Term Structure Modelling with Observable State Variables*

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Abstract

This paper proposes and implements a parsimonious three-factor model of the term structure whose dynamics is driven uniquely by observable state variables. The method allows comparing alternative views on the way state variables - macroeconomic variables, in particular - influence the yield curve dynamics, avoids curse of dimensionality problems commonly appearing in traditional models, and provides more reliable inference by using both the cross-sectional and the time series dimension of the data. I simulate the small-sample properties of the procedure and conduct in- and out-of-sample studies using a comprehensive set of US data. I show that even a parsimonious model where the level, slope and curvature factors of the term structure are driven by, respectively, measures of inflation, monetary policy and economic activity consistently outperforms the (latent-variable) benchmark model out-of-sample, when considering the five NBER-dated recessions of the last three decades.

Keywords: term structure of interest rates, yield curve estimation, dynamic term structure modelling.

JEL Classification: C5, E4, G1.

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1 Introduction

This paper proposes and implements a parsimonious three-factor model of the term structure whose dynamics is driven uniquely by observable state variables, as opposed to latent variables.[1] It builds upon a three-factor model describing the term structure behaviour first proposed in Nelson and Siegel (1987) and recetly reinterpreted by Diebold and Li (2006, DL) as a dynamic latent factor model. The Nelson-Siegel method is sufficiently flexible to approximate the changing shape of the yield curve yet parsimonious and easy to implement.

The interaction between the term structure of interest rates and macroeconomic variables has been extensively explored in a number of papers in the last decade or so, thanks in part to the fact that 'yields-only' models based on no-arbitrage were found to do well in fitting the cross-section of yields at a particular point in time (de Jong, 2000; Dai and Singleton, 2000), but poorly in describing the dynamics of the yield curve (Duffee, 2002; Brousseau, 2002). Although the techniques employed varied substantially, ranging from short-rate to VAR-based models, the aim of the papers jointly investigating yields and macro variables was to better understand (and forecast) the yield curve dynamics, whose aim the present paper shares.[2]

The intuition of the modelling strategy I adopt goes as follows. If the term structure moves as a result of changes in the economic fundamentals − here represented by a set of state variables − the term structure factors (and, by consequence, the term structure dynamics) should be somehow linked to these state variables. In this paper, I make this link explicit, so that the movements of the term structure are completely exerted by the underlying state variables.

The approach here proposed contributes to the literature from both the theoretical and the empirical viewpoints. First, the replacement of latent factors with observable state variables as the only drivers of the term structure factors allows comparing alternative views on the way state variables − macroeconomic variables, in particular − influence the yield curve dynamics.[3] Besides telling more about the economic fundamentals than latent variables, the use of observable variables might also provide guidance to the construction of theoretical models of the term structure dynamics. Additionally, the method enables testing hypotheses of economic interest − as a result, instead of

[1] In the text I usually refer to state variables for the sake of generality, but in the literature the set of variables that have been mostly used are macroeconomic variables.

[2] Contributions to the literature in the last decade include Fuhrer and Moore (1995), Rudebusch (1995, 2002), Evans and Marshall (1996), Fuhrer (1996), Dewachter and Lyrio (2002), Hördahl, Tristani and Vestin (2002), Wu (2002), Piazzesi (2005), Ang and Piazzesi (2003), and Bikbov and Chernov (2005).

[3] In a number of recent related studies, observable and latent factors coexist: Ang and Bekaert (2004) use one observable (inflation) and two latent factors; Rudebusch and Wu (2004) use two observable (GDP growth and inflation) and two latent; Hördahl, Tristani and Vestin (2004) use three observable (the short rate, GDP growth, and inflation) and one latent one; Ang, Dong, and Piazzesi (2005) use two observable (inflation and GDP growth) and one latent factor. Exceptions include Ang, Piazzesi, and Wei (2004), which use the short rate, the term spread, and GDP growth as their state variables, and Bekaert, Cho, and Moreno (2004), which uses the short rate, the output gap, and inflation. As opposed to what I present in the empirical exercise, these papers do not conduct model comparisons, pre-specifying the state variables they use − this important aspect of the paper is discussed in detail in the sequel.

pre-specifying the drivers of the yield curve dynamics, I compare alternative models and select the best among them. Moreover, the explicit link between term structure factors and observable state variables enables policy experiments to be performed. As a result, one can forecast the term structure by using forecasted variables, or perform stress testing of the term structure using scenarios constructed using the state variables. This feature is especially useful to bankers, who are interested in forecasting bond prices and might have a better idea of the expected state of the economy than the expected state of the yield curve. This feature is also of value to financial authorities, as a tool to assess financial stability.[4]

Second, the method is robust to curse of dimensionality problems commonly appearing in traditional models. The curse of dimensionality imposes constraints on the number of yields one can use and, in particular, results in poor measures of the term structure curvature.[5] Here, instead, the dimension of the parameter vector does not increase with the number of yields under study, just with the number of state variables explaining them, very much in the spirit of linear regression, where one loses degrees of freedom by including additional covariates, not more observations.

Third, the identification strategy comes out in a natural way. Essentially, the baseline model needs the state variables driving the term structure to be predetermined with respect to yields. When I incorporate a Taylor rule into the model, the identifying assumption made is also standard, requiring the state variables to be predetermined with respect to the monetary policy instrument.

Fourth, I conduct in- and out-of-sample studies using US data. The in-sample study uses a thorough set of macroeconomic variables to compare alternative specifications of the term structure dynamics and suggests two models which I then use in the out-of-sample exercise: a parsimonious model where the level, slope and curvature factors of the term structure are driven by, respectively, measures of inflation growth, monetary policy, and economic activity[6], and a richer specification where the level is driven by measures of inflation growth and economic activity, the slope by monetary policy and economic activity, and the curvature by fiscal policy growth.[7] The out-of-sample study shows that both specifications consistently outperform the (latent-variable) benchmark model in the study of the yield curve behaviour during the five NBER-dated recessions which occurred in the last three decades. Recessions are of interest not only for being bad states against which economic agents are willing to insure, but also for being periods

[4] Stress testing has become crucial in the risk management toolbox of financial institutions. It is defined in BIS (2000) as "a generic term describing various techniques used by financial institutions to gauge their potential vulnerability to exceptional but plausible events". Due to the fact that standard Value at Risk (VaR) models have been found to be of limited use in measuring exposures to extreme events, stress testing has been incorporated into the risk management routine of financial institutions, and has even been stressed during the ongoing Basel II process as a useful tool in assessing banks' internal models.

[5] One needs at least three yields for the curvature to be defined, but by relying on only five yields, as is often done in the literature, one is unlikely to obtain accurate measures of this factor.

[6] These are, respectively, the Consumer Price Index growth rate, the Fed Funds rate, and the Unemployment Rate.

[7] The level is driven by the Consumer Price Index growth rate and the Unemployment Rate, the slope by the Fed Funds rate and the Unemployment Rate, and the curvature by the growth rate of the ratio between Government deficit and Industrial Production – the proxy measure of GDP at the monthly frequency.

which tend to be preceded by the inversion of the term structure of interest rates, a feature usually difficult to be quickly captured – if at all – by term structure models.

The paper is organized as follows. Section 2 briefly reviews term structure estimation methods and recent developments of the Nelson-Siegel approach. Section 3 presents the model and discusses its identification and implementation. Section 4 presents simulation results, and Section 5 performs an empirical exercise using US data. The Appendix contains an empirical exercise using CRSP interest rate data as a robustness check and discusses strategies for incorporating spatial modelling into the model.

2 Yield Curve Estimation

2.1 Static Methods

When analyzing the evolution of the yield curve over time, one striking feature is the variety of shapes it can have. These vary from flat ones, where longer term rates are roughly the same as shorter ones, to upward-sloping ones, where longer term rates are higher, but also include 'hump-shaped', inverted, 'spoon-shaped' ones etc. As a result, yield curve fitting methods are expected to be flexible enough to match the different shapes the yield curve can have.

A number of approaches can be used to modelling the term structure of interest rates. First, one may consider models that make explicit assumptions about the evolution of state variables and use either equilibrium or arbitrage methods, which corresponds to modelling dynamic yield curves. According to this class of models, the evolution of the yield curve is modelled as depending linearly on a small number of (arbitrarily chosen) factors. Since in most of the cases the underlying state variable is the short term interest rate, they are frequently labeled as 'short-rate models'. The landmarks of this approach are the papers by Vasicek (1977) and Cox, Ingersoll, and Ross (1985, CIR), both of which use the short rate as the only underlying factor. Subsequent extensions to multi-factor models include the two-factor model of Longstaff and Schwartz (1992), and the three-factor one of Balduzzi, Das, Foresi and Sundaram (1996, BDFS). When it comes to fitting real data, one-factor models perform poorly: the yield curve corresponding to the Vasicek model does not allow a large range of shapes, whereas the ones corresponding to CIR and extensions allowing time-varying parameters such as Hull and White (1990) tend to evolve unrealistically over time. In what regards multi-factor models, there is an understanding that at least three factors are needed to generate a wide variety of yield curve shapes, although even so the fit close to the long end tends to be poor. Moreover, choosing the state variables involves both a certain degree of arbitrariness and a bit of art - direct factors may include the short rate, spot rates of various maturities, forward rates, swap rates, whereas indirect ones may include the short rate volatility, the mean short rate, the latter two rising issues such as the choice of the sample period involved in their calculations. Further, multi-factor models (such as the BDFS) usually lack of explicit formulae and are of difficult calibration to market prices.

Alternatively, one can smooth data obtained from asset prices to describe the static yield curve, usually without taking a view on the factors driving it. This corresponds to fitting, the yield curve as a whole. The analysis starts from information on asset prices, from which one extracts the corresponding yields. As there are only a few maturities available for which there are observations on prices (and, thus, yields), it is interesting to somehow 'connect' those points in order to evaluate instruments with maturities different from those of the yields one has already extracted, usually imposing some degree of smoothness. Among the estimation methods most widely used, there are (regression and smoothing) spline techniques, kernel methods, but also parametric classes of curves, broadly known as the Nelson-Siegel family of curves. Among regression splines one can find several sub-varieties - McCulloch (1971, 1975) used quadratic and cubic splines, Schaefer (1981) employed Bernstein polynomials, whereas Vasicek and Fong (1982) adopted exponential splines. Regression splines have some inconveniences though. One has to take into account the arbitrariness involved, first, in the choice of knot points, second, in the choice of basis functions. Thirdly, splines may oscillate too much and are too sensitive to modelling parameters, with the consequence of fitting poorly at too long and too short maturities. Fourthly, since splines are polynomials, they imply a discount function which diverges as maturity increases rather than converging to zero as required by theory - as a result, implied forward rates also diverge rather than converging to any fixed limit. Fifthly, there is no simple way to ensure that the discount function always declines with maturity i.e. that all forward rates are positive. Although exponential splines are appealing in theory, it is not clear that they perform better than standard splines in practice (Shea, 1985). As for smoothing splines (Fisher, Nychka and Zervos, 1995), they reduce the amount of curvature as one may well desire when uncomfortable with regression splines, but at the expense of a worse fit to the yield curve.

The class of curves first proposed in Nelson and Siegel (1987) is parsimonious and does well in capturing the overall shape of the yield curve, being popular among practitioners and central banks alike (BIS, 2000).[8] For a sample of N bonds measured at a given point in time, the yield curve as a function of time to maturity τ_i is written as

$$y(\tau_i) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_3 \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right) + u(\tau_i), i = 1, ..., N$$

providing a parsimonious representation of the term structure which is consistent with a well-behaved discount function i.e. continuous, positive and decreasing in τ, taking value 1 when $\tau = 0$ and approaching zero as τ grows large.[9] As we justify below, the parameters $\beta_1, \beta_2, \beta_3$ can be interpreted as, respectively, the level, (the negative of the) slope, and curvature components, whereas the parameter λ_t controls the exponential decay of the yield curve: small values produce slow decay and can better fit the curve at long maturities, while large values generate a fast decay and can better fit the curve at short maturities. Moreover, λ also determines where the loading on β_3 achieves its maximum. The loading on β_1 is a constant, implying that an increase in this factor increases all yields equally, which results in a change in the level of the yield curve. The loading on β_2 is a function that starts at 1 but decays monotonically to zero, implying that an increase in β_2 increases short yields more than long yields, resulting in a change in the slope of the yield curve. As for β_3, this is related to the curvature of the term

[8] Although one could argue that a three-factor model could be too much of a simplification, Diebold, Rudebusch and Aruoba (2005) find no evidence that extensions of Nelson-Siegel using four or five factors would do better, which is consistent with previous findings of Dahlquist and Svensson (1994)

[9] Equivalently, it guarantees positive forward rates at all horizons.

structure, as an increase in β_3 will have little effect on very short or very long yields, but will increase medium-term yields, thus resulting in an increase of curvature of the yield curve. As first described by Diebold and Li (2006), this representation can be related to a dynamic three-factor model of, respectively, level, slope, and curvature, which I describe in the following.

2.2 Nelson-Siegel and Beyond

The framework recently proposed in Diebold and Li (2006), and also used in Diebold, Rudebusch, and Aruoba (2006, DRA) reinterprets the Nelson and Siegel (1987) framework as a dynamic latent-factor model. Following Diebold and Li (2006), for every time period t, the yield curve is a function of time-to-maturity τ (or, rather, a combination of exponential functions thereof) and time-varying parameters interpreted as the level, slope, and curvature factors,

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau_i}}{\lambda_t\tau_i}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau_i}}{\lambda_t\tau_i} - e^{-\lambda_t\tau_i}\right) + u_t(\tau_i); i = 1, ..., N, t = 1, ..., T$$

Estimation could in principle be carried out using Nonlinear Least Squares (NLLS) although the usual practice since Nelson and Siegel (1987) – and also followed in Diebold and Li (2006) – has been to fix λ_t to a constant value, compute the factor loadings (regressors), and then use OLS to estimate $\{\beta_t\}$. The parameter λ_t determines the maturity τ^* at which the loading on the curvature factor achieves its maximum (usually between 2 and 3 years), and Diebold and Li (2006) simply pick a λ_t such that this maximum is achieved at the midpoint between these maturities – 30 months – and set $\lambda^* = \lambda_t = 0.0609$. After computing the sequence $\{\beta_t\}$ of factors and the pricing errors, they model the factors as a univariate AR(1) models and compare the forecasting power of the model out-of-sample with a number of alternatives, with reasonable performance, especially given the simplicity of the model.

