Trade in quality goods*

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Very preliminary and incomplete draft

Abstract

I develop a general Ricardian model of vertical product differentiation, consistent with previous models of product-cycle and quality goods. Whenever several countries supply the same product line, the higher the country's wage the higher the quality it will produce. Higher wages also imply a larger demand for high quality products. Thus, as predicted by Linder (1961), wealthy countries consume disproportionately more products from higher income countries. However, his more specific hypothesis - trade decreases with the difference between importer and exporter's per capita income - fails in general. My proof helps explain the apparent puzzle of the prevalence of quality goods in the world trade data combined with an elusive evidence for Linder's hypothesis. If a small country suffers a positive technology shock, then all consumers with income above a certain threshold are made strictly better off, while the remaining are indifferent or worse off. I use numerical simulations to analyze the impact of technology changes with wealth effects.

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1 Introduction

A series of recent empirical works provides strong evidence of widespread vertical product differentiation in the international trade (Hallak, 2005; Hummels and Klenow, 2004; Schott, 2004). Unit values vary positively and systematically with exporters' per capita GDP. This variation in the price of goods classified under the same product category suggests unobserved differences in the quality levels. The positive relation between price and quality suggests that high-income countries have a comparative advantage in the production of high-quality goods. It was also found that high and low-income countries export roughly the same product categories. Thus, the traditional hypothesis of specialization *across* products is rejected in detriment of a specialization *within* products. This finding also indicates that quality goods may very well be much more prevalent in the world trade than was originally thought.

The objective of the present work is to propose a Ricardian model to study trade in quality-differentiated markets, markets in which both demand and supply of high-quality goods tend to be concentrated in high-income regions. Most trade models neglect the role of demand in trade. By assuming preferences are homothetic, they presume goods being purchased in the same proportion in all countries. Two exceptions are Flam and Helpman (1987, FH henceforth) and Stokey (1991)'s models of North-South trade with vertically-differentiated goods. While these works also address the issues posed here, due to the usage of the tools developed by Eaton and Kortum (2002), my model has the advantage of being more comprehensive, which enables me to significantly extend their analysis.

I begin examining the broad patterns of trade by means of a general framework. Preferences are increasing in quality, and technologies aim to capture exclusively a notion of product cycles introduced by Vernon (1966) - the production of old low-quality products is standardized while that of new high-quality ones exhibits high variability. To support the generality claim, I show that the assumptions on technology are consistent with endogenous growth models (see Grossman and Helpman (1991)) and Kremer's (1993) O'Ring production function, as well as with FH and Stokey. This alone is enough to establish a very systematic pattern of specialization: whenever several countries supply the same product category, the higher a country's wage the higher the quality it will produce. In both FH and Stokey, the North produces high-quality goods and the South low-quality ones. Thus, this result not only shows that their find is robust to weaker assumptions, but it also generalizes it to numerous countries in a very orderly manner.

The aforementioned production configuration combined with a preference for quality implies that wealthy countries tend to produce and consume more high-quality products than poor ones do. This begs the question of whether trade is more intense among countries with similar income levels, as conjectured by Linder (1961). A compelling question for various reasons: (i) this is the only well-known hypothesis regarding the direction of trade in quality-differentiated markets; (ii) consumers' income and the quality aspect of manufactured goods constituted Linder's primary example, and (iii) no previous theoretical model, including FH and Stokey's, seriously addresses this theory by supporting, challenging or providing an alternative. In accordance with Linder's hypothesis, I prove that the fraction of goods purchased from high-income countries is strictly increasing in consumers' income. This is not to say, however, that trade increases as the importer's per capita income approaches that of the exporter's. For any choice of utility and production functions, I show that there exists an economy that violates this monotonicity. Further contradicting Linder, if goods are measured in expenditures rather than numbers, I also show the existence of an economy in which the fraction of spending devoted to goods preceding from wealthy countries is not increasing in consumers' income.

It is important to emphasize that the results above should not be taken as a serious objection to Linder's theory, as my model lacks some crucial pieces of his argument.[1] My results do nevertheless contribute to the empirical discussion that has arisen since the theory was first put forth.[2] In particular, the empirical validation of Linder's hypothesis remains controversial, and the counter-examples I provide in the proofs are rather plausible, thus raising some obstacles that potentially impede a transparent view of Linder's predictions in the data. For instance, his theory looses its power as two countries' income levels grow too far from each other. Hence, requiring trade volumes to be strictly decreasing in the distance between importer and exporter's per capita income *everywhere*, as most empirical tests do, may be excessively demanding.[3] To avoid this problem one could divide countries into broad income categories - e.g. low, middle and high. Unfortunately, my examples show that the assertion 'trade is more intense among countries with similar income levels' is sensitive to the choice of threshold dividing the different categories. In sum, the strong evidence of widespread quality-differentiated products in the world trade data combined with an at best elusive evidence of Linder's hypothesis should not be seen as a puzzle. All goods are quality-differentiated in the present set-up; the empirical results mentioned in the first paragraph always hold here, and yet Linder's predictions do not always hold. It is worth pointing out that I did not use FH and Stokey's models in the above analysis, as the theory is meaningless in a two country world, and they make very specific assumptions on the functional forms. Hence, even if Linder's predictions were confirmed there, one could not claim generality.

By assuming specific functional forms, I simplify the model greatly, which allows for more specific predictions on the direction of trade (see section 4) and for static analysis of technology changes. If a small country suffers a positive technology shock, then all consumers with income above a certain threshold are made strictly better off; the remaining are either indifferent or worse off. When I restrict my attention to two-country economies, I show that the present model is consistent with the results of FH and Stokey: the Northern welfare always improves after the South suffers a positive technology shock, but conversely the South can be worse off if technology advances in the North. This result, however, is true only if Hicks neutral technology changes are contemplated. By contrast in endogenous growth models and in Kremer (1993), technology shocks affect new products disproportionately. Following Vernon (1966), these works assume the possibilities for cost reduction are greater for new goods than for old standardized ones. I use numerical

[1]Linder argues that domestic demand is essential for the development of an industry. Only the surplus will be exported. In my model, technology is exogenous; it is not affected by demand.

[2]For surveys, see Deardorff (1984), Leamer and Levinsohn (1995), and McPherson et al. (2001).

[3]This is better explained in section 3. Briefly, the point here is that this assumption does not imply only that consumers' demand is shifted to products from high income countries as his wealth increases. It also makes strong requirements regarding the pace at which this shift to higher income producers occurs.

examples to prove that FH and Stokey's results break down if technological progress is biased towards high-quality goods. In this case, the North can indeed be made worse off due to a technology improvement in the South.

While my theoretical analysis is still incomplete (proposition 7 is particularly glaring), the most interesting extension to the work is empirical. One prediction of the general model is that unit prices vary positively with importer's per capita income, rather than only exporter's as found by the previously mentioned empirical works. Worth attempting is to calibrate the parameters of the simplified model to the world data, which would allow for static analysis of changes in technology, population and trade barriers.

The work is organized as follows. In section 2, I present the model, prove the existence of equilibria and relate its assumptions to existing theoretical theories (2.1). In section 3, I study the patterns of trade under the general set up, and in section 4 I present the simplified model with its corresponding results. Auxiliary figures are after the main text and all formal proofs are in the appendix.

2 The Model

There is a set of n countries, $N = \{1, ..., n\}$, with a continuum of individuals each. The measure of country i's population is denoted by $H_i \in \mathbb{R}_{++}$.[4] Each consumer is endowed with one unit of labor, which he supplies inelastically. There is no labor mobility across countries, but perfect mobility across sectors within the same country. There is a continuum of consumption goods indexed by $j \in [0, 1]$, each differentiated by quality levels $q(j) \in \mathbb{R}_+$. There are no transport costs, so that the price of goods do not vary across countries.

Consumers Consumers have identical preferences. They are satiated with one unit of each good j, and have preferences over consumption bundles $q = \{q(j)\}_{j\in[0,1]}$, which indicate the quality level of each good j. I assume utility is additively separable and symmetric over goods:

$$U(q) = \int_0^1 u(q(j))dj \tag{1}$$

I make assumption A1 on u:

Assumption 1 *The function u is continuously differentiable, strictly increasing and concave, with $u'(0) = \infty$, $u'(\infty) = 0$ and $u(0) = 0$.*

Production Production is perfectly competitive and uses labor as the unique input. There is constant returns to scale, and the labor requirement to produce each unit of a good is given by the function $C(q, t)$. It depends both on the quality level q and on a technology random variable $t \in \mathbb{R}$, which is country- and commodity-specific. (I assume below that C is strictly increasing in both arguments.) The unit cost of production is

[4]Throughout, I use subscript "++" to denote the strictly positive quadrants of $\mathbb{R}$ or $\mathbb{R}^n$.

$wC(q,t)$, where w is the country's wage. In other words, there is a set of technology curves $\{C(q,t)\}_{t \in \mathbb{R}}$. For each good, each country makes a draw t from the set. The curve $wC(q,t)$ is the country's unit cost for that particular good. Indexing country-specific variables with subscripts, the price faced by consumers is $\min_{i \in N}\{w_i C(q,t_i)\}$.

Assume commodities joint random variables $\{t_1(j), ..., t_n(j)\}_{j \in [0,1]}$ are distributed independently according to a joint cumulative distribution function F. (That is, I admit correlation across countries' technologies, but as in Eaton-Kortum not across goods.) In addition, F has a probability density function f that is continuous in its support τ, which is a compact convex subset of $\mathbb{R}^n$.

It is important to note that *there is no randomness in the present economy from the perspective of the agents.* They know all preferences and technologies, observe the prices of all commodities and then make their production and consumption decisions. Randomness comes from the perspective of an outside observer who arbitrarily draws a good in $[0,1]$ and reports its technology parameter t. The realization of t in this experiment is described by F. Therefore, the economy is perfectly deterministic, and F a measure function describing the spread of technology parameters t in the economy.

I sustain assumption A2 on C and explain it in subsection 2.1 below.

Assumption 2 *The function C is continuously differentiable in both arguments and satisfies:*
(i) $C(0,t) = 0$ *for all* $t \in \mathbb{R}$*;*
(ii) $\frac{\delta C(q,t)}{\delta q} > 0$ *and* $\frac{\delta^2 C(q,t)}{(\delta q)^2} \geq 0$ *for all* $(q,t) \in (\mathbb{R}_+ \times \mathbb{R})$*;*
(iii) $\frac{C(q,t)}{C(q,t')}$ *is strictly increasing in q for all $t, t' \in \mathbb{R}$ with $t > t'$, and*
(iv) $\lim_{q \to 0} \frac{C(q,t)}{C(q,t')} \geq 1$ *for all $t, t' \in \mathbb{R}$ with $t > t'$.*

Before proceeding, note that the utility function (1) is symmetric over all goods. Therefore, the only parameter that varies across them is their productivity t. It will then be convenient to re-label goods according to their parameter $t(j) \in \mathbb{R}^n$, referring to them henceforth as 'good t'. I rewrite (1) accordingly as

$$U(q) = \int_{\tau} u(q(t))dF(t) \tag{1'}$$

where t here is a vector, $t = (t_1, ..., t_n)$, so that the integral is over $\mathbb{R}^n$ (*i.e.* $\tau \subset \mathbb{R}^n$).

The present economy has an infinitum of goods and labor as the unique factor of production. Thus, as observed by Wilson (1980), it can be analyzed as a simple pure exchange economy in which countries' labor are treated as consumption goods. An equilibrium is therefore a set of wages and allocations such that consumers maximize their utility and the labor required from each country to produce the demanded goods equals its supply (*i.e.* its population H_i).

Proposition 1 *An equilibrium always exists.*

2.1 Assumptions and existing literature

Consistent with the introduction, I relate the assumptions on preferences to Linder (1961), and those on technologies to Vernon (1966) and followers.

2.1.1 Preferences

The focus of the present work is quality-differentiated markets. Each good t is best interpreted as a broad product category, such as "food" or "clothing". Linder argues that "only part of the higher income will be expressed in purely quantitative changes" and proceeds to claim the substantial change will occur in the quality dimension.[5] In my model, an increase in income features no changes in the quantities demanded, only in the quality level q. (The utility format, assumptions A1 and A2(i) and (ii) together imply that all goods are consumed in one unit.) This disregarding of quantity is also common to FH and Stokey.

2.1.2 Technologies

Parts (i) and (ii) of A2 are clear: no labor is required to produce quality zero of any good, and costs are strictly increasing and convex in quality. Since these are best related to the interpretation of goods and preferences, I focus below only on A2(iii) and (iv). The model is static. In relating the assumptions to dynamic product-cycles, I make the analogy between new (old) goods and high (low) quality versions of a product.

Vernon contended that the introduction of new products is characterized by a phase of high unpredictability followed by the process of eliminating uncertainties through standardization. Here, this "uncertainty" is captured by the variability across countries in the labor requirements to produce a certain good. Assumption A2(iv) is a mere normalization - efficiency in producing low-quality goods decreases with t. Whereas A2(iii) directly assumes that for any two distinct technologies ($t \neq t'$), the labor efficiencies become more uneven as q increases - $\frac{C(q,t)}{C(q,t')}$ departs from 1. Figure 1 illustrates how the notion of standardization of low-quality products is violated when A2(iii) or (iv) fail.

It is also important to understand the shape of the cost curves shown in figure 2. The cost of producing a certain product is lower in country i than in k if $\frac{w_i C(q,t_i)}{w_k C(q,t_k)} < 1$. By A2(iii) and (iv), the country specific technologies t_i and t_k are important determinants of the cost of the high-quality goods. However, they loose relevance with respect to wages as quality decreases and the technology gap between countries narrows. An extreme case of this occurs in endogenous growth models and in Kremer (below), where $\lim_{q\to 0} \frac{C(q,t)}{C(q,t')} = 1$. There, standardization is perfect, and the lowest cost producer of the low-quality goods will always be the lowest wage country - *i.e.* $\lim_{q\to 0}\left[\frac{w_i C(q,t)}{w_k C(q,t')}\right] = \frac{w_i}{w_k}$ for any t, t'.

