Price Discrimination in Input Markets

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Abstract

We analyze the short- and long-run implications of third-degree price discrimination in input markets where downstream firms differ in their efficiency. In contrast to the extant literature, where the supplier is typically an unconstrained monopolist, in our model input prices are constrained by the potential for demand-side substitution. This modification has far-reaching consequences. We show that more efficient firms receive lower input prices under price discrimination, and that the imposition of uniform pricing could stifle incentives to reduce own marginal costs. If downstream firms compete in the same market, we also find a "waterbed" effect, in that a reduction in a firm's own marginal costs not only reduces its own input price, but increases the input price of its competitors.

Keywords: Price Discrimination; Uniform Pricing; Input Market.

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1 Introduction

Whether to allow dominant upstream firms to price discriminate between different downstream buyers is an important policy question. Imposing non-discriminatory terms of supply is a frequent response towards dominant firms in regulated industries. For instance, under the European Union's new regulatory framework for electronic communications, nondiscrimination requirements represent one of the key options available to National Regulatory Authorities to regulate the wholesale market.[1] In the US, Section 251 of the Telecommunications Act of 1996 imposes upon incumbent Local Exchange Companies that interconnection with other carriers must be provided on rates, terms, and conditions that are "just, reasonable, and nondiscriminatory".[2] In non-regulated industries, suppliers' scope to discriminate between different buyers is also limited by various provisions in the antitrust laws such as the Robinson-Patman amendments to Section 2 of the Clayton Act in the US and Article 82(c) in the EU.

In accordance with its importance for regulation and antitrust policy, academic research has devoted much space towards analyzing the implications of prohibiting discriminatory pricing.[3] A smaller, but still substantial, fraction of this literature deals with third-degree price discrimination in input markets, which is where our own contribution lies. In our model, there is scope for price discrimination as some downstream firms are more efficient than others, as measured by their own marginal production costs. This setting is common in the literature, shared with, amongst others, DeGraba (1990) and Yoshida (2000). We analyze both the case where buyers serve separate markets and the case where they compete against each other.[4] In line with the

[1] The menu provided for in Articles 9-13 of the Access Directive lists nondiscrimination requirements next to the control of access and price, the imposition of accounting separation, and price and cost transparency. See Valletti (2003) for more details on the EU's regulatory regime.

[2] The Federal Communications Commission (FCC) notes that Section 251 imposes non-discrimination obligations that existed prior to the 1996 Act, and notably imports obligations from the Modification of Final Judgment when the Department of Justice settled the antitrust suit against AT&T. See "Notice of Inquiry Concerning a Review of the Equal Access and Nondiscrimination Obligations Applicable to Local Exchange Carriers", CC Docket No. 02-39, FCC 02-57, 2002.

[3] For authorative overviews see Varian (1989) and, more recently, Armstrong (2005) and Stole (2005).

[4] Admittedly, the case where buyers serve different markets is of less importance for competition policy given the scope and intention of the respective antitrust provisions. With respect to "secondary-line" injury, Section 2 of the Clayton Act and its Robinson-Patman amendments were passed to rein in larger businesses, which were thought to potentially enjoy substantial purchasing advantages. Likewise, Article 82(c) of the EU's Treaty

literature, our main interest lies in whether and when the imposition of uniform pricing may enhance welfare. As in DeGraba (1990), we analyze both the short run, namely the static implications for prices and output, and the long run, that is the dynamic implications for downstream firms' incentives to invest and innovate.

While the extant literature has by and large considered the case where the supplier can freely choose its optimal (monopoly) input prices, we explicitly recognize the potential for demand-side substitution as downstream firms may switch to a different, albeit potentially less efficient, source of supply. Importantly, though the supplier is no longer perfectly unconstrained, the prevailing input price may still lie substantially above the supplier's marginal costs.[5]

We find that, under price discrimination, a more efficient downstream firm receives a strictly lower input price than a less efficient firm, which is exactly the opposite to what one finds in the more standard setting where the supplier can act as an unconstrained monopolist. As a more efficient firm will also purchase a larger volume and, in case firms compete on the final market, will consequently have a larger market share than a less efficient firm, more competitive and larger buyers will thus receive a discount in our model. Moreover, we find that as one downstream firm becomes more efficient, under price discrimination this will not only reduce its own input price but increase that of its competitors. Input price discrimination thus tends to amplify differences in downstream firms' competitive position, strenghtening those firms that are already larger and further weakening those firms that are smaller and less competitive. Again, the opposite would be the case with an unconstrained monopolistic supplier, whose optimal input prices tend to disadvantage more efficient downstream firms.

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forbids practices by which a dominant firm would place trading parties at a competitive disadvantage. In the EU, however, the case of geographic price discrimination along the boundaries of member states has specific importance as the creation of an integrated market is one of the founding goals. To sanction geographic price discrimination, Community courts have relied both on a wider interpretation of Article 82(c) and, in particular, on prohibiting restrictions on parallel trade (on the grounds of Article 81), which are often a prerequisite to sustain meaningful price differentials in input markets. Price discrimination is infeasible if a supplier cannot control effectively the use of purchased quantities by limiting resale and preventing arbitrage. (See Gerardin and Petit (2005) for a more detailed account of the European law.)

[5] While in order to fall under the respective antitrust provision, a firm engaging in price discrimination must be to some extent shielded from effective competition, this does not require the firm to be able to charge (unconstrained) monopoly prices. Likewise, mandatory provisions in regulated industries, in particular in network industries, often apply to firms that do not enjoy (or do no longer enjoy) a fully monopolistic position.

The more standard setting where a supplier can act like an unconstrained monopolist seems more applicable in industries where there is a natural monopoly in the upstream level or where buyers are relatively fragmented and, given their choice of technology, highly locked into a relationship with a given supplier. In such an environment, the "hold-up" perspective, according to which a discriminating supplier extracts a higher price from more efficient firms and thus stifles incentives, seems more appropriate. In contrast, our setting should be more applicable to industries that are not monopolized at the upstream level and where such lock-in is less severe.

The imposition of uniform pricing has both short-run and long-run implications for consumer surplus and welfare. If buyers serve separate markets, we find that even though output unambiguously expands under uniform pricing, it dampens downstream firms' incentives to reduce their own costs. In case downstream firms are initially symmetric, the incentive effect dominates and consumer surplus is lower if price discrimination is banned. This stands again in contrast to what would happen if the supplier was an unconstrained monopolist, since price discrimination would then penalize more efficient firms and thus reduce incentives.[6] If buyers compete in the downstream market, the imposition of uniform pricing reduces the input price of the less efficient firm but increases that of the more efficient firm in the short run. While this shifts production to the less efficient firm, we can show with linear demand that total output typically expands in the short run. In the long run, however, uniform pricing is still likely to reduce incentives. By improving its own efficiency a firm may also benefit its competitor under uniform pricing, while under discriminatory pricing lower own marginal costs increase a competitor's input price under competition.

Our analysis also reveals the following long-run implication of imposing uniform pricing. We show that, under uniform pricing, both in the case of separate markets and in the case of competition, there is a natural tendency for firms to become asymmetric in their levels of efficiency and, consequently, also in their respective market shares. Intuitively, it is only the *ex-post* more efficient firm that puts a binding constraint on the supplier's uniform price, which

[6] The literature on input price discrimination has also considered alternative potential implications, from which we abstract. These include the impact of uniform pricing on alternative non-price decisions by downstream firms such as product differentiation (DeGraba (1987)) or inefficient backward integration (Katz (1987)) as well as its use to facilitate (tacit) collusion (e.g., Cooper (1986)). In addition, while we focus on downstream firms' incentives, it could also be asked how restrictions on the supplier's ability to extract profits via discriminatory pricing may affect its own incentives to invest and innovate.

creates additional incentives that are absent for the *ex-post* less efficient firm.

Our approach borrows heavily from Katz (1987) when we model the constraints faced by the upstream firm. There, a large buyer can sell into two different markets, where it competes against small buyers. While in principle both the large and the small buyers have the option to integrate backwards instead of purchasing the input from the supplier, the associated large fixed costs make this attractive only for the large buyer. We borrow from Katz (1987) the notion that switching away from a given supplier is costly, though we take a somewhat broader view in that buyers' alternative option could also originate from a competing supplier's offer. In contrast to Katz (1987), however, in our setting buyers differ with respect to their efficiency, which will endogenously create differences in size, while the alternative supply option will be sufficiently attractive for all and not only for a subset of buyers.

The rest of this paper is organized as follows. Section 2 introduces the basic model. Sections 3 and 4 analyze the cases where downstream firms operate in separate markets or where they compete in the same market. Section 5 concludes.

2 The Model

The basic set-up follows much of the literature comparing discriminatory and uniform pricing in an intermediary industry. We consider a single supplier that provides an input to the intermediary industry. Firms in the intermediary industry use the input to produce a homogeneous final good. All firms in the intermediary industry have identical production functions, where we specify that firms transform one unit of the input into one unit of the output.[7] The supplier produces at constant marginal costs, which we take to be zero to simplify our expressions. Also, without much loss in generality we restrict attention to two downstream firms, $i = 1, 2$. We denote firm i's respective own marginal costs of production by $k_i \geq 0$.

Our analysis distinguishes between the following two cases. In the first case, the two downstream firms serve two identical but independent market. For most of our analysis we assume that both markets are symmetric and characterized by the same inverse demand function $P(q)$. (See, however, Section 3.4.) Furthermore, P is continuously differentiable and satisfies $P(0) > k_i$ for both firms $i = 1, 2$. In the second case that we analyze, both firms $i = 1, 2$ are active in the

[7] Given symmetry of production functions, this specification is not important for our results. Retailing is a natural example where this specification is reasonable.

same market, offer homogeneous goods, and compete in quantities. There, the single homogeneous market is again characterized by the inverse demand $P(q)$. In both cases, we assume that $P(q)$ is strictly decreasing and twice continuously differentiable where $P(q) > 0$. Moreover, we employ throughout this paper the following standard (Cournot) assumption:

Assumption 1. $P' < \min\{0, -qP''\}$ *where* $P > 0$.[8]

The contractual game that we consider in this paper also follows closely the received literature. The supplier can make take-it-or-leave-it offers to the downstream firms. Under price discrimination, the supplier offers each firm a constant input price w_i, while under uniform pricing the same input price $w_i = w$ applies to both firms. Consequently, upon accepting the supplier's offer, a firm's total marginal cost equals $c_i := w_i + k_i$. Though our restriction to linear contracts is shared with much of the literature on third-degree price discrimination, in Section 3.4 and Section 4.4 we discuss this specification as well as alternatives in some detail.

Our main deviation from much of the literature is that the supplier, despite having the ability to make take-it-or-leave-it offers, is no longer an unconstrained monopolist. Here, we follow Katz (1987) and suppose that instead of seizing operations following rejection of the supplier's offer, a downstream firm can turn to an alternative source of supply. As in Katz (1987), this comes at costs $F > 0$ and allows the respective firm to obtain the input at constant marginal costs $\widehat{w} \geq 0$. Below we also discuss the possibility that the alternative input is only an imperfect substitute. Katz (1987) refers to the buyers' alternative as the option to integrate backwards, which represents a viable option for a very large buyer only, because of the high associated fixed costs. In contrast, we focus our analysis on the case where the supplier can not act as an unconstrained monopolist vis-à-vis any of the downstream firms. As an alternative interpretation, downstream firms could instead choose the competitive bid of another supplier, which is willing to sell at its own marginal cost of $\widehat{w}$. In this case, the (switching) costs F could arise either at the buyer or at the new supplier. Yet another but related interpretation could be that the costs F are incurred to adopt another technology under which a different input that is sold at the (competitive) market price of $\widehat{w}$ can be used instead. Section 3.4 discusses also the possibility that, after rejecting the supplier's offer, a buyer can reduce $\widehat{w}$ by expending more

[8] See, for instance, Vives (1999). This assumption ensures that under competition the firms' maximization problem is strictly concave. If the two firms serve independent markets, the weaker condition $2P' < \min\{0, -qP''\}$ would suffice.

resources, e.g., by searching for a lower offer or, in the case of vertical integration, by making higher investments.