The above framework is intuitive and easy to implement, but is still based on latent variables – despite the consensus that changes in the yield curve are exerted by changes in macroeconomic conditions (or, more generally, changes in state variables), the factors in the DL framework remain latent, whereas in DRA latent and observable factors (pre-specified by the researchers) coexist. As a result, it offers no room for comparing alternative views on the main drivers of the term structure dynamics. Moreover, the empirical implementations restrict the dynamics of the time-varying factors in ways that, despite their reasonability, are difficult to be either verified or refuted: whereas Diebold and Li (2006) model the parameters $\{\beta_p\}_{p=1}^3$ as univariate AR(1) processes, Diebold, Rudebusch, and Aruoba (2006), generalize it to a first-order vector autoregression. In any case, the estimation of the stochastic processes driving the level, slope, and curvature factors does not acount for the measurement error coming from the fact that $\{\beta_p\}_{p=1}^3$ are estimated rather than observed, so that any asymptotic statements are likely to be misleading.

3 Term Structure Modelling

This section proposes a term structure modelling approach building upon Nelson-Siegel and its reinterpretation by Diebold and Li (2006). The main contrast with respect to the DL model is that here the term structure dynamics is solely driven by the dynamics of observable state variables, as opposed to latent factors. The intuition behind this idea is that if the yield curve moves as a result of changes in relevant state variables, the factors should be somehow linked to these state variables. As a result, one can now, for instance, compare alternative hypotheses on the variables driving the term structure factors and state that level, slope and curvature factors are driven by, say, measures of economic activity, inflation, and monetary policy instrument, respectively.

3.1 A Model with State Variables

The DL model writes the yields at time t as a function of the maturity vector $\boldsymbol{\tau}$,

$$y_t(\boldsymbol{\tau}) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \boldsymbol{\tau}}}{\lambda_t \boldsymbol{\tau}}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \boldsymbol{\tau}}}{\lambda_t \boldsymbol{\tau}} - e^{-\lambda_t \boldsymbol{\tau}}\right) + u_t(\boldsymbol{\tau})$$

further assuming that λ_t is constant over time and setting it to a pre-specified value. Estimation of the parameter vector β_t for every period is carried out using linear least squares, which assumes that the error term does not depend on maturity. This results in a time series of the parameter vector $\{\beta_t\}$ whose dynamics is approximated by univariate first-order autoregressive processes for each of its components. The reasons for fixing λ_t to a pre-specified value are, according to the authors, its lack of straightforward economic intuition and the gains from the use of simple linear techniques when estimating the model. In fact, given the small cross-sectional dimension of the yields dataset they use, fitting a nonlinear model can be a very challenging exercise. The main take-away point is, however, the latent-variable character of the DL model.

The main point of departure from DL in this paper is the link between the dynamics of the latent variables to the one of observable state variables *predetermined* relative to $y_t(\tau)$, which I denote by M_{t-}.[10] This is done by decomposing the parameter vector $\theta_t := (\beta_t', \lambda_t)'$ as a sum of two components: the first, $\overline{\theta} := (\overline{\beta}', \overline{\lambda})'$, being a mean component, and the second being the combination of the state variables M_{t-} and parameters $\underline{\theta} := (\sigma_\beta', \sigma_\lambda')'$ measuring their impact on the latent variables. Thus,

$$\theta_t := \begin{bmatrix} \beta_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \end{bmatrix} + M_{t-}\begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \end{bmatrix}$$

Decomposing the time-varying parameters in such a way amounts to assuming that the movements of the yield curve are completely exerted by the movements of the underlying state variables, apart from the error term in the regression equation. Assuming a time-varying λ_t, which is now also to be estimated, complicates the problem, which can no longer be written as a linear regression model of the form $y_t(\tau) = X_t(\lambda^*)\beta_t + u_t$ due to the pre-specification of $\lambda^*$by the researcher, as in DL.

[10] In what follows, given two variables A and B, I write A_{t-} if A is predetermined with respect to B within period t.

The full model reads

$$\begin{aligned} y_t(\tau) &= X_t(\lambda_t)\beta_t + u_t(\tau) \\ \begin{bmatrix} \beta_t \\ \lambda_t \end{bmatrix} &= \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \end{bmatrix} + M_{t-} \begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \end{bmatrix} \end{aligned}$$

where the error term is a martingale difference sequence with respect to current and past covariate information and uncorrelated in the maturity domain i.e. $E[u_t(\tau)u_t(\tau)'] = \sigma^2\mathbf{I}$.[11] This model is more costly to be estimated from the numerical point of view, but this cost is offset by having the dynamics of $\{\beta_t\}$ driven by state variables. Moreover, there are also gains from modelling the dynamics of $\{\lambda_t\}$, apart from a pure generality argument. If the parameter β_{3t} governs the intensity of the curvature of the yield curve, the parameter λ_t governs the locus of its 'tilting point' or, alternatively, where the loading associated to the factor β_{3t} attains its maximum, thus making it unnatural to be disconnected to the analysis of the term structure curvature.

In what regards identification, the argument goes as follows. Data is observed at the monthly frequency, but recorded at different moments within a given month $-$ the state variables M_{t-} are observed at the beginning of each month, whereas the yields are observed at the end of the corresponding month.[12] As a result, the state variables M_{t-} are predetermined with respect to the yields.

Important features of the method are its robustness to errors in variables, its parsimony, and its robustness to the curse of dimensionality. First, as opposed to DL, where (i) the extraction of the $\{\beta_t\}$ sequence of parameters relies solely on the cross-sectional dimension of the data; (ii) the estimation of the AR(1) models for factor dynamics relies solely on the time series dimension of the data; and (iii) the estimation of the factor dynamics uses estimates of $\{\beta_t\}$ as if they were data, incurring in measurement error problems, estimation here relies on both the time series and the cross-sectional dimension of the data and is done in one step. Thus, by working on both T and N, the asymptotic results tend to be much more accurate. Moreover, the fact that the estimation is done simultaneously avoids the measurement error coming from the fact that $\{\beta_p\}_{p=1}^3$ are estimated rather than observed in DL.

Second, parsimony results from the fact that the ultimate parameters of interest are time-invariant. Third, as opposed to traditional VAR models such as in Evans and Marshall (2002), the number of parameters to be estimated does not increase with the number of yields, even after imposing zero restrictions that imply exogeneity of macro variables with respect to yields.

Finally, when it comes to simulate the movements of the term structure $-$ or out-of-sample forecasting, more generally $-$ one just needs to plug-in updated (or forecasted) values of M_{t-} and compute the resulting yields forecasts; alternative models, such as DRA, which contain both latent and observable factors, would need to rely on extra assumptions on the latent part to do so.

[11] The Appendix discusses how to relax the independence assumption of the error term with respect to maturity.

[12] In the empirical exercise using US data, the state variables are observed at the beginning of each month, whereas the yields are taken from the last working day of each month.

3.2 Implementation

In this section I discuss the implementation of the model

$$\begin{aligned} y_t(\boldsymbol{\tau}) &= X_t(\lambda_t)\beta_t + u_t(\boldsymbol{\tau}), t = 1, .., T \\ \begin{bmatrix} \beta_t \\ \lambda_t \end{bmatrix} &= \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \end{bmatrix} + M_{t-} \begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \end{bmatrix} \end{aligned}$$

where $y_t(\boldsymbol{\tau})$ is the vector of yields observed at date t, and $u_t(.)$ is the error term, both of dimension $N \times 1$, $X_t(.)$ is $N \times 3$, β_t and $\overline{\beta}$ are 3×1, λ_t and $\overline{\lambda}$ are scalars, σ_β and σ_λ are, respectively, $k_\beta \times 1$ and $k_\lambda \times 1$, and $M_{t-} = \begin{bmatrix} M_{\beta t-} & 0_{3\times k_\lambda} \\ 0_{1\times k_\beta} & M_{\lambda t-} \end{bmatrix}$ is $4 \times k(= k_\beta + k_\lambda)$.[13]

The model consists of N yield observations for each one of the T periods, k state variables per period, and $k + 4$ parameters to be estimated, regardless of the number of yields or time periods in the sample $-$ the dimension of the parameter vector grows only with the number of state variables in the model (say, at most three per factor, so that most likely $k \leq 12$). The nonlinearity of the model comes from the estimation of λ_t in the $N \times 3$ matrix of factor loadings at period t,

$$X_t(\lambda_t) = \begin{bmatrix} 1 & \frac{1-\exp(-\lambda_t\tau_1)}{\lambda_t\tau_1} & \frac{1-\exp(-\lambda_t\tau_1)}{\lambda_t\tau_1} - \exp(-\lambda_t\tau_1) \\ 1 & \frac{1-\exp(-\lambda_t\tau_2)}{\lambda_t\tau_2} & \frac{1-\exp(-\lambda_t\tau_2)}{\lambda_t\tau_2} - \exp(-\lambda_t\tau_2) \\ \ldots & \ldots & \ldots \\ 1 & \frac{1-\exp(-\lambda_t\tau_N)}{\lambda_t\tau_N} & \frac{1-\exp(-\lambda_t\tau_N)}{\lambda_t\tau_N} - \exp(-\lambda_t\tau_N) \end{bmatrix}$$

Since $\lambda_t = \overline{\lambda} + M_{\lambda t-}\sigma_\lambda$, one can write $X_t(\lambda_t) = X_t(\overline{\lambda}, \sigma_\lambda)$ but should bear in mind that both τ and $M_{\lambda_{t-}}$ are also arguments of $X_t(.)$ but are omitted for convenience.

The assumption that the error term $u_t(\boldsymbol{\tau})$ is a martingale difference sequence with respect to current and past covariate information implies conditional moment restrictions of the form $E[u_t(\boldsymbol{\tau})|W_t] = 0$, where W_t is a vector of instruments including current and past covariate information. In particular, for every period t, one can use unconditional moments of the form $E[W_t' u_t(\boldsymbol{\tau})] = 0$, whose sample counterpart is

$$\begin{aligned} 0 &= \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} u_{it}w_{it} \\ &= \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} \left(y_t(\tau_i) - X_t(\tau_i, \theta_\lambda)\overline{\beta} - X_t(\tau_i, \theta_\lambda)M_{\beta t-}\sigma_\beta\right) w_{it} \end{aligned}$$

with $\theta := (\theta_\beta'; \theta_\lambda')' = (\overline{\beta}', \sigma_\beta'; \overline{\lambda}, \sigma_\lambda')'$ and where it should be noted that $M_{\lambda t-}$ appears inside $X_t(.,.)$ and is omitted for convenience.

Before defining the estimation problem, stack the yields by period to form the $NT \times 1$ vector $y = [y_1(\tau)', y_2(\tau)', ..., y_T(\tau)']'$, the $X_t(.)$ and $M_{\beta t-}$ matrices to form the $NT \times 3$

[13] In particular, $M_{t-} = diag\{m_t^{\beta_1}, m_t^{\beta_2}, m_t^{\beta_3}, m_t^{\lambda}\}$. In the general case,

$M_{t-} = \begin{bmatrix} M_{\beta t-}, & M_{\lambda t-} \end{bmatrix}$ is $4 \times k(= k_\beta + k_\lambda)$.

matrix $X(\theta_\lambda) = [X_1(\theta_\lambda)', X_2(\theta_\lambda)', ..., X_T(\theta_\lambda)']'$ and the $NT \times k$ matrix $XM(\theta_\lambda) = [(X_1(\theta_\lambda)M_{\beta 1-})', (X_2(\theta_\lambda)M_{\beta 2-})', ..., (X_T(\theta_\lambda)M_{\beta T-})']'$, define $Z_t(\theta_\lambda) = [X_t(\theta_\lambda), X_t(\theta_\lambda)M_{\beta t-}]$ and its stacked version which is of dimension $NT \times (3+k)$, $Z(\theta_\lambda) = [Z_1(\theta_\lambda)', Z_2(\theta_\lambda)', ..., Z_T(\theta_\lambda)']'$, and let $W = [W_1', ..., W_T']$ be an instrument matrix of dimension $NT \times r (\geq k+4)$. I use a Generalized Method of Moments estimator $\widehat{\theta}$ of θ, which is such that the quadratic distance between $G_{NT}(\theta) = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} u_{it}w_{it}$ from zero is minimized:

$$\begin{aligned}
\widehat{\theta} &= \arg\min_{\theta\in\Theta} \left[G_{NT}(\theta)\right]' A_{NT} \left[G_{NT}(\theta)\right] \\
&= \arg\min_{\theta\in\Theta} \left[\frac{1}{NT}W'u\right]' A_{NT} \left[\frac{1}{NT}W'u\right] \\
&= \arg\min_{\theta\in\Theta} \left[\frac{1}{NT}W'\left[y - Z(\theta_\lambda)\theta_\beta\right]\right]' A_{NT} \left[\frac{1}{NT}W'\left[y - Z(\theta_\lambda)\theta_\beta\right]\right]
\end{aligned}$$

where A_{NT} is an $NT \times NT$, possibly random, positive semi-definite weighting matrix with rank at least $k+4$, and the last line shows that nonlinearity comes from the subset of parameters $\theta_\lambda = (\overline{\lambda}, \sigma_\lambda')'$ governing the locus of the tilting point of the yield curve.

One particular case of the above estimator is when $W_t = \left[Z_t(\theta_\lambda), \frac{\partial Z_t(\theta_\lambda)\theta_\beta}{\partial\theta_\lambda}\right]$ and $A_{NT} = \mathbf{I}_{NT}$, which results in the Nonlinear Least Squares estimator. Here, the associated covariance matrix is given by

$$\Omega = E\left(\nabla_\theta' A_0 \nabla_\theta\right)^{-1} E\left(\nabla_\theta' A_0 V_0 A_0 \nabla_\theta\right) E\left(\nabla_\theta' A_0 \nabla_\theta\right)^{-1}$$

where the asymptotic variance is $V_0 = \lim_{N,T\to\infty} \frac{1}{NT} Var\left(\frac{1}{\sqrt{NT}}\sum_{t=1}^{T}\sum_{i=1}^{N} u_{it}w_{it}\right)$, $\nabla_\theta := \frac{\partial G_{NT}(\theta)}{\partial\theta} = \left(\nabla_{\overline{\beta}}, \nabla_{\sigma_\beta}, \nabla_{\overline{\lambda}}, \nabla_{\sigma_\lambda}\right)$, with details given in the Appendix, and A_0 is a positive semi-definite matrix such that $A_{NT} \to^p A_0$.

The estimation problem raises a number of issues. First, the model is robust to curse of dimensionality issues, as the dimension of the parameter vector does not increase with the number of yields, but with the number of state variables, which is kept at a manageable size. As a result, one does need to restrain the number of yields used when estimating the yield curve, a fact which brings the undesirable consequence of poorly measuring the term structure curvature and, as a result, poorly estimating the connection between this factor and any state variables associated to it.

Second, more than just allowing the comparison of alternative specifications, one can test competing theories about variables driving the term structure dynamics using inference tools.

Finally, and in contrast with most of the literature, the estimation makes use of both the cross-sectional and time series dimensions of the data, resulting in much faster convergence of the parameter estimates.[14] This is of special interest given issues commonly raised against VAR models used in the analysis of monetary policy: Rudebusch (1998), for instance, points out that the use of quarterly data, together with the relatively frequent changes in monetary policy in the postwar period results in either short time series or misspecified VAR models, thus making inference unreliable: using quarterly data, the twenty years of the 'Greenspan era' correspond to only 80 observations.

