the expectations about multi-homing behaviour in the exhibitors' market. This basic property of multi-homing decisions in two-sided markets will prove useful in the analysis to follow.

In Gabszewicz and Wauthy (2003) we develop the analysis of duopoly price competition in a traditional vertically differentiated market when the joint purchase of the two variants is allowed. The analysis we develop therein is formally equivalent characterizing in the present model candidate-price equilibria when agents are allowed to multi-home. The equilibrium characterization we derive can be applied for describing optimal prices in the present framework, *conditional on expectations of the agents.* Consider for instance the market for visitors. Given their expectations $x_1 < x_2$ we may define the admissible values for x_3: $x_3 \in [x_2, x_2 + x_1]$. Recall that $x_3 = x_2$ can be interpreted as corresponding to the expectation that all exhibitors who rent a stand in fair 1 also do it in fair 2. Whenever $x_3 = x_1 + x_2$, it is expected that no exhibitor rents a stand in both fairs. It is obviously in this last case that the joint purchase option is mostly valued. In Gabszewicz and Wauthy (2003), we characterize the nature of the price equilibrium for all admissible values of x_3. We summarize these results in the next two lemmata where we assume for simplicity that $\overline{\gamma} = \overline{\theta} = 1$.

Lemma 1 *Assume $x_2 > x_1$. The open interval $]x_2, x_2 + x_1[$ can be divided into three non-degenerate sub-intervals such that*

1. *In the first sub-interval, there exists a unique price equilibrium with single-homing;*
2. *in the second, two price equilibria may co-exist: one with single-homing and one with multi-homing ; and, finally,*
3. *in the third one, there exists no equilibrium (in pure strategies).*

Proof: See Propositions 1-3 in Gabszewicz and Wauthy (2003, p.823-827)

The formal proof of the above lemma is mainly a matter of computations which are not useful to the argument we need in the sequel of the present analysis. However the intuition underlying the Lemma is easy to grasp. When consumers are allowed to buy simultaneously the low and the high quality product, the nature of price competition drastically changes. In particular the firm selling the low quality product faces two options: either it chooses to price "high" and sell a second product to the "rich" consumers who already bought the high quality product, or it chooses to price "low" in order to sell a first unit to "poor" types and a second unit to the "rich" ones. In all cases, only the "rich" consumers buy two units of the good, i.e. multi-home. Moreover, when the low quality firm prices "high", it actually behaves as a quasi-monopolist along a residual demand parametrized by the marginal gain attached to the second purchase of the good. If the high quality firm sets a high price, the low quality one will be aggressive and may or may not induce multi-homing. If the high quality firm is too agressive in prices, the low quality one will concentrate on the residual surplus it might extract from those who already bought the high quality good (at a low price). By way of consequence, the best reply of the low quality firm will be discontinuous and kinked, thereby leading either to multiple price equilibria, or lack of a pure strategy equilibrium. Obviously, the larger the added-value of a second purchase, the more profitable the quasi-monopolist strategy.

Lemma 2 *Assume $x_2 > x_1$. When $x_2 = x_3$ the vertical differentiation price equilibrium with single-homing prevails. When $x_3 = x_2 + x_1$, any active visitor multi-homes in equilibrium. Platform 1 and 2 set their monopoly prices in the visitors' market: $p_i(x_1, x_2) = \frac{x_i}{2}$.*

Proof: The first part of the Lemma is trivially satisfied. In order to prove the second part of the lemma, it is sufficient to notice that, in the case $x_3 = x_2 + x_1$, the utility derived from multi-homing by any visitor with type θ is $\theta(x_1 + x_2) - p_1 - p_2$, which is fully separable from the point of view of the platforms. Therefore, each platform acts as a monopolist. *QED.*

We are now in a position to state our main result.