The final part of our model is an initial investment stage. Following DeGraba (1990), each downstream firm can initially spend resources to reduce its own marginal cost k_i. For our general analysis, we specify that initially a firm's own marginal cost is equal to $\overline{k}_i > 0$, which the firm can reduce by Δ_k when incurring the expenditure $e(\Delta_k)$. We suppose that e is strictly increasing and continuously differentiable with $e'(0) = 0$ and $e'(\Delta_k) \to \infty$ as $\Delta_k \to \overline{k}_i$, which allows us to focus on interior solutions. The chosen investment levels and, consequently, the respective values of k_i are common knowledge.

3 Separate Markets

3.1 The Short Run

In this Section, we analyze the case where firms serve separate markets. We further stipulate that they are both monopolists in their respective markets.[9] For the short-run analysis, we take the firms' own marginal costs k_i as given and solve for the respective price equilibria, both under price discrimination and under uniform pricing. Section 3.2 analyzes the long run, solving for the first stage of our model, where downstream firms can invest in a reduction of own marginal costs.

Given total marginal costs of $c_i = k_i + w_i$ under discriminatory pricing, firm i optimally chooses the unique quantity

$$q(c_i) := \arg\max_q \{q [P(q) - c_i]\},$$

and realizes the profits

$$\pi(c_i) := q(c_i) [P(q(c_i)) - c_i],$$

where both q and π are strictly decreasing in c_i. The supplier's objective is to choose w_i to maximize $q(c_i)w_i$, where $c_i = w_i + k_i$ and where we use that the supplier has zero marginal costs. Take first the benchmark case where the supplier is an unconstrained monopolist. Assuming

[9] It is, however, straightforward to extend the analysis to the case where both firms face the same level of competition, that is as long as their competitors are not themselves purchasing from the same supplier.

that $q(c_i)w_i$ is strictly quasiconcave where $q > 0$, an unconstrained supplier would optimally set w_i equal to

$$w_i^{UC} := \arg\max_{w_i} \{q(c_i)w_i\}. \tag{1}$$

The distinctive feature of our model is that buyers' alternative supply options put a constraint on the supplier's pricing power. For firm i the value of the alternative supply option equals

$$V_i^A := \pi(\widehat{c}_i) - F, \tag{2}$$

where $\widehat{c}_i := \widehat{w} + k_i$ denotes the respective total marginal costs. Consequently, the supplier's offer to each firm i must satisfy the respective participation constraint

$$\pi(c_i) \geq V_i^A. \tag{3}$$

In what follows, we focus on the case where V_i^A is sufficiently attractive such that for each firm i the respective condition (3) constrains the supplier's optimal choice of w_i. This is the case if the monopolistic input price w_i^{UC} is high enough, i.e., if $\pi(c_i) < V_i^A$ holds for $c_i = k_i + w_i^{UC}$. It is in turn straightforward to show that this holds if both F and $\widehat{w}$ are not too large.[10] We obtain the following result.

Proposition 1. *In the case of separate markets and with price discrimination, a downstream firm's input price w_i is strictly lower the more efficient the firm is, i.e., the lower its own marginal costs k_i.*

Proof. See Appendix.

As noted in the Introduction, Proposition 1 is quite different to the results that are obtained in case the supplier can set its unconstrained optimal price $w_i = w_i^{UC}$. In the latter case, the strictly higher purchase volume of a more efficient downstream firm makes it optimal for the supplier to charge a strictly higher input price than for a less efficient firm. In standard terminology, going back to the seminal analysis of Robinson (1933), the more efficient downstream firm would represent for the supplier the "strong" market, where it is optimal to set a higher price than in the "weak" market. In contrast, with a constrained supplier we find that for a more efficient firm the respective participation constraint (3) becomes tighter, which only allows the supplier to charge a lower input price. Intuitively, under its alternative supply option, a more

[10] Below, we provide explicit conditions for the case with linear demand.

efficient firm will end up producing a larger quantity, namely $q(\widehat{c}_i)$, than a less efficient firm, which in turn allows it to "distribute" the fixed costs F over a larger volume. The "per-unit" total cost of the outside option is lower for a more efficient firm. To ensure that a more efficient firm's participation constraint is still satisfied, the supplier must thus lower its input price.

This argument relies crucially on the assumption that switching to the alternative source of supply comes at strictly positive costs $F > 0$. In contrast, for $F = 0$ the supplier's offer to both downstream firms would just match the marginal costs from their alternative supply option, $w_i = \widehat{w}$. More generally, we have the following immediate comparative results for the equilibrium input prices.

Corollary 1. *In the case of separate markets and with price discrimination, input prices are strictly increasing in both F and $\widehat{w}$. Moreover, if firm i is the more efficient firm, then the price differential $w_j - w_i > 0$ is strictly increasing in F.*

Proof. See Appendix.

Note finally that, as an immediate implication of Proposition 1, downstream firms that end up purchasing larger volumes will only pay a discounted input price.

We turn next to the case of uniform pricing, where the supplier must make any offer available to both downstream firms.[11] From Proposition 1 the following result is immediate.

Proposition 2. *In the case of separate markets and uniform pricing, the supplier offers both downstream firms the price that it would have offered to the more efficient firm under price discrimination. Consequently, compared to the case of price discrimination, under uniform pricing the input price for the more efficient firm stays unchanged, while that of the less efficient firm decreases.*

If the supplier was unconstrained, it is well known that uniform pricing would lead to an "average" price that lies strictly between the price of the more and that of the less efficient firm in the case of price discrimination. Hence, in the unconstrained case, the imposition of uniform pricing hurts the less efficient firm.

Next, in our case with a constrained supplier it follows immediately from Proposition 2

[11] Recall that we restrict consideration to linear contracts. This rules out the possibility that the supplier could make a more attractive offer to the more efficient firm, where incentive compatibility could be achieved via a sufficiently steep volume discount.

that uniform pricing increases output if $k_1 \neq k_2$. If $P(q)$ denotes the marginal valuation of a representative consumer, we then have the following result.[12]

Corollary 2. *In the case of separate markets and if downstream firms' own marginal costs k_i are exogenously given, uniform pricing unambiguously increases consumer surplus and welfare if $k_1 \neq k_2$.*

As we show next, however, if differences in the downstream firms' levels of efficiency arise *endogenously* through their previous investments, we can end up with exactly the opposite result, namely that the imposition of uniform prices makes consumers worse off and may also reduce welfare.

3.2 The Long Run

We now consider the first stage of our model, where downstream can firms invest in a reduction of their own marginal costs. As a helpful benchmark, suppose first that w_i was fixed and thus did not respond to a change in k_i. In this case, firm i's choice of the cost reduction Δ_k optimally trades off the marginal increase in profits $-\pi'(c_i)$ with the marginal investment costs $e'(\Delta_k)$. With discriminatory pricing and if w_i is, instead, strategically chosen by the supplier and thus varies with k_i, firm i's marginal benefits from reducing k_i are strictly higher. This follows as by Proposition 1 a reduction in k_i also leads to a lower input price w_i. Formally, the marginal benefits from a reduction of w_i in case of discriminatory pricing are given by

$$-\frac{d\pi(c_i)}{dk_i} = -\pi'(c_i)\left(1 + \frac{dw_i}{dk_i}\right), \tag{4}$$

where $dw_i/dk_i > 0$.

If $k_i \leq k_j$, then under uniform pricing the marginal benefits for firm i to decrease k_i are the same as in the case of discriminatory pricing and thus equal to (4). This follows immediately from our previous observation that the prevailing uniform input price is the same as the discriminatory price for the more efficient firm. In contrast, if $k_i > k_j$ then the marginal benefits are strictly

[12] Proposition 2 and Corollary 2 come with the caveat that under uniform pricing it is still optimal for the supplier to make an offer that is acceptable to both downstream firms. Alternatively, the supplier could decide to only offer the contract that satisfies the participation constraint of the less efficient firm. When analyzing the case of linear demand below, we derive explicitly the conditions for when either of the two cases applies. Intuitively, under uniform pricing the supplier will still make an offer that is acceptable to both downstream firms if their own marginal costs are not too different or always if F is sufficiently low.

smaller for firm i under uniform pricing. This follows again as given $k_i > k_j$, even after a marginal reduction of k_i firm i will still be the less efficient firm and will thus not affect the prevailing uniform input price.

Starting from initial levels $\overline{k}_i > 0$, we ask now about the marginal costs k_i that will prevail in a pure-strategy equilibrium of the investment game. Assuming that the expenditure function e is sufficiently convex, there is a unique equilibrium in the case of discriminatory pricing. Likewise, in the case of uniform pricing there are, at most, two different pure-strategy equilibria in which either firm $i = 1$ or firm $i = 2$ becomes the ex-post most efficient firm.[13] The following results follow then naturally from the previous arguments.

Proposition 3. *If markets are separate and if $k_i \leq k_j$, then the marginal benefits for firm i to further reduce k_i are the same under discriminatory as under uniform pricing. If instead $k_i > k_j$, then the marginal benefits for i are strictly lower under uniform pricing. Comparing pure-strategy equilibria of the investment game, we have that one firm chooses under uniform pricing the same level of investment as under discriminatory pricing, while the other firm chooses a strictly lower level of investment.*

Proof. See Appendix.

An immediate implication from our previous discussion is that under uniform pricing the two firms' marginal costs can not be equal in the long run. Formally, this follows as at $k_i = k_j$ the marginal benefits from reducing own marginal costs have a discontinuity, given that the uniform price only depends on the marginal costs of the (ex-post) more efficient firm. Hence, even if firms initially start out with the same costs $\overline{k}_i = \overline{k}_j$, in the long run one firm will be more efficient than the other.[14]

Corollary 3. *In the case with separate markets, under uniform pricing firms will always have different own marginal costs in the long run. This holds, in particular, also if firms are initially equally efficient.*

Proposition 3 can again be contrasted with the results if the supplier is unconstrained.

[13] This does not yet guarantee quasiconcavity of firms' profit functions and thus existence of a pure-strategy equilibrium. See the proof of Proposition 3.

[14] Interestingly, if firms are initially equally efficient then it is the ex-post less efficient firm that has higher total profits (net of initial investment costs). In essence, one firm free rides on the higher investment of the other firm, which bears alone the burden to push down the uniform input price.

There, the imposition of uniform pricing reduces a hold-up problem, which in turn arises as the unconstrained supplier "taxes away" some of the efficiency gains of downstream firms. The imposition of uniform pricing shields the investment of individual downstream firms.

From Proposition 3 it follows immediately that if firms are initially symmetric, then input prices for both firms will be the same under uniform and discriminatory pricing. This holds as, under uniform pricing, the input price is determined by the marginal cost of the more efficient firm, which invests the same as under discriminatory pricing. However, under uniform pricing one firm invests less so that its total marginal costs $c_i = w_i + k_i$ remain higher. Thus output and consumer surplus are strictly smaller than under price discrimination.

Corollary 4. *In the case with separate markets, if downstream firms are initially equally efficient and can invest in a reduction of own marginal costs, then total consumer surplus is strictly higher under price discrimination than under any pure-strategy equilibrium of the game with uniform pricing.*

In the linear example analyzed below, we show that the imposition of uniform pricing also lowers total welfare. Intuitively, we show that even under price discrimination firms have socially inefficient incentives to invest as they ignore consumer surplus. As uniform pricing leads to an even lower investment level, total welfare is consequently lower.

If firms' costs are initially different, say as $\overline{k}_j - \overline{k}_i > 0$, and if the firms' ordering in terms of efficiency remains the same such that $k_j - k_i > 0$,[15] then for the less efficient firm j there are now two different forces at work. While its own marginal costs k_j are strictly higher under uniform pricing, the uniform input price w is strictly lower than the respective discriminatory price w_j. Intuitively, however, we can show that if the initial difference is not too pronounced, then Corollary 4 still holds.

3.3 The Linear Case

For this Section, we suppose that demand is linear with $P(x) = 1 - x$.[16] This generates quadratic profits $\pi(c_i) = \left(\frac{1-c_i}{2}\right)^2$, which can be substituted into the binding participation constraint (3)

[15] Intuitively, if the difference $\overline{k}_j - \overline{k}_j$ is large, then this also the only possible pure-strategy equilibrium.