[14]The following Section illustrates the finite-sample proprties of the method and the convergence in both the maturity and time dimensions of the data.

4 Finite-Sample Performance

This section presents a simulation study investigating the finite-sample performance of the estimation method. To do so, I generate state variables M_{t-}, regressors X_t, population parameter values, and errors to generate the variables y_t. For every experiment, I compute the results of 500 replications, with time-series and cross section dimensions given by, respectively, T $(= 10, 50, 100)$ and N $(= 25, 50, 100)$.

The state variables M_{t-} are constructed by taking the exponent of independent standard Gaussian random variables, the regressors X_t are standard Gaussian random variables, whereas the error terms u_t are Gaussian variables with a variance of 0.2.

In what follows, I consider the model

$$\begin{aligned} y_t(\boldsymbol{\tau}) &= X_t(\lambda_t)\beta_t + u_t(\boldsymbol{\tau}) \\ \begin{bmatrix} \beta_t \\ \lambda_t \end{bmatrix} &= \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \end{bmatrix} + M_{t-} \begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \end{bmatrix} \end{aligned}$$

with each factor driven by one state variable. As in the empirical exercise, I make the curvature-related factors β_{3t} and λ_t i.e. the curvature intensity and the location where the curve tilts are driven by the same state variable, so that

$M_{t-} = diag\{m_{1t-}, m_{2t-}, m_{3t-}, m_{3t-}\}$. In all the experiments, $\overline{\beta} = (1, 1, 1)'$, $\sigma_\beta = (1, 1, 1)'$, $\overline{\lambda} = 0.05$, and $\sigma_\lambda = 0.01$.

[Table 1 about here]

The simulation results reported in Table 1 (with standard errors inside square brackets) show fast convergence of the θ parameter estimates to their population values, with increasing precision in both N and T. For the closest case to the smallest subset of data used in the empirical section, where $N = T = 50$, the biases are negligible. In what concerns precision, estimates for $\overline{\beta}_1$ and σ_{β_1} tend to be more precisely estimated than their counterparts because the corresponding factor loadings do not involve any parameters to be estimated, thus having no uncertainty.

5 Application

5.1 The Data

The data set used comprises end-of-month yields from US bonds from January, 1970 to December, 2003 and US macroeconomic variables obtained from the US Federal Reserve macroeconomic database − the FRED − and observed at the monthly frequency.[15] For every given period, the macroeconomic variables used are predetermined with respect to the interest data used.[16]

[15] The dataset is available from http://research.stlouisfed.org/fred2/.

[16] For instance, when using the yield curve of 31 March, 1970, I make sure I only use variables dated prior to that e.g. 1 March, 1970. In particular, the variables in level used date from 1 March, 1970, and the variables in growth rate are the increment from 1 February 1970 to 1 March, 1970.

5.1.1 Interest Rates

The interest rate data used consists on the December 2003 version of the unsmoothed Fama-Bliss yields described and thoroughly discussed in Bliss (1997).[17] It includes all available issues up to that date, implying that the range of available maturities from which the term structures are estimated will not be uniform throughout the sample period i.e. I use an unbalanced panel of yields ranging from 42 to 134 observations per period. The average number of yields for the full sample is 86.944, with a standard error of 26.854, the number of periods in the full sample is $T = 408$ months, and the longest maturity used in the study is 60 months. The main features in the data are the average upward-sloping yield curve, the fact that yield volatility tends to decrease with maturity whereas persistence tends to increase with maturity.

5.1.2 Macroeconomic Variables

Based on the existing literature, I consider a measures of inflation, economic activity, monetary policy, and fiscal policy. The inflation measures used are the CPI (Consumer Price Index For All Urban Consumers: All Items, seasonally adjusted), PPI1 (Producer Price Index: Finished Goods, seasonally adjusted), PPI2 (Producer Price Index: All Commodities, not seasonally adjusted), PPI3 (Producer Price Index: Industrial Commodities, not seasonally adjusted), and PCE (Personal Consumption Expenditures: Chain-type Price Index, seasonally adjusted) − all measured in growth rates.

The measures of economic activity used are HOUST (Housing Starts: Total: New Privately Owned Housing Units Started, seasonally adjusted), INDPRO (Industrial Production Index, seasonally adjusted), EMP (Civilian Employment, seasonally adjusted) − all measured in growth rates − plus TCU (Capacity Utilization: Total Industry, seasonally adjusted), HELP (Index of Help Wanted Advertising in Newspapers, seasonally adjusted) and UR (Unemployment Rate, seasonally adjusted), measured in levels.

The monetary policy instruments used are FF (Federal funds effective rate), NONBR (Non-Borrowed Reserves of Depository Institutions, seasonally adjusted − the monetary aggregate the Fed targeted during the period from October, 1979 to October, 1982), and M1 (Money Stock, in Billions of Dollars, seasonally adjusted).

[Table 2 about here]

All the above variables are recorded at the monthly frequency, and were obtained from the FRED database. Finally, following Dai and Philippon's (2005) recent finding that fiscal policy affects the term structure, I introduce the variable DEBT, which is their quarterly fiscal policy variable interpolated to the monthly frequency and divided by INDPRO, a *proxy* variable for GDP at the monthly frequency. Table 2 summarizes the macroeconomic variables used.

[17] I thank Robert Bliss for making his data available.

5.2 On The Economic Determinants of the Yield Curve

This section starts by selectively reviewing the literature addressing the relation between macroeconomic variables and the yield curve factors, thus paving the way for the empirical strategy I implement next. It goes without saying that with a set of macroeconomic variables as big as the one available from the FRED, there are countless alternative specifications to be compared ($15^3 = 3375$ using only the contemporaneous variables described above), so that a pragmatic starting point would be to consider specifications based on the existing literature and summarized in Table 3. The evidence documented in the literature is used to construct alternative configurations of M_{t-} which are then compared. For the sake of parsimony, I devote a section to single-variable (SV) specifications − the ones where each factor is driven by one state variable only − before addressing the general multi-variable (MV) case. I then use the 'best' SV and MV specifications in the out-of-sample comparison with the benchmark Diebold-Li model.

[Table 3 about here]

Much of the work in macro-finance gained momentum in the late 1990s (see Diebold, Piazzesi, and Rudebusch, 2005, and references therein for the latest account on the literature). One of the early papers is Evans and Marshall (1996) − to which Evans and Marshall (1998) also relates − where, using a VAR framework, the authors study the impact of shocks of measures of monetary policy, employment and inflation on the nominal term structure of interest rates. Their results suggest that the main effect of both employment and inflation measures is to induce a parallel shift of the yield curve, whereas (short-run) fluctuations in the slope and curvature of the yield curve are primarily attributed to the monetary policy shocks.

Also within the VAR framework, but imposing no-arbitrage restrictions, Ang and Piazzesi (2003) construct inflation and economic growth indices which they address as macro factors. By a factor representation of the pricing kernel they obtain a tractable way to examine how those macro factors affect the yield curve dynamics. However, in their study macro factors are able to explain only the short end and the middle of the yield curve. Due to difficulty to deal with the long end they introduce latent factors, now allowing the pricing kernel to be driven by both macro and latent factors. By relying on a Gaussian assumption and on the affine specification, they find that the slope and curvature factors can be explained by the macro factors, whereas the level factor can be only dealt with by using latent factors. In a related paper, but within a different framework, Piazzesi (2005) finds that monetary policy shocks change the slope of the yield curve, since they affect short rates more than long ones.

More recently, Diebold, Rudebusch and Aruoba (2006) examine the correlations between Nelson-Siegel factors and macroeconomic variables under a VAR framework and find that the level factor is highly correlated with inflation and the slope factor is highly correlated with real activity, whereas the curvature factor does not appear to be related to any of the macroeconomic variables used.

5.3 In-Sample Analysis

I start estimating SV specifications, where each factor is driven by one state variable only. These can be seen either as a parsimonious way of approaching the problem or as a first step before considering more complex (and difficult to compute) specifications for M_{t-}, besides providing additional out-of-sample benchmarks for those more complex specifications. A simplifying assumption made throughout the exercise is the curvature intensity β_{3t} and the parameter governing the location of the tilting point of the yield curve are the same.

Given two competing specifications with the same number of variables, I compare them using the Mean Absolute Error criterion (both the average and the median of the MAE's across time). The MAE is of special interest here for providing a model selection criterion, an idea of goodness-of-fit, and of mispricing of the specifications. Table 4 reports results of selected specifications from an exercise designed to select the best forecasting variables from the different categories.[18]

The preliminary results in Table 4 provide a number of insights on the forecasting ability of the state variables. First, the economic activity variable doing the best job at explaining the level factor is UR (see specifications 15-20), the unemployment rate; in what regards the inflation variables, their performance is less clear, but CPI and PCE tend to provide the lowest MAE's (see specifications 1-5).

As for explaining the slope, the best monetary policy variable is FF (see specifications 6-8), whereas the best economic activity variable is UR (see specifications 21-26). Finally, the best monetary policy variable explaining curvature is FF (see specifications 27-29), and the best economic activity variable is UR (see specifications 9-14).

Given the above findings plus the recent evidence that fiscal policy does play a role at explaining the curvature factor of the term structure (Dai and Philippon, 2005), we also include the variable DEBT in our empirical exercise together with the ones already mentioned. As a result, we estimate SV specifications using the three choices for the state variables explaining the level factor, the two choices explaining the slope factor, and the three choices explaining the curvature factor, being left with 18 alternative specifications to examine. Table 5 reports the results of the comparison using the MAE criterion.

[Table 4 about here]

Table 5 shows a clear dominance of the specifications for which inflation (either CPI or PCE) explains the level, monetary policy (FF) explains the slope, and economic activity (UR) explains the curvature of the term structure. Interestingly, the fact that inflation is the key driver of the level factor holds regardless whether CPI or PCE are used, although the literature tends to prefer the latter (see DRA and Duffee, 2005). However, although in line with the literature, it does not exactly match any of the papers listed above.

[18] The results are robust with respect to choice of selection criterion used − using the minimum value of the criterion function, the AIC or the BIC criteria gives the same results.

[Table 5 about here]

In what follows, I refer to the best SV specification (specification 7 in Table 5) as SV. The parameter estimates for the SV model show the positive impact of CPI on the level of the term structure, the impact of the monetary policy instrument FF on the slope (actually defined as $-\beta_{2t}$), and the impact of UR on both the intensity and the locus of the curvature, all of them found to be significant using Newey and West (1987) standard errors to account for the time dependence in the data. Interestingly, neither the CPI nor the UR are revised, which makes them even more attractive as predetermined variables with respect to yields. When coupled with the real time Taylor rule proposed in Evans (1997), the findings are consistent with what one would intuitively expect, in the sense that the yield curve tends to invert for values of FF above the Taylor rule, but remaining upward-sloping for values below the threshold.

[Table 6 about here]

Based on the findings in the literature, SV specifications are likely too simple to provide a satisfactory account of the term structure dynamics. The next step is thus to study the more general MV specifications. Based on the results reported in Table 4, I employ a general-to-specific approach starting with a specification where CPI, PCE and UR drive the level, FF and UR drive the slope, and DEBT, FF and UR drive the curvature. The alternative specifications compared in Table 7 show that several coefficients in the larger models are statistically insignificant. The model with the smaller BIC and with all of the parameters statistically significant is specification 7 $-$ which I from now on refer to as MV $-$, which has the level driven by CPI and UR, the slope by FF and UR, and the curvature by DEBT. Albeit more parsimonious than the full model the average MAE is only slightly larger.

[Table 7 about here]

The parameter estimates for the MV model are reported in Table 8. The findings reported in Table 8 are in line with previous results in that economic activity and inflation drive the level factor, economic activity and monetary policy drive the slope, and fiscal policy drives the curvature factor. However, the performance of the model in terms of MAE is very similar to the SV model.

[Table 8 about here]

The parameter estimates $-$ all of which significant $-$ show the upward impact of inflation on the term structure, as expected. The parameters related to the slope also have the expected sign, with FF affecting shorter rates more strongly, but UR having the opposite effect.

5.4 Incorporating Economic Relations

So far, the model presented considers only state variables which are predetermined with respect to the yield curve, not exploring (i) any interdependence among them; (ii) any forecasts of their future values, both of which are expected to play a role at explaining future realizations of the yield curve. In this section I discuss how to incorporate into the model information on the joint behaviour of the state variables. Intuitively, by informing the model that certain variables are related one should expect to get more accurate results, provided the relation imposed holds.

In this section I inform the model about the joint behaviour of the state variables using a feedback interest rule, or *Taylor rule*. Taylor (1993) suggested a simple formula describing how the US Federal Open Market committee has set the Federal funds rate since 1987 as a response to measures of inflation and output gaps – this relationship has been dubbed the Taylor rule and has been extensively studied and developed since then. Despite its simplicity, the Taylor rule has a number of appealing properties. Woodford (2001) shows how it incorporates several features of an optimal monetary policy in a class of optimizing models, and provides conditions under which the Taylor rule has a stabilizing effect on the economy. More recent developments such as Clarida, Galí and Gertler (2000) propose and estimate a Taylor rule incorporating both forward- and backward-looking elements. The former account for the fact that the monetary authority is considering future paths of the output and inflation gaps when setting the current value of the monetary policy instrument, whereas the latter arises as a consequence of interest rate smoothing conducted by the monetary policy authority.[19] In what follows, I estimate both forward- and backward-looking versions of the Taylor rule. Instead of using quarterly data, as in Clarida, Galí, and Gertler (2000), I use monthly observations and find that, by and large, their results follow through to the monthly frequency.

The results of this section provide the ground for alternative ways of computing out-of-sample forecasts, in the sense that one can plug into the model estimated quantities generated by a model inspired by (or consistent with) economic theory to obtain estimates of the future behaviour of the term structure.

5.4.1 Taylor Rules

In what follows I consider the following specification proposed and estimated (using quarterly US data) in Clarida, Galí, and Gertler (2000), which nests both forward- and backward-looking versions of the Taylor rule and states that the target rate each period is a linear function of the gaps between expected inflation and output and their respective target levels,

$$r_t^* = rr^* + \pi^* + \gamma^\pi \left[E\left(\pi_{t,l_\pi}|\psi_t\right) - \pi^*\right] + \gamma^g E\left(g_{t,l_g}|\psi_t\right)$$

where ψ_t is the information set available at time t, π_{t,l_π} denotes the percent change in the price level between periods t and $t + l_\pi$ (expressed in annual rates), π^* is the target for inflation, $rr^*(= r^* - \pi^*)$ is the long-run equilibrium real rate, with r^* being, by definition, the desired nominal rate when both output and inflation are at their target values. g_{t,l_g} is a measure of the average output gap between periods t and $t + l_g$, with

[19] See also Rudebusch (1995).

the output gap being defined as the percent deviation between actual GDP and the corresponding target.[20]

Following Clarida, Galí, and Gertler (2000), the *actual* Fed funds rate follows

$$r_t = \rho(L)r_{t-1} + (1-\rho)r_t^*$$

where $\rho(L) = \rho_1 + \rho_2 L + ... + \rho_{l_r} L^{l_r - 1}$ and $\rho = \rho(1) = \sum_{j=1}^{l_r} \rho_j$, which postulates a partial adjustment of the Fed funds rate to the target r_t^*, with ρ being an indicator of the degree of smoothing of interest changes by the monetary policy authority.