Proposition 3 *A configuration displaying multi-homing by all active agents in one market, and no multi-homing in the other market, is part of a Nash equilibrium. Firms set their monopoly price in the multi-homing market and sell at marginal cost in the other one. Furthermore, this equilibrium is the unique one exhibiting strictly positive profits for both firms.*

Proof: To start with we prove the first part of the proposition. The characterization of the equilibrium follows immediately from the condition of fullfilled expectations. Suppose that any active visitor actually visits the two fairs. This is optimal for them only to the extent they expect almost no exhibitor to hold a stand in both fairs. Conversely, it is rational for all exhibitors holding a stand in one fair to hold one in the other as well if only they expect almost no visitor to multi-home. Thus a necessary condition for generalized multi-homing in one market is lack of multi-homing in the other. In other words, a configuration where all active agents multi-home cannot be part of an equilibrium because it is not compatible with the fullfilled expectations condition. From Lemma 2 we know that in the market where generalized multi-homing prevails, the platforms set their monopoly prices. Accordingly, the resulting sizes of the networks are identical (and equal to $\frac{1}{2}$) so that, in the other market, products are viewed as homogeneous. Therefore, by a Bertrand-like argument, they are sold at marginal cost at equilibrium.

The proof of the second part ("uniqueness") is developed in 4 steps.

(i) Consider a vector of expectations (v_1, v_2, x_1, x_2) such that $v_3 \in]\max\{v_1, v_2\}, v_1 + v_2[$ and $x_3 \in]\max\{x_1, x_2\}, x_1 + x_2[$. These expectations give rise to two vertically differentiated markets. Applying Lemma 1, three possible situations may arise on each side: single-homing price equilibrium (which we denote by s), multi-homing price equilibrium (which we denote by m), or no price equilibrium at all. Formally, in order to characterize the equilibria in our game, we have to consider all possible combinations of these three price equilibrium configurations across the two markets.

(ii) First,we may rule out any configuration where equilibrium s prevails in one of the two markets. Indeed, if equilibrium s prevails, say in the market for visitors, the only expectation which is compatible with this equilibrium in the market for exhibitors is $v_3 = v_1 + v_2$, which does not belong to the admissible domain $]\max\{v_1, v_2\}, v_1 + v_2[$.

(iii) We also rule out any configuration of expectations which would lead to values x_3 and/or v_3 for which there would exist no price equilibrium. Indeed, our definition of a multi-homing equilibrium requires the existence of a price equilibrium in *both* markets.

(iv): The only case which remains to be considered is a vector of expectations in which in the two markets a multi-homing price equilibrium exists. Using Gabszewicz and Wauthy (2003), we may characterize candidate equilibrium configurations[5]. Assuming $x_2 > x_1$ and defining

$$K = \frac{(x_3 - x_2)(x_2 - x_1) + x_1(x_3 - x_1)}{x_1(x_2 - x_1)(x_3 - x_2)},$$

we obtain:

$$p_1(x_1, x_2) = \frac{3(x_2 - x_1)}{4(x_2 - x_1)K - 1},$$

$$p_2(x_1, x_2) = \frac{(2K(x_2 - x_1) + 1)(x_2 - x_1)}{4(x_2 - x_1)K - 1},$$

and

$$v_1(x_1, x_2) = 1 + \frac{p_2(x_1, x_2)}{x_2 - x_1} - p_1(x_1, x_2)K,$$

$$v_2(x_1, x_2) = 1 - \frac{p_2(x_1, x_2) - p_1(x_1, x_2)}{x_2 - x_1.}.$$

Replicating the same analysis for the exhibitors' side of the market under the assumption $v_2 > v_1$, we complete the characterization of the candidate equilibrium configuration. It then remains to solve for the fullfilled expectations conditions. Defining $x_3 = x_2 + z$ with $z \in]0, x_1[$, we obtain, as the unique valid solution,

$$\hat{x_1} = \frac{1}{12}(5 - 9z + \sqrt{25 + 30z + 81z^2}),$$

$$\hat{x_2} = -\frac{1}{6} - \frac{3z}{2} + \frac{1}{6}\sqrt{25 + 30z + 81z^2}.$$

[5] We consider the market for visitors, but obviously the symmetric characterization prevails for the exhibitors' market

Direct computations indicate however that $\hat{x_1} > \hat{x_2}$, which contradicts our initial assumption. Therefore, there exists no price quadruple which satisfies the fullfilled expectations conditions. We have thus ruled out all possible equilibrium configurations in the relevant interval. *QED*

A comparable equilibrium has been identified in Caillaud and Jullien (2003). Yet, their model assumes that agents on each side are homogeneous and active from the outset. Armstrong (2005) and Armstrong and Wright (2004) identify an equilibrium with similar features, with the multi-homing side being "exploited" and the other being targeted "agressively". A key-difference with us is that Armstrong (2005) *assumes* the homing structure (single-homing on one side, multi-homing in the other) and Armstrong and Wright (2004) cover a case where one side of the market only is heterogeneous. In our model, we obtain the multi-homing structure as an equilibrium outcome in a market where agents are heterogeneous on both sides. In light of these results, it seems that relaxing price competition is a very robust property of multi-homing.