Endogenous growth models - Typically, these models consist of two countries, N and S, and infinitely many varieties (quality levels here). At any point in time, there are two thresholds n_S and n_N with $0 \leq n_S \leq n_N$. Countries N and S share the same technology for all goods indexed below n_S; only N is able to make those in $(n_S, n_N]$, and varieties above n_N do not exist. In the present notation, this is translated into the

[5] For this point, see pages 94 and 95 in Linder (1961). The examples of "food" and "clothing" are his.

existence of a unique product whose technology t satisfies: $\frac{C(q,t_S)}{C(q,t_N)} = 1$ if $q \leq n_S$ and $\frac{C(q,t_S)}{C(q,t_N)} = \infty$ if $q \in (n_S, n_N]$. Thus, assumptions A2(iii) and (iv) only smoothen the "step" cost functions used in these models.

Kremer (1993) - The 'O-Ring' production function proposed by Kremer is consistent with both the present set up and a series of stylized facts in development and labor economics. Countries in his model differ in their levels of human capital, measured in the probability of a worker completing a task successfully (h_i). In turn, the complexity of a good is measured with the number of tasks involved in the production process - high-technology goods require more tasks. Thus, the labor needed to produce a good of technological (quality) level q is $\left(\frac{1}{h_i}\right)^q$. Clearly, for any $h_i < h_k$, we have (A2.iv) $\lim_{q\to 0} \frac{C(q,h_i)}{C(q,h_k)} = \lim_{q\to 0} \left(\frac{h_k}{h_i}\right)^q = 1$ and A2(iii) $\frac{C(q,h_i)}{C(q,h_k)} = \left(\frac{h_k}{h_i}\right)^q$ is strictly increasing in q.[6]

Stokey and Flam and Helpman - Both of these papers present North-South models in quality differentiated markets. Stokey explicitly assumes both that $\frac{C(q,t_S)}{C(q,t_N)}$ is strictly increasing in quality q (A2.iii) and that $\lim_{q\to 0} \frac{C(q,t_S)}{C(q,t_N)} > 1$ (A2.iv). The labor requirement curves in FH are $(e^{\gamma_N q}/A_N)$ and $(e^{\gamma_S q}/A_S)$ in the North and South, respectively. They assume that $\gamma_S > \gamma_N$ (A2.iii), and that A_S is not 'large enough' so that in equilibrium $w_S < w_N$ (A2.iv is equivalent to $A_S \leq A_N$).

Examples - Several sets of functions satisfy A2. A few examples are

$$\begin{aligned} C(q,t) &= q^{\beta_1} + tq^{\beta_2} \quad \text{, where } \beta_2 > \beta_1 \geq 1 \text{ and } t > 0; \\ C(q,t) &= (q+1)^t - 1\text{, where } t \geq 1; \\ C(q,t) &= e^{tq} - 1 \qquad \text{, where } t > 0. \end{aligned}$$

There is perfect standardization ($\lim_{q\to 0} \frac{C(q,t)}{C(q,t')} = 1$) in the first example, but not in the second and third.

3 Patterns of Trade

Summarizing the previous section, I have presented a Ricardian model of trade with a nonempty set of equilibria. I argued that the assumptions entailed exclusively: (i) preferences are increasing in quality, and (ii) the variability of labor efficiency across regions increases with quality. The main questions regarding patterns of trade that I will ask here are: if rich countries consume more high-quality goods relative to poor ones; if they have a comparative advantage in the production of high-quality goods, and if a positive answer to these is sufficient for trade to be more intense among regions with similar income levels, as predicted by Linder (1961).

Recall that the population is homogeneous within countries. Therefore, given an economy $(\{H_i\}_{i\in N}, F)$, an equilibrium wage rate $w = (w_1, ..., w_n)$ fully specifies the price of goods as well as the income of all consumers in each country. Wages are thus sufficient to determine the demand function (or correspondence).

[6] Variables q, h_i are equivalent to n, q_i in Kremer's notation. I ignored inequalities of human capital only for simplicity.

Proposition 2 *If countries $i, k \in N$ both produce the same good t and $w_i > w_k$, then the quality produced by i is strictly greater than the one produced by k. Moreover, if not all countries have the same wage in equilibrium, then the set of goods produced by at least two countries with different wages has a non-zero measure.*

Proposition 3 *For all goods t, the quality demanded is strictly increasing in the consumer's income.*

Proposition 2 confirms the prediction of FH and Stokey that poor countries (South) produce low-quality versions of the high-quality products made by the rich (North). Because of the generality of my model, there may be products such that all countries consume from the same source. There exists, nonetheless, a nonempty set of goods with several producers. Invariably, these will be such that the richer the country the higher the quality it will be producing. This is a very precise generalization of the trade pattern found in FH and Stokey to the case of several countries, goods and functional forms. Empirically, propositions 2 and 3 imply that the unit price of goods is strictly increasing in both *exporter*'s and *importer*'s per capita income. Evidence of the former result was found by Hallak (2005), Hummels and Klenow (2004), and Schott (2004), whereas the latter to my knowledge has not yet been tested.

As I turn to Linder's conjectures regarding the direction of trade, more notation is needed. A consumer with income Y chooses $q = \{q(t)\}_{t\in\tau}$ to maximize $U(q) = \int_\tau u(q(t))dF(t)$ subject to the budget constraint $\int_\tau P(w, q(t), t)dF(t) \leq Y$, where $P(w, q, t) = min_{i\in N}\{w_i C(q, t_i)\}$. Define $q^*(w, \lambda, t) = \arg\max_{q\in\mathbb{R}_+}\{u(q) - \lambda \min_{i\in N}\{w_i C(q, t_i)\}\}$ and $\hat{q}(w_i, \lambda, t_i) = \arg\max_{q\in\mathbb{R}_+}\{u(q) - \lambda w_i C(q, t_i)\}$. Since the utility maximization problem is additively separable, q^* is the quality of good t chosen by a consumer with shadow value of wealth λ when the prices are w; whereas $\hat{q}$ is his choice if he decides to buy the good from country i. (Note that t and w in the arguments of q^* are vectors, and t_i and w_i in $\hat{q}$ are numbers.) Also, let $\lambda(Y)$ be the function that delivers the Lagrangean multiplier of a consumer whose wealth is Y, and $G_i(w, Y) = \text{prob}(\{t \in \tau : \hat{q}(w_i, \lambda(Y), t_i) = q^*(w, \lambda(Y), t)\})$ is the measure of the set of goods that a consumer with income Y demands from country i when wages are w.[7]

Proposition 4 *Let $w_1 \leq ... \leq w_n$. For any $i \in N\backslash\{n\}$ such that $w_i < w_{i+1}$, $\left[\sum_{k\leq i} G_k(w, Y)\right]$ equals 0, 1 or is strictly decreasing in Y.*

The idea of the result is simple and captured by wealth expansion path of figure 3. Whenever a consumer is indifferent between buying a certain good from more than one source (*e.g.* countries 2 and 3, the second discontinuity in the bold line), all consumers poorer than him will strictly prefer one of the poorer producers (country 1 or 2). Therefore, proposition 4 follows - the representative consumers of wealthy countries buy proportionately more goods from high-income sources. So a very weak version of Linder's hypothesis is indeed linked unequivocally to Vernon's theory and a preference for quality. It is, nonetheless, the only version that my model upholds, which should not be surprising

[7]In appendix 5.1 I prove that $\lambda(Y)$ is a well-defined and strictly decreasing function for all $Y > 0$ and that $G_i(w, Y)$ is a continuous and well-defined function for all $(w, Y) \gg 0$.

as technology here is not governed by demand like Linder contended it was. Still, the series of negative results presented below raise some potential problems with analyzing the world trade data under a strict interpretation of Linder's hypothesis. I believe these remain valid even in a model where demand and supply are more tightly connected.

Assumption 3 *For all* $q > 0$, $\lim_{t\to\infty} C(q,t) = \infty$.

Assumption 4 *Technologies are independent across countries (i.e.* $F = \prod_{i\in N} F_i$*).*

Assumption A3 implies that there is no upper bound on the inefficiency of producing a certain good - it is satisfied for all functions and previous works mentioned in subsection 2.1. Assumption A4 only strengthens the results below. The format of propositions 5 through 7 is the same. I take C and U as primitives and define an economy by its set of countries N, their populations $\{H_i\}_{i\in N}$ and the distribution of technology across countries F. Countries frequently undergo population growths (increases in H_i) and technological advances (shifts in F). Thus, propositions 5 through 7 essentially state that an economy selected arbitrarily will not necessarily follow the patterns of trade predicted by Linder.

Proposition 5 *Let* C *and* U *be any cost and utility function satisfying A1 through A3. Then, there exists an economy* $(\{H_i\}_{i\in N}, F)$ *such that:*
(i) F *satisfies A4, and*
(ii) there exists a country $i \in N$ *such that* $G_i(w, Y)$ *strictly decreases and then increases in* Y *for some values of* $Y \in \{w_1, ..., w_n\}$ *and some equilibrium wage* $w \in \mathbb{R}^n_{++}$.

Linder's conjectures regarding trade patterns were more specific than the broad assertion of proposition 4: "The more similar the demand structures of two countries, the more intensive, potentially, is trade between these two countries" (p.94). In a world with no income inequalities within countries and frictionless borders as the present one, this conjecture is equivalent to the assertion that the percentage of goods demanded from a certain source is decreasing in the difference between the producer's and the buyer's per capita income.[8] According to proposition 5, there exists an economy where the demand for products from a certain source first decreases and then increases with per capita income. Therefore, no matter where this country is placed in the income scale, the monotonic relation between demand and differences in importer and exporter's per capita income cannot hold. Thus the contradiction to Linder's prediction.

Notwithstanding results 4 and 5, the natural measure of demand is *expenditures* rather than *number* of goods. Linder's hypothesis under this measure is addressed by propositions 6 and 7. In the first, I prove that the claim in P4 does not generally hold if 'number of goods' is substituted by 'expenditures'. The latter result is best understood after the proofs of propositions 5 and 6 are sketched. Define $L_i(w, Y)$ to be the labor demanded from country i by a consumer with income Y when the wage is $w = (w_1, ..., w_n)$. Then, the term $[w_i L_i(w, w_k)]$ in definition 1 is the expenditures in goods preceding from country i by the representative consumer of k.

[8]This is typical claim used in empirical tests of Linder's hypothesis. See footnote 2 for references.

Definition 1 *An economy* $(\{H_i\}_{i\in N}, F)$ *has* ***weak Linder property (WLP)*** *if for all equilibrium wages* w *(with* $w_1 \le w_2 \le ... \le w_n$*),*

$$\frac{\sum_{i'\le i} w_{i'} L_{i'}^*(w, w_k)}{w_k} \ge \frac{\sum_{i'\le i} w_{i'} L_{i'}^*(w, w_{k+1})}{w_{k+1}} \text{ for all } i, k \in N \setminus \{n\}. \quad (2)$$

Proposition 6 *Let* C *and* U *be any cost and utility function satisfying A1 through A3. Then, there exists an economy* $(\{H_i\}_{i\in N}, F)$ *such that* F *satisfies A4, and the WLP is violated.*

Consider an economy with at least three countries with equilibrium wages $w_1 < w_2 < w_3$. As per figure 3, whenever cost curves cross, as the consumer's income increases he switches the source of his demanded products from country 1 to 2 and later to 3. In a world with a variety of goods and countries, however, these changes may not follow any systematic pattern. And the absence of an orderly scheme drives the results in P5 and P6. To understand proposition 5, let the technology parameters in country 3 be t_3 for all goods and countries 1's and 2's parameters be (t_{1a}, t_{1b}) and (t_{2a}, t_{2b}), respectively, where $t_{ia} < t_{ib}$ for $i = 1, 2$. Assume their measures are $p(t_{1a}) = p(t_{1b}) = p(t_{2a}) = p(t_{2b}) = 1/2$. Now suppose three individuals p, m, r (poor, medium and rich, $w_p < w_m < w_r$) demand goods from countries 1, 2 and 3 as indicated in the table below:

	t_{2a}	t_{2b}
t_{1a}	$p, m \to 1;\ r \to 2$	$p, m \to 1;\ r \to \{1, 3\}$
t_{1b}	$p, m, r \to 2$	$p \to 2;\ m, r \to 3$

According to the table, $\frac{1}{2} = G_2(w, w_r) = G_2(w, w_p) > G_2(w, w_m) = \frac{1}{4}$ (*). This inequality is driven mostly by the NW and the SE quadrants of the table. Only r purchases good $t = (t_{1a}, t_{2a}, t_3)$ from the middle income country 2, whereas p and m prefer the poorer source 1. As t_1 and t_2 increase to t_{1a} and t_{2a}, both m and r shift to producer 3, and p shifts only to the middle income country 2. The remaining quadrants are chosen judiciously in order not to affect inequality (*). In the appendix, I show that a three-country economy with equilibrium wages w and distribution of t arbitrarily close to the one in the table always exists. The addition of small countries with the wages of p, m and r conclude the proof.

As for proposition 6, again let $N = \{1, 2, 3\}$ and $w_1 < w_2 < w_3$, and consider the fraction $\frac{w_1 L_1(w,Y) + w_2 L_2(w,Y)}{w_3 L_3(w,Y)}$. Suppose that as income increases from Y to $Y' > Y$, there is a large shift in consumption from country 1 to country 2, but the demand for 3's products remains fairly stable. Then, the change in income from Y to Y' significantly raises the numerator without altering the denominator much. Consequently, the fraction $\frac{w_1 L_1(w,Y) + w_2 L_2(w,Y)}{w_3 L_3(w,Y)}$ increases with income, instead of decreasing as required by the WLP. The proof is again completed by simply adding two small enough countries with wages Y and Y' to the three-country economy.