Combining the target rate and Fed funds equations results in the Taylor rule

$$r_t = (1-\rho)\left[rr^* + (1-\gamma^\pi)\pi^* + \gamma^\pi \pi_{t,l_\pi} + \gamma^g g_{t,l_g}\right] + \rho(L)r_{t-1} + \varepsilon_t$$

where $\varepsilon_t = (1-\rho)\left(\gamma^\pi\left[E(\pi_{t,l_\pi}|\psi_t) - \pi_{t,l_\pi}\right] + \gamma^g\left[E(g_{t,l_g}|\psi_t) - g_{t,l_g}\right]\right)$ is a linear combination of forecast errors, thus being orthogonal to any variable in the information set ψ_t. As one can only identify the term $rr^* + (1-\gamma^\pi)\pi^*$, but not rr^* or γ^π separately, and the inflation target is of interest, Clarida, Galí and Gertler (2000) assume that the equilibrium real rate rr^* equals its sample average. This specification allows a number of choices regarding the lead/lag periods of inflation and output, l_π and l_g, respectively, and lags for the Fed funds, l_r. The parameters of interest are π^*, γ^π, γ^g, $\{\rho_j\}_{j=1}^{l_r}$, so that the dimension of the parameter vector is $3 + l_r$. It also nests a number of specifications, as shown in Table 9.[21].

[Table 9 about here]

The regression equation above implies the set of moment conditions

$$E\left(\left[r_t - (1-\rho)\left[rr^* + (1-\gamma^\pi)\pi^* + \gamma^\pi \pi_{t,l_\pi} + \gamma^g g_{t,l_g}\right] - \rho(L)r_{t-1}\right] z_t\right) = 0$$

where z_t is a vector of instruments known when r_t is set ($z_t \in \psi_t$) and π_{t,l_π}, g_{t,l_g}, and r_{t-1} also belong in ψ_t.

The above moment conditions are used to obtain parameter estimates using the Generalized Method of Moments. As in Clarida, Galí, and Gertler (2000), I set the equilibrium rate rr^* to its sample average, so as to be able to identify the inflation target. To make the feedback rule consistent with the SV specification, I replace r with FF, π with CPI, and I also follow Evans (1997)'s implementation of the Taylor rule, replacing the output gap with the unemployment gap using Okun's law, besides setting the natural rate of unemployment to $UR_t^* = UR^* = 6$.[22] Moreover, I assume that current inflation and unemployment are not observed when setting the Fed Funds rate i.e. neither of them belongs in ψ_t. The moment conditions thus become

$$E\left(\varepsilon_t^* z_t\right) = 0$$

[20] Typically, the information set at time t contains past values of the Fed Funds rate and other economic variables, and usually no information on current inflation and output measures.

[21] Note that the Taylor rule is usually applied to quarterly data, whereas I consider monthly data.

[22] Arthur Okun observed that a one percent fall in the unemployment rate from its full employment level tended to produce a three percent increase in real GDP relative to trend. See Evans (1997) for discussion and robustness checks.

where $\varepsilon_t^* = [FF_t - (1-\rho)[rr^* + (1-\gamma^{CPI})CPI^* + \gamma^{CPI}CPI_{t-1,l_{CPI}} + +\gamma^{UR}3(6 - UR_{t-1,l_{UR}})] - \rho(L)FF_{t-1}]$.

Interestingly, given that in the SV specification the slope is driven by the Fed funds rate, the above specification can be linked to the interest-rate rule proposed in McCallum (1994), according to which the monetary authority reacts to term premia – the slope in particular – when setting the monetary policy instrument.[23]

[Table 10 about here]

The parameter estimates of the forward-looking Taylor rule are reported in Table 10. Although only the former is statistically significant, the responses to CPI inflation and unemployment rate are consistent with the results in Clarida, Galí, and Gertler (2000), which uses 1960:1-1996:4 data at the quarterly – as opposed to monthly – frequency. The closest inflation target level to their estimates is given by FWTR1, although not significant, and the interest rate smoothing parameter is more persistent than theirs. The goodness-of-fit of the specifications is very similar and none of them is rejected when testing for overidentifying restrictions.

Forward-looking Taylor rules might give accurate descriptions in-sample, but if the aim is to do out-of sample forecasting, one needs backward-looking ones. Table 11 reports estimates for alternative specifications of backward-looking Taylor rules regarding the choice of l_{CPI} and l_{UR}, the horizons at which the monetary policy authority looks when setting the monetary policy instrument.

[Table 11 about here]

The results for the backward-looking Taylor rules are robust to alternative horizons, and suggest that – at least at the monthly frequency – the monetary authority looks mostly at past inflation and past values of the monetary policy instrument when setting its current value. The persistence in the Fed funds rate is shown to be high, and even the non-significant parameters γ^{UR} and CPI^* tend to gravitate across a relatively narrow interval, at least for non-zero values of l_{CPI} and l_{UR}. The J-statistics suggest that the horizon at which the Fed looks is at least six months back. When compared to the forward-looking estimates, the responses to inflation seem to be tougher, and both the response to unemployment and the inflation target level are found not to be statistically significant.

[23] The McCallum interest-rate rule also allows rationalizing the empirical failure of the expectations hypothesis – see also Kugler (1997) and Gallmeyer, Hollifield, and Zin (2005).

5.5 Out-of-Sample Analysis

In this section I perform an out-of-sample study by considering five episodes of economic interest: the five NBER-dated US recessions which have entirely occurred during the period 1970-2003. Recessions are of economic interest *per se* being bad states of nature, characterized by reduced economic activity and increased lay-off of workers, thus being events against which economic agents are willing to insure. Moreover, within the term structure literature, recessions are of interest for being periods which tend to be preceded by the inversion of the yield curve, a feature often difficult to be quickly captured − if at all − by term structure models, making the exercise both more interesting and challenging. The recessions considered are described in Table 13.[24]

[Table 12 about here]

For every month in each of the five recessions, I compare the forecasts of the alternative specifications using two measures of accuracy . I also report results for specifications SV-TR and MV-TR, which incorporate the Taylor rule in an attempt to improve forecasting ability.

The ways I compute the out-of-sample forecasts are as follows:

For the macro-based specifications, assume the estimation sample has observations from periods $t = 1, ..., T$, where $t = 1$ is January, 1970 and $t = T$ is the month preceding the recession of interest. After obtaining parameter estimates using the estimation sample, the yield curve forecast for period $t^* > T$, denoted as $\widehat{y}_{t^*|T}$ are obtained either from observed or estimated values M_{t^*} of the state variables using the previously estimated parameters − in the case of the SV and MV specifications, I keep the parameter estimates fixed and keep updating the matrix M_{t^*} of state variables every period

When it comes to the SV-TR and MV-TR specifications, I estimate the Taylor rules as above, and just update the information on CPI, UR, and FF every period, thus obtaining a Taylor rule-based estimate of the value of FF the following period.

Finally, for the DL model, I estimate the model for every period $t = 1, ..., T$, compute the AR(1) processes describing the dynamics of each factor, and re-estimate the model at every period $t > t^*$.[25]

As a measure of 'overall accuracy', I compute average MAEs for the entire duration of each recession i.e. for every month t^* of a given recession, I compute

$$OA_{t^*} = \frac{1}{N_{t^*}} \sum_{j=1}^{N_{t^*}} \left| \widehat{y}_{t^*}(\tau_j) - y_{t^*}(\tau_j) \right|, t^* \in \text{Recession}$$

where N_{t^*} refers to the number of yield at period t^* in the recession. The results reported in Table 13.

[24] The NBER-dated recession going from December 1969 to November 1970 is not considered here since the dataset starts on January, 1970.

[25] See Diebold and Li (2005) for a thorough out-of-sample comparison of their model and previously existing ones.

[Table 13 about here]

The results in Table 13 show that the macro-based specifications consistently outperform the latent variable model. As a matter of fact, DL cannot beat its competitors for any month in recessions R3-5. When comparing SV and MV specifications, the former tends to perform better in the first two or three months of the recessions, being then outperformed by the latter. This suggests that it might take time for all the state variables to work in favour of the MV specification in such periods.

Panel A shows the dominance of the MV-TR model, especially during the second half of the recession. Its performance is followed by the SV-TR model, which suggests that Taylor rules convey information about the future state of the term structure.

Panel B shows the potential effect of a change in policy regime on the forecasting ability of the TR specifications – R2 was the first recession following the monetary policy experiment, right after its introduction.[26] As a result, the SV and MV-TR models perform closely, and the Taylor rule does not seem to provide a substantial gain to the models incorporating it.

Panels C-E show a clear dominance of the MV-TR specification, which might suggest two things. First, that SV specifications are way too simple to describe the term structure dynamics. Second, that incorporating Taylor rules does indeed play a role, improving the accuracy of the forecasts.

As a measure of 'maturity-disaggregated accuracy', I report time-averaged MAEs for fixed maturities i.e. for a given maturity τ_j I calculate

$$MDA_{\tau_j} = \frac{1}{\#t^*} \sum_{t^* \in \text{Recession}} \left| \widehat{y}_{t^*}(\tau_j) - y_t(\tau_j) \right|, j = 1, ..., N$$

where $\#t^*$ denotes the number of periods in the recession. The results are reported in Table 14.

[Table 14 about here]

The results reported in Table 14 confirm the view that macro-based specifications outperform the benchmark DL model. Panel A shows the superior performance of the MV-TR specification up to the 36-month maturity, after which the DL specification tends to do better.

Panels B and C show the superior performance of the MV and, to a lesser extent, SV specifications, most likely due to the change in the policy regime resulting from the monetary policy experiment. Panels D and E show a dominance of the MV specification, at least for maturities up to 10-12 months. In Panel D, the better performing specification from the 11-month maturity towards the long end of the curve is MV-TR, whereas in Panel E it is specification SV which performs better between the 24- to 60-month maturities.

Although it is not obvious which macro-based specification performs best throughout the exercises, all of them consistently outperform the latent-variable benchmark.

[26] See Clarida, Gali, and Gertler (2000) for a study of how the Taylor rule changed with this regime change.

Conclusion

This paper proposes a term structure model whose factors are uniquely driven by observable – as opposed to latent – state variables. The explicit link between the term structure factors and the state variables allows comparing alternative views on the drivers of its dynamics and competing economic hypotheses.

The method is robust to curse of dimensionality issues commonly appearing in the literature. This happens because instead of increasing with the number of observations (yields) used, the dimension of the parameter vector increases with the number of state variables, which is kept at a manageable size. As a result, the method is in a position to deliver more accurate measures of the curvature factor, thus better explaining intermediate maturities i.e. the 'belly' of the curve.

The estimation method uses both the cross-sectional and time series dimensions of the data, which results in faster convergence of the parameter estimates and more reliable inference. This is in stark contrast with VAR models, which are subject to the criticism that they make researchers choose between either short time series or misspecified models, thus making inference unreliable – a direct consequence of the frequent changes in policy regimes in the postwar period (Rudebusch, 1998).

The empirical exercise uses a comprehensible set of US macroeconomic data to compare alternative specifications of the term structure. In the in-sample study, the baseline (SV) specification is such that the level, slope and curvature factors are driven by, respectively, measures of inflation (CPI growth), monetary policy (the Fed Funds rate), and economic activity (the unemployment rate). The out-of-sample study compares macro-based models to a latent-variable benchmark model for the five NBER-dated recessions which occurred in the last three decades, showing that the former consistently outperforms the latter – a finding which is robust to alternative criteria.

This paper raises a number of questions for future research. First, how does the method perform using alternatives such as expectations variables obtained in consensus forecasts as state variables.

Second, how it performs as a risk management tool, making it appealing to both financial institutions and regulators, especially under the ongoing Basel II process.

Third, how it performs when coupled with VAR models feeding it with macroeconomic variables, or measures such as the Bernanke and Mihov (1998) monetary policy indicator.

Fourth, how it can be adapted to the study of credit risk, either at the country or the corporate level.

Finally, although the method relies on the Nelson-Siegel yield curve fitting method, it is by no means restricted to it. Nelson-Siegel is used here due to its intuitive appeal, well-known properties, and the common understanding that it is a reasonable first-order approximation to the yield curve. Alternative methods can be also used, and are left for future research.

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A Covariance Matrix Derivation

As in the text, $\nabla_\theta = \left(\nabla_{\overline{\beta}}, \nabla_{\sigma_\beta}, \nabla_{\overline{\lambda}}, \nabla_{\sigma_\lambda}\right)$, where

$$\nabla_{\overline{\beta}} = X(\theta_\lambda)$$

$$\nabla_{\sigma_\beta} = XM(\theta_\lambda)$$

$$\nabla_{\overline{\lambda}} = \frac{\partial X(\theta_\lambda)}{\partial \overline{\lambda}}\overline{\beta} = \begin{bmatrix} \frac{\partial X_1(\theta_\lambda)}{\partial \overline{\lambda}} \\ \\ \frac{\partial X_t(\theta_\lambda)}{\partial \overline{\lambda}} \\ \\ \frac{\partial X_T(\theta_\lambda)}{\partial \overline{\lambda}} \end{bmatrix} \overline{\beta}$$

with general element

$$\frac{\partial X_t(\theta_\lambda)}{\partial \overline{\lambda}} = \begin{bmatrix} 0 & \phi_{1t-} & \phi_{1t-} + \exp(-\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_1) \\ 0 & \phi_{2t-} & \phi_{2t-} + \exp(-\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_2) \\ ... & ... & ... \\ 0 & \phi_{Nt-} & \phi_{Nt-} + \exp(-\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_N) \end{bmatrix}$$

and

$$\phi_{it-} = \frac{\exp(-\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_i)}{\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_i}\overline{\lambda} - \frac{1 - \exp(-\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_i)}{\left[\left(\overline{\lambda} + M_{\lambda t-}\sigma_\lambda\right)\tau_i\right]^2}\tau_i, i = 1, ..., N$$

Finally,

$$\nabla_{\sigma_\lambda} = \frac{\partial X(\theta_\lambda)}{\partial \overline{\lambda}} \begin{bmatrix} M_{\beta_1}\sigma_\beta \\ M_{\beta_2}\sigma_\beta \\ ... \\ M_{\beta_N}\sigma_\beta \end{bmatrix} \begin{bmatrix} M_{\lambda_1} \\ M_{\lambda_2} \\ ... \\ M_{\lambda N} \end{bmatrix}$$

B Robustness Check Using the CRSP Data

In this Appendix I estimate a simplified version of the model on CRSP data and show that the SV and MV specifications still outperform the latent-variable benchmark even when pre-specifying the parameter λ, as in Diebold and Li (2005). This once again suggests that, besides the advantages discussed in the text, observable state variables do play a role out-of-sample.