[16] Linear demand is a frequent assumption in the literature on third-degree price discrimination (see, e.g., DeGraba (1990) and Yoshida (2000)).

to obtain the optimal discriminatory input prices

$$w_i = 1 - k_i - \sqrt{(1 - k_i - \widehat{w})^2 - 4F}. \tag{5}$$

From (5) it is immediate that $w_i > \widehat{w}$ is strictly increasing in both $\widehat{w}$ and F and that $dw_i/dk_i > 0$, which confirms Proposition 1. Furthermore, in the linear case the supplier's unconstrained optimum is $w_i^{UC} = (1 - k_i)/2$, from which we have that the outside option of firm i binds whenever

$$F \leq \left(\frac{1 - k_i - \widehat{w}}{2}\right)^2 - \left(\frac{1 - k_i}{4}\right)^2, \tag{6}$$

which is satisfied for any $\widehat{w} < w_i^{UC}$ as long as F is sufficiently small.

Under uniform pricing, the input price (5) for the more efficient firm applies to both firms. If firm $i = 1$ is more efficient, then it is still optimal to serve the more efficient firm in case

$$w_1(\frac{1 - w_1 - k_1}{2} + \frac{1 - w_1 - k_2}{2}) \geq w_2(\frac{1 - w_2 - k_2}{2}), \tag{7}$$

where we can substitute w_1 and w_2 from (5). Inspecting (7) reveals that this is in turn the case if either F is sufficiently low or, holding F fixed, if k_1 and k_2 are close enough. Returning again briefly to the case where the supplier is unconstrained, we find under uniform pricing that $w^{UC} = [1 - (k_1 + k_2)/2]/2$, which lies strictly between the higher discriminatory price for the more efficient firm and the lower discriminatory price for the less efficient firm.

For the long-run analysis, we follow DeGraba (1990) and take a quadratic investment cost function $e(\Delta_k) = t(\Delta_k)^2/2$, where we also set for now $t = 1$.[17] We further restrict the analysis to the case where firms are initially symmetric and relegate most of the calculations to Appendix B. There, we show that under price discrimination firms choose to reduce own marginal costs by

$$\Delta_k = \Delta^{PD} := 1 - \overline{k} - \widehat{w}. \tag{8}$$

Under uniform pricing, however, only one firm chooses Δ_k according to (8), while the other firm chooses

$$\Delta_k = \Delta^U := 2\sqrt{(\Delta^{PD})^2 - F} - \Delta^{PD}. \tag{9}$$

[17] Admittedly, as the derivative remains bounded as $\Delta_k \to \overline{k}_i$, this does not satisfy one of the stipulated requirements for the general case. In what follows, we will, however, impose sufficient conditions to ensure that the equilibrium is still interior. (See Appendix B for details.)

Comparing (9) and (8), we have from $2\sqrt{(\Delta^{PD})^2 - F} < 2\Delta^{PD}$ that $\Delta^U < \Delta^{PD}$. Finally, we show in Appendix B that total welfare is strictly quasiconcave in Δ_k and that Δ^{PD} is inefficiently low from the perspective of total welfare. Consequently, with linear demand we have the stronger result that, in the long run, uniform pricing reduces not only consumer surplus but also total welfare.

3.4 Discussion

In this Section, we discuss for the case with separate markets how we can relax various assumptions that we maintained in the general analysis. We first consider potential modifications to the buyers' alternative supply option. Subsequently, we examine modifications to the contractual game. Finally, we discuss the case where markets are of different size.

Firms' Alternative Supply Option

We stipulated that both buyers have the same alternative supply option. A different and equally plausible variant could be that a more efficient, and thus ultimately larger, firm would spend more resources to reduce marginal costs under its alternative supply option. More formally, suppose that next to incurring the fixed costs $F > 0$, each firm can invest $h(\Delta_w)$ to reduce the marginal costs under the alternative supply option from $\overline{w} > 0$ down to $\widehat{w}_i = \overline{w} - \Delta_w$.[18] Intuitively, as a firm with lower k_i will produce more also under the alternative supply option, it will consequently invest strictly more and thus end up also with a strictly lower $\widehat{w}_i$.

Importantly, changing the model in this way has no qualitative implications for our results. This holds for two reasons. First, when considering marginal changes in k_i, from the envelope theorem we can ignore the effect that a change in Δ_w and thus in $\widehat{w}_i$ has on the value of the alternative supply option and, as a result, on the buyer's participation constraint.[19] Second, when considering different levels of k_i, we can use that $k_i < k_j$ implies $\widehat{w}_i < \widehat{w}_j$, which further strenghtens our result that a more efficient buyer obtains a lower input price.

The Contractual Game

[18] The interpretation of Δ_w is straightforward if the option is to integrate backwards. Alternatively, we may think of $h(\Delta_w)$ as resources spent on locating a more attractive alternative supply option or on increasing production efficiency when using an alternative input.

[19] This presumes that the problem is smooth and leads to an interior solution for Δ_w.

Following much of the literature, we assumed that the input is sold under a linear contract.[20] We first want to provide some additional justification for this specification. As we argue below, with more complex contracts such as two-part tariffs, the *marginal* input price of each buyer would be the same.[21] While supply contracts are indeed often much more complex than the simple linear contracts we study, e.g., they can specify quantity discounts and a wide range of additional fees such as slotting fees, pay-to-stay fees, or display fees in retailing, the notion that the distribution of profits between up- and downstream firms has no impact on marginal input prices and thus on both quantities and prices on the final market seems equally extreme. Both casual evidence and more systematic data collection seem to suggest that discounts given to particular retailers are at least partially passed on to consumers. A fitting example are the "banana wars" between the UK's main retailing chains in 2002-2003. It is understood that following a huge volume discount negotiated by Asda, a fully owned subsidiary of Wal-Mart, with DelMonte, Asda started a prolonged price war by cutting the price of loose bananas from 1.08 to 0.94 pounds per kilo.[22] More systematic evidence for the UK was gathered in the Competition Commission's 2000 Supermarket report[23], documenting how large discounters received substantial discounts, which ultimately showed up in lower shelf prices.[24]

[20] The implications of uniform pricing (or, in a related fashion, "most-favored-customer clauses") under two-part tariff contracts have been studied, for instance, by O'Brien and Shaffer (1994), McAfee and Schwartz (1994), and DeGraba (1996). All of these papers deal, however, with the case where buyers compete downstream, which is treated in Section 4. For yet another variant, specifying linear tariffs but allowing for a more general bargaining setting, see also more recently O'Brien (2002).

[21] It should be recalled that the supplier has perfect information about the downstream firms' own marginal costs, implying that there is no need for the supplier to engage in second-degree price discrimination.

[22] See also "Concentration in Food Supply and Retail Chains", August 2004, WP Department for International Development (DFID), UK. The price of bananas plays an important role in grocery retailing both as bananas have become the most popular fruit in the UK and as they are a "known-value" item, on the basis of which consumers decide where to shop. The role of Asda's discount on bananas is also discussed in a recent inquiry by the UK's Competition Commission (see Competition Commission, "Safeway plc and Asda Group Limited (owned by Wal-Mart Stores Inc); Wm Morrison Supermarkets plc; J Sainsbury's plc; and Tesco plc: A Report on the Mergers in Contemplation," 2003).

[23] See Competition Commission, "Supermarkets: A Report on the Supply of Groceries from Multiple Stores in the United Kingdom," Report Cm-4842, 2000. Some of the key results have been recently summarized in Dobson (2005).

[24] From a theoretical perspective, two recent papers by Iyer and Villas-Boas (2003) and Milliou, Petrakis and Vettas (2004) support the assumption of linear contracts by showing that two-part tariffs may either aggravate

What are the implications if we allow for two-part tariffs contracts, stipulating a fixed fee τ_i next to a constant marginal input price ω_i? To rule out double marginalization, the supplier will then optimally always set $\omega_i = 0$ under price discrimination and likewise $\omega = 0$ under uniform pricing, using only the fixed fees τ_i and τ to extract profits. Consequently, the imposition of uniform pricing has no short-run effects on output and welfare. Using our previous arguments, we still have, however, that under price discrimination a more efficient firm obtains a discount, via a lower fixed fee τ_i.[25] This in turn implies that uniform pricing still stifles the investment incentives of the ex-post less efficient firm. As the marginal input price is now always equal to the supplier's marginal costs of zero, in the long run uniform pricing unambiguously reduces output and consumer surplus, irrespective of whether firms are initially symmetric or whether they initially differ in their own marginal costs.

Asymmetric Markets

We focused the analysis of price discrimination on differences in downstream firms' own marginal costs. None of our key results, namely that a reduction of k_i pushes down w_i under price discrimination and that uniform pricing lowers incentives, depends on this assumption. If markets are heterogenous, however, there is additional scope for price discrimination. For the sake of brevity we take only the case with linear demand, where with $P_i(q) = a_i - q$ a higher value of a_i corresponds to a larger market. A discriminating unconstrained supplier would choose $w_i = (a_i - k_i)/2$, setting again a higher price in the "stronger" market. This is again the opposite to what we find if the supplier is constrained.

To see this, observe that for a given input price w, which equals w_i under the supplier's offer and $\widehat{w}$ under the alternative supply option, the downstream firm's profits are now equal to $\pi = (a_i - k_i - w)^2/4$. The cross derivative of profits with respect to market size a_i and the input price w is strictly negative, which is just a formal restatement of the intuitive result that the firm's gains from a market expansion are larger the lower its input price. Given an input price $w_i > \widehat{w}$, this in turn implies that a (marginal) increase in a_i pushes up the value of the firm's alternative supply option V_i^A by strictly more than its profits under the supplier's offer. To still satisfy the firm's participation constraint, the supplier is then forced to reduce w_i.

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opportunism problems between suppliers and retailers or lead to excessive competition across vertical chains.

[25] It is also straightforward to show that a more efficient firm pays a strictly lower per-unit price $\tau_i/q(k_i)$ than a less efficient firm.

4 Competition

4.1 The Short Run

We now consider the case where both downstream firms compete in the same market. Our analysis produces two sets of results. First, we analyze to what extent our findings for the case of separate markets carry over to the case where buyers compete downstream, namely that i) under price discrimination a firm obtains a lower input price if it is more efficient, that ii) the imposition of uniform pricing is beneficial to the less efficient firm, and that iii) incentives to improve efficiency are weaker under uniform pricing. In addition, under competition some new issues arise, specifically the input prices of the two competing firms interact even in the case of price discrimination, and the imposition of uniform pricing affects the respective market shares of more and less efficient firms.[26] As in Section 3, we again consider first the short run, where firms' own marginal costs k_i are exogenously given.

Price Discrimination

With competition we have to be more specific about what a firm can observe regarding its competitor's offer. We find it most reasonable to assume that while firm i can observe whether its competitor j has rejected the supplier's offer and thus produces under its alternative supply option, it can not directly observe the particular offer w_j.[27]

As a benchmark, we first review the results of the case where the supplier is an unconstrained monopolist. In this case, the supplier chooses the pair of input prices (w_1, w_2) to maximize its total profits $w_1 q(c_1, c_2) + w_2 q(c_1, c_2)$, where $c_i = w_i + k_i$. If we continue to assume that the supplier's program is strictly quasiconcave and that there is an interior solution, then each input price w_i solves the respective first-order conditions and the supplier charges the more efficient firm a strictly higher price. The intuition is the same as in the case of separate markets.[28] That

[26] More precisely, without downstream competition the imposition of uniform pricing only affected buyers' share of the *upstream* market for the supplier's input, while now it affects buyers' share of the *downstream* market for final consumers.

[27] While we make these assumptions, it is important to note that our subsequent results do not depend on it. Most importantly, under binding participation constraints, an equilibrium pair of offers would have to satisfy the same pair of conditions (11) derived below if an offer to i was also observed by j and vice versa.