Notice that the equilibrium identified in our main proposition is a symmetric one. This is in sharp contrast with the single-homing case where the two firms are asymmetric. The possibility to register to several platforms actually allows customers to by-pass the asymmetry that might characterize networks. Accordingly, each platform prices according to its own size. The comparison between Proposition 2 and 3 neatly clarifies the main interest of multi-homing from the point of view of platforms, namely relaxing price competition drastically. What is actually surprising in the present case is the fact that the collusive outcome which prevails under mutli-homing obtains as the *unique* equilibrium of the game where the two firms enjoy positive profits.

5 Final Remarks

In this paper we have developed a model for markets characterized by cross network externalities and zero-stand alone value. The model we put forward is best seen as an application of the standard vertical differentiation model where the quality of the product on one side of the market is given by the size of the network on the other side of the market. Building on an heterogeneous valuation of the network effects by the agents, the model generates platforms' hierarchies as a natural equilibrium outcomes. This result is to be contrasted with the received literature where either monopolized or symmetric equilibrium outcomes are the rules. Our model also explains the extent of market coverage on the different sides of the market as a function of agents' heterogeneity. Last, the set-up also allows for an explicit analysis of multi-homing behaviour. In this respect, our results are in line with previous ones: multi-homing essentially relaxes competition between the platforms. In our model, a configuration where one side only multi-homes while being exploited by the platforms has been shown to be the unique equilibrium one where the two firms enjoy positive profits.

Although the analysis has been developed under particular assumptions, our qualitative results are robust to several generalizations. The assumption of uniform distributions and equal number of agents on the two sides is mainly instrumental in allowing us to obtain closed form solutions. Relying on the received literature on vertical product differentiation, we are confident that our qualitative results would hold for more general distributions, provided they remain reasonably regular. Our results have also been derived using a fullfilled expectations equilibrium. When relaxing this assumption, platforms are able to better internalize the externalities at work in the industry (i.e. when a platform decreases its price on the visitors' side, this could induce more exhibitors to patronize the platform given the price charged). A complete analysis of this case in on our research agenda.

References

[1] Armstrong M. (2005), Competition in Two-Sided Markets, University College London, mimeo.

[2] Armstrong M. and J. Wright (2004), Two-sided markets with multihoming and exclusive dealing, Mimeo, University College London

[3] Caillaud B. and B. Jullien (2003), Chicken and Eggs: Competition among Intermediation Service Providers, *Rand Journal of Economics*, 34-2, 309-328.

[4] Baake P. and A. Boom (2001), Vertical Product Differentiation, Network Externalities, and Compatibility Decisions, *International Journal of Industrial Organization*, 19, 267-284.

[5] Bental B. and M. Spiegel (1995), Network Competition, Product Quality and Market Coverage in the Presence of Network Externalities, *The Journal of Industrial Economics*, 43, 197-208

[6] Gabszewicz J. and X. Wauthy (2003), The Option of Joint Purchase in Vertically Differentiated Markets, *Economic Theory*, 22, 817-829.

[7] Matutes C. and X. Vives (1996), Competition for Deposits, Fragility and Insurance (1996), *Journal of Financial Intermediation*, 5, 184-216

[8] Mussa M. and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301-317

[9] Rochet J. and J. Tirole (2003), Platform Competition in Two-Sided Markets, *Journal of the European Economic Association*, 1, 990-1029

[10] Rochet J. and J. Tirole (2004), Two-Sided Markets: an Overview, mimeo, IDEI

[11] Wauthy X. (1996), Quality Choice in Models of Vertical Differentiation, *Journal of Industrial Economics*, XLIV, 3:345-353.