B.1 The Data

The data set used comprises end-of-month price quotes (bid-ask average) of US bonds from June, 1964 to March, 2000 collected by CRSP. Other than the bond yields, all remaining data are from the US Federal Reserve's macroeconomic database – the FRED –, observed at the monthly frequency.

B.1.1 Interest Rates

For every period I consider 17 maturities, going up to the 10-year maturity for a total of 430 months. The maturities used are as follows: 1 to 12 months, 24, 36, 48, 60, and 120 months. Although the analysis does not require the maturities to be fixed, this greatly simplifies the empirical exercise. Table A1 reports some sample statistics of the bond data.

TABLE A1 - Basic Statistics of Yields

	Mean	Std. Error	Min	Max	ACF(1)	ACF(9)
1mo	6.136	2.512	2.600	16.360	0.956^*	0.959^*
2mo	6.315	2.549	2.740	16.170	0.971^*	0.781^*
3mo	6.467	2.549	2.760	16.030	0.972^*	0.789^*
4mo	6.545	2.598	2.810	16.100	0.973^*	0.793^*
5mo	6.627	2.597	2.850	16.190	0.973^*	0.798^*
6mo	6.688	2.594	2.850	16.520	0.974^*	0.799^*
7mo	6.727	2.583	2.920	16.170	0.974^*	0.800^*
8mo	6.780	2.577	2.930	16.300	0.975^*	0.800^*
9mo	6.829	2.580	2.980	16.360	0.974^*	0.799^*
10mo	6.852	2.577	3.010	16.400	0.974^*	0.799^*
11mo	6.876	2.566	3.020	16.390	0.974^*	0.799^*
12mo	6.922	2.510	3.110	15.810	0.972^*	0.795^*
24mo	7.130	2.442	3.660	15.640	0.978^*	0.815^*
36mo	7.282	2.374	3.870	15.560	0.979^*	0.829^*
48mo	7.401	2.343	3.970	15.820	0.980^*	0.835^*
60mo	7.464	2.319	3.980	15.000	0.982^*	0.847^*
120mo	7.535	2.268	4.110	15.210	0.984^*	0.852^*

Note: Individual significance at the 5% level is denoted by a superscript *.

The main features in the data are the average upward-sloping yield curve, the fact that yield volatility tends to decrease with maturity whereas persistence tends to increase with maturity. The autocorrelations of all yields are individually significant up to lag nine (results available upon request).

B.1.2 Macroeconomic Variables

Based on the existing literature, I consider a number of measures of inflation, economic activity, monetary policy, and fiscal policy. The inflation measures used are the CPI (Consumer Price Index For All Urban Consumers: All Items), PPI1-3 (Producer Price Index: Finished Goods, All Commodities, and Industrial Commodities, respectively), and PCE (Personal Consumption Expenditures: Chain-type Price Index) - all measured in growth rates; the measures of economic activity used are HOUST (Housing Starts: Total: New Privately Owned Housing Units Started), INDPRO (Industrial Production Index), the HELP index (Index of Help Wanted Advertising in Newspapers), UR (Unemployment Rate), and EMP (Civilian Employment) - both HELP and UR are considered in levels and growth rates; the monetary policy instruments used are FF (Federal funds effective rate), NONBR (Non-Borrowed Reserves of Depository Institutions), and M1 (M1 Money Stock, in Billions of Dollars). All these variables are seasonally adjusted, of monthly frequency, and were obtained from the FRED database. Finally, following Dai and Philipon (2005)'s recent finding that fiscal policy affects the term structure, the fiscal policy variable used is DEBT (Outstanding Credit Market Debt of U.S. Government, State and Local Governments, and Private Nonfinancial Sectors).

B.2 In-Sample Analysis

B.2.1 Single-Variable Factor Specifications

The empirical implementation starts by investigating specifications where each factor is driven by one state variable only i.e. $M_{t-} = diag\{m_{1t-}, m_{2t-}, m_{3t-}\}$. This can be seen either as a parsimonious way of approaching the problem or as a first step before considering more complex specifications for M_{t-}. Table A2 shows that the best performing specification has PCE explaining the level, FF explaining the slope, and DEBT explaining the curvature.

TABLE A2 - Results for Alternative Single-Variable Factor Specifications

	$\widehat{\beta}$ [s.e.]		$\widehat{\sigma}$ [s.e.]		MAE
$L: CPI$	7.292	[0.061]	82.213	[10.585]	1.939
$S: FF$	−2.356	[0.275]	0.441	[0.030]	1.560
$C: M1$	−0.826	[0.119]	−46.889	[6.525]	
$L: PCE$	7.160	[0.063]	139.993	[14.686]	1.911
$S: FF$	−2.577	[0.252]	0.474	[0.026]	1.492
$C: M1$	−0.857	[0.111]	−42.270	[5.930]	
$L: PPI1$	7.595	[0.052]	66.639	[6.849]	1.981
$S: FF$	−2.200	[0.300]	0.420	[0.032]	1.690
$C: M1$	−0.847	[0.127]	−41.326	[6.838]	
$L: PPI2$	7.775	[0.056]	31.179	[4.215]	1.993
$S: FF$	−2.284	[0.300]	0.434	[0.031]	1.709
$C: M1$	−0.875	[0.127]	−38.170	[6.704]	
$L: PPI3$	7.758	[0.057]	36.596	[3.658]	2.009
$S: FF$	−2.220	[0.303]	0.427	[0.032]	1.759
$C: M1$	−0.883	[0.128]	−38.105	[6.850]	
$L: PCE$	7.356	[0.046]	115.975	[8.269]	1.890
$S: FF$	−2.751	[0.241]	0.496	[0.024]	1.433
$C: FFD$	−1.026	[0.103]	1.472	[0.394]	
$L: PCE$	7.088	[0.047]	126.068	[8.890]	$\underline{1.848}$
$S: FF$	−3.245	[0.210]	0.567	[0.019]	$\underline{1.344}$
$C: DEBT$	−0.728	[0.103]	−46.227	[5.782]	
$L: PCE$	5.512	[0.186]	510.668	[47.895]	1.946
$S: NONBR$	0.748	[0.309]	−13.162	[4.617]	1.584
$C: DEBT$	−1.264	[0.187]	33.981	[17.389]	
$L: PCE$	5.648	[0.126]	489.530	[24.113]	1.989
$S: M1$	1.162	[0.312]	−97.129	[18.479]	1.696
$C: DEBT$	−1.449	[0.185]	60.586	[17.319]	
$L: PCE$	6.002	[0.097]	456.802	[20.576]	2.015
$S: HELP$	−1.020	[0.447]	0.023	[0.004]	1.764
$C: DEBT$	−1.276	[0.179]	32.952	[16.000]	
$L: PCE$	5.126	[0.205]	621.969	[41.850]	2.058
$S: UR$	3.151	[0.548]	−0.405	[0.085]	1.844
$C: DEBT$	−1.405	[0.198]	53.034	[17.918]	

Note: Standard errors inside squared brackets. Non-significant estimates at the 5% significance level are marked with ∅. The underlined MAE values are the smallest ones in the Table.

B.2.2 Multi-Variable Factor Specifications

Based on the findings in the literature and the results obtained for the SV case, I now allow for more state variables to influence the term structure factors. The findings reported in Table A3 are in line with previous results in that inflation (actually, two measures of inflation, PCE and PPI1) drives the level factor, monetary policy drives the slope, and fiscal policy drives the curvature factor. The model is surprisingly similar to the SV specification previously obtained, as their differ only by the inclusion of the extra measure of inflation driving the level factor. Table A3 displays the results.

As opposed to previous findings in the literature, however, no inclusion of economic activity measures was found to improve on the best specification obtained improved the goodness-of-fit of the model. This finding could be rationalized by arguing that economic agents take into account some form of the Taylor rule when looking at the economic variables available to them and analyzing their impact on the yield curve. Hereafter we refer to the best specification for the multi-variable case (with level being driven by PCA and PPI1, slope driven by FF, and curvature being driven by DEBT) as the MV specification.

TABLE A3 - Results for Alternative Multi-Variable Factor Specifications

Panel A	$\widehat{\beta}$	[s.e.]	$\widehat{\sigma}$	[s.e.]	MAE
$L:PCE$			22.286	[17.454]	
$L:CPI$			189.352	[16.838]	
$S:FF$	6.892	[0.045]	0.431	[0.027]	2.027
$S:NONBR$	−2.035	[0.306]	−7.380	[2.173]	1.788
$S:M1$	−1.013	[0.140]	$-38.681^{\varnothing}$	[41.141]	
$C:DEBT$			$-7.043^{\varnothing}$	[7.062]	
$C:M1$			$-1.161^{\varnothing}$	[21.993]	
$L:PCE$	7.237	[0.044]	130.721	[9.040]	
$S:FF$	−3.104	[0.213]	0.553	[0.019]	1.889
$S:NONBR$	−0.817	[0.101]	−8.238	[2.179]	1.427
$C:DEBT$			−33.551	[5.243]	
$L:CPI$	6.922	[0.048]	177.428	[9.190]	
$S:FF$	−2.680	[0.228]	0.494	[0.022]	1.960
$S:NONBR$	−0.724	[0.110]	−10.821	[1.786]	1.624
$C:DEBT$			−45.088	[6.002]	
$L:PCE$			$-23.681^{\varnothing}$	[18.731]	
$L:CPI$	7.058	[0.044]	162.999	[16.663]	1.934
$S:FF$	−2.906	[0.217]	0.523	[0.021]	1.547
$S:NONBR$	−0.814	[0.103]	−9.357	[2.037]	
$C:DEBT$			−31.819	[5.356]	
$L:PCE$			132.707	[9.378]	
$S:FF$	7.243	[0.046]	0.553	[0.019]	
$S:NONBR$	−3.125	[0.214]	−5.391	[2.400]	1.892
$C:DEBT$	−0.749	[0.104]	−41.542	[5.912]	1.436
$C:FFD$			1.666	[0.398]	
$L:PCE$			133.358	[10.116]	
$S:FF$	7.214	[0.047]	0.492	[0.027]	
$S:NONBR$	−4.247	[0.242]	−6.997	[2.567]	1.924
$S:HELP$	−0.700	[0.110]	0.021	[0.003]	1.510
$C:DEBT$			−53.334	[6.599]	
$L:PCE$			193.354	[9.715]	
$S:FF$	7.057	[0.047]	0.464	[0.027]	
$S:NONBR$	−0.058	[0.402]	$-1.965^{\varnothing}$	[1.620]	2.058
$S:UR$	−0.911	[0.125]	−0.413	[0.035]	1.848
$C:DEBT$			−9.413	[7.939]	
$L:PCE$	7.059	[0.044]	161.146	[7.562]	
$S:FF$	−4.486	[0.244]	0.472	[0.029]	1.903
$S:HELP$	−0.674	[0.112]	0.025	[0.003]	1.458
$C:DEBT$			−54.253	[6.839]	

Panel B	$\widehat{\beta}$	[s.e.]	$\widehat{\sigma}$	[s.e.]	avg($\widehat{\rho}_t$)	MAE
$L : PCE$	5.789	[0.102]	485.700	[26.743]		
$S : NONBR$	−0.901	[0.415]	−8.658	[3.266]	0.973	2.027
$S : HELP$	−1.182	[0.177]	0.022	[0.004]		1.790
$C : DEBT$			$20.663^{\varnothing}$	[15.622]		
$L : PCE$			54.966	[12.670]		
$L : PPI1$	7.315	[0.054]	30.765	[4.586]		
$S : FF$	−3.204	[0.220]	0.567	[0.021]	0.954	1.866
$S : NONBR$	−0.732	[0.104]	−5.093	[2.374]		1.369
$C : DEBT$			−49.818	[5.753]		
$L : PCE$			237.420	[14.652]		
$L : PPI2$	6.925	[0.055]	−18.473	[3.607]		
$S : FF$	−2.996	[0.209]	0.540	[0.019]	0.957	1.887
$S : NONBR$	−0.811	[0.101]	−8.259	[2.342]		1.431
$C : DEBT$			−34.563	[5.454]		
$L : PCE$			170.695	[20.213]		
$L : PPI3$	7.106	[0.073]	$-1.926^{\varnothing}$	[4.842]		
$S : FF$	−2.839	[0.232]	0.519	[0.022]	0.961	1.892
$S : NONBR$	−0.716	[0.110]	−7.120	[2.707]		1.436
$C : DEBT$			−51.385	[6.372]		
$L : PCE$			58.423	[11.813]		
$L : PPI1$	7.395	[0.054]	16.708	[4.611]		
$S : FF$	−3.214	[0.222]	0.562	[0.021]	0.953	1.866
$S : NONBR$	−0.618	[0.108]	$2.252^{\varnothing}$	[2.593]		1.369
$C : DEBT$			−62.828	[6.214]		
$C : FFD$			2.962	[0.393]		
$L : PCE$	6.885	[0.042]	181.628	[8.315]		
$L : PPI1$	−3.172	[0.206]	21.672	[4.533]	0.957	$\underline{1.863}$
$S : FF$	−0.669	[0.104]	0.558	[0.018]		$\underline{1.356}$
$C : DEBT$			−55.522	[6.091]		
$L : PCE$	5.588	[0.155]	607.654	[47.835]		
$L : PPI1$	0.775	[0.304]	−54.640	[16.214]	0.978	1.999
$S : NONBR$	−1.283	[0.185]	−13.743	[4.512]		1.727
$C : DEBT$			36.041	[16.876]		
$L : PCE$			597.315	[51.068]		
$L : PPI1$	5.598	[0.163]	−55.628	[16.793]		1.980
$S : NONBR$	0.754	[0.305]	$-7.687^{\varnothing}$	[5.420]	0.978	1.685
$C : DEBT$	−1.220	[0.190]	$26.807^{\varnothing}$	[18.194]		
$C : FFD$			2.516	[0.950]		

Note: Standard errors inside squared brackets. Non-significant estimates at the 5% significance level are marked with $\varnothing$. The underlined MAE values are the smallest ones in the

Table.

B.2.3 In-Sample Comparison of Specifications

Once parameter estimates were obtained I am now in a position to compare the in-sample behaviour of the specifications. Table A4 reports results in terms of R^2 and MAE quantities according to which parsimony is well rewarded in our context, given how closely the measures of goodness-of-fit are.