[28] Incidentally, with linear demand, equilibrium input prices are the same if the two firms serve separate markets and if they compete in the same market.

is, as the more efficient firm will purchase more from the supplier for a given w_i, a marginal increase in w_i has a larger direct (positive) effect on the supplier's profits. Moreover, under additional assumptions that hold, in particular, in the linear case, a reduction in a firm's own marginal cost leads again to a higher input price. Finally, while the effect on the other firm's input price is in general ambiguous, with linear demand a firm's input price does not depend on its competitor's marginal costs.[29] (The results for the linear case are formally restated in Section 4.3.)

For the case where the supplier is constrained, we again have to set up first the firms' participation constraints. If individual offers are not directly observable, the chosen quantities and the respective profits will depend both on the actual input prices and on the set of rationally expected input prices. We denote for firm i the input price that its competitor j rationally anticipates by w_i^E, which gives rise to the respective total marginal costs c_i^E. We next denote the respective unique quantity of firm i by $q^E(c_i, c_j; c_i^E, c_j^E)$, which depends on both (c_i^E, c_j^E) and the true total marginal costs (c_i, c_j).[30] Profits are likewise denoted by π^E.[31] The value of firm i's alternative supply option is then $\pi^E(\widehat{c}_i, c_j; \widehat{c}_i, c_j^E) - F$, where we use that firm j's expectations about $\widehat{c}_i$ and its actual value always coincide. Consequently, the participation constraint for firm i becomes now

$$\pi^E(c_i, c_j; c_i^E, c_j^E) \geq V_i^A := \pi^E(\widehat{c}_i, c_j; \widehat{c}_i, c_j^E) - F. \tag{10}$$

We focus on the case where the supplier's offer to both firms is constrained by the respective participation constraint (10). Intuitively, this will again be the case whenever both F and $\widehat{w}$ are sufficiently small. It is then optimal for the supplier to choose the discriminatory input prices such that both participation constraints are satisfied with equality.[32] In addition, in an

[29] Intuitively, two conflicting forces are at work here. First, as a reduction in k_i is generally not fully offset by a higher w_i, the more efficient firm ends up with lower total marginal costs, which ceteris paribus reduces the other firm's quantity. This in itself should induce the supplier to lower w_j. On the other hand, given that w_i has been increased, the supplier optimally wants to ensure that more of the total industry output shifts to this firm.

[30] It is straightforward to show that under price discrimination the supplier will not find it optimal to exclude any firm.

[31] That for given (c_i^E, c_j^E) and (c_i, c_j) quantities and profits are unique follows again from Assumption 1.

[32] More formally, if one participation constraint was not binding, then from strict quasiconcavity of the supplier's payoff in w_i it would be optimal to increase the respective input price. (See also the proof of Lemma 1 below.) Note here that as the actual input price w_i is not observed by the competing firm, this adjustment does not affect

equilibrium it must also hold that $c_i = c_i^E$ for both $i = 1, 2$.

These two observations, namely that the two participation constraints bind and that $c_i = c_i^E$ holds in equilibrium, allow us to obtain a simple characterization of the equilibrium. For this we denote - in a slight abuse of the notation from Section 3 - by $q(c_i, c_j)$ the Cournot equilibrium quantities if the *known* marginal costs are c_i and c_j, respectively. The corresponding profits are $\pi(c_i, c_j)$. Given rational expectations and the fact that both participation constraints bind in equilibrium, we thus have that any equilibrium pair of offers (w_i, w_j) must satisfy the two equations

$$\pi(c_i, c_j) = \pi(\widehat{c}_i, c_j) - F \text{ for } i = 1, 2 \text{ and } j \neq i. \tag{11}$$

To simplify notation, we will from now one simply express the *equilibrium* input prices by (w_i, w_j) and the respective total marginal costs by (c_i, c_j).

Recall next that with separate markets we were able to show that a reduction in k_i affected the value of the alternative supply option *relatively* more, making the firm's participation constraint "tighter". As we show in the proof of Lemma 1, this still holds under competition if

$$\widehat{q}_i \left[1 - P'(\widehat{q}_i + \widehat{q}_j) \frac{d\widehat{q}_j}{dc_i} \right] > q_i \left[1 - P'(q_i + q_j) \frac{dq_j}{dc_i} \right], \tag{12}$$

where we abbreviated by $q_i = q(c_i, c_j)$ and $q_j = q(c_j, c_i)$ the respective quantities under the supplier's offer and by $\widehat{q}_i = q(\widehat{c}_i, c_j)$ and $\widehat{q}_j = q(c_j, \widehat{c}_i)$ the respective quantities that are chosen under firm i's alternative supply option.

To see whether and when condition (12) is satisfied, observe first that - though we will only make this formal below - the supplier will again always set its input price above $\widehat{w}$ such that $c_i > \widehat{c}_i$. Consequently, we have that $\widehat{q}_i > q_i$. Without competition, i.e., if $d\widehat{q}_j/dc_i = dq_j/dc_i = 0$, this would be sufficient for (12) to hold, which is just a restatement of our previous result from Proposition 1. Intuitively, the effect at work is that, if i produces a higher quantity as its total marginal cost is lower, then a further reduction in its own marginal cost has a larger "direct" effect on profits.

With competition, we also have to sign the difference between the terms $-P'(\widehat{q}_i + \widehat{q}_j)\frac{d\widehat{q}_j}{dc_i}$ and $-P'(q_i + q_j)\frac{dq_j}{dc_i}$ in (12). We can make the following observations. First, these two terms are identical under linear demand, as we will show below, implying that condition (12) holds. Second, suppose for a moment that demand was strictly concave, which is a common assumption.

its respective participation constraint.

Given that total quantity is higher if firm $i = 1$ produces at lower marginal costs, we would then have that $-P'(\widehat{q}_i + \widehat{q}_j) > -P'(q_i + q_j)$, which in turn would go in the "right direction".

Based on these observations, in what follows we make the assumption that (12) holds. Putting it more compactly, this transforms into the following requirement.

Assumption 2. $\pi_{11}(c_i, c_j) > 0$.

We ask next how the other firm j is affected by a change in firm i's marginal costs. While it is immediate that a reduction in c_i reduces the profits of the other firm j, i.e., that $\pi_2(c_j, c_i) > 0$, our interest lies mainly in how this affects $\pi(c_j, c_i)$ *relatively* to $\pi(\widehat{c}_j, c_i)$, which determines the impact on firm j's participation constraint. For this purpose, we now define in a slight abuse of the previous notation the quantities $\widehat{q}_i = q(c_i, \widehat{c}_j)$ and $\widehat{q}_j = q(\widehat{c}_j, c_i)$ for the case where firm j chooses its alternative supply option. In the proof of Lemma 1 we show that a change in k_i affects relatively more the value of firm j's alternative supply option if

$$\widehat{q}_j P'(\widehat{q}_i + \widehat{q}_j)\frac{d\widehat{q}_i}{dc_i} > q_j P'(q_i + q_j)\frac{dq_i}{dc_i}. \tag{13}$$

Putting it differently, if (13) holds, then a reduction of c_i *relaxes* the participation constraint of its competitor j. As in the case of condition (12), we have from $\widehat{q}_j > q_j$ again a first "direct" effect that supports condition (13). In words, under the alternative supply option, when firm j produces a larger volume, this firm is hurt more by a marginal reduction in price, which is in turn caused by an increase in output from the more efficient firm i. As in the discussion of condition (12), however, we also have to sign an additional term in (13), namely the difference between the expressions $P'(\widehat{q}_i + \widehat{q}_j)\frac{d\widehat{q}_i}{dc_i}$ and $P'(q_i + q_j)\frac{dq_i}{dc_i}$. Our observations here are similar. That is, in the linear case we can ignore these additional expressions as they are the same, while a strictly concave demand works in the "right" direction because $\widehat{q}_i + \widehat{q}_j$ exceeds $q_i + q_j$. In what follows, we thus assume that (13) holds, which can be restated more compactly as follows.

Assumption 3. $\pi_{12}(c_i, c_j) < 0$.

Before we proceed with our analysis, we note that Assumptions 2 and 3 are commonly used in the literature that studies strategic cost reduction - or, more generally, R&D - choices under competition. An early example is Katz (1986), while more recent references are listed in Amir, Evstigneev, and Wooders (2003).[33] We next formalize the preceding arguments in the following

[33] In this literature, these assumptions are typically invoked to sign the slope of the reaction functions in the R&D game.

Lemma.[34]

Lemma 1. *Given Assumptions 2 and 3, the following auxiliary results hold. First, holding w_j fixed, the participation constraint for firm i is relaxed if k_i increases, where $i \neq j$. Second, holding again w_j fixed, the participation constraint for firm j is relaxed if either w_i or k_i decrease, where $i \neq j$.*

Proof. See Appendix.

Assumptions 2 and 3 are not yet sufficient to ensure that the two binding participation constraints pin down a unique pair of prices (w_1, w_2). If the two equations (11) have multiple solutions, then in equilibrium the supplier offers the pair of prices that maximizes its own profits. From Lemma 1, we know that in any pair of solutions one input price must be lower and the other input price must be higher. In what follows, we want to exclude the possibility of having multiple solutions, which we can safely do for all sufficiently low values of F.

Proposition 4. *If firms compete and if there is price discrimination, we have the following results. Under Assumptions 2-3 and if F is sufficiently small, there is a unique pair of optimal offers (w_1, w_2), which prescribes a strictly lower input price for the more efficient firm.*

Proof. See Appendix.

In what follows, we assume that the conditions in Proposition 4 are always satisfied. The following result is now an immediate consequence of Lemma 1.

Corollary 5. *If one firm becomes more efficient as k_i decreases, then the supplier lowers the input price w_i for this firm and increases the input price w_j for the other firm $j \neq i$.*

Proof. See Appendix.

Corollary 5 is interesting in itself. It asserts that, in intermediary goods markets, if firm i becomes more efficient, this may reduce its competitor's profits by the following different channels: firstly, firm i's total marginal costs decrease as both k_i and w_i are lower; secondly, firm j's total marginal costs are higher as w_j increases. Putting it differently, the supplier's choice of input prices thus tends to amplify the differences in downstream firms' own marginal costs in

[34] It should be noted that the input prices w_i and w_j are *equilibrium* values and not possible deviating offers, which are not observed by the other firm.

two ways, namely via a reduction in the more efficient firm's input price and via an increase in the less efficient firm's input price.

Corollary 5 formalizes the frequently encountered conjecture that the exercise of buyer power on the procurement market may lead to higher input prices for other, possibly less powerful, buyers. This could then exacerbate the latter firms' competitive disadvantage in the downstream market. In the European Union, this has been recognized as a potential harm in antitrust decisions (see, for example, Faull and Nikpay (1999), para. 6.325). Typically, this "waterbed" effect has been rationalized by suppliers' attempt to recoup the "margin" they lose with the more powerful buyer.[35] While it is not clear to us why the respective price increase would not have been profitable even before the supplier had to give a discount to some buyers, Corollary 5 provides an alternative channel by which such a "waterbed" effect could operate.

Uniform Pricing

Clearly, the imposition of uniform pricing may only have an effect if one firm is more efficient. In the case with separate markets, the imposition of uniform pricing then simply pushed the less efficient firm's input price down to that of the more efficient firm. In contrast, with competition also the input price of the more efficient firm is affected.

To see this, suppose that $k_i < k_j$ and that, following the imposition of uniform pricing, the supplier would reduce w_j down to firm i's input price under price discrimination. We know from Lemma 1, however, that a reduction in w_j relaxes the participation constraint of firm i, which now allows the supplier to increase the uniform price $w_i = w_j = w$. We thus have the following result.

Proposition 5. *In case firms compete and have different marginal costs $k_i \neq k_j$, the imposition of uniform pricing leads to a unique input price that lies strictly above that of the more efficient firm under price discrimination and strictly below that of the less efficient firm under price discrimination.*

Proof. See Appendix.

[35] The UK's Competiton Commission stated in an investigation of a prospective supermarket merger that the resulting increase in buyer power would "have adverse effects on other, smaller, grocery retailers through the "waterbed" effect - that is, suppliers having to charge more to smaller customers if large retailers force through price reductions which would otherwise leave suppliers insufficiently profitable" (Competition Commission, "Safeway plc and Asda Group Limited (owned by Wal-Mart Stores Inc); Wm Morrison Supermarkets plc; J Sainsbury plc; and Tesco plc," Cm5950, HMSO, 2003; para. 2.218.).