There are two observations regarding the latter proof. First, it should be clear why this kind of counter example would not pose a problem when goods are measured in numbers - all the goods whose demand shifted from source 1 to 2 would receive the same weight under Y and Y' and hence they would not affect the numerator. Secondly, as the consumer's income increased from Y to Y' there was a shift to higher income sources. Intuitively, this should confirm rather than contradict Linder's hypothesis. Thus, the result is not at odds with his standings; it only shows that the validity of the claim that poor (rich) countries tend to spend disproportionately more in goods from other low (high) income sources is sensitive to the choice of threshold dividing rich from poor countries. Proposition 5, in turn, makes it evident that any attempt to recover Linder's hypothesis here with a pairwise comparison between countries is hopeless.

I therefore take an alternative approach in proposition 7. I focus economies with two countries, where absence of an orderly shift to higher income sources is not an issue. I then ask if the fraction of a expenditures dedicated to goods from the poor country is decreasing in consumer's wealth. Contrary to the WLP that only considers the wages of existing countries, the SLP looks at all income levels of the consumer.[9]

Definition 2 *An economy* $(\{H_i\}_{i\in N}, F)$ *has* ***strong Linder property (SLP)*** *if for all equilibrium wages* w *(with* $w_1 \leq w_2 \leq ... \leq w_n$*),*

$$\frac{\sum_{i'\leq i} w_{i'} L^*_{i'}(w,Y)}{Y} \quad \text{is decreasing in } Y \text{ for all } Y > 0. \tag{3}$$

Proposition 7 *Suppose the cost and utility functions* C *and* U *satisfy assumptions A1 through A3. If for all economies* $(\{H_i\}_{i\in N}, F)$ *with* $N = \{1, 2\}$ *and* F *satisfying A4 the SLP holds, then*

$$\frac{C(\hat{q}(w,\lambda,t),t)}{C(\hat{q}(w',\lambda,t'),t')} \text{ is decreasing in } \lambda \text{ for all } \lambda > 0, \text{ all } 0 < w < w', \text{ and all } t, t'. \tag{4}$$

(I think condition (4) is incompatible with A2, but have not yet had time to think about it. So, I do not explain anything here. If this is the case, then the statement of P7 would resemble those of P5 and P6. The term $C(\hat{q}(w,\lambda,t),t)$ *is the labor required to produce* $\hat{q}$ *in a country whose technology parameter is* t*.)*

A real world example illustrates proposition 7. Suppose there are two countries North and South with $w_N > w_S$ and consider two individuals with incomes $Y_{vp} < Y_p$ (*very poor* and *poor*). Let the income of both consumers be such that they consume most goods from the South, except for medicine which they both purchase from the North (take the South to be very inefficient at making medicine). As income moves from Y_{vp} to Y_p, the quality of the goods demanded from the South increase, thus increasing the demand for Southern labor. Suppose, however, that the same change in income barely alters the consumption of medicine. As a result, expenditures in goods from the South relative to those from the North will actually be smaller for the very poor consumer vp than for the richer one p. There is nothing extraordinary in this illustration; yet it violates SLP.

The above example accentuates a conflict between propositions 3 and 4. On the one hand, as the consumer's income increases, he shifts his consumption to goods from rich sources (P4). On the other, the price of the goods that he demands from poor countries

[9] Income levels different from the wages of the representative consumers may be viewed both as small countries or citizens of one of the existing countries with different labor endowments. Clearly, this is more interesting than considering monotonicity of expenditures only for the wages of the two countries.

increase (P3). Consequently, the net change in the ratio of spending in poor to rich countries' goods has no clear direction. In the proof, I provide an example in which the latter shift occurred at a faster pace than the former. Thus, the fraction of expenditures in goods from the poor country increases with the consumer's income (in some interval).

Discussion. I insist that the purpose of propositions 5 through 7 is not to argue against Linder's thesis, but against attempts to empirically verify it over the whole spectrum of countries. Summarizing Linder's theory, he argued that the proximity to the domestic market is a source of comparative advantage - industries generally develop to fulfill the local demand and export the surplus - thus generating two trends: (i) high-income households' consumption of high-quality goods provide rich countries with a comparative advantage in the production of high quality goods, and (ii) countries with an interest in importing the surplus of another country's production have demand patterns similar to that of the exporter. Contradicting endowment based trade theories, Linder concluded that trade should be more intense among similarly endowed countries.

As households get wealthier here they do switch to products from wealthier countries (P4). But an arbitrary choice of technologies will not necessarily lead to an orderly transition, and may consequently violate the hypothesis that trade volumes are decreasing in the difference between importer and exporter's per capita income (P5). The usage of Linder's arguments to justify strengthening the assumptions hereby made may be plausible for countries with similar income levels, but not to compare Thailand-US to Indonesia-Germany trade volumes as a worldwide empirical test does. This is the point of P5. One solution would be to divide countries into broad income categories and test whether the fraction of imports devoted to the higher income group is increasing in importer's per capita income. Proposition 6 shows that this monotonicity may be violated due to shifts to higher income sources within categories. A further difficulty is raised by P7. Countries do not produce only one quality of each product; before switching to Italian shoes consumers demand higher qualities of the Brazilian ones. Thus, the ratio of imports from poorer sources to the richer ones will not necessarily be strictly decreasing everywhere in importer's per capita income.

4 Specific model: assuming functional forms

In order to make the problem more tractable, I now assume specific functional forms for U, C and F. Let $U(q) = \int_0^1 q(j)dj$, and $C(q,t) = q(C + tq)$, where $C > 0$ is a constant. The parameter C captures the standardization of low-quality products: for all t, t', $\lim_{q\to 0} \frac{q(C+tq)}{q(C+t'q)} = 1$ and $\lim_{q\to\infty} \frac{q(C+tq)}{q(C+t'q)} = \frac{t}{t'}$. Country i's variables t_i are distributed independently according to a Fréchet distribution with c.d.f.

$$F_i(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 - \exp\left(-T_i t^{\theta}\right) & \text{otherwise} \end{cases} \tag{5}$$

where $\theta > 1$ and $T_i > 0$ for all i. Variables are independent across countries, so that the c.d.f. of the joint random variable $t = (t_1, ...t_n)$ is $F(t) = \prod_{i\in N} F_i(t_i)$. By equation (5), an increase in the country-specific technology parameter T_i increases its probability of low (efficient) draws of t. The parameter θ governs the spread of the distribution - the variance decreases with θ.

The set of equilibrium wages is the solution to the following system of equations (see appendix 5.9):

$$\sum_{k=1}^{n} w_k L_k(w, \lambda_i) - w_i = 0 \qquad \text{for all } i \in N, \text{ and}$$
$$\sum_{k=1}^{n} H_k L_i(w, \lambda_k) - H_i = 0 \qquad \text{for all } i \in N,$$

$$\text{where } L_i(w, \lambda) = \frac{\Gamma(1-1/\theta)}{4\lambda^2} \left\{ \frac{T_i w_i^{-1-\theta} \left(\min\{0, 1-\lambda w_i C\}\right)^{2\theta-1} (1+\lambda w_i C)}{\left[\sum_{k=1}^{n} T_k w_k^{-\theta} (\min\{0, 1-\lambda w_k C\})^{2\theta}\right]^{(1-1/\theta)}} \right\}, \tag{6}$$

and $\lambda \in \left(0, \max_i \{1/(w_i C)\}\right)^n$ and $w \in \mathbb{R}^n_{++}$ are the unknowns.

Albeit seemingly complicated, the system above presents some regularities. Three of them are pertinent. First, if countries are ordered as $\frac{T_1}{H_1} < \frac{T_2}{H_2} ... < \frac{T_n}{H_n}$, then equilibrium wages must satisfy $w_1 < ... < w_n$ (proposition 8(i) below). As in section 3, let the function $\lambda(Y)$ give the consumer's Lagrangean multiplier when his wealth is Y (*i.e.* the unique λ that satisfies $\sum_{i \in N} L_i(w, \lambda) = Y$). The function $\lambda(Y)$ decreases from $\left(\frac{1}{w_1 C}\right)$ to zero as Y increases from zero to infinity as shown in figure 4, and by equation (6) a consumer with income Y demands goods from country i if and only if $\left[\lambda(Y) < \frac{1}{w_i C}\right]$. Thus, the second point: the array of countries from which a consumer purchases products expands from the poorest to the richest as his income increases. This is a neat generalization of the demand pattern found in Stokey, where rich consumers demand goods from both the North and the South, and poor households only consume Southern goods. It is, however, different from FH. There, poor households do purchase only Southern goods, but rich ones do not demand goods from the South.[10] Also worth mentioning are the similarities to Matsuyama (2000). In his model, goods are ranked according to their level of essentiality, and the poorer the country the larger the comparative advantage in the more essential goods. Therefore, precisely as described in figure 4, as the consumer's income increases the set of countries supplying goods to him orderly expands from poorest to richest until all countries are included. Despite of the similarities, the two models are easily distinguishable empirically - in Matsuyama, the unit price of goods do not vary

[10]Here, rich households always demand some goods from low-income countries because the support of the F_i's overlap (equals R_{++} for all $i \in N$). Consequently, even the poorest country will be the most efficient producer of a small fraction of the goods, which all consumers buy from it. The choice of other distributions could clearly yield the pattern predicted by FH.
In FH and Stokey there is inequality within countries; so it is possible to have consumers purchasing only domestic goods and still have trade. To avoid confusion, I recall that in section 3 we had only generalized the pattern of production in FH and Stokey. Since their models are specific cases of mine (inequality does not change the results), Linder's hypothesis may also fail for an arbitrary choice of parameters there.

with importer's per capita income. The third remark is formalized by proposition 8. (Its weak version with $\frac{T_i}{H_i} \leq \frac{T_k}{H_k}$ also holds.)

Proposition 8 *Consider any economy* $\{H_i, T_i\}_{i \in N}$*, and let* w *be equilibrium wages. If* $i, k \in N$ *are such that* $\frac{T_i}{H_i} < \frac{T_k}{H_k}$*, then*

(i) $w_i < w_k$ *;*

(ii) $\frac{G_i(w,\lambda(Y))}{G_k(w,\lambda(Y))}$ *is strictly decreasing in* Y *whenever* $G_k(w, \lambda(Y)) > 0$*, and*

(iii) $\frac{w_i L_i(w,\lambda(Y))}{w_k L_k(w,\lambda(Y))}$ *is strictly decreasing in* Y *whenever* $L_k(w, \lambda(Y)) > 0$.

In words, contrary to the general set up of section 2, the fraction of goods purchased from a poor source relative to a rich one is strictly decreasing in consumer's income. This is true when goods are measured both in *numbers* (ii) and in *expenditures* (iii). Interestingly, the formulas of these ratios also have meaningful interpretations:

$$\frac{\text{number of goods from country } i}{\text{number of goods from country } k} = \frac{G_i(w,\lambda)}{G_k(w,\lambda)} = \frac{T_i w_i^{-\theta}(1-\lambda w_i C)^{2\theta}}{T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta}},$$

$$\frac{\text{expenditures in goods from country } i}{\text{expenditures in goods from country } k} = \frac{w_i L_i(w,\lambda)}{w_k L_k(w,\lambda)} = \frac{T_i w_i^{-\theta}(1-\lambda w_i C)^{2\theta-2}(1-(\lambda w_i C)^2)}{T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta-2}(1-(\lambda w_k C)^2)}.$$

where $\lambda = \lambda(Y)$. The terms $\left(\frac{T_i w_i^{-\theta}}{T_k w_k^{-\theta}}\right)$ are constant for all λ. The remaining $\left[\frac{(1-\lambda w_i C)^{2\theta}}{(1-\lambda w_k C)^{2\theta}}\right]$ and $\left[\frac{(1-\lambda w_i C)^{2\theta-2}(1-(\lambda w_i C)^2)}{(1-\lambda w_k C)^{2\theta-2}(1-(\lambda w_k C)^2)}\right]$ may be interpreted as a 'bias' towards consuming goods from the lower income country - they are both strictly greater than 1 whenever $w_i < w_k$. This bias is strictly increasing in λ (it is larger for poor consumers), and approaches 1 as λ tends to zero (consumer's income Y tends to infinity) or C tends to zero (standardization of low-quality products vanishes).

4.1 Welfare and Technology

Consider an increase in a small country's technology parameter T_i with the corresponding increase in its wage w_i to clear the labor market.[11] Observe from the cost function $C(q,t) = q(C + tq)$ that the gains in productivity will be biased towards the high-quality goods, capturing the notion discussed in section 2.1.2 that the production of old low-quality goods is already standardized and the learning possibilities exhausted. A decrease in t thus 'flattens' the higher end of the cost curve without significantly altering its lower end (arrow 1 in figure 5). Therefore, as the wage rises to adjust to the change (arrow 2), the cost of the low-quality products increases relative to the original cost curve $w^0 C^0$, and only the high-quality products remain cheaper. Consequently, if the change in (T_i, w_i) benefits a certain country different from i, it will also benefit all countries with income above it as stated in proposition 9.

[11] By "small", I assume that there are several larger countries with similar levels of income, so that the country in question is not the major consumer of any country's labor. This implies that all the wealth effects are negligible and consequently its wage must increase as a consequence of the technological improvement - $L_i(w, \lambda)$ is strictly increasing in T_i and strictly decreasing in w_i for all $\lambda < \frac{1}{w_i C}$.

Proposition 9 *Suppose country i is small relative to the rest of the countries. Then, if an increase in i's technology parameter increases the welfare of country $k \neq i$ it will increase the welfare of all countries $k' \geq k$.*

4.1.1 North-South trade

The static analysis of both FH and Stokey yielded the following results:

R1. A technology improvement in the South increases the welfare of both the North and the South.