TABLE A4 - Goodness-of-Fit of Alternative Models

Model	Average R^2	Median R^2	Average MAE	Median MAE
SV	0.704	0.878	1.848	1.344
MV	0.705	0.884	1.866	1.369

B.3 Out-of-Sample Analysis

In this section I perform a small out-of-sample study by considering three episodes. These episodes are of economic interest due to NBER-dated US recessions which have occurred during the periods December, 1969 to November, 1970; January to July, 1980; and July, 1990 to March, 1991. I estimate the DL, SV and MV specifications for three subsamples of the data, all of which starting from June, 1964. The first ends in December, 1969, the second in December, 1979, and the third in December, 1989.

TABLE A5 - Average MAEs Period-by-Period

	1970			**1980**			**1990**		
	DL	**SV**	**MV**	**DL**	**SV**	**MV**	**DL**	**SV**	**MV**
1st month	0.15	1.51	1.63	1.14	1.20	1.28	0.52	0.63	1.03
2nd month	1.18	0.50	0.26	3.79	3.20	3.23	0.73	0.10	0.52
3rd month	1.56	0.80	0.86	4.68	2.29	2.34	0.95	0.31	0.85
4th month	0.98	0.86	0.71	0.93	0.87	0.80	1.20	0.83	1.14
5th month	1.13	1.25	1.13	0.91	0.80	0.81	0.91	0.54	0.67
6th month	1.50	1.15	1.23	0.98	0.80	0.77	0.84	0.11	0.45
7th month	1.86	0.72	0.64	0.34	1.11	1.10	0.61	0.30	0.38
8th month	1.97	1.16	1.10	1.50	2.24	2.22	0.70	0.52	0.50
9th month	2.34	0.59	0.89	2.61	2.53	2.46	0.57	0.56	0.68
10th month	2.67	0.42	0.34	3.65	3.20	3.25	0.38	0.66	0.75
11th month	3.86	0.50	0.44	4.96	3.36	3.49	0.21	0.22	0.14
12th month	4.07	0.59	0.65	4.41	2.14	2.36	0.34	0.21	0.40

Note: The underlined quantities are the smaller values for a given time period and episode.

The results reported in Table A5 show the overall out-of-sample behaviour of the macro-based specifications tend to outperform the benchmark, although the DL model tends to perform better in one-month ahead forecasts in two out of the three episodes considered. Although the MV specification performs better for the first episode, the more parsimonious SV specification seems to be doing a very good job for the second and third episodes considered.

TABLE A6 - MAEs of Alternative Specifications

Panel A	**3rd month of event**			**6th month of event**			**9th month of event**		
1970	**DL**	**SV**	**MV**	**DL**	**SV**	**MV**	**DL**	**SV**	**MV**
1mo	1.09	$\underline{0.54}$	0.70	1.47	$\underline{0.33}$	0.42	1.82	$\underline{0.32}$	0.39
2mo	0.87	0.57	$\underline{0.56}$	1.31	$\underline{0.48}$	0.51	1.69	0.43	$\underline{0.42}$
3mo	0.92	$\underline{0.69}$	$\underline{0.69}$	1.18	0.61	$\underline{0.56}$	1.58	0.50	$\underline{0.49}$
4mo	0.92	0.69	$\underline{0.68}$	1.14	0.68	$\underline{0.63}$	1.55	0.55	$\underline{0.54}$
5mo	0.94	0.70	$\underline{0.65}$	1.14	0.71	$\underline{0.65}$	1.50	0.64	$\underline{0.62}$
6mo	0.95	0.74	$\underline{0.68}$	1.18	0.72	$\underline{0.65}$	1.51	0.68	$\underline{0.65}$
7mo	0.95	0.79	$\underline{0.74}$	1.11	0.83	$\underline{0.77}$	1.45	0.78	$\underline{0.76}$
8mo	0.97	0.83	$\underline{0.77}$	1.07	0.92	$\underline{0.85}$	1.39	0.88	$\underline{0.86}$
9mo	1.03	0.81	$\underline{0.76}$	1.08	0.96	$\underline{0.89}$	1.39	0.91	$\underline{0.89}$
10mo	1.06	0.82	$\underline{0.77}$	1.10	0.97	$\underline{0.90}$	1.43	0.91	$\underline{0.89}$
11mo	1.10	0.82	$\underline{0.77}$	1.13	0.98	$\underline{0.92}$	1.45	0.92	$\underline{0.90}$
12mo	1.11	0.85	$\underline{0.80}$	1.13	1.01	$\underline{0.95}$	1.46	0.94	$\underline{0.92}$
24mo	1.11	1.13	$\underline{1.10}$	$\underline{1.09}$	1.32	1.26	1.41	1.22	$\underline{1.20}$
36mo	1.06	1.29	$\underline{1.26}$	$\underline{1.02}$	1.47	1.43	$\underline{1.29}$	1.41	1.40
48mo	$\underline{0.83}$	1.55	1.53	$\underline{0.83}$	1.69	1.65	$\underline{1.09}$	1.64	1.63
60mo	$\underline{0.77}$	1.61	1.59	$\underline{0.79}$	1.73	1.69	$\underline{1.05}$	1.66	1.66
120mo	$\underline{0.79}$	1.52	1.50	$\underline{0.65}$	1.81	1.77	$\underline{0.88}$	1.75	1.74
Avg	0.97	0.94	$\underline{0.91}$	1.08	1.01	$\underline{0.97}$	1.41	0.95	$\underline{0.94}$

Panel B 1980	3rd month of event DL	SV	MV	6th month of event DL	SV	MV	9th month of event DL	SV	MV
1mo	3.15	1.37	1.51	2.27	1.55	1.57	2.00	1.36	1.38
2mo	3.56	1.86	1.99	2.39	1.41	1.43	1.98	1.32	1.33
3mo	3.69	2.06	2.19	2.44	1.37	1.37	2.13	1.41	1.41
4mo	3.81	2.26	2.38	2.51	1.47	1.46	2.18	1.53	1.52
5mo	3.88	2.40	2.51	2.51	1.49	1.49	2.19	1.58	1.57
6mo	3.85	2.44	2.54	2.48	1.48	1.48	2.17	1.59	1.58
7mo	3.78	2.44	2.53	2.44	1.45	1.44	2.16	1.58	1.57
8mo	3.63	2.35	2.44	2.37	1.36	1.41	2.11	1.54	1.57
9mo	3.76	2.55	2.63	2.46	1.44	1.54	2.17	1.62	1.67
10mo	3.66	2.50	2.58	2.37	1.40	1.49	2.11	1.60	1.65
11mo	3.69	2.59	2.66	2.37	1.44	1.53	2.10	1.63	1.68
12mo	3.35	2.31	2.37	2.16	1.32	1.31	1.96	1.52	1.50
24mo	2.86	2.34	2.34	1.84	1.48	1.50	1.67	1.74	1.74
36mo	2.33	2.14	2.11	1.42	1.59	1.58	1.40	1.88	1.85
48mo	1.98	2.02	1.96	1.16	1.67	1.64	1.23	1.97	1.93
60mo	1.87	2.07	1.99	1.05	1.90	1.86	1.20	2.20	2.14
120mo	1.66	2.22	2.09	0.97	2.13	2.05	1.11	2.35	2.26
Avg	3.21	2.23	2.28	2.07	1.53	1.54	1.88	1.67	1.67

Panel C 1990	3rd month of event DL	SV	MV	6th month of event DL	SV	MV	9th month of event DL	SV	MV
1mo	0.84	0.55	1.00	0.88	0.42	0.77	0.82	0.47	0.73
2mo	0.92	0.43	0.89	1.03	0.48	0.84	0.96	0.50	0.76
3mo	0.93	0.35	0.81	1.05	0.47	0.83	0.96	0.49	0.75
4mo	0.84	0.37	0.83	0.98	0.46	0.82	0.89	0.50	0.76
5mo	0.83	0.32	0.78	0.95	0.43	0.79	0.87	0.45	0.71
6mo	0.78	0.31	0.77	0.91	0.41	0.77	0.81	0.44	0.71
7mo	0.72	0.29	0.75	0.83	0.39	0.72	0.73	0.44	0.69
8mo	0.67	0.31	0.77	0.78	0.34	0.71	0.69	0.39	0.66
9mo	0.66	0.29	0.75	0.78	0.35	0.71	0.69	0.40	0.66
10mo	0.62	0.28	0.71	0.76	0.37	0.70	0.67	0.40	0.65
11mo	0.57	0.34	0.73	0.72	0.40	0.70	0.63	0.43	0.66
12mo	0.67	0.31	0.77	0.81	0.41	0.77	0.69	0.45	0.72
24mo	0.57	0.36	0.82	0.72	0.45	0.78	0.61	0.45	0.69
36mo	0.59	0.37	0.80	0.73	0.45	0.78	0.64	0.43	0.67
48mo	0.66	0.34	0.81	0.81	0.44	0.82	0.75	0.39	0.66
60mo	0.72	0.33	0.80	0.86	0.43	0.83	0.82	0.37	0.65
120mo	0.91	0.32	0.79	1.01	0.43	0.83	1.03	0.34	0.63
Avg	0.74	0.35	0.80	0.86	0.42	0.78	0.78	0.43	0.69

The results reported in Table A6 confirm the view that macro-based specifications tend to outperform the benchmark DL model. For Panel A, which reports the results for year 1970, this dominance occurs for 12-13 of the 17 maturities considered. Most notably, the cumulative average MAE across maturities of the DL specification for the nine-month horizon is 50% larger than the ones of the macro-based specifications. Overall, the fitting of the macro-based specifications is much better than the DL one for the shorter half of the yield curve by significant orders of magnitude, although this dominance is reversed in favour of the DL specification when it comes to the longer end. A candidate explanation for this fact is the higher persistence and lower volatility of longer yields, as discussed in Table A1.

The results reported in Panel B are qualitatively similar to the ones of Panel A. However, the goodness-of-fit for all specifications tends to be worse than before, probably due to the change in the way monetary policy was being conducted during that period. Finally, Panel C shows a clear dominance of the macro-based specifications and, in particular, of the parsimonious SV specification over the competing alternatives.

C Strategies for Spatial Modelling

In this Appendix I discuss two ways of relaxing assumption that the error terms $u_t(\tau)$ are uncorrelated in the maturity domain i.e. the assumption of no spatial correlation of the error terms used in the text. I start by discussing the Spatial Error Model, according to which the error term follows a first-order spatial autoregressive process, together with the corresponding estimator of Kelejian and Prucha (1999). I then discuss the Geostatistical approach proposed in Dubin (1988), according to which the spatial correlation matrix of the data is modelled directly as a function of their distances.

C.1 The Spatial Error Model

One intuitive way of modelling spatial dependence is by modelling the error term u_t as a first-order spatial autoregressive process, which can be understood as a spatial counterpart of a first-order autoregressive process, sharing with it most of its pros and cons. The field of spatial econometrics has already a relatively long tradition, with early contributions going back at least to Ord (1975)[27]. The spatial error model (SEM)[28] reads

$$u_t = \rho_t S u_t + e_t$$

where u_t and e_t are $N-$dimensional stacked error vectors, S is a $N \times N$ spatial weighting matrix i.e. a matrix of weights reflecting the pattern of spatial dependence among the yields, and ρ_t is the spatial autoregressive parameter.

In what regards the spatial weighting matrix S, given a sample of N observations, this is a matrix of dimension $N \times N$ (by convention, $[S]_{ij} = 0$ if $i = j$) symmetric, positive definite, and is usually standardized so that its rows sum to one, in order to make the spatial parameters comparable. Since there are $N(N-1)/2$ interactions being generated by N observations, one usually imposes some structure on S to make it a parsimonious representation of the spatial dependence of observations; classical examples include the contiguity spatial weighting matrix, $[S]_{ij} = 1\{|i-j| \leq 1\}$, where $1\{A\}$ is the indicator function[29], implying that the $(i, j)-$th element of the matrix takes value one whenever i and j are neighbours $-$ the normalized version of this matrix (with rows summing to one) was used in Case (1991) in the study of the demand for rice in Indonesia. Another standard matrix is the fixed distance neighbouring spatial matrix, with $[S]_{ij} = 1\{|i-j| <\text{critical distance}\}$, whereas measures of 'economic distance' have also been employed, such as $[S]_{ij} = s(|i-j|)$, where $s(.)$ is a function of the distance $-$ see DeLong and Summers (1991). More recently, Townsend (1994) specified the spatial weighting matrix with economic distance being proxied by measures of weather correlation between farms in Indian villages. Then, under regularity conditions on the error variance[30],

[27] Classical contributions were surveyed in Anselin (1988), and economic applications performed by Case (1991), DeLong and Summers (1991) and Townsend (1994), and more recent developments found in Conley (1999), Kelejian and Prucha (1998, 1999, 2004), Brett, Pinkse and Slade (2002), and Pinkse, Shen and Slade (2003).

[28] See Anselin (1988) for an introduction to this class of models.

[29] The indicator function $1\{A\}$ takes value one if the event A occurs, and zero otherwise.

[30] Typically, bounded variance and/or bounded fourth moments.

and dependence structure[31], it can be shown that ρ_t is a straightforwardly estimable parameter.

Estimation of this type of model is dealt with by Kelejian and Prucha (1998, 1999). It is worth mentioning that the estimation of ρ_t does not impose any cost on the estimation of β_t in the sense that one can do as well as estimating ρ_t or knowing its true population value when estimating β_t.