In the case with linear demand, we can show, at least for all sufficiently low F, that the uniform price is always low enough to ensure that total output and thus consumer surplus are higher under uniform pricing. While with separate markets this observation was also sufficient to show that total welfare is higher, this is no longer the case under competition. As uniform pricing lowers the input price of the less efficient firm and increases that of the more efficient firm, a larger fraction of total output is now produced by the less efficient firm. Interestingly, this is again the opposite from what would happen if the supplier was unconstrained, in which case the imposition of uniform pricing would increase the more efficient firm's market share.

Finally, we have the following additional comparative result, which will prove useful for the long-run analysis.

Corollary 6. *A marginal reduction of the more efficient firm's own marginal costs strictly reduces the uniform input price w, while a marginal reduction of the less efficient firm's own marginal costs increases w.*

Proof. See Appendix.

Both results in Corollary 6 follow intuitively from Lemma 1. Recall, in particular, that if firm j is the less efficient firm, a (marginal) reduction of k_j relaxes firm i's participation constraint. As the participation constraint for the more efficient firm i determines the uniform price w, this implies a higher input price for both firms.

4.2 The Long Run

We start by considering the incentives for firms to reduce marginal costs under discriminatory pricing. Differentiating firm i's profits, we find that a marginal reduction in k_i increases the firm's profits by

$$\begin{aligned} -\frac{d\pi(c_i, c_j)}{dk_i} &= -\left[\frac{\partial c_i}{\partial k_i}\pi_1(c_i, c_j) + \frac{\partial c_j}{\partial k_i}\pi_2(c_i, c_j)\right] \\ &= q_i\left[\left(1 + \frac{\partial w_i}{\partial k_i}\right)\left(1 - \frac{\partial q_j}{\partial c_i}P'\right) - \frac{\partial w_j}{\partial k_i}\frac{\partial q_j}{\partial c_j}P'\right], \end{aligned} \tag{14}$$

where we abbreviate $q_i = q(c_i, c_j)$ and $q_j = q(c_j, c_i)$. Recall from (4) that with separate markets the marginal benefits from a lower k_i were just $q_i\left(1 + \frac{dw_i}{dk_i}\right)$. Under competition, incentives to reduce own marginal costs arise now from two additional channels. First, firm i's lower marginal cost induces its competitor to lower the output q_j, which in turn increases the prevailing price.

Second, we know from Corollary 5 that a reduction of k_i leads to an increase of its rival's input price w_j, which in turn leads to a further reduction of the respective quantity q_j and thus to a higher price. The latter effect is accounted for by the final terms in (14), where we use that $\partial w_j/\partial k_i < 0$ and that $\partial q_j/\partial c_j < 0$.[36]

Under uniform pricing, where the same price w applies to both firms, the marginal benefits for firm i to reduce own marginal costs are generally given by

$$-\frac{d\pi(c_i, c_j)}{dk_i} = q_i \left[\left(1 + \frac{\partial w}{\partial k_i}\right) \left(1 - \frac{\partial q_j}{\partial c_i} P'\right) - \frac{\partial w}{\partial k_i} \frac{\partial q_j}{\partial c_j} P' \right]. \tag{15}$$

We take first the case where $k_i \leq k_j$. From Corollary 6, we then have that $\partial w/\partial k_i > 0$. The key difference compared to the incentives under price discrimination, as given by (14), is that a lower w is also shared by the firm's competitor, which reduces incentives. Formally, this is captured by the last term in (15), where we now have $\partial w/\partial k_i > 0$ instead of $\partial w_j/\partial k_i < 0$ as in (14). We can make these arguments more formally as follows.

Lemma 2. *If there is competition, then for $k_i \leq k_j$ the incentives for firm i to reduce own marginal costs k_i are strictly lower under uniform pricing than under discriminatory pricing.*

Proof. See Appendix.

It should be noted that this result is stronger than what we obtained under separate markets, where the incentives to marginally reduce k_i where unchanged if firm i was (currently) more efficient. Despite this observation, we can now, however, no longer assert that *in equilibrium* the long-run marginal costs of the ex-post more efficient firm will be lower under uniform pricing. This follows from the following observations. Incentives for the ex-post less efficient firm j are again lower under uniform pricing and as firms now compete in the same market, higher marginal costs of the competitor j increase the market share of firm i. As the quantity sold by firm i increases, this boosts firm i's incentives to reduce own marginal costs k_i.

In general, what holds still without ambiguity is that under uniform pricing firms have always different own marginal costs in the long run (that is, in any pure-strategy equilibrium of the investment game). Also, the ex-post less efficient firm invests strictly less than under price

[36] Interestingly, the strategy that we have just described, i.e., to invest in a reduction of the firm's own marginal costs, is orthogonal to the "raising rivals' costs" strategies considered in Salop and Scheffman (1983) and the follow-up literature to this. There, a dominant firm benefits from an increase in its own costs as this raises rivals' costs by sufficiently more.

discrimination. In fact, under competition the incentives for the ex-post less efficient firm are now additionally muted as, following from Corollary 6, a marginal reduction in its own marginal costs will increase the prevailing input price. If an increase in the shared input price w reduces firms' profits, which is a standard assumption and which we assume in what follows, this makes cost reductions even less attractive under uniform pricing.[37]

Proposition 6. *Under competition, there is no long-run pure-strategy equilibrium under uniform pricing where firms have the same marginal costs k_i and thus the same market share. This holds, in particular, also for the case where firms are initially symmetric, in which case it always holds that the ex-post less efficient firm will invest strictly less than under price discrimination.*

Proof. See Appendix.

Note finally that in analogy to Corollary 3 in the case of separate markets, the imposition of uniform pricing leads to ex-post asymmetries in long-run marginal costs, even if firms are initially symmetric. As firms now compete in the same downstream market, this translates into the creation of asymmetries in market shares.

4.3 The Linear Case

If demand is linear with $P(x) = 1 - x$, equilibrium quantities are $q(c_i, c_j) = (1 - 2c_i + c_j)/3$ and equilibrium profits are $\pi(c_i, c_j) = (1 - 2c_i + c_j)^2/9$. Taking first the benchmark case where the supplier is unconstrained, the two input prices w_1 and w_2 are chosen to maximize $w_1 q(c_1, c_2) + w_2 q(c_2, c_1)$. It is easily checked that this yields exactly the same input prices as in the case of separate markets, $w_i = (1 - k_i)/2$. Hence, with an unconstrained supplier a firm's input price does only depend on its own marginal costs but not on that of its competitor.

Note next that with linear demand the profit function $\pi(c_i, c_j)$ satisfies both Assumptions 2 and 3 as $\pi_{11} = 8/9 > 0$ and $\pi_{12} = -4/9 < 0$. With price discrimination, the binding participation constraint (10) for firm i can be transformed to

$$(w_i - \widehat{w})\left[1 - 2k_i + 2k_j - \widehat{w} - (w_i - w_j)\right] = \frac{9}{4}F. \tag{16}$$

[37] As noted, for instance, in Vives (1999, p. 105), Assumption 1 does not yet rule out the "less intuitive, or perhaps even perverse result" that a symmetric increase in input costs is beneficial to firms. Vives derives a set of sufficient conditions on the demand function that rule out this possibility.

While it is possible to solve explicitly the system of two quadratic expressions (16), it is more instructive to take the following alternative route.[38] We define the average price $W := (w_i + w_j)/2$ such that, together with $(w_i - w_j) = 2(w_i - W)$, equation (16) becomes

$$(w_i - \widehat{w})\,(1 - 2k_i + 2k_j - \widehat{w} - 2w_i + 2W) = \frac{9}{4}F,$$

implying that, for a given average price W, we have that

$$w_i = \frac{1 - 2k_i + k_j + \widehat{w} + 2W + A_i}{4}, \tag{17}$$

where $A_i = \sqrt{(1 - 2k_i + k_j - 3\widehat{w} + 2W)^2 - 18F}$. Adding up (17) for the two firms, the average price W solves

$$2 - k_i - k_j + 4\widehat{w} - 2W - A_i - A_j = 0. \tag{18}$$

Total output depends only on the average marginal costs under homogeneous Cournot competition , to study the short- and the long-run impact of imposing uniform pricing it is sufficient to compare the respective uniform price with the resulting average price W. Moreover, a quick inspection of w_i in (17) reveals immediately that $k_i < k_j$ must imply $w_i < w_j$.[39]

Turning next to uniform pricing, if firm i is more efficient then we obtain from its respective participation constraint (16) that

$$w = \widehat{w} + \frac{9F}{4(1 - 2k_i + k_j - \widehat{w})}. \tag{19}$$

As we show in Appendix B, at least for sufficiently small F, the uniform price w lies strictly below the average price W under price discrimination, implying that total output and thus consumer welfare is higher.[40] Recall that we obtained the same finding, though more generally, in the case of separate markets. In contrast to the case with separate markets, where it then followed trivially that also total welfare increased, with competition the shift of output to the less efficient firm provides now a countervailing force. For all examples that we studied we found, however, that also total welfare increased under uniform pricing. In Figure 1 we report an example where $\widehat{w} = 0.3$, $F = 0.01$, and $k_2 = 0.2$. The figure shows how a change in $k_1 \leq k_2$ affects the uniform price w (the solid line) and the average price under discrimination W (the

[38] This borrows from Valletti (2003b).

[39] More formally, this follows as holding all else constant, w_i in (17) is clearly strictly increasing in $k_j - 2k_i$.

[40] For fixed F, we can also show that W unambiguously exceeds w for all sufficiently low differences $k_j - k_i > 0$.

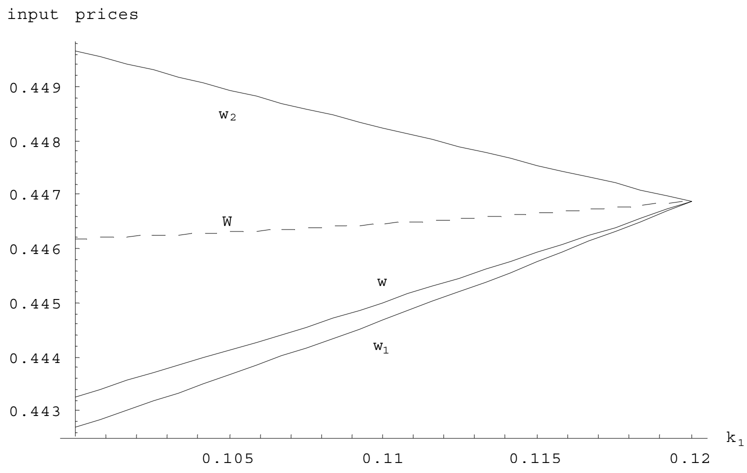

Figure 1: Input prices in the short run

dotted line). While the two prices clearly coincide when firms are symmetric, $k_1 = k_2$, the uniform price lies otherwise always strictly below the average price under price discrimination. For this example, it is also straightforward to verify that total welfare is strictly higher under uniform pricing for all $k_1 < k_2$, and that the difference is strictly increasing in the difference of marginal costs $k_2 - k_1$. Finally, it is interesting to note that the average price W in Figure 1 is almost flat. As k_1 decreases, the reduction of w_1 is compensated by an increase in the competitor's input price w_2.

For the long-run analysis, we again treat only the case of ex-ante identical firms. For all sufficiently convex investment cost functions, there is a unique symmetric equilibrium under discriminatory pricing.[41] From Proposition 6 we also know generally that under uniform pricing the equilibrium must be asymmetric and that the *ex-post* less efficient firm has a higher marginal cost than under discriminatory pricing. While we also know from Lemma 2 that for a *given* pair (k_i, k_j) the marginal benefits to reduce k_i are always lower under uniform pricing, generally this does not ensure that the equilibrium investment level will be lower for the *ex-post* more efficient firm than under discriminatory pricing. With linear demand, however, we can use explicit derivatives around $F = 0$ to show at least for all sufficiently low F that under uniform pricing the long-run marginal costs of *both* firms are strictly higher than under discriminatory pricing. While this already implies that uniform pricing reduces consumer surplus in the long

[41] In what follows, we focus on this symmetric equilibrium under price discrimination. In all numerical examples that we studied we have, however, not found an asymmetric equilibrium.

run, we show also for low F that total welfare is strictly lower, confirming the results that we obtained in the case with separate markets.