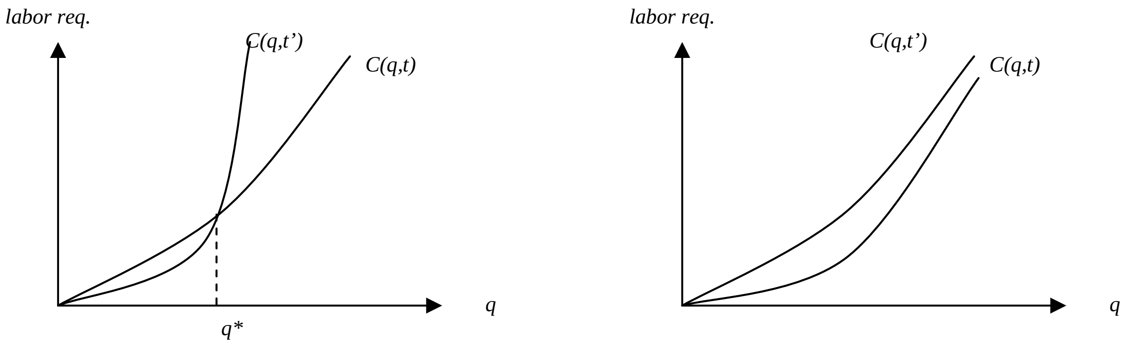
R2. A technology improvement in the North benefits the North, but may decrease the welfare of the South.

These results also hold in the present model, if technology changes are Hicks neutral. To see this, suppose there are only two countries, N (North) and S (South), with parameters $\frac{T_S}{H_S} < \frac{T_N}{H_N}$. Consider a positive technology shock that is not bias towards the high-quality goods. That is, let unit costs be $w_S C(q,t) = w_S A[q(C+qt)]$ and suppose the constant $A > 0$ decreases. As w_S increases to clear the market, the demand for labor increases faster in the North than in the South (the real incomes of both N and S have increased, so proposition 8.iii applies). Therefore, the change in w_S must be smaller than that of A - *i.e.* $(w_S A)$ decreases - making Southern goods cheaper after the shock and all consumers better off, as per result R1. Using a similar argument for the North, we find that the increase in w_N may be disproportionately larger than a positive technology shock in the North. Consequently, the net effect in the price of Northern goods from the perspective of Southern consumers is ambiguous yielding result R2.[12]

Nonetheless, the natural way to think about technology improvements here is through increases T_S and T_N, a change which confirms R1 and violates R2. This is not in conflict with FH and Stokey; they only consider Hicks neutral technology changes while the efficiency gains from increases in T are biased towards high-quality goods. An increase in T_S decreases the cost of goods typically produced in the North disproportionately (high-quality products). Contrary to the argument above, even as the wage in the South increases, consumers may still shift their consumption to Southern labor. Hence, the net effect of the change for Northern households is no longer clear.

For concreteness, table 1 shows a numeric example in which an increase in the technology parameter of the poor country decreases the welfare of the rich one (contradicting R2). The significance of this result is derived from the product cycle literature. Technological progress in endogenous growth models and in Kremer (1993) are biased towards goods associated to the high-quality ones here. In the first it occurs through the introduction of new products, and in the latter an increase in human capital causes a disproportional efficiency gain in the production of goods involving a lot of tasks (the high-technology ones). Thus, the finding that static welfare analysis of these technology changes contradict previous accord may be of interest.

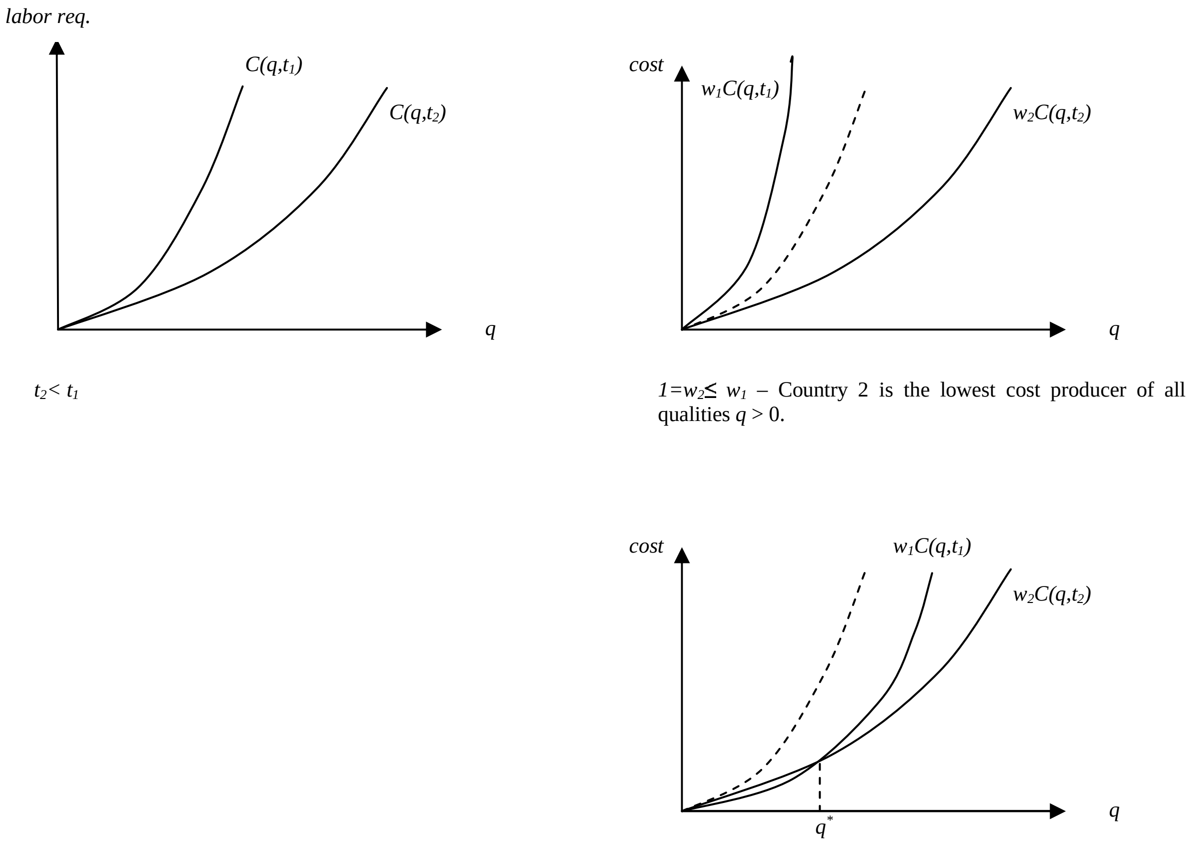
[12] As in FH, for the argument above I only consider the equilibrium where w_i is increasing in T_i for $i = S, N$, which always exists. Stokey finds conditions for the equilibrium to be unique.



A2(iv) holds, but not (iii): Both technology curves are identical (standardized) at quality q^*, but not for qualities lower than it.

A2(iii) holds, but not (iv): Technology curves become more similar as quality increases.

Figure 1:

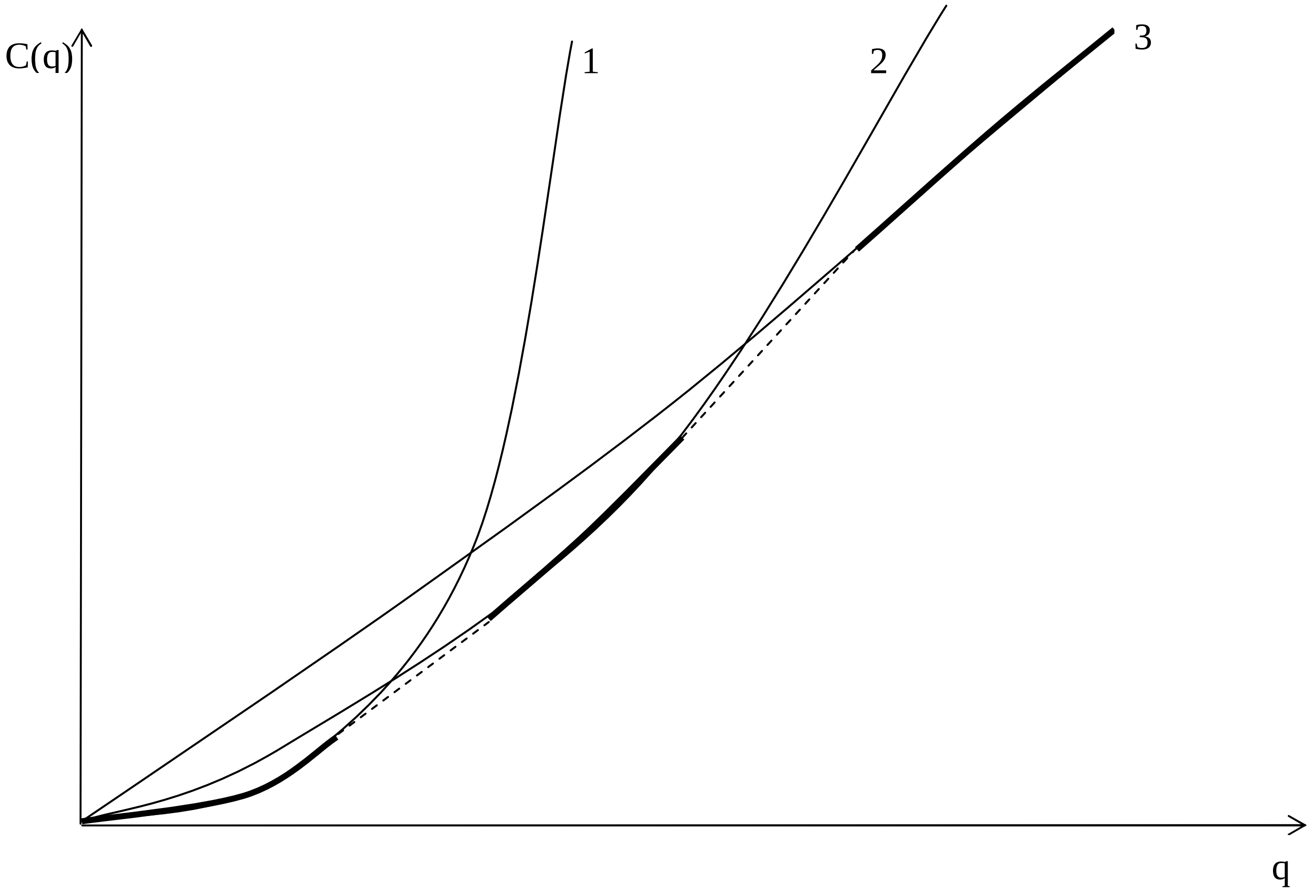


$t_2 < t_1$

$1 = w_2 \leq w_1$ – Country 2 is the lowest cost producer of all qualities $q > 0$.

$1 = w_2 > w_1$ – Country 2 is the lowest cost producer of all qualities $q > q^*$, and country 1 of all qualities $q < q^*$.

Figure 2:



The figure illustrates a typical wealth expansion path for some good with realization *t*. The thin lines indicate the cost curves of countries 1, 2 and 3, where $w_1 < w_2 < w_3$ and $t_1 > t_2 > t_3$. The thick line is the wealth expansion path, and the dotted intervals indicate its discontinuities. Note that as the consumer's wealth increases, his demanded quality increases and the lowest cost producer shifts from country 1 to 2 to 3.

Figure 3:

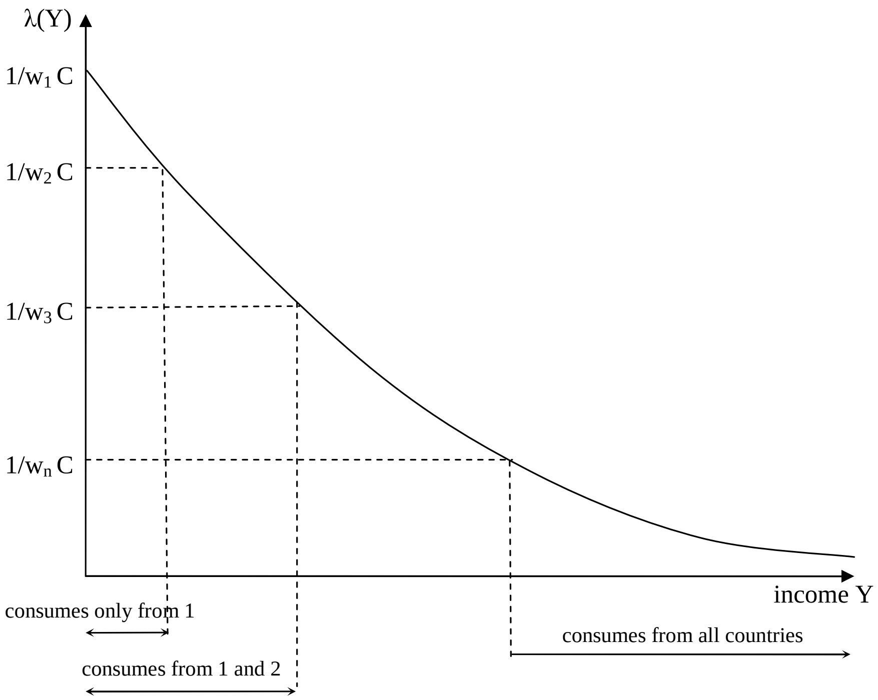


Figure 4:

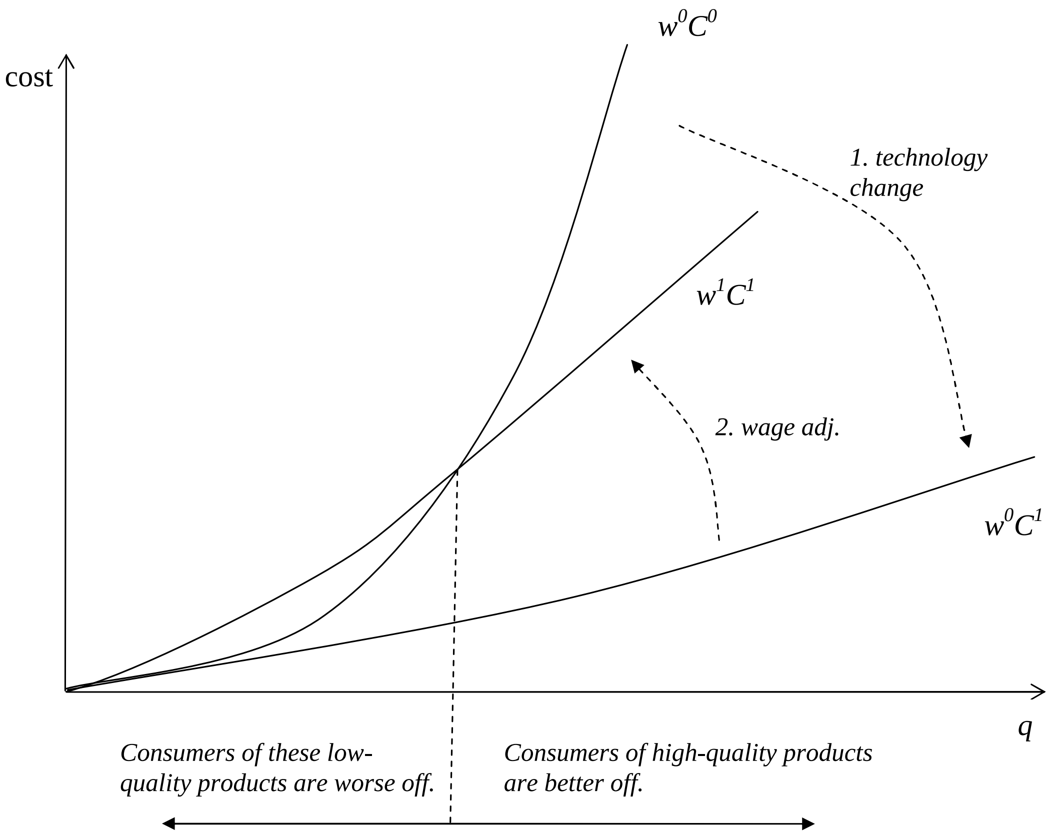


Figure 5:

C = 0.1 (low)

Tech. improvement in the South with small bias (TN=20)

TS	US	Δ	UN	Δ
3.00	1.96		2.72	
6.00	2.25	*0.29*	2.76	*0.04*
9.00	2.43	*0.18*	2.79	*0.03*
12.00	2.57	*0.14*	2.80	*0.02*
15.00	2.68	*0.11*	2.82	*0.01*
18.00	2.78	*0.09*	2.83	*0.01*
20.00	2.83	*0.06*	2.83	*0.01*

Tech. improvement in the North with small bias (TS=10)

T2	US	Δ	UN	Δ
12.00	2.46		2.53	
17.00	2.48	*0.02*	2.71	*0.17*
20.00	2.48	*0.01*	2.79	*0.08*
30.00	2.50	*0.02*	3.01	*0.22*
40.00	2.51	*0.01*	3.18	*0.17*
50.00	2.52	*0.01*	3.31	*0.13*
60.00	2.52	*0.00*	3.42	*0.11*

C = 0.5 (high)

Tech. improvement in the South with large bias (TN=20)

TS	US	Δ	UN	Δ
3.00	1.25		1.57	
6.00	1.37	*0.12*	1.57	*-0.001*
9.00	1.44	*0.07*	1.57	*-0.002*
12.00	1.48	*0.05*	1.57	*-0.002*
15.00	1.52	*0.04*	1.57	*-0.002*
18.00	1.55	*0.03*	1.56	*-0.002*
20.00	1.56	*0.02*	1.56	*-0.001*

Tech. improvement in the North with large bias (TS=10)

TN	US	Δ	UN	Δ
12.00	1.46		1.49	
17.00	1.46	*-0.003*	1.55	*0.05*
20.00	1.45	*-0.002*	1.57	*0.02*
30.00	1.45	*-0.005*	1.63	*0.06*
40.00	1.45	*-0.003*	1.67	*0.04*
50.00	1.44	*-0.003*	1.70	*0.03*
60.00	1.44	*-0.002*	1.72	*0.02*

The parameters not indicated in the table are $\theta = 2$, H1=H2=1.

The upper quadrants show technology increases in the South and in the North with a small bias - *C* is low so that there is little production standardization. All consumers are better off after the changes. In the lower quadrants *C* is large and thus technology improvements are bias to high-quality products. In both cases, the country that did not incur the technology change is made worse off.

Table 1: Comparative statics of technology changes

5 Appendix

5.1 Proof of proposition 1

As commented in the main text, we will look for the demand for labor from each country implicit in the consumers demand for individual goods, and find the market clearing wages.

Prices. For all wages $w \in \mathbb{R}^n$ and goods $t \in \mathbb{R}^n$, the equilibrium price curve is $P(w,q,t) = \min_{i \in N}\{w_i C(q,t_i)\}$. By A2, $P(w,q,t)$ is a continuous function differentiable almost everywhere, but it is in general not convex. (This is expected in a quality model; it is a direct implication of the fact the provider of the lowest cost product may change as quality increases.)

Utility maximization. Fix a wage rate $w \in \mathbb{R}^n_{++}$. A consumer with income $Y > 0$ chooses $q : \mathbb{R}^n \to \mathbb{R}_+$ to maximize $U(q) = \int_\tau u(q(t))dF(t)$ subject to the budget constraint $\int_\tau P(w,q(t),t)dF(t) \le Y$. Define $q^*(w,\lambda,t)$ as the quality of good $t = (t_1,...,t_n)$ chosen by the consumer when wages are $w = (w_1,...,w_n)$ and the Lagrangean multiplier of his maximization problem is λ. Since his problem can have multiple solutions, q^* is a correspondence.

Lemma 10 *The correspondence q^* satisfies: (i) it is a singleton and is continuous almost everywhere; (ii) it is strictly decreasing in λ, and (iii) $\lim_{\lambda\to 0} q^*(w,\lambda,t) = \infty$ and $\lim_{\lambda\to\infty} q^*(w,\lambda,t) = 0$ for all $(w,t) \in (\mathbb{R}^n_{++} \times \mathbb{R}^n)$.*

(Lemma 10 is proved below in subsection 5.1.2.) Substituting q^* in the budget constraint, we find λ:

$$\int_\tau P(w, q^*(w,\lambda,t),t)dF(t) = Y \tag{7}$$

By lemma 10, continuity of f and compactness of τ, the LHS of (7) is well-defined, continuous and strictly decreasing in λ and has limits ∞ and 0 as λ tends to 0 and ∞, respectively. Hence, for all $Y \in \mathbb{R}_{++}$, equation (7) has a unique fixed point $\lambda(Y)$, which is continuous and strictly decreasing in Y. We can then calculate the demand for each country's labor of a consumer with income Y facing wages w. For $i = 1,...n$, define $L_i^*(w,Y)$ as

$$L_i^*(w,Y) = \int_\tau L_i(w,Y,t)dF(t)$$

where $L_i(w,Y,t) = C(q^*(w,\lambda(Y),t),t_i)$ if $i \in \arg\max_{k\in N}\{w_k C(q^*(w,\lambda(Y),t),t_k)\}$ (good t is purchased from i) and 0 otherwise. By lemma 10, continuity of λ, continuity of f and compactness of τ, $L_i(w,Y)$ is a continuous function (as opposed to a correspondence).

Market clearing. Finally, $w \in \mathbb{R}^n_{++}$ is a set of equilibrium wages if and only if it clears the labor market:

$$\sum_{k\in N} H_k L_i^*(w,w_k) - H_i = 0 \text{ for all } i \in N.$$

Lemma 11 *A market clearing wage $w \in \mathbb{R}^n_{++}$ always exists.*

To conclude the proof we only need to prove lemmas 10 and 11. We start with the latter because the former will be used in the proofs of propositions 2 through 7.

5.1.1 Proof of lemma 11

Define $z : \mathbb{R}^n \to \mathbb{R}^n_{++}$ as the excess demand vector for the labor of each country for any set of strictly positive wages:

$$z_i(w) = \left(\sum_{k \in N} H_k L_i^*(w, w_k) - H_i \right) \quad \text{for } i = 1, ..., n$$

It is sufficient to prove that: (i) z is continuous; (ii) z is homogeneous of degree zero; (iii) $w \cdot z(w) = 0$ for all w; (iv) there is an $s > 0$ such that $z_i(w) > -s$ for all $i \in N$ and all $w \in \mathbb{R}^n_{++}$, and (v) if $w^m \to w$, where $w \neq 0$ and $w_i = 0$ for some i, then $\max\{z_1(w^m), ..., z_n(w^m)\} \to \infty$. (see Mas-Colell et al., 1995 p.585)

Property (i) comes continuity of L_i, (ii) from the fact that the consumer's problem remains unchanged if it is multiplied by a constant; (iii) is the sum of all the consumers' budget constraints. To prove (iv) let $s = \max\{H_i\}_{i \in N}$ and observe that $L_i^* \geq 0$. (v) From the definition of $\hat{q}$ and A1, we have that $\hat{q}(w_i^m, \lambda^m, t_i) \to \infty$ as $w_i^m \to 0$ if $\lambda^m \nrightarrow \infty$ (as it occurs to at least some consumer whose wealth does not converge to zero - $w \neq 0$). Therefore, $q^*(w^m, \lambda^m, t) \to \infty$ for all t. Since there is only a finite number of countries where q can be produced, we must have that $L_k^*(w^m, w_k^m) \to \infty$ for some producer j and some consumer k (for which $w_k \neq 0$). ■

5.1.2 Proof of lemma 10

The consumer's problem is additively separable. Hence, $q^*(w, \lambda, t)$ must solve $\max_q\{u(q) - \lambda P(w, q, t)\} = \max_q\{u(q) - \lambda \min_{i \in N}\{w_i C(q, t_i)\}\} = \max_{i \in N}\{\max_q\{u(q) - w_i C(q, t_i)\}\}$. In words, the consumer chooses the best quality conditional on buying good t from country i (inner bracket), and then selects the best producer (outer bracket). Define $V(w_i, \lambda, t_i)$ and $\hat{q}(w_i, \lambda, t_i)$ as the value and solution to the inner problem $\max_q\{u(q) - w_i C(q, t_i)\}$. By A1 and A2, $\hat{q}$ is unique and well-defined for all $(w_i, \lambda) \gg 0$; it is implicitly defined in the first order conditions (foc):

$$u'(q) - \lambda w_i C_1(q, t_i) \tag{8}$$

where C_1 is the derivative of C with respect to its first argument.

Property **(iii)** is immediate the facts that $\hat{q}(w_i, \lambda, t_i)$ has these limits for all i (by equation 8), and $q^*(w, \lambda, t) = \hat{q}(w_i, \lambda, t_i)$ for some i.

Properties **(i)** and **(ii)** will be proven together. **Note:** *Steps 1-5 below essentially prove that the wealth expansion path follows the description in figure 3. This will be taken for granted and extensively used in the succeeding propositions.*

Fix t and $w > 0$. Note that for all i, $\hat{q}(w_i, \lambda, t_i)$ are continuous functions. If countries' cost curves do not cross then the proof is immediate ($q^*(w, \lambda, t) = \hat{q}(w_i, \lambda, t_i)$ for all λ).

So, we let countries 1 and 2 be such that $w_1 < w_2$ and $[w_2 C(q, t_2) < w_1 C(q, t_1)]$ for some $q > 0$. To simplify the notation on this first part of the proof where t and w do not change, let $q_i(\lambda) = \hat{q}(w_i, \lambda, t_i)$, $C_i(q) = [w_i C(q, t_i)]$ and $V_i(\lambda) = V(w_i, \lambda, t_i)$ for $i \in N$. Also for simplicity let 1 and 2 be the only two countries.[13]

We will construct a minimum cost curve that makes the problem of maximizing $[u(q) - \lambda \min\{C_1(q), C_2(q)\}]$ convex.

step 1. *There exists a $\tilde{q} > 0$ such that $C_1(q) \leq C_2(q)$ if and only if $q \leq \tilde{q}$ (with equality only at $\tilde{q}$).* $[C_1(q) = C_2(q)] \equiv \left[\frac{C(q,t_1)}{C(q,t_2)} = \frac{w_1}{w_2}\right]$. By A2(iii) the LHS of the latter equation is strictly increasing in q (clearly $t_1 > t_2$ since $w_1 < w_2$ and the cost curves cross). Hence, it can only equal w_1/w_2 at most at a single value of q. By assumption such a value of q exists and $\tilde{q} > 0$.

step 2. *There exists a $\tilde{\lambda}$ such that $V_1(\tilde{\lambda}) = V_2(\tilde{\lambda})$.* Since $\lim_{\lambda\to\infty} q_i(\lambda) = 0$, for λ large enough we have $q_i(\lambda) < \tilde{q}$ for $i = 1, 2$. Thus, $C_1(q_i(\lambda)) < C_2(q_i(\lambda))$, and $V_1(\lambda) = u(q_1(\lambda)) - \lambda C_1(q_1(\lambda)) \geq u(q_2(\lambda)) - \lambda C_1(q_2(\lambda)) > u(q_2(\lambda)) - \lambda C_2(q_2(\lambda)) = V_2(\lambda)$, where the weak inequality comes from the fact that $q_1(\lambda)$ maximizes $[u(q) - \lambda C_1(q)]$. Analogously, $\lim_{\lambda\to 0} q_i(\lambda) = \infty$ implies that for a small enough λ, we have $C_2(q_i(\lambda)) < C_1(q_i(\lambda))$ for $i = 1, 2$, and thus $V_2(\lambda) > V_1(\lambda)$. The existence of $\tilde{\lambda}$ is then established by continuity.