C.1.1 The GM Estimator of Kelejian and Prucha

To derive the GM estimator of Kelejian and Prucha (1998, 1999), first define $u_S = Su$, $u_{SS} = SSu$ and so forth (the time subscript is omitted for the sake of clarity).[32] Letting $tr(.)$ denote the trace operator and using the relation $e = u - \rho Se$, Kelejian and Prucha obtain the following moment conditions: (i) $E(ee'/N) = \sigma^2$, by the definition of the variance; (ii) $E(e'S'Se) = \sigma^2(1/N)tr(S'S)$; and (iii) $E(e'Se/N) = 0$, since $tr(S) = 0$ by construction (the diagonal elements of the spatial matrix are all equal to zero). By rewriting the above relations in terms of u, one obtains the moment condition

$$G_S(\gamma) = \frac{1}{N}\begin{bmatrix} 2u_S'u_S & -u_S'u_S & N \\ 2u_{SSS}'u_{SS} & -u_{SSS}'u_{SSS} & tr(S'S) \\ (u_S'u_{SS} + u_{SS}'u_{SS}) & -u_{SS}'u_{SSS} & 0 \end{bmatrix}\gamma - \frac{1}{N}\begin{bmatrix} u_S'u_S \\ u_{SS}'u_{SS} \\ u_S'u_{SS} \end{bmatrix}$$

where $\gamma = (\rho, \rho^2, \sigma^2)'$. The corresponding criterion function is

$$(\widetilde{\rho}, \widetilde{\rho}^2, \widetilde{\sigma}^2) := \arg \min_{(\rho,\rho^2,\sigma^2)} G_S'G_S$$

To incorporating the GM Estimator into the term structure model, augment it to account for spatial dependence viz.

$$\begin{aligned} y_t(\boldsymbol{\tau}) &= X_t(\lambda_t)\beta_t + u_t(\boldsymbol{\tau}), t = 1, .., T \\ u_t &= \rho_t S u_t + e_t \\ \begin{bmatrix} \beta_t \\ \lambda_t \\ \rho_t \end{bmatrix} &= \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \\ \overline{\rho} \end{bmatrix} + M_{t-}\begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \\ \sigma_\rho \end{bmatrix} \end{aligned}$$

where $y_t(\boldsymbol{\tau})$ is the vector of yields observed at date t, and $u_t(\boldsymbol{\tau})$ is the error term, both of dimension $N \times 1$, $X_t(.)$ is $N \times 3$, β_t and $\overline{\beta}$ are 3×1, λ_t and $\overline{\lambda}$ are scalars, σ_β, σ_λ, and σ_ρ are, respectively, $k_\beta \times 1$, $k_\lambda \times 1$, and $k_\rho \times 1$, and $M_{t-} = \begin{bmatrix} M_{\beta t-} & 0_{3\times k_\lambda} & 0_{3\times k_\rho} \\ 0_{1\times k_\beta} & M_{\lambda t-} & 0_{1\times k_\rho} \\ 0_{1\times k_\beta} & 0_{1\times k_\lambda} & M_{\rho t-} \end{bmatrix}$ is $5 \times k(= k_\beta + k_\lambda + k_\rho)$. The matrix S is $N \times N$, and ρ_t is a scalar.

To obtain the corresponding moment conditions, define $u_{t,S} = Su_t$, $u_{t,SS} = SSu_t$ and so forth. The set of moment conditions of the baseline model is augmented with the

[31] Dependence being (a decreasing) function only of the distance between observations − a spatial analog of the notion of covariance-stationarity for time series.

[32] In this section I drop the time subscript for the sake of clarity.

inclusion of the moment conditions

$$G_S(\theta_\rho) = \frac{1}{NT}\sum_{t=1}^{T}\left(\Gamma_t\gamma_t - \begin{bmatrix} u'_{t,S}u_{t,S} \\ u'_{t,SS}u_{t,SS} \\ u'_{t,S}u_{t,SS} \end{bmatrix}\right)$$

where $\theta_\rho = (\overline{\rho}, \sigma_\rho)'$, $\Gamma_t = \begin{bmatrix} 2u'_{t,S}u_{t,S} & -u'_{t,S}u_{t,S} & N \\ 2u'_{t,SSS}u_{t,SS} & -u'_{t,SSS}u_{t,SSS} & tr(S'S) \\ (u'_{t,S}u_{t,SS} + u'_{t,SS}u_{t,SS}) & -u'_{t,SS}u_{t,SSS} & 0 \end{bmatrix}$, and $\gamma_t = \begin{bmatrix} \overline{\rho} + M_{\rho t}\sigma_\rho \\ (\overline{\rho} + M_{\rho t}\sigma_\rho)^2 \\ \sigma^2 \end{bmatrix}$.

As before, convergence is in both N and T, and the estimation is done simultaneously.

C.2 The Geostatistical Approach

Geostatistical models do not use spatial weighting matrices to summarize the spatial relationships, and no error generating process is specified. Instead, they impose a parametric structure on the spatial correlation of data up to a finite-dimensional parameter to be estimated. The correlation matrix of the data is assumed to be a function of the distances separating them, and needs to be pre-specified by the researcher. More formally, the covariance matrix of the error term of a regression is parameterized as $V = \sigma^2\Psi(d;\alpha)$, where d denotes the distance between observations and α is the parameter vector of interest.

C.2.1 The Dubin Estimator

Dubin (1988) proposes to parameterize the correlation between any two observations in a given sample as a negative exponential function of the distance between them i.e.

$$\Psi(d;\alpha) = 1\{d > 0\}[\alpha_1 \exp(-\alpha_2 d)] + 1\{d = 0\}$$

but other parameterizations, such as the Gaussian,

$$\Psi(d;\alpha) = 1\{d > 0\}\left[\alpha_1 \exp(-\alpha_2 d)^2\right] + 1\{d = 0\}$$

and the spherical,

$$\Psi(d;\alpha) = 1\{\alpha_2 > d > 0\}\left[\alpha_1\left(1 - \frac{3d}{2\alpha_2} + \frac{d^3}{2b_2^3}\right)\right] + 1\{d = 0\}$$

are also valid correlation functions.

In context of the linear regression model $y = X\beta + e$, the estimator is essentially a GLS estimator, with $\Psi(d;\alpha)$ entering as the weighting matrix in the least squares and error variance estimators, $\widetilde{\beta} = (X'\Psi(d;\alpha)^{-1}X)(X'\Psi(d;\alpha)^{-1}y)$ and $\widetilde{\sigma}^2 = (y - X\widetilde{\beta})'\Psi(d;\alpha)^{-1}(y - X\widetilde{\beta})/N$. Alternatively, under the normality assumption, estimation can be done via maximum likelihood (as in Dubin, 1988).

In the general nonlinear case, one can write the moment condition

$$G_S(\beta;\alpha) = \frac{1}{N} Z' \Psi(d;\alpha)(y - f(X;\beta))$$

and proceed using a two-step estimation method according to which β is estimated given preliminary estimates of α obtained from residuals from the first stage regression residuals. The corresponding criterion function is

$$\widetilde{\beta} = \arg\min_{\beta} G_S(\beta;\widetilde{\alpha})' A_N G_S(\beta;\widetilde{\alpha})$$

where $\widetilde{\alpha}$ denotes an estimate of α and A_N is a conformable positive semi-definite weighting matrix.

C.2.2 Incorporating the Dubin Estimator into the Term Structure Model

The augmented model now reads

$$\begin{aligned}
y_t(\tau) &= X_t(\tau,\lambda_t)\beta_t + u_t(\tau), t = 1,..,T \\
E(u_t(\tau)u_t(\tau)') &= \sigma_t^2 \Psi(d;\alpha_t), \\
\begin{bmatrix} \beta_t \\ \lambda_t \\ \alpha_t \end{bmatrix} &= \begin{bmatrix} \overline{\beta} \\ \overline{\lambda} \\ \overline{\alpha} \end{bmatrix} + M_{t-} \begin{bmatrix} \sigma_\beta \\ \sigma_\lambda \\ \sigma_\alpha \end{bmatrix}
\end{aligned}$$

where $y_t(\tau)$ is the vector of yields observed at date t, and u_t is the error term, both of dimension $N \times 1$, $X_t(.)$ is $N \times 3$, β_t and $\overline{\beta}$ are 3×1, λ_t and $\overline{\lambda}$ are scalars, σ_β, σ_λ, and σ_α are, respectively, $k_\beta \times 1$, $k_\lambda \times 1$, and $k_\alpha \times 1$, and $M_{t-} = \begin{bmatrix} M_{\beta t-}, & M_{\lambda t-} & M_{\alpha t-} \end{bmatrix}$ is $4 \times k (= k_\beta + k_\lambda + k_\alpha)$.

The corresponding moment condition is given by

$$G_{NT}(\theta) = \frac{1}{NT} \sum_{t=1}^{T} Z_t' \Psi(d;\alpha_t)(y_t(\tau) - X_t(\tau,\lambda_t)\beta_t)$$

with α_t, β_t, and λ_t defined above.

D Tables

TABLE 1 - Simulation Results for Single-Variable Factor Specification

Panel A	**T = 10**		
	N = 25	**N = 50**	**N = 100**
$\overline{\beta}_1$	1.075	0.987	0.993
	[0.642]	[0.323]	[0.112]
$\overline{\beta}_2$	0.925	1.016	1.013
	[0.675]	[0.389]	[0.208]
$\overline{\beta}_3$	0.786	0.942	0.946
	[1.136]	[0.765]	[0.628]
$\overline{\lambda}$	0.053	0.051	0.049
	[0.030]	[0.019]	[0.011]
σ_{β_1}	0.995	0.999	1.003
	[0.102]	[0.077]	[0.042]
$\sigma_{\beta 2}$	1.004	0.999	0.995
	[0.119]	[0.105]	[0.082]
σ_{β_3}	1.064	1.036	1.024
	[0.442]	[0.334]	[0.259]
σ_{λ}	0.011	0.010	0.010
	[0.011]	[0.006]	[0.004]

Panel B	**T = 50**		
	N = 25	**N = 50**	**N = 100**
$\overline{\beta}_1$	0.998	0.989	1.000
	[0.458]	[0.134]	[0.055]
$\overline{\beta}_2$	1.015	1.007	0.997
	[0.470]	[0.146]	[0.095]
$\overline{\beta}_3$	0.895	1.016	0.996
	[0.742]	[0.340]	[0.274]
$\overline{\lambda}$	0.050	0.050	0.050
	[0.015]	[0.006]	[0.005]
σ_{β_1}	1.000	1.002	1.000
	[0.040]	[0.028]	[0.014]
$\sigma_{\beta 2}$	0.996	0.999	1.000
	[0.045]	[0.033]	[0.025]
σ_{β_3}	1.033	0.995	1.002
	[0.165]	[0.094]	[0.075]
σ_{λ}	0.010	0.010	0.010
	[0.003]	[0.002]	[0.001]

Panel C	T = 100		
	N = 25	N = 50	N = 100
$\overline{\beta}_1$	1.003	1.000	0.998
	[0.367]	[0.108]	[0.045]
$\overline{\beta}_2$	0.999	1.002	1.004
	[0.373]	[0.129]	[0.075]
$\overline{\beta}_3$	0.956	0.985	0.985
	[0.577]	[0.269]	[0.203]
$\overline{\lambda}$	0.050	0.050	0.050
	[0.010]	[0.004]	[0.003]
σ_{β_1}	1.000	1.000	1.000
	[0.033]	[0.021]	[0.010]
$\sigma_{\beta 2}$	0.999	1.000	1.000
	[0.035]	[0.026]	[0.018]
σ_{β_3}	1.011	1.006	1.003
	[0.111]	[0.064]	[0.049]
σ_{λ}	0.010	0.010	0.010
	[0.002]	[0.001]	[0.001]

Note: Standard errors are reported within square brackets.

TABLE 2 - Macroeconomic Variables by Group

Economic Activity	**Inflation**	**Monetary Policy**	**Fiscal Policy**
UR$^{\mathcal{L}}$	CPI	FF$^{\mathcal{L}}$	DEBT
TCU$^{\mathcal{L}}$	PCE	NONBR	
HELP$^{\mathcal{L}}$	PPI1	M1	
IP	PPI2		
EMP	PPI3$^{\mathcal{N}}$		
HOUST			

Note: Variables in levels and not seasonally adjusted are marked with the superscripts $^{\mathcal{L}}$ and $^{\mathcal{N}}$, respectively. The remaining variables are measured in growth rates and are seasonally adjusted.

TABLE 3 - Macroeconomic Variables Driving Term Structure Factors

Reference	Level Factor	Slope Factor	Curvature Factor
Evans & Marshall (1996)	Employment Inflation	Monetary Policy	Monetary Policy
Ang & Piazzesi (2003)	–	Inflation	Output
Piazzesi (2005)	–	Monetary Policy	–
DRA (2005)	Inflation	Output	–

TABLE 4 - Preliminary Results for Single-Variable Specifications

Specification	Level	Slope	Curvature	Avg(MAE)	Med(MAE)
1	CPI	FF	M1	1.010	0.830
2	PCE	FF	M1	1.031	0.825
3	PPI1	FF	M1	1.012	0.836
4	PPI2	FF	M1	1.025	0.830
5	PPI3	FF	M1	1.023	0.833
6	PCE	FF	DEBT	1.069	0.869
7	PCE	NONBR	DEBT	1.822	1.395
8	PCE	M1	DEBT	1.823	1.415
9	CPI	FF	UR	0.833	0.696
10	CPI	FF	TCU	1.022	0.851
11	CPI	FF	HELP	1.058	0.893
12	CPI	FF	IP	1.036	0.873
13	CPI	FF	EMP	1.042	0.898
14	CPI	FF	HOUST	1.050	0.889
15	UR	FF	DEBT	0.877	0.736
16	TCU	FF	DEBT	1.041	0.854
17	HELP	FF	DEBT	1.069	0.873
18	IP	FF	DEBT	1.053	0.881
19	EMP	FF	DEBT	1.063	0.851
20	HOUST	FF	DEBT	1.062	0.865
21	PCE	UR	M1	1.629	1.284
22	PCE	TCU	M1	1.817	1.449
23	PCE	HELP	M1	1.668	1.279
24	PCE	IP	M1	1.815	1.448
25	PCE	EMP	M1	1.817	1.437
26	PCE	HOUST	M1	1.811	1.401
27	UR	FF	FF	0.854	0.738
28	UR	FF	NONBR	0.884	0.746
29	UR	FF	M1	0.872	0.739

Note: The last two columns report, respectively, the average and the median MAE across time.

TABLE 5 - Further Results for Single-Variable Factor Specifications

Specification	**Level**	**Slope**	**Curvature**	**Avg(MAE)**	**Med(MAE)**
1	CPI	FF	DEBT	1.048	0.886
2	PCE	FF	DEBT	1.069	0.869
3	UR	FF	DEBT	0.876	0.729
4	CPI	FF	M1	1.010	0.830
5	PCE	FF	M1	1.031	0.825
6	UR	FF	M1	0.872	0.729
7*	CPI	FF	UR	<u>0.833</u>	<u>0.696</u>
8	PCE	FF	UR	0.836	0.701
9	UR	FF	UR	0.873	0.727
10	CPI	UR	DEBT	1.048	0.886
11	PCE	UR	DEBT	1.468	1.203
12	UR	UR	DEBT	1.521	1.272
13	CPI	UR	M1	1.462	1.199
14	PCE	UR	M1	1.467	1.170
15	UR	UR	M1	1.526	1.287
16	CPI	UR	UR	1.462	1.211
17	PCE	UR	UR	1.467	1.179
18	UR	UR	UR	1.523	1.262

Note: The last two columns report, respectively, the average and the median MAE across time (the smaller quantities of every column are underlined). The superscript * indicates the best specification according to the MAE criterion.

TABLE 6 - Results for Best Single-Variable Factor Specification

Specification		$(\widehat{\overline{\beta}}', \widehat{\overline{\lambda}})'$		$(\widehat{\sigma}'_{\beta}, \widehat{\sigma}_{\lambda})'$		**Avg-Med(MAE)**
SV:	$\beta_{1t} : CPI$	10.655	[0.091]	55.820	[1.317]	$0.833 - 0.696$
	$\beta_{2t} : FF$	-10.148	[0.089]	0.892	[0.003]	
	$\beta_{3t} : UR$	-13.661	[0.169]	2.177	[0.019]	
	$\lambda_t : UR$	0.007	[0.001]	0.004	[0.001]	

Note: Newey-West standard errors with 12 lags are reported within square brackets.