For more general F and using quadratic costs $e(\Delta_{ki}) = t(\Delta_k)^2/2$, we provide next an illustrative example in Figure 2. Setting $t = 2, \widehat{w} = 0.3$, and $\overline{k}_i = \overline{k} = 0.4$, The left panel plots as a function of F the prevailing equilibrium investment levels under price discrimination (the dotted line) and under uniform pricing (the solid lines). In this example, as well as in all other examples that we studied, uniform pricing raises long-run marginal costs for both firms. The right panel plots the resulting welfare under price discrimination (the dotted line) and uniform pricing (the solid line). Welfare decreases with F in both cases, i.e., the less constrained by the outside option the supplier is. More importantly, what the figure also shows is that welfare is strictly higher in the long run under price discrimination than uniform pricing for all considered values $F > 0$.

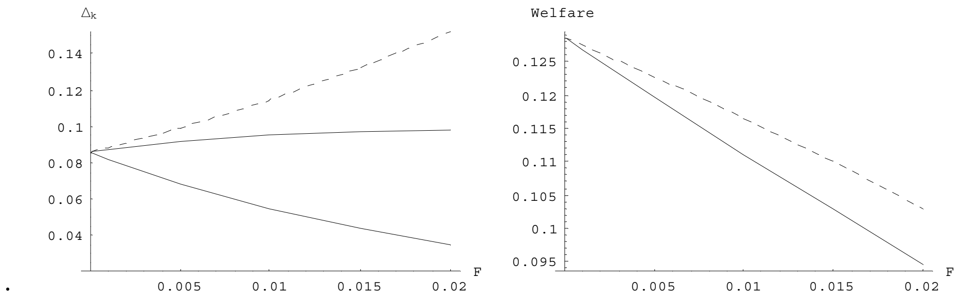

Figure 2: Investments (left panel) and welfare (right panel) in the long run

4.4 Discussion

Firms' Alternative Supply Option

With separate markets, we showed that results are not qualitatively changed if firms can invest in a reduction of their marginal costs $\widehat{w}_i$ under their alternative supply options. We now argue that this applies as well to the case where firms compete in the same market. To see why, observe first that the marginal costs $\widehat{w}_i$ do not affect the participation constraint of the competing firm j under discriminatory pricing. Moreover, as firm i will choose a higher quantity $q(\widehat{c}_i, c_j)$ the lower k_i, a more efficient firm will again invest more in a reduction of $\widehat{w}_i$, which increases the discount that a more efficient firm obtains under discriminatory pricing.

One implication from this observation is that, if $\widehat{w}_i$ becomes endogenous, then this tends to further widen the input price differential between a more and a less efficient firm. We should thus see a more pronounced price discrimination, as expressed by the difference between w_i and w_j, and potentially also a larger difference in market shares in industries where downstream firms that currently purchase from the same supplier would alternatively face different supply options. For instance, this could be the case if firms' only alternative is to vertically integrate backwards or to technically modify production so as to better accommodate an alternative input. Likewise, differences in $\widehat{w}_i$ may be more pronounced if firms must first locate feasible alternatives in a costly search process.

Exclusion

Under discriminatory pricing, it is still always optimal for the supplier to offer both firms an acceptable contract. This holds not only in our model where the supplier is constrained, but it is also true in case the supplier is an unconstrained monopolist. In contrast, as in the case of separate markets, the supplier may not want to offer a uniform price that is still acceptable to both firms.

It is straightforward to show that if either F or the difference in marginal costs $k_i - k_j$ is small, then the supplier's single offer is still acceptable to both firms. If, however, this is not the case, then in our model it is the more efficient firm that finds the supplier's offer not acceptable. In contrast, if the uniform price set by an unconstrained supplier is not acceptable to both firms, then it is the less efficient firm that will be excluded. If the supplier is an unconstrained monopolist, then the less efficient firm stays inactive and the downstream market is turned into a monopoly. In our model, instead, the firm that does not accept the buyer's offer turns to its alternative source of supply. As the switching firm is more efficient and as its resulting marginal input costs $\widehat{w}$ are always strictly lower than the supplier's offer w, we would predict that after the imposition of uniform pricing the supplier loses the more competitive buyer, i.e., the buyer that ultimately ends up with a larger share in the downstream market.

The Contractual Game

We finally comment on some of the assumptions in our contractual game. We already discussed previously the issue of whether an individual offer was observable to the other firm or not. Next, recall that after one firm i rejects the supplier's offer, we assumed that the other offer w_j is not renegotiated. This specification is not restrictive. While we know that the participation constraint for j is no longer binding in case firm i rejects, which follows from Lemma 1, firm j would clearly not be willing to settle for a higher input price.

We turn finally to the use of linear prices. With separate markets, two-part (discriminatory) tariffs allowed the supplier to extract a higher profit as it cut out double marginalization. From the analysis of the case where the supplier is unconstrained it is, however, well known that the introduction of two-part tariffs may reduce the supplier's profits if firms compete in the same market. Intuitively, two-part tariffs can encourage the supplier to behave opportunistically if individual reductions in the wholesale price ω_i, which can be accompanied by a larger fixed part τ_i, are not observed by the competing firm j. In this case, the imposition of a non-discrimination requirement can give the supplier more commitment power, resulting in higher prices and profits (see O'Brien and Shaffer (1994), McAfee and Schwartz (1994), or DeGraba (1996)).

It is straightforward to apply some of the seminal insights from this literature to the case with a constrained supplier. Passive beliefs, under which regardless of its own offer firm i always holds the same expectations about the offer to firm j, lead to marginal-cost pricing under discrimination: $w_i = w_j = 0$. Hence, a more efficient firm, which ultimately has still a larger share of the downstream market and thus commands over a larger purchasing volume, will again only obtain a discount via the fixed part τ_i. Imposing uniform pricing mitigates this opportunism problem, though, as in the case of an unconstrained supplier, results depend on how this requirement is specified in detail. Here, the literature has discussed various alternatives, ranging from the obligation to give all firms access to the same "menu" of two-part tariffs to imposing outright uniformity on the wholesale price or on each individual component of all offered tariffs. We must leave to future study a detailed comparison of these different forms of imposing uniform pricing on the one hand and discriminatory pricing on the other hand.

5 Conclusion

Price uniformity in input markets is often advocated by small firms that fear they might not be able to obtain the same purchasing conditions as bigger rivals. This argument is often backed by policy makers, too. The model presented in this paper, where differences in size arise endogenously from differences in costs and where the supplier is constrained by the threat of demand-side substitution, confirms this argument. In fact, if firms compete in the same market then we show that as competitors become more efficient and larger, this hurts smaller and less efficient firms both as their competitors' input price decreases and as their own input price increases.

From a welfare perspective, at least in the short run, where we take the levels of efficiency as given, price uniformity also tends to perform better than discriminatory pricing. Both when firms serve separate markets and when they compete, this holds as uniform pricing leads to an expansion of total output as it essentially allows also less efficient buyers to exploit the better outside option of a more efficient buyer. Importantly, this mechanism by which welfare is improved is markedly different from the mechanisms that are at work in case uniform pricing is imposed on an unconstrained monopolist, as it is often the case in the literature. In particular, with an unconstrained supplier and downstream competition, the main welfare benefits from imposing uniform pricing are to shift market share away from less efficient firms, which is in fact the opposite to what occurs in our model.

In the long run, however, the imposition of uniform pricing may reduce both consumer surplus and welfare as it stifles downstream firms' incentives to improve efficiency. This negative aspect of price uniformity is, in particular, different to the drawback of price uniformity that was emphasized in Katz (1987). There, it is shown that price discrimination can be beneficial when it allows to avoid inefficient backward integration by a larger buyer. In essence, this point is similar to the more general observation that price discrimination can be welfare improving when it allows to "open" new markets. In contrast, in our model we made sure that the outside option is never used in equilibrium. In other words, we concentrated only on the case where all markets would be served both under price discrimination and uniform pricing.

We also found that, once dynamics are taken into account, uniform prices can amplify differences among firms. In general, it is better for a small firm to wait for the bigger rival to invest, and then sit on its shoulders (i.e., exploit its lower input price). This finding points to possible unintended consequences of imposing uniform pricing. While they might be introduced by policy makers to create a "level playing field", they may end up creating differences endogenously.

6 Appendix A: Omitted Proofs

Proof of Proposition 1. Observe first that $q(c_i)$ is continuously differentiable and strictly decreasing in c_i, where strictly positive, while also profits are continuously differentiable with $\pi'(c_i) = -q(c_i)$, where $q > 0$. Recall next that $V_i^A > \pi(k_i + w_i^{UC})$ holds by assumption. Given continuity and strict monotonicity of $\pi(c_i)$ and given that the supplier's profits are strictly quasiconcave in w_i, there is a unique equilibrium offer w_i at which the constraint (3) for firm i

just binds.[42] By strict monotonicity of $\pi(c_i)$ it follows immediately that $w_i > \widehat{w}$. Next, implicit differentiation of the binding constraint (3) yields

$$\frac{dw_i}{dk_i} = \frac{\pi'(\widehat{c}_i) - \pi'(c_i)}{\pi'(c_i)} = \frac{q(\widehat{c}_i)}{q(c_i)} - 1. \tag{20}$$

Given that $w_i > \widehat{w}$ and as q is strictly decreasing, we have that $q(\widehat{c}_i) > q(c_i)$, which finally implies from (20) that $dw_i/dk_i > 0$. **Q.E.D.**

Proof of Corollary 1. Implicit differentiation of the binding participation constraint (3) yields $dw_i/dF = 1/q(c_i) > 0$, where we used that $\pi'(c_i) = -q(c_i)$. Given that $q(c_i) < q(c_j)$ for all $k_i > k_j$ and $F > 0$, this also implies that the difference $w_i - w_j > 0$ is strictly increasing in F. Finally, we have that $dw_i/d\widehat{w} = q(\widehat{c}_i)/q(c_i) > 0$. **Q.E.D.**

Proof of Proposition 3. Suppose first that $k_i \leq k_j$. Clearly, as $w = w_i$ holds under uniform pricing for all $k_i \leq k_j$, the marginal benefits to further reduce k_i are the same for i under uniform and under discriminatory pricing. Next, suppose that $k_i > k_j$. Under uniform pricing, a marginal reduction in k_i, which does not reverse the order between firms, generates the marginal benefits $-\pi_1(w + k_i)$. For the case of discriminatory pricing, instead of using (4) it is now more convenient to note that the profits of firm i are given by $\pi(\widehat{w} + k_i) - F$ such that the marginal benefits from a reduction of k_i are also equal to $-\pi_1(\widehat{w} + k_i)$. The assertion follows then from $\pi_{11} > 0$ and $\widehat{w} < w$.

For a characterization of the long-run equilibrium, we consider first the regime with discriminatory pricing. Given the assumptions imposed on $e'(\Delta_k)$ at the boundaries where $\Delta_k = 0$ and $\Delta_k = \overline{k}_i$ and using that $-\pi_1(\widehat{w} + k_i) = q(\widehat{c}_i)$ after substitution of $-\pi'(c_i) = q(c_i)$ and dw/dk_i from (20), a solution satisfies the first-order condition

$$e'(\Delta_k) = q(\widehat{w} + \overline{k}_i - \Delta_k). \tag{21}$$

We assume in what follows that the programme is strictly quasiconcave, yielding a unique solution characterized by (21).[43]

[42] Note that to ensure that (3) binds under the supplier's (constrained) optimal offer, strict quasiconcavity of the supplier's profits is a stronger requirement than what is needed. A sufficient requirement is that $d\left[q(c_i)w_i\right]/dw_i$ is strictly positive for all w_i between zero and the unique value w_i that solves (3) with equality. Inspection of $d\left[q(c_i)w_i\right]/dw_i$ reveals that this condition is, for instance, always satisfied if we choose F and $\widehat{w}$ sufficiently small.