step 3. $q_1(\tilde{\lambda}) < \tilde{q} < q_2(\tilde{\lambda})$. Suppose $q_1(\tilde{\lambda}) \geq \tilde{q}$. Then,

$$\begin{aligned} V_1(\tilde{\lambda}) &= u(q_1(\tilde{\lambda})) - \tilde{\lambda} C_1(q_1(\tilde{\lambda})) \\ &\leq u(q_1(\tilde{\lambda})) - \tilde{\lambda} C_2(q_1(\tilde{\lambda})) && (C_1(q) \geq C_2(q) \text{ for all } q \geq \tilde{q}) \quad (9) \\ &\leq u(q_2(\tilde{\lambda})) - \tilde{\lambda} C_2(q_2(\tilde{\lambda})) && (q_2(\tilde{\lambda}) \text{ maximizes } u - \tilde{\lambda} C_2) \quad (10) \\ &= V_2(\tilde{\lambda}). \end{aligned}$$

By definition of $\tilde{\lambda}$, $V_1(\tilde{\lambda}) = V_2(\tilde{\lambda})$ and thus all inequalities must be substituted by equalities. Hence, lines (9) and (10) imply that $q_2(\tilde{\lambda}) = q_1(\tilde{\lambda}) = \tilde{q}$. Substituting in the foc (equation 8 which defines $q_i(\lambda)$), we have $0 = u'(\tilde{q}) - \tilde{\lambda} C_1'(\tilde{q}) = u'(\tilde{q}) - \tilde{\lambda} C_2'(\tilde{q})$. Rearranging, $C_1'(\tilde{q}) = C_2'(\tilde{q})$. But by the definition of $\tilde{q}$ in step 1, we must have $C_1'(\tilde{q}) > C_2'(\tilde{q})$ which is a contradiction. The case where $q_2(\tilde{\lambda}) \leq \tilde{q}$ is analogous.

step 4. *Construct the cost curve $\tilde{C} \leq \min\{C_1, C_2\}$ that makes the consumer problem convex.* Define $\tilde{C} : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$\tilde{C}(q) = \begin{cases} C_1(q) & \text{for all } q \leq q_1(\tilde{\lambda}) \\ \frac{1}{\tilde{\lambda}}[u(q) - V_1(\tilde{\lambda})] & \text{for all } q_1(\tilde{\lambda}) \leq q \leq q_2(\tilde{\lambda}) \\ C_2(q) & \text{for all } q > q_2(\tilde{\lambda}) \end{cases} \quad (11)$$

By steps 1 and 3, to see that $\tilde{C}(q) \leq \min\{C_1(q), C_2(q)\}$ for all q, we only need to check $q \in [q_1(\tilde{\lambda}), q_2(\tilde{\lambda})]$. For all q in this interval, $V_1(\tilde{\lambda}) = u(q) - \tilde{\lambda}\tilde{C}(q)$. And by definition of $V_1(\tilde{\lambda})$, we have $[V_1(\tilde{\lambda}) \geq u(q) - \tilde{\lambda} C_i(q)]$ for all q and all $i \in \{1, 2\}$. Hence, $[u(q) - \tilde{\lambda}\tilde{C}(q) \geq u(q) - \tilde{\lambda} C_i(q)] \equiv [\tilde{C}(q) \leq C_i(q)]$.

[13] Extending the proof to an arbitrary number of countries is trivial. It only makes the text cumbersome by adding an ordering of the countries and considering different cases.

step 5. Now consider the consumer's modified problem to choose q to maximize $\tilde{V}(q,\lambda) = u(q) - \lambda\tilde{C}(q)$. By checking the first and second order conditions, one can easily verify that the solution $q^*(\lambda)$ will be:

$$q^*(\lambda) = \begin{cases} q_1(\lambda) & \text{if } \lambda > \tilde{\lambda}, \\ [q_1(\tilde{\lambda}), q_1(\tilde{\lambda})] & \text{if } \lambda = \tilde{\lambda}, \\ q_2(\lambda) & \text{otherwise.} \end{cases} \tag{12}$$

And the maximized function satisfies:

$$\tilde{V}(q^*(\lambda),\lambda) = \begin{cases} V_1(\lambda) & \text{if } \lambda > \tilde{\lambda}, \\ V_1(\tilde{\lambda}) = V_2(\tilde{\lambda}) & \text{if } \lambda = \tilde{\lambda}, \\ V_2(\lambda) & \text{otherwise.} \end{cases}$$

Note that the solution above is feasible in the maximization problem with the tighter constraint: $[u(q) - \lambda \min\{C_1(q), C_2(q)\}]$. Hence it must also solve the latter problem (with $q^*(\tilde{\lambda}) = \{q_1(\tilde{\lambda}), q_2(\tilde{\lambda})\}$). Observe also that the solution in (12) is strictly decreasing in λ ($q_1(\lambda)$ and $q_2(\lambda)$ are both strictly decreasing in λ by equation 8), thus establishing property (ii). It is also is a singleton everywhere except at $\tilde{\lambda}$, its only discontinuity point. By the same argument, these properties generalize to the case where there are n countries and possibly $(n-1)$ intersections (the number of discontinuities and values of λ for which the demand is not a singleton will be at most $(n-1)$).

To conclude the proof that $q^*(w,\lambda,t)$ is a singleton and continuous almost everywhere in the other variables, we now fix w and λ and focus on t. Recall that a consumer will buy a product with realization $t \in \tau^n$ from country i only if $V(w_i,\lambda,t_i) = \{u(\hat{q}(w_i,\lambda,t_i)) - \lambda\,[w_i C(\hat{q}(w_i,\lambda,t),t_i)]\} \geq V(w_k,\lambda,t_k)$ for all $k \in N$. By A2(iii) and (iv), $V(w_k,\lambda,t_k)$ is strictly decreasing in t_k. Hence, for all t_i and all $k \in N \setminus \{i\}$, there are only three possibilities: $V(w_i,\lambda,t_i) > V(w_k,\lambda,t_k)$ for all $t_k \in \tau$; $V(w_i,\lambda,t_i) < V(w_k,\lambda,t_k)$ for all $t_k \in \tau$, or there exists a $\tilde{t}_k \in \tau$ such that $V(w_i,\lambda,t_i) \leq V(w_k,\lambda,t_k)$ if and only if $t_k \leq \tilde{t}_k$ (with equality only at $\tilde{t}_k$). Since there are no mass points, the discontinuities in of $q^*(w,\lambda,t)$ in t must be of measure zero. This reasoning holds by replacing t with w. ■

Note that **proposition 3** has the same statement as lemma 10(iii), thus it has already been proven. Proposition 4 will be proven before 2.

5.2 Proof of proposition 4

If $w_i < w_{i+1}$, then $w_k < w_{k'}$ for any k,k' such that $k \leq i < k'$. Thus from the proof of lemma 10, there are two cases of realizations of $t \in \tau$. Either for all $\lambda > 0$, $q^*(w,\lambda,t) = \hat{q}(w_k,\lambda,t_k)$ for some $k \leq i$, or there exists a $\tilde{\lambda}$ such that

$$q^*(w,\lambda,t) = \begin{cases} \hat{q}(w_k,\lambda,t_k) \text{ for some } k \leq i & \text{if } \lambda > \tilde{\lambda}, \\ \bigcup_{k\in(K\bigcup K')}\{\hat{q}(w_k,\lambda,t_k)\} & \text{if } \lambda = \tilde{\lambda}, \\ \hat{q}(w_k,\lambda,t_k) \text{ for some } k > i & \text{otherwise.} \end{cases} \tag{13}$$

where K and K' are nonempty, and $K \subset \{1, ..., i\}$ and $K' \subset \{i+1, ..., n\}$. That is, if t is such that $q^*(w, \lambda, t) = \hat{q}(w_k, \lambda, t_k)$ for some $k \leq i$, then for all $\lambda' < \lambda$ there must be a $k' \leq i$ such that $q^*(w, \lambda', t) = \hat{q}(w_{k'}, \lambda', t_{k'})$. Since $\lambda(Y)$ is strictly decreasing in Y, $\left[\sum_{k \leq i} G_k(w, Y)\right]$ is non-increasing in Y.

To establish strict monotonicity, let $\left[\sum_{k \leq i} G_k(w, Y)\right] \in (0, 1)$. Then, there exist $t, t' \in \text{int}(\tau)$ and $k, l \in N$ with $k \leq i < l$ such that $V(w_k, \lambda, t_k) > V(w_m, \lambda, t_m)$ for all $m \in N \backslash \{k\}$ and $V(w_l, \lambda, t'_l) < V(w_m, \lambda, t'_m)$ for all $m \in N \setminus \{l\}$, where $\lambda = \lambda(Y)$. From continuity of V and convexity of τ, there exists a convex combination of t and t', $t'' \in \text{int}(\tau^n)$, such that $V(w_k, \lambda, t''_k) = V(w_l, \lambda, t''_l) > V(w_m, \lambda, t''_m)$ for some k, l, where $k \leq i < l$ (not necessarily the same as above), and for all $m \in N \setminus \{k, l\}$. Fix $Y' > Y$ so that $\lambda(Y') = \lambda' < \lambda$. Then,

$$V(w_l, \lambda', t''_l) > V(w_m, \lambda', t''_m) \quad \text{for all } m \leq i \text{ and some } l \leq i. \tag{14}$$

Define t''' as $t'''_k = t''_k - \varepsilon$ and $t'''_m = t''_m$ for all $m \neq k$. For $\varepsilon > 0$ small enough, $t''' \in \text{int}(\tau)$ and the strict inequalities in (14) hold. Moreover, for any $\varepsilon > 0$, $V(w_k, \lambda, t'''_k) > V(w_m, \lambda, t'''_m)$ for all $m \neq k$. By continuity of V, the inequalities must all be preserved in a small enough ball around t''', $B_\varepsilon(t''') \subset \text{int}(\tau)$. Hence, $\sum_{k \leq i} G_k(w, Y) \geq \sum_{k \leq i} G_k(w, Y') + \int_{B_\varepsilon(t''')} f(t)dt > \sum_{k \leq i} G_k(w, Y')$.■

5.3 Proof of proposition 2

Let $w_i > w_k$ and suppose both i and k produce the same product category. Then, following the same steps (and notation) of the proof of lemma 10, we have that all qualities q_i and q_k produced by i and k, respectively must satisfy $q_i \geq q_i(\tilde{\lambda}) > q_k(\tilde{\lambda}) \geq q_k$. To prove that this must occur for some goods, let w be the equilibrium wages of some economy such that $w_1 \leq w_2 \leq ... \leq w_n$ and $w_1 < w_n$. There are two cases: (i) consumers in n demand all their goods from countries with wages equal to w_n, or (ii) $\left[\sum_{k \leq i} G_k(w, w_n) \in (0, 1)\right]$ where $i = \max\{k : w_k < w_n\}$. In the first case, the proof is immediate - the demand for the other countries' labor must be strictly positive in equilibrium. The second follows from the proof of proposition 4: there exists a non-empty ball $B_\varepsilon(t) \subset \text{int}(\tau)$ such that consumers in n will purchase all goods in $B_\varepsilon(t)$ from a country $l \in N$ with $w_l = w_n$, and all consumers poorer then them will purchase it from some $l \in N$ with $w_l < w_n$. ■

5.4 Proof of proposition 5

We begin by presenting four lemmas which will be useful in proving propositions 5 through 7. We fix functions C and U satisfying A1 through A3 and refer to their related functions $\hat{q}$, q^*, V and L defined in the proof of proposition 1.

Lemma 12 $\lim_{t \to \infty} V(w, \lambda, t) = u(0) < V(w', \lambda, t')$ *for all* $(w, w', \lambda) \gg 0$ *and all* $t' \in \mathbb{R}$.

Lemma 13 *Let* $w < w'$ *and* $\lambda > \lambda'$. *Then, for all* $t' \in \mathbb{R}$, *there exists a* t *such that* $V(w', \lambda', t') > V(w, \lambda', t)$ *and* $V(w', \lambda, t') < V(w, \lambda, t)$.

Lemma 14 $\lim_{t \to \infty} C(\hat{q}(w, \lambda, t), t) = 0$ *for all* $(w, \lambda) \gg 0$.

Lemma 15 *Given N and F, let w be a set of wages such that $L_i(w, w_k) > 0$ for all $i, k \in N$. Then there exists a vector of populations $\{H_i\}_{i \in N}$ such that the labor market clears with wages w.*

proofs of lemmas. Lemma 12 follows trivially from A3. Lemma 15 is well stated since $L_i(w, w_k)$ depends exclusively on the set of countries and wages, not on the population of each country; its proof follows from a rationale similar to the one of finding equilibrium prices (omitted). Lemma 13 states that for any two consumers λ' richer than λ, two countries $w' > w$ and technology parameter of the richer country t', there exists a technology of the poorer country t such that: the rich consumer λ' strictly prefers to buy the good from the rich country w', and the poorer one λ strictly prefers the poorer source w. To prove it, just increase the value of t from t' to infinity. At $t = t'$, $V(w', \tilde{\lambda}, t') < V(w, \tilde{\lambda}, t')$ for $\tilde{\lambda} = \lambda, \lambda'$, and for t large enough by lemma 12 $V(w', \tilde{\lambda}, t') > V(w, \tilde{\lambda}, t)$ for $\tilde{\lambda} = \lambda, \lambda'$. From the proof of proposition 4, the richer consumer λ' switches his inequality first.