TABLE 7 - Results for Multi-Variable Factor Specifications

Specification	Level	Slope	Curvature	Avg(MAE)	BIC
1	CPI, PCE$^{\varnothing}$, UR	FF, UR	DEBT, FF$^{\varnothing}$, UR$^{\varnothing}$	0.759	49.191
2	CPI, UR	FF, UR	DEBT, FF$^{\varnothing}$, UR	0.759	45.097
3	CPI, PCE$^{\varnothing}$, UR	FF, UR	DEBT, UR$^{\varnothing}$	0.844	45.100
4	CPI, PCE, UR	FF, UR	DEBT$^{\varnothing}$, FF$^{\varnothing}$	0.800	45.099
5	CPI, UR	FF, UR	DEBT, FF	0.972	41.025
6	CPI, PCE$^{\varnothing}$, UR	FF, UR	DEBT, FF$^{\varnothing}$, UR$^{\varnothing}$	0.819	41.004
7*	CPI, UR	FF, UR	DEBT	0.820	36.910

Note: The last two columns report, respectively, the average MAE across time (the smaller quantities in every column are underlined) and Schwarz's BIC model selection criterion. The superscript $^{\varnothing}$ denotes non-significance of the corresponding parameter, whereas the superscript * indicates the best specification according to the BIC.

TABLE 8 - Results for Multi-Variable Factor Specification

Specification		$(\widehat{\overline{\beta}}', \widehat{\overline{\lambda}})'$		$(\widehat{\sigma}'_{\beta}, \widehat{\sigma}_{\lambda})'$		Avg-Med(MAE)
MV:	$\beta_{1t} : CPI$	-2.017	[0.304]	47.564	[1.330]	0.820
	$\beta_{1t} : UR$			1.791	[0.014]	
	$\beta_{2t} : FF$	2.094	[0.284]	0.886	[0.004]	
	$\beta_{2t} : UR$			-1.727	[0.017]	
	$\beta_{3t} : DEBT$	2.011	[0.551]	-6.029	[0.153]	
	$\lambda_t : DEBT$	0.031	[0.001]	-0.035	[0.002]	

Note: Newey-West standard errors with 12 lags are reported within square brackets.

Table 9 - Specifications Nested within the Interest Rate Feedback Rule

Specification	l_π	l_g	l_r	l_s
(Backward) Taylor rule	< 0	< 0	$-$	$-$
(Backward) Taylor rule with interest rate smoothing	< 0	< 0	< 0	$-$
Clarida-Galí-Gertler	> 0	> 0	< 0	$-$

Table 10 - Parameter Estimates for Forward-Looking Interest Rate Rule

	FWTR1	FWTR2	FWTR3	FWTR4
l_{CPI}	1	1	1	1
l_{UR}	1	1	1	1
l_{FF}	-1	-1	-1	-1
Instrument lags	1	2	3	4
γ^{CPI}	2.427*	1.929**	1.702**	1.605**
	[1.358]	[0.870]	[0.770]	[0.746]
γ^{UR}	0.454	0.473	0.618	0.615
	[0.498]	[0.381]	[0.394]	[0.404]
CPI^*	2.020	1.640	1.085	0.911
	[2.898]	[3.054]	[3.461]	[4.084]
ρ	0.965***	0.952***	0.948***	0.950***
	[0.002]	[0.002]	[0.002]	[0.002]
R^2	0.993	0.993	0.993	0.993
J-statistic	0.377	7.787	10.654	12.894
df	2	5	8	11

Note: Specification FWTR1 uses a constant, current values of CPI and UR, and lagged values of CPI, UR, and FF as instruments. Specification FWTR2-4 use the same instruments as FWTR1 plus 2-4 lagged versions of CPI, UR, and UR. Newey-West standard errors with 12 lags are reported inside square brackets. Significance at the 10, 5, and 1 percent levels is denoted by superscripts *, **, and ***, respectively.

Table 11 - Parameter Estimates for Backward-Looking Interest Rate Rule

	BWTR1	BWTR2	BWTR3	BWTR4
l_{CPI}	0	-2	-6	-10
l_{UR}	0	-2	-6	-10
l_{FF}	-1	-1	-1	-1
Instrument lags	1	1	1	1
γ^{CPI}	2.264	2.273	2.355*	2.866*
	[2.127]	[1.408]	[1.388]	[1.691]
γ^{UR}	1.158	0.410	0.598	0.352
	[1.380]	[0.504]	[0.431]	[0.416]
CPI^{*}	1.257	2.270	1.715	2.089
	[6.491]	[3.305]	[2.177]	[1.882]
ρ	0.986***	0.964***	0.951***	0.949***
	[0.001]	[0.002]	[0.002]	[0.002]
R^2	0.993	0.993	0.993	0.993
J-statistic	26.153***	6.326*	1.128	0.860
df	2	2	2	2

Note: Specification BWTR1 uses a constant, current values of CPI and UR, and lagged values of CPI, UR, and FF as instruments. Specification BWTR2-4 use the same instruments as BWTR1 plus 2-4 lagged versions of CPI, UR, and UR. Newey-West standard errors with 12 lags are reported inside square brackets. Significance at the 10, 5, and 1 percent levels is denoted by superscripts *, **, and ***, respectively.

TABLE 12 - NBER-Dated Recessions Considered

Recession Code	Start Date	End Date	Duration
R1	November, 1973	March, 1975	16 months
R2	January, 1980	July, 1980	6 months
R3	July, 1981	November, 1982	16 months
R4	July, 1990	March, 1991	8 months
R5	March, 2001	November, 2001	8 months

TABLE 13 - Overall Accuracy of Alternative Specifications

Panel A	**Recession R1**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1st month	1.31	1.56	*0.65*	1.71	1.00
2nd month	*0.81*	1.16	0.82	1.22	1.13
3rd month	*0.62*	1.17	0.79	1.16	1.08
4th month	0.60	0.56	*0.39*	0.68	0.59
5th month	1.09	0.45	*0.33*	0.52	0.41
6th month	1.48	0.95	0.85	0.87	*0.67*
7th month	1.37	1.21	1.13	*0.94*	1.00
8th month	1.57	1.67	1.60	1.39	*1.15*
9th month	1.74	1.69	1.67	1.32	*1.15*
10th month	1.88	1.12	1.13	0.83	*0.67*
11th month	1.14	1.05	1.06	0.92	*0.63*
12th month	1.38	0.71	0.79	0.61	*0.24*
13th month	1.05	0.34	0.38	0.40	*0.28*
14th month	0.77	*0.37*	0.40	0.57	0.43
15th month	0.99	0.71	*0.65*	1.79	1.07
16th month	*0.73*	1.09	1.03	1.68	1.65

Panel B	**Recession R2**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1st month	4.48	*0.88*	1.38	1.03	1.26
2nd month	4.57	*1.69*	1.77	1.82	*1.69*
3rd month	4.28	1.48	1.35	1.45	*1.28*
4th month	*1.99*	3.22	2.26	3.25	2.55
5th month	1.81	*0.81*	1.08	0.90	1.18
6th month	1.60	0.91	0.74	0.83	*0.66*

Panel C	Recession R3				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1st month	1.81	*0.88*	5.36	1.01	4.98
2nd month	3.50	2.47	2.58	2.47	*1.89*
3rd month	5.18	2.61	2.65	2.58	*2.25*
4th month	4.98	2.32	2.31	2.40	*2.05*
5th month	3.72	1.71	1.68	1.65	*1.52*
6th month	4.84	2.43	2.36	2.22	*2.14*
7th month	4.94	2.32	2.28	*1.93*	2.00
8th month	4.85	1.21	1.26	1.25	*0.90*
9th month	4.58	2.49	2.52	2.39	*2.30*
10th month	4.74	1.03	1.08	*0.91*	*0.91*
11th month	4.79	0.69	0.72	0.74	*0.59*
12th month	5.72	1.54	1.59	1.53	*1.40*
13th month	5.02	1.55	1.58	1.62	*1.51*
14th month	4.47	2.67	2.66	2.53	*2.49*
15th month	2.98	1.76	1.75	*1.59*	1.63
16th month	2.81	1.87	1.86	1.71	*1.69*

Panel D	Recession R4				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1st month	1.30	0.46	*0.35*	0.40	0.38
2nd month	0.97	0.90	0.93	0.91	*0.82*
3rd month	1.02	0.71	0.72	0.73	*0.64*
4th month	1.11	0.64	0.64	0.67	*0.59*
5th month	1.52	0.58	0.58	0.59	*0.53*
6th month	1.77	0.57	0.57	0.56	*0.51*
7th month	1.86	0.86	0.86	0.87	*0.82*
8th month	1.88	1.00	0.99	0.98	*0.95*

Panel E	Recession R5				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1st month	1.79	0.84	3.45	*0.69*	3.75
2nd month	2.52	0.64	0.65	0.60	*0.56*
3rd month	2.98	0.71	0.72	0.78	*0.55*
4th month	3.24	0.93	0.94	0.91	*0.74*
5th month	3.71	0.78	0.79	0.75	*0.71*
6th month	4.03	0.96	0.97	0.90	*0.86*
7th month	4.72	1.29	1.30	1.32	*1.21*
8th month	4.79	1.23	1.23	*1.17*	1.34

Note: The quantities in *italics* are the smaller values for a given time period and episode.

TABLE 14 - Maturity-Disaggregated Accuracy

Panel A	**Recession R1**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1mo	2.45	0.90	0.96	0.90	*0.73*
2mo	1.36	0.86	0.90	0.86	*0.70*
3mo	*0.63*	0.83	0.85	0.82	0.67
4mo	0.95	0.80	0.81	0.80	*0.65*
5mo	1.49	0.57	0.57	*0.54*	0.58
6mo	1.93	0.60	*0.57*	0.58	0.58
7mo	2.23	0.62	*0.56*	0.63	0.59
8mo	2.36	0.66	0.56	0.66	*0.55*
9mo	2.37	0.71	0.59	0.71	*0.58*
10mo	2.27	0.75	0.63	0.76	*0.61*
11mo	2.12	0.78	0.65	0.80	*0.63*
12mo	1.94	0.81	*0.66*	0.90	0.72
24mo	1.27	0.91	*0.76*	1.00	*0.76*
36mo	0.84	0.91	0.80	1.01	*0.79*
48mo	*0.70*	1.02	0.86	1.03	0.77
60mo	*0.93*	2.13	1.95	2.21	2.01

Panel B	**Recession R2**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1mo	2.89	1.61	2.32	*1.57*	2.17
2mo	2.57	1.60	2.30	*1.55*	2.14
3mo	2.81	1.57	2.26	*1.51*	2.09
4mo	3.09	1.52	2.20	*1.46*	2.02
5mo	3.42	1.45	2.12	*1.39*	1.95
6mo	3.67	1.33	1.98	*1.31*	1.81
7mo	3.94	1.25	1.86	*1.24*	1.69
8mo	4.12	*1.19*	1.74	*1.19*	1.58
9mo	4.19	1.15	1.63	*1.14*	1.49
10mo	4.21	1.13	1.54	*1.09*	1.40
11mo	4.22	*1.03*	1.49	1.10	1.37
12mo	4.22	*1.01*	1.47	1.13	1.33
24mo	3.06	1.02	0.97	1.04	*0.85*
36mo	2.80	*0.92*	0.96	0.93	1.03
48mo	1.89	1.85	*1.56*	1.89	1.68
60mo	4.00	3.38	2.95	3.45	*2.87*

Panel C	**Recession R3**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1mo	3.73	*1.44*	1.89	1.49	1.73
2mo	1.64	*1.43*	1.88	1.48	1.71
3mo	*0.79*	1.44	1.90	1.48	1.71
4mo	1.65	*1.46*	1.91	1.51	1.71
5mo	2.72	*1.40*	1.85	*1.40*	1.60
6mo	3.57	1.48	1.91	*1.45*	1.66
7mo	4.17	1.55	1.97	*1.50*	1.71
8mo	4.58	1.51	1.93	*1.43*	1.66
9mo	4.84	1.57	1.97	*1.51*	1.72
10mo	5.01	1.62	2.01	*1.58*	1.77
11mo	5.11	1.68	2.05	*1.65*	1.82
12mo	5.18	1.78	2.14	*1.71*	1.83
24mo	4.57	1.37	1.74	*1.24*	1.40
36mo	5.01	2.16	2.32	*2.02*	2.06
48mo	3.63	1.66	1.90	*1.61*	1.71
60mo	*4.28*	4.61	4.81	4.57	4.69

Panel D	**Recession R4**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1mo	6.90	0.68	0.79	*0.65*	0.75
2mo	5.77	0.71	0.82	*0.69*	0.78
3mo	4.83	0.71	0.82	*0.70*	0.78
4mo	4.07	0.70	0.80	*0.69*	0.76
5mo	3.47	*0.67*	0.78	*0.67*	0.72
6mo	3.00	*0.67*	0.77	*0.67*	0.71
7mo	2.64	*0.66*	0.70	*0.66*	*0.66*
8mo	2.35	0.39	0.45	*0.33*	0.39
9mo	2.13	0.39	0.45	*0.35*	0.39
10mo	1.93	0.40	0.44	*0.37*	0.38
11mo	1.77	0.41	0.44	0.41	*0.39*
12mo	1.63	0.43	0.45	0.44	*0.40*
24mo	1.35	0.57	0.56	0.53	*0.48*
36mo	0.98	*0.47*	0.50	0.49	0.50
48mo	*0.42*	1.25	1.17	1.26	1.11
60mo	0.84	0.69	0.61	0.70	*0.60*

Panel E	**Recession R5**				
	DL	**SV**	**SV-TR**	**MV**	**MV-TR**
1mo	7.01	1.36	1.73	*1.21*	2.02
2mo	6.45	1.42	1.79	*1.28*	2.07
3mo	5.98	1.45	1.82	*1.31*	2.09
4mo	5.59	1.47	1.84	*1.33*	2.10
5mo	5.28	1.47	1.87	*1.33*	2.09
6mo	5.03	1.51	1.87	*1.36*	2.10
7mo	4.84	1.50	1.47	*1.35*	2.08
8mo	4.69	1.11	1.47	*0.99*	1.71
9mo	4.57	1.10	1.47	*0.97*	1.69
10mo	4.45	1.08	1.47	*0.94*	1.65
11mo	4.35	1.08	1.44	*0.93*	1.63
12mo	4.24	1.06	1.42	*0.91*	1.59
24mo	3.33	*0.54*	0.87	0.61	0.87
36mo	2.79	*0.78*	0.95	0.80	*0.78*
48mo	2.73	0.87	1.24	*0.85*	1.02
60mo	2.35	*1.51*	1.85	1.59	1.64

Note: The quantities in *italics* are the smaller values for a given time period and episode.