[43] Firm i's program is strictly concave in Δ_k if $e''(\Delta_k) > -q'(\widehat{c}_i)$ holds over all relevant values Δ_k and thus $\widehat{c}_i$, i.e., in particular if this holds for all $\widehat{w} < \widehat{c}_i < \widehat{w} + \overline{k}_i$.

Under uniform pricing, we know that for $k_i \leq k_j$ the marginal benefit from reducing k_i is still $q(\widehat{c}_i)$, while for $k_i > k_j$ it equals $q(c_i)$. Given that $q(c_i) < q(\widehat{c}_i)$, under uniform pricing firm i's marginal benefits have a discontinuity at $k_i = k_j$. Incentives to reduce marginal benefits are strictly higher if already $k_i \leq k_j$ than if still $k_i > k_j$. An immediate consequence of this observation is that there can not be a pure-strategy equilibrium where firms are *ex-post* equally efficient. Moreover, in a pure-strategy equilibrium one firm i invests until (21) is satisfied, while the other ex-post less efficient firm's first-order condition becomes

$$e'(\Delta_k) = q(w + \overline{k}_j - \Delta_k). \tag{22}$$

Hence, in general there are two pure-strategy equilibrium candidates under uniform pricing, which are characterized by the first-order conditions (21) and (22) for the respective firms. The comparison between investment levels under uniform and discriminatory pricing follows then immediately from our previous observations and as we always have that $w > \widehat{w}$.

Finally, we deal with existence of a pure-strategy equilibrium under uniform pricing. Given our previous arguments, to ensure existence of a given candidate equilibrium it only remains to ensure that the ex-post more efficient i has no incentives to deviate to some value Δ_k such that $k_i > k_j$, i.e., where the order is reversed. If firm i chooses to deviate in this way, its optimal choice Δ_k solves the first-order condition (22) with the difference that this time the applied uniform w is equal to the discriminatory price of firm j at k_j. A sufficient condition for a pure-strategy equilibrium to exist is thus that for the ex-post more efficient firm the respective profit at this point is strictly less than the profit under the original strategy.[44] **Q.E.D.**

Proof of Lemma 1. Observe first that from (11) we have that $w_i > \widehat{w}$. From differentiating (11), we have next that an increase in own marginal costs k_i always relaxes the participation constraint of firm i if $\pi_1(c_i, c_j) > \pi_1(\widehat{c}_i, c_j)$. Given $w_i > \widehat{w}$ and thus $c_i > \widehat{c}$, this is always the case if $\pi_{11}(c_i, c_j) > 0$, as stipulated in Assumption 2. Finally, differentiating (11) with respect to the competitor's marginal costs c_j, we have that a higher c_j tightens the participation constraint of firm i if $\pi_2(c_i, c_j) < \pi_2(\widehat{c}_i, c_j)$. This holds from $c_i > \widehat{c}$ and Assumption 3. **Q.E.D.**

Proof of Proposition 4 and Corollary 5. It is convenient to prove at the same time Proposition 4 and Corollary 5. We argue first more formally that for sufficiently low F and $\widehat{w}$, which is what we assumed, an equilibrium pair of offers must solve the respective conditions

[44] It is straightforward, though notation-wise cumbersome, to express this condition more formally.

(11). As in the case of separate markets, denote by $w_i^{UC} > 0$ the unconstrained optimum, which is a unique pair of offers by strict quasiconcavity of the supplier's objective function. For all $\widehat{w} < \min_{i=1,2} w_i^{UC}$ and sufficiently low F we then have from the two participation constraints that all equilibrium offers $w_i = w_i^E$ must be strictly lower than the respective unconstrained optimum w_i^{UC}. From strict quasiconcavity of the supplier's program and as a deviation $w_i > w_i^E$ has no effect on the participation constraint of the other firm j, we have that both participation constraints must indeed bind in equilibrium. We show next that the system of equations given by (11) has a unique solution if F is sufficiently low.

We turn first to existence. The binding participation constraint for i gives us w_i as a function of $c_j = w_j + k_j$. If F is sufficiently small, then this has a solution for any c_j. (Choosing $F < \pi(\widehat{w} + k_i, \widehat{w} + k_j)$ is sufficient.) By $\pi_1 < 0$ the solution is also unique and also continuous in c_j, while for any choice of c_j the solution falls into some sufficiently large compact range $w_i \in [\widehat{w}, \overline{c}]$, where $\overline{c}$ solves $\pi(\overline{c}, 0) = 0$. Existence of a solution (w_1, w_2) is then guaranteed by Brouwer's fixed point theorem. To establish uniqueness, we show that the Jacobian matrix of the system (11) is positive semi-definite, which holds in turn if all principal minors are positive. For this, note that $-\pi_1(c_1, c_2) > 0$ and $-\pi_1(c_2, c_1) > 0$, while the determinant is given by

$$D = \pi_1(c_1, c_2)\pi_1(c_2, c_1) - [\pi_2(\widehat{c}_1, c_2) - \pi_2(c_1, c_2)]\,[\pi_2(\widehat{c}_2, c_1) - \pi_2(c_2, c_1)]\,. \tag{23}$$

Given that $\pi_1(c_1, c_2)\pi_1(c_2, c_1)$ is strictly positive and bounded away from zero in the relevant range, and given that $c_i - \widehat{c}_i > 0$ becomes arbitrarily close to zero as we lower F, we thus have together with continuity of profit functions that $D > 0$ holds surely for all sufficiently small $F > 0$.[45]

To complete the proof of Proposition 4, we still have to show that the more efficient firm obtains a strictly lower input price. Observe now that for Corollary 5 we need to show that a reduction in k_i strictly reduces w_i and strictly increases w_j, where $j \neq i$. As we can obtain any $k_i \neq k_j$ by gradually adjusting the respective marginal costs when starting from a symmetric situation $k_1 = k_2$ and as for $k_1 = k_2$ we have $w_1 = w_2$ in the unique solution, the final assertion

[45] It is straightforward to make the argument that $c_i - \widehat{c}_i \to 0$ as $F \to 0$ more formal. For this, observe that for any ε we can choose $\overline{F} > 0$ such that for $F < \overline{F}$ any solution must satisfy $w_i - \widehat{w} < \varepsilon$. If this was not the case, then we could construct a sequence of equilibria, indexed by α and along which $F(\alpha) \to 0$, such that $w_i(\alpha) - \widehat{w} > \varepsilon$ for all α. As all $w_i(\alpha)$ belong to a compact set as we argued to establish existence of an equilibrium, we can then choose a subsequence along which $w_i(\alpha) \to \overline{w}_i > \widehat{w}$. But it is then immediate to see that for high α and thus low $F(\alpha)$ this can not satisfy the participation constraint of i.

in Proposition 4 is implied by Corollary 5. To finally prove Corollary 5, totally differentiating (11) and using continuous differentiability, we obtain

$$\frac{\partial w_i}{\partial k_i} = \frac{D_i}{D},$$
$$\frac{\partial w_j}{\partial k_i} = \frac{D_j}{D},$$

where we substitute $D > 0$ from (23) and

$$\begin{aligned} D_i &= [\pi_1(\widehat{c}_i, c_j) - \pi_1(c_i, c_j)]\, \pi_1(c_j, c_i) \\ &\quad + [\pi_2(\widehat{c}_i, c_j) - \pi_2(c_j, c_i)]\, [\pi_2(\widehat{c}_j, c_i) - \pi_2(c_j, c_i)]\,, \\ D_j &= [\pi_2(\widehat{c}_j, c_i) - \pi_2(c_j, c_i)]\, \pi_1(c_i, c_j) \\ &\quad + [\pi_1(\widehat{c}_i, c_j) - \pi_1(c_i, c_j)]\, [\pi_2(\widehat{c}_i, c_j) - \pi_2(c_i, c_j)]\,. \end{aligned}$$

Using Assumptions 2 and 3, we thus have that $D_i > 0$, implying that $\partial w_i/\partial k_i > 0$, and that $D_j < 0$, implying that $\partial w_j/\partial k_i < 0$. **Q.E.D.**

Proof of Proposition 5. With a uniform price, that is known to both firms, the participation constraint of firm i becomes

$$\pi(w + k_i, w + k_j) \geq V_i^A = \pi(\widehat{w} + k_i, w + k_j) - F. \tag{24}$$

From Lemma 1, if $k_i < k_j$ then this is satisfied for firm j with strict inequality whenever it holds for firm i. Observe next that a reduction in w relaxes the constraint (24) whenever we have after differentiation with respect to w that

$$-\pi_1(c_i, c_j) + [\pi_2(\widehat{c}_i, c_j) - \pi_2(c_i, c_j)] > 0. \tag{25}$$

This holds from $\pi_1 < 0$ and from $\widehat{c}_i < c_i$ together with $\pi_{12} < 0$ (Assumption 3). By the argument in the proof of Proposition 4, this implies that for sufficiently low F the unique uniform input price solves (24) with equality for the more efficient firm.

We finally show by contradiction that for $k_i < k_j$ it holds that $w_i < w < w_j$. Suppose first that $w \leq w_i < w_j$, where we use from Proposition 4 that $w_i < w_j$ holds under price discrimination. If $w_i < w < w_j$ were to hold, then given Lemma 1 and as firm i's participation constraint is binding under uniform pricing, its participation constraint could not be binding under discriminatory pricing. Next, suppose that $w_i < w_j \leq w$. This is, however, not possible as by Lemma 1 and construction of (w_i, w_j) it is not possible that there are any two prices $w' > w_i$

and $w'' \geq w_j$, including thus prices $w' = w'' = w$, such that the participation constraints of both firms are still satisfied if we replace w_i by w' and w_j by w''. **Q.E.D.**

Proof of Corollary 6. Implicit differentiation of the binding participation constraint (24) yields

$$\frac{\partial w}{\partial k_i} = -\frac{\pi_1(\widehat{c}_i, c_j) - \pi_1(c_i, c_j)}{-\pi_1(c_i, c_j) + [\pi_2(\widehat{c}_i, c_j) - \pi_2(c_i, c_j)]} > 0, \tag{26}$$

where we use that the denominator is strictly positive by (25) in the proof of Proposition 5 and that the numerator is strictly negative by Assumption 2. Further, implicit differentiation yields

$$\frac{\partial w}{\partial k_j} = -\frac{\pi_2(\widehat{c}_i, c_j) - \pi_2(c_i, c_j)}{-\pi_1(c_i, c_j) + [\pi_2(\widehat{c}_i, c_j) - \pi_2(c_i, c_j)]} < 0, \tag{27}$$

where we now use Assumption 3. **Q.E.D.**

Proof of Lemma 2. It is convenient to calculate the marginal benefits from a reduction in k_i by considering a marginal change in the value $\pi(\widehat{c}_i, c_j) - F$ under both uniform and discriminatory pricing. (Recall that in either case the participation constraint for firm i binds if $k_i \leq k_j$.) Marginal benefits are then equal to

$$-\pi_1(\widehat{w} + k_i, w_j + k_j) - \pi_2(\widehat{w} + k_i, w_j + k_j)\frac{\partial w_j}{\partial k_i} \tag{28}$$

under discriminatory pricing and equal to

$$-\pi_1(\widehat{w} + k_i, w + k_j) - \pi_2(\widehat{w} + k_i, w + k_j)\frac{\partial w}{\partial k_i} \tag{29}$$

under uniform pricing. We show in what follows that (28) exceeds (29). Given $\pi_2 > 0$ as well as $\partial w_j/\partial k_i < 0$ and $\partial w/\partial k_i > 0$ from Corollary 6, this holds surely in case

$$\pi_1(\widehat{w} + k_i, w_j + k_j) \leq \pi_1(\widehat{w} + k_i, w + k_j). \tag{30}$$

Given $w_i < w < w_j$ from Proposition 5, (30) then follows as $\pi_{11} > 0$ holds by Assumption 2. **Q.E.D.**

Proof of Proposition 6. We show first that, as in the case with separate markets, under uniform pricing there is no pure-strategy equilibrium where firms are ex-post symmetric: $k_i = k_j$. For this we compare again the marginal benefits of firm i from a marginal reduction of k_i if either $k_i \leq k_j$ or $k_i > k_j$. In both cases, marginal benefits are given by (15), but for $k_i \leq k_j$ we have

$\partial w / \partial k_i > 0$ and for $k_i > k_j$ we have $\partial w / \partial k_i < 0$, where we can use (26) and (27). This implies that we have the same discontinuity as in the case with separate markets.