To establish lemma 14, fix $\lambda w > 0$. By A1 and A2(i) and (ii), for each t, there exists a $\overline{q}(w, \lambda, t)$ such that $\lambda w C(q, t) \leq [u(q) - u(0)]$ if and only if $q < \overline{q}(w, \lambda, t)$ with equality only at $\overline{q}$ ($\overline{q}$ is the point at which the curves $\lambda w C(q, t)$ and $[u(q) - u(0)]$ cross). By A3, $\lim_{t \to \infty} \overline{q}(w, \lambda, t) = 0$ (*). Since $\hat{q}(w, \lambda, t)$ maximizes $[u(q) - \lambda w C(q, t)]$, we must have $u(\hat{q}(w, \lambda, t)) - \lambda w C(\hat{q}(w, \lambda, t), t) \geq u(0)$. Thus, $\hat{q}(w, \lambda, t) \leq \overline{q}(w, \lambda, t)$ and $\lambda w C(\hat{q}(w, \lambda, t), t) \leq u(\hat{q}(w, \lambda, t)) - u(0) \leq u(\overline{q}(w, \lambda, t)) - u(0)$. By (*) the RHS of the latter inequality tends to zero as t tends to infinity, and thus so must the LHS as claimed.■

proof of proposition 5. Fix any six real numbers satisfying: $\lambda_p > \lambda_m > \lambda_r > 0$ and $0 < w_1 < w_2 < w_3$. The proof will follow three steps. First, we choose a distribution F of goods $t \in \mathbb{R}^3$ satisfying A4 such that if consumers with Lagrangean multipliers $\lambda_p, \lambda_m, \lambda_r$ were to face an economy with three countries with wages $w = (w_1, w_2, w_3)$ and technology F, then $G_2(w, w_m) < \min\{G_2(w, w_p), G_2(w, w_r)\}$ (*). In step 2 we construct an economy with equilibrium wages w, and technology distribution arbitrarily close to F so that (*) is not altered. In step 3, we add to this economy three small countries with equilibrium wages w_p, w_m, w_r, corresponding to the multipliers $\lambda_p, \lambda_m, \lambda_r$ in the referred economy.

Step 1. Suppose that country 1 has two different technology parameters (t_{1a}, t_{1b}); 2 has two (t_{2a}, t_{2b}), and 3 has only one t_3. We will choose the values of t appropriately so that consumers p, m, r will choose goods from countries 1, 2 and 3 according to the following matrix:

	t_{2a}	t_{2b}
t_{1a}	$p, m \to 1;\ r \to 2$	$p, m \to 1;\ r \to \{1, 3\}$
t_{1b}	$p, m, r \to 2$	$p \to 2;\ m, r \to 3$

where the arrows point from consumer to preferred producer, and the brackets $\{1, 3\}$ imply that the choice of (t_{1a}, t_{2a}, t_3) below is not enough to specify r's preferences for this good. In any case, if $\text{p}(t_{1a}) = \text{p}(t_{1b}) = \text{p}(t_{2a}) = \text{p}(t_{2b}) = \frac{1}{2}$; $\text{p}(t_3) = 1$, $\frac{1}{4} = G_2(w, w_m) < \min\{G_2(w, w_p), G_2(w, w_r)\} = \frac{1}{2}$ as desired in (*). Notice that the shadow value of wealth, λ and t are fully sufficient to specify the preferences of consumers p, m, r (*i.e.* $\max_{i \in \{1,2,3\}} \{V(w_i, \lambda_k, t_i) = \max_q \{u(q) - \lambda_k w_i C(q, t_i)\}\}$ for $k = p, m, r$). For simplicity, I will denote $V_{i\alpha}^k = V(w_i, \lambda_k, t_{i\alpha})$ for $k = p, m, r$; $i = 1, 2, 3$, and $\alpha = a, b$.

Finally, we choose the t's. **1 -** Pick $t_3 \in \mathbb{R}$ arbitrarily. **2 -** Choose t_{2b} such that $V_{2b}^p > V_3^p$ and $V_3^{m,r} > V_{2b}^{m,r}$, which exists by lemma 13 (SE quadrant ignoring t_{1b}). **3 -** Let $t_{2a} = t_3$ so that $V_{2b}^{p,m,r} > V_3^{p,m,r}$ (SW quadrant). **4 -** Using lemma 13 again, choose t_{1a} such that $V_{1a}^{p,m} > V_{2a}^{p,m}$ and $V_{2a}^r > V_{1a}^r$ (NW quadrant). Automatically, this implies that preferences in the NE quadrant also hold since $t_{2b} > t_{2a}$. **5 -** Finally, just pick t_{1b} large enough so that $V_{2b}^{p,m,r} > V_{1b}^{p,m,r}$, which exists by lemma 12.

Step 2. We now construct a three country economy $\{(H_1, H_2, H_3), F\}$ such that w is an equilibrium wage and technologies follow step 1. Let the distribution of technologies be $\mathrm{p}(t_{1a}) = \mathrm{p}(t_{1b}) = \mathrm{p}(t_{2a}) = \mathrm{p}(t_{2b}) = (1-\Delta)/2$; $\mathrm{p}(t_3) = (1-\Delta)$, and $\mathrm{p}(t_{1h}) = \mathrm{p}(t_{2h}) = \mathrm{p}(t_{3h}) = \Delta$, where $\Delta > 0$ be small enough so that irrespective of the choices of p, m and r for goods the new goods t_{ih}, (*) still holds in this modified economy.

By lemma 15, we only need to choose (t_{1h}, t_{2h}, t_{3h}) such that $L_i(w, w_k) > 0$ for all $i, k \in N$. Note that as we increase t_{ih} the economy changes, and hence so does the function $\lambda(Y)$, which provides the Lagrangean multiplier for each income level Y. However, by lemma 14, as t_{ih} tends to infinity, these changes become marginal. Therefore, there exist (t_{1h}, t_{2h}, t_{3h}) large enough such that all consumers purchase from country i all goods with technology parameter $t_i \neq t_{ih}$ (one of the parameters chosen in step 1) and $t_k = t_{kh}$ for all $k \in N \setminus \{i\}$. Since for all countries $i \in N$, the set of these goods is of measure $\Delta(1-\Delta)^2 > 0$, we have found a distribution of technologies in which $L_i(w, w_k) > 0$.

Step 3. By the same argument, we can always choose t_{ih} high enough to satisfy two conditions: (i) $V(w_i, \lambda_k, t_i) > V(w_l, \lambda_k, t_{lh})$ for all $t_i \neq t_{ih}$, $i, l = 1, 2, 3$ and $k = p, m, r$, and (ii) there exists a $t_{rl} \in \mathbb{R}$ such that $V(w_r, \lambda_i, t_{rl}) > V(w_k, \lambda_i, t_{kh})$ for all $k \in \{1, 2, 3\}$ and all $i \in \{1, 2, 3, p, m, r\}$ where $\lambda_i = \lambda(w_i)$ for $i = 1, 2, 3$ and $w_r = \lambda^{-1}(\lambda_r)$.[14] Condition (ii) assures the existence of a t_{rl} low enough so that all consumers prefer to buy a good $t = (t_{1h}, t_{2h}, t_{3h}, t_{rl})$ from a country with wages w_r and parameter t_{rl} than from countries 1, 2, 3. Clearly, if it holds for r it must also hold for p and m, since $w_p < w_m < w_r$.

We only show how to add consumer r to the economy from step 2. The others follow from the same procedure. By A3, there exists t_{rh} such that $V(w_r, \lambda_i, t_{rh}) < V(w_k, \lambda_i, t_{k\alpha})$ for all $k = 1, 2, 3$, $\alpha = a, b, h$ and $i \in N = \{1, 2, 3, r\}$. Let the country r's distribution of parameters be $\mathrm{p}(t_{rl}) = \Delta'$ and $\mathrm{p}(t_{rh}) = (1-\Delta')$. As before, all consumers will buy good t from i whenever $t_i \neq t_{ih}$ and $t_k = t_{kh}$ for all $k \in N \setminus \{i\}$. Thus, $L_k(w, w_i) > 0$ for all $i, k \in N$. In addition, note that labor from r will be strictly positive only when $t_r = t_{rl}$. Hence, for $\Delta' > 0$ small enough, the disturbance in the function λ will be small enough (by lemma 12 its changes are bounded even as $t_{rh} \to \infty$) and not reverse any of the preferences over production sources established above, nor the inequality $G_2(w, w_m) < \min\{G_2(w, w_p), G_2(w, w_r)\}$.■

5.5 Proof of proposition 6

1. Fix arbitrarily $0 < w_1 < w_2 < w_3$, $\lambda > 0$ and $t_{3l} \in \mathbb{R}$. By lemma 13, we can also choose t_{1l} and t_{2l} such that $V(w_3, \lambda, t_{3l}) > V(w_2, \lambda, t_{2l}) = V(w_1, \lambda, t_{1l})$.

[14] The discreteness of the distribution F implies that the function λ may not be invertible everywhere. But this occurs only in the cases where $\max_i\{V(w_i, \lambda, t_i)\}$ has more than one solution. This can always be avoided to occur for λ_r, λ_m and λ_p in the construction above by simply trembling the values of t.

2. We construct an economy with $N = \{1, 2, 3\}$ and $\{H_i, F_i\}_{i\in N}$ such that (w_1, w_2, w_3) chosen in step 1 are equilibrium wages. Define the distribution over technologies as $\text{prob}(t_{il}) = \text{prob}(t_{ih}) = \frac{1}{2}$ for $i = 1, 2, 3$, where t_{1l}, t_{2l}, t_{3l} are the values defined in step 1. By lemmas 12 and 13, we can choose t_{1h}, t_{2h}, t_{3h} as to satisfy: (i) that $V(w_1, \lambda, t_{1h}) > \max\{V(w_2, \lambda, t_{2h}), V(w_3, \lambda, t_{3h})\}$ and $V(w_3, \lambda, t_{il}) > \max_i\{V(w_i, \lambda, t_{ih})\}$ for all $i \in N$, (ii) and consumers with wages w_1, w_2 and w_3 always purchase good t from country i whenever $t_i = t_{il}$ and $t_k = t_{kl}$ for all $k \in N \setminus \{i\}$. Hence, $L_i(w, w_k) > 0$ for any $i, k \in N$ which by lemma 15 suffices to conclude our construction.

3. Consider the problem $\max_i \{\max_q\{u(q) - \lambda w_i C(q, t_i)\}\}$; that is, the source of production most preferred by a consumer with multiplier λ, chosen in step 1. By construction, if t is such that $t_3 = t_{3l}$, , then $i = 3$ solves the problem, otherwise $t_3 = t_{3h}$ and the solutions are as follows:

	t_2^l	t_2^h
t_1^l	1 and 2	1
t_1^h	2	1

Since $V(w_2, \lambda, t_{2l}) = V(w_1, \lambda, t_{1l})$, $q^*(w, \lambda, t) = \{\hat{q}(\lambda, w_1, t_{1l}), \hat{q}(\lambda, w_2, t_{2l})\}$ for all goods $t = (t_{1l}, t_{2l}, t_{3h})$, which have measure $\frac{1}{8}$ (*i.e.* λ is the value for which the wealth expansion path "jumps" in figure 3). Hence, there is a range of incomes $[Y, Y']$ with $Y < Y'$ all with the shadow value of wealth equal to λ. As income increases in this region, the demand for qualities of $t \neq (t_{1l}, t_{2l}, t_{3h})$ is constant, and $q(t_{1l}, t_{2l}, t_{3h})$ will equal $\hat{q}(\lambda, w_1, t_{1l})$ for some proportion α of these goods and $\hat{q}(\lambda, w_2, t_{2l})$ for $(1-\alpha)$, where $\alpha \to 0$ (or $\to 1$) as $Y" \downarrow Y$ (or $Y" \uparrow Y'$). Therefore,

$$\frac{\sum_{i\leq 2} w_i L_i^*(w, Y)}{w_3 L_3^*(w, Y)} = \frac{\frac{1}{8} w_1 C(\hat{q}(w_1, \lambda, t_{1l}), t_{1l}) + \frac{1}{8}\sum_{t\in T} P(w, q^*(w, \lambda, t), t)}{\frac{1}{2} w_3 C(\hat{q}(w_3, \lambda, t_{3l}), t_{3l})}$$

$$< \frac{\frac{1}{8} w_2 C(\hat{q}(w_2, \lambda, t_{2l}), t_{2l}) + \frac{1}{8}\sum_{t\in T} P(w, q^*(w, \lambda, t), t)\}}{\frac{1}{2} w_3 C(\hat{q}(w_3, \lambda, t_{3l}), t_{3l})} \quad (15)$$

$$= \frac{\sum_{i\leq 2} w_i L_i^*(w, Y')}{w_3 L_3^*(w, Y')} \quad (16)$$

where $T = \{t : t_3 = t_{3h} \text{ and } t \neq (t_{1l}, t_{2l}, t_{3h})\}$. The inequality comes from the fact that by steps 1 and 3 of proof of lemma 10, we have $w_1 C(\hat{q}(w_1, \lambda, t_{1l}), t_{1l}) < w_2 C(\hat{q}(w_2, \lambda, t_{2l}), t_{2l})$, which are the only terms that change in (15). Using the budget constraints, we get that $\frac{\sum_{i\leq 2} w_i L_i^*(w,Y)}{Y} < \frac{\sum_{i\leq 2} w_i L_i^*(w,Y')}{Y'}$. Arbitrarily small countries with wages Y and Y' can be added to the economy following step 3 of the proof of proposition 5.■

5.6 Proof of proposition 7

Suppose not. Suppose there exist $(t_1, t_2, w_1, w_2, \lambda, \lambda')$ with $w_1 < w_2$, $\lambda > \lambda'$ and t_1, t_2 such that

$$\frac{C(\hat{q}(w_1, \lambda, t_1), t_1)}{C(\hat{q}(w_2, \lambda, t_2), t_2)} > \frac{C(\hat{q}(w_1, \lambda', t_1), t_1)}{C(\hat{q}(w_2, \lambda', t_2), t_2)}. \quad (17)$$

The values of λ and λ' can be chosen so that either $V(w_1, \tilde{\lambda}, t_1) > V(w_1, \tilde{\lambda}, t_2)$ for both $\tilde{\lambda} = \lambda, \lambda'$, or $V(w_1, \tilde{\lambda}, t_1) < V(w_1, \tilde{\lambda}, t_2)$ for both $\tilde{\lambda} = \lambda, \lambda'$. For simplicity, we assume the former holds (the proof of both cases are identical). Consider the following distribution of technologies of countries 1 and 2: $p(t_1) = p(t_{1h}) = p(t_2) = p(t_{2h}) = \frac{1}{2}$. By lemmas 12, 14 and 15, there exists a $\underline{t}$ such that for all $t_{1h} = t_{2h} \geq \underline{t}$, there exists an economy (H_1, H_2, F) with equilibrium wages (w_1, w_2). That is, for t_h large enough, $L_i(w, w_k) > 0$ for all $i, k \in \{1, 2\}$. In addition, in all these economies, there exist $Y < Y'$ such that $Y = \lambda^{-1}(\lambda)$ and $Y' = \lambda^{-1}(\lambda')$. From inequality (17) and $lim_{t_{1h} \to \infty} C(\hat{q}(w, \lambda, t_{1h}), t_{1h}) = 0$ (lemma 14), for $t_{1h} = t_{2h}$ large enough,

$$\begin{aligned}\frac{w_1 L_1(w, Y)}{w_2 L_2(w, Y)} &= \frac{\frac{1}{2} w_1 C(\hat{q}(w, \lambda, t_1), t_1) + \frac{1}{4} w_1 C(\hat{q}(w, \lambda, t_{1h}), t_{1h})}{\frac{1}{4} w_2 C(\hat{q}(w, \lambda, t_2), t_2)} \\ &> \frac{\frac{1}{2} w_1 C(\hat{q}(w, \lambda', t_1), t_1) + \frac{1}{4} w_1 C(\hat{q}(w, \lambda', t_{1h}), t_{1h})}{\frac{1}{4} w_2 C(\hat{q}(w, \lambda', t_2), t_2)} \quad = \frac{w_1 L_1(w, Y')}{w_2 L_2(w, Y')}. \blacksquare\end{aligned}$$

5.7 Proof of proposition 8

(i) Suppose $\frac{T_i}{H_i} < \frac{T_k}{H_k}$, and $w_i > w_k$. If $\lambda \leq 1/(w_i C)$, then $L_i(w, \lambda) = 0$; otherwise

$$\frac{L_i(w, \lambda)}{L_k(w, \lambda)} = \frac{T_i w_i^{-1-\theta} (1 - \lambda w_i C)^{2\theta - 2} (1 - (\lambda w_i C)^2)}{T_k w_k^{-1-\theta} (1 - \lambda w_k C)^{2\theta - 2} (1 - (\lambda w_k C)^2)} < \frac{T_i}{T_k} \text{ if } w_k < w_i.$$

Summing over all consumers and substituting in the market clearing conditions for i and k, we get $\frac{H_i}{H_k} = \frac{\sum_{l \in N} H_l L_i(w, \lambda_l)}{\sum_{l \in N} H_l L_k(w, \lambda_l)} < \frac{T_i}{T_k}$ which is contradiction.

The derivation of $G_i(w, \lambda)$ is very similar to that of $L_i(w, \lambda)$ in subsection 5.9, and hence omitted. Parts **(ii)** and **(iii)** are both done by taking derivatives with respect to λ. I only do (ii). Let $w_i < w_k$ and $\lambda < \frac{1}{C w_k}$, which is equivalent to $G_i(w, \lambda) > 0$. Then,

$$\frac{d(G_i/G_k)}{d\lambda} = \underbrace{\frac{T_i w_i^{-\theta} (1 - \lambda w_i C)^{2\theta - 1}}{T_k w_k^{-\theta} (1 - \lambda w_k C)^{2\theta + 1}}}_{>0} C \underbrace{\left(-w_i (1 - \lambda w_k C) + w_k (1 - \lambda w_i C)\right)}_{>0 \text{ if } w_i < w_k} > 0.$$

Since $\lambda(Y)$ is strictly decreasing in Y, the result follows.$\blacksquare$

5.8 Proof of proposition 9

First note that if $T_k/H_k = T_{k'}/H_{k'}$ for all $k, k' \in N \setminus \{i\}$, then the proposition holds trivially. In the general case, not only the relative wages $w_k/w_{k'}$ of countries $k, k' \neq i$ must not change after the increase in T_i, but also the Lagrangean multiplier of their representative consumers. The reason is that country k's demand for labor of two countries different from i must not be affected by changes in the parameters of a small country. That is, if $w_k \neq w_{k'}$ remain unchanged after the technology advance in i, then in order to keep

$$\frac{L_k^*(w,Y)}{L_{k'}^*(w,Y)} = \frac{T_k w_k^{-1-\theta}(1-\lambda w_k C)^{2\theta-2}(1-(\lambda w_k C)^2)}{T_{k'} w_{k'}^{-1-\theta}(1-\lambda w_{k'} C)^{2\theta-2}(1-(\lambda w_{k'} C)^2)}$$

constant, λ must also stay constant.

Now consider a consumer in a country other than i. Denote his income by Y, the corresponding Lagrangean multiplier by λ and the highest indexed country whose labor he demands by $\tilde{n}$. His utility before the change is given by

$$U = \int q(t)dF(t) = \int q(t)dF(t) - \lambda\left[\int \min_k\{C_k(q)\}dF(t) - Y\right] \tag{18}$$

$$= \frac{\Gamma(1-1/\theta)}{4\lambda}\left[\sum_{k=1}^{\tilde{n}} T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta}\right]^{1/\theta} + \lambda Y \tag{19}$$

where the bracket in 18 is the budget constraint, which is equal to zero in equilibrium, and the algebra to attain (19) is similar to subsection 5.9. By applying the implicit function theorem to equation (19), we can calculate the change in w_i that makes the consumer above indifferent to a marginal increase in T_i. That is, the consumer is better off if and only if the increase in w_i is smaller than $\frac{dw_i}{dT_i} = -\frac{U_{T_i}}{U_{w_i}}$. Therefore, to prove the proposition, we only need to show that $\frac{dw_i}{dT_i}$ is decreasing in λ, which is equivalent to being increasing in Y. From (19),

$$U_{T_i} = \frac{\Gamma(1-1/\theta)}{4\lambda}\frac{1}{\theta}\left[\sum_{k=1}^{\tilde{n}} T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta}\right]^{1/\theta-1} w_i^{-\theta}(1-\lambda w_i C)^{2\theta} \qquad \text{and}$$

$$U_{w_i} = \frac{\Gamma(1-1/\theta)}{4\lambda}\frac{1}{\theta}\left[\sum_{k=1}^{\tilde{n}} T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta}\right]^{1/\theta-1}$$

$$\cdot T_i w_i^{-\theta-1}(1-\lambda w_i C)^{2\theta-1}\left(-\theta(1-\lambda w_i C) - 2\theta\lambda w_i C\right)$$

$$= -\frac{\Gamma(1-1/\theta)}{4\lambda}\left[\sum_{k=1}^{\tilde{n}} T_k w_k^{-\theta}(1-\lambda w_k C)^{2\theta}\right]^{1/\theta-1} T_i w_i^{-\theta-1}(1-\lambda w_i C)^{2\theta-1}(1+\lambda w_i C)$$

$\Rightarrow \frac{dw_i}{dT_i} = \frac{w_i(1-\lambda w_i C)}{\theta T_i(1+\lambda w_i C)}$, which is clearly decreasing in λ as we wanted to prove.■

5.9 Solution to the linear utility case

In this appendix, we solve the special case of section 4. A consumer with income Y chooses $q : \tau \to \mathbb{R}_+$ to maximize

$$U(q) = \int_\tau u(q(t))dF(t) \tag{20}$$

$$\text{subject to } \int_\tau \min_i\{w_i q(t)(C + t_i q(t))\}dF(t) \le Y$$

where $\tau = \mathbb{R}^n_{++}$, the support of F. Function $\hat{q}$ defined in equation (8) becomes:

$$\hat{q}(w_i, \lambda, t_i) = \begin{cases} 0 & \text{if } \lambda \geq 1/(w_i C) \\ \frac{1-\lambda w_i C}{2\lambda w_i t_i} & \text{otherwise} \end{cases} \tag{21}$$

Let $\hat{L}$ be the labor required to fulfill the demand for $\hat{q}$ and V, as defined in 5.1, be its contribution to the Lagrangean function. Then,

$$\hat{L}(w_i, \lambda, t_i) = \hat{q}_i(C + \hat{q}_i) = \begin{cases} 0 & \text{if } \lambda \geq 1/(w_i C) \\ \frac{\left(1-(\lambda w_i C)^2\right)}{4(\lambda w_i)^2 t_i} & \text{otherwise} \end{cases} \tag{22}$$

$$V(w_i, \lambda, t_i) = \hat{q}_i - \lambda w_i \hat{L}_i = \begin{cases} 0 & \text{if } \lambda \geq 1/(w_i C) \\ \frac{(1-\lambda w_i C)^2}{4\lambda w_i t_i} & \text{otherwise} \end{cases} \tag{23}$$

where subscript i is used to denote the omitted arguments of the functions (w_i, λ, t_i). The consumer will choose to buy a good with technology parameter t from country i if and only if

$$V(t_i, w_i, \lambda) \geq V(t_k, w_k, \lambda) \quad \Leftrightarrow \quad t_k \geq t_i \frac{a_k}{a_i} \text{ for all } k \in N$$

where $a_k = (1 - \lambda w_k C)^2 w_k^{-1}$. Clearly, if $\lambda \geq \max_i\{\frac{1}{w_i C}\}$, then $\hat{q}(w_i, \lambda, t) = 0$ for all $t \in \mathbb{R}_+$ and all $i \in N$, which cannot occur if $Y > 0$. Hence, $\lambda < \max_i\{\frac{1}{w_i C}\}$. Without loss of generality, let $w_1 \leq w_2 \leq ... \leq w_n$ and $n_k \in N$ be the largest index country $k \in N$ such that $\lambda < \frac{1}{w_k C}$. Denote by $L_i(w, \lambda)$ the consumer's demand for country i's labor. If $i > n_k$ then $L_i(w, \lambda) = 0$, otherwise

$$\begin{aligned} L_i(w, \lambda) &= \int_0^\infty \hat{L}(w_i, \lambda, t) \prod_{k \neq i, k \leq n_k} \left[1 - F_k\left(t\frac{a_k}{a_i}\right)\right] dF_i(t) \\ &= \frac{(1-(\lambda w_i C)^2)}{4\lambda^2 w_i^2} \int_0^\infty t^{-1} \left[\prod_{k \neq i, k \leq n_k} \exp\left(-T_k\left(t\frac{a_k}{a_i}\right)^\theta\right)\right] (-T_i)\theta t^{\theta-1} \exp(-T_i t^\theta) dt \\ &= \frac{(1-(\lambda w_i C)^2)}{4\lambda^2 w_i^2} \int_0^\infty -\theta t^{\theta-2} \exp\left\{-\left[a_i^{-\theta} \sum_{k \leq n_k} T_k a_k^\theta\right] t^\theta\right\} dt \\ &= \frac{(1-(\lambda w_i C)^2)}{4\lambda^2 w_i^2} \int_0^\infty x^{-1/\theta} \left[a_i^{-\theta} \sum_{k \leq n_k} T_k a_k^\theta\right]^{-1+1/\theta} \exp(-x) dx \\ &= \frac{(1-(\lambda w_i C)^2)}{4\lambda^2 w_i^2} \left[a_i^{-\theta} \sum_{k \leq n_k} T_k a_k^\theta\right]^{-1+1/\theta} \Gamma(1-1/\theta) \int_0^\infty \frac{x^{(1-1/\theta)-1}}{\Gamma(1-1/\theta)} dx \\ &= \frac{\Gamma(1-1/\theta)}{4\lambda^2} \left\{\frac{T_i w_i^{-1-\theta} (1-\lambda w_i C)^{2\theta-2} (1-(\lambda w_i C)^2)}{\left[\sum_{k=1}^{n_k} T_k w_k^{-\theta} (1-\lambda w_k C)^{2\theta}\right]^{(1-1/\theta)}}\right\} \end{aligned} \tag{24}$$

where line (24) is obtained by changing the variable t to $x = \left[a_i^{-\theta} \sum_{k \leq n_k} T_k a_k^{\theta}\right] t^{\theta}$. Substituting $\{L_i(w, \lambda)\}_{i \in N}$ into the budget constraint, we have

$$\frac{\Gamma(1-1/\theta)}{4\lambda^2} \left\{ \frac{\sum_{i \in N} T_i w_i^{-1-\theta} \left(\min\{0, 1-\lambda w_i C\}\right)^{2\theta-1} (1+\lambda w_i C)}{\left[\sum_{i \in N} T_i w_i^{-\theta} (\min\{0, 1-\lambda w_i C\})^{2\theta}\right]^{(1-1/\theta)}} \right\} = Y \tag{25}$$

Following the same steps as lemma 10, the LHS of (25) is continuous and strictly decreasing in λ for all $\lambda \in (0, \min_i\{1/w_i C\})$. Its limits are 0 and infinity as λ approaches $\min_i\{1/w_i C\}$ and 0, respectively. Thus, λ is implicitly defined in (25) as a strictly decreasing function of $Y > 0$. The equilibrium conditions of the whole economy is characterized by the following system of equations:

$$\sum_{i=1}^{n} w_i L_i(w, \lambda_k) - w_k = 0 \qquad \text{for all } k \in N. \tag{26}$$

$$\sum_{k=1}^{n} H_k L_i(w, \lambda_k) - H_i = 0 \qquad \text{for all } i \in N \setminus \{1\}. \tag{27}$$

Equations (26) are the budget constraints for the representative consumers of each country, and (27) the labor market clearing conditions. These form a system of $(2n-1)$ equations in $(2n-1)$ unknowns $\{\lambda_k, w_k\}_{k=1}^{n}$ with $w \in \Delta_{(n-1)}$ normalized. By proposition 1 the system has a solution.[15]

[15] Although U does not satisfy assumption 1, the proof holds here without any changes.

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