Take now the second claim. We denote by k_i^{PD} and k_i^U the respective equilibrium marginal costs under the two pricing regimes. We also take as given a symmetric equilibrium under price discrimination such that $k_1^{PD} = k_2^{PD} = k^{PD}$. Suppose now that i is the ex-post less efficient firm. We argue to a contradiction and suppose that $k_i^U < k^{PD}$, which then also implies that $k_j^U < k^{PD}$. It is the latter implication that will lead to a contradiction. To obtain this result, it is sufficient to show that, holding for firm i the choices k^{PD} and k_i^U fixed, the marginal benefits to reduce k_j under price discrimination are always larger than under uniform pricing at $k_j = k_i^U$.[46] To show this, it is convenient to calculate as in the proof of Lemma 2 the marginal benefits from reducing k_j by

$$-\pi_1(\widehat{w} + k_j, w_i + k^{PD}) - \pi_2(\widehat{w} + k_j, w_i + k^{PD}) \frac{\partial w_i}{\partial k_j} \tag{31}$$

under price discrimination and by

$$-\pi_1(\widehat{w} + k_j, w + k_i^U) - \pi_2(\widehat{w} + k_j, w + k_i^U) \frac{\partial w}{\partial k_j} \tag{32}$$

under uniform pricing. We want to show that (31) exceeds (32) for all k_i. Given that $\partial w / \partial k_j > 0$ and $\partial w_i / \partial k_j < 0$, it is sufficient to show that $\pi_1(\widehat{w} + k_j, w + k_i^U) > \pi_1(\widehat{w} + k_j, w_i + k^{PD})$, which from Assumption 3 holds if $w + k_i^U < w_i + k_i$. As it holds by assumption that $k_i^U < k^{PD}$, we thus only need to show that $w < w_i$. To finally see that this holds, recall that by assumption we have that $k_j \le k_i^U < k^{PD}$. Note also that the discriminatory price for i would be smaller if instead of k^{PD} firm i would choose $k_i = k_i^U$, in which case we would now compare the uniform price with the less efficient firm's discriminatory price, holding each firm's marginal costs fixed. That the less efficient firm i pays then a strictly lower price under uniform pricing follows immediately from Proposition 5. **Q.E.D.**

[46] Incidentally, the following argument establishes this also for all $k_j \le k_i^U$. Note that we use again that the respective programs are strictly quasiconcave.

7 Appendix B: Calculations for the Linear Case

7.1 Separate Markets

The static analysis is reported in the main text, therefore we deal here only with the dynamic case. We take a quadratic investment cost function $e(\Delta_{ki}) = t(\Delta_{ki})^2/2$. To simplify expressions with set in what follows $t = 1$. Also, in the case of price discrimination we drop any subscript i. In the second stage, the equilibrium profit of a downstream firm is $\pi(c) = (1-\overline{k}-\widehat{w}+\Delta_k)^2/4-F$. In the first stage, the firm maximizes $\pi(c) - \Delta_k^2/2$, which is concave in Δ_k. The solution is simply[47]

$$\Delta_k = \Delta^{PD} := 1 - \overline{k} - \widehat{w}, \tag{33}$$

which yields a total profit of[48]

$$\Pi^{PD} = \frac{(1-\overline{k}-\widehat{w})^2}{2} - F.$$

We next calculate the marginal change in total welfare as Δ_k increases, where total welfare Ω is given by

$$\Omega = \frac{4-4(\overline{k}-\Delta_k)-\sqrt{(1-\overline{k}+\Delta_k-\widehat{w})^2-4F}}{8}\sqrt{(1-\overline{k}+\Delta_k-\widehat{w})^2-4F}-\frac{\Delta_k^2}{2},$$

which is strictly quasiconcave in Δ_k when F is small enough and has the derivative

$$\frac{d\Omega}{d\Delta_k} = \frac{(2\Delta_k+2(1-\overline{k}-\widehat{w})+\widehat{w})(\Delta_k+1-\overline{k}-\widehat{w})-4F}{2\sqrt{(\Delta_k+1-\overline{k}-\widehat{w})^2-4F}} - \frac{5\Delta_k+1-\overline{k}-\widehat{w}}{4}.$$

After substituting from (33) that $1-\overline{k}-\widehat{w}$ is just equal to Δ^{PD}, we thus have that

$$\left.\frac{d\Omega}{d\Delta_k}\right|_{\Delta_k=\Delta^{PD}} = \sqrt{(\Delta^{PD})^2-F} + \frac{\Delta^{PD}(2\Delta^{PD}+\widehat{w})}{2\sqrt{(\Delta^{PD})^2-F}} - \frac{3\Delta^{PD}}{2} > 0, \tag{34}$$

where the final step follows after some simple but tedious transformations.[49]

[47] Note that by imposing the condition that $\overline{k} < 1-\widehat{w}$ we can ensure that the investment is interior.

[48] One can show that this is always strictly positive whenever F is sufficiently small such that the outside option indeed constrains the supplier.

[49] Precisely, we can show that (34) is strictly convex in Δ^{PD} in the admissible range $(\Delta^{PD})^2/2 > F$, first decreasing and then increasing. Therefore, the entire expression has a minimum, which arises when $\sqrt{(\Delta^{PD})^2-F} = \frac{4(\Delta^{PD})^3-F\widehat{w}-6F\Delta^{PD}}{3((\Delta^{PD})^2-F)}$. After substitution, the value that (34) takes at this minimum is $\frac{2F^2+\widehat{w}(\Delta^{PD})^3}{2\left((\Delta^{PD})^2-F\right)^{3/2}} > 0$.

Under uniform pricing, we know that there is no symmetric long-run equilibrium. The ex-post more efficient firm i chooses the same investment as under price discrimination, namely $\Delta_{ki} = \Delta^{PD}$, while the ex-post less efficient firm chooses

$$\Delta_{kj} = \sqrt{(1 - \overline{k} - \widehat{w} + \Delta^{PD})^2 - 4F} - \Delta^{PD}, \tag{35}$$

realizing total profits of $(\Delta_{kj})^2/2$. To ensure existence, we know from the proof of Proposition 3 that it only remains to verify that firm i could not profitably deviate to some $\Delta_k < \Delta_{kj}$, where using (35) it would optimally choose a deviation

$$\Delta_k^D = \sqrt{(1 - \overline{k} - \widehat{w} + \Delta_{kj})^2 - 4F} - \Delta_{kj}.$$

Comparing profits, we find that this deviation is not profitable if

$$(\Delta^{PD})^2 - F > \Delta^{PD}\sqrt{(\Delta^{PD})^2 - 2F},$$

which is satisfied for all $F > 0$. Finally, since $\Delta_{kj} < \Delta^{PD}$, firm j invests less than under price discrimination. The underinvestment problem of (34) is thus exacerbated under uniform pricing.

7.2 Competition

We compare the average discriminatory price W from (18) with w from (19). For this, it is helpful to compute the "average" binding constraint, i.e., the left-hand side of (18), evaluated at $W = w$, yielding

$$2 - k_i - k_j - 2\widehat{w} - \frac{9F}{1 - 2k_i + k_j - \widehat{w}} - A_i - A_j.$$

After some simple but tedious manipulations, this transforms to

$$\frac{216F(k_j - k_i)[(1 - 2k_i - k_j - \widehat{w})^2 - 9F](2 - k_i - k_j - 2\widehat{w})}{(1 - 2k_i - k_j - \widehat{w})^2} - \frac{8748F^3(k_j - k_i)}{(1 - 2k_i - k_j - \widehat{w})^3},$$

of which the second term goes to zero faster than the first term as F goes to zero. From this we can immediately conclude that the average price under price discrimination is higher than under uniform pricing at least for all sufficiently low F.

For the long run, we again only treat the case with initially symmetric firms. Under discriminatory pricing, the benefits from a marginal reduction in k_i is given by (14), which in the linear case simplifies to

$$\begin{aligned} & \frac{2}{3}q_i\left[2 + 2\frac{dw_i}{dk_i} - \frac{dw_j}{dk_i}\right] \\ = \quad & \frac{4(1 - k_i - \widehat{w})^2 - 9F}{144(1 - k_i - \widehat{w})^3[2(1 - k_i - \widehat{w})^2 - 9F]}\left[32(1 - k_i - \widehat{w})^4 + 36F(1 - k_i - \widehat{w})^2 - 81F^2\right]. \end{aligned} \tag{36}$$

In the uniform case, suppose firm $i = 1$ will be ex-post more efficient, in which case we obtain the marginal benefits

$$\frac{2}{3}q_1\left[2+\frac{dw}{dk_1}\right] = \frac{2}{9}\left[1-2k_1+k_2-\widehat{w}-\frac{9F}{4(1-2k_1+k_2-\widehat{w})}\right]\left[2+\frac{9F}{2(1-2k_1+k_2-\widehat{w})^2}\right], \tag{37}$$

while incentives for the ex-post less efficient firm $j = 2$ are given by

$$\frac{2}{3}q_2\left[2+\frac{dw}{dk_2}\right] = \frac{2}{9}\left[1+k_1-2k_2-\widehat{w}-\frac{9F}{4(1-2k_1+k_2-\widehat{w})}\right]\left[2-\frac{9F}{4(1-2k_1+k_2-\widehat{w})^2}\right]. \tag{38}$$

We specify a quadratic investment cost, $e'(\Delta_k) = t\Delta_k^2/2$. Equating marginal costs $e'(\Delta_k) = t\Delta_k$ to marginal benefits (36) under discrimination, and to (37) and (38) and under uniform pricing, we can obtain numerically the investment levels discussed in the main text.

We consider here the limiting situation when F tends to zero. Recall that in this case input prices are always equal to $\widehat{w}$, while we find that there is a unique optimal level of cost reduction Δ_k^* solving[50]

$$t\Delta_k^* = \frac{4(1-\overline{k}-\widehat{w})}{9}.$$

With the quadratic investment function, we can show that a sufficient condition for the investment problem to be strictly concave for low F is that $t = e''(\Delta_k) \geq 2$. To simplify expressions in the remainder, we fix $t = 2$. We can also show that in the limit $F = 0$ the investment Δ_k^* is inefficiently low as welfare is strictly concave in the symmetric Δ_k and maximized at $\Delta_k = [4(1-\overline{k})-\widehat{w}]/9$.

For $F > 0$ but small, we consider first the case with price discrimination. The symmetric equilibrium investment level is obtained from equating (36) to marginal costs investment costs. Implicit differentiation of the first-order condition obtains

$$\left.\frac{dk_i}{dF}\right|_{F=0} = -\frac{27}{28(1-k^*-\widehat{w})},$$

where $k^* = \overline{k}-\Delta_k^*$. Under uniform pricing, the first-order conditions are obtained from (37) and (38). Implicit differentiation of the system of equation results in

$$\left.\frac{dk_i}{dF}\right|_{F=0} = \frac{-9}{14(1-k^*-\widehat{w})},$$

[50]Here and in what follows, we stipulate again that the solution is interior, which holds for low F if $\overline{k} < 1-\widehat{w}$.

$$\left.\frac{dk_j}{dF}\right|_{F=0} = \frac{45}{28(1-k^* - \widehat{w})},$$

where firm i is the ex-post more efficient firm. By $9/14 < 27/28$ we thus have that around $F = 0$ both firms have strictly lower long-run costs under price discrimination. We thus have that around $F = 0$ both firms' marginal costs and also input prices are strictly higher under uniform prices. As at $F = 0$ there was underinvestment in cost reduction, we hence know that consumer surplus and welfare are strictly lower in the long run under uniform pricing.

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